

1 **An automatic graph-based method for characterizing**  
2 **multichannel networks**

3 Yanhui Liu<sup>a,b,e</sup>, Paul A. Carling<sup>b,c,d</sup>, Yuanjian Wang<sup>a</sup>, Enhui Jiang<sup>a</sup>,  
4 Peter M. Atkinson<sup>b,c,f,\*</sup>

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6 <sup>a</sup> *Yellow River Institute of Hydraulic Research, Zhengzhou 450003, China*

7 <sup>b</sup> *Lancaster Environment Centre, Lancaster University, Bailrigg, Lancaster LA1 4YR, UK*

8 <sup>c</sup> *Geography and Environmental Science, University of Southampton, Highfield,*

9 *Southampton SO17 1BJ, UK*

10 <sup>d</sup> *State Key Laboratory of Geohazard Prevention and Geoenvironment Protection,*

11 *Chengdu University of Technology, Chengdu, Sichuan 610059, China*

12 <sup>e</sup> *College of Water Conservancy and Hydropower Engineering, Hohai University, Nanjing*

13 *210098, China*

14 <sup>f</sup> *Institute of Geographic Sciences and Natural Resources Research, Chinese Academy*

15 *of Sciences, 11A Datun Road, Beijing 100101, China*

16

17 **ABSTRACT:** Assessment and quantitative description of river morphology  
18 using widely recognized river planview measures (e.g., length, width and  
19 sinuosity of channels, bifurcation angles and island shape) for multichannel  
20 rivers are regarded as fundamental parts of the toolkit of geomorphologists and  
21 river engineers. However, conventional assessment methods including field  
22 surveys or existing algorithms for the extraction of multichannel planviews might  
23 be suboptimal. More recently, the potential for the application of complex  
24 network analysis to the study of river morphology has led to emphasis on the  
25 accurate characterization and definition of multichannel network topology.  
26 Therefore, we developed a novel algorithm called RivMACNet (River  
27 Morphological Analysis based on Complex Networks) that enables the  
28 extraction of multichannel network topology using satellite sensor images as  
29 the input. We applied RivMACNet to a meandering reach of the Yangtze River  
30 and a strongly anastomosing reach of the Indus River to construct their network

\* Corresponding author. Lancaster Environment Centre, Lancaster University,  
Bailrigg, Lancaster LA1 4YR, UK

E-mail address: [pma@lancaster.ac.uk](mailto:pma@lancaster.ac.uk) (Peter M. Atkinson)

31 topologies, and then calculated a series of common topological measures  
32 including weighted degree (WD), clustering coefficient (CC) and weighted  
33 characteristic path length (WCPL). The network analysis indicated that both  
34 networks exhibit poor transitivity with small clustering coefficients. The  
35 topological properties of the Indus at the reach scale are independent of flow  
36 conditions, while they vary across space at the subnetwork scale. In addition,  
37 comparison between RivMACNet and an alternative common river network  
38 analysis engine (RivaMap) demonstrated that RivMACNet is superior in terms  
39 of representation accuracy and network connectivity and, thus, is more suitable  
40 for multichannel fluvial systems with complex planviews. RivMACNet is, thus, a  
41 useful tool to support further investigation of multichannel river networks using  
42 graph theory.

43 **Keywords:** multichannel network, remote sensing, complex network analysis,  
44 river network topology, graph theory.

45

## 46 **1. Introduction**

47 Fluvial systems are generally regarded as linear features that can be  
48 divided into two distinct groups based on current river channel classification  
49 patterns (Nanson and Knighton, 1996): (i) single channel networks such as  
50 straight rivers and meandering rivers (Parker et al., 2011), and (ii) multichannel  
51 networks (Carling et al., 2014) defined as river planviews composed of more  
52 than one interlinked channel forming inosculate patterns, such as braiding and  
53 anabranching rivers (Leopold and Wolman, 1957; Parker 1976; Rust, 1978;  
54 Bridge 2009; Jansen and Nanson, 2010; Meshkova and Carling, 2013;  
55 Kleinhans et al., 2019; Hiatt et al., 2020). The largest rivers on Earth often  
56 exhibit a network of multiple channels and, thus, can be regarded as naturally  
57 occurring forms of a generic class of network structures (Gupta, 2008). Different  
58 channel planforms are thought to reflect differences in river behaviour, and  
59 planform assessment remains central to all modern river channel classification  
60 schemes (Carling et al., 2014). However, quantitative assessment of river  
61 planviews is considered a challenging task in river channel analyses, inclusive  
62 of channel evolution, migration and bank erosion (Miller, 1988; Osman and  
63 Thorne, 1988; Richardson, 2002; Smith and Pain, 2009; Kleinhans et al., 2013;  
64 Grabowski et al., 2014; Yousefi et al., 2016; Li et al., 2017; Shahrood et al.,  
65 2020).

66 For over half a century, researchers have quantified different elements of  
67 channel planviews via metrics including the braiding index, bifurcation angle,  
68 channel width, length, sinuosity and migration distance, as well as island and  
69 sand bar shapes (Parker and Anderson, 1975; van den Berg, 1995; Chew and  
70 Ashmore, 2001; Tooth and Nanson, 2004; Xu, 2004; Harrison et al., 2011;

71 Shwenk, 2016; Ashour et al., 2017; Yukawa et al., 2019; Liu et al., 2021).  
72 However, such conventional quantitative geometrical metrics of fluvial systems  
73 are unlikely to be sufficient to define, or discriminate between, channel types  
74 (Carling et al., 2014). Meanwhile, the definition of river network topologies  
75 (Dodds and Rothman, 2000; Rodriguez and Rinaldo, 2000) and their stream  
76 ordering laws (Tokunaga, 1966; Williams and Rust, 1969; Bai et al., 2015)  
77 demonstrates that river networks can be treated as real-world, non-random  
78 networks of varying complexity. In this view, channel bifurcations (whether  
79 divergent or convergent) are nodes, with the individual channels between  
80 nodes regarded as links. With the development of complex network analysis  
81 (Watts and Strogatz, 1998; Newman, 2003; Rubinov and Sporns, 2010), the  
82 topological properties of multichannel networks, which could highlight emergent  
83 and novel spatial and temporal relations at some local or reach scales for river  
84 channels, have attracted interest from researchers. Despite some success in  
85 the quantification of river network topology and some common topological  
86 measures such as Betweenness Centrality (BC) (Marra et al., 2014), physical  
87 or hydraulic explanations for such topological properties within multichannel  
88 networks have been limited. One reason is that no efficient tools were proposed  
89 for multichannel network construction and the subsequent extraction of a range  
90 of potentially useful metrics including both geometrical and topological  
91 measures.

92 Conventional field surveys and manual inspections of remote sensing  
93 images are prohibitively expensive and laborious for defining and constructing  
94 multichannel topologies and are subject to operator errors (Gupta et al, 2013;  
95 Guo et al., 2017). Increasingly, developments in remote sensing and image

96 processing provide the possibility of reliable automated algorithms or software  
97 packages to extract some of the aforementioned river networks. Examples  
98 include: RivWidth (Pavelsky and Smith, 2008) and RivaMap for river width  
99 (Isikdogan et al., 2017); PyRIS (Monegaglia et al., 2018) and RiMARS for river  
100 network morphology analysis (Shahrood et al., 2020); and RovMAP for river  
101 migration (Schwenk, 2016), as well as other methods for constructing the  
102 topology of river networks (Chen et al., 2019; Schwenk and Hariharan, 2021).  
103 However, gaps remain in terms of methods for the construction of river network  
104 representations, especially for multichannel networks as follows: (i) most  
105 methods adopt a channel mask that differentiates those areas that are within  
106 the river boundary (including islands or sand bars) and those areas outside the  
107 river boundary (Pavelsky and Smith, 2008), but ignore islands or sand bars  
108 located within rivers, which is unacceptable for multichannel networks as island  
109 presence and shape plays an important role in defining multichannel networks  
110 (Meshkova and Carling, 2013); (ii) it is difficult to guarantee the connectivity of  
111 the output river channels when the method for delineating the river network  
112 relies on centerlines (Shahrood et al., 2020) and, thus, such methods result in  
113 extra bifurcation nodes and links being identified, and; (iii) geometrical  
114 measures of individual channels including length, width, and sinuosity are  
115 poorly quantified during the process of multichannel network construction  
116 (Chen et al., 2019), such that whether the river network topology is related to  
117 river behaviour remains unknown.

118 The objectives of the research reported herein were to develop a novel  
119 river morphological analysis method based on complex networks, called  
120 RivMACNet, for multichannel topology construction and assessment, as well as

121 extraction of a range of geometrical and topological measures. The remainder  
122 of this paper is organized as follows. In section 2, the algorithms and methods  
123 used in RivMACNet and some common topological and geometrical measures  
124 of multichannel networks are introduced. Section 3 presents two selected study  
125 reaches, part of the Yangtze and Indus multichannel networks. Section 4  
126 presents the results of the case study in detail including its topological and  
127 geometrical measures at the reach and sub-network scales, in which  
128 RivMACNet is tested and validated. In section 5, we discuss the advantages of  
129 RivMACNet for quantifying multichannel networks by comparing RivMACNet  
130 with another conventional method: RivaMap. Section 6 ends the paper with a  
131 conclusion.

## 132 **2. Methods**

### 133 2.1 River network topology construction

134 The general methodology for constructing a river network topology using  
135 remote sensing comprises the following steps: (i) water body extraction; (ii) river  
136 channel delineation; (iii) node detection; and, (iv) derivation of the river network  
137 connectivity matrix (Chen, 2019). Each of these steps in RivMACNet is  
138 introduced systematically in this section, with particular attention given to  
139 improvements over conventional methods and algorithms. Our proposed  
140 software tools were developed in MATLAB, and are freely available at:  
141 <http://github.com/lyh444/RivMACN.git>.

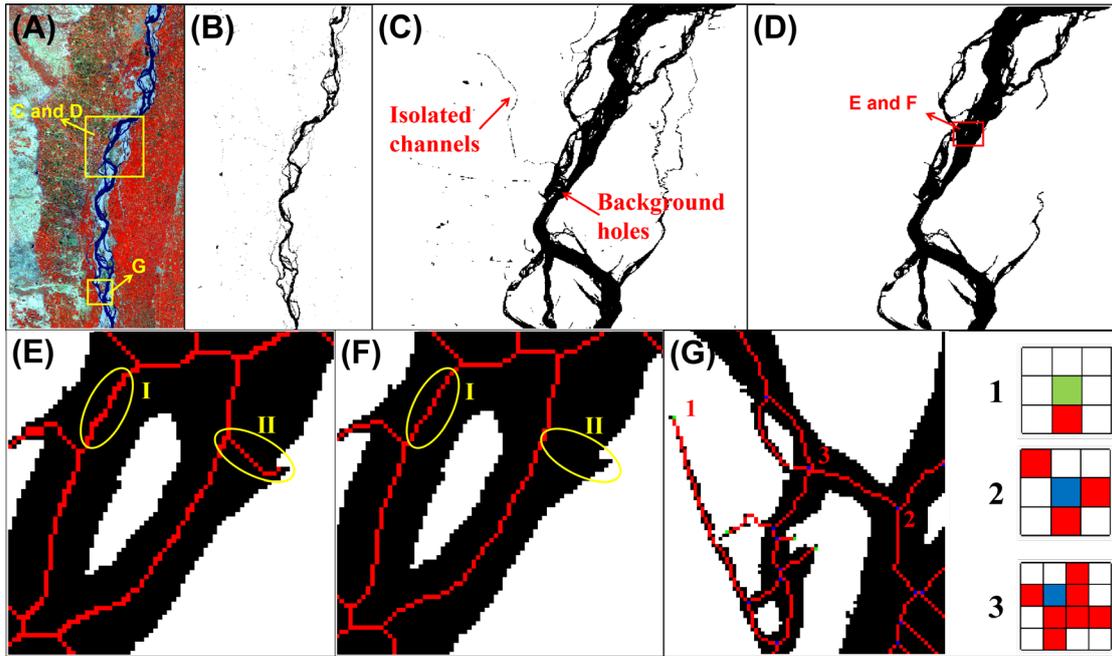
142 Water body extraction: Various reliable algorithms and methods can be  
143 used for extracting singular objects like rivers from remote sensing images  
144 (McFeeters, 1996; Xu, 2006; Petropoulos et al, 2012; Zhu et al., 2015). In  
145 RivMACNet, we employed a widely accepted index called the Modified

146 Normalized Difference Water Index (MNDWI) (Xu, 2006) to extract the water  
147 bodies, which can be expressed as follows:

$$148 \quad \text{MNDWI} = \frac{\text{Green} - \text{MIR}}{\text{Green} + \text{MIR}} \quad (1)$$

149 where Green is a green band, such as band 2 for Landsat 5, while MIR is a  
150 middle infrared band, such as band 5 for Landsat 5. Water bodies have greater  
151 positive MNDWI values, so that a simple thresholding method (the threshold is  
152 0 in this paper unless otherwise stated) can be used for extraction. In this  
153 manner, the extracted water bodies (Fig. 1B) are represented by a binary  
154 image: 1 (river network pixels) and 0 (background pixels). However, although  
155 sporadic discrete water bodies like ponds, lakes, and isolated channels (these  
156 are correctly classified as water, but are not of interest) can be removed by  
157 saving only the largest portion of the extracted water bodies, some remaining  
158 noise is inevitable, particularly in a group of misclassified pixels which we refer  
159 to as small 'background holes' located in the river channel (e.g., false sand bars  
160 or island objects caused by bridges or rivercraft) (Fig. 1C). Such noise cannot  
161 actually affect the river morphology, but can lead to discontinuity in the river  
162 topology or the miscounting of bifurcations and channels. To fill these holes  
163 within the extracted water bodies, we apply a convolutional filter window to the  
164 entire binary image. This window, shown in Fig. 2A, employs a variable size  $\sigma$   
165 ( $\sigma=3, 5, 7\dots$ ) with edge pixels set to 1 and the rest 0. For background pixels, if  
166 the convolution results are equal to  $(4\sigma-4)$ , they are recorded as 'holes' that  
167 need to be filled. The size of the convolution kernel is increased gradually and  
168 the above steps are repeated. In this manner, the final river network with noise  
169 removed can be derived (Fig. 1D).

170



171

172 **Fig. 1.** (A) false colour composite of Landsat 5 TM image derived from Indus River in  
 173 Pakistan. (B) water bodies extracted in (A) based on *MNDWI*. (C) zoomed image of (B).  
 174 (D) water bodies without noise after the process of noise elimination in (C). (E) and (F)  
 175 comparison of thinning results: (E) original Zhang-Suen algorithms and (F) the revised  
 176 algorithms in RivMACNet. Red pixels represent channel skeletons. (G) illustration of the  
 177 node detection in RivMACNet. Nodes 1, 2 and 3 are examples of end nodes (green pixels)  
 178 and bifurcation nodes (blue pixels) with different patterns in (G), respectively.  
 179

180 Delineating the river channels: The aforementioned water bodies are  
 181 usually reduced to a set of single-resolution lines (herein termed 'river  
 182 representatives') to define the links and nodes in the multichannel network  
 183 (Schaefer and Pelletier, 2020). In contrast to conventional river representations  
 184 such as geometrical channel centerlines (EGIS, 2002; Mount et al., 2003), river  
 185 skeletons (Hasthorpe and Mount, 2007) are defined as the refined curves with  
 186 the same geometrical characteristics as the river channels. This approach has  
 187 the advantage of maintaining the connectivity of the refined curves and greater  
 188 enforcement efficiency (Shen et al., 2017; Chen et al., 2019). We adopt the  
 189 revised version of the classic Zhang-Suen fast parallel thinning algorithm  
 190 presented by Chen et al. (2012) in RivMACNet to produce one-pixel wide river  
 191 skeletons (e.g., I in Fig.1F). This procedure avoids unwanted spurs caused by

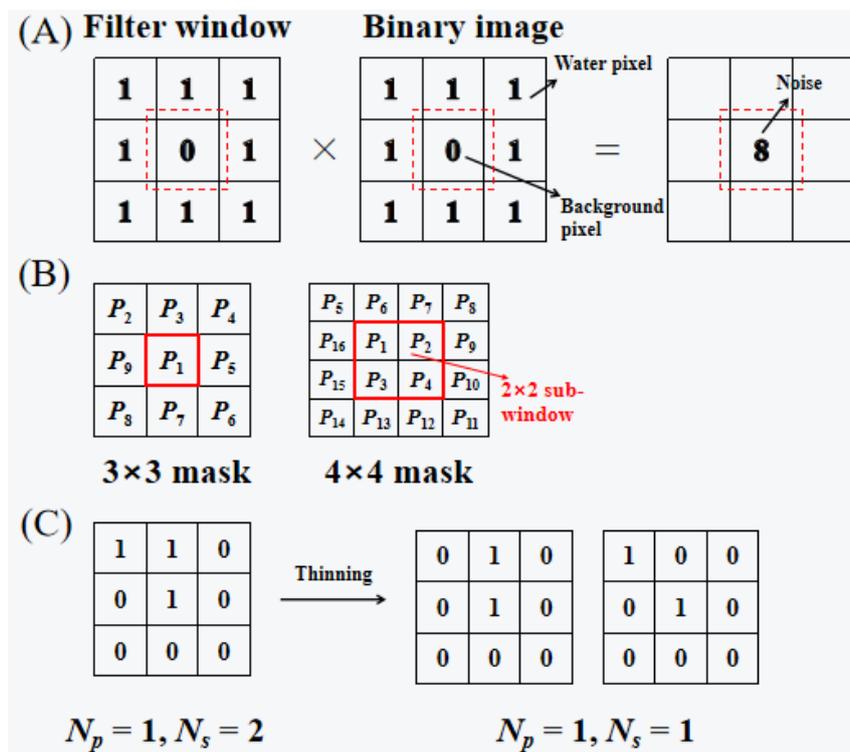
192 local convex water pixels (e.g., II in Fig. 1E) in contrast to the original algorithm  
 193 (Zhang and Suen, 1984).

194 Node detection: The river skeleton image is convolved with a mask for  
 195 node detection in RivMACNet. The conventional method (Olsen et al., 2011),  
 196 using a simple 3×3 mask (Fig. 2B), can detect only eight end nodes and 18  
 197 bifurcation node structures (e.g., node 1 and 2 in Fig. 1G) in river skeletons, but  
 198 ignores the structure formed by two adjacent bifurcation nodes (and one node  
 199 connecting four or more links) due to the limitation of the mask size (e.g., node  
 200 3 in Fig. 1G). A higher order 4×4 mask with a 2×2 sub-window (Fig. 2B) is added  
 201 in RivMACNet to detect the remainder of the nodes because of the extendibility  
 202 of the method. Each skeleton pixel within the image is traversed using the 3×3  
 203 mask and 4×4 mask, in turn. The former mask has 1 center pixel  $P_i$  ( $i = 1$ ) and  
 204 8 edge pixels  $P_j$  ( $j = 2, 3, \dots, 9$ ), while the latter has 4 center pixels  $P_i$  ( $i = 1, 2,$   
 205  $3, 4$ ) and 12 edge pixels  $P_j$  ( $j = 5, 6, \dots, 16$ ). Center pixels and edge pixels in  
 206 both masks are all sorted clockwise (Fig. 2B). Pixels satisfying the following  
 207 conditions are defined as end or bifurcation nodes:

$$208 \quad P \in \begin{cases} \text{end node,} & \text{if } N_p=1 \text{ and } N_s=1; \\ \text{bifurcation node connecting to 3 links,} & \text{if } N_p=1 \text{ and } N_s=3; \\ \text{bifurcation node connecting to more than 3 links,} & \text{if } N_p > 2 \text{ and } N_s > 3; \end{cases} \quad (2)$$

209 where  $N_s$  and  $N_p$  are defined as the number of edge pixels  $P_j$  and middle pixels  
 210  $P_i$  within two masks that belong to skeletons (Fig. 2B). In addition, RivMACNet  
 211 omits the node structures where an individual channel ends with a bend (e.g.,  
 212 Fig. 2C):  $N_p = 1$ ,  $N_s = 2$ . Such output would not be generated by the thinning  
 213 algorithm, as it would be further refined into a single pixel-wide end node  
 214 skeleton.

215 In the above manner, all detected nodes can be recorded as the node  
 216 matrix **Node** =  $\{X_i, Y_i\}$  ( $X_i$  and  $Y_i$  are the pixel coordinates of the  $i$ -th node in  
 217 water body binary images), where nodes are sorted in order from upstream to  
 218 downstream within the multichannel network according to the Euclidean  
 219 distances between them and the start point of the multichannel centerline (see  
 220 Fig. 4A). The order of nodes in connectivity matrices has no effect on the  
 221 computation of network measures (Rubinov and Sporns, 2010). Specifically, for  
 222 two nodes  $i$  and  $j$  with the same distance from the start of the channel network,  
 223  $i < j$  when (i)  $X_i < X_j$ , or (ii)  $Y_i < Y_j$  if  $X_i = X_j$ .(Fig. 4A)



224

225 **Fig. 2.** (A) illustration of the convolution between the filter window structure (take  $\sigma = 3$  as  
 226 an example) and binary images. (B) two node detection masks with different sizes. (C)  
 227 examples of end node structures: (left)  $N_p = 1, N_s = 2$ ; (right)  $N_p = 1, N_s = 1$ .  
 228

229 Derivation of the river network connectivity matrix: the connectivity matrix

230 **A**={ $a_{ij}$ } shown in Fig. 3B plays an important role in the calculation of river

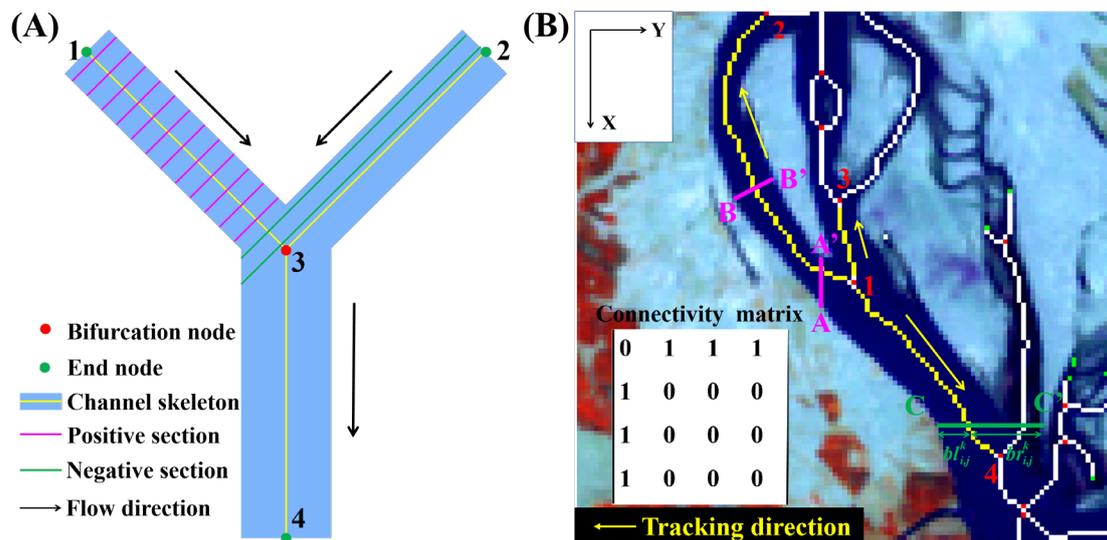
231 network topological measures (Rubinov and Sporns, 2010). Its rows and

232 columns denote nodes, while matrix entries denote links.  $a_{ij} = 1$  if node  $i$  is

233 connected to node  $j$ , while 0 if they are not connected. A tracking algorithm from  
 234 one node to another was presented in RivMACNet to construct the river network  
 235 topology automatically, summarized as follows:

236 (i) Define the zero matrix  $\mathbf{A}=\{a_{i,j}\}_{n \times n}$  ( $n$  is the node numbers) and traverse  
 237 each detected node in the **Node** matrix.

238 (ii) Take node  $i$  as the starting pixel and track each skeleton connecting to  
 239 node  $i$  by pixels, in turn, until another node  $j$  at the other side of the skeleton is  
 240 reached. Then,  $a_{i,j}=a_{i,j}+1$ . If all skeletons connecting to  $i$  have been tracked,  
 241 then move to the next node pixel (Fig. 3B).



**Fig. 3** (A) Examples of the positive and negative cross-sections in a 4-node multichannel network. (B) Illustration of the tracking process from node 1 to others (nodes 2 - to - 4) as well as their connectivity matrix. Red numbers represent nodes, while magenta and green lines are positive and negative cross-sections located on the individual channels, respectively. The value of the pixel coordinate  $X$  increases in the downstream direction of the river network in RivMACNet.

## 250 2.2 Channel planview measures

251 Two groups of channel planview measures including geometrical and  
 252 topological properties of multichannel networks are introduced in this section.  
 253 RivMACNet establishes the bridge between such geometrical and topological  
 254 measures to provide certain physical and hydraulic bases for complex network

255 analysis of multichannel networks. The extraction process of channel  
 256 geometrical measures in the multichannel network actually occurs during the  
 257 tracing from one node  $i$  to another node  $j$  in the river network.

258 Two kinds of channel length are considered in RivMACNet: the curve  
 259 length  $l_{i,j}^c$  and the straight line distance  $l_{i,j}^s$  of the individual channel between  
 260 two consecutive bifurcations  $i$  and  $j$ , which can be expressed as follows:

$$261 \quad l_{i,j}^c = IR \times \sum_{k=1}^K \sqrt{(X_{i,j}^{k+1} - X_{i,j}^k)^2 + (Y_{i,j}^{k+1} - Y_{i,j}^k)^2}, \quad (3)$$

$$262 \quad l_{i,j}^s = IR \times \sqrt{(X_i - X_j)^2 + (Y_i - Y_j)^2}, \quad (4)$$

263 where  $X_{i,j}^k$  and  $Y_{i,j}^k$  are the  $(X, Y)$  location coordinates of the  $k$ -th pixel in the  
 264 skeleton connecting nodes  $i$  and  $j$ , while  $K$  is the total pixel number of the  
 265 skeleton, and  $IR$  (in units of m) indicates the image resolution.

266 Channel sinuosity  $s_{i,j}$  can be defined as the ratio of the aforementioned two  
 267 kinds of lengths of the corresponding skeleton connecting nodes  $i$  and  $j$ :

$$268 \quad s_{i,j} = \frac{l_{i,j}^c}{l_{i,j}^s}. \quad (5)$$

269 The individual channel width could be considered equal to the mean  
 270 lengths of a set of measurement cross-sections (Fig. 3A) (Howard et al., 1970)  
 271 separated by approximately equal distance (spaced one pixel apart in this  
 272 paper) along its skeleton. Each cross-section is set orthogonal to the local  
 273 orientation of the channel skeleton in RivMACNet:

$$274 \quad \text{slop}_{i,j}^k = \frac{Y_{i,j}^{k+1} - Y_{i,j}^{k-1}}{X_{i,j}^{k+1} - X_{i,j}^{k-1}}, \quad (6)$$

275 where  $\text{slop}_{i,j}^k$  is the local orientation of the channel skeleton connecting nodes  
 276  $i$  and  $j$  at the  $k$ -th pixel. A special case is  $X_{i,j}^{k+1} = X_{i,j}^{k-1}$ , in other words, the

277 denominator is zero. In this case, the orientation of the corresponding cross-  
278 section line is set to vertical in RivMACNet (e.g., A-A' in Fig. 3B).

279 However, not all cross-sections contribute when calculating the individual  
280 channel width, especially those near nodes. For example, the green sections in  
281 Fig. 3A measure not only the width of the individual channel connecting nodes  
282 1 and 3, but also the length of the individual channel connecting nodes 2 and  
283 3. Such cross-sections affected by other individual channels are defined as  
284 'negative sections' and, thus, omitted in RivMACNet when calculating the  
285 channel width. Conversely, the 'positive sections' are roughly bisected by  
286 channel skeletons (e.g., B-B' in Fig. 3B), playing an important role in measuring  
287 channel widths. In this context, another coefficient called width gate  $\Delta b$  was  
288 introduced to distinguish between 'positive and negative sections'. The cross-  
289 section that intersects the  $k$ -th pixel of the channel skeleton connecting nodes  
290  $i$  and  $j$  belongs to the 'positive section' when the following condition is true:

$$291 \quad \Delta b \geq |bl_{i,j}^k - br_{i,j}^k|, \quad (7)$$

292 where  $bl_{i,j}^k$  and  $br_{i,j}^k$  represent lengths of the left and right sub-sections (e.g., C  
293 - C' in Fig. 3B) divided by the channel skeleton, respectively. As a result, the  
294 width  $b_{i,j}$  of the individual channel connecting to nodes  $i$  and  $j$  is calculated as  
295 follows:

$$296 \quad b_{i,j} = \frac{1}{K'} \sum_{k=1}^{K'} b_{i,j}^k, \quad (8)$$

297 where  $K'$  is the total number of the 'positive sections' located on the channel  
298 skeleton, while  $b_{i,j}^k$  represent the length of the cross-section that intersects  $k$ -  
299 th skeleton pixel of the skeleton.

300 RivMACNet also highlights the calculation of common river network  
 301 topological properties (Table 1), including degree, clustering coefficient, and the  
 302 characteristic path length, based on the connectivity matrix **A** (Rubinov and  
 303 Sporns, 2010).

304 **Table 1** Expressions of multichannel network topological measures

Measure	Unweighted expression <sup>a</sup>	Weighted expression <sup>b</sup>
Degree	$k_i = \sum_{j=1}^n a_{i,j}$	$k_i^w = \sum_{j=1}^n w_{i,j}^k$
Average neighbour degree <sup>c</sup>	$k_{nn,i} = \frac{\sum_{j=1}^n \alpha k_j}{k_i}$	$k_{nn,i}^w = \frac{\sum_{j=1}^n \alpha k_j^w}{k_i^w}$
Cluster coefficient	$C_i = \frac{\sum_{j,h \in n} a_{ij} a_{ih} a_{jh}}{k_i(k_i-1)}$	/
The characteristic path length <sup>d</sup>	$l = \frac{1}{\frac{1}{2}n(n+1)} \sum_{i \neq j} d_{i,j}$	$l^w = \frac{1}{\frac{1}{2}n(n+1)} \sum_{i \neq j} w_{i,j}^l$

305 <sup>a</sup>.  $a_{ij}$  represents values in the connectivity matrix A, while  $n$  is the total number of nodes.

306 <sup>b</sup>.  $w_{i,j}^k$  and  $w_{i,j}^l$  are weights for degree and the characteristic path length, respectively.

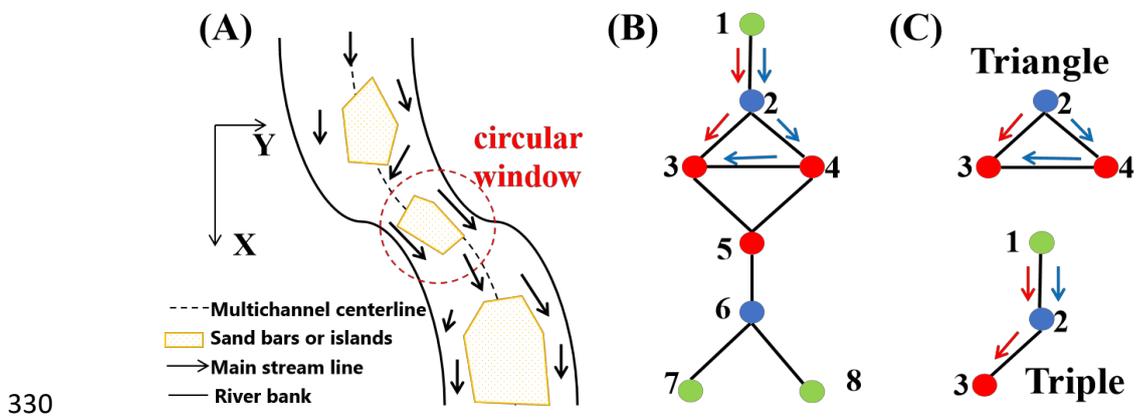
307 <sup>c</sup>.  $\alpha$  is 1 if node  $i$  and  $j$  are neighbours, or 0 if they are not connected.

308 <sup>d</sup>. The characteristic path length of the network in RivMANCnet is calculated using the  
 309 Floyd-Warshall (1962) algorithm.

310

311 In a river network, the degree  $k_i$  quantifies how many individual channels  
 312 are connected to the bifurcation node  $i$  (divergent or convergent), while mean  
 313 neighbour degree  $k_{i,nn}$  measures the mean number of individual channels  
 314 connecting to its neighbour nodes. In this manner, nodes within river networks  
 315 can be divided roughly into three types (Fig. 4): (i) ‘end nodes’ ( $k_i = 1$ ) indicate  
 316 the upstream inlet and the downstream outlet of the river network as well the  
 317 terminations of other channels; (ii) ‘simple bifurcation nodes’ ( $k_i > 1$  and  $k_{i,nn} <$   
 318  $3$ ) indicate divergent or convergent points because of the inflow and outflow of  
 319 other streams; (iii) ‘island or sand bar nodes’ ( $k_i > 1$  and  $k_{i,nn} \geq 3$ ) indicate  
 320 divergent or convergent points caused by enclosing sand bars or islands  
 321 located in the river network.

322 Given the topological structure of the network there could be a probability  
 323 that node  $i$  is connected to node  $j$  if both of them are neighbours of node  $k$ , a  
 324 condition termed transitivity, or clustering of the network (Newman, 2003). The  
 325 clustering coefficient  $C_i$  of node  $i$  (Table 1) is defined to quantify this  
 326 neighbourhood property based on the ratio of the number of triangles (formed  
 327 by islands and sand bars) and triples around node  $i$  (Fig. 4C). Specifically,  $C_i$   
 328 lies in the range  $[0, 1]$ , with the maximum value of 1 if nodes in the river network  
 329 connect to each other.



331 **Fig. 4.** (A) basic elements for multichannel networks, and red circle indicates circle raster  
 332 for assessment of the network topology at a sub-network scale. (B) illustration of a river  
 333 network topology in (A). Arrows indicate the flow orientation, while green, blue and red  
 334 nodes indicate end, simple bifurcation and island/sand bar nodes, respectively. Two  
 335 different paths from node 1 to 3 are shown by red, and blue arrows, respectively. (C)  
 336 illustration of the 'triangle' around node 2 and the 'triple' structure centered on node 2 in  
 337 (B). Node 2 has one triangle and three triples and, thus, its clustering coefficient is  $1/3$ .  
 338

339 For multichannel networks, water and sediment could be transported from  
 340 one node  $i$  to another  $j$  though different paths (e.g., the two paths from node 1  
 341 to node 3 in Fig. 4B). Therefore, the characteristic path length  $l$  is defined as  
 342 the mean value of the shortest path length  $d_{i,j}$  between all pairs of nodes though  
 343 the multichannel network.  $l$  is a connectivity measure of the multichannel  
 344 network, and its minimum value is 1 if all divergent or convergent points connect  
 345 to each other.

346 Multichannel networks are hydraulically complex and, thus, the  
347 aforementioned unweighted (channel links are equivalent in network analysis)  
348 topological properties cannot be related adequately to the controlling processes  
349 and variables of river channels. For example, the longitudinal slope, bankfull  
350 discharge, channel depth, and median diameter of bed material usually are  
351 unknown when interrogating remote-sensing images and these are widely  
352 considered to be important parameters in determining channel form and  
353 behaviour. However, link width and length often are considered to be related to  
354 bankfull discharge and channel slope, respectively, and bifurcation angles  
355 reflect well-studied hydrodynamic controls as well as the constraints imposed  
356 by the width of the macrochannel. In this context, the multichannel network  
357 topological properties were weighted in this paper to reflect unmeasured  
358 controls such as a slope and depth (Table 1). The weight for degree  $w_{i,j}^k$  is the  
359 ratio of individual channel width and length, indicating that nodes with a larger  
360 weighted degree play a more important role in multichannel networks because  
361 they participate in more water and sediment transport and redistribution, while  
362 the weight for characteristic path length  $w_{i,j}^l$  is the ratio of the length of the  
363 individual channel and the mean length of all channel links and is, thus,  
364 proportional to spatial distances for water and sediment transport. Additionally,  
365 no weight was set for the clustering coefficient, which is a density measure for  
366 the occurrence of sand bars and islands in multichannel networks. Expressions  
367 for these topological measures of unweighted and weighted multichannel  
368 networks are listed in Table 1.

### 369 2.3 Spatial evolution of topological measures at the sub-network 370 scale

371 The planviews of multichannel networks vary due to the influence of  
372 upstream flow and boundary conditions. In addition to assessment of the global  
373 network properties, RivMACNet also provides methods to explore the spatial  
374 distributions of topological properties along the multichannel centerline by using  
375 a circular moving window (Fig. 4A) with an adjustable radius  $R$  instead of the  
376 macrochannel (van Niekerk et al., 1995) cross lines. In this manner, the local  
377 topological measures  $f(x_0)$  at  $x_0$  km from the most upstream extent of the  
378 multichannel network can be expressed as follows:

$$379 \quad f(x_0) = \frac{1}{n_x} \sum f(x)_i, \quad (9)$$

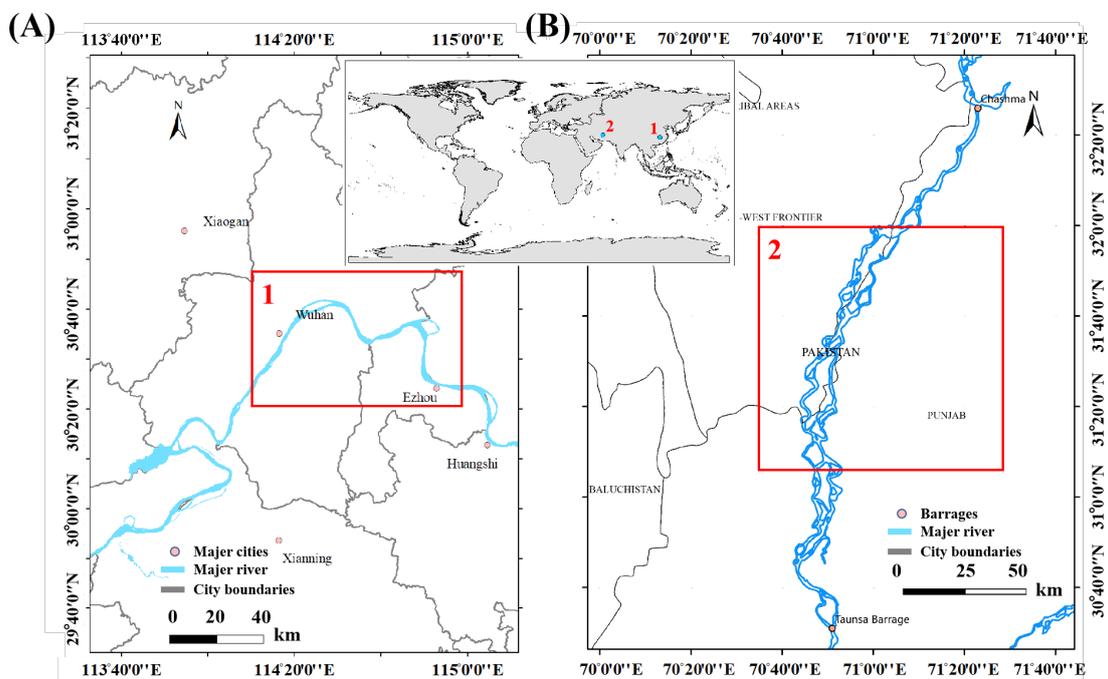
380 where  $n_x$  is the total number of nodes in the circular window, while  $f(x)_i$  is the  
381 measure value of the  $i$ -th node.

### 382 **3. Study area**

383 To test the practical utility and reliability of RivMACNet, we selected two  
384 regions as study areas. Region I: the Yangtze River (Chen et al., 2019) near  
385 Wuhan, China (Fig. 5A); Region II: the Indus River (Inam et al., 2007; Ali, 2013;  
386 Syvitski and Brackenridge, 2013; Kale, 2014; Carling et al., 2018) between the  
387 Chashma and Taunsa barrages located in the middle of the Indus Basin in  
388 Pakistan (Fig. 5B). The former case is a meandering river reach, while the latter  
389 exhibits an anastomosed river pattern composed of sand bars, islands, wet  
390 channels, and dry channels.

391 Landsat 5/8 TM images of the two study reaches with a spatial resolution  
392 of 30/15 m were downloaded from Earth Explorer  
393 (<http://earthexplorer.usgs.gov>). The MNDWI was calculated for each of the  
394 pixels within the images, for use as the input to the RivMACNet algorithm. We  
395 selected three images to test our model. The first is the Landsat 8 TM image of

396 Region I in April 2019 with a size of 2705×1792 pixels. The other two are  
 397 Landsat 5 TM images of Region II representing the low flow (LF) period in  
 398 March 2011 and the high flow (HF) period in October 2011, to illustrate the  
 399 reliability of the proposed method for identifying the river network topology  
 400 between different flow conditions. Both images are 3000×3000 pixels. The  
 401 ground data on water bodies were derived from the corresponding false colour  
 402 composite of Landsat TM images by supervised classification using the support  
 403 vector machine (SVM), which was executed in ENVI (Oliver, 2008). The training  
 404 samples for each class were selected manually based on their colours.  
 405 Although some error might be associated with the ground data, these data can  
 406 be considered as a control group of constructed river channels when comparing  
 407 RivMACNet with other methods because the error in the ground data is small  
 408 relatively (Chen et al, 2019).



409

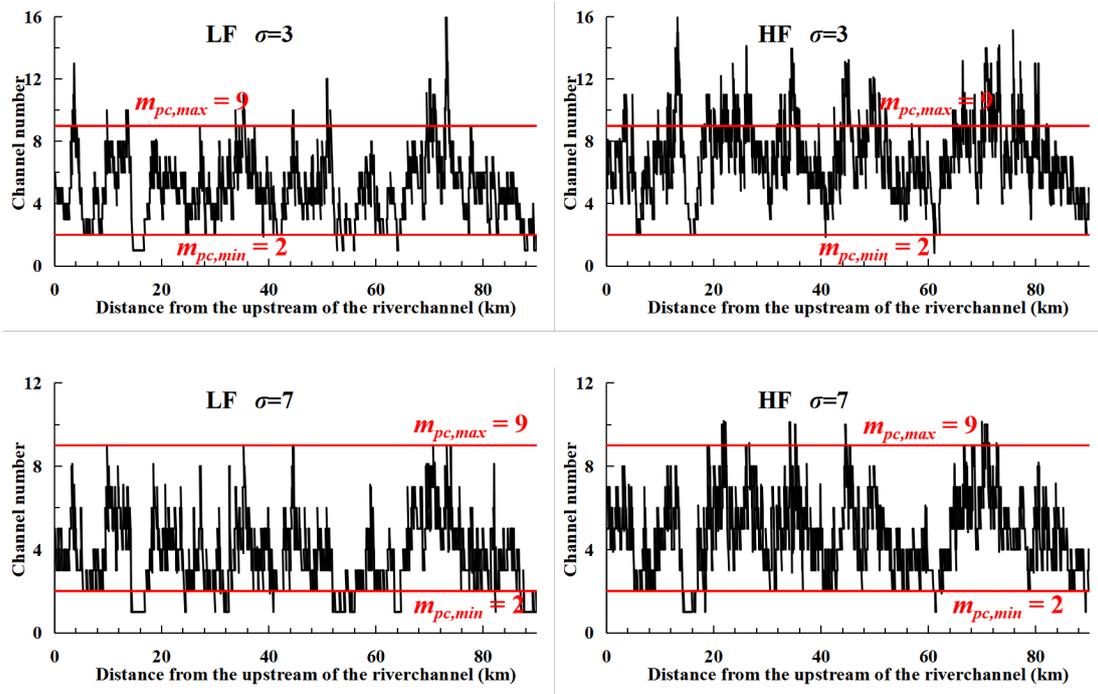
410 **Fig. 5.** Maps of the study river reaches on (A) the Yangtze River near Wuhan, and (B) the  
 411 Indus River located between the Chashma and Taunsa barrages, Pakistan.

412

## 413 4. Results

### 414 4.1 Parameter settings

415 Before presenting results for the river network topologies and geometrical  
416 properties, essential parameter settings including the size of the noise removal  
417 window  $\sigma$  and the threshold of the width gate  $\Delta b$  need to be considered  
418 because different parameter values will lead to variable outputs. We set these  
419 parameters using data of the Indus. A larger  $\sigma$  makes RivMACNet insensitive  
420 to islands and sand bars. Fig. 6 illustrates the numbers of individual channels  
421 detected in the Indus network with different  $\sigma$  values during low and high flow  
422 periods, respectively. We tested different  $\sigma$  values and found that the output  
423 of individual channel numbers per macrochannel cross-section ( $m_{pc}$ ) when  $\sigma$   
424 = 7 is consistent with observation (by visual interpretation) that the Indus  
425 network has a minimum of two ( $m_{pc,min} = 2$ ) and a maximum of nine ( $m_{pc,max} =$   
426 9) channels (Carling et al., 2018). Furthermore, the choice of threshold of  $\Delta b$   
427 is a trade-off between the width-extraction accuracy and the number of  
428 individual channels. Thus, we employed a sensitivity analysis to determine its  
429 optimal value (Fig. 7). In Fig. 7, although a smaller threshold can strictly  
430 guarantee the accuracy of the detected river width it leads to the loss of  
431 channels. The latter phenomenon gradually improved as the threshold  
432 increased, and became stable at the threshold of  $\Delta b$  larger than 2. In this  
433 context, we set the threshold of  $\Delta b$  to 3 indicating that this optimal value can  
434 not only achieve a high measurement accuracy of the channel width, but also  
435 maximize the number of channels with positive width. We set the parameters  
436 to the aforementioned recommended values in all experiments reported in this  
437 paper.

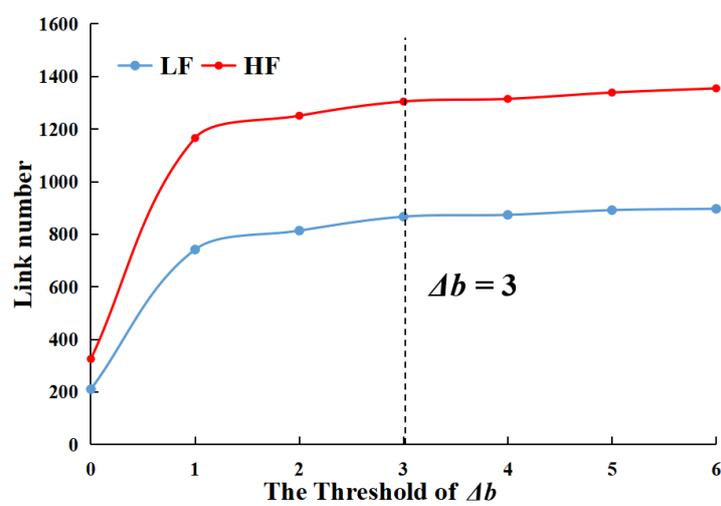


438

439

440 **Fig. 6.** Numbers of individual channels detected in the Indus network using different  $\sigma$   
 441 values during high flow (HF) and low flow (LF) periods.

442



443

444 **Fig. 7.** Sensitivity analysis of the threshold of  $\Delta b$ , showing the number of links plotted  
 445 against the threshold of  $\Delta b$  during low flow and high flow periods.

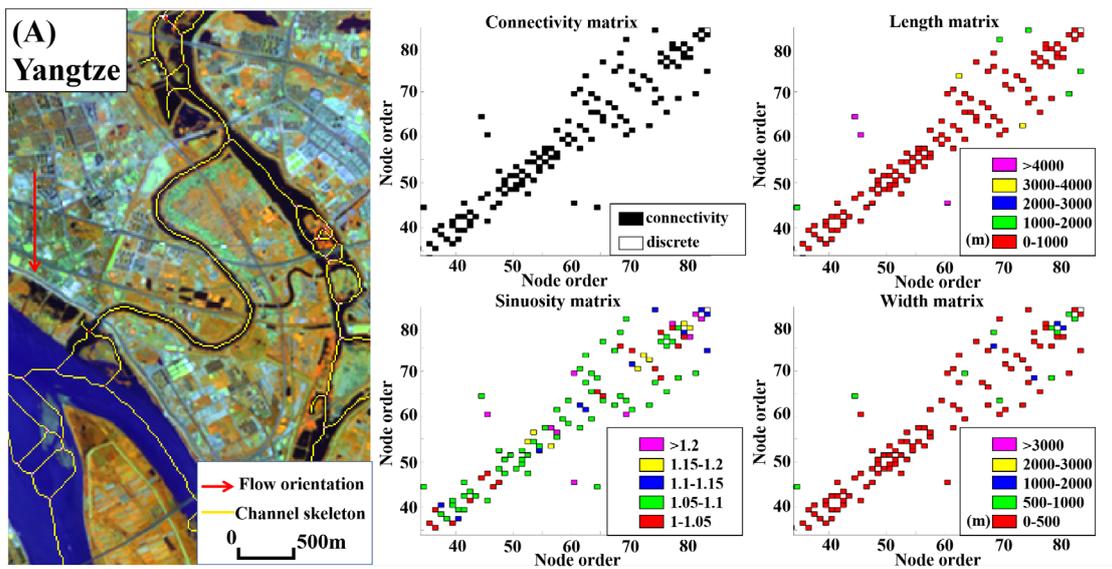
446

447 **4.2 River network topology**

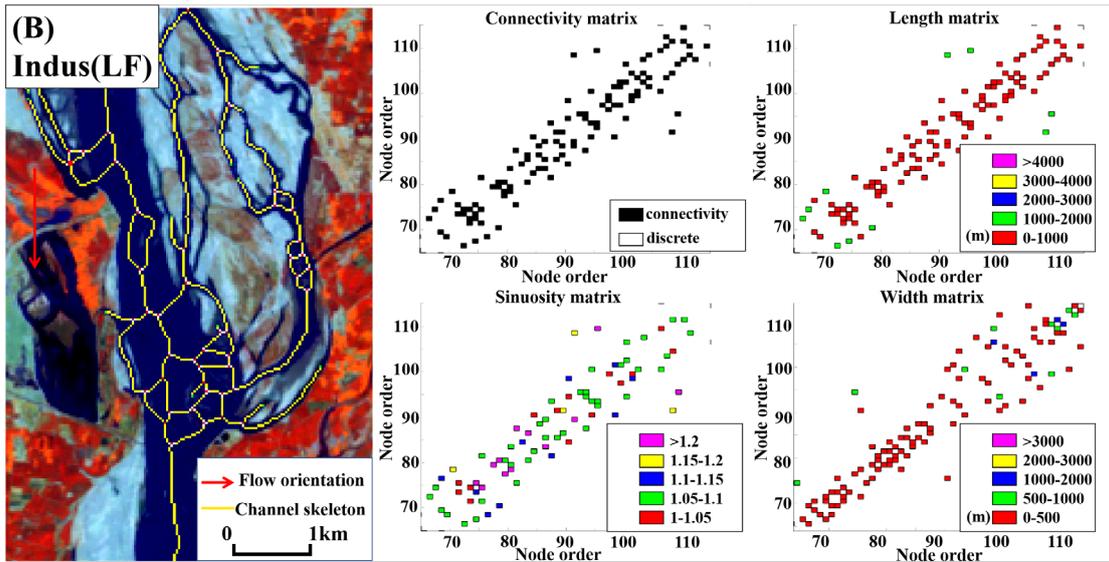
448 The river network topologies of the Yangtze and Indus were constructed  
 449 after parameter ( $\sigma$  and the threshold of  $\Delta b$ ) settings in RivMACNet. Fig. 8  
 450 illustrates the connectivity matrix **A** of the study reach, as well as a series of

451 geometrical measures vectorized for visualization including the channel width  
452 matrix  $\mathbf{B} = \{b_{i,j}\}$ , length matrix  $\mathbf{L} = \{l_{i,j}^c\}$  and sinuosity matrix  $\mathbf{S} = \{s_{i,j}\}$ . In matrix  $\mathbf{A}$ ,  
453 nodes within both two networks exhibit strong linear distributions and tend to  
454 connect to their neighbours in geographical space because multichannel  
455 networks can be described as ‘slender’ networks (Marra et al., 2014) with  
456 limited lateral extension in space, and their lengths are much larger than the  
457 multichannel widths. RivMACNet produced fewer nodes  $n$  and links  $m$  in the  
458 Yangtze network ( $n_1 = 281$ ,  $m_1 = 339$ ) than in the Indus network during the high  
459 flow ( $n_{2,HF} = 1205$ ,  $m_{2,HF} = 1339$ ) and low flow ( $n_{2,LF} = 826$ ,  $m_{2,LF} = 892$ ) periods,  
460 which is related to the reach length and the river pattern (Van den Berg, 1995;  
461 Xu, 2004). Additionally, to further compare the differences in links of the Indus  
462 between the low and high flow periods, statistics describing the geometrical  
463 properties of individual channels in the Indus network are shown in Fig. 9. On  
464 average, individual channel lengths during high flow periods are smaller than  
465 during low flow periods ( $\overline{l_{2,HF}} < \overline{l_{2,LF}}$ ). This result can be explained by space-  
466 filling considerations: the development of a new channel emanating from a node  
467 in a space-filling network inevitably intersect neighboring channels and  
468 consequently decrease individual channel lengths (Meshkova and Carling,  
469 2014). Although river channels would expand during high flow periods (for  
470 example, the number of individual channels with width larger than 1000 m  
471 increases in Fig. 9C), an opposite result is observed for the mean widths of the  
472 individual channels ( $\overline{b_{2,HF}} < \overline{b_{2,LF}}$ ) because more narrow ( $b_{2,HF} < 250$  m)  
473 channels were generated during high flow periods. For the sinuosity, the  
474 number of channels where  $s_{2,HF} > 1.2$  decreased significantly during high flow  
475 periods, and more straight channels ( $s_{2,HF} < 1.05$ ) appeared. Nonetheless, the

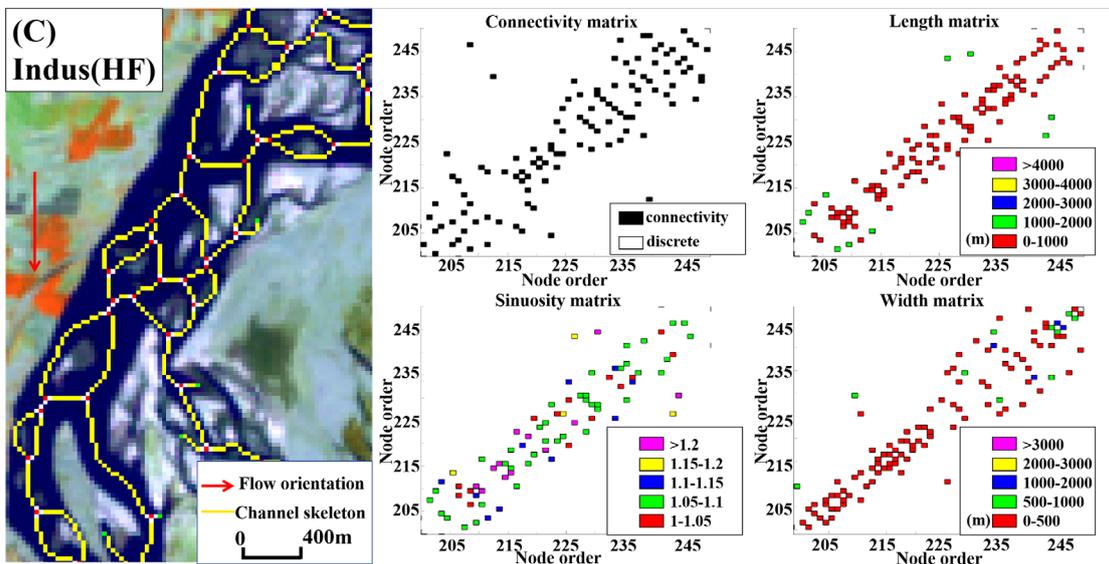
476 mean values of sinuosity during the high and low flow periods are still close to  
 477 each other (Fig. 9B). In this context, the increase in the scale (the numbers of  
 478 nodes and links) of network topology ( $n_{2,HF} > n_{2,LF}$ ,  $m_{2,HF} > m_{2,LF}$ ) implies that  
 479 the Indus network exhibits more complex planviews during high flow periods,  
 480 and most of these new individual channels during high flow periods tend to be  
 481 short, narrow, and straight.



482



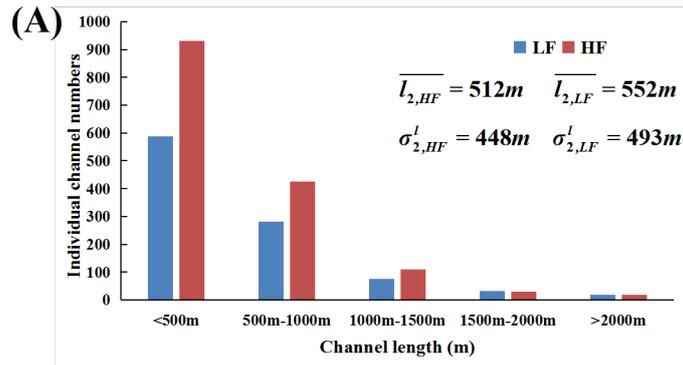
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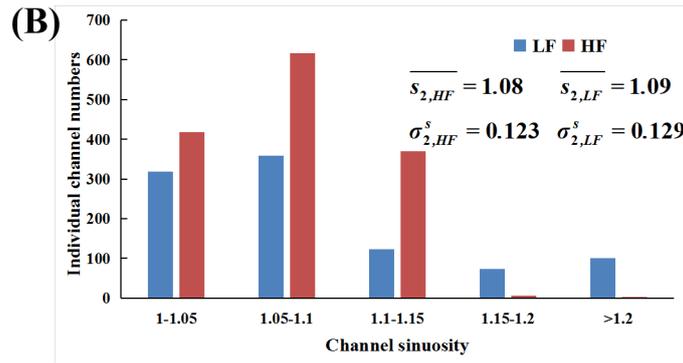
484

485 **Fig. 8.** (Right) An illustrative set of topological and geometrical matrices extracted from (A)  
 486 the Yangtze network, and the Indus network during (B) LF periods and (C) HF periods.  
 487 (Left) the water bodies are planviews of the sub-reaches shown by these matrices. For  
 488 interpretation of the colours in the right matrices see the individual legends. A 50 – node  
 489 matrix is used to provide the best visual impression of the different colors in each matrix.  
 490

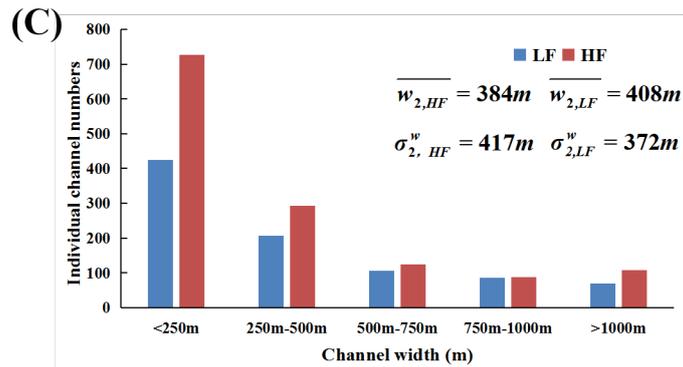
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492



493



494 **Fig. 9.** Distributions of the geometrical properties (A) length, (B) sinuosity and (C) width of  
 495 the individual channels in the Indus network during high and low flow periods.  
 496

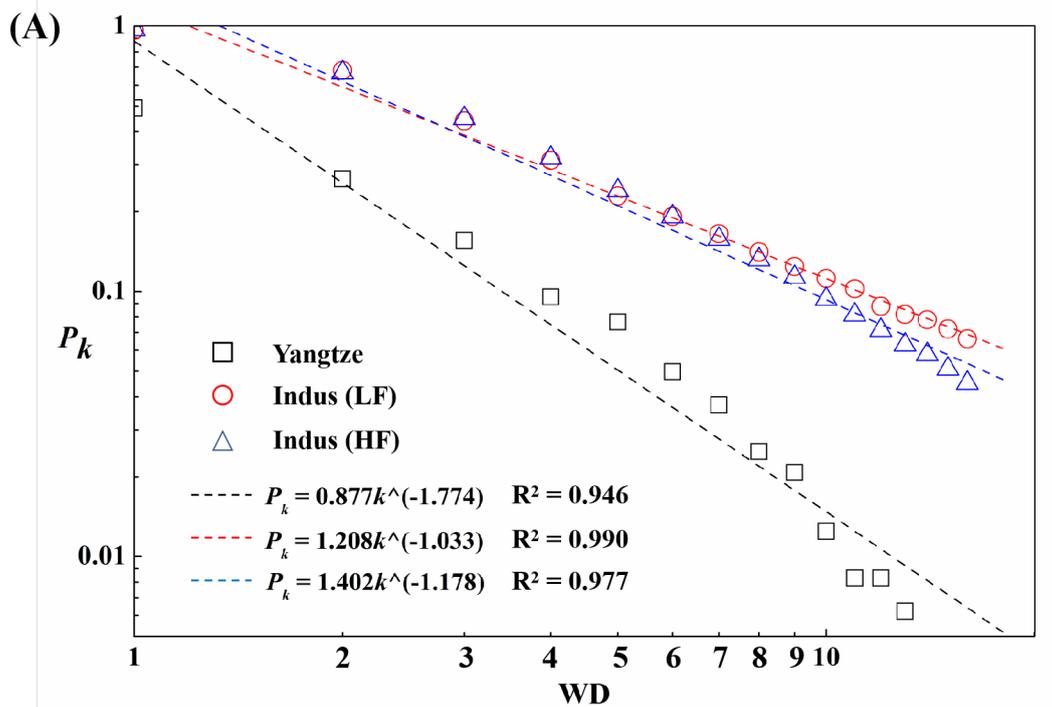
497 **4.3 Multichannel network topological measures**

498 Three global topological measures including the weighted degree (WD),  
 499 clustering coefficient (CC) and the weighted characteristic path length (WCPL)  
 500 of the two study areas are reported in Table 2. Values of these three weighted  
 501 topological measures are different between two different river networks, but  
 502 close when comparing different flow conditions in the same study area. For  
 503 example, the WD value of the Yangtze network considering the length/width

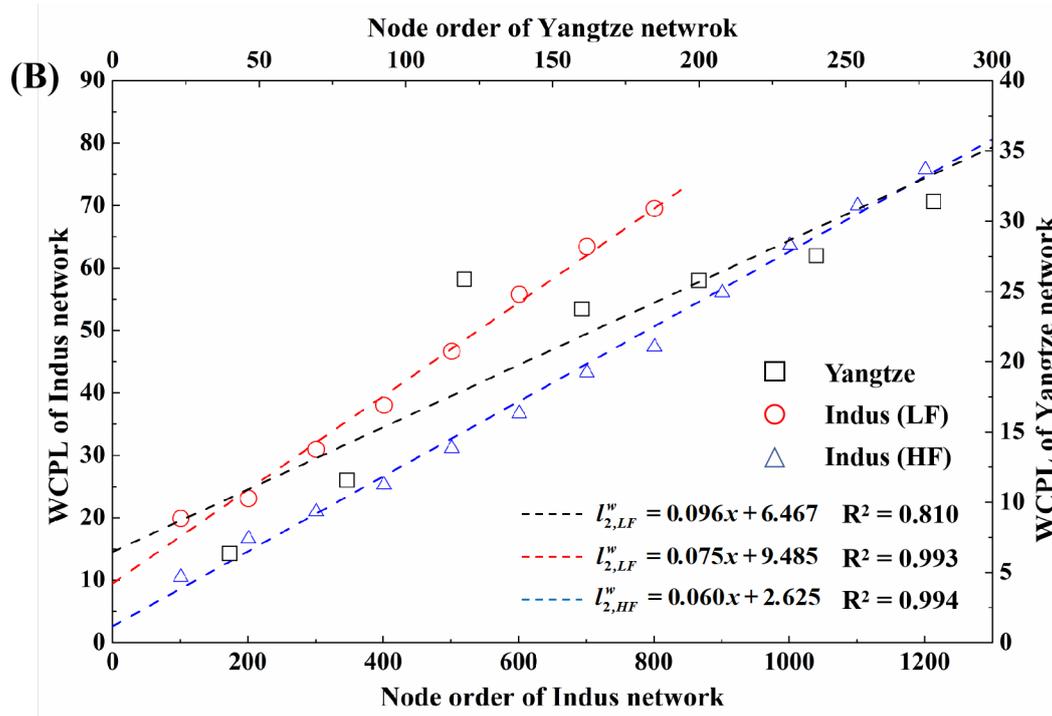
504 ratio of individual channels as connection strengths is 1.601, smaller than that  
505 of the Indus network during LF and HF periods ( $\overline{k_{2,HF}^w} \approx \overline{k_{2,LF}^w} \approx 2.7$  ).  
506 Furthermore, the cumulative distribution of weighted degree  $P_k$ , representing  
507 the probability that one node has WD value greater than or equal to  $k$ , was  
508 calculated and plotted in Fig. 10A. The distributions in both Yangtze and Indus  
509 networks follow the power-law ( $P_k \propto k^{-\lambda}$ ) distribution and nodes with low  
510 weighted degree values ( $k_i^w < \overline{k^w}$ ) account for the largest proportion (70.2% for  
511 Yangtze network, and 68.8% and 68.7% for Indus network during HF and LF  
512 periods, respectively), followed by a positively skewed long tail (Fig. 10A).

513 **Table 2.** Topological measures of the Yangtze and Indus networks at the  
514 whole reach scale.

Network	Date	WD	CC	WCPL
Yangtze	April 2019	1.601	0.043	31.516
Indus	March 2011 (LF)	2.667	0.039	71.313
	October 2011 (HF)	2.689	0.026	75.722



515

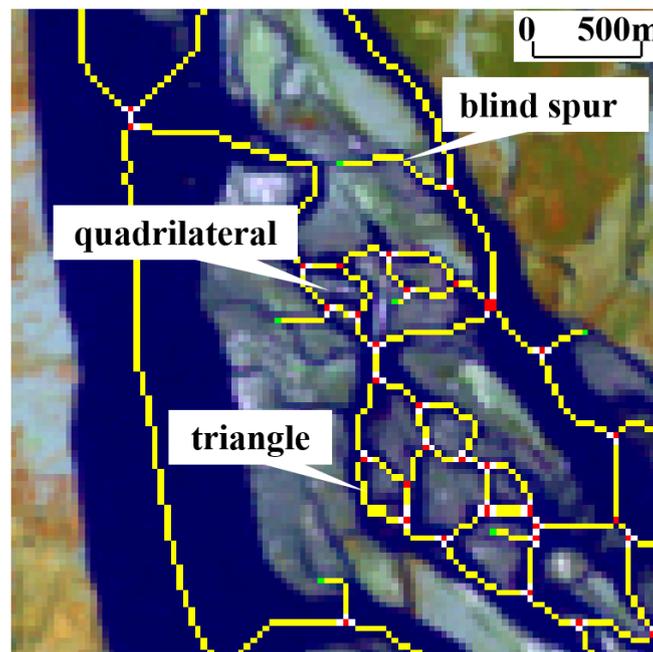


516

517 **Fig. 10.** (A) The cumulative distributions of weighted degree (WD) of the Yangtze and  
 518 Indus networks during high and low flow periods. (B) The relationship between the  
 519 weighted characteristic path length (WCPL) and node number for the Yangtze and Indus  
 520 networks. The step size of the node numbers  $\Delta n$  are 40, and 100 for Yangtze and Indus,  
 521 respectively.  
 522

523 In contrast to the maximum clustering coefficient (CC) value of 1, global  
 524 CC values for both Yangtze and Indus networks are small, implying that these

525 two river networks have a poor transitivity (Newman, 2003). This outcome may  
 526 be due to the existence of a considerable number of end nodes caused by ‘blind  
 527 spurs’ with a CC value of 0 (Fig. 11). Moreover, the clustering coefficient  
 528 assesses only the density of ‘triangle patterns’ defining sand bars and islands,  
 529 but ignores higher order structures like ‘quadrilaterals’ in multichannel networks  
 530 (Fig. 11). Due to limited computational resources, we considered only CC  
 531 values of quadrilaterals (cycles of length 4),  $C_i^4 = \frac{\sum_{j,h,k \in n} a_{ij} a_{jh} a_{hk} a_{ki}}{k_{i,nn} k_i (k_i - 1)}$ , based on the  
 532 extendibility of the expression in Table 1 (Caldarelli et al., 2004). As a result,  
 533 similar to the third-order CC, fourth order CC values of both Yangtze and Indus  
 534 networks were also small; 0.006 for the former, while 0.015 and 0.012 for the  
 535 latter during LF and HF periods.



536

537 **Fig. 11.** Illustration of blind spurs, triangles, and quadrilaterals located in the multichannel  
 538 network.  
 539

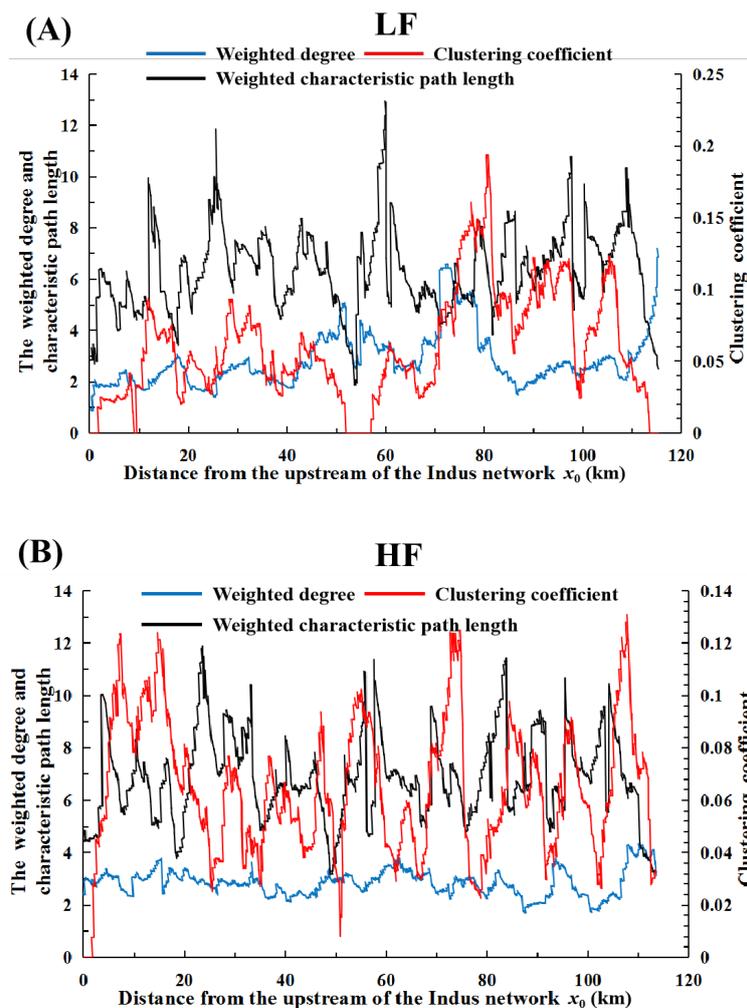
540 The weighted characteristic path length (WCPL) serves as a measurement  
 541 of the mean shortest water and sediment transport distance between pairs of  
 542 bifurcation nodes within a river network. In order to explore the relationship

543 between the WCPL value and the node numbers within river networks, an  
544 ordered subset of the nodes was used to create the sub-networks. The ordered  
545 subset starts from the most upstream node of the multichannel network, and  
546 the number of nodes in the subset increases gradually by a given step size  $\Delta n$ .  
547 The corresponding WCPL value of each sub-network of the Yangtze and Indus  
548 networks was calculated (Fig. 10B). The results indicate that WCPL remarkably  
549 scales linearly with the network scale in any study area ( $I^W \propto m$ ), although  
550 values of WCPL varies greatly between two river networks (Table 2).  
551 Additionally, as shown in Fig. 10B, the WCPL of the Indus network during high  
552 flow periods is smaller than that during low flow periods with the same number  
553 of nodes due to the larger proportion of short channels during high flow periods  
554 (Fig. 9A).

#### 555 4.4 Spatial evolution of topological measures at a sub-network scale

556 RivMACNet also examined the spatial evolution of the multichannel  
557 network (i.e., Indus network in this study) topology. The radius of the circular  
558 moving window  $R$  for assessment of local topological properties was set to 3  
559 km, slightly larger than the multichannel width of the Indus network to prevent  
560 nodes from being ignored. Fig. 12 illustrates the spatial evolution of topological  
561 measures at the subnetwork scale ( $R = 3$  km) along the multichannel centerline  
562 of the Indus during both high and low periods. In contrast to the global measures,  
563 three local measures vary along the Indus network, indicating that the Indus  
564 network topology is irregular. Furthermore, the trends in Fig. 12 are likely to be  
565 the sum of a series of sine functions of varying periods and amplitudes, rather  
566 than monotonic. Thus, a continuous wavelet transform (CWT) was used to  
567 analyze the dominant periods  $T_a$  in the spatial evolution of the Indus network

568 topology. For brevity this method is not explained here, but is reported in detail  
 569 by Kharitonenko et al. (2002) and Liang et al. (2010). The wavelet coefficients  
 570  $W_f(a, b)$  and their variance values  $Var(a)$  of the three topological measures  
 571 are illustrated in Fig. 13, which shows that the aforementioned topological  
 572 measures exhibit similar spatial evolution periods under the same flow  
 573 conditions. Although no clear long-distance trends were observed, 8 - to - 12  
 574 km periods during high flow, and 15 - to - 37 km periods during low flow are  
 575 identified shown by the white horizontal lines, implying that the Indus network  
 576 exhibits beaded planforms such that, multiple channel reaches are interspersed  
 577 with reaches with fewer channels.

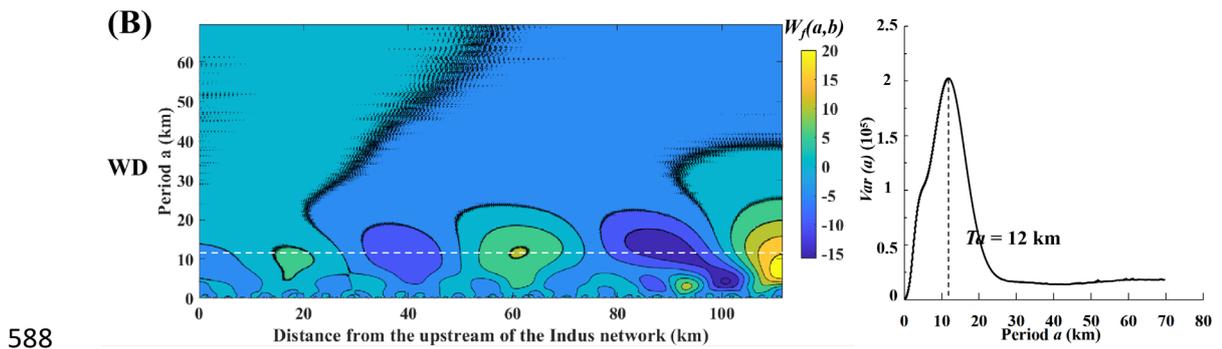
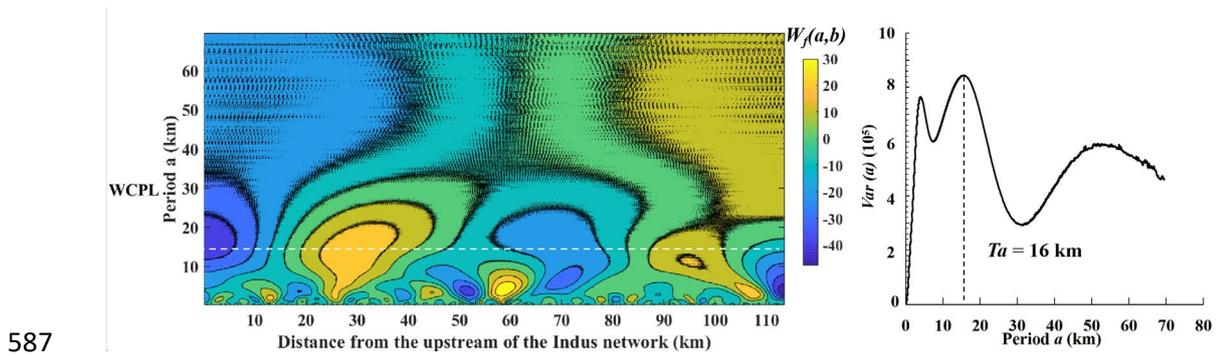
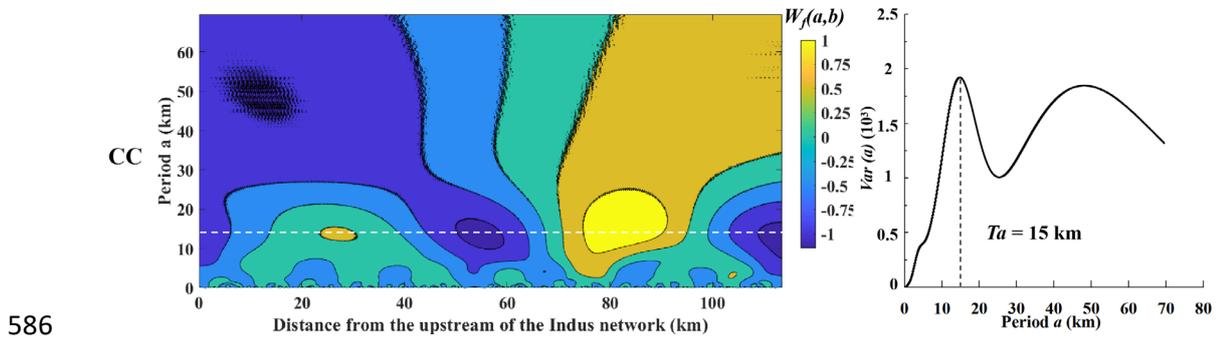
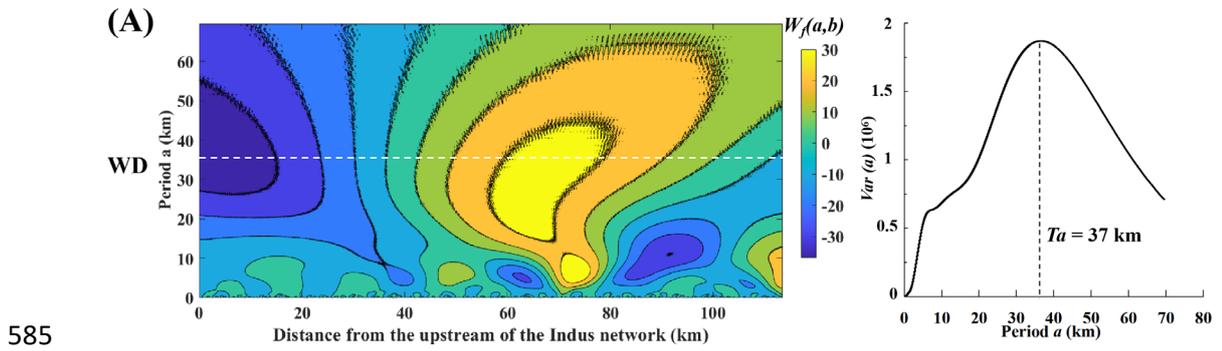


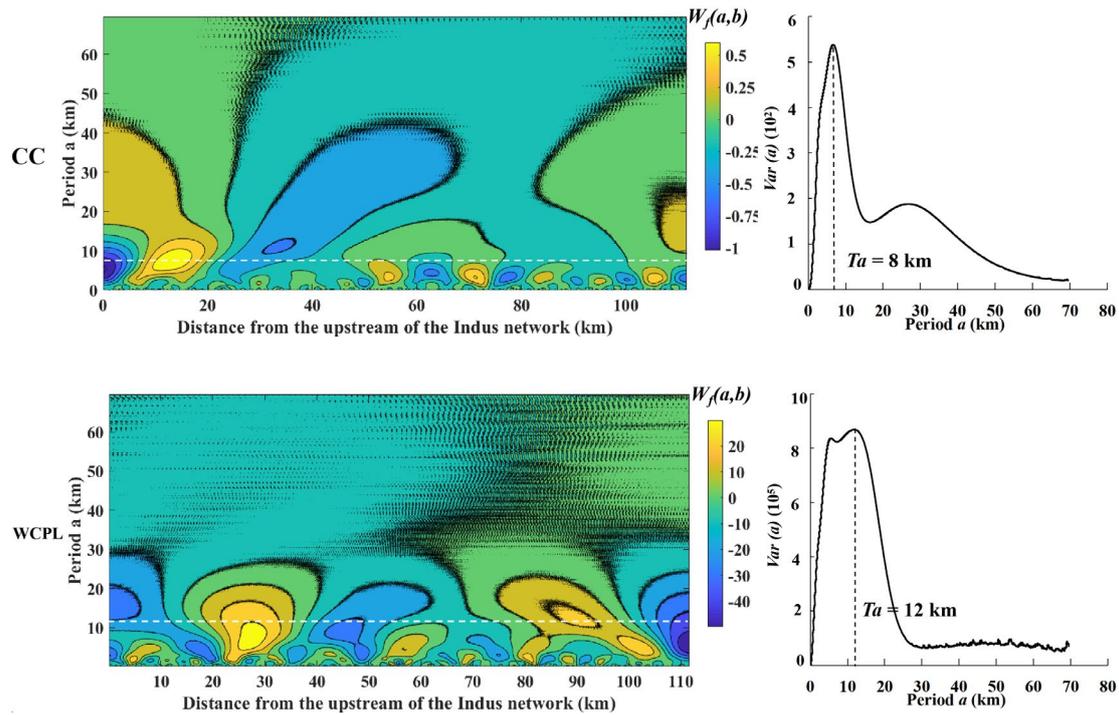
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579

580 **Fig. 12.** Spatial distributions of three topological measures, including weighted degree,  
 581 cluster coefficient and the weighted characteristic path length, at the subnetwork scale ( $R$

582 = 3 km) along the multichannel centerline of the Indus network during (A) low and (B) high  
 583 flow periods.  
 584





589

590

591 **Fig. 13.** (Left) Contours of the wavelet coefficient  $W_f(a, b)$  and (Right) its variance values  
 592  $Var(a)$  against the period of the local topological measures: weighted degree (WD),  
 593 clustering coefficient (CC) and weighted characteristic path length (WCPL) in the Indus  
 594 network during (A) low and (B) high flow periods. The dominant periods  $T_a$  in the spatial  
 595 evolution can be reflected by the maximum values of  $Var(a)$ , and marked by white  
 596 horizontal lines in the contours of  $W_f(a, b)$ .

597

## 598 5. Discussion

### 599 5.1 Reliability of RivMACNet

600 To assess the reliability and performance of RivMACNet, another popular  
 601 river analysis engine RivaMap (Isikdogan et al., 2017) was applied to the  
 602 Yangtze network and the Indus network during high flow periods (see  
 603 <http://live.ece.utexas.edu/research/rivamap/>). The three following issues were  
 604 considered when comparing these two methods:

605 1) Computation complexity. We executed the RivMACNet and RivaMap on  
 606 MATLAB R2016a using a PC (CPU: Intel Core i5-4590T at 2 GHz, RAM: 8 GB,  
 607 Windows10). It took 595s and 1527s for RivaMap to construct the topologies of  
 608 the Yangtze and Indus networks. This time period is longer than for RivMACNet

609 which took 254s and 629s, respectively. The difference between the two  
610 methods suggests that RivMACNet has a significantly higher computational  
611 efficiency and because the total time consumed will increase with the scale of  
612 river network, this difference is likely to be larger in practice.

613 2) Comparison of network topological measures. 281 nodes and 339 links  
614 were detected in the Yangtze network in RivMACNet, larger than numbers (less  
615 than 220 nodes and 240 links in the same study area) reported by Chen et al.  
616 (2019). These differences are caused by the MNDWI threshold and node  
617 detection method. Additionally, the constructed maps of the study reaches were  
618 derived using the line with length  $b_{i,j}^k$  (expression (8)) orthogonal to the channel  
619 local orientation  $slop_{i,j}^k$  (expression (6)) at each skeleton pixel (or centerline  
620 point in RivaMap) in RivMACNet. These constructed maps are shown in Fig.  
621 14 with ground reference data on water bodies presented as background. Given  
622 the ground-reference images, we calculated and compared the precision and  
623 recall of the channel images constructed by the two methods (Table 3):

$$624 \quad Precision = \frac{TP}{TP+FP}; \quad (10)$$

$$625 \quad Recall = \frac{TP}{TP+FN}; \quad (11)$$

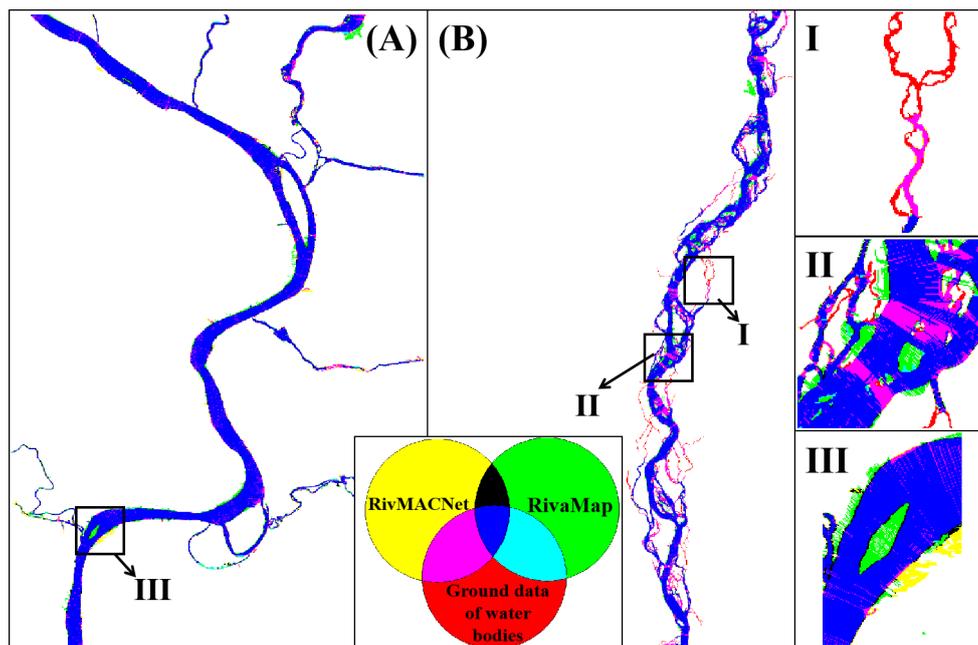
626 where  $TP$  indicates the number of pixels considered as water bodies in both  
627 ground - reference images and RivMACNet (or RivaMap), while  $FP(FN)$   
628 indicate the number of pixels considered as water bodies (non-water bodies) in  
629 ground - reference images, but non-water bodies (water bodies) in RivMACNet  
630 or RivaMap. For the Yangtze network, the precision values for RivMACNet and  
631 RivaMap are close, and the main false positives refer to isolated channels  
632 caused by the small  $MNDWI$  (red pixels in I shown in Fig. 14). However,

633 RivaMap produced a low precision in the constructed map of the Indus, missing  
 634 large slices of channels (pink pixels in II shown in Fig. 14). This lack of precision  
 635 is because RivaMap is unable to guarantee the connectivity of the constructed  
 636 channels, especially for multichannel networks with a large number of  
 637 bifurcations (Isikdogan et al, 2017). An individual channel may be cut into  
 638 several short and discontinuous channels, of which some small connected  
 639 areas were mistakenly regarded as noise and then omitted when regenerating  
 640 river channels. Additionally, values of recall of RivMACNet are slightly larger  
 641 than that of RivaMap, indicating that RivMACNet is more sensitive to the  
 642 presence of small islands and sand bars (green pixels in III shown in Fig. 14),  
 643 which play important roles in the multichannel network study.

644 **Table 3.** Precision and recall of RivMACNet and RivaMap.

	<i>Precision (%)</i>		<i>Recall (%)</i>	
	Yangtze	Indus	Yangtze	Indus
RivMACNet	98	95	91	91
RivaMap	92	71	88	87

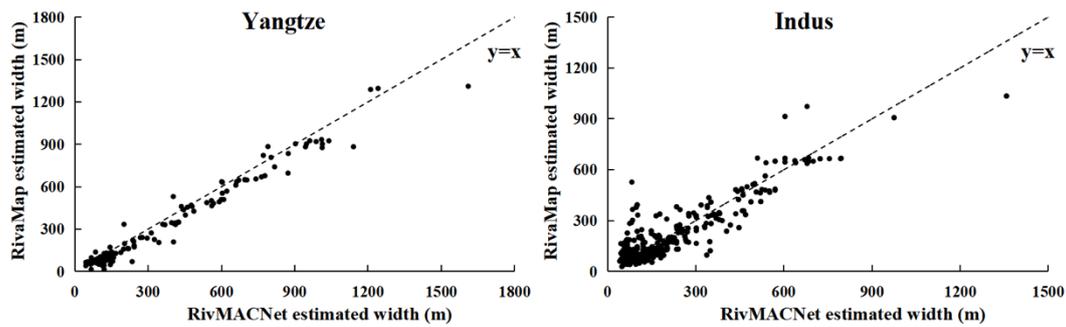
645



646

647 **Fig. 14.** Comparison between the constructed (A) Yangtze and (B) Indus maps produced  
648 using RivMACNet and RivaMap using ground reference data on water bodies as  
649 background. (I), (II), and (III) are zoomed images of (A) and (B), respectively. Sporadic  
650 water bodies in size  $Area$  ( $Area < 0.05 \times M \times N$  in this paper) are considered as noise, and  
651 have been omitted in RivaMap constructed maps and the ground data.  
652

653 3) Comparison of network planview measures. Clearly, the ability of  
654 RivaMap to derive reliable individual channel lengths and sinuosity, as well as  
655 node and link counts is limited by its poor performance in maintaining river  
656 connectivity. In this context, we considered the individual channel widths and  
657 compared these between RivaMap and RivMACNet because the extraction  
658 process is almost unaffected by network connectivity. In contrast to the  
659 centerlines of individual channels in RivaMap, the channel skeleton is applied  
660 for channel width extraction in RivMACNet. Thus, we computed the average of  
661 the width estimates for the centerline points in RivaMap that were within a given  
662 distance (herein referred to as one resolution unit) from the skeletons to ensure  
663 the same individual channels in RivMACNet were compared. In this manner,  
664 we examined 254 and 1141 individual channels of the Yangtze and Indus  
665 network, respectively, and then calculated the Spearman correlation coefficient  
666 (Spearman, 1987) of individual channel widths produced by RivMACNet and  
667 RivaMap. A significant correlation (Spearman correlation coefficients of 0.988  
668 and 0.915 for the Yangtze and Indus networks, respectively) between the two  
669 channel width datasets produced by RivMACNet and RivaMap was observed  
670 (Fig. 15), implying that river network measures (e.g., individual channel width)  
671 extracted by RivMACNet are similar to those produced by RivaMap.



672

673 **Fig. 15.** Comparison of the estimates of river width produced by RivMACNet and RivaMap.

## 674 5.2 Limitations

675 Although RivMACNet has been demonstrated to be superior in  
 676 guaranteeing the connectivity of multichannel networks, the RivMACNet results  
 677 remain limited by the quality of the input data and methods used. First, user  
 678 decisions are a central component of RivMACNet and include selecting  
 679 parameters such as a set of thresholds for *MNDWI*, the noise removal window  
 680 size  $\sigma$ , width gate  $\Delta b$ , and sub-network radius  $R$ . Although these decisions  
 681 could be made based on prior knowledge, any uncertainty associated with  
 682 these parameter values could be transferred to the outputs. Second,  
 683 RivMACNet poorly detects individual channels with width close to or less than  
 684 the spatial resolution of the input images. The same limitation also apply to  
 685 other routines such as RivaMap. Third, in contrast to field surveys, errors and  
 686 uncertainty associated with geospatial data also are important issues that need  
 687 to be considered in the river network analysis (Downward et al., 1994). The  
 688 likely sources of errors and uncertainty in RivMACNet can be summarised as  
 689 follows: (i) errors caused by transforming the longitude and latitude of the river  
 690 network in the real-world to the corresponding  $X$  and  $Y$  pixel coordinates in the  
 691 digital maps; (ii) uncertainty caused by delineating the boundaries of river  
 692 channel extent in digitized maps (Leonard et al., 2020); (iii) errors due to the

693 definition of the river network as a directed and weighted network, with the  
694 directions of its links determined simply by the distances from nodes connected  
695 to them to the upstream of the multichannel network. A 3D representation  
696 achieved using a digital elevation model (DEM) could increase accuracy,  
697 especially for lateral individual channels in river networks.

## 698 **6. Conclusions**

699 We presented a new automatic multichannel network analysis method  
700 called RivMACNet for: (i) constructing multichannel network topology; (ii)  
701 calculating geometrical measures including individual channel lengths, widths  
702 and sinuosities and (iii) calculating topological measures including the weighted  
703 degree (WD), clustering coefficient (CC) and weighted characteristic path  
704 length (WCPL) at the reach and subnetwork scales. The method used, as input,  
705 satellite sensor images of *MNDWI*, although other variable inputs are possible.  
706 We tested RivMACNet on the meandering reach of the Yangtze River near  
707 Wuhan, and the braided reach of the Indus River, Pakistan, and analyzed their  
708 topological properties at different scales.

709 Comparison between RivMACNet and other alternative conventional  
710 methods demonstrated that RivMACNet is a reliable tool for assessing and  
711 analyzing multichannel topology because: (i) RivMACNet is more sensitive to  
712 islands and sand bars located in multichannel rivers; (ii) RivMACNet has a  
713 higher computational efficiency and precision and (iii) RivMACNet can maintain  
714 network connectivity.

715 Network analysis of reaches of the Yangtze and Indus Rivers indicated that  
716 multichannel networks exhibited a strong linear, but beaded (Meshkova and  
717 Carling, 2013) planview such that reaches with multiple parallel channels are

718 interspersed with reaches with fewer, or only one channel. The topological  
719 measures (*e.g.*, WD, CC and WCPL in this study) at the reach scale were found  
720 to be independent of discharge. The small CC values imply poor transitivity in  
721 both Yangtze and Indus networks. Additionally, the dominant topological scale  
722 of the Indus network varied periodically along the river reach (8 - to-12 km for  
723 HF periods, and 15 – to - 37 km for LF periods).

724 The proposed RivMACNet method has considerable application prospect  
725 for the analysis of complex river networks, providing a new lens through which  
726 to analyze river network behaviour. In the future, research should focus on other  
727 multichannel networks using time-series datasets and compare the similarities  
728 and differences between topological measures characterizing these  
729 multichannel networks in nature, with the general aim to discover the physical  
730 bases of river networks.

### 731 **Authorship statement**

732 LYH developed the conception and design of study, wrote the necessary  
733 scripts, performed analysis, and wrote the manuscript. PMA helped in the  
734 conception and design of study, guided the methodological analysis, helped  
735 write the manuscript, and revised the manuscript critically for important  
736 intellectual content. PAC originated the conception and design of study, guided  
737 the analysis, helped write the manuscript, and revised the manuscript critically  
738 for important intellectual content. WYJ acquired the data, and participated in  
739 the analysis. JEH acquired the data, and participated in the analysis.

### 740 **Declaration of competing interest**

741 The authors declare that they have no known competing financial interests  
742 or personal relationships that could have appeared to influence the work  
743 reported in this paper.

#### 744 **Computer code availability**

745 RivMACNet are available at <http://github.com/lyh444/RivMACN.git>. The  
746 code developer was Yanhui Liu (Address: Hohai University, Nanjing, China.  
747 Contact number: + 86 - 15850553774. E-mail address: [liuyanhui@hhu.edu.cn](mailto:liuyanhui@hhu.edu.cn).)  
748 RivMACNet is implemented and tested in MATLAB R2016a. Everyone is  
749 granted permission to copy, modify and redistribute this code, but under the  
750 condition that the original algorithm copyright is preserved.

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