The Kőszegi-Rabin Expectations-based Model and Risk-apportionment Tasks for Elicitation of Higher Order Risk Preferences

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June 25, 2024

Abstract

This paper examines the predictions of expectations-based reference-dependent models for risk-apportionment tasks that elicit higher-order risk attitudes. We consider some of the most commonly used specifications of Kőszegi and Rabin (2006,2007) and disappointment aversion models. Our analysis reveals that higher order risky choices exhibited by decision makers defined by those model specifications depend on whether risks to be apportioned in these tasks are symmetric or asymmetric, whether they include small probability outcomes, and on the level of loss aversion. We highlight that some of the predicted choice behaviour in the risk-apportionment tasks differs from the ones in alternative models of decision under risk. We employ experimental data to examine whether choice patterns in the risk apportionment tasks are in line with the predictions of the model specifications described here. We find that only a small proportion of them are consistent with those predictions.

Keywords: higher-order risk preferences, reference-dependent preferences, prudence, temperance, decision under risk.

JEL codes: D81, C91

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Ivan Paya acknowledges financial support from Conselleria de Innovación, Universidades, Ciencia y Sociedad Digital de la Generalitat Valenciana (CIPROM/2021/060).

1 Introduction

Since Markowitz (1952) first postulated that the utility function should be defined in terms of gains and losses relative to "customary wealth," and Kahneman and Tversky (1979) combined utility over gains and losses with probability weighting, many models within the decision theory literature include reference-dependent preferences. A few examples include cumulative prospect theory (CPT) (Tversky and Kahneman, 1992), rank dependent utility (Quiggin, 1981, 1982), disappointment models (see Bell, 1985; Loomes and Sugden, 1986; Gul, 1991; Delquié and Cillo, 2006), regret theory (Bell, 1982; Loomes and Sugden, 1982), and salience theory (Chetty et al., 2009; Bordalo et al., 2012). Models with reference-dependent preferences have become influential in decision theory and behavioural economics. These include models with reference points that are deterministic or stochastic, prospect or choice specific, and that use linear or nonlinear probability weighted functions. Significant progress has been made in the modelling and interpretation of empirical and experimental evidence in areas such as finance (e.g. Benartzi and Thaler, 1995; Chen, 2016 ch.8 and 9; Wang et al., 2017), risk taking (e.g. Bordalo et al., 2012; Loomes and Sugden, 1982), consumer choice (e.g. Heidhues and Kőszegi, 2014; Munro and Sugden, 2003), labour supply (Camerer et al., 1997; Farber, 2008; DellaVigna et al., 2017), sports (Pope and Schweitzer, 2011; Allen et al., 2017), and health (e.g. Bleichrodt et al., 2001, Bleichrodt et al. 2003).

Kőszegi and Rabin (2006, 2007) contributed to this literature with the development of two expectations-based reference-dependent models, the *choice-acclimating personal equilibrium model* (CPE) and the *preferred-personal equilibrium* (PPE) model. These models were later applied to a number of issues in economics. For instance, Pagel (2015, 2017) applied the Kőszegi and Rabin framework to life-cycle consumption phenomena and asset pricing, Balzer and Rosato (2021) to the analysis of bidding in auctions, and Herweg et al. (2010) to contract theory. Furthermore, Abeler et al. (2011) and Crawford and Meng (2011) find evidence consistent with Kőszegi and Rabin in the context of labour supply, while Gill and Prowse (2012) find supporting evidence in tournaments, and Card and Dahl (2011) on human conflict. Alternative empirical evidence does not find a significant role for rational expectation-based reference points in general (e.g. Allen et al., 2017)) or the the Kőszegi and Rabin preferences in particular (e.g. Barseghyan et al., 2013; Heffetz and List, 2014). Additionally, the supporting evidence previously found can also be consistent with other reference-dependent models (see Baillon et al., 2020).

Despite recent attention to expectations-based reference-dependent models in general, and the Kőszegi and Rabin specification in particular (see e.g., in addition to the references in previous paragraph, Cerulli-Harms et al. 2019; Freeman, 2017,2019; Knetsch and Wong, 2009; Meisner and von Wangenheim 2023; O'Donoghue and Sprenger, 2018; Sprenger, 2015), an area where their properties have not yet been examined is risk apportionment tasks for elicitation of risk attitudes of higher order. Theoretical research has demonstrated the important role that higher-order risk preferences, particularly *prudence* and *temperance*, play in economic models of risky choice such as health, precautionary savings, asset pricing, contests, auctions and several other (see Trautmann and van de Kuilen (2018) for a review of this literature). Eeckhoudt and Schlesinger (2006) demonstrated how these higher order preferences could be elicited in experimental research on the basis of the choices between particular lottery pairs -the so called risk-apportionment tasks, that we consider here. Subsequently, a number of studies have utilised this experimental design in order to elicit the higher order risk preferences of decision makers (e.g. Maier and Rüger, 2012; Deck and Schlesinger, 2010; 2014; and 2018, Ebert and Wiesen, 2014; Noussair et al., 2014; Heinrich and Mayrhofer, 2018; Brunette and Jacob, 2019; and Bleichrodt and van Bruggen, 2022). While empirical and experimental studies about higher order risk attitudes have been growing over the last two decades, the literature on the predictions of risky choice models on behaviour in these choice tasks to elicit higher order risk preferences is still scarce (see Eeckhoudt and Schlesinger, 2006; Deck and Schlesinger, 2014; Paya et al., 2023; Georgalos et al., 2023). There is no research to date which, within the context of risk-apportionment tasks for elicitation of higher-order risk attitudes, theoretically examines choice behaviour assuming expectations-based reference dependent models such as the ones developed by Kőszegi and Rabin or disappointment aversion models.[1] That is the purpose in this paper.

We examine the utility associated with choices in risk-apportionment tasks that elicit higher-order risk preferences for the most typically employed CPE model specification of Kőszegi and Rabin and for the disappointment aversion model, which we will denote as KR and DA, respec-

[1]Baillon et al. (2020) empirically identify which reference point decision makers use. Allowing for a number of alternative reference points, they report that up to 20% of the risky choices were consistent with expectations-based reference point models. The CPE model of Kőszegi and Rabin is included as only one of those models. However, the lottery choices in that study were all designed to elicit second order risk preferences.

tively. We are able to demonstrate the new findings that a KR decision maker (DM) exhibits lottery choices consistent with prudence, imprudence or prudence neutrality, depending on the symmetry of the risk to be apportioned in the risk-apportionment tasks and on whether those prospects include small-probability outcomes. For instance, for the widely-used elicitation tasks with symmetric risk, a KR DM would make choices consistent with prudence neutrality. With respect to risk attitude of order four, a loss averse KR DM exhibits intemperance, although her choices would imply the reverse preference if she was gain seeking. We also obtain that the predictions under symmetric zero-mean risk are the same for the PPE model specification of Kőszegi and Rabin. Likewise, we demonstrate choice behaviour for those elicitation tasks within the DA specification. The predictions are in most cases similar to the ones for KR preferences, although, they can differ, such as when the lotteries accommodate small probability outcomes. Finally, our findings into the risk profile of a DM are employed to examine whether experimental data is in line or not with preferences exhibited by a KR or DA DMs. We find that a relatively small proportion, a maximum of 13%, of experimental subjects choosing risk apportionment tasks covering all the cases examined in this paper are consistent with KR and DA.

The reminder of the paper proceeds as follows. In Section 2 we describe the specifications of the Kőszegi and Rabin and disappointment aversion expectations-based reference dependent models we examine in the paper. In Section 3 we first set out the method of elicitation of higher order risk preferences considered here. We subsequently present the analyses and predictions regarding third and fourth order risky choices for the previously presented model specifications. In Section 4 we contrast the predictions set out in this paper with the ones of alternative risky choice models. This section also presents an experimental analysis of those predictions employing data from three prominent studies. Finally, Section 5 presents some concluding remarks.

2 Expectations-based reference dependent models

In reference-dependent models, choices over risky outcomes are evaluated from a reference point. However, we will employ the term 'referent' rather than reference point, and we will denote it by R, to describe the location around which losses and gains are defined. This is helpful because it emphasises that, within the Kőszegi and Rabin framework, there is not a single reference point.

In their approach the referent is prospect specific (i.e. can differ between the prospects in the choice set), hence it is stochastic and based on expectations of the entire distribution of expected outcomes.

We follow the review paper of O'Donoghue and Sprenger (2018) to introduce expectations-based reference-dependent models in general, and the ones of Kőszegi and Rabin and disappointment aversion, in particular. Consider prospect L with payoffs $x_1, x_2, ..., x_N$ and corresponding probabilities $p_1, p_2, ..., p_N$, where $\sum_{n=1}^{N} p_n = 1$, and is represented by $L \equiv (p_1 : x_1, p_2 : x_2, ..., p_N : x_N)$. For the general case of prospects over N potential outcomes, the DM evaluates lottery L according to the following preferences

$$U(L|R) \equiv \sum_{n=1}^{N} p_n \left[u\left(x_n\right) + v\left(x_n|R\right)\right], \tag{1}$$

where $u\left(x_n\right)$ denotes the intrinsic utility from outcome x_n, and $v\left(x_n|R\right)$ denotes the gain-loss utility associated with getting outcome x_n given referent R. Kőszegi and Rabin assume that the comparisons of gains and losses are aggregated by the DM using the weights that correspond to the probabilities in the reference lottery R.

To model the determination of expectations-based reference points within the context of preferences described in (1), Kőszegi and Rabin introduced three solution concepts: personal equilibrium (PE), preferred personal equilibrium (PPE) and choice-acclimating personal equilibrium (CPE). A choice will constitute a PE if the DM would want to make it given that she expects it.

Definition 1 *Given a choice set $\mathcal{L}$, a lottery $L \in \mathcal{L}$ is a personal equilibrium (PE) if $U(L|L) \geq U(L'|L)$ for all $L' \in \mathcal{L}$.*

There may be a multiplicity of PE within choice set $\mathcal{L}$. Since the DM can decide which of those to choose it is reasonable to assume the DM would make the choice that yields the maximum utility ex-ante and that will constitute the PPE.

Definition 2 *Given a choice set $\mathcal{L}$, a lottery $L \in \mathcal{L}$ is a preferred personal equilibrium (PPE) if it is a PE and if for any other PE L' it holds that $U(L|L) \geq U(L'|L')$.*

The third equilibrium concept, CPE, uses the criteria used in the PPE except that the DM does not restrict herself to choose among PE alternatives.

Definition 3 *Given a choice set $\mathcal{L}$, a lottery $L \in \mathcal{L}$ is a choice-acclimating personal equilibrium(CPE) if it holds that $U(L|L) \geq U(L'|L')$ for all $L' \in \mathcal{L}$.*

Kőszegi and Rabin motivate the use of the three alternative equilibrium concepts according to the time elapsed between the choice and the resolution of the uncertainty. They argue that PE and PPE are appropriate for decisions where the DM commits to the choice made only shortly before the uncertainty is resolved. However, CPE is applicable when the DM commits well in advance to the resolution of uncertainty. The CPE concept is arguably more tractable and less complicated to work with (see Baillon et al., 2020 and a review of applications of the Kőszegi and Rabin models in O'Donoghue and Sprenger, 2018). Moreover, the results we obtain are qualitatively similar for the case of symmetric risk, which was the only case within our framework where the PPE was unique. For those reasons, in the paper, we will focus on the CPE model, and we will make the analysis with the PPE model available in an appendix.

As with any other risky choice decision model, there are in principle a wide range of model specifications that could be used. We will employ the ones used by Kőszegi and Rabin and in most of the literature that have followed from their work (e.g. Cerulli-Harms et al. 2019; Herweg et al. 2010; Sprenger, 2015; Pagel, 2015, 2017). In particular, they assume that preferences are linear in probabilities, that intrinsic utility is linear, i.e. $u(x_n) = x_n$, and that the reference-dependent gain-loss utility $\mu(\cdot)$ is two-part linear where gains and losses are defined in terms of intrinsic utilities with parameter λ capturing the degree of loss aversion when comparing gains to losses and that it is assumed to be $\lambda > 1$.

$$v(x|R) = \mu\left(u(x) - u(R)\right) \text{ where } \mu(z) = \begin{cases} z \text{ for } z \geq 0 \\ \lambda z \text{ for } z < 0 \end{cases} \tag{2}$$

We therefore characterise the implications of loss aversion without diminishing sensitivity, as presented in Kőszegi and Rabin (2007).[2] We further assume, as it is typically done in experimental work, that the subject's initial wealth remains constant and final wealth is equated with outcome (e.g. Sprenger, 2015; Cerulli-Harms et al. 2019; Baillon et al., 2020).

[2] $\mu(\cdot)$ satisfied that $\mu(0) = 0$. It is also common to scale the two-part linear gain-loss utility (2) by a parameter η, where η would represent the weight attached to gain-loss utility and it is the same for all dimensions of consumption. However, given that η would not play any significant role in our derivations we have omitted it in our analysis. We also note that we will discuss below cases where diminishing sensitivity in the functional form $\mu(\cdot)$ is taken into consideration.

Following from (1), (2), and Definition 3, the CPE preferences (denoted as KR hereafter) become

$$\begin{aligned} U(L|L) &\equiv \sum_{n=1}^{N} p_n \left[u\left(x_n\right) + v\left(x_n|L\right) \right] \\ &\equiv \sum_{n=1}^{N} p_n \left[u\left(x_n\right) + \sum_{m=1}^{N} p_m \mu\left(u\left(x_n\right) - u\left(x_m\right)\right) \right] \\ &\equiv \sum_{n=1}^{N} p_n u\left(x_n\right) + \sum_{n=1}^{N} \sum_{m=1}^{N} p_n p_m \mu\left(u\left(x_n\right) - u\left(x_m\right)\right). \end{aligned} \tag{3}$$

The Kőszegi and Rabin models mentioned above are close to models of disappointment aversion, some of them introduced two decades earlier (see Bell, 1985; Loomes and Sugden, 1986; Gul, 1991; Delquié and Cillo, 2006). We note that the model of Delquié and Cillo (2006) is like the CPE model discussed above. Masatlioglu and Raymond (2016) discuss the link between this model and models of disappointment aversion and EUT. From the perspective of our analysis, the relevant difference between KR and disappointment aversion models is the way the referent is specified.

Disappointment aversion models assume that the lottery outcomes (x_n) are compared to a single summary statistic from the reference lottery R. In particular these models assume $v\left(x_n|R\right) = \mu\left(u\left(x_n\right) - \sum_{m=1}^{N} p_m u\left(x_m\right)\right)$.[3] Assuming utility is linear, $u\left(x_n\right) = x_n$, the intrinsic utility of the outcomes is compared to the mean of the reference lottery (L), and the preferences (denoted as DA hereafter) become

$$U(L|L) \equiv \sum_{n=1}^{N} p_n u\left(x_n\right) + \sum_{n=1}^{N} p_n \mu\left(u\left(x_n\right) - \sum_{m=1}^{N} p_m u\left(x_m\right)\right). \tag{4}$$

[3] We note that in Gul's (1991) formulation, the referent is the certainty equivalent of the lottery including disappointment, while in Bell (1985) and Loomes and Sugden (1986) the referent is effectively the certainty equivalent using only intrinsic utility.

3 Experimental elicitation of higher-order risk preferences using risk apportionment tasks and expectations-based reference dependent models

The original definitions of higher order preferences were based on utility functions. The term prudence was coined by Kimball (1990) and derived from the importance of the third derivative of utility in determining demand for precautionary savings. Kimball et al. (1992) introduced the concept of temperance relating the fourth derivative of the utility function to behavioural aspects of investors. Within the EUT framework, the risk attitude of prudence is equivalent to an aversion to increases in downside risk as defined by Menezes et al. (1980), and the risk attitude of temperance is equivalent to an aversion to an increase in outer risk (Menezes and Wang, 2005).

Eeckhoudt and Schlesinger (2006) provide a behavioural model-free definition of higher order risk preferences through simple lottery pairs. Experimental work has, since then, mainly employed the link between observable lottery choices in the suggested risk-apportionment tasks and higher order risk attitudes to assess the risk profile of DMs.[4] We now introduce the lottery forms most commonly used in experimental research —the so-called risk-apportionment lotteries—to elicit third and fourth order risk preferences (see Eeckhoudt and Schlesinger, 2006; Deck and Schlesinger, 2010, 2014; Maier and Rüger, 2012; Ebert and Weisen, 2014; Noussair et al., 2014; Heinrich and Mayrhofer, 2018; Brunette and Jacob, 2019; Bleichrodt and van Bruggen, 2022). In subsequent subsections we will specify various forms of lottery structures that have been employed to elicit higher order risk preferences accounting for issues such as asymmetric risks (Ebert and Wiesen, 2014), small probabilities (Bleichrodt and van Bruggen, 2022), or equal moments other than the order of the preference being examined (Ebert and Wiesen, 2011; Bleichrodt and van Bruggen, 2022).

Lottery $[x, y]$ denotes outcomes x and y are each received with probability 0.5. A zero-mean binary random variable with two outcomes of opposite sign and equal absolute amount e is denoted as $\widetilde{\varepsilon}$, i.e, $\widetilde{\varepsilon} = [e, -e]$. The DM is endowed with the preference relation $\succeq$. The lottery designed to elicit prudence is $B_3 = [x, c + \widetilde{\varepsilon}]$, i.e., outcome x is received with probability 0.5, outcome $c + e$

[4] Higher-order preferences have also been measured using alternative approaches to the one of risk apportionment tasks. For instance, directly through elicitation of utility functions and certainty equivalents, or indirectly through financial and economic decisions or via survey measures. This will be discussed in more detailed in Section 4 below.

with probability 0.25, and outcome $c - e$ with probability 0.25. We note that $c > x > e$. The lottery designed to elicit the reverse preference of imprudence is $A_3 = [c, x + \widetilde{\varepsilon}]$. The risk preference of prudence, $B_3 \succeq A_3$, is determined by whether the zero-mean risk $\widetilde{\varepsilon}$ is allocated (apportioned) to the state of higher wealth (c). Therefore, the DM has in this case a preference for combining 'good' with 'bad' and disaggregating the 'harms', as in B_3, rather than combining 'bad' with 'bad' as in A_3, where the zero-mean risk $\widetilde{\varepsilon}$ is allocated to the state of lower wealth (x).[5]

The risk preference of temperance, consistent with risk apportionment of order four, is a preference for disaggregating two independent zero-mean risks, $\widetilde{\varepsilon_1}$ and $\widetilde{\varepsilon}_2$. The lottery pair designed to elicit such risk preference is $B_4 = c + [\widetilde{\varepsilon_1}, \widetilde{\varepsilon_2}]$ and $A_4 = c + [0, \widetilde{\varepsilon_1} + \widetilde{\varepsilon_2}]$, where $\widetilde{\varepsilon_1}$ and $\widetilde{\varepsilon}_2$ are two independent zero-mean risks. A DM is temperate if she prefers to combine the relatively 'good' and 'bad' outcomes (B_4) instead of combining the two 'bad' outcomes together (zero-mean risks $\widetilde{\varepsilon}_1$ and $\widetilde{\varepsilon}_2$) as it is the case in A_4. Hence, temperance implies $B_4 \succeq A_4$. Intemperance implies the reverse preference.

The analytical derivations of the utility of each lottery from their corresponding referent are presented in the appendices. In particular, Appendix A provides the calculations related to KR's predictions about choices in the risk-apportionment tasks of order 3 and 4. Likewise, Appendix B presents the calculations for DA, while Appendix C shows the ones related to the PPE specification of Kőszegi and Rabin.

3.1 Analysis of third order risk preferences

In this section we consider various forms of lottery pairs, introduced by Eeckhoudt and Schlesinger, 2006; Ebert and Weisen, 2014; and Bleichrodt and van Bruggen, 2022, that have been employed in the experimental literature to elicit prudence. First we consider the lottery pair B_3 and A_3, and find that the value of both lotteries for KR is equal, hence the DM is indifferent between them (see expressions (5) and (6) where the values of $U(B_3|B_3)$ and $U(A_3|A_3)$ have been obtained). Therefore, the first prediction is that a KR DM would exhibit prudent neutral choice when the zero-mean risk

[5] The proposed '*combining good with bad*' framework by Eeckhoudt et al. (2009) assumed risk averse preferences, hence the terms 'good', 'bad', and 'harm' relate to that behavioural trait. Crainich et al. (2013) extended this framework to risk seekers and kept the same terminology to examine risk lovers' utility function. See Deck and Schlesinger (2014) for a further discussion of the '*combining good with bad*' framework, models of EUT that can accommodate such behaviour, as well as experimental evidence on higher order risk attitudes. We also note that, within an EUT framework, lottery A_3 can also be interpreted as a downside risk increase of lottery B_3 (see Menezes et al., 1980), and lottery A_4 described in the next paragraph is an outer risk increase of lottery B_4 (see Menezes and Wang, 2005).

to be apportioned is symmetric.[6] The prediction is the same under the PPE model approach (see Appendix C). This prediction also holds for DA (see expressions (20) and (21) in Appendix B).

PREDICTION 1: *Assuming preferences are determined by either KR or DA, the DM would be indifferent between lotteries B_3 and A_3, therefore implying prudence neutrality.*

This is the first model we are aware of that, whilst being consistent with aversion to risk, it implies neutrality in risk apportionment of order 3. This prediction differs from the most commonly used models of decision under risk whose choice behaviour in risk-apportionment tasks for eliciting higher order risk attitudes have been previously examined, such as CPT or EUT (see Eeckhoudt and Schlesinger, 2006, Deck and Schlesinger, 2014, Paya et al., 2023, and Bleichrodt and van Bruggen, 2022).

Whilst the lottery pair considered above has been widely used in previous empirical studies to elicit risk apportionment of order 3, other forms of lottery pairs have been employed to account for skewed risk, small probabilities or higher order even moments. We now examine whether any of those forms impact on the prediction made above.

First, we consider a modification to the design of B_3 and A_3 introduced by Ebert and Wiesen (2014) to account for asymmetric risks. In this case, the zero-mean risk $\widetilde{\varepsilon}'$ is skewed rather than symmetric, where outcome $-e$ is obtained with probability q, and outcome $\frac{q}{1-q}e$ is obtained with probability $1-q$, that is, $\widetilde{\varepsilon}' = \left(1-q : \frac{q}{1-q}e, q : -e\right)$. The risk preference exhibited by the DM will depend on the skewness of the risk and the lottery structure. For positively skewed risk, i.e., $q > 0.5$, the loss averse ($\lambda > 1$) KR and DA DMs will make the prudent lottery choice (see expressions 7, 8, 22 and 23), although in some specific case they will be indifferent between the lottery pair (see expressions (9) and 24). For negatively skewed risk, the KR and DA DMs will make the imprudent lottery choice (see expressions 11, 12, 26, 27), although in a specific case they would be indifferent between B_3 and A_3 (see expressions 10 and 25).

This feature of the model contrasts with the fact that a change in the asymmetry of the background risk does not alter the choice within the elicitation task of order 3 in other models such as

[6] We note that this is the case regardless of the functional form of the gain-loss utility. This restriction could therefore make a difference in terms of the preferences exhibited by the KR DM if departures from linear intrinsic utility were considered. The first term in (3) would then determine the risk preference exhibited by the DM. For instance, under power utility with exponent less than unity the first term in (3) would imply that prudence would prevail in this case. This remark also applies to subsequent results where the prediction holds irrespective of the functional form of $\mu(\cdot)$.

the CPT and EUT models indicated above.

PREDICTION 2: *The lottery pair B_3 and A_3 is defined such that the zero-mean risk is asymmetric,* $\widetilde{\varepsilon}' = \left(1-q : \frac{q}{1-q}e, q : -e\right)$.

If $q > 0.5$, *a loss averse KR DM would choose lottery B_3 over A_3 except when* $e < (c-x)$ *and* $\frac{q}{1-q}e < (c-x)$ *that she would be indifferent between them. If* $q < 0.5$, *a loss averse KR DM would choose lottery A_3 over B_3 except when* $e < (c-x)$ *that she would be indifferent between them.*

If $q > 0.5$, *a loss averse DA DM would choose lottery B_3 over A_3 except when* $e < 0.5\,(c-x)$ *and* $\frac{q}{1-q}e < 0.5\,(c-x)$ *that she would be indifferent between them. If* $q < 0.5$, *a loss averse DA DM would choose lottery A_3 over B_3 except when* $e < 0.5\,(c-x)$ *that she would be indifferent between the lottery pair.*

The second type of lottery pairs we consider is the one recently introduced by Bleichrodt and van Bruggen (2022). They extend the lottery design previously used in the literature to measure higher order risk preferences such that small probabilities could be accounted for. To do so, they use the lottery $(p : x, p : y, 1-2p : z)$ to denote outcomes x and y are given with probability p each, and outcome z with probability $1-2p$. This lottery can be rewritten as $L = (2p : [x, y], 1-2p : z)$ (see Bleichrodt and van Bruggen, 2022, Appendix C). Within this lottery structure, the lottery that elicits prudence in their small probability gain treatment is $B_{3,sp} = (2p : [x, c+\widetilde{\varepsilon}], 1-2p : z)$, that is, outcome x is given with probability p, outcome $c+e$ with probability $\frac{p}{2}$, outcome $c-e$ with probability $\frac{p}{2}$, and outcome z with probability $1-2p$. The lottery that elicits the reverse preference is $A_{3,sp} = (2p : [c, x+\widetilde{\varepsilon}], 1-2p : z)$. Constants c, x, e, z are such that $c > x > e > z$. The lottery outcomes will fall within the gain or loss domain depending on whether c is larger or smaller than $x+e$. Our calculations in the appendix reveal that both cases yield the same qualitative result. We also assume that $x > e+z$ since Bleichrodt and van Bruggen (2022) argued that payoff z is considered to be small relative to the other payoffs in the lottery, although in Appendix A we comment on the result if this assumption does not hold. We note that stochastic dominance preferences imply risk apportionment of any order not only for the 50-50 lotteries discussed above but also for these extended definitions to account for small probabilities (see Bleichrodt and van Bruggen (2022) for a detailed discussion on this issue).

This lottery pair, $B_{3,sp}$ and $A_{3,sp}$, has the property that the central moments of mean and variance are equal, skewness is always greater in $B_{3,sp}$ and kurtosis can be higher or lower in $B_{3,sp}$. Our derivations presented in Appendix A demonstrate that the value of both lotteries under KR are equal (see expressions (13)-(16)), and the DM is indifferent between the them.[7] DA does not, in this case, yield an unambiguous prediction, and the lottery choice exhibited by the DM will depend on the lottery payoff structure (see expressions (28) and (29)). However, a restricted version of this type of lottery pairs to elicit prudence provides a specific prediction. In particular, if condition $z = \frac{x+c}{2}$ is imposed, the first, second and fourth central moments of the two lotteries are equal, whilst skewness is positive in the lottery consistent with prudence and skewness seeking, and is negative in the lottery consistent with the reverse preference, with the same absolute amount.[8] The utilities of these two lotteries with equal kurtosis labelled $B_{3,sk}$ and $A_{3,sk}$, are also presented in Appendix B. Expressions (30), (31) reveal that the DA DM would, in both cases, be indifferent between the two lotteries.

PREDICTION 3: *The lottery pair to elicit risk apportionment of order 3 that account for small probabilities outcomes is $B_{3,sp}$ and $A_{3,sp}$, and the KR DM would be indifferent between them. However, the choice of the DA DM depends on the lottery payoff structure. For the case that the lottery pair to account for small probabilities outcomes is designed to have the same first, second and fourth central moments, $B_{3,sk}$ and $A_{3,sk}$, the DA DM would be indifferent between the two lotteries.*

3.2 Analysis of fourth order risk preferences

In this section, we examine choice behaviour of the DM in risk apportionment tasks to elicit fourth order risk attitudes. For the case of two independent and symmetric zero-mean risks $\widetilde{\varepsilon_1} = [e_1, -e_1]$ and $\widetilde{\varepsilon}_2 = [e_2, -e_2]$, our analysis described in Appendix A shows that the prediction of the lottery choice made by a loss averse KR DM is intemperate, $A_4 \succeq B_4$. This is the case if either the size of the two risks is different ($e_1 > e_2$) or equal ($e_1 = e_2$) (see expressions (17), (18) and (19), respec-

[7]We note here that if diminishing sensitivity is taken into consideration, then computation reveals that the lottery choice exhibited by the DM depends on the functional form of the intrinsic utility function. For instance, let us consider the Tversky and Kahneman (1992) parameters for loss aversion ($\lambda = 2.25$) and coefficient of the power utility function ($\alpha = 0.88$), and with values of $p = 0.06$, $x = 44$, $c = 75$, $e = 32$, $z = 1$. In this case, the utility of each lottery is $U(B_{3,sp}) = -4.7208$ and $U(A_{3,sp}) = -4.7206$. Therefore the second term in (3) would imply imprudent choice. However, we now also need to consider the first term in (3) representing expected utility. Under power (exponent less than unity) utility function, this term would imply prudence, yielding, overall, a prudent lottery choice.

[8]We note that, theoretically, prudence implies skewness seeking, but not the other way around. Given that in this lottery pair, the fourth order moment is equal, the lottery choice is also a test for skewness seeking (see Ebert and Wiesen, 2011).

tively). DA implies the same risky choice for those tasks (see expressions (32) and (33)). Similarly, the prediction for the case of symmetric risk under the KR PPE model specification yields the same prediction (see Appendix C). The analytic solutions for the cases of small probability gains and asymmetric risk are complex and we have not included them in this analysis.

PREDICTION 4: *Assuming either KR or DA, the two independent symmetric zero-mean risks are either of equal or different size, the loss averse DM would choose lottery A_4 over B_4, $A_4 \succeq B_4$, therefore implying intemperance ((anti)risk apportionment of order 4).*

Prediction 4 differs from EUT models with power utility with exponent less than unity, EUT models of mixed risk aversion, and also from CPT models assuming the status quo reference point and lotteries in the gains domain (see Deck and Schlesinger, 2014; Paya et al., 2023; Bleichrodt and van Bruggen, 2022). It is also worth noting that, although this predicted lottery choice would not differ from the one made by an EUT mixed-risk lover (see Deck and Schlesinger, 2014), it would for the lottery choices eliciting risk apportionment of order 3, as stated in the previous subsection.

Prediction 4 hinges on the assumption of loss aversion, i.e., parameter $\lambda > 1$. Whilst there is overwhelming evidence supporting loss aversion in experimental research, some studies have reported that gain-seeking preferences could occur over smaller losses (e.g. Gal and Rucker, 2018; and Gächter et al., 2022) (see Wakker (2010) for a discussion of the concept of gain-seeking). If the DM was gain-seeking, implying $\lambda < 1$, the prediction would be reversed and the DM would make the temperate choice. This predicted link between loss aversion and fourth order risky choices is also a feature that could potentially characterise KR or DA from other alternative specifications such as CPT (see Paya et al., 2023).

4 Experimental Evidence

We have demonstrated the cases where, in experimental research relying on risk-apportionment tasks to elicit higher order preferences, a DM with expectations-based reference dependent preferences would make choices consistent with risk or (anti)risk apportionment of orders 3 and 4, depending upon the precise lottery pair structure employed in the experimental design. We summarise those results in Table 1 where we have also included the predictions of other commonly used models of decision under risk and uncertainty, in particular, EUT models of mixed risk-averse and mixed risk-loving, as well as CPT specifications under two alternative reference points (see Deck and Schlesinger, 2014, Paya et al., 2023, Bleichrodt and van Bruggen, 2022). For instance, in the first row, we report that a DM consistent with EUT mixed-risk-averse preferences would make prudent and temperate choices regardless of whether the zero-mean risk is symmetric or asymmetric. Likewise, a DM consistent with CPT preferences under the status quo reference point and symmetric zero-mean risk would make, over gains (third row), prudent and temperate choices, while, over losses (fourth row), prudent and intemperate ones. Therefore, the results in this paper underline a differential choice in experimental elicitation of higher-order risk preferences using risk apportionment tasks between preferences defined by KR, DA and alternative risky choice models. Below, we highlight experimental results reported in the literature on higher order risk attitudes and relate them to the predictions presented here.

The studies of Maier and Rüger (2012) and Deck and Schlesinger (2014) employ the 50-50 lottery pair B_3 and A_3 with symmetric risk described in Section 3 above. We note that experimental research employing these risk-apportionment tasks typically employs the number of choices made by DMs to classify them as prudent, imprudent or prudent neutral. Neutrality usually implies the number of choices of lottery B_3 a subject made in the experiment is around the mean of tasks designed to elicit third order risk preferences, which is equivalent to the behaviour of choosing B_3 with probability $\frac{1}{2}$ (see Deck and Schlesinger, 2014). The option of indifference between lotteries is not explicitly available. In practice, there is no statistical test carried out at the subject level with the null hypothesis of random choice or indifference. Therefore, experimental evidence suggesting neutrality in third, likewise fourth, order risk preferences should be taken with caution.[9] We will interpret such evidence as risky choice behaviour that might be in line with preferences exhibited by a particular preference specification, e.g. KR or DA, rather than explicitly *in favour* of those preferences.

For instance, Deck and Schlesinger (2014, p.1940) report that 35% of subjects were classified as prudent neutral. Maier and Rüger (2012, p20) reported "*Another large part of the subject pool is not distinguishable from being prudence neutral*."[10] Whilst there seems to be potentially a non-negligible

[9]Future research might shed light on this issue if, for instance, elicitation methods, experimental design and subject-level tests of random choice made it possible to explicitly test for neutral higher order risk preferences.

[10]Maier and Rüger (2012) do not provide a specific criterion to define prudent neutral subjects. Using the 'rule of

share of choices reported in these experimental studies that would be in line with the third order risky choice exhibited by KR or DA, we note that the predominant third order preference revealed in experimental studies is that of prudence as illustrated in the review paper by Trautmann and van de Kuilen (2018). This is also supported by studies that measure prudence employing methods that do not employ risk-apportionment tasks. For instance, Schneider and Sutter (2021) report predominantly prudent behaviour using a method based on the elicitation of utility points through the use of certainty equivalents with equally likely outcomes. An earlier study relying on direct elicitation through certainty equivalents is Tarazona-Gomez (2004) who find support for modest prudence. Another approach is to infer prudence via the precautionary savings motive based on EUT models (e.g. Dynan, 1993; Carroll and Kimball, 2008). Alternatively, the prevalence and intensity of prudence has been inferred indirectly from subjects' choices in financial and economic decisions such as auctions (Kocher et al. 2015), bargaining (Embrey et al. 2014), savings and investment tasks in the laboratory (Bostian and Heinzel, 2011; Xu et al. 2016), or asset markets (Huber et al. 2014).

The evidence in the literature regarding fourth order risk attitudes is, however, less clear cut. There is not a consensus view whether a preference for temperate over intemperate lotteries is prevalent for the majority of the population (see Trautmann and van de Kuilen 2018). In this regard, there is considerable experimental evidence that would correspond with the prediction of intemperate behaviour (e.g. Deck and Schlesinger 2010; Bleichrodt and van Bruggen 2022). However, the predictions outlined in our paper show that not all subjects making choices consistent with prudence neutral or intemperance in tasks designed to elicit risk apportionment would necessarily be consistent with KR and DA. Only those that exhibited risky choices consistent with the predictions regarding both third and fourth order risk apportionment tasks would be. Considering the predictions on both third and fourth order risky choices, the evidence for subjects that behave both as prudent neutral and intemperate as implied by the expectations-based reference-dependent models presented here finds weaker support. For instance, Deck and Schlesinger (2014) report that only 11 out of 150 subjects would be consistent with these two risk attitudes.[11] In the

thumb' of 25% of the choices, in their case, 7 out of 28, falling into the middle range, that is, subjects making between 11 and 17 prudent choices would be classified as prudent neutral, and by looking at their Figure 2, one would infer around 50% of subjects could be classified as prudent neutral.

[11]The study of Maier and Rüger (2012) does not provide the number of subjects at the intersection of third and fourth order risk preferences. Hence, one can only assume that there would be an upper bound to the proportion of subjects

next section, we examine this issue in more detail employing data from three experimental studies.

Table 1. Predicted choice in risk-apportionment tasks for models commonly used in decision theory under risk and uncertainty

Model	Ref. Point	Risk	Domain	3rd order	4th order
Expected Utility (Mixed Risk Averse)	-	Sym/Asym	G	P	T
Expected Utility (Mixed Risk Loving)	-	Sym/Asym	G	P	IT
Cumulative Prospect Theory (CPT)	SQ	Sym	G	P	T
			Ls	P	IT
			G(SP)	P	IT
		Asym	G	P	T/IT
			Ls	P	T/IT
	EV	Sym	G/Ls	P/IP	T/IT
		Asym	G/Ls	P/IP	T/IT
Kőszegi-Rabin (KR)	L	Sym	G/Ls	PN	IT
			G(SP)	PN	-
		Asym	G/Ls	P/IP/PN	-
Disappointment Aversion (DA)	EV	Sym	G/Ls	PN	IT
			G(SP/SK)	PN	-
		Asym	G/Ls	P/IP/PN	-

Notes: Column 2 specifies the reference point: status quo (SQ), expected value (EV), or the lottery outcomes (L). Column 3 describes whether the zero-mean risk is assumed to be symmetric (Sym) or asymmetric (Asym). Column 4 denotes the domain of lottery payoffs: gains (G), losses (Ls), small probability gains (G(SP)) or small probability gains with same kurtosis (G(SP/SK)). Column 5 reports the predicted risky choice of order 3: prudent (P), imprudent (IP), prudent neutral (PN). Column 6 reports the predicted risky choice of order 4: temperate (T), intemperate (IT). Assuming loss aversion parameter $\lambda > 1$.

classified as both risk neutral and intemperate that would correspond to the lower proportion of each of those two risk attitudes, which in their case would be around 15%.

4.1 Evidence from three experimental studies

We employ three experimental datasets designed to elicit higher-order risk preferences to examine the theoretical predictions set out in the previous section. The reason of using those datasets, namely the ones from Bleichrodt and van Bruggen (2022), Ebert and Wiesen (2014) and Heinrich and Mayrhofer (2018), is that, in addition to using the 50-50 probability lotteries with symmetric risk, the former study is the only one that employs small probability outcomes, and the latter two studies the only ones that use asymmetric zero-mean risks. With those three datasets we can therefore test all four predictions derived in section 3. In particular, we utilise the data from Bleichrodt and van Bruggen (2022) to jointly test the two predictions involving 50-50 probabilities, one about third order and the other one about fourth order, that is, Predictions 1 and 4, as well as the one involving small probability outcomes, Prediction 3. Data from Ebert and Wiesen (2014) and Heinrich and Mayrhofer (2018) will be used to jointly test predictions about third order choices with both symmetric and asymmetric risks and fourth order risky choices, that is, Predictions 2 and 4.

The Bleichrodt and van Bruggen (2022) dataset consists of three treatments, two involving gains and one involving losses which we will not use as we only focus on the gain treatments. These two gain treatments involve a between-subjects design with subjects being randomly allocated either to the 50-50 gain treatment, which involves the standard 50-50 risk apportionment tasks to elicit higher order risk preferences, or to the small probabilities treatment, which involves the small probability lotteries, as discussed in section 3.1. All those tasks are listed in Appendix D. There are in total data for 121 subjects in the 50-50 treatment and 124 subjects in the small-probabilities one. In each treatment subjects had to provide their choices in pairwise lotteries for all three higher order risk preferences (risk aversion, prudence and temperance). Within each order, there were in total 12 tasks with lotteries consisting of a risk apportionment option and its reverse. We follow Bleichrodt and van Bruggen (2022) and classify subjects as risk averse, prudent or temperate if they made lottery choices consistent with those traits at least 10 out of the 12 tasks within each order. Similarly, subjects who made those choices for no more than 2 tasks are classified as risk loving, imprudent or intemperate while if an option has been chosen between 3 and 9 times, the subject is classified as neutral, for that particular order. Furthermore, to take into account the presence of stochasticity in choice, we adopt their Maximum Likelihood Estimation (MLE) approach, and estimate the distribution of different risk apportionment types allowing for errors. Specifically, assuming three DM types within a given order (averse, neutral, and loving), one can estimate a mixed binomial distribution and derive population proportion estimates for each type, denoted as π_s, π_n, and π_o. Each type with strict preferences (i.e. averse or loving) is allowed to make errors. That is, in any task, a subject who satisfies risk apportionment chooses the corresponding option with probability $1 - \omega_s$, and with probability $\omega_s < 1/2$ she chooses the opposite one. Similarly for subjects who satisfy the opposite of risk apportionment, we assume that they make a mistake with probability $\omega_o < 1/2$, with $\omega_o \neq \omega_s$. Neutral subjects are by definition indifferent between the two options.

The probability of choosing the option indicating a given risk apportionment x out of 12 times can be represented by the following density, within a given order:

$$f(x) = \binom{12}{x} \left[\pi_s (1 - \omega_s)^x \omega_s^{12-x} + \pi_n \frac{1}{2}^x \frac{1}{2}^{12-x} + \pi_0 \omega_o^x (1 - \omega_0)^{12-x} \right]$$

with $\pi_s + \pi_n + \pi_o = 1$.

We have predictions about third and fourth order risky choices and, therefore, we are interested in the frequencies of combinations of third and fourth-order choices, which give in total 9 possible combinations. Therefore, we extend the above specification to include the joint probabilities of subjects belonging to a specific combination (e.g. prudent-neutral and intemperate), in addition to the four order-specific error terms $\omega_s, \omega_o, \xi_s$ and ξ_o for the prudent, imprudent, temperate and intemperate types respectively. The estimated proportions are presented in Figure 1. Using the estimated proportions, we can jointly test predictions 1 and 4. The majority (28%) of the subjects is classified as preferring the prudent neutral and temperate neutral lotteries, while only 6% of the subjects behave according to the predictions of KR and DA.

Finding 1 *6% of the subjects make lottery choices consistent with prudence neutrality and intemperance, in line with the predictions of KR and DA for the case of 50-50 lotteries with symmetric risk.*

To test prediction 3 related to small probabilities outcomes, we repeat the same estimation using the data from the small-probabilities treatment. We do not have a prediction of fourth-order risky choice for small probabilities. We therefore complement the analysis by incorporating the

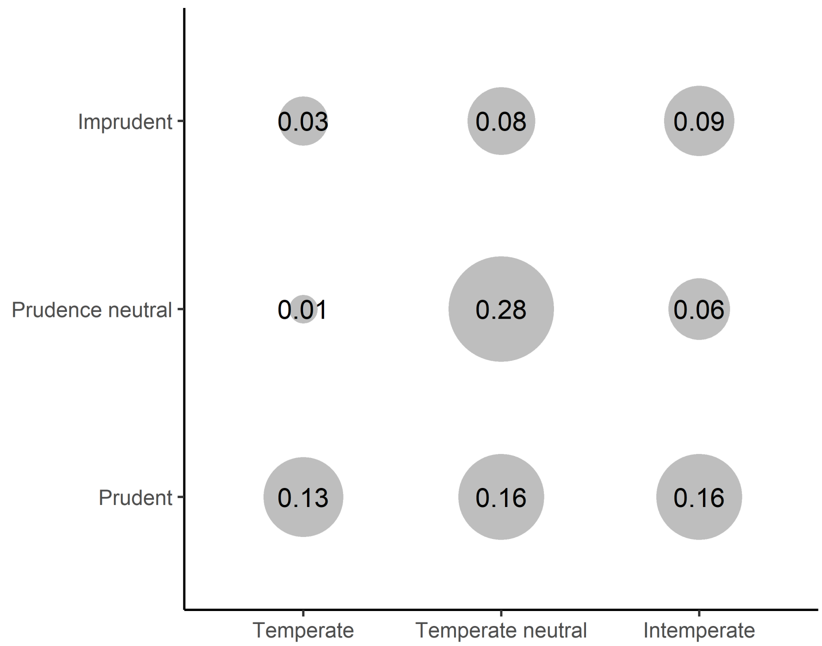


Figure 1: MLE Combinations of Third- and Fourth-Order Risk Preferences 50-50 Gains Treatment. Estimated error rates $\hat{\omega}_s = 0.081$ (prudent), $\hat{\omega}_o = 0.009$ (imprudent), $\hat{\xi}_s = 0.050$ (temperate) and $\hat{\xi}_o = 0.022$ (intemperate).

second order choice that would be consistent with KR which in this case is that of risk aversion. Figure 2 shows the estimated proportions between all combinations of second and fourth-order risk preferences in the small-probabilities treatment. The majority of the subjects (61%) is classified as prudent and risk averse, while 13% of the subjects behave according to the predictions of KR (prudence neutrality and risk aversion).

Finding 2 *13% of the subjects make lottery choices consistent with prudence neutrality and risk aversion in line with the predictions of KR for the case of small probabilities outcome lotteries.*

Finally, to test prediction 2 related to risk apportionment tasks with asymmetric zero-mean risk, we employ the data from Ebert and Wiesen (2014) and Heinrich and Mayrhofer (2018) because, to the best of our knowledge, those are the only studies that deviate from the common practice of eliciting higher order risk preferences using exclusively risk apportionment tasks with symmetric zero-mean risk. The experiment of Ebert and Wiesen (2014) consists of 127 subjects (119 once those who switched multiple times are discarded) making their choices from a menu

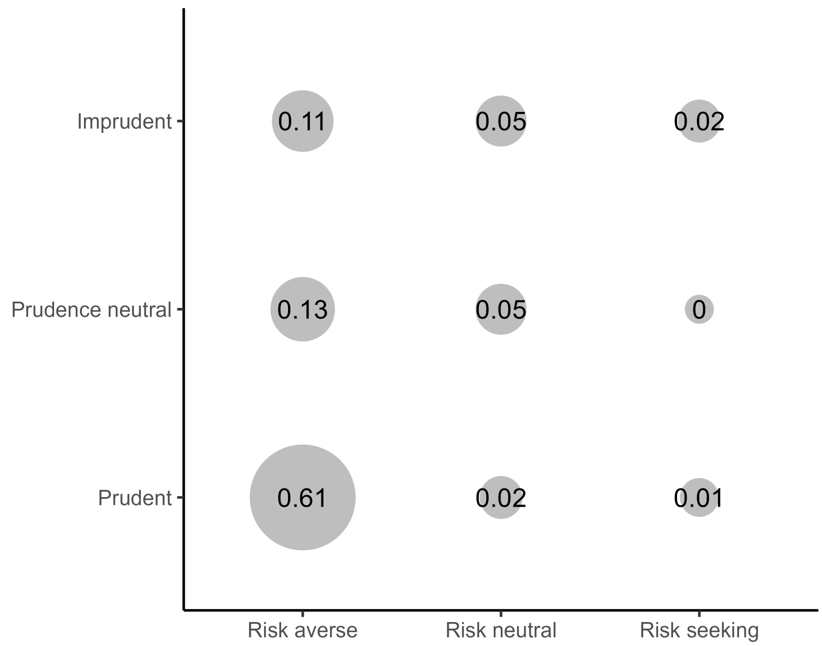


Figure 2: MLE Combinations of Second- and Fourth-Order Risk Attitudes Small Probabilities Gains Treatment. Estimated error rates $\hat{\omega}_s = 0.052$ (prudent), $\hat{\omega}_o = 0.034$ (imprudent), $\hat{\xi}_s = 0.047$ (temperate) and $\hat{\xi}_o = 0.048$ (intemperate).

of pairwise lotteries designed to elicit risk preferences. There were in total 6 risk apportionment tasks per subject, one to elicit second order, three to elicit third order, and two to elicit fourth order risk preferences. Within the tasks to elicit third order risk preferences, one task is designed with symmetric zero-mean risk (PRU_1) while the other two with asymmetric zero-mean risk (PRU_2 and PRU_3). Similarly, in the tasks designed to elicit risk apportionment of order four, one task includes symmetric zero-mean risk (TE_1) and the other one asymmetric risk (TE_2). The full list of tasks is provided in Appendix E.

The aim of the experiment was to elicit compensations for n-th order risk preferences and, in particular, to identify the smallest amount for which a DM would choose the seeking option over the averse one. A compensation of zero would indicate neutral attitude, a negative amount would classify a subject as n-th order risk loving, while a positive one would indicate n-th order risk averse behaviour. This methodology was designed to yield model-independent intensity measures of risk attitudes of different orders. The experiment included various treatments varying a number of elements such as sequence effects, width of the grid interval of potential compensations, as well as coarseness of the grid. Heinrich and Mayrhofer (2018) adopt the same experimental design to study higher order preferences in a social setting. In their experiment, subjects had to make decisions in isolation and in a social setting where their choices had an influence on others' payoffs. We use the data from the individual decision making part of their experiment, which was identical to Ebert and Wiesen (2014) treatments that adopted a fine compensation grid. Their dataset consists of the choices of 312 subjects, where 35 subjects had been discarded because of multiple switch in the choices.

We follow the analysis method adopted in these studies, and classify subjects according to the mean value of the compensation they demanded within each order. KR and DA predict that when the zero-mean risk is symmetric the agent would make choices consistent with prudence neutral and intemperance within the lotteries designed to elicit risk apportionment of order three and four, while in the case of asymmetric zero-mean risk this might not be the case. In particular, for task PRU_2 the predicted lottery choice for both KR and DA would be the imprudent one given the risk is negatively skewed and $e > (c - x)$. On the other hand, for task PRU_3, the predicted lottery choice for both KR and DA would be the prudent one given the risk is positively skewed and $e < (c - x)$ (but $e > 0.5(c - x)$) and $\frac{q}{1-q}e > c - x$. According to this, our hypothesis is that, within the subjects that make lottery choices consistent with intemperance in task TE_1, the proportion of those who prefer the imprudent lotteries in tasks PRU_1 and PRU_2, is lower compared to the proportion of subjects who prefer the imprudent lotteries considering only task PRU_2. Likewise, within the subjects that make lottery choices consistent with intemperance in task TE_1, the proportion of those who prefer the prudent lotteries in tasks PRU_1 and PRU_3 should be lower compared to the proportion of subjects who prefer the prudent lotteries considering only task PRU_3.

In the Ebert and Wiesen (2014) sample, we find that, out of the experimental subjects that make the intemperate choice in task TE_1, the number of subjects who make imprudent choices when considering tasks PRU_1 and PRU_2 and when considering only task PRU_2 is the same. We also find that the number of subjects who make prudent choices when considering tasks PRU_1 and PRU_3 is actually two less than the number obtained when we only consider task PRU_3. In the Heinrich and Mayrhofer (2018) experimental sample, we find that only one more subject makes imprudent choices in task PRU_2 relative to those considering both tasks PRU_1 and PRU_2. We also find that 18 subjects make the prudent choices in tasks PRU_1 and PRU_3, while 16 subjects choose the prudent lotteries in task PRU_3. All those differences are statistically insignificant. This experimental evidence does not support the predictions implied by the KR and DA specifications that different third order choices should be observed in the presence of asymmetric zero-mean risk relative to symmetric risk.

Finding 3 *There is no significant difference in third order risky choices when subjects face risk apportionment lotteries with symmetric or asymmetric zero-mean risk.*

Overall, the experimental findings in this section employing three different datasets on risk apportionment tasks for elicitation of higher-order risk preferences suggest that a relatively small proportion of DMs, 13% as the upper limit, make choices consistent with KR or DA.

5 Conclusions

This is the first paper to examine choice behaviour in risk-apportionment tasks to elicit higher order risk attitudes exhibited by a DM defined by some of the most popular expectations-based reference-dependent models. We have demonstrated the cases where, in experimental research using risk-apportionment tasks, a DM defined by a Kőszegi and Rabin or a disappointment aversion model specification may exhibit prudent, imprudent or prudent neutral choices. We illustrated how the precise structure of the lotteries impacted on these choices. In particular whether the lotteries exhibited symmetric or asymmetric risks, small probabilities of outcomes and equal higher order moments. We also illustrated that the assumption of loss aversion implies that a DM defined by either the KR or DA model would make risky choices consistent with anti-risk apportionment of order four. Our analysis has implications for the interpretation of experimental research. In particular, some of the predictions for elicitation tasks of higher-order risk preferences differ from those in models of decision under risk such as CPT or EUT. The ability of the model specifications outlined in this paper to describe risky choices found in the literature is also examined employing data from three influential experimental studies. We find that only a small proportion of subjects make risky choices consistent with the Kőszegi and Rabin and disappointment aversion model specifications.

Appendix A The Kőszegi and Rabin model: derivations of the utility of lotteries to elicit higher order risk preferences

In this Appendix, we provide the calculations that are employed as the basis of the predictions presented in the main text of the paper. We note that, under the assumptions described in section 2, the first term in (3) represents the DM's expected value of the lottery. Given that all the lottery pairs presented below to elicit the risk preferences of prudence and temperance share the same expected value, they will cancel each other when computing the difference between the lottery pair. Therefore, to simplify notation, we have omitted the first term in (3) from the calculations.

We now present the utility of the lotteries designed to elicit prudence and the reverse preference, $B_3 = [x, c + \widetilde{\varepsilon}]$ and $A_3 = [c, x + \widetilde{\varepsilon}]$, respectively, and recall that $\widetilde{\varepsilon} = [e, -e]$ and that $[x, y]$ denotes outcomes x and y are each received with probability 0.5.

$$\begin{aligned}
U(B_3|B_3) &= 0.5\left(0.5\mu(0) + 0.25\mu(c - x + e) + 0.25\mu(c - x - e)\right) \\
&+ 0.25\left(0.5\mu(-(c - x) - e) + 0.25\mu(0) + 0.25\mu(-2e)\right) \\
&+ 0.25\left(0.5\mu(e - (c - x)) + 0.25\mu(2e) + 0.25\mu(0)\right) \\
&= 0.5\left(0.25\mu(c - x + e) + 0.25\mu(c - x - e)\right) + 0.25\left(0.5\mu(-(c - x) - e) - 0.5\mu(e)\right) \\
&+ 0.25\left(0.5\mu(e - (c - x)) + 0.5\mu(e)\right) \qquad (5)
\end{aligned}$$

$$\begin{aligned}
U(A_3|A_3) &= 0.5\left(0.5\mu(0) + 0.25\mu(e - (c - x)) + 0.25\mu(-e - (c - x))\right) \\
&+ 0.25\left(0.5\mu(c - x - e) + 0.25\mu(0) + 0.25\mu(-2e)\right) \\
&+ 0.25\left(0.5\mu(c - x + e) + 0.25\mu(2e) + 0.25\mu(0)\right) \\
&= 0.5\left(0.25\mu(e - (c - x)) + 0.25\mu(-(c - x) - e)\right) + 0.25\left(0.5\mu(c - x - e) - 0.5\mu(e)\right) \\
&+ 0.25\left(0.5\mu(c - x + e) + 0.5\mu(e)\right). \qquad (6)
\end{aligned}$$

We note that both expressions (5) and (6) yield the same outcome. Moreover, both expressions have been left in terms of the form of the gain-loss utility function $\mu(\cdot)$ defined in section (2), to demonstrate that this result holds regardless of the specific functional form assumed for $\mu(\cdot)$.[12]

We now consider a departure from the assumption of symmetry of the zero-mean background risk. We assume it is asymmetric, that is, $\widetilde{\varepsilon}' = \left(1-q : \frac{q}{1-q}e, q : -e\right)$. The predicted choice behaviour will depend on the skewness of the risk and the relative size of e and $(c-x)$. Recall that $c > x > e$.

Let us now compute the utility of the lottery pair. First, lottery B_3

$$\begin{aligned} U(B_3|B_3) &= 0.5\left(0.5\mu(0) + 0.5\left(1-q\right)\mu(c-x+\frac{q}{1-q}e) + 0.5q\mu(c-x-e)\right) \\ &+ 0.5\left(1-q\right)\left(0.5\mu(-\left(c-x\right)-\frac{q}{1-q}e) + 0.5\left(1-q\right)\mu(0) + 0.5q\mu(-\frac{e}{1-q})\right) \\ &+ 0.5q\left(0.5\mu(e-(c-x)) + 0.5\left(1-q\right)\mu(\frac{e}{1-q}) + 0.5q\mu(0)\right) \end{aligned}$$

The value of $U(B_3|B_3)$ depends on the lottery parameters and there are two cases:

$e > c - x$

$$\begin{aligned} U(B_3|B_3) &= 0.25q\left(1-\lambda\right)\left(e-(c-x)\right) + 0.25\left(1-q\right)\left(1-\lambda\right)\left(c-x+\frac{q}{1-q}e\right) \\ &+ 0.25q\left(1-q\right)\left(1-\lambda\right)\left(\frac{e}{1-q}\right) \end{aligned}$$

$e < c - x$

$$\begin{aligned} U(B_3|B_3) &= 0.25(1-q)(1-\lambda)\left(c-x+\frac{q}{1-q}e\right) + 0.25q\left(1-\lambda\right)\left(c-x+-e\right) \\ &+ 0.25q\left(1-q\right)\left(1-\lambda\right)\left(\frac{e}{1-q}\right) \end{aligned}$$

Second, we derive the calculations for lottery A_3

[12]Departing from linear utility in $u(\cdot)$ would imply that the difference in utility between B_3 and A_3 would be determined by the difference in the first term of (3). In this case, that first term would represent the expected utility of the lotteries rather than their expected value. Therefore, they may not cancel out when computing their difference in utility, and if, for instance, the utility function was power with exponent less than unity, the DM would exhibit prudence.

$$
\begin{aligned}
U(A_3|A_3) &= 0.5\left(0.5\mu(0)+0.5\,(1-q)\,\mu(\frac{q}{1-q}e-(c-x))+0.5q\mu(-(c-x)-e)\right) \\
&+ 0.5\,(1-q)\left(0.5\mu\left(c-x-\frac{q}{1-q}e\right)+0.5\,(1-q)\,\mu(0)+0.5q\mu(-\frac{e}{1-q})\right) \\
&+ 0.5q\left(0.5\mu(c-x+e)+0.5\,(1-q)\,\mu(\frac{e}{1-q})+0.5q\mu(0)\right)
\end{aligned}
$$

.

The value of $U(A_3|A_3)$ also depends on the lottery parameters and there are two cases:

$\frac{q}{1-q}e < c-x$

$$
\begin{aligned}
U(A_3|A_3) &= 0.25q\,(1-\lambda)\,(c-x+e)+0.25\,(1-q)\,(1-\lambda)\left(c-x-\frac{q}{1-q}e\right) \\
&+ 0.25\,(1-q)\,q\,(1-\lambda)\left(\frac{e}{1-q}\right).
\end{aligned}
$$

$\frac{q}{1-q}e > c-x$

$$
\begin{aligned}
U(A_3|A_3) &= 0.25q\,(1-\lambda)\,(c-x+e)+0.25\,(1-q)\,(1-\lambda)\left(\frac{q}{1-q}e-(c-x)\right) \\
&+ 0.25\,(1-q)\,q\,(1-\lambda)\left(\frac{e}{1-q}\right).
\end{aligned}
$$

The predicted risky choices will therefore be the following. Let us first assume the risk is positively skewed, i.e., $q > 0.5$.

If $e > c-x$ (and therefore it also holds that $\frac{q}{1-q}e > c-x$)

$$
\begin{aligned}
U(B_3|B_3)-U(A_3|A_3) &= 0.25(1-q)(1-\lambda)(2c-2x)+0.25q(1-\lambda)(2x-2c) \\
&= -0.5(1-q)(\lambda-1)(c-x)+0.5q(\lambda-1)(c-x) \qquad (7)
\end{aligned}
$$

which is positive for $q > 0.5$ and therefore, the loss averse DM makes the prudent choice.

If $e < c - x$ and $\frac{q}{1-q}e > c - x$

$$\begin{aligned} U(B_3|B_3) - U(A_3|A_3) &= 0.25(1-q)(1-\lambda)(2c-2x) + 0.25q(1-\lambda)(-2e) \\ &= 0.5(\lambda-1)eq - 0.5(1-q)(\lambda-1)(c-x) \\ &= 0.5(\lambda-1)(q(e+c-x)+x-c) \end{aligned} \tag{8}$$

which is positive since $eq > (1-q)(c-x)$ which holds since $\frac{q}{1-q}e > c-x$, therefore the DM prefers the prudent choice.

If $e < c - x$ and $\frac{q}{1-q}e < c - x$

$$\begin{aligned} U(B_3|B_3) - U(A_3|A_3) &= 0.25(1-q)(1-\lambda)(\frac{2q}{1-q}e) + 0.25q(1-\lambda)(-2e) \\ &= 0.5(1-\lambda)\left((1-q)\frac{q}{1-q}e - qe\right) \\ &= 0 \end{aligned} \tag{9}$$

Therefore, the DM is prudent neutral in this case.

Let us now assume the risk is negatively skewed, i.e., $q < 0.5$.

If $e < c - x$ (and therefore it also holds that $\frac{q}{1-q}e < c - x$)

$$\begin{aligned} U(B_3|B_3) - U(A_3|A_3) &= 0.5(1-q)(1-\lambda)\frac{q}{1-q}e - 0.5q(1-\lambda)e \\ &= 0.5q(\lambda-1)e - 0.5(1-q)(\lambda-1)\frac{q}{1-q}e \\ &= 0 \end{aligned} \tag{10}$$

and the DM is prudent neutral.

If $e > c - x$ and $\frac{q}{1-q}e < c - x$

$$\begin{aligned} U(B_3|B_3) - U(A_3|A_3) &= 0.25(1-q)(1-\lambda)\left(\frac{2q}{1-q}e\right) + 0.25q(1-\lambda)(2x-2c) \\ &= 0.5q(\lambda-1)(c-x) - 0.5(1-q)(\lambda-1)\frac{q}{1-q}e \\ &= c-x-e \end{aligned} \tag{11}$$

which is negative and therefore the loss averse DM makes the imprudent choice.

If $e > c - x < e$ and $\frac{q}{1-q}e > c - x$

$$\begin{aligned} U(B_3|B_3) - U(A_3|A_3) &= 0.25(1-q)(1-\lambda)(2c-2x) + 0.25q(1-\lambda)(2x-2c) \\ &= 0.5(\lambda-1)(c-x)(2q-1) \end{aligned} \tag{12}$$

The loss averse DM makes again the imprudent choice since $q < 0.5$.

We now present the calculations of the expected utility of the lotteries designed to elicit third order risk attitudes accounting for small probability outcomes as described in the paper, that is, lottery pair $B_{3,sp}$, and $A_{3,sp}$.

$$\begin{aligned} U(B_{3,sp}|B_{3,sp}) = \; & \frac{p}{2}\left(\frac{p}{2}\mu(0) + \frac{p}{2}\mu(-2e) + p\mu(x-c-e) + (1-2p)\mu(z-c-e)\right) \\ & + \frac{p}{2}\left(\frac{p}{2}\mu(2e) + \frac{p}{2}\mu(0) + p\mu(x-c+e) + (1-2p)\mu(z-c+e)\right) \\ & + p\left(\frac{p}{2}\mu(c+e-x) + \frac{p}{2}\mu(c-e-x) + p\mu(0) + (1-2p)\mu(z-x)\right) \\ & + (1-2p)\left(\frac{p}{2}\mu(c+e-z) + \frac{p}{2}\mu(c-e-z) + p\mu(x-z) + (1-2p)\mu(0)\right) \end{aligned}$$

$$\begin{aligned} U(A_{3,sp}|A_{3,sp}) = \; & \frac{p}{2}\left(\frac{p}{2}\mu(0) + \frac{p}{2}\mu(-2e) + p\mu(c-x-e) + (1-2p)\mu(z-x-e)\right) \\ & + \frac{p}{2}\left(\frac{p}{2}\mu(2e) + \frac{p}{2}\mu(0) + p\mu(c-x+e) + (1-2p)\mu(z-x+e)\right) \\ & + p\left(\frac{p}{2}\mu(x+e-c) + \frac{p}{2}\mu(x-e-c) + p\mu(0) + (1-2p)\mu(z-c)\right) \\ & + (1-2p)\left(\frac{p}{2}\mu(x+e-z) + \frac{p}{2}\mu(x-e-z) + p\mu(c-z) + (1-2p)\mu(0)\right). \end{aligned}$$

Recall that the gain-loss utility function is a two-part linear where λ is the coefficient of loss aversion and that $c > x > e > z$. The lottery outcomes will fall within the gain or loss domain depending on whether c is larger or smaller than $x + e$. Our calculations reveal that both cases yield the same qualitative result. First, we consider the case where $c > x + e$.[13] Expanding and removing

[13]Note that we also assume that $x > e + z$ given that, as argued by Bleichrodt and van Bruggen (2022), payoff z is considered to be small relative to the other payoffs in the lottery. However, if that assumption was dropped and it was assumed that $x < e + z$ then $U(B_{3,sp}|B_{3,sp}) > U(A_{3,sp}|A_{3,sp})$ and the DM would exhibit prudence (calculations available upon request). This is intuitive given that, in that case, there would be a relatively large mass associated to

common terms in $U(B_{3,sp}|B_{3,sp})$ and $U(A_{3,sp}|A_{3,sp})$, we find that the difference in expected utility between the two lotteries depends on the evaluation of the difference between

$$\begin{aligned} U(B_{3,sp}|B_{3,sp}) &= -\lambda p\left((c-z)-p\left(x+c\right)+2pz\right) \\ &\quad +p\left(p(c-x)-\lambda(1-2p)(x-z)\right) \\ &\quad +(1-2p)\left(p(c-z)+p(x-z)\right) \end{aligned} \tag{13}$$

$$\begin{aligned} U(A_{3,sp}|A_{3,sp}) &= p\left(p(c-x)-\lambda(1-2p)(x-z)\right) \\ &\quad -\lambda p\left(p(c-x)+(1-2p)(c-z)\right) \\ &\quad +(1-2p)\left(p(x-z)+p(c-z)\right). \end{aligned} \tag{14}$$

It can easily be seen that in this case $U(B_{3,sp}|B_{3,sp}) = U(A_{3,sp}|A_{3,sp})$.

Second, we consider the case where $c < x + e$. Expanding and removing common terms in $U(B_{3,sp}|B_{3,sp})$ and $U(A_{3,sp}|A_{3,sp})$, we find that

$$\begin{aligned} U(B_{3,sp}|B_{3,sp}) &= \frac{p}{2}\left(1-\lambda\right)\left((c+e-z)+(c-e-z)\right) \\ &\quad +p\left(1-\lambda\right)\left((x-z)\right) \end{aligned} \tag{15}$$

$$\begin{aligned} U(A_{3,sp}|A_{3,sp}) &= \frac{p}{2}\left(1-\lambda\right)\left((x+e-z)+(x-e-z)\right) \\ &\quad +p\left(1-\lambda\right)\left((c-z)\right) \end{aligned} \tag{16}$$

It can easily be seen that also in this case $U(B_{3,sp}|B_{3,sp}) = U(A_{3,sp}|A_{3,sp})$.

We now consider the value of the lottery pair designed to elicit risk apportionment of order

the large probability $(1-2p)$.

4. We assume that both independent zero-mean risks are symmetric, i.e., $\widetilde{\varepsilon_1} = [e_1, -e_1]$ and $\widetilde{\varepsilon}_2 = [e_2, -e_2]$. First, we consider the general case where the size of the risks differ, $e_1 > e_2$.

$$\begin{aligned} U(B_4|B_4) &= 0.25\,(0.25\mu(0) + 0.25\mu(-2e_1) + 0.25\mu(e_2 - e_1) + 0.25\mu(-e_1 - e_2)) \\ &\quad +0.25\,(0.25\mu(2e_1) + 0.25\mu(0) + 0.25\mu(e_2 + e_1) + 0.25\mu(-e_2 + e_1)) \\ &\quad +0.25\,(0.25\mu(e_1 - e_2) + 0.25\mu(-e_1 - e_2) + 0.25\mu(0) + 0.25\mu(-2e_2)) \\ &\quad +0.25\,(0.25\mu(e_1 + e_2) + 0.25\mu(-e_1 + e_2) + 0.25\mu(2e_2) + 0.25\mu(0)) \end{aligned}$$

$$\begin{aligned} U(A_4|A_4) &= 0.5\,(0.5\mu(0) + 0.125\mu(e_1 + e_2) + 0.125\mu(e_1 - e_2) + 0.125\mu(-e_1 + e_2) + 0.125\mu(-e_1 - e_2)) \\ &\quad +0.125\,(0.5\mu(-e_1 - e_2) + 0.125\mu(0) + 0.125\mu(-2e_2) + 0.125\mu(-2e_1) + 0.125\mu(-2e_1 - 2e_2)) \\ &\quad +0.125\,(0.5\mu(-e_1 + e_2) + 0.125\mu(2e_2) + 0.125\mu(0) + 0.125\mu(-2e_1 + 2e_2) + 0.125\mu(-2e_1)) \\ &\quad +0.125\,(0.5\mu(e_1 - e_2) + 0.125\mu(2e_1) + 0.125\mu(2e_1 - 2e_2) + 0.125\mu(0) + 0.125\mu(-2e_2)) \\ &\quad +0.125\,(0.5\mu(e_1 + e_2) + 0.125\mu(2e_1 + 2e_2) + 0.125\mu(2e_1) + 0.125\mu(2e_2) + 0.125\mu(0))\,. \end{aligned}$$

Removing the common terms in $U(B_4|B_4)$ and $U(A_4|A_4)$, we find that the difference in expected utility between the lottery pair depends on the difference between these two expressions:

$$U(B_4|B_4) = \frac{1}{16}\,(\mu(2e_1) + \mu(-2e_1)) + \frac{1}{16}\,(\mu(2e_2) + \mu(-2e_2))$$

$$\begin{aligned} U(A_4|A_4) &= \frac{1}{32}\,(\mu(2e_1) + \mu(-2e_1)) + \frac{1}{32}\,(\mu(2e_2) + \mu(-2e_2)) + \\ &\quad + \frac{1}{64}\,(\mu(2e_1 + 2e_2) + \mu(-2e_1 - 2e_2)) + \frac{1}{64}\,(\mu(2e_1 - 2e_2) + \mu(-2e_1 + 2e_2))\,. \end{aligned}$$

The difference in expected utility between the lottery pair depends on

$$U(B_4|B_4) = \frac{1}{16}\,(2e_1 - 2\lambda e_1 + 2e_2 - 2\lambda e_2) = -\frac{(\lambda - 1)}{8}\,(e_1 + e_2) \tag{17}$$

$$U(A_4|A_4) = -\frac{(\lambda-1)}{8}e_1 - \frac{(\lambda-1)}{16}e_2. \tag{18}$$

Assuming $\lambda > 1$, these expressions imply that $U(B_4|B_4) < U(A_4|A_4)$.

We now consider the particular case that the zero-mean risks are of equal size, $e_1 = e_2$. It is easy to see that if the two zero-mean independent risks have payoffs of equal size then the difference in utility reduces to

$$U(B_4|B_4) - U(A_4|A_4) = -\frac{(\lambda-1)}{8}(e_1) + \frac{(\lambda-1)}{16}(e_1). \tag{19}$$

Assuming $\lambda > 1$, this difference is negative and therefore the DM also exhibits in this case choice consistent with intemperance ((anti)risk apportionment of order 4).

Appendix B Disappointment Aversion (DA) model: derivations of the utility of lotteries to elicit higher order risk preferences

We recall that, like in Appendix A, the reference-dependent gain-loss utility $\mu(\cdot)$ is two-part linear with loss aversion parameter λ. The lotteries to elicit risk apportionment of order 3 are $B_3 = [x, c+\widetilde{\varepsilon}]$ and $A_3 = [c, x+\widetilde{\varepsilon}]$, where $\widetilde{\varepsilon} = [e, -e]$ and $c > x > e$. The referent is in this case the expected value of each of the lotteries, $0.5(x+c)$. We note that the expected utility of the lottery pair will depend on the relative size of the risk (e) and the expected value ($0.5(c-x)$). However, the qualitative result is the same in both cases, and we therefore only present below one of these cases, in particular, $e > 0.5(c-x)$.

$$\begin{aligned} U(B_3|B_3) &= 0.5\mu(x - 0.5(x+c)) + 0.25\mu(c + e - 0.5(x+c)) \\ &= 0.25\lambda(c-x) + 0.25(c-x) + 0.25e - 0.25\lambda(e - 0.5(c-x) \\ &= -0.125(\lambda-1)(c-x) - 0.25(\lambda-1)e \end{aligned} \tag{20}$$

$$
\begin{aligned}
U(A_3|A_3) &= 0.5\mu\left(c-0.5\left(x+c\right)\right)+0.25\mu\left(x+e-0.5\left(x+c\right)\right)+0.25\mu\left(x-e-0.5\left(x+c\right)\right)\\
&= 0.25\left(c-x\right)+0.125\left(x-c\right)+0.25e-0.25\lambda e+0.125\lambda\left(x-c\right)\\
&= -0.125\left(\lambda-1\right)\left(c-x\right)-0.25\left(\lambda-1\right)e. \qquad (21)
\end{aligned}
$$

Therefore, $U(B_3|B_3)=U(A_3|A_3)$ and the DM would in this case exhibit indifference between the lottery pair designed to elicit risk apportionment of order 3.

If background risk is assumed to be asymmetric, that is, $\widetilde{\varepsilon}' = \left(1-q : \frac{q}{1-q}e, q : -e\right)$, the predictions can be drawn depending on the relative size of the risk (e) and the expected value of the lotteries ($0.5\left(c+x\right)$), and on the skewness of the zero-mean risk determined by q.

The value of $U(B_3|B_3)$ and $U(A_3|A_3)$ depend on the lottery parameters and we have the following cases:

First, if $e > 0.5(c-x)$

$$
\begin{aligned}
U(B_3|B_3) &= -0.25\lambda\left(c-x\right)+0.25\left(1-q\right)\left(c-x\right)+0.5(1-q)\frac{q}{1-q}e-0.5q\lambda\left(e-0.5\left(c-x\right)\right)\\
&= 0.25\left(1-\lambda\right)\left(c-x\right)\left(1-q\right)+0.5q\left(1-\lambda\right)qe\\
&= -0.25\left(\lambda-1\right)\left(c-x\right)\left(1-q\right)-0.5q\left(\lambda-1\right)e
\end{aligned}
$$

$$
\begin{aligned}
U(A_3|A_3) &= 0.25\mu(c-x)+0.5(1-q)\mu(x+\frac{q}{1-q}e-0.5(c+x))+0.5q\mu(x-e-0.5(c+x))\\
&= 0.25\mu(c-x)+0.5(1-q)\mu\left(\frac{q}{1-q}e-0.5(c-x)\right)+0.5q\mu(-0.5(c-x)-e)
\end{aligned}
$$

In this case, if $q > 0.5$ then it always holds that $\frac{q}{1-q}e > 0.5(c-x)$ and therefore

$$
\begin{aligned}
U(A_3|A_3) &= 0.25(c-x)+0.5qe-0.25(1-q)(c-x)-0.25q\lambda(0.5(c-x)+e)\\
&= 0.25q(1-\lambda)(c-x)+0.5q(1-\lambda)e
\end{aligned}
$$

If, on the other hand, $q < 0.5$ then $\frac{q}{1-q}e < e$ and there are two cases

if $\frac{q}{1-q}e > 0.5(c-x)$ then the result is the same as above, i.e., $U(A_3|A_3) = 0.25q(1-\lambda)(c-x) + 0.5q(1-\lambda)e$

if $\frac{q}{1-q}e < 0.5(c-x)$ then

$$\begin{aligned}U(A_3|A_3) &= 0.25(c-x) - 0.5(1-q)\lambda(0.5(c-x) - \frac{q}{1-q}e) - 0.25q\lambda(c-x) - 0.5qe \\ &= 0.25(1-\lambda)(c-x)\end{aligned}$$

Second, if $e < 0.5(c-x)$

$$\begin{aligned}U(B_3|B_3) &= -0.25\lambda(c-x) + 0.25(1-q)(c-x) + 0.5qe + 0.25q(c-x) - 0.5qe \\ &= 0.25(c-x)(-\lambda + (1-q) + q) \\ &= 0.25(1-\lambda)(c-x)\end{aligned}$$

$$U(A_3|A_3) = 0.5\mu(0.5(c-x)) + 0.5(1-q)\mu(x + \frac{q}{1-q}e - 0.5(x+c)) + 0.5q\mu(x - e - 0.5(x+c))$$

In this case, if $q < 0.5$ then it always holds that $\frac{q}{1-q}e - 0.5(c-x) < 0$ and

$$\begin{aligned}U(A_3|A_3) &= 0.5\mu(0.5(c-x)) + 0.5(1-q)\mu(x + \frac{q}{1-q}e - 0.5(x+c)) + 0.5q\mu(x - e - 0.5(x+c)) \\ &= 0.25(c-x) - 0.5(1-q)\lambda(0.5(c-x) - \frac{q}{1-q}e) - 0.5q\lambda(0.5(c-x) + e) \\ &= 0.25(1-\lambda)(c-x)\end{aligned}$$

If, on the other hand, $q > 0.5$ then $\frac{q}{1-q}e > e$ and there are two cases

if $\frac{q}{1-q}e < 0.5(c-x)$ then the result is the same as above, i.e., $U(A_3|A_3) = 0.25(1-\lambda)(c-x)$.

if $\frac{q}{1-q}e > 0.5(c-x)$ then

$$\begin{aligned}
U(A_3|A_3) &= 0.25(c-x)+0.5(1-q)(\frac{q}{1-q}e-0.5(c-x))-0.5q\lambda(0.5(c-x)+e) \\
&= 0.25(c-x)+0.5qe-0.25(1-q)(c-x)-0.25q\lambda(c-x)-0.5q\lambda e \\
&= 0.25q(1-\lambda)(c-x)+0.5qe(1-\lambda)
\end{aligned}$$

Once we have computed the value of $U(B_3|B_3)$ and $U(A_3|A_3)$ for all possible cases, we obtain the predicted risky choices. Let us first assume the risk is positively skewed, i.e., $q > 0.5$.

If $e > 0.5(c-x)$ (and it always holds that $\frac{q}{1-q}e > 0.5(c-x)$)

$$\begin{aligned}
U(B_3|B_3)-U(A_3|A_3) &= 0.25(c-x)(1-q)(1-\lambda)+0.5qe(1-\lambda) \\
&- 0.25q(1-\lambda)(c-x)-0.5qe(1-\lambda) \\
&= 0.25(c-x)(\lambda-1)(2q-1) > 0 \qquad (22)
\end{aligned}$$

therefore, the loss averse DM makes the prudent choice.

If $e < 0.5(c-x)$ and $\frac{q}{1-q}e > 0.5(c-x)$

$$\begin{aligned}
U(B_3|B_3)-U(A_3|A_3) &= 0.25(1-\lambda)(c-x)-0.25q(1-\lambda)(c-x)-0.5qe(1-\lambda) \\
&= -0.25(\lambda-1)\left((c-x)(1-q)-2qe\right) > 0 \qquad (23)
\end{aligned}$$

since $((c-x)(1-q)-2qe) < 0$ and the loss averse DM makes the prudent choice.

If $e < 0.5(c-x)$ and $\frac{q}{1-q}e < 0.5(c-x)$ then

$$\begin{aligned}
U(B_3|B_3)-U(A_3|A_3) &= 0.25(1-\lambda)(c-x)-0.25(1-\lambda)(c-x) \qquad (24) \\
&= 0
\end{aligned}$$

and the DM is indifferent between the lottery pair.

Let us now assume that the risk is negatively skewed, i.e., $q < 0.5$.

If $e < 0.5(c-x)$ (and therefore it always holds that $\frac{q}{1-q}e < 0.5(c-x)$)

$$\begin{aligned} U(B_3|B_3) - U(A_3|A_3) &= 0.25(1-\lambda)(c-x) - 0.25(1-\lambda)(c-x) \\ &= 0 \end{aligned} \tag{25}$$

and therefore, the DM exhibits prudent neutral choice.

If $e > 0.5(c-x)$ and $\frac{q}{1-q}e < 0.5(c-x)$

$$\begin{aligned} U(B_3|B_3) - U(A_3|A_3) &= 0.25(c-x)(1-q)(1-\lambda) + 0.5qe(1-\lambda) - 0.25q(1-\lambda)(c-x) \\ &= 0.25q(\lambda-1)\left((c-x)-2e\right) < 0 \end{aligned} \tag{26}$$

and therefore, the loss averse DM exhibits imprudent choice.

If $e > 0.5(c-x)$ and $\frac{q}{1-q}e > 0.5(c-x)$

$$\begin{aligned} U(B_3|B_3) - U(A_3|A_3) &= 0.25(c-x)(1-q)(1-\lambda) + 0.5qe(1-\lambda) \\ &- 0.25q(1-\lambda)(c-x) - 0.5qe(1-\lambda) \\ &= 0.25(c-x)(\lambda-1)(2q-1) \end{aligned} \tag{27}$$

and given that in this case $q < 0.5$, the loss averse DM exhibits the imprudent choice.

We now present the calculations of the expected utility of the lotteries designed to elicit third order risk attitudes accounting for small probability outcomes as described in the paper, that is, lottery pair $B_{3,sp}$, and $A_{3,sp}$.

$$\begin{aligned} U(B_{3,sp}|B_{3,sp}) &= p\mu\left(x - px - pc - (1-2p)z\right) \\ &+ \frac{p}{2}\mu\left(c + e - px - pc - (1-2p)z\right) \\ &+ \frac{p}{2}\mu\left(c - e - px - pc - (1-2p)z\right) \\ &+ (1-2p)\mu\left(z - px - pc - (1-2p)z\right) \end{aligned} \tag{28}$$

$$
\begin{aligned}
U(A_{3,sp}|A_{3,sp}) &= p\mu\,(c - px - pc - (1-2p)z) \\
&+ \frac{p}{2}\mu\,(x + e - px - pc - (1-2p)z) \\
&+ \frac{p}{2}\mu\,(x - e - px - pc - (1-2p)z) \\
&+ (1-2p)\mu\,(z - px - pc - (1-2p)z)
\end{aligned} \tag{29}
$$

Even though the last term can be ignored as it is common in both lotteries, there is not an unambiguous choice in this case between $B_{3,sp}$ and $A_{3,sp}$.

We also present the calculations of the expected utility of the lotteries designed to elicit prudence and the reverse preference accounting for equal kurtosis, $B_{3,sk}$ and $A_{3,sk}$, (i.e. restriction $z = \frac{x+c}{2}$ is imposed).

$$
\begin{aligned}
U(B_{3,sk}|B_{3,sk}) &= p\mu\,(x - 0.5(x+c)) \\
&+ \frac{p}{2}\mu\,(c + e - 0.5(x+c)) \\
&+ \frac{p}{2}\mu\,(c - e - 0.5(x+c))
\end{aligned} \tag{30}
$$

$$
\begin{aligned}
U(A_{3,sk}|A_{3,sk}) &= p\mu\,(c - 0.5(x+c)) \\
&+ \frac{p}{2}\mu\,(x + e - 0.5(x+c)) \\
&+ \frac{p}{2}\mu\,(x - e - 0.5(x+c))
\end{aligned} \tag{31}
$$

where it holds $U(B_{3,sk}|B_{3,sk}) = U(A_{3,sk}|A_{3,sk})$ and the DM is indifferent between the two.

We now present the case of the lottery pair designed to elicit risk apportionment of order 4, i.e., $B_4 = c + [\widetilde{\varepsilon_1}, \widetilde{\varepsilon_2}]$ and $A_4 = c + [0, \widetilde{\varepsilon_1} + \widetilde{\varepsilon_2}]$, where the two independent and symmetric zero-mean risks are $\widetilde{\varepsilon_1} = [e_1, -e_1]$ and $\widetilde{\varepsilon_2} = [e_2, -e_2]$. In this case, the expected value of both lotteries is the same and it is equal to c. The calculation of the expected utilities will depend on whether the size of the two independent risks are the same or not. However, the results are qualitatively the same and we only present below the case where they differ, i.e. $e_1 > e_2$.

$$\begin{aligned} U(B_4|B_4) &= 0.25\mu\,(e_1) + 0.25\mu\,(-e_1) + 0.25\mu\,(e_2) + 0.25\mu\,(-e_2) \\ &= -0.25\,(\lambda-1)\,e_1 - 0.25\,(\lambda-1)\,e_2 \end{aligned} \tag{32}$$

$$\begin{aligned} U(A_4|A_4) &= 0.5\mu\,(0) + 0.125\mu\,(e_1+e_2) + 0.125\mu\,(e_1-e_2) + 0.125\mu\,(-e_1+e_2) + 0.125\mu\,(-e_1-e_2) \\ &= -0.25\,(\lambda-1)\,e_1. \end{aligned} \tag{33}$$

Assuming $\lambda > 1$, the result that $U(B_4|B_4) < U(A_4|A_4)$ implies the DM would exhibit (anti)risk apportionment of order 4 and lottery choice consistent with intemperance.

Appendix C Predictions under the KR preferred personal equilibrium (PPE)

We recall Definitions 1 and 2

Definition 1 Given a choice set $\mathcal{L}$, a lottery $L \in \mathcal{L}$ is a *personal equilibrium* (PE) if $U(L|L) \geq U(L'|L)$ for all $L' \in \mathcal{L}$.

Definition 2 Given a choice set $\mathcal{L}$, a lottery $L \in \mathcal{L}$ is a *preferred personal equilibrium* (PPE) if it is a PE and if for any other PE L' it holds $U(L|L) \geq U(L'|L')$.

C.1 Risk apportionment of order three

$$\begin{aligned}
U(B_3|B_3) &= \frac{1}{2}\left(\frac{1}{2}\mu(0)+\frac{1}{4}\mu(c+e-x)+\frac{1}{4}\mu(c-e-x)\right) \\
&+ \frac{1}{4}\left(\frac{1}{2}\mu(x-c-e)+\frac{1}{4}\mu(0)+\frac{1}{4}\mu(-2e)\right) \\
&+ \frac{1}{4}\left(\frac{1}{2}\mu(x-c+e)+\frac{1}{4}\mu(2e)+\frac{1}{4}\mu(0)\right) \\
&= \frac{1}{2}\left(\frac{1}{4}(c+e-x)+\frac{1}{4}(c-e-x)\right) \\
&+ \frac{1}{4}\left(-\frac{1}{2}\lambda(c-x+e)-\frac{1}{4}\lambda 2e\right) \\
&+ \frac{1}{4}\left(-\frac{1}{2}\lambda(c-x-e)+\frac{1}{4}2e\right) \\
&= \frac{c}{4}-\frac{x}{4}-\frac{\lambda c}{4}+\frac{\lambda x}{4}+\frac{e}{8}-\frac{\lambda e}{8}
\end{aligned}$$

$$\begin{aligned}
U(A_3|A_3) &= \frac{1}{2}\left(\frac{1}{2}\mu(0)+\frac{1}{4}\mu(x+e-c)+\frac{1}{4}\mu(x-e-c)\right) \\
&+ \frac{1}{4}\left(\frac{1}{2}\mu(c-e-x)+\frac{1}{4}\mu(0)+\frac{1}{4}\mu(-2e)\right) \\
&+ \frac{1}{4}\left(\frac{1}{2}\mu(c+e-x)+\frac{1}{4}\mu(2e)+\frac{1}{4}\mu(0)\right) \\
&= \frac{1}{2}\left(-\frac{1}{4}\lambda(c-x-e)-\frac{1}{4}\lambda(c-x+e)\right) \\
&+ \frac{1}{4}\left(\frac{1}{2}(c-x-e)-\frac{1}{4}\lambda 2e\right) \\
&+ \frac{1}{4}\left(\frac{1}{2}(c+e-x)+\frac{1}{4}2e\right) \\
&= \frac{c}{4}-\frac{x}{4}-\frac{\lambda c}{4}+\frac{\lambda x}{4}+\frac{e}{8}-\frac{\lambda e}{8}
\end{aligned}$$

$$
\begin{aligned}
U(A_3|B_3) &= \frac{1}{2}\left(\frac{1}{2}\mu(c-x)+\frac{1}{4}\mu(e)+\frac{1}{4}\mu(-e)\right)\\
&+ \frac{1}{4}\left(\frac{1}{2}\mu(e)+\frac{1}{4}\mu(x-c+2e)+\frac{1}{4}\mu(x-c)\right)\\
&+ \frac{1}{4}\left(\frac{1}{2}\mu(-e)+\frac{1}{4}\mu(x-c)+\frac{1}{4}\mu(x-c-2e)\right)\\
&= \frac{1}{2}\left(\frac{1}{2}(c-x)+\frac{1}{4}e-\frac{1}{4}\lambda e\right)\\
&+ \frac{1}{4}\left(\frac{1}{2}e+\frac{1}{4}(x-c+2e)-\frac{1}{4}\lambda(c-x)\right)\\
&+ \frac{1}{4}\left(-\frac{1}{2}\lambda e-\frac{1}{4}\lambda(c-x)-\frac{1}{4}\lambda(c-x+2e)\right)\\
&= \frac{3c}{16}-\frac{3x}{16}-\frac{\lambda 3c}{16}+\frac{\lambda 3x}{16}+\frac{3e}{8}-\frac{3\lambda e}{8}
\end{aligned}
$$

$$
\begin{aligned}
U(B_3|A_3) &= \frac{1}{2}\left(\frac{1}{2}\mu(x-c)+\frac{1}{4}\mu(e)+\frac{1}{4}\mu(-e)\right)\\
&+ \frac{1}{4}\left(\frac{1}{2}\mu(e)+\frac{1}{4}\mu(c-x+2e)+\frac{1}{4}\mu(c-x)\right)\\
&+ \frac{1}{4}\left(\frac{1}{2}\mu(-e)+\frac{1}{4}\mu(c-x)+\frac{1}{4}\mu(c-x-2e)\right)\\
&= \frac{1}{2}\left(-\frac{1}{2}\lambda(c-x)+\frac{1}{4}e-\frac{1}{4}\lambda e\right)\\
&+ \frac{1}{4}\left(\frac{1}{2}e+\frac{1}{4}(c-x+2e)+\frac{1}{4}(c-x)\right)\\
&+ \frac{1}{4}\left(-\frac{1}{2}\lambda e+\frac{1}{4}(c-x)-\frac{1}{4}\lambda(x-c+2e)\right)\\
&= \frac{3c}{16}-\frac{3x}{16}-\frac{\lambda 3c}{16}+\frac{\lambda 3x}{16}+\frac{3e}{8}-\frac{3\lambda e}{8}
\end{aligned}
$$

Combining the relations together we get that both lotteries A_3 and B_3 are *personal equilibria* when it holds that:

$$U(A_3|A_3) \geq U(B_3|A_3)$$

and

$$U(B_3|B_3) \geq U(A_3|B_3)$$

This is satisfied when

$$
\begin{aligned}
U(A_3|A_3) \geq U(B_3|A_3) \quad &\Rightarrow \quad \frac{c}{4} - \frac{x}{4} - \frac{\lambda c}{4} + \frac{\lambda x}{4} + \frac{e}{8} - \frac{\lambda e}{8} \\
&\geq \quad \frac{3c}{16} - \frac{3x}{16} - \frac{\lambda 3c}{16} + \frac{\lambda 3x}{16} + \frac{3e}{8} - \frac{3\lambda e}{8} \\
&\Rightarrow \quad \frac{1}{4} \geq \frac{3}{16}
\end{aligned}
$$

which always holds (similarly for B_3). Therefore, the preferred personal equilibrium would predict that the decision maker is indifferent between B_3 and A_3, since $U(B_3|B_3) = U(A_3|A_3)$ and therefore, CPE and PPE yield the same prediction.

C.2 Risk apportionment of order four

Here we work with the hypothesis that $e_1 > e_2$.

$$
\begin{aligned}
U(B_4|B_4) \quad &= \quad \frac{1}{4}\left(\frac{1}{4}\mu(0) + \frac{1}{4}\mu(-2e1) + \frac{1}{4}\mu(e2 - e1) + \frac{1}{4}\mu(-e1 - e2)\right) \\
&+ \quad \frac{1}{4}\left(\frac{1}{4}\mu(2e1) + \frac{1}{4}\mu(0) + \frac{1}{4}\mu(e1 + e2) + \frac{1}{4}\mu(e1 - e2)\right) \\
&+ \quad \frac{1}{4}\left(\frac{1}{4}\mu(e1 - e2) + \frac{1}{4}\mu(-e1 - e2) + \frac{1}{4}\mu(0) + \frac{1}{4}\mu(-2e2)\right) \\
&+ \quad \frac{1}{4}\left(\frac{1}{4}\mu(e1 + e2) + \frac{1}{4}\mu(e2 - e1) + \frac{1}{4}\mu(2e2) + \frac{1}{4}\mu(0)\right) \\
&= \quad \frac{1}{4}\left(-\frac{1}{4}\lambda 2e_1 - \frac{1}{4}\lambda(e1 - e2) - \frac{1}{4}\lambda(e1 + e2)\right) \\
&+ \quad \frac{1}{4}\left(\frac{1}{4}2e1 + \frac{1}{4}(e1 + e2) + \frac{1}{4}(e1 - e2)\right) \\
&+ \quad \frac{1}{4}\left(\frac{1}{4}(e1 - e2) - \frac{1}{4}\lambda(e1 + e2) - \frac{1}{4}\lambda 2e2\right) \\
&+ \quad \frac{1}{4}\left(\frac{1}{4}(e1 + e2) - \frac{1}{4}\lambda(e1 - e2) + \frac{1}{4}2e2\right) \\
&= \quad -\frac{6\lambda e1}{16} + \frac{6e1}{16} + \frac{2e2}{16} - \frac{2\lambda e2}{16}
\end{aligned}
$$

$$
\begin{aligned}
U(A_4|A_4) &= \frac{1}{8}\left(\frac{1}{8}\mu(0)+\frac{1}{8}\mu(e1+e2)+\frac{1}{8}\mu(e1-e2)+\frac{1}{8}\mu(e2-e1)+\frac{1}{8}\mu(-e1-e2)\right)\\
&+ \frac{1}{8}\left(\frac{1}{8}\mu(-e1-e2)+\frac{1}{8}\mu(0)+\frac{1}{8}\mu(-2e2)+\frac{1}{8}\mu(-2e1)+\frac{1}{8}\mu(-2e1-2e2)\right)\\
&+ \frac{1}{8}\left(\frac{1}{8}\mu(-e1+e2)+\frac{1}{8}\mu(2e2)+\frac{1}{8}\mu(0)+\frac{1}{8}\mu(-2e1+2e2)+\frac{1}{8}\mu(-2e1)\right)\\
&+ \frac{1}{8}\left(\frac{1}{8}\mu(e1-e2)+\frac{1}{8}\mu(2e1)+\frac{1}{8}\mu(2e1-2e2)+\frac{1}{8}\mu(0)+\frac{1}{8}\mu(-2e2)\right)\\
&+ \frac{1}{8}\left(\frac{1}{8}\mu(e1+e2)+\frac{1}{8}\mu(2e1+2e2)+\frac{1}{8}\mu(2e1)+\frac{1}{8}\mu(2e2)+\frac{1}{8}\mu(0)\right)\\
&= \frac{1}{8}\left(\frac{1}{8}(e1+e2)+\frac{1}{8}(e1-e2)-\frac{1}{8}\lambda(e1-e2)-\frac{1}{8}\lambda(e1+e2)\right)\\
&+ \frac{1}{8}\left(\frac{1}{8}\lambda(e1+e2)-\frac{1}{8}\lambda 2e2-\frac{1}{8}\lambda 2e1-\frac{1}{8}\lambda(2e1+2e2)\right)\\
&+ \frac{1}{8}\left(-\frac{1}{8}\lambda(e1-e2)+\frac{1}{8}2e2-\frac{1}{8}\lambda(2e1-2e2)-\frac{1}{8}\lambda 2e1\right)\\
&+ \frac{1}{8}\left(\frac{1}{8}(e1-e2)+\frac{1}{8}2e1+\frac{1}{8}(2e1-2e2)-\frac{1}{8}\lambda 2e2\right)\\
&+ \frac{1}{8}\left(\frac{1}{8}(e1+e2)+\frac{1}{8}(2e1+2e2)+\frac{1}{8}2e1+\frac{1}{8}2e2\right)\\
&= -\frac{6\lambda e1}{16}+\frac{6e1}{16}+\frac{e2}{16}-\frac{\lambda e2}{16}
\end{aligned}
$$

In what follows we assume that $2e_2 - e_1 \geq 0$ which means that $e_1 - 2e_2 < 0$ (the result is symmetric if one assumes the reverse).

$$
\begin{aligned}
U(B_4|A_4) &= \frac{1}{2}\left(\frac{1}{4}\mu(e1)+\frac{1}{4}\mu(-e1)+\frac{1}{4}\mu(e2)+\frac{1}{4}\mu(-e2)\right) \\
&+ \frac{1}{8}\left(\frac{1}{4}\mu(-e2)+\frac{1}{4}\mu(-2e1-e2)+\frac{1}{4}\mu(-e1)+\frac{1}{4}\mu(-2e2-e1)\right) \\
&+ \frac{1}{8}\left(\frac{1}{4}\mu(e2)+\frac{1}{4}\mu(-2e1+e2)+\frac{1}{4}\mu(2e2-e1)+\frac{1}{4}\mu(-e1)\right) \\
&+ \frac{1}{8}\left(\frac{1}{4}\mu(2e1-e2)+\frac{1}{4}\mu(-e2)+\frac{1}{4}\mu(e1)+\frac{1}{4}\mu(e1-2e2)\right) \\
&+ \frac{1}{8}\left(\frac{1}{4}\mu(2e1+e2)+\frac{1}{4}\mu(e2)+\frac{1}{4}\mu(e1+2e2)+\frac{1}{4}\mu(e1)\right) \\
&= \frac{1}{2}\left(\frac{1}{4}e1-\frac{1}{4}\lambda e1+\frac{1}{4}e2-\frac{1}{4}\lambda e2\right) \\
&+ \frac{1}{8}\left(-\frac{1}{4}\lambda e2-\frac{1}{4}\lambda(2e1+e2)-\frac{1}{4}\lambda e1-\frac{1}{4}\lambda(2e2+e1)\right) \\
&+ \frac{1}{8}\left(\frac{1}{4}e2-\frac{1}{4}\lambda(2e1-e2)+\frac{1}{4}(2e2-e1)-\frac{1}{4}\lambda e1\right) \\
&+ \frac{1}{8}\left(\frac{1}{4}(2e1-e2)-\frac{1}{4}\lambda e2+\frac{1}{4}e1-\frac{1}{4}\lambda(2e2-e1)\right) \\
&+ \frac{1}{8}\left(\frac{1}{4}(2e1+e2)+\frac{1}{4}e2+\frac{1}{4}(e1+2e2)+\frac{1}{4}e1\right) \\
&= \frac{10e1}{32}+\frac{10e2}{32}-\frac{10\lambda e1}{32}-\frac{10\lambda e2}{32}
\end{aligned}
$$

$$
\begin{aligned}
U(A_4|B_4) &= \frac{1}{4}\left(\frac{1}{2}\mu(-e1)+\frac{1}{8}\mu(e2)+\frac{1}{8}\mu(-e2)+\frac{1}{8}\mu(e2-2e1)+\frac{1}{8}\mu(-2e1-e2)\right)\\
&+ \frac{1}{4}\left(\frac{1}{2}\mu(e1)+\frac{1}{8}\mu(e2+2e1)+\frac{1}{8}\mu(2e1-e2)+\frac{1}{8}\mu(e2)+\frac{1}{8}\mu(-e2)\right)\\
&+ \frac{1}{4}\left(\frac{1}{2}\mu(-e2)+\frac{1}{8}\mu(e1)+\frac{1}{8}\mu(e1-2e2)+\frac{1}{8}\mu(-e1)+\frac{1}{8}\mu(-e1-2e2)\right)\\
&+ \frac{1}{4}\left(\frac{1}{2}\mu(e2)+\frac{1}{8}\mu(e1+2e2)+\frac{1}{8}\mu(e1)+\frac{1}{8}\mu(-e1+2e2)+\frac{1}{8}\mu(-e1)\right)\\
&= \frac{1}{4}\left(\frac{1}{2}\lambda e1+\frac{1}{8}e2-\frac{1}{8}\lambda e2-\frac{1}{8}\lambda(2e1-e2)-\frac{1}{8}\lambda(2e1+e2)\right)\\
&+ \frac{1}{4}\left(\frac{1}{2}e1+\frac{1}{8}(e2+2e1)+\frac{1}{8}(2e1-e2)+\frac{1}{8}(e2)-\frac{1}{8}\lambda e2\right)\\
&+ \frac{1}{4}\left(-\frac{1}{2}\lambda e2+\frac{1}{8}(e1)-\frac{1}{8}\lambda(2e2-e1)-\frac{1}{8}\lambda e1-\frac{1}{8}\lambda(e1+2e2)\right)\\
&+ \frac{1}{4}\left(\frac{1}{2}e2+\frac{1}{8}(e1+2e2)+\frac{1}{8}e1+\frac{1}{8}(-e1+2e2)-\frac{1}{8}\lambda e1\right)\\
&= \frac{10e1}{32}+\frac{10e2}{32}-\frac{10\lambda e1}{32}-\frac{10\lambda e2}{32}
\end{aligned}
$$

Combining the relations together we get that both lotteries A_4 and B_4 are *personal equilibria* when it holds that:

$$U(A_4|A_4) \geq U(B_4|A_4)$$

and

$$U(B_4|B_4) \geq U(A_4|B_4)$$

condition which is satisfied whenever $\lambda > 1$. Therefore, the preferred personal equilibrium would predict that the decision maker prefers lottery A_4, since $U(A_4|A_4) > U(B_4|B_4)$ whenever $\lambda > 1$, and therefore, CPE and PPE yield the same prediction.

Appendix D Tasks from Bleichrodt and van Bruggen (2022)

Order	Task	Risk Apportionment Option	Reverse Option
	1	[4, 11 + [3, -3]]	[11, 4 + [3, -3]]
	2	[3, 9 + [2, -2]]	[9, 3 + [2, -2]]
	3	[5, 8 + [4, -4]]	[8, 5 + [4, -4]]
	4	[5, 10 + [3, -3]]	[10, 5 + [3, -3]]
	5	[3, 8 + [1, -1]]	[8, 3 + [1, -1]]
3	6	[5, 9 + [4, -4]]	[9, 5 + [4, -4]]
	7	[6, 12 + [5, -5]]	[12, 6 + [5, -5]]
	8	[6, 10 + [5, -5]]	[10, 6 + [5, -5]]
	9	[5, 10 + [4, -4]]	[10, 5 + [4, -4]]
	10	[4, 6 + [3, -3]]	[6, 4 + [3, -3]]
	11	[2, 6 + [1, -1]]	[6, 2 + [1, -1]]
	12	[3, 6 + [2, -2]]	[6, 3 + [2, -2]]
	1	7 + [[2,-2],[4,-4]]	7 + [0,[2,-2] + [4,-4]]
	2	7 + [[3,-3],[3,-3]]	7 + [0,[3,-3] + [3,-3]]
	3	5 + [[1,-1],[2,-2]]	5 + [0,[1,-1] + [2,-2]]
	4	5 + [[1,-1],[3,-3]]	5 + [0,[1,-1] + [3,-3]]
	5	8 + [[2,-2],[3,-3]]	8 + [0,[2,-2] + [3,-3]]
4	6	9 + [[2,-2],[6,-6]]	9 + [0,[2,-2] + [6,-6]]
	7	8 + [[3,-3],[4,-4]]	8 + [0,[3,-3] + [4,-4]]
	8	8 + [[2,-2],[5,-5]]	8 + [0,[2,-2] + [5,-5]]
	9	10 + [[3, -3], [6, -6]]	10 + [0, [3, -3] + [6, -6]]
	10	10 + [[4, -4], [5, -5]]	10 + [0, [4, -4] + [5, -5]]
	11	8 + [[1,-1],[6,-6]]	8 + [0,[1,-1] + [6,-6]]
	12	5 + [[2,-2],[2,-2]]	5 + [0,[2,-2] + [2,-2]]

Table D.1: Tasks used in the 50-50 Gains treatment.

Order	Task	Risk Apportionment Option	Reverse Option
	1	(0.14: [20, 50 + [15, -15]], 0.86: 1)	(0.14: [50, 20 + [15, -15]],0.86:1)
	2	(0.16: [22, 80 + [16, -16]], 0.84: 1)	(0.16: [80, 22 + [16, -16]],0.84:1)
	3	(0.18: [30, 70 + [25, -25]], 0.82: 1)	(0.18: [70, 30 + [25, -25]],0.82:1)
	4	(0.10: [30, 60 + [24, -24]], 0.90: 2)	(0.10: [60, 30 + [24, -24]],0.90:2)
	5	(0.10: [54, 96 + [50, -50]], 0.90: 2)	(0.10: [96, 54 + [50, -50]],0.90:2)
3	6	(0.08: [60, 100 + [46, -46]], 0.92: 1)	(0.08: [100, 60 + [46, -46]], 0.92: 1)
	7	(0.06: [42, 88 + [38, -38]], 0.94: 2)	(0.06: [88, 42 + [38, -38]], 0.94: 2)
	8	(0.04: [34, 64 + [20, -20]], 0.96: 2)	(0.04: [64, 34 + [20, -20]], 0.96: 2)
	9	(0.04: [60, 125 + [45, -45]], 0.96: 2)	(0.04: [125, 60 + [45, -45]], 0.96: 2)
	10	(0.02: [55, 150 + [30, -30]], 0.98: 2)	(0.02: [150, 55 + [30, -30]], 0.98: 2)
	11	(0.20: [40, 60 + [28, -28]], 0.80: 1)	(0.20: [60, 40 + [28, -28]], 0.80: 1)
	12	(0.12: [44, 75 + [32, -32]], 0.88: 1)	(0.12: [75, 44 + [32, -32]], 0.88: 1)
	1	(0.08: 70 + [[20, -20], [45, -45]], 0.92: 1)	(0.08: 70 + [0, [20, -20] + [45, -45]], 0.92: 1)
	2	(0.10: 70 + [[28, -28], [30, -30]], 0.90: 2)	(0.10: 70 + [0, [28, -28] + [30, -30]], 0.90: 2)
	3	(0.10: 65 + [[24, -24], [36, -36]], 0.90: 2)	(0.10: 65 + [0, [24, -24] + [36, -36]], 0.90: 2)
	4	(0.12: 32 + [[14, -14], [14, -14]], 0.88: 2)	(0.12: 32 + [0, [14, -14] + [14, -14]], 0.88: 2)
	5	(0.12: 60 + [[22, -22], [26, -26]], 0.88: 2)	(0.12: 60 + [0, [22, -22] + [26, -26]], 0.88: 2)
4	6	(0.14: 75 + [[25, -25], [30, -30]], 0.86: 1)	(0.14: 75 + [0, [25, -25] + [30, -30]], 0.86: 1)
	7	(0.16: 35 + [[10, -10], [20, -20]], 0.84: 1)	(0.16: 35 + [0, [10, -10] + [20, -20]], 0.84: 1)
	8	(0.06: 100 + [[30, -30], [50, -50]], 0.94: 1)	(0.06: 100 + [0, [30, -30] + [50, -50]], 0.94: 1)
	9	(0.04: 85 + [[34, -34], [40, -40]], 0.96: 2)	(0.04: 85 + [0, [34, -34] + [40, -40]], 0.96)
	10	(0.02: 68 + [[18, -18], [42, -42]], 0.98: 1)	(0.02: 68 + [0, [18, -18] + [42, -42]], 0.98: 1)
	11	(0.20: 55 + [[5, -5], [38, -38]], 0.80: 1)	(0.20: 55 + [0, [5, -5] + [38, -38]], 0.80: 1)
	12	(0.08: 58 + [[14, -14], [34, -34]], 0.92: 2)	(0.08: 58 + [0, [14, -14] + [34, -34]], 0.92: 2)

Table D.2: Tasks used in the Small Probabilities Gains treatment.

Appendix E Tasks from Ebert and Weisen (2014).

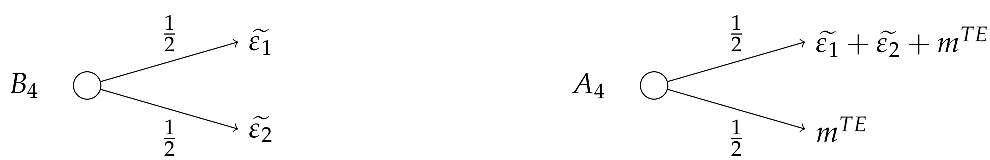
Risk Aversion

with endowment $x = 25$.

Prudence

with different zero-mean risks $\widetilde{\varepsilon}$ for tasks PR_1, PR_2 and PR_3, i.e. $\widetilde{\varepsilon_1} = [0.5, 7; 0.5, -7]$ for PR_1, $\widetilde{\varepsilon_1} = [0.8, 3.5; 0.2, -14]$ for PR_2, and $\widetilde{\varepsilon_1} = [0.8, -3.5; 0.2, 14]$ for PR_3, as well as $x = 20$ for all tasks.

Temperance



with different zero-mean risks $\widetilde{\varepsilon_1}$ and $\widetilde{\varepsilon_2}$ for tasks TE_1 and TE_2, i.e. $\widetilde{\varepsilon_1} = [0.5, 7; 0.5, -7]$, $\widetilde{\varepsilon_2} = [0.5, 3.5; 0.5, -3.5$ for TE_1, and $\widetilde{\varepsilon_1} = [0.8, -2.8; 0.2, 11.1]$, $\widetilde{\varepsilon_2} = [0.8, 2.8; 0.2, -11.1]$ for TE_2, as well as $x = 17.5$ for both tasks.

There were four grids with four different ranges (i) $[-2.5, 2.25]$ and (ii) $[-0.5, 4.25]$ both with an increment of 0.25, as well as (iii) $[-5, 4.5]$ and (iv) $[-3, 6.5]$ with an increment of 0.50. Heinrich and Mayrhofer (2018) used only grids (i) and (ii).

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