Generalized Disappointment Aversion and the Variance Term Structure☆

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Abstract

Contrary to leading asset pricing theories, recent empirical evidence indicates that financial markets compensate only short-term equity variance risk. An equilibrium model with generalized disappointment aversion risk preferences and rare events reconciles salient features of the variance term structure. In addition, a calibration explains the variance and skew risk premiums in equity returns and the implied volatility skew of index options while capturing standard moments of fundamentals, equity returns, and the risk-free rate. The key intuition for the results stems from substantial countercyclical risk aversion induced by endogenous variation in the probability of disappointing events in consumption growth.

Keywords: Generalized Disappointment Aversion, Learning, Rare Events, Price of Variance Risk, Variance and Skew Risk Premiums, Implied Volatility Skew
JEL: D81, E32, E44, G12

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1 Introduction

The consumption-based asset pricing literature has been recently revived by generalized models of long-run risks and rare disasters to capture many characteristics of the equity and derivatives markets. Yet leading theories fail to explain the timing of variance risk. Contrary to most successful asset pricing models, Dew-Becker, Giglio, Le, and Rodriguez (2017) show that it has been costless to hedge future variance at horizons longer than two months, whereas only unexpected realized variance was significantly priced.[1,2] The term structure of variance risk possess a challenge to models featuring time-varying expected growth and volatility (Bansal and Yaron, 2004) or disaster risk (Rietz, 1988; Barro, 2006).

I illustrate the challenge in Figure 1 by showing the empirical Sharpe ratios and prices for forward variance claims, which are swap contracts that pay the owner the realized stock market variance during a particular future period.[3] The figure shows the term structure of forward claims on future variance up to one year. The average prices are upward-sloping at the short end and quickly flatten with the horizon. Sharpe ratios are significantly negative for short maturities, suggesting investors are willing to hedge short-term variance risk. Puzzling, however, is that future variance from three to 12 months is unpriced. Well-known asset pricing theories predict a strongly upward-

[1]Dew-Becker, Giglio, and Kelly (2021) show that it is highly costly to hedge realized volatility but not forward-looking uncertainty across different markets. Berger, Dew-Becker, and Giglio (2019) provide new empirical evidence that shocks to future uncertainty have no significant effect on the economy. Also, Dew-Becker and Giglio (2019) find that investors do not view shocks to cross-sectional uncertainty as bad.

[2]Also, van Binsbergen, Brandt, and Koijen (2012) and van Binsbergen, Hueskes, Koijen, and Vrugt (2013) document a downward-sloping term structure of equity risk premia and volatility, which is at odds with leading asset pricing models.

[3]For instance, a payoff (realized variance) of n-month variance forward equals the sum of daily squared stock market returns in month n from today.

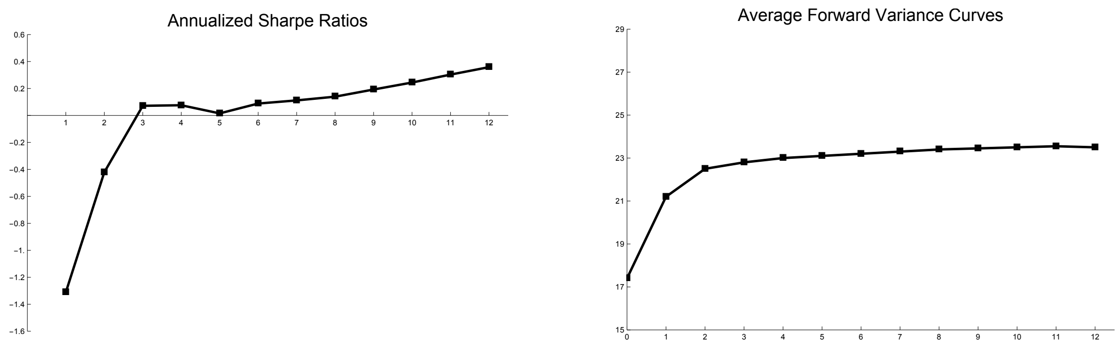

Figure 1. Average prices and annualized Sharpe ratios for forward variance claims
The figure plots annualized Sharpe ratios and average prices for forward variance claims in the US data from 1996 to 2013. The prices are reported in annualized volatility terms. The data are from Dew-Becker et al. (2017).

sloping term structure of forward variance prices and, hence, imply the negative and significant Sharpe ratios at future horizons, counter to what we observe empirically.

I capture the observed variance term structure by introducing asymmetric preferences into a model with learning about consumption depressions.[4] Disaster risk generates the upward-sloping term structure of return variance, however, I demonstrate that asymmetric preferences cancel the increasing effect in the long term. The reason is that, in bad times, forward variance becomes higher in the short term than in the long term with asymmetric preferences, which flattens the increasing pattern at longer horizons on average. The properties of forward return variance translate into empirically consistent variance forward prices. This mechanism also implies negative Sharpe ratios on short-term variance forwards and positive and increasing ratios at longer maturities.

Formally, I consider an exchange economy with generalized disappointment aversion (GDA) risk preferences (Routledge and Zin, 2010) and rare events. Consumption growth follows a hidden two-state Markov chain where a rare "depression" is calibrated

[4]The ingredients are empirically motivated. A number of studies provide micro-level evidence that investors dislike losses more than they enjoy gains (Choi, Fisman, Gale, and Kariv, 2007). Also, Hansen (2007) argues that the assumption of the investor's full information about the model structure is extreme.

to the US Great Depression. The agent filters the hidden state probabilities. GDA preferences amplify the impact on the pricing kernel of disappointing beliefs corresponding to utilities below a scaled certainty equivalent. The amplification of lower-tail shocks yields strongly countercyclical risk aversion, which helps capture the variance term structure.

The economic mechanism is as follows. Following Veronesi (1999), the conditional volatility of equity return is a hump-shaped function of a posterior probability of expansion, π_t (GDA in Figure 4). The economy is in a good state for most of the periods, in which case π_t is high and close to 1. A good piece of news reinforces investor's beliefs that the current regime is the expansion. In this case, the risk of future disasters generates an upward-sloping term structure of forward variance. A bad piece of news decreases π_t and leads to a spike in return variance initially. Bad news could be due to a disaster and hence the investor will learn times are bad in the future ($\pi_t \approx 0$). Bad innovations could also be due to idiosyncratic consumption risk in expansion and hence the investor will update beliefs to reflect times are still good ($\pi_t \approx 1$). In both cases, return variance will decrease quickly when π_t approaches 0 or 1, implying the inversion in forward variance. Unconditionally, the investor is always willing to hedge high realized variance in the short term. In the long term, however, the inversion in bad times dominates the upward-sloping effect of disaster risk in good periods, flattening the forward variance curve. Variance claims inherit the properties of forward variance. Thus, the unconditional term structure of prices is upward-sloping at the short end and flattens out quickly in maturity. The inversion in prices yields positive Sharpe ratios on variance forwards at longer horizons on average.

Intuitively, the inversion in forward variance in response to bad news happens because high volatility is short-lived in the economy.[5] Indeed, the conditional volatility peaks within a narrow range of beliefs and sharply diminishes outside this interval. When beliefs change, return volatility spikes but does not persist. Mechanically, sizable countercyclical risk aversion induced by GDA preferences yields strong and weak price sensitivities to belief changes in good and bad times (Veronesi, 1999). This difference in sensitivities implies return volatility should be higher following a bad piece of news in good times than a good piece of news in bad times. As a result, an asymmetric effect on the price sensitivity to news leads to a skewed shape of conditional volatility.

Next, I compare GDA preferences with nested utility functions. I show that the term structure of variance risk can be replicated with GDA preferences due to a sufficiently countercyclical risk aversion.[6] Interestingly, not only can nested preference specifications be rejected by the unconditional term structure, but they are also inconsistent with the conditional dynamics of the variance term structure.

I first compare GDA preferences to a disappointment aversion utility function (Gul, 1991) and Epstein-Zin preferences (Epstein and Zin, 1989). First, a disappointment-averse agent increases the pricing kernel for disappointing utilities, defined as being below the certainty equivalent. Compared to Routledge and Zin (2010), Gul's preferences increase the disappointment threshold. This generates a large number of disappointing events and a large risk aversion in two states. Thus, price sensitivities are similar in good and bad times, generating a symmetric shape of return volatility (DA in Figure

[5] This mechanism is consistent with Dew-Becker et al. (2017) showing that, during consumption disasters and financial crisis, realized volatility spikes for one month only and then reverts quickly.

[6] The countercyclical risk aversion can rationalize the equity premium puzzle (Melino and Yang, 2003). I show that, in my setting, a sufficiently countercyclical risk aversion induced by generalized disappointment aversion can further explain the variance term structure.

4). Second, a model with Epstein-Zin preferences generates a slightly skewed shape of return volatility (EZ in Figure 4). However, conditional volatility remains elevated for a wide range of beliefs in both models. When investor's beliefs change, high variance persists in the long term. This generates the upward-sloping term structure of forward variance and prices.

I also look at the conditional dynamics of the term structures. I assume the investor holds a median belief (normal times). I then study the impact of one positive and three negative consumption innovations. First, at the one-month maturity, the average Sharpe ratios in the GDA economy are pro-cyclical, meaning more (less) negative in bad (good) times, consistent with Aït-Sahalia, Karaman, and Mancini (2020). The reason is that GDA preferences generate a beliefs-dependent pricing kernel with higher marginal utility in low consumption states, increasing the hedge against high realized variance associated with low-utility states.[7] At longer maturities, the Sharpe ratios remain close to zero in response to small shocks, whereas they become upward-sloping and positive in response to large negative news. The reason is that small shocks are not priced due to a low disappointment threshold. In contrast, lower-tail shocks place the posterior belief within the interval of the highest return variance and, hence, the variance tends to be lower in later periods. The variance claims are priced accordingly, making the short-term variance forwards more expensive. The inversion in prices generates positive Sharpe ratios at longer horizons.

Second, in the disappointment aversion model, the conditional variance forward

[7] Routledge and Zin (2010) and Bonomo, Garcia, Meddahi, and Tédongap (2011) provide a similar analysis of GDA stochastic discount factor with alternative consumption processes. Also, the beliefs-dependent effective risk aversion of my paper echoes the mechanism of Berrada, Detemple, and Rindisbacher (2018) with learning and a beliefs-dependent utility function.

prices remain strongly upward-sloping, implying negative Sharpe ratios across all economic conditions. The reason is that high variance is persistent due to the shape of the conditional return variance and, therefore, variance risk concentrates in the long term. Third, in the Epstein-Zin economy, prices of variance forwards remain markedly increasing in the horizon for most economic conditions and become mildly decreasing only when consumption growth is extremely low. The mild inversion is too weak to dampen the upward-sloping effect at other times. Thus, the conditional Sharpe ratios remain strongly negative.

Finally, the GDA model shows the superior performance when confronted with other asset pricing facts. It captures salient features of the equity variance and skew risk premiums and a volatility skew implied by index option prices.[8] In contrast, other frameworks generate too small variance and skew risk premiums and flat implied volatility curves. In a comparative analysis, I show that my results are robust to different calibrations of key parameter values. Following Pohl, Schmedders, and Wilms (2018) and Lorenz, Schmedders, and Schumacher (2020), I check that global projection methods provide highly accurate numerical solutions.

This paper is related to several strands of the literature. First, it contributes to the growing literature on the term structures of equity and variance claims (van Binsbergen, Brandt, and Koijen, 2012; van Binsbergen, Hueskes, Koijen, and Vrugt, 2013; Dew-Becker, Giglio, Le, and Rodriguez, 2017). A number of studies (Croce, Lettau, and Ludvigson, 2014; Belo, Collin-Dufresne, and Goldstein, 2015; Favilukis and Lin, 2015; Hasler and

[8] Also, see Choi, Mueller, and Vedolin (2017) and Londono and Zhou (2017) for bond and currency variance risk premiums.

Márfe, 2016; Márfe, 2017; Ai, Croce, Diercks, and Li, 2018; Hasler, Khapko, and Márfe, 2019) explain the downward-sloping term structure of equity risk premia and return volatility.[9] I complement these papers by explaining the variance term structure.

Second, this study builds on the literature exploring asset pricing properties of GDA preferences. These preferences have been used to explain stock market returns (Bonomo, Garcia, Meddahi, and Tédongap, 2011, 2015; Liu and Miao, 2014; Schreindorfer, 2020), sovereign spreads (Augustin and Tédongap, 2016), portfolios (Dahlquist, Farago, and Tédongap, 2016), the cross section of stock returns (Delikouras, 2017; Farago and Tédongap, 2018; Delikouras and Kostakis, 2019), and the term structure of interest rates (Augustin and Tédongap, 2021). I employ GDA preferences to explain the variance forward prices and returns. This paper is, to my knowledge, the first to reconcile the variance term structure. It does so while jointly explaining equity returns, variance and skew premiums, and option prices. Also, the extant literature studies GDA preferences in long-run risk models, while this paper examines a rare event model with learning.

Third, this paper is related to leading asset pricing theories focusing on the variance premium and option prices. These include the extensions of equilibrium models with habit (Du, 2011), rare disasters (Liu, Pan, and Wang, 2005; Benzoni, Collin-Dufresne, and Goldstein, 2011; Seo and Wachter, 2019), and long-run risks (Eraker and Shaliastovich, 2008; Bollerslev, Tauchen, and Zhou, 2009; Drechsler and Yaron, 2011; Drechsler, 2013; Zhou and Zhu, 2014; Shaliastovich, 2015). My paper is distinct from this literature because it points out the importance of the investor's generalized disappointment aversion for the variance term structure.

[9] See van Binsbergen and Koijen (2017) for a review of the literature on term structures of equity claims.

Finally, this paper connects to hidden Markov switching models (David, 1997; Veronesi, 1999, 2000).[10] The recent literature extends this approach to learning about unknown volatility (Weitzman, 2007) and persistence (Cogley and Sargent, 2008; Gillman, Kejak, and Pakos, 2015; Andrei, Hasler, and Jeanneret, 2019) as well as to a multidimensional learning problem (Collin-Dufresne, Johannes, and Lochstoer, 2016; Johannes, Lochstoer, and Mou, 2016; Babiak and Kozhan, 2020, 2021). This paper contributes to the learning literature by investigating how state uncertainty is priced in the presence of GDA preferences with a particular emphasis on the pricing of the variance risk.

The remainder of the paper is organized as follows. Section 2 describes the economy. Section 3 outlines the equilibrium conditions. Section 4 provides asset pricing results and sensitivity analysis. Section 5 concludes. Internet Appendix provides supporting analysis and additional results.

2 Model

2.1 Generalized Disappointment Aversion Risk Preferences

The environment is an infinite-horizon, discrete-time exchange economy with a representative agent. Following Epstein and Zin (1989), the agent's utility V_t is defined by

$$V_t = \left[(1-\beta)C_t^{\rho} + \beta\mathcal{R}_t^{\rho}\right]^{1/\rho}, \tag{1}$$

in which C_t is consumption, $0 < \beta < 1$ is the subjective discount factor, $1/(1-\rho) > 0$ is the elasticity of intertemporal substitution (EIS), and $\mathcal{R}_t = \mathcal{R}_t(V_{t+1})$ is the certainty equivalent.

The certainty equivalent captures the generalized disappointment aversion (GDA)

[10] See Pastor and Veronesi (2009) for a survey of the early literature on learning in financial markets.

risk of Routledge and Zin (2010). GDA preferences put more weight on "disappointing" events compared to the expected utility, similar to disappointment aversion risk preferences of Gul (1991). For Gul's model, however, an outcome is viewed as disappointing when it is below the certainty equivalent, whereas for Routledge and Zin's specification a disappointing outcome is below a constant fraction of the implicit certainty equivalent. Formally, the certainty equivalent of GDA preferences is implicitly defined by

$$\frac{[\mathcal{R}_t(V_{t+1})]^\alpha}{\alpha} = \mathbb{E}_t\left[\frac{V_{t+1}^\alpha}{\alpha}\right] - \theta\mathbb{E}_t\left[\mathbb{I}\left(\frac{V_{t+1}}{\mathcal{R}_t(V_{t+1})} \leqslant \delta\right)\left(\frac{[\delta\mathcal{R}_t(V_{t+1})]^\alpha}{\alpha} - \frac{V_{t+1}^\alpha}{\alpha}\right)\right], \qquad (2)$$

in which $\mathbb{I}(\cdot)$ is the indicator function, $1-\alpha > 0$ is the relative risk aversion, $\delta \leq 1$ is the disappointment threshold, and $\theta \geq 0$ is disappointment aversion. GDA preferences enable one to control the disappointment threshold by changing δ. Routledge and Zin's preferences nest two specifications. The expected utility is obtained by setting $\theta = 0$. Setting $\theta \neq 0$ and $\delta = 1$ reduces GDA preferences to the disappointment aversion utility.

2.2 *Endowments and Inference Problem*

I consider a Markov switching model for aggregate consumption growth

$$\Delta c_{t+1} = \mu_{s_{t+1}} + \sigma\varepsilon_{t+1}, \quad \varepsilon_{t+1} \sim N(0,1).$$

where Δc_{t+1} is log consumption growth, s_{t+1} is a hidden two-state Markov chain with a state space $\mathcal{S} = \{1,\ 2\}$ and a transition matrix $\mathcal{P} = \left(\pi_{ij}\right)$, in which $\pi_{11} = 1 - \pi_{12}$ and $\pi_{22} = 1 - \pi_{21}$ are transition probabilities, $\mu_{s_{t+1}}$ is the state-dependant mean growth rate, and σ is the constant consumption volatility. I assume $\mu_2 < \mu_1$ to identify $s_{t+1} = 1$ and $s_{t+1} = 2$ as expansion and recession, respectively.[11]

[11] The application of a regime-switching framework is a popular paradigm in the asset pricing literature. These models are flexible to embed business cycle fluctuations (Cecchetti et al., 1990; Veronesi, 1999; Ju

The motivation for constructing a two-state model is twofold. First, I want to maintain parsimony for the sake of convenient interpretation. Second, I do not introduce additional risks to isolate the impact of learning and GDA preferences. A model with additional ingredients would certainly make the framework more flexible. However, I show that a tightly calibrated GDA model with a single state variable can already reproduce the variance term structure with a wide array of salient features of the equity and derivatives markets.

I seek to price a levered consumption claim with log dividend growth:

$$\Delta d_{t+1} = g_d + \lambda \Delta c_{t+1} + \sigma_d e_{t+1}, \quad e_{t+1} \sim N(0,1),$$

in which λ is a leverage ratio on expected consumption growth. I use g_d to equalize long-run dividend and consumption growth rates, and σ_d to match the empirical dividend growth volatility. In addition, the chosen value of λ allows me to match the observed correlation between annual consumption and dividend growth rates.

The investor knows the true parameters and distribution of shocks but does not observe the state. At time t, the agent updates the probability of expansion $\pi_t = \mathbb{P}(s_{t+1} = 1|\mathcal{F}_t)$ conditional on the history of consumption growth rates denoted by $\mathcal{F}_t$. I assume a Bayesian agent who updates his belief through Bayes' rule:

$$\pi_{t+1} = \frac{\pi_{11} f(\Delta c_{t+1}|1)\pi_t + (1-\pi_{22}) f(\Delta c_{t+1}|2)(1-\pi_t)}{f(\Delta c_{t+1}|1)\pi_t + f(\Delta c_{t+1}|2)(1-\pi_t)}, \tag{3}$$

$$f(\Delta c_{t+1}|i) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(\Delta c_{t+1}-\mu_i)^2}{2\sigma^2}}, \quad i = 1, 2.$$

and Miao, 2012; Johannes et al., 2016; Collin-Dufresne et al., 2016), the "peso problem" in the mean (Rietz, 1988; Barro, 2006; Backus et al., 2011; Gabaix, 2012) or persistence (Gillman et al., 2015), long-run risks (Bonomo et al., 2011, 2015), and economic recoveries (Hasler and Márfe, 2016) in endowments.

3 Equilibrium

3.1 Equilibrium and Pricing Kernel

Following Routledge and Zin (2010), I show (see Appendix B) that the gross return $R_{i,t+1}$ on the i-th traded asset satisfies the condition

$$\mathbb{E}_t\left[M_{t+1}R_{i,t+1}\right] = 1, \tag{4}$$

in which M_{t+1} is the stochastic discount factor (SDF) of the GDA economy defined as

$$M_{t+1} = \underbrace{\beta\left(\frac{C_{t+1}}{C_t}\right)^{\rho-1}}_{M_{t+1}^{CRRA}} \cdot \underbrace{\left(\frac{V_{t+1}}{\mathcal{R}_t(V_{t+1})}\right)^{\alpha-\rho}}_{M_{t+1}^{EZ}} \cdot \underbrace{\left(\frac{1+\theta\mathbb{I}(V_{t+1}\leqslant\delta\mathcal{R}_t(V_{t+1}))}{1+\theta\delta^{\alpha}\mathbb{E}_t\Big[\mathbb{I}(V_{t+1}\leqslant\delta\mathcal{R}_t(V_{t+1}))\Big]}\right)}_{M_{t+1}^{GDA}}. \tag{5}$$

The first component M_{t+1}^{CRRA} is the SDF of the power utility. The second multiplier M_{t+1}^{EZ} is the adjustment of Epstein-Zin preferences, which separate the coefficient of risk aversion and EIS. The third component M_{t+1}^{GDA} represents the GDA adjustment. When the agent's utility is below a predefined fraction of the certainty equivalent, more weight is attached to the SDF, magnifying the countercyclical dynamics of the pricing kernel. For a better understanding of the key role of GDA, I shut down the Epstein-Zin adjustment in SDF for the models with (generalized) disappointment aversion by setting $\alpha = \rho$. Thus, the pricing kernel simplifies to

$$M_{t+1} = \beta\left(\frac{C_{t+1}}{C_t}\right)^{\rho-1} \cdot \left(\frac{1+\theta\mathbb{I}(V_{t+1}\leqslant\delta\mathcal{R}_t(V_{t+1}))}{1+\theta\delta^{\alpha}\mathbb{E}_t\Big[\mathbb{I}(V_{t+1}\leqslant\delta\mathcal{R}_t(V_{t+1}))\Big]}\right).$$

3.2 Model Solution

The latest long-run risk models generate significant nonlinearities, which, coupled with the log-linearization of equilibrium quantities, can generate economically significant numerical errors (Pohl et al., 2018). Hence, I solve the model numerically using global solution methods to accurately capture the nonlinear nature of the model under consideration. The model solution boils down to approximating the return on the wealth portfolio R^{ω}_{t+1} and the equity return $R_{e,t+1}$ implicitly defined by Eq. (4). Denoting the investor's wealth and equity price by W_t and P^e_t, we obtain

$$R^{\omega}_{t+1} = \frac{W_{t+1}}{W_t - C_t} = \frac{\frac{W_{t+1}}{C_{t+1}}}{\frac{W_t}{C_t} - 1} \cdot e^{\Delta c_{t+1}} \quad \wedge \quad R^e_{t+1} = \frac{P^e_{t+1} + D_{t+1}}{P^e_t} = \frac{\frac{P^e_{t+1}}{D_{t+1}} + 1}{\frac{P^e_t}{D_t}} \cdot e^{\Delta d_{t+1}}.$$

I conjecture that $\frac{W_t}{C_t} = G(\pi_t)$ and $\frac{P^e_t}{D_t} = H(\pi_t)$ are functions of π_t. I substitute R^{ω}_{t+1} and R^e_{t+1} into Eq. (4) and apply the projection method (Judd, 1992) to approximate $G(\pi_t)$ and $H(\pi_t)$. I discuss the numerical solution and its accuracy in Appendix C and provide the model-generated asset prices in Appendix D.

4 Data and Quantitative Results

4.1 Data

I construct annual real per capita consumption growth from January 1930 to December 2016 using the US National Income and Product Accounts. I then retrieve data from the Center for Research in Security Prices to obtain aggregate equity market dividends and asset returns. To discipline quantitative analysis, I tightly calibrate each model in this paper to closely match the key moments of fundamentals and equity returns.

In addition to standard asset pricing moments, I study the implications of different models for the high moment risk premiums and option prices. The variance premium is the difference between expectations of stock market return variance under the risk-neutral $\mathbb{Q}$ and actual physical $\mathbb{P}$ probability measures.[12] Formally, a τ-month variance

[12] In the model, the Radon-Nikodym derivative is defined as $\frac{d\mathbb{Q}}{d\mathbb{P}} = \frac{M_{t+1}}{\mathbb{E}_t(M_{t+1})}$ and allows one to compute the risk-neutral moments.

Table 1. Summary statistics: variance and skew risk premiums

This table reports monthly descriptive statistics for the conditional variance vp_t and skew sp_t premiums. Mean, Median, SD, Max, Skewness, and Kurtosis report the sample average, median, standard deviation, maximum, skewness, and kurtosis, respectively. The empirical statistics of the variance and skew risk premiums are for the US data from January 1990 to December 2016 and from January 1996 to December 2016, respectively.

	vp_t	sp_t
Mean	10.24	−42.12
Median	7.50	−68.11
SD	10.49	82.11
Max	83.70	447.37
Skewness	2.62	3.57
Kurtosis	14.15	16.26

premium at time t is $vp_t = \mathbb{E}_t^{\mathbb{Q}}\left[\text{Return Variation}(t, t+\tau)\right] - \mathbb{E}_t^{\mathbb{P}}\left[\text{Return Variation}(t, t+\tau)\right]$, in which the total return variation is calculated over the period t to $t+\tau$. The quantity vp_t corresponds to the expected profit of a variance swap, which pays the equity's realized variance over the term of the contract. Like the variance premium, I follow Kozhan, Neuberger, and Schneider (2013) and define a τ-month skew risk premium at time t as $sp_t = \frac{\mathbb{E}_t^{\mathbb{P}}\left[\text{Return Skewness}(t,t+\tau)\right]}{\mathbb{E}_t^{\mathbb{Q}}\left[\text{Return Skewness}(t,t+\tau)\right]} - 1$, in which the total return skewness is calculated from t to $t+\tau$. The quantity sp_t corresponds to the excess return on a skew swap, which pays the equity's realized skewness over the term of the contract. The literature has mainly focused on the variance premium, while the skew premium has received little attention, especially from theoretical research.

The data for the variance premium covers the period from January 1990 to December 2016 and is from Chicago Board of Options Exchange (CBOE). For the skew risk premium and implied volatility surface, I use European options written on the S&P 500 index and traded on the CBOE. The options data cover the period from January 1996 to December 2016 and are from OptionMetrics.[13] Table 1 shows summary statistics for

[13] I present the empirical methodology in Appendix A and the model-based asset prices in Appendix D.

one-month variance and skew risk premiums.[14] Figure 2 shows the implied volatility curves. The size of the variance and skew premiums as well as the level and the slope of implied volatility curves remain a challenge for asset pricing models. This paper shows that a model with GDA preferences and learning about rare depressions jointly captures standard moments of equity returns, high moment premiums, and option prices with new evidence about the variance term structure.

4.2 Calibration

To better understand the role of generalized disappointment aversion, I consider three frameworks: a model with generalized disappointment aversion preferences (GDA), an economy with disappointment aversion preferences (DA), and a specification with Epstein-Zin preferences (EZ). The comparison of GDA and DA isolates the contribution of disappointment aversion, while the comparison of GDA and EZ illustrates the impact of the agent's preference for early resolution of uncertainty. Having solved the model numerically, I generate 10,000 simulations of each calibration and report model-based 5^{th}, 50^{th} and 95^{th} percentiles of sample moments of cash flows and asset prices across all simulations.[15] In line with the data, the model-implied cash flows and returns are based on simulations with depressions, while the model-based variance forwards, moment risk premiums, and option prices correspond to simulations without depressions. The results are robust to the inclusion of rare events, which are excluded to eliminate the impact of large consumption declines and to highlight the role of learning and generalized

[14]The estimates are consistent with Bakshi et al. (2003), Bollerslev et al. (2009), and Kozhan et al. (2013).

[15]The previous version of the paper reported model population moments. For a convenient exposition of tables and figures, those results are not reported but are available upon request. In those results, I check that the fact the model explains the variance term structure and other moments is not a finite-sample phenomenon.

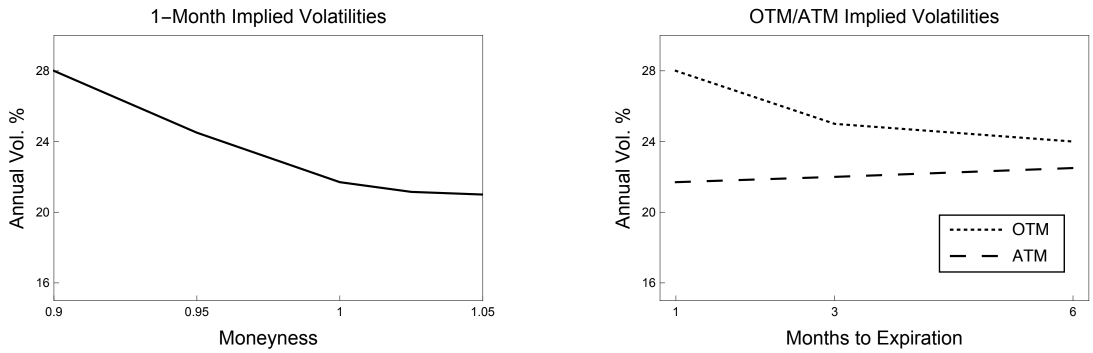

Figure 2. Implied volatilities

The left panel plots the empirical 1-month implied volatility curve as a function of moneyness. The right panel plots the empirical implied volatility curves for ATM and OTM options as functions of the time to maturity (in months). All curves are for the US data from January 1996 to December 2016.

disappointment aversion.

Table 2 reports the parameter values. As in Bansal and Yaron (2004), I make the model's time-averaged consumption statistics consistent with observed annual log consumption growth. As in Collin-Dufresne et al. (2016), I calibrate the recession state to a consumption decline in the US during the Great Depression.[16] Specifically, I set $\pi_{11} = 1151/1152$ and $\pi_{22} = 47/48$. These numbers imply an average duration of the high-growth state of $(1 - \pi_{11})^{-1} = 96$ years and the depression state of $(1 - \pi_{22})^{-1} = 4$ years. The unconditional probability of expansion is $\overline{\pi}_{11} = (1 - \pi_{22})/(2 - \pi_{11} - \pi_{22}) = 0.96$ and hence the economy experiences one four-year depression per century, consistent with the historical data. Consumption declines on average at the annual rates of $\mu_2 \times 12 = -4.6\%$ in the depression state, which is equal to an average annual decline in the real, per capita log consumption growth during the Great Depression.

I now calibrate parameters in the dividend process. To compare my results to prior

[16] The Great Depression is the only example of a consumption disaster in US history for the period considered in my paper. Thus, I naturally calibrate the recession state to this observation following Collin-Dufresne et al. (2016). Furthermore, Nakamura, Steinsson, Barro, and Ursua (2013) note that rare disasters tend to unfold over multiple years. Instead of assuming extreme instantaneous consumption disasters, I choose a milder depression with an average duration corresponding to four years of the Great Depression.

Table 2. Parameter values

This table reports parameter values in the cash-flow processes and the three models: GDA, DA, and EZ.

Parameter	Description	Value		
π_{11}	Transition probability from expansion to expansion	1151/1152		
π_{22}	Transition probability from recession to recession	47/48		
$\mu_1 \times 12$	Consumption growth in expansion	2.06		
$\mu_2 \times 12$	Consumption growth in recession	−4.6		
$g_d \times 12$	Mean adjustment of dividend growth	−2.87		
$\sigma \times \sqrt{12}$	Std. deviation of consumption growth shock	2.6		
$\sigma_d \times \sqrt{12}$	Std. deviation of dividend growth shock	11.41		
λ	Leverage ratio	2.6		
		GDA	DA	EZ
β^{12}	Discount factor	0.99	0.99	0.99
$1/(1-\rho)$	EIS	1.5	1.5	1.5
$1-\alpha$	Risk aversion	1/1.5	1/1.5	6.0
θ	Disappointment aversion	8.41	0.6	0
δ	Disappointment threshold	0.930	1	

studies, particularly the disaster literature, I set the leverage ratio $\lambda = 2.6$, the value used in Seo and Wachter (2019).[17] I further follow the literature and set g_d to equalize the long-run dividend and consumption growth. The standard deviation of the dividend process σ_d is used to generate large annual dividend volatility observed in the data.

Table 2 further summarizes the values of GDA, DA, and EZ preferences. I set $\beta^{12} = 0.99$ and $1/(1-\rho) = 1.5$ in all cases. In the GDA model, the coefficient of relative risk aversion is $1-\alpha = 1/1.5$. This cancels the Epstein-Zin adjustment in SDF as shown in Section 3 and also deletes one degree of freedom caused by extra GDA parameters. I jointly set $\theta = 8.41$ and $\delta = 0.930$ to match the high equity premium. The calibrated disappointment aversion is consistent with the empirical estimates from 3.29 to 8.41 (Delikouras, 2017). Note that the variance term structure, the variance and skew premiums, and the implied volatility surface are not directly targeted during the model

[17] I regress the annual dividends on the annual consumption covering the period 1930-2016 and find the leverage ratio is around 2.5, a number within an interval of commonly used values from 1.5 to 4.5. The leverage ratio is an important parameter for two reasons. First, it controls the volatility of dividends in normal times. Second, it determines the decline of dividends in the depression state. Consequently, a larger leverage parameter would increase the payoff of put options, conditional on the depression realization.

calibration.

In the DA model, I set $1 - \alpha = 1 - \rho = 1/1.5$ to eliminate the impact of a relative risk aversion parameter on SDF. I also shut down the generalized disappointment aversion channel by setting $\delta = 1$. This inevitably generates larger effective risk aversion in good times due to an increased number of disappointing events, significantly distorting equity moments in the DA model. Thus, I decrease the disappointment aversion parameter $\theta = 0.6$ to match the observed equity premium. The remaining parameters are fixed at the initial values. For the EZ model, I turn off disappointment aversion by setting $\theta = 0$. The model operates only through the risk aversion channel with the coefficient of relative risk aversion of $1 - \alpha = 6$. In this case, the agent has a preference for early resolution of uncertainty, a workhorse in the asset pricing literature. Other parameters correspond to those in the GDA model.

4.3 Endowments and Equity Returns

Panel A in Table 3 compares the annualized consumption and dividends moments of the data with those implied by the calibration. A two-state regime-switching process matches the key empirical statistics well. Panel B in Table 3 reports the annualized moments of equity returns for the three specifications. All three models do a good job of accounting for salient features of equity returns, as all predict the low risk-free rate, the large equity premium, and the volatility of excess returns. Also, the volatility of the risk-free rate and the level of the log price-dividend ratio correspond well to the empirical estimates. The shortcoming of the three models is too low volatility of the log price-dividend ratio.

Table 3. Cash flows and stock market returns

Panel A reports moments of consumption and dividend growth denoted by Δc and Δd. Panel B reports moments of the log risk-free rate r_f, the excess log equity returns $r_e - r_f$, and the log price-dividend ratio pd. The entries are annualized statistics except for autocorrelation and correlation. The moments are for the data and the three models: GDA, DA, and EZ. The empirical moments are for the US data from January 1930 to December 2016. For each model, I simulate 10,000 economies at a monthly frequency with a sample size equal to its empirical counterpart and report percentiles of sample statistics. The model-implied results are based on the simulations with consumption disasters, consistent with the historical data. I use common notations for mean E, volatility σ, autocorrelation $ac1$, and correlation $corr$.

	Data	GDA			DA			EZ		
		5%	50%	95%	5%	50%	95%	5%	50%	95%
Panel A: Cash flows										
$E(\Delta c)$	1.83	0.91	1.85	2.40	0.91	1.85	2.40	0.91	1.85	2.40
$\sigma(\Delta c)$	2.22	1.90	2.28	3.19	1.90	2.28	3.19	1.90	2.28	3.19
$ac1(\Delta c)$	0.50	0.09	0.30	0.62	0.09	0.30	0.62	0.09	0.30	0.62
$E(\Delta d)$	1.44	−1.10	1.91	4.44	−1.10	1.91	4.44	−1.10	1.91	4.44
$\sigma(\Delta d)$	11.04	9.51	11.05	12.97	9.51	11.05	12.97	9.51	11.05	12.97
$ac1(\Delta d)$	0.19	0.09	0.27	0.46	0.09	0.27	0.46	0.09	0.27	0.46
$corr(\Delta c, \Delta d)$	0.55	0.38	0.55	0.71	0.38	0.55	0.71	0.38	0.55	0.71
Panel B: Returns										
$E(r_f)$	0.81	−0.13	0.86	1.49	0.68	1.14	1.20	0.22	1.03	1.41
$\sigma(r_f)$	1.87	1.48	2.52	3.51	0.04	0.25	1.22	0.73	1.50	2.34
$E(r_e - r_f)$	5.22	3.67	6.10	8.35	3.43	6.04	8.47	3.50	5.89	8.19
$\sigma(r_e - r_f)$	19.77	15.58	19.22	23.11	13.03	16.02	20.34	14.64	18.69	23.49
$E(pd)$	3.11	2.96	3.03	3.05	2.90	2.97	2.98	2.95	3.04	3.06
$\sigma(pd)$	0.33	0.04	0.08	0.18	0.01	0.05	0.18	0.03	0.08	0.22

4.4 The Price of Variance Risk

Figure 3 compares the empirical and model-based term structure of variance swap prices and returns. The left plot shows that the GDA model does a good job of matching the overall shape of annualized Sharpe ratios. In particular, it generates a curve that is negative and steep at shorter horizons and becomes positive and upward-sloping at longer maturities. The figure also shows that both DA and EZ specifications fail to reconcile the concave and upward shape of the term structure. Consistent with Dew-Becker et al. (2017), the calibration with Epstein-Zin preferences underprices variance risk in the short term and overprices future variance in the long term. The DA model implies even more negative Sharpe ratios at longer horizons, while the one-month forwards earn a similar risk premium as in the EZ model. The right panel plots the average prices of

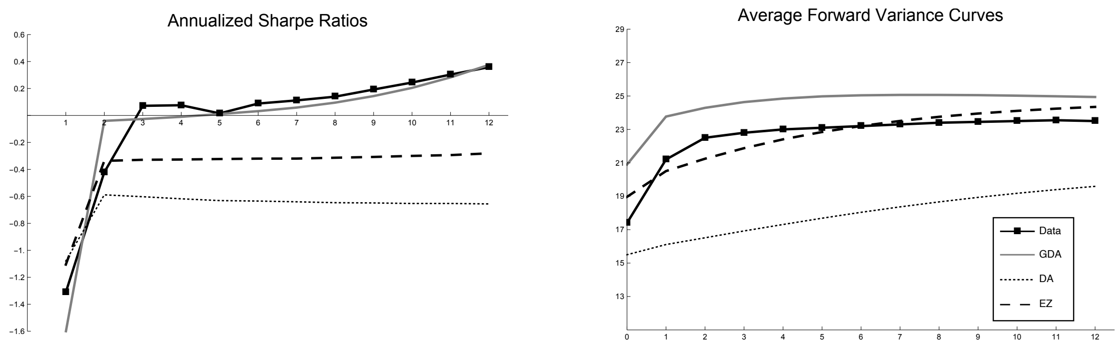

Figure 3. Sharpe ratios and forward variance claim prices

The figure plots annualized Sharpe ratios and average prices for forward variance claims for the data and the three models: GDA, DA, and EZ. The prices are reported in annualized volatility terms. The empirical lines are from Dew-Becker et al. (2017) and correspond to the US data from 1996 to 2013. For each model, I simulate 10,000 economies at a monthly frequency with a sample size equal to its empirical counterpart and report medians of sample statistics. The model-implied results are based on the simulations without consumption disasters, consistent with the historical data.

forward variance claims for different maturities in the data and the three models. The empirical curve is steep and concave at the very short end and it flattens significantly at the long end. In contrast, the DA and EZ specifications predict strongly upward-sloping term structures at all horizons. Although the GDA model generates slightly higher prices of variance claims, it captures the concave shape and the flatness of the curve at longer maturities.

Table 4 augments the results in Figure 3 by reporting the p-values of annualized Sharpe ratios with respect to their finite-sample distribution. For each model, it shows the fraction of samples across 10,000 simulations of the economy satisfying one of the conditions. For the first three conditions, simulated average Sharpe ratios for one-, three-, and 12-month horizons should be respectively smaller, larger, and larger than the empirical estimates. One can interpret these fractions as p-values for a one-sided test of the model generating as negative or as positive average Sharpe ratios for a particular maturity as in the data. For the last condition, simulated statistics should jointly satisfy the first three requirements. This corresponds to the p-value for a test of the model

Table 4. Model tests using annualized Sharpe ratios for forward variance claims

The entries are for the three models: GDA, DA, and EZ. For each model, I simulate 10,000 economies at a monthly frequency with a sample size equal to the length of the variance swap data. In each simulation, I calculate average annualized Sharpe ratios for forward variance claims with one-, three-, and 12-month maturities. For each model, the first row shows fractions of samples in which the simulated Sharpe ratios are at least as small as the empirical one-month estimates. The second and third rows present the fraction of samples in which the simulated Sharpe ratios are at least as large as the empirical three-month and 12-month estimates, respectively. The entries of the bottom row are the fraction of samples in which all three conditions are satisfied simultaneously. The model-implied results are based on the simulations without consumption disasters, consistent with the historical data.

	p-value		
	GDA	DA	EZ
Simulated 1mo/SR $\leq$ empirical SR	0.91	0.26	0.27
Simulated 3mo/SR $\geq$ empirical SR	0.38	< 0.01	0.03
Simulated 12mo/SR $\geq$ empirical SR	0.48	< 0.01	< 0.01
Joint test: 1mo/SR $\leq$ data $\wedge$ 3mo/SR $\geq$ data $\wedge$ 12mo/SR $\geq$ data	0.32	< 0.01	< 0.01

replicating the observed upward-sloping shape of the term structure.

Table 4 shows that we cannot reject any of the three models based on the one-month variance forward returns only. Specifically, one would expect to see as small average one-month Sharpe ratios as observed empirically in 91%, 26%, and 27% of the time in the GDA, DA, and EZ specifications, respectively. At longer maturities, however, one can reject at the 5% level the null hypothesis that the DA or EZ frameworks generate the variance swap data. The GDA model instead generates large p-values for all tests and cannot be rejected. In particular, the models with disappointment aversion or Epstein-Zin preferences would predict positive Sharpe ratios at longer maturities as in the data in fewer than 3% of simulations, while the likelihood of replicating the overall shape is less than 1%. This is in stark contrast to the GDA model, which captures negative Sharpe ratios at the short end and positive ones at the long end in 32% of the simulations.

To gain a better understanding of the results, Figure 4 illustrates annualized return volatility as a function of the posterior probability of the expansion in the three models. The volatility has a pronounced humped shape and is maximized at an interior point

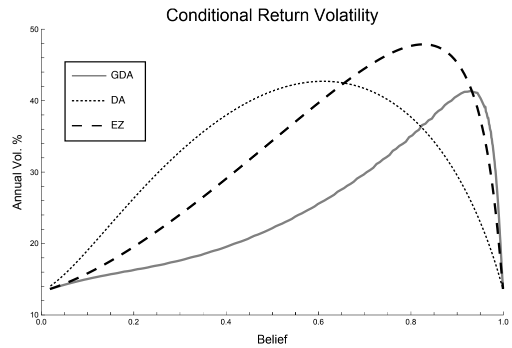

Figure 4. Return volatility

The panel plots equity return volatility as a function of a posterior belief for the three models: GDA, DA, and EZ. Quantities are reported in annualized volatility terms, $100 \times \sqrt{12 \times var_t(r_e)}$.

of the probability simplex in all cases. The GDA model generates highly skewed conditional volatility. The DA specification yields a symmetric shape of the volatility curve. The volatility line in the EZ economy is roughly located in the middle of the two. For Epstein-Zin and especially disappointment aversion preferences, return volatility is high for a wide range of beliefs and becomes low only when the investor has full confidence in the state. As the investor's beliefs tend to change slowly over time, high return volatility persists in the long term and hence increases the hedge against long-term volatility risk. As a result, this generates the upward-sloping term structure of variance claim prices, which is inconsistent with the data.[18]

In contrast, return volatility in the GDA model is high within a narrow range of beliefs and quickly diminishes outside this interval. When the investor's beliefs become pessimistic, return volatility initially spikes but does not persist in the long term, implying a larger amount of variance risk in the short term. In equilibrium, the properties of return variance transmit to variance forwards, which generates the inversion in their

[18]This intuition also applies to the models with full information. If the state is observable, the high stock market variance happens during consumption disasters and hence persists on average for four years, the average duration of a depression state. As a result, high variance risk will concentrate in the long term, and forward variance will be upward-sloping, counter to what we observe empirically.

prices in bad times. In the short term, the inversion increases the hedge against realized variance. In the long run, it kills the upward-sloping effect of time-varying disaster risk and produces the flat unconditional term structure of prices. I also show that the inversion is strong enough to produce on average positive and slightly increasing Sharpe ratios at longer maturities.

The mechanism determining the conditional return volatility is as follows. Veronesi (1999) demonstrates that, in the endowment economy with two hidden regimes, price sensitivity to news is strongly driven by the risk aversion component stemming from the investor's degree of risk aversion. In the DA model, risk aversion is equally large in the expansion and depression states because a high disappointment threshold implies a large number of disappointing outcomes in the two regimes. Thus, price sensitivities are similar across states, resulting in symmetric conditional return volatility. In the EZ economy, equity prices are more sensitive to consumption shocks in good times than in bad times, although Epstein-Zin preferences do not generate a significant difference in price sensitivities in the two regimes. In the GDA model, instead, substantial countercyclical risk aversion leads to a stronger overreaction of stock prices to bad news in good times, whereas equity prices are substantially less sensitive to good news in bad times. This asymmetry in price sensitivities leads to strongly skewed return volatility in the GDA model.

As an additional exercise, Figure 5 provides impulse responses of the conditional term structure of Sharpe ratios and average prices. The investor holds a median belief (normal times). I then study conditional dynamics of the term structures next period when consumption growth is a 1.5 standard deviation above and 1.5, 2.2, and 2.5 stan-

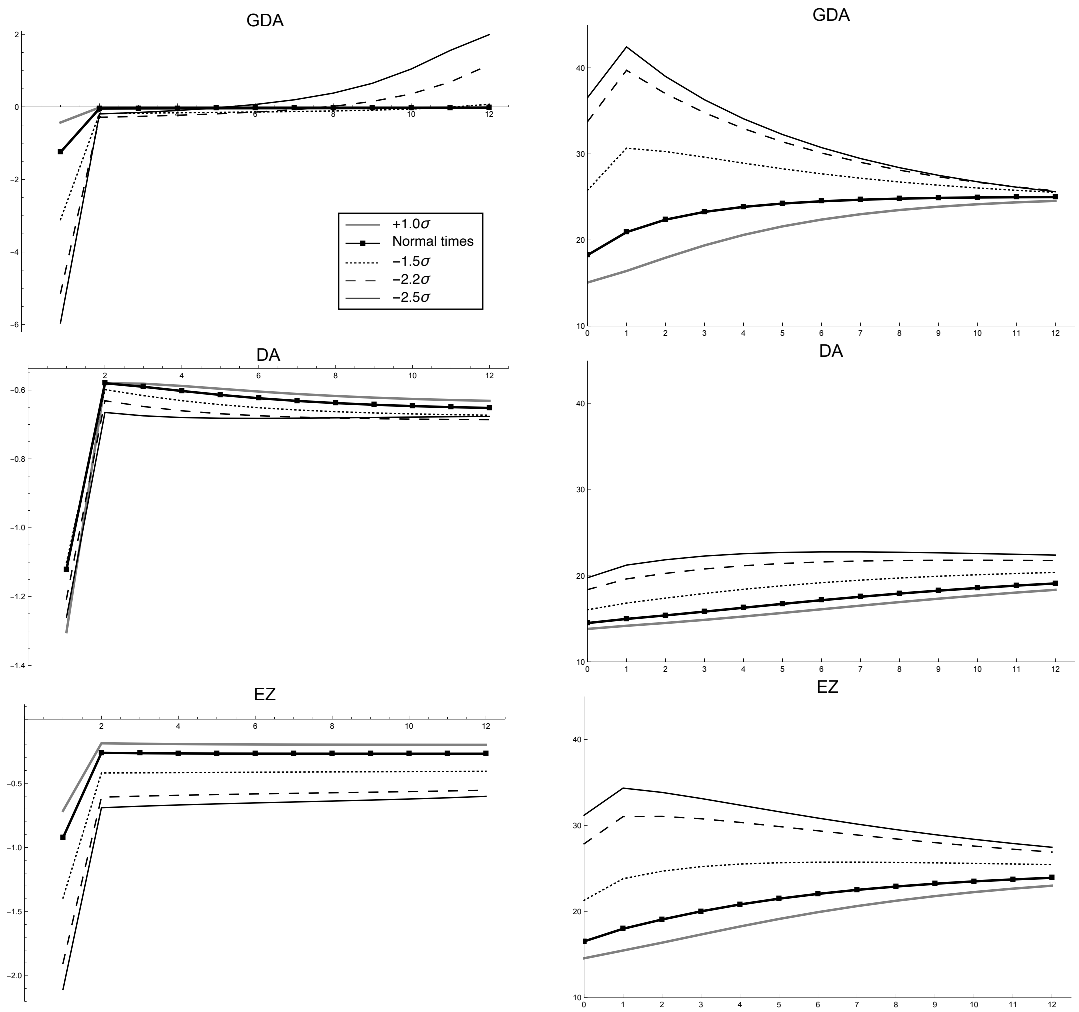

Figure 5. Conditional Sharpe ratios and forward variance claim prices
The figure plots annualized Sharpe ratios and average prices for forward variance claims for the three models: GDA, DA, and EZ. Each panel shows the term structures in good, normal, and bad times. The economy is initially in normal times, corresponding to a median posterior belief. In good (bad) times denoted by "+1.0σ" ("-1.5σ", "-2.2σ", and "-2.5σ"), consumption growth is a 1.0 (1.5, 2.2, and 2.5) standard deviation(s) above (below) an average growth in expansion.

dard deviations below average growth in the expansion (good and bad times). Figure 5 shows that the DA and EZ models predict negative Sharpe ratios for all economic conditions. Contrary to the empirical evidence, average Sharpe ratios become more negative in the upside scenario under disappointment aversion. The economy with GDA preferences generates a procyclical and steep curve for short-term claims, consistent with Aït-Sahalia et al. (2020). Furthermore, the term structure of Sharpe ratios is insignificant for maturities longer than two months in good and normal times as well as bad but not depression-like states and is steep and positive in response to large consumption declines. The latter feature of the GDA model enables to match the sign and shape of the unconditional curves.

Figure 5 further depicts impulse responses of variance claim prices. The average curve for the DA model remains upward-sloping in all scenarios. This explains negative average returns on variance forwards. For the EZ economy, the term structure of prices switches from strongly increasing in normal and good times to slightly increasing in bad (but not severe) times, and it even becomes weakly downward-sloping in very bad times. Nevertheless, this amplification of short-term prices is too weak to generate on average positive returns on holding a variance forward. In contrast, generalized disappointment aversion inverts the term structure in all bad scenarios, and this inversion is substantially stronger than in the EZ economy. Thus, GDA preferences strongly amplify the short-term variance risk in bad times that enables one to replicate empirical term structures.

Next, I conduct a sensitivity analysis to examine the robustness of key results to alternative calibrations of preference specifications and to address the concern that the findings are driven by a particular choice of parameters. Specifically, I change one key parameter in each of the three preference specifications, while holding the remaining parameters as in the original calibration. In the GDA model, I consider smaller and larger values of disappointment aversion and threshold parameters. In the DA model, I decrease or increase disappointment aversion compared to the original calibration. In the EZ model, I consider smaller and larger relative risk aversion coefficients.

Figure 6 depicts Sharpe ratios and prices for variance forwards in various cali-

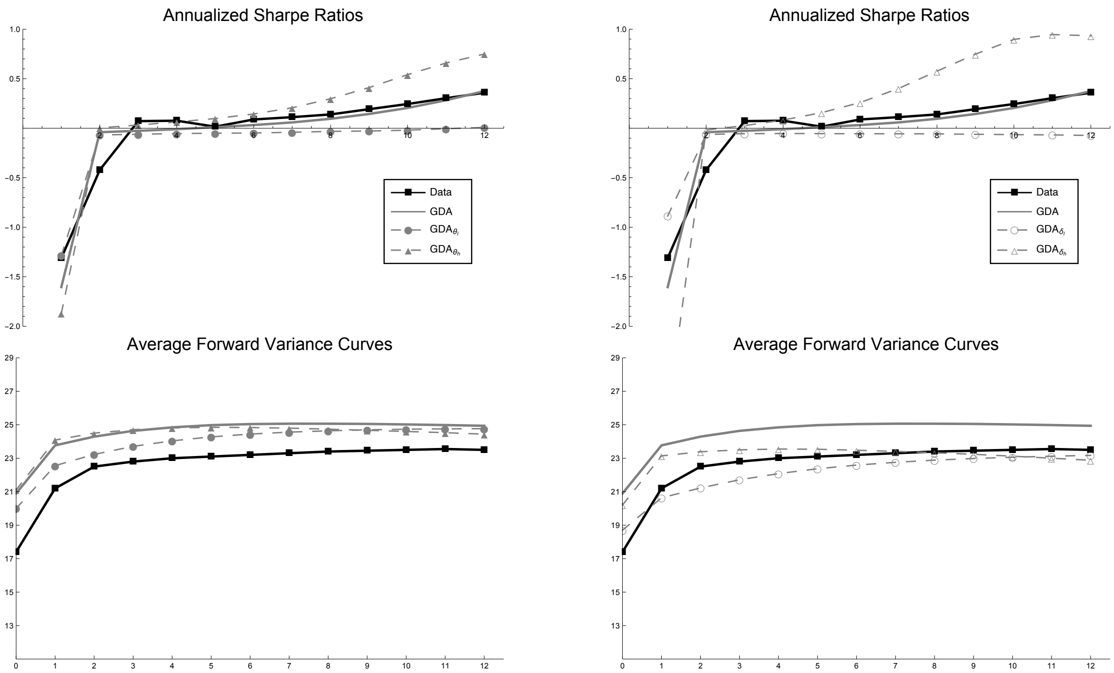

Figure 6. Sensitivity of Sharpe ratios and forward variance claim prices: GDA

The figure plots annualized Sharpe ratios and average prices for forward variance claims for different model calibrations with generalized disappointment aversion preferences. GDA corresponds to the original GDA model. In GDA$_{\theta_l}$ and GDA$_{\theta_h}$, $\theta_l = 6.41$ and $\theta_h = 10.41$. In GDA$_{\delta_l}$ and GDA$_{\delta_h}$, $\delta_l = 0.920$ and $\delta_h = 0.940$. If not stated otherwise, the remaining parameters are set at the original values in the GDA model. For each model, I simulate 10,000 economies at a monthly frequency with a sample size equal to its empirical counterpart and report medians of sample statistics. The model-implied results are based on the simulations without consumption disasters, consistent with the historical data.

brations of GDA preferences. The shape of variance forward prices flattens and the term structure of Sharpe ratios becomes upward-sloping with the higher disappointment threshold or disappointment aversion. Intuitively, variance risk is amplified more in the short term than in the long term in bad times. As a result, this generates downward- and upward-sloping patterns in prices and Sharpe ratios, respectively. In normal and good times, average prices are slightly increasing in the horizon. However, only short-term variance risk earns a significant premium as measured by large and negative Sharpe ratios for one and two months but insignificant ratios for longer horizons. Since higher disappointment risk reinforces the first effect, the higher disappointment threshold or disappointment aversion implies flatter and steeper term structures of prices and Sharpe

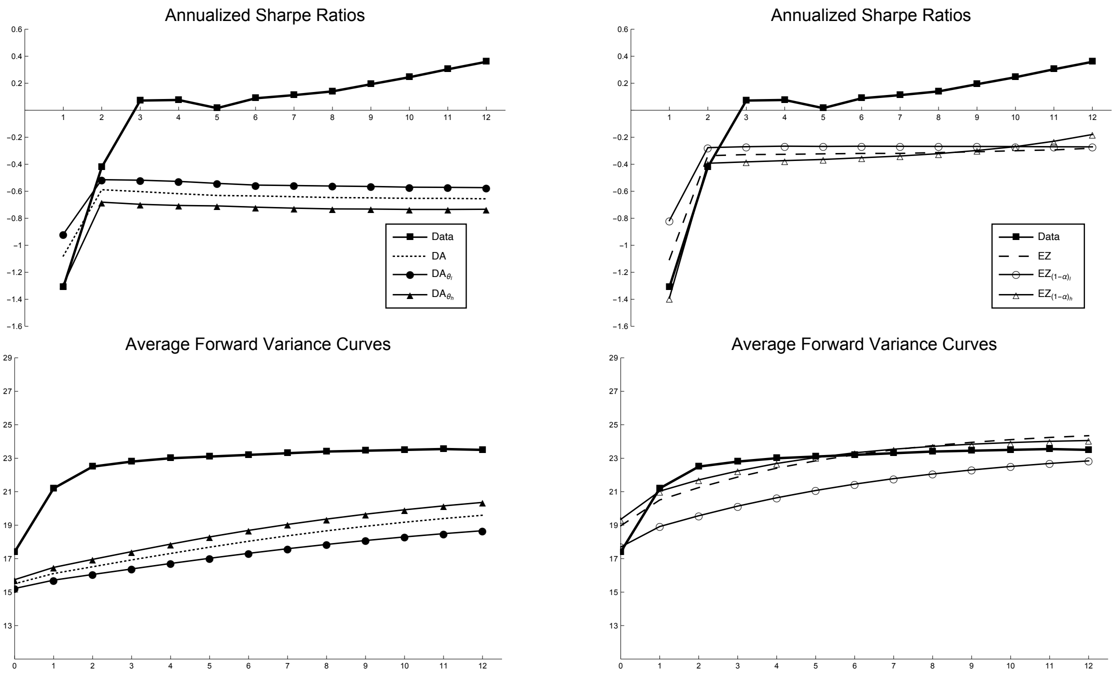

Figure 7. Sensitivity of Sharpe ratios and forward variance claim prices: DA and EZ
The figure plots annualized Sharpe ratios and average prices for forward variance claims for different model calibrations with disappointment aversion and Epstein-Zin preferences. DA and EZ correspond to the original DA and EZ models. In DA_{θ_l} and DA_{θ_h}, $\theta_l = 0.5$ and $\theta_h = 0.7$. In $\text{EZ}_{(1-\alpha)_l}$ and $\text{EZ}_{(1-\alpha)_h}$, $(1-\alpha)_l = 5$ and $(1-\alpha)_h = 7$. If not stated otherwise, the remaining parameters are set at the original values in the DA and EZ models. For each model, I simulate 10,000 economies at a monthly frequency with a sample size equal to its empirical counterpart and report medians of sample statistics. The model-implied results are based on simulations without consumption disasters, consistent with the historical data.

ratios, respectively.

Figure 7 examines the impact of disappointment and risk aversion parameters on the variance term structures in the models with Gul and Epstein-Zin preferences. In the DA specification, the slope of forward variance prices is increasing in disappointment aversion. The reason is that the disappointment-averse investor strongly dislikes low and high variance. Thus, stronger disappointment aversion increases already high insurance premia against shocks to realized and future volatility. In the EZ economy, the slope of forward variance prices is decreasing in risk aversion. To generate a close-to-zero slope at least after the ten-month maturity, the risk aversion should be at least 7. For this value, however, the model would generate a Sharpe ratio of less than -2.0 for the one-month claim compared to -1.3 in the data. Moreover, with this value of relative risk aversion, the mean equity premium has a median value of 8% in the EZ model, well above the empirical estimate of around 5%. Raising risk aversion even more would only worsen the model fit with the variance term structure at the one-month maturity and with equity moments and higher-moment risk premiums (see Appendix E). Thus, one cannot reconcile the variance term structure in the EZ framework by increasing risk aversion.

In sum, this sensitivity analysis confirms that the pricing kernel, necessary to reconcile the empirical variance term structure, is consistent with generalized disappointment aversion and cannot be supported by parameter values in alternative preferences. Appendix E augments a comparative statics exercise by reporting the remaining results in alternative model calibrations. It demonstrates that the DA and EZ specifications with different parameter choices are unable to capture the higher-moment risk premiums and implied volatility curves.

4.5 Variance and Skew Risk Premiums

Panel A in Table 5 collects moments of the variance premium and related measures in the data and models. The GDA economy is able to generate a large and volatile variance premium. It also qualitatively respects the non-normality of the variance premium distribution, although the sample skewness and kurtosis statistics are smaller relative to the data. The GDA model accounts for the variance premium with empirically consistent conditional return variances under both probability measures. In particular, it predicts that return variance is more volatile under the risk-neutral distribution and that both variances are persistent, as they are in the data.

Table 5. Variance premium and predictability

Panel A reports moments of variance premium vp, market return variances $var_t^{\mathbb{P}}(r_e)$ and $var_t^{\mathbb{Q}}(r_e)$ under physical $\mathbb{P}$ and risk-neutral $\mathbb{Q}$ probability measures. The Panel A entries are monthly statistics. Panel B reports results of the predictive regression of h-month future excess log equity returns constructed as $r^{ex}_{t+1\to t+h} = \sum_{i=1}^{h}\left(r_{e,t+i} - r_{f,t-1+i}\right)$ on the lagged variance premium vp_t. Specifically, the slope estimates $\beta(h)$ and $R^2(h)$ are based on the linear projection:

$$100 \times r^{ex}_{t+1\to t+h} = \text{Intercept} + \beta(h) \times vp_t + \varepsilon_{t+h}, \quad h = 1, 3, 6.$$

The moments and regression outputs are for the data and the three models: GDA, DA, and EZ. The empirical statistics are for the US data from January 1990 to December 2016. For each model, I simulate 10,000 economies at a monthly frequency with a sample size equal to its empirical counterpart and report percentiles of sample statistics based on these series. The model-implied results are based on the simulations without consumption disasters, consistent with the historical data. I use common notations for mean E, volatility σ, autocorrelation $ac1$, skewness $skew$, and kurtosis $kurt$.

	Data	GDA			DA			EZ		
		5%	50%	95%	5%	50%	95%	5%	50%	95%
Panel A: Variance premium										
$E(vp)$	10.27	8.13	12.32	17.14	1.34	2.11	3.39	3.15	4.92	7.23
$\sigma(vp)$	10.87	12.22	15.99	18.93	1.53	3.11	5.25	4.79	7.46	10.91
$skew(vp)$	2.33	0.91	1.49	2.25	0.49	2.76	4.17	−0.62	1.71	2.98
$kurt(vp)$	10.90	2.34	4.06	7.67	5.88	12.03	24.96	4.00	7.27	13.59
$\sigma(var_t^{\mathbb{P}}(r_e))$	29.32	17.34	25.44	32.68	5.02	14.25	36.48	13.00	25.50	40.47
$ac1(var_t^{\mathbb{P}}(r_e))$	0.79	0.70	0.81	0.88	0.61	0.79	0.92	0.66	0.82	0.91
$\sigma(var_t^{\mathbb{Q}}(r_e))$	33.76	29.58	40.60	49.11	6.55	17.24	38.91	17.79	31.46	45.00
$ac1(var_t^{\mathbb{Q}}(r_e))$	0.80	0.69	0.79	0.85	0.62	0.80	0.92	0.66	0.82	0.89
$skew(var_t^{\mathbb{Q}}(r_e))$	3.53	0.86	1.47	2.21	2.40	3.73	5.75	1.47	2.30	3.34
$kurt(var_t^{\mathbb{Q}}(r_e))$	21.47	2.30	4.13	7.78	8.46	19.47	45.54	3.87	8.24	16.18
Panel B: Predictability of excess returns										
$\beta(1m)$	0.76	0.19	0.75	1.38	−1.43	1.81	5.56	−0.95	0.93	2.77
$R^2(1m)$	2.70	0.15	2.39	6.99	0.02	1.09	4.19	0.01	1.31	6.01
$\beta(3m)$	0.83	0.18	0.63	1.09	−1.31	1.55	4.02	−0.73	0.87	2.11
$R^2(3m)$	8.61	0.47	5.57	15.24	0.02	2.39	8.67	0.03	3.28	12.87
$\beta(6m)$	0.57	0.15	0.50	0.82	−1.05	1.24	3.06	−0.63	0.74	1.60
$R^2(6m)$	7.55	0.68	7.78	20.79	0.08	3.32	13.14	0.04	4.67	16.95

I now examine return predictability by the variance premium documented by prior literature. I regress the one-, three-, and six-month cumulative excess log returns (expressed in percentages) on the lagged monthly variance premium. Panel B in Table 5 reports positive and slightly decreasing regression coefficients and increasing R^2s with the horizon. The GDA model matches the magnitude and monotonicity of coefficients and R^2 statistics.

Table 5 shows that the model with disappointment aversion preferences produces

Table 6. Skew premium

This table reports moments of skew premium sp, market return skewness $skew_t^{\mathbb{P}}(r_e)$ and $skew_t^{\mathbb{Q}}(r_e)$ under the physical $\mathbb{P}$ and risk-neutral $\mathbb{Q}$ probability measures. The entries are monthly statistics. The moments are for the data and the three models: GDA, DA, and EZ. The empirical statistics are for the US data from January 1996 to January 2016. For each model, I simulate 10,000 economies at a monthly frequency with a sample size equal to its empirical counterpart and report percentiles of sample statistics based on these series. The model-implied results are based on the simulations without consumption disasters, consistent with the historical data. I use common notations for mean E, volatility σ, skewness $skew$, and kurtosis $kurt$.

	Data	GDA			DA			EZ		
		5%	50%	95%	5%	50%	95%	5%	50%	95%
$E(sp)$	−42.20	−39.11	−34.58	−30.78	26.58	34.88	56.36	−22.79	−19.34	−12.84
$\sigma(sp)$	81.81	11.23	26.42	46.52	24.44	29.53	377.79	3.03	21.31	91.65
$skew(sp)$	3.57	−4.32	3.28	8.56	−3.37	1.24	13.98	−11.70	3.23	13.69
$kurt(sp)$	16.26	1.93	43.48	112.04	3.10	4.65	215.60	2.04	80.40	219.37
$ar1(sp)$	0.04	−0.12	0.15	0.62	−0.01	0.61	0.70	−0.27	0.11	0.58
$E(skew_t^{\mathbb{P}}(r_e))$	−87.52	−42.55	−39.99	−33.83	−20.49	−15.82	−12.34	−38.00	−33.98	−29.43
$\sigma(skew_t^{\mathbb{P}}(r_e))$	173.59	8.99	11.79	22.21	7.68	11.41	15.56	12.03	13.61	23.97
$E(skew_t^{\mathbb{Q}}(r_e))$	−177.73	−70.44	−64.13	−53.83	−17.39	−13.22	−9.82	−47.60	−42.42	−36.45
$\sigma(skew_t^{\mathbb{Q}}(r_e))$	92.33	23.20	28.27	41.96	7.91	11.14	15.54	16.44	18.68	28.90

a mean and volatility of the variance premium that are more than five times smaller than with the generalized utility function. Turning off the generalized disappointment aversion channel also leads to a significant reduction in the volatility of return variance in the DA model. As the variance premium decreases, its predictive power for the excess log returns also suffers. This is manifested in the lower R^2s and empirically inconsistent regression coefficients. Next, I turn off any source of (generalized) disappointment aversion and consider a representative agent with Epstein-Zin preferences. The EZ model leads to around a two-fold increase in the mean and volatility of the variance premium relative to the DA model, but sample statistics are less than half of the numbers in the GDA model. A smaller variance premium is due to the reduced volatility in conditional variances. A smaller variance premium in the DA and EZ models results in excessively-high regression coefficients and too small R^2s in the predictive regressions.

Table 6 reports summary statistics of the skew risk premium in the data and models. The GDA model produces a sizeable skew premium, which corresponds well to the historical value and generates positive skewness and excess kurtosis statistics. The conditional mean of the return skewness under both measures is significantly negative, although the model cannot fully capture the size observed in the data. The main drawback of the GDA model is lower volatility of the skew premium, realized and implied skew. Since conditional dynamics of the model are driven by a single state, allowing the model to operate through other channels (time-varying expected growth and volatility, jumps in consumption, etc.) would make the economy more flexible to jointly match all moments.

Table 6 also shows that disappointment aversion predicts the wrong sign of the skew premium. The DA model also predicts the smallest first and second moments of return skewness across the three models. In the EZ model, the risk-neutral return density becomes more distorted towards the left tail; however, the model generates less than half of the average skew premium in the data. Although the EZ model predicts the correct sign, it significantly understates the magnitude. Overall, generalized disappointment aversion better explains salient features of the skew premium than nested preferences.

4.6 The Term Structure of Implied Volatilities

Figure 8 compares the implications of all models for equity index options. The implied volatilities are expressed as a function of moneyness. The empirical implied volatilities decline in moneyness, a pattern known as the implied volatility skew. The DA implied volatilities for the 1-month maturity are flat and approximately equal to the realized stock market volatility. One apparent candidate to generate a steep volatility skew is high risk aversion. Although raising risk aversion in Epstein-Zin preferences

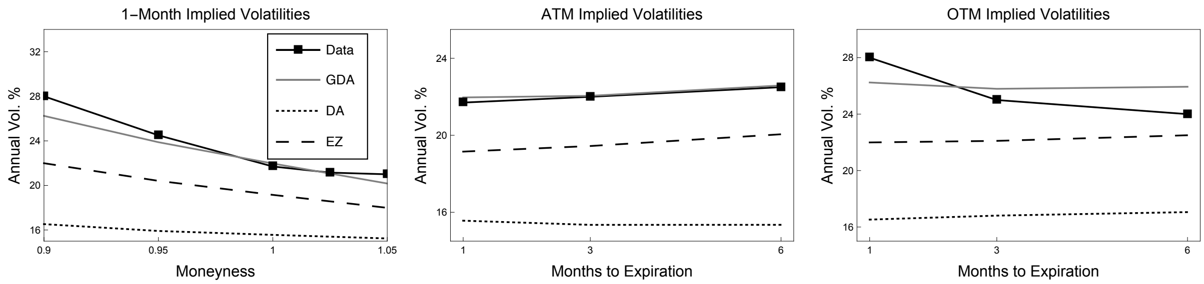

Figure 8. Implied volatilities

The left panel plots the 1-month implied volatility curve as a function of moneyness for the data and the three models: GDA, DA, and EZ. The middle and right panels plot the empirical and model-based implied volatility curves for ATM and OTM options as functions of the time to maturity (in months). The empirical statistics are for the US data from January 1996 to December 2016. The model-based curves are calculated for option prices using the annualized model-implied interest rate and dividend yield. For each model, I simulate 10,000 economies at a monthly frequency with a sample size equal to its empirical counterpart and report the medians of sample statistics. The model-implied results are based on the simulations without consumption disasters, consistent with the historical data.

improves the model's performance, this cannot fully account for the level of implied volatilities. In contrast, the GDA framework can fit the option prices much better. Figure 8 additionally presents implied volatilities for ATM and 0.90 OTM options. In the data, ATM (OTM) volatilities slightly increase (decrease) over the horizon. Neither DA nor EZ specification can match the level of the empirical curves. In contrast, generalized disappointment aversion can explain overall patterns and magnitudes of the empirical implied volatilities.

5 Conclusion

I build an equilibrium model with GDA preferences and rare events in consumption growth. I show that the combination of the investor's tail aversion and fluctuating economic uncertainty due to learning about a hidden depression state explains a wide variety of asset pricing phenomena. Most notably, the model rationalizes the variance term structure, a new stylized fact of the variance swap data. In particular, the model predicts large and negative Sharpe ratios on one-month variance forwards and produces a slightly positive term structure for maturities longer than two months. Furthermore, the model accounts for the large variance and skew risk premiums, and generates a realistic volatility surface implied by index options, while simultaneously matching the salient features of equity returns and the risk-free rate. I show that the success of the model is attributable to generalized disappointment aversion by comparing GDA preferences to nested utilities: disappointment aversion and Epstein-Zin preferences. Although three specifications can reasonably match equity moments, only GDA preferences can explain the variance term structure, moment risk premiums, and option prices.

There are several interesting avenues for future research. First, my paper highlights the importance of the specific values of the disappointment threshold and disappointment aversion. Although Delikouras (2017) provides the empirical estimate of a disappointment aversion parameter in Gul (1991), joint estimation of the parameters in Routledge and Zin (2010) has not been addressed yet. Second, it is fruitful to explore the implications of the richer model for the term structure of dividend strips and interest rates. For instance, the extension with post-depression recoveries (Hasler and Márfe, 2016) has the potential to jointly explain the term structures of interest rates, equity and variance risk. Third, generalized disappointment aversion is likely to have additional asset pricing implications for risk premia with a multi-dimensional learning problem (Johannes et al., 2016) or rational parameter learning (Collin-Dufresne et al., 2016). Finally, it is interesting to study GDA preferences with other behavioral biases (Brandt, Zeng, and Zhang, 2004).

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Internet Appendix

"Generalized Disappointment Aversion and the Variance Term Structure"

Abstract

This appendix provides a detailed description of the data, the numerical methods used to solve different models, model-based asset prices as well as additional results not included in the main body of the paper.

Contents

A Data

A.1 Consumption, Dividends, and Market Returns

I follow Bansal and Yaron (2004) and construct real per capita consumption growth series (annual, due to the frequency restriction) for the longest sample available, 1930-2016. In the literature, consumption is defined as a sum of personal consumption expenditures on nondurable goods and services. I download the data from the US National Income and Product Accounts (NIPA) as provided by the Bureau of Economic Analysis. I apply the seasonally adjusted annual quantity indexes from Table 2.3.3. (Real Personal Consumption Expenditures by Major Type of Product, Quantity Indexes, A:1929-2016) to the corresponding series from Table 2.3.6. (Real Personal Consumption Expenditures by Major Type of Product, Chained Dollars, A:1995-2016) to obtain real personal consumption expenditures on nondurable goods and services for the sample period 1929-2016. I further retrieve mid-month population data from NIPA Table 7.1. to convert real consumption series to per capita terms.

I measure the total market return as the value-weighted return including dividends, and the dividends as the sum of total dividends, on all stocks traded on the NYSE, AMEX, and NASDAQ. The dividends and value-weighted market return data are monthly and are retrieved from the Center for Research in Security Prices (CRSP). To construct the monthly nominal dividend series, I use the CRSP value-weighted returns including and excluding dividends of CRSP common stock market indexes (NYSE, AMEX, NASDAQ, ARCA), denoted by RI_t and RE_t, respectively. Following Hodrick (1992), I construct the price series P_t by initializing $P_0 = 1$ and iterating recursively $P_t = (1 + RI_t)P_{t-1}$. Next, I compute normalized nominal monthly dividends $D_t =$

$(RI_t - RE_t)P_t$. The proxy of the risk-free return $R_{f,t+1}$ is the 1-month nominal Treasury bill. The nominal annualized dividends are constructed by summing the corresponding monthly dividends within the year. Finally, I retrieve the inflation index from CRSP to deflate all quantities to real values.

A.2 Variance Premium Data

For the variance risk premium, I closely follow Bollerslev et al. (2009), Bollerslev et al. (2011), Drechsler and Yaron (2011) and Drechsler (2013). Under the no-arbitrage assumption, the risk-neutral conditional expectation of the return variance is equal to the price of a variance swap, which is a forward contract on the realized variance of the asset. Since the CBOE calculates the VIX index as a measure of the 30-days ahead risk-neutral expectation of the variance of the S&P 500 index, I use the VIX index as a proxy for the risk-neutral expectation of the market's return variation. The VIX is quoted in an annualized standard deviation. Hence, I first take it to a second power to transform it to variance units and then divide it by 12 to obtain monthly frequency. Thus, I obtain a new series defined as $[\mathrm{VIX}]_t^2 = \frac{\mathrm{VIX}_t^2}{12}$. I further use the last available observation of $[\mathrm{VIX}]_t^2$ in a particular month as a measure of the risk-neutral expectation of return variance in that month.

For the objective expectation of return variance, a second component in the variance premium, I calculate a one-step-ahead forecast from a simple regression similar to Drechsler and Yaron (2011) and Drechsler (2013). I first calculate the measure of the realized variance by summing the squared daily log returns on the S&P 500 futures and S&P 500 index obtained from the CBOE. The constructed series are denoted by FUT_t^2

and IND_t^2, respectively. Subsequently, I estimate the following regression:

$$\text{FUT}_{t+1}^2 = \beta_0 + \beta_1 \cdot \text{IND}_t^2 + \beta_2 \cdot [\text{VIX}]_t^2 + \varepsilon_{t+1}. \tag{A.1}$$

The actual expectation is measured as the one-period ahead forecast given by (A.1). I refer to the resulting series as the realized variance and denote it by RV_t. Theoretically, the variance premium should be non-negative in each period. Thus, I truncate the difference between the implied series of $[\text{VIX}]_t^2$ and RV_t from below by 0.

For the empirical strategy above, I obtain the daily data series of the VIX index, S&P 500 index futures, and the S&P 500 index from the CBOE. The main restriction on the length of the constructed monthly variance premium is the VIX index, reported by the CBOE from January 1990. Using high-frequency data would provide a finer estimation precision of the quantities in the variance premium, but my estimates remain largely consistent with the numbers reported by the existing literature.

A.3 Options Data for the Skew Premium and Implied Volatility Skew

The empirical strategy and key definitions of the skew risk premium are in line with Bakshi et al. (2003) and Kozhan et al. (2013). For the empirical analysis of the skew risk premium and implied volatility surface, I use European options written on the S&P 500 index and traded on the CBOE. The options data set covers the period from January 1996 to December 2016 and is from OptionMetrics. Options data elements include the type of options (call/put) along with the contract's variables (strike price, time to expiration, Greeks, Black-Scholes implied volatilities, closing spot prices of the underlying) and trading statistics (volume, open interest, closing bid and ask quotes), among other details. The empirical estimates of the conditional skew risk premium are computed in line with Kozhan et al. (2013). The empirical strategy consists of calculating fixed and floating legs for the skew swap, which correspond to the risk-neutral and physical expectations of the return skewness. For a detailed description of the methodology, see Kozhan et al. (2013).

To construct the empirical implied volatility curves, I first compute the moneyness for each observed option using the daily S&P 500 index on a particular trading day. I filter out all data entries with non-standard settlements. I use the remaining observations to construct the implied volatility surface for a range of moneyness and maturities. In particular, I follow Christoffersen and Jacobs (2004) and perform polynomial extrapolation of volatilities in the maturity time and strike prices. This strategy makes use of all available options and not only those with a specific maturity time. The fitted values are further used to construct the implied volatility curves.

B Representative Agent's Maximization Problem

A representative agent starts with an initial wealth denoted by W_0. Each period t, the agent consumes C_t consumption goods and invests in N assets traded on the competitive market. Denote the fraction of the total t-period wealth W_t invested in the i-th asset with gross real return $R_{i,t+1}$ by $\omega_{i,t}$. Then, the agent's budget constraint in period t takes the form:

$$W_{t+1} = (W_t - C_t) R^{\omega}_{t+1} \tag{B.1}$$

$$\sum_{i=1}^{N} \omega_{i,t} = 1 \text{ and } R^{\omega}_{t+1} = \sum_{i=1}^{N} \omega_{i,t} R_{i,t+1}. \tag{B.2}$$

The agent chooses $\{C_t, \omega_{1,t}, ..., \omega_{N,t}\}$ in period t to maximize the utility subject to (B.1)-(B.2).

The Bellman equation becomes:

$$J_t = \max_{C_t,\omega_{1,t},\ldots,\omega_{N,t}} \left\{(1-\beta)C_t^{\rho} + \beta\left[\mathcal{R}_t(J_{t+1})\right]^{\rho}\right\}^{1/\rho}$$

subject to (B.1) and (B.2). I guess the optimal value function of the form $J_t = \phi_t W_t$. Using this conjecture of J_t and the form of $\mathcal{R}_t$ from (2), I rewrite the Bellman equation as:

$$\phi_t W_t = \max_{C_t,\omega_{1,t},\ldots,\omega_{N,t}} \left\{(1-\beta)C_t^{\rho} + \beta\left[\mathbb{E}_t\left[(\phi_{t+1}W_{t+1})^{\alpha}\mathcal{K}(\phi_{t+1}W_{t+1})\right]\right]^{\rho/\alpha}\right\}^{1/\rho},$$

$$\mathcal{K}(x) = \frac{1+\theta\mathbb{I}\{x \leqslant \delta\mathcal{R}_t(x)\}}{1+\theta\delta^{\alpha}\mathbb{E}_t\left[\mathbb{I}\{x \leqslant \delta\mathcal{R}_t(x)\}\right]}.$$

Note that the function $\mathcal{K}$ defined above is homogeneous of degree zero.

The Return on the Aggregate Consumption Claim Asset. I further conjecture that the consumption C_t is homogeneous of degree one in wealth at the optimum, that is $C_t = b_t W_t$. Then, I obtain the Bellman equation:

$$\phi_t^{\rho} = \left\{(1-\beta)\left(\frac{C_t}{W_t}\right)^{\rho} + \beta\left(1-\frac{C_t}{W_t}\right)^{\rho}\left[\mathbb{E}_t\left[(\phi_{t+1}R_{t+1}^{\omega})^{\alpha}\mathcal{K}(\phi_{t+1}R_{t+1}^{\omega})\right]\right]^{\rho/\alpha}\right\} \quad \text{(B.3)}$$

or equivalently

$$\phi_t^{\rho} = \left\{(1-\beta)b_t^{\rho} + \beta\left(1-b_t\right)^{\rho} y_t^*\right\} \quad \text{(B.4)}$$

$$y_t^* = \left[\mathbb{E}_t\left[(\phi_{t+1}R_{t+1}^{\omega})^{\alpha}\mathcal{K}(\phi_{t+1}R_{t+1}^{\omega})\right]\right]^{\rho/\alpha}.$$

Taking the FOC of the right side of a simplified Bellman equation (B.3) with respect to C_t, I find:

$$(1-\beta)\left(\frac{C_t}{W_t}\right)^{\rho-1} = \beta\left(1-\frac{C_t}{W_t}\right)^{\rho-1} y_t^*.$$

or using the notations:

$$(1-\beta)b_t^{\rho-1} = \beta(1-b_t)^{\rho-1}y_t^*. \quad \text{(B.5)}$$

Solving for y_t^* from the last equation and substituting it into (B.4), I deduce:

$$\phi_t = (1-\beta)^{\frac{1}{\rho}} b_t^{\frac{\rho-1}{\rho}} = (1-\beta)^{\frac{1}{\rho}} \left(\frac{C_t}{W_t}\right)^{\frac{\rho-1}{\rho}}$$

Shifting one period ahead the formula for ϕ_t and substituting ϕ_{t+1} into (B.5), I obtain:

$$(1-\beta)C_t^{\rho-1} = \beta(W_t - C_t)^{\rho-1} \left[\mathbb{E}_t \left[(1-\beta)^{\alpha/\rho} \left(\frac{C_{t+1}}{W_{t+1}}\right)^{\alpha\frac{\rho-1}{\rho}} \left(R_{t+1}^{\omega}\right)^{\alpha} \mathcal{K}\left(\phi_{t+1} R_{t+1}^{\omega}\right)\right]\right]^{\rho/\alpha}.$$

Then, I rewrite the equation above as:

$$C_t^{\rho-1} = \beta\mathbb{E}_t \left[\left(\frac{C_{t+1}}{\frac{W_{t+1}}{(W_t - C_t)}}\right)^{\alpha\frac{\rho-1}{\rho}} \left(R_{t+1}^{\omega}\right)^{\alpha} \mathcal{K}\left(\left(\frac{C_{t+1}}{\frac{W_{t+1}}{W_t - C_t}}\right)^{\frac{\rho-1}{\rho}} R_{t+1}^{\omega}\right)\right]^{\rho/\alpha}.$$

and derive the asset pricing restriction for the return on the total wealth R_{t+1}^{ω} :

$$\mathbb{E}_t \left[\left\{\underbrace{\left(\beta\left(\frac{C_{t+1}}{C_t}\right)^{\rho-1} R_{t+1}^{\omega}\right)^{1/\rho}}_{z_{t+1}}\right\}^{\alpha} \mathcal{K}\left(\underbrace{\left(\beta\left(\frac{C_{t+1}}{C_t}\right)^{\rho-1} R_{t+1}^{\omega}\right)^{1/\rho}}_{z_{t+1}}\right)\right]^{1/\alpha} = 1.$$

Define R_{t+1}^c the return on the consumption endowment. In equilibrium, $R_{t+1}^c = R_{t+1}^{\omega}$ and, as in Routledge and Zin (2010), using the definition of the certainty equivalent and the function $\mathcal{K}$, the return R_{t+1}^c should satisfy the equation:

$$\mathcal{R}_t(z_{t+1}) = 1, \quad z_{t+1} = \left(\beta\left(\frac{C_{t+1}}{C_t}\right)^{\rho-1} R_{t+1}^c\right)^{1/\rho}. \tag{B.6}$$

Rewriting R_{t+1}^c in the form:

$$R_{t+1}^c = \frac{W_{t+1}}{W_t - C_t} = \frac{\frac{W_{t+1}}{C_{t+1}}}{\frac{W_t}{C_t} - 1} \cdot \frac{C_{t+1}}{C_t} = \frac{\xi_{t+1}}{\xi_t - 1} \cdot \frac{C_{t+1}}{C_t},$$

the wealth-consumption ratio $\xi_t = \frac{W_t}{C_t}$ can be found from the equation:

$$\mathbb{E}_t \left[\beta^{\frac{\alpha}{\rho}} \left(\frac{C_{t+1}}{C_t} \right)^{\alpha} \cdot \left(\frac{\xi_{t+1}}{\xi_t - 1} \right)^{\frac{\alpha}{\rho}} \cdot \mathcal{K}(z_{t+1}) \right] = 1.$$

The Return on the Aggregate Dividend Asset. Following Routledge and Zin (2010), the portfolio problem for the obtained values ϕ_{t+1} reads as follows:

$$\max_{\omega_{1,t},\ldots,\omega_{N,t}} \mathcal{R}_t(\phi_{t+1} R^{\omega}_{t+1}),$$

subject to the constraints $\sum_{i=1}^{N} \omega_{i,t} = 1$ and $R^{\omega}_{t+1} = \sum_{i=1}^{N} \omega_{i,t} R_{i,t+1}$. Taking the FOC with respect to the weight $\omega_{i,t}$, I derive:

$$\mathbb{E}_t \left[\phi^{\alpha}_{t+1} (R^{\omega}_{t+1})^{\alpha-1} [1 + \theta \mathbb{I}(\phi_{t+1} R^{\omega}_{t+1} < \delta \mathcal{R}_t)] R_{i,t+1} \right] = 0.$$

Taking the difference between the i-th and j-th FOCs, I thus obtain:

$$\mathbb{E}_t \left[\phi^{\alpha}_{t+1} (R^{\omega}_{t+1})^{\alpha-1} [1 + \theta \mathbb{I}(\phi_{t+1} R^{\omega}_{t+1} < \delta \mathcal{R}_t)] (R_{i,t+1} - R_{j,t+1}) \right] = 0.$$

Multiplying the last equation by $\omega_{j,t}$ and summing over j, I further obtain:

$$\mathbb{E}_t \left[\phi^{\alpha}_{t+1} (R^{\omega}_{t+1})^{\alpha-1} [1 + \theta \mathbb{I}(\phi_{t+1} R^{\omega}_{t+1} < \delta \mathcal{R}_t)] R_{i,t+1} \underbrace{\sum_{j=1}^{N} \omega_{j,t}}_{=1} \right] =$$

$$= \mathbb{E}_t \left[\phi^{\alpha}_{t+1} (R^{\omega}_{t+1})^{\alpha-1} [1 + \theta \mathbb{I}(\phi_{t+1} R^{\omega}_{t+1} < \delta \mathcal{R}_t)] \underbrace{\sum_{j=1}^{N} R_{j,t+1} \omega_{j,t}}_{=R^{\omega}_{t+1}} \right]$$

$$\mathbb{E}_t\left[\phi_{t+1}^{\alpha}(R_{t+1}^{\omega})^{\alpha-1}[1+\theta\mathbb{I}(\phi_{t+1}R_{t+1}^{\omega}<\delta\mathcal{R}_t)]R_{i,t+1}\right]=$$

$$=\mathbb{E}_t\left[\phi_{t+1}^{\alpha}(R_{t+1}^{\omega})^{\alpha}[1+\theta\mathbb{I}(\phi_{t+1}R_{t+1}^{\omega}<\delta\mathcal{R}_t)]\right]. \tag{B.7}$$

Following Epstein and Zin (1989), it is straightforward to show that $\phi_{t+1}=\frac{z_{t+1}}{R_{t+1}^{\omega}}$ holds in equilibrium. Using these equilibrium conditions and the definition of $\mathcal{R}_t$, I have:

$$\mathbb{E}_t\left[\phi_{t+1}^{\alpha}(R_{t+1}^{\omega})^{\alpha}[1+\theta\mathbb{I}(\phi_{t+1}R_{t+1}^{\omega}<\delta\mathcal{R}_t)]\right]=\mathbb{E}_t\left[z_{t+1}^{\alpha}[1+\theta\mathbb{I}(z_{t+1}<\delta\mathcal{R}_t)]\right]=$$

$$\mathbb{E}_t\left[1+\theta\delta^{\alpha}\mathbb{I}(z_{t+1}<\delta\underbrace{\mathcal{R}_t(z_{t+1})}_{=1})]\right]\underbrace{\mathcal{R}_t(z_{t+1})^{\alpha}}_{=1}=\mathbb{E}_t\left[1+\theta\delta^{\alpha}\mathbb{I}(z_{t+1}<\delta\right]. \tag{B.8}$$

Combining (B.7)-(B.8) and using the equilibrium condition $R_{t+1}^{c}=R_{t+1}^{\omega}$, I finally obtain the asset pricing restriction for the gross return $R_{i,t+1}$:

$$\mathbb{E}_t\left[\frac{z_{t+1}^{\alpha}(R_{t+1}^{c})^{-1}(1+\theta\mathbb{I}(z_{t+1}<\delta)R_{i,t+1}}{1+\theta\delta^{\alpha}\mathbb{E}_t\left[\mathbb{I}(z_{t+1}<\delta)\right]}\right]=1, \tag{B.9}$$

Moreover, the pricing kernel M_{t+1} is:

$$M_{t+1}=\frac{z_{t+1}^{\alpha}(R_{t+1}^{c})^{-1}(1+\theta\mathbb{I}(z_{t+1}<\delta))}{1+\theta\delta^{\alpha}\mathbb{E}\left[\mathbb{I}(z_{t+1}<\delta)\right]}.$$

Rewriting $R_{i,t+1}$ in the form:

$$R_{i,t+1}=\frac{P_{i,t+1}+D_{i,t+1}}{P_{i,t}}=\frac{\frac{P_{i,t+1}}{D_{i,t+1}}+1}{\frac{P_{i,t}}{D_{i,t}}}\cdot\frac{D_{i,t+1}}{D_{i,t}}=\frac{\lambda_{t+1}+1}{\lambda_t}\cdot\frac{D_{i,t+1}}{D_{i,t}},$$

the price-dividend ratio of the i-th asset $\lambda_t=\frac{P_{i,t}}{D_{i,t}}$ can be found from the equation:

$$\mathbb{E}_t\left[\beta^{\frac{\alpha}{\rho}}\left(\frac{C_{t+1}}{C_t}\right)^{\alpha-1}\frac{D_{i,t+1}}{D_{i,t}}\cdot\left(\frac{\xi_{t+1}}{\xi_t-1}\right)^{\frac{\alpha}{\rho}-1}\cdot\mathcal{K}(z_{t+1})\cdot(\lambda_{t+1}+1)\right]=\lambda_t.$$

C Numerical Solution

Following the notation from the paper, aggregate consumption growth is

$$\Delta c_{t+1} = \mu_{s_{t+1}} + \sigma \varepsilon_{t+1}, \quad \varepsilon_{t+1} \sim N(0,1).$$

The consumption volatility σ is constant, whereas the mean growth rate $\mu_{s_{t+1}}$ is driven by a two-state Markov-switching process s_{t+1} with a state space:

$$\mathcal{S} = \{1 = \text{expansion},\ 2 = \text{recession}\},$$

a transition matrix

$$\mathcal{P} = \begin{pmatrix} \pi_{11} & 1-\pi_{11} \\ 1-\pi_{22} & \pi_{22} \end{pmatrix}$$

and transition probabilities $\pi_{ii} \in (0,1),\ i = 1,2$. Let

$$\mathcal{X}(y_1, y_2, y_3) = \frac{1 + \theta \mathbb{I}\left\{\beta e^{\rho y_1}\left(\frac{y_2}{y_3 - 1}\right) \leqslant \delta^{\rho}\right\}}{1 + \theta \delta^{\alpha} \mathbb{E}_t\left[\mathbb{I}\left\{\beta e^{\rho y_1}\left(\frac{y_2}{y_3 - 1}\right) \leqslant \delta^{\rho}\right\}\right]},$$

then, the wealth-consumption ratio $\xi_t = \frac{W_t}{C_t}$ satisfies the equation:

$$\mathbb{E}_t\left[\beta^{\frac{\alpha}{\rho}} e^{\alpha \Delta c_{t+1}} \cdot \left(\frac{\xi_{t+1}}{\xi_t - 1}\right)^{\frac{\alpha}{\rho}} \cdot \mathcal{X}\left(\Delta c_{t+1}, \xi_{t+1}, \xi_t\right)\right] = 1, \tag{C.1}$$

and the price-dividend ratio $\lambda_t = \frac{P_t}{D_t}$ of the asset with a gross return R_{t+1} (I skip the subscript i for convenience) is given by:

$$\mathbb{E}_t\left[\beta^{\frac{\alpha}{\rho}} e^{(\alpha-1)\Delta c_{t+1} + \Delta d_{t+1}} \cdot \left(\frac{\xi_{t+1}}{\xi_t - 1}\right)^{\frac{\alpha}{\rho} - 1} \cdot \mathcal{X}\left(\Delta c_{t+1}, \xi_{t+1}, \xi_t\right) \cdot \frac{\lambda_{t+1} + 1}{\lambda_t}\right] = 1. \tag{C.2}$$

C.1 Projection Method

Following Pohl et al. (2018), I apply a projection method of Judd (1992) to solve for the equilibrium pricing functions defined by (C.1) and (C.2). The model solution consists of two steps. First, I find the wealth-consumption ratio from Equation (C.1). Second, I use the wealth return from the first step and substitute it into (C.2) to find the price-dividend ratio for the equity claim.

The Return on the Aggregate Consumption Claim Asset. I conjecture the wealth-consumption ratio of the form $\xi_t = G(\pi_t)$, in which π_t is the posterior belief. I seek to approximate the functional form of $G(\pi_t)$ by a basis of complete Chebyshev polynomials $\Psi = \{\Psi_k(\pi_t)\}_{k=0}^{n}$ of order n with coefficients $\psi = \{\psi_k\}_{k=0}^{n}$:

$$G(\pi_t) = \sum_{k=0}^{n} \psi_k \Psi_k(\pi_t) \quad \pi_t \in [1-p,\ q]. \tag{C.3}$$

I further define the function:

$$\begin{aligned} \Gamma(\pi_t; j) &= \mathbb{E}_{t,j}\left[\beta^{\frac{\alpha}{\rho}} e^{\alpha \Delta c_{t+1}} \cdot \left(\frac{\xi_{t+1}}{\xi_t - 1}\right)^{\frac{\alpha}{\rho}} \cdot \mathcal{X}\Big(\Delta c_{t+1}, \xi_{t+1}, \xi_t\Big)\right] = \\ &= \beta^{\frac{\alpha}{\rho}} \int e^{\alpha y} \left(\frac{G(B(y,\pi_t))}{G(\pi_t) - 1}\right)^{\frac{\alpha}{\rho}} \cdot \mathcal{X}\Big(y, G(B(y,\pi_t)), G(\pi_t)\Big) f(y,j) dy, \end{aligned} \tag{C.4}$$

$$B(y, \pi_t) = \frac{(1-q) f(y,1)(1-\pi_t) + p f(y,2)\pi_t}{f(y,1)(1-\pi_t) + f(y,2)\pi_t},$$

$f(y, j)$ is the probability density function of a normal distribution $N(\mu_{s_t}, \sigma^2)$ conditional on $s_t = 1, 2$. I further apply the Gauss-Hermite quadrature to calculate expectations in (C.4). Substituting $G(\pi_t)$ from (C.3) and $\Gamma(\pi_t; j)$ from (C.4) into (C.1), I obtain:

$$R^c(\pi_t; \psi) = (1-\pi_t)\Gamma(\pi_t, 1) + \pi_t \Gamma(\pi_t, 2) - 1.$$

The objective is to choose the unknown coefficients ψ to make $R^c(\pi_t; \psi)$ close to zero $\forall \pi_t \in [1-p,\ q]$. I apply the orthogonal collocation method. Formally, I evaluate the residual function in the collocation points $\{r_k\}_{k=1}^{n+1}$ given by the roots of the $n+1$

order Chebyshev polynomial and then solve the system of $n+1$ equations:

$$R^c(r_k;\psi) = 0 \quad k = 1, ..., n+1$$

for $n+1$ unknowns $\psi = \{\psi_k\}_{k=0}^n$. Let $\tilde{\xi}_t = \tilde{G}(\pi_t) = \sum_{k=0}^{n} \tilde{\psi}_k \Psi_k(\pi_t)$ denote an approximation of the wealth-consumption ratio, which will be used in the second step.

The Return on the Aggregate Dividend Asset. I conjecture the price-dividend ratio of the form $\lambda_t = H(\pi_t)$. Now, I seek to approximate the functional form of $H(\pi_t)$, which solves Equation (C.2). I approximate $H(\pi_t)$ by a basis of complete Chebyshev polynomials $\Upsilon = \{\Upsilon_k(\pi_t)\}_{k=0}^n$ of order n with coefficients $v = \{v_k\}_{k=0}^n$:

$$H(\pi_t) = \sum_{k=0}^{n} v_k \Upsilon_k(\pi_t) \quad \pi_t \in [1-p,\ q]. \tag{C.5}$$

I define the function:

$$\begin{aligned} \Lambda(\pi_t; j) &= \mathbb{E}_{t,j}\left[\beta^{\frac{\alpha}{\rho}} e^{(\alpha-1)\Delta c_{t+1} + \Delta d_{t+1}} \left(\frac{\tilde{\xi}_{t+1}}{\tilde{\xi}_t - 1}\right)^{\frac{\alpha}{\rho}-1} \cdot \mathcal{X}\left(\Delta c_{t+1}, \tilde{\xi}_{t+1}, \tilde{\xi}_t\right) \cdot \frac{\lambda_{t+1}+1}{\lambda_t}\right] = \\ &= \beta^{\frac{\alpha}{\rho}} \iint e^{(\alpha+\lambda-1)y + g_d + z} \left(\frac{\tilde{G}(B(y,\pi_t))}{\tilde{G}(\pi_t) - 1}\right)^{\frac{\alpha}{\rho}-1} \cdot \mathcal{X}\left(y, \tilde{G}(B(y,\pi_t), \tilde{G}(\pi_t)\right) \cdot \\ &\quad \cdot \frac{H(B(y,\pi_t))}{H(\pi_t) - 1} f(y,j) g(z,j) dy dz, \end{aligned} \tag{C.6}$$

in which $f(y,j)$ and $g(z,j)$ are probability density functions of normal distributions $N(\mu_{s_{t+1}}, \sigma)$ and $N(g_d, \sigma_d)$, respectively, conditional on $s_{t+1} = 1, 2$. Substituting $H(\pi_t)$ from (C.5) and $\Lambda(\pi_t; j)$ from (C.6) into (C.2), I obtain:

$$R^d(\pi_t; v) = (1-\pi_t)\Lambda(\pi_t, 1) + \pi_t \Lambda(\pi_t, 2) - 1.$$

Again, I apply the orthogonal collocation method. Formally, I evaluate $R^d(\pi_t; \psi)$ in the collocation points $\{s_k\}_{k=1}^{n+1}$ given by the roots of the $n+1$ order Chebyshev polynomial and solve the system of $n+1$ equations

$$R^d(s_k; v) = 0 \quad \forall k = 1, ..., n+1$$

for $n+1$ unknowns $v = \{v_k\}_{k=0}^{n}$.

C.2 Implementation in Matlab

This paper implements a one-dimensional projection method for solving functional equations. I approximate unknown functions using Chebyshev polynomials of the first kind and compute them recursively as:

$$T_0(z) = 1, \quad T_1(z) = z, \quad T_k(z) = 2zT_k(z) - T_{k-1}(z), \quad k = 2, ..., n \ \wedge \ z \in [-1, 1].$$

I adjust the domain of Chebyshev polynomials to the state space of pricing ratios and use modified polynomials in the approximation. Thus, the following equalities hold on the interval $[\pi_{\min}, \pi_{\max}] = [1-p,\ q]$:

$$\Psi_k(\pi_t) = \Upsilon_k(\pi_t) = T_k\left(2\left[\frac{\pi_t - \pi_{\min}}{\pi_{\max} - \pi_{\min}}\right] - 1\right), \quad k = 0, ..., n.$$

I present the results based on the collocation method. For this purpose, I evaluate residual functions in a set of nodes corresponding to $n+1$ zeros of the $(n+1)$-order Chebyshev polynomial, which are formally defined as:

$$z_k = \cos\left(\frac{2k+1}{2n+2}\pi\right), \quad k = 0, ..., n.$$

I adjust the nodes $z_k \in [-1, 1]$ to the domain of the state variable π_t :

$$\pi_k = \pi_{\min} + \frac{\pi_{\max} - \pi_{\min}}{2}(1 + z_k), \quad k = 0, ..., n.$$

The numerical algorithm, which requires solving a system of nonlinear equations, is efficiently programmed in Matlab. I experiment with different nonlinear solvers to achieve better performance of the code. Initially, I used the simple solver "fsolve". Then I found the solution of the system of nonlinear equations through minimizing a constant subject to the system of nonlinear functions. I apply the nonlinear programming solver "fmincon" with the SQP algorithm for this purpose. Similar to Pohl et al. (2018), I found that "fmincon" provides faster running of the code and a more accurate solution compared to "fsolve". Thus, I present all results based on the "fmincon" approach.

Additional numerical details involve the choices of an order of Chebychev polynomials used in the approximation of unknown functions (n), a number of Gauss-Hermite quadrature points used in the numerical integration of expectations in the residual functions (N_{GH}), and a number of draws used in Monte-Carlo simulations to compute model-based European put prices (N_{MC}). I report the results of all models in the main text based on the numerical solution, in which $n = 400$, $N_{GH} = 150$, and $N_{MC} = 4,000,000$. The next section performs a sensitivity analysis of alternative approximation choices.

C.3 Accuracy of Numerical Solution

To better assess the numerical accuracy, I first calculate the root mean squared error (RMSE) in the residual function for the wealth-consumption ratio. I evaluate $R^c(\pi_t; \psi)$ on a dense grid of points $\{\pi_i\}_{i=1}^{N_{\text{RMSE}}}$ that are equally spaced on the interval $[\pi_{\min}, \pi_{\max}]$.

Table A1. Euler errors: GDA, DA, and EZ

The table reports the RMSE for different models. For each specification, it shows the results for two different degrees of Chebyshev polynomials n and two different numbers of Gauss-Hermite quadrature points N_{GH}. The Euler errors are computed using Equation (C.7) with 10,000 points equally spaced on the interval $[\pi_{\min}, \pi_{\max}]$.

Model	$n = 200$ $N_{GH} = 100$	$n = 200$ $N_{GH} = 150$	$n = 400$ $N_{GH} = 100$	$n = 400$ $N_{GH} = 150$
GDA	4.71e-07	4.18e-07	1.83e-07	1.47e-07
GDA_{δ_l}	3.40e-07	2.83e-07	1.25e-07	1.16e-07
GDA_{δ_h}	5.01e-07	4.40e-07	1.83e-07	1.75e-07
GDA_{θ_l}	4.28e-07	4.12e-07	1.61e-07	1.53e-07
GDA_{θ_h}	5.42e-07	4.76e-07	1.86e-07	1.84e-07
DA	1.28e-08	9.48e-09	3.97e-09	3.47e-09
DA_{θ_l}	8.82e-09	7.95e-09	3.16e-09	2.52e-09
DA_{θ_h}	1.30e-08	1.17e-08	4.63e-09	3.53e-09
EZ	7.59e-14	7.37e-14	9.15e-14	9.32e-14
$\text{EZ}_{(1-\alpha)_l}$	5.57e-14	4.95e-14	6.85e-14	6.58e-14
$\text{EZ}_{(1-\alpha)_h}$	9.94e-14	9.86e-14	1.34e-13	1.26e-13

I choose $N_{\text{RMSE}} = 10,000$ of these points. The RMSE is calculated as:

$$\text{RMSE}^c = \sqrt{\frac{1}{N_{\text{RMSE}}} \sum_{k=1}^{N_{\text{RMSE}}} \left[R^c(\pi_k; \psi) \right]^2}, \tag{C.7}$$

$$\pi_k = \pi_{min} + \frac{\pi_{\max} - \pi_{\min}}{N_{\text{RMSE}} - 1}(k-1), \quad k = 1, ..., N_{\text{RMSE}}.$$

I consider four pairs of $(n, N_{GH}) : (200, 100), (200, 150), (400, 100), (400, 150)$. For each pair, I solve different model calibrations of this paper and compute the RMSE.

Table A1 reports the Euler errors implied by various approximation and integration choices. Several observations are noteworthy. First, the numerical solution technique is highly accurate, producing errors consistently below 6e-7 for all cases. Second, the projection method generates smaller RMSE for the models with Epstein-Zin preferences relative to the calibrations with disappointment aversion and generalized disappointment aversion utility functions. This result is expected in light of nonlinearities in the pricing

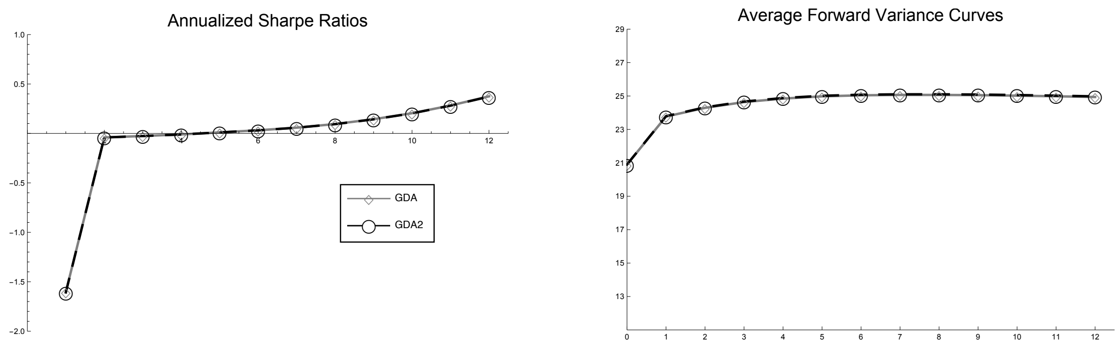

Figure A1. Accuracy of the projection method: Sharpe ratios and forward variance claim prices

The figure plots annualized Sharpe ratios and average prices for variance forwards for the original GDA calibration, which is solved and simulated with different precisions. ”GDA” denotes the results of the original solution. ”GDA2” shows the results of the original calibration, which is solved with a twice larger order of Chebyshev polynomials.

kernel implied by disappointing outcomes in consumption growth. Third, increasing either the degree of Chebyshev polynomials or the number of quadrature points generally leads to a better approximation precision.

Figure A1 conducts further robustness checks. It compares the results of the two solutions of the original GDA calibration. First, the "GDA" lines correspond to the variance term structures as presented in the main text. Second, the "GDA2" curves represent the results of the same calibration, which is solved with a twice larger order of Chebyshev polynomials. The panels in Figure A1 show that the results across the two solutions are very similar, confirming the high-precision solution obtained by the projection method.

D Asset Prices

The empirical evidence concerning the variance term structure and higher moment risk premiums is based on the data at the daily frequency and is then expressed in monthly terms. The risk-neutral neutral expectation of return variance can be synthesized using options data in a model-free way or proxied by synthetic variance swap rates (Britten-Jones and Neuberger, 2000; Bakshi et al., 2003; Carr and Wu, 2009; Dew-Becker et al., 2017). The ex-post total return variance is commonly estimated by a sum of squared daily returns. The ex-ante expectation of total return variance under the physical measure requires using the high-frequency data to compute ex-post return variation and then forecasting the future return variance using lagged realized variance or additional predictors. Kozhan et al. (2013) further extend these approaches for computing the risk-neutral and physical expectations of return skewness.

Turning to the model-based asset prices, one needs to calibrate the model at a daily frequency in order to exactly follow the procedure used to obtain empirical estimates. Bonomo et al. (2015) build a discrete-time model with the daily interval. I want to be as close as possible to the existing long-run risk and rare disaster models in discrete time, particularly Drechsler and Yaron (2011) and Gabaix (2012), which fail to replicate the variance term structure, a key focus of my paper, as shown by Dew-Becker et al. (2017). Therefore, I calibrate my framework at the monthly frequency and present the model-based asset prices in this section.

D.1 Prices and Returns of Variance Claims

Consider an n-month variance swap, a claim to realized variance over months $t+1$ to $t+n$. Given the discrete nature of the model, the total variance of the return is equal to the sum of conditional variances RV_{t+i} in each subperiod. Following Dew-Becker et al. (2017), the price of an n-month variance swap is

$$VS_t^n = \mathbb{E}_t^Q \left[\sum_{i=1}^{n} RV_{t+i} \right].$$

In turn, the price of a zero coupon forward claim on realized variance is

$$F_t^n = \mathbb{E}_t^{\mathbb{Q}}\left[RV_{t+n}\right].$$

Thus, F_t^n is equal to the risk-neutral expectation of return variance during the n-th month from the current period. F_t^0 is naturally defined as the realized variance in the current period. Next, I define the return on the n-month variance forward as a return on the trading strategy in which investors buy the n-month forward at the time t and sell it in the next period as a forward claim with maturity $n-1$. The proceeds from selling the forward are then used to purchase a new n-month variance at price F_{t+1}^n. Formally, the excess return of an n-period variance forward is

$$R_{t+1}^n = \frac{F_{t+1}^{n-1} - F_t^n}{F_t^n}.$$

Using the law of iterated expectations and the Radon-Nikodym derivative defined as $\frac{d\mathbb{Q}}{d\mathbb{P}} = \frac{M_{t+1}}{\mathbb{E}_t(M_{t+1})}$, I recursively compute the prices and returns of variance forwards for different maturities.

D.2 Variance and Skew Risk Premiums

The focus of this paper is on the monthly variance and skew risk premiums associated with equity returns. Since I calibrate the economy at the monthly frequency, the t-time monthly variance premium vp_t is defined as the difference between risk-neutral and physical expectations of the total return variance between t and $t+1$. The monthly decision horizon of a discrete-time model considered in this paper implies that the variance premium simply equals

$$vp_t = var_t^{\mathbb{Q}}(r_{e,t+1}) - var_t^{\mathbb{P}}(r_{e,t+1}), \tag{D.1}$$

in which $var_t^{\mathbb{Q}}(r_{e,t+1})$ and $var_t^{\mathbb{P}}(r_{e,t+1})$ are t-period conditional variances of the log return $r_{e,t+1} = \ln(R_{e,t+1})$ under the risk-neutral $\mathbb{Q}$ and physical $\mathbb{P}$ probability measures, respectively. Drechsler and Yaron (2011) call the definition (D.1) as the level difference. Furthermore, they argue that calibrating the model at a higher frequency would imply

$$vp_t = \mathbb{E}_t^{\mathbb{Q}}\left[\sum_{i=1}^{n-1} var^{\mathbb{Q}}_{t+\frac{i-1}{n}}\left(r_{e,t+\frac{i-1}{n},t+\frac{i}{n}}\right)\right] - \mathbb{E}_t^{\mathbb{P}}\left[\sum_{i=1}^{n-1} var^{\mathbb{P}}_{t+\frac{i-1}{n}}\left(r_{e,t+\frac{i-1}{n},t+\frac{i}{n}}\right)\right], \tag{D.2}$$

in which $var_{t+\frac{i-1}{n}}\left(r_{e,t+\frac{i-1}{n},t+\frac{i}{n}}\right)$ denotes the conditional variance of the market return between $t+\frac{i-1}{n}$ and $t+\frac{i}{n}$. Following Equation (D.2) for calibrations at the higher frequency, Drechsler and Yaron (2011) define the variance premium as

$$vp_t = \mathbb{E}_t^{\mathbb{Q}}(var^{\mathbb{Q}}_{t+1}(r_{e,t+2})) - \mathbb{E}_t^{\mathbb{P}}(var^{\mathbb{P}}_{t+1}(r_{e,t+2})), \tag{D.3}$$

in which vp_t is the sum of the level difference and the drift difference defined as:

$$\text{drift difference} = \left[\mathbb{E}_t^{\mathbb{Q}}(var^{\mathbb{Q}}_{t+1}(r_{e,t+2})) - var^{\mathbb{Q}}_t(r_{e,t+1})\right] - \left[(\mathbb{E}_t^{\mathbb{P}}(var^{\mathbb{P}}_{t+1}(r_{e,t+2})) - var^{\mathbb{P}}_t(r_{e,t+1}))\right]. \tag{D.4}$$

As I compare the predictions of our model with those implied by Drechsler and Yaron (2011), I similarly define the variance premium by Eq. (D.3). However, in the unreported results, I confirm that the main results related to the variance premium are robust to the alternative formulation in Eq. (D.4) because the drift difference strongly dominates vp_t, the finding also reported by Drechsler and Yaron (2011) and Lorenz et al. (2020).

The t-time monthly skew premium is defined as a return on a skew swap, a contract paying the realized skew of the return between time t and $t+1$. Following Kozhan et al.

(2013), I define the skew premium as

$$sk_t = \frac{\mathbb{E}_t^{\mathbb{P}}(skew_{t+1}^{\mathbb{P}}(r_{e,t+2}))}{\mathbb{E}_t^{\mathbb{Q}}(skew_{t+1}^{\mathbb{Q}}(r_{e,t+2}))} - 1,$$

in which $skew_{t+1}^{\mathbb{Q}}(r_{e,t+2})$ and $skew_{t+1}^{\mathbb{P}}(r_{e,t+2})$ are $(t+1)$-period conditional skewness of the log return $r_{e,t+2} = \ln(R_{e,t+2})$ under the risk-neutral $\mathbb{Q}$ and physical $\mathbb{P}$ probability measures, respectively. Note that Kozhan et al. (2013) define the skew premium as the ratio between the risk-neutral and physical expectation of return skewness, unlike the difference between the expectations in the case of the variance premium. This leads to an economic interpretation that is different from the variance premium: the skew premium measures the average return on a skew swap, a synthetic instrument with a price equal to the risk-neutral expectation of return skewness that pays off the realized return skewness. I want to be consistent with Kozhan et al. (2013) and their estimates, therefore, I follow their definition of the skew premium.

D.3 Option Prices and Implied Volatilities

I now describe how I compute model-based option prices and solve for their Black-Scholes implied volatilities. Consider a European put option written on the price of the equity that is traded in the economy. Note that the equity price should not include dividend payments; that is, options are written on the ex-dividend stock price index. Using the Euler condition (4), the relative price $\mathcal{O}_t(\pi_t, \tau, K) = \frac{P_t^o(\pi_t,\tau,K)}{P_t^e(\pi_t)}$ of the τ-period European put option with the strike price K, expressed as a ratio to the initial price of the equity P_t^e, should satisfy

$$\mathcal{O}_t(\pi_t, \tau, K) = \mathbb{E}_t\left[\prod_{k=1}^{\tau} M_{t+k} \cdot \max\left(K - \frac{P_{t+\tau}^e}{P_t^e}, 0\right)\right]. \tag{D.5}$$

Note that a put price P_t^o depends on the equity price P_t^e, whereas the normalized price $\mathcal{O}_t$ does not. One can express the ratio $\frac{P_{t+\tau}^e}{P_t^e}$ in terms of dividend growth rates and price-dividend ratios on the equity and hence the state belief π_t provides sufficient information for the calculation of the option prices. Specifically, I compute model-based European put prices $\mathcal{O}_t = \mathcal{O}_t(\pi_t, \tau, K)$ via Monte Carlo simulations. I convert them into Black-Scholes implied volatilities with a properly annualized continuous interest rate $r_t = r_t(\pi_t)$ and dividend yield $q_t = q_t(\pi_t)$. Thus, given the maturity τ, the strike price K, the risk-free rate r_t, and dividend yield q_t, the implied volatility $\sigma_t = \sigma_t^{\mathrm{BS}}(\pi_t, \tau, K)$ solves the equation:

$$\mathcal{O}_t = e^{-r_t\tau} \cdot K \cdot N(-d_2) - e^{-q_t\tau} \cdot N(-d_1), \tag{D.6}$$

$$d_{1,2} = \left[-\ln(K) + \tau\left(r_t - q_t \pm \sigma_t^2/2\right)\right] / \left[\sigma_t\sqrt{\tau}\right].$$

E Sensitivity Analysis

This appendix presents additional results of alternative calibrations of GDA, DA, and EZ specifications.

E.1 Equity Returns and Moment Risk Premiums

Figure A2 provides sensitivity results for the risk-free rate, the equity premium, the price-dividend ratio, and the moment risk premiums for a broad range of parameter choices in the three models. In particular, I change a key parameter in each of the three preference specifications, while holding the remaining parameters at the values in the original calibration. In the GDA model, I vary the disappointment threshold between 0.915 and 0.945. In the DA model, I change the disappointment aversion parameter between 0.45 and 0.75. In the EZ model, the results are provided for the coefficient of relative risk aversion ranging from 4.5 to 7.5. The panels in Figure A2 present the model-based average statistics implied by the GDA, DA, and EZ frameworks. The asset pricing moments are expressed as a function of a varying parameter, which is indicated on the corresponding axis.

Figure A2 shows that the risk-free rate decreases with the disappointment threshold, disappointment aversion, and relative risk aversion in the GDA, DA, and EZ models, respectively. Further, the equity premium increases and equity prices decline in δ, θ, and $1-\alpha$. Intuitively, when the agent faces more disappointing outcomes or becomes more averse to low consumption growth rates, he demands larger premiums in expected returns for bearing the additional risk in consumption growth. The impact of δ and $1-\alpha$ on the volatility of asset prices is similar across the GDA and EZ models: a higher disappointment threshold or a higher risk aversion leads to a more volatile risk-free rate, while the volatility of equity returns and the price-dividend ratio exhibits a hump-shaped pattern with a maximum approximately in the middle of the parameter intervals considered. In the DA model, raising disappointment aversion slightly increases the volatility of the risk-free rate, equity returns, and prices. Overall, the magnitude of changes in the risk-free rate, equity returns, and the price-dividend ratio is quite comparable across the three models, especially when looking at the GDA and EZ frameworks. These findings suggest that all three preference specifications can reasonably explain the first and second moments of equity returns by adjusting a key preference parameter. In contrast, the four bottom panels in Figure A2 indicate the crucial importance of generalized disappointment aversion for generating significant risk premiums in higher moments of equity returns.

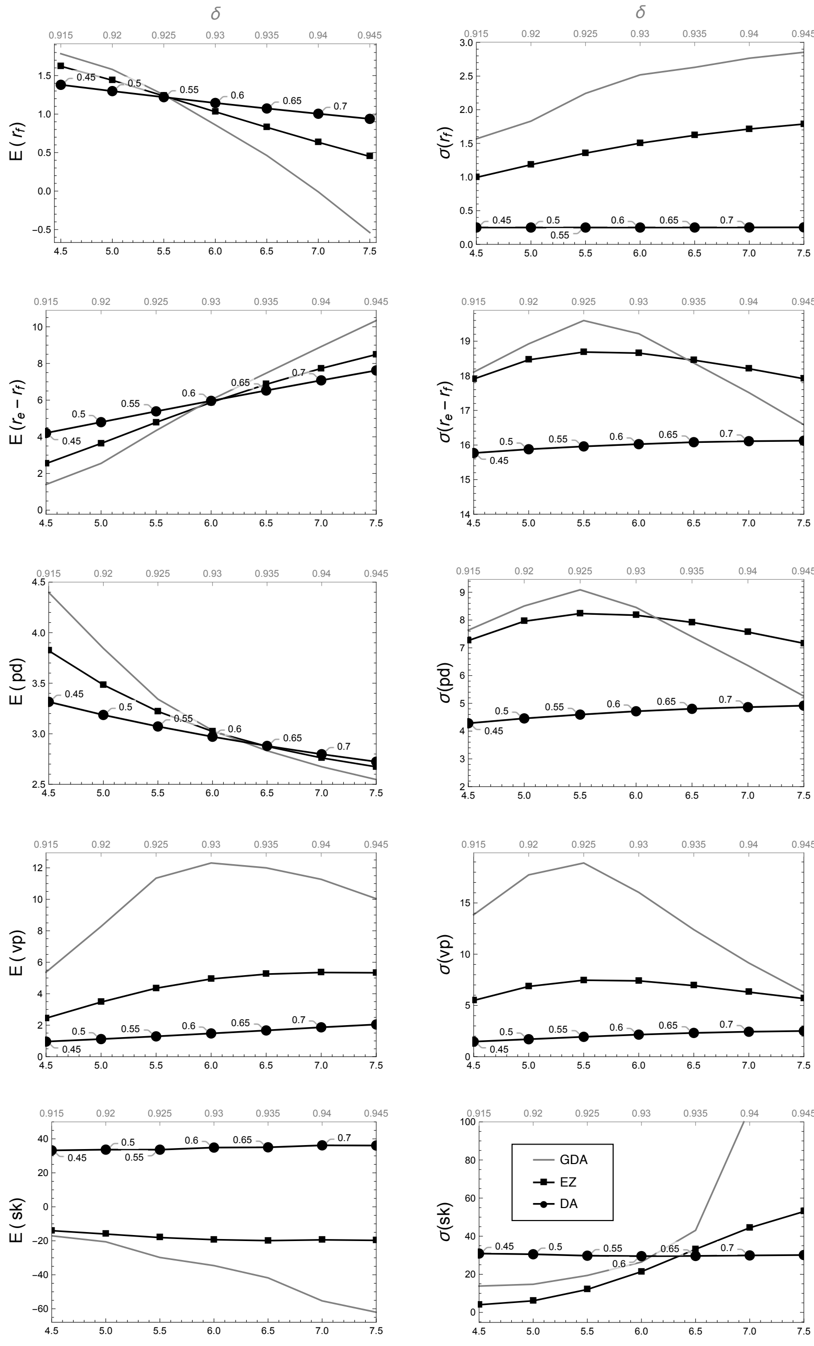

Figure A2. Sensitivity of asset prices: GDA, DA, and EZ

The figure plots asset pricing moments in the GDA, DA, and EZ models, in which a single parameter is changed while others are fixed at the original values. Specifically, I change the disappointment threshold, the disappointment aversion parameter, and the coefficient of risk aversion in the original GDA, DA and EZ models, respectively, over a range of values. For each calibration, I simulate 10,000 economies at a monthly frequency with a sample size equal to its empirical counterpart. The entries of the figure are medians of sample statistics (annualized for the risk-free rate, the equity premium and the price-dividend ratio; monthly for the variance and skew risk premiums). The model-implied results for equity returns (moment risk premiums) are based on the simulations with (without) consumption disasters, consistent with the historical data. I use common notations for mean E and standard deviation σ.

Figure A2 shows that, in the DA setting, changing the disappointment aversion for a wide range of values does not improve the model's performance, as the variance and skew risk premium moments are not very sensitive to changes in θ. Moreover, no value of the disappointment aversion parameter can support the negative skew premium. Figure A2 also shows that Epstein-Zin preferences provide a better fit of the model with the data. In particular, when the risk aversion increases from 4.5 to 6, the average variance premium increases from less than 2 to around 5, while the skew premium declines from around -10% to -20%. However, the mean and volatility of the variance premium actually start to decline at some point, and thus the higher risk aversion will move the model away from the data. Finally, the comparative analysis with respect to the disappointment threshold in the GDA model generates patterns in the variance and skew risk premiums similar to those predicted by different risk aversion parameters in the EZ economy. However, with generalized disappointment aversion, the magnitude and time-variation of variance and skew risk premiums are significantly amplified.

E.2 Implied Volatilities

Figures A3 and A4 further provide comparative statics of the implied volatility curves in the three preference specifications. Several observations are noteworthy. First, in all economies, the implied volatility curve for one-month options is not very sensitive to a further increase in effective risk aversion. In all cases, an incremental increase is less than 1% for any particular maturity and moneyness. Second, in the model with Epstein-Zin preferences, the slope of the ATM and OTM volatilities stays the same for higher risk aversion. In the DA economy, even though ATM volatilities for longer maturities seem to increase more in response to raising disappointment aversion, the levels are

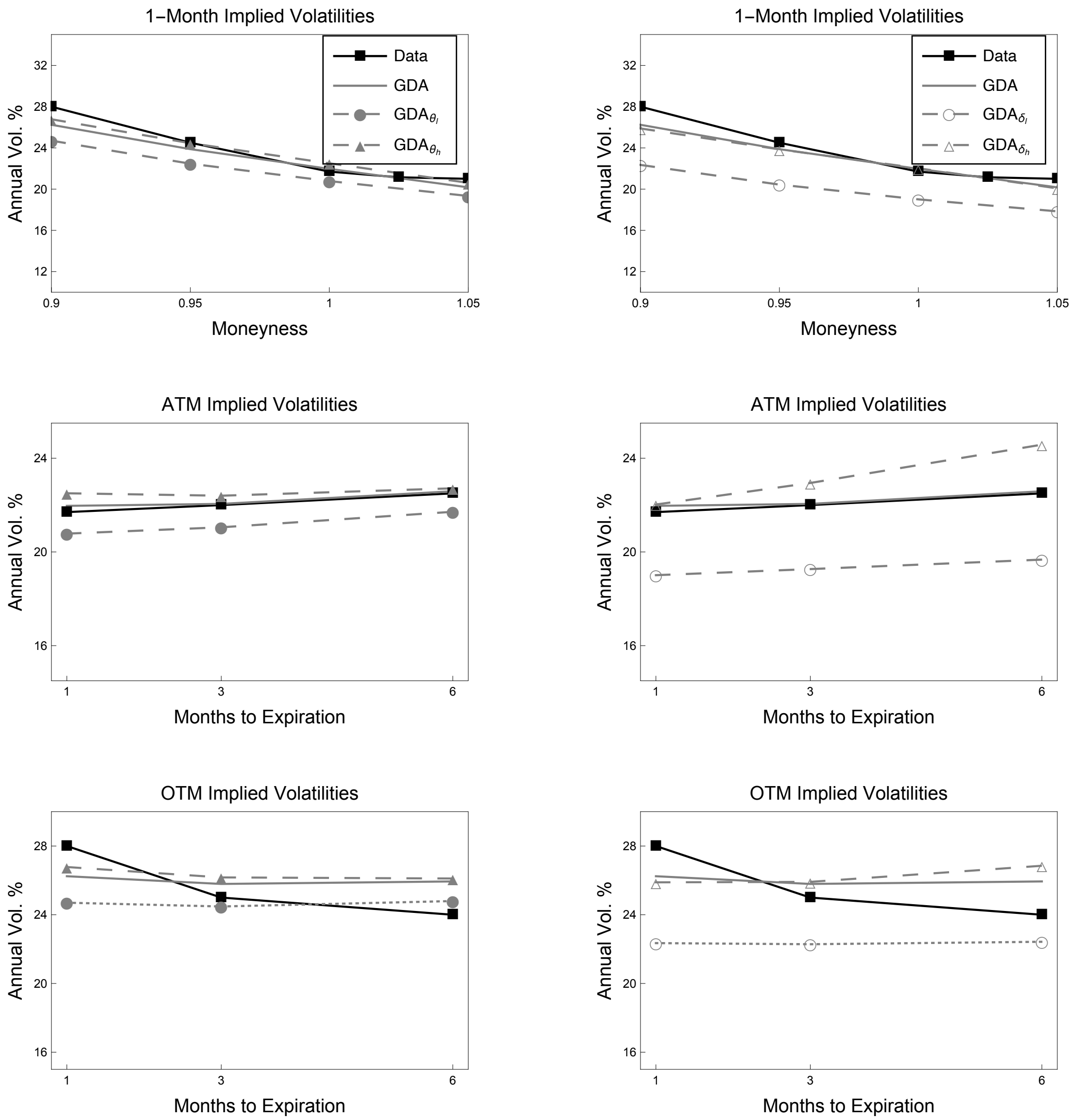

Figure A3. Sensitivity of implied volatilities: GDA

The figure plots the 1-month implied volatility curve (top) as a function of moneyness, and implied volatility curves for ATM (middle) and OTM (bottom) options as functions of the time to maturity (in months) for different model calibrations with generalized disappointment aversion preferences. GDA corresponds to the original GDA model. In GDA$_{\theta_l}$ and GDA$_{\theta_h}$, the disappointment aversion parameters are $\theta_l = 6.41$ and $\theta_h = 10.41$, respectively. In GDA$_{\delta_l}$ and GDA$_{\delta_h}$, the disappointment threshold parameters are $\delta_l = 0.920$ and $\delta_h = 0.940$, respectively. If not stated otherwise, the remaining parameters in all specifications are set at the original values in the GDA model. For each model, I simulate 10,000 economies at a monthly frequency with a sample size equal to its empirical counterpart and report medians of sample statistics based on these series. The model-implied results are based on the simulations without consumption disasters, consistent with the historical data.

significantly below the empirical curves. In the GDA economy, changes in θ and δ have a larger impact on the term structure of ATM and OTM volatilities. Specifically, Figure

A3 suggests that a higher disappointment threshold increases the prices of options with longer maturities, helping to explain a slightly upward-sloping shape in ATM volatilities. Meanwhile, a higher disappointment aversion parameter seems to increase prices of short-term OTM options more than those with longer maturities, helping to explain a slightly downward-sloping pattern in OTM volatilities. Therefore, in the setting of my model, simultaneously increasing θ and decreasing δ could allow one to keep the one-month implied volatilities close to the empirical curves while even better matching the salient statistics of ATM and OTM volatilities. Finally, a lower degree of effective risk aversion implies that the implied volatility curves become flatter and shift down in all models, especially in the economies with GDA and Epstein-Zin preferences.

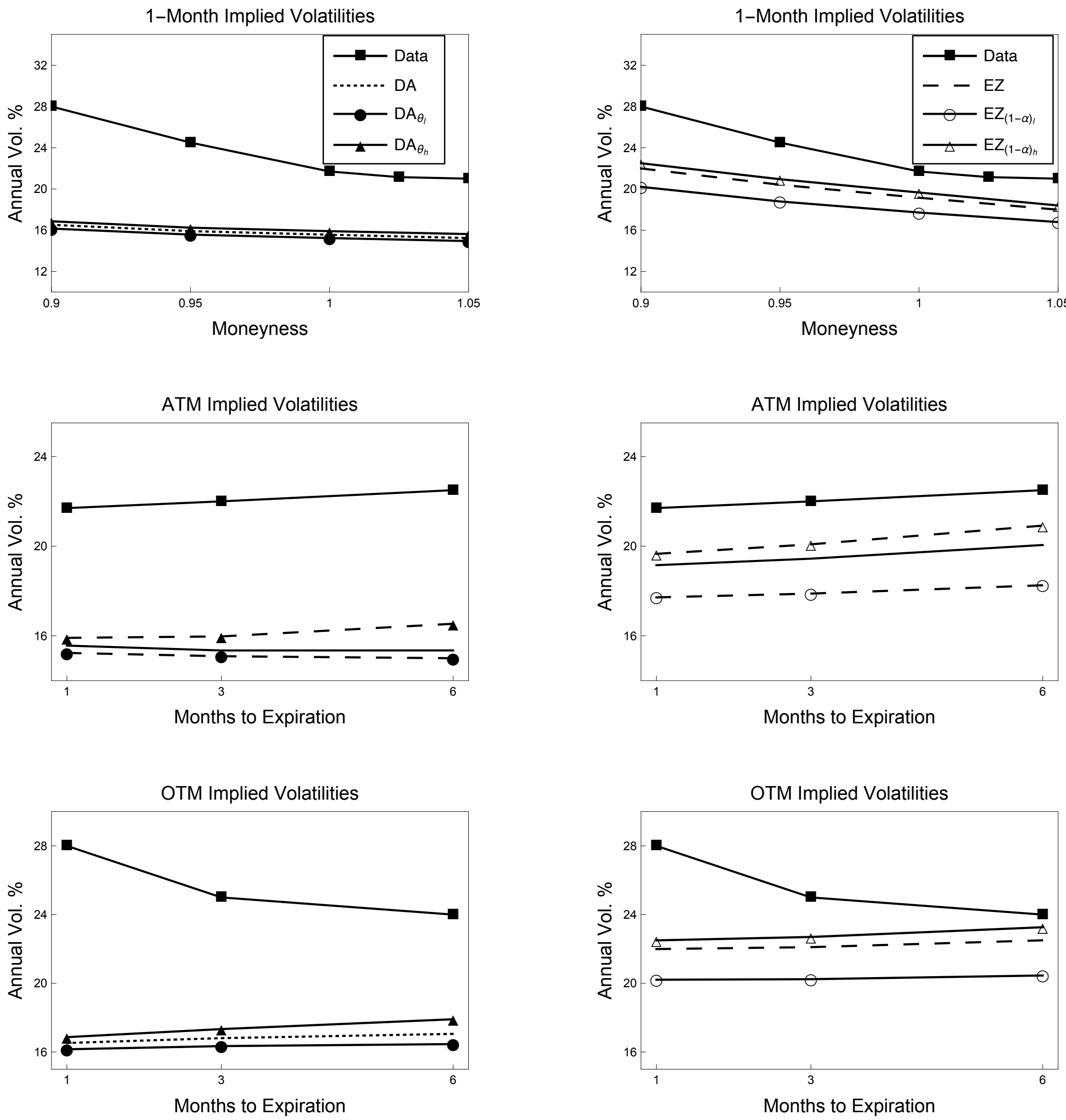

Figure A4. Sensitivity of implied volatilities: DA and EZ

The figure plots the 1-month implied volatility curve (top) as a function of moneyness, implied volatility curves for ATM (middle) and OTM (bottom) options as functions of the time to maturity (in months) for different model calibrations with disappointment aversion and Epstein-Zin preferences. DA and EZ correspond to the original DA and EZ models. In DA_{θ_l} and DA_{θ_h}, the disappointment aversion parameters are $\theta_l = 0.5$ and $\theta_h = 0.7$, respectively. In $EZ_{(1-\alpha)_l}$ and $EZ_{(1-\alpha)_h}$, the risk aversion parameters are $(1-\alpha)_l = 5$ and $(1-\alpha)_h = 7$, respectively. If not stated otherwise, the remaining parameters in all specifications are set at the original values in the DA and EZ models. For each model, I simulate 10,000 economies at a monthly frequency with a sample size equal to its empirical counterpart and report medians of sample statistics based on these series. The model-implied results are based on the simulations without consumption disasters, consistent with the historical data.