The spring-and-lever balancing mechanism, George Carwardine and the Anglepoise lamp

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Abstract: Two short and direct methods of exact analysis of the spring-and-lever balancing mechanism are presented. It is shown that perfect balance can be achieved by the use of a close-coiled spring whose free length is effectively zero and whose stiffness is chosen appropriately. The two-degree-of-freedom balancing mechanism, commonly seen in desk lamps but useful in many other situations, is then analysed. The treatment is extended to allow for the weight of the components of the mechanism itself.

The originator of these devices was George Carwardine, whose patents over the period 1931–35 show the evolution of his ideas on this subject. A short biographical note is included as an appendix.

Keywords: balancing mechanism, equilibrator, constant force, Carwardine

NOTATION

a, a_1, a_2	overall lengths of the springs
b, b_1, b_2	lengths of the spring-operating arms
c, c_1, c_2	lengths of the fixed vertical arms
F, F_1, F_2	tensions in the springs
g	acceleration due to gravity
k, k_1, k_2	spring stiffnesses
l_0	unstretched length of the spring
m	mass of the supported load
m_1, m_3	masses of the arms
m_2	mass of the outer arm, including the supported load
P	notional added force
r, r_1, r_2	lengths of the arms
R_1, R_2, R_3	radii to the centres of gravity
θ, θ_1, θ_2	angles

1 INTRODUCTION

It is often desired to support the weight of an object such as a microphone, a power tool, an opening window or a desk lamp, and yet to allow it to be moved readily. There are numerous devices, sometimes called ‘equilibrators’, capable of supporting such loads, e.g. counterbalance weights, constant-force (‘negator’) springs or suitably designed spring-and-lever mechanisms.

The spring-and-lever type of equilibrator is light and compact, has low inertia and deserves to be more widely understood and used. Perhaps the best-known example is to be found in the Anglepoise lamp (Fig. 1), a series of elegant designs by the engineer George Carwardine, which appeared for the first time over 60 years ago in 1933–35 and have since become classics.

This paper sets out a new account of the working principle of the counterbalancing system and how to apply it, including Carwardine’s ingenious extension of the basic element to two degrees of freedom. As far as is known, the first published treatment of this problem, other than Carwardine’s patents, is that in the first edition (1971) of reference [**1**].

Earlier, Hain [**2**] had studied similar problems but had only devised techniques for a limited number of precision points, or for an infinite number of precision points using a snail of general form and a fusee. He appears to have overlooked Carwardine’s solution.

Nathan [**3**], evidently unaware of Carwardine’s work, derived the solution and extended it to two degrees of freedom. He proposed several methods of achieving a spring of effectively zero free length (an essential requirement for exact balance, as shown in Section 2), e.g. by combining a pair of tension and compression springs, but he did not mention the possibility of using a close-coiled spring.

Streit and Gilmore [**4**] apparently read [**1**] but failed to note that it included the solution (perhaps because reference [**1**] is a textbook in which this is set as a problem exercise). In reference [**4**], and again in reference [**5**], an analysis was presented of a balancing mechanism with any number of springs which need not lie in a

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Fig. 1 Anglepoise lamp of the original design

vertical plane. The use of springs that do not lie in a vertical plane introduces undesirable side loads and requires ball-joints instead of pins at the ends of springs; thus it is difficult to see why anyone would want to use this method.

2 ANALYSIS OF THE SINGLE-DEGREE-OF-FREEDOM MECHANISM

Figure 2a is a schematic diagram of a simple spring-and-lever balancing mechanism. A mass m is at point Z on the end of an arm CAZ, pivoted at A to a member AB. The whole mechanism is in a vertical plane, and AB is held in a vertical position. The aim is to support the weight of the mass m accurately over a wide range of positions θ. Two methods of solution of this problem are presented.

2.1 Method 1

It would be possible to balance the arm by means of a constant, vertically downward force at C equal to mgr/b (see Fig. 2a for the lengths a, b, c and r). The balancing spring CB needs to exert the same moment about A as this vertical force, for all positions θ of the arm.

Let a force P in the direction AC be added to the spring force F such that the resultant of these two forces is vertical. Then, since P has no moment about A, this vertical resultant has the same moment about A as force F has; for exact balance the resultant must always be equal to mgr/b.

Now, consider ABC as a triangle of forces used to obtain this resultant. The resultant, represented by AB, is vertical and will be constant if the force in the spring is proportional to CB, i.e. to the length of the spring. Thus the free length of the spring, i.e. the length when there is no tension, must be zero (see Section 2.3).

Finally, to make the resultant equal to mgr/b, the scale of the triangle of forces must be chosen so that the length c represents the force mgr/b; then an extension c of the spring would give this force, and the stiffness k is given by

$$k = \frac{mgr}{bc} \tag{1}$$

2.2 Method 2

An alternative solution is as follows:

$$\text{Area of triangle ABC} = \tfrac{1}{2}\,\text{BC} \times (\text{perpendicular distance of A from line BC}) \tag{2}$$

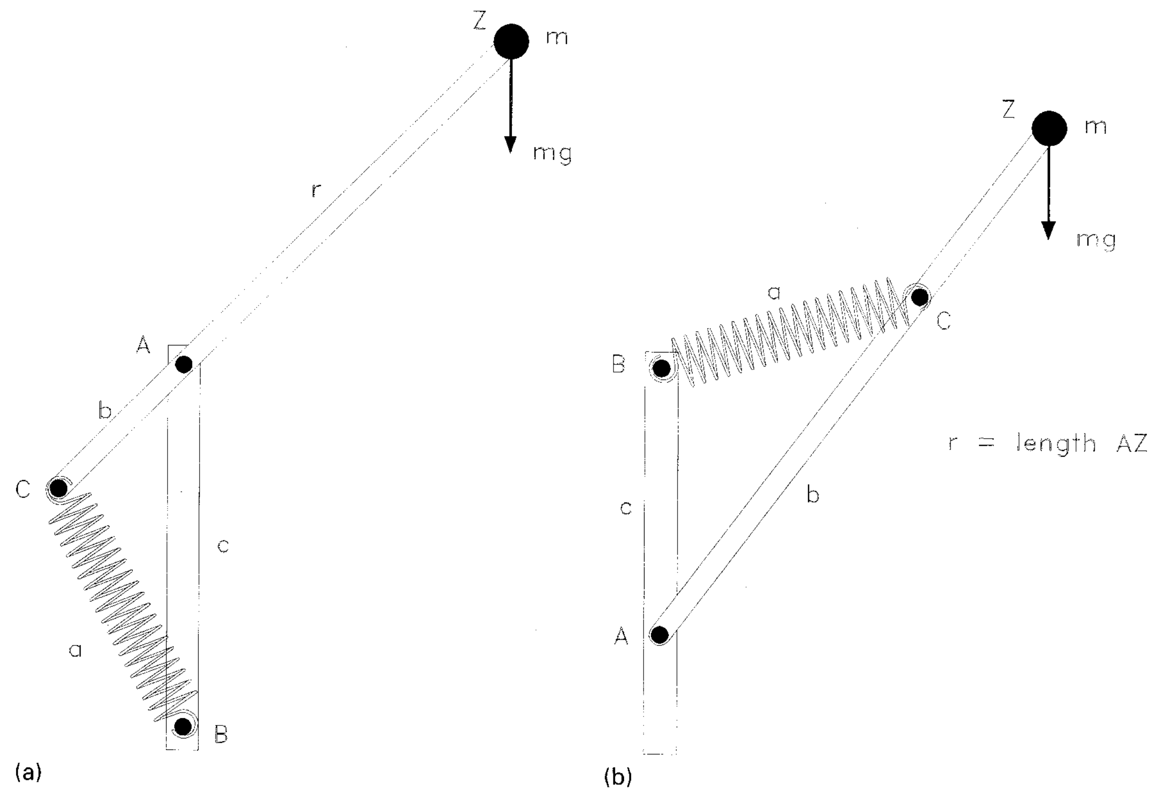

Fig. 2 Simple spring-and-lever balancing mechanism

Therefore the arm about A of the force F in the spring BC, in other words the perpendicular distance of A from the line BC, is

$$\frac{2 \times \text{area of triangle ABC}}{\text{BC}} \tag{3}$$

However, the area of the triangle ABC is equal to

$$\tfrac{1}{2} bc \sin\theta \tag{4}$$

Thus the arm about A of the force F is

$$\frac{bc \sin\theta}{a} \tag{5}$$

The required moment is $mgr \sin\theta$; therefore, for balance,

$$\frac{Fbc \sin\theta}{a} = mgr \sin\theta \tag{6}$$

or

$$F = \frac{mgr}{bc} a \tag{7}$$

as before. In other words, the tension F in the spring is proportional to its overall length a; the spring stiffness is $k = mgr/bc$. For the tension F to be brought to zero, a must also go to zero, i.e. the spring has zero free length.

Should the reader be in any doubt about this, put l_0 equal to the unstretched length of the spring. Then

$$F = k(a - l_0) \tag{8}$$

Therefore, for exact balance, substituting for F in equation (7),

$$k(a - l_0)bc = mgra \tag{9}$$

or

$$a = \frac{kl_0}{k - (mgr/bc)} \tag{10}$$

The length a clearly varies with θ, but all the quantities in the fraction on the right-hand side of equation (10) are constants. The only way that equation (10) can hold for all θ is if the right-hand side is indeterminate, i.e. if l_0 and $k - (mgr/bc)$ are both zero.

2.3 Zero-free-length springs

It is well known that a pre-tension in a spring can be induced by imposing a twist on the wire as the spring is wound [**6**, **7**]. The coils of the spring then remain tightly closed until the applied force reaches a certain value, and the effective unstretched length is less than the initial close-wound length. It is quite possible to reduce the effective unstretched length to zero, or even to a negative value.

2.4 Alternative arrangement with the spring above A

If desired, C may be placed on the same side of A as Z, but then B must be above A (Fig. 2b). It can be shown by similar methods that, for perfect balance, the requirements are the same, namely that the unstretched length l_0 of the spring must be zero, and the stiffness of the spring must be given by $k = mgr/bc$.

3 TWO-DEGREE-OF-FREEDOM DESIGNS

3.1 Configuration with springs on the arms

A two-degree-of-freedom system may be made using a parallelogram (or equivalent) mechanism to carry the vertical reference from the inner to the outer joint. One such arrangement is shown in Fig. 3a, where the basic scheme is that of Fig. 2b, i.e. with C on the same side of A as the load in both cases. $A_1L_1B_1$ and $A_2L_2B_2$ are congruent rigid links, and the parallelogrammatic four-bar chain $L_1L_2A_2A_1$ maintains A_2B_2 parallel to A_1B_1, i.e. vertical. The spring B_2C_2 for the outer system needs to be designed according to the requirements set out in Section 2 above.

As far as the inner system is concerned, the weight mg at Z can be replaced by a downward force mg at A_2 together with a clockwise couple $mgr_2 \sin\theta_2$. The vertical force mg at A_2 is supported by the spring B_1C_1, which must have zero free length and stiffness k_1 calculated as set out in equation (1). The couple $mgr_2 \sin\theta_2$ is carried entirely by tension and compression in the arms L_1L_2 and A_1A_2 respectively, and imposes no load on the spring B_1C_1. This is evident since the line A_2B_2 remains vertical; therefore, the couple does no work as the angle θ_1 is varied. Hence, once the stiffness of the inner spring has been calculated to bring the force mg at A_2 to equilibrium, the linkage is balanced for all values of the angles θ_1 and θ_2.

3.2 Configuration with all the springs at the base

The Anglepoise design has the more elegant two-degree-of-freedom system shown in Fig. 3b, with all the springs at the base and only one link AB. The four-bar chain HJED is parallelogrammatic and the lengths C_2D and AE are equal, so that the points C_2 and A, although not directly linked, lie on a line parallel to DE and HJZ. The lengths a_1 and a_2* and therefore the tensions in the two springs then depend only on θ_1 and θ_2 respectively; thus the two arms C_1AEJ and HJZ are effectively decoupled as far as their static equilibrium is concerned. This is borne out by the argument below.

By method 2 of Section 2.2, the arms of the two springs about point A are respectively

$$\frac{b_1c_1\sin\theta_1}{a_1} \quad \text{and} \quad \frac{b_2c_2\sin\theta_2}{a_2} \tag{11}$$

By moments about point A, for balance,

$$F_1\frac{b_1c_1\sin\theta_1}{a_1} + F_2\frac{b_2c_2\sin\theta_2}{a_2} = mg(r_1\sin\theta_1 + r_2\sin\theta_2) \tag{12}$$

where F_1 and F_2 are the tensions in springs 1 and 2 respectively. Equation (12) is to hold for all values of θ_1 and θ_2, but this can only be so if the terms in the equation involving θ_1 always balance and the terms involving θ_2 also always balance. Therefore

$$F_1 = \frac{mgr_1}{b_1c_1}a_1, \qquad F_2 = \frac{mgr_2}{b_2c_2}a_2 \tag{13}$$

(There are other ways of approaching this which are equally good, e.g. using energy methods.)

As in Section 2.2, the tension in each spring is proportional to its overall length a, i.e. it must have zero unstretched length. The stiffnesses of the two springs are

$$k_1 = \frac{mgr_1}{b_1c_1}, \qquad k_2 = \frac{mgr_2}{b_2c_2} \tag{14}$$

4 DISTRIBUTED MASS

So far the weights of the arms have been neglected. In practice these are often relatively small, but they may not be insignificant.

For design of the single-degree-of-freedom mechanism, referring again to Fig. 2a, the centre of mass, G, of the arm and load combined should be used in place of the end of the arm, Z. Points C, A and G should be in a straight line, and r is now the distance AG. The stiffness of the spring, k, is equal to mgr/bc, and its free length is to be zero, as before.

Figure 4 shows the two-degree-of-freedom mechanism with the weights of the principal parts included. For exact balance the outer part of the mechanism must have points H, J and G_2 in a straight line, just as for the single-degree-of-freedom mechanism.

In the figure it has been assumed (as is likely) that the centre of gravity, G_1, of arm C_1AEJ lies on the straight line through these four points. Similarly, G_3 has been assumed to lie on the line C_2DH.

*a_1, a_2, b_1, b_2, c_1 and c_2 have similar meanings to a, b and c in Fig. 2a.

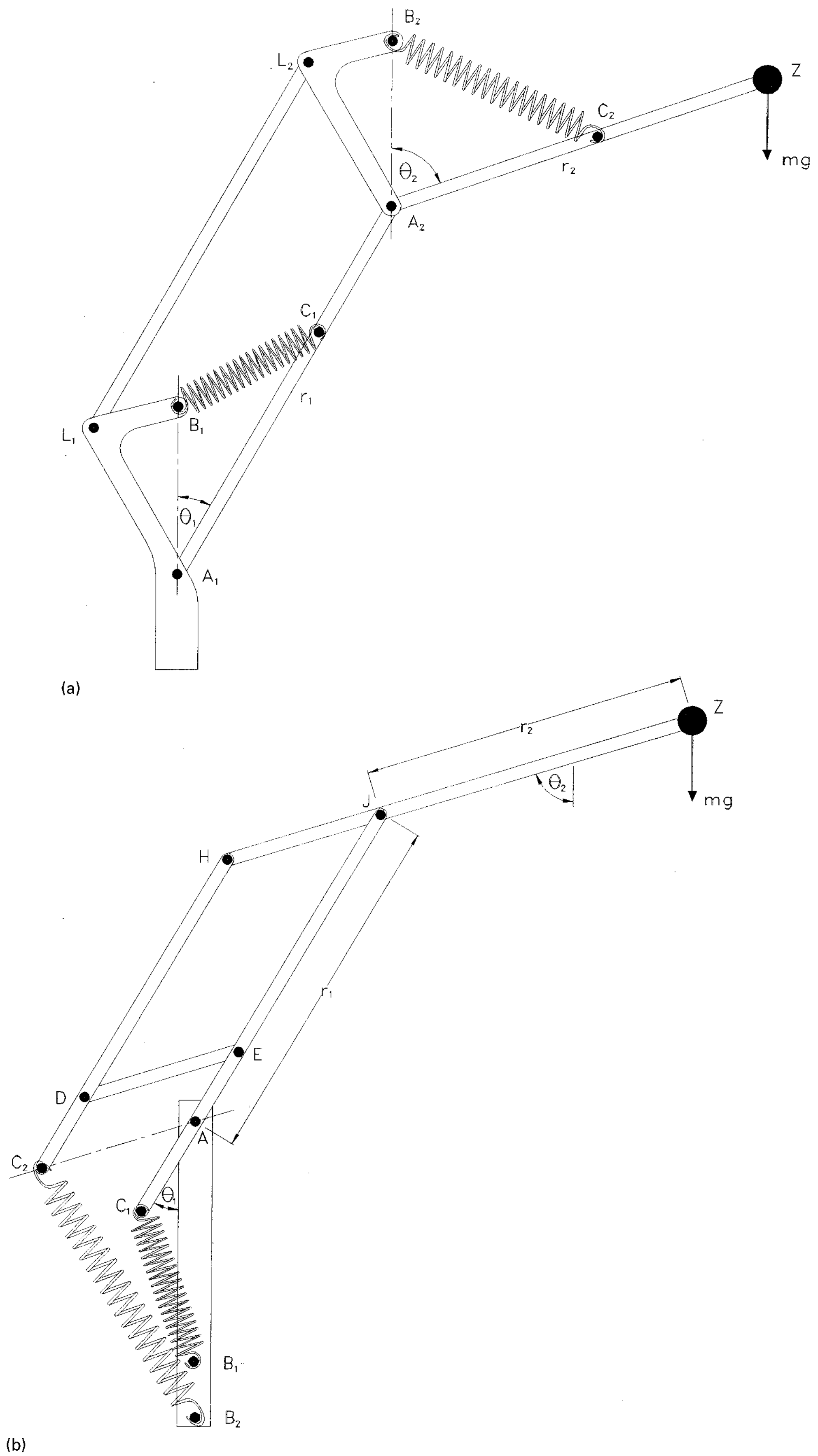

Fig. 3 Two-degree-of-freedom balancing mechanisms

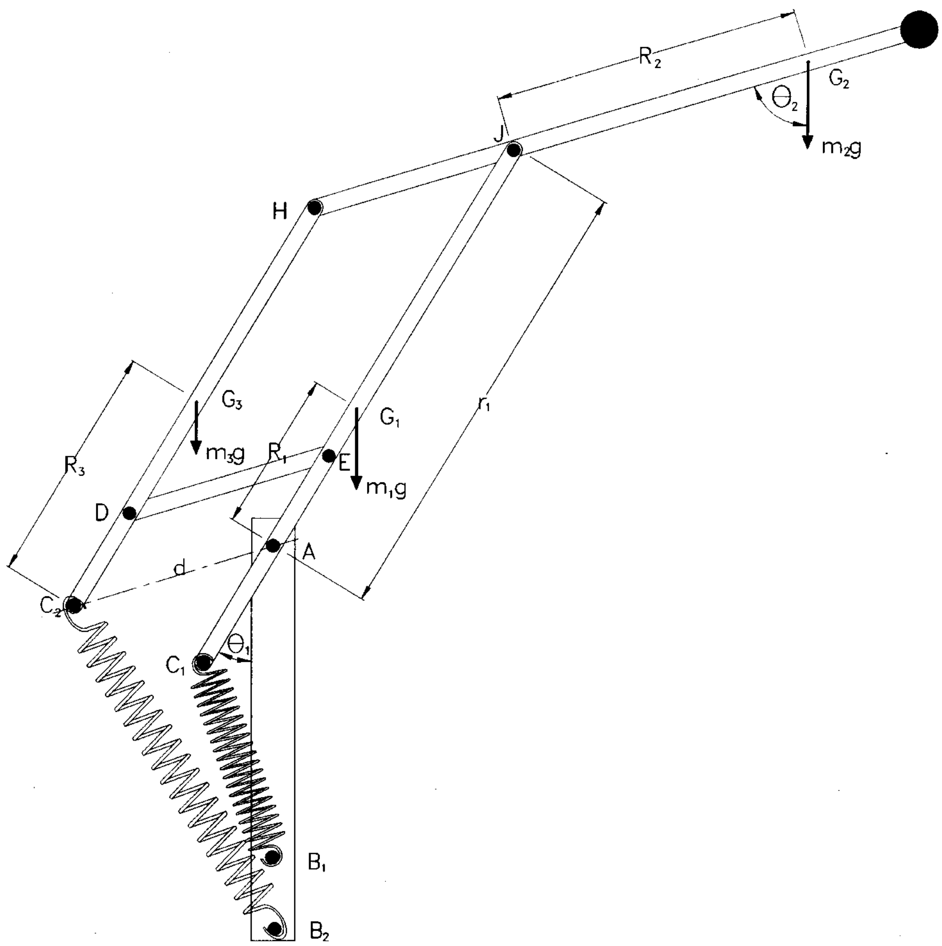

Fig. 4 Two-degree-of-freedom mechanism with weights of the arms included

Moments can be taken about the point A, as in Section 3.2. The moments of the three weight forces are

$$(m_1): \qquad m_1 g(R_1 \sin\theta_1) \tag{15a}$$

$$(m_2): \qquad m_2 g(r_1 \sin\theta_1 + R_2 \sin\theta_2) \tag{15b}$$

$$(m_3): \qquad -m_3 g(d \sin\theta_2 - R_3 \sin\theta_1) \tag{15c}$$

Summing these and equating to the moments of the two spring tensions give

$$\begin{aligned} F_1 \frac{b_1 c_1 \sin\theta_1}{a_1} + F_2 \frac{b_2 c_2 \sin\theta_2}{a_2} &= (m_1 R_1 + m_2 r_1 + m_3 R_3) g \sin\theta_1 \\ &\quad + (m_2 R_2 - m_3 d) g \sin\theta_2 \end{aligned} \tag{16}$$

By the same argument as was used in Section 3.2, the terms involving θ_1 in this equation must balance, and those involving θ_2 must also balance. Hence

$$F_1 = \left[\frac{(m_1 R_1 + m_2 r_1 + m_3 R_3) g}{b_1 c_1}\right] a_1$$

$$F_2 = \left[\frac{(m_2 R_2 - m_3 d) g}{b_2 c_2}\right] a_2 \tag{17}$$

Once again, each spring is to exert a force proportional to its length, i.e. the free length of the spring is to be zero. The stiffnesses of the two springs are given by the expressions in square brackets.

If the centres of gravity of the arms C_1AEJ and C_2DH were to lie off the straight lines through the pivots, it can readily be shown that a term in $\cos\theta_1$ would be introduced into equation (16); then the equation would be exactly satisfied only for a single position θ_1.

In most later copies of the Anglepoise lamp, it seems that precision has been sacrificed to cheapness of manufacture, since the position of the centroid G_2 varies as the lamp is rotated. In the original Anglepoise, however, the lamp is mounted in trunnions on a yoke so that the position of its own centroid remains unchanged, and exact balance is maintained.

5 CONCLUSIONS

The two-degree-of-freedom spring balancing mechanism patented by Carwardine in 1933 and made famous in the classic Anglepoise lamp design has been analysed, including the effects of the weight of the mechanism itself, and the conditions for exact balance established. The simplicity of the design of this mechanism and the perfection of its action recommend it for use in many applications.

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APPENDIX

George Carwardine and the Anglepoise lamp

George Carwardine (pronounced 'Car-*war*-deen') was born in Bath on 4 April 1887. The second youngest of 12 surviving children, he attended Bath Bluecoat School on a scholarship but left when aged 14 (R. Raven, 1995, personal communication).

From 1901 to 1905 Carwardine served his apprenticeship at the Whiting Auto Works in Bath. Following this he was employed in a succession of the town's engineering workshops, all the while gaining formal qualifications through study at home. As a young man, he also studied for the Ministry with the intention of joining his elder brother Charles as a missionary in China, but a bout of illness left him unfit to follow this vocation.

In 1912 he joined the Horstmann Car Company in Bath as chargehand, rising by 1916 to the position of both Works Manager and Chief Designer. Second only to the entrepreneurial inventor Sidney Horstmann, Carwardine was largely responsible for the design of all cars manufactured there, travelling on occasion to the Brooklands racetrack for trials.

In about 1924 he set up his own business, Cardine Accessories, in Locksbrook Road, Bath. Here he designed and manufactured various items and components, most notably automobile suspension systems. It has been alleged (but never proved) that his successful design for independent front suspension was pirated by General Motors. The late 1920s saw Cardine Accessories cease operations, no doubt a victim of the Depression, and Carwardine's brief return to Horstmann. In 1931 he became a freelance consulting engineer and inventor.

He had already displayed interest in developing versatile counterbalancing devices, intended to support a weight in any position in three dimensions. The stream of patents registered in his name between 1931 and 1934 demonstrates clearly how his ideas developed.

The first patent in this series [**8**] that relates directly to the spring-and-lever balancing mechanism describes, as one possible embodiment, a special form of the mechanism in which (referring to the letters used in Fig. 2) the lengths AB and AC are equal. As we have seen in Section 2, this is an unnecessary requirement. The zero-free-length spring was to be achieved by arranging a pivoted slide so that (a) the centre-line of the spring always passed through the pivot point and (b) the free end of the spring when relaxed was opposite the pivot. Carwardine does not seem to have been aware when he filed the patent that AB and AC do not need to be equal, nor that a spring could be wound so that it had a tension in it in the close-coiled condition.

The next patent [**6**], filed on 4 July 1932, has the essential features of the Anglepoise mechanism: two cranks, not necessarily of equal length, connected by a spring of zero free length. The patent briefly describes the means of producing such springs. A two-degree-of-freedom linkage to support an electric lamp is shown, with the springs on the arms as in Fig. 3a.

UK Patent 417 970 [**9**], filed in October 1933, relates to a means of supporting table mirrors, pictures and the like so that they can be tilted at any desired angle and still remain in equilibrium. The invention includes the close-wound spring whose tension is proportional to its overall length and is similar in essence to the arrangement shown in Fig. 2b. A difficulty in this case is the absence of a fixed vertical reference, which Carwardine overcomes by adding a third member pivoted to the back of the mirror and sliding on a pin in the supporting strut so that, although not fixed, it remains vertical.

The Anglepoise configuration as shown in Fig. 1, with all the springs at the base, was introduced in UK Patent 433 617 [**10**]. The drawings that illustrate the patent are clearly of the final design of the lamp. The essential features of the equipoising mechanism, namely the close-wound springs of zero free length and the parallelogram linkage, are clearly described, and Carwardine noted that the lamp itself is mounted in a fork so that the axis passes through the centre of gravity of the lamp and the balance is maintained as the lamp is rotated. He also notes that friction cannot be entirely eliminated from the mechanism, so that the links may remain in position even in presence of small errors such as deviations from linearity in the springs.

Carwardine's designs have been and continue to be copied in many products throughout the world.

