PAPER • OPEN ACCESS

Jointly modelling Cosmic Inflation and Dark Energy

View the article online for updates and enhancements.

Jointly modelling Cosmic Inflation and Dark Energy

Konstantinos Dimopoulos

Physics Department, Lancaster University, LA1 4YB, U.K.

E-mail: `k.dimopoulos1@lancasaster.ac.uk`

Abstract.   Quintessential inflation utilises a single scalar field to account for the observations of both cosmic inflation and dark energy. The requirements for modelling quintessential inflation are described and two explicit successful models are presented in the context of α-attractors and Palatini modified gravity.

1. Introduction

The 14 Gy history of the Universe requires special initial conditions, which are arranged by cosmic inflation [1]. Cosmic inflation is defined as a period of accelerated (superluminal) expansion in the early Universe. Inflation produces a Universe that is large and uniform according to observations.

In particle physics, inflation is typically modelled through the inflationary paradigm, which considers that the Universe undergoes inflation when dominated by the potential energy density of a scalar field, called the inflaton field. The inflaton field is homogenised by the rapid expansion.

The equation of motion of a (minimally coupled, canonically normalised) homogeneous scalar field ϕ is[1]

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0\,, \tag{1}$$

where the dots denote time derivatives and the prime denotes derivative with respect to the field. It is evident that the above equation is identical to the one describing the roll of a body (but in field space) down its potential $V(\phi)$ under friction due to H, which is the rate of the Universe expansion (Hubble parameter). Therefore, to assist our intuition, we can visualise the system as a ball rolling down the potential.

Since the inflationary paradigm requires the field to be potentially dominated during inflation, its kinetic energy density should be negligible, which means that the field slow-rolls a flat patch of $V(\phi)$, which is called the inflationary plateau. At some point, the potential becomes steep and curved so the kinetic energy density builds up and inflation is terminated at a critical value $\phi_{\rm end}$.

After the end of inflation, the inflaton field rushes down to its vacuum expectation value (VEV) and oscillates around it. These coherent oscillations have a particle interpretation; they correspond to massive particles (inflatons), which decay into the standard models particles that comprise the primordial plasma. The process is called reheating.

---

[1] We use natural units where $c = \hbar = k_B = 1$ and $8\pi G = m_P^{-2}$, with $m_P = 2.43 \times 10^{18}$ GeV being the reduced Planck mass.

As mentioned already, the Universe is homogenised by inflation. However, this uniformity is not perfect. Indeed, perturbations in the density of the material filling the Universe are necessary in order for structures (like galaxies and galactic clusters) to eventually form. Fortunately, inflation also provides these essential primordial density perturbations (PDPs). In the context of the inflationary paradigm, inflation does this as follows.

The superluminal expansion of space during inflation amplifies the quantum fluctuations of the inflaton field and renders them into classical perturbations of this field, through a process called quantum decoherence. This means that the critical value ϕ_{end} is reached at slightly different times at different points in space. Consequently, inflation continues a little more in some locations that in others. This results to primordial curvature perturbations which, through the Einstein equations, give rise to corresponding density perturbations.

Do we have any evidence of this amazing scenario? Indeed we do, because the PDPs reflect themselves onto the temperature perturbations of the Cosmic Microwave Background (CMB) radiation (a kind of afterglow of the Big Bang explosion) through the Sachs-Wolfe effect. These perturbations have been observed at the level of $\sim 10^{-5}$. In fact there is impressive agreement with the detailed CMB observations. From the observations, the main characteristics of PDPs are the following.

Firstly, the PDPs are predominantly adiabatic, which means that they are originally at the same level ($\sim 10^{-5}$) in all components of the Universe content (neutrinos, dark matter etc.). This strongly suggests that the primordial plasma is comprised by the decay products of a single degree of freedom, which could be the inflaton field. The second characteristic of the PDPs is that they are mainly Gaussian, which reflects the inherent randomness of the inflaton's quantum fluctuations. Finally, the PDPs are almost scale-invariant.

This approximate scale-invariance suggests that inflation is of a special type, called quasi-de Sitter inflation, where the energy density during inflation is almost constant. What does this translate into, in the inflationary paradigm? Well, a homogeneous scalar field can be modelled as a perfect fluid with barotropic parameter

$$w \equiv \frac{p}{\rho} = \frac{\rho_{\mathrm{kin}} - V}{\rho_{\mathrm{kin}} + V}\,, \tag{2}$$

where $\rho_{\mathrm{kin}} \equiv \frac{1}{2}\dot{\phi}^2$ is the kinetic energy density of the scalar field. During inflation we have potential domination, $V \gg \rho_{\mathrm{kin}}$, which means that $w \approx -1$. Indeed, the expectation value of a slow-rolling inflaton is roughly constant because ts kinetic energy density is negligible. This means that their potential energy density remains constant so that $\rho \simeq V \simeq \mathrm{constant}$ and inflation is quasi-de Sitter.

A lot of emphasis is put on the PDPs because they can discriminate between different inflationary models. Apart from the PDP amplitude, which is $\sim 10^{-5}$, there are two other observables which focus the attention of observationalists and theorists alike. They are the spectral index of the scalar perturbations n_s and the tensor to scalar ratio r. At the moment, observations suggest [2]

$$n_s = 0.968 \pm 0.006\ (1\text{-}\sigma) \quad \text{and} \quad r < 0.06\ (2\text{-}\sigma)\,. \tag{3}$$

In terms of the spectrum of scalar perturbations, the spectral index is $\mathcal{P}_\zeta(k) \propto k^{n_s(k)-1}$, where k is the momentum scale. Notice that $\mathcal{P}_\zeta(k)$ is not necessarily a power-law since $n_s = n_s(k)$. It is evident that when $n_s = 1$ then the dependence of $\mathcal{P}_\zeta(k)$ on k disappears and the spectrum is scale-invariant. From Eq. (3), we see that the observed values of n_s are close to unity but also significantly away from it (see also Fig, 1).

Inflation generates tensor perturbations (gravitational waves) in a similar manner that it does scalar perturbations, which give rise to the PDP. The ratio of the spectra of the tensor
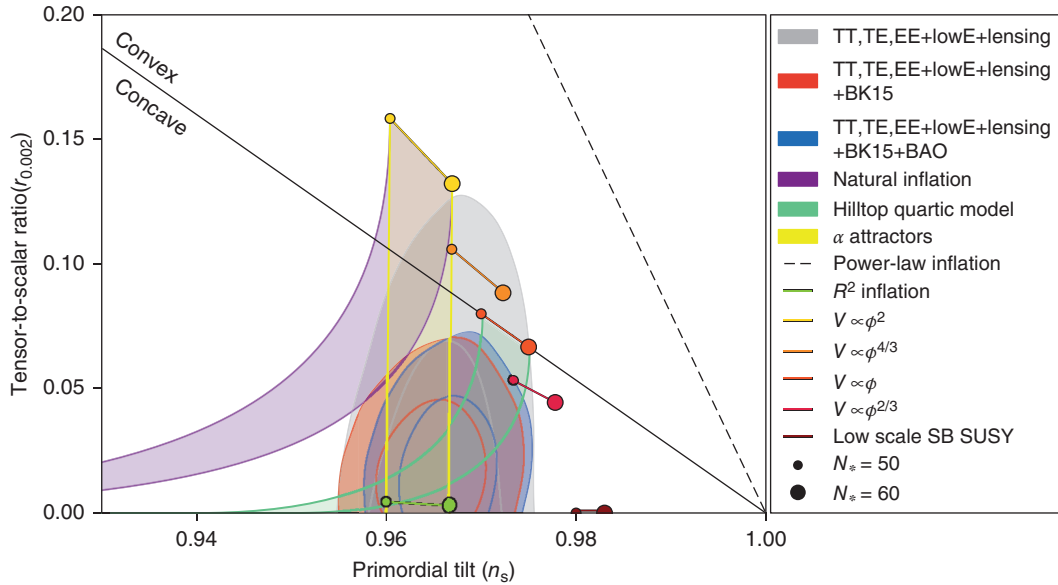
Figure 1. Planck sattelite observations of the spectral index n_s vs the tensor to scalar ratio r. The blobs correspond to particular inflation models (see legend) considering $N = 60$ [$N = 50$] efolds (big [small] blob) after the cosmological scales exit the horizon.

perturbations $\mathcal{P}_t(k)$ over the scalar perturbations $\mathcal{P}_\zeta(k)$ is $r \equiv \mathcal{P}_t/\mathcal{P}_\zeta$, which is bounded from above at 6%, cf. Eq. (3). The latest observations of n_s and r from the Planck satellite mission are shown in Fig. 1. The blobs in the graph correspond to particular inflationary models. We discuss some of them below.

2. Examples of inflation models

2.1. Quartic hilltop inflation

The name of hilltop inflation was coined in 2005 by Lotfi Boubekeur and David H. Lyth [3]. The potential is

$$V(\phi) = V_0 - \lambda\phi^4 + \cdots, \tag{4}$$

where V_0 is a constant density scale and $\lambda > 0$ is a self-coupling constant, where the ellipsis denotes higher order terms, which are needed to stabilise the potential. These terms are negligible during inflation. The inflationary observables are analytically related as [4]

$$r = \frac{8}{3}(1 - n_s)\left\{1 - \frac{\sqrt{3[2(1-n_s)N - 3]}}{(1 - n_s)N}\right\}, \tag{5}$$

where N is the number of exponential expansions (efolds) after the cosmological scales are pushed out of the horizon due to the superluminal expansion during inflation. The value of N depends on the reheating process. For prompt reheating $N \simeq 60$, but if reheating is very inefficient it can be decreased down to $N \simeq 50$. The predictions of the model correspond to the green band in Fig. 1, as specified in the legend. It is evident that this band overlaps significantly with the 1-σ contour of the observations.

In order to produce the correct amplitude for the PDP, we need $\lambda \sim 10^{-12}$. As a result, the VEV of the inflaton is super-Planckian, which undermines the perturbative origin of the potential (since all the Planck-suppressed operators become important).

2.2. Starobinsky inflation

This was proposed in 1980 by Alexei A. Starobinsky [5]. It is the first inflation model (the name was not even coined yet) and it's fair so say that it is still the most successful one. The model is

$$\mathcal{L} = \frac{1}{2}m_P^2 R + \beta R^2 \,, \tag{6}$$

where R is the scalar curvature (Ricci scalar). The above is a modified gravity theory. The first term on the right-hand-side is the usual Einstein-Hilbert term. The importance of the higher order term is gauged by the coefficient β. The theory is not a Taylor expansion, so β is not perturbative. Quadratic (R^2) gravity introduces an additional degree of freedom which can be revealed in the form of a scalar field (scalaron), when transforming from the modified gravity frame (Jordan frame) to the general relativity frame (Einstein frame). This conformal transformation redefines the metric as $g_{\mu\nu} \to \Omega^2 g_{\mu\nu}$, where Ω^2 is the conformal factor, given by $\Omega^2 = \exp(\sqrt{\frac{2}{3}}\,\phi/m_P)$, where ϕ is the scalaron field. In this theory, the conformal factor is $\Omega^2 = 1 + 4\beta R/m_P^2$. Transforming to the Einstein frame renders the theory as

$$\mathcal{L} = \frac{1}{2}m_P^2 R + \frac{1}{2}(\partial\phi)^2 - V(\phi) \,, \tag{7}$$

where $(\partial\phi)^2 \equiv -\partial_\mu\phi\,\partial^\mu\phi$ and R corresponds to the new metric. The potential is

$$V(\phi) = \frac{m_P^4}{16\beta}\left(1 - e^{-\sqrt{\frac{2}{3}}\,\phi/m_P}\right)^2 \,. \tag{8}$$

The potential approaches a constant $V \simeq m_P^4/16\beta$ for large vales of the inflaton, which is generating the inflationary plateau. The predictions of the model are

$$n_s = 1 - \frac{2}{N} \quad \text{and} \quad r = \frac{12}{N^2} \,. \tag{9}$$

As shown in Fig. 1, the predictions of the model fall near the sweet spot of the Planck observations, especially for $N \simeq 60$. In order to produce the correct amplitude of PDPs we require $\beta = 5.5225 \times 10^8$.

2.3. α-attractors

A much more recent proposal generates the inflationary plateau not by considering modified gravity, but by introducing a suitable non-minimal kinetic term. In 2013, Renata Kallosh, Andrei Linde and Diederik Roest have introduced the model [6]

$$\mathcal{L} = \frac{1}{2}m_P^2 R + \frac{\frac{1}{2}(\partial\varphi)^2}{\left[1 - \frac{1}{6\alpha}\left(\frac{\varphi}{m_P}\right)^2\right]^2} - V(\varphi) \,, \tag{10}$$

where the kinetic term features poles whose location in field space is determined by the value of the parameter $\alpha > 0$. The above can be obtained in supergravity with a non-trivial Kähler manifold but can also originate in conformal theory. We can switch to a canonically normalised field ϕ using the transformation

$$\frac{\mathrm{d}\varphi}{1 - \frac{1}{6\alpha}\left(\frac{\varphi}{m_P}\right)^2} = \mathrm{d}\phi \quad \Rightarrow \quad \frac{\varphi}{m_P} = \sqrt{6\alpha}\,\tanh\left(\frac{1}{\sqrt{6\alpha}}\frac{\phi}{m_P}\right) \,. \tag{11}$$

The canonical field satisfies an equation of the form shown in Eq. (7).

The remarkable characteristic of these models is that the inflationary predictions are independent from the particular form of the potential as long as it does not feature the same poles. Indeed, we have

$$n_s = 1 - \frac{2}{N} \quad \text{and} \quad r = \frac{12\alpha}{N^2} \,. \tag{12}$$

The above predictions are identical with Starobinsky inflation (cf. Eq. (9)) if $\alpha = 1$. Modulating α we can transpose the predictions vertically in Fig. 1 without affecting the value of n_s (the yellow lines in the figure). This is the reason why these models are called "attractors". Obviously, α cannot be too large ($\alpha < 18$).

3. Quintessence as dark energy

Observations suggest that the late Universe is also undergoing accelerated expansion, attributed to an exotic substance called dark energy, which accounts for almost 70% of the Universe content at present. Dark energy could correspond simply to a positive cosmological constant $\Lambda > 0$. However, this requires incredible fine-tuning of the order of 10^{-120}, which has been called "the worst fine-tuning in physics" (Lawrence Krauss). Moreover, $\Lambda > 0$ violates the swampland conjectures which postulate that there are no de Sitter vacua in the string landscape.

This is why alternative suggestions for explaining dark energy (while assuming zero vacuum density) have been put forward. One of the most prominent ones is quintessence [7], so called because it is the fifth element after normal (baryonic) matter, dark matter, photons (mainly CMB) and neutrinos.

Quintessence is a scalar field, much like the inflaton field, slow-rolling down a runaway potential, called quintessential tail. Thus, it can be said that the Universe is currently engaging in a bout of late time inflation. This inflation is also quasi-de Sitter, since the observations suggest that the barotropic parameter for dark energy is $w = 1.006 \pm 0.045$ if constant or $w \in [-1, 0.95]$ if w is variable [8].

Of particular interest is thawing quintessence, which begins frozen at some value at its quintessential tail, only to start unfreezing today when it starts to become dominant. However, being a dynamical degree of freedom, quintessence has to have its initial conditions explained. In particular, for thawing quintessence, the initial frozen value must be such that the potential density is comparable with the current energy density of the Universe. This is called, the coincidence problem.

4. Quintessential inflation

One way to account for the coincidence problem of quintessence is to connect the latter with inflation. In the end of the last century, P. Jim E. Peebles and Alex Vilenkin have proposed quintessential inflation [9], where the quintessence field is identified with the inflaton. This is a natural idea because both are scalar fields. It is also economic as it utilises one degree of freedom to explain the history of the early and the late Universe. Moreover, it models this history in a common theoretical framework. A successful model of quintessential inflation must satisfy the observations of both cosmic inflation and dark energy, which is rather difficult but not impossible. Finally, the initial conditions of quintessence are fixed by the inflationary attractor, so the coincidence problem is overcome.

Typically, in quintessential inflation the scalar potential features two flat regions, the inflationary plateau and the quintessential tail, connected through a sharp potential cliff. Because the inflaton field must survive until today to become quintessence, it cannot decay into the primordial plasma, as in the standard inflationary paradigm. Therefore, reheating has to occur by other means. Fortunately, there are many mechanisms which can generate the primordial radiation without the decay of the inflaton, e.g. gravitational reheating [10], instant

preheating [11], curvaton reheating [12], Ricci reheating [13] and warm quintessential inflation [14] to name but some.

Below two different models of Quintessential inflation are presented, to demonstrate how such constructions are feasible in modern theory.

4.1. Quintessential inflation with α-attractors

Quintessential inflation with α-attractors has been studied in Refs. [15, 16, 17]. The Lagrangian density is the one given in Eq. (10) with the potential of the non-minimal inflaton being $V(\varphi) = V_0 \exp(-\kappa\varphi/m_P)$ and we also add a negative cosmological constant term given by $V_\Lambda = -V_0 e^{\kappa\sqrt{6\alpha}}$ in order to ensure that the vacuum density is zero (as postulated to overcome the cosmological constant problem well before the observation of dark energy). Note that a negative cosmological constant is natural in string theory, which considers many anti-de Sitter vacua.

Switching to the canonical field via the transformation in Eq. (11) the scalar potential becomes

$$V(\phi) = e^{-n}V_0 \left\{ \exp\left[n \left(1 - \tanh\frac{\phi}{\sqrt{6\alpha}m_P} \right) \right] - 1 \right\}, \tag{13}$$

where $n = \kappa\sqrt{6\alpha}$. It might not be evident, but the above potential has the desired form.

Inflation occurs on the inflationary plateau in the limit $\phi \to -\infty$ where the potential is reduced to

$$V(\phi) \simeq e^n V_0 \left(1 - 2n e^{\sqrt{\frac{2}{3\alpha}}\,\phi/m_P} \right), \tag{14}$$

which, when $\alpha = 1$, is identical with Starobinsky inflation along the inflationary plateau, as evident by comparing with Eq. (8).

The quintessence limit corresponds to $\phi \to +\infty$ in which the potential is reduced to

$$V(\phi) \simeq 2n e^{-n} V_0 \, e^{-\sqrt{\frac{2}{3\alpha}}\,\phi/m_P}, \tag{15}$$

which is a standard exponential quintessential tail.

As shown in Ref. [16], the model works for $\alpha \simeq 0.3$ and $\kappa \simeq 60$, which means that the non-canonical inflaton in the original exponential potential is suppressed by the scale of grand unification.

4.2. Quintessential inflation in Palatini gravity

Recently, the framework of Palatini modified gravity has been utilised to study quintessential inflation in Ref. [18]. The Lagrangian density is

$$\mathcal{L} = \frac{1}{2}m_P^2 R + \beta R^2 + \frac{1}{2}(\partial\varphi)^2 - V(\varphi). \tag{16}$$

The first part of the above is identical with the Starobinsky model in Eq. (6), but the crucial difference is that, in Palatini gravity, the R^2-term does not introduce an extra degree of freedom (no scalaron) so that a scalar field has to be explicitly introduced, as shown in Eq. (16). After a conformal transformation, we can switch to Einstein gravity, where the Lagrangian density is

$$\mathcal{L} = \frac{1}{2}m_P^2 R + \frac{\frac{1}{2}(\partial\varphi)^2}{1 + 16\beta\frac{V(\varphi)}{m_P^4}} - \frac{V(\varphi)}{1 + 16\beta\frac{V(\varphi)}{m_P^4}}. \tag{17}$$

The above demonstrates that the scalar potential has become $U = \frac{V}{1+16\beta V/m_P^4}$, which is very useful in designing quintessential inflation models, because all is needed is a runaway potential
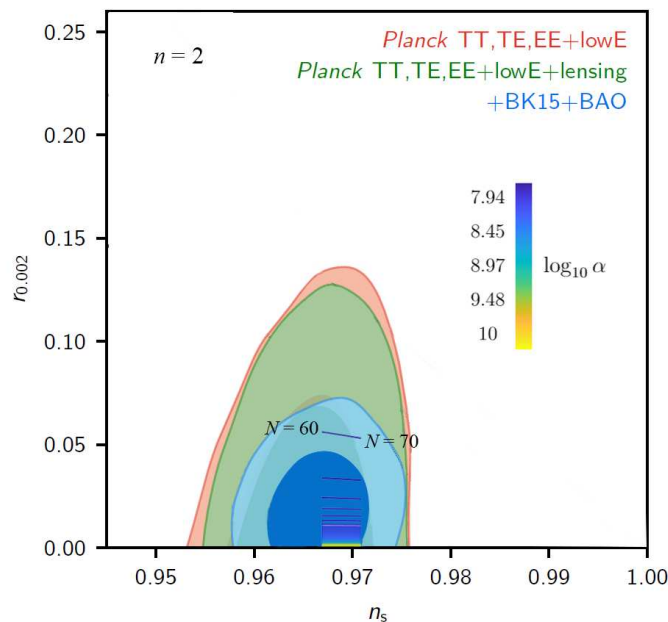
Figure 2. The inflationary predictions of the model in Eq. (19) in Palatini modified gravity theory in Eq. (16). We see that the model performs very well for $\beta \gtrsim 10^8$ (confusingly, β is denoted as α in the figure).

V (e.g. of racetrack type). When V becomes large, the unity term in the denominator of U becomes negligible and the potential U plateaus with $U \simeq m_P^4/16\beta$, which is exactly the Starobinsky inflationary plateau. In the opposite limit, when V is small, the denominator in U approaches unity. This is also true for the kinetic term, so the field becomes canonically normalised. Thus, any successful (thawing) quintessence model can, in principle, work for modelling quintessential inflation in this setup, because Palatini gravity "flattens" the runaway quintessence potential at large values, creating thereby the inflationary plateau.

In Ref. [18] a generalisation of the original quintessential inflation model has been considered, where the potential is

$$V(\varphi) = \begin{cases} \frac{\lambda^n}{m_P^{n-4}}(\varphi^n + M^n) & \text{when } \varphi < 0 \\[2ex] \frac{\lambda^n}{m_P^{n-4}} \frac{M^{n+q}}{\varphi^q + M^q} & \text{when } \varphi \geq 0 \,, \end{cases} \tag{18}$$

where $\lambda = \text{constant}$ and n, q are positive integers of order unity. It was found that the best results are obtained when $(n, q) = (2, 4)$. Then, the above becomes

$$V(\varphi) = \begin{cases} m^2(\varphi^2 + M^2) & \text{when } \varphi < 0 \\[2ex] \frac{m^2 M^6}{\varphi^4 + M^4} & \text{when } \varphi \geq 0 \,, \end{cases} \tag{19}$$

where $m = \lambda m_P$.

When studying inflation, it was found that the correct amplitude of the PDPs is obtained with $m \simeq 8.8 \times 10^{12}\,\text{GeV}$. For the spectral index and the tensor-to-scalar ratio, we find $n_s \simeq 0.9708$
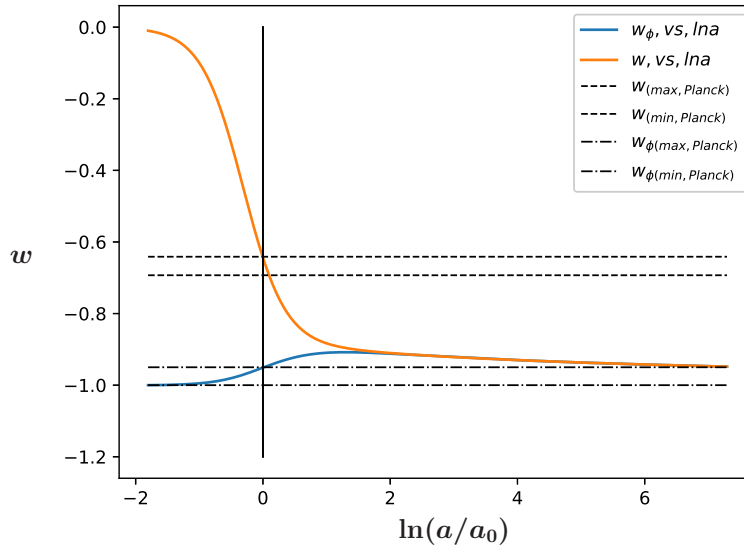
Figure 3. The running of the barotropic parameter of quinessence (lower curve -blue) and of the Universe (upper curve - orange) in the model in Eq. (19) with respect to the scale factr of the Universe. The vertical line corresponds to the present time, while the horizontal lines correspond to the observationl bounds. Originally quintessence is frozen with $w_\phi = -1$, while the Universe is matter dominated with $w = 0$. As quintessence unfreezes, its barotropic parameter w_ϕ grows. Simultaneously, quintessence begins to dominate the Universe so that the overal barotropic parameter w decreases. In the future, quintessence fully dominates the Universe so $w = w_\phi$ and the two curves merge.

and $r \lesssim 0.05$. In more detail, our findings are shown in Fig. 2, where it is shown that successful inflation is obtained with $\beta \gtrsim 10^8$, similar to the Starobinsky values. By varying β one moves vertically in the $n_s - r$ graph of Fig. 2, similarly to the case of α-attractors (cf. Fig. 1).

When studying quintessence, we find that coincidence is attained when $M \sim 10\,\mathrm{GeV}$. No incredible fine tuning needed, in contrast to ΛCDM. The barotropic parameter of thawing quintessence is variable. In this limit, the model is reduced to inverse power-law quartic quintessence, which has been studied in Ref. [16]. The behaviour of the barotropic parameters of quintessence w_ϕ and of the Universe w is shown in Fig. 3, plotted against the scale factor $a(t)$, which parametrises the Universe expansion (it grows with time).

A varying dark energy barotropic parameter is parametrised as [19] (CPL parametrisation)

$$w_\phi = w_0 + \left(1 - \frac{a}{a_0}\right) w_a \,, \tag{20}$$

which is obtained by Taylor expansion of $w_\phi(a/a_0)$ around the present time, when $a = a_0$. In the above, w_0 and $w_a \equiv -(\mathrm{d}w_\phi/\mathrm{d}a)_0$ are constants, soon to be measured by forthcoming observations of the EUCLID and wFIRST (Nancy Grace Roman) satellites. At present, the observational bounds from the Planck satellite are $-1 \le w_0 \le -0.95$ and $w_a = -0.28^{+0.31}_{-0.27}$ [8]. Varying w_0 in the allowed region, we find $-0.0659 < w_a < 0$, which is well in agreement with current observations and is to be tested in the near future.

5. Conclusions

Cosmic inflation determines the initial conditions of the history of the Universe and leads to a large and uniform Universe, as observed. Inflation also generates the primordial density perturbations, which seed galaxy formation and are reflected on the observed CMB anisotropy. The Universe today engages into a late inflationary period, which may be due to quintessence, a form of dark energy. Quintessential inflation assumes that a single field drives both inflation and quintessence, thereby allowing the study of the early and late Universe in the context of a common theoretical framework. Quintessential inflation leads to distinct observational signatures, such as a varying dark energy barotropic parameter and a spike in primordial gravitational waves [20] soon to be tested by observations. For more details, see Ref. [21].

Palatini modified gravity is a natural framework for model-building quintessential inflation because if "flattens" a runaway scalar potential to generate the desired inflationary plateau. It was demonstrated that successful quintessential inflation in Palatini modified gravity can be achieved with a runaway potential which interpolates between quadratic at high energies to inverse quartic at low energies. Concrete predictions of successful Palatini quintessential inflation may be soon tested by observations.

Acknowledgments

References

[1] Guth A H 1981 *Phys. Rev.* D **23** 347-56
[2] Akrami Y *et al.* [Planck] 2020 *Astron. Astrophys.* **641** A10
[3] Boubekeur L and Lyth D H 2005 *JCAP* **07** 010
[4] Dimopoulos K 2020 *Phys. Lett.* B **809** 135688.
[5] Starobinsky A A 1980 *Phys. Lett.* B **91** 99-102
[6] Kallosh R, Linde A and Roest D 2013 *JHEP* **11** 198
[7] Caldwell R R, Dave R and Steinhardt P J 1998 *Phys. Rev. Lett.* **80** 1582-5
[8] Aghanim N *et al.* [Planck] 2020 *Astron. Astrophys.* **641** A6
[9] Peebles J P E and Vilenkin A 1999 *Phys. Rev.* D **59** 063505
[10] Ford L H 1987 *Phys. Rev.* D **35** 2955
[11] Felder G N, Kofman L and Linde A D 1999 *Phys. Rev.* D **59** 123523
[12] Feng B and Li M Z 2003 *Phys. Lett.* B **564** 169-74
    Bueno Sanchez J C and Dimopoulos K 2007 *JCAP* **11** 007
[13] Dimopoulos K and Markkanen T 2018 *JCAP* **06** 021
    Opferkuch T, Schwaller P and Stefanek B A 2019 *JCAP* **07** 016
[14] Dimopoulos K and Donaldson Wood L 2019 *Phys. Lett.* B **796** 26-31
    Rosa J G and Ventura L B 2019 *Phys. Lett.* B **798** 134984
[15] Dimopoulos K and Owen C 2017 *JCAP* **06** 027
[16] Dimopoulos K, Donaldson Wood L and Owen C 2018 *Phys. Rev.* D **97** no.6 063525
[17] Akrami Y, Kallosh R, Linde A and Vardanyan V 2018 *JCAP* **06** 041
[18] Dimopoulos K and Sánchez López S 2021 *Phys. Rev.* D **103** no.4 043533
[19] Chevallier M and Polarski D *Int. J. Mod. Phys.* D **10** 213-24
    Linder E V 2003 *Phys. Rev. Lett.* **90** 091301
[20] Tashiro H, Chiba T and Sasaki M 2004 *Class. Quant. Grav.* **21** 1761-72
[21] Dimopoulos K 2021 *Introduction to Cosmic Inflation and Dark Energy* (London: CRC Press/Taylor & Francis) pp 232-43