Solving Optimal Transmission Switching Problem via DC Power Flow Approximation

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***Abstract*—The objective of the Optimal Transmission Switching (OTS) problem is to identify a topology of the power grid that minimizes the total energy production costs, while satisfying the operational and physical constraints of the power system. The problem is formulated as a non-convex mixed-integer nonlinear program, which poses extraordinary computational challenges. A common approach to solve the OTS problem is to replace its non-convex non-linear constraints with some linear constraints that turn the original problem into a mixed-integer linear programming, named DC OTS. Although there is plenty of work studying solution methods for the DC OTS in the literature, whether and how solutions of the DC OTS are actually useful for the original OTS problem is often overlooked. In this work, we investigate to what extent DC OTS solutions can be used as a fast heuristic to compute feasible solutions for the original OTS problem. Computational experiments on a set of PGLib benchmark instances highlighted that the optimal solution of the DC OTS is rarely feasible for the original OTS problem, which is consistent with the literature. However, we also find that easy-to-implement modifications of the solution procedure help to address this issue. Therefore, we suggest using DC OTS solutions as a complementary option to state-of-the-art heuristics to compute feasible solutions of the original OTS problem.**

***Index Terms*—AC Optimal Transmission Switching, DC Optimal Transmission Switching, Heuristics**

I. Introduction

The increasing cost of energy demands a more economical dispatch of electric power on grids. Therefore, more efficient operations of electric power transmission are of utmost practical importance. The optimal (alternative current) power flow (OPF) problem computes a minimum-cost dispatching of electric power to meet power demand that satisfies the physical laws - such as Kirchhoff's Laws and Ohm's Laws - as well as other operational limits imposed by the grid. From the mathematical modeling standpoint, the OPF problem is a non-convex quadratic constrained quadratic programming problem (first proposed by Carpentier [1]), which is strongly NP-hard [2].

The optimal transmission switching (OTS) problem is the OPF problem augmented with the option of switching off transmission lines, which is modeled by introducing binary variables. The resulting model is a mixed-integer non-convex non-linear program, which poses extraordinary computational challenges. Under some realistic operational assumptions, the OTS problem is often approximated by a simplified mixed integer linear/quadratic program (MILP/MIQP) [3], commonly referred to as Direct-Current (DC) OTS, which is amenable to faster solution approaches. Indeed, the DC OTS can be solved efficiently by taking advantage of modern commercial MILP/MIQP solvers, e.g., Gurobi and CPLEX among others. Due to its favorable computational performance, DC OTS has received substantial interest from the scientific community [3]–[9]. However, the use of DC OTS solutions to retrieve good quality solutions of the original problem is often criticized in the literature. Indeed, DC OTS optimal solutions often lead to infeasible solutions of the original OTS problem [10]. As pointed out in [11], network topologies with positive economic impact in the DC OTS setting can actually be misleading and economically inefficient in the original domain (i.e., the AC power flow). However, in this paper, we show that the DC OTS problem and its solutions can be effectively used to compute feasible solutions of the AC OTS problem. We propose a simple heuristic procedure that compute good quality solutions. These solutions are comparable - in many instances - with solutions computed by the Mixed Integer NonLinear Programming solver Bonmin, which is considered as the state-of-the-art heuristic for the AC OTS problem [12], [13]. It is important to highlight that the proposed procedure does not provide any (theoretical) guarantee on the quality of the computed solution, which is in the nature of any heuristic algorithm, thus motivating the comparison with similar state-of-the-art algorithms (i.e., Bonmin).

The rest of the paper is organized as follows: Section II provides the mathematical programming formulation of the original OTS problem as well as the simplified DC OTS approximation. In Section III, we describe the proposed simple DC OTS-based heuristic. The computational results are reported in Section V. Finally, Section VI draws the conclusion of this work and mentions possible future research directions.

II. Formulations of Optimal Transmission Switching Problem

Nomenclature

δ_{ij} the voltage angle difference of branch (i, j)
θ_i the voltage angle at bus i

G_{ii}/B_{ii} the shunt conductance / susceptance of branch (i,j) at the sending end
G_i/B_i the shunt conductance / susceptance at bus i
p_i^g, q_i^g the active, reactive power injection at bus i
p_{ij}, q_{ij} the active, reactive power flow across branch (i,j)
x_{ij} binary variable representing on/off status of transmission line (i,j)
$\bar{S}_{ij}$ the thermal limit of branch (i,j)
$\overline{P}_i, \underline{P}_i$ the active power production upper and lower limits at generators connected to bus i
$\overline{Q}_i, \underline{Q}_i$ the active power production upper and lower limits at generators connected to bus i
$\overline{V}_i, \underline{V}_i$ the upper and lower bounds of voltage magnitude at bus i
$\underline{\Phi}_{ij}, \overline{\Phi}_{ij}$ the minimum/maximum voltage angle difference of branch (i,j)
B_{ij} the susceptance of branch (i,j)
C_{2i}, C_{1i}, C_{0i} the coefficients for power production costs
D_i^p, D_i^q the active, reactive power demand at bus i
G_{ij} the conductance of branch (i,j)

Consider a power transmission grid $D=(N,E)$, where N denote the set of buses (i.e. nodes in the terminology of graph theory), and E denotes the set of branches linking the buses (high-level abstraction of transmission lines, transformers and so on). Note that branches are asymmetric, so (i,j) and (j,i) are treated separately. Generation units (i.e., electric power generators) are connected to a subset of buses. We assume that there is electric demand, also called load, at every bus. The aim of the OTS problem is to satisfy demand at all buses at the minimum total production costs.

The mathematical programming formulation of the OTS problem is shown in the sequel (1). Note that we use capital letters to represent parameters and lower-case letters to represent variables unless otherwise specified. Moreover, $N(i)$ and $G(i)$ denote the set of buses and the set of generators connected to bus i respectively.

$$\min_{p,p^g,q,q^g,v,\theta,\delta} \quad \sum_{g\in G} C_{2g}\cdot(p_g)^2 + C_{1g}\cdot p_g + C_{0g} \tag{1a}$$

$$\text{s.t.} \quad G_i v_i^2 + \sum_{j\in N(i)} p_{ij} = \sum_{g\in G(i)} p_g - D_i^p,\ \forall i\in N \tag{1b}$$

$$-B_i v_i^2 + \sum_{j\in N(i)} q_{ij} = \sum_{g\in G(i)} q_g - D_i^q,\ \forall i\in N \tag{1c}$$

$$\delta_{ij} = \theta_i - \theta_j,\ \forall (i,j)\in E \tag{1d}$$

$$p_{ij} = x_{ij}\big(G_{ii}v_i^2 + G_{ij}v_iv_j\cos(\delta_{ij}) + B_{ij}v_iv_j\sin(\delta_{ij})\big), \forall (i,j)\in E \tag{1e}$$

$$q_{ij} = x_{ij}\big(-B_{ii}v_i^2 - B_{ij}v_iv_j\cos(\delta_{ij}) + G_{ij}v_iv_j\sin(\delta_{ij})\big),\ \forall (i,j)\in E \tag{1f}$$

$$p_{ij}^2 + q_{ij}^2 \le x_{ij}(\overline{S}_{ij})^2,\ \forall (i,j)\in E \tag{1g}$$

$$\underline{P}_g \le p_g \le \overline{P}_g,\ \forall g\in G \tag{1h}$$

$$\underline{Q}_g \le q_g \le \overline{Q}_g,\ \forall g\in G \tag{1i}$$

$$\underline{V}_i \le v_i \le \overline{V}_i,\ \forall i\in N \tag{1j}$$

$$M_{ij}\underline{\Phi}_{ij}(1-x_{ij}) + \underline{\Phi}_{ij}x_{ij} \le \delta_{ij},\ \forall (i,j)\in E \tag{1k}$$

$$\delta_{ij} \le x_{ij}\overline{\Phi}_{ij} + M_{ij}\overline{\Phi}_{ij}(1-x_{ij}),\ \forall (i,j)\in E \tag{1l}$$

$$x_{ij}\in\{0,1\},\ \forall (i,j)\in E \tag{1m}$$

where M_{ij} is length of the longest path from bus i to bus j in the network D.

Constraints (1b) and (1c) ensure the conservation of active and reactive power flows at each bus, respectively (i.e. whatever electric power flows into the bus must flow out unless consumed). Constraints (1e) and (1f) express how active and reactive power flow across a transmission line is determined by the voltages at its two delimiting busses, respectively. Constraints (1g) to (1k) are operation limits associated with the power system.

The DC power flow approximation is based on the following assumptions on the power system network:

1) the voltage angle difference δ_{ij} between any pair of adjacent buses is small. For typical instances of the Power Grid Library (PGLib) [14], $-\frac{\pi}{6} \le \delta_{ij} \le \frac{\pi}{6}$;
2) the voltage magnitude v_i is close to 1 for all the buses;
3) $B_{ij} \gg G_{ij}$;
4) $G_{ii} \approx -G_{ij}$.

With the aforementioned assumptions, following simplifications on the power flow, we can approximate the active power flow of the transmission line (i,j) as $p_{ij} \approx B_{ij}(\theta_i - \theta_j)x_{ij}$, while ignoring the reactive power flow (q_{ij}) because much smaller than the active power flow (i.e., $p_{ij} \gg q_{ij}$). For more details on DC power flow approximation, we referred the readers to [15].

Note that power transmission loss is neglected in the DC power flow approximation. Therefore, in the DC OTS formulation, we assume symmetry of branches, meaning that branch (i,j) and (j,i) are identical. We denote with E' the set of symmetric (undirected) branches in DC OTS formulation (2).

$$\min_{p^g,p,\theta,x} \quad \sum_{g\in G} C_{2g}\cdot(p_g)^2 + C_{1g}\cdot p_g + C_{0g} \tag{2a}$$

$$\text{s.t.} \quad \sum_{(j,i)\in E'} p_{ji} - \sum_{(i,j)\in E'} p_{ij} = D_i^p - \sum_{g\in G(i)} p_i^g,\ \forall i\in N \tag{2b}$$

$$(1-x_{ij})M_{ij}\underline{\Phi}_{ij}B_{ij} \le p_{ij} - B_{ij}(\theta_i-\theta_j),\ \forall (i,j)\in E' \tag{2c}$$

$$p_{ij} - B_{ij}(\theta_i-\theta_j) \le (1-x_{ij})M_{ij}\overline{\Phi}_{ij}B_{ij},\ \forall (i,j)\in E' \tag{2d}$$

$$-\bar{S}_{ij}\cdot x_{ij} \le p_{ij} \le \bar{S}_{ij}\cdot x_{ij},\ \forall (i,j)\in E' \tag{2e}$$

$$\underline{P}_g \le p_i^g \le \overline{P}_g,\ \forall i\in G \tag{2f}$$

$$x_{ij}\in\{0,1\},\ \forall (i,j)\in E' \tag{2g}$$

III. DC OTS AS A FAST HEURISTIC FOR THE ORIGINAL OTS PROBLEM

The solution approach herein proposed -based on DC OTS solutions - is underpinned by the following observations. First, when solving instances of the DC OTS problem without any limit on the number of branches that can be feasibly switched off, the optimal solution often suggests switching off many branches. This behavior, in addition to being impractical, affects the likelihood of retrieving feasible solutions of the original problem from DC OTS solutions. The second observation is that the optimal solution of DC OTS often overestimates the production cost savings. This results in branches' switching strategies that lead to infeasible OTS problems. Therefore, we suggest using a set K of candidate (sub-optimal) DC OTS solutions, to retrieve the best feasible solution of the original problem. In view of these observations, we first solve the DC OTS problem with a cardinality constraint on the maximum number of switchable branches L (in formula, $\sum_{(i,j)\in E'}(1 - x_{ij}) \leq L$), then for each of the best $|K|$ solutions of the DC OTS problem, we solve an OPF problem on the modified grid (with branches switched off as suggested by the considered DC OTS solution). A diagram of our procedure is depicted in Figure 1.

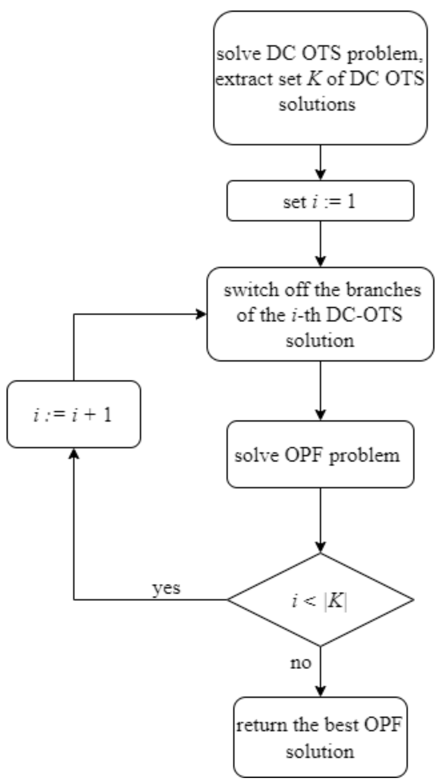

Fig. 1. The proposed solution procedure

IV. COMPUTATIONAL EXPERIMENTS

To assess the viability of the proposed heuristic, we compare and contrast the proposed approach with Bonmin on a selected set of benchmark instances. Indeed, Bonmin is reported to find high-quality solutions of the OTS problem, thus motivating our choice, see for instance [12], [13].

A. Experiments' settings

- Thirty instances extracted from the Power Grid Library (PGLib) [14], representing different features and/or operating conditions of the electric power system, are used. In particular, ten instances represent typical operating conditions. Ten other instances are characterized by binding power demands (denoted by "api"), and the last group of ten instances considers binding limit on the voltage magnitude difference (denoted by "sad").
- The experiments are conducted on a laptop with CPU i7-8750H and 16 GB of RAM. All computer programs are written in Julia programming language (version 1.6.3). The package JuMP (version 1.3.1) [16] is used as a mathematical programming modeling language.
- Gurobi 9.0.1 is used to solve the DC OTS problem. The parameter "Heuristics" of Gurobi is set to its maximum value so that the solver will try as many primal heuristics as possible to increase the quantity and quality of feasible solutions. The stopping MIP optimality gap of Gurobi is set to 0.5%.
- The solver Bonmin (version 1.8.8) is used to compute heuristic solutions of the original OTS problem. We call the solver Bonmin using Julia package "AmplNLWriter.jl" (version 1.0.1). Bonmin has different nested algorithms. We follow the developers' advice and use its branch-and-bound algorithm (setting attribute "algorithm" to "B-BB") to solve the OTS problem. We also set Bonmin attribute "honor_original_bounds" to "yes". All the other attributes remain at their default value.
- The maximum time for solving integer programs (including Gurobi and Bonmin) is set to two hours.
- IPOPT [17] is used as a heuristic for the OPF problem. In this work, we use IPOPT via the Julia package "Ipopt.jl" (version 1.1.0)

V. COMPUTATIONAL RESULTS

Table I shows the number and the ratio - between parenthesis - of instances for which the proposed DC OTS-based heuristic was able to compute a feasible solution of the original OTS problem. More specifically, it displays the results for different combinations of the parameters L and K. It is important to observe that our results are in agreement with the findings in [10] when we do not impose any limit on the parameter L (i.e., $L = \infty$). In fact, in this case, we are able to retrieve feasible solutions to the original OTS problem from DC OTS optimal solutions in only two out of the thirty instances. This behavior is only partially alleviated by considering a larger set K of candidate solutions.

However, when we consider tighter restrictions on the number of switchable branches (i.e., smaller values of L), the DC OTS-based heuristic is drastically more effective. The performance obviously improves by increasing the size of set K. Indeed, the DC OTS-based heuristic is able to compute a feasible solution in 21 instances (combining results with both $L = 5$ and $L = 3$).

TABLE I
NUMBER(RATIO) OF INSTANCES FOR WHICH DC OTS-BASED HEURISTIC FINDS A FEASIBLE SOLUTION OF THE ORIGINAL OTS PROBLEM.

	$L = \infty$	$L = 5$	$L = 3$
$K = 1$	2 (6.7%)	4 (13.4%)	8 (26.8%)
$K = 3$	3 (10%)	11 (36.7%)	14 (46.7%)
$K = 10$	5 (16.7%)	15 (50%)	19 (63.3%)

Table II displays the computational results of the heuristic procedure herein proposed. More specifically, it reports the best result of the procedure with the following parameters' settings: $L = 3, 5, \infty$ and $K = 10$. For all the benchmark instances, listed in the first column, the table displays the following statistics. The second column reports the value of the optimal solution of the AC OPF problem, which is computed with IPOPT solver. Columns 3–7 display the statistics of the proposed heuristic algorithm. More specifically, column 3 reports the value of the heuristic solution while columns 4, 5 and 6 report the computational time of the MIP solver to compute the set K of DC OTS solutions, the time to solve the restricted OPF problem and the total time respectively. The number of DC OTS solutions obtained from the MIP solver (size of the set K) is reported in column 7 (# sol). The third and the second to last columns report the computational performance of Bonmin, in terms of both objective function value and solution time. Finally, the last column reports the percentage difference (GAP) between the best solution computed by the heuristic procedure herein proposed and Bonmin. "INF" means that no feasible solution of the original OTS problem is found. "TL" means that the time limit is reached, i.e., the solution procedure hits the 2-hour time limit of computation. Therefore, the solution process is halted and the best feasible solution found within the time limit is reported.

Overall, the DC OTS-based heuristic is very fast. For many instances, the total computational time is a fraction of a second with very few cases (two) requiring a computational time that is in the order of a couple of dozen of minutes. The DC OTS-based heuristic is considerably faster than Bonmin. In some instances, the speed-up factor is of several orders of magnitude. Indeed, in eight instances of the considered set Bonmin reaches the time limit of 2 hours of computation. However, Bonmin provides better quality solutions, as also highlighted by the GAP statistics.

In terms of solution quality, the DC OTS-based heuristic performs reasonably well on instances representing typical operating conditions. For only one of these instances, the DC OTS-based heuristic fails to retrieve a feasible solution to the original problem due to the infeasibility of the underlying DC OTS problem. On average, the heuristic solutions are close to the one provided by Bonmin, with an average gap of 0.4%. On the set of instances with binding power demand, the DC OTS-based heuristic is not always able to retrieve a feasible solution, even for those instances, e.g., "case89_pagase__api", for which the underlying DC OTS problem is feasible. Overall, the performance of the DC OTS-based heuristic in terms of the quality of solutions computed deteriorates with respect to Bonmin, with an average gap between the two approaches that amount to 4.5%, considering only the instances for which a feasible solution is available for both the methods.

Finally, we observe that the DC OTS-based heuristic fails to compute feasible solutions for half of the instances with binding voltage angle difference limit because the underlying DC OTS is infeasible. The reader should not be surprised by such behavior. In the DC power flow approximation, the voltage magnitude on each bus is set to 1, and the voltage angle difference is the only determinant for transferring power along branches of the grid. Therefore, limiting the voltage angle difference may be too restrictive thus turning the problem into an infeasible one. For completeness, this class of instances is also challenging for Bonmin. Finally, we would like to highlight instances in which the DC OTS-based heuristic outperforms Bonmin. Indeed, the DC OTS-based heuristic computes comparable solutions to Bonmin in "case30_ieee" and "case24_ieee_rts__sad" but in a much shorter computational time. We also observe that for instances "case30_as__api" and "case73_ieee_rts__sad", DC OTS-based heuristic finds significantly better solutions than Bonmin. Indeed, Bonmin is not able to compute a solution of instance "case30_pagase__api" within the imposed time limit, while the heuristic solution is even 44% smaller than the AC OPF solution. This is a significant improvement and, once again, highlights the potential benefit of modifying the grid topology by switching off branches.

All in all, we can recommend considering the DC OTS heuristic as a complementary option to Bonmin, which is a state-of-the-art heuristic for OTS problems, to compute feasible solutions to the original OTS problem, especially for instances representing nominal operating conditions.

VI. Conclusion

In this paper, we study to what extent solutions of the DC OTS problem can be used to retrieve feasible solutions of the original OTS problem, thus motivating the development of a fast DC OTS-based heuristic. We found that the key for DC OTS-based heuristic to compute high-quality feasible solutions of the original OTS problem is to restrict the number of switched-off branches and use sub-optimal solutions of DC OTS problem. With these easy-to-implement modifications, DC OTS can be significantly more effective and together with its unparallel speed, it can be a useful complementary alternative to Bonmin, which can be deemed as a state-of-the-art heuristic for the AC OTS problem.

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TABLE II
HEURISTIC SOLUTIONS FOR THE OTS PROBLEM.

	AC OPF	DC OTS Heuristic					Bonmin		
			TIME						GAP
instance	OF	OF	MIP	NLP	TOT	# sol	OF	time	(%)
case5_pjm	17551.9	15174	0.22	2.2	2.4	5	15174	1.2	0.00
case14_ieee	2178.1	2178.4	0.03	0.2	0.2	1	2178.1	0.9	0.01
case30_as	803.1	806.7	0.03	0.2	0.2	1	803.1	1.8	0.45
case30_ieee	8208.5	7617.4	0.04	0.3	0.3	2	7579	5.9	0.51
case57_ieee	37589.3	37595.1	0.03	0.4	0.4	1	37559.5	26.1	0.09
case60_c	92693.7	INF	0.02	–	0.02	0	92679.9	361.6	–
case73_ieee_rts	188862.3	188980	1.07	0.7	1.8	2	188862.3	17.3	0.06
case89_pegase	110183.2	110183.2	0.15	1	1.1	1	108505.3	TL	1.55
case118_ieee	97213.6	97424.8	0.07	0.8	0.8	1	96590.2	194.9	0.86
case162_ieee_dtc	107412.1	107260.6	4.16	1	5.2	8	103189.6	TL	3.95
case14_ieee__api	5999.4	INF	0.04	0.2	0.3	2	5999.4	0.6	–
case24_ieee_rts__api	134508.4	124204.5	0.26	0.4	0.6	6	119306.1	15.9	4.11
case30_ieee__api	18043.9	18043.9	0.04	0.3	0.3	2	17936.5	13.2	0.60
case30_as__api	4996.2	2797.7	0.08	0.2	0.3	7	INF	0.6	–
case57_ieee__api	49290.4	49294.8	0.03	0.4	0.4	1	49273.8	59.6	0.04
case60_c__api	185239	INF	0	–	0	0	181863.7	69.7	–
case73_ieee_rts__api	421209.3	408012	78.4	0.9	79.3	10	384113	618.2	6.22
case89_pegase__api	143620.3	INF	0.64	1.9	2.5	7	102618.37	2356.4	–
case118_ieee__api	242236.8	198949.6	2337.46	0.9	2338.4	10	183736.5	TL	8.28
case162_ieee_dtc__api	120238.4	120234.4	0.76	1.2	2	5	116119.5	1010	3.54
case14_ieee__sad	2776.8	INF	0.02	–	0.02	0	2727.5	6.8	–
case24_ieee_rts__sad	76365.9	75288.4	0.39	0.3	0.7	10	75108.6	195.2	0.24
case30_ieee__sad	8208.5	INF	0.03	–	0.03	0	8265	12.9	–
case30_as__sad	897.4	INF	0.03	–	0.03	0	893.9	12.2	–
case57_ieee__sad	38663.3	38725.3	0.1	0.5	0.6	2	38597.9	95.3	0.33
case60_c__sad	113498.8	INF	0.02	–	0.6	0	113824.6	TL	–
case73_ieee_rts__sad	226432.1	220991.6	45.45	0.7	46.2	10	226458.8	TL	-2.41
case89_pegase__sad	110183.2	INF	0	–	0	0	108735.4	TL	–
case118_ieee__sad	105155	99563.7	329.06	0.8	329.8	7	96993.8	3331.9	2.65
case162_ieee_dtc__sad	108013.2	107666	2492.58	1.1	2493.6	5	103203.6	TL	4.32

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