

SUPPLEMENTARY MATERIAL

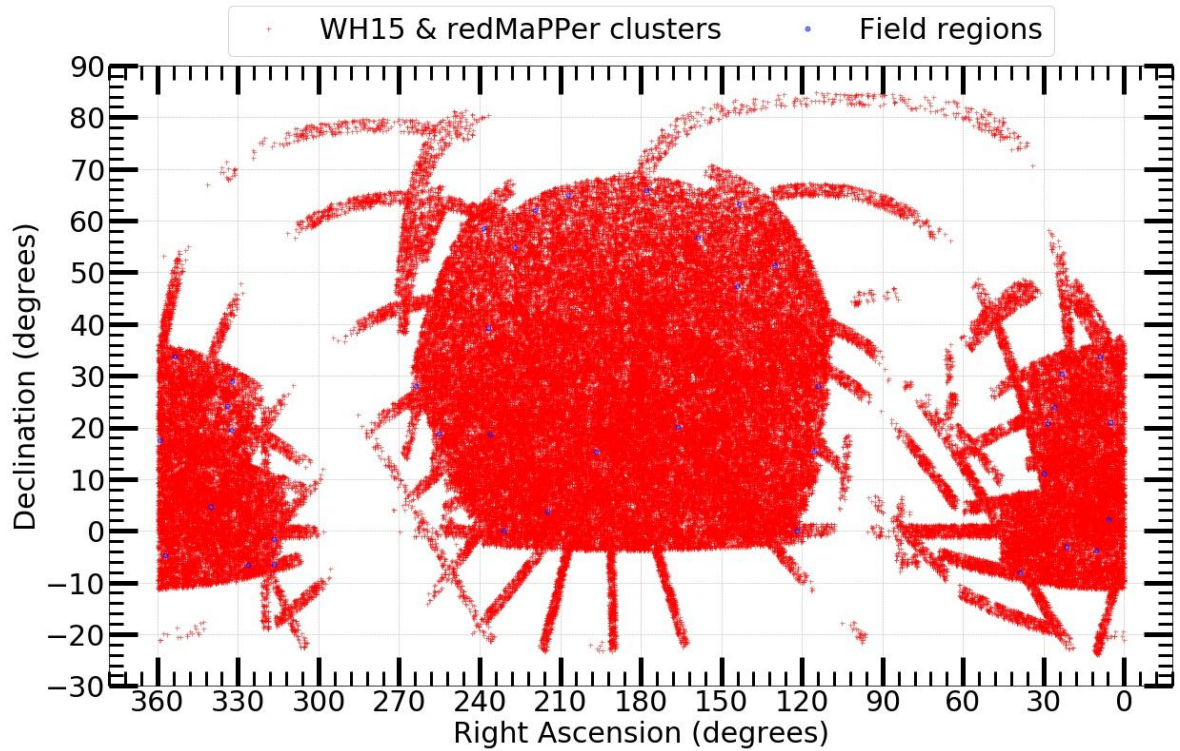


Figure S1. This figure displays a sky map of the astronomical coordinates (J2000) for the WH15 and redMaPPer clusters (red cross) as well as the astronomical coordinates of our forty different proposed ‘field’ regions (blue circle).

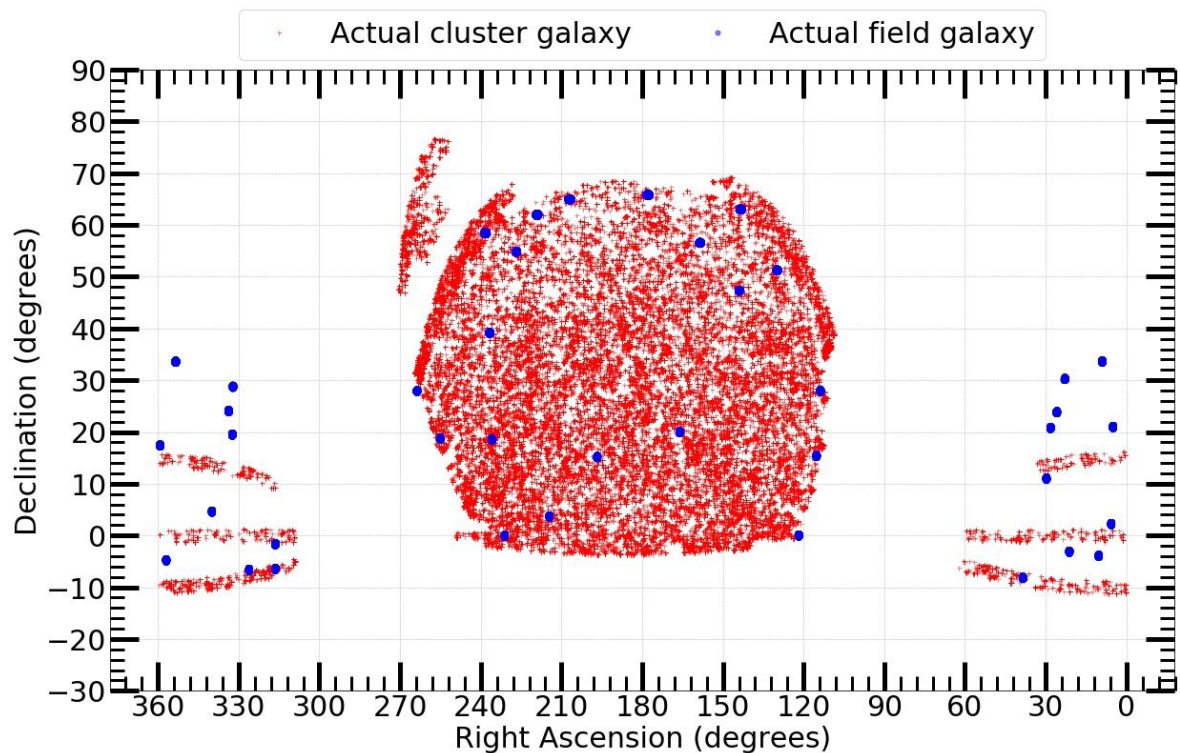


Figure S2. This figure displays a sky map of the astronomical coordinates (J2000) for the cluster (red cross) and field (blue circle) galaxies that had been cross-matched with galaxies observed within SDSS-IV DR16.

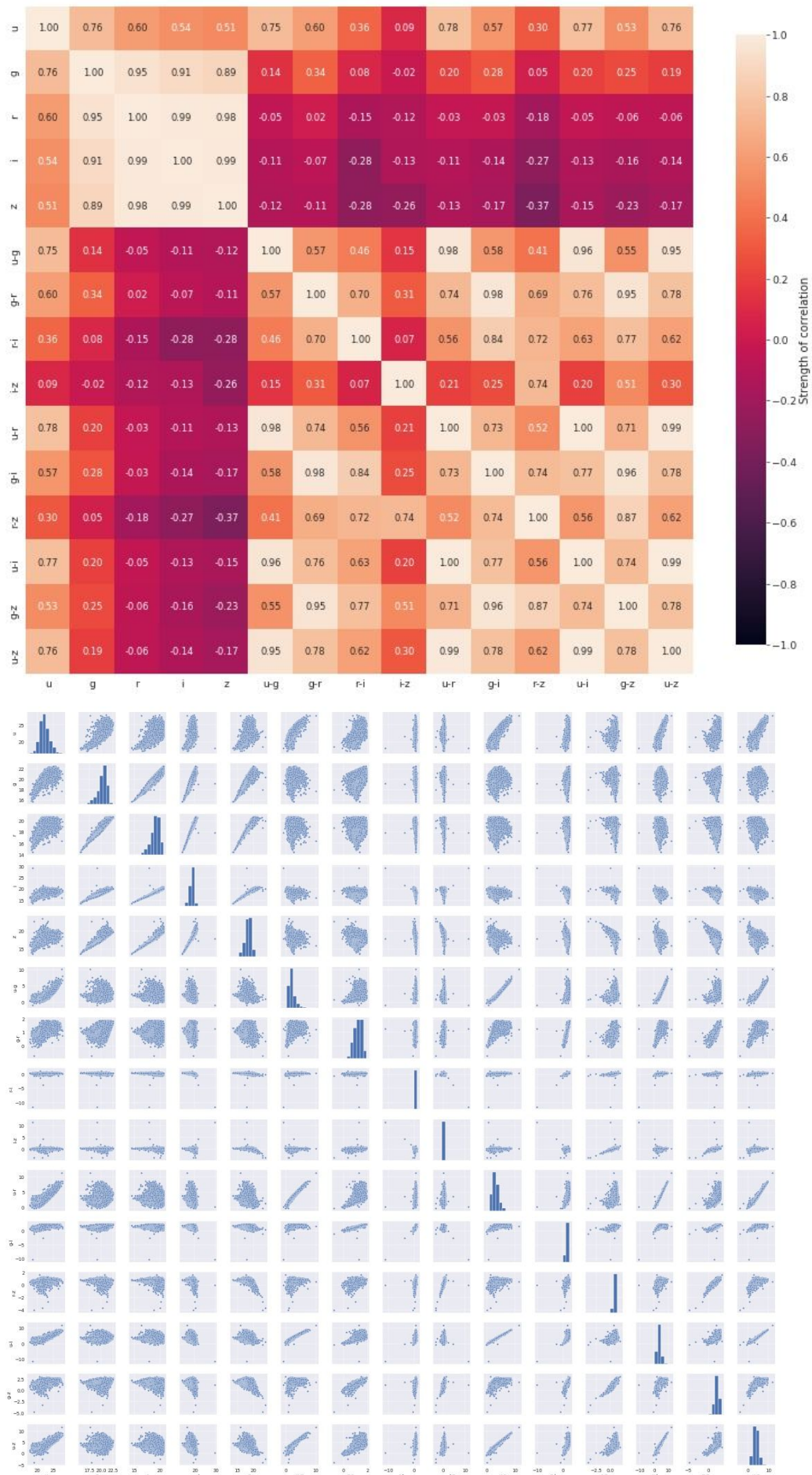


Figure S3. This figure displays a correlation matrix heatmap (top image) and scatterplots (bottom image) of features (i.e. filters and colours) from optical photometry data of galaxies in our cluster galaxy sample. The colourbar for the correlation matrix heatmap represents the strength and direction of the linear correlation between features.

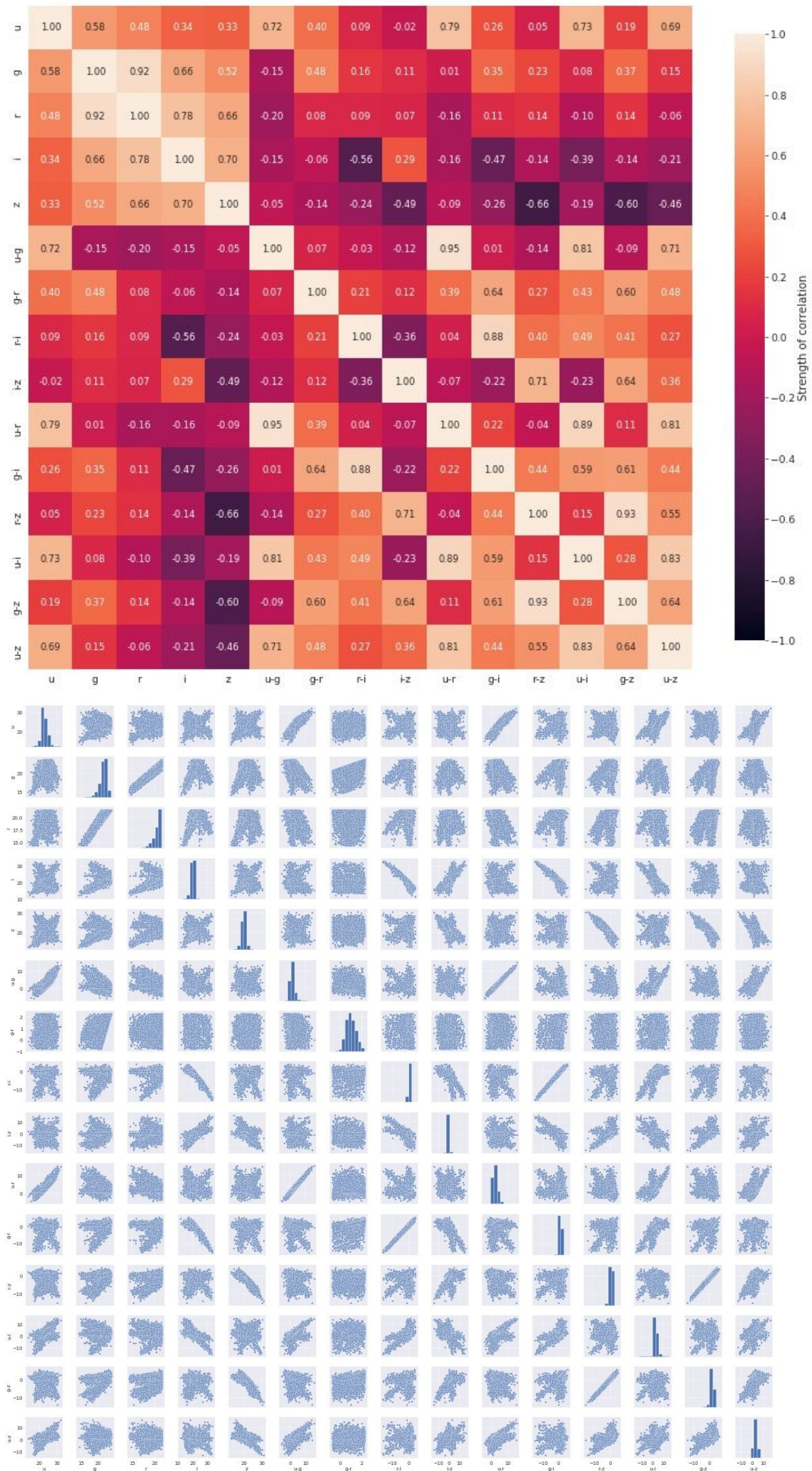


Figure S4. This figure is the same as Figure S3 except it is showing galaxies in our field galaxy sample.

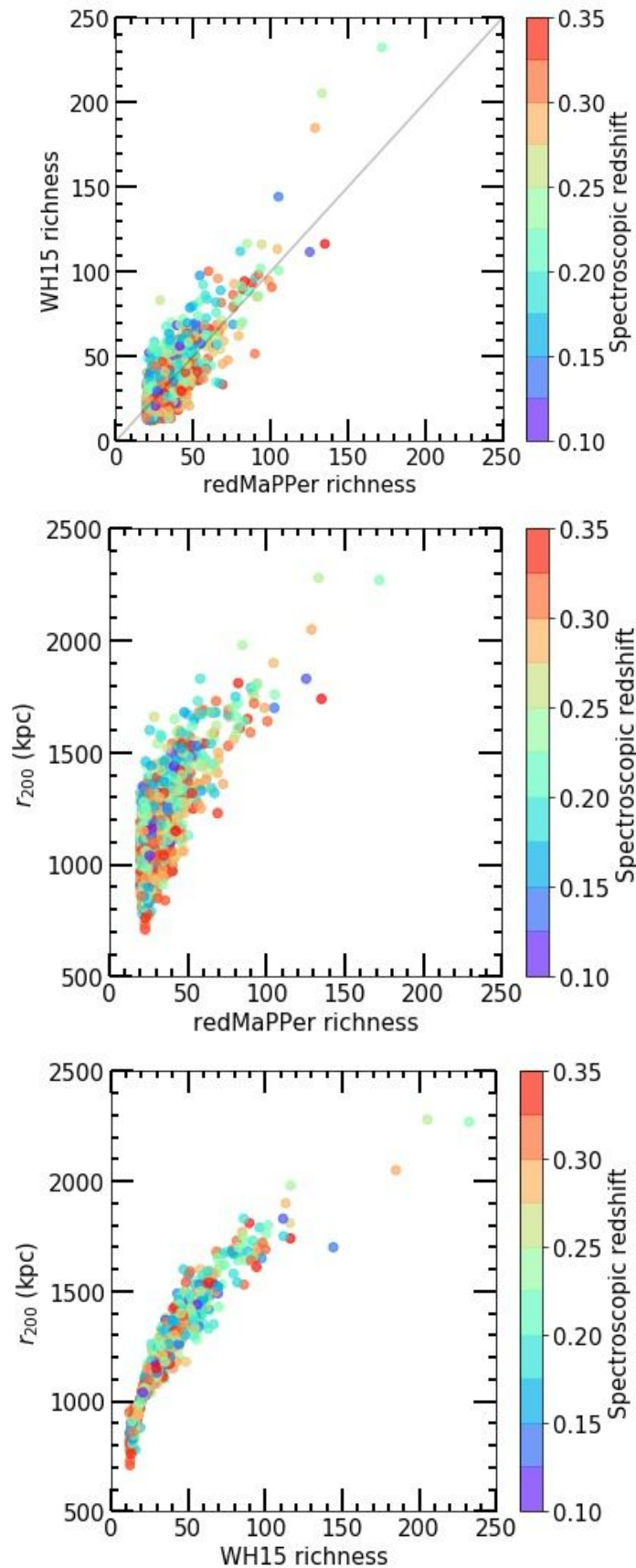


Figure S5. Top image: this figure displays a direct comparison between redMaPPer and WH15 richness for a subsample of cross-matched WH15 and redMaPPer clusters between a redshift range of $0.1 \leq z \leq 0.35$. Middle image: this figure displays a direct comparison between redMaPPer richness and r_{200} values determined by WH15 for a subsample of cross-matched WH15 and redMaPPer clusters between a redshift range of $0.1 \leq z \leq 0.35$. Bottom image: this figure displays a direct comparison between WH15 richness and r_{200} values determined by WH15 for a subsample of cross-matched WH15 and redMaPPer clusters between a redshift range of $0.1 \leq z \leq 0.35$.

Random search iteration	Batch size	Learning rate	Optimiser algorithm	Architecture layout	AUCPR (per cent) [mean]	AUCPR (per cent) [standard deviation]
1	512	0.0001	'RMSprop'	5	39.87	1.52
2	2048	0.01	'Adadelata'	1	34.03	2.46
3	2048	0.001	'Adam'	5	38.93	1.27
4	1024	0.1	'Adadelata'	5	38.94	2.14
5	512	0.001	'SGD'	5	38.41	3.71
6	2048	0.0001	'Adagrad'	3	33.68	1.22
7	256	0.001	'Adam'	1	39.95	1.41
8	2048	0.0001	'Adadelata'	1	34.02	0.35
9	256	0.1	'Adadelata'	1	37.74	2.95
10	256	0.0001	'Adadelata'	1	34.15	0.37
11	1024	0.001	'Adadelata'	1	35.29	1.97
12	1024	0.0001	'Nadam'	1	39.28	2.05
13	512	0.0001	'RMSprop'	1	38.93	2.38
14	256	0.1	'Adadelata'	5	39.18	1.30
15	1024	0.0001	'Nadam'	5	39.23	2.36
16	256	0.01	'Adagrad'	1	39.17	2.47
17	512	0.0001	'Nadam'	3	38.65	1.63
18	512	0.1	'Nadam'	1	34.08	0.34
19	1024	0.1	'Adam'	3	35.82	3.34
20	2048	0.1	'SGD'	5	38.48	1.51
21	1024	0.01	'Adagrad'	3	39.00	3.01
22	256	0.1	'Nadam'	5	34.08	0.34
23	1024	0.1	'Adamax'	5	37.88	3.12
24	1024	0.0001	'Adadelata'	3	33.74	0.98
25	2048	0.001	'RMSprop'	1	39.50	2.07
26	1024	0.01	'Nadam'	1	38.99	2.12
27	256	0.01	'Adadelata'	3	38.89	1.69
28	512	0.001	'Nadam'	3	39.51	1.43
29	512	0.01	'Adagrad'	1	38.52	3.34
30	256	0.0001	'Adamax'	3	38.05	1.25
31	512	0.001	'SGD'	3	38.20	1.90
32	1024	0.1	'Adagrad'	5	37.83	2.89
33	512	0.01	'Adamax'	1	38.51	2.72
34	1024	0.01	'Adam'	3	37.15	2.08
35	256	0.001	'Adagrad'	1	36.44	2.80
36	1024	0.1	'Adadelata'	3	38.08	2.24
37	2048	0.001	'Adadelata'	1	33.83	2.25
38	256	0.1	'Adam'	1	34.08	0.34
39	1024	0.0001	'Adagrad'	5	34.09	1.48
40	512	0.01	'Adadelata'	1	37.93	2.83
41	1024	0.0001	'Adamax'	5	40.03	1.24
42	2048	0.0001	'Adam'	5	39.26	2.00
43	256	0.01	'Nadam'	5	39.15	2.27
44	2048	0.0001	'RMSprop'	3	40.24	1.85
45	1024	0.1	'Nadam'	5	34.08	0.34
46	512	0.0001	'Adam'	5	37.64	2.09
47	2048	0.001	'Nadam'	1	39.38	2.45
48	1024	0.1	'Nadam'	3	34.08	0.34
49	512	0.1	'Adadelata'	3	39.29	1.38
50	1024	0.0001	'RMSprop'	1	39.36	2.19
51	2048	0.0001	'RMSprop'	1	38.92	2.20
52	256	0.0001	'Adagrad'	1	34.73	1.61
53	256	0.001	'SGD'	1	37.09	2.16
54	1024	0.01	'RMSprop'	1	37.56	3.23
55	512	0.001	'Adamax'	5	38.17	2.19
56	1024	0.0001	'SGD'	3	33.55	2.83
57	512	0.01	'SGD'	5	38.79	2.05
58	1024	0.001	'SGD'	1	34.61	3.00
59	1024	0.01	'Adadelata'	5	37.68	1.74
60	256	0.01	'RMSprop'	5	38.93	1.80

Table S1. This table displays the randomly selected hyper-parameter (i.e. batch size, learning rate, optimiser algorithm and architecture layout) combinations for tuning our background subtraction model on galaxies in our validation set using sixty iterations of random search. We also display the mean and standard deviation of the resultant AUCPR from performing ten iterations of Monte Carlo cross-validation on each random search iteration.

Class probability threshold	F1 score (per cent)
0	40.00
0.01	42.43
0.02	42.66
0.03	42.81
0.04	42.96
0.05	43.07
0.06	43.26
0.07	43.38
0.08	43.49
0.09	43.59
0.1	43.67
0.11	43.76
0.12	43.87
0.13	44.03
0.14	44.19
0.15	44.36
0.16	44.54
0.17	44.77
0.18	44.97
0.19	45.15
0.2	45.32
0.21	45.62
0.22	45.99
0.23	46.32
0.24	46.83
0.25	47.47
0.26	47.96
0.27	48.31
0.28	48.69
0.29	48.92
0.3	48.69
0.31	45.18
0.32	19.17
0.33	0.00
0.34	0.00
0.35	0.00
0.36	0.00
0.37	0.00
0.38	0.00
0.39	0.00
0.4	0.00
0.41	0.00
0.42	0.00
0.43	0.00
0.44	0.00
0.45	0.00
0.46	0.00
0.47	0.00
0.48	0.00
0.49	0.00
0.5	0.00
0.51	0.00
0.52	0.00
0.53	0.00
0.54	0.00
0.55	0.00
0.56	0.00
0.57	0.00
0.58	0.00
0.59	0.00
0.6	0.00
0.61	0.00
0.62	0.00
0.63	0.00
0.64	0.00
0.65	0.00
0.66	0.00
0.67	0.00
0.68	0.00
0.69	0.00
0.7	0.00
0.71	0.00
0.72	0.00
0.73	0.00
0.74	0.00
0.75	0.00
0.76	0.00
0.77	0.00
0.78	0.00
0.79	0.00
0.8	0.00
0.81	0.00
0.82	0.00
0.83	0.00
0.84	0.00
0.85	0.00
0.86	0.00
0.87	0.00
0.88	0.00
0.89	0.00
0.9	0.00
0.91	0.00
0.92	0.00
0.93	0.00
0.94	0.00
0.95	0.00
0.96	0.00
0.97	0.00
0.98	0.00
0.99	0.00
1	0.00

Table S2. This table displays the resultant F1 scores on galaxies in our validation set when using different class probability thresholds with the optimal hyper-parameter combination for our background subtraction model. It should be noted that an F1 score of 0 signifies that no cluster galaxies were identified at the given class probability threshold.

Bin size	Chi-square fitting error	Number of bins with galaxies	M^*	M^* [standard deviation]	n^*	n^* [standard deviation]	α	α [standard deviation]
0.01	102.140920976604	147	-25.3072697006017	7.28783936301292	0.164112608931114	1.38199927866705	-2	0.466629082092991
0.02	76.5453069917579	87	-22.8960081024292	1.09379460105731	4.71921420366664	9.20338546553196	-2	0.580632607165434
0.03	68.1745019597015	65	-22.4908977078703	0.790075482406013	12.70439476778	18.952694149263	-1.99999999999989	0.599104919143905
0.04	61.266861269483	50	-22.415789304908	0.756339770593328	19.3568690083877	27.8274793059772	-1.99999999999979	0.608973124211291
0.05	39.6185650543927	43	-22.6222052707545	0.672513724488882	18.3176186296316	23.2838977629391	-1.99999999999999	0.486524958545581
0.06	36.5625019331505	37	-22.6258973735442	0.626947096373978	21.9653980090031	26.0819814894214	-1.99999999999946	0.458571758087398
0.07	33.5905001761644	33	-22.6099206948986	0.602242351450481	26.3737283301809	30.2193978963726	-1.99999999999887	0.451033803392379
0.08	29.6385878283548	28	-22.5816910642751	0.591756403834509	31.7971766662677	35.8707702408829	-1.99999999999999	0.452169800589196
0.09	20.3637951180156	27	-22.6746874027806	0.521101827728886	31.5718547496863	31.5686519817016	-1.99999999999869	0.393056307595776
0.1	16.5530148685729	23	-22.6582443109673	0.569061669900563	36.3423806202571	39.4465302657616	-1.99999999999955	0.421022202247245
0.11	13.0767631976711	21	-22.6755620489245	0.55251309291653	39.2543259599577	41.4152576582025	-1.99999999999977	0.408316979552766
0.12	15.2459769970052	19	-22.7247184161576	0.591396789212753	39.7619946638302	44.3787960605664	-2	0.411200351509468
0.13	9.14827055635872	19	-22.6183758110969	0.458502287321535	52.1384807705458	45.3748880546628	-1.95453486711378	0.371591371552821
0.14	11.7492730266906	18	-22.643331608495	0.462615820139636	52.7951370756613	47.0475807096035	-1.98924101376699	0.367764744434002
0.15	10.9037429665667	16	-22.4923685847581	0.448002369078026	74.7956455459639	61.8368875874148	-1.88377867043106	0.397289759070223
0.16	16.772783477289	15	-22.5384337422092	0.475641293653476	70.5894407833445	64.5502898072399	-1.97587961393189	0.40429719863603
0.17	4.15402355091424	15	-22.7292157905515	0.466124335935932	56.9967978748963	51.1443097694934	-2	0.351042953646424
0.18	12.7790690539623	14	-22.6591309218703	0.490607883863879	65.8069565315428	62.2084888857685	-1.99999999999984	0.381498515390209
0.19	10.1521887967407	13	-22.7126392343591	0.531049352235773	64.521848195881	65.4523309529134	-2	0.389521040297862
0.2	7.99147466406507	12	-22.7208482829558	0.537766203637089	67.4775596294566	69.1927939236387	-2	0.390698636062222
0.21	4.29404454893266	12	-22.778047368944	0.507161177536059	65.6184421027745	63.563335472533	-2	0.363575936524285
0.22	5.67230887264036	11	-22.7623890236569	0.525989016830309	70.1952555544012	70.3336011457905	-2	0.374685241450145
0.23	3.92256129130857	11	-22.705216487009	0.489026439161547	80.1076053301875	75.3087082850019	-2	0.369199150776928
0.24	8.17416297511997	10	-22.5708243936802	0.46728589789064	104.425526335234	92.3190354948186	-1.94542652350871	0.390175180596096
0.25	4.29832445617372	10	-22.7749629261976	0.504034834908083	78.5599805972613	75.5810214476751	-1.99999999999975	0.360888114340128
0.26	1.5900013241842	10	-22.7543979409332	0.456121860305321	84.6673328318602	74.3374090694452	-2	0.340403198275327
0.27	4.97043601393654	9	-22.7326239407067	0.537509637953644	90.2958645221638	92.7197761899969	-2	0.390545267175811
0.28	4.10694483459387	9	-22.7361583932848	0.475351264466895	94.4841506940494	85.5761639766845	-1.97909263203428	0.356655435560158
0.29	2.43440108474398	9	-22.7735764454752	0.483748588363934	91.7522170908038	85.109797159416	-2	0.353263598483103
0.3	5.1520525754007	8	-22.7918350993829	0.571519770757192	92.7936083899066	99.5684604796216	-2	0.393296494943888
0.31	5.90218198017966	8	-22.7496055699693	0.512441830655374	101.027295842528	99.0607465472664	-1.99999999999949	0.374980540977817
0.32	10.104409330581	8	-22.7068155761029	0.46673483659344	110.104060272333	99.3073073368942	-2	0.361982802750744
0.33	5.96846341699309	8	-22.7391063048754	0.46925404584864	109.201691285243	98.8710176390871	-2	0.356791209942564
0.34	1.12105000461229	8	-22.7478245333539	0.440292188347238	112.250370963957	95.4874446889144	-2	0.336758675291583
0.35	3.12523238008784	7	-22.7822285687626	0.546318661436639	109.623965612747	113.882723095951	-1.99999999999999	0.387253819700969
0.36	3.03061687353762	7	-22.7241788039726	0.52765200413997	122.841382114879	123.931590655044	-2	0.390054298402819
0.37	1.48984677624025	7	-22.7309176072148	0.46076140879309	125.422731293451	111.639466908686	-2	0.354493289723339
0.38	2.36115567803497	7	-22.758776999429	0.469319697928797	123.406631037008	111.329064686043	-2	0.350817563549449
0.39	0.603516773400222	7	-22.6964112181343	0.413245262968147	140.371839664555	112.444496886239	-1.98962179171752	0.334505631972704
0.4	0.637438766276678	6	-22.7621355562676	0.547164384340818	130.14248212713	135.853063636089	-2	0.394672140845081
0.41	1.03851432178477	6	-22.7388960765666	0.539100941022473	137.925294842631	142.459296913682	-1.99999999999998	0.398317582214352
0.42	0.617019282021179	6	-22.7836403833345	0.52063968702413	132.412600884289	131.908715973817	-2	0.377789372068514
0.43	1.00222006125957	6	-22.7669424142399	0.497083941926808	138.84417570389	132.560878756468	-2	0.368534172557065
0.44	1.3854524348229	6	-22.735461750642	0.442508674389626	149.866239382466	127.730806309675	-1.98775443119698	0.344888791011023
0.45	2.09325476913197	6	-22.7542533809697	0.455514311827121	147.815629869087	129.964796940122	-2	0.347242980067736
0.46	2.06708478166107	6	-22.7198920761451	0.449235034517127	159.112883886869	138.59783627731	-2	0.353667237181105
0.47	1.91277317747975	6	-22.7134175644432	0.4158846753342	164.305815020088	133.021223122312	-1.99999999999996	0.33502131248639
0.48	3.09959702941975	5	-22.7133128234714	0.526534407663765	168.666330230149	170.028242094597	-1.991839331825	0.400161540375978
0.49	2.99396275206369	5	-22.7586216420838	0.530865270015062	160.296570599804	163.006668703465	-1.99999999999998	0.391406599797717
0.5	3.01068130899773	5	-22.7996514653902	0.532221118709986	154.04407380113	156.7841994340023	-1.999999999999812	0.383433470402977
0.51	1.10735348985882	5	-22.8207986936304	0.529542207845286	153.028828033463	154.869239126894	-1.99999999999999	0.37851641868231
0.52	0.566621255611028	5	-22.8082232056862	0.504804314046596	159.821770266413	154.624246072466	-1.99360356962263	0.370844073513688
0.53	0.804364552288225	5	-22.5216510211879	0.418951220094654	274.838166240616	202.702584893226	-1.77308302024065	0.39148468263154
0.54	2.39612065033725	5	-22.5437634518665	0.441778546861592	265.78170850412	210.927359077781	-1.8183753728052	0.401173098435923
0.55	1.46317886420762	5	-22.4701482130265	0.382373057404958	310.083447783411	206.11140981392	-1.74205611806367	0.377492189289949
0.56	1.51574912980039	5	-22.5086962590862	0.3905000766944	297.455379636805	203.562572401559	-1.75836724883236	0.375135903611389
0.57	1.45663254988623	5	-22.5602264810274	0.402236411508689	278.002178944076	199.230027703658	-1.79306653623351	0.372083907644912
0.58	0.719369820743466	5	-22.6910262019285	0.426822999134624	223.608144930381	178.401952889802	-1.90202692521294	0.3585602913609
0.59	0.589469793678187	5	-22.603110886294	0.376524243377872	267.158181157209	183.285827177277	-1.83213557762778	0.345949650208558

Table S3. This table displays the computed Chi-square fitting errors when fitting the Schechter function to a composite luminosity distribution that consisted of a subsample of identified cluster galaxies from our CMWR- r_{200} training set that visually appeared to have high completeness. We also display the best fit parameter values for M^* , n^* and α as well as their respective standard deviations when using different r filter absolute magnitude bin sizes. In addition, we display the number of bins that contain at least one identified cluster galaxy. It should be noted that we do not display the results of bin sizes that have fewer than five bins with identified cluster galaxies nor do we display the results of bin sizes that do not have successful fits.