

Theorem: With the notation as above

$$\begin{aligned} c_0 &= f(a_{m-1}, \dots, a_0, b_{m-1}, \dots, b_0) \\ c_1 &= f(a_0, a_{m-1}, a_{m-2}, \dots, a_1, b_0, b_{m-1}, b_{m-2}, \dots, b_1) \\ &\vdots \\ c_{m-1} &= f(a_{m-2}, \dots, a_0, a_{m-1}, b_{m-2}, \dots, b_0, b_{m-1}) \end{aligned} \quad (4)$$

The result is easy to show since $f(a_{m-2}, \dots, a_0, a_{m-1}, b_{m-2}, \dots, b_0, b_{m-1})$ is the first bit of $\log_2(\sigma(\alpha) + \sigma(\beta)) = \log_2(\sigma(\gamma)) = (\sigma(c))_2$, the first bit of which is c_{m-1} . The argument is repeated for the other bits. This shows that the same f can be used to calculate each bit of the logarithmic representation of $\gamma = \alpha + \beta$. f can easily be implemented with logic gates or be stored in a ROM. The concept is similar to the Massey-Omura multiplier for elements represented in the normal basis expansion. The structure of the adder is similar to that of the Massey-Omura multiplier [1, 2]. It is particularly well suited for a parallel-input-serial-output implementation.

f has no known general analytical form. It satisfies

- (i) $f(a_{m-1}, \dots, a_0, b_{m-1}, \dots, b_0) = f(b_{m-1}, \dots, b_0, a_{m-1}, \dots, a_0)$
- (ii) $f(a_{m-1}, \dots, a_0, a_{m-1}, \dots, a_0) = 1$
- (iii) $f(a_{m-1}, \dots, a_0, 1, \dots, 1) = f(1, \dots, 1, a_{m-1}, \dots, a_0) = a_0$.

Example: Consider the field $\mathbb{F}_{2^3} \cong \mathbb{F}_2(\zeta)$ where $\zeta^3 = \zeta + 1$. ζ is primitive and can be used as the basis for the logarithmic representation. For example $(111)_2 = 7$ represents the element $0 \in \mathbb{F}_{2^3}$ and $(101)_2 = 5$ represents the element $\zeta^5 = \zeta^2 + \zeta + 1 \in \mathbb{F}_{2^3}$ in the logarithmic representation. For the addition, the Massey-Omura type adder uses the function

$$\begin{aligned} f(a_2, a_1, a_0, b_2, b_1, b_0) &= a_2b_0b_2 + a_1b_0b_2 + b_0a_0a_2 + b_0b_2a_0 \\ &\quad + b_0b_1a_0 + b_0b_1 + a_2b_0b_1 + a_1b_0b_1 \\ &\quad + b_0a_0a_1 + b_0b_2 + b_0a_2 + b_1a_0a_1 + b_1 \\ &\quad + a_2b_1 + a_0a_2b_1 + a_1 + a_1b_2 + a_0a_1b_2 \\ &\quad + b_2a_0 + a_0a_2b_2 + a_0a_2 + a_0a_1 + 1 \end{aligned} \quad (5)$$

found by a computer search. It can be easily verified that the function satisfies all three properties listed above. f can be optimised and implemented with 27 conventional logic gates. Hence, to add ζ^5 with ζ^6 ($5 = (101)_2$ and $6 = (110)_2$), we evaluate

$$\begin{aligned} f(101, 110) &= 1 = c_0 \\ f(011, 101) &= 0 = c_2 \\ f(110, 011) &= 0 = c_1 \end{aligned} \quad (6)$$

from which the sum is found to be $(001)_2 = 1$, i.e. $\zeta^5 + \zeta^6 = \zeta$. The Massey-Omura adder computes the result without transforming between the logarithmic representation and an \mathbb{F}_2 -basis expansion representation.

Conclusion: The architecture presented performs the sum of two elements in the logarithmic representation. An application requiring an electronic circuit to perform many multiplications, inversions, divisions and exponentiations but few additions on elements of \mathbb{F}_{2^m} would benefit from using the logarithmic representation.

In this Letter integers are represented in the conventional binary 2 expansion (unsigned). Other integer representations may yield simpler circuitry. The integer representation must satisfy the generalised Massey-Omura condition with respect to the addition [3]. By analogy with the Massey-Omura multiplier this corresponds to representing the elements of $GF(2^m)$ in an optimal (or at least a minimal) normal basis [4].

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Shunt bootstrapping technique to improve bandwidth of transimpedance amplifiers

C. Hoyle and A. Peyton

The input capacitance of transimpedance amplifiers can limit their response time. One novel technique based on a shunt bootstrap method is shown to be of advantage to op-amp based circuits with a high transimpedance gain.

Introduction: Transimpedance amplifiers using voltage feedback op-amps are widely used in applications when a moderate/high bandwidth and a high sensitivity are required, such as photodiodes, photomultipliers, electron multipliers and capacitive sensors. The conventional circuit, outlined in Fig. 1, presents a virtual earth and thus almost zero load impedance to the current source. At high frequencies and especially for high gain circuits, the total input capacitance C_s can severely limit the available bandwidth from the circuit [1] or even cause a lightly damped/unstable dynamic response. A simple expression for the transimpedance gain of the circuit can be readily derived by assuming that the source resistance R_s is infinite and that the op-amp has a single pole transfer function (pole frequency ω_n , and DC gain of A_o):

$$v_o = \frac{-i.R_f}{\frac{s^2(C_s+C_f)R_f}{A_o\omega_n} + s\left(C_f R_f + \frac{1}{A_o\omega_n} + \frac{(C_s+C_f)R_f}{A_o}\right) + \frac{1}{A_o} + 1} \quad (1)$$

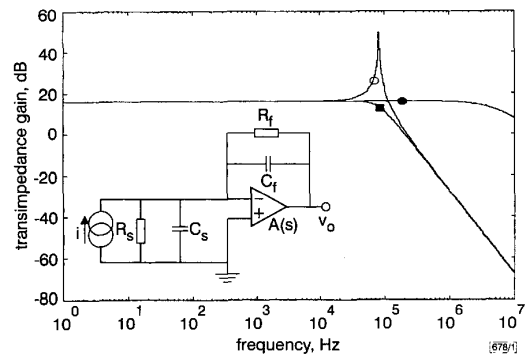


Fig. 1 Op-amp based transimpedance amplifier and its frequency response showing effects of circuit capacitances

- $C_s = 100$ pF, $C_f = 0$ pF
- $C_s = 100$ pF, $C_f = 2.8$ pF
- $C_s = 0$ pF, $C_f = 0$ pF

Using eqn. 1, typical plots are also shown in Fig. 1, with $R_f = 1$ M Ω , $C_s = 100$ pF, $f_n = 40$ Hz and $f_o = 4$ MHz. With no feedback capacitance C_f , the response has a resonant peak as a direct consequence of the source capacitance. This peak represents a lightly damped or unstable dynamic response. Lag compensation by introducing feedback capacitance ($C_f = 2.8$ pF) can be used to give a critically damped frequency response. However, this approach does not permit the full gain-bandwidth characteristic of the op-amp to be fully exploited. An effective reduction in source capacitance results in less feedback capacitance being required to obtain a critically damped response. The limit case, with $C_s = C_f = 0$, is also plotted.

Bootstrap approach: Bootstrapping techniques for reducing the input capacitance have been previously reported [2, 3] and several papers have described successful examples of transimpedance circuits using series bootstrapping [4]. The basic bootstrapping principle is to use an additional buffer amplifier to actively charge and discharge to the input capacitance as required. Consequently, the effective source capacitance is reduced, enabling the overall bandwidth of the circuit to be increased. There are four possible bootstrap configurations (series or shunt bootstrapping modes, with either floating or grounded sources) which can be applied to the basic circuit. Previous published techniques have employed only the series technique, for example in the case of a photodiode bootstrapped by the source of the input FET [5, 6]. In this Letter, we now describe a successful application of the shunt approach.

Shunt mode bootstrap: The bootstrap principle was applied to the photodiode preamplifier circuit shown in Fig. 2, using an AC coupled bootstrap path ($C_c = 47\text{nF}$). The frequency responses of the circuit with and without the bootstrap loop are compared in Fig. 2.

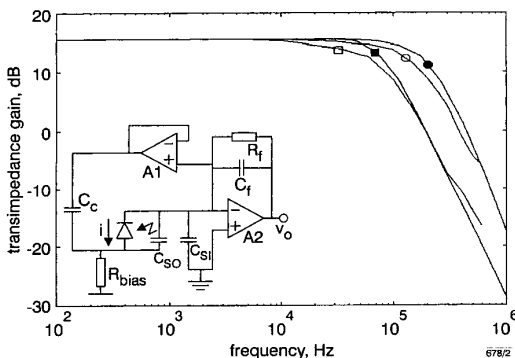


Fig. 2 Shunt bootstrap photodiode preamplifier and frequency responses of circuit with and without bootstrap loop

- theoretical critically damped bootstrap response
- experimental near critically damped bootstrap response
- theoretical critically damped standard response
- experimental near critically damped standard response

An LF353N op-amp, $1\text{M}\Omega$ feedback resistor and a large area photodiode (type 0SD15-5T) were selected and the circuit was tested with an optical signal source. In the bootstrap circuit, the only feedback capacitance required to produce a near critically damped response was the parasitic capacitance possessed by the layout and feedback resistor. Without the bootstrap loop the circuit required an additional 1.8pF of feedback capacitance to give a near critically damped response. It can be seen that the bootstrap circuit has twice the bandwidth of the standard circuit. Also included in Fig. 2 are the theoretical critically damped frequency responses of the two circuits using eqn. 1 and the small signal transfer function for the bootstrap circuit given below in eqn. 2. To obtain the small signal transfer function (eqn. 2) the source resistance was considered infinite and A1 and A2 were considered to be of the same type of op-amp with a single pole transfer function (pole frequency ω_a , unity gain frequency ω_o and DC gain of A_o) giving

$$v_o = \frac{-i.R_f}{\text{characteristic equation}} \quad (2)$$

where the characteristic equation is

$$\frac{s^2(C_{S0} + C_{S1} + C_f)R_f}{\omega_o} + \frac{s(C_{S0} + C_{S1} + C_f)R_f}{A_o} + sC_fR_f + \frac{s}{\omega_o} - \frac{s(s + \omega_a)C_{S0}R_f}{s + (\omega_a + \omega_o)} + \frac{1}{A_o} + 1 \quad (3)$$

The responses predicted by eqn. 1 and eqn. 2 were used with values for the op-amp and photodiode ($C_{S0} \approx 80\text{pF}$, $C_{S1} = 20\text{pF}$, $R_f = 1\text{M}\Omega$, $f_0 \approx 4\text{MHz}$, $f_a \approx 40\text{Hz}$, $A_0 \approx 10^5$). Sufficient feedback capacitance was added in both models to obtain a critically damped response. The bootstrap model required a feedback capacitance of 1.4pF compared to 2.8pF in the standard model. The increase in bandwidth exhibited by the bootstrap circuit can

be attributed to this reduction in feedback capacitance, indicating the effect of bootstrapping in effectively reducing the source capacitance. Frequency responses for a range of op-amps and bootstrap circuits have been considered and these have been extensively compared to SPICE simulations with very good agreement. Finally, the step responses of the standard and bootstrap circuits are compared in Fig. 3. The improvement in settling time can be clearly seen. The waveform at the output of A1 is included revealing the transients associated with the charge injection action of the bootstrap loop. It can be seen that the standard circuit requires additional feedback capacitance in order to obtain a similarly damped response to the bootstrap circuit.

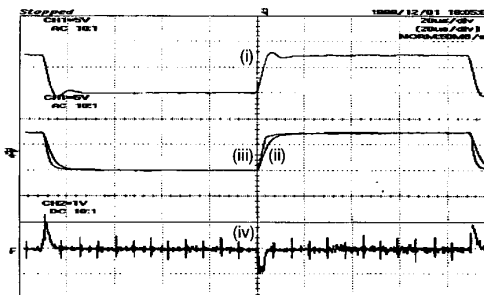


Fig. 3 Step responses of bootstrap and standard circuits

- (i) Standard response with parasitic feedback capacitance only
- (ii) Standard response with parasitic $+1.8\text{pF}$ feedback capacitance
- (iii) Bootstrap response with parasitic feedback capacitance only
- (iv) Output of bootstrap amplifier, A1

Conclusions: In this Letter we have presented a simple example of a new shunt bootstrap amplifier based on two op-amps of the same type and shows that the techniques can be used to realise a faster response than is possible with a single op-amp alone. The bootstrap method may provide a viable design option for applications with high gain (i.e. high R_f) and requiring a wide bandwidth (where the bandwidth is limited by the parasitic effects of the input capacitance).

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