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Simultaneous Measurement of Distance and Thickness of a Thin Metal Plate with an Electromagnetic Sensor Using a Simplified Model

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Abstract – ***This paper presents a simplified model which can describe the inductance change when an air-core coil is placed next to a thin nonmagnetic metallic plate. The model has two independent parameters and is valid for a range of thickness, conductivity, and lift-offs. Use of this new relationship provides a fast and accurate method to measure the distance and thickness simultaneously. Measurements made for a sample coil next to thin copper and aluminium plates of various thicknesses verified the theory and the proposed method.***

I. INTRODUCTION

Quality inspection and thickness measurement of thin metal plates using eddy current methods are affected by lift-off variations. Furthermore, simultaneous thickness and distance measurements of metal layers are required for a range of technological applications such as coating and surface treatment [1][2]. A variety of coil designs have been used in these applications. For an air-core coil, the general analytical solution for inductance change caused by a thin layer of non-magnetic metal was described by Dodd and Deeds[3]. This paper evaluated the analytical solutions of Dodd and Deeds for nonmagnetic plates of different thickness, conductivity and lift-offs. It has been found that the solution can be approximated by a simple scaling relation.

$$\Delta L(\omega) = \Delta L_0 \cdot L^*(\omega / \omega_0) \tag{1}$$

Here, ΔL_0 is the overall strength of the inductance change that depends on lift-off for a given coil, and is independent of the thickness and conductivity, ω_0 is the characteristic angular frequency at which the imaginary part of ΔL_0 is a maximum. The frequency ω_0 depends on the conductivity and thickness of the plate and is virtually independent of life-off. The function $L^*(\omega/\omega_0)$ is

$$L^*(\frac{\omega}{\omega_0}) = (j\frac{\omega}{\omega_0})/(1 + j\frac{\omega}{\omega_0} + 0.15j\sqrt{\frac{\omega}{\omega_0}}) \tag{2}$$

Measurements were made for a sample coil next to thin copper and aluminium plates of various thicknesses using an impedance analyser. These results can be approximated by (1) and (2) to reasonable precision. Comparison between the calculated and measured peak $Im\Delta L_0$ and ω_0 provides the lift-off and thickness simultaneously.

II. METHODS

This section describes the procedures used to determine the simplified model. First, the two limiting cases for $\Delta L(\omega)$ with $\omega=0$ and ∞ will be considered with a physical explanation of the results. Then, the calculated and measured results will be displayed. Further, the procedure to simplify the analytic solution of Dodd and Deeds will be given. Finally, simultaneous thickness and lift-off inferred from measurements are listed.

A. Analytic solution

The difference in the complex inductance is $\Delta L(\omega) = L(\omega) - L_A(\omega)$, where $L(\omega)$ is the coil inductance above a plate, and $L_A(\omega)$ is the inductance in free space. Fig. 1 is a schematic diagram of the model.

The formulas of Dodd and Deeds are:

$$\Delta L(\omega) = K \int_0^\infty \frac{P^2(\alpha)}{\alpha^6} A(\alpha)\phi(\alpha)d\alpha \tag{3}$$

where

$$\phi(\alpha) = \frac{(\alpha_1 + \alpha)(\alpha_1 - \alpha) - (\alpha_1 + \alpha)(\alpha_1 - \alpha)e^{2\alpha_1 c}}{-(\alpha_1 - \alpha)(\alpha_1 - \alpha) + (\alpha_1 + \alpha)(\alpha_1 + \alpha)e^{2\alpha_1 c}} \tag{4}$$

$$\alpha_1 = \sqrt{\alpha^2 + j\omega\sigma\mu_0} \tag{5}$$

$$K = \frac{\pi\mu_0 N^2}{(l_1 - l_2)^2 (r_1 - r_2)^2} \tag{6}$$

$$P(\alpha) = \int_{\alpha r_1}^{\alpha r_2} xJ_1(x)dx,\ A(\alpha) = (e^{-\alpha l_1} - e^{-\alpha l_2})^2 \tag{7}$$

Where μ_0 denotes the permeability of free space. N denotes the number of turns in the coil; r_1 and r_2 denote the inner and outer radii of the coil, while l_1 and l_2 denote the height of the bottom and top of the coil; and c denotes the thickness of the plate.

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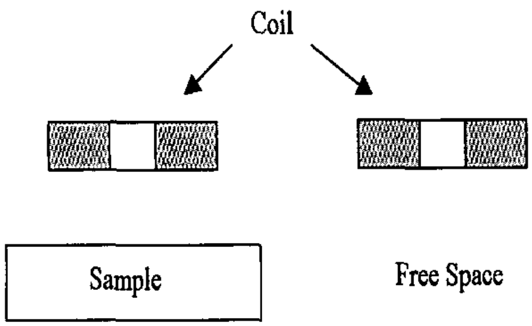
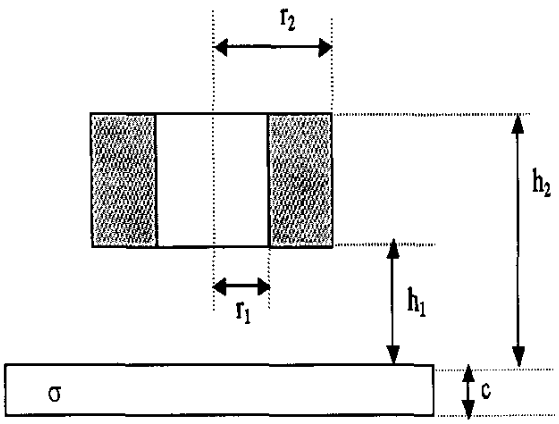


(a)



(b)

Fig. 1. (a). The schematic diagram of the model (b);the geometry of the coil

B. $\Delta L(\omega)$ *in two limiting cases* ω*=0 and* ∞

Setting ω=0 gives the inductance change for zero-frequency. The real part and imaginary part of the inductance change are zero, which means the non-magnetic plate causes no inductance change and the magnetic flux penetrates the plate as in free space.

In the limit of arbitrary large frequency, the inductance change is given by $\Delta L = -\Delta L_0$, where $\Delta L_0 = K\int \frac{P^2(\alpha)}{\alpha^6} A(\alpha) d\alpha$. ΔL_0 is dependent on lift-off for a given coil, and is independent of the thickness and conductivity, which corresponds to the situation that the incident magnetic flux is totally excluded from the plate.

C. The scaling relation with various calculated results and measured results

The frequency-dependent inductance change for a coil defined by r_1= 20*mm*, r_2=20.1*mm*, *h*=2*mm*, and *N*=10 is plotted in Figure 2. The conductivity of the plate was chosen to be 5.8×10^7 (copper) and thickness was chosen to be 22μm, 22×2μm, ... 22×6μm. The lift-off was chosen to be 1mm.

On examination of Figure 2, $Im\Delta L_0$ peaks at a characteristic frequency ω_0, which is seen to increase as thickness decreases. Figure 3 shows the same calculations scaled by $1/\Delta L_0$ versus the normalized frequency. All the calculations fall on the same curve. On trials over a range of conductivities and thicknesses, it was found that this common scaling relation could also be applied to calculations for plates with different conductivity (e.g. 3.82×10^7 Ω for aluminium) and different lift-offs (1*mm*, 1.1*mm*...2.0*mm*), the existence of this common scaling relation was also verified by measurements from a coil (parameters defined as above) with aluminium and copper foil samples (see Fig. 4).

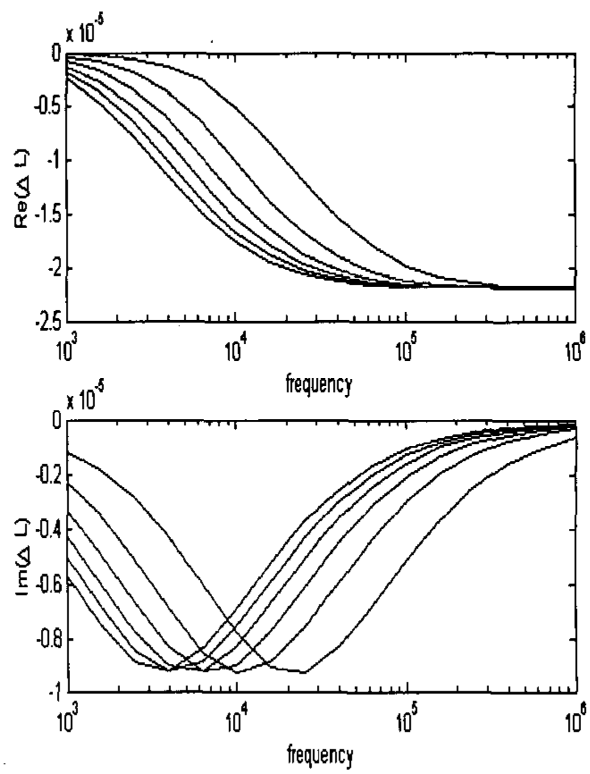


Fig. 2. The real and imaginary parts of ΔL_0 for copper plates with thickness 22μm, 44μm, ...22×6μm

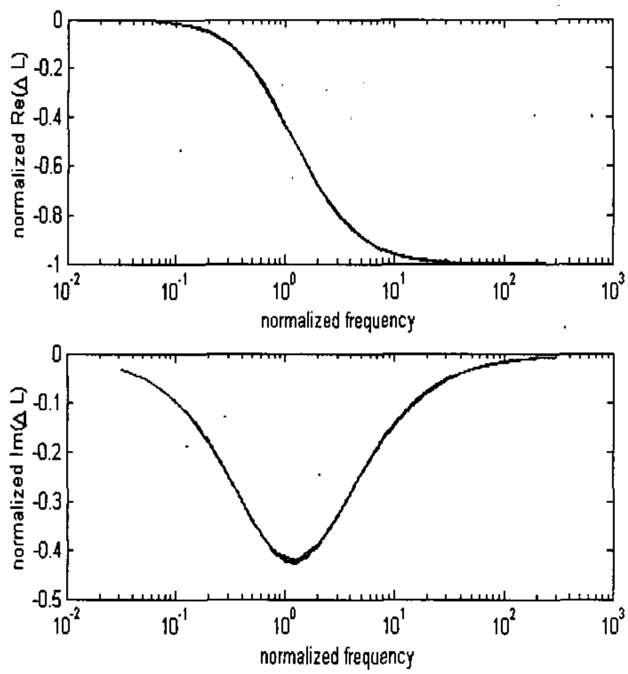


Fig. 3. The six curves in Fig.2 overlap identically when plotted verse normalized frequency.

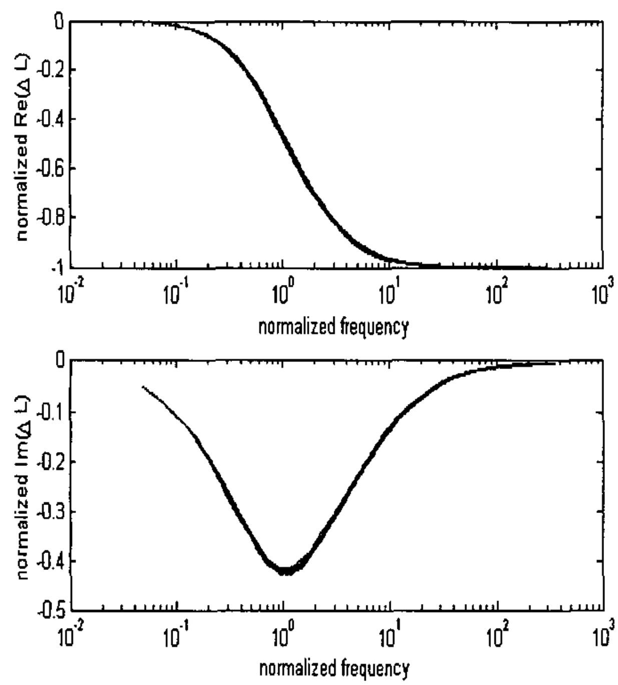


Fig. 4. Measurements for aluminium and copper foil samples overlap identically when plotted verse normalized frequency.

D. The simplification of the analytic solution

Two approximations can be used to simplify the analytical solution.

Substituting $e^{2\alpha_1 c}$ with $1+2\alpha_1 c$, and considering Equation (5), Equation (4) becomes,

$$\phi(\alpha) \approx \frac{j\omega\sigma\mu_0 c}{j\omega\sigma\mu_0 c + 2\alpha^2 c + 2\alpha + 2\alpha\alpha_1 c} \tag{8}$$

The next approximation originates from the fact that $\phi(\alpha)$ varies slowly with α compared to the rest of the integrand, which reaches its maximum at a characteristic spatial frequency α_0. α_0 is defined to be one over the smallest dimension of the coil. The approximation is to evaluate $\phi(\alpha)$ at α_0 and take it outside of the integral.

$$\Delta L(\omega) = \phi(\alpha_0)\Delta L_0 \tag{9}$$

Letting $\omega_0 = \dfrac{2\alpha_0^2 c + 2\alpha_0}{\sigma\mu_0 c}$, equation (8) can be expressed as

$$\phi(\alpha) = \frac{j\omega/\omega_0}{j\omega/\omega_0 + 1 + 2\alpha_0\alpha_1 c/(2\alpha_0^2 c + 2\alpha_0)} \tag{10}$$

Noticing that $\alpha_0 c \ll 1$ is well satisfied for the coil and plate dimension, we have

$$\omega_0 = \frac{2\alpha_0}{\sigma\mu_0 c} \tag{11}$$

$$\phi(\alpha_0) = \frac{j\omega/\omega_0}{j\omega/\omega_0 + 1 + \alpha_1 c} \tag{12}$$

It was found that Equation (12) can be approximated by

$$\phi(\alpha_0) = (j\frac{\omega}{\omega_0})/(1 + j\frac{\omega}{\omega_0} + 0.15j\sqrt{\frac{\omega}{\omega_0}}) \tag{13}$$

Note that 0.15 in Equation (13) is an empirical coefficient obtained via trials. It is dependent on the coil geometry, but independent on lift-off and the conductivity of the test plate.

From (11), it can be concluded that the characteristic angular frequency increases if the conductivity and thickness become smaller, which agrees with observations.

III. RESULTS

Figure 5 shows plots, which provide some verification of the simplified model with analytic solution and measurements. As can be seen, the simple model approximates the frequency-dependent inductance change to a reasonable precision.

Simultaneous thickness and lift-offs inferred from a number of measurements are given table 1, which verified our model and measurement method. The difference between the actual value and inferred value is due to the imperfect modeling of the coil and the error caused by the estimation of peak $Im\Delta L_0$ and ω_0 from measurements at a finite number of frequency points.

Measurement method based on a comprehensive model normally requires calculation and measurement of the inductance change over a large range of frequency. Its principle is, to find in a two-dimensional table a set of parameters (i.e. thickness and lift-off) for which the calculation is as close as possible to the measured data in a least-squared sense. Therefore, it requires a large storage capability to preserve the calculated data and large computation power to do the search and comparison. The significance in using such a simplified model is that it only requires measurements over a relative small range of frequency (i.e. near the characteristic frequency) and that only one-dimensional search is required. These features may *facilitate the realization of this method in small portable* instruments employing micro-controllers with limited memory and computation power.

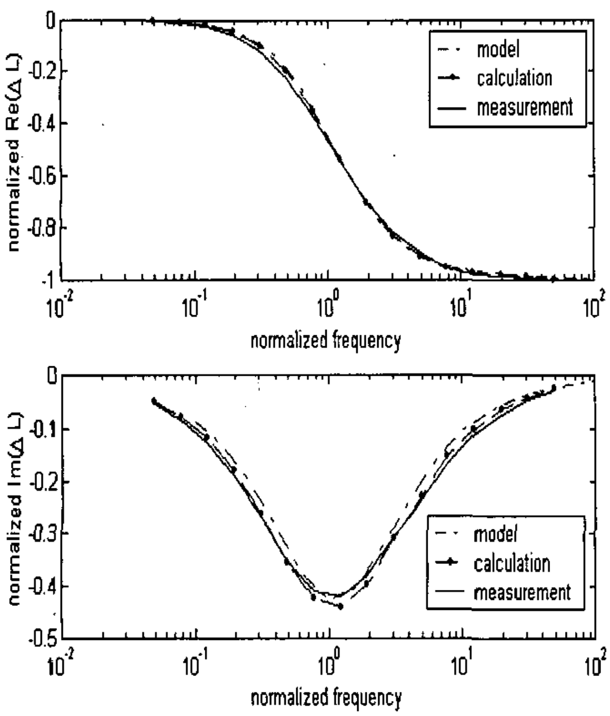


Fig. 5. The comparison of the real and imaginary parts of ΔL_0 from the simplified model, the analytic solution and actual measurements

Table 1. The actual and inferred thickness and lift-off

Plate	Actual		Inferred	
	Thickness	Lift-off	Thickness	Lift-off
Copper	22μm	1mm	20-23μm	0.92-1.02mm
	66μm	1.5mm	60-69μm	1.41-1.54mm
	110μm	2mm	102-116μm	1.88-2.07mm
Aluminium	18μm	1mm	16-21μm	0.94-1.02mm
	72μm	1.5mm	64-71μm	1.43-1.53mm
	108μm	2mm	97-113μm	1.89-2.04mm

IV. CONCLUSION

Based upon the described simplified model and the measurement method, it is possible to realize a small portable measuring instrument that is capable of providing real-time distance and thickness simultaneously. Future work will involve modeling other coil geometries, configurations (e.g. coils with rectangular cross-sections and ferrite-cored coils) and frequency-dependent response for magnetic plates. The field performance of the sensor will be evaluated.

V. REFERENCES

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