Transaction Cost-Optimized Equity Factors Around the World*

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November 21, 2023

Abstract

Firm characteristics like value, momentum, or quality help explain the cross-section of stock returns and have become core pillars in the practice of factor investing. However, when practically implementing factor strategies, transaction costs can significantly impact the corresponding factor portfolios' performances. Using proprietary trading data from a large institutional asset manager, we construct a realistic transaction cost model to investigate how to optimally implement factor portfolios with transaction costs. We provide a framework to optimize factor performance net of transaction costs, but do not overly sacrifice factor exposure at the expense of lower transaction costs. We show that our analysis can be readily extended to a multi-factor setting.

Keywords: Factor investing, Transaction costs, Portfolio optimization.

JEL Classification: C51, C61, G11.

*We thank David Blitz, Lars ter Braak, Khuram Chaudhry, Bart van der Grient, Iman Honarvar, Tobias Hoogteijling, Clint Howard, Maarten Jansen, Stefan Mittnik, Nick Mutsaers, Martin van der Schans, Dmitri Tikhonov, Weili Zhou, Machiel Zwanenburg, participants at the 13th CEQURA Conference on Advances in Financial and Insurance Risk Management in Munich, the J.P. Morgan Macro Quant and Derivatives Conference in London, and the Robeco Quant Research Seminar. This paper expresses the authors' views, which may not coincide with those of Robeco.

1 Introduction

We explore the impact of transaction costs on single and multi-factor equity portfolios, consisting of the six factors momentum, value, quality, size, low volatility and short-term reversal. Our aim is not only to learn about the net profitability of these strategies but to present a portfolio optimization framework for constructing transaction-cost-optimized factor portfolios. The extant literature has mostly focused on producing factors with lower transaction costs, but little has been done to verify if such portfolios retain the desired factor properties. Hence, our framework is centred around the concept of a target factor portfolio and allows tuning how much transaction cost optimization can deviate from the target. The resulting factor portfolios have a better transaction cost profile and preserve their factor integrity. Novy-Marx and Velikov (2016) argue that many high-turnover factors deliver a return spread net of transaction costs that is not statistically significant. However, by properly taking transaction costs into account when building factor portfolios, transaction costs can be substantially reduced. Although value-weighted portfolios mitigate some of the transaction cost concerns faced by equally weighted portfolios, this route is sub-optimal as market capitalization is not a precise measure of liquidity. Moreover, value-weighted factor portfolios come with high portfolio concentration, which works against harvesting the factor's premium in a diversified manner.

In this light, this paper makes five contributions to the factor investing literature. First, we use factor portfolio trading data from a large institutional asset manager to measure transaction costs. Like Frazzini, Israel, and Moskowitz (2018), we show that one can thus more accurately estimate realistic transaction costs, as measured by a trade's implementation shortfall (Perold, 1988). In addition, we consider shorting costs as well as other fixed fees such as tax, foreign exchange and brokerage fees. Indeed, based on these transaction cost estimates we note that the gross benefits of most academic factors are eroded by transaction costs.

Second, we utilize a simple model to optimize the net performance of factor strategies. Our approach to constructing cost-efficient factor portfolios focuses on mean-variance factor portfolio optimization augmented by transaction cost penalties. We thus avoid the expensive trades and excessive turnover that academic factors typically require. Overall, our methodology is flexible with respect to the target portfolios, in terms of both the underlying characteristic and weighting scheme. After optimizing for transaction costs, a high turnover strategy like momentum increases from a net return of -1.52% per annum to 2.18% per annum. For a low turnover strategy like low volatility, the net returns increase from 4.48% to 6.31% per annum. This shows that our approach is not only useful for high turnover strategies, but also for more conservative ones, and one can thus improve net performance across the full spectrum of factor strategies.

Third, the transaction-cost-optimized factor portfolios preserve their factor integrity. For this purpose, we employ a set of measures to monitor how (dis)similar the optimized factor portfolios are relative to the original factors. We find that optimized factors preserve around 80% of their initial factor exposure. Also, the correlation of the optimized factors' portfolio weights with respect to the original factors are above 90% in most cases and the corresponding tracking errors are typically below 2%. Fourth, we explore the factor portfolio performance for different fund size assumptions and document factor portfolios to retain their profitability and factor integrity at different fund sizes.

Lastly, we show that these US results carry over to other regions. In particular, we find that transaction-cost-optimized factors in Europe and Emerging Markets deliver risk-adjusted returns net of transaction costs that are larger than that of the original factors, which are less liquid than those in the US. Considering transaction costs in portfolio construction is even more important for these markets.

The transaction costs literature branches into two segments (Patton and Weller, 2020). The first group resorts to estimating transaction costs using market-wide data; see, for instance, Korajczyk and Sadka (2004) and Novy-Marx and Velikov (2016). The authors use Trade and Quote (TAQ) data to estimate transaction cost model parameters and determine the after-cost performance of relevant asset pricing factors. The second group estimates transaction costs using proprietary data, as performed in the work of Engle, Ferstenberg, and Russell (2012), Frazzini et al. (2018) and de Rossi, Hoch, and Steliaros (2022). The goal of our paper coincides with that of Frazzini et al. (2018), however, there are important differences. Our methodology allows us to explore how portfolios are impacted by transaction cost considerations. Exploring a mean-variance optimization setting with a changing transaction cost penalty presents different portfolios which we can analyze to observe the impact of transaction cost considerations on different strategies. In contrast, Frazzini et al. (2018) minimize transaction costs given a tracking error constraint. We find that using a single metric like tracking error for gauging factor integrity can be misleading. Even though tracking error relative to the original factor is an important consideration, we find it to be too dependent on the factor in question. For example, low volatility can retain its factor score as tracking error increases, whereas the momentum factor sees a large decline in factor score as tracking error increases with higher transaction cost penalties. Hence, our methodology allows us to achieve portfolios with varying transaction cost profiles without restrictions on any given factor integrity metric.

Aiming to improve net performance of factor strategies, our work addresses the concerns raised by Chen and Velikov (2023) and Novy-Marx and Velikov (2016). Chen and Velikov (2023) consider transaction costs in single-factor portfolios and show their impact on 204 stock market anomalies. They argue that the average investor should expect tiny net profits from investing in any individual factor. They attribute the lack of net performance to two key factors, the post-publication performance decline and transaction costs. However, as argued in Frazzini et al. (2018) and observed in our data, actual transaction costs are different from what one might infer when using TAQ-based (or similar) estimates.

Furthermore, Novy-Marx and Velikov (2016) focus on mitigating costs of a single-factor at a time. In contrast, we focus on a standard set of factors and propose a methodology to mitigate the cost of both single and multi-factor portfolios. Instead of defining a set of mechanical transaction cost mitigating procedures, we opt to reduce transaction costs by applying a transaction cost penalty in our mean-variance utility. Combining multiple factors helps to mitigate the impact of transaction costs due to trading diversification across factors. In particular, DeMiguel, Martín-Utrera, Nogales, and Uppal (2020) show that combining characteristics reduces transaction costs since trades in the underlying stocks required to rebalance different factor portfolios can cancel out. As a result, they observe an increase in the number of relevant factor characteristics that explain the cross-section. The multi-factor portfolio framework of DeMiguel et al. (2020) optimizes the factor weights to minimize the impact of transaction costs. Similar to DeMiguel et al. (2020), we also construct transaction-cost optimal multi-factor portfolios, however, we propose a methodology to optimize the stock weights rather than factor weights.

The importance of optimizing factors with respect to transaction costs is further highlighted in the work of Brière, Lehalle, Nefedova, and Raboun (2019) and de Groot, Huij, and Zhou (2012). In Brière et al. (2019), single-factor portfolio performances are calculated net of transaction costs. They use institutional trading data to construct both a parametric and a non-parametric transaction cost model. In the case of the parametric transaction cost model, they build on the work of Frazzini et al. (2018), which closely relates to our approach of modeling transaction costs. While we focus on constructing more efficient factor portfolios, Brière et al. (2019) instead focus on calculating the capacities of factor strategies. de Groot et al. (2012) show that short-term reversal strategies' profitability can largely be attributed to excessive trading in small capitalization stocks. These trades generate large transaction costs, diminishing the net profitability of such strategies. They further show that these costs can be reduced by limiting the stock universe to large caps as well as applying a more sophisticated portfolio construction algorithm to lower turnover.

Notably, while the literature agrees that transaction costs are an important consideration, there is no consensus on the profitability of factor portfolios once we account for transaction costs. On the one hand, we have Chen and Velikov (2023) positing that most factor portfolios have very little returns after publication and transaction costs. For instance, Lesmond, Schill, and Zhou (2004) argue that the momentum factor trades disproportionately in expensive stocks, concluding that the magnitude of the abnormal returns associated with momentum are illusory. On the other hand, Korajczyk and Sadka (2004) use non-proportional transaction costs to show that momentum retains profitability after transaction costs. Also, de Groot et al. (2012) manage to construct profitable reversal strategies, and Novy-Marx and Velikov (2016) obtain similar results for different factor portfolios. More recently, de Rossi et al. (2022) show that, for realistic portfolio sizes, the typical costs of rebalancing a single-factor portfolio are unlikely to erode the factor premium. Taking all these findings into account, one cannot help but wonder what is causing this apparent dissonance in the literature. To shed light on the matter, we show that transaction-cost-optimized factors deliver important economic gains relative to the original factor without jeopardizing the factor integrity. To achieve this, we develop a methodology of constructing transaction-cost efficient portfolios that is independent of the notion of factor integrity. Combining this with a set of factor integrity measures such as factor score, tracking error, or active share, we can better determine what constitutes a portfolio that is transaction-cost efficient as well as consistent with its factor integrity.

The remainder of this paper is organized as follows. Section 2 describes the stock and transaction cost data used to construct our factor portfolios and transaction cost model. Section 3 presents the methodology used in creating factor portfolios, estimating a transaction cost model and the optimization procedure. Section 4 presents our main results comparing the performances of optimized factor portfolios to their unoptimized counterpart. Section 5 compares our findings to Novy-Marx and Velikov (2016). Section 6 concludes.

2 Factor and transaction cost data

2.1 Factor investment universes

Our analysis covers three international equity markets: the United States (US), Europe (EU) and Emerging Markets (EM). Our universe of stocks for the US and Europe are constituents of the Standard and Poor's (S&P) Global Broad Market Index (BMI). This is a broad market index designed to capture the global investable opportunity set by including all listed companies with float-adjusted market capitalization greater than $100 million. This data spans over more than 50 developed and emerging markets, tracking over 99% of each constituent country's available market capitalization. The S&P Global BMI is regionally split into several broad indices. The one used to define our US and European universe is the S&P Developed BMI. The resulting universe consists of 9,834 stocks in the US with a median cross-section of 2,999 stocks dating from January 1980 to March 2023. For Europe, the resulting universe consists of 7,671 stocks with a median cross-section of 2,594 stocks also dating from January 1980 to March 2023.

For emerging markets, we use the constituents of the MSCI Emerging Markets Index. The MSCI Emerging Markets Index captures large and mid capitalization representation across 24 emerging markets countries. With 1,379 constituents as of March 2023, the index covers approximately 85% of the free float-adjusted market capitalization in each country. This results in a total of 3,438 stocks with a median cross-section of 715 stocks dating January 1995 to March 2023. We obtain characteristics, return and volume data from Compustat (for the US) and from Worldscope (for Europe and EM).

2.2 Transaction cost data

We utilize transaction cost data from a large institutional asset manager, covering 269,836 factor investment trades over the period from July 2017 to March 2023. The underlying assets are equities traded on various markets. Each trade is executed by the trading desk in multiple executions and the relevant information is then aggregated on all the executions done for the trade. This includes stock identifier and timestamps of the beginning and the end of the trade, where the beginning of the trade is its arrival time to the trading desk and the end of the trade timestamp is created once the last execution is done and the trade is completed.

To measure the cost of executing a certain trade, we refer to the work of Perold (1988) where implementation shortfall (IS) is defined as the sum of execution cost and opportunity cost. Since all trades are fully executed, we can omit the opportunity cost and define implementation shortfall of a single trade as:

$$IS(q) = \sum_{i=1}^{m} q_i(p_i - p_b), \tag{1}$$

where the total numbers of shares traded, q, is assumed to be executed over m different executions. Each execution trades q_i shares at price p_i. p_b is the initial price of the underlying stock when the trade was submitted. Finally, we calculate the ratio between this cost and the total value of the trade, qp_b, to obtain relative implementation shortfall:

$$IS_{rel} = \frac{IS(q)}{qp_b} \tag{2}$$

where qp_b represents the initial value (size) of the trade in USD. We use the term implementation shortfall throughout the text when referring to the relative implementation shortfall.

The main parameter used in our transaction cost analysis is the trade size as a percentage of median daily volume (MDV) which we calculate over the last 25 trading days. Another important variable is the range volatility, measured as the variance of r_t^{HL} over the last 15 days, where:

$$r_t^{HL} = \ln(H_t) - \ln(L_t), \tag{3}$$

where H_t and L_t are the highest and lowest prices on day t. We also include spreads in our transaction cost considerations which are sourced from FactSet. Missing spread observations are replaced by the cross-sectional median value.

In Table 1, we present the summary statistics for the transaction cost data split across regions. Note that the median trade size is similar to the one reported in Frazzini et al. (2018), with the median traded size for the US being 0.17% of MDV. We also observe that trade size differs substantially across regions, with Europe having larger median MDV of a trade at 0.37%, and EM having the highest at 1.24%. The median transaction cost is also largest for the EM region with a median of 4, 3 and 9 basis points for the US, Europe and EM respectively.

Table 1: Transaction cost data summary statistics across regions

We present the summary statistics of our trading data across regions. The data consists of 269,836 trades from a large institutional asset manager, covering the period from July 2017 to March 2023. The trading data is fairly evenly distributed across regions, with 99,420 trades for the US, 46,905 trades for Europe and 123,511 for EM. We include the trade size as a percentage of median daily volume (MDV), volatility and implementation shortfall. Median daily volume is calculated as the last 25 trading days median of daily volume. Range volatility is calculated using the past 15 days, and we only consider observations with range volatility between 5 and 100%. Implementation shortfall is calculated in accordance to Perold (1988) and is reported as a fraction of the original trade value and capped at 10%.

	United States			Europe			Emerging Markets		
	% MDV	Vol (%)	IS (%)	% MDV	Vola (%)	IS (%)	% MDV	Vola (%)	IS (%)
min	0.00	5.15	-9.94	0.00	5.00	-9.51	0.00	5.01	-9.79
q25	0.04	19.50	-0.69	0.09	18.25	-0.36	0.31	20.02	-0.56
Median	0.17	25.75	0.04	0.37	23.64	0.03	1.24	27.43	0.09
q75	0.61	34.40	0.78	1.33	31.25	0.46	4.84	37.73	0.82
max	296.86	100.00	9.86	470.18	99.99	9.13	476.52	99.98	9.99
Mean	0.90	29.04	0.05	1.91	26.52	0.05	4.76	30.56	0.15
SD	3.07	14.06	1.48	6.78	12.35	0.88	10.82	14.58	1.46
# of obs.	99,420			46,905			123,511		

3 Factor portfolio construction and transaction costs

In this section, we introduce a mean-variance approach to constructing optimized equity factor portfolios, explaining each of the components of the utility function in detail. Furthermore, we define the transaction cost and shorting cost models used and report the obtained parameters.

3.1 Academic factor portfolios

Assume a market with N stocks and K factors. A single-factor portfolio is a 100% long and 100% short portfolio that is typically obtained in a two-step procedure. First, all stocks considered are ranked by the underlying factor characteristic to identify the top and bottom stocks to populate the long and short leg of the factor portfolio. Second, a weighting scheme is assigned, usually with little regard to more than the ranks and size of the stocks chosen to be included in the portfolio. Common choices would be equal-, value- and characteristic rank-weighted portfolios. As we aim to exploit the entire cross-section, we opt for characteristic rank-weighted single factor portfolios. Characteristic rank-weighted portfolios are typically obtained by sorting the stock universe on the characteristic, assigning a rank to each security, and applying a linear weighting scheme across ranks. Then, define the return of the k-th single factor long-short portfolio at time t, denoted by r_t^k as:

$$r_t^k = w_t^{k\top} r_t = w_{1,t}^k r_{1,t} + w_{2,t}^k r_{2,t} + \ldots + w_{N,t}^k r_{N,t}, \tag{4}$$

with

$$\forall k \ \forall t \quad \sum_{i=1}^{N} w_{i,t}^k = 0, \ \sum_{i=1}^{N} |w_{i,t}^k| = 2, \tag{5}$$

where $w_{i,t}^k$ is the weight of stock i in the k-th single-factor long-short portfolio and $r_{i,t}$ the return on stock i at time t. w_t^k is a vector of the k-th single-factor long-short portfolio weights $w_{i,t}^k$, and r_t is the vector of stock returns $r_{i,t}$ at time t.

3.2 Mean-variance framework with transaction costs

Having constructed the standard academic single factor portfolios, we introduce a mean-variance optimization framework for constructing the weights of the optimized single-factor long-short portfolios, $w_{i,t}^k$. Define a mean-variance utility maximization problem with transaction costs on a single-factor long-short portfolio as:

$$\underset{w_{i,t}^k}{\operatorname{argmax}} \ \mathbb{E}_{t-1}\Big[w_t^{k\top} r_t\Big] - \frac{\gamma}{2} Var_{t-1}\Big[w_t^{k\top} r_t\Big] - \delta \mathbb{E}_{t-1}\Big[\Delta w_t^{k\top} TC\Big(\Delta w_t^k\Big)\Big], \tag{6}$$

subject to

$$\forall k \ \forall t \quad \sum_{i=1}^{N} w_{i,t}^k = 0, \ \sum_{i=1}^{N} |w_{i,t}^k| = 2, \tag{7}$$

where $\mathbb{E}_{t-1}\Big[w_t^{k\top} r_t\Big]$ is the expected return of the i-th single-factor portfolio, $Var_{t-1}\Big[w_t^{k\top} r_t\Big]$ its variance, $\mathbb{E}_{t-1}\Big[\Delta w_t^{k\top} TC(\Delta w_t^k)\Big]$ the expected transaction costs, Δw_t^k is the difference in weights defined as $\Delta w_t^k = w_t^k - w_{t-1}^k$, γ the risk aversion coefficient, and δ is the transaction cost parameter.

For a multi-factor portfolio, we can define the same standard mean-variance utility maximization problem with transaction costs, but use expected returns, risk and transaction costs for multi-factor portfolios instead. Equations (6) and (7) defines the entire portfolio construction setup, leaving us with defining the elements of the utility function.

3.2.1 Estimating transaction costs

Transaction cost modeling

Transaction cost modeling has evolved significantly in recent history. Moving away from estimating effective bid-ask spreads and Kyle lambdas, most transaction cost models imply a price impact curve between constant and linear. The price impact curve denotes the change in relative price as a function of quantity bought, usually as a percentage of median daily volume (MDV). We define a transaction cost model following the work best described in Kissell (2014), and the associated I-Star market impact model is widely used in practice today. As a measure of transaction costs we thus consider relative implementation shortfall, that is, implementation shortfall as a fraction of total value of a given trade. Specifically, we estimate a transaction cost model of the following form:

$$TC(\Delta w_{i,t}) = a_1 \sigma_{i,t} \left(\frac{AuM \times \Delta w_{i,t}}{MDV_{i,t}} \right)^{a_2} + \frac{1}{2} s_{i,t} + \epsilon_{i,t} \tag{8}$$

where TC is the relative implementation shortfall, $s_{i,t}$ is the spread, $\sigma_{i,t}$ is the range volatility of stock i at time t, and AuM is the assumption of the assets under management, which we will keep constant and investigate different values. The product $AuM \times \Delta w_{i,t}$ gives us the total amount traded in USD, which we scale by MDV (based on the last 25 days).

Another important aspect of long-short portfolio construction are shorting fees. However, data regarding shorting fees is not publicly available. Cohen, Diether, and Malloy (2007) use proprietary data on shorting fees from a large institutional investor to examine the link between the shorting market and stock prices. For their sample period of September 1999 to August 2003, they show that the mean shorting fee was 2.60% per annum with a median of 1.82%. More recently, Muravyev, Pearson, and Pollet (2022) report that the mean borrow fee is 1.67% per annum with a median of 0.38%, first percentile of 0.25%, 10^{th} percentile of 0.28%, 90^{th} percentile of 3% and 99^{th} percentile of 30% for the sample period of July 2006 to December 2020. Since the sample period of shorting fee data is substantially larger in Muravyev et al. (2022), we will use their reported percentiles to fit a model for shorting costs, see the appendix for estimation details.

Transaction cost function in optimization

In order to make the optimization problem computationally more feasible, we estimate both the model given in Equation (8) together with a simplified transaction cost model, where we will fix $a_2 = 1$. Hence, the only parameter left to be estimated is a_1 which we denote as b_1 in the case of the simplified transaction cost model. The model in Equation (8) will be used to compute the realised transaction costs whereas the simplified model will be used in optimization. For the transaction cost model in Equation (8), a_2 is estimated to be 0.67, see Table 2. This suggests price impact (the rate at which we move the price with respect to traded amount) to be between square root and linear, often found in the literature. For example, Frazzini et al. (2018) use both a square root term and a linear term in their model. Comparing a_1 to b_1, we see that b_1 is higher which would imply higher costs with a linear price impact, meaning lower transaction costs for small trades, and higher for large trades compared to the model in Equation (8). Looking at the R^2, we see a marginal increase from 3.8% to 4.5% in favour of the model in Equation (8). Similar to de Rossi et al. (2022), we impose a cap on transaction costs by assuming that transaction costs are maximized at MDV= 500%; in other words, trading more than 500% of MDV will not further increase the associated transaction costs.

Table 2: Transaction cost model estimation

We present the parameters of the transaction cost model in Equation (8) and the parameter of the transaction cost model used in optimization. The data used consists of 269,836 trades from a large institutional asset manager, covering the period from July 2017 to March 2023. We estimate the model using nonlinear least squares, and report the coefficients obtained. Panel A gives results for the unrestricted model, and Panel B gives the results for the restricted model with a_1 set to 1.

Panel A: Transaction cost model

Parameter	Value	St. Dev.	t-stat	P-value	95% CI	
a_1	0.35	0.01	23.83	0.00	0.32	0.38
a_2	0.67	0.02	32.73	0.00	0.63	0.72

R^2(%)	χ^2	Red. χ^2
4.5	38.57	0.00

Panel B: Simplified transaction cost model with $a_2 = 1$

Parameter	Value	St. Dev.	t-stat	P-value	95% CI	
a_1	0.43	0.02	18.11	0.00	0.38	0.48

R^2(%)	Adj. R^2(%)	$F-stat$	$p(F-stat)$	Durbin-Watson	Jarque-Bera	$p(JB)$
3.8	3.8	245.30	0.00	2.00	182,799.62	1.00

3.2.2 Expected risk and return inputs

Factor portfolio risk model

As the cross-section of our universe is very large, constructing a variance-covariance matrix for each element is unfeasible. Therefore, we reduce the dimension of our problem using a simple linear factor model. Let

$$r_{i,t} = X_{i,t}^{\top} r_t^f + u_{i,t}, \tag{9}$$

describe the factor model where r_t^f is the vector of factor portfolio returns at time t, $r_t^f = \left[r_t^1, r_t^2, \ldots, r_t^K\right]$, $X_{i,t}$ denotes the sensitivity of stock i to factors, and $u_{i,t}$ is the stock specific excess return. Stock variances can then be estimated as

$$Var_{t-1}[r_{i,t}] = X_{i,t}^{\top} \Sigma_t^f X_{i,t} + U_{i,t}, \tag{10}$$

where Σ_t^f is the covariance matrix of factors and $U_{i,t} = Var_{t-1}(u_{i,t})$ the specific risk variance of stock i at time t. Let X_t denote the matrix of all stock sensitivities $X_{i,t}$ and U_t the diagonal matrix of $U_{i,t}$. Then, the factor portfolio variance becomes

$$Var_{t-1}\left[r_t^k\right] = Var_{t-1}\left[w_t^{k\top} r_t\right] = w_t^{k\top} X_t^{\top} \Sigma_t^f X_t w_t^k + w_t^{k\top} U_t w_t^k. \tag{11}$$

Expected factor portfolio returns

Since we aim to construct optimal factor portfolios with respect to transaction costs, our utility function should produce weights equal to the ones of the long-short standard factor portfolio when transaction costs are ignored. To this end, we simply extract the implied expected returns from a given factor portfolio which would return this very factor portfolio as the optimal solution in an unconstrained mean-variance optimization. Adding a transaction cost penalty as an additional friction then helps isolating the corresponding ramifications to the factor profile. By properly managing the impact of the transaction cost penalty, we can thus improve the factors' risk-return profile whilst still targeting the underlying exposures as best as possible. Since we have defined the risk term of utility, and assuming that $w_{t,target}^k$ are the factor weights of a given factor portfolio, we can directly calculate the implied expected returns by considering the following reverse optimization problem. Since $w_{t,target}^k$ are the solutions to Equation (12) below, we have

$$w_{t,target}^{k} = \underset{w_t^k}{\text{argmax}}\, \mathbb{E}_{t-1}\Big[w_t^{k\top} r_t\Big] - \frac{\gamma}{2} Var_{t-1}\Big[w_t^{k\top} r_t\Big], \tag{12}$$

where the variance term is estimated using Equation (11). The first order conditions of the above optimization problem are

$$\mathbb{E}_{t-1}^{k}[r_t] - \gamma w_{t,target}^{k\top} X_t^{\top} \Sigma_t^{f} X_t = 0, \tag{13}$$

which implies that the expected return vector for factor k is given by

$$\mathbb{E}_{t-1}^{k}[r_t] = \gamma w_{t,target}^{k\top} X_t^{\top} \Sigma_t^{f} X_t, \tag{14}$$

where $\mathbb{E}_{t-1}^{k}[r_t]$ denotes the implied expected return vector of the k-th factor portfolio.

4 Transaction-cost-optimized factors

In this section, we introduce factor portfolios and their optimized counterparts for all three regions, US, Europe, and Emerging Markets. We showcase the efficacy of our methodology in mitigating transaction costs while preserving the integrity of factors.

4.1 Single factor portfolios

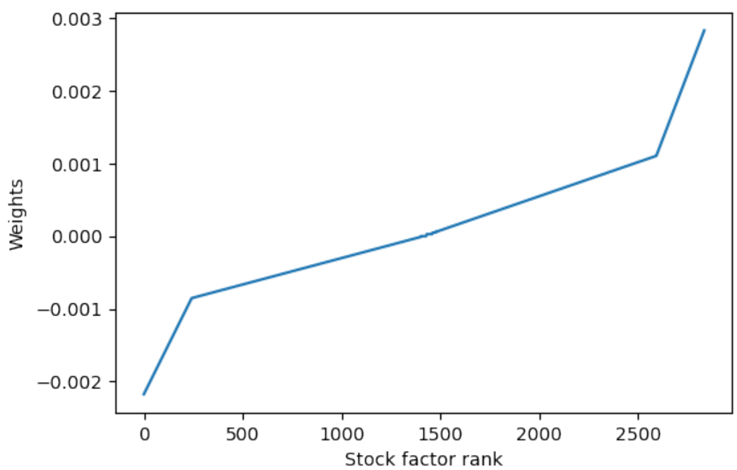
We first explore optimizing single-factor long-short portfolios. Starting from defining a target portfolio, we look into the performance of optimized portfolios and show how our methodology impacts transaction costs.

4.1.1 Academic target factor portfolios

As argued by Chordia, Goyal, and Shanken (2017), firm characteristics are deemed good proxies of expected returns, further substantiated in Kagkadis, Nolte, Nolte, and Vasilas (2023) at the portfolio level. With this in mind, we follow a similar approach to that of Kozak, Nagel, and Santosh (2020) and apply a linear rank-weighting scheme across two size groups, large and small. Large stocks are those in the top 90% of the market capitalization for the region, and small stocks are those in the bottom 10%. To generate market-beta-neutral long-short portfolios for each factor we lever one of the two legs based on their respective market betas. Then, we rescale both legs to retain the total leverage of 2. The resulting portfolio is market neutral but does not have a net investment of zero, i.e, the weights do not sum to zero. This procedure is reiterated each month. The assumed net asset value of the position of our portfolios is $1 billion USD, i.e., $500 million long and short before beta adjustment. An exemplary weighting scheme is depicted in Figure 1.

Figure 1: Target factor portfolio weights

We illustrate the weighting scheme for constructing the target long-short factor portfolios based on a given factor characteristic for the US universe. To depict the weighting scheme, we show the momentum weights for May 2001. The weights are representative of what one would expect for an average portfolio of any factor.



In Table 3, we collect the summary statistics for six salient US long-short factor portfolios following the above construction. We find that momentum is the highest gross yielding factor portfolio, with an annualized gross return of 7.71%. The return of short-term reversal (STR) is second highest at 7.66%, and low volatility is next with 7.22%. Conversely, size is the lowest yielding portfolio, with a return of 0.81%. Value is next in line, with an annualized gross return of 2.49%, about half of quality with 6.25%. Turning from gross to net factor performance, the associated transaction costs follow intuition in many cases. For example, momentum profits are erased, seemingly confirming the notions of Lesmond et al. (2004) that trading costs eliminate the profits to momentum portfolios at small fund sizes. Momentum and STR entail the highest turnover due to their fast-paced nature, with turnover figures of 534.6% and 1,659.2%, respectively. In turn, this turnover results in high annualized transaction costs of 7.19% and 31.58%. Although size has the lowest turnover at 134.8%, it still comes with considerable transaction costs at 2.08%, clearly showcasing that overweighting the smaller side of the universe comes at a hefty cost. The cheapest factors to trade were quality and low volatility, both due to their low turnovers and tendency to trade relatively larger stocks. Similar to Muravyev et al. (2022), we find momentum and quality to have the highest shorting costs at 1.89% and 1.76%, respectively, whereas value and size observe the lowest shorting costs at 0.88% and 0.18%. We find fixed fees (which include taxes, brokerage fees and foreign exchange fees) to mainly depend on turnover, with little difference between factor portfolios with similar turnover.

Our transaction cost decomposition in Panel B of Table 3 showcases how significant the two components, market impact and spread, are in estimating transaction costs. The ratio between spread and market impact clearly has a trade size dependency, further emphasizing how expensive trading illiquid stocks is. For the very liquid market portfolio, the market impact becomes small, showing that the spread component of transaction costs is not to be ignored when considering portfolios investing in liquid assets. Since the target factor portfolios are constructed with no regard to transaction costs, this naive portfolio construction proves detrimental to the resulting net performance. Apart from low volatility and quality, each academic factor portfolio observes a negative net return. STR reverses its profits to an average loss of −25.39% and the other factors, apart from low volatility, end up around zero. Low volatility becomes the best performing factor portfolio with a net Sharpe ratio of 0.39.

4.1.2 How to monitor factor integrity

Before optimizing factors, we elaborate on the notion of factor integrity and our approach to measuring it. Portfolio optimization with respect to transaction cost typically focuses on reducing turnover and the associated transaction costs to ultimately improve net (risk-adjusted) returns. While this is a reasonable goal, it might result in failing to capture the very factors' nature. In order to measure the closeness to the original target factor portfolio, we employ several measures. First, a key measure is the *factor score*, computed as the weighted average of the percentile characteristic rank of all stocks, normalized by the weighted average of the percentile characteristic rank of all stocks for the target portfolio. Second, the deviation from the target factor portfolio weights can be gauged by the *active share*, which is the

Table 3: Long-short factor portfolio summary statistics: US

We present the gross and net performance of the US market portfolio as well as of each long-short US factor portfolio. The considered sample period is January 1985 to March 2023. For each portfolio we present the gross and net return, Sharpe ratio, volatility, turnover, shorting costs, fixed fees and transaction costs decomposed into two components, spread and market impact. All numbers are annualized.

	Market	Momentum	Value	Quality	Size	Lowvol	STR
Panel A: Gross performance							
Gross return %	11.78	7.71	2.49	6.25	0.81	7.22	7.66
Volatility %	15.36	11.35	10.85	7.66	10.79	11.59	11.34
Gross Sharpe	0.77	0.68	0.23	0.82	0.08	0.62	0.68
Panel B: Transaction costs							
Turnover %	8.30	534.6	222.8	200.2	134.8	143.8	1,659.2
Transaction costs %	0.04	7.19	2.95	2.50	2.08	1.48	31.58
TC Spread %	0.03	2.17	0.95	0.82	0.59	0.59	6.69
TC MI %	0.01	5.02	2.00	1.68	1.49	0.89	24.89
Shorting Costs %		1.89	0.88	1.76	0.18	1.22	0.99
Fixed Fees %	0.01	0.15	0.06	0.06	0.04	0.04	0.48
Panel C: Net performance							
Net return %	11.73	-1.52	-1.40	1.93	-1.49	4.48	-25.39
Net Sharpe	0.76	-0.13	-0.12	0.26	-0.13	0.39	-2.24

absolute difference of weights from the target portfolio; it ranges from zero to two, with zero being an identical portfolio and two a completely different portfolio with no overlap in positions. Third, a related measure is the *transfer coefficient*, which is the correlation of optimized portfolio weights with target portfolio weights. We also report *tracking error* and *number of effective names* to track important portfolio characteristics. Obviously, the higher the transaction cost penalty, the harder it is to stay true to a given factor, and these metrics enable monitoring the degree of factors' integrity.

4.1.3 Optimized factor portfolios

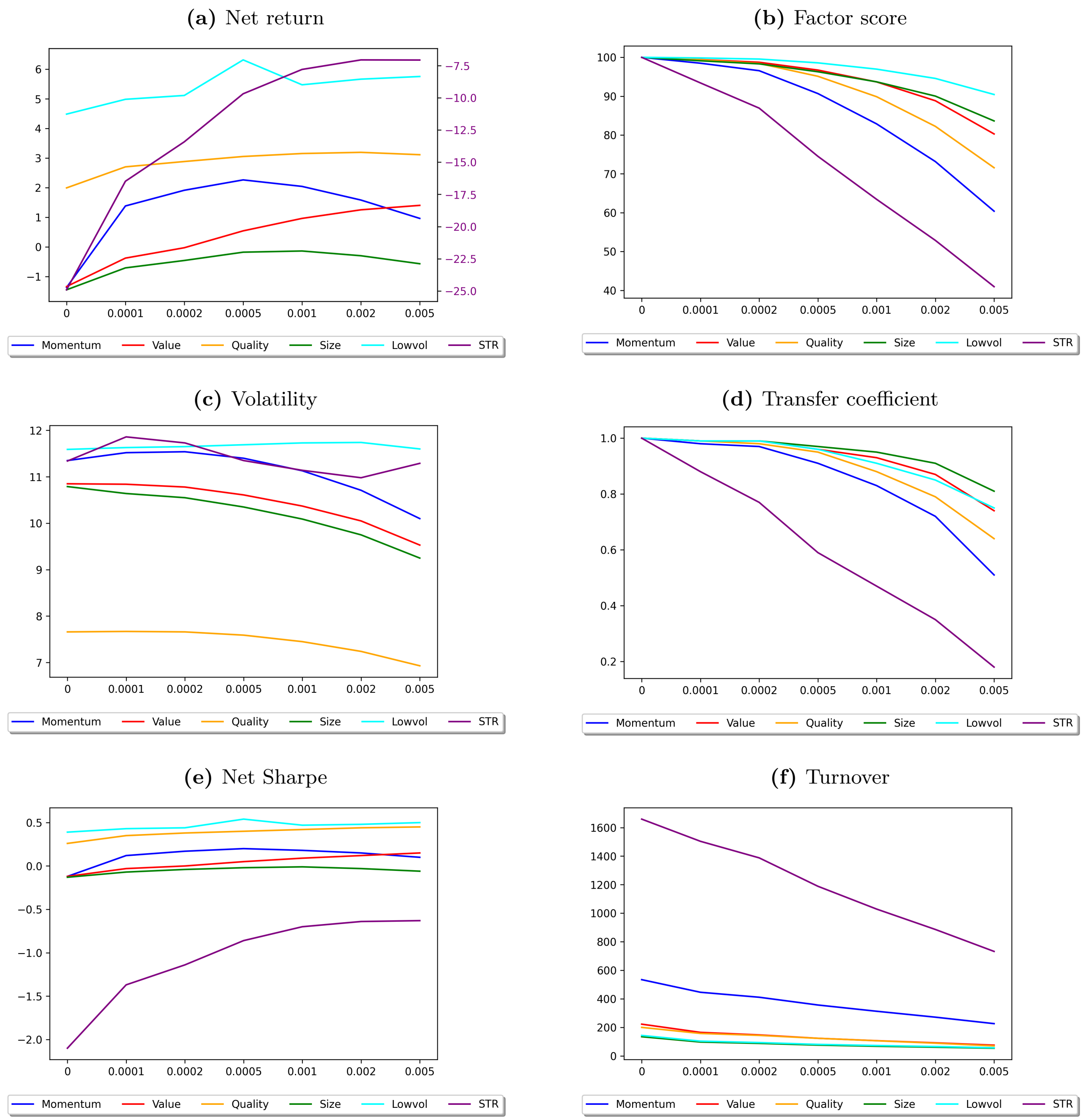
In Table 4, we investigate the optimization of factors with respect to different transaction cost penalty parameters, denoted δ in Equation (6). There are several notable observations that apply across factors. The constructed optimal factor portfolios show a decline in gross returns when δ increases. This outcome is in line with Frazzini, Israel, and Moskowitz (2014) and can be rationalized by the factor score that is dropping with increasing transaction cost penalties. In an ideal scenario, the reduction in gross returns is made up for by the gains in net returns, see, e.g., the momentum factor (with exceptions in the higher δ values). After a certain δ threshold, the gross return simply deteriorates beyond the improvement on the transaction costs side. This results in a concave-shaped net return across deltas, facilitating the choice of the appropriate transaction cost parameter. For most factors, we observe close to constant volatility across different deltas, with a slight decline for $\delta > 0.0005$. As a result, Sharpe ratios yield to similar conclusions as returns, with a constant drop for gross portfolios, and a concave shape for the net portfolios. We experience a sharp decline in turnover and transaction costs across all factors, which comes with an increase in net returns as we increase the transaction cost parameter, see Figure 2.

Judging by net performance alone, one might be tempted to keep increasing the transaction cost penalty, yet factor integrity might be compromised as a result. Indeed, we observe a sharp decline in factor score and transfer coefficient for $\delta > 0.0005$ for most factors. Furthermore, the number of effective names drops (Table 4). These are important guideposts to gauging whether one has drifted too far from the target factor portfolios. Next, we will discuss the specific outcome of transaction cost optimization for each of the six factors.

Momentum For momentum, we observe a rapid decline in gross returns as well as a decay in the factor score which is relatively large compared to other factors. This is consistent with a high-turnover factor like momentum, for which transaction cost penalties are expected to have more of an impact. Frazzini et al. (2014) also find that momentum observes the largest drop in gross return after optimization and the highest net returns out of the three factors (momentum, value, size) they consider. Notably, momentum observes a peak net return and Sharpe ratio for $\delta = 0.0005$, while its factor integrity is still intact; the optimized portfolio is also well diversified, the factor score is above 90%, tracking error and active share are still reasonably low and the transfer coefficient is at 0.91. The top performing portfolio gains 3.70% in net returns over the original target factor portfolio, showing that sacrificing some momentum exposure for reducing costs can yield much better net outcomes. We see that the transaction costs are drastically reduced from 7.19% to 2.70% for the optimized momentum factor. Furthermore, momentum observes the highest shorting costs among all factors, starting from 1.89% for the target portfolio and going down to 1.54% for the optimal portfolio.

Figure 2: Key US factor characteristics by transaction cost penalty

We plot the net return, volatility, net Sharpe, factor score, transfer coefficient and turnover across different deltas, transaction cost penalty parameter from Equation (6), for the US factor portfolios. The period considered is January 1985 to March 2023. Net return, volatility and turnover are reported as percentages and annualized Net Sharpe ratio is also annualized. Net return and Sharpe ratios are calculated using the gross return and estimated transaction costs from our transaction cost model. Factor score is calculated as the weighted average of individual stock scores and given as a fraction of the respective value for the target portfolio. Transfer coefficient is calculated as the correlation between optimal weights for a given δ and target factor weights. Turnover for a given month ranges from 0 to 100%, which equates to a maximum of 1,200% per annum. Since short-term reversal exhibits very low net returns, we plot it on a separate right-hand-side y-axis in Figure 2a.



Value The optimized value factors display very different behaviour when compared to momentum. Their gross returns and factor scores decay much slower, but start from a lower point of 2.49% going down to 2.30% in the optimal case of $\delta = 0.001$. Transaction costs are though significantly reduced from 2.95% for the target portfolio down to 0.67% for the optimized value factor portfolio. We also observe improvements in net Sharpe ratio, going from -0.12 to 0.09 in the optimized case. Both tracking error and active share are comparable to that of the momentum factors. Shorting fees are relatively low at 0.88% and 0.67% for the target and optimized portfolios, respectively. Increasing δ further, net performance increases as well. Yet, for larger values of δ, we observe too low factor scores and too high tracking errors, making value a great example of how overly reducing transaction costs can create a portfolio that no longer represents the targeted factor portfolio.

Quality Quality is one of the lower turnover strategies and offers the least room for improvement as a result. Notwithstanding, we can reduce the required transaction costs from 2.50% to 0.63% in the optimized case ($\delta = 0.001$). The gross performance is reduced by 1.25%, but the net return is still significantly increased, going from 1.93% to 3.11%. The optimized quality portfolio retains a low tracking error of 1.49% and a high transfer coefficient of 0.88. The associated shorting fees are very high (second only to the ones of momentum), standing at 1.76% and 1.22% for the target and optimal factor portfolios, respectively.

Size The size factor observes the lowest target factor gross return out of all factors considered (0.81%). Its turnover is the lowest of all factors, starting at 134.8% and going down to 68.0% in the optimized case ($\delta = 0.001$). Consistent with Brière et al. (2019), we find size to incur lower transaction costs, second only to low volatility. However, when comparing size to other factors of similar turnover, its transaction costs to turnover ratio is high which is a consequence of frequently trading in small stocks. Its net performance increases from the initial -1.49% to -0.17% for the optimized case, rendering the size factor unattractive after transaction costs. Shorting costs are tiny compared to other factors, as size only short-sells large stocks with low shorting costs. We also see that the shorting costs are almost identical across all deltas. This is expected, as the transaction cost penalties apply mostly on the long (and less liquid) leg of the portfolio.

Table 4: Optimized US factor portfolios

We present the summary statistics of different optimized US factor portfolios. The optimized portfolios differ by the transaction cost parameter used in the utility function: $\delta = 0$ corresponds to the long-short target portfolios. Where applicable, all measures are annualized and quoted in percentages. All relative measures such as tracking error and active share are benchmarked to the $\delta = 0$ (target) portfolio. The considered sample period is January 1985 to March 2023.

Momentum

	$\delta = 0$	$\delta = 0.0001$	$\delta = 0.0002$	$\delta = 0.0005$	$\delta = 0.001$	$\delta = 0.002$	$\delta = 0.005$
Panel A: Performance							
Gross return %	7.71	7.57	7.27	6.51	5.52	4.40	3.17
Net return %	-1.52	1.28	1.83	2.18	1.97	1.52	0.91
Volatility %	11.35	11.52	11.54	11.40	11.13	10.71	10.10
Gross Sharpe	0.68	0.66	0.63	0.57	0.50	0.41	0.31
Net Sharpe	-0.13	0.12	0.17	0.20	0.18	0.15	0.10
Panel B: Transaction costs							
Turnover %	534.6	446.2	411.4	357.0	313.2	271.8	226.4
Transaction costs %	7.19	4.43	3.67	2.70	2.10	1.66	1.29
TC Spread %	2.17	1.64	1.44	1.15	0.94	0.77	0.62
TC MI %	5.02	2.79	2.23	1.56	1.16	0.88	0.67
Shorting Costs %	1.89	1.76	1.69	1.54	1.38	1.17	0.91
Fixed Fees %	0.15	0.10	0.08	0.08	0.07	0.06	0.05
Panel C: Factor integrity							
Factor score %	100.00	98.49	96.56	90.67	82.86	73.16	60.38
Tracking error %	0.00	0.56	0.84	1.38	1.97	2.71	3.93
Active Share	0.00	0.25	0.39	0.68	0.97	1.30	1.73
Transfer coefficient	1.00	0.98	0.97	0.91	0.83	0.72	0.51
# of effective names	1489	1448	1426	1376	1279	1119	782

Size

	$\delta = 0$	$\delta = 0.0001$	$\delta = 0.0002$	$\delta = 0.0005$	$\delta = 0.001$	$\delta = 0.002$	$\delta = 0.005$
Panel A: Performance							
Gross return %	0.81	0.57	0.60	0.65	0.55	0.28	-0.08
Net return %	-1.49	-0.75	-0.50	-0.21	-0.17	-0.33	-0.60
Volatility %	10.79	10.64	10.55	10.35	10.09	9.75	9.25
Gross Sharpe	0.08	0.05	0.06	0.06	0.05	0.03	-0.01
Net Sharpe	-0.13	-0.07	-0.04	-0.02	-0.01	-0.03	-0.06
Panel B: Transaction costs							
Turnover %	134.8	97.6	88.2	76.2	68.0	61.4	54.4
Transaction costs %	2.08	1.09	0.88	0.64	0.50	0.40	0.31
TC Spread %	0.59	0.37	0.33	0.27	0.23	0.20	0.17
TC MI %	1.49	0.72	0.56	0.38	0.27	0.20	0.14
Shorting Costs%	0.18	0.18	0.18	0.18	0.18	0.18	0.19
Fixed Fees %	0.04	0.04	0.04	0.03	0.03	0.03	0.03
Panel C: Factor integrity							
Factor score %	100.00	99.11	98.32	96.29	93.69	90.02	83.62
Tracking error %	0.00	0.48	0.73	1.20	1.72	2.37	3.36
Active Share	0.00	0.14	0.22	0.38	0.55	0.77	1.13
Transfer coefficient	1.00	0.99	0.99	0.97	0.95	0.91	0.81
# of effective names	1489	1479	1469	1434	1379	1283	1077

Value

	$\delta = 0$	$\delta = 0.0001$	$\delta = 0.0002$	$\delta = 0.0005$	$\delta = 0.001$	$\delta = 0.002$	$\delta = 0.005$
Panel A: Performance							
Gross return %	2.49	2.02	2.02	2.15	2.30	2.35	2.28
Net return %	-1.40	-0.43	-0.07	0.50	0.92	1.22	1.37
Volatility %	10.85	10.84	10.78	10.61	10.37	10.05	9.53
Gross Sharpe	0.23	0.19	0.19	0.20	0.22	0.23	0.24
Net Sharpe	-0.12	-0.03	-0.00	0.05	0.09	0.12	0.15
Panel B: Transaction costs							
Turnover %	222.8	165.2	147.4	123.8	107.0	92.0	75.6
Transaction costs %	2.95	1.57	1.24	0.88	0.67	0.51	0.37
TC Spread %	0.95	0.62	0.52	0.40	0.33	0.27	0.21
TC MI %	2.00	0.95	0.72	0.48	0.34	0.24	0.16
Shorting Costs %	0.88	0.83	0.80	0.73	0.67	0.60	0.51
Fixed Fees %	0.06	0.05	0.04	0.04	0.04	0.03	0.03
Panel C: Factor integrity							
Factor score %	100.00	99.42	98.73	96.72	93.67	88.82	80.27
Tracking error %	0.00	0.49	0.73	1.21	1.77	2.49	3.48
Active Share	0.00	0.17	0.26	0.46	0.67	0.94	1.34
Transfer coefficient	1.00	0.99	0.99	0.96	0.93	0.87	0.74
# of effective names	1527	1510	1495	1452	1384	1268	1046

Lowvol

	$\delta = 0$	$\delta = 0.0001$	$\delta = 0.0002$	$\delta = 0.0005$	$\delta = 0.001$	$\delta = 0.002$	$\delta = 0.005$
Panel A: Performance							
Gross return %	7.22	6.92	6.83	6.76	6.72	6.72	6.59
Net return %	4.48	4.98	5.11	6.31	5.47	5.66	5.75
Volatility %	11.59	11.63	11.65	11.69	11.73	11.74	11.60
Gross Sharpe	0.62	0.60	0.59	0.58	0.57	0.57	0.57
Net Sharpe	0.39	0.43	0.44	0.54	0.47	0.48	0.50
Panel B: Transaction costs							
Turnover %	143.8	103.8	93.8	81.0	73.1	66.0	57.2
Transaction costs %	1.48	0.76	0.61	0.44	0.36	0.30	0.24
TC Spread %	0.59	0.35	0.30	0.24	0.20	0.17	0.15
TC MI %	0.89	0.41	0.31	0.21	0.16	0.12	0.09
Shorting Costs %	1.22	1.14	1.08	0.97	0.86	0.73	0.57
Fixed Fees %	0.04	0.04	0.03	0.03	0.03	0.03	0.03
Panel C: Factor integrity							
Factor score %	100.00	99.84	99.56	98.58	96.96	94.57	90.43
Tracking error %	0.00	0.33	0.53	0.96	1.39	1.88	2.55
Active Share	0.00	0.16	0.26	0.48	0.71	0.97	1.29
Transfer coefficient	1.00	0.99	0.99	0.96	0.91	0.85	0.75
# of effective names	1,332	1,313	1,293	1,231	1,136	1,016	847

Quality

	$\delta = 0$	$\delta = 0.0001$	$\delta = 0.0002$	$\delta = 0.0005$	$\delta = 0.001$	$\delta = 0.002$	$\delta = 0.005$
Panel A: Performance							
Gross return %	6.25	5.84	5.65	5.30	5.00	4.69	4.25
Net return %	1.93	2.65	2.83	3.01	3.11	3.15	3.08
Volatility %	7.66	7.67	7.66	7.59	7.45	7.24	6.93
Gross Sharpe	0.82	0.76	0.74	0.70	0.67	0.65	0.61
Net Sharpe	0.26	0.35	0.38	0.40	0.42	0.44	0.45
Panel B: Transaction costs							
Turnover %	200.2	157.8	144.2	123.6	106.4	89.6	70.6
Transaction costs %	2.50	1.50	1.22	0.86	0.63	0.45	0.30
TC Spread %	0.82	0.57	0.49	0.38	0.30	0.22	0.16
TC MI %	1.68	0.93	0.73	0.48	0.33	0.22	0.14
Shorting Costs %	1.76	1.64	1.55	1.39	1.22	1.05	0.85
Fixed Fees %	0.06	0.05	0.05	0.04	0.04	0.04	0.03
Panel C: Factor integrity							
Factor score %	100.00	99.37	98.42	95.12	89.86	82.23	71.57
Tracking error %	0.00	0.37	0.58	1.02	1.49	2.05	2.91
Active Share	0.00	0.18	0.30	0.56	0.84	1.16	1.56
Transfer coefficient	1.00	0.99	0.98	0.95	0.88	0.79	0.64
# of effective names	1522	1497	1476	1419	1336	1207	972

STR

	$\delta = 0$	$\delta = 0.0001$	$\delta = 0.0002$	$\delta = 0.0005$	$\delta = 0.001$	$\delta = 0.002$	$\delta = 0.005$
Panel A: Performance							
Gross return %	7.66	5.61	4.46	3.15	2.20	1.08	-0.28
Net return %	-25.39	-16.88	-13.77	-10.02	-8.08	-7.31	-7.29
Volatility %	11.34	11.86	11.73	11.35	11.14	10.98	11.29
Gross Sharpe	0.68	0.47	0.38	0.28	0.20	0.10	-0.02
Net Sharpe	-2.10	-1.37	-1.14	-0.86	-0.70	-0.64	-0.63
Panel B: Transaction costs							
Turnover %	1,659.2	1,503.8	1,387.8	1,188.8	1,029	886.2	732.4
Transaction costs %	31.58	21.23	16.99	12.02	9.26	7.38	5.81
TC Spread %	6.69	5.69	4.98	3.89	3.15	2.58	2.09
TC MI %	24.89	15.54	12.02	8.14	6.12	4.80	3.72
Shorting Costs %	0.99	0.85	0.89	0.82	0.73	0.76	0.98
Fixed Fees %	0.48	0.40	0.35	0.33	0.28	0.25	0.22
Panel C: Factor integrity							
Factor score %	100.00	93.41	86.92	74.51	63.43	52.86	40.94
Tracking error %	0.00	1.50	1.93	2.83	3.66	4.40	6.69
Active Share	0.00	0.51	0.75	1.17	1.53	1.87	2.29
Transfer coefficient	1.00	0.88	0.77	0.59	0.47	0.35	0.18
# of effective names	1454	1047	705	406	326	292	143

Low volatility Low volatility has the second lowest turnover, with an annual turnover of the target portfolio at 143.8%. It also exhibits the lowest transaction costs of all factors, which is in line with de Rossi et al. (2022). When optimizing transaction costs its net performance is increased by more than one percent for the $\delta = 0.002$ case. Its volatility, however, slightly increases as well, moderating the improvement in Sharpe ratio. Interestingly, factor score is still above 90% for the last case of $\delta = 0.005$, yet active share is 1.29 and tracking error is at 2.55%. This highlights the importance of using different metrics when considering how close one tracks the target portfolio. Transaction costs are reduced from 1.48% to 0.30%, making the low volatility factor the cheapest optimized factor portfolio to trade.

Short-term reversal The short-term reversal factor exhibits a huge turnover of $1,659.2\%$, leading to a vast transaction cost figure of 31.58% for the target factor portfolio. The corresponding gross portfolio return of 7.66% is thus reduced to a net return of -25.39%, highlighting the need to optimize the cost of implementing such fast-paced strategies. For the case of $\delta = 0.0005$, we observe a net performance increase of about 15% points, with a tracking error of 2.83%. Yet, despite cutting all these costs, the factor remains deeply in the negative as a standalone long-short portfolio. Similarly, de Rossi et al. (2022) also find that short-term reversal observes negative net returns even in the case of the smallest portfolio size. This is further substantiated in Blitz, Grient, and Honarvar (2023a), where they argue using a single signal to construct short-term reversal results in strategies with negative net returns even under a low transaction cost assumption.

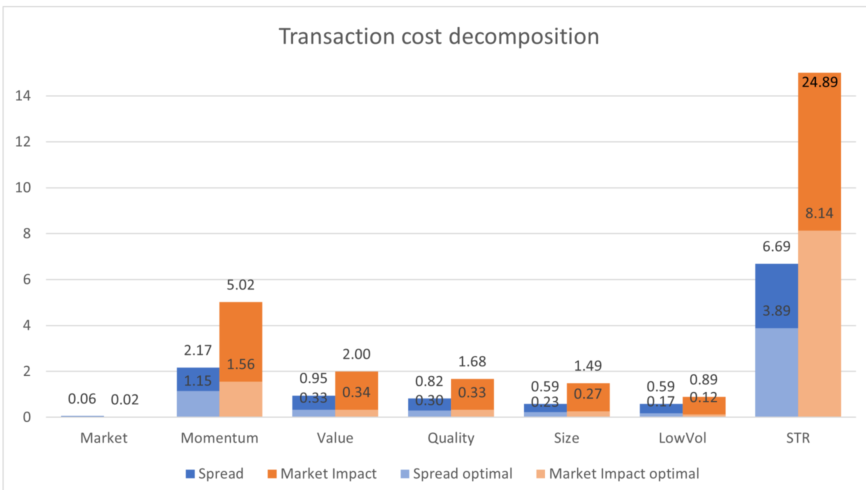
4.1.4 Decomposing transaction costs

Notably, our framework does not require to penalize transaction costs in factor portfolios with equal transaction cost penalties. For a high turnover signal like momentum, it can be more reasonable to penalize transaction costs less, as imposing too strict of a transaction cost penalty would result in a significant loss in factor score and thus exposure. Indeed, momentum shows the second fastest decline in factor score. Conversely, one can be more strict on some factors in terms of penalizing transaction costs but still conserving the integrity of factors.

Clearly, optimizing single-factor portfolios comes with sacrificing some turnover, and thus some factor exposure, in favor of lowering the cost of trading. What makes this optimization setting using a transaction cost model so beneficial is how effective it is in reducing expensive turnover, see Figure 3 which depicts the transaction cost decomposition for all target and optimized factor portfolios. We note the effect of reducing large relative trade sizes as the market impact component experiences a minimum threefold reduction in all cases. Naturally, the spread component can also be reduced by reducing turnover in stocks with higher spread, but not as effectively as the market impact component. This is a consequence of market impact growing with a larger exponent of amount traded than spread, since spread is a fixed cost per share.

Figure 3: Transaction cost decomposition for the US factor portfolios

We decompose the transaction cost of all single factors into the spread and market impact components for the target and optimized portfolios. Transaction costs of the market portfolio and each single factor portfolio is first split into two columns, spread (blue) and market impact (orange). Then, each column presents the transaction costs of a target factor portfolio and the optimized factor portfolio. This is illustrated by a lighter shade of the respective color for the optimized factor portfolio, i.e., light-blue and light-orange correspond to the spread and market impact of the optimal portfolio, respectively. In the case of short-term reversal, the value of market impact (orange) presented is lower than the actual value for better readability. All transaction cost numbers are quoted in percentages. The underlying sample period is January 1985 to March 2023.



4.2 International evidence

The performance improvement we find for US factor portfolios is only amplified for Europe and Emerging Markets. To illustrate, Figure 4 visualizes the net performance improvements that come from using the optimal transaction cost penalties for all six factors. Each bar starts from the theoretical maximum, i.e., the gross return of the target factor portfolio. Accounting for transaction costs, the net performance of the target portfolio is then contrasted with the net performance improvement of the optimal target portfolio. Across regions, we can successfully enhance net returns over the market returns for most factors, with some factors going from negative to positive territory.

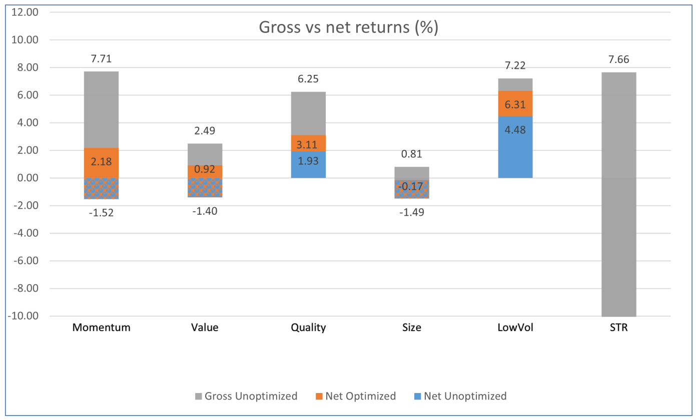
In Europe, the results are similar to that of the US but on a larger scale. The gross and net returns of the momentum target factor portfolios are 11.36% and −0.10%, respectively. After optimization, the net return of the optimized momentum portfolio is 4.25%. For EM, the gross return of the momentum target factor portfolio is close to the US one at 6.95% with a negative net return of −0.26%. After optimization, the net return goes up to 0.87% which is significantly lower than that of the US momentum factor, showcasing how expensive EM stocks are to trade frequently compared to the US. Consistent with de Rossi et al. (2022), the value factor in Europe observes a negative gross return in the target factor portfolio. Conversely, value in EM has a 2.99% gross return. However, the associated net returns stand at −1.67% which can be pushed to 0.02% only (after optimization). Therefore, unlike in the US, we cannot retain the value factor premium by avoiding expensive trades in the EM universe.

Quality factor performance is quite comparable across markets, with Europe observing the highest gross target factor portfolio returns. Net returns of the optimized quality factor portfolio go from 1.10% to 2.68% and 1.02% to 2.38% for the European and EM case, respectively. In contrast to the US, the size factor observes larger gross returns in European and Emerging markets. In both regions, we manage to retain about two thirds of the gross returns after optimization. Despite low volatility being one of the highest performing factors across all markets, this is where the US outperforms the other regions. Looking at the EM results, we again observe the increase of gross returns in the optimized case compared to the target factor portfolio, seemingly resulting in almost very low transaction costs. This again shows how important optimization is in the EM universe. Lastly, short-term reversal leaves a lot to be desired for across markets. For both Europe and EM, the costs of trading this turnover-heavy strategy are just too high to overcome. Such finding is consistent with Blitz et al. (2023a) who argue that the classic short-term reversal factor needs refinement to become profitable. Specifically, the authors suggest controlling for industry and factor momentum effects. Also, the combination of different short-term signals is found an effective means to improving signal efficacy after transaction costs, see Blitz, Hanauer, Honarvar, Huisman, and van Vliet (2023b).

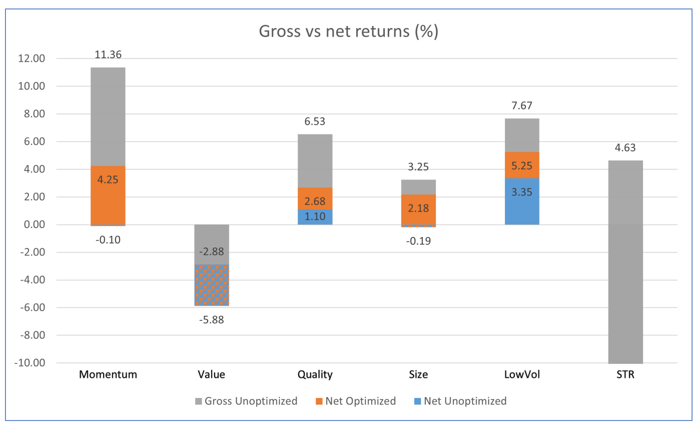
Figure 4: Target vs. optimized factor performance

We plot the gross and net return of target factor portfolios with the net return increase of the optimal portfolio across regions. For each factor portfolio, we show the gross performance of the target factor portfolio (gray), the net performance of the target factor portfolio (blue) and the net performance increase from the target factor portfolio to the optimal factor portfolio (orange). Overlapping areas are colored in dual colors. The underlying sample period is January 1985 to March 2023 for the US and Europe, and January 2000 to March 2023 for Emerging markets.

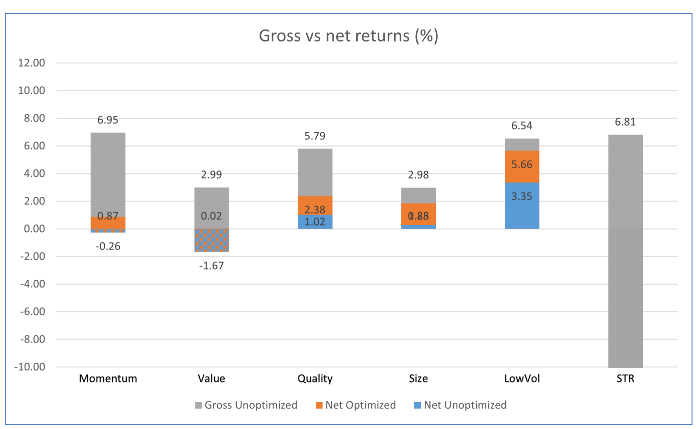
(a) US



(b) Europe



(c) Emerging Markets



4.3 Robustness with respect to fund size

While our main results are based on an assumed fund size of $1 billion USD net asset value, Table 5 next showcases our framework's efficacy using two alternative NAV assumptions, $500 million USD and $2 billion USD. Apart from short-term reversal and value in emerging markets, we see positive net returns across factors and markets for the smaller NAV case of $500 million USD. For the $2 billion USD case, we see some factors becoming unprofitable even in the optimized case, for example momentum and value in emerging markets. This is mainly a result of change in market impact, as spread, shorting costs and fixed fees remain mostly unchanged.

Table 5: Long-short single-factor portfolios under different NAV assumptions

The table presents gross and net performance of optimized single-factor portfolios under three different net asset value assumptions: $500 million USD, $1 billion USD and $2 billion USD. The table is split into three regions: US, Europe and Emerging markets. The underlying sample period is January 1985 to March 2023 for the US and Europe, and January 2000 to March 2023 for Emerging markets. All numbers are quoted in percentages and annualized.

US	Momentum	Value	Quality	Size	Lowvol	STR
Gross return %	7.71	2.49	6.25	0.81	7.22	7.66
Net return % ($ 500 mil.)	3.88	1.22	3.45	0.09	6.53	-4.23
Net return % ($ 1 bil.)	2.18	0.92	3.11	-0.17	6.31	-10.02
Net return % ($ 2 bil.)	0.82	0.35	2.63	-0.48	6.12	-13.35
Europe	Momentum	Value	Quality	Size	Lowvol	STR
Gross return %	11.36	-1.00	6.53	3.25	7.67	4.63
Net return % ($ 500 mil.)	6.22	-2.10	3.63	2.84	5.82	-6.34
Net return % ($ 1 bil.)	4.25	-2.88	2.68	2.18	5.25	-10.79
Net return % ($ 2 bil.)	1.12	-3.81	1.32	1.29	4.52	-17.35
Emerging markets	Momentum	Value	Quality	Size	Lowvol	STR
Gross return %	6.95	2.99	5.79	2.98	6.54	6.81
Net return % ($ 500 mil.)	2.63	0.71	2.80	2.19	5.89	-4.98
Net return % ($ 1 bil.)	0.87	0.02	2.38	1.88	5.66	-11.43
Net return % ($ 2 bil.)	-1.86	-0.64	1.99	1.67	5.31	-18.49

4.4 Multi-factor portfolios

After constructing optimized single-factor portfolios, we next turn to the multi-factor case. It is natural to expect multi-factor portfolios to see some reduction in turnover (relative to the average turnover of considered factors) because of trading cost diversification. We show that our methodology naturally carries over to this case such that one can preserve the balance across the contributing factors. Similar to the single-factor portfolio case, we start by defining a multi-factor target portfolio. For simplicity, we choose the multi-factor target portfolio to be an equally weighted average of all six single-factor target portfolios. As a result, the multi-factor portfolio is a long-short portfolio based on characteristic-rank weighting of each factor. Not surprisingly, we find that combining factor portfolios increases the gross performance compared to individual factor portfolios, and we will dive into the regional multi-factor evidence next.

4.4.1 US

The results for the optimized US multi-factor portfolios across different transaction cost parameter δ are shown in Table 6. The multi-factor target portfolio has a gross performance of 14.90% against that of the market of 11.78%. However, the net performance of the multi-factor portfolio is -1.05%, as the massive turnover (938.8%) and subsequent transaction costs erode all of the gross performance. The volatility for the US multi-factor target portfolio is 10.23%, lower than that of the market at 15.36%. This results in a slightly negative net Sharpe ratio of -0.10. As in the single-factor portfolios, after optimization, we observe a decline in gross returns, going from 14.90% to 9.13% when increasing δ. Volatility remains similar across all tested portfolios, resulting in gross Sharpe ratios declining similarly to gross returns. Looking at net performance, we find that the $\delta = 0.0005$ portfolio comes with almost the best net return of 6.83%, an improvement of almost 8% over the target portfolio. In terms of staying true to the target portfolio, the tracking error stands at 1.69% with an active share of 0.75. In addition with a high transfer coefficient of 0.88, we conclude that factor integrity of the multi-factor target portfolio is kept.

Table 6: Long-short optimized multi-factor portfolios: US

We present the summary statistics of different optimized multi-factor portfolios. The optimized portfolios differ by the transaction cost parameter used in the utility function: $\delta = 0$ corresponds to the long-short target portfolio. Where applicable, all measures are annualized and quoted in percentages. All relative measures such as tracking error and active share are benchmarked to the $\delta = 0$ (target) portfolio. For each factor, we consider one target and seven optimized portfolios and present gross return, net return, transaction costs, turnover, volatility, gross Sharpe ratio, net Sharpe ratio, factor score, tracking error, active share, transfer coefficient and number of effective names. The sample period ranges from January 1985 to March 2023.

	Market	$\delta = 0$	$\delta = 0.0001$	$\delta = 0.0002$	$\delta = 0.0005$	$\delta = 0.001$	$\delta = 0.002$
Panel A: Performance							
Gross return %	11.78	14.90	13.35	12.75	11.92	11.16	10.19
Net return %	11.73	-1.05	4.46	5.65	6.83	7.13	6.88
Volatility %	15.36	10.23	10.33	10.36	10.40	10.38	10.30
Gross Sharpe	0.77	1.46	1.29	1.23	1.15	1.08	0.99
Net Sharpe	0.76	-0.10	0.43	0.55	0.66	0.69	0.67
Panel B: Transaction costs							
Turnover %	8.30	938.8	761.4	687.6	579.6	503.6	438.0
Transaction costs %	0.04	15.15	8.20	6.43	4.49	3.47	2.80
TC Spread %	0.03	3.78	2.73	2.32	1.76	1.42	1.18
TC MI %	0.01	11.37	5.46	4.11	2.73	2.05	1.62
Shorting Costs %		0.71	0.61	0.58	0.54	0.49	0.45
Fixed Fees %	0.01	0.09	0.08	0.08	0.07	0.07	0.07
Panel C: Factor integrity							
Factor score %		100.00	96.70	95.14	91.96	88.12	83.15
Tracking error %		0.00	0.81	1.14	1.69	2.11	2.60
Active Share		0.00	0.30	0.46	0.75	1.02	1.33
Transfer coefficient		1.00	0.98	0.95	0.88	0.80	0.68
# of effective names		1,511	1,484	1,447	1,320	1,160	921

4.4.2 Europe

The gross performance of the European target portfolio is better than that of the US, with a gross Sharpe ratio of 1.69. However, the transaction costs are considerably higher at 22.13%, resulting in a negative net Sharpe ratio of -0.95. As we increase δ, we again observe the loss of gross returns, which gradually declines from 14.62% to 9.60%. Volatility remains flat for most cases, resulting in Sharpe ratio patterns resembling net returns. The largest net return is 6.52% in the case of $\delta = 0.001$. Looking at tracking error, we see that the optimized portfolios quickly drift apart from the target multi-factor portfolio as we reduce transaction costs. Considering the rapidly increasing tracking error and active share, we conclude that using a higher transaction cost parameter leads to optimized portfolios too different from the target portfolio. We will further evidence this when analyzing the exposure to individual factors.

Table 7: Long-short optimized multi-factor portfolios: Europe

We present the summary statistics of different optimized multi-factor portfolios for Europe. The optimized portfolios differ by the transaction cost parameter used in the utility function: $\delta = 0$ corresponds to the long-short target portfolio. Where applicable, all measures are annualized and quoted in percentages. All relative measures such as tracking error and active share are benchmarked to the $\delta = 0$ (target) portfolio. For each factor, we consider one target and seven optimized portfolios and present gross return, net return, transaction costs, turnover, volatility, gross Sharpe ratio, net Sharpe ratio, factor score, tracking error, active share, transfer coefficient and number of effective names. The sample period ranges from January 1985 to March 2023.

	Market	$\delta = 0$	$\delta = 0.0001$	$\delta = 0.0002$	$\delta = 0.0005$	$\delta = 0.001$	$\delta = 0.002$
Panel A: Performance							
Gross return %	10.64	14.62	13.82	13.41	12.77	11.97	11.05
Net return %	10.38	-8.25	2.17	4.13	6.00	6.52	6.49
Volatility %	17.33	8.67	8.71	8.63	8.60	8.59	8.65
Gross Sharpe	0.61	1.69	1.59	1.55	1.49	1.39	1.28
Net Sharpe	0.60	-0.90	0.25	0.48	0.70	0.76	0.75
Panel B: Transaction costs							
Turnover %	16.27	980.0	768.2	694.2	588.6	512.8	448.2
Transaction costs %	0.23	22.13	11.06	8.72	6.26	4.97	4.11
TC Spread %	0.06	3.97	2.65	2.24	1.71	1.36	1.09
TC MI %	0.17	18.16	8.41	6.48	4.55	3.61	3.03
Shorting Costs %		0.60	0.48	0.45	0.42	0.39	0.36
Fixed Fees %	0.02	0.14	0.11	0.11	0.10	0.10	0.09
Panel C: Factor integrity							
Factor score %		100.00	95.26	92.91	87.97	81.76	74.14
Tracking error %		0.00	1.44	2.33	3.20	4.01	4.59
Active Share		0.00	0.44	0.64	1.00	1.34	1.69
Transfer coefficient		1.00	0.96	0.92	0.84	0.73	0.58
# of effective names		572	509	386	262	193	175

4.4.3 Emerging markets

For Emerging markets, the results are somewhere in between the US and Europe, starting with a gross Sharpe ratio of 1.58. The turnover is the highest ($1,035.6\%$) resulting in transaction costs of 16.07%. The associated net Sharpe ratio is -0.15. On the other hand, the transaction cost-optimized portfolio construction yields even better outcomes in Emerging markets. We see familiar patterns for gross returns, volatility and net returns with one important difference, the pace of decay. For larger deltas, portfolios observe lower tracking errors and active shares, and higher transfer coefficients. Taking all of this into account, we find that the $\delta = 0.001$ portfolio gives the best return profile while retaining factor integrity to a great extent with the exception of a higher tracking error.

Table 8: Long-short optimized multi-factor portfolios: EM

We present the summary statistics of different optimized multi-factor portfolios for the EM region. The optimized portfolios differ by the transaction cost parameter used in the utility function: $\delta = 0$ corresponds to the long-short target portfolios. Where applicable, all measures are annualized and quoted in percentages. All relative measures such as tracking error and active share are benchmarked to the $\delta = 0$ (target) portfolio. For each factor, we consider one target and seven optimized portfolios and present gross return, net return, transaction costs, turnover, volatility, gross Sharpe ratio, net Sharpe ratio, factor score, tracking error, active share, transfer coefficient and number of effective names. The sample period ranges from January 2000 to March 2023.

	Market	$\delta = 0$	$\delta = 0.0001$	$\delta = 0.0002$	$\delta = 0.0005$	$\delta = 0.001$	$\delta = 0.002$
Panel A: Performance							
Gross return %	8.15	15.42	15.53	15.56	15.44	15.08	14.46
Net return %	7.70	-1.42	1.16	2.48	4.61	6.20	7.52
Volatility %	22.36	9.73	9.72	9.73	9.75	9.72	9.63
Gross Sharpe	0.36	1.58	1.60	1.60	1.58	1.55	1.50
Net Sharpe	0.35	-0.15	0.12	0.26	0.48	0.65	0.79
Panel B: Transaction costs							
Turnover %	30.30	1,035.6	982.2	947.5	873.4	791.4	689.6
Transaction costs %	0.43	16.07	13.64	12.37	10.15	8.22	6.32
TC Spread %	0.09	2.28	2.08	1.96	1.72	1.48	1.20
TC MI %	0.34	13.79	11.56	10.41	8.43	6.74	5.12
Shorting Costs%		0.62	0.60	0.58	0.56	0.54	0.51
Fixed Fees %	0.02	0.16	0.14	0.13	0.12	0.12	0.10
Panel C: Factor integrity							
Factor score %		100.00	99.27	98.74	97.39	95.54	92.50
Tracking error %		0.00	0.89	1.18	1.72	2.28	3.00
Active Share		0.00	0.11	0.18	0.32	0.49	0.71
Transfer coefficient		1.00	0.99	0.99	0.97	0.95	0.90
# of effective names		483	482	481	478	469	441

4.4.4 Factor balance

While it is comforting to see that the overall transfer of the multi-factor target is successful, we are also mindful of investigating as to how much of the underlying single factor exposures the multi-factor portfolios are able to transfer. Given the different single factor dynamics, we are eager to learn if the targeted equal factor balance can actually be maintained in a transaction-cost-optimized multi-factor portfolio. We perform such factor loading attribution by regressing the optimized multi-factor portfolio weights on the underlying target single-factor portfolio weights. Figure 5 collects the ensuing factor loadings and the overall decomposition by factor. Naturally, the multi-factor target portfolio can be fully replicated in absence of transaction costs ($\delta = 0$, leftmost stacked bars), hence we observe a fully balanced (i.e., equally weighted) factor loadings decomposition. As the transaction cost penalty δ increases we see the decay of short-term reversal and momentum exposures which we expect to suffer the most as expensive turnover is reduced. Conversely, low volatility is the most persistent factor with little loss in factor exposure across all deltas.

Focusing on US evidence, we see the preservation of factor balance even at higher deltas, reinforcing our choice of $\delta = 0.0005$. We find that the optimized multi-factor portfolio only loses exposure to the short-term reversal and momentum factors, as expected. Looking at higher deltas, we find that value and quality are reduced but retain significant presence in the portfolio. Size and low volatility remain almost unaffected. Still, the regression fit quickly deteriorates, seeing R^2 below 50% for δ higher than 0.01.

In contrast, the European multi-factor portfolios experience a more rapid decay in factor exposure as we increase the transaction cost penalty. Momentum, quality, value and short-term reversal gradually lose exposure with quality decaying the quickest. Size loses a lot of exposure in the higher deltas. Low volatility increases in exposure significantly in the middle range of deltas. The rapid decay in factor exposure is further evidenced by the faster decline of R^2. This outcome can be further rationalized when considering the fact that the European multi-factor portfolio is considerably less diversified than the US or EM portfolios in terms of number of effective names, making it relatively harder to sustain factor exposure at higher transaction cost penalties (and thus complicating the substitution of costly high factor exposure names with cheaper ones).

Against this backdrop, we see very persistent EM multi-factor loadings with little decay, similar to the US evidence. Naturally, we also observe the faster decay of short-term reversal and retention of the low volatility factor. The remaining four factors decay gradually and in a similar manner. This is further supported by a high R^2 across most deltas. The observed factor balance reinforces our choice of $\delta = 0.001$ as the best multi-factor portfolio for the EM region.

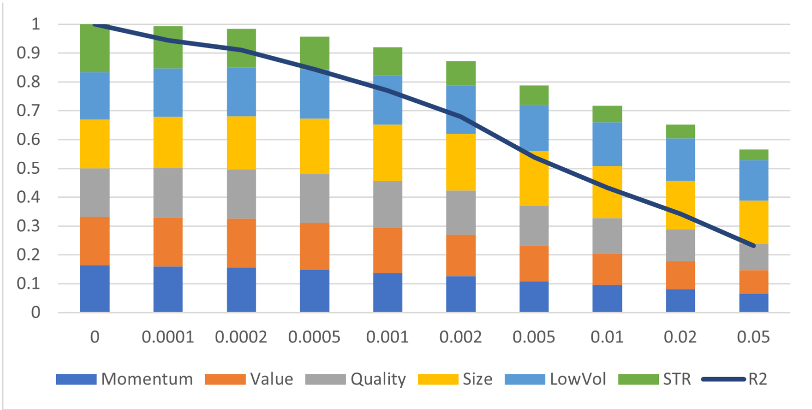
4.5 VMQL portfolios

When constructing a multi-factor portfolio, the approach was to equal-weight all six single-factor portfolios. However, as argued in Blitz and Hanauer (2021), the size factor is weak as a stand-alone factor (even after controlling for quality-versus-junk exposures) and is hardly

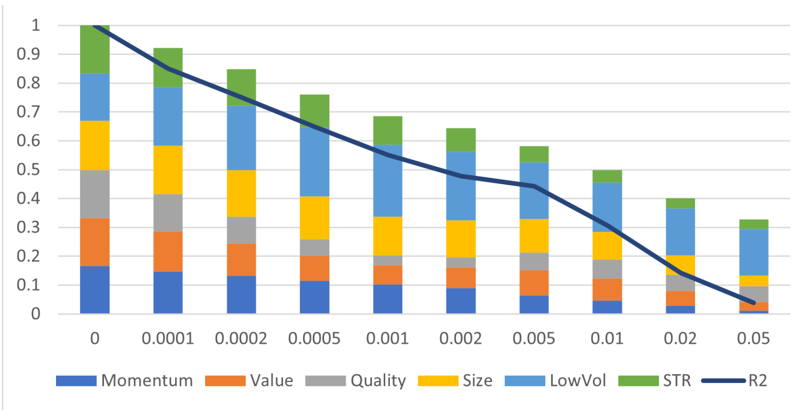
Figure 5: Factor balance across optimal multi-factor portfolios

We regress the multi-factor portfolio weights on the individual factor weights for each optimal portfolio. Each bar shown represents the average factor exposure per factor. Starting from a balanced 1/6 target portfolio, we observe how multi-factor portfolios reduce exposure in some factors quickly while other factors remain unaffected or decay slowly. The underlying sample period is January 1985 to March 2023 for the US and Europe, and January 2000 to March 2023 for Emerging markets.

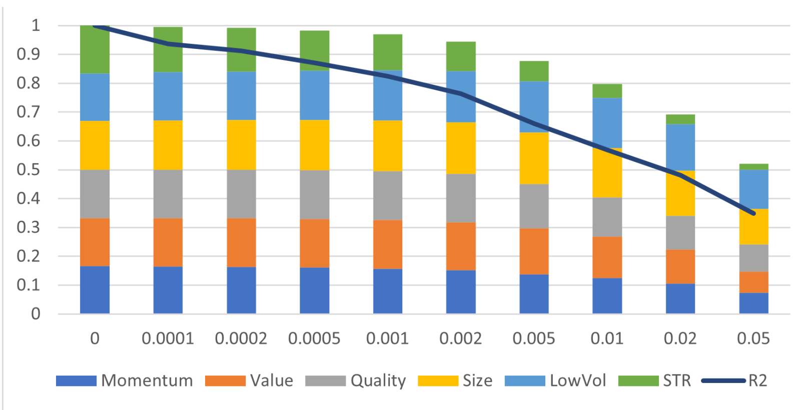
(a) US



(b) Europe



(c) EM



explicitly targeted in factor investing propositions. Similarly, the short-term reversal factor has weakened over time when the underlying characteristic used is last month return (Blitz et al., 2023a). Indeed, our single factor results confirm the poor performance of short-term reversal and size, and thus we investigate a more common choice of multi-factor portfolio that focuses on the four factors value, momentum, quality and low volatility. We showcase the performance of these VMQL portfolios in Table 9. As expected, while removing short-term reversal results in a decrease in gross Sharpe, we observe a sharp increase in net Sharpe. The decrease in gross Sharpe becomes less pronounced in the optimized case, as the exposure to short-term reversal is usually considerably reduced as shown in Figure 5. Looking at factor integrity, VMQL portfolios generally observe larger factor scores, substantially lower tracking errors and active shares, when compared to the 6-factor portfolios.

Importantly, the removal of size and short-term reversal make the multi-factor performance more resilient with respect to larger fund sizes as well, see Panel D of Table 9 that shows multi-factor portfolio performance in two more cases, $500 million USD and $2 billion USD.

5 Benchmarking transaction cost-optimized factor portfolios

In order to mitigate transaction costs, Novy-Marx and Velikov (2016) suggest several turnover-reducing methods, concluding that the buy/hold spread rule is a highly effective cost mitigation technique. For example, their 10%/20% buy/hold spread rule for the long side implies buying a stock when the stock enters the top 10% based on the underlying characteristic, but only selling it when it falls below 20% and thus avoiding flip-flopping of a given name. They also argue that only factor portfolios with low to moderate turnover generate significant net returns, which deviates from our conclusions. To rationalize, we replicate the buy/hold methodology employed in Novy-Marx and Velikov (2016) based on our factor data and transaction cost model, see Table 10. Instead of using their value-weighting scheme, we use equally-weighted market-beta-neutral factor portfolios, rendering both factor construction methods more comparable. While the resulting portfolios yield higher gross returns than our factor portfolios, we note that this is driven by their considerably higher volatility, resulting in relatively lower Sharpe ratios. Furthermore, the

Table 9: Multi-factor vs. VMQL performance

The table presents the gross and net performance of optimized multi-factor portfolios under two different constructions. The first is the multi-factor portfolio using all six factors we investigate. VMQL is the multi-factor portfolio restricted to only four factors: momentum, value, quality and low volatility. All relative metrics such as tracking error and factor score are benchmarked to the respective unoptimized portfolios and all numbers are quoted in percentages and annualized. Panel D presents the gross and net performance of optimized multi-factor and VMQL portfolios under three different net asset value assumptions: $500 million USD, $1 billion USD and $2 billion USD. The underlying sample period is January 1985 to March 2023 for the US and Europe, and January 2000 to March 2023 for Emerging markets. All numbers are quoted in percentages and annualized.

	US		EU		EM	
	6-factor	*VMQL*	*6-factor*	*VMQL*	*6-factor*	*VMQL*
Panel A: Performance						
Gross return %	11.92	12.27	11.97	12.51	15.08	13.58
Net return %	6.83	10.73	6.52	8.82	6.20	10.87
Volatility %	10.40	12.24	8.59	9.31	9.72	9.22
Gross Sharpe	1.15	1.00	1.39	1.34	1.55	1.47
Net Sharpe	0.66	0.88	0.76	0.95	0.65	1.18
Panel B: Transaction costs						
Turnover %	579.6	500.9	512.8	551.6	791.4	529.92
Transaction costs %	4.49	1.01	4.97	3.13	8.22	2.17
TC Spread %	1.76	0.44	1.36	0.73	1.48	1.09
TC MI %	2.73	0.57	3.61	2.40	6.74	1.08
Shorting Costs%	0.54	0.48	0.39	0.35	0.54	0.43
Fixed Fees %	0.07	0.05	0.10	0.09	0.12	0.10
Panel C: Factor integrity						
Factor Score %	91.96	94.56	81.76	92.56	95.54	94.68
Tracking error %	1.69	1.06	4.01	1.82	2.28	1.49
Active Share	0.75	0.54	1.34	0.50	0.49	0.50
Transfer coefficient	0.88	0.89	0.73	0.94	0.95	0.93
# of effective names	1,320	1,075	193	370	469	424
Panel D: Performance under different NAV assumptions						
Gross return %	11.92	12.27	11.97	12.51	15.08	13.58
Net ret. % ($ 500 mil.)	9.15	11.15	8.47	9.87	9.73	11.34
Net ret. % ($ 1 bil.)	6.83	10.73	6.52	8.82	6.20	10.87
Net ret. % ($ 2 bil.)	4.44	10.21	4.02	7.45	2.47	9.90

net returns of these portfolios are much lower than those of our portfolios due to the large transaction costs they incur. The turnover is often very high for the equally-weighted starting portfolios, and is substantially reduced in the more restrictive h = 0.5 case. While their approach is effective at reducing turnover, the resulting reduction in transaction costs is much less rewarding when compared to our approach. Take momentum as an example: our approach reduces turnover from 534.6% to 357.0% in the optimal case and thus cuts transaction costs from 7.19% down to 2.70%. On the other hand, the buy/hold approach reduces turnover from 761.5% down to 313.6% in the h = 0.5 case. Despite having similar turnover to our approach, the buy/hold spread rule sees more than twice the transaction costs (5.65%), clearly showing that we can achieve much better cost mitigation at similar turnover values.

To illustrate, consider a scenario with a 10%/50% buy/hold spread portfolio where a stock enters the top decile in the first month, drops at the 30^{th} percentile next month, and then again drops at the 50^{th} the month after. In the buy/hold spread portfolio, we enter into a long position in the stock the first month, hold it for two months and finally sell it as it hits the 50^{th} percentile. In both our unoptimized and optimized portfolios, the stock is gradually traded off as the underlying characteristic reduces, where the trades are done even more gradually in the optimal case due to the transaction cost penalty. This results in very similar turnovers but the market impact incurred is substantially reduced. Importantly, in stark contrast to our approach, the buy/spread rule leads to a considerable divation from the starting factor portfolio, as reflected in high active share and tracking error figures. As a result, we observe lower transfer coefficents, and thus the buy/hold spread rule struggles to stay true to the original factor given relatively low factor scores either. To summarize, the buy/spread rule is too ad hoc and leaves ample room for further optimizing a given factor's transaction cost-factor integrity tradeoff.

6 Conclusion

We present a framework to optimize the transactions cost of implementing salient long-short equity factors. The framework is independent of factor choice and can be applied to any weighting scheme or variance-covariance matrix. Using real trading data, we construct a tractable parametric transaction cost model to both evaluate and optimize factor portfolios. While many academic factor portfolios are characterized by underwhelming net performances, these factor portfolios can be optimized such that their net performance becomes economically relevant. Specifically, sacrificing less than half of the turnover in most cases, we can greatly reduce the incurred transaction costs. This is more noticeable in the market impact component, as it grows with a higher exponent than the spread component.

Table 10: Comparison to Novy-Marx and Velikov (2016)

We present the summary statistics of factor portfolios created using the buy/hold spread approach of Novy-Marx and Velikov (2016). The portfolios differ by the hold threshold used. Equal-weighted corresponds to the equal-weighted long-short decile market-beta-neutral portfolios and $h = 0.2$ and 0.5 correspond to a hold threshold of 20 and 50%. Non-optimized and Optimized columns correspond to the $\delta = 0$ and optimal δ cases from Table 4. The considered sample period is January 1985 to March 2023. Where applicable, all measures are annualized and quoted in percentages. All relative measures such as tracking error and active share are benchmarked to the equal-weighted portfolio.

Momentum	Equal-weighted	h = 0.2	h = 0.5	Non-optimized	Optimized
Panel A: Performance					
Gross return %	11.30	10.11	7.21	7.71	6.51
Net return %	-10.58	-3.14	-0.94	-1.52	2.18
Volatility %	24.35	21.86	17.94	11.35	11.40
Gross Sharpe	0.46	0.46	0.40	0.68	0.57
Net Sharpe	-0.43	-0.15	-0.06	-0.12	0.20
Panel B: Transaction costs					
Turnover %	761.5	483.1	313.6	534.6	357.0
Transaction costs %	18.82	10.46	5.65	7.19	2.70
TC Spread %	3.12	1.97	1.27	2.17	1.15
TC MI %	15.70	8.49	4.38	5.02	1.56
Shorting Costs %	2.87	2.66	2.42	1.89	1.54
Fixed Fees %	0.20	0.14	0.09	0.15	0.08
Panel C: Factor integrity					
Factor score %	100.00	94.75	79.30	100.00	90.67
Tracking error %	0.00	3.67	8.21	0.00	1.38
Active Share	0.00	0.96	1.93	0.00	0.68
Transfer coefficient	1.00	0.87	0.72	1.00	0.91
# of effective names	414	544	809	1489	1376

Size	Equal-weighted	h = 0.2	h = 0.5	Non-optimized	Optimized
Panel A: Performance					
Gross return %	3.44	4.03	2.85	0.81	0.55
Net return %	-8.31	-1.31	0.22	-1.49	-0.17
Volatility %	15.65	13.42	10.78	10.79	10.09
Gross Sharpe	0.22	0.30	0.26	0.08	0.05
Net Sharpe	-0.53	-0.09	0.02	-0.13	-0.01
Panel B: Transaction costs					
Turnover %	262.6	146.5	112.5	134.8	68.0
Transaction costs %	11.49	5.09	2.42	2.08	0.50
TC Spread %	1.24	0.68	0.51	0.59	0.23
TC MI %	10.25	4.41	1.91	1.49	0.27
Shorting Costs %	0.18	0.18	0.18	0.18	0.18
Fixed Fees %	0.08	0.07	0.04	0.04	0.03
Panel C: Factor integrity					
Factor score %	100.00	95.14	82.66	100.00	93.69
Tracking error %	0.00	4.16	8.40	0.00	1.72
Active Share	0.00	1.05	1.90	0.00	0.55
Transfer coefficient	1.00	0.86	0.72	1.00	0.95
# of effective names	542	731	1,014	1,489	1,379

Value	Equal-weighted	h = 0.2	h = 0.5	Non-optimized	Optimized
Panel A: Performance					
Gross return %	1.51	-0.39	1.56	2.49	2.30
Net return %	-14.61	-10.91	-6.34	-1.40	0.92
Volatility %	23.80	22.03	18.48	10.85	10.37
Gross Sharpe	0.06	-0.02	0.08	0.23	0.22
Net Sharpe	-0.62	-0.49	-0.33	-0.12	0.09
Panel B: Transaction costs					
Turnover %	579.8	407.9	346.1	222.8	107.0
Transaction costs %	14.85	9.29	6.74	2.95	0.67
TC Spread %	2.34	1.65	1.48	0.95	0.33
TC MI %	12.51	7.63	5.26	2.00	0.34
Shorting Costs %	1.18	1.14	1.11	0.88	0.67
Fixed Fees %	0.10	0.08	0.05	0.06	0.04
Panel C: Factor integrity					
Factor score %	100.00	94.29	79.10	100.00	93.67
Tracking error %	0.00	8.49	14.21	0.00	1.77
Active Share	0.00	1.13	2.13	0.00	0.67
Transfer coefficient	1.00	0.84	0.66	1.00	0.93
# of effective names	348	479	743	1527	1384

Lowvol	Equal-weighted	h = 0.2	h = 0.5	Non-optimized	Optimized
Panel A: Performance					
Gross return %	14.22	12.61	11.56	7.22	6.76
Net return %	1.76	3.50	4.05	4.48	6.31
Volatility %	24.06	22.51	20.33	11.59	11.69
Gross Sharpe	0.59	0.56	0.57	0.62	0.58
Net Sharpe	0.08	0.16	0.20	0.39	0.54
Panel B: Transaction costs					
Turnover %	448.4	364.7	329.5	143.8	66.0
Transaction costs %	10.65	7.41	5.92	1.48	0.30
TC Spread %	1.76	1.44	1.35	0.59	0.17
TC MI %	8.89	5.97	4.57	0.89	0.12
Shorting Costs %	1.72	1.63	1.54	1.22	0.73
Fixed Fees %	0.08	0.07	0.05	0.04	0.03
Panel C: Factor integrity					
Factor score %	100.00	94.29	80.51	100.00	94.57
Tracking error %	0.00	6.92	11.03	0.00	1.88
Active Share	0.00	1.07	1.94	0.00	0.97
Transfer coefficient	1.00	0.85	0.70	1.00	0.85
# of effective names	306	415	598	1,332	1,016

Quality	Equal-weighted	h = 0.2	h = 0.5	Non-optimized	Optimized
Panel A: Performance					
Gross return %	14.40	12.02	10.54	6.25	5.00
Net return %	0.39	1.13	0.98	1.93	3.11
Volatility %	21.50	18.25	17.65	7.66	7.45
Gross Sharpe	0.67	0.66	0.60	0.82	0.67
Net Sharpe	0.02	0.06	0.06	0.26	0.42
Panel B: Transaction costs					
Turnover %	502.1	413.5	382.7	200.2	106.4
Transaction costs %	11.70	8.69	7.51	2.50	0.63
TC Spread %	1.99	1.65	1.53	0.82	0.30
TC MI %	9.71	7.04	5.97	1.68	0.33
Shorting Costs %	2.22	2.12	1.99	1.76	1.22
Fixed Fees %	0.09	0.08	0.06	0.06	0.04
Panel C: Factor integrity					
Factor score %	100.00	95.15	82.00	100.00	89.86
Tracking error %	0.00	8.05	12.94	0.00	1.49
Active Share	0.00	0.96	1.78	0.00	0.84
Transfer coefficient	1.00	0.87	0.73	1.00	0.88
# of effective names	347	457	633	1,522	1,336

STR	Equal-weighted	h = 0.2	h = 0.5	Non-optimized	Optimized
Panel A: Performance					
Gross return %	9.34	9.33	10.68	7.66	3.15
Net return %	-53.78	-47.46	-29.69	-25.39	-10.02
Volatility %	26.30	26.17	25.29	11.34	11.35
Gross Sharpe	0.36	0.36	0.42	0.68	0.28
Net Sharpe	-2.04	-1.81	-1.17	-2.10	-0.86
Panel B: Transaction costs					
Turnover %	2,180.7	2,016.5	1,562.9	1,659.2	1,188.8
Transaction costs %	61.42	55.19	39.00	31.58	12.02
TC Spread %	8.84	8.18	6.38	6.69	3.89
TC MI %	52.57	47.00	32.62	24.89	8.14
Shorting Costs %	1.06	1.04	0.95	0.99	0.82
Fixed Fees %	0.65	0.57	0.42	0.48	0.33
Panel C: Factor integrity					
Factor score %	100.00	97.85	81.45	100.00	74.51
Tracking error %	0.00	7.45	15.36	0.00	2.83
Active Share	0.00	0.46	1.45	0.00	1.17
Transfer coefficient	1.00	0.94	0.78	1.00	0.59
# of effective names	332	362	502	1,454	406

We further show that these results can be achieved for various fund size assumptions. In addition to the standard US universe, we explore the performance of our framework in Europe and Emerging Markets. We find our results to remain consistent across those regions, resulting in larger improvements in the performance of the optimized portfolios when compared to the US.

Importantly, we document the relevance of monitoring the trade-off of reducing factor turnover and transaction costs versus preserving factor integrity. Specifically, some factors exhibit high net returns when constructed with a very high transaction cost penalty, yet bear little to no similarity with the original factors. This puts an emphasis on monitoring factor exposure as turnover is reduced.

We optimize a diversified multi-factor portfolio, where one does not only care about preserving the underlying factor's integrity but also about preserving the targeted factor balance. Indeed, the constructed optimized multi-factor portfolio observes increased net performance while retaining a well-balanced factor exposure. We show that our factor portfolio construction is robust to fund size assumption and choice of multi-factor portfolio constituents.

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A Appendix: Transaction costs

A.1 Transaction cost data cleaning

The data is cleaned in several stages. First, we only retain trades that were executed on the market by brokers. This ensures that market impact is reflected on the underlying share price and can be measured using implementation shortfall. Additionally, if the time delay between the trade submission and the start of the execution of the trade is longer than one day for US and Europe and two days for Emerging markets, the trades are filtered out. Omitting this step would result in additional noise in implementation shortfall because of potentially large differences in price at the start of the execution and the prevailing benchmark price when the trade was submitted. Next, if there are multiple trades being executed within the same period, their transaction costs should be measured as a single trade. For this purpose, we merge trades that have overlapping arrival to end of trade dates. The merged trades' columns are then recalculated appropriately. For example, the trade size of the merged trade as a percentage of MDV is calculated as the sum of the individual trade sizes. IS is the value weighted average of the single trade IS, with volatility using the same approach.

A.2 Estimating shorting fees

This appendix explains how we compute reasonable shorting fee cost estimates based on the reported shorting fee information in Muravyev et al. (2022). In the sample period of July 2006 to December 2020, they document a mean borrow fee of 1.67% per annum with a median of 0.38%. The first percentile is 0.25%, the 10^{th} percentile is 0.28%, the 90^{th} percentile is 3% and the 99^{th} percentile is 30%. We estimate a quantile function based on the four observations given above, and we define the shorting fee quantile function as

$$\ln SF(q_i) - 0.1 = \ln a + bq_i^5 + \epsilon_i \qquad (15)$$

where q is a quantile ranging from 0 to 1 and 0.1% the smallest shorting fee assumed. Using log-linear least squares, we estimate the parameters a and b shown in Table 11.

This functional form is motivated by the relatively low mean compared to function values in the higher quantiles, implying high skewness as mentioned in Muravyev et al. (2022). To

Table 11: Shorting fees quantile function

We present the parameter of the shorting cost quantile function. Parameters are obtained based on quantile reports in Muravyev et al. (2022). The underlying sample period is July 2006 to December 2020. These values will be used as estimates for shorting fees based on a size rank.

Parameter	Value	St. Dev.	t-stat	P-value	95% CI	
$\ln(a)$	-1.73	0.24	-7.35	0.02	-2.74	-0.72
b	5.21	0.42	12.42	0.01	3.41	7.02
$R^2(\%)$	Adj. $R^2(\%)$	$F-stat$	$p(F-stat)$	Jarque-Bera	$p(JB)$	# of obs.
98.7	98.1	154.2	0.01	0.52	0.77	4

further substantiate the validity of our quantile model in Equation (15), we estimate the mean shorting fee using inverse transform sampling and drawing 10 million samples from our distribution. The obtained mean of the sample is 1.97%. We plot the fitted model against the data in Figure 6. Upon obtaining the quantile function, we can then rank stocks based on size and apply the shorting fee quantile function to obtain an estimate for shorting fees, since size is shown to have a considerable impact on shorting fees as shown in Cohen et al. (2007) Table I.

Figure 6: Shorting costs model

We plot the shorting costs model estimates against the quantiles reported in Muravyev et al. (2022).