Sequential Contests with Incomplete Information: Theory and Experimental Evidence

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Abstract

We investigate behavior in two-player sequential-move contests with complete and incomplete information about the value of the prize. First, we describe a Bayesian equilibrium in which both players have private prize values. Then, we test our predictions in the experimental laboratory. We analyze three settings: symmetric prize valuations with complete information, asymmetric prize valuations with complete information, and asymmetric prize valuations with incomplete information. We find that subjects' behavior is less consistent with theory and more in line with simple mental shortcuts. Our data supports a simple investment heuristic for each player type. On average, first-movers invest half of their own valuation and second-movers, regardless of their prize valuation, invest frequently in one of the following ways: drop out of the contest or invest at or just above the first-movers' investment. We add to the growing literature by showing that experimental contest data can be better explained by simple heuristics.

Keywords: contest, sequential moves, heterogeneity, incomplete information, investment heuristic
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1 Introduction

Contests, introduced by Tullock (1980), are competitive settings in which individuals make costly and irreversible investments (e.g., resources, effort, or time) with the goal of outperforming other contestants to obtain a valuable prize (e.g., monetary reward, promotion, or funding).[1] As contests are complex multidimensional settings involving various types of information structures, rules, uncertainty, player heterogeneity, timing of moves, and so on, it is of course not surprising that the first settings studied in the literature are those exploring the consequences that different environmental factors have on investment behavior in static one-shot environments with *simultaneous moves*.[2] While simultaneous-move contests are an important class to study, extending these models to dynamic settings and exploring behavior in such settings empirically is a natural progression, especially given that the majority of naturally occurring contests are better expressed dynamically.[3] Indeed, the dynamics of sequential contests are echoed in various real-life economic strategic interactions such as internal labor markets inside firms, oligopolies, public goods provision, tragedy of the commons, rent-seeking (e.g., via lobbying activity), R&D races, advertising, and sporting events (Deng et al., 2024; Hinnosaar, 2024).

In this paper, we theoretically and experimentally explore behavior in *sequential contests*—contests in which players make investments sequentially across time—under various information structures and player heterogeneity configurations. Specifically, we study how player heterogeneity, modeled as differences in winning (continuously distributed) valuations across players, and the privateness of such valuations affect investment behavior in a class of two-player two-stage sequential contests. While we consider a rich set of player heterogeneity and information conditions, our sequential contest framework is otherwise straightforward.

In the benchmark setting, two players with symmetric prize valuations compete in a two-stage winner-take-all Tullock lottery contest (Tullock, 1980). In stage 1, player 1—a.k.a. first-mover—makes an investment. In stage 2, after observing player 1's investment, player 2—a.k.a. second-mover—makes their investment decision, and then a single prize is awarded probabilistically, i.e., the probability of winning the prize is proportional to own investment and inversely proportional to opponent investment. We consider two important extensions of the benchmark setting. In the first extension, we relax the assumption of valuation symmetry, allowing each player to value the prize differently. Second, we build on the asymmetric valuation case by assuming that prize valuations are the private information of each player, and thus, opponent valuations are unknown.

[1] For comprehensive surveys of the theoretical contest literature see, e.g., Nitzan (1994), Szymanski (2003), Congleton, Hillman and Konrad (2008), Konrad (2009), Connelly et al. (2014), and Corchón and Serena (2018).

[2] See Dechenaux, Kovenock and Sheremeta (2015) for a survey of the empirical contest literature.

[3] There are a number of dynamic contest settings that are worth mentioning. In many of these settings, players simultaneously make investments in a number of stages, and the winner is determined by the sum of investments (see, e.g., Schmitt et al., 2004; Aoyagi, 2010; Ederer, 2010), highest investment level achieved (see, e.g., Taylor, 1995; Terwiesch and Xu, 2008; Seel and Strack, 2013), the last contestant standing after a series of elimination rounds (see, e.g., Rosen, 1986; Gradstein, 1998; Moldovanu and Sela, 2006; Fu and Lu, 2012), or the first to reach a specific target investment level (see, e.g., Harris and Vickers, 1985; Konrad and Kovenock, 2009).

Our main contributions are summarized as follows. First, our incomplete information setting contributes to the theoretical contest theory literature. We extend the binary type model of Linster (1993) to a continuous support and completely characterize the subgame perfect equilibrium. Unlike previous work by Hinnosaar (2024) and Kahana and Klunover (2018), our contribution focuses on private information on the prize valuation that follows a continuous distribution. The addition of incomplete information in the sequential contest is important because valuation information is often partial or incomplete information. For instance, political campaigns evolve and announce voting advances sequentially months before the election. At the same time, the value of winning the elections (prizes) can be a function of the candidate's cost structure (that can be private information), or related to the candidate's intrinsic preference and motivation to win. Contests are more relevant to real-world scenarios in which effort increases the likelihood of winning but does not secure the win (like all-pay auctions).[4]

Second, we add to the literature on sequential-move contests and present the first empirical investigation of behavior in imperfectly discriminating contests with sequential moves and various information structures. In a comprehensive survey of the empirical contest and tournament literature, Dechenaux, Kovenock and Sheremeta (2015) cite 231 references. A total of 9 papers (approximately 4%) involve sequential moves, and only 6 of those studies are empirical. In our analysis, we focus our attention on understanding how each player type behaves in a sequential contest, and compare behavior across settings with varying information sets empirically.

Finally, we provide new evidence supporting "investment heuristic play," whereby each player type makes investment decisions using simple rules of thumb rather than playing according to equilibrium predictions. The most intriguing aspect of studying sequential-move contests is that, under full disclosure of stage 1 investment, *all strategic uncertainty and ambiguity is removed from the second-mover's decision environment*. Hence, one would expect *ex ante* for second-mover behavior in our laboratory experiment to be more "rational" than first-mover behavior. Consistent with this conjecture, we find that player 1 uses past contest play to guide the current investment decisions and player 2, consistent with theory, conditions behavior on valuation and player 1 investment. Player 2's advantaged situation leads them to bid in the following way: "drop out," i.e., bid sufficiently low to secure a probability of winning below 10%, or just outbid the investment of player 1 so as to secure a probability of winning at or just above 50%. Given that player 1 faces an even more complex decision environment, we searched for and even simpler heuristic: bid half of their valuation. Our data strongly supports this investment heuristic. This finding, coupled with the investment heuristic results found in prior studies (Sheremeta, 2011; Brookins, Lightle and Ryvkin, 2015; Rodet and Smyth, 2020), suggests that subjects tend to bid according to simple heuristics as opposed to theoretical predictions.

Our paper is organized as follows. The relevant literature is discussed in Section 2. In Section 3, we present the model and theoretical predictions. In Section 4 we discuss our experimental design and procedures. We present our main findings in Section 5, followed by a discussion and

[4]Related theoretical papers on all-pay auctions are discussed in the next section.

conclusion in Section 6.

2 Related literature

The existing theoretical literature studying contests with asymmetries in valuations has focused on discrete values.[5] Linster (1993) was the first to analyze two-stage sequential-move Tullock contests with symmetric and asymmetric valuations, considering both complete and incomplete information. We generalize Linster (1993) by allowing for private valuations to be drawn from an arbitrary distribution with continuous support. The standard backwards induction approach to extensive form games allows us to completely characterize the subgame-perfect equilibrium in closed form for the uniform distribution of valuations used for our experiment, which is typically not possible in private information contests with simultaneous moves (see, e.g., Fey, 2008; Ryvkin, 2010). Linster (1993)'s analysis, coupled with our incomplete information extension, generate the predictions for all settings in our experimental design.

More recent theoretical work has addressed additional gaps within the contest literature. Hinnosaar (2024) considers generalized sequential Tullock contests featuring various combinations of sequential and simultaneous moves, public information on the prize valuation, and with disclosure of the players' aggregate efforts following each stage. In concurrent work to Hinnosaar (2024), Kahana and Klunover (2018) theoretically prove that a unique subgame perfect equilibrium in pure strategies exists in sequential lottery contests with an arbitrary number of players, i.e., beyond the frequently studied two-player case. While both studies feature theoretically challenging generalizations of sequential-move Tullock-style contests, neither considers player heterogeneity, and hence, incomplete information regarding players' types is not possible. Restricting the number of players to two, we extend this line of work by considering contests with asymmetric and publicly available prize valuations, as well as the incomplete information counterpart.

Certain theoretical studies within the all-pay auction literature share some similarities with this paper while also exhibiting distinct differences. Chen, Kuang and Zheng (2017) investigate a one-sided asymmetric information setup: one player's prize valuation is public information, while the other's is private (either high or low). The authors explore many interesting aspects of sequential-move contests, such as the designer's optimal disclosure policy, and also discuss how results change, if at all, when considering the Tullock (lottery) contest success function. Our incomplete information setting complements their work, but note that there is no asymmetry in terms of information regarding the prize valuation, *ex ante*, between the two players in the two-sided incomplete information setting we consider. Furthermore, we consider a continuously distributed prize valuation as opposed to discrete, both of which are important classes to study. Deng et al. (2024) also characterize the equilibrium of a multi-player incomplete information sequential all-pay auction and examine earlier-/later-mover advantage. However as Deng et al.

[5] For additional theoretical studies of Tullock contests with sequential moves see Leininger (1993), Morgan (2003), and Serena (2017).

(2024) argue, sequential all-pay auctions are radically distinct from sequential Tullock contests by definition, which results in entirely different predictions.[6] In an all-pay auction, the theoretical prediction is trivial for player 2, who simply outbids their competitor by ϵ if doing so generates a non-negative payoff. In our setup, outbidding player 1 does not guarantee the win which makes player 2's decision less straightforward and an arguably better setting to study decision heuristics. At the same time, while we vary the information set on the prize valuation, player 2 has the same best response function in theory, making it an intriguing setting to explore the influence of uncertainty on bidding behavior empirically.

While the sequential-move contest literature is sparse relative to the simultaneous-move literature, several studies share some features of our experimental design. Most of existing related work compares behavior between sequential-move and simultaneous-move contests (Fonseca, 2009; Weimann, Yang and Vogt, 2000; Nelson, 2020). Fonseca (2009) investigates valuation asymmetries in simultaneous and sequential settings. However, he only considers cases with complete information. Furthermore, in his asymmetric value treatment the value of the prize is always greater for player 1. Another study that is more remotely related to ours, but more similar to Fonseca (2009), is Weimann, Yang and Vogt (2000). In their parameter setup, the subgame-perfect equilibrium is where player 1 bids high enough to deter player 2 from participating in the contest. Their main finding is that first-movers bid either very aggressively or very tamely. Second-movers either bid zero (their equilibrium strategy) or very high amounts that yield negative expected payoffs for both players. We find similar behavior from player 2 in our experiment, and show that this behavior persists regardless of the valuation. Nelson (2020) studied behavior in sequential contests between two and three players. The goal of his design was to test the preemptive investment prediction of Linster (1993), whereby player 1 bids aggressively so as to crowd player 2 out of the market, i.e., force the player to drop out of the contest.

Within the experimental all-pay auction literature, the study by Jian, Li and Liu (2017) shares some similarities with this paper. Jian, Li and Liu (2017) examine the equilibrium of the n-player n-stage sequential all-pay contest under incomplete information and compare behavior to the simultaneous-move benchmark. However, the goal of their study and their environment differ from ours. Unlike this study, they aim to compare investments in sequential and simultaneous move-contests and they model contests as incomplete information all-pay auctions (Moldovanu and Sela, 2001). Complete information is not considered and, as discussed above, there are key differences in investment behavior across all-pay and Tullock contest settings. To the best of our knowledge, this is the first study that experimentally compares behavior in stochastic sequential contests under incomplete information, as well as complete information with and without valuation heterogeneity.

[6] For instance, Deng et al. (2024) find a later-mover advantage in all-pay auctions while Hinnosaar (2024) finds an earlier-mover advantage in sequential Tullock contests (with complete information).

3 Model

Consider two risk-neutral players, who compete for their prizes in a sequential two-stage contest. In stage 1, player 1 makes investment $x_1 \geq 0$. In stage 2, player 2 observes the choice of player 1 and makes investment $x_2 \geq 0$. Given investments of both players, player $i \in \{1, 2\}$ wins the contest with probability $\frac{x_i}{x_1+x_2}$.[7]

3.1 Complete information

Suppose that players' prize valuations $v_1 > 0$ and $v_2 > 0$ are common knowledge. Then the subgame-perfect equilibrium (SPE) of the sequential two-stage contest can be calculated using backward induction. In stage 2, player 2 observes the investment choice of player 1, x_1, and solves the following maximization problem

$$\max_{x_2 \geq 0} v_2 \frac{x_2}{x_1 + x_2} - x_2. \tag{1}$$

The best response of player 2 is

$$x_2 = \max\left\{\sqrt{v_2 x_1} - x_1, 0\right\}. \tag{2}$$

In stage 1, player 1 expects that player 2 will respond according to the best-response (2) function and, therefore, solves the following maximization problem:

$$\max_{x_1 \geq 0} \left\{ v_1 \frac{x_1}{x_1 + \max\left\{\sqrt{v_2 x_1} - x_1, 0\right\}} - x_1 \right\}.$$

The solution is

$$x_1^{CA}(v_1, v_2) = \begin{cases} \frac{v_1^2}{4v_2}, & if \quad v_2 > \frac{v_1}{2}, \\ v_2, & if \quad v_2 \leq \frac{v_1}{2}. \end{cases} \tag{3}$$

From equations (2) and (3), we get

$$x_2^{CA}(v_1, v_2) = \begin{cases} \frac{v_1}{2} - \frac{v_1^2}{4v_2}, & if \quad v_2 > \frac{v_1}{2}, \\ 0, & if \quad v_2 \leq \frac{v_1}{2}. \end{cases} \tag{4}$$

It is easy to see that individual investments are weakly increasing in their own values.

The expected payoff to player $i = 1, 2$ given valuations (v_1, v_2) is given by

$$\pi_i^{CA}(v_1, v_2) = v_i \frac{x_i^{CA}(v_1, v_2)}{x_1^{CA}(v_1, v_2) + x_2^{CA}(v_1, v_2)} - x_i^{CA}(v_1, v_2). \tag{5}$$

We use CA to indicate the solution and payoffs with complete information about the asymmetric

[7] We assume that each player wins the contest with probability $1/2$, if $x_1 + x_2 = 0$.

valuations of the two players. To summarize:

Proposition 1. *Suppose that players have prize valuations $v_1 > 0$ and $v_2 > 0$, in the two-player sequential contest with public information. Then, in the SPE players 1 and 2 investments are described by expressions (3) and (4).*

If both players have the same prize valuation then from proposition 1, we have the following result.

Corollary 1. *Suppose that both players have the same prize valuation, $v_1 = v_2 = v > 0$, in the two-player sequential contest with public information. Then, in the SPE, both players invest the same amount:*

$$x_1^{CS}(v, v) = x_2^{CS}(v, v) = \frac{v}{4}, \tag{6}$$

yielding expected payoff

$$\pi_1^{CS}(v, v) = \pi_2^{CS}(v, v) = \frac{v}{4}. \tag{7}$$

We use CS to indicate the solution and payoffs with complete information about the symmetric valuations of the two players.

3.2 Incomplete information

Suppose that prize valuations v_i are independently and identically distributed from continuous distribution function G on the interval $[a, b]$, where $i \in \{1, 2\}$, $G(a) = 0$, $G(b) = 1$, and $0 < a < b$. Each player knows their own prize valuation, the distribution function G, and the interval $[a, b]$.

In stage 2, player 2 observes the investment of player 1, $x_1 \geq 0$, and solves maximization problem (1). Hence, player 2's investment is again described by the best-response function (2).

In stage 1, player 1 expects that player 2 will best respond according to function (2), but player 1 does not know the private valuation v_2 of player 2. Therefore, player 1 solves the following maximization problem

$$\max_{x_1 \geq 0} \quad v_1 \int_a^b \frac{x_1 dG(v)}{x_1 + \max\{\sqrt{vx_1} - x_1, 0\}} - x_1. \tag{8}$$

Expression (8) can be rewritten as

$$\max_{x_1 \in [0,b]} \quad v_1 \left(\int_a^{\max\{x_1,a\}} \frac{x_1 dG(v)}{x_1 + \max\{\sqrt{vx_1} - x_1, 0\}} + \int_{\max\{x_1,a\}}^b \frac{x_1 dG(v)}{x_1 + \max\{\sqrt{vx_1} - x_1, 0\}} \right) - x_1. \tag{9}$$

It simplifies to

$$\max_{x_1 \in [0,b]} \quad v_1 \left(\int_a^{\max\{x_1,a\}} dG(v) + \sqrt{x_1} \int_{\max\{x_1,a\}}^b \frac{dG(v)}{\sqrt{v}} \right) - x_1. \tag{10}$$

Consider two cases now. First, suppose that $x_1 \leq a$, then, the maximization problem (10) becomes

$$\max_{x_1\in[0,a]} \quad v_1\left(\sqrt{x_1}\int_a^b \frac{dG(v)}{\sqrt{v}}\right) - x_1. \tag{11}$$

The first order condition is

$$\frac{v_1}{2\sqrt{x_1}}\int_a^b \frac{dG(v)}{\sqrt{v}} - 1 = 0. \tag{12}$$

Therefore,

$$x_1 = \frac{v_1^2}{4}\left(\int_a^b \frac{g\,(v)\,dv}{\sqrt{v}}\right)^2, \tag{13}$$

where $g(v) = G'(v)$. The expected payoff of player 1 is

$$\pi_1 = \frac{v_1^2}{4}\left(\int_a^b \frac{g\,(v)\,dv}{\sqrt{v}}\right)^2. \tag{14}$$

Second, suppose that $x_1 > a$, then, maximization problem (10) becomes

$$\max_{x_1\in[a,b]} \quad v_1\left(\int_a^{x_1} dG(v) + \sqrt{x_1}\int_{x_1}^b \frac{dG(v)}{\sqrt{v}}\right) - x_1. \tag{15}$$

Differentiating expression (15) with respect to x_1, we obtain the following first-order condition

$$\frac{v_1}{2\sqrt{x_1}}\int_{x_1}^b \frac{dG(v)}{\sqrt{v}} - 1 = 0. \tag{16}$$

The second order condition is

$$-\frac{v_1}{4}x_1^{-\frac{3}{2}}\int_{x_1}^b \frac{dG(v)}{\sqrt{v}} - \frac{v_1}{2x_1}g(x_1) \leq 0. \tag{17}$$

Define

$$A(x_1; v_1) = \frac{v_1^2}{4}\left(\int_{x_1}^b \frac{g\,(v)\,dv}{\sqrt{v}}\right)^2. \tag{18}$$

Then, from the first order condition (16), player 1's investment $x_1 > a$ is the solution of the following equation

$$x_1 = A(x_1; v_1). \tag{19}$$

Note that the left (right) hand side in equation (19) is increasing (decreasing) in x_1. Therefore, there exists a unique equilibrium which we describe in the following proposition.

Proposition 2. *In the Perfect Bayesian equilibrium of a two-player sequential contest with private information, player 2 invests as in equation (2), or*

$$x_2^I(v_1, v_2) = \max\left\{\sqrt{v_2 x_1^I(v_1)} - x_1^I(v_1), 0\right\}; \tag{20}$$

player 1 believes that player 2 will respond according to (2) *and chooses*

$$x_1^I(v_1) = \begin{cases} A(x_1^I; v_1), & if \quad A(a; v_1) > a, \\ A(a; v_1), & if \quad A(a; v_1) \leq a, \end{cases} \tag{21}$$

where $A(x_1; v_1)$ is defined in equation (18).

Again, as we have already seen in the case of complete information, individual investments are weakly increasing in their own values.

The expected payoff to player 1 with valuation v_1 is therefore

$$\pi_1^I(v_1) = v_1 \int_a^b \frac{x_1^I(v_1)}{x_1^I(v_1) + x_2^I(v_1, v_2)} dG(v_2) - x_1^I(v_1), \tag{22}$$

and the expected payoff to player 2 given valuations (v_1, v_2) is given by

$$\pi_2^I(v_1, v_2) = v_2 \frac{x_1^I(v_1)}{x_1^I(v_1) + x_2^I(v_1, v_2)} - x_2^I(v_1, v_2). \tag{23}$$

We use I to indicate the solution and payoffs with incomplete information about the valuations of the two players.

The following example illustrates Proposition 2 and provides our experimental predictions.

Example 1. *Suppose that the distribution function is a uniform distribution on the interval $[a, b]$, or*

$$G(x) = \frac{x - a}{b - a},$$

and

$$g(x) = \frac{1}{b - a}.$$

Then,

$$A(x_1; v_1) = v_1^2 \left(\frac{\sqrt{b} - \sqrt{x_1}}{b - a} \right)^2. \tag{24}$$

Hence, the first player selects x_1 such that

$$x_1^I(v_1) = \begin{cases} v_1^2 \left(\frac{\sqrt{b} - \sqrt{x_1^*}}{b-a} \right)^2, & if \quad \left(\frac{v_1}{\sqrt{b}+\sqrt{a}} \right)^2 > a, \\ \left(\frac{v_1}{\sqrt{b}+\sqrt{a}} \right)^2, & if \quad \left(\frac{v_1}{\sqrt{b}+\sqrt{a}} \right)^2 \leq a, \end{cases} \tag{25}$$

or

$$x_1^I(v_1) = \begin{cases} \left(\frac{\sqrt{b} v_1}{v_1 + (b-a)} \right)^2, & if \quad \left(\frac{v_1}{\sqrt{b}+\sqrt{a}} \right)^2 > a, \\ \left(\frac{v_1}{\sqrt{b}+\sqrt{a}} \right)^2, & if \quad \left(\frac{v_1}{\sqrt{b}+\sqrt{a}} \right)^2 \leq a. \end{cases} \tag{26}$$

The following corollary follows from Example 1, and summarizes the main predictions of our experiment.

Corollary 2. *Suppose that the distribution function is a uniform distribution on the interval* $[100, 900]$*. In the Perfect Bayesian equilibrium, player 2 invests* x_2 *in* (2)*; player 1 believes that player 2 will respond according to* (2) *and chooses*

$$x_1^I(v_1) = \begin{cases} \left(\frac{30v_1}{v_1+800}\right)^2, & if \quad v_1 > 400, \\ \left(\frac{v_1}{40}\right)^2, & if \quad v_1 \le 400. \end{cases} \tag{27}$$

For comparability across our three settings, it will be convenient to introduce average investment and payoff functions,[8] as well as investments and payoffs averaged over all possible realizations of v_1 and v_2. In CA, average investment and payoff functions are constructed as follows. As in setting I, assume that valuations v_i are drawn from $G(v)$. Averaging over all realizations of the opponent player, we obtain the *average investment functions*

$$x_1^{CA}(v) = \int_a^b x_1^{CA}(v, v')dG(v') \tag{28}$$

$$x_2^{CA}(v) = \int_a^b x_2^{CA}(v', v)dG(v'), \tag{29}$$

where $x_1^{CA}(v, v')$ and $x_2^{CA}(v', v)$ are defined in equations (3) and (2), respectively, and the *average payoff functions*

$$\pi_1^{CA}(v) = \int_a^b \pi_1^{CA}(v, v')dG(v') \tag{30}$$

$$\pi_2^{CA}(v) = \int_a^b \pi_2^{CA}(v', v)dG(v'), \tag{31}$$

where $\pi_1^{CA}(v, v')$ and $\pi_2^{CA}(v', v)$ are defined in equation (5). In setting I, player 1's equilibrium investment and expected payoff are already solely functions of v_1, and hence, their average investment function is equivalent to their equilibrium investment in equation (21) and average payoff function is equivalent to their expected payoff in equation (22). Average investment and payoff functions for player 2 in I are constructed in a similar way as (28) and (30), respectively; in this case, the average investment function is

$$x_2^I(v) = \int_a^b x_2^I(v', v)dG(v'), \tag{32}$$

where $x_2^I(v', v)$ is defined in equation (20), and the average payoff function is defined as

$$\pi_2^I(v) = \int_a^b \pi_2^I(v', v)dG(v'), \tag{33}$$

where $\pi_2^{CA}(v', v)$ is defined in equation (23). Finally, we define *average investment* and *average*

[8] Such bidding functions have been proven useful in several prior studies (Konrad and Kovenock, 2010; Brookins and Ryvkin, 2014).

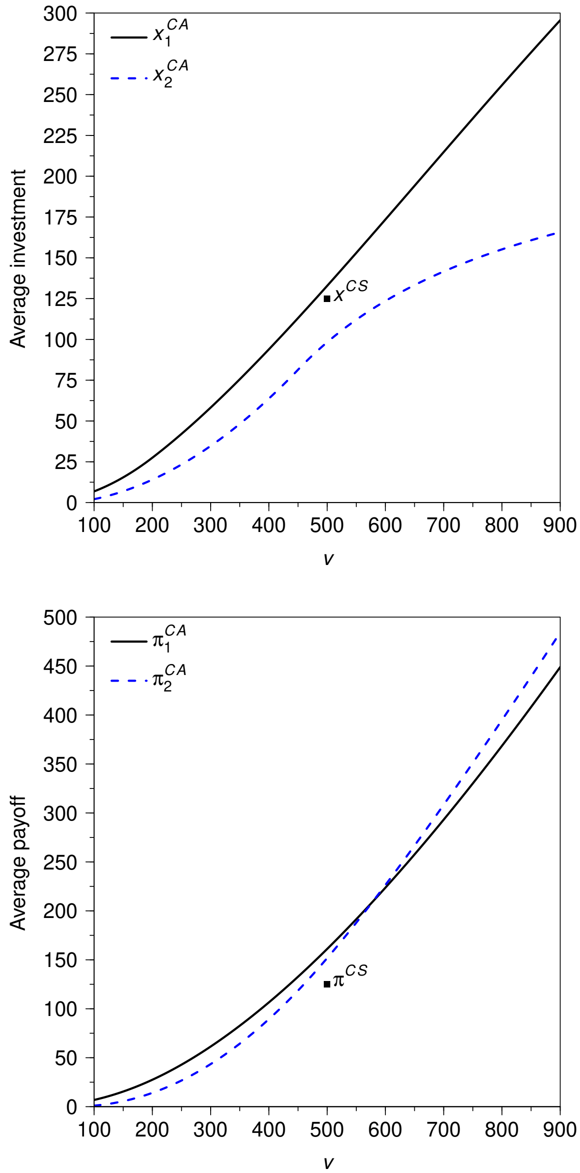

Figure 1: Average investment function (top: CA left, I right) and average payoff function (bottom: CA left, I right) for each treatment and player type. Valuations are uniformly drawn on $[100, 900]$.

payoff as the average equilibrium investment and payoff, averaged over all realizations of own and opponent valuation, or

$$x_i^t = \int_a^b x_i^t(v')dG(v'), \tag{34}$$

and

$$\pi_i^t = \int_a^b \pi_i^t(v')dG(v'), \tag{35}$$

for $i = 1, 2$ and $t \in \{CA, I\}$. These investment and payoff functions allow us to compare investment behavior and welfare, on average, across treatments and player types.

The top panels of Figure 1 show average investment functions across player types and treatment (CA left, I right), and the bottom panels of Figure 1 show average payoff functions across player types and treatment (CA left, I right).

According to Figure 1, in CA, on average, there exists a unique valuation $\hat{v}^{CA}$ such that for all $v < \hat{v}^{CA}$ a first-mover advantage exists in CA; otherwise, player 2 has the advantage. In treatment I, a similar single crossing property exists, with the exception that the first-mover only emerges for sufficiently large values, i.e., $v > \hat{v}^{I}$. Therefore, even though the average investment level is higher for player 1 as compared to player 2 for all valuations, the first-mover advantage eventually vanishes for sufficiently high v. These single crossing observations are exceptions rather than a rule in these settings. We provide intuition below for the first- and second-mover advantage. In general, these effects depend on the distribution function of evaluations.

Consider the dynamics between the two players in a scenario with incomplete information. If player 1 possesses a low value, she anticipates that a high-value player 2 will make a substantial investment, leading to a reduction in investment from a low-value player 1. Player 2, whether holding a lower or higher value, observes this behavior from the low-value player 1 and adjusts their strategy accordingly. Consequently, the lower-value player 1 invests less, diminishing her chances of winning the contest. These influences exert opposing effects on the expected payoff of the low-value player 1. Simultaneously, the low-value player 2 gains an advantage over the low-value player 1. However, the low-value player 2 refrains from competing against a high-value player 1. This implies that the low-value player 2 engages solely with the low-value player 1, where she holds an advantage. Once more, the impact on the expected payoff of this player 2 remains uncertain.

When player 1 possesses a high value, she foresees that making only a modest investment would dissuade a low-value player 2 from competing. Consequently, there is minimal investment, ensuring a definite victory against the low-value player 2 but diminishing the likelihood of success against a high-value player 2. As a result, the overall impact on the expected payoff remains uncertain. Now, the high-value player 2 holds an advantage over the player 1, regardless of their type. Investing less, she receives greater odds of winning and thus increases her expected payoff. Yet, considering that players can possess either low or high values, the advantage of being the first or second-mover hinges on the distribution of private values.

Note that our observations in the CA and I cases depend on the distribution function of evaluations. Figure 1 presents the single crossing conditions for the uniform distribution. In this case, the effect that the low-value player 2 refrains from competing against a high-value player 1 gives the first-mover advantage when values are relatively small in the complete information situation. At the same time, the effect of making only a modest investment by the high-value player 1 creates the first-mover advantage when values are sufficiently large in the incomplete information situation.

4 Experimental design

The experiment follows a between-subjects design by varying valuation information and symmetry. Specifically, subjects made decisions in one of the three sequential contest settings described in Section 3: complete and symmetric (CS), complete and asymmetric (CA), or incomplete (I) information. All experimental sessions were conducted at the Behavioral Research Lab located in the Darla Moore School of Business at the University of South Carolina (USC). A total of 176 subjects (63.1% of them female) were randomly recruited by email from a sub-population of USC students who consented to receive announcements about participating in economics experiments. Table 1 summarizes the number of sessions, subjects, and independent matching groups for each treatment. The experiment was implemented in z-Tree Fischbacher (2007). Each participant was in exactly one session, and none of our participants were experienced in our environment. Sessions lasted approximately 75 minutes.

Treatment	Sessions	Subjects	Groups
CS	4	48	6
CA	5	56	7
I	5	72	9
Total	14	176	22

Table 1: Summary of experimental treatments.

Each session of the experiment consisted of three parts. At the beginning of each session, subjects were informed that the experiment would consist of several parts, but they were not informed about the number of parts or the nature of each part until it was announced; payment information in each part was withheld until the end of the experiment. Prior to the start of each part, paper instructions were distributed and read out loud while subjects followed along (see Appendix H for instructions). All monetary figures in the experiment were denominated in points. The exchange rate was 200 points for $1. The average amount earned from all three parts, including a $5 show-up fee, was $20.83.

In Part 1, subjects were assigned to play either as player 1 or player 2. Their role remained the same for the entire experiment. Subjects were matched randomly in groups of 2 and went through 30 decision rounds of the corresponding contest game. After each round, subjects were re-matched (in pairs) within a matching group of 8 subjects. In each round, subjects received an endowment of 1000 points. They were able to invest any integer number of points $x_i \in \{0, \dots, 1000\}$ into a project. Players made investment decisions sequentially in two stages. In stage 1, player 1 made their investment decision. In stage 2, player 2 was informed about player 1's investment and then made their investment decision. One of the two subjects in each group was successful with probability $p_i = x_i/(x_i+x_j)$ (or $p_i = 1/2$ if both players invested zero). The successful subject received the value of their project minus their investment to the project, and the unsuccessful subject lost their investment. At the end of each round, after making their decisions, subjects were informed whether their projects were successful and shown their payoffs from that round. At the end of part 1, 3 rounds were chosen randomly to base subjects' actual earnings from this part. The only difference in Part 1 across treatments was the nature of valuation symmetry and information. In the CS treatment, valuations were known to each player and equal to 500 points. In the CA and I treatments, valuations were randomly drawn from the set $\{100, \ldots, 900\}$, with each value equally likely. Valuations were common knowledge in treatment CA, but subjects only knew their own valuation in treatment I.

In Part 2, subjects' risk attitudes were measured using an incentivized multiple choice list procedure similar to Holt and Laury (2002) and Sutter et al. (2013).[9] Finally, in Part 3, subjects were asked their sex and asked to self-assess their risk.[10]

5 Results

Throughout the analysis, unless specified otherwise, standard errors are adjusted to correct for arbitrary correlations at the matching group level and p-values are obtained using two-sided Wald tests.

5.1 Aggregate Behavior

Table 2 contains the theoretical SPE predictions alongside the observed averages for investment and individual payoff by treatment and player type. To investigate whether subjects are adjusting behavior or *learning* across time, we obtain summary statistics using all data (rounds 1-30), only data from the first half of the experiment (rounds 1-15), and only data from the second half (rounds 16-30). Note, that the data and predictions of investment and payoffs in treatments CA and I are averaged across all configurations of own and opponent valuation, and hence, represent the empirical *average* investment and payoff.

One of the most widespread findings in the empirical contest literature is that of rampant "overbidding," or investment behavior significantly exceeding equilibrium predictions (see Sheremeta (2013) for a summary). It is clear from Table 2 that, whether considering all rounds or the rounds from each half separately, average investment levels are significantly higher than the SPE predictions. This includes all treatments and both player types. Such overbidding significantly reduces average payoffs for each player type in each treatment, with the exception of player 2 in CS.[11] It is worth noting that, in theory, player 2 makes investment x_2 according

[9] Specifically, subjects were presented with a list of 21 choices between a sure amount of money and a lottery with two outcomes. The sure amounts of money changed gradually from the top to the bottom of the list. Subjects were asked to select a row where they were willing to switch from preferring a lottery to preferring a sure amount. The lottery was $(0, \$2; 0.5, 0.5)$ and sure amounts changed between \$0 and \$2 in increments of \$0.10. Instructions for this part are straightforward and available from the authors upon request.

[10] We included a self-assessed measure for risk preferences because it is easy for subjects to understand the question and its validity is heavily supported by Dohmen et al. (2011).

[11] Using data from rounds 16-30, Player 1 investment vs. theory: CS ($p = 0.009$), CA ($p = 0.004$), and I ($p = 0.008$); Player 2 investment vs. theory: CS ($p = 0.034$), CA ($p < 0.0001$), and I ($p < 0.0001$); Player 1 payoff vs. theory: CS ($p = 0.004$), CA ($p = 0.001$), and I ($p = 0.005$); Player 2 payoff vs. theory: CS ($p = 0.159$), CA ($p = 0.007$), and I ($p = 0.001$).

	Player 1				Player 2			
	Rd. 1-30	Rd. 1-15	Rd. 16-30	SPE	Rd. 1-30	Rd. 1-15	Rd. 16-30	SPE
Investment								
CS	351.43	452.32	250.53	125.0	281.08	353.76	208.40	125.0
	(21.09)	(34.33)	(30.43)		(26.88)	(31.50)	(28.70)	
CA	275.57	307.79	243.35	138.27	248.10	288.42	207.79	89.40
	(13.52)	(14.72)	(22.79)		(20.46)	(28.35)	(18.63)	
I	283.20	326.02	240.39	128.43	239.98	281.82	198.14	104.0
	(23.15)	(18.57)	(31.55)		(9.98)	(11.26)	(18.05)	
Payoff								
CS	902.74	819.90	985.58	1125.0	964.76	874.02	1055.49	1125.0
	(26.30)	(38.53)	(27.34)		(29.73)	(29.07)	(41.99)	
CA	1003.99	969.54	1038.43	1182.94	1041.20	1000.42	1081.97	1182.94
	(21.03)	(23.87)	(26.01)		(15.27)	(21.58)	(24.84)	
I	1003.72	950.80	1056.63	1173.63	1015.09	976.57	1053.61	1166.80
	(21.69)	(18.98)	(30.79)		(19.18)	(20.06)	(22.93)	

Table 2: Average investment and payoff by treatment and player type, with cluster adjusted standard errors in parentheses, and theoretical predictions. Averages are obtained using all data (rounds 1-30), only the first half of data (rounds 1-15), and only the second half of data (rounds 16-30).

to the best response (see equation (2)) and after observing the investment of player 1. Therefore, the overbidding behavior of player 1 should lead player 2 to invest below the equilibrium prediction. However, we fail to see such adjustments. In other words, on average, not only is the investment of player 2 higher than the predicted best response, but it is also higher than the equilibrium prediction. Thus, overbidding persists despite the absence of strategic uncertainty faced by player 2. We look more closely at the difference between player 2 investment and the predicted best response in section 5.2.

Result 1. *Whether using data from rounds 1-30, only rounds 1-15, or only rounds 16-30, there is substantial "overbidding," measured as the difference between observed investment and the SPE prediction, in all treatments. Overbidding significantly reduces average payoffs except for player 2 under complete information and symmetric prize valuations.*

While overbidding is present using any subset of data, we find that there is a significant difference in average investment and payoff between the first and second half of the experiment. Specifically, subjects' investments are significantly lower and payoffs are significantly higher in the second half of the experiment compared to the first. Ten out of twelve of the differences are significant at a 5% significance level and the remaining two are significant at a 10% significance level.[12] This is indicative of substantial learning across rounds, i.e., subjects learn to make lower

[12]Two-sided p-values are obtained following the Wald test for comparisons of behavior using all data versus

investments across time, a common finding in many contest experiments (see e.g. Sheremeta (2013)). Learning can be, at least partially, attributed to the feedback subjects were given after every round including both players' investments, their own probability of winning, and payoff. Since we are primarily interested in analyzing converged or "stable" behavior of experienced subjects, we focus the remainder of our analysis using only data from rounds 16-30.[13]

Result 2. *On average, both player types in all treatments have significantly lower investments and significantly higher payoffs in the second half of the experiment as compared to the first.*

In the CS treatment, theory predicts identical investment levels across player types, and hence, identical expected payoffs. While we find that player 1 makes significantly higher investments as compared to player 2 ($p = 0.036$), the relatively higher rates of overbidding by player 1 do not lead to a significant payoff advantage ($p = 0.155$). In fact, on average, player 1 earns approximately 70 points *less* than player 2, albeit not a statistically significant difference. Therefore, consistent with theory, we do not observe evidence of a first-mover advantage, which replicates findings from prior experimental studies exploring behavior in symmetric sequential move contests (Fonseca, 2009; Nelson, 2020).

Result 3. *In the CS treatment, the average investment of player 1 is significantly higher than player 2, thus contradicting theory. Despite the difference in average investment between players, we find no evidence of a first-mover advantage in expected payoff as predicted.*

Table 2 also sheds light on comparisons of average payoff between player types in CA and I, an indicator of which player has an advantage in the contest. In treatment CA, theory predicts that payoffs are equalized across players, whereas in treatment I, theory predicts that player 1 should experience a payoff advantage over player 2. Our findings are in partial agreement with theory, as we find no significant difference in average payoff between player 1 and player 2 in neither CA (1038.43 vs. 1081.97; $p = 0.207$) nor I (1056.63 vs. 1053.61; $p = 0.915$). Theory also predicts that, on average, player 1 invests more than player 2. We find no significant difference between player types in CA (243.35 vs. 207.79; $p = 0.178$), however, in agreement with theory, we find marginally significantly higher investment by player 1 relative to player 2 in I (240.39 vs. 198.14; $p = 0.085$).

Result 4. *Consistent with theory, we find no significant difference in payoffs, averaged over all realizations of values, across player types under complete information and asymmetric valuations. However, contrary to the theoretical prediction that player 1 has a higher average payoff than player 2 under incomplete information, we find no significant difference.*

only the second half, with standard errors clustered by matching group. Player 1 investment: CS ($p = 0.009$), CA ($p = 0.056$), I ($p = 0.006$); Player 2 investment: CS ($p = 0.003$), CA ($p = 0.0.018$), I ($p = 0.006$); Player 1 payoff: CS ($p = 0.010$), CA ($p = 0.043$), I ($p = 0.005$); Player 2 payoff: CS ($p = 0.007$), CA ($p = 0.059$), I ($p = 0.004$).

[13] This approach is used often in the literature (see, e.g., Brookins, Lightle and Ryvkin, 2018; Boosey, Brookins and Ryvkin, 2020), especially when a large degree of learning is observed.

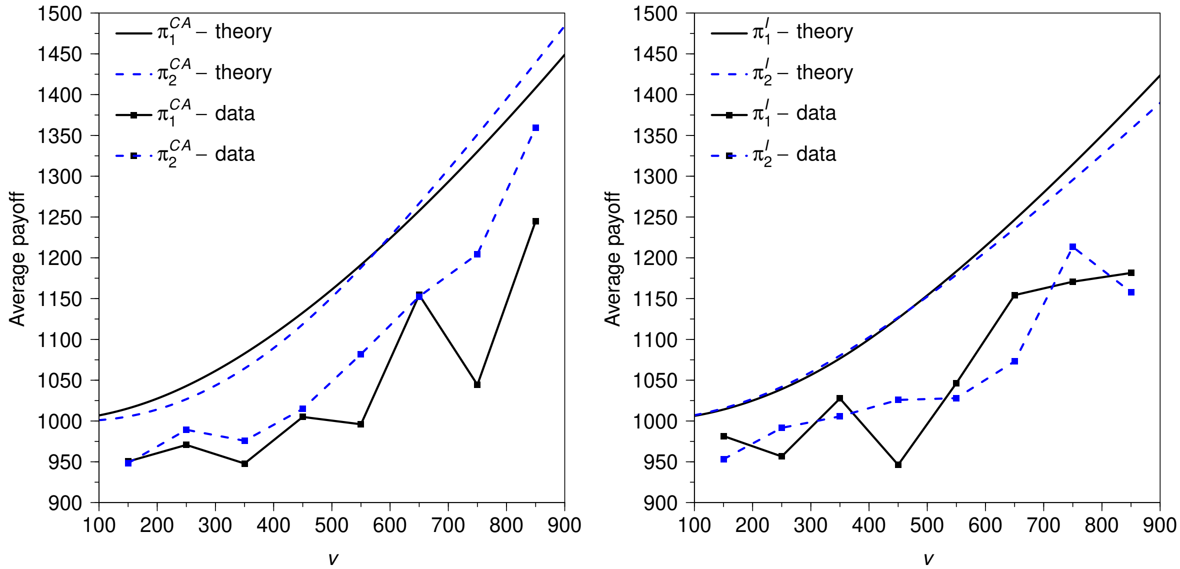

Figure 2: Theoretical (solid curves) and observed (connected with squares) average payoff functions in treatment CA (left) and I (right) across player types.

In order to compare the first-mover advantage theoretically and empirically *taking into account players' valuations*, we plot the theoretical (solid curves) and observed (curves with squares) average payoff functions in CA (left) and I (right) across player types in Figure 2. Throughout the rest of the paper, we "bin" valuations as follows. There are 8 valuation bins, organizing players with similar valuations from lowest to highest as follows: $[100, 200]$, $[201, 300]$, $\ldots$, $[801, 900]$. Empirical investment and payoff functions therefore consist of 8 data points, each created by averaging over all realizations within a valuation bin. Consistent with Table 2, average payoff functions are below predictions for all player types in CA and I. Out of 16 comparisons, one for each valuation bin across treatments, only 2 comparisons of average payoff are significantly different across players: valuation bin $[100, 200]$ for I, and bin $[701, 800]$ for CA.[14] Thus, contrary to theory, we fail to find a unique and sufficiently high valuation $\hat{v}$ such that one player has an advantage over the other. This further confirms result 4, i.e., that we find no difference in payoffs across player types in CA and I.

Result 5. *Under complete information and asymmetric valuations and incomplete information, for each valuation bin, we do not observe a difference in average payoffs between player 1 and player 2. Hence, contrary to theory, there does not exist a unique valuation separating the case when the first-mover or second-mover has an advantage.*

Comparing average behavior across treatments is interesting, but such an analysis is limited.

[14]For each panel in Figure 2, and for each valuation bin, we compare average payoff across players for CA and I. From lowest to highest valuation bin, p-values in CA: $p = 0.946, 0.484, 0.293, 0.814, 0.189, 0.979, 0.058, 0.116$; in I: $p = 0.085, 0.226, 0.642, 0.198, 0.696, 0.346, 0.659, 0.824$.

For the remainder of the analysis we focus on player behavior as a function of own and opponent valuation. We begin by analyzing the behavior of player 2, followed by player 1.

5.2 Player 2 Behavior

The second-mover's decision problem is unambiguously easier than the first-mover's, as strategic uncertainty *and* ambiguity are removed. Simply put, subjects in the role of player 2 have everything they need to compute expected payoff: x_1 and v_2.

x_2^{CA}	(1)	(2)	(3)	(4)
v_1	0.039	-0.031	-0.035	-0.033
	(0.061)	(0.059)	(0.052)	(0.051)
v_2		0.359***	0.378***	0.377***
		(0.068)	(0.063)	(0.064)
x_1		0.206*	0.199*	0.194*
		(0.094)	(0.094)	(0.094)
RA			-12.088*	-12.085*
			(5.828)	(5.837)
Female			17.434	17.543
			(39.230)	(39.268)
t				-2.130
				(1.391)
Constant	188.297***	-0.912	84.910	134.900*
	(35.119)	(39.593)	(52.327)	(66.800)
N	420	420	420	420

Table 3: Columns (1)-(4) report OLS coefficient estimates of player 2 investment on player 1 valuation, using player 2 data in rounds 16-30 in treatment CA. Standard errors in parentheses are clustered at the matching group level. Significance levels: *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$.

Recall that, regardless of treatment, player 2 invests according to best-response function (2), which depends directly on own valuation and player 1's investment, and indirectly on player 1's valuation. Therefore, in treatment CA, we should not find a significant correlation between x_2 and v_1, since player 2 should only condition behavior on v_2 and x_1. In Table 3, we report OLS coefficient estimates of player 2 investment on player 1 valuation for treatment CA. In column (1), we find a positive coefficient on v_1, but the magnitude is small and the estimate is not statistically significant. Column (2) additionally controls for player 2 valuation and player 1 investment. While now negative, we again see no statistical relationship between x_2^{CA} and v_1. This result continues to hold when RA and a Female dummy are added in column (3), and they are robust when controlling for time trend t; see column (4).[15] Consistent with theory,

[15] Female takes the value of 1 if the subject is female and 0 if male. Risk aversion RA $\in \{0, 1, \ldots, 21\}$ with 0

and not surprisingly, an increase in player 2's valuation leads to higher investments, as the coefficient estimate on v_2 is positive and statistically significant in columns (2)–(4). There is a positive and significant relationship between x_2 and x_1 in columns (2)–(4), but the magnitude of the coefficient estimate on x_1 is small and marginally significant; a more detailed player 2 best-response analysis is contained below. Finally, we find that more risk-averse player 2's bid significantly less, a common finding in the literature.[16] Altogether, consistent with theory, player 2 in CA does not condition investment behavior on irrelevant information, i.e., the valuation of player 1.[17] From a behavioral economics standpoint, we do not observe an anchoring effect driven by the valuation of player 1.[18]

Result 6. *Consistent with theory, under complete information and asymmetric valuations, on average, there is a positive and statistically significant correlation between player 2's valuation and investment level. Holding fixed own valuation and player 1 investment, on average, there is no statistically significant relationship between player 2 investment and player 1 valuation.*

Given that player 2's best response is a function of v_2 and player 1's investment x_1, we "bin" both valuations and player 1 investment as follows. Valuation bins are $[100, 200]$, $[201, 300]$, $\ldots, [801, 900]$ and player 1 investment bins are $[0, 100]$, $[101, 200]$, $\ldots$, $[901, 1000]$. Therefore, for each valuation bin, the empirical best response function of player 2 contains up to 10 points, one for each of player 1's investment bins. Figure 3 displays theoretical and empirical best response behavior of player 2 across treatments. Panels 1-8 (arranged from left-to-right and top-to-bottom) contain the average best response prediction (red curves), defined as $\bar{x}_2 = \frac{1}{|V|}\sum_{v \in V} x_2(x_1; v)$, where $V = [\underline{v}, \overline{v}]$ is the set of values within a given valuation bin as well as the corresponding empirical best responses and estimated best response fit for CA and I. For each valuation bin, the empirical best response fit is obtained using the coefficient estimates resulting from the regression $x_{2it} = \beta_1\sqrt{x_{1it}} + \beta_2 x_{1it} + \epsilon_{it}$, with standard errors clustered at the matching group level. Panel 9 contains theoretical best response $x_2(500)$, empirical best response, and estimated best response fit for CS.

Out of the 17 estimated fits to player 2's empirical best response, we observe very few cases where player 2 exhibits the "correct" best response. However, accounting for the significant overbidding reported in Section 5.1, the majority of fitted best responses are nonmonotonic in x_1, and hence, the fitted best response functions are qualitatively in line with theoretical

indicating the most risk averse and 21 the least risk averse. Time trend t is equivalent to experimental round.

[16] Generally speaking, the effects of risk aversion on investment in Tullock contests is ambiguous and depends on players' preferences. A negative effect of risk aversion on investment is consistent with, for example, a model where individuals' preferences are represented by a utility function exhibiting constant absolute risk aversion.

[17] Table 6 in Appendix B reports coefficient estimates obtained using a random effects model for each specification in Table 3. The regression results are similar, but there is no longer a statistically significant relationship between x_2 and x_1.

[18] In Tversky and Kahneman (1974) and Ariely, Loewenstein and Prelec (2003) it has been found that subjects' willingness to pay for various consumption goods can be significantly impacted by irrelevant information, such as a random number drawn by the experimenter or the last 2 digits of the subjects' social security number. Bergman et al. (2010) later replicated these findings suggesting that cognitive skills can reduce the existence of anchoring. Fudenberg, Levine and Maniadis (2012) and Luccasen (2012) find no anchoring effects in binary lotteries and public good games, respectively.

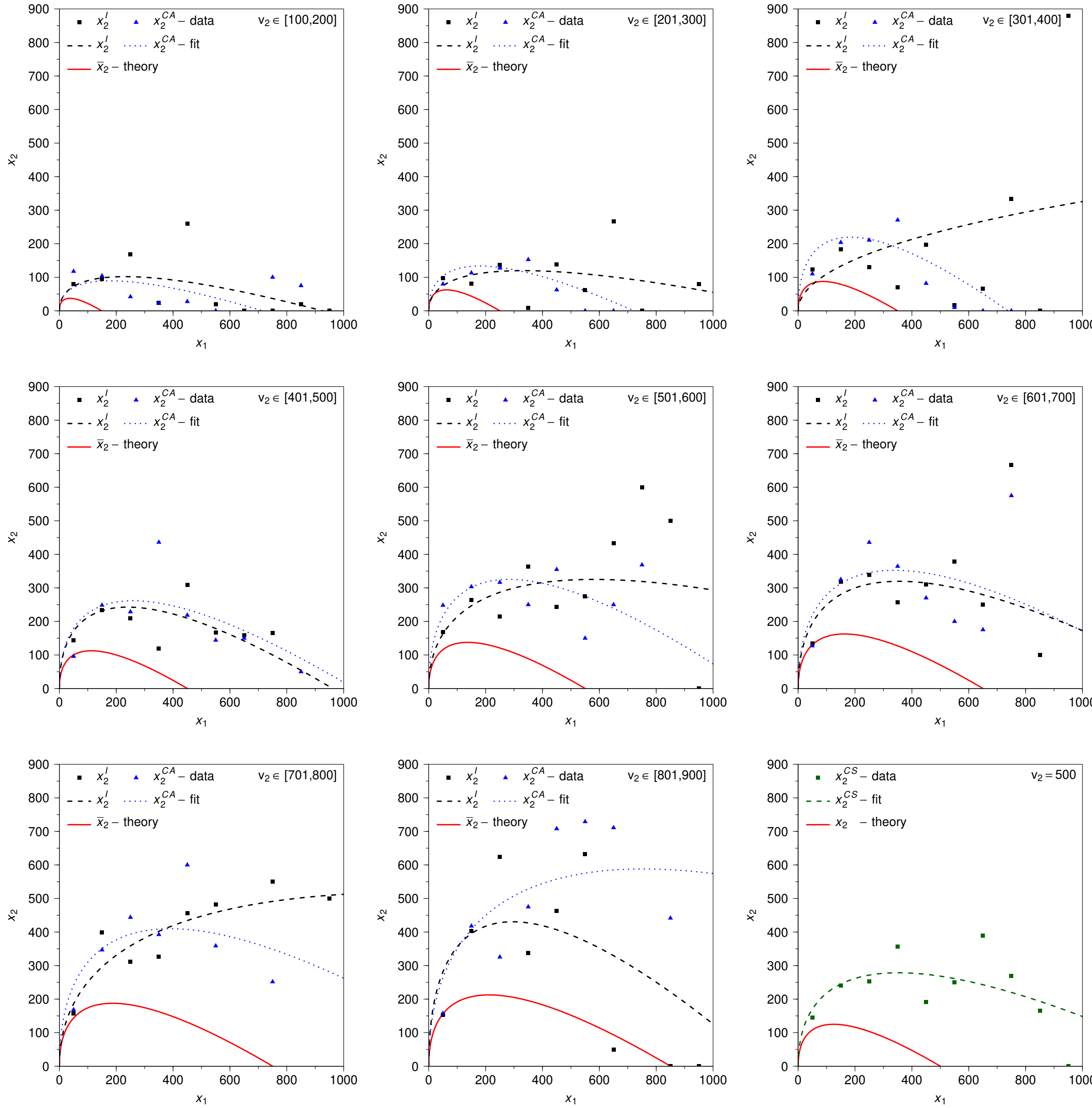

Figure 3: Player 2 empirical and theoretical best response function. Panels 1-8 contain the average best response prediction (red curves), empirical best response (solid blue and black squares for CA and I, respectively), and second-order polynomial best response fit (dashed blue and black curves for CA and I, respectively). Panel 9 contains theoretical best response x_2^{CS} (red curve), empirical best response (green solid squares), and second-order polynomial best response fit (green dashed curve) for CS.

predictions. That is, in most cases, second-movers begin to decrease investment when the first-mover's investment is sufficiently high. In terms of our estimated coefficients, there should be a negative coefficient on x_1 and a positive coefficient on $\sqrt{x_1}$. In CA, all coefficients on x_1 are negative, but half of them are statistically significant and above the predicted $\hat{\beta}_1$ (two at 10%, 1 at 5%, and 1 at 1%). All 8 coefficient estimates on $\sqrt{x_1}$ in CA are negative as predicted, but one is significantly below the predicted $\hat{\beta}_2$, albeit only marginally so. In treatment I, all coefficients on x_1 are negative, with five statistically significant (2 at 10% and 3 at 1%), and all coefficients on $\sqrt{x_1}$ are positive, with only 1 statistically significant (at 10%). Panel 9 demonstrates a similar story for treatment CS, and both coefficients are in line with the theory. To summarize, accounting for overbidding, player 2's best response is qualitatively in agreement with the theoretical best response given by equation (2) in most, but not all, cases. The main disagreement with theory is $\hat{\beta}_1$ above and $\hat{\beta}_2$ below the predicted value, respectively, indicating player 2 overbids and is less responsive to "aggressive" player 1 investment. These findings are in line with previous studies exploring behavior in symmetric and asymmetric sequential contests (Fonseca, 2009). A complete analysis of comparisons of estimated versus theoretical best response for player 2 is presented in Appendix C.

Result 7. *On average, and accounting for overbidding, player 2's empirical best response is qualitatively similar to the predicted best response in most, but not all, cases across treatments and valuations.*

Figure 3 also seems to hint at a positive relationship between overbidding and valuation, averaged across player 1's investment. To investigate this relationship further, we define a measure of *absolute overbidding* by player 2 relative to the equilibrium prediction as the difference between subjects' empirical and predicted best response, $\Delta x_2 = x_2^{data} - x_2^{theory}$. Note that our measure is the absolute difference between observed behavior and the theoretically derived best response prediction given observed v_2 and x_1, which contrasts to most other studies who simply define overbidding as the absolute difference between observed investment and the equilibrium prediction. We believe our measure is more appropriate given that subjects in the role of player 2 can fully condition their behavior on v_2 and the investment of player 1, and hence, a better measure of player 2 "rationality."

In Table 4, we report OLS coefficient estimates of Δx_2 on valuation, risk aversion (RA), sex (Female), and round (t). Columns (1)-(3) refer to CA and columns (4)-(6) refer to I. First, controlling only for v_2 as in columns (1) and (4), we observe a significant and positive effect of v_2 on Δx_2 in CA ($p = 0.024$) and I ($p = 0.001$), confirming the observation that player 2's absolute overbidding is an increasing function of valuation. When considering controls for sex and risk aversion in columns (2) and (5), significance levels and coefficient estimate magnitudes do not change much in CA (0.011) and I ($p = 0.001$). Consistent with most prior studies, we see a positive effect of being female on absolute overbidding and a negative effect of risk aversion; however, the only significant result is risk aversion in CA, albeit only marginally significant ($p = 0.084$). The results reported in columns (2) and (5) are robust to adding time trend; see columns (3) and (6). The time trend is negative, but only marginally significant in CA. Finally, it is also evident from Table 4 that absolute overbidding rates are similar across CA and I.[19] In

[19]Table 8 in Appendix D reports coefficient estimates obtained using a random effects model for each specification in Table 4. The regression results are similar, with the exception that the coefficient estimate on Female in I in columns (5) and (6) are marginally significant (both $p = 0.065$).

Δx_2	*CA* (1)	(2)	(3)	*I* (4)	(5)	(6)
v_2	0.167**	0.188**	0.186**	0.145***	0.142***	0.141***
	(0.056)	(0.051)	(0.053)	(0.028)	(0.027)	(0.029)
RA		-12.151*	-12.143*		-3.171	-3.172
		(5.871)	(5.868)		(8.560)	(8.569)
Female		25.367	25.329		52.044	52.056
		(36.832)	(36.836)		(28.338)	(28.348)
t			-3.178*			-3.210
			(1.299)			(1.759)
Constant	62.803	140.157**	213.992**	51.971**	50.709	125.014
	(36.023)	(47.410)	(65.219)	(17.096)	(79.870)	(81.892)
N	420	420	420	540	540	540

Table 4: Columns (1)-(6) report OLS coefficient estimates of Δx_2 on valuation, using player 2 data in rounds 16-30. Column (1)-(3) correspond to treatment CA, and (4)-(6) to treatment I. Standard errors in parentheses are clustered at the matching group level. Significance levels: *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$.

a simple regression of Δx_2 on treatment dummies, including data for all treatment, all pairwise coefficient estimate comparisons are insignificant (all $p > 0.41$). Therefore, averaging across all valuations, absolute overbidding is indistinguishable across treatments.

Result 8. *(a) Average player 2 overbidding, measured as the difference between observed investment and predicted best response, under complete information and asymmetric valuations, and incomplete information is increasing in the prize valuation; (b) Player 2 overbidding, averaged over all realizations of valuations, is indistinguishable across treatments.*

Since player 2 can easily control their desired probability of winning given v_2 and x_1, it is interesting to explore the distribution of their winning probability, p_2, controlling for their valuation. Figure 4 presents histograms of winning probability for player 2 across treatments: panels 1-8 contain data from treatments CA and I and panel 9 contains data from CS. In CS, theory predicts that each player type wins with $p_2 = 0.5$. Looking at the observed frequencies, a few observations are immediate. First, $p_2 \leq 0.1$ for 86 out of 360 observations, with $p_2 = 0$ for 64 of those observations. This is a typical finding in both simultaneous and sequential contests experiments, and suggests a high degree of "dropout."[20] This is also consistent with the finding in Section 5.1 that player 1's average investment is higher than player 2, indicating that player 1 is behaving "preemptively," or investing sufficiently high so as to lead to player 2 to drop out. Next, we find that $p_2 \in (0.5, 0.6]$ is the second most frequent outcome, with a total of 57 observations.[21] Finally, we see $p_2 = 0.5$ for 9.2% of the observations, which accounts for most

[20] See Fallucchi, Niederreiter and Riccaboni (2021) for a detailed analysis of dropout behavior in Tullock contests.

[21] Interestingly, "dropping out" ($p_2 \leq 0.1$) and bidding slightly over player 1 ($p_2 \in (0.5, 0.6]$) is observed by the

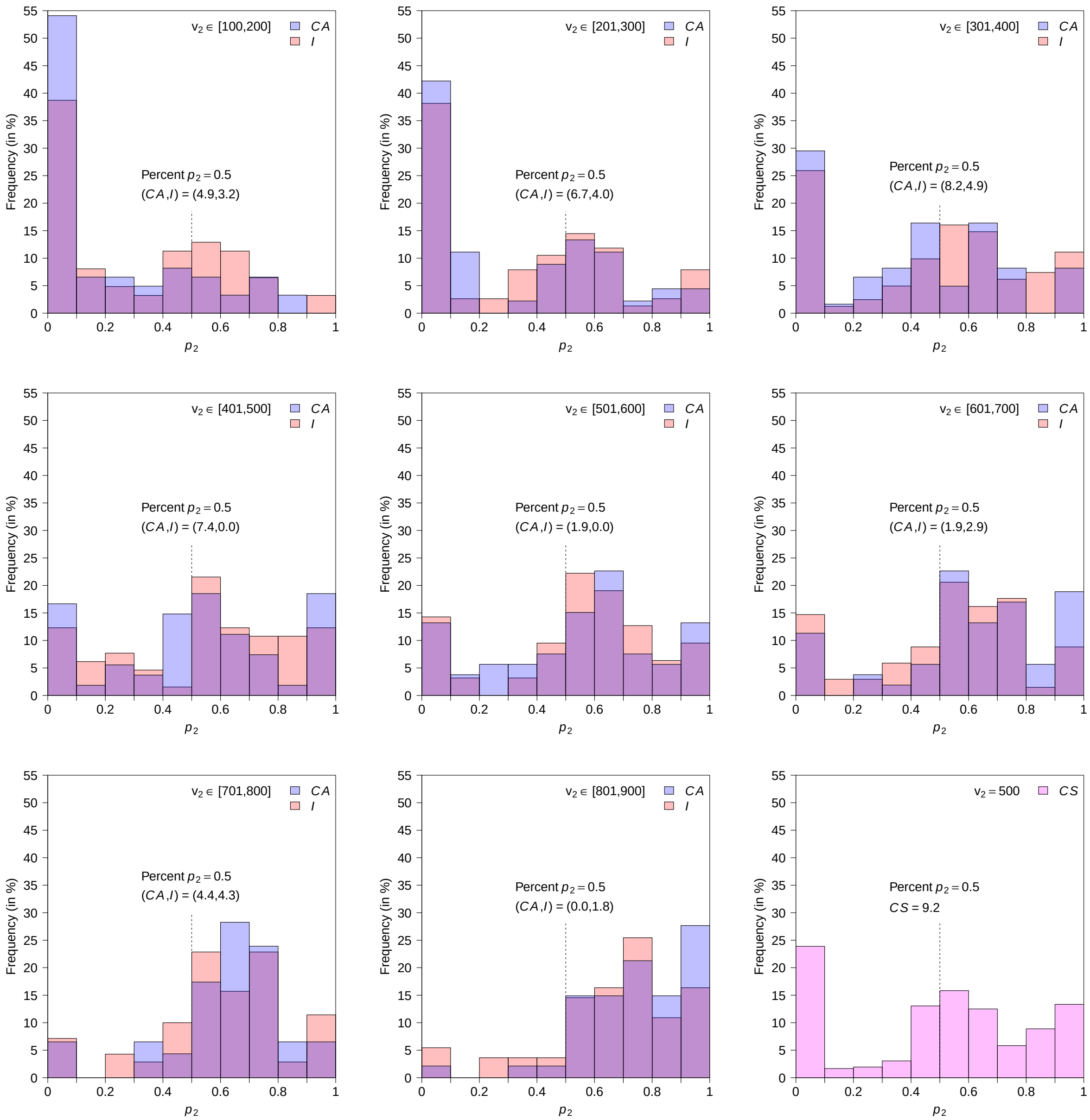

Figure 4: Histograms of player 2 probability of winning, p_2, by treatment and valuation bin. Panels 1-8 contain frequencies (in %) for CA and I for each of the 8 valuation bins, and panel 9 contains frequencies for CS. Text at the end of vertical dotted lines denotes the observed frequency of $p_2 = 0.5$ for each treatment within in valuation bin.

of the data in the $p_2 \in (0.4, 0.5]$ frequency bin (33 of 47 obs.). Taken together, these findings suggest that player 2 uses the following investment heuristic: drop out completely or choose an investment level so as to secure at least a 50% chance of winning the contest. Of course, due to a high degree of dispersion in p_2 for CS, many other outcomes are realized. That said, the

majority of our subjects, about 75% and 83% of our sample, respectively, and not by a few outliers who engage in such behavior often. The same is true in CA and I.

distribution seems to be left-skewed and in favor of $p_2 \geq 0.5$. This finding is in agreement with Fonseca (2009), who finds that strong first-movers behave preemptively to deter weak second-movers from entering the contest, while a substantial amount of second-movers often responded aggressively.

Turning our attention to panels 1-8, we first observe that the distributions of p_2 within each valuation bin have a high degree of dispersion and are statistically indistinguishable across CA and I. Interestingly, just as with CS, we observe a high frequency of $p_2 \leq 0.1$ and $p_2 \in (0.5, 0.6]$ in *all* valuation bins. Not surprisingly, dropout is highest in the lowest valuation bin and steadily decreases as player 2's value increases. While not always the second most observed p_2 bin, we consistently observe a large number of p_2 observations in $(0.5, 0.6]$; there are also many observations at or near $p_2 = 0.5$. Thus, *regardless of the valuation bin*, second-movers tend to use a similar investment heuristic as in the CS treatment. That is, they choose an investment level in a way that secures a probability of winning close to 50%; otherwise, they drop out of the contest completely. Similar results can also be drawn from figure 7 of Appendix E where we present the kernel density estimates for player 2's probability of winning in CA and I.

Given the high degree of overbidding by player 1, the empirical best response for player 2 should be zero quite often; hence, we should observe many observations in the $p_2 \leq 0.1$ bin. In Table 9 in Appendix F, we compare observed and predicted p_2 across valuation bins for treatments CA and I satisfying $p_2 \leq 0.1$. In almost every single valuation bin, in both treatments, the theoretical best response for player 2 given x_1 resulting in $p_2 \leq 0.1$ is higher than the observed frequency; however, the difference is only significant for some comparisons in valuation bins with $v \leq 500$. This finding coupled with overbidding further supports the tendency for player 2's to drop out. Table 10 in Appendix F presents the same analysis but for $p_2 \in [0.5, 0.6]$. 11 out of 16 comparisons are significantly different, with the observed frequency of $p_2 \in [0.5, 0.6]$ higher than predicted. This further supports the patterns seen in Figure 4, suggesting a tendency for player 2's to match or slightly outbid player 1's investment.

Result 9. *While there is a large dispersion in the probability of winning for player 2 in all treatments, there is a large degree of "dropping out," i.e., investments at zero or low enough to generate a probability of winning less than or equal to 10%, and a large degree of investments that secure a probability of winning at or just above 50%.*

5.3 Player 1 Behavior

Compared to player 2, player 1's decision problem is much more complex. This is especially the case for player 1 in I, where they do not know the valuation of the player 2 who they are matched with in a given round. That is, in CA, player 1 can at least condition their investment on the valuation of the player 2 who they are matched with. That begs the question: How does player 1 choose their investment, and how does this differ across treatments? One possibility, which we explore in detail below, is that player 1 uses a combination of reinforcement learning (e.g., Roth and Erev, 1995) and directional learning (e.g., Selten and Stoecker, 1986), whereby individuals adjust behavior after each round following the feedback they receive about investments and payoffs.[22] Specifically, evidence of reinforcement learning would come in the form of a strong degree of investment persistence across rounds, i.e., subjects adjust investment from round-to-round within a sufficiently narrow neighborhood in search for a higher payoff and avoiding strategies that lead to a reduction in payoffs. In our contest setting, evidence of directional learning would come in the form of reactions to winning and losing as follows: if $\text{Win}_{t-1} = 1$, then adjust investment downward to avoid "wasting" resources; otherwise, if $\text{Win}_{t-1} = 0$, then adjust investment upward in hopes to secure the prize. Finally, note that such a history-dependent learning approach such as directional learning is less likely to be used by player 2, since our analysis in Section 5.2 demonstrated that those players tended to try to match or just outmatch the investment of player 1, i.e., they conditioned their behavior on current round information.

Table 5 reports the results of dynamic regressions of investment on various controls for all treatments. Columns (1)-(3) contain player 1 observations and columns (4)-(6) contain observations for player 2. All specifications control for player valuation v, treatment dummies CA and I, and round t to account for time trends. First, we find that all coefficient estimates on v are positive and significant (all $p < 0.01$), suggesting that a 1-point increase in own valuation leads to an investment increase of about 0.5 points for player 1 and an increase of about 0.4 points for player 2, on average. The coefficient on t is negative and significant in columns (1)-(4) (p-values: 0.000, 0.01, 0.01, 0.045), but not in columns (5) and (6) (p-values: 0.116, 0.114) when controls other than treatment dummy and valuation are added to the regression. Therefore, even though average investment levels are significantly lower in the second half of the experiment compared to the first, there are still downward adjustments for player 1, but not for player 2 when considering controls other than treatment and round. Thus, this is the first piece of evidence that history-dependent learning is more prevalent for player 1 compared to player 2.

Columns (2) and (5) add a lagged dummy variable for whether the subject won in the previous round, Win_{t-1}, and a lagged variable for own investment in the previous round, x_{t-1}. The coefficients on x_{t-1} are positive and highly significant (all $p < 0.001$). These estimates are similar to individual investment fixed effects and, due to their positive sign, indicate strong persistence in investment for both player types. The coefficients on Win_{t-1} are negative and statistically significant for player 1, but positive and not statistically significant for player 2. Interestingly, this supports the existence of directional learning for player 1, and the lack of such adjustments by player 2. The negative coefficient on Win_{t-1} suggests that player 1 invests lower in the current round following a win and vice versa following a loss.[23] These findings are robust when accounting for risk and sex; see columns (3) and (6).

[22]See Grosskopf (2003) for an in-depth analysis of reinforcement and directional learning in the ultimatum game.

[23]We also ran a regression similar to the specification in column (2) where we interacted Win_{t-1} with treatment. The negative effect of Win_{t-1} is statistically significant in CA and I, but we do not find a difference across those treatments.

	Player 1			Player 2		
x_t	(1)	(2)	(3)	(4)	(5)	(6)
v	0.48***	0.47***	0.47***	0.35***	0.37***	0.37***
	(0.04)	(0.03)	(0.03)	(0.03)	(0.03)	(0.03)
CA	-4.26	3.54	1.66	4.64	6.12	4.63
	(34.93)	(20.58)	(20.76)	(31.90)	(22.88)	(20.56)
I	-7.02	2.39	-0.94	-6.42	-2.99	-1.39
	(41.74)	(24.58)	(24.35)	(31.07)	(21.92)	(19.59)
Win_{t-1}		-36.02***	-35.96***		7.98	8.93
		(11.32)	(11.36)		(9.88)	(9.81)
x_{t-1}		0.44***	0.43***		0.30***	0.28***
		(26.01)	(0.06)		(0.08)	(0.08)
RA			0.67			-3.73
			(2.67)			(2.42)
Female			15.91			31.96**
			(14.52)			(15.19)
t	-4.53***	-2.52**	-2.53**	-2.32**	-1.45	-1.48
	(1.09)	(0.90)	(0.90)	(1.09)	(0.88)	(0.90)
Constant	113.49*	-23.48	-36.64	88.36**	-9.64	6.18
	(48.27)	(40.47)	(44.58)	(40.85)	(36.63)	(48.70)
N	1,320	1,320	1,320	1,320	1,320	1,320

Table 5: OLS regression results of investment on various controls using data from rounds 16-30. Columns (1)-(3) contain player 1 data and columns (4)-(6) contain player 2 data. Standard errors in parentheses are clustered at the matching group level. Significance levels: *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$.

Result 10. *Winning in the previous round significantly and negatively affects player 1 investment in all treatments, which is consistent with directional learning theory. There is no such finding for player 2. Consistent with reinforcement-type learning, we find a strong persistence in investment across player types in all treatments.*

Given the complexity of player 1's problem, subjects in this position may rely on heuristics to avoid the cognitive challenge of choosing the optimal bid and reduce the mental effort in their decision making. Heuristics are often considered to be sub-optimal mental shortcuts that can lead to systematic errors and cognitive illusions.[24] Because subjects are more inclined to rely on them in complicated rather than simple decision environments, heuristics are very relevant in our context for player 1. In fact, several experimental papers exploring behavior in

[24] Tversky and Kahneman (1974) introduced the availability, representativeness, anchoring and adjustment heuristics. Depending on the context, subjects may rely on such heuristics to lower the mental effort required to make a decision. Following this line of research, Gigerenzer and Goldstein (1996), DeMiguel, Garlappi and Uppal (2009), and Gigerenzer and Gaissmaier (2015) among others discuss different heuristics as well as their accuracy and effectiveness in decision making.

a variety of contest settings have found evidence of frequent heuristic use. Sheremeta (2011) finds an endowment size effect, whereby investment proportionately increases with the size of the endowment. Brookins, Lightle and Ryvkin (2015) find that subjects, who differ in the marginal cost of effort, bid in such a way that total effort costs are equalized across player types, suggesting that players use a bidding heuristic that guarantees that a sufficiently high amount of the endowment is uninvested in the contest and therefore paid out with certainty. Finally, Rodet and Smyth (2020) explore behavior in a finitely repeated contest environment with endogenous endowment carryover and exogenously changing prizes (framed as market demand). As compared to theory, their data is better supported by two independent bidding heuristics: investing a fixed percentage of the prize forecast or investing a fixed percentage of the endowment. The common theme among these papers is that the investment heuristic depends on the size of the endowment, or in the case of Rodet and Smyth (2020), on the size of the endogenously determined prize.

In our setting, we find no effect of an endowment/investment heuristic, but like Rodet and Smyth (2020), we do find an interesting and simple prize/investment heuristic: *player 1 bids half of their valuation*. In CA and I, player 1 receives a new valuation draw each period, which likely rules out endowment/investment heuristics of the likes previously mentioned as the decision problem remains complex across time. Thus, subjects may find an alternative heuristic which depends on the only changing piece of information in each round—the valuation—and bidding half of this may seem like a reasonable *focal* course of action. A closer look at player 1 data from Table 5 highlights the possibility of that investment heuristic. Indeed, the coefficient on v is approximately 0.5 in all three specifications and for both CA and I. What about treatment CS? In Table 2, average player 1 investment is 250.53, which is in remarkable agreement with our hypothesized investment heuristic! Moreover, we find no significant difference between average investment and 250 in each treatment, suggesting that, on average, the heuristic is supported by our data.

It is arguably more interesting to disaggregate the data in CA and I to determine whether the heuristic is supported across valuation bins. Figure 5 includes the theoretical bidding functions, the empirical average investment functions, and the half of the valuation heuristic function (red dotted line). The left panel is for CA and the right for I.[25] we find a strong agreement between our data and the heuristic for all valuation bins in each treatment. In fact, the only statistically significant difference between theory and our data is in valuation bin $[801, 900]$ for CA ($p = 0.052$), and only marginally so.

Result 11. *For all treatments, on average, our data significantly and strongly supports the following investment heuristic: Player 1 invests an amount equal to half of their valuation.*

[25]For completeness, Figure 8 in Appendix G includes bidding functions for both player types.

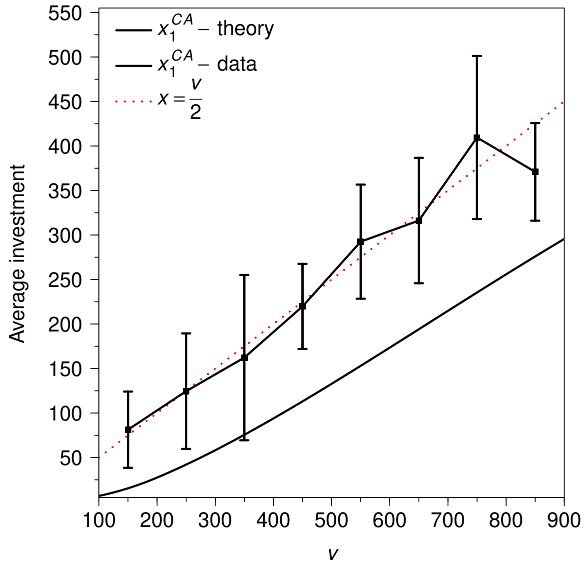

Figure 5: Theoretical (solid curves) and observed (connected squares) average investment functions in treatment CA (left) and I (right) for player 1. The dotted red line represents investment levels equal to one half of the valuation, $x = \frac{v}{2}$. Error bars represent the 95% confidence interval of the mean, clustered by matching group.

6 Discussion and Conclusion

We theoretically and experimentally explored behavior in three two-player sequential-move Tullock contest settings: symmetric valuations and complete information, asymmetric valuations and complete information, and incomplete information. For the latter environment, we add to the theoretical contest literature as we completely characterize the subgame-perfect equilibrium in closed-form for the case when private valuations are drawn from a continuous distribution.

Our main goal was to understand how rational each player type was in each setting, i.e., how close the empirical contest investments were to the theoretical predictions. Given that most prior experimental studies report extreme overbidding, i.e., average investments far exceeding the Nash equilibrium predictions, we expected to find similar results, but with one caveat. That is, in all settings, second-movers observe the investment level chosen by first-movers, and therefore face no strategic uncertainty nor ambiguity. Second-movers' decision is simple: condition investment solely on their valuation and the first-movers investment so as to achieve a desired probability of winning. From an experimental economics design standpoint, we are giving individuals the best shot for behaving optimally in the contest, and hence, if subjects are far from equilibrium and best-response play in that decision environment, then what are the odds that individuals play according to predictions? The story is less clear for first-movers, especially those first-movers who have incomplete information about their opponents' valuation. Thus, we expected the decision environment for first-movers to be relatively more complex, potentially involving substantial learning and experimentation across experimental contest rounds.

Summary of second-mover behavior

1) Despite knowing the investment level of the first-mover, there is substantial overbidding by second-movers.

2) Accounting for overbidding, the estimated best response fit to our data is nonmonotonic in first-mover investment and qualitatively matches the theoretical predictions in most cases across treatments. This suggests that, as predicted, second-movers are responsive to high, aggressive bidding by first-movers. Moreover, second-movers do not suffer from anchoring effects, i.e., they ignore irrelevant information such as first-mover valuations when available.

3) Regardless of valuation and treatment, second-movers frequently invest in such a way that guarantees a winning probability below 10% or near 50%, suggesting two types of second-movers in our data: those who "drop out" of the contest completely and those who "fight fire with fire" in response to aggressive or preemptive first-mover investment levels.

Summary of first-mover behavior

1) Contrary to second-movers, first-movers face a difficult decision environment, strategic uncertainty, and ambiguity. Therefore, we expected investment in current periods to depend on feedback and realizations of previous contest rounds. We find evidence that first-movers behave in a way consistent with reinforcement and directional learning theories.

2) In treatments with asymmetric valuations, first-movers receive a new draw each round, thereby making it difficult for them to converge on a personal investment strategy. In all treatments, we find strong support that first-movers use the following simple investment heuristic: investment and amount equal to half of the valuation. This is consistent with the prior literature showing that individuals resort to simple rules of play when the decision environment is challenging.

As outlined in the introduction, contests find application across various real-world scenarios, with R&D races being a common example in the literature. In a scenario aligning with our model of incomplete information, consider a situation where technology startups compete for a prestigious grant to advance their research projects, with a limited understanding of their competitors' strategies for securing the grant. Both startups devote irreversible efforts to refine their research projects, unveiling them sequentially through conferences, working papers, or journal publications. While their efforts increase their chances of receiving a grant, winning is not guaranteed. Our study findings suggest that the startup initiating action could invest effort equivalent to half the value of winning the grant. Conversely, the second startup may opt to exit the competition entirely or match the effort of the first-mover, thereby leveling the playing field.

In such real-life scenarios, ambiguity and uncertainty can be even more prevalent, which in turn could enhance subjects' reliance on decision heuristics. However, further empirical support is needed to ensure this is the case.

To summarize, we find weak support that second-movers behave consistent with theory. Instead, they follow simple rules most frequently, such as matching or slightly outbidding the first-mover. As with the analysis of second-mover behavior, we again find weak support for equilibrium and best response play. Instead, it seems more plausible that each player type resorts to simple heuristics, which would be consistent with several previous experimental contest studies (Sheremeta, 2011; Brookins, Lightle and Ryvkin, 2015; Rodet and Smyth, 2020). In light of these heuristic findings, we suggest that future experiments carefully search for similar heuristics and accompany such analyses by eliciting qualitative data, such as "Did you use a particular investment strategy?" or something similar. After all, if we cannot explain departures from theory, then what's the use?

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A Difference in average payoff between players

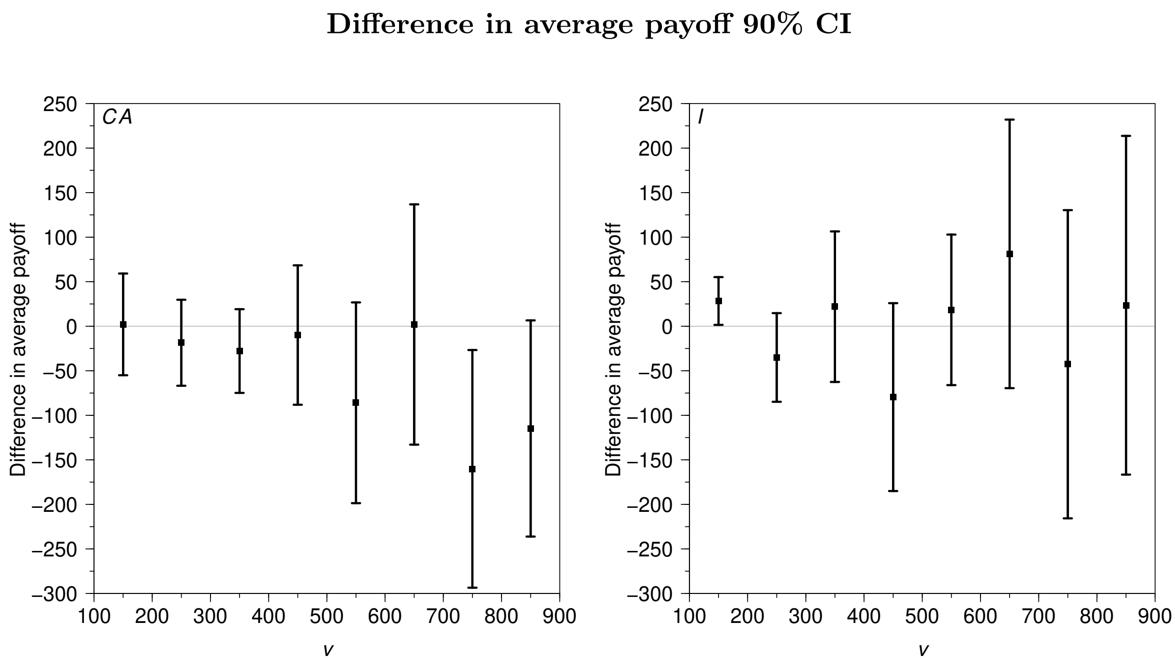

Figure 6: Difference in average payoff between players, $\pi_1 - \pi_2$, for each valuation bin in CA and I. Error bars represent the 90% confidence interval of the mean, clustered by matching group.

B RE coefficient estimates of x_2 on v_1 in CA

x_2^{CA}	(1)	(2)	(3)	(4)
v_1	0.070	0.031	0.029	0.031
	(0.068)	(0.057)	(0.057)	(0.055)
v_2		0.378***	0.382***	0.380***
		(0.066)	(0.066)	(0.067)
x_1		0.131	0.131	0.125
		(0.111)	(0.111)	(0.110)
RA			-11.960*	-11.957*
			(6.113)	(6.124)
Female			20.504	20.644
			(39.141)	(39.180)
t				-2.361*
				(1.409)
Constant	173.447***	-23.036	65.152	120.612*
	(39.699)	(23.549)	(48.114)	(67.757)
N	420	420	420	420

Table 6: Columns (1)-(4) report random effects model coefficient estimates of player 2 investment on player 1 valuation, using player 2 data in rounds 16-30 in treatment CA. Standard errors in parentheses are clustered at the matching group level. Significance levels: *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$.

C Estimated player 2 best response

	$x_2 = \hat{\beta}_1\sqrt{x_1} + \hat{\beta}_2 x_1$			
	CA		I	
$[\underline{v}, \overline{v}]$	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\beta}_1$	$\hat{\beta}_2$
$[100, 200]$	13.37	-0.50*	13.39	-0.44***
	(4.71)	(0.23)	(2.88)	(0.09)
$[201, 300]$	19.89	-0.74	13.13	-0.36***
	(4.93)	(0.19)	(3.12)	(0.15)
$[301, 400]$	32.33*	-1.19	11.25	-0.03***
	(5.79)	(0.19)	(4.89)	(0.28)
$[401, 500]$	32.52**	-1.01	31.31	-1.01
	(4.30)	(0.18)	(6.06)	(0.31)
$[501, 600]$	38.69***	-1.15	26.99	-0.56
	(3.63)	(0.17)	(6.54)	(0.36)
$[601, 700]$	38.29	-1.04	33.92	-0.90
	(9.47)	(0.58)	(6.95)	(0.37)
$[701, 800]$	41.49	-1.05	29.17	-0.41*
	(10.80)	(0.65)	(5.52)	(0.28)
$[801, 900]$	42.83*	-0.78	50.15*	-1.46*
	(5.73)	(0.23)	(6.67)	(0.30)

Table 7: This table reports OLS coefficient estimates of x_2 on $\sqrt{x_1}$ and x_1 without a constant. Standard errors in parentheses are clustered at the matching group level. Significance levels (*** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$) pertain to Wald tests of equality between estimated and predicted coefficients.

Table 7 shows coefficient estimates used to create the empirical best response fits which appear in figure 3. For each valuation bin in CA and I, the empirical best response fit is obtained using the coefficient estimates resulting from the regression $x_{2it} = \beta_1\sqrt{x_{1it}} + \beta_2 x_{1it} + \epsilon_{it}$, with standard errors clustered at the matching group level; the same specification is used for CS. The lack of "stars" in this table shows that the estimated best response is not significantly different than predicted in most cases.

D RE coefficient estimates of Δx_2 on v_2 in CA

	CA			*I*		
Δx_2	(1)	(2)	(3)	(4)	(5)	(6)
v_2	0.192***	0.195***	0.193***	0.133***	0.132***	0.131***
	(0.054)	(0.054)	(0.055)	(0.030)	(0.030)	(0.030)
RA		-12.187**	-12.178**		-3.176	-3.177
		(5.881)	(5.878)		(8.568)	(8.577)
Female		25.528	25.489		52.162*	52.175*
		(36.901)	(36.903)		(28.301)	(28.310)
t			-3.169**			-3.218*
			(1.296)			(1.754)
Constant	50.703*	137.076***	210.710***	58.143***	55.486	130.007
	(29.259)	(45.637)	(63.486)	(21.006)	(80.713)	(83.532)
N	420	420	420	540	540	540

Table 8: Columns (1)-(6) report random effects model coefficient estimates of Δx_2 on valuation, using player 2 data in rounds 16-30. Column (1) and (2) correspond to treatment CA, and (3) and (4) to treatment I. Standard errors in parentheses are clustered at the matching group level. Significance levels: *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$.

E Kernel densities for player 2's probability of winning

Figure 7 graphs the results of kernel density estimates (KDE) for player 2's probability of winning in CA (left) and I (right) by valuation level. Compared to the histograms in graph 4, KDE are smooth and independent of the choice of origin (or the chosen bins in a histogram). Therefore, KDE can be used as an additional check to visualize the distribution of the probability of winning for player 2. KDE present similar results to the histograms. At low valuations, there is a high chance of dropout. At high valuations, players invest in order to have a very high chance to win the prize, and at every valuation bin there is a high mass around the 60% chance. KDE also make it easier to see the shift of investment depending on valuations.

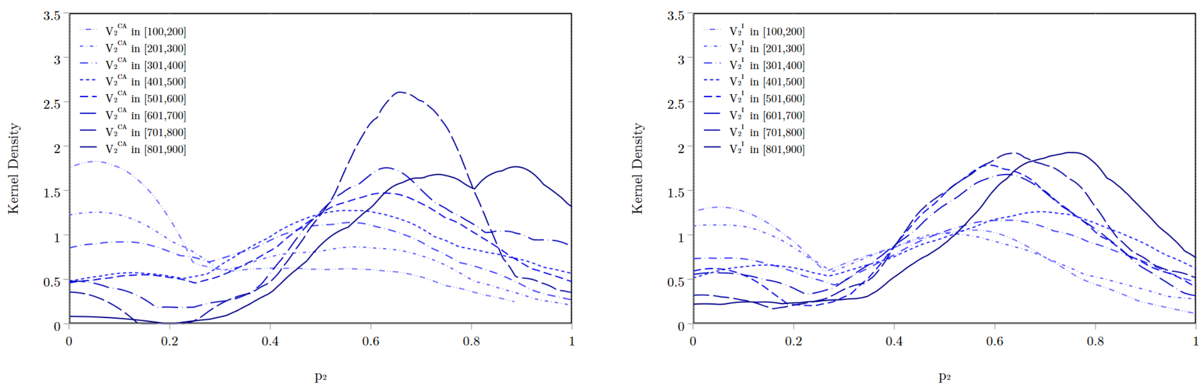

Figure 7: Kernel density estimates for CA (left) and I (right) across valuation bins for player 2.

F Player 2 observed and theoretical probability of winning

In this appendix, we test whether there are significantly more (or less) players 2's in the $[0, 0.1]$ and $[0.5, 0.6]$ probability bins than predicted by theory. Table 9 refers to the $[0, 0.1]$ and table 10 to the $[0.5, 0.6]$ probability bin. Note that in table 9 testing for differences in the $[701, 800]$ valuation bin for both CA and I and in $[801, 900]$ valuation bin for treatment I is not possible as there is not enough variation between the observed behavior and the theoretical prediction. The observed behavior and theoretical prediction are different only for a handful of observations which does not allow for significance testing. In fact, in the $[801, 900]$ valuation bin for treatment I the percent of observations in $[0, 0.1]$ matches exactly the percent predicted by theory.

	CA		I	
$[\underline{v}, \overline{v}]$	Obs.	Theory	Obs.	Theory
$[100, 200]$	54%	82%	39%	60%
	-28%***		-21%**	
	(0.000)		(0.038)	
$[201, 300]$	42%	60%	38%	45%
	-18%		-7%	
	(0.115)		(0.534)	
$[301, 400]$	30%	46%	26%	36%
	-16%*		-10%*	
	(0.074)		(0.093)	
$[401, 500]$	17%	31%	12%	20%
	-14%**		-8%	
	(0.030)		(0.244)	
$[501, 600]$	13%	23%	14%	17%
	-10%		-3%	
	(0.346)		(0.605)	
$[601, 700]$	11%	19%	15%	18%
	-8%		-3%	
	(0.149)		(0.646)	
$[701, 800]$	7%	7%	7%	7%
	0%		0%	
	(1.000)		(1.000)	
$[801, 900]$	2%	4%	5%	5%
	-2%		0%	
	(0.304)		(-)	

Table 9: This table reports OLS mean comparisons of the percent of player 2's who bid such that their probability of winning is less or equal to 0.1. p-values in parentheses, standard errors are clustered at the matching group level. Significance levels (*** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$).

$[\underline{v}, \overline{v}]$	*CA* Obs.	*CA* Theory	*I* Obs.	*I* Theory
$[100, 200]$	11%	0%	16%	5%
	11%***		11%	
	(0.001)		(0.137)	
$[201, 300]$	20%	2%	18%	7%
	18%***		11%*	
	(0.002)		(0.086)	
$[301, 400]$	13%	5%	21%	10%
	8%*		11%*	
	(0.050)		(0.090)	
$[401, 500]$	26%	6%	22%	11%
	20%***		11%*	
	(0.000)		(0.095)	
$[501, 600]$	17%	11%	22%	14%
	6%		8%	
	(0.374)		(0.136)	
$[601, 700]$	25%	11%	24%	6%
	14%**		18%***	
	(0.011)		(0.003)	
$[701, 800]$	22%	7%	27%	9%
	15%**		18%**	
	(0.039)		(0.018)	
$[801, 900]$	15%	17%	16%	18%
	-2%		-2%	
	(0.779)		(0.880)	

Table 10: This table reports OLS mean comparisons of the percent of player 2s' who bid such that their probability of winning is between 0.5 and 0.6, inclusive. p-values in parentheses, standard errors are clustered at the matching group level. Significance levels (*** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$).

G Bidding functions across player types for CA and I

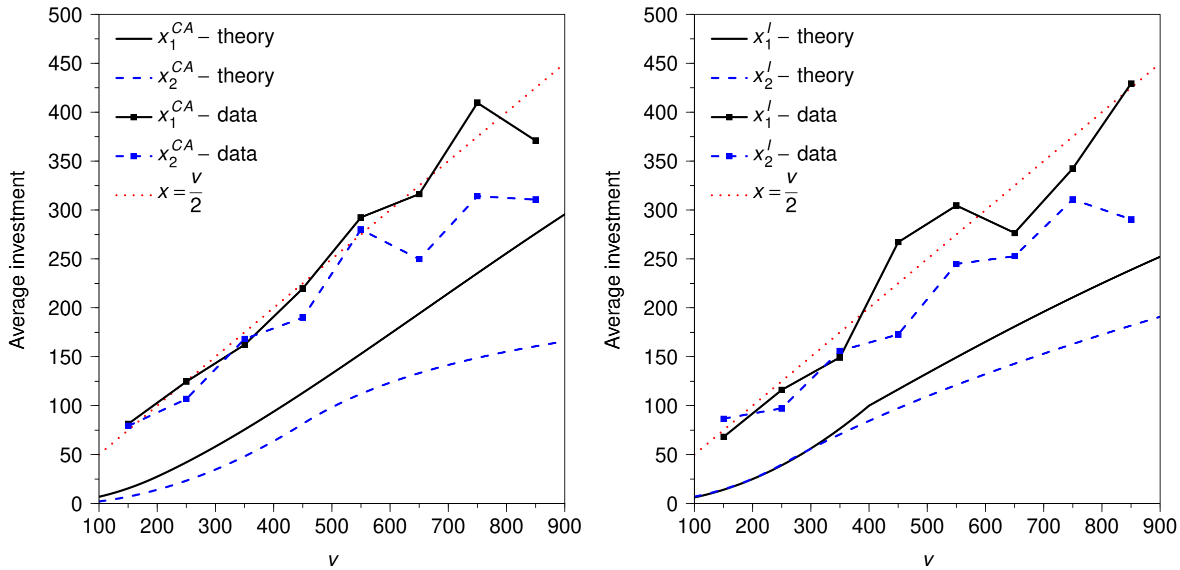

Figure 8: Theoretical (solid curves) and observed (connected squares) average investment functions in treatment CA (left) and I (right) across player types. The dotted red line represents investment levels equal to one half of the valuation, $x = \frac{v}{2}$.

H Experimental instructions

In this section we copy the main instructions for the complete and asymmetric (CA) information treatment. In the footnotes we describe the different wording used in the complete and symmetric (CS) treatment and the incomplete information (I) treatment.

Instructions

All amounts in this part of the experiment are expressed in points. The exchange rate is 200 points = $1.

This part of the experiment consists of a sequence of 30 decision rounds.

Matching and roles

At the beginning of this part of the experiment, you will be randomly assigned one of the following roles: Participant 1 or Participant 2. You will remain in your assigned role for all 30 rounds. At the beginning of each round, you will be randomly matched with another participant who does not share the same role as you. That is, if you are Participant 1 you will always be matched to a Participant 2, and if you are Participant 2 you will always be matched to a Participant 1.

Values

In each round, you will be randomly assigned a value. Your value can be any integer number of points between 100 and 900 (100 and 900 inclusive). Each number is equally likely to be drawn. The other participant you are matched with will also be randomly assigned a value between 100 and 900. These values are drawn independently and can be different from yours. You will know your value and the value of the other participant you are matched with.[26]

Endowment and investment

In each round, you will be given an endowment of 1000 points. You can invest any number of these points from 0 to 1000 into a project. Any points you do not invest, you get to keep. The project can either succeed or fail. If your project succeeds, you will receive a number of points equal to your value. If your project fails, you will not receive any revenue for the round.

[26] In the CS treatment the text of the "values" section reads "In each round, you will be assigned a value of 500 points. The other participant you are matched with will also be assigned a value of 500 points each round." In the I treatment the text of the "values" section reads "In each round, you will be randomly assigned a value. Your value can be any integer number of points between 100 and 900 (100 and 900 inclusive). Each number is equally likely to be drawn. The other participant you are matched with will also be randomly assigned a value between 100 and 900. These values are drawn independently and can be different from yours. You will know your value, but not the value of the other participant."

Timing of investments within a round

In each round, there are two stages: Stage 1 and Stage 2.

<u>Stage 1</u>

In this stage, only Participant 1 will make their investment decision. Participant 2 will wait until Participant 1 is through making their decision.

<u>Stage 2</u>

In this stage, only Participant 2 will make their investment decision, while Participant 1 waits. At the time of making their decision, Participant 2 will be informed about Participant 1's investment decision from Stage 1.

What is the likelihood that your project succeeds?

After both participants have made their investment decisions, the outcome of your project will be determined. Only one participant can have a successful project: either you or the other participant you are matched with. The **probability that your project succeeds** is given by:

$$\frac{\text{Your investment}}{\text{Your investment} + \text{Other participant's investment}}$$

For example, if you invested 100 points and the other participant you are matched with invested 300 points, then the probability that your project succeeds is

$$\frac{100}{100+300} = \frac{100}{400} = \frac{1}{4} = 25\%$$

and the probability that the other participant's project succeeds is

$$\frac{300}{300+100} = \frac{300}{400} = \frac{3}{4} = 75\%.$$

If both participants make an investment of 0 points, then each participant's project has a 50% probability of success.

Payoff calculation

After determining the probability that your project succeeds, the software program will randomly determine whether your project succeeds or not, according to the calculated probability.

Your **individual payoff** for the round is determined as follows:

If your project succeeds:	If your project fails:
$+1000$ (endowment)	$+1000$ (endowment)
$+Value$ (your value)	$+0$ (no revenue)
$-Investment$ (points you invested)	$-Investment$ (points you invested)
$1000 + Value - Investment$	$1000 - Investment$

Feedback at the end of each round

At the end of each round, you will be provided with the following feedback: you and the other participant's project investment, success and outcome, and your individual payoff.

How are your earnings from this part determined?

You will participate in a series of 30 decision rounds. At the end of the series, **3** of these rounds will be chosen randomly (with all rounds being equally likely to be chosen). At the end of the experiment, you will be informed about which three rounds were chosen and your payoff from each of those three rounds. Then, your earnings from this part will be the sum of your payoffs from the three randomly selected rounds.