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Belief Distortions and Disagreement about Inflation*

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Abstract

This paper investigates the macroeconomic effects of a belief distortion shock—an unexpected increase in the wedge between household and professional forecaster inflation expectations. Using survey and macro data alongside machine-learning techniques, we identify this shock and examine its effects within and outside the ZLB, while conditioning on the degree of inflation disagreement. The shock increases unemployment during normal times, whereas it reduces it in the ZLB, when the monetary stance is accommodative. Inflation disagreement instead dampens the expansionary effects of the shock. A New Keynesian model with belief distortion shocks replicates these dynamics and reproduces the inflation disagreement empirical patterns.

Keywords: Inflation; Belief Distortion Shock; Inflation Disagreement; Households Expectation; Machine Learning; Local Projections; New Keynesian model; Monetary Policy, ZLB.

JEL Codes: E31, C220, D840, C320

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1 Introduction

Understanding how expectations shape macroeconomic outcomes has become increasingly important in a world of persistent inflation surprises and heightened uncertainty.

Most of the literature on inflation expectations primarily focuses on explaining survey microdata and understanding the formation, determinants, and heterogeneity of inflation expectations (see Coibion, Gorodnichenko and Weber, 2022, for a survey). While examining the endogenous formation of agents' beliefs is crucial, our paper takes a different yet complementary approach by assessing the macroeconomic effects of a belief distortion shock. We define this shock as an unexpected increase in the wedge between household and professional forecaster expectations.

Recently, D'Acunto et al. (2024) revealed significant deviations from traditional assumptions about rationality in the formation of consumer expectations. They show that households act on their inflation beliefs, though in heterogeneous ways that may depart from the predictions of conventional economic models. On average, household expectations deviate noticeably from Central Bank targets and contrast with expert forecasts, such as those from the Survey of Professional Forecasters, particularly during recessions. Bhandari, Borovička and Ho (2024) show that, consistent with survey evidence, an increase in pessimism generates upward biases in unemployment and inflation forecasts, leading to lower economic activity. By developing a theory of subjective beliefs that departs from rational expectations, they demonstrate that biases in household beliefs have quantitatively large negative effects on macroeconomic aggregates.

Our paper speaks to this concern by providing new empirical and theoretical insights into how belief distortions and inflation disagreement affect unemployment—especially during crisis scenarios when conventional monetary policy is constrained at the Zero Lower Bound (ZLB).

On the empirical side, we identify a belief distortion shock and assess its macroeconomic effects for the US economy in normal times and during deep recessions, particularly focusing on the Global Financial Crisis recession and the COVID-19 recession, both with the policy rates at the ZLB. On the theoretical side, we provide a model capable of reproducing the state-dependent effects of inflation belief distortion shocks found in the empirical part and generating dynamics of inflation disagreement consistent with those in the data.

The empirical part of the paper presents two main contributions. First, we compute a measure of inflation disagreement based on a functional principal component analysis of the distribution of household inflation expectations. Second, we identify an inflation belief distortion shock starting from a measure of inflation belief wedge—defined as the difference between consensus household inflation expectations from the Michigan Survey of Consumers (MSC) and those from the Survey of Professional Forecasters (SPF). Then, to further isolate households' inflation beliefs from a corresponding best forecast (based on observed fundamental changes), we depart from Bhandari, Borovička and Ho (2024) and apply a ridge machine learning algorithm to the belief wedges, controlling for a broad set of macroeconomic and financial variables from the Fred-QD database. The estimated residual serves as our identified inflation belief distortion shock.

We use this shock to run linear local projections and examine its transmission to key macroeconomic variables, including our estimated measure of inflation disagreement. The impulse response functions show that a positive belief distortion shock—an unexpected increase in the wedge between household and professional forecaster expectations—leads to a significant and persistent increase in inflation, inflation disagreement, and unemployment. By highlighting the effects of belief shocks on unemployment, we relate to the recent literature on the interplay between expectations-driven aggregate fluctuations and labour market (Pappa, Ravn and Sterk, 2023).

We extend our analysis to investigate the effects of the belief distortion shock under state-dependent scenarios. After identifying regimes characterized by high levels of inflation disagreement, we find that these regimes partially overlap with periods when the economy is constrained by the ZLB. We therefore conduct a state-dependent local projection analysis to examine the effects of the shock across different states—both within and outside the ZLB—while conditioning on the degree of inflation disagreement. Our results show that the shock increases unemployment during normal times. In contrast, when the Central Bank is constrained by the ZLB and maintains a prolonged accommodative policy stance, the shock becomes expansionary and reduces unemployment. However, once we control for inflation disagreement under ZLB conditions, the expansionary effect of the shock on unemployment is attenuated. We interpret this as evidence that disagreement has a negative effect on the economy, dampening the expansionary effect of the belief distortion shock.

A higher wedge between household and professional forecaster expectations plays a key role in the ZLB. As inflation expectations increase, the real interest rate lowers, producing expansionary effects that accelerate the recovery.

The literature has also sought to explain the increase in households' inflation beliefs by examining unfunded fiscal shocks, which—by raising inflation expectations in the absence of monetary policy adjustments—reduce real interest rates and help lift the economy out of recessions, particularly in the aftermath of the Global Financial Crisis and the COVID-19 downturn (Bianchi and Melosi, 2017, 2019; Bianchi, Faccini and Melosi, 2023).[1]

In the second part of the paper, we consider a simple New Keynesian model with search and matching frictions, where inflation expectations slightly depart from rationality due

[1]This result is also consistent with Coibion, Gorodnichenko and Ropele (2020) who provide micro-level evidence of the possibility of expansionary effects from rising inflation expectations at the ZLB, using firm level survey data from the Italian economy.

to the presence of an exogenous belief distortion shock. Although not derived from explicit microfoundations, our exogenous shock to short-term inflation expectations falls into the broader category of belief shocks.[2]

By taking a third-order approximation of our simple model, we construct a measure of the expected variance of inflation expectations, which we take as the model counterpart of inflation disagreement.[3] We show that our model is able to rationalize the dynamics of inflation, unemployment, and inflation disagreement observed in the empirical part, both in normal times and during crises accompanied by ZLB for monetary policy.

A key validation of our approach lies in the model's ability to generate simulated data that closely match the empirical results. By feeding the identified belief distortion shock into our theoretical framework, we obtain simulated paths for inflation disagreement that, particularly during the last two crises, align with those observed in the historical decomposition from the empirical analysis. Finally, we show that, consistent with the empirical counterpart, inflation disagreement per se increases the unemployment rate in the ZLB, thus contributing to dampen the expansionary effect of the belief distortion shock.

Related literature. The paper draws primarily on three strands of the literature. First, it refers to studies that examine distortions in inflation expectations and the role of belief distortion shocks as a source of business cycle fluctuations.

Different papers provide evidence of deviations from rationality for households and firms (Bachmann, Berg and Sims, 2015; D'Acunto et al., 2023, 2021; D'Acunto, Malmendier and Weber, 2021; Bassanin, Faia and Patella, 2021; Bhandari, Borovička and Ho, 2024; Diegel and Nautz, 2021). Capistrán and Timmermann (2009) and D'Acunto, Malmendier and Weber (2023) show that inflation expectations are upwardly biased. Binder and Kamdar (2022) examine the relationship between expected and realized inflation on an historical perspective. Coibion and Gorodnichenko (2015) and Bordalo et al. (2020) test the predictability of forecast errors, and find aggregate underreaction and individual-level overreaction, respectively. Angeletos, Huo and Sastry (2020) reconcile the two views with a theory that combines dispersed information with over-extrapolation. The literature is not decisive about the impact of belief distortion shocks in the data [Fève and Guay (2019), Chahrour and Jurador (2018), Enders, Kleemann and Müller (2021), Lagerborg, Pappa and Ravn (2023), Adams and Barrett (2024)] and in the models [Lorenzoni (2009); Benhabib, Wang and Wen (2015); Angeletos, Collard and Dellas (2018); Ascari

[2] In the same vein, Angeletos, Collard and Dellas (2018) introduce a shock to short-run expectations of real variables as a reduced-form shock to capture sentiment shifts due to higher-order beliefs. They emphasize that their short-run expectational shock differs from optimism/pessimism in the news shock literature, which typically applies to expected TFP increases in the distant future.

[3] Since the model is based on a representative agent framework, expectation disagreement may be captured by the expected volatility of inflation.

et al. (2023)]. We contribute here by using a machine learning regularization algorithm to identify a belief distortion shock entailing a deviation from rational expectations. Its effects are highly nonlinear in normal and crisis times.

Second, our paper connects with the literature documenting disagreement in agents' expectations (Mankiw, Reis and Wolfers, 2003; Branch, 2004; Capistrán and Timmermann, 2009; Coibion and Gorodnichenko, 2012; Malmendier and Nagel, 2016; D'Acunto, Malmendier and Weber, 2023; Weber, Gorodnichenko and Coibion, 2022; Andrade et al., 2019) and its effects on macroeconomic aggregates, such as price-setting decisions (Meeks and Monti, 2023) and monetary policy effectiveness (Falck, Hoffmann and Hürtgen, 2021; Sastry, 2022; Diegel and Nautz, 2021; Pfäuti and Seyrich, 2022). The literature has proposed alternative theories to the full-information rational expectations. These include sticky information rational expectations (Mankiw and Reis, 2002; Reis, 2006; Andrade et al., 2016); noisy information rational expectations (Sims, 2003; Woodford, 2001); and subjective models with belief distortion shocks in deviations from objective data-generating processes (Giacomini, Skreta and Turen, 2020; Andre et al., 2022; Meeuwis et al., 2022; Angeletos, Huo and Sastry, 2020). We first compute a functional measure of inflation disagreement and then use a general equilibrium model with inflation belief distortion shocks and endogenous uncertainty, as to match both the unconditional and conditional properties of inflation disagreement in response to the belief distortion shock.

At last, we relate to the vast literature enhancing the role of nonlinearities in the transmission of macroeconomic shocks. Auerbach and Gorodnichenko (2012, 2013) and Ramey and Zubairy (2018) estimate different fiscal multipliers in recessions and expansions. Jo and Zubairy (2025) establish interaction states between inflation and unemployment to discipline the state dependence of fiscal multipliers with respect to the demand/supply nature of recessions. Bianchi and Ilut (2017) and Bianchi and Melosi (2017, 2019) show that monetary and fiscal policy interaction regimes (including the ZLB) significantly shape inflationary and macroeconomic outcomes. They recommend a temporary fiscal dominance regime, increasing inflation expectations, as an exit strategy from crises. Falck, Hoffmann and Hürtgen (2021) evidence that monetary policy effectiveness changes with the degree of disagreement in inflation expectations. Our work empirically and theoretically shows that inflation expectation shocks help escape ZLB states, where we condition the recovery on inflation disagreement. It dampens the expansionary impulse of the shock.

The remainder of the paper is organized as follows. Section 2 presents preliminary evidence on inflation expectations and introduces a measure of inflation disagreement based on a functional principal component analysis of the distribution of household inflation expectations. Section 3 identifies the belief distortion shock (Section 3.1), conducts an impulse response analysis via linear local projections (Section 3.2), and introduces a state-dependent analysis by examining the role of ZLB periods and the contribution of inflation disagreement (Section 3.3). Finally, Section 4 presents the theoretical model and its implied conditional dynamics in response to the belief distortion shock. Section 5 concludes. Robustness and technical details are left in the Appendix at the bottom of the paper.

2 Inflation Expectations and Disagreement

This section describes the series used to construct our measure of inflation disagreement and to identify the belief distortion shock. First, we refer to the mean difference between households' and professional forecasters' 1-year-ahead inflation expectations as the belief wedge. Household expectations are taken from the Michigan Survey of Consumer Attitudes (MSC), whereas the professional forecasters' from the Survey of Professional Forecasters (SPF) by the Philadelphia FED.[4]

We refer to the average expectation of the professional forecaster as a proxy for rational expectations, as professional forecasters have greater access to data, rely on sophisticated forecasting techniques (Bonham and Dacy, 1991), and therefore are considered good benchmarks (Faust and Wright, 2013; Bhandari, Borovička and Ho, 2024).

Figure 1 compares the dynamics of the mean inflation expectations of the MSC and the SPF, taken at the quarterly frequency over the 1981:Q4-2024:Q2 sample. The figure shows that household inflation forecasts tend to systematically exceed the forecasts of professional forecasters, resulting in predictable errors. The literature shows that economic agents tend to associate the prospect of higher future inflation with adverse macroeconomic outcomes. Households anticipating higher inflation also expect higher levels of unemployment (Bhandari, Borovička and Ho, 2024), while reduced inflation expectations are associated with an increase in the consumption of durable goods (Kumar, Gorodnichenko and Coibion, 2023). Coibion, Gorodnichenko and Ropele (2020) reach similar conclusions for firms. On the basis of this evidence, we interpret positive deviations from rational expectations as indicating consensus pessimism, while negative ones as signaling consensus optimism.

The difference (or vertical gap) between the average household forecast and that of the professional forecasters represents the belief wedge. It provides a measure of the degree of belief distortion regarding inflation. Belief distortions, defined on the inflation belief wedge, display significant temporal variation, with the two main episodes of distortion coinciding with major economic disruptions: the 2007-2009 Global Financial Crisis and

[4]The MSC survey is a rotating panel of approximately 500 to 1500 households, with respondents from the general public being either new participants or returning respondents from the previous 6 and 12 months. Data are collected monthly, and the second month of each quarter is selected to create a quarterly series. The Survey of Professional Forecasters collects data on a quarterly basis in the second month of each quarter, with respondents ranging from 9 to 83 professionals, including economists, bankers, and forecasters.

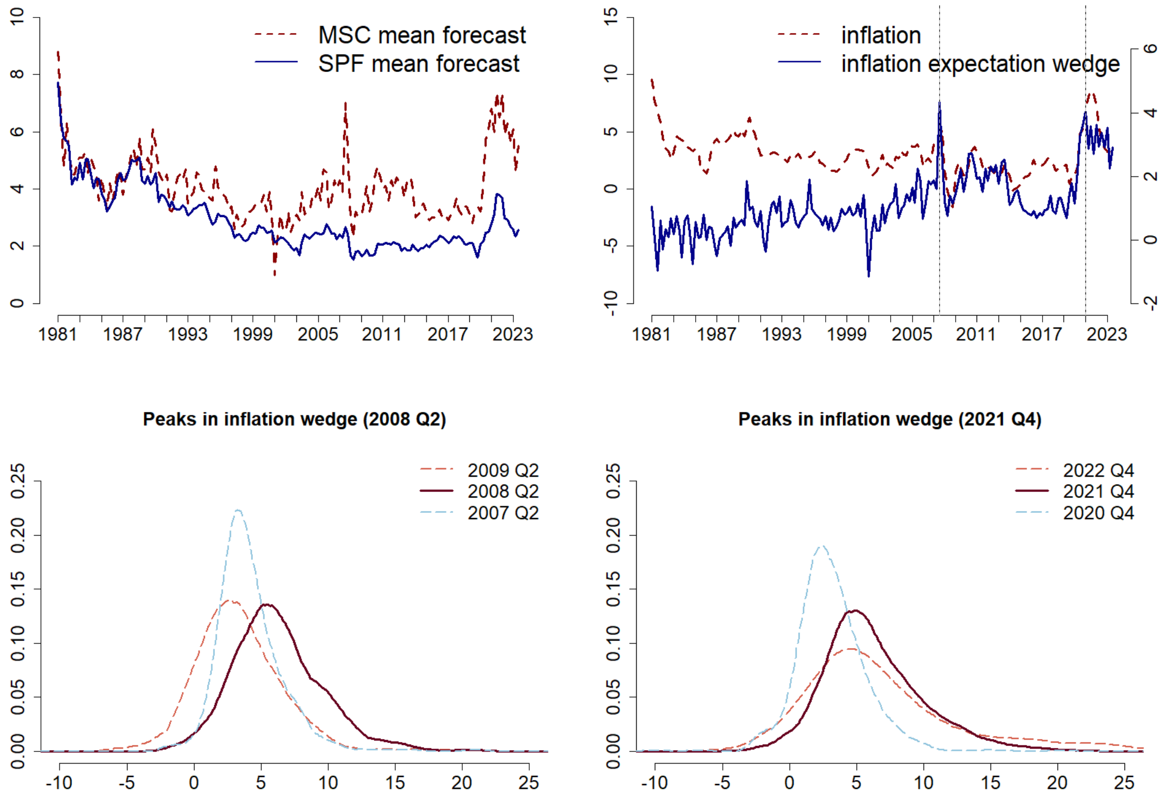


Figure 1: **Inflation expectations: the belief wedge and the distribution.** The top-left panel shows the path of the average 1-year-ahead inflation expectations for households (MSC) and professional forecasters (SPF). The difference defines the inflation belief wedge, which is plotted against the time series of CPI inflation in the top-right panel. The vertical dotted lines highlight the two peaks of the inflation belief wedge. The bottom panel shows the estimated probability density functions for the distribution of households' expectations at distinct dates, distinguishing the quarters one year before and after the two belief wedge peaks in 2008Q2 (left) and 2021Q4 (right). Forecast densities are estimated by nonparametric kernel methods. *Source:* Michigan Survey of Consumers, Philadelphia Survey of Professional Forecasters, Bureau of Labor Statistics. *Sample:* 1981Q4-2024Q2.

the post-2020 Covid-19 crisis. In particular, both correspond to inflationary pressures, as shown in the top right panel of Figure 1.

Then, to further explore the information contained in the expectations' data, the bottom panels in Figure 1 report the estimated probability densities of the MSC micro-data on inflation expectations for each period around two notable peaks in the belief wedges (2008Q2 and 2021Q4). Forecast densities are estimated using kernel methods. We show that the distribution of inflation expectations largely moves around the belief wedge peaks. Specifically, it tends to shift toward higher values, consistently with larger mean belief wedges. In both cases, this shift is followed by more dispersed densities, often with pronounced right tails.

Overall, this evidence shows correlated dynamics of belief wedges, shifts in the distribution of household expectations, and inflation. In the next sections, we compute a functional measure of the distribution of households' beliefs about future inflation to then assess its response to a belief distortion shock.

2.1 Functional measure of Inflation Disagreement

This section focuses on measuring disagreement in the distribution of households' expected inflation. Several recent papers argue that considering the distribution of expected inflation helps explaining macroeconomic outcomes (Meeks and Monti, 2023). Falck, Hoffmann and Hürtgen (2021) find that the dispersion in inflation expectations affects monetary policy transmission.

To obtain a measure of the dispersion among MSC forecasters, we adopt a functional principal component analysis approach. This non-parametric method, extracting higher moments of the entire distribution, offers several advantages over taking second moments or the interquartile range estimated directly from the cross-section of responses. It captures the dominant modes of variation directly from the data structure, reducing sensitivity to deviations from normality and separating systematic variability from random noise. Another advantage of this approach is that by perturbing the scores we can reconstruct the responses of the probability density functions to the shock.

Details of the procedure we follow are in Appendix B.[5] As a first step, we estimate the probability density functions from the cross-section of the responses through non-parametric kernel density estimation. We center them around the distributional mean.[6] Second, we construct a smooth functional approximation of the log densities via spline smoothing, following Ramsay and Silverman (2005). The log-transformation enforces the non-negativity of the densities, and the approximation is controlled through a roughness penalty smoothing parameter. The smoothed log-densities are re-exponentiated and normalized to recover valid density functions. Finally, in the third step, we apply functional principal component analysis by projecting the smooth centered densities onto a basis of orthonormal eigenfunctions. These eigenfunctions, or functional principal components, summarize the main patterns of variation across time. The associated scores quantify the extent to which each individual density deviates from the mean density.

In Figure 2, we select three functional principal components. The scree plot shows that, together, they explain about 95% of the total variability in the data, with the first component alone accounting for approximately 76%. This component places more weight on the tails of the distribution. Its score is also highly correlated with standard measures of expectation dispersion, such as the cross-sectional standard deviation of responses (correlation = 0.89) and the interquartile range (correlation = 0.79).[7] We consider the first principal component score to be a suitable functional measure of households' disagreement about expected inflation, with the added advantage of being consistent with the measure of inflation disagreement used in the theoretical model. We label this score as

[5]Further details on the methodology can be found in (Kneip and Utikal, 2001).

[6]This step removes location effects, ensuring that the principal component analysis captures variations in the shape rather than shifts in central tendency.

[7]The second score correlates with the skewness of the distribution (correlation = 0.69).

"inflation disagreement" and denote it by $\hat{\Delta}_t$.[8]

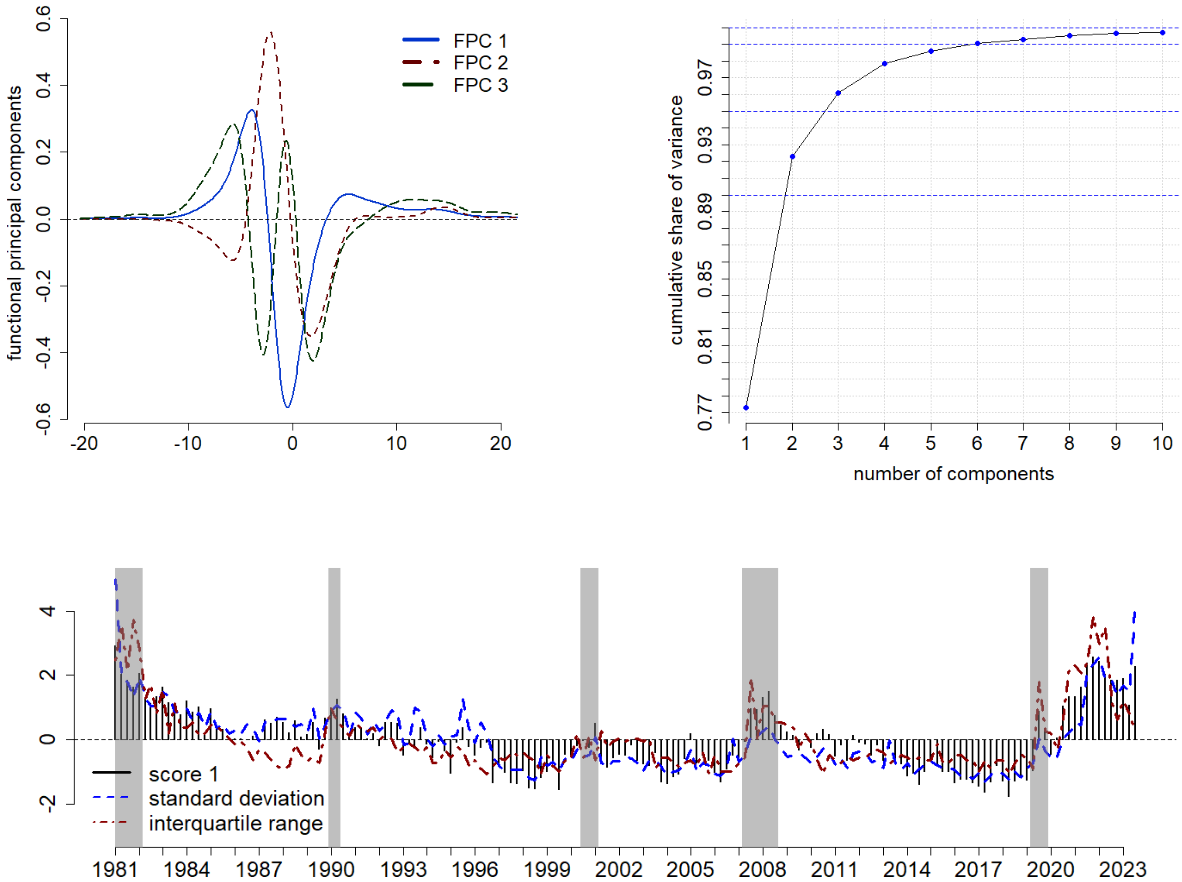


Figure 2: **Functional principal component analysis and Inflation Disagreement.** The top-left panel plots the first three functional principal components. The scree plot in the top-right panel highlights the cumulative share of variance explained by each functional principal component. The bottom panel displays the path of the first functional principal component score, plotted against the standard deviation and the interquartile range of the cross-section of responses. All measures are standardized. Shaded areas represent NBER recessions. *Source:* Michigan Survey of Consumers. *Sample:* 1981Q4-2024Q2.

3 Impulse responses to a belief distortion shock

In this section, we describe how we estimate the belief distortion shock and study its impact on the economy both unconditionally—in a linear environment—and conditionally, in the presence of ZLB states, after controlling for the effects of inflation disagreement. Specifically, Section 3.1 details the methodology used to estimate the belief distortion shock, including an evaluation of its correlation with a set of macroeconomic and policy shocks from the literature. Section 3.2 presents impulse responses generated by our identified belief distortion shock using a linear local projections analysis. Section 3.3 deepens the analysis by providing a nonlinear investigation, highlighting the presence of high and

[8]We show that the main results of the paper remain unchanged when controlling for the other scores. For the sake of space, we leave these results available upon request.

low inflation disagreement—features that partially coincide with ZLB periods. The same section illustrates the state-dependent transmission of the belief distortion shock, showing that its effects are nonlinear and conditional on ZLB states and the level of inflation disagreement.

3.1 Inflation Belief Distortion shock

To identify an inflation belief distortion shock, we proceed in two steps. In the first step, we compute a belief wedge for inflation, defined as the difference between the average 1-year-ahead inflation forecasts by MSC households and those by professional forecasters from the SPF at the same horizon. The latter is taken as a proxy for rational expectations, since professionals rely on more sophisticated methods to generate predictions (Bonham and Dacy, 1991; Faust and Wright, 2013).[9] Thus, the belief wedge is expressed as differences in first-moment beliefs, as follows:

$$y_{x,t} = \tilde{\bar{E}}_t(x_{t+h}) - \bar{E}_t(x_{t+h})$$

where x_{t+h} is the generic forecasted variable, namely inflation, $\tilde{\bar{E}}_t(x_{t+h})$ is the cross-sectional mean of households' distorted $h = 4$-period-ahead forecast of x, and $\bar{E}_t(x_{t+h})$ is the rational counterpart. The belief wedge collapses to zero in the absence of distortions in belief formation, so that $\tilde{E}_t(x_{t+h}) = \bar{E}_t(x_{t+h})$.

The extent to which the belief wedge generates a pure belief distortion shock depends on our ability to clean it from variations attributable to predictable or observable changes in macroeconomic variables, including fluctuations associated with the business cycle. For this reason, in the second step, we adopt a machine learning regularization method and predict changes in the belief wedge using a large set of regressors from the FRED-QD dataset.[10] This serves to establish orthogonality conditions based on the residuals of a high-dimensional regression.[11]

Ridge regression minimize the residual sum of squares augmented by a penalty term that shrinks the coefficients toward zero by penalizing their squared magnitudes:

[9] Crump et al. (2021), document that a large-scale Bayesian vector autoregressive model generates out-of-sample forecasts that are very close to those of professional forecasters. Our results are also robust to computing machine-based machine forecasts as the rational benchmark (Bianchi, Ludvigson and Ma, 2022).

[10] See Appendix A for data description and sources. Regressors include both contemporaneous and dynamic variables (8 lags each). Non-stationary regressors are transformed into annual percentage changes. All regressors are standardized.

[11] We choose the ridge regression method because, unlike LASSO and elastic net regression, it penalizes high-dimensional regression coefficients without performing variable selection (shrinking some coefficients to 0). Conceptually, it can be interpreted as a Bayesian OLS regression with a normal prior centered at 0 on the coefficients. This results in a dense model rather than a sparse one, which is more appropriate for economic data with highly correlated regressors (Giannone, Lenza and Primiceri, 2021).

$$L(\beta) = \|y - \tilde{\mathrm{X}}\beta\|_2^2 + \lambda\|\beta\|_2^2 \tag{1}$$

where y is the inflation belief wedge, $\tilde{\mathrm{X}}$ is the matrix of high-dimensional regressors, and the hyperparameter λ controls the degree of shrinkage in the model. If $\lambda = 0$, the constraint imposed in the minimization problem is not binding, and the ridge model is equivalent to OLS. The higher is λ, the stronger is the shrinkage. Its optimal value is selected using time-series cross-validation, a standard method in the machine learning literature for validating a model's predictive ability when dealing with time-series data.[12]

Results from the ridge regression model are reported in Figure 3. Panel (a) shows that the model performs well in forecasting the inflation belief wedge, with an adjusted $R^2 = 0.81$. Panel (b) displays the residuals from the ridge regression. This residual represents the unpredictable component of the belief wedge, which we interpret as the belief distortion shock. The model effectively centers the wedge around zero and yields a distribution of the shock that more closely resembles normality, as shown in Panel (c). Panel (d) reports the correlograms of the raw inflation wedge and the belief distortion shock, showing that the model successfully removes autocorrelation from the latter.

To assess robustness, we consider an alternative shock specification that includes information related to expected unemployment dynamics. We construct the belief wedges for unemployment and inflation.[13] Then, we take their first principal component to obtain a joint belief wedge. While this approach is similar to that of Bhandari, Borovička and Ho (2024), our implementation differs in that we recover the unpredictable innovation in the joint belief wedge by treating it as the dependent variable in the ridge regularization algorithm. We find that including expected unemployment into the analysis does not add much relevant information. As shown in Figure C.3 in Appendix C, the shock we obtain by orthogonalizing the inflation belief wedge and the one derived from the joint wedge are nearly identical, and display a correlation of 0.99. These evidences support our belief distortion shock solely based on the inflation belief wedge, and justify modelling it as a shock to expected inflation in a theoretical framework.

[12]This technique ensures that the data temporal structure is preserved by training the model only on past observations and validating it on future observations. Starting from an initial time window, at each iteration the training set expands to include all previous observations up to the current split, while the test set shifts forward. The model is trained on a fine grid of hyperparameter values. The optimal value of λ is chosen to minimize the RMSE.

[13]As in Bhandari, Borovička and Ho (2024), the unemployment rate consensus forecast is inferred from categorical responses using the procedure suggested by Carlson and Parkin (1975).

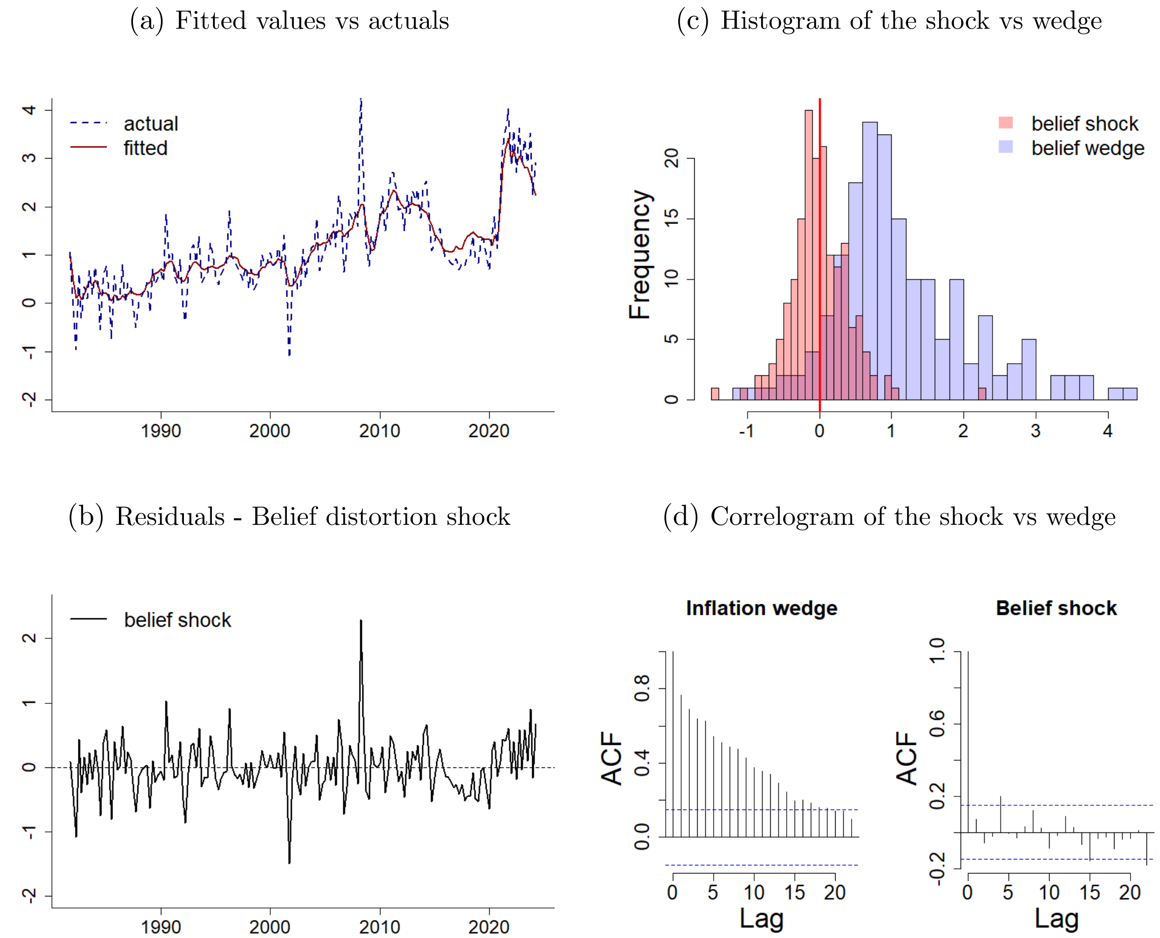


Figure 3: **Identification of the Belief distortion shock.** The figure presents histograms comparing the uncleaned belief wedge and the cleaned belief distortion shock, with the latter being the residual of a ridge regression on the belief wedge. It also displays the fitting regression performance, by comparing actual and fitted values, the estimated residuals, and autocorrelation functions (ACF) for the uncleaned wedge and cleaned shock. *Source:* Michigan Survey of Consumers, Survey of Professional Forecasters, FRED-QD. *Sample:* 1981Q4–2024Q2.

Correlation with other shocks. Following Enders, Kleemann and Müller (2021), we perform a correlation analysis on the belief distortion shock, based on a database of identified policy and non-policy shocks from the literature. Specifically, we regress the residual of the ridge regression model on its 4 lags and the contemporaneous values of some structural innovations. We consider the monetary policy and Central Bank information shocks of Jarociński and Karadi (2020), the monetary policy shock of Romer and Romer (2004), the conventional and unconventional monetary policy shocks of Jarociński (2024), the oil price news shock of Känzig (2021), the oil shocks of Baumeister and Hamilton (2019), the oil supply shocks of Kilian (2008) and Kilian (2009), the oil demand and supply shocks of Caldara, Cavallo and Iacoviello (2019), the "pure" oil price expectation shock of Baumeister (2023), the uncertainty shock of Bloom (2009), the exogenous tax shock of Romer and Romer (2010), the defence news shock of Ben Zeev and Pappa (2017), and the technology shock of Francis et al. (2014). Table 1 shows that most of the estimated correlations are not statistically significant or display small values, e.g. around 0.10 with the oil news shocks of Känzig (2021). However, such low correlations across shock series are to be expected, as these shocks are generated regressors and are subject to measurement errors stemming from model misspecification in the identification processes.

Table 1: **Correlations of the belief distortion shock with known structural shocks.**

Innovation	Sample	Coeff.	Innovation	Sample	Coeff.
Monetary policy shock Jarocinski and Karadi (2020)	1990Q1 - 2024Q2	0.0326	Oil price news Kanzig (2021)	1981Q4 - 2022Q2	0.1014**
Monetary policy shock Romer and Romer (2004)	1981Q4 - 2007Q4	0.0102	Oil supply Baumeister and Hamilton (2019)	1981Q4 - 2024Q2	-0.0684*
Standard monetary policy Jarocinski (2024)	1991Q1 - 2024Q2	0.0231	Oil supply shock Killian (2009)	1981Q4 - 2009Q4	-0.0339
Odyssean forward guidance Jarocinski (2024)	1991Q1 - 2024Q2	0.0556**	Oil supply shock Killian (2008)	1981Q4 - 2007Q4	-0.0598
Long-term asset purchase Jarocinski (2024)	1991Q1 - 2024Q2	0.0493**	Oil supply Caldara et al. (2019)	1987Q1 - 2015Q4	-0.024
Delphic forward guidance Jarocinski (2024)	1991Q1 - 2024Q2	-0.0075	Oil demand Caldara et al. (2019)	1987Q1 - 2015Q4	-0.0099
Central bank information Jarocinski and Karadi (2020)	1990Q1 - 2024Q2	0.0134	Oil demand Baumeister and Hamilton (2019)	1981Q4 - 2024Q2	0.0476
Oil inventories Baumeister and Hamilton (2019)	1981Q4 - 2024Q2	-0.0222	Pure oil price expectation Baumeister (2023)	1986Q1 - 2023Q1	-0.033
Uncertainty Bloom (2009)	1981Q4 - 2017Q4	0.0121	Exogenous tax shock Romer and Romer (2010)	1981Q4 - 2007Q4	0.006
Defense news shock Ben Zeev and Pappa (2017)	1981Q4 - 2007Q4	0.0051	Technology shock Francis et al. (2014)	1981Q4 - 2009Q4	-0.0846**

Notes: The table reports the coefficients estimated regressing the belief distortion shock on its 4 lags and the contemporaneous values of some structural innovations. The regression coefficients are standardized so that they capture the change in the belief distortion shock associated with a one-standard-deviation change in the structural innovations.

3.2 Linear Local Projections Analysis

To estimate the effects of an increase in inflation belief distortions, we run lag-augmented local projections (Jordà, 2005; Montiel Olea and Plagborg-Møller, 2021).[14] Hence, assuming that the shock follows an $AR(p)$ process, we estimate a collection of $H+1$ forecasting regressions:

$$x_{t+h} = \beta_x(h)\tilde{\theta}_t + controls + \epsilon_{t+h}, \quad h = 0, 1, 2, ..., H \tag{2}$$

where h is the horizon at which we run each regression, x_{t+h} is the response variable, $\tilde{\theta}_t$ denotes the belief distortion shock, ϵ_t is the forecast error. We specify x_t as among

[14] Several advantages motivate our approach over standard vector autoregressions (VARs). Specifically, impulse responses are estimated from the Wold representation of the VAR process, which requires invertibility of parameter estimates. This is justified only if the model is not misspecified. Local projections abstract from assumptions about the data-generating process, to estimate robust impulse responses. Another advantage is their flexibility: local projections conveniently accommodate nonlinearities and allow for the computation of state-dependent responses.

$\{\hat{\Delta}_t, \pi_t, u_t\}$, where $\hat{\Delta}_t$ is the obtained measure for inflation disagreement, π_t is the annualized consumer inflation, u_t is the unemployment rate. The sample period spans from 1981:Q4 to 2024:Q2. As *controls*, we include a constant, up to four lags of the shock and all dependent variables. We also include a decaying COVID indicator that takes a value of 1 in 2020:Q2 and decreases linearly to 0 over the subsequent quarters.

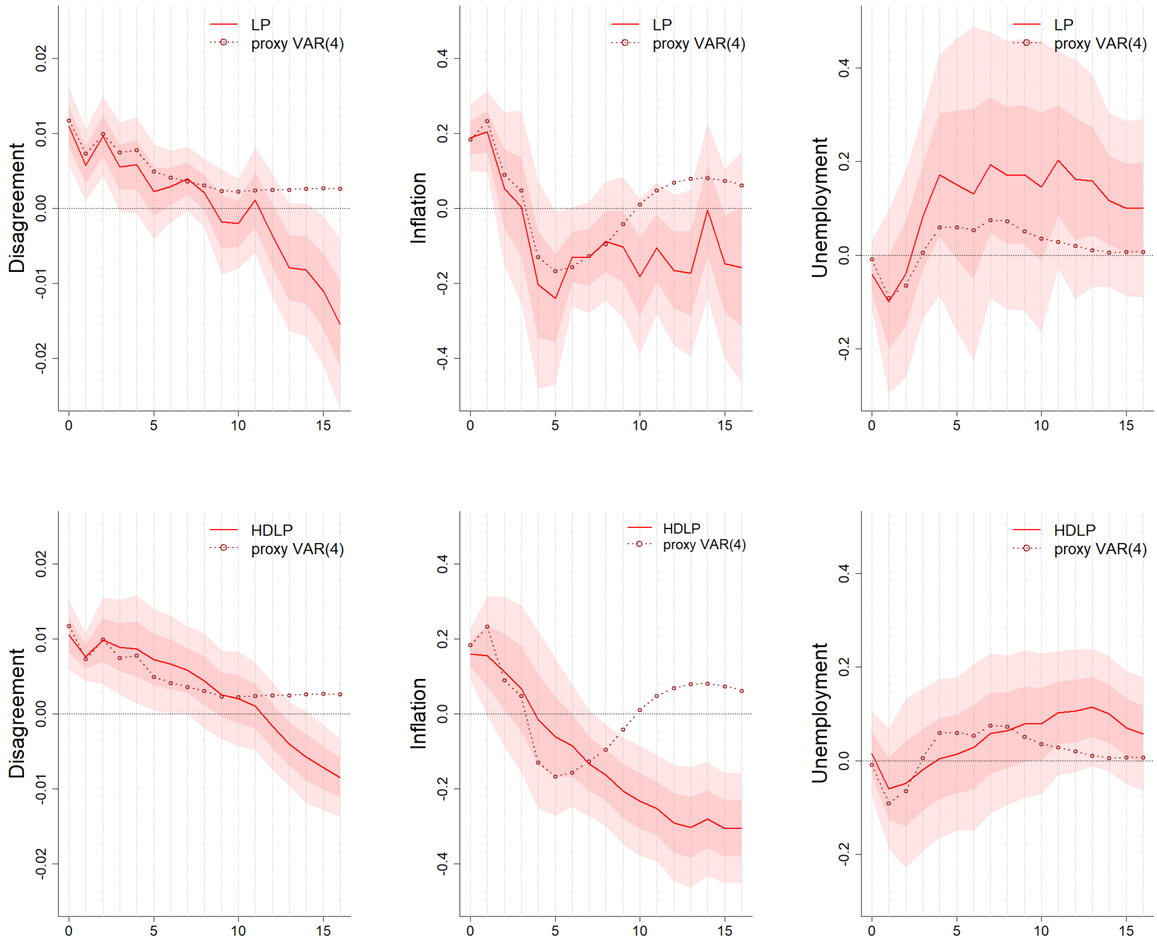


Figure 4: **Impulse responses to a belief distortion shock in linear local projections.** The figure shows the impulse responses of inflation disagreement, inflation, and unemployment to a one standard deviation belief distortion shock. Responses in the top row are generated via lag-augmented local projection with four lags for the control variables. In the bottom row, we estimate the responses through a one-step approach based on high-dimensional local projections (HDLP)(Adamek, Smeekes and Wilms, 2024). The bands show the 95% and 68% confidence intervals based on Newey-West standard errors. The responses are compared with the ones generated by a recursive proxy-internalizing VAR(4). *Sample*: 1981Q4-2024Q2.

We test the robustness of the lag-augmented local projection analysis with two exercises. First, we specify a Proxy VAR(4) in the vector of variables $\hat{y}_t' = (\tilde{\theta}_t, \hat{\Delta}_t, \pi_t, u_t)'$, imposing a recursive structure (Bruns and Lütkepohl, 2022).[15] We choose a lag structure for the Proxy VAR that replicates the local projections, and set four lags for all variables

[15] According to Plagborg-Møller and Wolf (2021), the advantage of this approach over the equivalent SVAR-IV method of Stock and Watson (2018), besides its simplicity, is that the estimated impulse responses are valid even if the shock of interest is non-invertible.

(Plagborg-Møller and Wolf, 2021). Second, we study impulse response functions generated through a one-step approach based on high-dimensional local projections (Adamek, Smeekes and Wilms, 2024). In this case, we directly regress the response variable on the belief wedge and control for the large set of regressors used to identify the shock in the ridge regression model, along with eight of their lags.[16] We follow the identification strategy proposed by Bernanke, Boivin and Eliasz (2005), classifying controls into "fast" and "slow" moving variables, depending on whether in the local projections they are allowed to respond contemporaneously to the shock.[17]

Figure 4 shows the impulse response functions to a one standard deviation belief distortion shock obtained from the lag-augmented local projection (top row), and from the high-dimensional local projections (bottom row). In both columns, the median responses from the Proxy VAR are plotted as dotted-circled lines.

We find that a belief distortion shock significantly and persistently increases the level of disagreement among households—it remains positively affected for up to a two-year horizon at the median. Inflation surges on impact but quickly reverts, partly due to the persistent effects triggered by the ensuing recession. Unemployment is muted on impact but increases after a few periods and remains persistently high, highlighting the recessionary aspect of the belief distortion shock.

Similar dynamics are fairly robust across impulse response functions generated from the Proxy VAR and high-dimensional local projections. Regarding the Proxy VAR, our evidence confirms the findings of Plagborg-Møller and Wolf (2021), as the lag-augmented local projection and the VAR produce equivalent responses at short horizons. As for the high-dimensional local projections, the similar patterns suggest that the results are robust to the inclusion of additional information from the data and confirm the recessionary nature of the belief distortion shock once its effects are investigated within linear specifications.

3.3 State-dependent Analysis

Belief distortions—defined as the wedge between households' inflation expectations and those of professional forecasters, as shown in the upper panels of Figure 1—exhibit significant temporal variation, with notable spikes during the Global Financial Crisis and the post-COVID19 period, when the U.S. economy reached the zero lower bound (ZLB) for nominal interest rates in response to the unfolding impact of the crises. The bottom panels of the same figure show that these periods were also characterized by a more dispersed

[16] Lloyd and Manuel (2024) show that the widely used two-step approach for estimating impulse response functions—in which the researcher identifies the shock externally and then estimates the response by regressing the variable of interest on the identified shock—may, in some circumstances, generate inflated confidence bands and biased responses. We adopt a one-step procedure to mitigate these concerns.

[17] In Appendix A, we report which variables of the FRED-QD database we classify as fast moving.

distribution of households' inflation expectations—a measure of inflation disagreement. Given the contemporaneous movements in inflation disagreement around the peaks of the belief wedge—which also coincide with periods of economic disruption—it is natural to extend the analysis and investigate whether the effects of a belief distortion shock exhibit state dependence. More specifically, we aim to verify whether the recessionary and disruptive effects on the labour market caused by the belief distortion shock, as observed in the linear local projections in Section 3.2, may result from confounding patterns or from limitations of the empirical model, which, being linear, may be unable to distinguish between different states of the economy.

3.3.1 High Disagreement and ZLB states

We investigate the presence of distinct states in the distribution of inflation beliefs, i.e., inflation disagreement. States of high and low disagreement are defined based on the dispersion of inflation expectations among forecasters, measured by the first functional principal component score, $\hat{\Delta}_t$. Following Falck, Hoffmann and Hürtgen (2021), we smooth the inflation disagreement measure using a seven-period backward-looking weighted moving average filter, which we scale by average inflation expectations in order to account for high inflation periods.[18]

The transition between states of high and low inflation disagreement is governed by probabilities defined by a logistic function $S(\hat{\Delta}_{t-1}) \in [0, 1]$, as follows:

$$S(\hat{\Delta}_{t-1}) = \frac{exp\left(\eta \frac{\hat{\Delta}_{t-1}-\mu}{\sigma_{\hat{\Delta}}}\right)}{1 + exp\left(\eta \frac{\hat{\Delta}_{t-1}-\mu}{\sigma_{\hat{\Delta}}}\right)} \tag{3}$$

where η determines the curvature of the function—hence, how strongly the state probabilities react to changes in inflation disagreement—while μ and $\sigma_{\hat{\Delta}}$ represent the median and standard deviation of the state variable.[19]

As a robustness check, we compare these probabilities with states defined by using a "clear cut-off" rule, according to which the economy is in a high disagreement state $S(\cdot)$ with probability 1 if $\hat{\Delta}_{t-1} > 0$, and 0 otherwise.

Figure 5 plots—on the left axis in both panels—the probabilities of being in the high disagreement state, comparing the smooth transition specification with the clear-cut rule. In the same figure but on the right axis, the smoothed state variable, namely inflation disagreement, is plotted in the upper panel, and the smoothed inflation belief distortion shock is plotted in the bottom panel.

Probabilities of being in the high disagreement state exhibit a clear time-varying

[18] Therefore, we lose the first 7 observations and the state-dependent local projections are run over the 1983Q2-2024Q2 period.

[19] As in Falck, Hoffmann and Hürtgen (2021), we set $\eta = 5$.

pattern: they are high in the first part of the sample up to the early 1990s, during the 2008–09 crisis, persist for a few years thereafter, and rise again with the most recent post-COVID inflationary surge. Unsurprisingly, inflation disagreement—in blue solid line in the upper panel—follows a similar pattern. Notably, the last two high-disagreement episodes—coinciding with the Global Financial Crisis and the recovery following the post-COVID19 pandemic—partially align with periods when the federal funds rate reached the ZLB, limiting Central Bank operations.

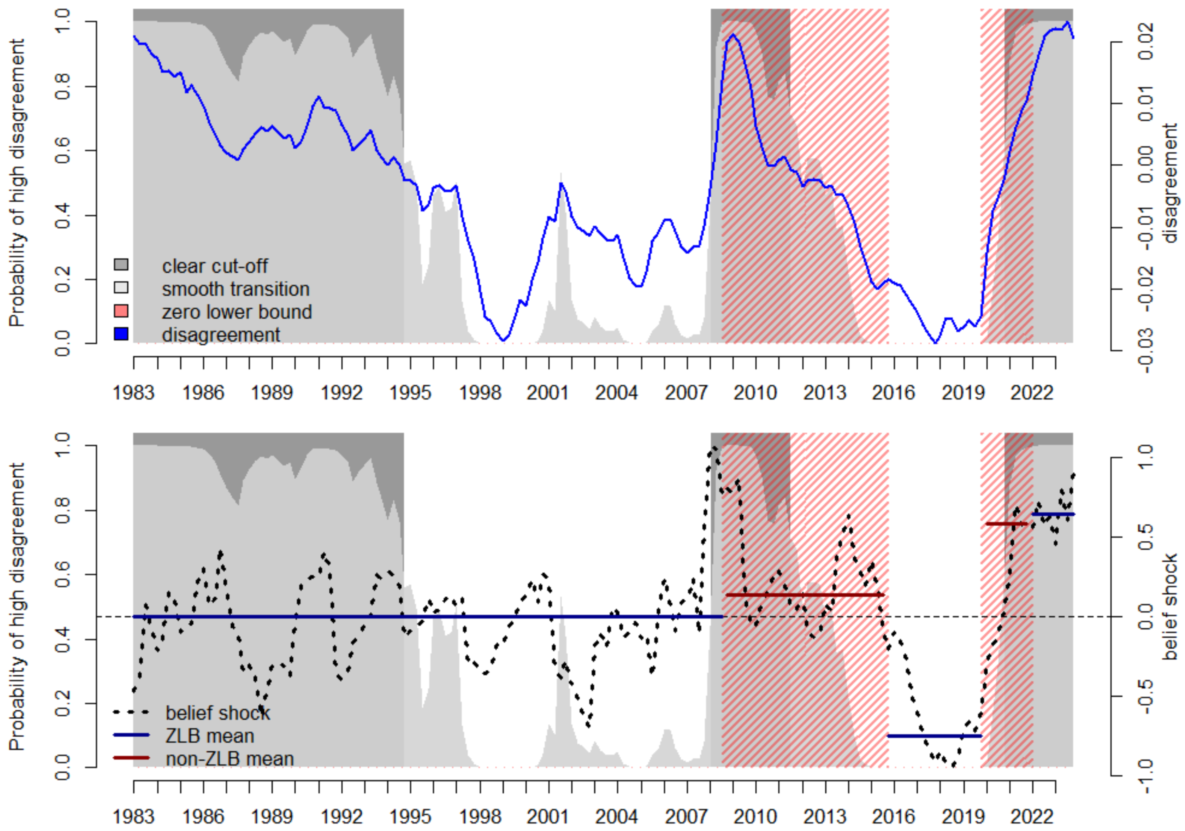


Figure 5: **High disagreement and ZLB states.** The top panel displays the estimated probability of the high disagreement state, as defined by the smooth transition logistic function (light gray areas) and the clear cut-off rule (dark gray areas). The blue line denotes the smoothed measure of inflation disagreement, while red shaded regions indicate ZLB periods. The bottom panel plots the belief distortion shock, smoothed via a moving average filter. The horizontal lines represent the conditional mean of the shock, at the ZLB (red) and outside the ZLB (blue). Sample: 1983Q2-2024Q2.

The belief distortion shock series—in black dashed line in the bottom panel—exhibits state-dependency over the sample period. This is captured by the conditional mean of the structural shocks, which is close to zero in the first part of the sample, but it is higher during both ZLB periods as a result of more positive inflation belief shocks. The conditional mean of the shocks is negative after the first ZLB episode, while remains elevated and positive after the second one, which coincides with the COVID-19 pandemic.

We provide an economic rationale based on movements in the shock series. When the ZLB binds and the Central Bank is constrained, households may expect special intervention by policymakers to restore economic activity, resulting in rising inflation. Positive belief distortion shocks may therefore reflect households' anticipation of *unfunded* expansionary shocks, possibly triggered by fiscal policies that increase inflation but are not offset by the monetary authority through interest rate hikes (Bianchi, Faccini and Melosi, 2023). In contrast, professional forecasters, who may base their expectations on medium-term dynamics beyond the immediate impact of expansionary shocks, may have been more conservative in revising their inflation expectations upward during ZLB periods. The different impact of anticipated measures on households versus professional forecasters may help explain the positive mean of the belief distortion shock during ZLB episodes.

When the economy is constrained at the ZLB, an increase in inflation expectations—such as the one caused by the belief distortion shock—leads to a decrease in the ex-ante real interest rate, stimulating the economy and generating an expansionary effect. In such a case, we would observe an increase in inflation and a decrease in unemployment due to the interest rate channel. This result is consistent with Coibion, Gorodnichenko and Ropele (2020), who provide micro-level evidence of the possibility of expansionary effects from rising inflation expectations at the ZLB. The authors use firm survey data to document that firm profitability increased during the ZLB period. This result is also in line with the work of Bianchi and Melosi (2017, 2019), who propose fiscal strategies aimed at raising inflation expectations to lift the economy out of the ZLB, resulting in negative real rates, higher inflation, and lower debt accumulation.

Given the potential impact of higher inflation expectations at the ZLB, we test whether the transmission of the belief distortion shock exhibits state dependence. However, as Figure 5 shows, ZLB episodes are also characterized by high inflation disagreement, indicating that the overlap of states may influence our results. Hence, we run local projections analysis, which departs from the linear specification in Equation 2 and resorts to nonlinear specifications to study the effects of the belief distortion shock across different states—within and outside the ZLB—while conditioning on the level of inflation disagreement.

3.3.2 State-dependent local projection

We amend the linear specification in Equation 2 and run state-dependent local projections by incorporating both the probability of high disagreement, $S(\hat{\Delta}_{t-1})$, as specified in Figure 5, and a state variable defined as $Z(FFR_{t-1})$, which takes the value 1 when the federal funds rate, FFR_{t-1}, is constrained at the zero lower bound, and 0 otherwise.[20] We label the two states as *ZLB* and *Normal Times*, respectively.

The baseline specification of the state-dependent local projections, labeled as *Baseline*,

[20]Estimating the responses using a state variable for inflation disagreement defined by a "clear-cut" threshold rule yields similar but attenuated results. We use the smooth transition probability as a benchmark to better capture the variability in inflation expectation dispersion within ZLB states.

are given by the following set of regressions for each horizon $h = 0, 1, 2, ..., H$:

$$
\begin{aligned}
x_{t+h} = & \left[\beta_x^Z(h)\tilde{\theta}_t + \mathit{controls}(x,Z) + \sum_{l=0}^{j} \gamma_{\text{int},t-l}^{Z} \tilde{\theta}_{t-l} S(\hat{\Delta}_{t-1-l})\right] Z(FFR_{t-1}) \\
& + \left[\beta_x^{\bar{Z}}(h)\tilde{\theta}_t + \mathit{controls}(x,\bar{Z}) + \sum_{l=0}^{j} \gamma_{\text{int},t-l}^{\bar{Z}} \tilde{\theta}_{t-l} S(\hat{\Delta}_{t-1-l})\right] \bar{Z}(FFR_{t-1}) \\
& + \epsilon_{t+h}, \quad \text{with } \bar{Z} \equiv 1 - Z(FFR_{t-1})
\end{aligned}
\tag{4}
$$

Consistent with the linear specification, h denotes the horizon at which each regression is run, x_{t+h} is the response variable, $\tilde{\theta}_t$ represents the belief distortion shock, and ϵ_t is the forecast error. As *control* variables, we include a state-specific constant and lagged values of both the shock and the response variable. For the sake of tractability, in the baseline specification, we avoid including lagged values of additional variables beyond the one for which we estimate the response to the shock, x.[21] We allow the number of lags for the control variables to be both variable- and state-dependent, as determined by the Akaike Information Criterion.

In the state-dependent local projections of Equation (4), the approach results in a triple interaction term in each state.[22] The triple interaction captures the contribution of inflation disagreement to the dependent variable when the shock hits the economy in the ZLB or in Normal Times.

Given our interest in verifying the recessionary nature and the disruptive effects on the labour market triggered by the belief distortion shock, as indicated by the linear local projection in Section 3.2, we estimate the state-dependent local projections of Equation (4) for unemployment only.[23]

The first two panels in Figure 6 show the response of unemployment to the belief shock—namely, the sequence of β_x in panel a)—and the response to the triple interaction term—namely, the sequence of γ_{int} in panel b), respectively. The responses are state-dependent: the responses during Normal Times are shown in blue, while those during ZLB periods are shown in red.

While in Normal Times, the unemployment response to the belief distortion shock captured by the coefficients $\beta_x^{\bar{Z}}$ is positive—and therefore consistent with the linear local projection specification described in Section 3.2, suggesting a detrimental effect of the shock on the economy—the response has the opposite sign under ZLB states. The

[21]We verify that the empirical findings are robust to the inclusion of additional controls, specifically the variables used in the lag-augmented linear local projection in Section 3.2.

[22]$\tilde{\theta}_t S(\hat{\Delta}_{t-1}) Z(FFR_{t-1})$ and $\tilde{\theta}_t S(\hat{\Delta}_{t-1}) \bar{Z}(FFR_{t-1})$, respectively.

[23]The Akaike Information Criterion suggests 4 lags for the shock, 2 lags for the dependent variable, and $j = 3$ lags for the interaction term in the ZLB state (Z), while in Normal Times (state $\bar{Z}$), it suggests 1 lag for the shock, 4 lags for the dependent variable, and $j = 1$ lag for the interaction term.

response of unemployment to the belief distortion shock under ZLB, captured by the coefficient β_x^Z, reverts in the sign becoming negative and larger in magnitude than that under Normal Times. This suggests a favorable outcome in the labour market when households form higher inflation expectations at the ZLB. This is the case when the *pure* effect of the shock is isolated, i.e., when it is separated from the effects triggered by a high disagreement.

In contrast, when analyzing the estimated response of unemployment to the interaction term under the two states, we observe opposite dynamics. Under the ZLB, the contribution of inflation disagreement is captured by the coefficient γ_{int}^Z. It is positive and statistically significant, providing evidence in favor of an attenuation of the expansionary effect of the belief distortion shock because of the disagreement in households' inflation expectations. The estimated response to the interaction term is negative under Normal Times captured by the coefficient $\gamma_{\text{int}}^{\bar{Z}}$.

Our empirical evidence supports the interpretation of belief distortion shocks under the ZLB as *unfunded* expansionary shocks, as perceived by households (Bianchi, Faccini and Melosi, 2023). Given the distressing times during which they occur, belief distortion shocks raise inflation expectations. When combined with high disagreement among households, they tend to impair the expansionary effect of the shock alone as captured by the positive response of unemployment to the interaction term. In contrast, outside the ZLB, the combined effect of the shock and high disagreement is different, resulting in a decline in unemployment.

Based on the evidence obtained from the state-dependent local projections of Equation (4), we test our findings using an alternative specification, which we label as *Misspecified*. This specification is identical to the *Baseline* but omits the triple interaction term among the belief distortion shock, the probability of high disagreement, and the state of the economy. The *Misspecified* specification is given by the following set of regressions for each horizon $h = 0, 1, 2, \ldots, H$:

$$\begin{aligned} x_{t+h} = & \left[\tilde{\beta}_x^Z(h)\tilde{\theta}_t + \mathit{controls}(x, Z)\right] Z(FFR_{t-1}) \\ & + \left[\tilde{\beta}_x^{\bar{Z}}(h)\tilde{\theta}_t + \mathit{controls}(x, \bar{Z})\right] \bar{Z}(FFR_{t-1}) \\ & + \epsilon_{t+h}, \quad \text{with } \bar{Z} \equiv 1 - Z(FFR_{t-1}) \end{aligned} \tag{5}$$

The terms in the regression equation are the same as in the *Baseline*; however, the control variables no longer include any lags of the interaction term, as this is not considered here. Panel c) in Figure 6 shows the response of unemployment to the belief shock under the *Misspecified* state-dependent local projections—represented by the sequence of coefficients $\tilde{\beta}_x$. The responses of unemployment in ZLB and Normal Times follow a pattern similar to those in panel a), which depicts the *Baseline* specification's responses via the sequence of coefficients β_x from Equation (4). However, while the positive response during Normal Times remains comparable to that in panel a), the negative response under ZLB conditions is notably attenuated in the *Misspecified* case—shifting from approximately -1.5 in the *Baseline* to slightly above -0.4.

These findings underscore the importance of isolating the effect of the belief distortion shock from the effects of disagreement under ZLB conditions. During periods of economic distress, disagreement exerts a disruptive effect on unemployment, and the expansionary impact of the belief distortion shock cannot be fully appreciated without disentangling the two.

(a) Baseline (b) Disagreement contribution (c) Misspecified

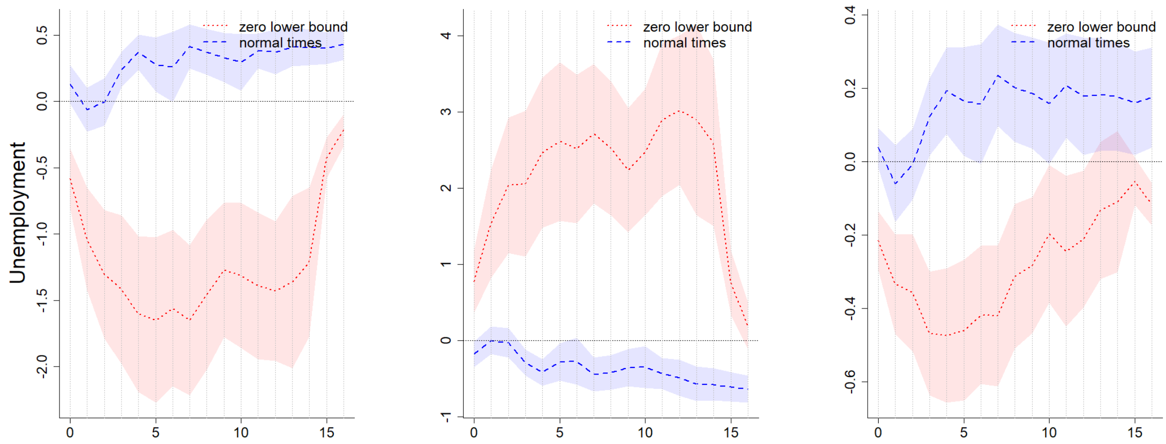


Figure 6: **Unemployment state-dependent response to a belief distortion shock.** Panels (a) and (b) report results from the *baseline* local projections in Equation (4), while panel (c) is based on the *misspecified* specification in Equation (5). Estimation follows the lag-augmentation method of Montiel Olea and Plagborg-Møller (2021), with lag length selected via AIC. Shaded bands represent 68% confidence intervals based on Newey-West standard errors. *Sample: 1983Q3–2024Q2.*

To provide economic reasoning behind the expansionary effect of the belief distortion shock at the ZLB, when controlling for inflation disagreement, we examine the state-dependent transmission of the shock to additional variables. We focus on the vacancy rate and the Shiller price-to-earnings ratio (CAPE), which serves as a measure of firm profitability.[24] The vacancy rate helps assess whether dynamics similar to those observed for unemployment are common to the broader labour market. In contrast, tracking the response of a profitability measure provides insight into state-contingent firm behavior. Figure D.5 in Appendix D.2 shows the impulse responses obtained from the two state-dependent local projection specifications, as for unemployment in Figure 6.

[24] We retrieve data on the vacancy rate provided by Barnichon (2010). As for the Shiller price-to-earnings ratio (CAPE), it is computed as the ratio of the S&P500 index to average inflation-adjusted earnings over the past 10 years, reflects the price investors are willing to pay per dollar of average inflation-adjusted earnings, implicitly incorporating market expectations of future profitability. Data on the Shiller CAPE ratio are freely available in the Shiller database. Source: https://shillerdata.com/.

The results for both the vacancy rate and the CAPE ratio align with those for unemployment under both the *Baseline* and *Misspecified* specifications. These results indicate expansionary effects when the pure contribution of the belief distortion shock is considered. However, the interaction between the shock and disagreement under ZLB conditions generates recessionary effects, confirming the detrimental implications of high disagreement during times of economic distress.

Finally, the *Misspecified* state-dependent local projections deliver results for vacancy and CAPE that are consistent with those for unemployment: the effect of the belief distortion shock is state-dependent and positive under ZLB conditions, though smaller than in the *Baseline* specification, as the contribution of disagreement is not disentangled in the alternative local projections equation.

3.3.3 Validity of State-dependent Local Projections

Recently, Gonçalves et al. (2024) raised concerns about endogeneity and the potential bias of impulse response functions in the context of state-dependent local projections. Moreover, the same contribution highlights the role of large shocks as a leading factor driving the results.

We address these concerns in three steps. First, we note that while the probabilities of high-disagreement states tend to increase with inflation, there is significant co-movement between these probabilities and the identified belief distortion shock. Thus, in our case, the local projection estimator may still yield marginal impulse responses that accurately approximate average conditional responses. Second, we standardize the empirical shock series entering the state-dependent local projection. This ensures that the magnitude of the shock, relative to the standard deviation of the empirical series, remains small. Third, as a preliminary analysis in a nonlinear framework, we test the contribution of large shocks by running a nonlinear local projections in the spirit of Caravello and Martinez-Bruera (2024), to evaluate whether larger shocks may have disproportionately different effects.[25]

Figure D.4 in Appendix D compares the impulse response functions obtained from nonlinear but state-independent local projections. The results show that no size asymmetry in the transmission of the shock prevails in our case, further alleviating concerns that large shocks may bias the results. However, we do observe sign asymmetry in the transmission of the shock. Specifically, positive shocks are relatively stronger than negative shocks for real variables such as unemployment. This finding aligns with our interpretation of positive—and potentially relevant—shocks as those occurring during ZLB periods, as indicated by the conditional mean of our shock series.

[25] Caravello and Martinez-Bruera (2024) demonstrate that, under the assumption of symmetric shock distributions—which we verify holds for the belief distortion shock—introducing nonlinear transformations within the local projection framework effectively captures both size and sign asymmetries. Specifically, even-order nonlinear transformations capture sign asymmetries, while odd-order nonlinear transformations capture size asymmetries.

4 Theoretical Model

The model considered is a basic New Keynesian model featuring search and matching frictions (SAM) in the labour market, labelled as NKSAM and modified to embed belief distortion shocks. The economy is populated by households, firms, and a monetary policy authority. Households consist of a continuum of worker members. They consume a basket of differentiated retail-goods and their consumption is characterized by internal habits formations. They own a continuum of firms, each of which uses one worker to produce an intermediate-good under monopolistic competition and flexible prices. The labour market is characterized by search and matching frictions. In each period, a fraction of workers is unemployed and searches for jobs. Firms post vacancies at a fixed cost. The number of successful matches is produced with a Cobb-Douglas matching technology. Real wages are determined by Nash bargaining between firms and workers. Real wages are however sticky and adjust slowly to their Nash optimal value. The government finances workers' unemployment benefits through lump-sum taxes. Retail sector firms compete under monopolistic competition and set their prices using quadratic Rotemberg (1982) adjustment costs. Finally, the monetary policy is described by the following standard Taylor rule, where the nominal interest rate responds to deviations of inflation from their long-run target.

The model equations are reported in Table 2. We now present a very brief description of the model, underlying the way in which we model inflation expectations and belief distortion shocks.

Inflation Expectations and Belief Distortion Shocks The primary shock analyzed in the model economy is a shock to inflation expectations. We slightly deviate from the standard rational expectation assumption by positing that agents' forecast for the next period, Π^e_{t+1}, follows the rational expectation hypothesis but is subject to a shock, as:

$$\Pi^e_t \equiv E_t[\Pi_{t+1}]e^{\varepsilon_{bd,t}}, \tag{6}$$

with $E_t[\Pi_{t+1}]$ being the rational expectation in t of *t+1* gross inflation and $\varepsilon_{\pi,t}$ being an exogenous process that allows inflation expectations to deviate from their rational expectation solution. By taking the logs of the previous equation and subtracting the agent expectation from the rational expectation we get a measure of the belief distortion, BD, which is entirely exogenous and function of the shock:

$$BD = \pi^e_t - E_t\pi_{t+1} = \varepsilon_{bd,t}, \tag{7}$$

The belief distortion shock has zero mean and a constant standard deviation equal to σ_{bd}:

$$\varepsilon_{bd,t} = \rho_{bd}\varepsilon_{bd,t-1} + \sigma_{bd}u_{\pi^e,t}. \tag{8}$$

Endogenous Inflation Disagreement A novel part of our analysis is that we focus on how shocks to inflation expectations affect uncertainty around agents' expected inflation in the model. To this scope, as in Basu and Bundick (2017) and Mumtaz and Theodoridis (2020), we alternatively use the term volatility of expected variables or measured uncertainty to refer to the heteroskedastic response of a variable. In the model, we define the inflation disagreement as:

$$\hat{\sigma}_{\pi,t} = 100\log\left(\frac{\sigma_{\pi,t}}{\sigma_\pi}\right), \qquad \text{where} \qquad \sigma_{\pi,t} = var_t(\pi_t) = E_t[\pi_{t+1} - E_t\pi_{t+1}]^2, \tag{9}$$

Despite our focus on the belief distortion shock, we aim to examine the transmission of this shock under both normal times and distress periods, characterized by the zero-lower bound constraint in the interest rate. For this reason, we also introduce a risk premium shock, as specified in Smets and Wouters (2007). This shock allows the economy to enter a recession and enables monetary policy to reach the zero lower bound.

In particular, we denote the risk premium by $\varepsilon_{rp,t}$, which has a zero mean and a constant standard deviation equal to σ_{rp}. The AR(1) process characterizing the risk premium shock is presented in Table 2, along with the other equations describing the model.

Model Calibration The model is calibrated using standard values from the literature. The discount factor ($\beta = 0.99$) is set accordingly, while the intertemporal elasticity of substitution is unitary. The steady-state unemployment rate is $U = 0.064$ close to the mean value in the US over the sample period considered in the empirical analysis. Labor market parameters include a vacancy filling probability ($q^v = 0.7$), a Nash bargaining weight ($\chi = 0.5$), and a real wage rigidity parameter ($\gamma = 0.8$). The separation rate is set at $\rho = 0.1$, while vacancy posting costs are given by $\kappa = 0.02(Y/v) = 0.14$. The elasticity of substitution among varieties is $\eta = 6$, and price adjustment costs are $\Omega_P = 50$, which corresponds to a probability of resetting price every three quarters. Monetary policy follows a Taylor rule with inflation weight ($\rho_\Pi = 1.5$). The steady-state gross inflation rate is $\Pi = 1.005$.

For shock processes, the autoregressive parameter is set to 0.9 for both shocks ($\rho_{\pi^e} = \rho_\varepsilon = 0.9$) and the steady-state standard deviations to 0.01 ($\sigma_{\pi^e} = \sigma_\varepsilon = 0.01$).

Table 2: **Model Equations**

Description	**Equation**
Euler Equation	$\frac{1}{R_t} = \beta e^{\varepsilon_{rp,t}} E_t \left[\frac{\Lambda_{t+1}}{\Lambda_t} \frac{1}{\Pi_t^e}\right]$
Nash Bargaining Real Wage	$w_t = \chi \left(q_t Z_t + (1-\rho)\beta E_t \frac{\Lambda_{t+1}}{\Lambda_t} \frac{\kappa v_{t+1}}{u_{t+1}}\right) + (1-\chi)\left(\frac{\chi}{\Lambda_t} + \phi\right)$
Production Function	$Y_t = Z_t N_t$
Resource Constraint	$\left(1 - \frac{\Omega_P}{2}\left(\frac{\Pi_t}{\Pi} - 1\right)\right)^2 Y_t = C_t + \kappa v_t$
Phillips Curve	$q_t = \frac{\eta-1}{\eta} + \frac{\Omega_P}{\eta}\left(\frac{\Pi_t}{\Pi} - 1\right)\frac{\Pi_t}{\Pi} - \frac{\Omega_P}{\eta}\beta E_t \left[\frac{\Lambda_{t+1}}{\Lambda_t}\frac{Y_{t+1}}{Y_t}\left(\frac{\Pi_t^e}{\Pi} - 1\right)\frac{\Pi_t^e}{\Pi}\right]$
Job Creation Condition	$\frac{\kappa}{q_t^v} = q_t Z_t - w_t + (1-\rho)\beta E_t \left\{\frac{\Lambda_{t+1}}{\Lambda_t}\frac{\kappa}{q_{t+1}^v}\right\}$
Matching Function	$m_t = \mu u_t^\alpha v_t^{1-\alpha}$
Job Finding Probability	$q_t^u = \frac{m_t}{u_t}$
Vacancy Filling Probability	$q_t^v = \frac{m_t}{v_t}$
Employment Law of Motion	$N_t = (1-\rho)N_{t-1} + m_t$
Unemployment Rate	$U_t = 1 - N_t$
Job Searcher Population	$u_t = 1 - (1-\rho)N_{t-1}$
Risk Premium Shock	$\varepsilon_{rp,t} = \rho_{rp}\varepsilon_{rp,t-1} + \sigma_{rp}u_{b,t}$
Belief Distortion Shock	$\varepsilon_{bd,t} = \rho_{db}\varepsilon_{bd,t-1} + \sigma_{bd}u_{bd,t}$
Government Budget Constraint	$(1 - N_t)\,\phi = T_t$
Taylor Rule	$\frac{R_t}{R} = max\left(1, \left(\frac{\Pi_t}{\Pi}\right)^{\rho_\Pi}\right)$

4.1 Impulse Response Functions

In this section, we take the third-order approximation of the model economy and show the impulse response function of unemployment, inflation, and inflation disagreement to a belief distortion shock that hits the economy in normal times and in a recessionary scenario, where the Central Bank faces the ZLB constraint. In the latter case, we hit the economy by a belief distortion shock together with a sequence of risk premium shocks that drag the economy into a liquidity trap for several periods.[26]

Figure 7 shows the contribution of a belief distortion shock in both normal times and during the ZLB scenario. As for the empirical findings, the response of the rate unemployment is state-dependent. During normal times, the belief distortion shock behaves like a negative supply shock: it is both inflationary and recessionary, leading to higher inflation disagreement and increased unemployment. Although the shock remains inflationary when the economy is at the ZLB, the increase in inflation disagreement is significantly higher than in normal times, while the unemployment response turns negative, resulting in an economic boom. Therefore, unemployment follows a similar state-dependent dynamic as observed in the data, confirming the model's ability to replicate the empirical evidence described in Section 3.2.

The responses to the belief distortion shock under the ZLB in Figure 7 are obtained by isolating its contribution from that of the recessionary risk-premium shocks that drive the

[26] To solve the model in a stochastic environment, we use Dynare and the OBC toolbox developed by Tom Holden. See https://github.com/tholden/dynareOBC.

economy to the ZLB. To enhance clarity, we generate impulse response functions (IRFs) for inflation disagreement, inflation, and unemployment in response to a risk-premium shock that brings the nominal interest rate to the ZLB, as well as IRFs for the same shock combined with a one standard deviation increase in the belief distortion shock. The responses to the belief distortion shock under the ZLB, shown in Figure 7, are then derived as the difference between the IRFs generated by both shocks together and those generated by the risk-premium shock alone.

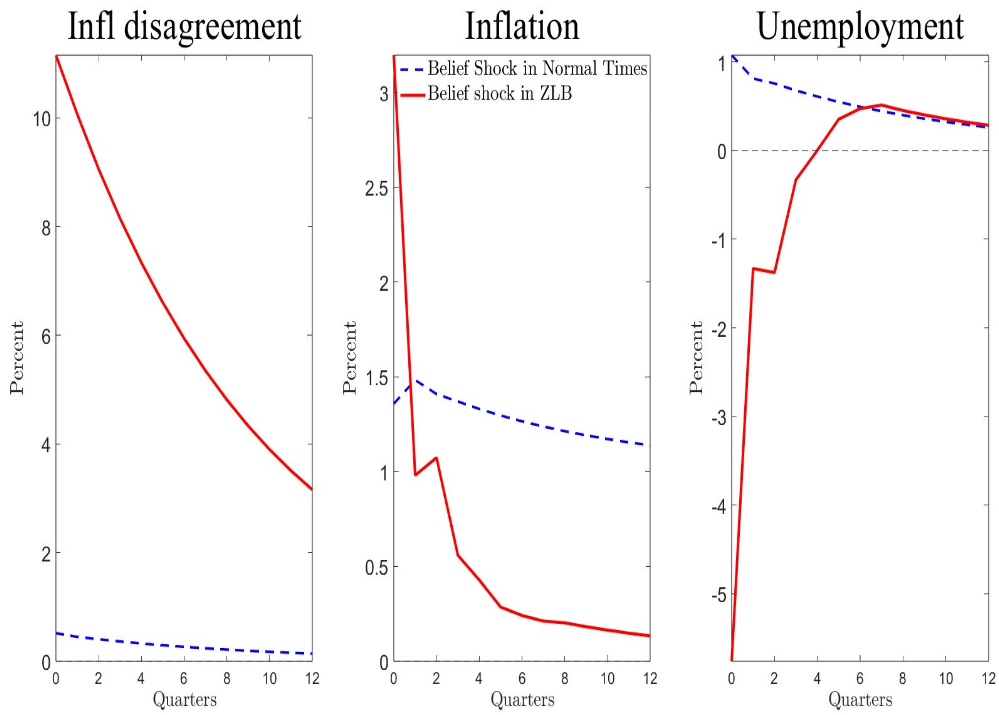


Figure 7: **Model Dynamics.** Simulated responses to a one standard deviation in *belief distortion shock* in Normal Times (blue dashed lines) and in ZLB (red solid lines).

Figure 8 presents the responses of unemployment and the nominal interest rate to the risk-premium shock that drives the economy to the ZLB, with (solid lines) and without (dashed lines) the additional positive belief distortion shock. The belief distortion shock mitigates the severity of the recession, as indicated by the smaller increase in unemployment (left panel). Overall, the belief distortion shock supports an earlier exit from the ZLB, as reflected in the nominal interest rate remaining at the lower bound for fewer periods when the shock is present (right panel).

The key driver of this result is inflation expectations. An increase in inflation expectations, induced by our identified belief distortion shock in ZLB, lowers the ex-ante real interest rate, stimulating the economy and generating expansionary effects that accelerate the recovery. As policymakers—specifically the Central Bank in this case—adopt a prolonged accommodative stance that does not offset the effects triggered by elevated inflation expectations, the conditional effect of the belief distortion shock becomes ex-

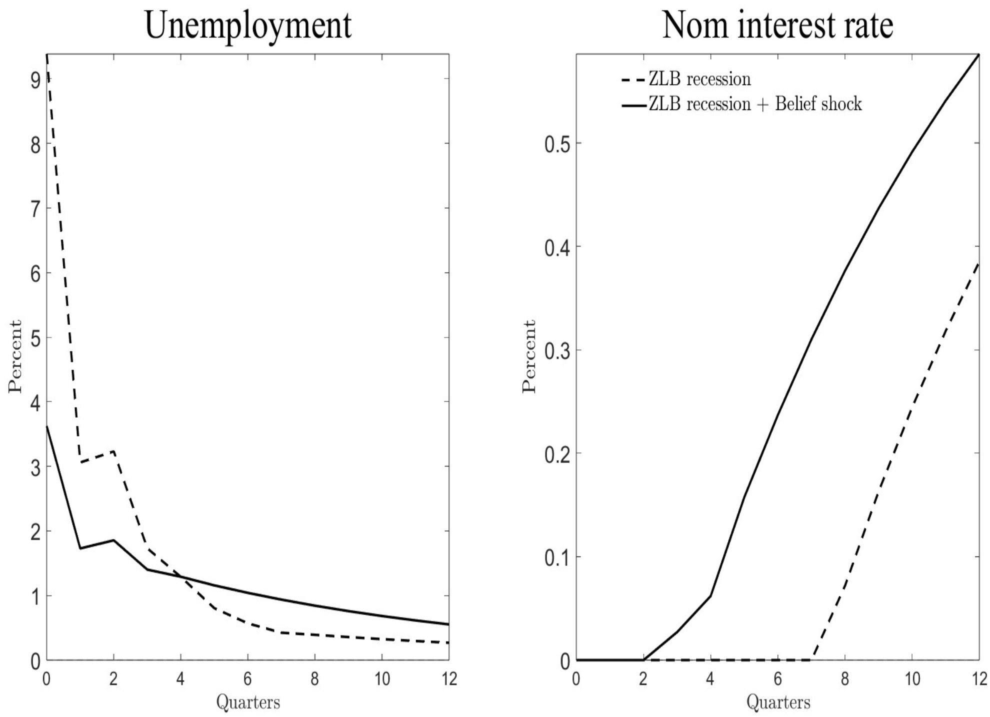


Figure 8: **Model Dynamics.** Simulated responses to a one standard deviation increase in the *risk premium shock* that brings the economy into the ZLB (black dashed lines); simulated responses to the same same risk premium shock combined with a one standard deviation in *belief distortion shock* (black solid lines).

pansionary, resulting in lower unemployment in ZLB.

4.2 Model Validation

In this section, we evaluate the model's ability to generate endogenous disagreement consistent with its empirical counterpart and to reproduce a state-dependent response of the unemployment rate to shocks. To achieve this, we feed the model with empirical values of the belief distortion shock and generate model-implied simulated data for the sample 1983Q3-2024Q2. For consistency with the data, we hit the economy by a risk premium shock in order to get the same number of ZLB periods that the US economy has experienced during the Global Financial Crisis and Covid-19 crises.

Simulated vs. Empirical Disagreement To validate the model's ability to generate inflation disagreement in line with empirical data, we simulate the third-order solution of the model using identified belief distortion shocks. We obtain the simulated path for inflation disagreement and compare it with the shock contribution from the historical decomposition of the proxy VAR model estimated in Section 3.2. Figure 9 shows that the simulated disagreement closely aligns with the empirical data with a correlation of 0.82.

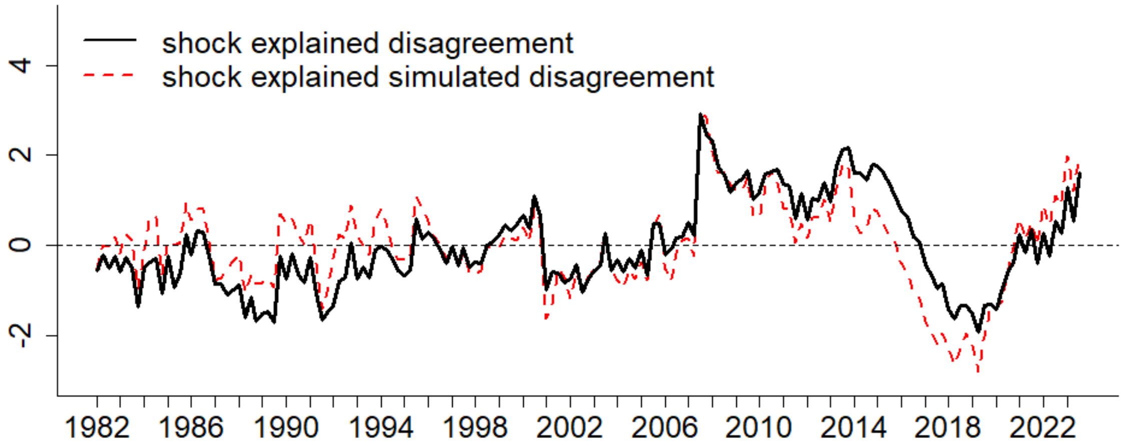
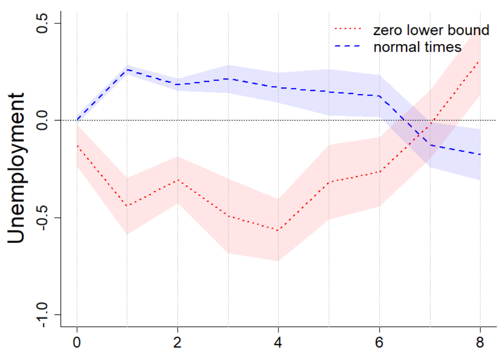


Figure 9: **Simulated versus Empirical Disagreement.** Disagreement from simulated data obtained by feeding the theoretical model with the empirical belief distortion shocks (red dashed lines); Shock contribution to Disagreement obtained from the historical decomposition of a proxy-VAR (black solid lines). Both measures are scaled for comparability. *Sample: 1982Q4-2024Q2.*

State-Dependent Local Projection with Simulated Data To further validate the performance of the model, we now use the simulated data from the model to run state-dependent local projections. As in the empirical part, we estimate Equation 4 using the simulated data for unemployment. Figure 10 (panel a) shows the response β_x^Z, which captures the effect of the shock on the unemployment rate at the ZLB, and $\beta_x^{\bar{Z}}$, which represents the effect of the shock on unemployment when the economy is outside the ZLB period.

(a) Baseline



(b) Disagreement contribution

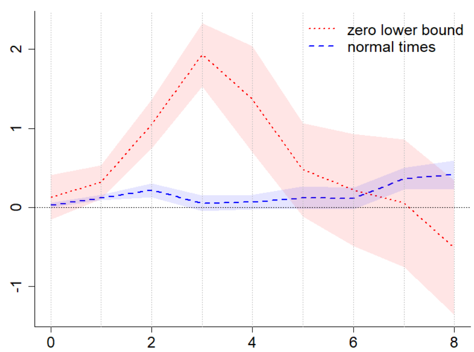


Figure 10: **Unemployment response to a belief distortion shock under Normal Times and ZLB.** Local projections are estimated on model-simulated data using the lag-augmentation method of Montiel Olea and Plagborg-Møller (2021) and the specification in Equation (4), where lags are selected by minimizing the Akaike Information Criterion. It suggests, in the Zero Lower Bound (ZLB) state (Z), the inclusion of 3 lags of the shock, 4 lags of the dependent variable, and $j = 4$ lag for the interaction term. In normal times (state $\bar{Z}$), the AIC recommends 2 lags of the shock, 2 lags of the dependent variable, and $j = 2$ lags for the interaction term. The bands show the 68% confidence intervals, based on Newey-West standard errors. *Sample: 1983Q3-2024Q2.*

Figure 10 (panel b) instead reports the disagreement contribution, which is given by the coefficient γ_{int}^{Z} at the ZLB and by $\gamma_{\text{int}}^{\bar{Z}}$ outside the ZLB period. It shows that, consistently with the empirical counterpart, inflation disagreement per se increases the unemployment rate in the ZLB, thus contributing to dampen the expansionary effect of the belief distortion shock.

5 Conclusions

This paper examines the macroeconomic effects of a belief distortion shock, defined as an unexpected rise in the wedge between household and professional forecaster expectations. Using survey and macroeconomic data alongside machine learning techniques, we identify this shock and study its impact through linear and state-dependent local projections. We find that it significantly raises inflation disagreement and has state-dependent effects: it is inflationary and recessionary in normal times, but expansionary during crises when monetary policy is accommodative and at the ZLB.

To interpret these findings, we develop a New Keynesian model incorporating belief distortion shocks. The model replicates the observed dynamics in inflation, unemployment, and disagreement, confirming that these shocks depress activity in normal times but support recovery in deep recessions—such as the Global Financial Crisis and the COVID-19 downturn—by lowering real rates. A key validation of our results comes from the model's ability to generate simulated data that closely match empirical results. By feeding the identified belief distortion shock into our theoretical framework, we obtain simulated paths for inflation disagreement that align with those observed in the historical decomposition from the empirical analysis.

Our study shows that belief distortions are a key source of inflation disagreement and business cycle fluctuations. These findings have important policy implications: Central Banks should account for belief heterogeneity and its state-dependent effects when responding to inflation. Future research could explore optimal policy design and communication strategies that target belief dispersion and its macroeconomic impact.

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Appendices

A Data

A.1 Survey expectations data

To measure households' inflation expectations, we use US data from the Michigan Survey of Consumer Attitudes (MSC), conducted by the Survey Research Center at the University of Michigan, over the period 1981Q4-2024Q2. The MSC survey is a rotating panel of approximately 500 to 1500 households, with respondents from the general public being either new participants or returning respondents from the prior six and twelve months. Data are collected monthly, with the second month of each quarter selected to create a quarterly series.

One-period-ahead inflation forecasts are derived from responses to the question: "By about what percent do you expect prices to go (up/down), on the average, during the next 12 months?".[27] We follow the data preparation approach of Curtin (1996), applying a truncation rule of $[-5, 50]$ to remove outliers. We exclude responses without directional information on expected inflation. Categorical responses indicating only an increase/decrease without a specific rate are imputed, proportionally distributed across positive and negative values in the sample.

For rational expectations, we utilize CPI inflation and unemployment rate annual forecasts from the Survey of Professional Forecasters (SPF) of the Philadelphia Fed.[28] These forecasts are collected quarterly during the second month of each quarter, with respondents ranging from 9 to 83 professionals, including economists, bankers, and forecasters. Given their access to advanced forecasting methods, we consider these professionals to approximate rational agents relative to households (Bonham and Dacy, 1991).

A.2 Fred-QD data

In order to identify the belief distortion shock through a ridge regression approach as shown in Section 3.1, we control for a large set of variables from the FRED-QD (Federal Reserve Economic Data - Quarterly Data) dataset.[29] It is a comprehensive macroeconomic dataset sourced from the Federal Reserve Bank of St. Louis, and it is widely used in empirical macroeconomics for forecasting and structural analysis. As reported by McCracken and Ng (2020), it provides a large set of real-time updated macroeconomic and financial data, and it is built to keep a comparable coverage as Stock and Watson (2012). The dataset consists of 248 variables, some of which are not trivially obtained through the FRED database. Nominal series contained in the dataset are made real by adjusting them through a deflator. Most of the series are available starting from 1951-Q1.

[27] Michigan Survey of Consumers Questionnaire.
[28] Philadelphia FED Survey of Professional Forecasters.
[29] The dataset is freely accessible at http://research.stlouisfed.org/econ/mccracken/sel/

The series are divided in 14 groups: NIPA; Industrial Production; Employment and Unemployment; Housing; Inventories, Orders, and Sales; Prices; Earnings and Productivity; Interest Rates; Money and Credit; Household Balance Sheets; Exchange Rates; Other; Stock Markets; and Non-Household Balance Sheets. The portion of the dataset we consider spans from 1981-Q4 until 2024-Q2. Because of the lack of observations, we drop some of the variables contained in the dataset, ending up with 224 series in total.[30]

In the high-dimensional local projections (Adamek, Smeekes and Wilms, 2024), we consider as fast-moving the following variables: Group 4: Housing; Group 8: Interest Rates; Group 9: Money and Credit; Group 11: Exchange Rates; Group 12: Other; Group 13: Stock Markets plus Shares of gross domestic product: Gross private domestic investment: Change in private inventories (A014RE1Q156NBEA); Real Manufacturing and Trade Inventories (INVCQRMTSPL); Total Business Inventories (BUSINVx); Total Business: Inventories to Sales Ratio (ISRATIOx); Real Crude Oil Prices: West Texas Intermediate (WTI) - Cushing, Oklahoma (OILPRICEx); Nonrevolving consumer credit to Personal Income (CONSPIx).

B Functional principal component analysis

B.1 Densities of Survey inflation Expectations

We approximate the disagreement among economic agents by using the distribution of inflation expectations. Hence, we estimate the kernel density for the responses of 1-year-ahead inflation expectations of households. This methodology consists of a non-parametric approach aimed at estimating the probability density function of a random variable by using kernel functions as weights. In line with Chang, Gómez-Rodríguez and Hong (2022), we adopt the rule-of-thumb proposed by Silverman (1986) to set the bandwidth parameter λ:

$$\lambda = \left(\frac{4}{3}\right)^{\frac{1}{5}} \hat{\sigma} n^{-1/5} \tag{B.10}$$

We then use the Epanechnikov (1969) kernel $K(x)$ as our weighting function, as it minimizes the mean integrated square error (MISE), defined as $\text{MISE} = E\int[\hat{f}(x) - f(x)]^2 dx$. The Epanechnikov kernel is given by:

$$K(x) = \frac{3}{4}(1 - x^2) \quad \text{for } |x| \leq 1 \tag{B.11}$$

[30] Because of the lack of observations within the sample, we exclude from the manufacturing sector the hours worked per capita, real output, real manufacturing sector new orders for durable goods, real value of new orders for consumer goods, oil price, the real value unfilled orders for durable goods, the real value of new orders for unfilled goods, the real compensation per hour, real output per capita per hour, the unit labor cost. Further, we exclude real retail and food services sales, the spread between the 3-month commercial paper and the corresponding treasury bill, net percentage of U.S. bank loan officers reporting increased willingness to issue consumer installment loans, the S&P/Case-Shiller 10 and 20 City Composite Home Price Index, the value of the U.S. dollar relative to a basket of advanced foreign currencies, weighted by trade volume, the US-Euro exchange rate, the interest rate on 3-month commercial paper issued by AA financial institutions, the spread between the 3-month commercial paper rate and the federal funds rate and the owners' equivalent rent of residences.

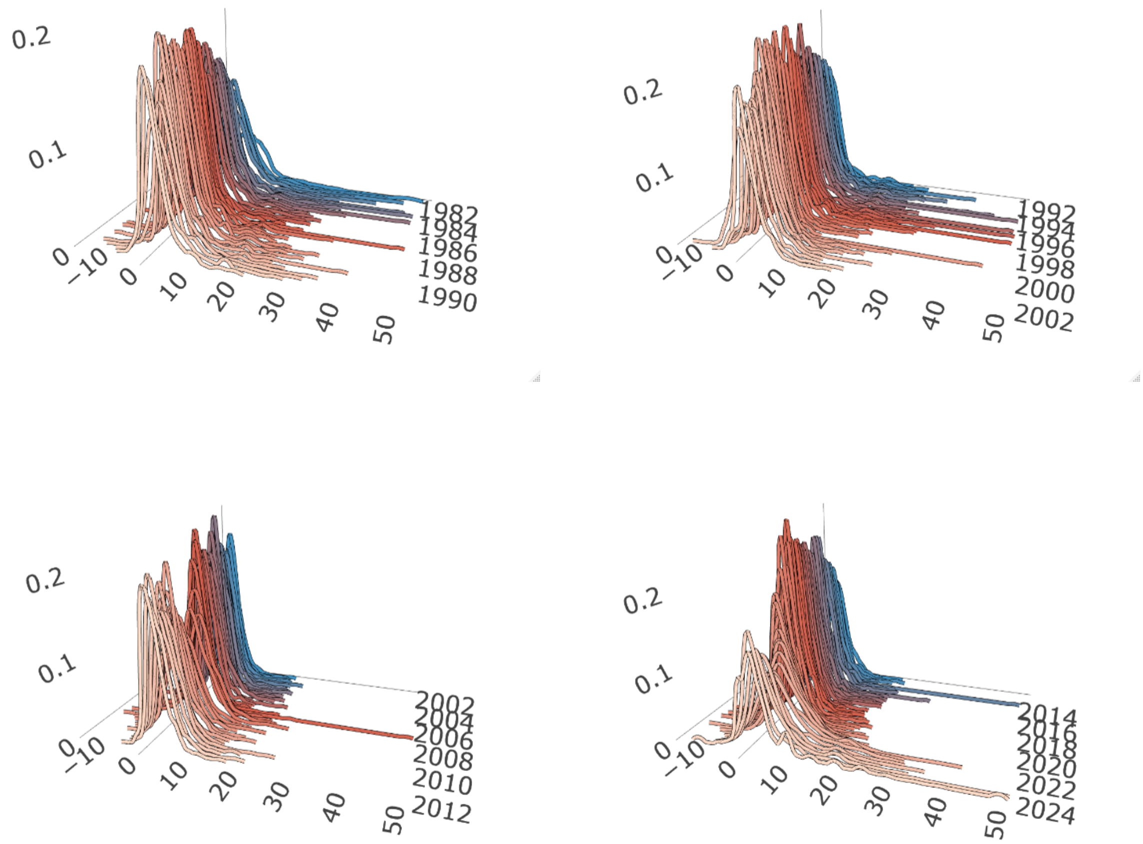

Figure B.1: **Estimated densities for households' inflation expectations.** The figure plots the estimated probability density functions for 1-year-ahead households inflation expectations, for four subsets of our sample. Estimation is based on Epanechnikov's kernel density and Silverman (1986)'s rule-of-thumb. Densities are centered around a common mean. *Source: Michigan Survey of Consumers.*

Hence, the estimated probability density function for each quarter will be:

$$\hat{p}_t(x) = \frac{1}{\lambda n} \sum_{i=1}^{n} K\left(\frac{x - x_i}{\lambda}\right) \tag{B.12}$$

Results are reported in Figure B.1. We observe that the distributional dispersion declined steadily from 1990 until 2020, but increased sharply with the onset of the COVID-19 pandemic, when the densities flattened and became more right-skewed (Weber, Gorodnichenko and Coibion, 2022).

B.2 Functional curves via spline smoothing

Starting from the set of discrete probability density functions obtained via kernel density estimation, we aim to estimate an equal number of functional curves. To get this approximation we follow a spline smoothing approach, as by Ramsay and Silverman (2005). The functional approximation of the probability density functions has to satisfy a non-negativity constraint. For this reason, we operate on the log densities. We can reconstruct the original curves by exponenting the log probability density functions and normalizing them ex-post in order to have them integrating to 1 as in Chang, Chen and Shorfeide (2024).

Given a discrete probability density function $p_t(x)$, we define the corresponding log density as $l_t(x) = \ln p_t(x)$. The original density can then be recovered through the following relation:

$$p_t(x) = \frac{exp\{l_t(x)\}}{\int exp\{l_t(x)\}dx} \tag{B.13}$$

We take a set of K known basis functions, given by cubic splines, allowing us to approximate any given function through a linear combination of them. A given log density $l_t(x)$ is then represented through its basis expansion as follows:

$$l_t^K(x) = \sum_{k=1}^{K} c_{k,t}\phi(x) = c_t'\phi(x) \tag{B.14}$$

where $\phi(x) = (\phi_1(x), \phi_2(x), ..., \phi_K(x))'$ is the vector of basis functions, $c_t = (c_{1,t}, c_{2,t}, ..., c_{K,t})'$ is the corresponding vector of basis coefficients for time t. The superscript K indicates the number of basis functions used in the basis expansion. In terms of matrix notation, we want to estimate the following model:

$$L^J = \Phi C + \epsilon \tag{B.15}$$

where L^J is the matrix of observed log-densities evaluated on a fine grid of J points across T time periods. Each column of this matrix corresponds to a specific $l_t(x)$. Φ is the design matrix containing evaluations of the K cubic splines for each gridpoint and C is the matrix of unknown basis coefficients to be estimated. We assume the residuals to be i.i.d with zero mean and constant variance, across gridpoints and time. To control the smoothness of the estimated log-density functions, we introduce a roughness penalty. The penalty is controlled by a smoothing parameter λ, defined over the interval $[0, +\infty)$, and it is based on the integrated squared second derivative of each log-density function — a common measure of curvature or "acceleration". Hence, for each log density $l_t(x)$ the penalty is given by:

$$PEN(l_t) = \int [l_t''(x)]^2 dx = c'Pc \tag{B.16}$$

where P is the penalty matrix that depends on the basis functions and their second derivatives. Then, the coefficient matrix C is estimated by minimizing the sum of squared errors, penalized by the roughness of the fitted curves. The objective function is:

$$PENSSE_\lambda = (L^J - C\Phi)'(L^J - C\Phi) + \lambda\mathbf{trace}(C'PC) \tag{B.17}$$

the closed form solution is given by:

$$\hat{C} = (\Phi'\Phi + \lambda P)^{-1}\Phi' L^J \tag{B.18}$$

and the smoothed fitted log-density matrix is given by:

$$\hat{L^J} = \Phi\hat{C} = \Phi(\Phi'\Phi + \lambda P)^{-1}\Phi' L^J = S_{\lambda,\Phi} L^J \tag{B.19}$$

and S_λ is the smoother matrix.

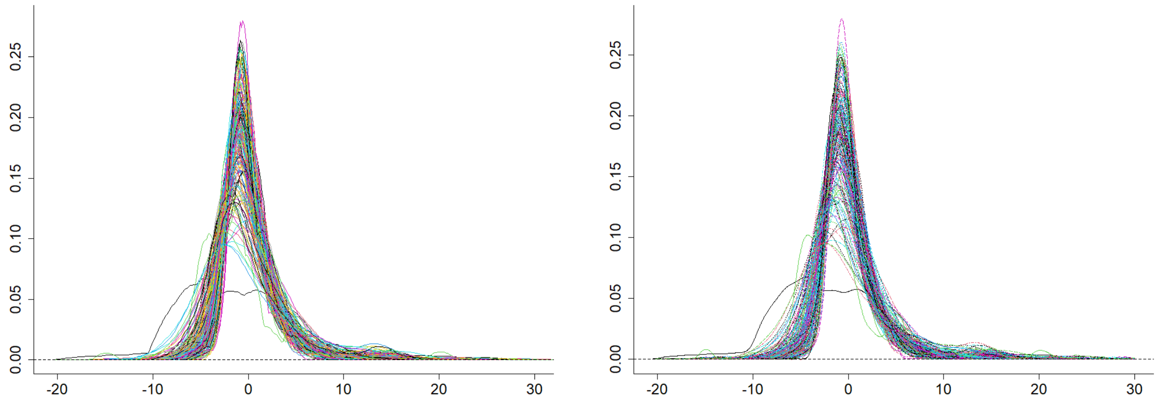

Figure B.2: **Registered densities and their smooth functional approximation.** The figure on the left represents the centered probability density functions for household's 1-year-ahead inflation expectations estimated by using kernel density estimation with the Epanechnikov's kernel function and the rule-of-thumb value for the bandwidth parameter. The figure on the right represents the smooth functional curves derived through spline smoothing. *Source: Michigan Survey of Consumers.*

The estimation proceeds in two steps. In the first step, we set $\lambda = 0$ and we estimate the functional approximation of the log densities for different K. The choice of the optimal K is done by selecting the number of basis functions for which we reach a plateau in the RMSE computed by comparing the probability density functions with their approximation. After choosing the optimal K, we select the value of the smoothing parameter λ that minimizes the GCV criterion:

$$GCV = \frac{n^{-1} SSE}{[n^{-1}\text{trace}(I - S_{\lambda,\Phi})]^2} = \frac{n}{n - \text{df}(\lambda)} \frac{SSE}{n - \text{df}(\lambda)} \tag{B.20}$$

where n is the number of observations, SSE is the sum of the square of the errors, $S_{\lambda,\Phi}$ is the smoothing matrix and $\text{df}(\lambda)$ indicates the degrees of freedom associated with the smoothing parameter. This procedure leads us to select $K = 128$ and $\lambda = 0.1$ Then, we perform spline smoothing. The smooth log densities are then exponentiated and normalized to integrate to 1 to recover a smooth approximation of the distribution of survey responses about the 1-year-ahead inflation expectations of households, as from Figure B.2.

$$p_t^K(x) = \frac{exp\{l_t^K(x)\}}{\int exp\{l_t^K(x)\}dx} \tag{B.21}$$

At last, we center the obtained functional curves by subtracting the following mean curve:

$$\hat{\mu}(x) = \frac{1}{T}\sum_{t=1}^{T} p_t(x) \tag{B.22}$$

We transform the normalized density curves back into a functional data object by projecting them onto the original spline basis. Now, we can perform functional principal component analysis.

B.3 Functional principal component analysis

Functional principal component analysis is a dimensionality reduction technique that allow us to capture the primary modes of variation in the functional representation of the densities (Ramsay and Silverman, 2005). In order to perform it, we define the first functional principal component score $s_{1,t}$ as follows:

$$s_{1,t} = \int \xi_1(x) X(x)\, dx \tag{B.23}$$

where the weight function $\xi_1(x)$ is the first functional principal component, capturing the primary mode of variation in the functional data $X(x)$. To determine $\xi_1(x)$, we solve the following maximization problem, which aims to find $\xi_1(x)$ that maximizes the variance of $s_{1,t}$, subject to the constraint that the functional principal component has a unit norm:

$$\max \quad T^{-1} \sum_{t=1}^{T} \left(\int \xi_1(x) X_t(x)\, dx \right)^2 \tag{B.24a}$$

$$\text{s.t.} \quad \int \xi_1^2(x)\, dx = \|\xi_1\| = 1 \tag{B.24b}$$

Here, N is the number of functional observations, and $X_i(x)$ represents the i-th observation of the functional data. After finding the first functional principal component $\xi_1(x)$, the subsequent ones are determined by solving a similar maximization problem, ensuring each is orthogonal to the previously identified components:

$$\max \quad T^{-1} \sum_{t=1}^{T} \left(\int \xi_k(x) X_t(x)\, dx \right)^2 \tag{B.25a}$$

$$\text{s.t.} \quad \int \xi_k^2(x)\, dx = \|\xi_k\| = 1, \tag{B.25b}$$

$$\int \xi_k(x) \xi_j(x)\, dx = 0 \quad \text{for all } j < k \tag{B.25c}$$

where k denotes the index of the current principal component. Each functional principal component explains a progressively smaller portion of the variance in the data. We select 3 functional principal components, as they together explain approximately 95% of the cumulative variance of the data: the first accounts for about 75% of the total variance, 15% the second, and 5% the third. As we point out in the main text, the first score is strongly associated with the densities' dispersion, while the second one is strongly associated with skewness.

C Belief Shock

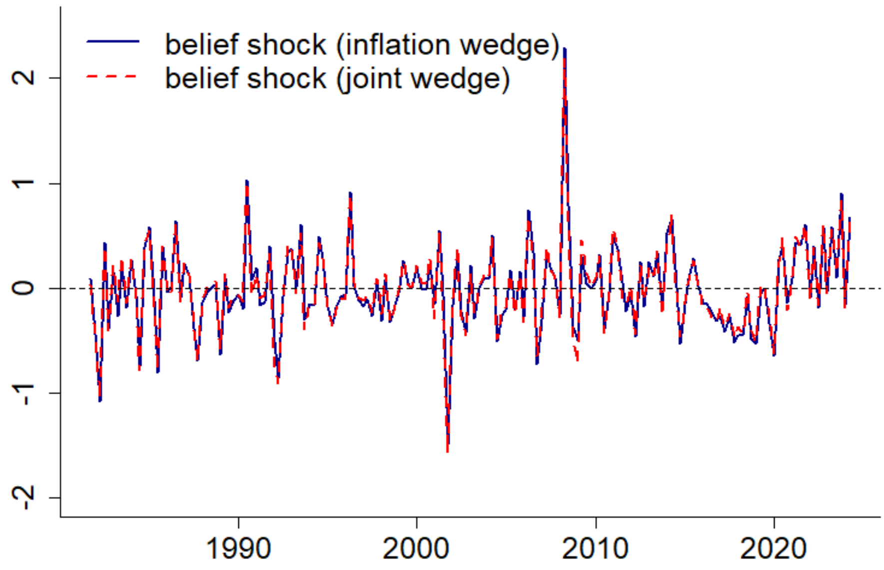

Figure C.3: **Belief shock from inflation wedge vs Belief shock from joint inflation and unemployment wedge.** Belief shock obtained by applying the ridge regression to predict the inflation wedge (blue solid line); Belief shock obtained by applying the ridge regression on the first principal component of the inflation and unemployment wedge (red dashed line).

D Local Projections

D.1 Size and Sign Asymmetry

In this section, we extend our analysis using nonlinear local projections, following Caravello and Martinez-Bruera (2024). They show that, under symmetric shock distributions, nonlinear transformations in local projections can capture both **sign** and **size** asymmetries: *even* functions capture sign asymmetries, while *odd* functions capture size asymmetries.

Following Gonçalves et al. (2024), we first assess whether larger belief distortion shocks have disproportionate effects before turning to state-dependent projections. After testing for shock symmetry using the triples test Randles et al. (1980), we estimate the following for each horizon $h = 0, 1, \ldots, H$:

$$x_{t+h} = \beta_{1,x}(h)\tilde{\theta}_t + \beta_{2,x}(h)f_1(\tilde{\theta}_t) + controls(x) + \epsilon_{t+h}, \tag{D.26}$$

where $f_1(\tilde{\theta}_t) = |\tilde{\theta}_t|$ captures **sign asymmetries**, or alternatively $f_1(\tilde{\theta}_t) = \tilde{\theta}_t^3$ to capture **size asymmetries**. We also consider both simultaneously:

$$x_{t+h} = \beta_{1,x}(h)\tilde{\theta}_t + \beta_{2,x}(h)|\tilde{\theta}_t| + \beta_{3,x}(h)\tilde{\theta}_t^3 + controls(x) + \epsilon_{t+h}, \tag{D.27}$$

In Equation (D.26) and Equation (D.27), x_{t+h} denotes the response variable (inflation disagreement $\hat{\Delta}_t$, inflation π_t, or unemployment u_t), and $\tilde{\theta}_t$ is the belief distortion shock. The sample spans 1981:Q4–2024:Q2. Controls include a constant, four lags of the shock, and all dependent variables.

Figure D.4 shows IRFs from these models with 95% confidence intervals:

- The first column shows the sequence of $\beta_{1,x}(h)$, capturing the linear effect of the shock. The shaded areas represent the linear specification without nonlinear terms, while the lines within each panel show the median of $\beta_{1,x}(h)$ under alternative nonlinear specifications.
- The second column displays the responses of $\beta_{2,x}(h)$ from Equation (D.26) when $f_1(\tilde{\theta}_t) = |\tilde{\theta}_t|$, highlighting **sign asymmetries**.
- The third column shows $\beta_{2,x}(h)$ from Equation (D.27) when both absolute value and cubic terms are included, isolating the effect of the absolute value term and highlighting **sign asymmetries** when controlling for size asymmetries.
- The fourth column presents the response of $\beta_{3,x}(h)$ from Equation (D.26) when $f_1(\tilde{\theta}_t) = \tilde{\theta}_t^3$, to assess **size asymmetries**.

The response of the linear coefficients aligns with the findings from standard local projections, indicating increasing disagreement, inflation, and unemployment in response to belief distortion shocks. However, when focusing on nonlinear effects, increases in belief distortion appear to cause more pronounced effects than decreases. This is evident in the second and third columns, where the coefficients on the nonlinear term $|\tilde{\theta}_t|$ are positive for disagreement and unemployment, while the estimates are less clear for inflation. This suggests that at least for disagreement and unemployment, positive shocks generate stronger responses than negative ones.

In contrast, the fourth column provides no evidence of size asymmetry: the coefficients on the cubic term $\tilde{\theta}_t^3$ are not statistically significant for any variable. We interpret this as evidence that the magnitude of the shock (size) does not matter beyond its sign; i.e., the responses are symmetric in size but asymmetric in sign.

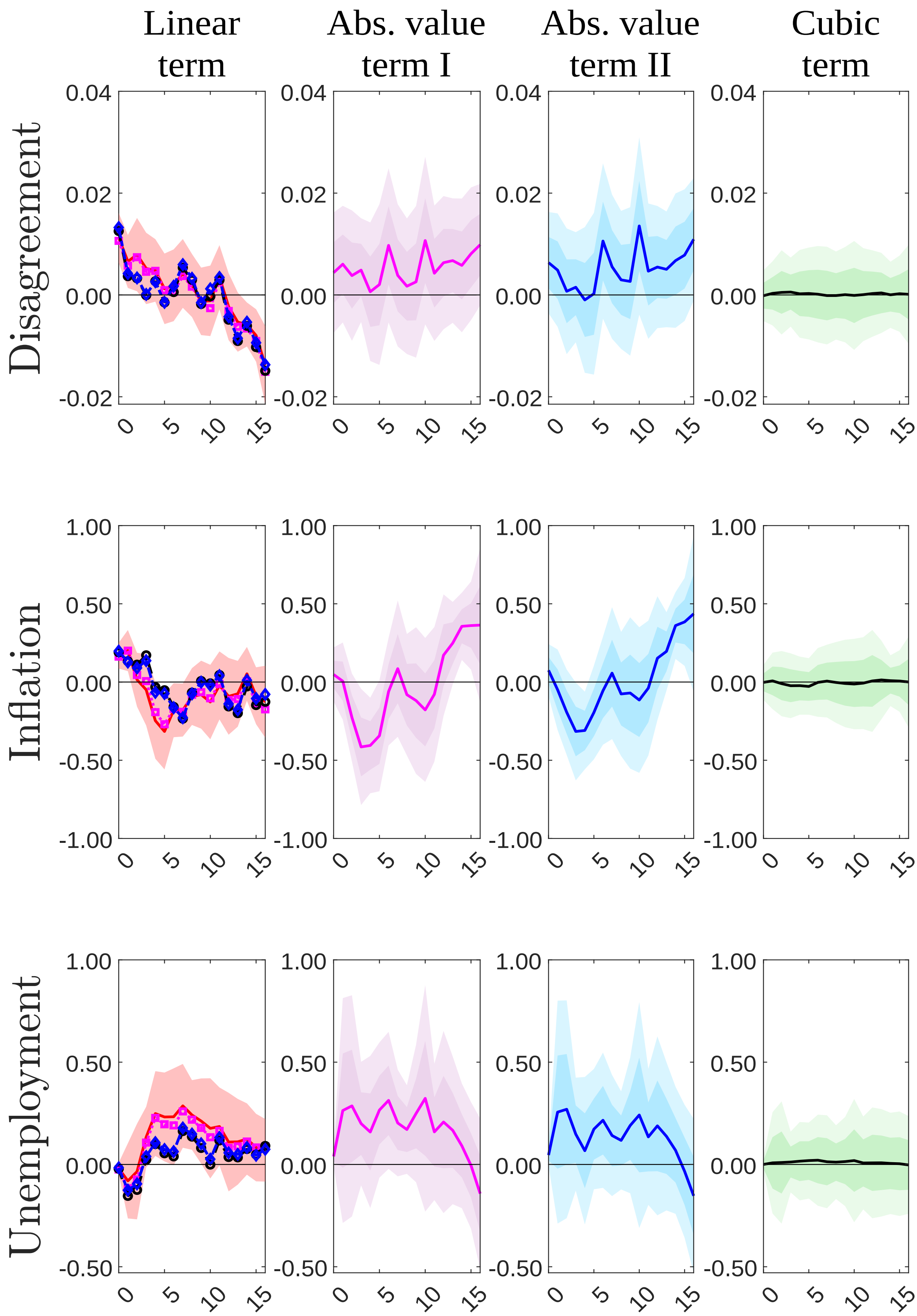

Figure D.4: **LPs to belief distortion shocks.** Column 1 shows β_i on ε_t, Column 2 shows β_i on $|\varepsilon_t|$, Column 3 shows β_i on $|\varepsilon_t|$ when a cubic term is included as regressor, Column 4 shows β_i on $(\varepsilon_t)^3$; Shaded areas are 68% and 95% confidence intervals.

D.2 State-dependent local projections: vacancy and CAPE

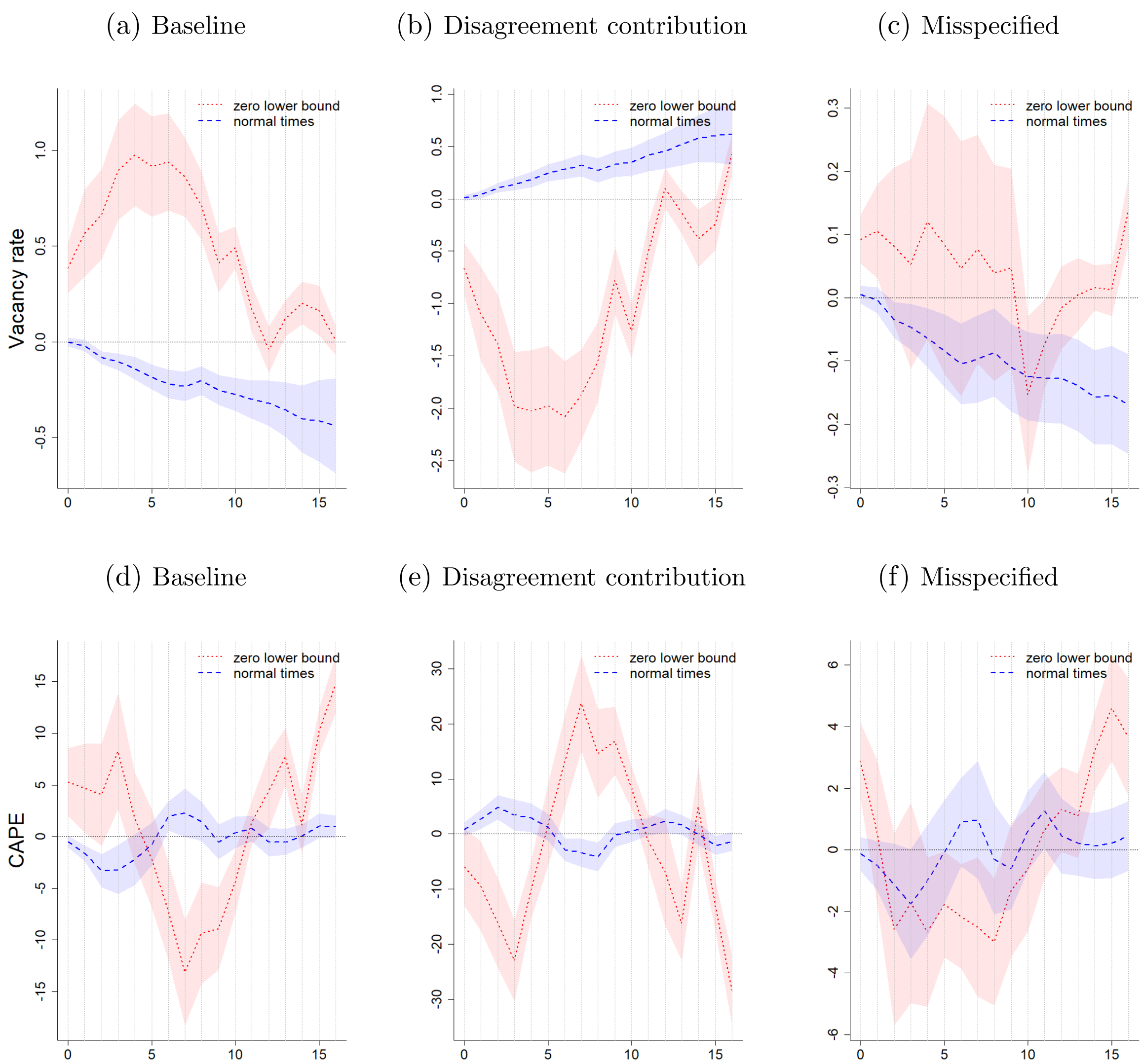

Figure D.5: **Vacancy rate and CAPE state-dependent responses to a belief distortion shock.** Panels (a),(b), (d), and (e) report results from the *baseline* local projections in Equation (4), while panels (c) and (f) are based on the *misspecified* specification in Equation (5). Local projections are estimated using the lag-augmentation method of Montiel Olea and Plagborg-Møller (2021), where lags are selected by minimizing the AIC. In the baseline local projections, for the *vacancy rate*, in the ZLB state (Z), we select 4 lags of the shock, 4 lags of the dependent variable, and $j = 1$ lag for the interaction term. In normal times (state $\bar{Z}$), 2 lags of the shock, 2 lags of the dependent variable, and $j = 2$ lags for the interaction term. For the *CAPE*, we select 4 lags of the shock, 2 lags of the dependent variable, and $j = 3$ lags for the interaction term in the ZLB state (Z), whereas in normal times ($\bar{Z}$), 2 lags for both the shock and the dependent variable, and $j = 2$ lags for the interaction term. In the misspecified local projections, for the vacancy rate, we select 4 lags of the shock and 2 lags of the dependent variable in the ZLB state, and 1 lag of the shock and 4 lags of the dependent variable in normal times. For the CAPE, 4 lags of the shock and 2 lags of the dependent variable in the ZLB state, and 1 lag of the shock and 4 lags of the dependent variable in normal times. The bands show the 95% and 68% confidence intervals, based on Newey-West standard errors. *Sample: 1983Q3-2024Q2.*