A Robust Distributed Observer Design for Lipschitz Nonlinear Systems With Time-Varying Switching Topology

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***Abstract*—This paper deals with state estimation for a class of Lipschitz nonlinear systems under a time-varying disconnected communication network. A distributed observer consists of some local observers that are connected to each other through a communication network. We consider a situation where a communication network does not remain connected all the time, and the network may be caused by intermittent communication link failure. Moreover, each local observer has access to a local measurement, which may be insufficient to ensure the system's observability, but the collection of all measurements in the network ensures observability. In this condition, the purpose is to design a distributed observer where the estimated state vectors of all local observers converge to the state vector of the system asymptotically, while local observers exchange estimated state vectors through a communication network and use their local measurements. According to theoretical analysis, a nonlinear and a robust nonlinear distributed observer exist when in addition to the union of all communication topologies being strongly connected during a time interval, the component of each communication graph is also strongly connected during each subinterval. The existence conditions of the distributed observers are derived in terms of a set of linear matrix inequalities (LMIs). Finally, the effectiveness of the presented method is numerically verified using some simulation examples.**

***Index Terms*—Distributed state estimation, Nonlinear distributed observer, LMIs, Switching topology.**

I. Introduction

RECENTLY, much attention has been paid to the distributed state estimation to solve the distributed control problems in multi-agent systems [1]–[3]. Distributed estimation algorithms can be used for sensor networks, target tracking, or state estimation of large-scale systems where the output of the system is required to be observed by multiple sensors [4]. The focus on distributed estimation is because, in many practical systems, not enough measurements can be made in one place to provide an asymptotic estimate of the system state, and using a centralized method may be costly or impractical.

A distributed observer consists of N local observers called agents that are connected to each other through a communication network. The main challenge in designing a distributed observer is the lack of local observability, since the output vector of the system is divided between the agents, leading to the rank deficiency in the observability matrix of each agent. This is why each agent requires to update its local estimates based on its local measurement as well as the information shared between its neighboring agents in the network. In this way, a consensus of the estimation can be reached on the unobservable part of all local observers.

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Most recent research on the distributed observer problem has focused on linear time-invariant systems, see in [5]–[8]. One of the main contributions was made in [9] where it was suggested to apply the observability decomposition to each agent and this idea made designs for the locally observable and the unobservable subspaces independent from each other. Then, [9] introduced a distributed design of the Luenberger-type state observer for continuous-time LTI systems by designing two observer gains. [10] extended the work done in [9] based on some LMIs under the strongly connected network. A more general structure of [9] is designed in [11] where each agent has its own coupling gain that can be different from the others, leading to the decentralized design.

In contrast to the above work which designed distributed observers for linear systems, [12] and [13] introduced distributed observers for nonlinear systems. [12] considers the state and output matrices to be diagonal blocks, which limits the application of the proposed observation method. As stated in [13], the subsystem only has one output system for each agent, and each local observer can only receive one output measurement.

Although the above-mentioned papers concentrated on steady and fixed communication, the first extension for time-varying networks were created in [14]. The communication networks used in [14] must always be strongly connected for all time, while in practice, a network's disconnectedness may result from infrequent communication link failures or environmental changes. So, further research on the distributed estimation problem using jointly connected switching networks which can be disconnected at any time is more engaging and difficult. Considering a time-varying version of [11], [15] showed an asymptotic state estimate of linear systems over a jointly connected switching network. Under the same condition as [15], [16] developed the asymptotic convergence results of [15] to the exponential convergence. However, in both [15] and [16], the topology of the communication network is described by an undirected switching graph.

The problem of distributed observer design for the linear system under time-varying disconnected communication topologies is investigated in [17]. In this work, some necessary constraints for achieving omniscience asymptomatically are applied. When the LTI system is subject to external disturbances and measurement noise, [18] solved the problem of distributed observer design for the linear system under switching communication topology.

Since most practical systems are inherently nonlinear, the necessity of designing a distributed observer for nonlinear systems was raised. the only point is that the nonlinearity in this work is Lipschitz-type. Note that the Lipschitz nonlinear systems usually arise in practical applications due to increasing in the domain of attraction. Moreover, by applying nonlinear transformation on many dynamic systems, one can achieve some canonical form e.g. Brunovsky form, which involves linear and nonlinear parts, where the nonlinearity involves the Lipschitz condition.

The observer synthesis problem becomes more challenging when the nonlinear system dynamics involve disturbance. By adding a bounded disturbance to the introduced nonlinear system, our proposed nonlinear distributed observer loses its efficiency. Then, taking the advantages of the sliding mode observer in [19]–[21], we extended the proposed nonlinear distributed observer to a robust distributed observer for Lipschitz nonlinear systems involving matching uncertainty. If the matching condition is met, the disturbance effect disappears and the asymptotic convergence can be achieved.

The connection between agents may not be stable due to several issues including barriers, communication device malfunctions, etc. So, the connection links may break and then be rebuilt at various intervals, and the communication topology may change over time. This problem affects state estimation and may possibly lead to the failure of the entire estimate system. In this situation, a distributed estimating strategy must be developed to ensure state estimation even when communication links are interrupted. Therefore, in this paper, we investigate the asymptotic omniscience of distributed observers (which is equivalent to the consensus of error system) for Lipschitz nonlinear systems with switched disconnected topologies.

The main contribution of this paper is to propose a distributed observer such that if each communication graph's component is strongly connected during each subinterval and the union/joint of all communication topologies are strongly connected during a time interval, the state vector of the system is estimated by each local observer. So, we first extend the linear distributed observer to the nonlinear one under switching topology, and then, by taking advantage of the observability decomposition of the local observers, we analyzed the structure of the design parameters. Next, by using an auxiliary undirected graph during each subinterval for each agent, a bank of LMIs is derived to calculate the design parameters of our distributed observer.

We tried to design our proposed observers without using complex structures, or applying any kind of restrictions mentioned in [12] and [13], and compared with existing approaches, our proposed approaches are more resilient to unreliable communication.

Notations: Throughout the paper, $\mathbb{R}^n$ indicates the n-dimensional space and N is the number of local observers. For a given matrix X, X^T shows its transpose, and X^{-1} denotes its inverse. The symbol $\text{sym}(X)$ is represented as $\text{sym}(X) := X + X^T$. I_n is the identity matrix, and $\mathbf{1}_N$ is the N-dimensional column vector with all entries equal to one. $P > 0$ ($P < 0$) means that P is a positive (negative) definite matrix. The expression $\text{col}(X_1, X_2, \cdot\cdot\cdot, X_N)$ refers to the matrix $[X_1^T, X_2^T, \cdot\cdot\cdot, X_N^T]^T$. The Kronecker product is expressed as $X_1 \otimes X_2$.

II. Preliminaries

A. Graph Theory

The topology of the communication network for N agents is described by a disconnected switching graph $\mathcal{G}(t) = (\mathcal{V}(t), \xi(t), \mathcal{A}(t))$, where $\mathcal{V}(t) = \{1, 2, ..., N\}$ is the set of nodes and $\xi(t) \subseteq \mathcal{V}(t) \times \mathcal{V}(t)$. Each node in $\mathcal{V}(t)$ corresponds to an agent in the network and $(i, j) \in \xi(t)$ if and only if the ith agent can exchange information with the jth agent at the time instant t. $\mathcal{A}(t) = [\alpha_{ji}(t)] \in \mathbb{R}^{N\times N}, i, j \in \mathcal{V}(t)$ is the nonnegative weighted adjacency matrix of graph $\mathcal{G}(t)$ at the instant of time t where $\alpha_{ii}(t) = 0$. We have $\alpha_{ji}(t) \neq 0$ if and only if $(i, j) \in \xi_(t)$, otherwise $\alpha_{ji}(t) = 0$. The index set is denoted by $\varpi = \{0, 1, ..., \tau_k - 1\}$, and $\sigma(t) : \mathbb{R}^+ \to \varpi$ is the piecewise constant switching signal.

Let the Laplacian matrix of the disconnected graph $\mathcal{G}(t)$ at the instant of time t be $\mathcal{L}(t) = [l_{ij}(t)] \in \mathbb{R}^{N\times N}$. The diagonal elements of $\mathcal{L}(t)$ are equal to the degree of the nodes, and the off-diagonal elements of $\mathcal{L}(t)$ are -1 if the node i is adjacent to node j, otherwise equal to 0. The Laplacian matrix of a connected graph is always a semi-positive definite matrix (by having a zero eigenvalue), however, if $\mathcal{G}(t)$ is a disconnected graph the Laplacian matrix will have multiple zero eigenvalues.

B. System Description

In this note, we regard the following nonlinear system

$$\dot{x} = Ax + f(t, x, u) + \Gamma d(t) + Bu \tag{1}$$

$$y = Cx, \tag{2}$$

where $x \in \mathbb{R}^n$ is the state vector, $u \in \mathbb{R}^m$ denotes the vector of known inputs, and $y \in \mathbb{R}^p$ is the measurement output. $f(t, x, u)$ is a known smooth vector field with dimension n. The bounded disturbance input is represented with $d(t) \in \mathbb{R}$

such that $\|d\| \leq D$ where $D > 0$ is a known constant. The matrices $A \in \mathbb{R}^{n\times n}, B \in \mathbb{R}^{n\times m}, C \in \mathbb{R}^{p\times n}, \Gamma \in \mathbb{R}^n$ are the state, input, output, and disturbance matrices, respectively.

It is assumed that the output y in (2) has been segmented into N parts y_i, so that $y_i \in \mathbb{R}^{p_i}$ for $i = \{1, 2, ..., N\}$ and $\sum_{i=1}^{N} p_i = p$. Consequently, the sub-matrix $C_i \in \mathbb{R}^{p_i\times n}$ is a partition of output matrix C. The portion $y_i = C_i x \in \mathbb{R}^{p_i}$ is the only data that can be received by the node i. Now, we will give three basic assumptions of this paper, and throughout this paper, we assume that (1) and (2) satisfy the following assumption.

Assumption 1: The system (1) and (2) is observable.

Assumption 2: The nonlinear function $f(t, x, u)$ is a Lipschitz nonlinear function with respect to $x \in \mathbb{R}^n$ and a positive Lipschitz constant ζ i.e.

$$\|f(t, \hat{x}_i, u) - f(t, x, u)\| \leq \zeta\|(\hat{x}_i - x)\|, \ \forall i \in \mathcal{V}(t), \quad (3)$$

where $f(t, \hat{x}_i, u)$ is the nonlinear function of state estimated by node i.

Assumption 3: We consider an infinite time sequence $\{t_0, t_1, t_2, ...\}$ such that $t_0 = 0$, and the intervals $[t_k, t_{k+1}), k = \{0, 1, 2, ...\}$ are bounded and in each interval, the graph is switching in a way that it is *jointly connected* (see definition in [22]). It is assumed that there is a finite time sequence $\{t_k^0, t_k^1, ..., t_k^{\tau_k-1}\}$ in the interval $[t_k, t_{k+1})$ so that $\mathcal{G}(t)$ switches to $\mathcal{G}_k^\omega$ at $t = t_k^j$, ω and $j \in \varpi$ and $\mathcal{G}_k^\omega$ stays steady during the subinterval $[t_k^j, t_k^{j+1})$. It is worth noting that $\mathcal{G}_k^\omega$ can be a disconnected graph with isolated nodes, however, the remaining part consists of a single strongly connected subgraph.

Remark 1. The system (1) is a generalization of a linear system in which a nonlinear term has been added to the system to increase the field of practical application. Therefore, the linear term cannot be zero in this class of system. A wide range of practical nonlinear systems can be indicated in the form of (1),(2) provided that $f(t, x, u)$ is differentiable with respect to the system state variables [23].

Assuming that in system (1), $\Gamma d(t) = 0$, and the function $f(t, x, u)$ has a Lipschitz constant $\zeta = 0$ in (3), [10] can be used to create a distributed Luenberger observer for the resulting linear system. In this case, the dynamic of the local observer at node i under fixed strongly connected communication topology has the following form

$$\dot{\hat{z}}_i = A\hat{z}_i + \urcorner_i(y_i - C_i\hat{z}_i) + \gamma\mathcal{M}_i \sum_{j=1}^{N} \alpha_{ij}(\hat{z}_j - \hat{z}_i) + Bu, \quad (4)$$

where $\hat{z}_i \in \mathbb{R}^n$ is the state estimated by the ith agent, and α_{ij} is the (i, j)th entry of the adjacency matrix $\mathcal{A}$ of the network. Observer gain $\urcorner_i \in \mathbb{R}^{n\times p_i}$, coupling gain $\gamma \in \mathbb{R}^+$ and weighted matrix $\mathcal{M}_i \in \mathbb{R}^{n\times n}$ are the designing parameters which need to be designed.

Remark 2. Assuming that $\mathcal{L}_k^j$ is the Laplacian matrix associated with the graph $\mathcal{G}_k^\omega$, then according to Assumption 3, $\mathcal{L}_k^j$ can be disconnected in general and hence $rank(\mathcal{L}_k^j) < N - 1$. Let $\mathcal{I}_k^j \subset \mathcal{V}(t)$ be the set of nodes associated with the strongly connected subgraph of $\mathcal{G}_k^\omega$ referred to as $\tilde{\mathcal{G}}_k^\omega$ and $\tilde{\mathcal{L}}_k^j$ is the associated Laplacian matrix. Obviously, $\tilde{\mathcal{L}}_k^j$ is a semi-positive definite matrix that has only one zero eigenvalue (i.e., $\tilde{\mathcal{L}}_k^j \mathbf{1}_{N_k^j} = 0_{N_k^j}$), $N_k^j \in \mathcal{I}_k^j$. It should be noted that N_k^j is the number of nodes connected to each other in the subinterval $[t_k^j, t_k^{j+1})$.

Now we review the following lemma for strongly connected graphs.

Lemma 1. *Consider $\tilde{\mathcal{G}}_k^\omega$ is the strongly connected subgraph of $\mathcal{G}_k^\omega$ and $\tilde{\mathcal{L}}_k^j$ as the Laplacian matrix associated with $\tilde{\mathcal{G}}_k^\omega$. In other words, $\tilde{\mathcal{L}}_k^j$ is obtained by omitting the rows and columns corresponding to the isolated nodes in $\mathcal{L}_k^j$ at subinterval $[t_k^j, t_k^{j+1})$. There exists a unique positive row vector $\theta_k^j = [\theta_{k1}^j, \cdots, \theta_{kN_k^j}^j]$ such that $\theta_k^j \tilde{\mathcal{L}}_k^j = 0$ and $\theta_k^j \mathbf{1}_{N_k^j} = N_k^j$. Define $\Theta_k^j := diag\ \{\theta_{k1}^j, \cdots, \theta_{kN_k^j}^j\}$. Then $\hat{\mathcal{L}}_k^j := \Theta_k^j \tilde{\mathcal{L}}_k^j + (\tilde{\mathcal{L}}_k^j)^T \Theta_k^j$ is the Laplacian of the balanced digraph obtained from $\tilde{\mathcal{G}}_k^\omega$ by ignoring the directions of the edges. It is concluded that $\hat{\mathcal{L}}_k^j$ is a positive semi-definite matrix (i.e., $\mathbf{1}_{N_k^j}^T \hat{\mathcal{L}}_k^j = 0$ and $\hat{\mathcal{L}}_k^j \mathbf{1}_{N_k^j} = 0$) [10].*

Before giving the main results of the paper, the following definition is introduced.

Definition 2 [9]. For the linear system given by

$$\dot{x} = Ax \quad (5)$$

$$y = Cx, \quad (6)$$

the distributed observer (4) achieves omniscience asymptotically if for all initial conditions $\hat{z}_i(0)$ we have:

$$\lim_{t\to\infty} \|\hat{z}_i(t) - x(t)\| = 0, \ \forall i \in \mathcal{V}(t). \quad (7)$$

Equation (7) indicates that each local observer state $\hat{z}_i(t)$ converges to the system state $x(t)$ asymptotically. The objective of this paper is to propose sufficient conditions for the asymptotic omniscience of robust and nonlinear distributed observers under switching topology.

In order to achieve the observability decomposition for the pairs (C_i, A), the orthogonal transformation $\mathcal{T}_i \in \mathbb{R}^{n\times n}$ is introduced. The orthogonal transformation matrix $\mathcal{T}_i$ for $i \in \mathcal{V}(t)$ is designed such that it decomposes the matrices A and C_i into the canonical form as

$$A = \mathcal{T}_i \begin{bmatrix} A_{io} & 0 \\ A_{ir} & A_{iu} \end{bmatrix} \mathcal{T}_i^T, C_i = \begin{bmatrix} C_{io} & 0 \end{bmatrix} \mathcal{T}_i^T, \quad (8)$$

where $C_{io} \in \mathbb{R}^{pi\times vi}$, $A_{io} \in \mathbb{R}^{vi\times vi}$, $A_{ir} \in \mathbb{R}^{(n-vi)\times vi}$, $A_{iu} \in \mathbb{R}^{(n-vi)\times(n-vi)}$, vi is the dimension of the observable subspace of the pair (C_i, A) and the pair (C_{io}, A_{io}) is observable.

According to Lemma 4 of [10], if we let $\tilde{\mathcal{L}}_k^j \in N_k^j \times N_k^j$ be the Laplacian matrix associated with the strongly connected subgraph $\tilde{\mathcal{G}}_k^\omega$, for all $g_i > 0, i \in \mathcal{I}_k^j$, there exists $\epsilon > 0$ such that

$$(\mathcal{T}_k^j)^T(\tilde{\mathcal{L}}_k^j \otimes I_n)\mathcal{T}_k^j + G_k^j > \epsilon I_{nN_k^j}, \quad (9)$$

where

$$\mathcal{T}_k^j = diag\{\mathcal{T}_i\},$$

$$G_k^j = diag\{G_i\}, G_i = \begin{bmatrix} g_i I_{vi} & 0 \\ 0 & 0_{n-vi} \end{bmatrix}, \forall i \in \mathcal{I}_k^j. \quad (10)$$

Remark 3. For building $\mathcal{T}_k^j$, and G_k^j we ignore the transformation matrices corresponding to the isolated nodes at the sub-interval $[t_k^j, t_k^{j+1})$. Therefore, in forming the matrices in (10) for each switch, we discard $\mathcal{T}_i$s corresponding to the isolated nodes.

III. Main Results

A. Distributed Nonlinear Observer without Disturbance

As the first step, for the dynamic system (1) , (2) without disturbance, i.e. $\Gamma d(t) = 0$, we suggest a nonlinear distributed observer with N local observers, having the following dynamics for each local observer at node i:

$$\begin{aligned}\dot{\hat{x}}_i = A\hat{x}_i + \daleth_i(y_i - C_i\hat{x}_i) + \gamma\mathcal{M}_i \sum_{j=1}^{N_k^j} \alpha_{ij}(t)(\hat{x}_j - \hat{x}_i) \\ + f(t, \hat{x}_i, u) + Bu, \ \forall i \in \mathcal{I}_k^j.\end{aligned} \quad (11)$$

where $\hat{x}_i \in \mathbb{R}^n$ is the state estimation of node i and $\daleth_i$, $\mathcal{M}_i$, γ are as defined in (4), and $\alpha_{ij}(t)$ is the element of adjacent matrix $\mathcal{A}(t)$ of the time-dependence communication graph. Note the difference between (11) and classic distributed Luenberger observer is that $\alpha_{ij}(t)$ is time-varying.
The estimation error of each local observer is defined as $e_i := \hat{x}_i - x$, then the following error dynamic of each local observer is obtained by combining (1) and (11):

$$\dot{e_i} = (A - \daleth_i C_i)e_i + \gamma\mathcal{M}_i \sum_{j=1}^{N_k^j} \alpha_{ij}(t)(e_j - e_i) + \tilde{f}_i, \forall i \in \mathcal{I}_k^j. \quad (12)$$

where $\tilde{f}_i = f(t, \hat{x}_i, u) - f(t, x, u)$. By stacking the estimation error of the local observers in the column vector $e :=\text{col}(e_1^T, e_2^T, \cdots, e_{N_k^j}^T)$, the dynamics of the concatenated error can be defined as (13) below

$$\dot{e} = (\Delta - \gamma\bar{\mathcal{M}}(\mathcal{L} \otimes I_n))e + \tilde{F}, \quad (13)$$

where $\Delta = diag\{A - \daleth_1 C_1, \cdots, A - \daleth_{N_k^j} C_{N_k^j}\}$,

$$\bar{\mathcal{M}} = diag\{\mathcal{M}_1, \cdots, \mathcal{M}_{N_k^j}\} \quad , \quad \tilde{F} = col(\tilde{f}_1^T, \tilde{f}_2^T, \cdots, \tilde{f}_{N_k^j}^T).$$

Now, under the Assumptions 1 to 3, for the dynamic system (1) and (2) without disturbance, the following theorem is introduced which assures the existence of the nonlinear distributed observer (11).

Theorem 1. *The distributed observer (11) can achieve omniscience asymptotically under the switching topologies $\mathcal{G}_k^\omega, \omega \in \varpi$ if there exists a positive coupling gain $\gamma \in \mathbb{R}^+$, positive definite symmetric matrices $\mathcal{P}_{io} \in \mathbb{R}^{v_i \times v_i}$, $\mathcal{P}_{iu} \in \mathbb{R}^{(n-v_i)\times(n-v_i)}$, and the matrix $\mathcal{W}_i \in \mathbb{R}^{v_i \times p_i}$, such that the following LMI holds for $i \in \mathcal{I}_k^j$*

$$\begin{bmatrix} \Phi_i + \zeta^2\eta I_{vi} & A_{ir}^T\mathcal{P}_{iu} & \mathcal{P}_{io} & 0 \\ \mathcal{P}_{iu}A_{ir} & \Psi_i + \zeta^2\eta I_{n-vi} & 0 & \mathcal{P}_{iu} \\ \mathcal{P}_{io} & 0 & -\eta I_{vi} & 0 \\ 0 & \mathcal{P}_{iu} & 0 & -\eta I_{n-vi} \end{bmatrix} < 0, \quad (14)$$

where, $\Phi_i \in \mathbb{R}^{vi \times vi}$ and $\Psi_i \in \mathbb{R}^{(n-vi)\times(n-vi)}$ are defined as below
$\Phi_i = sym(\mathcal{P}_{io}A_{io}) - sym(\mathcal{W}_i C_{io}) + \frac{\gamma}{\theta_{ki}^j} g_i I_{vi} - \frac{\gamma}{\theta_{ki}^j}\epsilon I_{vi} + 2\rho\mathcal{P}_{io}$
and,
$\Psi_i = sym(\mathcal{P}_{iu}A_{iu}) - \frac{\gamma}{\theta_{ki}^j}\epsilon I_{n-vi} + 2\rho\mathcal{P}_{iu}$, and $\eta \in \mathbb{R}^+$ is a positive balance factor.

Then, the gain matrices are obtained from the following equations:

$$\daleth_i := \mathcal{T}_i \begin{bmatrix} \daleth_{io} \\ 0 \end{bmatrix}, \mathcal{M}_i := \mathcal{T}_i \begin{bmatrix} \mathcal{P}_{io}^{-1} & 0 \\ 0 & \mathcal{P}_{iu}^{-1} \end{bmatrix} (\mathcal{T}_i)^T, \quad (15)$$

where $\daleth_{io} = \mathcal{P}_{io}^{-1}\mathcal{W}_i, \ i \in \mathcal{I}_k^j$.

The solvability of the LMI (14) is a requirement for the existence of the nonlinear distributed observer (11).

The design algorithm for the nonlinear distributed observer problem stated in (11) is listed in Table 1.

TABLE I
Algorithm to design nonlinear distributed observer (11)

1	Select an orthogonal matrix $\mathcal{T}_i$ for $i \in \mathcal{I}_k^j$ s.t. (8) holds.
2	Under each switching topology, compute the positive row vector $\theta_k^j = [\theta_{k1}^j, \cdots, \theta_{kN_k^j}^j]$ s.t. $\theta_k^j \tilde{\mathcal{L}}_k^j = 0$ and $\theta_k^j \mathbf{1}_{N_k^j} = N_k^j$.
3	Consider $g_i > 0$, $i \in \mathcal{I}_k^j$ and choose $\epsilon > 0$ s.t. (9) holds.
4	Solve LMI (14) to obtain $\mathcal{P}_{io}, \mathcal{P}_{iu}, \mathcal{W}_i$ and γ for $i \in \mathcal{I}_k^j$.
5	Compute $\daleth_{io} = \mathcal{P}_{io}^{-1}\mathcal{W}_i$, and get $\daleth_i$ and $\mathcal{M}_i$ as in (15), $i \in \mathcal{I}_k^j$.

Proof. The Lyapunov function candidate is defined as

$$V_k^\omega(e) = \sum_{i=1}^{N_k^j} \theta_{ki}^j e_i^T \mathcal{P}_i e_i \quad (16)$$

with $\mathcal{P} = diag\{\mathcal{P}_1, \cdots, \mathcal{P}_{N_k^j}\}$ and $\mathcal{P}_i = \mathcal{M}_i^{-1}, i \in \mathcal{I}_k^j$. By differentiating the Lyapunov function (16) along the error dynamic trajectory (13) yields

$$\begin{aligned}\dot{V}_k^\omega(e) = e^T[(\Theta_k^j \otimes I_n)(\Delta^T\mathcal{P} + \mathcal{P}\Delta) - \gamma\hat{\mathcal{L}}_k^j \otimes I_n]e \\ + 2e^T(\Theta_k^j \otimes I_n)\mathcal{P}\tilde{F}.\end{aligned} \quad (17)$$

Considering the Lipschitz condition (3), the following inequality will result:

$$\begin{aligned}\tilde{f}_i^T\mathcal{P}_i e_i + e_i^T\mathcal{P}_i\tilde{f}_i &\leq \eta^{-1}e_i^T\mathcal{P}_i\mathcal{P}_i e_i + \eta\tilde{f}_i^T\tilde{f}_i \\ &\leq \eta^{-1}e_i^T\mathcal{P}_i\mathcal{P}_i e_i + \eta\zeta^2\|\hat{x}_i - x\|^2 \\ &\leq e_i^T(\eta^{-1}\mathcal{P}_i\mathcal{P}_i + \eta\zeta^2)e_i, i \in \mathcal{I}_k^j\end{aligned} \quad (18)$$

Combining (18) and (17) yields

$$\dot{V}_k^{\omega}(e) \le e^T[(\Theta_k^j \otimes I_n)(\Delta^T \mathcal{P} + \mathcal{P}\Delta + \zeta^2 \eta I_{nN_k^j} + \eta^{-1}\mathcal{P}\mathcal{P}) - \gamma \hat{\mathcal{L}}_k^j \otimes I_n]e. \tag{19}$$

By applying the Schur complement Lemma to (14), we get the following inequality

$$\begin{bmatrix} \Phi_i + \zeta^2 \eta I_{vi} + \eta^{-1}\mathcal{P}_{io}\mathcal{P}_{io} & A_{ir}^T \mathcal{P}_{iu} \\ \mathcal{P}_{iu} A_{ir} & \Omega_i \end{bmatrix} < 0. \tag{20}$$

Here $\Omega_i = \Psi_i + \zeta^2 \eta I_{n-vi} + \eta^{-1}\mathcal{P}_{iu}\mathcal{P}_{iu}$, Φ_i and Ψ_i are defined as in (14) in Theorem 1. Moreover, by putting (20) and (9) together the following inequality is derived

$$diag\{\mathcal{Q}_1, \cdots, \mathcal{Q}_{N_k^j}\} - (\mathcal{T}_k^j)^T \gamma (\hat{\mathcal{L}}_k^j \otimes I_n)\mathcal{T}_k^j \le 0, \tag{21}$$

where

$$\mathcal{Q}_i = \theta_{ki}^j(\begin{bmatrix} \phi_i + \zeta^2 \eta I_{vi} + \eta^{-1}\mathcal{P}_{io}\mathcal{P}_{io} & A_{ir}^T \mathcal{P}_{iu} \\ \mathcal{P}_{iu} A_{ir} & \chi_i \end{bmatrix}), \tag{22}$$

$\chi_i = \psi_i + \zeta^2 \eta I_{n-vi} + \eta^{-1}\mathcal{P}_{iu}\mathcal{P}_{iu}$,
$\phi_i = \mathrm{sym}(\mathcal{P}_{io}A_{io}) - \mathrm{sym}(\mathcal{W}_i C_{io}) + 2\rho \mathcal{P}_{io}$,
$\psi_i = \mathrm{sym}(\mathcal{P}_{iu}A_{iu}) + 2\rho \mathcal{P}_{iu}$.
Pre- and post-multiplying of (21) by $\mathcal{T}_k^j$ and its transpose yields the following inequality

$$(\Theta_k^j \otimes I_n)(\Delta^T \mathcal{P} + \mathcal{P}\Delta + \zeta^2 \eta I_{nN_k^j} + \eta^{-1}\mathcal{P}\mathcal{P} + 2\rho\mathcal{P}) - \gamma \hat{\mathcal{L}}_k^j \otimes I_n \le 0. \tag{23}$$

It can be readily inferred from (23) that $\dot{V}_k^{\omega}(e) < -2\rho V_k^{\omega}(e)$. □

As a result, the nonlinear distributed observer (11) obtains asymptotic omniscience, and all the calculations of the error dynamic (13) converge to zero asymptotically with the convergence rate of at least $\rho > 0$.

Remark 4. Both inequalities (9) and (14) are LMIs that can be mathematically addressed with the LMI Toolbox. The feasibility of solving these LMIs depends on the nonlinear Lipschitz constant and cannot be solved if it exceeds the practical limit. It should also be emphasized that the only constraint related to the nonlinear term of the described system (1) is the Lipschitz condition (3), and the only limitation related to the linear term is the observability. Therefore, the unobservability problem of the linear part of the system cannot be solved by working on the nonlinear term.

B. Robust Nonlinear Distributed Observer

Referring to the nonlinear system (1) and (2) with external disturbance, i.e. $\Gamma d(t) \ne 0$, it is possible to ensure the convergence of the estimation error in the suggested robust nonlinear observer by adding the following assumptions.

Assumption 4: Associated with each node for a given positive definite symmetric matrix $\mathcal{P}_i$, there exists a vector $\mathcal{Z}_i^T \in \mathbb{R}^{p_i}, \forall i \in \mathcal{I}_k^j$ such that

$$\Gamma^T \mathcal{P}_i = \mathcal{Z}_i C_i. \tag{24}$$

This assumption is known as the matching condition [20] where C_i and Γ are output and disturbance matrices with known values, respectively. The proposed robust distributed observer for each node is defined as

$$\dot{\hat{x}}_i = A\hat{x}_i + \daleth_i(y_i - C_i\hat{x}_i) + \gamma \mathcal{M}_i \sum_{j=1}^{N_k^j} \alpha_{ij}(t)(\hat{x}_j - \hat{x}_i) + f(t, \hat{x}_i, u) + \int_i + Bu, \ \forall i \in \mathcal{I}_k^j, \tag{25}$$

where $\int_i \in \mathbb{R}^n$ is a discontinuous function defined as [19]

$$\int_i = \begin{cases} -\frac{\mathcal{P}_i^{-1} C_i^T \mathcal{Z}_i^T \mathcal{Z}_i C_i e_i D}{\|\mathcal{Z}_i C_i e_i\|} & \|\mathcal{Z}_i C_i e_i\| \ne 0 \\ 0 & \|\mathcal{Z}_i C_i e_i\| = 0. \end{cases} \tag{26}$$

By subtracting (1) from (25), the error dynamic of each local observer is given by

$$\dot{e}_i = (A - \daleth_i C_i)e_i + \gamma \mathcal{M}_i \sum_{j=1}^{N_k^j} \alpha_{ij}(t)(e_j - e_i) + \tilde{f}_i + \int_i - \Gamma d(t). \tag{27}$$

By stacking the estimation error of the local observers in a column vector as in (13), the global estimation error dynamic can be found as

$$\dot{e} = \Delta e - \gamma \bar{\mathcal{M}}(\hat{\mathcal{L}}_k^j \otimes I_n)e + \tilde{F} + \mathcal{S} - \Lambda d(t). \tag{28}$$

In (28), $\mathcal{S} := col(\int_1^T, \int_2^T, \cdots, \int_{N_k^j}^T)$ and $\Lambda \in R^{n.N_k^j}$ which is formed by superimposing N_k^j same distribution matrix.

Theorem 2. *For the dynamic system (1) and (2) with the external disturbance under the Assumptions 1 to 4, let $\theta_{ki}^j > 0$, $i \in \mathcal{I}_k^j$ be defined as in Lemma 1, $g_i > 0, i \in \mathcal{I}_k^j$, and $\epsilon > 0$ be chosen such that the inequality (9) is satisfied. For a given error convergence rate $\rho > 0$, assuming that the coupling gain $\gamma \in \mathbb{R}^+$, the matrix $\mathcal{W}_i \in \mathbb{R}^{v_i \times p_i}$, $\mathcal{Z}_i \in \mathbb{R}^{1 \times p_i}$, and the positive definite symmetric matrices $\mathcal{P}_{io} \in \mathbb{R}^{v_i \times v_i}$, and $\mathcal{P}_{iu} \in \mathbb{R}^{n-v_i \times n-v_i}$ can be found to satisfy the set of LMIs in (14). Then, there exists matrices $\daleth_i$ and $\mathcal{M}_i$, $i \in \mathcal{I}_k^j$, the coupling gain $\gamma \in \mathbb{R}^+$, and the discontinuous function $\int_i \in \mathbb{R}^n$, $i \in \mathcal{I}_k^j$, such that the error dynamic (28) converges to zero with convergence rate of ρ.*

Proof. Differentiating the Lyapunov function (16) along the system error dynamics (28) yields

$$\dot{V}_{\omega}(e) = e^T[(\Theta_k^j \otimes I_n)(\Delta^T \mathcal{P} + \mathcal{P}\Delta) - \gamma \hat{\mathcal{L}}_k^j \otimes I_n]e + 2e^T(\Theta_k^j \otimes I_n)\mathcal{P}(\tilde{F} + \mathcal{S} - \Lambda d(t)). \tag{29}$$

Considering (24) and the discontinuous function (26), it can be shown that

$$\begin{aligned} 2e_i^T \mathcal{P}_i \int_i - 2e_i^T \mathcal{P}_i \Gamma d(t) &\le -2\frac{e_i^T C_i^T \mathcal{Z}_i^T \mathcal{Z}_i C_i e_i D}{\|\mathcal{Z}_i C_i e_i\|} + 2\|\Gamma^T \mathcal{P}_i e_i\| D \\ &\le 2(-\|\mathcal{Z}_i C_i e_i\| + \|\mathcal{Z}_i C_i e_i\|)D = 0. \end{aligned} \tag{30}$$

According to (30), similar to the proof of Theorem (1) we can achieve (23) and finally $\dot{V}_k^{\omega}(e) < -2\rho V_k^{\omega}(e)$. □

As a result, the robust nonlinear distributed observer (25) obtains asymptotic omniscience, converging all the calculations of the error dynamic (28) to zero asymptotically with a convergence rate of at least $\rho > 0$.

Remark 5. To design a robust distributed observer, the matching condition is a constraint and the magnitude of the disturbance should not be excessively exceeded.

Remark 6. To prove the stability of the overall system, it is required to define a comprehensive Lyapunov function that is a union of all Lyapunov functions for each sub-interval defined in Theorem 2. The issue with stability, when the switch happens, is that the system will experience some level of uncertainty. Since this uncertainty is in the form of a sign function it is considered a bounded uncertainty and appears in the Laplacian matrix next to other uncertainties in relation (27). Since the distributed observer design proposed in this paper is designed to be robust against such bounded uncertainties, the stability of the closed-loop system can still be guaranteed.

IV. Simulation Examples

Example 1: We consider a nonlinear system with four measurement nodes. In this case, the dynamic system (1) and (2) can be represented with the following matrices

$$A = \begin{bmatrix} -1 & 0 & 0 & 0 & 0 & 0 \\ -1 & 1 & 1 & 0 & 0 & 0 \\ 1 & -2 & -1 & -1 & 1 & 1 \\ 0 & 0 & 0 & -1 & 0 & 0 \\ -8 & 1 & -1 & -1 & -2 & 0 \\ 4 & -0.5 & 0.5 & 0 & 0 & -4 \end{bmatrix}, B = 0,$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 2 & 0 & 0 \\ 2 & 0 & 0 & 1 & 0 & 0 \\ 2 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 2 & 0 & 0 \\ 1 & 0 & 2 & 0 & 0 & 0 \\ 2 & 0 & 4 & 0 & 0 & 0 \end{bmatrix}$$

$$C_1 = \begin{bmatrix} 1 & 0 & 0 & 2 & 0 & 0 \\ 2 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

$$C_2 = \begin{bmatrix} 2 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

$$C_3 = \begin{bmatrix} 0 & 0 & 0 & 2 & 0 & 0 \end{bmatrix}$$

$$C_4 = \begin{bmatrix} 1 & 0 & 2 & 0 & 0 & 0 \\ 2 & 0 & 4 & 0 & 0 & 0 \end{bmatrix}$$

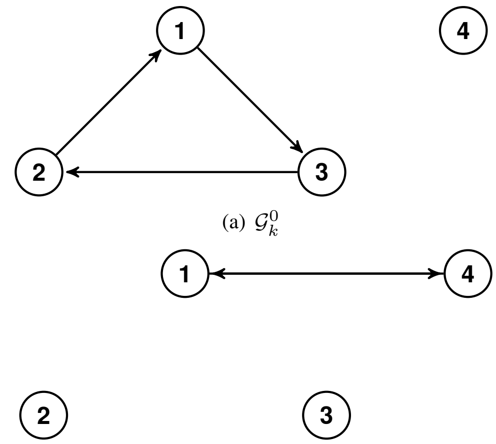
and

$$f(x) = \begin{bmatrix} cos(x_1) \\ sin(x_3) \\ x_1^2 \\ 0 \\ 0 \\ 0 \end{bmatrix}, d(t) = 0.$$

These four nodes transmit information over a communication network described through a switching communication network $\mathcal{G}_{\sigma(t)}$ illustrated by two digraphs in Figs 1, and governed by the following switching signal:

$$\sigma(t) = \left\{ \begin{bmatrix} 0, & if \quad kT_k \leq t < (k+\frac{1}{3})T_k \\ 1, & if \quad (k+\frac{1}{3})T_k \leq t < (k+1)T_k. \end{bmatrix} \right. \quad (31)$$

where $k = \{0, 1, 2, ...\}$, and $T_k = 3$.



(a) $\mathcal{G}_k^0$

(b) $\mathcal{G}_k^1$

Fig. 1. Switching Topology of the Graph used for Example 1.

It can be seen from Fig.1 that the subgraphs of $\mathcal{G}_k^0$ and $\mathcal{G}_k^1$ are strongly connected, and the union of $\mathcal{G}_k^0$ and $\mathcal{G}_k^1$ is uniformly strongly connected. Therefore, Assumption 3 is satisfied. By choosing the decay rate $\rho = 1$ and solving LMI (14), the coupling gain is $\gamma = 190.9510$. The gain matrices of the three local observers in the first switch are computed as:

$$\daleth_1 = \begin{bmatrix} 136.4019 & -59.5304 \\ 0 & 0 \\ 0 & 0 \\ 82.7251 & -148.0247 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \daleth_2 = \begin{bmatrix} 4.5737 \\ -23.9247 \\ -4.1396 \\ -4.1417 \\ -18.5646 \\ 2.1322 \end{bmatrix},$$

$$\daleth_3 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 8.4388 \\ 0 \\ 0 \end{bmatrix},$$

$$\mathcal{M}_1 = 10^{-2} \begin{bmatrix} 18.5 & 0 & 0 & 0 & 0 & 0 \\ 0 & 55.0 & -28.9 & 0 & 36.0 & 16.5 \\ 0 & -28.9 & 72.6 & 0 & -0.02 & -85.2 \\ 0 & 0 & 0 & 18.5 & 0 & 0 \\ 0 & 36.0 & -0.02 & 0 & 360.1 & -225 \\ 0 & 16.5 & -85.2 & 0 & -225 & 475 \end{bmatrix}$$

$$\mathcal{M}_2 = 10^{-3} \begin{bmatrix} 4.8 & 1.1 & -10.9 & -10.7 & -7.1 & 2.9 \\ 1.1 & 43.5 & -9.6 & -15.4 & 19.9 & -2.1 \\ -10.9 & -9.6 & 27.6 & 26.2 & 13.3 & -7.1 \\ -10.7 & -15.4 & 26.2 & 30.2 & 10.0 & -6.2 \\ -7.1 & 19.9 & 13.3 & 10.0 & 24.8 & -7.2 \\ 2.9 & -2.1 & -7.1 & -6.2 & -7.2 & 4.7 \end{bmatrix}$$

$$\mathcal{M}_3 = 10^{-2} \begin{bmatrix} 18.7 & 5.1 & -6.3 & 0 & 11.8 & 0.7 \\ 5.1 & 58.3 & -25.1 & 0 & 57.2 & -19.7 \\ -6.3 & -25.1 & 64.9 & 0 & -16.1 & -59.4 \\ 0 & 0 & 0 & 0.01 & 0 & 0 \\ 11.8 & 57.2 & -16.1 & 0 & 342.2 & -250 \\ 0.7 & -19.7 & -59.4 & 0 & -250 & 459 \end{bmatrix}$$

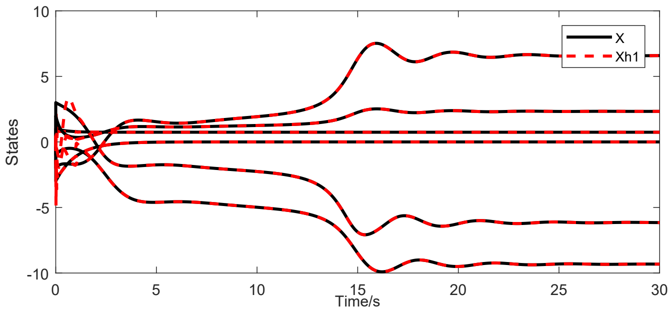


Fig. 2. The real and estimated value of state x by Node 1.

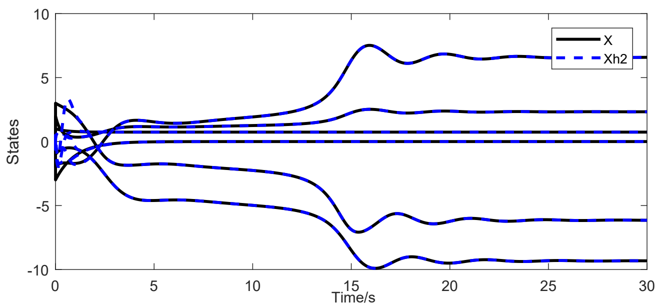


Fig. 3. The real and estimated value of state x by Node 2.

Note that $\theta^0_{k1} = \theta^1_{k1}$, since the positive vectors $\theta^0_k = [1, 1, 1]$ and $\theta^1_k = [1, 1]$. Therefore, the gain matrices of Node 1 are not changed in the second switch. The gain matrices of Node 4 in the second switch are computed as follows:

$$\daleth_4 = \begin{bmatrix} 0.0114 & 0.0480 \\ -5.2081 & -8.4560 \\ 0.7703 & 1.2551 \\ 2.0676 & 3.2027 \\ -2.6674 & -4.1432 \\ 0.6896 & 1.2903 \end{bmatrix},$$

$$\mathcal{M}_4 = 10^{-3} \begin{bmatrix} 2.2 & 1.5 & -1.1 & -6.1 & -7 & 1.9 \\ 1.5 & 15.9 & -2.9 & -11 & 2.9 & 0.4 \\ -1.1 & -2.9 & 1.2 & 3.9 & 2.3 & -0.9 \\ -6.1 & -11 & 3.9 & 21.8 & 17.9 & -5.7 \\ -7 & 2.9 & 2.3 & 7.9 & 31.6 & -8.8 \\ 1.9 & 0.4 & -0.9 & -5.7 & -8.8 & 4.2 \end{bmatrix},$$

The simulation results are plotted in Figs. 2-5. The figures are showing the real and estimated states by each agent.

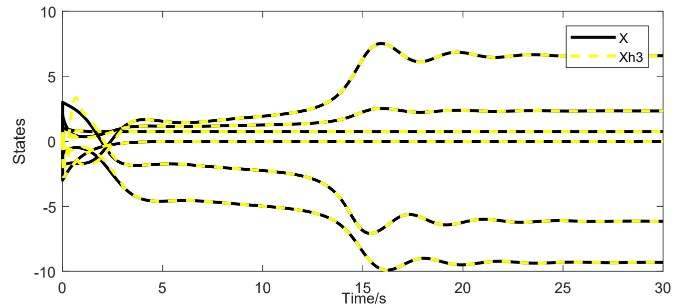


Fig. 4. The real and estimated value of state x by Node 3.

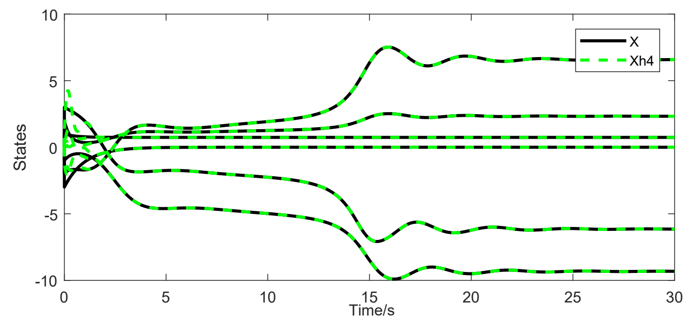


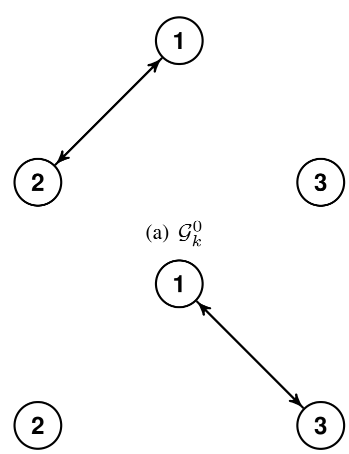
Fig. 5. The real and estimated value of state x by Node 4.

Example 2: In the second numerical example we assume the system matrices in (1) and (2) are defined as

$$A = \begin{bmatrix} -1 & 0 & 1 \\ 1 & -2 & 0 \\ 0 & 0 & -1 \end{bmatrix}, C = \begin{bmatrix} -2 & 2 & 1 \\ 1 & 0 & 1 \\ 1 & -1 & 1 \end{bmatrix} = \begin{bmatrix} C_1 \\ C_2 \\ C_3 \end{bmatrix}$$

$$B = 0 \quad , \quad f^T(x) = [\ 0 \quad 0 \quad \sin(x_1)\ ].$$

These three nodes transmit information over a communication network described by the switching graph in Fig. 6, which is dictated by the switching signal introduced in (31). Subgraphs of $\mathcal{G}^0_k$ and $\mathcal{G}^1_k$ in Fig. 6 are strongly connected as presented in Example 1, and the union of $\mathcal{G}^0_k$ and $\mathcal{G}^1_k$ is uniformly strongly connected. Therefore, Assumption 3 is



(a) $\mathcal{G}^0_k$

(b) $\mathcal{G}^1_k$

Fig. 6. Switching Topology of the Graph used for Example 2.

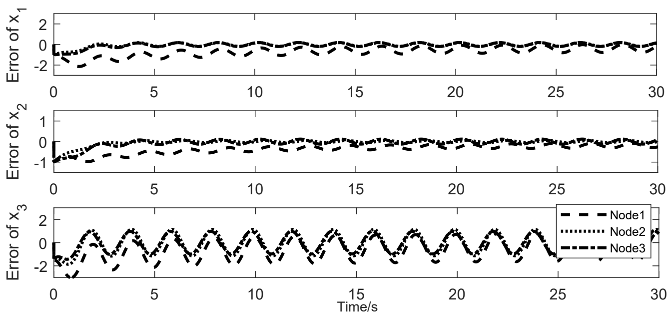


Fig. 7. The estimation error of states without applying $\int_i, i \in \mathcal{N}$.

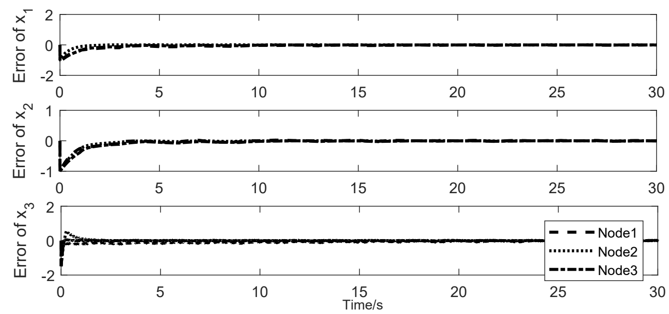


Fig. 8. The estimation error of states after applying $\int_i, i \in \mathcal{N}$.

satisfied. The disturbance signal is defined as $d = \sin(\pi t)$ and the disturbance input matrix Γ is given by Γ^T=$[0, 0, 4]$. The initial conditions for all local observers are set to zero. By applying the synthesis method proposed in Theorem 2 and under the matching condition (24), the set of LMIs (14) can be solved and $\gamma = 0.0353$. The gain matrices of the robust nonlinear distributed observer (25) for Nodes 1 and 2 in the first switch are calculated as:

$$\daleth_1 = \begin{bmatrix} -0.0334 \\ 0.0334 \\ -0.0753 \end{bmatrix}, \daleth_2 = \begin{bmatrix} 1.1790 \\ 0 \\ 1.4978 \end{bmatrix},$$

$$\mathcal{M}_1 = \begin{bmatrix} 124.3712 & 124.0414 & 0.3158 \\ 124.0414 & 124.3712 & -0.3158 \\ 0.3158 & -0.3158 & 0.7024 \end{bmatrix},$$

$$\mathcal{M}_2 = \begin{bmatrix} 0.5892 & 0 & -0.3328 \\ 0 & 50.6320 & 0 \\ -0.3328 & 0 & 0.7304 \end{bmatrix},$$

$$\mathcal{Z}_1 = 0.0020, \mathcal{Z}_2 = 0.0031.$$

The gain matrices of the Node 3 in the second switch are computed as follows:

$$\daleth_3 = \begin{bmatrix} 0.0272 \\ -0.0272 \\ 0.0638 \end{bmatrix}, \mathcal{M}_3 = \begin{bmatrix} 53.2666 & 52.9255 & 0.3039 \\ 52.9255 & 53.2666 & -0.3039 \\ 0.3039 & -0.3039 & 0.7199 \end{bmatrix},$$

$$\mathcal{Z}_3 = 0.0018.$$

Note $\theta^0_{k1} = \theta^1_{k1}$, since the positive vectors $\theta^0_k = [1, 1]$ and $\theta^1_k = [1, 1]$. Therefore, the gain matrices of Node 1 are not changed in the second switch. The results of state estimation error before and after applying the robust term in the nonlinear distributed observer (25) are plotted in Figs. 7 and 8, respectively. It shows the efficacy of the proposed robust distributed observer in eliminating the effect of sinusoidal disturbance.

V. CONCLUSIONS

This paper investigates the design problem of a robust and nonlinear distributed observer for a class of Lipschitz nonlinear dynamic systems in the presence of nonlinear uncertainty and bounded disturbance. The topology assumed for the communication network of the distributed system does not remain connected all the time and the network may go under temporary communication failure. All local observers are not necessarily observable, however, it is assumed that the whole system is collectively observable. The design problem is formulated in the form of an LMI to calculate the observer gains under time-varying switching topology so that the observer error dynamics become asymptotically omniscient. The effect of nonlinear uncertainties and norm-bounded disturbance are mitigated by introducing a discontinuous robust term. The illustrative numerical results prove the efficacy of the proposed robust distributed observer method when matched uncertainty affects the dynamics of each individual agent. In future work, we are going to apply the proposed method for estimating the states of cooperative robots.

REFERENCES

[1] T. Liu and J. Huang, "A distributed observer for a class of nonlinear systems and its application to a leader-following consensus problem," *IEEE Transactions on Automatic Control*, vol. 64, no. 3, pp. 1221–1227, 2019.

[2] X. Bu, P. Zhu, Z. Hou, and J. Liang, "Finite-time consensus for linear multi-agent systems using data-driven terminal ilc," *IEEE Transactions on Circuits and Systems II: Express Briefs*, vol. 67, no. 10, pp. 2029–2033, 2020.

[3] S. Bhowmick and S. Panja, "Leader–follower bipartite consensus of linear multiagent systems over a signed directed graph," *IEEE Transactions on Circuits and Systems II: Express Briefs*, vol. 66, no. 8, pp. 1436–1440, 2019.

[4] S. Park and N. C. Martins, "Necessary and sufficient conditions for the stabilizability of a class of lti distributed observers," in *2012 IEEE 51st IEEE Conference on Decision and Control (CDC)*. IEEE, 2012, pp. 7431–7436.

[5] H. Zhu, K. Liu, J. Lü, Z. Lin, and Y. Chen, "On the cooperative observability of a continuous-time linear system on an undirected network," in *2014 International Joint Conference on Neural Networks (IJCNN)*. IEEE, 2014, pp. 2940–2944.

[6] A. Mitra and S. Sundaram, "Distributed observers for LTI systems," *IEEE Transactions on Automatic Control*, vol. 63, no. 11, pp. 3689–3704, 2018.

[7] L. Wang, J. Liu, and A. S. Morse, "A distributed observer for a continuous-time linear system," in *2019 American Control Conference (ACC)*, 2019, pp. 86–91.

[8] K. Liu, H. Zhu, and J. Lü, "Cooperative stabilization of a class of lti plants with distributed observers," *IEEE Transactions on Circuits and Systems I: Regular Papers*, vol. 64, no. 7, pp. 1891–1902, 2017.

[9] T. Kim, H. Shim, and D. D. Cho, "Distributed luenberger observer design," in *2016 IEEE 55th Conference on Decision and Control (CDC)*, 2016, pp. 6928–6933.

[10] W. Han, H. L. Trentelman, Z. Wang, and Y. Shen, "A simple approach to distributed observer design for linear systems," *IEEE Transactions on Automatic Control*, vol. 64, no. 1, pp. 329–336, 2019.

[11] T. Kim, C. Lee, and H. Shim, "Completely decentralized design of distributed observer for linear systems," *IEEE Transactions on Automatic Control*, vol. 65, no. 11, pp. 4664–4678, 2020.

[12] S. Battilotti and M. Mekhail, "Distributed estimation for nonlinear systems," *Automatica*, vol. 107, pp. 562–573, 2019. [Online]. Available: https://www.sciencedirect.com/science/article/pii/S0005109819303206

[13] H. Xu, J. Wang, H. Wang, B. Wang, and M. Bai, "Distributed observer design for omniscience asymptotical aimed at a class of nonlinear system*," in *2019 IEEE 58th Conference on Decision and Control (CDC)*, 2019, pp. 3303–3308.

[14] L. Wang, J. Liu, and A. S. Morse, "A distributed observer for a continuous-time linear system with time-varying network," *arXiv preprint arXiv:2003.02134*, 2020.

[15] L. Zhang, M. Lu, F. Deng, and J. Chen, "Distributed state estimation of linear systems under uniformly connected switching networks," in *2021 60th IEEE Conference on Decision and Control (CDC)*. IEEE, 2021, pp. 4008–4013.

[16] T. Liu and J. Huang, "Distributed exponential state estimation of linear systems over jointly connected switching networks," *arXiv preprint arXiv:2205.00218*, 2022.

[17] H. Xu, J. Wang, H. Wang, S. Zhao, and H. Lin, "Distributed observer design for achieving asymptotical omniscience over time-variant disconnected networks," *IFAC-PapersOnLine*, vol. 53, no. 2, pp. 3565–3570, 2020.

[18] G. Yang, H. Rezaee, A. Alessandri, and T. Parisini, "State estimation using a network of distributed observers with switching communication topology," *Automatica*, vol. 147, p. 110690, 2023.

[19] Y. K. Shirkadehi, M. Ekramian, and M. Ataei, "Sliding mode observer for uncertain nonlinear systems based on generalized lipschitz condition," in *2018 6th RSI International Conference on Robotics and Mechatronics (IcRoM)*, 2018, pp. 100–105.

[20] B. Walcott and S. Zak, "State observation of nonlinear uncertain dynamical systems," *IEEE Transactions on automatic control*, vol. 32, no. 2, pp. 166–170, 1987.

[21] M. Ekramian, "Observer-based controller for lipschitz nonlinear systems," *International Journal of Systems Science*, vol. 48, no. 16, pp. 3411–3418, 2017.

[22] W. Ni and D. Cheng, "Leader-following consensus of multi-agent systems under fixed and switching topologies," *Systems & control letters*, vol. 59, no. 3-4, pp. 209–217, 2010.

[23] M. Sajjadi, M. Chahari, H. N. Pishkenari, and G. Vossoughi, "Designing nonlinear observer for topography estimation in trolling mode atomic force microscopy," *Journal of Vibration and Control*, vol. 28, no. 23-24, pp. 3890–3905, 2022.