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Large-Scale Primordial Magnetic Fields from Inflation and Preheating [1]

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Abstract. We consider models in which the (transverse) photon mass is non-zero during inflation and drops to zero non-adiabatically at the end of inflation. Through this process, vacuum fluctuations of the photon field are converted into physical, long-wavelength modes with high amplitude. The resulting spectrum of the field strength is approximately $B_\ell \sim \ell^{-1}$, where ℓ is the relevant coherence scale. With a reasonable model of field evolution we obtain, on comoving galactic scales, a magnetic field strong enough to seed a dynamo mechanism and generate the observed galactic magnetic fields.

INTRODUCTION

Spiral galaxies typically contain magnetic fields with strength of about 10^{-6} G that are aligned with the spiral density waves [1]. A plausible explanation is that these fields have been produced via a galactic dynamo mechanism, in which a weak seed field was amplified by the differential rotation of the galaxy in conjunction with magnetohydrodynamic turbulence. The required strength of the seed field is subject to large uncertainties; for a flat, critical-density universe, estimates lie in the range 10^{-23}–10^{-19} G at the time of completed galaxy formation. This lower bound can be relaxed [2] to about 10^{-30} G for a universe with a dark-energy component (e.g. a cosmological constant or quintessence), which appears to be favoured by recent results from supernova observations and balloon experiments.

For the dynamo to work, the galactic seed field should be correlated on a scale of ~ 100 pc, corresponding to the largest turbulent eddy. Because the magnetic field strength B and correlation length ℓ scale as $B \sim \ell^{-2} \sim \rho^{2/3}$ during the collapse of matter into a galaxy, where ρ is the matter density, the required comoving

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correlation length is $\sim$ 10 kpc [2]. On this scale, the lower bounds for the seed field are 2×10^{-27} G for a critical-density universe, and 2×10^{-34} G for a flat, dark-energy dominated, low-density universe.

Many proposals have been put forward regarding the origin of the seed field [3]. Among the most interesting is the possibility that the field is primordial and was generated in the early Universe through one of several field-theoretic mechanisms that occur in models of particle physics. Such a mechanism must occur out of equilibrium, which limits the choice to either inflation or a phase transition. In phase transitions, however, it is difficult to explain the large correlation length. For instance, the comoving scale corresponding to the *horizon* at any time before the last (QCD) phase transition is much smaller than 10 kpc, so the seed field is unlikely to have been produced by *causal* processes in a phase transition in the early universe.

Inflationary models solve problems with causality (such as the horizon problem) by stretching space itself, without changing the speed of light rays relative to a local, inertial frame. For example, it is well known that long-wavelength vacuum fluctuations of scalar fields grow during inflation and provide the seeds for large-scale structure in the form of density perturbations [4]. In a similar way, one might expect inflation to be able to produce magnetic fields correlated on scales that are beyond causality bounds.

EVOLUTION OF GAUGE FIELDS IN INFLATION

In a conformal metric $ds^2 = a^2(\tau)[d\tau^2 - d\vec{x}^2]$ (which characterises de Sitter inflation and flat FRW eras), gauge fields A_μ are conformally invariant and do not couple gravitationally. In fact, the equation of motion can be reduced to $\Box A_\mu = 0$, where $\Box = \eta^{\mu\nu}\partial_\mu\partial_\nu$ is the Minkowski-space d'Alembertian. This equation has only harmonic solutions, and thus the amplitude of vacuum fluctuations of A_μ is a constant function of (conformal) time τ.

In order to have a changing amplitude of vacuum fluctuations, one must somehow break conformal invariance [5]. A simple way would be to add, by hand, a mass term $m^2 g^{\mu\nu} A_\mu A_\nu/2$ to the Lagrangian. This gives rise to the following equation for a (transverse) Fourier mode $\mathcal{A}_{\vec{k}}$:

$$\left(\partial_\tau^2 + k^2 + \frac{m^2}{H_\mathrm{I}^2 \tau^2}\right) \mathcal{A}_{\vec{k}} = 0 \ , \tag{1}$$

where we have used that the scale factor $a = -1/H_\mathrm{I}\tau$ in de Sitter inflation with Hubble parameter H_I. It is obvious that the addition of such a positive mass term will not lead to growth in the amplitude $\mathcal{A}_{\vec{k}}$; on the contrary, the amplitude of long-wavelength vacuum fluctuations decreases during inflation.

To see this in detail, we investigate the exact solutions of Eq. (1), which are

$$\mathcal{A}_{\vec{k}}^{(i)} = \frac{1}{2}\sqrt{-\pi\tau}\, H_\nu^{(i)}(-k\tau) \ , \qquad i = 1, 2, \qquad \nu = \frac{1}{2}\sqrt{1 - \frac{4m^2}{H_\mathrm{I}^2}} \ , \tag{2}$$

where $H_\nu^{(i)}$ are Hankel functions. Towards the beginning of inflation ($\tau \to -\infty$), the solutions behave as $(2k)^{-1/2}\exp(\pm ik\tau)$ [up to a phase] such that $|\mathcal{A}_{\vec{k}}^{(i)}| \sim (2k)^{-1/2}$ and $|\partial_\tau \mathcal{A}_{\vec{k}}^{(i)}| \sim (k/2)^{1/2}$. At the end of inflation, which in our parametrisation occurs at $\tau = -1/H_{\mathrm{I}}$, we have for long wavelengths ($k/H_{\mathrm{I}} \ll 1$) that $|\mathcal{A}_{\vec{k}}^{(i)}| \sim (2k)^{-1/2}\Gamma(\nu)(k/2H_{\mathrm{I}})^{1/2-\nu}$ and $|\partial_\tau \mathcal{A}_{\vec{k}}^{(i)}| \sim (k/2)^{1/2}\Gamma(\nu)(1-2\nu)(2H_{\mathrm{I}}/k)^{1/2+\nu}/4$. Because $\nu < 1/2$, the amplitude has decreased. Note, however, that the electric-field amplitude $|\partial_\tau \mathcal{A}_{\vec{k}}^{(i)}|$ has increased enormously for very long wavelengths $k \ll H_{\mathrm{I}}$. This is a consequence of the Heisenberg uncertainty relation. We shall see next how such a large electric field is converted into a magnetic field when the mass m suddenly drops to zero at the end of inflation.

NON-ADIABATICITY

In order to understand the concept of non-adiabaticity, consider a quantum particle in the ground state of a one-dimensional infinite well. If the walls of the well are suddenly moved apart (so quickly that the particle has no time to exchange energy with its surroundings), the particle will find itself in a quantum state which is, in general, a linear superposition of excited states of the new, larger well. In the case of a quantum field, such an excited state corresponds to a physical observable field, distinct from the vacuum.

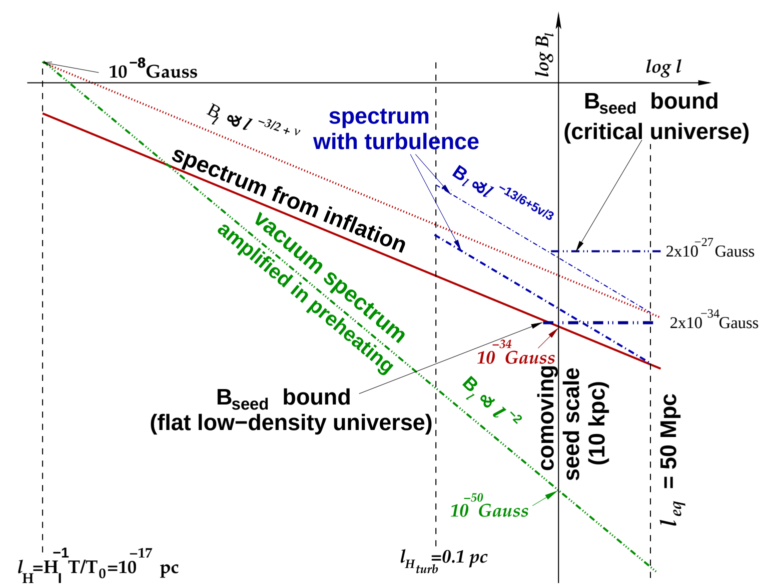
In our scenario [6], we propose that the photon mass m goes non-adiabatically to zero at the end of inflation, so that the massive-photon mode functions described by Eq. (2) are suddenly projected onto mode functions $(2k)^{-1/2}\exp(\pm ik\tau)$ that are solutions of the massless equation. In the quantum language, this can be expressed as a Bogoliubov transformation $\tilde{a}_{\vec{k}} = \alpha_{\vec{k}} a_{\vec{k}} + \beta^*_{\vec{k}} a^\dagger_{\vec{k}}$ between annihilation operators $\tilde{a}_{\vec{k}}$ and $a_{\vec{k}}$ for the quantum fields with $m \neq 0$ and $m = 0$, respectively. The values of $\alpha_{\vec{k}}$ and $\beta_{\vec{k}}$ are found by matching the quantum field and its time derivative at the transition. Through this matching, the large time derivative of the massive mode is converted into a large amplitude of the massless mode. A non-zero value of $\beta_{\vec{k}}$ indicates particle creation. In our case, $\beta_{\vec{k}} \sim (H/k)^{\nu+1/2}$ for $k \ll H_{\mathrm{I}}$, showing that long-wavelength modes are created with large amplitude. This occurs also if m goes to zero smoothly [6], provided that it does so non-adiabatically, i.e. $|\omega''/2\omega^3 - 3\omega'^2/4\omega^4| > 1$, where $\omega^2(\tau) = k^2 + (m/H_{\mathrm{I}}\tau)^2$.

Let us now consider several models which exhibit this behaviour [7]. First, consider inflation with two scalar fields, s real and ϕ complex, where ϕ couples minimally to the photon field A_μ, giving it a mass $m_A^2 = 2e^2|\phi|^2$. The potential is $V_s(s^2) + m_\phi^2|\phi|^2/2 + \lambda_\phi|\phi|^4/4 - gs^2|\phi|^2/2$ where V_s is an increasing function. During inflation, s and $|\phi|$ decrease as they roll along the curve $|\phi| = \sqrt{(gs^2 - m_\phi^2)/\lambda_\phi}$ until $s < m_\phi/\sqrt{g}$, after which $|\phi| = 0$. In order for our mechanism to be efficient, we require $m_A^2 \approx 10^{-2}H_{\mathrm{I}}^2$ and thus $g/\lambda_\phi \approx 10^{-12}$. Such values arise naturally in supergravity models,[3] where $g \sim \alpha H_{\mathrm{I}}^2/M_{\mathrm{P}}^2$, $\alpha \approx 0.1$.

[3] We thank A. Riotto and W. Porod for independently pointing this out.

The second possibility is through the back reaction of the vacuum fluctuations of a scalar field, $\langle\Phi^\dagger\Phi\rangle$, on the equation of motion of a minimally coupled gauge field. We know that $\langle\Phi^\dagger\Phi\rangle$ grows during inflation, and thus would give a mass $2e^2\langle\Phi^\dagger\Phi\rangle$ to the photon. The mass goes to zero if Φ decays soon after the end of inflation, which would be the case if Φ is, e.g., a heavy squark field, or if Φ is the electroweak Higgs field and the electroweak symmetry is restored by reheating. In the latter case, the electroweak Z field would play the role of the gauge field. There is some controversy about this; in a different talk at this conference, Shaposhnikov argues that the effective transverse (so-called magnetic) mass of the gauge field is zero both during inflation and after inflation. However, when a quantum operator evolves non-adiabatically, one should employ out-of-equilibrium methods to calculate its expectation value, and there is no reason that it should be zero.

All these mechanisms result in a gauge-field spectrum $\mathcal{A}_{\vec{k}} \propto k^{-1-\nu} \sim k^{-3/2}$, corresponding to a magnetic field $B_\ell \propto \ell^{-3/2+\nu} \sim \ell^{-1}$, where ℓ is the relevant coherence scale and $\nu \approx 1/2$. This spectrum is displayed below. Pairs of parallel lines indicate spectra with and without amplification by a factor 10^5 during preheating. We see that the lower bounds for the dynamo seed field can be met with our mechanism.



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