A Money-Financed Fiscal Stimulus: Redistribution and Social Welfare

By CHIARA PUNZO and LORENZA ROSSI*

April 28, 2022

Abstract

We analyze the redistribution channel of a money-financed (MFFS) versus debt-financed fiscal stimulus (DFFS) in a Borrower-Saver framework. The redistribution channel is larger when we consider a MFFS and borrowers are the main beneficiaries. A liquidity trap scenario amplifies the differences between a MFFS and a DFFS. The redistribution channel makes a MFFS effective at having an expansionary effect in the medium run, despite the adverse scenario. We show, however, that a MFFS increases the consumption gap between the two agents by redistributing income from savers to borrowers. Thus, a MFFS results detrimental for welfare when the welfare function is approximated around the efficient steady state.

*Chiara Punzo, Banque de France - e-mail: chiara.punzo@banque-france.fr and Lorenza Rossi, Department of Economics and Management, University of Pavia, via San Felice, 5, 27100 Pavia, Italy, e-mail: lorenza.rossi@unipv.it (correspoding author). The authors thank Guido Ascari, Florin Bilbiie, Andrea Boitani, Fabrice Collard, Davide Debortoli, Jordi Galì, Tommaso Monacelli and Salvatore Nisticò, two anonymous referee and seminar participants at the 22nd Conference on "Theories and Methods in Macroeconomics", the 24th Conference on "Computing in Economics and Finance", the 5th Macro, Banking and Finance Workshop, the Conference on "Finance and Economic Growth in the Aftermath of the Crisis", the 22nd Annual Workshop on Economic Science with Heterogeneous Interacting Agents held at the Università Cattolica del Sacro Cuore, the Catalan Economic Society Conference, the 8th RCEA Macro-Money-Finance Workshop and the XXV Rome Conference on Money, Banking and Finance for valuable insights. The views expressed in this paper are those of the authors and do not reflect those of the Banque de France.

JEL CODES: E3; E5; E62

KEYWORDS: Borrowers-Savers, Fiscal Stimuli, Welfare, Fiscal Multipliers

1 INTRODUCTION

The current health and economic crisis has called for a urgent fiscal intervention in a scenario in which debt ratios were already large despite policy rates hit their zero lower bound for a relatively long time now.

Giavazzi and Tabellini (2020) proposed the issue of irredeemable or very long maturity Eurobonds backed by the ECB to keep the financing burden low. Galì (2020b) proposed to provide struggling firms with unrepayable central bank funding, without raising their financial liabilities. In the same vein, Galì (2020a) and English et al. (2017) analyzed a MFFS. Galì (2020a) analyzed the effects of a MFFS and compared them with those resulting from a conventional DFFS, showing the stronger effectiveness of a MFFS. Also it showed that the difference in effectiveness of the two stimuli persists but it is much smaller in a zero lower bound (ZLB) scenario. In the same framework, English et al. (2017) highlighted how money-financed fiscal programs, if communicated successfully and credibly, could provide a significant stimulus in a Representative Agent New-Keynesian model (RANK, hereinafter).

The above mentioned papers are mainly concerned by the aggregate effects of a MFFS and are confined to a representative agent setting. They therefore ignore the potential redistribution channel of a MFFS. However, empirical evidence[1] on monetary stimuli shows large redistributive effects if they are not

[1] Sterk and Tenreyro (2018) provide empirical evidence of the non-Ricardian effects of unconventional monetary policy and fiscal policy interaction, due to the redistribution channel. They document a substantial response of public debt to a monetary policy shock. Kaplan et al.(2018) argue that the aggregate effect of monetary policy shocks depends on the type of fiscal policy reaction. In addition, if low-income agents more than proportionately benefit

compensated by effects of opposite sign triggered by a fiscal stimulus.

The main interest of the paper lies in the redistribution channel of a money-financed versus debt-financed fiscal stimulus in a heterogenous agent economy, how this channel can alter the results in an adverse scenario (see Galì 2020a) and how it influences welfare. Indeed, policy debate in the aftermath of the Covid crisis was whether to implement a MFFS in exceptional periods, with very low output and a binding ZLB constraint on nominal interest rate.

We model the redistribution channel via Borrower-Saver framework à la Bilbiie et al. (2013). The two agents differ in their degree of impatience, they are both intertemporal maximizers so that borrowing and lending take place in equilibrium, and financial markets are imperfect. Borrowers face a suitable defined borrowing limit, and it is important to highlight that, differently from the standard rule of thumb framework, the distribution of debt/saving across agents is endogenous. Thus, the paper analyzes the redistribution effects of a money-financed versus debt-financed government expenditure increase - which has more uniform effects on the two agents than tax cuts would have - and it investigates how these effects can influence welfare. To isolate the redistribution channel, we also keep as benchmark the analysis of such stimuli in a standard RANK model (Galì, 2020a).

We compare the redistribution channel of a MFFS versus the one of a DFFS - both in normal times and at the zero lower bound and, for the welfare analysis, we derive the second order approximation of the welfare-loss function using the Linear Quadratic method of Woodford (2002) and Benigno and Woodford (2003). In line with Ferrero et al. (2018), we show that, in a Borrowers-Savers framework, the loss function not only depends on output gap and inflation

from increases in aggregate income - as suggested by Coibion et al.(2017) - the earnings heterogeneity channel also amplifies the effects of a monetary and fiscal interaction, as a MFFS. See also Auclert (2019), Doepke and Schneider (2006) and Adam and Zhu (2016) among others.

but also on the consumption gap between borrowers' and savers' consumption. This implies that any redistributive policy may either reduce or increase welfare depending on its effect on the consumption gap. Ceteris paribus, as long as the policy is able to reduce the consumption gap, welfare increases. If instead the policy increases the gap, welfare reduces. We show that a MFFS increases the consumption gap between the two agents by redistributing income from savers to borrowers. Thus, a MFFS results detrimental for welfare when the welfare function is approximated around the efficient steady state and the two agents have the same steady state consumption[2].

More in details, we find that a MFFS is able to redistribute from savers to borrowers around one and a half times what a DFFS does. This result can be explained by two effects that compose the redistribution channel, the wealth effect and the substitution effect. The unexpected increase in income - due to the fiscal stimulus - is higher in a MFFS than in a DFFS, because of the different behavior of real interest rate. In a MFFS, the decrease of real rate generates both a positive wealth effect in favor of borrowers and a positive substitution effect on the saver's consumption as well as on the borrower's consumption. Instead, in a DFFS, the increase of real rate has both a negative wealth effect on borrowers and a negative substitution effect on saver's consumption.

Finally, differently from Galì (2020a), we show that a liquidity trap scenario amplifies the differences between MFFS and DFFS because, differently from normal times, both interest and inflation paths implied by the two financing regimes are much different. The wealth and substitution effects explain how the redistribution channel influences the effectiveness of policy combinations. The redistribution channel makes a MFFS effective at having an expansionary effect in the medium run, despite the adverse scenario. In fact, under a

[2] Otherwise, the loss function would be different and it would include some linear term related to steady state consumption inequality [see Bilbiie and Ragot (2020)].

money-financing regime, the presence of borrowers exerts an upward pressure on inflation that is absent in a RANK model (see Galì, 2020a).

The outline of the paper is as follows. Section 2 sets out the baseline model. In Section 3, we formally present the alternative combinations of monetary and fiscal policies that are object of our analysis. Section 4 presents the comparison between the redistributive dynamics of a MFFS and the ones of a DFFS in normal times and in liquidity trap, while Section 5 provides an analytical and numerical welfare analysis of these stimuli.

2 THE MODEL ECONOMY

We build a DSGE model that follows closely Bilbiie et al. (2013)[3]: it features heterogeneous agents, who differ in their degree of impatience, and imperfect financial markets. Both agents are intertemporal maximizers - so that borrowing and lending take place in equilibrium - but a fraction of agents face a suitably defined borrowing limit. In addition, the distribution of debt/saving across agents is endogenous. This setup is labelled Borrower-Saver model[4]. Below we introduce the key details of the model. All the equations characterizing the equilibrium of the economy are reported in Table(1) .

2.1 *Households*

All households have preferences defined over private consumption, $c_{\iota,t}$, real balances, $m_{\iota,t} = M_{\iota,t}/P_t$,and labor services, $n_{\iota,t}$, according to the following sepa-

[3]This model is a variant of the RBC-type borrower-saver framework proposed by Kiyotaki and Moore(1997), and extended to a New Keynesian environment by Iacoviello(2005) and Monacelli(2009). See also, Eggertsson and Krugman(2012) and Monacelli and Perotti(2011).

[4]See also Mankiw (2000) for a slightly different model in which only one agent optimizes intertemporally, and coexists with a *myopic* agent, who merely consumes her income - it is labelled savers-spenders model for fiscal policy. The classic savers-spenders model has been extended by, among others, Galì et al.(2007) and Bilbiie(2008) to include nominal rigidities and other frictions to study questions ranging from the effects of government spending to monetary policy analysis and equilibrium determinacy.

rable period utility function,

$$\ln\left(c_{\iota,t}\right)-\frac{n_{\iota,t}^{1+\varphi}}{1+\varphi}-\frac{\chi_{\iota}}{1+\sigma}\left(\bar{x}-\frac{M_{\iota,t}}{c_{\iota,t}^{a}}\right)^{1+\sigma}, \text{ with } \varphi>0 \text{ and } \sigma>0.$$

The agents differ only in their discount factors $\beta_{\iota} \in (0,1)$ and weight of real balances $\chi_{\iota} \geq 0$. Specifically, we assume that there are two types of agents $\iota = s, b$, $\beta_s > \beta_b$ and $\chi_s > \chi_b = 0$[5]. Following English et al. (2017), the final term of the equation implies that real balances - expressed as a ratio to ι's consumption - are valued at the margin until reaching a stochastic bliss point of $\bar{x}$. The scaling factor is aggregate consumption of each type of agents, $c_{\iota,t}^{a}$, which is taken as given by household; this formulation implies that the consumption Euler equation doesn't depend on the level of real balances, consistent with most empirical analysis.

$1-\lambda$ is the share of patient households: we label them savers, discounting the future at β_s.Consistent with the equilibrium outcome (discussed below) that patient agents are savers (and hence will hold the bonds issued by impatient agents), we impose that patient agents also hold all the shares in firms and money holdings. Each saver chooses consumption, hours worked, money holdings[6] and asset holdings (bonds and shares), solving the intertemporal problem subject to the sequence of constraints.

[5] The assumption $\chi_b = 0$ ensures that impatient agents are net borrowers at all times. The working paper version derives endogeneouly the borrowers money demand and shows that it is equal to zero.

[6] Given the opportunity cost of holding money balances when the (net) interest rate is positive, real money demand (expressed relative to consumption) is less than its satiation level $\bar{x}$. As in Eggertsson and Woodford (2003), the money demand function is continuous at $i_t = 0$ with $\frac{M_{s,t}}{C_{s,t}} \geq \bar{x}$ if $i_t = 0$. Under log utility over consumption, real money balances vary directly with consumption with a unit coefficient.

$$
\begin{aligned}
c_{s,t} + b^h_{s,t} + a_{s,t} + \Omega_{s,t} v_t + m_{s,t} \quad \leq \quad & \frac{1+i_{t-1}}{\pi_t} b^h_{s,t-1} + \frac{1+i_{t-1}}{\pi_t} a_{s,t-1} \\
& + \Omega_{s,t-1} \left(v_t + \Pi_t \right) \\
& + \frac{m_{s,t-1}}{\pi_t} + w_t n_{s,t} - \tau_{s,t}
\end{aligned}
$$

where w_t is the real wage, $a_{s,t}$ is the real value of total private assets (π_t is the gross inflation rate), a portfolio of one-period bonds issued in $t-1$ on which the household receives nominal interest rate, i_t. v_t is the real market value at time t of shares in intermediate good firms, Π_t are real dividend payoffs of these shares, $\Omega_{s,t}$ are share holdings, $\tau_{s,t}$ are per capita lump-sum taxes paid by the saver, and $b^h_{s,t-1}$ are the savers' holdings of real public bonds which deliver the same nominal interest rate as private bonds.

The rest of the households on the $[0, \lambda]$ interval are impatient (and will borrow in equilibrium, hence we index them by b for borrowers). They face the intertemporal constraint:

$$
c_{b,t} + a_{b,t} + m_{b,t} \leq \frac{1+i_{t-1}}{\pi_t} a_{b,t-1} + \frac{m_{b,t-1}}{\pi_t} + w_t n_{b,t} - \tau_{b,t}
$$

as well as the additional borrowing constraint[7] (on borrowing in real terms) at all times t :

$$
-a_{b,t} \leq \bar{d}.
$$

[7] The Lagrangian multiplier associated to the borrowing constraint, ψ_t, takes a positive value whenever the constraint is binding. Indeed, because of our assumption on the relative size of the discount factors, the borrowing constraint will bind in steady state.

2.2 *Firms*

There are infinitely many firms indexed by z on the unit interval [0,1], and each of them produces a differentiated variety of goods. Following Rotemberg(1982), we assume that firms face quadratic price-adjustment costs, $\frac{\theta}{2}\left(\frac{p_t(z)}{p_{t-1}(z)}-1\right)^2$, expressed in the units of consumption goods and $\theta \geq 0$[8]. Assuming that firms discount at the same rate as savers implies that $Q_{t,t+i} = \beta^s \frac{c_{s,t}}{c_{s,t+i}\pi_{t+i}}$. Each firm faces the following demand function: $y_t(z) = \left(\frac{p_t(z)}{p_t}\right)^{-\varepsilon} y_t^d$, where y_t^d is aggregate demand and it is taken as given by firm z.

2.3 *The fiscal and monetary policy framework*

The government - henceforth understood as combining the fiscal and monetary authority, acting in a coordinated way - is assumed to finance its expenditures through three sources: (i) lump-sum taxes, (ii) the issuance of riskless one-period bonds with a nominal yield i_t, which are held only by savers and (iii) the issuance of (non-interest bearing) money[9]. Let $\hat{b}_t = \frac{B_t - B}{Y}, \hat{g}_t = \frac{G_t - G}{Y}$, and $\hat{\tau}_t = \frac{\tau_t - \tau}{Y}$ denote, respectively, deviations of government debt, government purchases, and taxes from their steady state values, expressed as a fraction of steady state output. In what follows we interpret B as an exogenously given long run debt target (denoted by $b \equiv B/Y$ when expressed as a share of steady state output). Thus, we introduce a fiscal rule, according to which tax variation is endogenous and varies in response to deviations of the debt ratio from its long run target.

2.4 *Equilibrium*

In an equilibrium of this economy, all agents take prices as given (with the exception of monopolists who reset their price in a given period), as well as the

[8] The benchmark of flexible prices can easily be recovered by setting the parameter $\theta = 0$.

[9] $\left(M_{t+1} - \frac{M_t}{\pi_t}\right)$ represents period t's seigniorage, i.e. the purchasing power of newly issued money

evolution of exogenous processes. A rational expectations equilibrium is then (as usual) a sequence of processes for all prices and quantities introduced above such that the optimality conditions hold for all agents and all markets clear at any given time t. Private debt is in zero net supply $\int_0^1 a_{\iota,t} = 0$, and hence, since agents of a certain type make symmetric decisions: $\lambda a_{b,t} + (1-\lambda)\, a_{s,t} = 0$. Equity market clearing implies that share holdings of each saver are: $\Omega_{s,t+1} = \Omega_{s,t} = \Omega = \frac{1}{1-\lambda}$. Finally, by Walras' Law the goods market also clear. All bonds issued by the government will be held by savers. And, considering that $\chi_b = 0$, all money issued by the government will be also held by savers.

3 MONEY *VERSUS* DEBT-FINANCED FISCAL STIMULUS

We use the model to analyze the effects of an increase in government purchases under two financing schemes: debt and money financing.

The intervention takes the form of an exogenous increase in government purchases. We assume that such fiscal stimulus follows the exogenous AR(1) process:

$$\hat{g}_t = \rho_g \hat{g}_{t-1} + \varepsilon_t^g. \tag{1}$$

.where $\hat{g}_t = \frac{G_t - G}{Y}$ denotes the deviation of government purchases from its steady state value, expressed as a fraction of steady state output. ρ_g measures the persistence of the exogenous fiscal stimulus, and ε_t^g is its innovation.

Before undertaking that analysis, it is useful to note that, in the special case of fully flexible prices, $\theta = 0$, and separable real balances, the effects of a fiscal stimulus on real variables (other than real balances) are independent of the financing method, when either labour supply is inelastic or steady-state consumption of savers and borrowers are equal - regardless of how high the fraction

Description	*Equations*
Budget Constraint, S	$c_{s,t} + b^h_{s,t} + a_{s,t} + \Omega_{s,t} v_t + m_{s,t}$
	$\leq \frac{1+i_{t-1}}{\pi_t} b^h_{s,t-1} + \frac{1+i_{t-1}}{\pi_t} a_{s,t-1}$
	$+\Omega_{s,t-1}\left(v_t + \Pi_t\right) + \frac{m_{s,t-1}}{\pi_t}$
	$+w_t n_{s,t} - \tau_{s,t}$
Euler equation for bond, S	$c^{-1}_{s,t} = \beta_s E_t \left(\frac{1+i_t}{\pi_{t+1}} c^{-1}_{s,t+1}\right)$
Euler equation for share holdings, S	$v_t = \beta_s E_t \left[\frac{c_{s,t}}{c_{s,t+1}}\left(v_{t+1} + \Pi_{t+1}\right)\right]$
Labor supply, S	$n^{\varphi}_{s,t} c_{s,t} = w_t$
Money demand, S	$\chi\left(\bar{x} - \frac{m_{s,t}}{c_{s,t}}\right)^{\sigma} = \frac{i_t}{1+i_t}$
Labor supply, B	$n^{\varphi}_{b,t} c_{b,t} = w_t$
Euler equation, B	$c^{-1}_{b,t} = \beta_b E_t \left(\frac{1+i_t}{\pi_{t+1}} c^{-1}_{b,t+1}\right) + \psi_t$
Production function	$y_t = n_t$
Firm's profits	$\Pi_t = y_t - w_t n_t - \frac{\theta}{2}\left(\frac{\pi_t}{\pi_{t-1}} - 1\right)^2$
Labor demand	$mc_t = w_t$
Phillips curve	$\pi_t\left(\pi_t - 1\right) = \beta^s E_t \left[\frac{C_{s,t}}{C_{s,t+1}} \pi_{t+1}\left(\pi_{t+1} - 1\right)\right]$
	$+\frac{\varepsilon N_t}{\theta}\left(mc_t - \frac{\varepsilon - 1}{\varepsilon}\right)$
Government's *consolidated* real budget constraint	$g_t + \frac{1+i_{t-1}}{\pi_t} b_{t-1} = b_t + \tau_t + \left(m_t - \frac{m_{t-1}}{\pi_t}\right)$
Fiscal rule	$\hat{\tau}_t = \phi_B \hat{b}_{t-1}$
Labor market clearing condition	$n_t = \lambda n_{b,t} + \left(1 - \lambda\right) n_{s,t}$
Resource constraint	$y_t = c_t + g_t + \frac{\theta}{2}\left(\pi_t - 1\right)^2$
Aggregate consumption	$c_t \equiv \lambda c_{b,t} + \left(1 - \lambda\right) c_{s,t}$
Aggregate tax	$\tau_t \equiv \lambda \tau_{b,t} + \left(1 - \lambda\right) \tau_{s,t}$
Market clearing for money	$\left(1 - \lambda\right) m_{s,t} = m_t$
Market clearing for public debt	$\left(1 - \lambda\right) b^h_{s,t} = b_t$

Table 1: Summary of the Model

of borrowers λ and how tight the debt constraint $\bar{d}$ are[10]. That irrelevance result is a consequence of Ricardian equivalence, given the assumption of lump-sum taxes, combined with money neutrality, which follows from price flexibility and separability of real balance (see Bilbiie et al., 2013 and Galì, 2020a). In our sticky price model instead the Ricardian equivalence does not hold.

3.1 *Money-financed fiscal stimulus (MFFS)*

In the present paper we analyze the redistribution channel of a MFFS, and its impact on the aggregate effectiveness of the stimulus itself. The stimulus we investigate requires neither an increase in the stock of government debt nor higher taxes, current or future. Thus, following Galì(2020a), we define our MFFS as a regime in which seigniorage is adjusted every period in order to keep *real debt* b_t unchanged. In terms of the notation above, this requires

$$\hat{b}_t = 0 \tag{2}$$

Note that, combined with the fiscal rule, taxes need not be adjusted as a result of an increase in government purchases relative to their initial level, neither in the short run nor in the long run.

3.2 *Debt-financed fiscal stimulus (DFFS)*

As an alternative to the fiscal monetary regime described above, and with the purpose of having a benchmark, we also analyze the effects of a debt-financed fiscal stimulus in a (more conventional) environment in which the central bank

[10] In the limiting case, with flexible prices and representative agent framework, given Ricardian equivalence, the response of aggregate output to a fiscal stimulus is not affected by the path of the money supply or the nominal interest rate, and hence it is independent of the extent to which the fiscal stimulus is money-financed (see Galì, 2020a).

follows a simple interest rate rule given by

$$\log\left(\frac{1+i_t}{1+i}\right) = \phi_\pi \log E_t\left(\frac{\pi_{t+1}}{\pi}\right) \qquad \text{for all } t \tag{3}$$

where $\phi_\pi = 1.5$[11] determines the strength of the central bank's response of inflation deviations from the zero long-term target. An interest rate like (3) gives the central bank a tight control over inflation in response to a fiscal stimulus, through its choice of coefficient ϕ_π.The fiscal authority, on the other hand, issues debt in order to finance the fiscal stimulus, eventually adjusting the path of taxes in order to attain the long run debt target B, as implied by the fiscal rule. Bilbiie et al. (2013) show that, in this borrower-saver framework, with sticky prices, Ricardian equivalence always fails. This is the reason why the magnitude of the effects of a DFFS depend non-trivially on the speed of debt repayment, ϕ_B, as long as $\phi_B \in [1-\beta_s, 1]$.

4 MODEL DYNAMICS

This section is divided in four parts. First, it reports the parameterization. Second, it reports an analysis on the implied inequality in steady state. Third, it compares the model dynamics implied by a money-financed fiscal stimulus to the model dynamics implied by a debt-financed fiscal stimulus, and their respective fiscal multipliers, in normal times. Fourth, we compare the two alternative financing regimes of a government expenditure increase in liquidity trap.

[11] The coefficient ϕ_π in the interest rate rule could play a key role in these comparison. This is the reason why in Appendix A, we analyze the results under alternative values of the parameter ϕ_π.

4.1 *Parameterization*

We solve the model by taking a first order approximation around the steady state. The model parameterization is summarized in Table (2). We assume the following settings for the household related parameters in line with those of Bilbiie et al.(2013): discount factors of borrowers and savers are set respectively $\beta_b = 0.95$ and $\beta_s = 0.99$. Analogously, as in Bilbiie et al. (2013), we set the borrowing constraint $\bar{d} = 0.5$. Parameter λ, denoting the share of impatient agents, is set to 0.25.

The remaining parameters are kept at their baseline values. We assume the elasticity of substitution among goods $\varepsilon = 6$ and the curvature of labor disutility $\varphi = 1$. The model's main frictions are given by price stickiness and market power in goods market. We assume a baseline setting of $\alpha = 0.75$, an average price duration of four quarters, a value consistent with much of the empirical micro and macro evidence[12]. Further, we assume the following setting for the parameters related to money demand in utility function. The weight of real balances in utility function is set $\chi = 0.018$, in line with Annicchiarico et al.(2012). The specification of money demand implies a unitary long-run elasticity with respect to consumption. We impose a short-run interest rate semi-elasticity of money demand equal to 2.5 (when expressed at an annual rate), in line with English et al.(2017).

The fiscal parameters are in line with Galì (2020a). We set the tax adjustment parameter, ψ_b, equal to 0.02. That calibration can be seen as a rough approximation to the fiscal adjustment speed required for euro area countries, as established by the so-called *fiscal compact* adopted in 2012. With regard to the target/steady state debt ratio, b, we assume a baseline setting of 2.4, which

[12] That parameter, α, is the Calvo price parameter. In our model, we adopt Rotemberg price stickiness. That's the reason why we derive θ, Rotemberg's parameter, in function of the well-estimated $\alpha : \theta = \frac{\alpha(\varepsilon-1)Y}{(1-\alpha)(1-\alpha\beta)}$.

	Description	Value
NK Model		
β_b	Borrower's discount factor	0.95
β_s	Saver's discount factor	0.99
$\bar{d}$	SS private debt	0.5
λ	Share of impatient agents	0.25
χ	Weight of money in utility function	0.018
$\bar{x}$	Money satiation level	1
σ	short run interest rate semi-elasticity of money demand	2.5
γ	SS share government purchases in output	1/5
ρ_g	Fiscal stimulus persistence	0.5
$\hat{b}^H$	Steady state debt ratio (quarterly)	2.4
ϵ	Elasticity of substitution (goods)	6
α	Index of price rigidities	0.75
Φ_B	Debt feedback coefficient	0.02

Table 2: Baseline parameters

is consistent with the 60 percent reference value specified in EU agreements. Finally, with regard to the persistence parameter ρ_g, we choose 0.5 as a baseline setting, while the steady state share of government purchases in output equals 0.2.

4.2 *Steady State*

We focus on a deterministic steady state where inflation is zero. As the constraint binds in steady state ($\psi = c_b^{-1}\left[1 - \frac{\beta_b}{\beta_s}\right]$ whenever $\beta_s > \beta_b$), patient agents are net borrowers and steady-state private debt is $a_b \leq -\bar{d}$; by debt market clearing, then the patient agents are net lenders and their private bond holdings are $a_s \leq \frac{\lambda \bar{d}}{1-\lambda}$.

It implies that savers work less than borrowers in steady state, $n_b > n_s$; the labor inequality, $n_b - n_s$, is equivalent to 0.17 and the implied steady state share of borrowers' labor in total labor is 30%. At the same time, it implies that borrowers consume less than savers, $c_b < c_s$, and the consumption inequality, $c_s - c_b$, is equivalent to 0.13. The implied steady state share of borrowers'

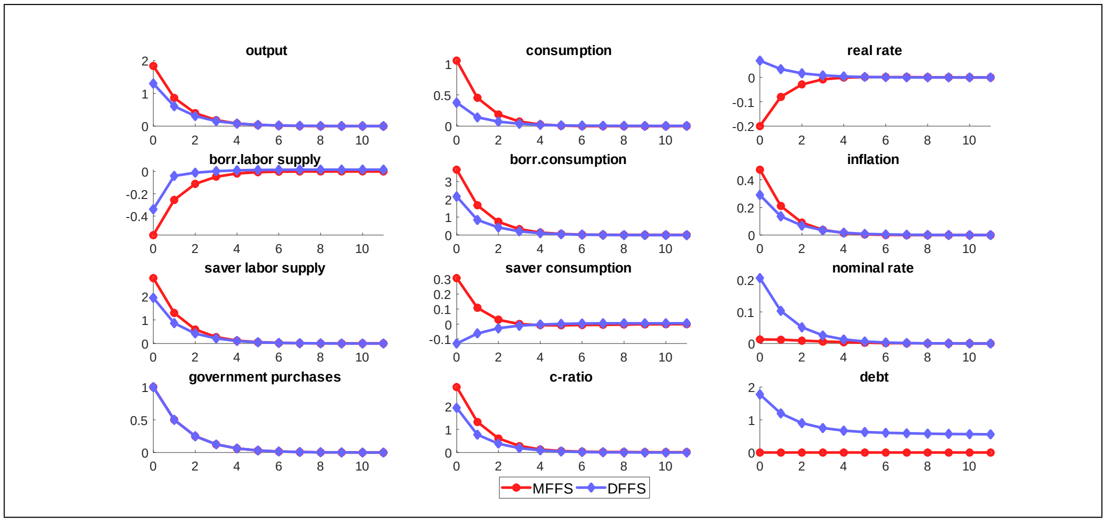

Figure 1: Dynamic Effects of an Increase in Government Purchases: Debt vs. Money Financing

consumption in total consumption is 21%.

4.3 *Normal times*

Figure (1) shows the response over time of output, inflation, debt and other relevant variables to an exogenous increase in government purchases, under the baseline calibration introduced above and ignoring the ZLB constraint. The red lines with circles show the responses under the money financing (MFFS) scheme, while the blue lines with diamonds show the response under the debt financing (DFFS).

We can observe how the redistribution channel influences the effectiveness of these combinations, particularly of our benchmark MFFS. In relative terms, if we measure the redistribution channel as the ratio of borrower's consumption over saver's consumption (c-ratio), we can observe that a MFFS has a larger effect than a DFFS. In any case, both the stimuli bring about a redistribution from savers to borrowers. However, a MFFS is able to redistribute around

150% of what a DFFS is able to do. This result can be explained by two effects that compose the redistribution channel, the wealth effect and the substitution effect. The unexpected increase in income - due to the fiscal stimulus - is higher in a MFFS than in a DFFS, because of the different behavior of real interest rate. In a MFFS, the decrease of real rate generates both a positive wealth effect in favor of borrowers and a positive substitution effect on the saver's consumption as well as on the borrower's consumption. Instead, in a DFFS, the increase of real rate has both a negative wealth effect on borrowers and a negative substitution effect on saver's consumption. If the monetary authority is assumed to pursue an independent price stability mandate or, in other words, the money supply adjusts endogenously in order to bring about the interest rate required to stabilize prices, as the blue line with diamonds show, the real interest rate will increase on impact because the inflation targeting interest rate rule implies that the nominal interest rate increases more than one to one with inflation. This is the reason why the consumption of savers declines after a DFFS differently from a MFFS.

Consequently, the expansionary effects of a MFFS are larger than those of a DFFS. Debt increases in the latter case, returning to its initial value slowly, as guaranteed by fiscal policy rule (through higher taxes). Under the money financing scheme, the larger expansion in output and consumption leads to an increase of inflation which reinforces the expansion in aggregate demand by lowering the real rate.

As shown in Figure (2), if we compare the response of output and consumption in a Two-Agent New Keynesian (TANK, hereinafter) model with the same responses in a RANK model, both a MFFS and a DFFS are more effective in a TANK model, particularly a MFFS.

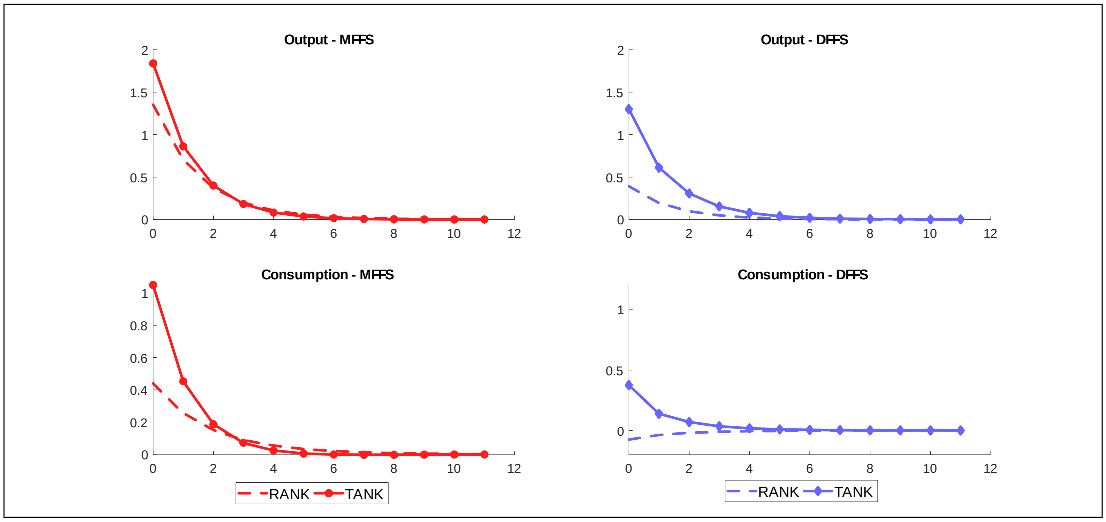

Figure 2: Dynamic Effects of an Increase in Government Purchases: Representative Agent (dotted lines) vs. Two-Agent (solid lines)

4.3.1 Multipliers

Next we discuss the sensitivity of some of the qualitative findings on the effectiveness of fiscal policy, particularly focusing on the share of borrowers. We firstly compute the fiscal multipliers of output and consumption associated to an increase in government purchases under the money-financing and the debt-financing regimes presented above. Then, in order to understand the role played by the financial constraint, we evaluate the same multipliers under different values of the steady state borrowing limit, $\bar{d}$.

To differentiate between the immediate impact of a change in fiscal spending and its long-run implications for the economy, we compute both the instantaneous and the cumulative fiscal multiplier, following Uhlig (2010).

The Instantaneous Fiscal Multiplier (IFM, hereinafter) measures, in each period, the percentage deviation of a generic variable X_t from its steady state in response to a change in government purchases that, on impact, amounts to one percent of the SS value of output. That is:

$$IFM(\hat{x}) \equiv \frac{\hat{x}_t}{\hat{g}_T \frac{\bar{G}}{\bar{Y}}}, \forall t \geq T$$

where $\hat{x}_t = \frac{x_t - \bar{x}}{\bar{x}}, \hat{g}_T = \frac{G_T - \bar{G}}{\bar{G}}$ with t being the time index for the periods following the initial fiscal shock in period T. $\bar{G}$ and $\bar{Y}$ are, respectively, the steady state values of government spending and output. In particular, we will consider the instantaneous multipliers associated to $t = T$, and we refer to them as Instantaneous Multipliers of $\hat{x}$.

As stressed in Uhlig (2010), policymakers cannot solely rely on the instantaneous multiplier since it can be misleading as it ignores the cumulated impact of the initial fiscal policy measure on the economy over time. Thus, in order to capture the cumulative impact on the variable of interest of the fiscal shock, we consider also the cumulative fiscal multiplier according to Uhlig (2010).

The Cumulative Fiscal Multiplier (CFM, hereinafter) identifies, in each period, the discounted cumulative change of a variable x_t measured in terms of percentage deviation from its steady state with respect to the discounted cumulative deviation of government spending from its steady state value. That is,

$$CFM(\hat{x}) \equiv \frac{\sum_{s=T}^{t} \bar{R}^{-(s-t)} \hat{x}_s}{\frac{\bar{G}}{\bar{Y}} \sum_{s=T}^{t} \bar{R}^{-(s-t)} \hat{g}_s}, \forall t \geq T$$

where R being the steady state of the nominal interest rate used as discount rate.

The impact and cumulative fiscal multipliers of output related to a MFFS are always greater than one, while those ones related to a DFFS need at least 20% of borrowers to be larger than one. Further, all multipliers considered increase exponentially as the share of borrowers, λ, increases, particularly the impact

λ	**MFFS**	**DFFS**
IFMC	24%	43%
CFMC	29%	46%

Table 3: Share of borrowers associated to unitary fiscal multipliers of consumption

multipliers. The multipliers of consumption, on the other hand, are larger than one for values which depend on the type of multiplier that we analyze (instantaneous or cumulative) and on the regime analyzed. Table (3) summarizes the results.

In all cases, fiscal multipliers increase as $\bar{d}$ increases. By relaxing the borrowing constraint in steady state, the borrower's consumption can increase more, and it generates higher fiscal multipliers. To sum up, under both financing schemes, fiscal multipliers are an increasing function of the share of borrowers and of the steady state borrowing limit.

4.4 In a liquidity trap

Next we explore the effectiveness of a money-financed fiscal stimulus in stabilizing the economy in face of a temporary adverse shock. The latter is assumed to be large enough to prevent the central bank from fully stabilizing output, due to a zero lower bound (ZLB) constraint on nominal interest rate. That MFFS is compared to a DFFS.

Note that under the notation introduced above the ZLB constraint takes the form $1 + i_t \geqslant 1$ for all t. The baseline experiment assumes that, $\varepsilon_t^d = \xi < 1$ for $t = 0, 1, 2, ...T$ and $\varepsilon_t^d = 0$ for $t = T + 1, T + 2, ..$In words, this describes a temporary adverse demand shock that brings the natural rate into negative territory up to period T. After period T,the shock vanishes and the natural rate returns to its initial (positive) value. The shock is assumed to be fully unanticipated but, once it is realized, the trajectory of ε_t^d and the corresponding policy responses are known with certainty.

In the case of a MFFS, the ZLB constraint can be incorporated formally in the set of equilibrium conditions listed in Table (1) by replacing the saver's money demand with the complementarity slackness condition:

$$\left(1 + i_t - \beta_s^{-1}\right)\left[\chi\left(\bar{x} - \frac{m_{s,t}}{c_{s,t}}\right)^{\sigma} - \frac{i_t}{1+i_t}\right] = 0$$

for all t, where

$$1 + i_t \geqslant 1$$

is the ZLB constraint and

$$\chi\left(\bar{x} - \frac{m_{s,t}}{c_{s,t}}\right)^{\sigma} - \frac{i_t}{1+i_t} \geqslant 0 \tag{4}$$

represents the demand for real balances. As long as the nominal rate is positive, (4) holds with equality (but it with inequality once the nominal rate reaches the ZLB and real balances overshoot their satiation level).

By contrast, in the case of a DFFS, condition (3) must be replaced with

$$(1 + i_t - 1)\left[\log\left(\frac{1+i_t}{1+i}\right) - \phi_\pi \log E_t\left(\frac{\pi_{t+1}}{\pi}\right)\right] = 0 \qquad \text{for all } t \tag{5}$$

together with

$$\left[\log\left(\frac{1+i_t}{1+i}\right) - \phi_\pi \log E_t\left(\frac{\pi_{t+1}}{\pi}\right)\right] = 0 \qquad \text{for } t = T+1, T+2, .. \tag{6}$$

Thus, the zero inflation target is assumed to be met once the shock vanishes; until that happens the nominal rate is assumed to be kept at the ZLB, i.e. $1 + i_t = 1$ for $t = 0, 1, 2, ...T$

Money-financed and debt-financed scenarios are analyzed next as a response to the demand shock described above. We assume $\varepsilon_t^d = -0.02$[13] and $T = 6$. Thus, the experiment considered corresponds to an unanticipated drop of the natural interest rate to -1% for five quarters, and a subsequent reversion back to the initial value of $+1\%$.

We start by considering the benchmark case of no fiscal response to the shock (i.e. $\hat{g}_t = 0$, for $t = 0, 1, 2, ...$) and with monetary policy described by (5) and (6). The solid black line (with crosses) in Figure (3) shows the economy's response to the adverse demand shock in the absence of a fiscal response. The ZLB constraint prevents the central bank from lowering the nominal rate to match the decline in the natural rate. As a result, the adverse demand shock triggers a significant drop in output and inflation. Note also that real debt increases considerably due to the rise in real interest rates, which increases the government's financial burden. Once the natural rate returns to its usual value, inflation and the output gap are immediately stabilized at their zero target, with debt gradually returning to its initial value through the (endogenous) increase in taxes as implied by the government's consolidated real budget constraint.

The blue line (with diamonds) and red line (with circles) show the equilibrium paths for the different variables when the fiscal authority responds to the adverse demand shock by increasing government expenditure, financing the resulting deficit through debt or money issuance, respectively. In either case the size of the government expenditure is assumed to amount to 1 percent of steady state output, and to last for the duration of the shock ($\hat{g}_t = 0.01$, for $t = 0, 1, 2, ...5$). We see that a debt-financed increase in government purchases is not very effective at dampening the negative effects of the adverse demand shock on output and inflation. The presence of borrowers amplifies the negative

[13] With the exception of a two-agent model with money-financed fiscal stimulus, where the shock must be equal to $\varepsilon_t^d = -0.07$ and $T = 6$, to generate the same fall in the interest rate analyzed in the other cases for comparative purposes.

effect on the economy of an adverse demand shock for two reasons, as shown on the right side of Figure (**4**). The unexpected decrease in the price level, on one side, revalues nominal balance sheets with nominal creditors gaining and nominal debtors losing (wealth effect) and, on the other side, it causes an increase in real rates which makes the borrowing constraint even more stringent (substitution effect). When the increase in government purchases (of the same size) is money-financed its impact on output and inflation is larger than under the debt-financing case, after the second quarter[14]. In a opposite way, the greater effectiveness of money-financing in the liquidity trap scenario can be traced to the unexpected increase in the price level, due to the accumulated liquidity resulting from the money-financing rule and the upward pressure on inflation exerted by borrowers that is absent in a RANK model, as shown on the left side of Figure (**4**). It revalues nominal balance sheets with nominal creditors losing and nominal debtors gaining (wealth effect) and, on the other side, it causes a decrease in real rate which makes the borrowing constraint less stringent (substitution effect). The interest and inflation paths implied by the two financing regimes are much different in a liquidity trap, while only the interest path implied by the two financing regimes is much different in normal times. The wealth and substitution effects explain how the redistribution channel (approximated by c-ratio) influences the effectiveness of policy combinations. We can observe that, after the second quarter, a MFFS redistributes from savers to borrower, even when the natural rate returns to its usual value. By contrast, a DFFS redistributes at all times from borrowers to savers and the consumption gap is closed when the natural rate returns to its usual value. The redistribution channel makes a MFFS more effective than a DFFS at dampening the negative effects of an adverse demand shock and implies an expansionary effect in the

[14] In the first quarter, $b_t^h = 0$ and the decline on real wage lead savers to decrease their labor supply on impact (wealth effect).

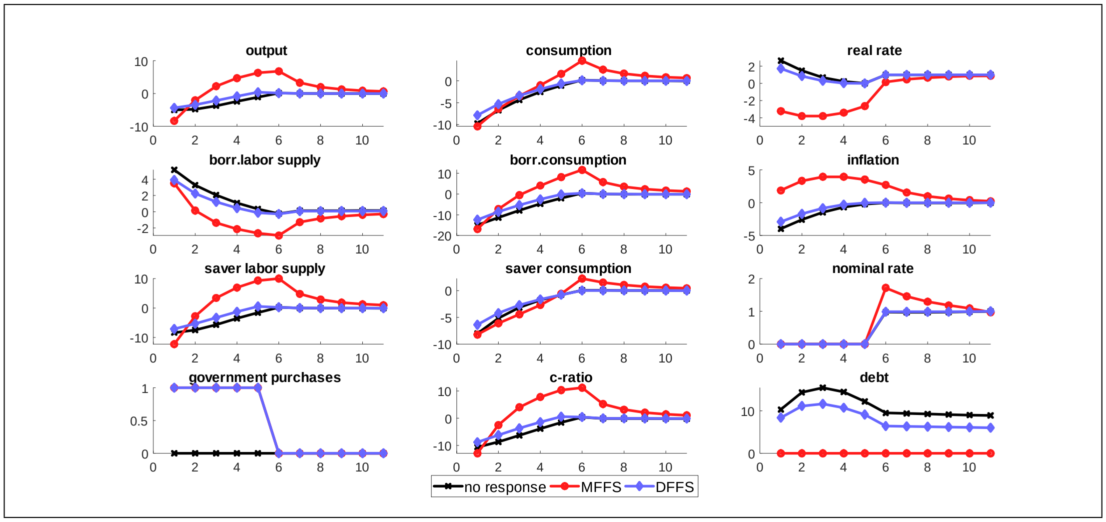

Figure 3: Dynamic Effects of an Increase in Government Purchases in a Liquidity Trap: Debt vs. Money Financing

medium run.

We can therefore summarize the result as follows. *A liquidity trap scenario amplifies the differences between MFFS and DFFS. A MFFS in a TANK model is able of having an expansionary effect, despite the adverse scenario. This is not valid in a RANK model (see Galì, 2020).*

5 WELFARE

This section provides a welfare evaluation of the two alternative financing regimes of a government expenditure increase, the money-financing regime and the debt-financing. A formal evaluation of the performance of a MFFS with respect to a DFFS requires the use of some quantitative criterion. Following the seminal work of Woodford (2002) and Benigno and Woodford (2003), we adopt a welfare-based criterion, relying on a second-order approximation of the utility losses due to the deviations from the efficient allocation. In particular, in line

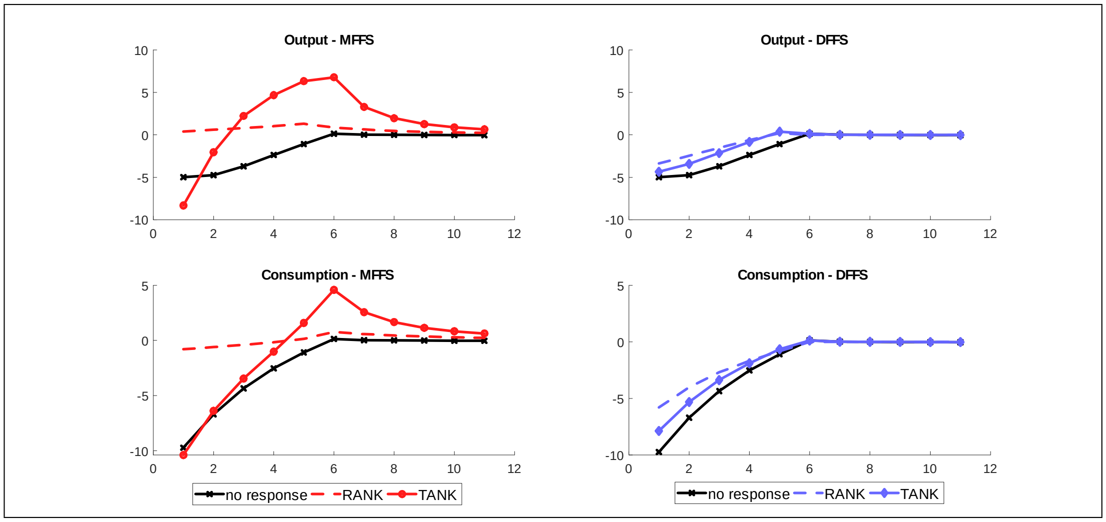

Figure 4: Dynamic Effects of an Increase in Government Purchases in a Liquidity Trap: Representative Agent vs. Two-Agent Model.

with Ferrero et al. (2018), we derive the welfare-based loss function of the average per-period utility functions of borrowers and savers in an utilitarian perspective, by weighting the utility of each type of agent according to their share in the population. Further, we assume that the policy maker discounts the future by using savers' discount factor. The second order approximation yields the following welfare-loss function:[15]

$$\widetilde{W}_t \simeq \frac{1}{2} E_0 \beta_s^t \sum_{t=0}^{\infty} \left[\gamma_x \widetilde{x}_t^2 + \gamma_C \widetilde{c}_t^2 + \gamma_\pi \pi_t^2 \right] + t.i.p. \quad (7)$$

where welfare losses are expressed in terms of the equivalent permanent consumption, measured as a fraction of steady state consumption. Notice that, as in Ferrero et al. (2018), the loss function not only depends on output gap and inflation but also on the consumption gap between borrowers' consumption and savers' one. The term $\widetilde{c}_t = \widehat{c}_t^b - \widehat{c}_t^s$ is indeed the gap between borrowers' consump-

[15] Technical details on the derivation of the objective function are left to the Appendix.

tion and savers' one; $\widetilde{x}_t = y_t - y_t^{Eff}$ measures the output gap between the actual output and the efficient equilibrium output[16] and π_t measures the inflation gap between the actual inflation and the long run rate, set equal to 1 in gross terms. Since all terms are squared terms, the larger the gaps the higher will be the implied welfare losses. The coefficients $\gamma_x = (\sigma + \phi)$, $\gamma_C = \frac{\lambda(1-\lambda)\sigma}{(\varphi+\sigma)}\left(\frac{1+\sigma+\varphi}{1+\varphi}\right)$ and $\gamma_\pi = \phi_P$ represent the weights attached respectively to output gap, consumption gap and inflation gap.

The average welfare loss per period is thus given by the following linear combination of the variances of output gap, inflation and consumption gap:

$$\mathcal{L} = \frac{(\varphi+\sigma)}{2}\left[\gamma_x var(\widetilde{x}_t) + \gamma_C var\left(\widetilde{c}_t\right) + \gamma_\pi var\left(\pi_t\right)\right]. \tag{8}$$

Given our particular policy regimes (MFFS versus DFFS) and the calibration of the model's parameters, one can determine the implied variance of inflation, output gap, consumption gap and the corresponding welfare losses associated with each policy regime.

Table 4 displays some statistics for the three alternative regimes: i) a MFFS; 2) a DFFS with a Central Bank implementing a standard Taylor rule with $\phi_\pi = 1.5$; iii) a DFFS with a Central Bank implementing a strict inflation targeting rule, $\pi_t = 0$. The remaining parameters are calibrated at their baseline values as in the rest of the paper. For each policy regime, Table 4 shows the implied standard deviations of output gap, inflation and consumption gap, expressed in percent terms, as well as the welfare losses resulting from the associated deviations from the efficient allocation, expressed as a fraction of steady state consumption. The same statistics are shown for the baseline model and for the

[16] This is true when $y_t^{Eff} = \frac{1+\varphi}{1+\sigma}a_t$ is the efficient output under the assumption that $y_t = A_t n_t$ and A_t is an exogenous productivity shock.

RANK model. In the latter case, the average welfare loss function becomes:

$$\mathcal{L}_{RANK} = \frac{1}{2}\left[\gamma_x var(\widetilde{x}_t) + \gamma_\pi var\left(\pi_t\right)\right] \tag{9}$$

Several results stand out. First, in a way consistent with the analysis presented in Section 3 on IRFs and consumption and income multipliers, a MFFS generates larger fluctuations than a DFFS in the output gap, the inflation gap and the consumption gap. Consequently, a MFFS generates larger welfare losses than a DFFS. The losses are moderate (6 percent of steady state consumption) under a DFFS with a standard Taylor rule and they are 1 percent of steady state consumption under a DFFS with a strict inflation targeting rule. They become larger (more than 25 percent of steady state consumption) when the increase in public expenditure is financed through money, that is under a MFFS. Similar results hold in a standard representative agent economy. Also in that case a MFFS is welfare detrimental with respect to a DFFS. An aspect was not considered in Gali (2020a). In a borrower-saver framework, the redistribution channel of the stimuli contribute to enlarge the difference between a MFFS and a DFFS. For example, consumption losses increase by 20 percentage points in a MFFS with respect to a DFFS in a two-agent economy. In a standard representative agent economy, the increase in the consumption loss is instead of 11 percentage points. We can therefore state that the redistributive effects of the stimuli are strongly welfare detrimental.

It is worth to notice that the welfare analysis here presented is derived assuming that the steady state is efficient and thus that there is no consumption inequality in steady state. Indeed, if there were consumption inequality in steady state, the loss function would be different and it would include a linear term related steady state consumption inequality. As shown by Bilbiie and Ragot (2020) in this case inflation and the volatility of aggregate demand would count

	MFFS	**DFFS** ($\phi_\pi = 1.5$)	**DFFS**($\pi_t = 0$)
RANK			
$\sigma(\widetilde{x}_t)$	1.58	1.09	0.64
$\sigma(\pi_t)$	0.40	0.17	0
$\mathcal{L}$	0.15	0.04	0.008
TANK			
$\sigma(\widetilde{x}_t)$	1.6	0.82	0.47
$\sigma(\widetilde{c}_t)$	1.9	1.57	1.44
$\sigma(\pi_t)$	0.55	0.24	0
$\mathcal{L}$	0.26	0.06	0.01

Table 4: Welfare analysis

for welfare, beyond their direct effects and, intuitively, a MFFS could be preferable to a DFFS also in terms of welfare. An issue that deserves attention in the future reseach.

6 CONCLUSION

The current health and economic crisis has called for a urgent fiscal intervention in a scenario in which debt ratios are already large despite policy rates hit their zero lower bound for a relatively long time now. Empirical evidence shows large redistributive effects of monetary stimuli, if they are not compensated by effects of opposite sign triggered by a fiscal stimulus. In order to understand better these dynamics, we compare the redistribution effects of a MFFS versus the ones of a DFFS - both in normal times and at the zero lower bound.

We find that the redistribution from savers to borrowers is larger when we consider a MFFS. A corollary of this result is that a MFFS implies always higher impact and cumulative fiscal multipliers than a DFFS. Under both financing schemes, fiscal multipliers are an increasing function of the share of borrowers and of the steady state borrowing limit. However, MFFS generates also larger fluctuations than a DFFS in the output gap, inflation gap and consumption gap. Consequently, a MFFS generates larger welfare losses than a DFFS, particularly in a borrower-saver framework due to the additional presence of the consumption gap with respect to a RANK model. To sum up, the redistributive effects are welfare detrimental. This the reason why only a borrower-saver framework can highlight the trade-off faced by a government to finance a fiscal stimulus by money creation, if it is not forbidden by the legislation. A MFFS has larger redistributive effects than a DFFS but it also implies a larger welfare loss due to the redistributive effect itself, when the welfare function is derived around an efficient steady state.

In addition, a liquidity trap scenario amplifies the differences between MFFS and DFFS. A MFFS is able of having an expansionary effect, despite the adverse scenario, in a TANK model. This is not valid in a RANK model (see Galì, 2020a).

Debortoli and Galì (2017) show that TANK models can provide a good and tractable approximation of the HANK models. They show that a TANK model approximates well, both quantitatively and qualitatively the dynamics of an HANK model in response to aggregate shocks. For this reason we believe that our results will be robust to the introduction of a more structured HANK model, even though studying the effects of this stimulus by using the latest generation of HANK models is also part of our research agenda. Considering a non-Walrasian labor market or investigating the effects of a MFFS in a medium scale model, as well as considering the possibility of relaxing the assumption of rational expectations are all important research questions that are left to future research. Finally, following Bilbiie and Ragot (2020) a further investigation on the welfare effect of a MFFS when consumptions of the two agents are not equal in the steady state, is worth to be considered in future research.

References

[1] Adam, Klaus and Junyi Zhu (2016). Price Level Changes and the Redistribution of Nominal Wealth Across the Euro Area, *Journal of the European Economic Association*, 14(4), 871-906.

[2] Annicchiarico, Barbara, Nicola Giammarioli, and Alessandro Piergallini (2012). Budgetary Policies in a DSGE Model with Finite Horizons, *Research in Economics*, 66(2).

[3] Auclert, Adrien. (2019). Monetary Policy and the Redistribution Channel, *American Economic Review*, 109, 2333-2367.

[4] Benigno, Pierpaolo and Michael Woodford (2003). Optimal Monetary and Fiscal Policy: A Linear-Quadratic Approach in M. Gertler and K. Rogoff, eds., *NBER Macroeconomics Annual 2003,* Cambridge (US), MIT Press, 18, 271-333.

[5] Bilbiie, Florin.O.(2008). Limited asset market participation, monetary policy, and (inverted) aggregate demand logic, *Journal of Economic Theory*, 140(1), 162-196.

[6] Bilbiie, Florin O., Tommaso Monacelli, and Roberto Perotti (2013). Public Debt and Redistribution with Borrowing Constraints, *Economic Journal*, 123, 64-98.

[7] Bilbiie, Florin O. and Xavier Ragot (2021). Optimal Monetary Policy and Liquidity with Heterogeneous Households, *Review of Economic Dynamics,* 41, 71-95.

[8] Coibion, Olivier, Yuriy Gorodnichenko, Lorenz Kueng, and John Silvia (2017). Innocent Bystanders? Monetary policy and Inequality in the U.S., *Journal of Monetary Economics*, 88, 70-89.

[9] Debortoli, Davide, and Jordi Galí (2017). Monetary policy with heterogeneous agents: Insights from TANK models, Economics Working Papers 1686, Department of Economics and Business, Universitat Pompeu Fabra.

[10] Doepke, Matthias, and Martin Schneider (2006). Aggregate Implications of Wealth Redistribution: The Case of Inflation, *Journal of the European Economic Association*, 4, 493-502.

[11] Eggertsson, Gauti B, and Paul Krugman (2012). Debt, Deleveraging, and the Liquidity Trap: a Fisher-Minsky-Koo Approach, *The Quarterly Journal of Economics*, 127(3), 1469-1513.

[12] English William B., Christopher J. Erceg, and David Lopez-Salido. (2017). Money-Financed Fiscal Programs: A Cautionary Tale, FEDS Working Paper No. 2017-060.

[13] Ferrero, Andrea, Richard Harrison and Ben Nelson (2018). Concerted efforts? Monetary policy and macro-prudential tools, Bank of England working papers 727.

[14] Giavazzi, Francesco and Guido Tabellini (2020). Covid Perpetual Eurobonds: Jointly guaranteed and supported by the ECB, *VoxEu.org*, 24 March 2020.

[15] Galì, Jordi (2020a). The effects of a money-financed fiscal stimulus, *Journal of Monetary Economics*, 115, 1-19.

[16] Galì, Jordi (2020b). Helicopter money. The time is now, *VoxEu.org*, 17 March 2020.

[17] Galì, Jordi, David Lopez-Salido, and Javier Vallés (2007). Understanding the Effects of Government Spending on Consumption, *Journal of the European Economic Association*, 5(1), 227-270.

[18] Iacoviello, Matteo (2005). House Prices, Borrowing Constraints and Monetary Policy in the Business Cycle, *American Economic Review*, 95(3), 739-764.

[19] Kaplan, Greg, Benjamin Moll, and Giovanni L.Violante (2018). Monetary Policy According to HANK, *American Economic Review*, 108(3), 697-743.

[20] Kiyotaki, Nobuhiro, and John Moore (1997). Credit cycles, *Journal of Political Economy*, 105, 211-248.

[21] Mankiw, N.Gregory (2000). The Savers-Spenders Theory of Fiscal Policy, *American Economic Review*, 90(2), 120-125.

[22] Monacelli, Tommaso (2009). New Keynesian models, durable goods, and collateral constraints, *Journal of Monetary Economics*, 56(2), 242-254.

[23] Monacelli, Tommaso, and Roberto Perotti (2011). Redistribution and the Multiplier, *IMF Economic Review*, 59(4), 630-651.

[24] Rotemberg, Julio J. (1982). Sticky Prices in the United States, *Journal of Political Economy,* 90(6), 1187-1211.

[25] Sterk, Vincent, and Silvana Tenreyro. (2018). The Transmission of Monetary Policy Operations through Redistributions and Durable Purchases, *Journal of Monetary Economics*, 99, 124-137.

[26] Uhlig, Harald. (2010). Some Fiscal Calculus, *American Economic Review*, 100(2), 30-34.

[27] Woodford, Michael (2002). Inflation Stabilization and Welfare, *The B.E. Journal of Macroeconomics*, 2(1), 1-53.

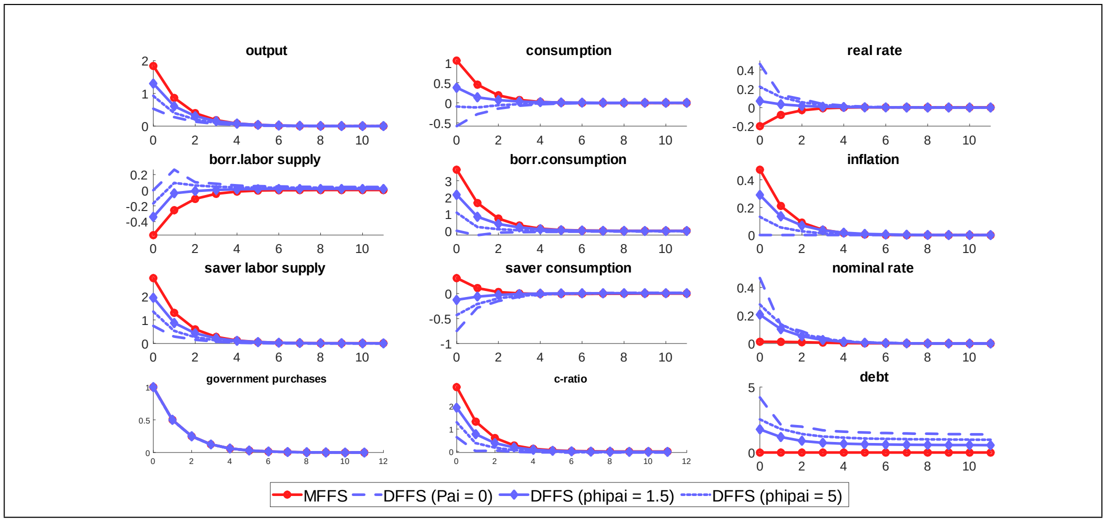

Figure 5: Redistribution channel: The Role of IT rule

A SENSITIVITY ANALYSIS

Under debt financing, monetary policy is assumed to pursue an inflation targeting mandate implying Equation (3) .However, a key parameter in these comparisons is the coefficient in the interest rate rule ($\phi_\pi = 1.5$ is our benchmark) . Next, we discuss the sensitivity of the redistribution channel of a DFFS regarding that coefficient.

We consider two alternative values with respect to our benchmark: i) $\phi_\pi = 5$;ii) $\phi_\pi \to \infty$, as in Galì (2019). Fig. (5) displays the response over time of output, inflation, debt and other macroeconomic variables of interest, as well as the disaggregated consumption, the disaggregated labor supply and the consumption ratio (C_b/C_s) to an exogenous increase in government purchases, under the baseline calibration. The red lines with circles display the responses under the money-financing scheme, while the blue lines with diamonds show the response under the debt-financing scheme when $\phi_\pi = 1.5$; the dotted blue lines when $\phi_\pi = 5$ and the dashed blue lines when $\phi_\pi \to \infty$,or in other words, $\pi_t = 0$.

The higher the coefficient in the interest rate rule, the higher is the increase in nominal rates. It creates a consumption crowding-in solely when $\phi_\pi = 1.5$ (and in a money-financing regime). The higher ϕ_π, the higher the crowding out effect on aggregate consumption will be (and the money demand collapse). The higher ϕ_π, the lower redistribution channel is, also if whatever combination we analyze it brings about a redistribution from savers to borrowers. However, an IT rule with $\phi_\pi \to \infty$ is able to redistribute 30% of what an IT rule with $\phi_\pi = 1.5$ is able to do, because of the interest rate exposure effect. The latter is increasing with the size of ϕ_π, but it redistributes from borrowers to savers. In other words, the higher ϕ_π, the higher is the ability of the interest rate exposure effect of compensating the Fisher effect and minimizing it.

This is the reason why, as shown in Figure (6), the interest rate exposure effect is perfectly able to compensate the Fisher channel when $\phi_\pi \to \infty$.

B WELFARE DERIVATIONS

B.1 Derivation of the Efficient Steady State

Let us to consider the steady state efficient equilibrium. It establishes the conditions under which a zero inflation ($\pi = 1$) steady state is efficient. Indeed, it measures the subsidy/tax needed in the decentralized equilibrium in order to obtain the efficiency of the steady state allocation. First of all, we consider a Social Planner that maximizes the following welfare function in steady state

$$U = \widetilde{\lambda} U\left(c^b, n^b\right) + \left(1 - \widetilde{\lambda}\right) U\left(c^s, n^s\right) \tag{10}$$

where $\widetilde{\lambda}$ is a Pareto weight $\widetilde{\lambda} \in [0, 1]$ and where $U\left(c^j, n^j\right)$ is the per-period utility function of type $j = \{b, s\}$ household. As in the numerical analysis, we assume that real balances have a negligible weight in utility relative to consumption or

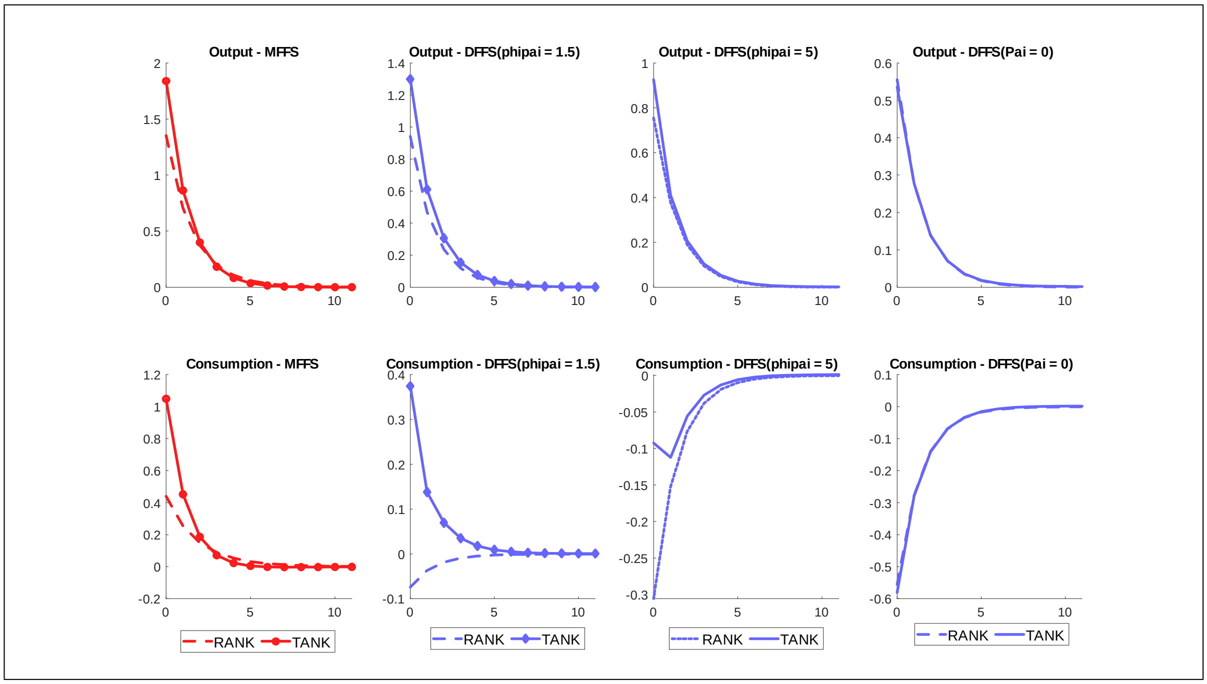

Figure 6: RANK vs TANK: The role of IT rule

employment, so that they do not affect welfare results.[17] The Social Planner maximizes the welfare function under the constraints given by the production function,

$$y = n; \tag{11}$$

the resource constraint,

$$c + g = y; \tag{12}$$

and the aggregations of consumption and labor given respectively by:

$$\begin{aligned} c &= \lambda c^b + (1-\lambda) c^h, & (13) \\ n &= \lambda n^b + (1-\lambda) n^s. & (14) \end{aligned}$$

Combining the constraints in a unique constraint, we get

$$\lambda c^b + (1-\lambda) c^h + g = \lambda n^b + (1-\lambda) n^s. \tag{15}$$

The Lagrangian implied is

$$U = \widetilde{\lambda} U\left(c^b, n^b\right) + \left(1 - \widetilde{\lambda}\right) U\left(c^s, n^s\right) - \mu_1 \left[\lambda c^b + (1-\lambda) c^h + g - \lambda n^b - (1-\lambda) n^s\right]. \tag{16}$$

Taking the first order conditions with respect to c^b, n^b, c^s, n^s, we get

$$\begin{aligned} \widetilde{\lambda} U_c^b &= \mu_1 \lambda, \\ \widetilde{\lambda} U_n^b &= \mu_1 \lambda, \\ \left(1 - \widetilde{\lambda}\right) U_c^s &= -\mu_1 (1-\lambda), \quad \text{and} \\ \left(1 - \widetilde{\lambda}\right) U_n^s &= -\mu_1 (1-\lambda). \end{aligned}$$

[17] We assume that $\chi \to 0$. We do not want that welfare results on MFFS are driven by the presence of real balances in the utility function. We adopt a conservative assumption.

Notice that FOCs imply

$$\begin{aligned} U_c^b &= U_c^b \\ U_n^b &= U_n^b \end{aligned}$$

and that

$$\begin{aligned} c^b &= c^s = c, \quad \text{and} \\ n^b &= n^s = n. \end{aligned}$$

Hence,

$$\frac{U_n^b}{U_c^b} = \frac{U_n^s}{U_c^s} = -\frac{y}{n} = -1$$

that comes from $y = n$.

It can be shown that the standard subsidy applies. Indeed, in the decentralized equilibrium of the labor market, the labor supply choices are given by the following equations

$$\begin{aligned} w &= -\chi_b \frac{U_n^b}{U_c^b} \\ w &= -\chi_s \frac{U_n^s}{U_c^s} \end{aligned}$$

while the labor demand is given according to

$$w = mc = \frac{\epsilon}{\epsilon - 1}.$$

The equilibrium in the labor market implies

$$\frac{\epsilon}{\epsilon - 1} = \lambda \left(-\chi_b \frac{U_n^b}{U_c^b} \right) + (1 - \lambda) \left(-\chi_s \frac{U_n^s}{U_c^s} \right).$$

And, if as we have assumed in our model $\chi_b = \chi_s = 1$, then

$$-\frac{\epsilon}{\epsilon - 1} = \lambda \left(\frac{U_n^b}{U_c^b} \right) + (1 - \lambda) \left(\chi_s \frac{U_n^s}{U_c^s} \right).$$

To get the efficient equilibrium, it must hold that $\frac{\epsilon}{\epsilon - 1} = 1$. Thus, a standard employment subsidy is sufficient to get the result, so that in the decentralized equilibrium, the labor demand becomes

$$w(1 - \tau_L) = mc = \frac{\epsilon - 1}{\epsilon}. \tag{17}$$

It implies that the decentralized equilibrium will be equal to the efficient one if:

$$-1 = -\frac{\epsilon - 1}{\epsilon (1 - \tau_L)} = \lambda \left(\frac{U_n^b}{U_c^b} \right) + (1 - \lambda) \left(\chi_s \frac{U_n^s}{U_c^s} \right)$$

implying

$$1 = -\frac{\epsilon - 1}{\epsilon (1 - \tau_L)}$$

and thus, solving for τ_L we get the optimal subsidy,

$$\tau_L = 1 - \frac{\epsilon - 1}{\epsilon} = \frac{1}{\epsilon}. \tag{18}$$

B.2 Derivation of the Welfare Based Loss Function

We can now move to the second order approximation of the household utility function

$$W_0 = E_0 \left(\sum_{t=0}^{\infty} \beta_s^t U_t \right) \tag{19}$$

where

$$U_t = \widetilde{\lambda} U \left(c_t^b, n_t^b \right) + \left(1 - \widetilde{\lambda} \right) U \left(c_t^s, n_t^s \right) \tag{20}$$

Following Woodford (2002), we take the second order approximation around the efficient steady state, ignoring terms of order three and higher, and also exogenous terms, so that

$$\begin{aligned}
U_t - U &\simeq \widetilde{\lambda}\left[U_c^b\left(c_t^b - c^b\right) + \frac{1}{2}U_{cc}^b\left(c_t^b - c^b\right)^2\right] + \\
&+ \left(1-\widetilde{\lambda}\right)\left[U_c^s\left(c_t^s - c^s\right) + \frac{1}{2}U_{cc}^s\left(c_t^s - c^s\right)^2\right] \\
&+ \widetilde{\lambda}\left[U_n^b\left(n_t^b - n^b\right) + \frac{1}{2}U_{nn}^b\left(n_t^b - n^b\right)^2\right] + \\
&+ \left(1-\widetilde{\lambda}\right)\left[U_n^s\left(n_t^s - n^s\right) + \frac{1}{2}U_{nn}^s\left(n_t^s - n^s\right)^2\right]
\end{aligned}$$

Now, factoring out the marginal utility of consumption and labor for each type of household:

$$\begin{aligned}
U_t - U &\simeq \widetilde{\lambda}U_c^b\left[\left(c_t^b - c^b\right) + \frac{1}{2}\frac{U_{cc}^b}{U_c^b}\left(c_t^b - c^b\right)^2\right] + \left(1-\widetilde{\lambda}\right)U_c^s\left[\left(c_t^s - c^s\right) + \frac{1}{2}\frac{U_{cc}^s}{U_c^s}\left(c_t^s - c^s\right)^2\right] \\
&+ \widetilde{\lambda}U_n^b\left[\left(n_t^b - n^b\right) + \frac{1}{2}\frac{U_{nn}^b}{U_n^b}\left(n_t^b - n^b\right)^2\right] + \left(1-\widetilde{\lambda}\right)U_n^s\left[\left(n_t^s - n^s\right) + \frac{1}{2}\frac{U_{nn}^s}{U_n^s}\left(n_t^s - n^s\right)^2\right].
\end{aligned}$$

By using the FOCs of the efficient steady state, that is for

$$\begin{aligned}
\widetilde{\lambda}U_c^b &= \mu_1\lambda \\
\widetilde{\lambda}U_n^b &= \mu_1\lambda \\
\left(1-\widetilde{\lambda}\right)U_c^s &= -\mu_1\left(1-\lambda\right) \\
\left(1-\widetilde{\lambda}\right)U_n^s &= -\mu_1\left(1-\lambda\right)
\end{aligned}$$

it becomes

$$
\begin{aligned}
U_t - U \simeq\ & \lambda\mu_1 \left[\left(c_t^b - c^b\right) + \frac{1}{2}\frac{U_{cc}^b}{U_c^b}\left(c_t^b - c^b\right)^2 \right] + (1-\lambda)\,\mu_1 \left[\left(c_t^s - c^s\right) + \frac{1}{2}\frac{U_{cc}^s}{U_c^s}\left(c_t^s - c^s\right)^2 \right] \\
& -\lambda\mu_1 \left[\left(n_t^b - n^b\right) + \frac{1}{2}\frac{U_{nn}^b}{U_n^b}\left(n_t^b - n^b\right)^2 \right] - (1-\lambda)\,\mu_1 \left[\left(n_t^s - n^s\right) + \frac{1}{2}\frac{U_{nn}^s}{U_n^s}\left(n_t^s - n^s\right)^2 \right].
\end{aligned}
$$

Given the preferences in the period utility of each household

$$
\begin{aligned}
\frac{U_{cc}^b}{U_c^b} &= \frac{U_{cc}^s}{U_c^s} = -\frac{\sigma}{C} \\
\frac{U_{nn}^b}{U_n^b} &= \frac{U_{nn}^s}{U_n^s} = \frac{\varphi}{n},
\end{aligned}
$$

substituting above and collecting first order terms, we obtain:

$$
\begin{aligned}
U_t - U \simeq\ & \mu_1 \left[\lambda \left(c_t^b - c^b\right) + (1-\lambda)\left(c_t^s - c^s\right) \right] \\
& -\mu_1 \left[\lambda \left(n_t^b - n^b\right) + (1-\lambda)\left(n_t^s - n^s\right) \right] \\
& -\mu_1 \frac{1}{2}\frac{\sigma}{C} \left[\lambda \left(c_t^b - c^b\right)^2 + (1-\lambda)\left(c_t^s - c^s\right)^2 \right] \\
& -\mu_1 \frac{1}{2}\frac{\varphi}{n} \left[\lambda \left(n_t^b - n^b\right)^2 + (1-\lambda)\left(n_t^s - n^s\right)^2 \right].
\end{aligned}
$$

By considering aggregate consumption, $c_t = \lambda c_t^b + (1-\lambda)\, c_t^s$, and taking the first order approximation around the efficient steady state, the previous objective function can be rewritten as

$$
\begin{aligned}
U_t - U \simeq\ & \mu_1 \left(c_t - c\right) \\
& -\mu_1 \left[\lambda \left(n_t^b - n^b\right) + (1-\lambda)\left(n_t^s - n^s\right) \right] \\
& -\mu_1 \frac{1}{2}\frac{\sigma}{C} \left[\lambda \left(c_t^b - c^b\right)^2 + (1-\lambda)\left(c_t^s - c^s\right)^2 \right] \\
& -\mu_1 \frac{1}{2}\frac{\varphi}{n} \left[\lambda \left(n_t^b - n^b\right)^2 + (1-\lambda)\left(n_t^s - n^s\right)^2 \right].
\end{aligned}
$$

Given the resource constraint implied by the Rotemberg model, the second order approximation of that constraint implies

$$C\left(\widehat{c}_t+\frac{1}{2}\widehat{c}_t^2\right)=y\left(\widehat{y}_t+\frac{1}{2}\widehat{y}_t^2\right)-\frac{\phi_P}{2}y\pi_t^2$$

Then, under the efficient steady state[18],

$$c_t-c=\widehat{c}_t+\frac{1}{2}\widehat{c}_t^2=\widehat{y}_t+\frac{1}{2}\widehat{y}_t^2-\frac{\phi_P}{2}y\pi_t^2$$

the welfare function becomes

$$\begin{aligned}U_t-U \quad &\simeq \quad \mu_1 y\left(\widehat{y}_t+\frac{1}{2}\widehat{y}_t^2-\frac{\phi_P}{2}y\pi_t^2\right)\\ &\quad -\mu_1\left[\lambda\left(n_t^b-n^b\right)+(1-\lambda)\left(n_t^s-n^s\right)\right]\\ &\quad -\mu_1\frac{1}{2}\frac{\sigma}{C}\left[\lambda\left(c_t^b-c^b\right)^2+(1-\lambda)\left(c_t^s-c^s\right)^2\right]\\ &\quad -\mu_1\frac{1}{2}\frac{\varphi}{n}\left[\lambda\left(n_t^b-n^b\right)^2+(1-\lambda)\left(n_t^s-n^s\right)^2\right],\end{aligned}$$

from

$$\begin{aligned}n_t^b-n^b &= n^b\left(\widehat{n}_t^b+\frac{1}{2}\left(\widehat{n}_t^b\right)^2\right)=n\left(\widehat{n}_t^b+\frac{1}{2}\left(\widehat{n}_t^b\right)^2\right)\\ n_t^s-n^s &= n^s\left(\widehat{n}_t^s+\frac{1}{2}\left(\widehat{n}_t^s\right)^2\right)=n\left(\widehat{n}_t^s+\frac{1}{2}\left(\widehat{n}_t^s\right)^2\right)\end{aligned}$$

and then

$$\begin{aligned}U_t-U \quad &\simeq \quad \mu_1 y\left(\widehat{y}_t+\frac{1}{2}\widehat{y}_t^2-\frac{\phi_P}{2}\pi_t^2\right)\\ &\quad -\mu_1\left[\lambda n\left(\widehat{n}_t^b+\frac{1}{2}\left(\widehat{n}_t^b\right)^2\right)+(1-\lambda)\,n\left(\widehat{n}_t^s+\frac{1}{2}\left(\widehat{n}_t^s\right)^2\right)\right]\\ &\quad -\mu_1\frac{1}{2}\frac{\sigma}{C}\left[\lambda\left(c_t^b-c^b\right)^2+(1-\lambda)\left(c_t^s-c^s\right)^2\right]\\ &\quad -\mu_1\frac{1}{2}\frac{\varphi}{n}\left[\lambda\left(n_t^b-n^b\right)^2+(1-\lambda)\left(n_t^s-n^s\right)^2\right]\end{aligned}$$

[18] As in Benigno and Woodford (2003), we can omit exogenous terms like g_t. Also notice that its steady state is zero in the efficient steady state.

or

$$
\begin{aligned}
\mu_1 (U_t - U) \quad \simeq \quad & y\left(\left(\widehat{y}_t + \frac{1}{2}\widehat{y}_t^2\right) - \frac{\phi_P}{2}\pi_t^2\right) \\
& -n\left(\lambda \widehat{n}_t^b + (1-\lambda)\,\widehat{n}_t^s\right) - \frac{1}{2}n\left[\lambda\left(\widehat{n}_t^b\right)^2 + (1-\lambda)\left(\widehat{n}_t^s\right)^2\right] \\
& -\frac{1}{2}\frac{\sigma}{c}\left[\lambda\left(c_t^b - c^b\right)^2 + (1-\lambda)\left(c_t^s - c^s\right)^2\right] \\
& -\frac{1}{2}\frac{\varphi}{n}\left[\lambda\left(n_t^b - n^b\right)^2 + (1-\lambda)\left(n_t^s - n^s\right)^2\right]
\end{aligned}
$$

Knowing that in steady state $c^b = c^s = c = y$ and $n^b = n^s = n = y$ and from

$$
\begin{aligned}
\left(c_t^b - c^b\right)^2 &= \left(c^b\right)^2\left(\widehat{c}_t^b\right)^2 = (c)^2\left(\widehat{c}_t^b\right)^2 \\
\left(c_t^s - c^s\right)^2 &= \left(c^s\right)^2\left(\widehat{c}_t^s\right)^2 = (c)^2\left(\widehat{c}_t^s\right)^2 \\
\left(n_t^b - n^b\right)^2 &= \left(n^b\right)^2\left(\widehat{n}_t^b\right)^2 = (n)^2\left(\widehat{n}_t^b\right)^2 \\
\left(n_t^s - n^s\right)^2 &= \left(n^s\right)^2\left(\widehat{n}_t^s\right)^2 = (n)^2\left(\widehat{n}_t^s\right)^2
\end{aligned}
$$

Substituting and rearranging

$$
\begin{aligned}
\frac{\mu_1 (U_t - U)}{y} \quad \simeq \quad & \left(\widehat{y}_t + \frac{1}{2}\widehat{y}_t^2\right) - \frac{\phi_P}{2}\pi_t^2 \\
& -\left(\lambda \widehat{n}_t^b + (1-\lambda)\,\widehat{n}_t^s\right) - \frac{1}{2}\left[\lambda\left(\widehat{n}_t^b\right)^2 + (1-\lambda)\left(\widehat{n}_t^s\right)^2\right] \\
& -\frac{1}{2}\sigma\left[\lambda\left(\widehat{c}_t^b\right)^2 + (1-\lambda)(y)^2\left(\widehat{c}_t^s\right)^2\right] \\
& -\frac{1}{2}\varphi\left[\lambda\left(\widehat{n}_t^b\right)^2 + (1-\lambda)\left(\widehat{n}_t^s\right)^2\right].
\end{aligned}
$$

Further, from the production function we know that

$$
\widehat{y}_t = \widehat{n}_t = \lambda \widehat{n}_t^b + (1-\lambda)\,\widehat{n}_t^s
$$

and therefore, simplifying and collecting terms, the objective function is

$$
\begin{aligned}
\frac{\mu_1 \left(U_t - U\right)}{y} \quad \simeq \quad & \frac{1}{2}\widehat{y}_t^2 - \frac{\phi_P}{2}\pi_t^2 - \frac{1}{2}\sigma \left[\lambda \left(\widehat{c}_t^b\right)^2 + (1-\lambda)\left(\widehat{c}_t^s\right)^2\right] \\
& -\frac{1}{2}\left(1+\varphi\right)\left[\lambda \left(\widehat{n}_t^b\right)^2 + (1-\lambda)\left(\widehat{n}_t^s\right)^2\right].
\end{aligned}
$$

Notice that, at this point, the welfare-based loss function is fully quadratic. Following Ferrero et al.(2018), we rewrite it to obtain terms with a more meaningful economic interpretation. Hence, we combine terms in output and consumption, as follows

$$
\begin{aligned}
\frac{\mu_1 \left(U_t - U\right)}{y} \quad \simeq \quad & -\frac{1}{2}\left\{\sigma \left[\lambda \left(\widehat{c}_t^b\right)^2 + (1-\lambda)\left(\widehat{c}_t^s\right)^2\right] - \widehat{y}_t^2\right\} \\
& -\frac{1}{2}\left(1+\varphi\right)\left[\lambda \left(\widehat{n}_t^b\right)^2 + (1-\lambda)\left(\widehat{n}_t^s\right)^2\right] - \frac{\phi_P}{2}\pi_t^2 -
\end{aligned}
$$

Now we rewrite the objective function adding and subtracting $\frac{1}{2}\left(\sigma+\varphi\right)\widehat{y}_t^2$

$$
\begin{aligned}
\frac{\mu_1 \left(U_t - U\right)}{y} \quad \simeq \quad & -\frac{1}{2}\left\{\sigma \left[\lambda \left(\widehat{c}_t^b\right)^2 + (1-\lambda)\left(\widehat{c}_t^s\right)^2\right] - \widehat{y}_t^2\right\} - \frac{1}{2}\left(1+\varphi\right)\left[\lambda \left(\widehat{n}_t^b\right)^2 + (1-\lambda)\left(\widehat{n}_t^s\right)^2\right] \\
& +\frac{1}{2}\left(\sigma+\varphi\right)\widehat{y}_t^2 - \frac{1}{2}\left(\sigma+\varphi\right)\widehat{y}_t^2 - \frac{\phi_P}{2}\pi_t^2 + t.i.p.
\end{aligned}
$$

where $t.i.p$ collects all terms independent of policy. We can put $\frac{1}{2}\sigma\widehat{y}_t^2$ into the consumption terms and $\frac{1}{2}\left(1+\varphi\right)\widehat{y}_t^2$ into the labor terms

$$
\begin{aligned}
\widetilde{U}_t \quad \simeq \quad & -\frac{1}{2}\left\{\sigma \left[\lambda \left(\widehat{c}_t^b\right)^2 + (1-\lambda)\left(\widehat{c}_t^s\right)^2\right] + \sigma\widehat{y}_t^2 + \frac{1}{2}\left(1+\varphi\right)\left[\lambda \left(\widehat{n}_t^b\right)^2 + (1-\lambda)\left(\widehat{n}_t^s\right)^2 - \widehat{y}_t^2\right] - \left(\sigma+\varphi\right)\widehat{y}_t^2 - \right\} + \\
& -\frac{\phi_P}{2}\pi_t^2 - \frac{\phi_P}{2}\pi_t^2 + t.i.p.
\end{aligned}
$$

where $\widetilde{U}_t = \frac{\mu_1 (U_t - U)}{y}$.

Now notice that

$$
\lambda \left(\widehat{c}_t^b\right)^2 + (1-\lambda)\left(y\right)^2\left(\widehat{c}_t^s\right)^2 - \widehat{y}_t^2 = \lambda\left(\left(\widehat{c}_t^b\right)^2 - \widehat{y}_t^2\right) + (1-\lambda)\left(\left(\widehat{c}_t^s\right)^2 - \widehat{y}_t^2\right)
$$

using again the resource constraint to replace the differences between each type's consumption and output, we can rewrite

$$\lambda \left(\widehat{c}_t^b\right)^2 + (1-\lambda)(y)^2 \left(\widehat{c}_t^s\right)^2 - \widehat{y}_t^2 = \lambda (1-\lambda) \left(\widehat{c}_t^b - \widehat{c}_t^s\right)^2$$

Then from the labor supply conditions,

$$\begin{aligned} w_t &= \left(n_t^b\right)^{\varphi} \left(c_t^b\right)^{\sigma} \\ w_t &= \left(n_t^s\right)^{\varphi} \left(c_t^s\right)^{\sigma} \end{aligned}$$

then

$$\left(n_t^b\right)^{\varphi} \left(c_t^b\right)^{\sigma} = \left(n_t^s\right)^{\varphi} \left(c_t^s\right)^{\sigma},$$

and also from

$$\begin{aligned} w_t n_t^b &= \left(n_t^b\right)^{1+\varphi} \left(c_t^b\right)^{\sigma} \\ w_t n_t^s &= \left(n_t^s\right)^{1+\varphi} \left(c_t^s\right)^{\sigma} \end{aligned}$$

then, the first order approximation gives us:

$$\begin{aligned} (1+\varphi)\,\widehat{n}_t^b + \sigma \widehat{c}_t^b &= w_t + \widehat{n}_t^b \\ (1+\varphi)\,\widehat{n}_t^s + \sigma \widehat{c}_t^s &= w_t + \widehat{n}_t^s \end{aligned}$$

Then, by aggregating

$$\begin{aligned} \lambda \left(w_t + \widehat{n}_t^b\right) + (1-\lambda)\left(w_t + \widehat{n}_t^s\right) &= w_t + \widehat{n}_t = (1+\varphi)\,\widehat{n}_t + \sigma \widehat{c}_t \\ \lambda \left(w_t + \widehat{n}_t^b\right) + (1-\lambda)\left(w_t + \widehat{n}_t^s\right) &= w_t + \widehat{n}_t = (1+\varphi)\,\widehat{n}_t + \sigma \widehat{y}_t \end{aligned}$$

and given that $\widehat{n}_t = \lambda \widehat{n}_t^b + (1-\lambda)\,\widehat{n}_t^s$,

$$w_t + \widehat{n}_t = (1+\varphi)\,\widehat{n}_t + \sigma \widehat{y}_t = (1+\varphi)\left(\lambda \widehat{n}_t^b + (1-\lambda)\,\widehat{n}_t^s\right) + \sigma \widehat{y}_t$$

consequently,

$$(1+\varphi)\left(\lambda \widehat{n}_t^b + (1-\lambda)\,\widehat{n}_t^s\right) + \sigma \widehat{y}_t = (1+\varphi)\,\widehat{n}_t^b + \sigma \widehat{c}_t^b$$

and by collecting terms in $\widehat{n}_t^b$

$$\widehat{n}_t^b = \widehat{n}_t^s - \frac{\sigma}{1+\varphi(1-\lambda)}\left(\widehat{c}_t^b - \widehat{y}_t\right)$$

and

$$\widehat{n}_t^s = \frac{\left(\widehat{n}_t - \lambda \widehat{n}_t^b\right)}{1-\lambda}$$

and by substituting the last one into the previous one and solving for $\widehat{n}_t^b$,

$$\widehat{n}_t^b = \widehat{n}_t - \frac{\sigma}{1+\varphi}\left(\widehat{c}_t^b - \widehat{y}_t\right) = \widehat{y}_t - \frac{\sigma}{1+\varphi}\left(\widehat{c}_t^b - \widehat{y}_t\right)$$

Similarly, we find

$$\widehat{n}_t^s = \widehat{y}_t - \frac{\sigma}{1+\varphi}\left(\widehat{c}_t^s - \widehat{y}_t\right)$$

Then, using the first order approximation of the resource constraint we can rewrite

$$\begin{aligned}
\widehat{n}_t^b &= \widehat{y}_t - \frac{\sigma}{1+\varphi}(1-\lambda)\left(\widehat{c}_t^b - \widehat{c}_t^s\right) \\
\widehat{n}_t^s &= \widehat{y}_t + \frac{\sigma}{1+\varphi}\lambda\left(\widehat{c}_t^b - \widehat{c}_t^s\right)
\end{aligned}$$

substituting everything into the objective function

$$\widetilde{U}_t \simeq -\frac{1}{2}\left\{\begin{array}{c} \sigma\lambda\left(1-\lambda\right)\left(\widehat{c}_t^b-\widehat{c}_t^s\right)^2+ \\ +\frac{1}{2}\left(1+\varphi\right)\left[\begin{array}{c} \lambda\left(\widehat{y}_t-\frac{\sigma}{1+\varphi}\left(1-\lambda\right)\left(\widehat{c}_t^b-\widehat{c}_t^s\right)\right)^2+ \\ +\left(1-\lambda\right)\left(\widehat{y}_t+\frac{\sigma}{1+\varphi}\lambda\left(\widehat{c}_t^b-\widehat{c}_t^s\right)\right)^2-\widehat{y}_t^2 \end{array}\right] \\ -\left(\sigma+\varphi\right)\widehat{y}_t^2-\frac{\phi_P}{2}\pi_t^2+t.i.p. \end{array}\right\}$$

Let us to consider only the terms in the squared brackets

$$\left[\lambda\left(\widehat{y}_t-\frac{\sigma}{1+\varphi}\left(1-\lambda\right)\left(\widehat{c}_t^b-\widehat{c}_t^s\right)\right)^2+\left(1-\lambda\right)\left(\widehat{y}_t+\frac{\sigma}{1+\varphi}\lambda\left(\widehat{c}_t^b-\widehat{c}_t^s\right)\right)^2-\widehat{y}_t^2\right]$$

and expand the two squared terms

$$\begin{array}{c} \lambda\left(\widehat{y}_t^2+\left[\frac{\sigma}{1+\varphi}\left(1-\lambda\right)\left(\widehat{c}_t^b-\widehat{c}_t^s\right)\right]^2-2\frac{\sigma}{1+\varphi}\left(1-\lambda\right)\left(\widehat{c}_t^b-\widehat{c}_t^s\right)\widehat{y}_t^2\right)+ \\ +\left(1-\lambda\right)\left(\widehat{y}_t^2+\left(\frac{\sigma}{1+\varphi}\lambda\left(\widehat{c}_t^b-\widehat{c}_t^s\right)\right)^2+2\frac{\sigma}{1+\varphi}\lambda\left(\widehat{c}_t^b-\widehat{c}_t^s\right)\widehat{y}_t^2\right)-\widehat{y}_t^2. \end{array}$$

By simplifying, we obtain:

$$\begin{aligned} & \widehat{y}_t^2+\lambda\left(1-\lambda\right)\left(1-\lambda\right)\left[\frac{\sigma}{1+\varphi}\left(\widehat{c}_t^b-\widehat{c}_t^s\right)\right]^2+\left(1-\lambda\right)\lambda\lambda\left(\frac{\sigma}{1+\varphi}\left(\widehat{c}_t^b-\widehat{c}_t^s\right)\right)^2-\widehat{y}_t^2 \\ = \; & \lambda\left(1-\lambda\right)\left[\frac{\sigma}{1+\varphi}\left(\widehat{c}_t^b-\widehat{c}_t^s\right)\right]^2 \end{aligned}$$

and by substituting in the objective function and by collecting terms

$$\widetilde{U}_t \simeq -\frac{1}{2}\left\{\left(\varphi+\sigma\right)\widehat{y}_t^2+\lambda\left(1-\lambda\right)\sigma\left(\frac{1+\sigma+\varphi}{1+\varphi}\right)\left(\widehat{c}_t^b-\widehat{c}_t^s\right)^2\right\}-\frac{\phi_P}{2}\pi_t^2+t.i.p.$$

where *t.i.p.* indicates terms independent from policy. In particular, with a production function where $y_t = A_t n_t$ and A_t representing an exogenous TFP shocks, the implied welfare function would be

$$\widetilde{U}_t \simeq -\frac{(\varphi+\sigma)}{2} E_0 \beta_s^t \sum_{t=0}^{\infty} \left[(\varphi+\sigma)\, \widetilde{x}_t^2 + \lambda\,(1-\lambda)\,\sigma \left(\frac{1+\sigma+\varphi}{1+\varphi} \right) \widetilde{c}_t^2 + \phi_P \pi_t^2 \right] + t.i.p. \tag{21}$$

where, we define $\widetilde{c}_t = \widehat{c}_t^b - \widehat{c}_t^s$ as the consumption gap we have defined $\widetilde{x}_t = \widehat{y}_t - y_t^{Eff}$, with $y_t^{Eff} = \frac{1+\phi}{\sigma+\phi} a_t$ and $a_t = \ln(A_t/A)$ the log-deviation of the TFP from its steady state. Otherwise, in the absence of this shock, as in our particular model economy, $\widetilde{x}_t = \widehat{y}_t$. Multiplying everything by -1, the Loss function becomes

$$\widetilde{W}_t \simeq \frac{1}{2} E_0 \beta_s^t \sum_{t=0}^{\infty} \left[(\varphi+\sigma)\, \widetilde{x}_t^2 + \lambda\,(1-\lambda)\,\sigma \left(\frac{1+\sigma+\varphi}{1+\varphi} \right) \widetilde{c}_t^2 + \phi_P \pi_t^2 \right] + t.i.p. \tag{22}$$

Notice that as in Ferrero et al (2018) the welfare function depends, not only on standard output gap and inflation but also on the consumption gap between borrower and saver consumption. Since all terms are squared terms, the larger the gaps the higher will be the welfare loss. To interpret our numerical results in table (4) , it is indeed important to analyze the role played by the redistributive channel of each stimulus in affecting not only inflation and output but also the consumption gap, which is indeed crucial to explain the results.