GALACTIC MAGNETIC FIELDS AS A CONSEQUENCE OF INFLATION

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Abstract The generation of a magnetic field in the Early Universe is considered, due to the gravitational production of the Z-boson field during inflation. Scaled to the epoch of galaxy formation this magnetic field suffices to trigger the galactic dynamo and explain the observed galactic magnetic fields. The mechanism is independent of the inflationary model.

Keywords: Galactic Magnetic Fields, Early Universe, Inflation

1. Introduction

Magnetic fields permeate most astrophysical systems (Kronberg 1994). In particular galaxies carry magnetic fields of the order of $\sim \mu$Gauss (Beck et al. 1996). In spirals such fields follow closely the density waves, which strongly suggests that galactic magnetic fields are sustained by a dynamo mechanism (Kulsrud et al. 1997). The galactic dynamo combines the turbulent motion of ionized gas with the differential galactic rotation to amplify exponentially a week seed field up to dynamical equipartition value. This seed field should be coherent over the dimensions of the largest turbulent eddy ($\sim$100 pc) or else it may destabilize the dynamo action (Kulsrud & Anderson 1992). Moreover, in order to produce the observed galactic fields the seed field has to be stronger that a critical value. For a spatially-flat, dark-energy dominated Universe the required strength may be as low as $B_{\rm seed} \sim 10^{-30}$Gauss (Davis et al. 1999). However, the origin of such a field remains elusive.

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Attempts to generate $B_{\rm seed}$ astrophysically via vorticity (Rees 1987) or battery (Subramanian 1996, Colgate et al. 2000) effects require large-scale separation of charges i.e. substantial ionization, which is hard to realize as late as galaxy formation. Thus, the origin of the seed field is most likely primordial. Since it breaks isotropy the generation of large-scale magnetic fields has to occur out of thermal equilibrium. Therefore, before decoupling, magnetogenesis is possible either *at phase transitions* or *during inflation* (for a review see Grasso & Rubinstein 2000).

Because phase transitions occur very early in the Universe history the comoving size of the particle Horizon is rather small. Thus, since magnetogenesis mechanisms are causal, the resulting magnetic field is too incoherent. On the other hand, inflation is possibly the only way one can achieve superhorizon correlations. However, the conformal invariance of electromagnetism forces the magnetic field to satisfy flux conservation *during* inflation (Turner & Widrow 1988). As a result the strength of the generated magnetic field is exponentially suppressed due to the rapid inflationary expansion of the Universe.

We show that natural magnetogenesis during inflation can occur due to the breaking of the conformal invariance of the Z-boson field of the Standard Model, which also contributes to the formation of large-scale magnetic fields. The resulting seed field is sufficient to trigger the galactic dynamo and explain the observations in a model-independent way.

We will use a negative signature metric and units such that $c = \hbar = 1$.

2. Inflation

Inflationary theory is so successful that it may be considered as part of the standard model of Cosmology. Indeed, inflation manages with a single stroke to solve the Horizon, Flatness and Monopole problems of the Standard Hot Big Bang (SHBB), while successfully providing the seeds for the formation of Large Scale Structure (i.e. the distribution of galactic clusters and superclusters) and for the observed anisotropies of the Cosmic Microwave Background Radiation (CMBR).

The basic picture. According to inflationary theory, at some time in the early stages of its evolution, the Universe was dominated by false-vacuum energy density, which played the role of an effective cosmological constant leading to a period of superluminal accelerated expansion.

In the simplest case one can consider that, during inflation, the energy density is $\rho_{\rm inf} = \Lambda_{\rm eff}/8\pi G = $ constant, which suggests, by means of the Friedman equation: $H^2 = (8\pi G/3)\rho_{\rm inf}$, that the Hubble parameter

$H \equiv \dot{a}/a$ is constant and, therefore, the scale factor is $a \propto \exp(Ht)$, i.e. the Universe engages into a de-Sitter exponential expansion phase.

During the inflationary expansion any pre-existing thermal bath is drastically diluted and the temperature of the Universe is $T \propto a^{-1} \to 0$, i.e. the Universe is supercooled. When inflation ends there is enormous entropy production and almost all the false vacuum energy is given to a thermal bath of newly created particles via a process called reheating. In typical inflationary models (e.g. chaotic, hybrid, natural) the reheating temperature is, $T_{\rm reh} \sim \rho_{\rm inf}^{1/4} \sim 10^{16}$GeV, i.e. at grand unification scale.

Magnetic Fields in Inflation. Magnetogenesis during inflation is based on the fact that the inflated quantum fluctuations of gauge fields become classical, long-range gauge fields with *superhorizon* correlations. Unfortunately, this is not effective for conformally invariant gauge fields, such as the photon, since they do not couple to the inflating gravitational background. However, this is not so for the Z-boson of the Standard Model (SM), which may also contribute into magnetic field generation.

Indeed, due to supercooling, the electroweak (EW) symmetry is broken during inflation and, therefore, the Z-boson field is massive. The existence of a non-zero mass M_Z breaks conformal invariance and, consequently, Z is gravitationally generated on superhorizon scales. However, at the end of inflation reheating typically restores the EW-symmetry ($T_{\rm reh} > 100$ GeV) and the Z-boson is projected onto the Hypercharge giving rise to a hypermagnetic field with superhorizon correlations. After the EW-transition the latter becomes a regular magnetic field.

3. Z-boson production in Inflation

Initial amplitude at Horizon crossing. During inflation all fields with masses smaller than H are gravitationally produced (unless they are conformally invariant) because their Compton wavelength is larger than the Horizon size $\sim H^{-1}$ and, therefore, their quantum fluctuations can reach the Horizon before dying out, i.e. the uncertainty principle allows the existence of virtual particles long enough for them to exit the Horizon. After their exit, the fluctuations cease to be causally self-correlated and cannot collapse back into the vacuum, that is, they become from virtual, real classical objects. The energy of such particle generation is provided by the false vacuum energy driving inflation.

Consider such a Z-boson fluctuation. Since the fluctuation is quantum-generated it is subject to the uncertainty relation,

$$\Delta\mathcal{E} \cdot \Delta t \simeq 1 \tag{1}$$

Because, typically, $M_Z \ll H$ the fluctuation is not suppressed before reaching the Horizon and, also, the field can be considered to be effectively massless. Thus, the energy density of its fluctuation is mainly kinetic, $\Delta\mathcal{E} \sim [\partial_t(\delta Z)]^2 \Delta V$. Now, at Horizon crossing $\Delta V \sim H^{-3}$. Also, the time required for the fluctuation to reach and exit the Horizon is $\Delta t \sim H^{-1}$. Finally, because of the random nature of quantum fluctuations (there is no coherent motion) we may identify $\partial_t \sim \Delta t^{-1}$. The above suggest, that $(\delta Z)_H \sim H$.

In fact the amplitude of the fluctuation when exiting the Horizon is set by the Gibbons-Hawking temperature, $T_H \simeq H/2\pi$. This can be understood if the particle Horizon during inflation is viewed as the event horizon of an inverted (i.e. inside-out) black hole, in the sense that nothing can escape being "sucked" *out*. The above suggest,

$$|Z(t_{\rm x})| = (\delta Z)_H \simeq H/2\pi \quad \text{and} \quad |\dot{Z}(t_{\rm x})| = (\delta Z)_H/\Delta t \simeq H^2/2\pi \tag{2}$$

where $t_{\rm x}$ is the moment of Horizon crossing. The amplitude of the fluctuation at Horizon crossing is independent of $t_{\rm x}$ due to the self-similarity of de-Sitter spacetime (all dynamical scales, such as H, stay constant).

Superhorizon evolution during Inflation. The subsequent evolution of the Z-fluctuation during inflation, after Horizon crossing, is classical and described by the equation of motion,

$$[\partial_\mu + (\partial_\mu \ln \sqrt{-D_g})][g^{\mu\rho} g^{\nu\sigma} (\partial_\rho Z_\sigma - \partial_\sigma Z_\rho)] + M_Z^2 g^{\mu\nu} Z_\mu = 0 \tag{3}$$

where $D_g \equiv \det(g_{\mu\nu})$. Using a Friedman-Robertson-Walker metric we get,

$$\partial_t^2 Z_i - \partial_t \partial_i Z_t + H(\partial_t Z_i - \partial_i Z_t) + a^{-2}(\partial_j \partial_j Z_i - \partial_i \partial_j Z_j) + M_Z^2 Z_i = 0 \tag{4}$$

Since the fluctuation in question is quantum-generated inside the Horizon it is causally connected at birth. Therefore, it can be taken to be smooth and homogeneous, when exiting the Horizon. Its subsequent, superhorizon evolution should not affect its comoving spatial distribution because of the symmetries of the metric (Remember that, after exiting the Horizon, the fluctuation becomes causally disconnected). Thus, the initial homogeneity is expected to be preserved during the superhorizon evolution. So, we can take $\partial_i Z_\mu = 0$ which recasts (4) as,

$$\ddot{Z} + H\dot{Z} + M_Z^2 Z = 0 \tag{5}$$

Solving this with $M_Z \simeq$ const. and the initial conditions of (2) we find,

$$Z(t) = -\frac{H}{4\pi\nu}\Big(\frac{1}{2} - \nu\Big)\, e^{-H\Delta t(\frac{1}{2}-\nu)} \;+\; \frac{H}{4\pi\nu}\Big(\frac{1}{2} + \nu\Big)\, e^{-H\Delta t(\frac{1}{2}+\nu)} \tag{6}$$

where, $\Delta t = t - t_{\rm x}$ with $t_{\rm x} \leq t_{\rm end}$ and $\nu \equiv \sqrt{\frac{1}{4} - (M_A/H)^2}$.

The *physical* momentum-scale k of fluctuation in question behaves as $k(t) \propto a^{-1}$. Thus, because $k(t_{\rm x}) = 2H$ and $a \propto e^{Ht}$ we have,

$$k(t) = \frac{a(t_{\rm x})}{a(t)}\, 2H \Rightarrow e^{-H\Delta t} = \frac{k(t)}{2H} \tag{7}$$

Inserting this into (6) and considering $M_Z \ll H$ we find,

$$Z(k) = -\frac{H}{2\pi}\Big(\frac{M_Z}{H}\Big)^2\Big(\frac{k}{2H}\Big)^{(M_Z/H)^2} + \frac{H}{2\pi}\Big(\frac{k}{2H}\Big) \tag{8}$$

The photon versus the *Z*-boson. During inflation M_Z is not the bare mass of the Z-boson but it is given by the magnitude of the EW-Higgs field condensate, $M_Z = g_{\rm z}\sqrt{\langle\Psi^\dagger\Psi\rangle}$, where $g_{\rm z} \sim 0.6$ is the gauge coupling of the Z with the EW-Higgs field Ψ. Because Ψ is also effectively massless during inflation, every e-folding (= exponential expansion) creates a superhorizon fluctuation of order $H/2\pi$, i.e. much larger than the VEV of the EW-Higgs field. The quantity $\sqrt{\langle\Psi^\dagger\Psi\rangle}$ represents an accumulative "memory" of these fluctuations corresponding to a random walk in the inner-space of Ψ with number of steps given by the elapsing e-foldings, $\sim \ln[a(t_{\rm x})/a(t_{\rm i})]$, where $t_{\rm i}$ denotes the onset of inflation. Thus,

$$(M_Z/H)^2 = (\frac{g_{\rm z}}{2\pi})^2 \ln(k_{\rm i}/k) \tag{9}$$

For the scales of interest $(M_Z/H)^2 \sim 0.05 \gg (k/2H)$ and the first term of (8) is by far dominant for superhorizon scales, because $k \ll 2H$. Thus, for the superhorizon spectrum of Z we have,

$$|Z(k)| \simeq \frac{H}{2\pi}\Big(\frac{M_Z}{H}\Big)^2 \tag{10}$$

i.e. the Z-boson has an *almost scale invariant superhorizon spectrum* (plus a logarithmic tilt). In contrast, the photon A_μ is not coupled to any scalar field and its mass is exactly zero. Therefore, if we set A in place of the Z in (8), then $M_A = 0$ gives that, $A(k) = k/4\pi \ll Z(k)$. Thus, on superhorizon scales the amplitude of the Z-boson is much larger that the conformal invariant photon field, as shown in Fig. 1.

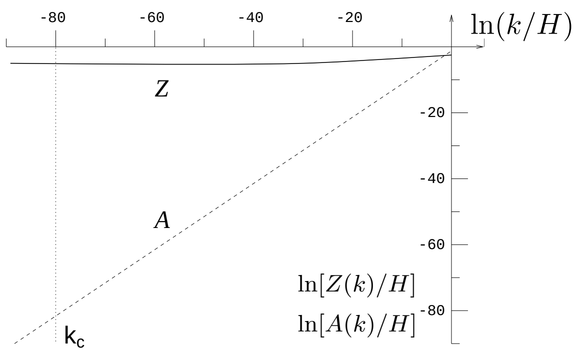

Figure 1 The superhorizon spectrum of the conformally invariant photon A versus the one of the gravitationally generated Z-boson field. At the scale of interest k_c the difference is over 30 orders of magnitude.

4. The Magnetic Field at Galaxy Formation

Evolution during the Hot Big Bang. At the end of inflation reheating restores the EW-symmetry and the photon A_μ merges with the Z_μ to form the Hypercharge,

$$Y_\mu = \cos\theta_W A_\mu - \sin\theta_W Z_\mu \tag{11}$$

where for the Weinberg angle of the SM, $\sin^2\theta_W \simeq 0.231$. Now, since the photon production during inflation is negligible, *the Hypercharge spectrum is simply a projection of the Z-spectrum*, i.e. $Y_\mu \simeq \sin\theta_W Z_\mu$.

The Hypercharge is a massless, Abelian gauge field, which obeys the analog of Maxwell's equations. The associated hypermagnetic field is defined as $\boldsymbol{B}^Y \equiv \boldsymbol{\nabla}\times\boldsymbol{Y}$, so that, at the end of inflation we have,

$$B^Y_{\rm rms} \simeq k(t_{\rm end})\, Y_{\rm rms} \simeq k(t_{\rm end})\, \sin\theta_W\, Z_{\rm rms} \tag{12}$$

Due to the high conductivity of the reheated plasma, *the hypermagnetic field gets frozen in and evolves satisfying flux conservation*, i.e. $B^Y_\mu \propto a^{-2}$. On the other hand, the relevant hyperelectric component decays being Debye screened. Furthermore, the magnetic field associated with the three W^α_μ gauge fields of the SM, which are also gravitationally produced during inflation, is screened by the existence of a thermal mass, due to the self-coupling of the non-Abelian W-boson fields.

As the Universe continues to expand it cools down. When the temperature drops below the EW energy scale the EW-symmetry is broken again and the photon is formed by Y_μ and the non-Abelian W^3_μ-boson,

$$A_\mu = \sin\theta_W W^3_\mu + \cos\theta_W Y_\mu \tag{13}$$

Since the W-bosons are screened, their amplitude is negligible compared to the Hypercharge. Thus, the spectrum of the photon reflects that of the

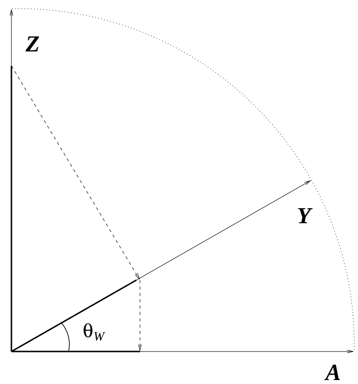

Figure 2 The formation of a magnetic field from the generation of the Z-boson has to pass through the intermediate Hypercharge stage similarly to light when crossing a set of orthogonal polarizers, which needs an intermediate third polarizer at some angle θ in order to go through.

Hypercharge, i.e.$A_\mu \simeq \cos\theta_W Y_\mu$. Therefore, *at the EW-phase transition, the hypermagnetic field transforms into a regular magnetic field* as,

$$B_\mu = \cos\theta_W B^Y_\mu \tag{14}$$

At first sight, it may strike as unlikely that a magnetic field is obtained from the gravitational production of the Z-boson, which is orthogonal to the photon. In fact, the situation is analogous to that of light polarizers (Fig. 2). Light cannot go through two orthogonal polarizers. However, when a third polarizer is inserted at an angle θ, photons do cross.

The magnitude of the Seed Field. $k(t_{\rm end})$ scaled until today is,

$$k(t_{\rm end}) = \frac{2\pi}{\ell}(\frac{T_{\rm reh}}{T_{\rm CMB}}) \tag{15}$$

where ℓ is the scale of the mode in question at present, $T_{\rm CMB}$ is the temperature of the CMBR at present, $T_{\rm reh} \simeq T(t_{\rm end})$ (prompt reheating) and we used that $a \propto T^{-1}$ at all times. Assuming that the field remains frozen until galaxy formation and using (12), (14) and (15) we find,

$$B^{\rm gf}_{\rm rms} = \pi\sin(2\theta_W)(1+z_{\rm gf})^2\frac{T_{\rm CMB}}{\ell}\frac{Z_{\rm rms}}{T_{\rm reh}} \tag{16}$$

where $z_{\rm gf}$ is the redshift that corresponds to galaxy formation. The collapse of matter into galaxies amplifies the above by a factor given by the fraction of the galactic matter density today to the present critical density, $(\rho_{\rm gal}/\rho_0)^{2/3} \approx 5\times10^3$. In view of this (16) becomes,

$$B^{\rm gf}_{\rm rms} = 7.3\times10^{-27}\left(\frac{1\ {\rm Mpc}}{\ell}\right)\frac{Z_{\rm rms}}{T_{\rm reh}}\,{\rm Gauss} \tag{17}$$

where $T_{\rm CMB} = 2.4 \times 10^{-13}$ GeV, $z_{\rm gf} \simeq 4$ and $\sin(2\theta_W) \approx 0.84$.

The superhorizon spectrum of $Z_{\rm rms}$ is approximately scale invariant with $Z_{\rm rms} \sim g_z^2 H/(2\pi)^3$ and $H = H(t_{\rm end})$. In typical inflationary models $T_{\rm reh} \sim 10^{16}$GeV and $H(t_{\rm end}) \sim \sqrt{G}\, T_{\rm reh}^2 \sim 10^{13}$GeV. Considering that the scale of the largest turbulent eddy corresponds to the comoving scale of $\ell_c \simeq 10$ kpc before the gravitational collapse of the protogalaxy, we find,

$$B_{\rm seed} \sim 10^{-30}\text{Gauss} \tag{18}$$

This is sufficient to trigger the galactic dynamo in the case of a spatially flat, dark-energy dominated Universe (Davis et al. 1999). Extra amplification ($\sim 10^3$) may be achieved by preheating (Davis et al. 2001). Also, additional enhancement is possible, when considering turbulent helicity phenomena (Son 1999, Field & Carroll 2000).

5. Conclusions

We have shown that all inflationary models of grand unification scale create magnetic fields of enough strength and coherence to trigger successfully the galactic dynamo. Since this is a *model-independent magnetogenesis mechanism* it can be thought of as a feature of inflationary theory itself. Thus, *accounting for the observed galactic magnetic fields can be considered as another generic success of inflation.*

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References

R. Beck R., Brandenburg A., Moss D., Shukurov A.A. and Sokoloff D., Ann.Rev.Astron.Astrophys. **34**(1996)155

Colgate S.A., Li H. and Pariev V., (astro-ph/0012484)

Davis A.C., Dimopoulos K., Prokopec T. and Törnkvist O., Phys.Lett.B **501**(2001)165

Davis A.C., Lilley M. and Törnkvist O., Phys.Rev.D **60**(1999)021301

Field G.B. and Carroll S.M., Phys.Rev.D **62**(2000)103008

Grasso D. and Rubinstein H.R., (astro-ph/0009061) to appear in Phys.Rep.

Kronberg P.P., Rep.Prog.Phys. **57**(1994)325

Kulsrud R.M. and Anderson S.W., Ap.J. **396**(1992)606

Kulsrud R.M., Cowley S.C., Gruzinov A.V. and Sudan R.N., Phys.Rep. **283**(1997)213

Rees M.J., Quart.Jl.R.Astron.Soc. **28**(1987)197

Son D.T., Phys.Rev.D **59**(1999)063008

Subramanian K., (astro-ph/9609123)

Turner M.S. and Widrow L.M., Phys.Rev.D **37**(1988)2743