Another look at Contagion across US and European financial markets: Evidence from the CDS markets

Journal:	*International Journal of Finance and Economics*
Manuscript ID	IJFE-20-0555
Wiley - Manuscript type:	Research Article
Keywords:	Co-kurtosis, Co-skewness, Co-volatility, Financial contagion, Financial crisis

Another look at Contagion across US and European financial markets: Evidence from the CDS markets

Mike G. Tsionas* Nicholas Apergis†

Abstract

This paper proposes a novel model that allows time variation in volatility, skewness and kurtosis based on the multivariate stable distributions. The analysis looks at set of bank sector CDS, insurance sector CDS, sovereign bonds, equity and volatility indices. Our new findings corroborate their results and indicate significant evidence of contagion, especially through the channels of co-skewness and co-kurtosis. In addition, it establishes a higher order channel of causality between co-skewness and co-kurtosis. Finally, a significant novelty of the paper is that it documents that after splitting the sample over the period prior and after the 2008 financial crisis, the results are dominated by those obtained over the latter period. Overall, the strong empirical results have important implications for international capital flows, the efficiency of international portfolio diversification, as well as for market efficiency.

Keywords: Co-kurtosis; Co-skewness; Co-volatility; Financial contagion; Financial crisis.

JEL Classifications: C11, C13, C58, G15, G01, F36.

Data availability: The data will be made available on request.

*Lancaster University Management School, LA1 4YX, United Kingdom, m.tsionas@lancaster.ac.uk
†University of Derby, School of Business, Law and Social Sciences, UK, , n.apergis@derby.ac.uk

1 Introduction

The goal of this paper is to provide a model with time-varying variance, skewness and kurtosis in a principled way. Previous attempts, based on the multivariate skew-t distribution have the drawback that the degrees of freedom of the multivariate student-t should exceed 4, which excludes several cases of empirical interest. A prominent approach has been provided by Wilhelmsson (2009).

In a recent paper, Apergis, Christou and Kynigakis (2019) investigate whether contagion occurs during the 2007/08 global financial crisis across European and US financial markets. They use the linear (mean) channel of contagion by employing the correlation-based contagion tests proposed by Forbes and Rigobon (2002). Furthermore, they use the asymmetric dependence test developed by Fry et al. (2010) to study the co-skewness channel of contagion, as well as the Fry-McKibbin and Hsiao (2018) extremal dependence tests to explore the co-kurtosis and co-volatility channels.

Our work uses the family of multivariate stable distributions which allows separation of volatility, skewness and kurtosis, but it has no second or higher moments, in agreement with the evidence of fat tails in financial data. Although the family of multivariate stable distributions is quite suited for this task, it has never been used before, due, perhaps, to computational difficulties. The new results confirm Apergis, Christou and Kynigakis (2019) and they open up the way for summary tools in the analysis of co-kurtosis, co-volatility, and co-skewness.

Broda et al. (2013) recommend an approach based on mixtures of stable Paretian distributions. The tests of Forbes and Rigobon (2002), Fry et al. (2010), and Fry-McKibbin and Hsiao (2018) focus on bivariate comparisons, without explicitly considering or conditioning on all other variables as they should have. Our analysis corrects this by using the family of multivariate stable distributions. Overall, a couple of novelties are provided by this study. First, our findings corroborate those reached in the international literature under the new corrective methods.

Our modeling approach also provides supportive evidence on the direction of contagion across CDS and other financial markets between US and Europe under a new modeling approach. While our new results provide supportive evidence to those reached by Apergis et al. (2019), the current analysis splits the overall time period into that prior and after the 2008 crisis event to clearly highlight that the similarity of the obtained results with those in the literature is due to the dominance of the latter period which increased the occurrence of contagion effects across all markets under investigation, as well as across all new contagion metrics used by the empirical analysis. The results are expected to be of high importance since contagion per se is a policy-relevant issue given that it results in an inefficient allocation of risks.

Contagion induces people not to consider the effect of their actions on others, thus, raising the overall level of risk in the economy, which calls from intervention actions by policymakers, such as regulatory actions, to stabilize financial and real markets in terms of efficiency. Moreover, contagion usually results in an overall destabilization of the financial system with detrimental effects on economic growth. In this case, explicit intervention macroeconomic policies are needed to be implemented so as to mitigate negative spillovers to the real economy arising from a destabilized financial system.

In addition, the presence of strong contagion phenomena may substantially undermine the rationale for international diversification across assets, while the risk of negative shocks hitting asset markets may significantly reduce financial flows across markets and/or across countries, even in cases where the fundamentals of the economies are strong (Sun and Zhang, 2009).

2 Literature review

There has been great interest in the empirical literature on cross-country and cross-market contagion events. Many empirical studies have been undertaken to measure the extent of financial spillovers to mature and emerging markets, as well as to detect specific channels of transmission of shocks to other markets and/or countries.

The presence of financial contagion phenomena has been stronger following the liberalisation of financial markets which allowed stronger mobility for information and capital flows, thus intensifies international asset prices and volatility linkages. The literature has explicitly emphasized the importance of asset prices in the transmission of idiosyncratic shock across markets and countries (Kindleberger and Aliber, 2011).

Our work is closely related to the strand of the literature that focuses on investor-based contagion (the so called 'pure contagion' hypothesis) (Kumar and Persaud, 2002); according to this literature, the presence of shock propagation is unrelated to fundamentals, but it can still induce changes in investors' behavior. Theoretical explanations put forward are associated with theories of multiple equilibria due to changes in investors' self-fulfilling expectations.

Diamond and Dybvig (1983) present a bank run model in which many bank customers suddenly withdraw their deposits if they expect that the bank might become insolvent. In this modelling approach depositors form expectations about the behavior of other depositors. If the others run, the former will also run, thus, showing a type of herd behavior.

This bank running phenomenon is expected to exhaust a bank's liquid assets, which leads to more withdrawals and, eventually, to bank bankruptcy. In addition, contagion phenomena could be explained by portfolio rebalancing. More specifically, negative shocks in one economy may lead to the deterioration in the value of leveraged investors' collateral, inducing them to liquidate assets even in unaffected economies so as to meet margin calls.

According to the portfolio theory, such portfolio rebalancing effects can be explained by that international investors decide how much to invest in a risky foreign country by weighing their expected returns against the associated risks.

If a shock occurs in any part of the economy, portfolios change to reflect the new equilibrium prices of risk. In relevant to this explanation, contagion phenomena could be caused by information asymmetries and herding behavior. Under the assumption of uncertainty conditions and that investors do not have a complete picture of a country's or market's fundamentals, they are prompted to reach investment decisions based on the actions of other investors, thus causing herding behavior or financial panic. Calvo and Mendoza (2000) illustrate that it is expensive to collect and process country-specific information; in that case, less informed or uninformed investors tend to watch and obtain beneficial informational strings by observing informed investors who act early in adjusting their portfolios.

If the latter group moves to a bad equilibrium, then the former group causes another bad equilibrium. Forbes and Rigobon (2002) also introduce the term "shift-contagion' and classify the theoretical approaches explaining the occurrence of shifts as crisis-contingent theories. Accordingly, contagion is nothing else but a substantial increase in cross-market linkages after a negative shock hits a country or a market.

The empirical literature has explored contagion events through probability models, such as probit and logit models, without, however, assuming the presence of structural breaks in cross-market linkages (Caramazza et al., 2000; Haile and Pozo, 2008). In particular, this strand provides estimates about the probability of spreads of financial crises, while identifying specific channels through which contagion occurs. Additionally, another sub-group in this empirical literature identifies contagion as volatility spillovers by using the GARCH methodological approach, according to which conditional variances of financial variables could be potentially linked to each other across asset classes / markets (Chancharoenchai and Dibooglu, 2006; Tanai and Lin, 2013). A major drawback of this approach, however, is that it does not consider the presence of any structural break in the data generating process caused by the crisis. A very popular method in the literature to test for contagion comes from the family of correlation breakdown tests; in this approach, the correlation coefficients of asset returns are estimated over crisis and non-crisis periods; the test explores whether there has been any significant change in correlations across regimes.

King and Wadhwani (1990) first applied this approach to explore structural changes in cross-market linkages in the US, the UK and Japan after the 1987 stock market crash. Their findings document that contagion increased during and immediately after the crash event as higher volatility; in that sense, there is a transmission channel that cannot be explained by a model based on fundamentals. These correlation breakdown tests suffer from serious heteroskedasticity problems, since the correlation across asset returns is affected by their corresponding volatilities, which tend to be extremely high during crisis events (Forbes and Rigobon 2002; Rigobon 2003).

Within the same strand, Corsetti et al. (2005) argue that increases in variances associated with the crisis market may be caused by both idiosyncratic components and non-observable variables, yielding high biases to estimated coefficients. They recommend weighting the increasing factor for each component of shocks. Another problem with these methods has to do with the fact that they provide estimates only through bivariate testing.

Dungey et al. (2005) resolve the problem by recommending a multivariate version of the Forbes and Rigobon test by scaling the asset returns and correcting for endogeneity bias. In another part of the empirical strand and based on theoretical arguments in favour of the presence of multiple equilibria caused by changes in investors' expectations during a crisis, the literature stresses the need the underlying distribution of asset returns not to be multimodal, thus avoiding discontinuities in the data-generating distribution.

Ismail and Rahman (2009) assess an Markov Switching (MS) model to study the link between US and Asian stock markets; their findings clearly display the pre-eminence of non-linear MS-VAR over linear VAR in modeling asset return interactions across countries. Similarly, by using an MS-VAR framework, Guo and Stepanyan (2011) study contagion effects between stock markets, real estate markets, CDS markets, and energy markets in the US. Their results document contagion effects from these markets, across two distinct regimes. In that sense, all financial markets respond highly significantly to economic shocks when a highly volatile regime is dominant.

3 Univariate and multivariate stable distributions

Stable distributions have received quite a lot of attention in econometrics, statistics and finance over the past few decades (Samorodnitsky and Taqqu, 1994). Their empirical application is still hampered by the fact that their density is not available in closed form, despite advances in Bayesian computation using Markov Chain Monte Carlo (MCMC).

Buckle (1995) and Tsionas (1999) provided Gibbs sampling schemes for general and symmetric stable distributions, respectively. The problem is that the conditional posterior distributions of certain latent variables are cumbersome to deal with and require careful tuning. This work falls squarely within recent advances in the econometrics of stable distributions. Dominicy and Veredas (2012) propose a method of quantiles to fit symmetric stable distributions. Since the quantiles are not available in closed form they are obtained using simulation resulting in the method of simulated quantiles or MSQ.

Hallin, Swan, Verdebout and Veredas (2012) propose an easy-to-implement R-estimation procedure which remains $\sqrt{n}$-consistent contrary to least squares with stable disturbances. Broda, Haas, Krause, Paolella and Steude (2012) propose a new stable mixture GARCH model that encompasses several alternatives and can be extended easily to the multivariate asset returns case using independent components analysis.

Ogata (2012) uses a discrete approximation to the spectral measure of multivariate stable distributions and proposes estimating the parameters by equating the theoretical and empirical characteristic function in a generalized empirical likelihood / GMM framework. Regarding (Bayesian) indirect inference for the parameters of univariate stable distributions, we provide useful results that can be used to implement MCMC for any data set when draws from the posterior distribution of normal mixtures are available. Classical indirect inference has been examined by Garcia, Renault and Veredas (2011) among others, using the skewed Student-$t$as auxiliary model. As the authors mention this “appears as a good candidate since it has the same number of parameters as the α-stable distribution, with each parameter playing a similar role”.

Lombardi and Veredas (2007) used a multivariate Student-t to perform indirect estimation for for elliptical stable distributions based on the same argument.

A random variable X is called (strictly) stable if for all n, $\sum_{i=1}^{n} X_i \sim c_n X$, for some constant c_n, where $X_1, ..., X_n$ are independently distributed with the same distribution as X. It is known that the only possible choice is to have $c_n = n^{1/\alpha}$, for some $\alpha \in (0, 2]$. General non-symmetric stable distributions are defined via the log of the characteristic function (CF) which is given by the following expression (Samorodnitsky and Taqqu, 1994, and Zolotarev, 1986):

$$\log \varphi(\tau; \alpha, \beta) \triangleq \log \mathbb{E} \exp(\iota \tau X) = \begin{cases} \iota \mu \tau - \sigma^\alpha |\tau|^\alpha \left[1 - \iota \beta sgn(\tau) \tan \frac{\pi \alpha}{2}\right], & \alpha \neq 1, \\ \iota \mu \tau - \sigma |\tau| \left[1 + \iota \beta sgn(\tau) \frac{2}{\pi} \log |\tau|\right], & \alpha = 1, \end{cases} \tag{1}$$

for any $\tau \in \mathbb{R}$, where μ and σ are the location and scale parameters of the distribution, respectively, $\iota = \sqrt{-1}$, $\alpha \in (0, 2]$, $-1 \leq\leq \beta \leq 1$. The tail index is α and β measures asymmetry. We denote a general stable random variable by: $X \sim S_{\alpha,\beta}(\mu, \sigma)$. The density is given by:

$$f(x; \alpha, \beta) = \tfrac{1}{2\pi} \int_{-\infty}^{\infty} \exp(-\iota \tau x) \varphi(\tau; \alpha, \beta) \, d\tau. \tag{2}$$

The density is not available in closed form making it difficult to implement maximum likelihood or Bayesian MCMC procedures. Buckle (1995) and Tsionas (1999) considered scale mixture representation of stable distributions for Bayesian analysis in the general and symmetric class respectively. Tsionas (1999) exploited the fact that for symmetric laws, that is $X \sim S_{\alpha,0}(\mu, \sigma)$ we have: $X = W^{1/2} Z$, where $Z \sim \mathcal{N}(0, 1)$, and independently $W \sim S_{\frac{\alpha}{2},1}(0, 1)$.

Since this is a scale mixture of normal distributions, Bayesian numerical procedures are greatly facilitated. Buckle (1995) used another representation due to Zolotarev (1986, pp. 65-66):

$$X = \frac{\sin \alpha U}{(\cos \alpha U)^{1/\alpha}} \left[\frac{\cos(\alpha - 1) U}{E}\right]^{(1-\alpha)/\alpha}, \tag{3}$$

for $\alpha \neq 1$, where U is uniformly distributed in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$, and E is standard exponential. Approximate computation of the general stable densities is facilitated by the Fast Fourier Transform (FFT), see Mittnik, Doganoglu, and Chenyao (1999) and Mittnik, Rachev, Doganoglu, and Chenyao (1999)[1]. The integral representation in (2) is computed at $\overline{N}$ equally spaced points with distance h, that is $x_k = \left(k - 1 - \frac{\overline{N}}{2}\right) h$, $k = 1, ..., \overline{N}$. If $\tau = 2\pi\omega$, and we omit the conditioning of density and CF on α, β, the integral becomes

$$f\left(\left(k - 1 - \tfrac{\overline{N}}{2}\right) h; \alpha, \beta\right) = \int_{-\infty}^{\infty} \varphi(2\pi\omega; \alpha, \beta) \exp\left(-\iota 2\pi\omega \left(k - 1 - \tfrac{\overline{N}}{2}\right) h\right) d\omega,$$

[1] See also Matsui and Takemura (2006).

which can be approximated using the rectangle rule as:

$$f\left(\left(k-1-\tfrac{\overline{N}}{2}\right)h;\alpha,\beta\right)\cong \tfrac{1}{h\overline{N}}\left(-1\right)^{k-1-\frac{\overline{N}}{2}}\sum_{n=1}^{\overline{N}}\left(-1\right)^{n-1}\varphi\left(2\pi\left(n-1-\tfrac{\overline{N}}{2}\right)h/\overline{N};\alpha,\beta\right)\exp\left(-\iota 2\pi\left(n-1\right)\right)\left(k-1\right)/\overline{N}\right). \tag{4}$$

In turn, this is equivalent to performing a FFT to the sequence:

$$\left(-1\right)^{n-1}\varphi\left(2\pi\left(n-1-\tfrac{\overline{N}}{2}\right)h/\overline{N};\alpha,\beta\right),\quad n=1,...,\overline{N}.$$

A fairly accurate procedure results when $\overline{N}=2^{16}$, and $h=10^{-4}$. Accuracy of the FFT has been examined in detail by Tsionas (2012) in a different context. In the case of symmetric stable distributions, $S_{\alpha,0}\left(\mu,\sigma\right)$, McCulloch (1998) developed a more efficient procedure without sacrificing accuracy.

In connection with **multivariate stable Paretian distributions**, even the computation of the CF becomes complicated because they are only defined through their spectral measure, an object that is needed in order to retain the equivalence between the density and the characteristic function (Tsionas, 2016). As this computation is quite difficult, we use the following representation. Let $\varepsilon=[\varepsilon_1,...,\varepsilon_n]'$ and $u=[u_1,...,u_n]'$. Define:

$$\varepsilon=Cu,\ u_i\sim iid\,S_{\alpha_i,\beta_i}\left(0,1\right),\ i=1,...,n, \tag{5}$$

so we take a linear combination of $u=[u_1,...,u_n]'$ where the u_is follow independent $S_{\alpha_i,\beta_i}\left(0,1\right)$ distributions. The scale matrix is defined as $\Sigma=CC'$ although it does not represent a covariance matrix, unless $\alpha=2$ (for any value of β). Moreover, α_i and β_i are directly related to fat tails and skewness respectively. The important feature of this multivariate stable distribution is that it allows for different α and β for each time series.

We denote $\alpha=[\alpha_1,...,\alpha_n]'$ and $\beta=[\beta_1,...,\beta_n]'$. Their time-varying versions are denoted by α_t and β_t. We denote the n-dimensional stable distribution by $\mathbb{S}_{n,\alpha,\beta}\left(\Sigma\right)$, assuming the location parameter is a zero vector.

4 Time-varying scale, skewness and kurtosis

For the tail coefficient we first set $\tilde{\alpha}_i=-\ln\left(\frac{2}{\alpha_i}-1\right)$, which gives $\frac{\alpha_i}{2}=\frac{1}{1+e^{-\tilde{\alpha}_i}}$, viz. the standard logistic distribution function, so that $\tilde{\alpha}_i\in\mathbb{R}$.

$$\tilde{\alpha}_t=\gamma_1+\Gamma_1\tilde{\alpha}_{t-1}+\Gamma_2\tilde{\beta}_{t-1}+v_{t1}. \tag{6}$$

For the skewness coefficient, it is convenient to define $\tilde{\beta}_i=\frac{1}{2}\ln\frac{1+\beta_i}{1-\beta_i}=\text{arctanh}(\beta_i)$, the well-known Fisher's transformation so that $\tilde{\beta}_i\in\mathbb{R}$, and assume

$$\tilde{\beta}_t = \gamma_2 + \Gamma_3 \tilde{\alpha}_{t-1} + \Gamma_4 \tilde{\beta}_{t-1} + v_{t2}. \quad (7)$$

For other approaches to modeling skewness, see Theodossiou (1998, 2000) and Theodossiou and Savva (2015). In (6) and (7) both the tail and skewness coefficients follow a vector autoregressive (VAR) scheme. The specification of v_{t1} and v_{t2} is detailed below. We denote all elements in γ_j, Γ_js by γ. The initial conditions in (6) and (7) are treated as unknown parameters.

Suppose $u_t = [u_{t1}, ..., u_{tn}]', t = 1, ..., T$, is a vector of random variables distributed as $\mathbb{S}_{n,\alpha,\beta}(\Sigma)$. Recall that, so far, we have assumed that u_{ti}s have unit scale. As Σ is positive definite, it has n positive eigenvalues λ. Letting U denote the orthogonal matrix of orthonormal eigenvectors of Σ and $\Lambda = \text{diag}(\lambda)$, we can write

$$\Sigma = U \Lambda U'. \quad (8)$$

Define $L = \Lambda^{1/2} U'$, where $\Lambda^{1/2}$ denotes a diagonal matrix which contains the square roots of the eigenvalues along the main diagonal. In turn, we get a factorization of $\Sigma = L'L$ based on the spectral decomposition, and we define that

$$\varepsilon_t = L' u_t, \quad (9)$$

follows a multivariate SGT distribution. Based on (8), we can use the Givens parametrization (Appendix A) which uses the eigenvalues of Σ directly in the definition of a parameter vector δ. The Givens parametrization is based on the spectral decomposition of Σ given in (8). In summary, the Givens decomposition provides $\frac{n(n+1)}{2}$ parameters (whose domain is $\mathbb{R}$) so that using elementary operations (described in Appendix A) we can recover the scale matrix Σ_t. To these parameters we must, of course, add the two equations in (6) and (7).

Since we need the scale matrix to be stochastic and time-varying, say Σ_t, it is enough to make the parameters δ in (A.3) of Appendix A, time-varying and stochastic, as follows:

$$\delta_{it} = \theta_{i0} + \theta_{i1} \delta_{i,t-1} + \sigma_i \xi_{it}, i = 1, ..., d = n + \tfrac{n(n-1)}{2}, \quad (10)$$

where

$$\xi_{it} \sim iid\, \mathcal{N}(0, 1). \quad (11)$$

Notice that in (10) we do not use a VAR as in (6) and (7), since all elements of the scale matrix Σ_t will be time-varying through the Givens decomposition. To these equations, we add (6) and (7), so in total, we have $d = \frac{n(n+1)}{2} + 2$ elements in $\delta_t = [\delta_1, ..., \delta_d, \tilde{\alpha}_t, \tilde{\beta}_t]'$ and $3d$ parameters in (10) plus six additional parameters from (6) and (7). Moreover, these two additional equations introduce:

$$v_t = [v_{t,\frac{n(+1)}{2}+1}, v_{t,\frac{n(+1)}{2}+2}]', \tag{12}$$

which we represent as:

$$v_{t1} = \sigma_{\frac{n(n+1)}{2}} \xi^*_{t1},\ v_{t2} = \sigma_{\frac{n(n+1)}{2}} \xi^*_{t2}, \tag{13}$$

and

$$\xi^*_{t1}, \xi^*_{t2} \sim iid\,\mathcal{N}(0,1). \tag{14}$$

The multivariate model has a large number of parameters but, on the other hand, it is quite flexible for empirical purposes. Our parameters are:

$$\theta = [\gamma', \theta_0', \theta_1', \sigma_1, ..., \sigma_d]' \in \mathbb{R}^{10+3d}, \tag{15}$$

where $\theta_0 = [\theta_{i0}, i = 1, ..., d]'$, and $\theta_1 = [\theta_{i1}, i = 1, ..., d]'$.

We use a recent advance in sequential Monte Carlo methods known as the Particle Gibbs (PG) sampler, see Andrieu et al. (2010). The algorithm allows us to draw paths of the state variables in large blocks. Particle filtering is a simulation based algorithm that sequentially approximates continuous, marginal distributions using discrete distributions.

This is performed by using a set of support points called "particles" and probability masses; see Creal (2012) for a review. The PG sampler draws a single path of the latent or state variables from this discrete approximation. As the number of particles M goes to infinity, the PG sampler draws from the exact full conditional distribution.

Here, we follow Creal and Tsay (2015). As mentioned in Creal and Tsay (2015, p. 339): "The PG sampler is a standard Gibbs sampler but defined on an extended probability space that includes all the random variables that are generated by a particle filter. Implementation of the PG sampler is different than a standard particle filter due to the "conditional" resampling algorithm used in the last step. Specifically, in order for draws from the particle filter to be a valid Markov transition kernel on the extended probability space, Andrieu et al. (2010) note that there must be positive probability of sampling the existing path of the state variables that were drawn at the previous iteration. The preexisting path must survive the resampling steps of the particle filter. The conditional resampling step within the algorithm forces this path to be resampled at least once.

We use the conditional multinomial resampling algorithm from Andrieu et al. (2010), although other resampling algorithms exist, see Chopin and Singh (2013)."

Suppose the posterior is $p(\theta, \delta_{1:T}|\mathbf{y}_{1:T})$ where $\delta_{1:T}$ denotes the latent variables whose prior can be described by $p(\delta_t|\delta_{t-1}, \theta)$. In the PG sampler we can draw the structural parameters $\theta|\delta_{1:T}, \mathbf{y}_{1:T}$ as usual, from their posterior conditional distributions. This is important because, in this way, we can avoid mixture approximations or other

Monte Carlo procedures that need considerable tuning and may not have good convergence properties. As for posterior conditional distributions of parameters in θ we omit details as all of them are quite standard. Provided $u_t \sim iid\mathcal{N}(\mathbf{0}, \Sigma_t)$, $t = 1, ..., T$, under (10) estimation is possible by using Sequential Monte Carlo (SMC) techniques, also known as particle filtering.

When $u_t \sim \mathbb{S}_{n,\alpha,\beta}(\Sigma_t)$, the major obstacle is that the density function of $S_{\alpha,\beta}(0,1)$ is not available in closed form (notice that $\mathbb{S}_{n,\alpha,\beta}(\Sigma)$ depends, essentially, on computing n times the density function of $S_{\alpha_i,\beta_i}(0,1)$). If Σ is a constant matrix, this computation is not exceedingly difficult provide we use the FFT. When Σ_t is time varying, the situation is more involved. For drawing the latent variables, the details are in Appendix B.

5 Data

The analysis uses the same data set obtained from Apergis, Christou and Kynigakis (2019). The data set is composed of daily observations of the equity, volatility, government bond, insurance sector CDS, and bank sector CDS indices for Europe and the US. As the authors mention: "[T]he two equity indices are the Euro Stoxx 50 index (EUEQ) and the S&P 500 index (USEQ) for Europe and the US, respectively. The VSTOXX (EUVOL) is a volatility index based on option prices on the Euro Stoxx 50 index, while the CBOE volatility index or VIX (USVOL) is based on options written on the S&P 500 index. The bond indices for the European Monetary Union (EMU) and the US (EMUGB and USGB, respectively) are based on five-year sovereign bonds.

Finally, the credit default swaps indices are based on Thomson Reuter's five-year CDS data for the European and US bank (EUBCDS and USBCDS) and insurance sectors (EUICDS and USICDS)" (p. 5). All returns are computed as $r_{it} = \log(p_{it}/p_{i,t-1})$, where p_{it} is the corresponding price. The time span is from January 15, 2004 to January 14, 2012 (2,088 observations). The crisis event is set to September 15, 2008, which is the date when Lehman Brothers filed for Chapter 11 bankruptcy.

Apergis, Christou, and Kynigakis (2019) split the sample into two periods at the date when the shock occurred; the pre-crisis period from January 15, 2004 to September 15, 2008 and the post-crisis period from September 16, 2008 to January 14, 2012, for a total of 1,044 observations for each sub-period. In contrast, in our study we do not split the sample. However, results based on splitting the sample are reported in Section 6.2. The daily market returns are filtered with a 29-lag VAR model and the residuals are used, as in Apergis, Christou, and Kynigakis (2019).

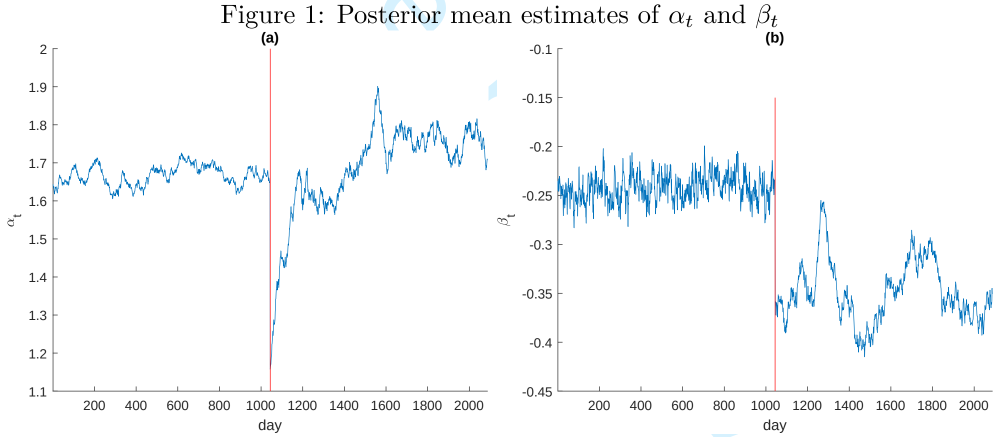
6 Results

6.1 Results based on the whole sample

Figure 1 presents the first principal component of α_t and β_t across all series. The first principal component accounts, in both cases, for over 90% in total variation of α_t and β_t. Clearly, there is a break around the date of the sub-prime crisis: from an average of 1.63, the characteristic exponent drops to 1.2 and reverts back to an average of 1.60 after, roughly, 400 days.

Towards the end of the sample, the characteristic exponent is close to 1.7. The skewness coefficient drops from an average of -0.25 to -0.37 and remains well below the pre-crisis level thereafter. This indicates higher skewness towards negative values for stock returns and fatter tails relative to the pre-crisis level for 400 trading days after the sub-prime crisis. In summary, although tails are somewhat thinner compared to the pre-crisis period, negative skewness prevails, and has substantial time persistence. If we ignore the time-varying nature of α_t and β_t and assume that they are constant over time and the same across all series, we obtain posterior densities as shown in Figure 2.

Figure 1: Posterior mean estimates of α_t and β_t

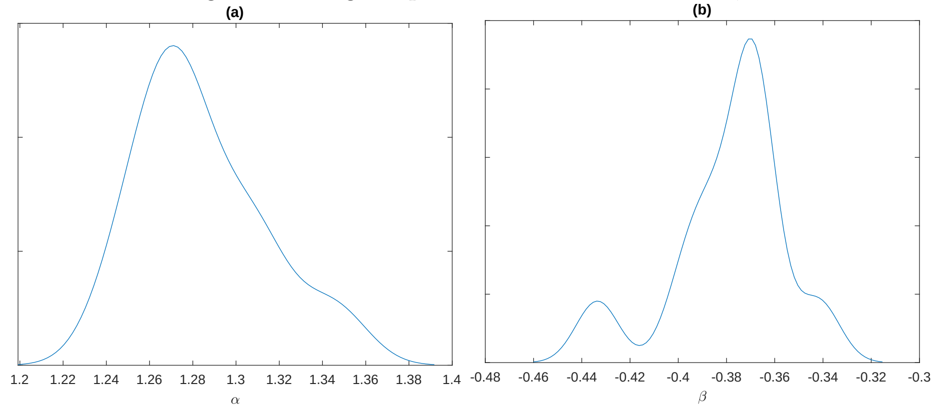


In Figure 1, we present the first principal component of α_t and β_t across all series. The first principal component accounts, in both cases, for over 90% in total variation of α_t and β_t. Clearly, there is a break around the date of the sub-prime crisis: From an average of 1.63, the characteristic exponent drops to 1.2 and reverts back to average of 1.60 after, roughly, 400 days.

Towards the end of the sample, the characteristic exponent is close to 1.7. The skewness coefficient drops from an average of -0.25 to -0.37 and remains well below the pre-crisis level thereafter. This indicated higher skewness towards negative values for stock returns and fatter tails relative to pre-crisis level for 400 trading days after the sub-prime crisis. In summary, although tails are somewhat thinner compared to the pre-crisis period, negative skewness prevails, and has substantial time persistence.

If we ignore the time-varying nature of α_t and β_t and assume that they are constant over time and the same across all series, we obtain posterior densities as shown in Figure 2.

Figure 2: Marginal posterior densities of α and β

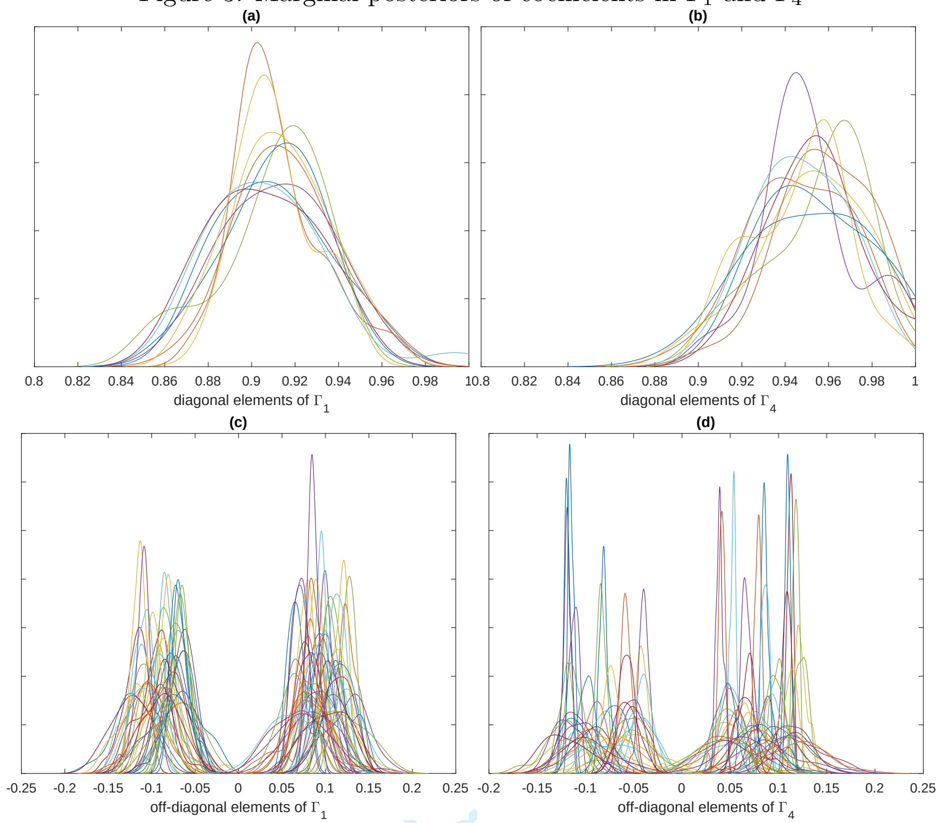


From these densities, it turns out that the characteristic exponent is 1.3 on the average, ranging from 1.2 to 1.4, and the distribution has positive skewness. The posterior density of the skewness parameter is clearly multimodal, with a dominant mode around -0.38 and ranging from -0.46 to -0.31. Given that this model is more restrictive relative to the previous model, the extent of misspecification that time-varying parameters can eliminate, is quite important.

Next, we present some evidence regarding co-skewness and co-kurtosis. In Figure 3, we provide marginal posterior densities of the diagonal elements of Γ_1 and Γ_4 (panels (a) and (b)), as well as marginal posterior densities of the off-diagonal elements of these matrices (panels (c) and (d)). All diagonal elements of Γ_1 and Γ_4 show that there is substantial persistence in α_t and β_t with evidence of unit roots in β_t as there is considerable probability concentration around unity. The evidence is corroborated by the marginal posterior densities of parameters in Γ_3 and Γ_4.

From the marginal posterior densities in panels (a) and (b) of Figure 4, it is clear that all the off-diagonal elements are away from zero so, this provides direct evidence in favor of co-kurtosis and co-skewness. Notice that there are 100 elements in Γ_3 and Γ_4 so we plot 100 different posteriors in each case. For the majority of off-diagonal elements in panels (c) and (d) of Figure 3 their marginal posterior densities are away from zero but certain coefficients are close to zero, in a probabilistic sense.

Figure 3: Marginal posteriors of coefficients in Γ_1 and Γ_4



As certain off-diagonal elements are zero it is important to find precisely which series are involved. A contagion test can be performed using the relevant lags of each pair in the VAR. We call it "C". To examine whether there is co-volatility for a given pair ("V") we compute the generalized impulse response function (Koop, Pesaran, and Potter, 1996) Σ_t implied by (10) along with its 95% highest posterior density interval.

For co-skewness (S12 and S21) and co-kurtosis (K12 and K21) we rely on the elements of Γ_4 and Γ_1, respectively. In all cases, except "V", we rely on a Bayes factor against the hypothesis that there is no contagion, no co-skewness, and no co-kurtosis. The Bayes factors are computed easily using the Verdinelli and Wasserman (1995) approximation without the need to apply MCMC for each instance.

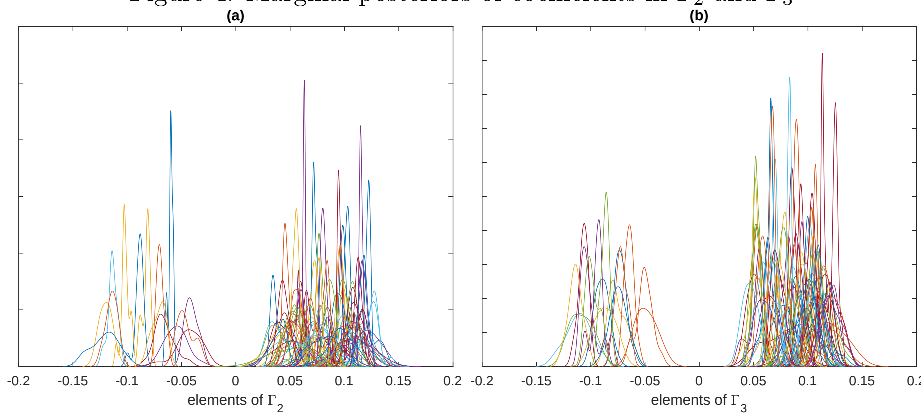
The results are presented in Table 1 which is in the same spirit as in Tables 1–4 of Apergis, Christou, and Kynigakis (2019).

Table 1: Contagion, Co-volatility, co-skewness and co-kurtosis summaries

Notes: EUBCDS, USBCDS and EUICDS, USICDS are the European and US bank sector CDS indices, and the European and US insurance sector CDS indices, respectively. EMUGB and USGB are the EMU and the US sovereign bond indices. EUEQ, USEQ and EUVOL, USVOL are the European and US equity and volatility indices, respectively. C stands for contagion, S12 and S21 for co-skeweness, K12 and V for co-kurtosis and V stands for co-volatility. The first column indicates the source market, while the first row indicates the recipient market. The symbols refer to results from Apergis, Christou and Kynigakis (2019) as follows:
● CS12 in their Table 2, † CS21 , □ CK13 in their Table 3a, ○ CK31 in their Table 3b, ↗ CV22 in their Table 4.

	EUBCDS	EUICDS	EMUGB	EUEQ	EUVOL	USBCDS	USICDS	USGB	USEQ	USVOL
EUBCDS	–	S12 S21 K12 K21 V ● † □ ○ ↗	S21 K21 † ○	C S12 S21 K21 V ● † □ ○ ↗	S12 S21 K12 K21 V ● † □ ○ ↗	S12 S21 K12 K21 V ● † □ ○ ↗ † □ ○ ↗	C S12 S21 K12 K21 ● † ○	S12 S21 K12 K21 V ● † ○ ↗	C S12 S21 K21 ● † ○	S21 K12 K21 V ○ † □
EUICDS	C S12 S21 K12 K21 V ● † ○ ↗	–	S21 K21	C S12 S21 K21 V † ○	S21 K12 K21 V † ○ ↗	S12 S21 K12 K21 V ● † ○ ↗	C S12 S21 K12 K21 V ● † □ ○ ↗	S12 K12 K21 V ● † □ ○ ↗	S12 S21 K21 V ● † ○ ↗	S21 K12 K21 ○ † □
EMUGB	S12 S21 K12 K21 ● □	S12 K12 K21 ● □	–	C S12 K21 V ● ○ □	S12 K12 V □	S12 S21 K12 K21 V ● † □ ○	V K21 □ ○	C S12 S21 K12 K21 V ● † □ ○	S21 K12 K21 V † □ ○	S12 K12 V ● □ ↗
EUEQ	C S12 S21 K12 K21 V ● † □ ○ ↗	C S12 K12 K21 V ● † □ ○ ↗	S21 K12 K21 V † □ ○ ↗	–	S21 K12 K21 V ● † □ ○ ↗	S12 K12 K21 V ● □ ○ ↗	S12 K12 K21 V ● □ ○ ↗	S21 K12 K21 V ● † □ ○ ↗	S21 K12 V † □ ○ ↗	S21 K21 V † ○ ↗
EUVOL	S12 S21 K21 V ● † □ ○ ↗	S12 K21 ● □ ○ ↗	S21 K21 V ○ ↗	S12 S21 ● †	–	S12 K12 K21 V ● □ ○ ↗	C S12 K12 ● □ ↗	S12 K12 V □ ○ ↗	S12 K12 V □ ↗	S12 K12 K21 V ○ ● □ ↗
USBCDS	S12 S21 K12 K21 V ● † □ ○ ↗	S12 K12 K21 V ● † □ ○ ↗	S12 S21 K21 V ● † □ ○ ↗	C S21 K12 K21 V † □ ○ ↗	S21 K21 V † ○ ↗	–	C S12 S21 K12 V V ● † □ ○ ↗	S21 K12 K21 V ● † □ ○ ↗	S21 K12 V K21 ● † □ ○ ↗	S21 K12 K21 V † □ ○ ↗
USICDS	C S12 S21 K12 V ● † □ ○	C K12 V K21 ● † □ ○ ↗	C S21 K21 V ○ ↗	S21 K21 V † □ ○ ↗	C S21 K12 K21 † □ ○ ↗	C S12 S21 K12 K21 V ● † □ ○ ↗	–	C S21 K12 K21 V ● † □ ○ ↗	K12 V □ ↗	C K12 K21 V □ ○ ↗
USGB	S12 S21 K12 K21 V ● † □ ○ ↗	V K21 † □ ○ ↗	C S12 S21 K21 V ● † □ ○ ↗	C S12 S21 K12 K21 V ● † ○ ↗	V K21 ↗	S12 S21 K12 K21 V ● † □ ○ ↗	C S12 K12 K21 V ● † □ ○ ↗	–	S21 K12 K21 V † □ ○ ↗	K12 K21 V □ ○ ↗
USEQ	S12 S21 K12 ● † □	S12 K12 V ● † □	S12 S21 V ●	S12 V K21 ● □	K12 K21 □	S12 S21 K12 V ● † □	K12 K21 V □	K12 K21 V ● □	–	K12 V □ ↗
USVOL	S12 K12 K21 V ● □ ○	S12 K12 K21 ● □ ○ ↗	S21 V K21 † ○ ↗	C S12 V K21 ● □ ○ ↗	K12 [illegible] K21 † □ ○ ↗	S12 K12 K21 V ● □ ○ ↗	C K12 K21 V □ ○ ↗	K12 K21 V □ ○ ↗	K12 K21 V □ ○ ↗	–

Figure 4: Marginal posteriors of coefficients in Γ_2 and Γ_3



Our results are very similar to those presented by Apergis, Christou and Kynigakis (2019). We summarize the findings as follows:

a) Within Europe, there exists contagion between equity and bank and insurance CDSs. Furthermore, significant contagion also occurs from insurance CDSs to bank CDSs, as well as from government bonds to equities. Within the US, it is evident that shocks propagate between insurance CDSs and all other indices, with the exception of equities, which are affected only by bank CDSs. In terms of regional contagion, the findings document that it runs from US insurance CDSs to all European indices, with the exception of equities. By contrast, US insurance CDSs are affected by both European CDSs and volatility, while contagion is transmitted to the European equities, running from US bank CDSs, sovereign bonds and volatility. Finally, contagion spreads from European bank CDSs to the US equities, and between sovereign bonds in both regions.

b) For co-skewness, in terms of the European markets, contagion originates from sovereign bonds and volatility towards equities, while, in terms of the S12 and S21 statistics, there are significant volatility (contagion) spillovers between the equity markets and all the remaining markets on both sides of the Atlantic. Sovereign bonds remain unaffected by the rest of the European indices. In terms of the US markets, contagion is found running from equities to sovereign bonds. Moreover, contagion runs from the European and US banking and insurance CDSs to the majority of the remaining indices; more specifically, within Europe, the analysis detects significant contagion running from equities to all other markets, as well as from volatility to bank CDSs and equities. By contrast, EMU sovereign bonds do not affect any other market CDSs. Within the US, contagion also occurs running from sovereign bonds to all indices, volatility, as well as from equities to bank CDSs. In terms of contagion across the two regions, the findings highlight contagion running from both sovereign bonds and European equities, as indicated by tests C12 and C21.

c) Regarding co-kurtosis the findings clearly illustrate that based on the CK13 test (which captures contagion from the expected returns of the source market to the skewness of the recipient market), European bank CDSs, insurance CDSs and volatility are all affected by all European indices, while in term of the CK31 test (which detects contagion from the skewness of the source market to the expected return of the recipient market), within Europe, contagion runs from bank CDSs, insurance CDSs, equities and volatility, with bonds affecting only equities. Contagion is widespread across all financial markets in the US, but US insurance CDSs do not affect any equity and volatility, respectively. Contagion spreads from the European and US bank and insurance CDSs towards the financial markets across both regions, while contagion occurs from the European equities to all US markets, as well as from US government bonds and volatility to all European CDSs. In contrast, the effects of contagion are weaker when EMU sovereign bonds, European volatility and US equities are considered as origins. In addition, from the evidence in Figure 4, it turns out that all elements of Γ_2 and Γ_3 are away from zero, indicating that there is an additional channel of contagion, running from skewness to kurtosis, and vice versa. The tests in Apergis, Christou and Kynigakis (2019) leave this channel unexplored as existing tests in the literature do not allow to examine cross co-skewness to co-kurtosis (panel (a)) and cross co-kurtosis to co-skewness (panel (b)).

This channel is highly significant as no parameter in Γ_2 or Γ_3 turns out to be close to zero. To put it differently, there appears to exist bi-directional causality between skewness and kurtosis indices in (6) and (7). To develop *a measure of co-kurtosis* we focus on (6). We need to evaluate the hypothesis $H : \mathring{\Gamma}_1 = \mathbf{O}$, where for any square matrix $\mathbf{A}$, we define $\mathring{\mathbf{A}}$ to contain all its off-diagonal elements. From Figure 3 it is clear that there is no support for H. To develop, however, a summary measure for each series, we estimate the full model as well as the model under H, for the entire sample using consecutive windows of length 50. In turn, we report the Bayes factor in favor of the full model and against H. Bayes factors in excess of, roughly, 10, indicate significant evidence in favor of the full model and against H. For absence of co-skewness we need $H : \mathring{\Gamma}_4 = \mathbf{O}$. For *absence of co-volatility* we need $H : \theta_{i1} = 0 \, \forall i = 1, ..., d$ in (10). Notice that the Bayes factors we derive are *conservative* in the sense that we do allow for stochastic variation in multivariate volatility, skewness and kurtosis. In Figure 5, we report marginal posterior densities of co-volatility, co-skewness, and co-kurtosis measures for the entire data set.

Figure 5: Posterior densities of co-volatility, co-skewness, and co-kurtosis measures

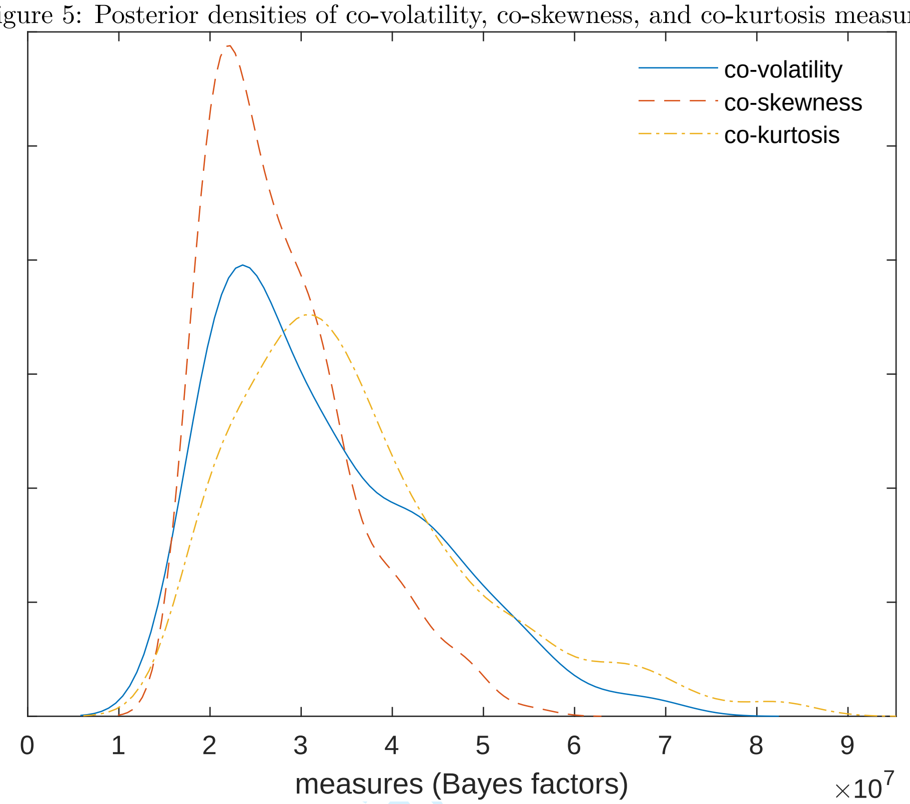
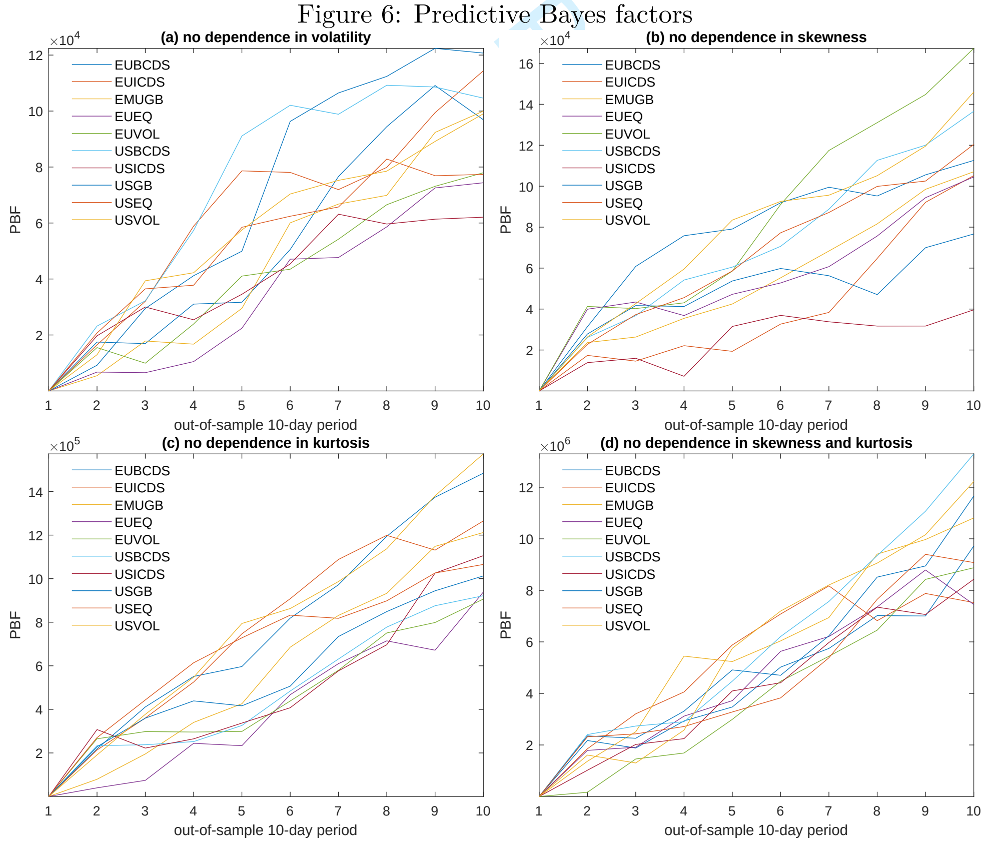


Figure 6: Predictive Bayes factors



Notes: EUBCDS, USBCDS and EUICDS, USICDS are the European and US bank sector CDS indices, and the European and US insurance sector CDS indices, respectively. EMUGB and USGB are the EMU and the US sovereign bond indices. EUEQ, USEQ and EUVOL, USVOL are the European and US equity and volatility indices, respectively.

In Figure 6 we present predictive Bayes factors (PBF) in favor of our general model and against (i) a model without co-volatility, (ii) a model without co-skewness, (iii) a model without co-kurtosis, and (iv) a model without co-skewness *and* co-kurtosis.[2] We leave a total of 100 observations out of the sample and we re-estimate the models using a rolling window of 10 days. We normalize the PBF for period 1 to unity for ease of comparison. The figure provides additional evidence in favor of the fact that our full model is essential in terms of out-of-sample performance, as the PBFs against more restricted versions are overwhelming, and tend to increase over time.

As noticed in Apergis, Christou, and Kynigakis (2019), the findings of co-skewness indicate the systemic importance of the banking and insurance sectors (Gropp and Moerman, 2004; Allen and Carletti, 2006, respectively), with contagion affects being prominent among bank and insurance CDS and also between the CDS and the majority of the remaining indices, for both EU and the US.

6.2 Splitting the sample

Given the evidence on breaks documented above it is more reasonable to split the same and re-examine the robustness of the findings. For the pre-crisis period, we were not able to document quantitatively and qualitatively important differences. For the period in the aftermath of the subprime crisis, we summarize our evidence in Table 2, which has the same structure as Table 1. Overall, the presence of strong contagion effects across markets clearly show that the results reported in Table 1 are dominated by those occurred in the aftermath of the financial crisis. More specifically:

[2]Marginal likelihood and, therefore, Bayes factors are a direct byproduct of Sequential Monte Carlo / Particle Filtering, see Appendix B.

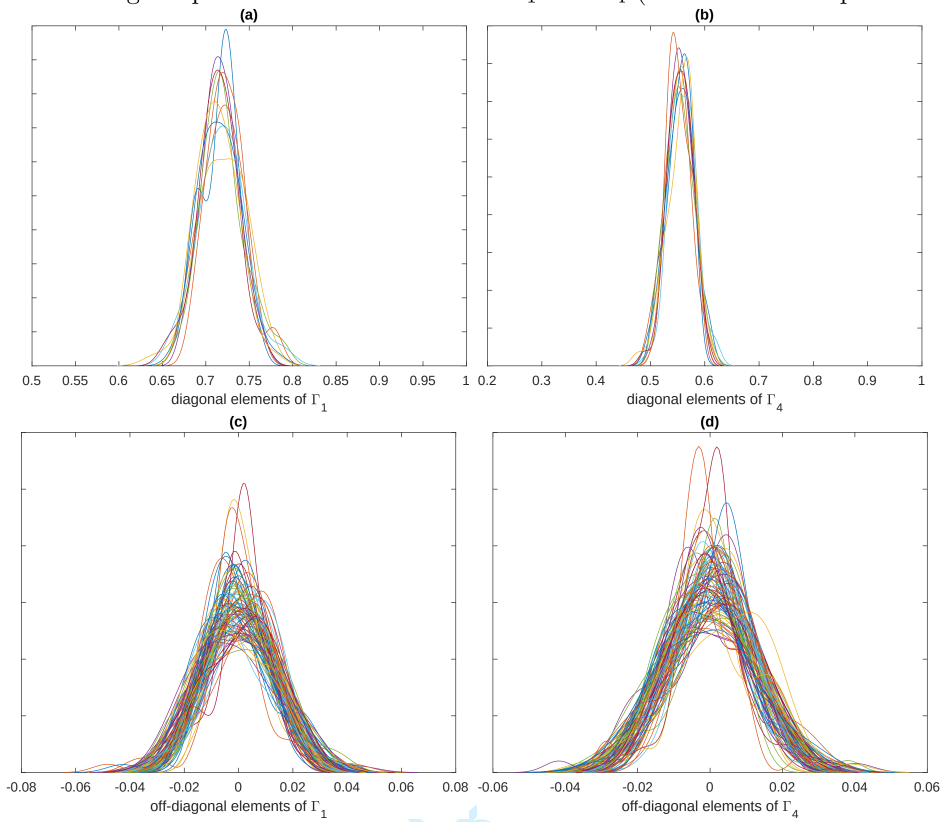
Table 2: Contagion, Co-volatility, co-skewness and co-kurtosis summaries in the aftermath of the subprime crisis

Notes: EUBCDS, USBCDS and EUICDS, USICDS are the European and US bank sector CDS indices, and the European and US insurance sector CDS indices, respectively. EMUGB and USGB are the EMU and the US sovereign bond indices. EUEQ, USEQ and EUVOL, USVOL are the European and US equity and volatility indices, respectively. C stands for contagion, S12 and S21 for co-skeweness, K12 and V for co-kurtosis and V stands for co-volatility. First column indicates the source market, while the first row indicates the recipient market. The symbols refer to results from Apergis, Christou and Kynigakis (2019) as follows:

	EUBCDS	EUICDS	EMUGB	EUEQ	EUVOL	USBCDS	USICDS	USGB	USEQ	USVOL
EUBCDS	–	V		C V	V	V	K21	V	C K21	V
EUICDS	C V	–	V	C V	V	S12 S21 K12 K21 V	C V	V	V	
EMUGB			–	C V	V	K21 V	V	C S V		V
EUEQ	C V	C V	S21 K21 V	–	S21 K21 V	S12 V	S12 V	S21 V	S21	S21
EUVOL	V		V	S21	–	K21 V	S12 K12	K12 V	K12 V	V
USBCDS	K21 V	K21 V	V	V	V	–	V	V	V K21	K21 V
USICDS	V					V	–	V	V	V
USGB	V	V	C V	C S12 V	V	S12 S21	C S12	–	S21	K21 V
USEQ		V	V	K21		V	V	V	–	K12 V
USVOL	V	V	V	C V K21	V K21	V	C V	V	V	–

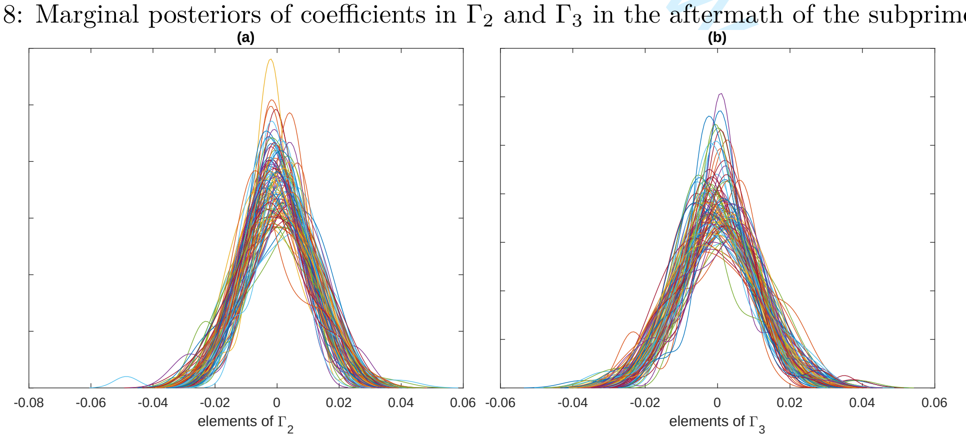
Even a cursory look at Table 2 shows that there are important differences relative to Table 1. To trace out the differences we look again into marginal posterior densities as in Figures 3 and 4. The new marginal posteriors of elements of the Γ−matrices are reported in Figures 7 and 8.

Figure 7: Marginal posteriors of coefficients in Γ_1 and Γ_4 (aftermath of subprime crisis)



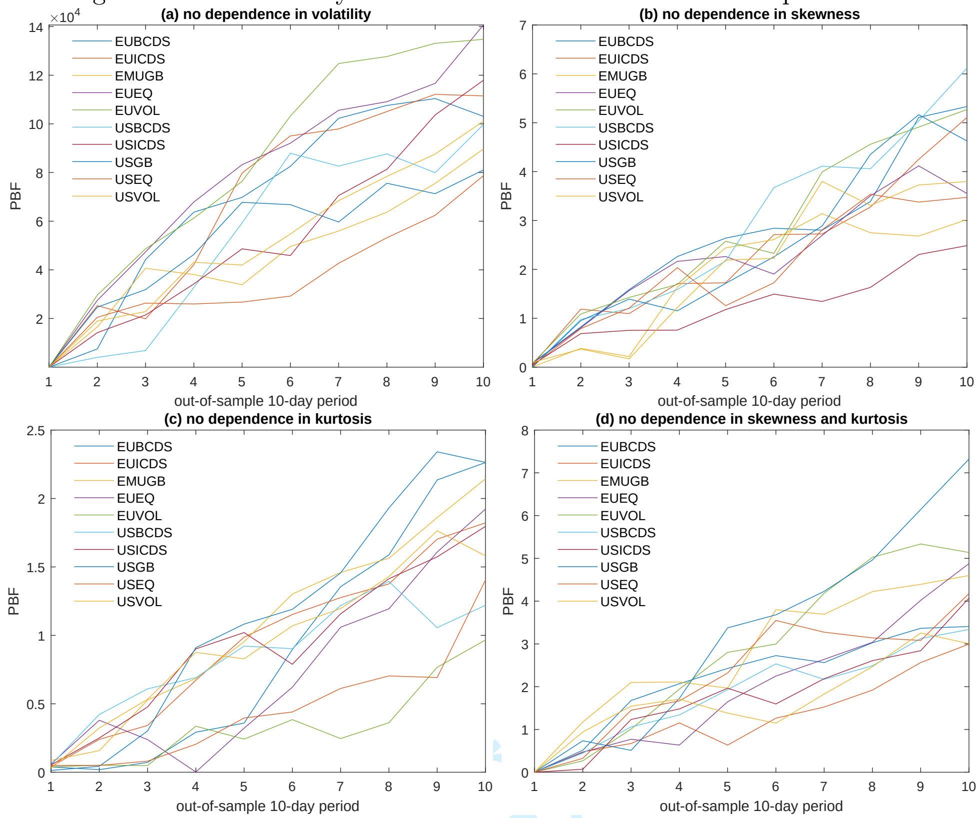
Surprisingly, perhaps, the diagonal elements of Γ_1 and Γ_4 are much lower (approximately, 0.73 and 0.55 on the average). The off-diagonal elements are also more concentrated around zero. The same conclusion holds, broadly, for the marginal posteriors of coefficients in Γ_2 and Γ_3, reported in Figure 8. Evidently, this change in marginal posterior densities affects in a substantive way the conclusions that lead to Table 2.

Figure 8: Marginal posteriors of coefficients in Γ_2 and Γ_3 in the aftermath of the subprime crisis



Predictive Bayes factors, similar to the ones reported in Figure 6 are reported in Figure 8. Although the Bayes factors still favor dependence in volatility, dependence in skewness is favored to a lesser extent when compared to Figure 6). Also, Bayes factors do not favor, at least by a large margin, dependence in kurtosis (panel(c)) or jointly skewness and kurtosis (panel (d)).

Figure 9: Recursive Bayes factors in the aftermath of the subprime crisis



We summarize our results as follows.

a) Within Europe, there exists contagion running from insurance CDSs to both equity markets on both sides of the Atlantic. Furthermore, significant contagion also occurs from European insurance CDSs to European bank CDSs and equities, as well as to US insurance CDS. In addition, contagion also is present running from European bonds to European equity markets, as well as to US bond markets. In terms of the equity markets, contagion also runs from these markets to the European bank and insurance markets. Within the US, it is evident that shocks propagate mainly from the US bond markets to European bond and equity markets, as well as to the US insurance markets. In terms of US volatility, contagion also occurs onto the US equity and insurance market. The results seem to be online with a number of studies from the CDS literature. For instance, Belke and Gokus (2011) suggest that volatility takes a significant higher level in times of crisis, which is particularly evident in the variances of stock returns and CDS spread changes. Furthermore, they provide evidence that correlations and covariances also increase after the outbreak of the 2008 crisis, indicating stronger dependency among the examined variables. Similar findings are provided by Jacobs, Karagozoglu and Peluso (2010) who document that risk aversion, as well as the uncertainty about credit products (e.g., bonds and CDS) increases after the crisis in a sense that borrowers paid higher compensations to potential investors for bearing default risks.

b) For co-skewness, in terms of the European markets, there is mutual contagion between European insurance and US bank CDS markets. In addition, European equity markets have received substantial contagion from all markets in Europe and in the US, while they exert significant contagion in both US bank and insurance CDS markets. In terms of the US markets, contagion is found running from sovereign bonds to equities, as well as to both bank and insurance CDS markets. The findings corroborate those provided by Norden and Weber (2009), Schreiber et al. (2009), and Rhaman (2009) who persuasively assert that the correlations between CDS spreads of different institutions, as well as those between CDS, bond and equity markets increase in turbulent times which indicates contagion effects. c) Regarding co-kurtosis the findings clearly illustrate that based on the CK13 test (which captures contagion from the expected returns of the source market to the skewness of the recipient market), European bank CDSs, insurance CDSs and volatility are all affected by all European indices. Contagion is widespread across all financial markets in the US. Contagion spreads from the European and US bank and insurance CDSs towards the financial markets across both regions, while contagion occurs from the European equities to all US markets, as well as from US government bonds and volatility to all European CDSs. This time, the effects of contagion remain even when EMU sovereign bonds, European volatility and US equities are considered as origins. These results receive support from certain studies in the relevant literature. For instance, Heinz and Sun (2014) in an effort to analyse sovereign CDS drivers show that bank CDSs and sovereign bonds are closely related in terms of volatility, especially after the financial crisis event.

Similarly, Acharya et al. (2013) present cross-country evidence about the potential for banks to trigger fiscal crises and, thus, ignite contagion effects. Their evidence documents that since 2010, sovereign risk has become a major concern with spillover effects from and to financial risk, due to the fact that banks have been heavily exposed to the sovereign risk.

Concluding remarks

This study used the family of multivariate stable distributions to provide measures of co-volatility, co-skewness and co-kurtosis in a principled way, avoiding the shortcomings of recent tests that assume the presence of higher moments, and perform only pairwise comparisons. As in Apergis, Christou and Kynigakis (2019), the analysis looked at set of bank CDSs, insurance CDSs, sovereign bonds, equity and volatility indices. The findings corroborated the results by Apergis, Christou and Kynigakis (2019) and indicated significant evidence of contagion, especially through the channels of higher order moments.

Following these authors, the subprime crisis was selected to be the decisive crisis event, using data on bank CDSs, insurance CDSs, sovereign bonds, equity and volatility indices, spanning the period 2004-2012. The evidence corroborated Apergis, Christou and Kynigakis (2019) in that not only co-volatility, but also co-skewness and co-kurtosis showed that these are important additional channels of contagion, mainly in the bank and insurance CDS markets. Although the literature had identified that contagion phenomena do not occur very frequently, our results provided evidence that contagion effects are very prevalent across important (developed) financial markets. As a result, policymakers should adopt policies to maintain financial stability, thus, minimizing the likelihood of contagion events that harmfully impact markets and economies confidence. In that sense, they should establish stable macroeconomic environments, as well as resilient domestic financial systems. Moreover, there must be some extensive collaborating efforts between domestic and international supervisory authorities that would guarantee efficient financial surveillance methods ending up with low negative spillovers running from financial markets to the real economy.

Appendix A. Givens decomposition

The Givens parametrization is based on the spectral decomposition of Σ given in (8) and the fact that the eigenvector matrix U can be represented by $n(n-1)/2$ angles, used to generate a series of Givens rotation matrices (Thisted, 1988, §3.1.6.6., Pinheiro and Bates, 1996). Specifically, we have:

$$U = G_1 G_2 \dots G_{n(n-1)/2}, \tag{A.1}$$

where

$$G_i(j,k) = \begin{cases} \cos(\psi_i), & \text{if } j = k = m_1(i) \text{ or } j = k = m_2(i), \\ \sin(\psi_i), & \text{if } j = m_1(i), k = m_2(i), \\ -\sin(\psi_i), & \text{if } j = m_2(i), k = m_1(i), \\ 1, & \text{if } j = k \neq m_1(i) \text{ and } j = k \neq m_2(i), \\ 0, & \text{otherwise,} \end{cases} \tag{A.2}$$

where $m_1(i) < m_2(i) \in \{1, ..., n\}$ subject to: $i = m_2(i) - m_1(i) + \frac{(m_1(i)-1)(n-m_1(i))}{2}$. In turn, assume that the eigenvalues of Σ have been sorted in ascending order. We use the Jupp (1978) parametrization:

$$\begin{aligned} \delta_i &= \ln(\lambda_i - \lambda_{i-1}), \; i = 1, ..., n, \; \lambda_0 = 1, \\ \delta_{n+i} &= \ln\left(\frac{\psi_i}{\pi - \psi_i}\right), \; i = 2, ..., n(n-1)/2. \end{aligned} \tag{A.3}$$

To ensure uniqueness of the Givens parametrization we must have $\psi_i \in (0, \pi), i = 1, ..., n(n-1)/2$. The main advantage of the Givens parametrization is that the first n elements of δ provide information about the eigenvalues of Σ directly. Unsurprisingly, one cannot relate δ to the other elements of Σ in a straightforward way. However, in (A.3) the δ_is are unrestricted so, we can recover λ_is and ψ_is to use in (A.2), then in (A.1), and, finally, in (8).

Appendix B. Sequential Monte Carlo

Suppose we have $\delta_{1:T}^{(1)}$ from the previous iteration. The particle filtering procedure consists of two phases.

Phase I: Forward filtering (Andrieu et al., 2010).

- Draw a proposal $\delta_{it}^{(m)}$ from an importance density $q(\delta_{it}|\delta_{i,t-1}^{(m)},\theta), m = 2, ..., M$.
- Compute the importance weights:

$$w_{it}^{(m)} = \frac{p(y_{it};\delta_{it}^{(m)},\theta)p(\delta_{it}^{(m)}|\delta_{i,t-1}^{(m)},\theta)}{q(\delta_{it}|\delta_{i,t-1}^{(m)},\theta)}, m = 1, ..., M. \tag{B.1}$$

- Normalize the weights: $\tilde{w}_{it}^{(m)} = \frac{w_{it}^{(m)}}{\sum_{m'=1}^{M} w_{it}^{(m')}}, m = 1, ..., M$.
- Resample the particles $\{\delta_{it}^{(m)}, m = 1, ..., M\}$ with probabilities $\{\tilde{w}_{it}^{(m)}, m = 1, ..., M\}$.

In the original PG sampler, the particles are stored for $t = 1, ..., T$, and a single trajectory is sampled using the probabilities from the last iteration. An improvement upon the original PG sampler was proposed by Whiteley (2010), who suggested drawing the path of the latent variables from the particle approximation using the backwards sampling algorithm of Godsill et al. (2004). In the forwards pass, we store the normalized weights and particles and we draw a path of the latent variables as we detail below (the draws are from a discrete distribution).

Phase II: Backward filtering (Chopin and Singh, 2013, Godsill et al., 2004).

- At time $t = T$ draw a particle $\delta_{iT}^{*} = \delta_{iT}^{(m)}$.
- Compute the backward weights: $w_{t|T}^{(m)} \propto \tilde{w}_{t}^{(m)} p(\lambda_{i,t+1}^{*}|\delta_{it}^{(m)},\theta)$.
- Normalize the weights: $\tilde{w}_{t|T}^{(m)} = \frac{w_{t|T}^{(m)}}{\sum_{m'=1}^{M} w_{t|T}^{(m')}}, m = 1, ..., M$.
- Draw a particle $\delta_{it}^{*} = \delta_{it}^{(m)}$ with probability $\tilde{w}_{t|T}^{(m)}$.

Therefore, $\delta_{i,1:T}^{*} = \{\delta_{i1}^{*}, ..., \delta_{iT}^{*}\}$ is a draw from the full conditional distribution. The backwards step often results in dramatic improvements in computational efficiency. For example, Creal and Tsay (2015) find that $M = 100$ particles is enough. There remains the problem of selecting an importance density $q(\delta_{it}|\delta_{i,t-1},\theta)$. We use an importance density implicitly defined by $\delta_{it} = a_{it} + \sum_{p=1}^{P} b_{it}\delta_{i,t-1}^{p} + h_{it}\xi_{it}$ where ξ_{it} follows a standard (zero location and unit scale) Student-t distribution with $\nu = 5$ degrees of freedom. That is, we use polynomials in $\lambda_{i,t-1}$ of order P. We select the parameters a_{it}, b_{it} and h_{it} during the burn-in phase (using $P = 1$ and $P = 2$) so that the weights $\{\tilde{w}_{it}^{(m)}, m = 1, ..., M\}$ and $\{\tilde{w}_{t|T}^{(m)}, m = 1, ..., M\}$ are approximately not too far from a uniform distribution.

Chopin and Singh (2013) have analyzed the theoretical properties of the PG sampler, and proved that the sampler is uniformly ergodic. They also prove that the PG sampler with backwards sampling strictly dominates the original PG sampler in terms of asymptotic efficiency.

Alternatively, when the dimension of the state vector is large, we can draw $\delta_{i,1:T}$, conditional on all other paths $\delta_{-i,1:T}$ that are not path i. Therefore, we can draw from the full conditional distribution $p(\delta_{i,1:T}|\delta_{-i,1:T}, \mathbf{y}_{1:T}, \theta)$.

References

[1] Acharya, V., Dreschlet, I., Schnabl, P. (2013). A pyrrhic victory: bank bailouts and sovereign credit risk. Mimeo.

[2] Allen, F., Carletti, E. (2006). Credit risk transfer and contagion. Journal of Monetary Economics 53, 89–111.

[3] Andrieu, C., Doucet, A., Holenstein, R. (2010). Particle Markov chain Monte Carlo methods (with discussion). Journal of the Royal Statistical Society Series B 72 (2), 1–33.

[4] Apergis, N., C. Christou and I. Kynigakis (2019). Contagion across US and European financial markets: Evidence from the CDS markets. Journal of International Money and Finance 96, 1–12.

[5] Arellano-Valle, R. B., A. Azzalini (2013). The centred parameterization and related quantities of the skew-t distribution. Journal of Multivariate Analysis 113, 73–90.

[6] Arellano-Valle, R. B., M. G. Genton (2010). Multivariate extended skew-t distributions and related families. Metron - International Journal of Statistics LXVIII (3), 201–234.

[7] Basso R.M., Lachos, V. H., Cabral, C. R. B., Ghosh, P. (2010) Robust mixture modeling based on scale mixtures of skew-normal distributions. Computational Statistics and Data Analysis 54, 2926–2941.

[8] Belke, A., Gokus, C. (2011). Volatility patterns of CDS, bond and stock markets before and during the financial crisis: evidence from major financial institutions. Working Paper, No. 243, Ruhr Economic Papers.

[9] Branco, M.D., Dey, D.K., (2001). A general class of multivariate skew elliptical distributions. Journal of Multivariate Analysis 79, 99–113.

[10] Cabral C., Lachos V., Prates, M. (2012) Multivariate mixture modeling using skew-normal independent distributions. Computational Statistics and Data Analysis 56:126– 142.

[11] Broda, S.A., Haas, M., Krause, J., Paolella, M.S., and S.C. Steude (2013). Stable mixture GARCH models, Journal of Econometrics 172 (2), 292–306.

[12] Buckle, D.J. (1995). Bayesian inference for stable distributions, Journal of the American Statistical Association 90, 605-613.

[13] Chopin, N., Singh, S.S. (2013). On the particle Gibbs sampler. Working paper, ENSAE. http://arxiv.org/abs/1304.1887.

[14] Creal, D.D. (2012). A survey of sequential Monte Carlo methods for economics and finance. Econometric Reviews 31 (3), 245–296.

[15] Creal, D., and R. Tsay (2015). High dimensional dynamic stochastic copula models. Journal of Econometrics 189 (2), 335–345.

[16] Forbes, K., Rigobon, R. (2002). No contagion, only interdependence: measuring stock market comovements. Journal of Finance 57, 2223–2673.

[17] Fruhwirth-Schnatter S., Pyne, S. (2010). Bayesian inference for finite mixtures of univariate and multivariate skew-normal and skew-t distributions. Biostatistics 11, 317–336.

[18] Fry, R., Martin, V.L., Tang, C. (2010). A new class of tests of contagion with applications. Journal of Business and Economic Statistics 28, 423–437.

[19] Fry-McKibbin, R., Hsiao, C.Y., Martin, V.L. (2018). Joint tests of contagion with applications. Quantitative Finance 19, 473–490.

[20] Godsill, S.J., Doucet, A., West, M. (2004). Monte Carlo smoothing for nonlinear time series. Journal of the American Statistical Association, 99 (465), 156–168.

[21] Gropp, R., Moerman, G. (2004). Measurement of contagion in banks' equity prices. J. Int. Money Financ. 23, 405-459.

[22] Heinz, F., Sun, Y. (2014). Sovereign CDS spreads in Europe: the role of global risk aversion, economic fundamentals, liquidity, and spillovers. IMF Working Paper, No.17.

[23] Jacobs, J., Karagozoglu, A.K., Peluso, C.M. (2010). Measuring credit risk: CDS spreads vs. credit ratings. Working Paper submitted for presentation at the 2010 FMA Meetings in New York.

[24] Jupp, D. L. B. (1978) Approximation to data by splines with free knots. SIAM Journal of Numerical Analysis 15 (2), 328–43.

[25] Hansen, B. E. (1994). Autoregressive Conditional Density Estimation. International Economic Review 35, 705–730.

[26] Koop, G., Pesaran, M. H. and S. M. Potter (1996). Impulse Response Analysis in Nonlinear Multivariate Models, Journal of Econometrics, 74, 119–147.

[27] Magnus, J. R. and H. Neudecker (1979). The commutation matrix: some properties and applications. The Annals of Statistics 1979, Vol. 7, No. 2, 381–394.

[28] McCulloch, J. H. (1998). Numerical Approximation of the Symmetric Stable Distribution and Density, in R. Adler, R. Feldman, and M. Taqqu (eds), A practical guide to heavy tails: Statistical techniques for analyzing heavy tailed data, Boston, Birkhauser, 489–500.

[29] Mittnik, S., T. Doganoglu, and D. Chenyao (1999). Computing the probability density function of the stable Paretian distribution, Mathematical and Computer Modelling 29, 235-240.

[30] Mittnik, S., S.T. Rachev, T. Doganoglu, and D. Chenyao (1999). Maximum likelihood estimation of stable Paretian models, Mathematical and Computer Modelling 29, 275-293.

[31] Norden, L., Weber, M. (2004). Informational efficiency of credit default swap and stock markets: the impact of credit rating announcements. Journal of Banking and Finance, 28, 2813-2843.

[32] Pinheiro, J. C., and D. M. Bates (1996). Unconstrained parametrizations for variance-covariance matrices. Statistics and Computing, 6, 289-296.

[33] Rahman, D. (2009). Are banking systems increasingly fragile? Investigating financial institutions' CDS returns extreme co-Movements. Working Paper, No. 34, Université Paris X Nanterre.

[34] Samorodnitsky, G., and M. S. Taqqu (1994). Stable non-Gaussian random processes, Chapman and Hall, New York.

[35] Schreiber, I., Müller, G., Klüppelberg, C., Wagner, N. (2009). Equities, credits and volatilities: a multivariate analysis of the European market during the sub-prime crisis. Working Paper, TUM, University of Passau, Germany.

[36] Sun, T., Zhang, X. (2009). Spillovers of the U.S. subprime financial turmoil to mainland China and Hong Kong SAR: evidence from stock markets. IMF Working Paper, No.166, Washington: International Monetary Fund.

[37] Theodossiou, P. (1998) Financial data and the skewed Generalized T distribution. Management Science 44 (12), 1650–1661.

[38] Theodossiou P. (2000) Skewed generalized error distribution of financial assets and option pricing. SSRN working paper, http://papers.ssrn.com/sol3/papers.cfm?abstract_id=219679.

[39] Theodossiou, P., Savva, C. S. (2015). Skewness and the Relation Between Risk and Return. Management Science 62 (6), 1598–1609.

[40] Thisted, R. A. (1988) Elements of Statistical Computing. Chapman & Hall, London. Tsionas, LGRM. G., 1999, Monte Carlo inference in econometric models with symmetric stable disturbances, Journal of Econometrics 88, 365-401.

[41] Tsionas, M. G. (2012). Maximum likelihood estimation of stochastic frontier models by the Fourier transform, Journal of Econometrics 170, 234–248.

[42] Tsionas, M. G. (2016). Bayesian analysis of multivariate stable distributions using one-dimensional projections Journal of Multivariate Analysis, 143, 185–193.

[43] Verdinelli, I. and Wasserman, L. (1995). Computing Bayes Factors Using a Generalization of Savage-Dickey Density Ratio, Journal of the American Statistical Association 90, 614-618.

[44] Wilhelmsson, A. (2009). Value at Risk with time varying variance, skewness and kurtosis—the NIG-ACD model. Econometrics Journal 12, 82–104.

[45] Zolotarev, V. M. (1986). One-dimensional stable distributions, AMS Translations of Mathematical Monographs, vol. 65, AMS, Providence, Long Island.