Extensions of the Pesaran, Shin and Smith (2001) bounds testing procedure

Abstract

We replicate the Pesaran, Shin and Smith (2001) bounds testing procedure (BTP), and extend it with 6 new cases, 4 of which involve a quadratic trend. We provide critical values for the BTP of the lagged regressors in levels under the framework of unrestricted error-correction models (UECMs) to account for degenerate cases of co-integration. Further, we extend the BTP with 11 cases for the quantile UECMs of Cho, Kim and Shin (2015), and present critical values for interdecile and interquartile BTPs for the unrestricted cases. Moreover, we extend the Shin, Yu and Greenwood-Nimmo (2014) methodology to account for non-linear, or asymmetric, responses of the dependent variables to its covariates (NARDL) and for distributional, or location, asymmetry (QARDL of Cho, Kim and Shin; 2015) of the dependent variable. This is the quantile non-linear ARDL, or QNARDL. We provide codes that generate critical values for different sample sizes of the BTPs. These critical values are utilized in an empirical application of a dynamic equity valuation model for the S&P Global Index. Misspecifying a non-linear relationship as linear produces misleading results and policy implications. There is strong evidence of (i) trading activity based on fundamentals and (ii) the existence of a stable equilibrium relationship for the price-to-book (PB) ratio of the market index and its fundamentals. During periods of high *PB* relative to its fundamental values, convergence to equilibrium is faster than during periods of relatively low *PB*. There is also evidence of momentum trading, i.e. of traders that rely on positive feedback.

JEL classification: C12; C22; E32; G15; G40;

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1. Introduction

In this paper we first replicate the bounds testing procedure (BTP) of Pesaran, Shin and Smith (2001, hereafter PSS) using unrestricted error-correction models (UECMs) based on the autoregressive distributed lag (ARDL) models of Pesaran and Shin (1999).[1] Second, we extend PSS with 6 new cases, as well as introduce this BTP to the quantile framework of Koenker and Bassett (1978). Furthermore, we present new critical values for interdecile and interquartile BTP and test whether there is within-quantile evidence of co-integration. We also introduce the term quantile non-linear ARDL, or QNARDL, in the literature of the ARDL family.

Our contribution to the relevant literature (see e.g. Koenker and Bassett 1978; PSS; Narayan 2005; Xiao, 2009; Shin, Yu and Greenwood-Nimmo 2014, hereafter SYG; Cho, Kim and Shin 2015, hereafter CKS; Jordan and Philips 2018; McNown et al. 2018) is fivefold:

1. We introduce 2 new cases with intercept and linear trend bringing the total number of cases involving linear trend to 4 along with those examined by PSS. Further, we introduce 4 new cases with quadratic trend (in the model there are linear and quadratic trend terms, as well as a constant term). These cases involve the following restrictions when we test for the co-integrating relationship in levels between the dependent and the independent variables: restricted intercept (there is a constant term in the long-run equation), restricted trends (there are trends in the long-run equation), restricted intercept and trends (both intercept and trends are included in the long-run equation), and no restrictions at all (unrestricted case, where no constant term or trends are involved in the long-run equation). Hence, for a total number of 11 cases, we provide critical values for the PSS BTP.
2. We provide critical values (lower and upper bounds) for testing the lagged-once levels of the stochastic regressors in unrestricted error-correction models (UECMs) both for joint tests (*F*-statistics) along with any deterministic factors (intercept, linear trend,

[1] UECMs are also known as conditional, or structural, error-correction models.

quadratic trend) and individual tests (t-statistics). With these values we can check whether there is a degenerate case of co-integration (see PSS, p. 304) when the lagged-once level of the dependent variable has a negative and statistically significant coefficient, and the lagged-once levels of the dependent and stochastic variables along with any deterministic factors (intercept, linear trend, quadratic trend) are jointly statistically significant, while the lagged-once regressors in levels are either jointly statistically insignificant along with any deterministic factors (intercept, linear trend, quadratic trend), or individually statistically insignificant.

3. We repeat 1. and 2. for the quantile UECMs of CKS under the quantile framework of Koenker and Bassett (1978). We provide lower and upper bounds of the critical values required for the quantile version of PSS BTP.
4. We provide critical values (Wald statistics or W-statistics) for interdecile, i.e. the 10^{th}, 20^{th}, … , 90^{th} percentiles, and for interquartile, i.e. the 25^{th}, 50^{th} and 75^{th} percentiles, BTP a la PSS for 3 cases. These cases involve no deterministic restrictions under the null of no co-integration when we test for the long-run relationship in levels between the dependent and independent variables, i.e. the equilibrium relationship does not include an intercept or trends. We also allow for a constant term in the model. Specifically, we are interested in the cases where the model includes (i) an intercept, (ii) an intercept and linear trend, or (iii) an intercept, a linear and quadratic trend. Alternatively, we test for these cases to see whether there is evidence of quantile-dependent co-integration. Specifically we test first, whether the coefficient on the lagged dependent variable in levels is jointly equal across the quantiles and the lagged covariates in levels are jointly equal across the quantiles, second whether the coefficient on the lagged dependent variable in levels implies convergence rates subject to quantile-oriented heterogeneity and third, whether the lagged-once covariates in levels are location shifters.

5. Finally, we present an empirical application, which allows for both non-linear responses in the spirit of SYG and for distributional, or location, asymmetry employing conditional quantile regressions (CQREGs). Therefore, we contribute to the growing literature that focuses on quantile-varying relationships with co-integration analysis. The literature, which shows evidence in favor of quantile-varying or time-varying co-integrating relationships, has been growing fast lately (see Xiao 2009; Lee and Zeng 2011; Tsong and Lee 2013; CKS; Zhu et al. 2016; Lahiani et al. 2017; Benkraiem et al. 2018; Shahbaz et al. 2018; Lahiani et al. 2019; Mensi et al. 2019).

We provide computer codes that generate critical values for different sample sizes (lower and upper bounds) for ARDL models with up to 13 regressors. These simulated critical values ignore terms that do not matter asymptotically (e.g. lagged differences). Naturally, this may have some limitations when dealing with finite samples, in particular when dealing in an empirical application with "short" time-series and when there is a rich lag structure in the estimated ARDL model.

Our empirical application demonstrates the new critical values using an asymmetric variant of Bertsatos and Sakellaris (2016) Dynamic Dividend Discount Model (3DM) of equity valuation applied to the global stock market, proxied by the S&P Global Index. We find evidence of non-linearities in the price-to-book ratio (PB), which, if neglected, mask the existence of co-integration between *PB* and its fundamental determining factors. We provide ample evidence of co-integration once we allow for asymmetric responses both in OLS and CQREG setups. However, there are differences between the two estimation procedures.

With OLS we find that, on average, there is a long-run equilibrium relationship for every period, or state, of the PB ratio. In the quantile framework, however, we uncover time-varying co-integrating relationships that depend on the conditional distribution of the PB ratio and do not hold for every period.[2] Specifically, during periods of high *PB* relative to its fundamental values (25^{th} or lower percentile) and very low *PB* relative to its fundamentals (90^{th} or higher percentiles), *PB* shares no long-run relationship with its covariates and exhibits persistency. In contrast, in periods that *PB* takes values in its middle range relative to its fundamental values, we find that *PB* is driven by its fundamental factors. Finally, we find no evidence for equality of the coefficient of the error-correction term within quantiles or for the lagged fundamental factors in levels within quantiles using the interdecile and interquartile BTP.[3]

The rest of this paper is organized as follows: Section 2 is dedicated to the bounds testing procedure and is divided in three subsections. In 2.1 we present the original cases and new cases, in 2.2 we provide some motivation for the empirical relevance of the new cases, and in 2.3 we provide the replication of PSS and analyze the extensions. Section 3 presents the empirical application on the global stock market and Section 4 offers concluding remarks. Section 5 contains declarations and Section 6 provides the references. Also, there is an Online Appendix with supplementary information.

2. Bounds testing

The PSS BTP is one the most influential and cited paper in time-series analysis and co-integrating relationships. The BTP is a series of tests, which imply the existence of co-integration once their null hypotheses are jointly rejected. This method relies on the unrestricted version of an error-correction model (ECM) that comes from an ARDL model (see

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[2] Xiao (2009) is the first to note time-varying co-integrating relationships with the help of CQREGs. We discuss this later in the text.

[3] The coefficient of the ECT in an UECM is equal to the coefficient of the lagged-once dependent variable in levels, i.e. y_{t-1}.

Pesaran and Shin 1999). Banerjee et al. (1993 and 1998) state the benefits of using unrestricted error-correction models (UECM) in relation to the 2-step method of Engle and Granger (1987).

In brief, the 2-step procedure is shown to be inefficient (see e.g. Pagan 1984; and CKS) and allows only $I(1)$ variables to enter the long-run equation.[4] The long-run multipliers from the first step are super consistent but their t-ratios are not interpretable since their distribution is unknown.[5] The second-stage regression involves only stationary variables and so, standard inference can be applied.

UECMs on the other hand, allow for both stationary and non-stationary variables to enter the co-integrating equation, while they estimate both the short-run and long-run coefficients.[6] Also, the ARDL models have nice small-sample properties (see Pesaran and Shin 1999), which are transferred to the respective UECMs. Hassler and Wolters (2006) show the equivalence between an ARDL in levels and an ARDL in error-correction form, i.e. UECM.

Let us have a quick look at the BTP of PSS. Suppose we get in practice an ARDL (1, 1) in levels from an information criterion of our choice:

$$y_t = c + \lambda \cdot y_{t-1} + \beta_0 \cdot x_t + \beta_1 \cdot x_{t-1} + \upsilon_t \qquad (1)$$

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[4] This involves the problems associated with pre-testing and unit-root tests (increased "type II" error, i.e. failing to reject the null when it does not hold, or low statistical power).

[5] Three estimators have been proposed in the literature to deal with the inference of the long-run coefficients: fully-modified OLS (FMOLS) of Phillips and Hansen (1990), canonical co-integrating regression (CCR) of Park (1992) and dynamic OLS (DOLS) of Saikkonen (1992), and Stock and Watson (1993).

[6] To see this better, let all variables be $I(1)$ and one regressor be $I(0)$. Also, suppose you test for an economic theory suggesting a model with all these variables. The Engle-Granger approach sets the long-run multiplier of the stationary variable to zero, which is a quite restrictive scenario, while within the ARDL framework all variables are included in the model.

This is equivalent to the following ARDL (0, 0) in unrestricted error-correction form:[7]

$$y_t - y_{t-1} = c + (\lambda - 1) \cdot y_{t-1} + (\beta_0 + \beta_1) \cdot x_{t-1} + \beta_0 \cdot \Delta x_t + \upsilon_t \quad (2)$$

$$\Delta y_t = c + \varphi \cdot (y_{t-1} - \theta \cdot x_{t-1}) + \beta_0 \cdot \Delta x_t + \upsilon_t \quad (3)$$

where, $\varphi = (\lambda - 1)$ is the coefficient of the error-correction term (ECT), $\beta = \beta_0 + \beta_1$ and $\theta = -\frac{\beta}{\varphi}$ is the long-run coefficient of x_t. The speed of adjustment (SOA) is equal to $-\varphi$. PSS suggest for Case III that if (i) φ and β are not jointly equal to 0, i.e. the hypothesis of $\varphi = \beta = 0$ (F-test is employed) is rejected over the alternative that both φ and β, or one of them, are not 0, and (ii) φ is not statistically insignificant, i.e. the hypothesis of $\varphi = 0$ (t-test is employed) is rejected over the alternative $\varphi < 0$ then, a co-integrating relation running from x_t to y_t exists. However, PSS also argue that this could be a degenerate case of co-integration when β is statistically insignificant. Furthermore, with ARDLs, y_t is allowed to Granger-cause x_t only in the short run through the error-projection mechanism (see Pesaran and Shin, 1999, p. 385; PSS, p. 293; SYG, p. 289; and CKS, p. 293, for more details), however x_t may Granger-cause y_t both in the short run and the long run implying that at most one co-integrating vector exists. With this mechanism, ARDLs deal with the problem of endogenous regressors (see also PSS, p. 308, footnote 18).

[7] In Section 4 of the Online Appendix, we point out differences between an ARDL in unrestricted error-correction form and a model in first differences.

2.1 Original cases and new cases

We follow PSS for the data generation process (DGP) of the simulations. We expand this DGP first, with the inclusion of the quadratic trend, and second, by transferring it to the quantile environment. We employ the following estimation methods:

$$\begin{aligned}&OLS:\\ &\Delta y_t = \mathbf{\Phi}' \cdot \mathbf{z}_{t-1} + \mathbf{a}' \cdot \mathbf{w}_t + \xi_t, \quad t = 1, 2, \ldots, T\end{aligned} \tag{4}$$

$$\begin{aligned}&CQREG:\\ &\Delta y_t = \mathbf{\Phi}'(\tau) \cdot \mathbf{z}_{t-1} + \mathbf{a}'(\tau) \cdot \mathbf{w}_t + \xi_t(\tau), \quad t = 1, 2, \ldots, T\end{aligned} \tag{5}$$

where, $\mathbf{z}_{t-1} = \left(y_{t-1}, \mathbf{x}_{t-1}'\right)'$, $\mathbf{x}_t = \left(x_{1t}, \ldots, x_{kt}\right)'$, $\mathbf{w}_t = \left[1, t, t^2\right]'$, the variables y_t and $\mathbf{x}_t$ are generated from $y_t = y_{t-1} + \varepsilon_{1,t}$ and $\mathbf{x}_t = \mathbf{P}\mathbf{x}_{t-1} + \boldsymbol{\varepsilon}_{2,t}$ with $y_0 = 0$ and $\mathbf{x}_0 = \mathbf{0}$, and $\boldsymbol{\varepsilon}_t = \left(\varepsilon_{1t}, \boldsymbol{\varepsilon}_{2t}'\right)'$ is drawn as $(k+1)$ independent standard normal variables. $\tau \in (0, 1)$ is the user-specified percentile and Equation (5) is the quantile counterpart of Equation (4). T is the size of sample, or the number of observations, and y_t contains a unit root. If $\mathbf{x}_t$ is purely $I(1)$ then, $\mathbf{P} = \mathbf{I}_k$, while if $\mathbf{x}_t$ is purely $I(0)$ then, $\mathbf{P} = \mathbf{0}$. The same data generation process for y_t and $\mathbf{x}_t$ holds in the quantile environment, i.e. $y_t = y_{t-1} + \varepsilon_{1,t}$ and $\mathbf{x}_t = \mathbf{P}\mathbf{x}_{t-1} + \boldsymbol{\varepsilon}_{2,t}$ with $y_0 = 0$ and $\mathbf{x}_0 = \mathbf{0}$, and $\boldsymbol{\varepsilon}_t = \left(\varepsilon_{1t}, \boldsymbol{\varepsilon}_{2t}'\right)'$ is drawn as $(k+1)$ independent standard normal variables, where $\tau \in (0,1)$ is the user-specified percentile and $\xi_t(\tau) = \Delta y_t - Q_{\Delta y_t}(\tau \mid \Theta_{t-1})$ with $\Theta_{t-1} = \sigma\{\mathbf{z}_{t-1}, \mathbf{w}_t\}$ being the smallest sigma field generated by $\{\mathbf{z}_{t-1}, \mathbf{w}_t\}$.[8]

[8] We follow CKS for the definition of the error term in the quantile framework. However, we could also follow Xiao (2009) and rewrite Equation (5) as:

$Q_{\Delta y_t}(\tau) = \mathrm{A}_{\xi}^{-1}(\tau) + C + A(\tau) \cdot t + B(\tau) \cdot t^2 + \mathbf{\Phi}'(\tau) \cdot \mathbf{z}_{t-1} = \mathbf{\Phi}'(\tau) \cdot \mathbf{z}_{t-1} + \mathbf{a}'(\tau) \cdot \mathbf{w}_t$, where

The raw vectors $\mathbf{\Phi}'$ consists of $\left(\varphi_y, \boldsymbol{\varphi}_{\mathbf{x}}'\right) = \left(\varphi_y,\ \varphi_1, \ldots, \varphi_k\right)$ and $\mathbf{a}' = \left(c,\ \alpha,\ b\right)$, where c is the constant term, α the coefficient of the linear trend and b is the coefficient of the quadratic trend. PSS provide critical values for $k = 0,\ldots,10$ and $T = 1{,}000$ for Cases I, II, III, IV and V of Table 1, where k stands for the number of stochastic regressors $\mathbf{x}_t$ and is the row size of $\mathbf{x}_t$. Table 1 contains also new cases that we introduce and motivate in the next subsection.

Table 1: Cases of the bounds testing procedure

$\mathbf{z}_{t-1} = \left(y_{t-1}, \mathbf{x}_{t-1}'\right)'$ and $w_t = 0$	Case I
$\mathbf{z}_{t-1} = \left(y_{t-1}, \mathbf{x}_{t-1}', 1\right)'$ and $w_t = 0$	Case II
$\mathbf{z}_{t-1} = \left(y_{t-1}, \mathbf{x}_{t-1}'\right)'$ and $w_t = 1$	Case III
$\mathbf{z}_{t-1} = \left(y_{t-1}, \mathbf{x}_{t-1}', t\right)'$ and $w_t = 1$	Case IV
$\mathbf{z}_{t-1} = \left(y_{t-1}, \mathbf{x}_{t-1}'\right)'$ and $w_t = \left(1, t\right)'$	Case V
$\mathbf{z}_{t-1} = \left(y_{t-1}, \mathbf{x}_{t-1}', 1\right)'$ and $w_t = t$	Case VI
$\mathbf{z}_{t-1} = \left(y_{t-1}, \mathbf{x}_{t-1}', 1, t\right)'$ and $w_t = 0$	Case VII
$\mathbf{z}_{t-1} = \left(y_{t-1}, \mathbf{x}_{t-1}', 1, t, t^2\right)'$ and $w_t = 0$	Case VIII
$\mathbf{z}_{t-1} = \left(y_{t-1}, \mathbf{x}_{t-1}', t, t^2\right)'$ and $w_t = 1$	Case IX
$\mathbf{z}_{t-1} = \left(y_{t-1}, \mathbf{x}_{t-1}', 1\right)'$ and $w_t = \left(t, t^2\right)$	Case X
$\mathbf{z}_{t-1} = \left(y_{t-1}, \mathbf{x}_{t-1}'\right)'$ and $w_t = \left(1, t, t^2\right)$	Case XI

Notes: Cases I to V are presented in PSS, while cases VI to XI are introduced in this paper. We estimate Equations (4) and (5), respectively, with OLS and conditional quantile regressions (CQREG) of Koenker and Bassett (1978).

$\mathbf{a}'(\tau) = \left[C(\tau),\ A(\tau),\ B(\tau)\right] = \left[\mathrm{A}_{\xi}^{-1}(\tau) + C, A(\tau), B(\tau)\right]$ and is $\mathrm{A}(.)$ is the cumulative distribution function of ξ_t.

The tests involved in each case are: (i) $\mathbf{\Phi}' = \mathbf{0}$ (F_{yx}-test), where $\mathbf{\Phi}'$ equals to $\mathbf{\Phi}' = \left(\varphi_y, \boldsymbol{\varphi}'_{\mathbf{x}}\right)$, includes the coefficients of $\mathbf{z}_{t-1}$ according to Table 1 and φ_y is the coefficient of y_{t-1}, (ii) $\varphi_y = 0$ (t_y-test), (iii) $\boldsymbol{\varphi}'_{\mathbf{x}} = \mathbf{0}$ (F_x-test) and (iv) $\varphi_j = 0$ (t_x-test), where $j = 1$ to k. However, the t_y-test is the same for Cases II and III (we keep Case III), Cases IV to VII (we keep Case V), and Cases VII to XI (we keep Case XI). The same also holds for the t_x-test. The t_y-test is left oriented and the t_x-test is two tailed. For the calculation of the critical values we have created 3 codes in EViews (9th edition) based on 50,000 replications: one for the BTP in the OLS framework, one for the BTP in the CQREG environment and one for the interpercentile BTP (for the 3 quartiles and the 9 deciles).

2.2 Motivation for the new cases

Why do we expand the PSS BTP with more cases? Let us review first the 5 cases for the BTP introduced by PSS. The first one (Case I) does not incorporate either an intercept or a linear trend in the model to be estimated. The next two incorporate a constant term in the model and specifically, one case (Case II) includes the intercept in the long-run equation and the other case (Case III) excludes the constant term from the co-integrating relationship. Next, there are 2 more cases, where PSS add a linear trend in the model. In one case (Case IV) the trend term is included in the steady-state equation, whereas in the other case (Case V) no intercept and no trend is incorporated in the co-integration relation. However, what if in the long-run equilibrium relationship both a linear trend and an intercept, or a just a constant term should be included? We extend the cases involving a linear trend below.

We start with 2 new cases (Cases VI and VII) that involve two scenarios not examined by PSS in their specification with a linear trend. As stated earlier, Case IV and V of PSS incorporate UECMs with intercept and linear trend. Case IV suggests that the equilibrium relationship includes a linear trend and no constant term. So, under the null of no co-integration the dependent variable in levels involves just an intercept. Furthermore, Case V suggests that the equilibrium relationship does not include either a constant term or a linear trend. So, under the null of no co-integration the dependent variable in levels involves both an intercept and a linear trend. The newly introduced Case VI under the null of no co-integration allows for the dependent variable, in levels, to evolve in time with only a linear trend. Another scenario would be that the dependent variable does not incorporate either a linear trend or an intercept (Case VII) under the null of no co-integration.[9] Alternatively, Case VI suggests that in the

[9] So, testing for Cases IV or V, while the true null hypothesis is depicted in Cases VI or VII then, this could involve distortion of the power of the tests.

long-run equilibrium there should be an intercept, and Case VII suggests that the steady-state relationship should include both a constant term and a linear trend. So, we introduce 2 new cases with linear trend that complement these of PSS (Cases VI and VII).

Moreover, if the key variable of interest, i.e. the dependent variable, is best fit by a trend polynomial of second order and the regressors are a mix of $I(0)$ and $I(1)$ variables then, specifying the ARDL model with intercept and/or linear trend could produce severe estimation and inference problems. A quadratic trend should be included in the model to be estimated. A quadratic trend accounts for unobserved factors that are increasing, or decreasing, at either an increasing, or a decreasing, rate. A linear time trend captures unobserved factors rising, or falling, at a constant rate in a model (first derivative is constant). In the same spirit, a quadratic trend can capture unobserved increasing, or decreasing, patterns at either an increasing or decreasing rate ("rising part" of a concave function, "falling part" of a concave function, "falling part" of a convex function, "rising part" of a convex function).[10]

Motivated by the above, we introduce 4 cases (VIII, IX, X and XI) where in the model we allow for an intercept, a linear trend and a quadratic trend. Specifically, Case VIII includes in the long-run equation a constant term, a linear and quadratic trend, Case IX is like Case VIII but there is no constant in the equilibrium relationship and Case X is like Case VIII but there are no trend terms in the steady-state equation. Finally, Case XI is the unrestricted case in the quadratic model since in the *F*-test for the BTP neither a constant term nor trend terms are incorporated into the long-run equilibrium equation. Alternatively, under the null of no co-integration Case VIII suggests there are no trends and constant term in the evolution of the dependent variable, Case IX suggests that the dependent variable incorporates just a constant

[10] Harvey et al. (2011) argue that a quadratic trend allows for an evolution from an upward linear trend at the beginning of the time series to a downward linear trend at the end of the time series, or the opposite, i.e. from a downward to an upward trend.

term, Case X suggests that the dependent variable evolves in time with the components of linear and quadratic trend and lastly, according to Case XI the dependent variable does not incorporate neither an intercept nor linear and quadratic trend terms. To sum up, with these new cases researchers and practitioners have several options to test for co-integration given the presence of a second-order trend polynomial, or unobserved factors as those described in the previous paragraph.

Examples of data with quadratic trend could be these in Charalambakis et al. (2017, see Figure 1), Toudas et al. (2017, see Figure 1), and Anastasiou et al. (2019, see Figures 3 and 4). These papers study the NPLs in the case of Greece and empirically it seems that the levels of NPLs evolve in time incorporating a quadratic trend. The first paper examines the NPLs by type of loans (business, consumers and mortgages) in aggregate level, while the second paper studies the NPLs for the 4 systemic banks in Greece. Finally, the third paper focuses on the governance indicators and aggregated NPLs in Greece. So, re-examining the case of the Greek NPLs with ARDL modelling and quadratic trend sounds like a promising project with fruitful discussion.

Next, we pose a second question: Why is it interesting to develop a quantile version of the PSS BTP? Quantile regressions are attractive because they provide robust estimates even under the presence of extremely low, or high, values in the dependent variable. Further, they allow the researcher/practitioner to explore the whole distribution of the dependent variable and not only the mean like the OLS estimator, which is sensitive to outliers. Also, they allow for a shift of the location, scale and shape of the distribution of the dependent variable (see Xiao, 2009; and CKS).[11] Moreover, the quantile autoregressive distributed lag (QARDL) model of

[11] However, there is a potential problem with quantile regressions when the sample size becomes smaller than the total number of estimated coefficients across different percentiles. To alleviate this, a researcher, or practitioner, could (i) either employ a simple model with few regressors so that he/she can compare the coefficients

CKS is the quantile extension of the ARDL models of Pesaran and Shin (1999), and it is a promising tool for modelling quantile-dependent co-integrating relationships from single-equation models. QARDL accounts for potential multiple thresholds, as determined by the quantiles (see Lahiani et al., 2019), and inherits all the advantages of the standard ARDL methodology; it allows for a mix of regressors, i.e. $I(0)$ and $I(1)$ or mutually co-integrated ones, and estimates both short-run and long-run responses of the dependent variable in one step.

Furthermore, regarding quantile non-linear ARDL, every application that has been performed by (N)ARDL models could be reexamined with Q(N)ARDL should the main variable exhibits non-linear and distributional asymmetries. Therefore, there is plenty of room for reexamination of many empirical findings, as well as for promising results on new projects. In the introductory section (5th bullet point) we present some papers, where QARDL models are employed. It would be interesting to check if the results are robust to allowing non-linearities under QNARDL modelling. In the empirical application in this paper, the results are sensitive to non-linearities. With ARDL and QARDL no co-integration exists, while with the non-linear ARDL (NARDL) and QNARDL there is ample evidence of co-integration.

Regarding the quantile version of ARDLs and co-integration in the quantile environment, Xiao (2009) proposes a quantile version of the dynamic OLS (DOLS) estimator, i.e. a quantile co-integrating technique for the estimation of the long-run multipliers using the CQREGs of Koenker and Bassett (1978). CKS introduce in the literature of co-integration the QARDL model and, in their concluding remarks, encourage future researchers to invest on completing the PSS BTP under the quantile framework. CKS also employ the estimation technique of Koenker and Bassett (1978) and suggest a complete way of running the QARDL model. Our paper shows how to execute the BTP both at a given percentile as well as across percentiles.

across different percentiles at the cost of a probable misspecification bias or, (ii) estimate a full model, as in the OLS case, only at specific quantiles (e.g. the 50th).

We also contribute to the literature of quantile co-integrating estimators (see Xiao 2009; and CKS) by adding a new way to test for the statistical significance of the long-run multipliers, where in the same step short-run responses of the dependent variable are also estimated along with long-run coefficients.

Another contribution is that we empirically extend the SYG methodology of non-linear modeling, under the quantile framework. SYG extend the ARDL model to the non-linear ARDL, where the regressors are allowed to affect differently the dependent variable.[12] Particularly, the dependent variable may respond differently to increases of a regressor rather than to equal-size decreases, i.e. non-linear responses of the dependent variable are allowed.[13] Also, SYG talk about a pragmatic BTP and they argue that the BTP introduced by PSS is valid in the NARDL framework.

So, we account for non-linear, or asymmetric, responses of the dependent variables to its covariates (NARDL) and for distributional, or location, asymmetry (QARDL) of the dependent variable. Thus, we introduce the term quantile non-linear ARDL, or QNARDL, in the literature of the ARDL family. A concrete example of non-linear and quantile non-linear ARDLs is given in the empirical application of this paper in Section 3.

[12] In ARDL modelling, increases and equal-size decreases affect the dependent variable equally. That is, the responses of the dependent variable to shocks of the regressors are linear.

[13] If at least one of the two coefficients matching to the increases and decreases of x_t is statistically significant then, this is the case of asymmetry (full asymmetry with two significant and unequal coefficients, and partial asymmetry with one significant coefficient). On the other hand, if y_t equally responds to increases and decreases of x_t then, no asymmetry exists. In the Online Appendix file, Section 5 describes in detail each of the 13 cases of a possible asymmetric behavior.

2.3 Replication of PSS and extensions

PSS find for Cases I to V (see Table 1) the bounds of the critical values of *t*-statistics for the null hypothesis $\varphi_y = 0$, where φ_y is the coefficient of y_{t-1}, and the bounds of *F*-statistics for the null hypothesis $\mathbf{\Phi}' = 0$, where $\mathbf{\Phi}'$ is equal to $\mathbf{\Phi}' = \left(\varphi_y, \boldsymbol{\varphi}'_{\mathbf{x}}\right)$ and includes the coefficients of $\mathbf{z}_{t-1}$ according to Table 1, using a sample of 1,000 observations with 40,000 replications in GAUSS. However, as Narayan (2005) points out, these critical values depend heavily on the size of the sample and thus, the two-step testing procedure (*F*-test first and then, the *t*-test) of PSS (see PSS, p. 304) for the existence of co-integration between y_t and the vectors $\mathbf{x}_t$ can be misleading.[14] We replicate the BTP of PSS using EViews and our simulations provide almost identical results to PSS (all our codes are available in the journal's website).[15]

We go beyond replicating PSS and introduce 6 new cases for the *F*-statistics, namely Cases VI, VII, VIII, IX, X and XI, (see Table 1) which are added to Cases I to V of PSS, and 1 new case for the *t*-statistics, namely Case XI, which is added to Cases I, III and V of PSS.[16] We

[14] So, Narayan (2005) borrows the GAUSS codes from Pesaran and Shin, and calculates new critical values for the *F*-statistics for sample sizes ranging from 30 to 40 observations with a step of 1 observation (that is in Narayan 2004) and from 40 to 80 observations with a step of 5 observations (that is in Narayan 2005, where he shows 30 to 80 with a step of 5) for Cases II to V for $k = 0, \ldots, 7$. Following Narayan (2005), Jordan and Philips (2018) provide a program in STATA and R for the BTP of PSS for Cases I to V with critical values for the *F*-tests.

[15] We compute the critical values using 50,000 simulations. For example, when k = 0 our code generates *t*-statistics that are either exactly the same with those of PSS or differ by 0.02 in the worst case, and when k = 10 it generates *F*-statistics that are either the same with those of PSS or differ by 0.03 in the worst case. To ensure comparability of our replication and the work of PSS, we round the critical values at the 2nd decimal as PSS do. This program as well as two more programs and an Online Appendix are in the website of the journal as supportive material to this paper.

[16] When in the right-hand side there are only lags of the dependent variable, the critical values of *t*-statistics (in the OLS setup) of Cases I, III and V correspond to the Dickey-Fuller (DF, 1979) unit-root *t*-statistics. Also, Case

also introduce a new set of critical values for the t-statistics of the null hypothesis $\varphi_j = 0$ for $j = 1,\ldots,13$ and F-statistics of the null hypothesis $\boldsymbol{\varphi}'_{\mathbf{x}} = \mathbf{0}$, where $\boldsymbol{\varphi}'_{\mathrm{x}}$ includes the coefficients of $\mathbf{z}_{t-1}$ except for that of y_{t-1}.[17] With these critical values we shed light on the degenerate case, as PSS argue (see PSS, p. 304). That is when the following two conditions are met. First, the F-test (H_0: $\boldsymbol{\Phi}' = \mathbf{0}$, where $\boldsymbol{\Phi}'$ includes the coefficients of $\mathbf{z}_{t-1}$ according to Table 1) and t-test (H_0: $\varphi_y = 0$) show evidence in favor of the existence of a co-integrating relationship between y_t and the vector $\mathbf{x}_t$, and second, the t-test (H_0: $\varphi_j = 0$), or F-test (H_0: $\boldsymbol{\varphi}'_{\mathbf{x}} = \mathbf{0}$) show that the $\mathbf{x}_t$ variables in model (1) are statistically insignificant.[18] We apply this exercise both in OLS and CQREG setups.

When we depart from the OLS setup to the quantile framework, we also provide critical values for interdecile and interquartile BTP for Cases III, V and XI with Wald tests.[19] We find that these critical values also depend on sample size, number and integration order of regressors, and the inclusion of intercepts and trends (see also Narayan and Narayan 2005, for the OLS framework). With these tests we can check whether, φ_y and $\boldsymbol{\varphi}'_{\mathbf{x}}$ (W_{yx}-test), or φ_y (W_{y}-test), or finally, $\boldsymbol{\varphi}'_{\mathbf{x}}$ (W_{x}-test), are common within percentiles. The interdecile test, tests the slope equality of the lagged dependent variable in levels and/or lagged stochastic regressors

XI is a new case to the (augmented) DF framework, where a second-order trend polynomial is taken into consideration.

17 These t-statistics are not normally distributed. However, they exhibit no skewness and are symmetrically distributed.

18 McNown et al. (2018) also notice this degenerate case, where the lagged levels of the explanatory variables are statistically insignificant, and they employ simulations based on an eight-step bootstrap procedure to deal with it.

19 We study the unrestricted cases only because the deterministic regressors (intercept, linear and quadratic trends) are not likely to be equal within the quantiles and thus, the test statistic may be biased towards the rejection of the null hypothesis.

in levels for the 9 deciles, i.e. the 10th, 20th, … , 90th percentiles, while the interquartile test for the 3 quartiles, i.e. the 25th, the 50th and the 75th percentiles. [20] So, these tests give us the opportunity to explore whether there is evidence of time-varying estimates or quantile-dependent co-integration.

Regarding the critical values for different sample sizes, first we simulate the data according to Equations (4) and (5), and Table 1. Subsequently we compute the statistics for the tests employed (*t*-tests and *F*-tests for the BTP in OLS and CQREG setup, and *W*-tests for the interpercentile BTP) and the empirical quantiles of these statistics become the critical values. This is also how PSS compute their critical values for the case of 1,000 observations. Regarding our codes, we first generate y as $I(1)$, and $\mathbf{x}$ as $I(0)$, $I(1)$, or a mixture of stationary and non-stationary variables. Then, we run regressions as in Equations (4) and (5), and compute the *t*-statistics and *F*-statistics according to Table 1 (11 Cases) for the BTP in OLS and CQREG setup, and the *W*-statistics for Case III, V and XI for the interpercentile BTP. We repeat this 50,000 times and subsequently, we compute the empirical test statistics at 1%, 2.5%, 5% and 10% levels of statistical significance.

3. Empirical application

In our empirical application we examine whether there is a stable long-run relationship of the market PB valuation of the S&P Global Index with its fundamentals. Such a stable long-run relationship was found in Bertsatos and Sakellaris (2016), and Bertsatos et al. (2017) for the

[20] CKS suggest that the standard Wald tests are asymptotically valid for testing the equality of parameters at the 3 quartiles, i.e. 25th, 50th and 75th percentiles (jointly or for two of them, i.e. 4 Wald tests), in a setup of a QARDL (1, 1) model in levels with intercept and one $I(1)$ regressor, i.e. $k = 1$. Specifically, they deal first, with the equality of the autoregressive terms of the dependent variable in levels, Y, second, with the sum of the coefficients (of current and previous period) of the stochastic regressor X and third, the long-run response of Y to X.

case of the largest systemic U.S. banks. If such a relationship exists, what can we infer about the existence and influence of momentum/contrarian traders and fundamentalists? This question has not been addressed before. The economic hypothesis we are going to test is that PB shares an equilibrium relationship with risk, growth and cash flow characteristics for the case of the S&P Global Index:

~\~\~ a *PB* steady-state relationship exists for the S&P Global Index ~/~/~	(6)

Bertsatos and Sakellaris (2016) developed an equity valuation model using the price-to-book (PB) ratio as the main variable of interest. They named it Dynamic Dividend Discount Model or 3DM. The fundamental factors determining *PB* according to 3DM are the cost of equity, expected growth of net income and dividend payout ratio. These variables represent the risk, growth and cash flow characteristics of a firm's stock.

In this paper, we use 3DM allowing for asymmetric responses of *PB* in the spirit of SYG, i.e. we let the increases and the decreases of the right-hand-side variables to affect *PB* in a different way, where there is a "positive" and a "negative" regime. Thus, our model incorporates a regime-switching co-integrating relation, where the regimes depend on whether a positive, or a negative, shock occurs in the covariates of *PB* (see also SYG, footnote 4). Also, we replace cost of equity with the z-score variable (more details in the "Data description" below).

<u>The 3D model</u>

In our empirical application, we adopt an asymmetric variant of the Bertsatos and Sakellaris (2016) model. We employ ARDL and QARDL models (see Equations 7 and 8). Further down in this section we also employ NARDL and QNARDL modelling strategies (see Equations 12 and 13). The models belonging to the ARDL family allow for one co-integrating vector running from $\mathbf{x}_t$ to y_t, while in the short run y_t can also affect $\mathbf{x}_t$. We assume that *PB* affects

ZSCORE, *DPR* and *GROWTH* in the short run. However, *PB* responds to shocks of *ZSCORE*, *DPR* and *GROWTH* both in the short run and in the long run.

$$\begin{aligned}\Delta PB_t &= \sum_{i=1}^{6} \lambda_i \cdot \Delta PB_{t-i} + \sum_{i=0}^{6} a_{i\,1} \cdot \Delta ZSCORE_{t-i} + \sum_{i=0}^{6} a_{i\,2} \cdot \Delta DPR_{t-i} \\ &+ \sum_{i=0}^{6} a_{i\,3} \cdot \Delta GROWTH_{t-i} + w \cdot DC + c + \gamma_1 \cdot t + \gamma_2 \cdot t^2 + \varphi_y \cdot PB_{t-1} \\ &+ \varphi_1 \cdot ZSCORE_{t-1} + \varphi_2 \cdot DPR_{t-1} + \varphi_3 \cdot GROWTH_{t-1} + \varepsilon_t\end{aligned} \quad (7)$$

$$\begin{aligned}\Delta PB_t &= \sum_{i=1}^{6} \lambda_i(\tau) \cdot \Delta PB_{t-i} + \sum_{i=0}^{6} a_{i\,1}(\tau) \cdot \Delta ZSCORE_{t-i} + \sum_{i=0}^{6} a_{i\,2}(\tau) \cdot \Delta DPR_{t-i} \\ &+ \sum_{i=0}^{6} a_{i\,3}(\tau) \cdot \Delta GROWTH_{t-i} + w(\tau) \cdot DC + c(\tau) + \gamma_1(\tau) \cdot t \\ &+ \gamma_2(\tau) \cdot t^2 + \varphi_y(\tau) \cdot PB_{t-1} + \varphi_1(\tau) \cdot ZSCORE_{t-1} + \varphi_2(\tau) \cdot DPR_{t-1} \\ &+ \varphi_3(\tau) \cdot GROWTH_{t-1} + \varepsilon_t(\tau)\end{aligned} \quad (8)$$

where, *PB* is the price-to-book ratio, *ZSCORE* is the z-score variable, *DPR* is the dividend payout ratio, *GROWTH* is the expected growth of net income, *DC* is a dummy variable for the recent financial crisis and τ represents the user-specified percentiles.

Data description

We use data of the S&P Global Index provided by Datastream for the period 1982:Q1-2017:Q1, i.e. 141 observations. Table 2 provides some descriptive statistics, where one can observe that all main variables exhibit skewness and this is a primal source of asymmetry.

Table 2: Descriptive statistics

	PB	***ZSCORE***	***DPR***	***GROWTH***
Mean	12.141	12.420	20.893	21.602
Median	5.020	8.851	10.830	19.162
Std. deviation	23.931	12.399	31.794	52.010
Skewness	4.171	1.575	4.215	-8.714
Kurtosis	20.965	5.708	21.260	94.078

Notes: Std. corresponds to standard. *PB* is the price-to-book ratio, *ZSCORE* is the z-score variable, *DPR* is the dividend payout ratio and *GROWTH* is the expected growth of net income. We round to the 3rd decimal point.

The price-to-book (PB) ratio is the ratio of market value of equity to the book value of equity. The z-score variable is constructed as:

$$ZSCORE = \frac{ROE + 1}{\sigma_{ROE}} \tag{9}$$

where, *ROE* is the return of equity and σ_{ROE} is the standard deviation of *ROE*. We estimate *ROE* as the ratio of current earnings times 100 to the average of the first and second lag of book value of equity, and σ_{ROE} as the standard deviation of current *ROE* and its first seven lags.

ZSCORE indicates the number of standard deviations that a firm's ROE can fall in a single period before it becomes insolvent. High values represent a low probability of default. Alternatively, small values imply great risk. For example, a value of 5 in *ZSCORE* indicates that a firm becomes insolvent when its ROE falls more than 5 times its standard deviation in a single period. In the case of a market index, *ZSCORE* portrays distress rate of that market and in our case, it is a measure of the overall financial health of the global market. The dividend payout ratio (DPR) is calculated as the share of current dividends to the previous value of book value of common equity in percentage points:

$$DPR_t = \frac{DIV_t}{Equity_{t-1}} \cdot 100 \tag{10}$$

where, *DIV* is the dividends distributed to the shareholders. We calculate the expected growth in percentage points as the product of the retention ratio (*RR*) and return on equity.

$$GROWTH_t = \left(1 - \frac{DPR_t}{100}\right) \cdot ROE_t = RR_t \cdot ROE_t \tag{11}$$

where, 1 minus *DPR*/100 is equal to *RR* and shows the amount of *ROE* that is invested back in the firm for future projects, and *ROE* is the return of equity in percentage points. The dummy for the recent financial crisis takes the value of 1 for the period 2007:Q1-2009:Q4.[21]

Regarding the "positive" and "negative" asymmetries of a variable, or the increases and decreases, we follow the definitions of SYG, and Greenwood-Nimmo and Shin (2013, hereafter GS). These variables are required for the NARDL and QNARDL models. Specifically, given a variable AB_t, the "positive" first differences, ΔAB_t^+, are defined as $\max\left[AB_t, 0\right]$ and the "negative" first differences, ΔAB_t^-, as $\min\left[AB_t, 0\right]$. Correspondingly, AB_t^+ and AB_t^- are calculated as $\sum_{j=1}^{T} \Delta AB_j^+$ and $\sum_{j=1}^{T} \Delta AB_j^-$, i.e. the partial sums of the "positive" and "negative" first differences, respectively.

Also, the regime probabilities of the asymmetric variables in first differences, i.e. ΔAB_t^+ and ΔAB_t^-, are defined as the effective number of observations as a share of the total number of observations. The effective number for ΔAB_t^+ (ΔAB_t^-) is defined as the total number of observations net of the zero values in ΔAB_t^+ (ΔAB_t^-). SYG and GS suggest that these probabilities should be close to each other, i.e. near 50%. Otherwise, researchers and practitioners should worry about the validity of their estimates, as well as their inference. For

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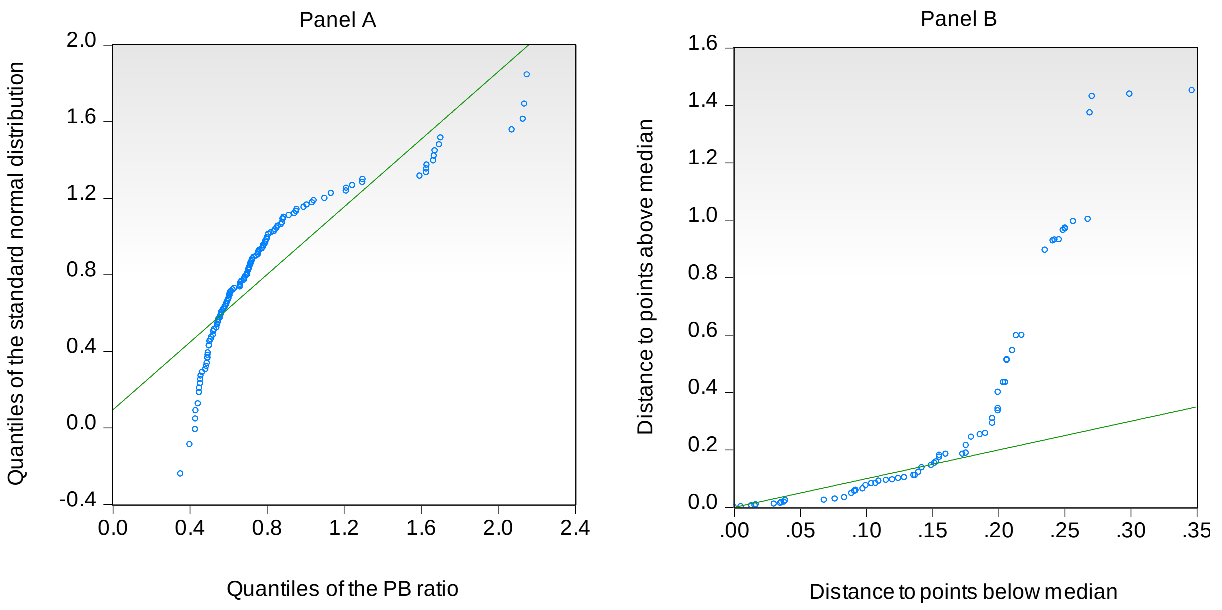
[21] The number of "1's" for the crisis dummy over the total number of observations is near 9%. PSS argue that the dummy variables should bear a ratio close to zero such that the simulated critical values for the BTP remain valid. In their empirical application, PSS use two dummy variables, where one dummy has a ratio of 7.6% and the other 19.2%. Therefore, considering a maximum threshold of 20% for dummy variables from PSS, we have no reason to worry in our estimation about the critical values for the BTP, as well as other researchers and practitioners that augment their specification with dummy variables under this condition. Also, one could include such a dummy variable in the long-run relationship however, he/she should increase *k* by 1 for the critical values for the BTP.

example, in GS, the empirical probabilities in the "positive" and "negative" regime have roughly a "60-to-40" ratio in favor of the "positive" regime and the authors claim there are no worries for estimation and inference.

PB exhibits positive skewness (see Table 2), 4.171, and excess kurtosis, 17.965, so the assumption of linear responses does not seem to be reasonable. Panel A of Figure 1 indicates signs of asymmetry for *PB* and in Panel B we see that *PB* exhibits indeed asymmetric characteristics. Furthermore, "negative" and "positive" PB ratios display different measures of skewness (-3.590 vs 2.803) and excess kurtosis (13.564 vs 6.982). "Negative" *PB* and "positive" *PB* are defined as partial sums of negative and positive-first differences of *PB*, respectively. *ZSCORE* and *DPR* are also positively skewed (the respective numbers are 1.575 and 4.215) and the corresponding excess kurtosis is 2.708 and 18.260, while *GROWTH* displays negative skewness (-8.714) and excess kurtosis (91.078). The empirical probabilities in the "positive" and "negative" regimes for *ZSCORE* are 53% and 47%. The corresponding ones for *DPR* are 42% and 58%, and for *GROWTH* are 51% and 49%.[22]

[22] Hence, according to the "60-to-40" ratio of GS we do not have to worry about estimation and inference issues.

<u>Figure 1: Quantile-Quantile (QQ) plots of the PB ratio (in logs)</u>



<u>Notes:</u> Panel A shows the empirical plot of the standard normal distribution with the logged PB ratio. A normally-distributed variable would exhibit points very close to the straight line. Panel B shows the empirical symmetry plot of the logged PB ratio. Being close to the straight line implies symmetry. If not close then, asymmetries are present. The data of the PB ratio for the S&P Global Index comes from Datastream.

Estimation

We reserve the first 8 observations to construct the lag differences, and the partial sums of positive and negative first differences in order to ensure comparability of the results during the lag-length selection. We allow up to 6 maximum lags in the UECMs, i.e. 7 maximum lags for the ARDL in levels. The length of the time series is 133 observations and the reference period becomes 1984:Q1-2017:Q1. So, the common sample consists of 133 observations for all the models (ARDL, NARDL, QARDL and QNARDL) across the lag-selection procedure. The optimally selected OLS model is chosen among $6 \cdot 7^3 = 2{,}058$ models for the linear specification and among $6 \cdot 7^6 = 705{,}894$ models for the non-linear specification. We rely on the Schwarz (1978) information criterion (SIC) and test whether there are serially correlated errors (we use the Breusch-Godfrey LM test) in the OLS-based models before we continue our analysis.[23, 24]

First, we confirm that none of the variables is $I(2)$ so that the ARDL method is valid, and then, that the dependent variable and at least one of the regressors is $I(1)$ so that the regression is well-balanced.[25] To answer the question posed in the null hypothesis (6), we are going to rely on linear and non-linear modelling. We start with linear models of ARDL and QARDL. Regarding the deterministic factors, we follow the practice of Harvey et al. (2011), where the authors suggest the inclusion of a linear-trend-only fit. A quadratic fit would be very suggestive for the eyes and does not allow the reader to judge on the necessity of the quadratic trend. We observe in Figure 2 that the PB ratio does not seem to evolve into time following a path of linear trend. Therefore, we add a second-order trend polynomial in our specifications.[26]

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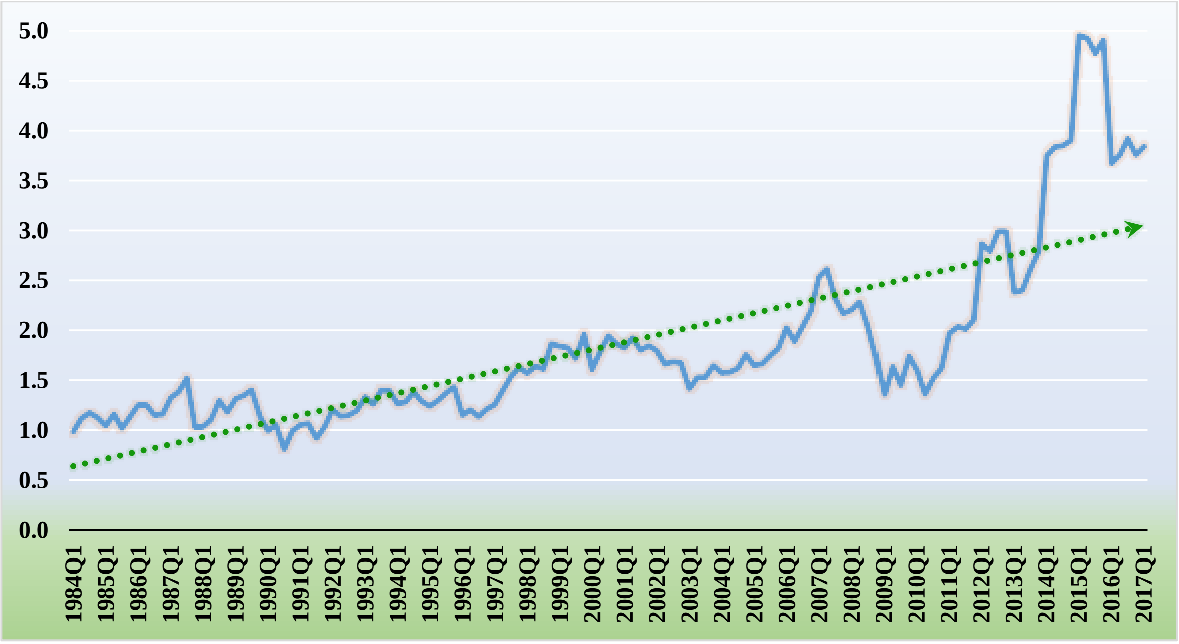
[23] As PSS state, there is a dilemma regarding the choice of the lags. A small number does not always guarantee a free-of-serial-correlation model (a crucial assumption for the validity of ARDL models), while with a large number one may end up with an over-parameterized model. We use the LM test to test for 1st order to 13th order serial correlation. In our OLS-based specifications we find no evidence of either short-run or long-run serially correlated errors.

[24] The Hannan and Quinn (1979) information criterion (HQIC) gives the same lag structure with SIC in our empirical application.

[25] Details and results of unit-root tests are in the Online Appendix (see Section 2 there).

[26] We also employ an augmented DF test with quadratic trend for the first time as far as we know, where we use the simulated critical values for $k = 0$. In this way, we address any concerns that the dependent variable could be modelled as a stationary process around a quadratic trend. The PB ratio persists to be a $I(1)$ variable. See Tables 7 and 8 in the Online Appendix for more details. Moreover, a Perron-Yabu (2009) test for the presence of a quadratic trend could also be preferred but this type of particular test is not documented in their series of papers.

Figure 2: Time path of the PB ratio (in logs)



Notes: We use the logged PB ratio for illustration and presentation reasons. The time-series ranges from 1984:Q1 to 2017:Q1, i.e. 133 quarterly observations. The data of the PB ratio for the S&P Global Index comes from Datastream.

We run the linear ARDL with OLS (see Equation 7), which generates a positive error-correction coefficient, i.e. $\varphi_y > 0$. This indicates a divergent behavior and an explosive relationship. Switching to the quantile framework, we run a QARDL with the same lag order as in the OLS specification to ensure that our results are comparable (see Equation 8).[27, 28] We find positive error-correction coefficients across the quantiles of the conditional distribution of *PB* implying that no steady-state equilibrium relationships exist.[29]

[27] See CKS, p. 293, for an excellent discussion for the transition from a (conditional) mean to a conditional quantile model. Moreover, the mean-based models are used as a benchmark for the quantile-oriented models.

[28] We estimate both Equations (8) and (13) at the 9 deciles, as well as at the 3 quartiles.

[29] We do not report the models' estimates with the linear specification since the speed of adjustments (SOAs) are positive denoting divergence and thus, the relationships are explosive.

To sum up, we conclude that the hypothesis posed earlier (see 6) for the S&P Global Index is rejected assuming linear responses of *PB* to risk, cash flow and growth as measured by the ZSCORE, DPR and GROWTH variables.

Does the existence of explosive relationships with linear modelling imply that *PB* adopts a random-walk behavior? The answer is no. Dropping the quite restrictive assumption of linear responses and adopting the assumption of non-linear responses may be helpful. SYG argue that linear adjustment could be extremely restrictive in a plethora of economically interesting situations, especially in the presence of non-negligible transaction costs and policy interventions by authorities.[30] We now adopt asymmetric non-linear modelling and we follow GS and SYG. Therefore, we focus on the non-linear versions of ARDL and QARDL, i.e. NARDL and QNARDL, and explore whether the hypothesis (6) we examine holds in our empirical application.

Our specifications for the NARDL and QNARDL models are the following:

$$\begin{aligned}\Delta PB_t = &\sum_{i=1}^{6}\lambda_i\cdot\Delta PB_{t-i}+\sum_{i=0}^{6}a_{i\,1}^{-}\cdot\Delta ZSCORE_{t-i}^{-}+\sum_{i=0}^{6}a_{i\,1}^{+}\cdot\Delta ZSCORE_{t-i}^{+}+\sum_{i=0}^{6}a_{i\,2}^{-}\cdot\Delta DPR_{t-i}^{-}\\&+\sum_{i=0}^{6}a_{i\,2}^{+}\cdot\Delta DPR_{t-i}^{+}+\sum_{i=0}^{6}a_{i\,3}^{-}\cdot\Delta GROWTH_{t-i}^{-}+\sum_{i=0}^{6}a_{i\,3}^{+}\cdot\Delta GROWTH_{t-i}^{+}\\&+w\cdot DC+c+\gamma_1\cdot t+\gamma_2\cdot t^2+\phi_y\cdot PB_{t-1}+\phi_1^{-}\cdot ZSCORE_{t-1}^{-}+\phi_1^{+}\cdot ZSCORE_{t-1}^{+}\\&+\phi_2^{-}\cdot DPR_{t-1}^{-}+\phi_2^{+}\cdot DPR_{t-1}^{+}+\phi_3^{-}\cdot GROWTH_{t-1}^{-}+\phi_3^{+}\cdot GROWTH_{t-1}^{+}+\varepsilon_t\end{aligned}\tag{12}$$

$$\begin{aligned}\Delta PB_t = &\sum_{i=1}^{6}\lambda_i(\tau)\cdot\Delta PB_{t-i}+\sum_{i=0}^{6}a_{i1}^{-}(\tau)\cdot\Delta ZSCORE_{t-i}^{-}+\sum_{i=0}^{6}a_{i1}^{+}(\tau)\cdot\Delta ZSCORE_{t-i}^{+}\\&+\sum_{i=0}^{6}a_{i2}^{-}(\tau)\cdot\Delta DPR_{t-i}^{-}+\sum_{i=0}^{6}a_{i2}^{+}(\tau)\cdot\Delta DPR_{t-i}^{+}+\sum_{i=0}^{6}a_{i3}^{-}(\tau)\cdot\Delta GROWTH_{t-i}^{-}\\&+\sum_{i=0}^{6}a_{i3}^{+}(\tau)\cdot\Delta GROWTH_{t-i}^{+}+w(\tau)\cdot DC+c(\tau)+\gamma_1(\tau)\cdot t+\gamma_2(\tau)\cdot t^2\\&+\phi_y(\tau)\cdot PB_{t-1}+\phi_1^{-}(\tau)\cdot ZSCORE_{t-1}^{-}+\phi_1^{+}(\tau)\cdot ZSCORE_{t-1}^{+}+\phi_2^{-}(\tau)\cdot DPR_{t-1}^{-}\\&+\phi_2^{+}(\tau)\cdot DPR_{t-1}^{+}+\phi_3^{-}(\tau)\cdot GROWTH_{t-1}^{-}+\phi_3^{+}(\tau)GROWTH_{t-1}^{+}+\varepsilon_t(\tau)\end{aligned}\tag{13}$$

[30] For an excellent analysis of this topic and a review of relative works, see the introductory part in SYG.

where, *PB* is the price-to-book ratio, *ZSCORE* is the z-score variable, *DPR* is the dividend payout ratio, *GROWTH* is the expected growth of net income, *DC* is dummy variable for the recent financial crisis and τ represents the user-specified percentiles. To the best of our knowledge this is the first time (i) that non-linear responses are introduced to *PB* modelling and (ii) that the PB ratio is used for analysis at a market-index level.

Once we allow for non-linearities (NARDL, see Equation 12) or both non-linearities and distributional asymmetries (QNARDL, see Equation 13) into the specification of 3DM we get significant results.[31] We thus, confirm Shin (2009) that (i) misclassifying a non-linear asymmetric process as linear can be misleading and (ii) being unable to detect a linear co-integrating relationship when there truly exists a stable non-linear, or regime-switching, relationship can be misleading. Also, we witness that the choice of estimation strategy (OLS vs CQREGs) does contribute to the differentiation of the models' estimates; the coefficients at the 30th to 50th percentile of the conditional distribution of *PB* are greatly different from those of OLS when we allow for asymmetric responses to *PB*. Finally, using the interdecile and interquartile BTP we find no support of the equality of the ECT and/or its lagged fundamental factors in levels.

GS and SYG refer to a pragmatic BTP and show that the BTP of PSS is valid in the framework of NARDL modelling, where complex interdependencies could arise due to the nature of partial sum decompositions. They argue that the long-run inference can be achieved through the BTP shown in PSS regardless of the integration order of the variables. The underlying regressors can be $I(0)$, $I(1)$ or mutually co-integrated. Also, SYG advise that the number of explanatory variables, $\mathbf{x}_t$, should be calculated before the decomposition of the partial sums to ensure conservatism. In our case, $k = 3$, and the BTP is executed with $k = 3$ for

[31] These results are presented below in Table 3 and its discussion.

both linear and non-linear models. Furthermore, the long-run and short-run symmetry can be tested with the standard Wald tests.

Moreover, given the existence of co-integration we can claim that there are investors, who heavily rely on fundamental valuation. However, the presence of fundamentalists cannot be explained (i) during periods of high market PB ratios relative to fundamental (or predicted) PB ratios, i.e. during an overheated financial market and when overvaluation status prevails, and (ii) during periods of low market PB ratios relative to fundamental (or predicted) PB ratios, i.e. when increased financial stress and economic mayhem triumph over the markets. Alternatively, the evidence of no co-integration at percentiles lower than the 25th percentile or higher than the 90th percentile, i.e. during "boom" and "bust" periods respectively, does not justify the presence of traders relying on grounds of fundamental analysis.

Table 3: Estimates of the NARDL and QNARDL models

	Momentum	*Z-score*_neg(-1)	*Z-score*_pos(-1)	*DPR*_neg(-1)	*DPR*_pos(-1)	*Growth*_neg(-1)	*Growth*_pos(-1)	ECT
(i) **Panel A: NARDL, Unrestricted Error-Correction Model**								
	9.387*	-0.274*	0.538*	-1.498*	-0.437****	0.498*	1.122*	-1.469****
Long-run values	-	-0.186**	0.366*	-1.020*	-0.298****	0.339*	0.764*	-
(ii) **Panel B: QNARDL, Unrestricted Error-Correction Model**								
Quantile	Momentum	*Z-score*_neg(-1)	*Z-score*_pos(-1)	*DPR*_neg(-1)	*DPR*_pos(-1)	*Growth*_neg(-1)	*Growth*_pos(-1)	ECT
0.3	3.760*	-0.043	0.252*	-0.790*	-0.158^{ga}	0.184*	0.374*	-0.701***
Long-run values	-	-0.062	0.360*	-1.127*	-0.225	0.262*	0.534*	-
0.4	2.941*	-0.049	0.229*	-0.806*	-0.096	0.104^{ga}	0.300*	-0.604^{ga}
Long-run values	-	-0.081	0.380*	-1.335**	-0.160	0.172****	0.496*	-
0.5	4.314*	-0.040	0.181***	-0.697*	-0.136	0.163**	0.303*	-0.788**
Long-run values	-	-0.051	0.229*	-0.885**	-0.173	0.207**	0.385*	-
0.6	7.277*	-0.073	0.334**	-1.352*	-0.275^{ga}	0.333*	0.687*	-1.327**
Long-run values	-	-0.055	0.251*	-1.019*	-0.207	0.251*	0.518*	-
0.7	10.338*	-0.147^{ga}	0.468*	-1.370*	-0.346^{ga}	0.590*	1.037*	-1.652**
Long-run values	-	-0.089	0.283*	-0.830**	-0.209	0.357*	0.628*	-
0.75	12.057*	-0.132	0.559*	-1.469*	-0.386****	0.691*	1.157*	-1.912*
Long-run values	-	-0.069	0.293*	-0.768*	-0.202	0.362*	0.605*	-
0.8	11.643*	-0.169^{ga}	0.498*	-1.278*	-0.511***	0.648*	1.160*	-1.777**
Long-run values	-	-0.095	0.280*	-0.719**	-0.287****	0.365*	0.653*	-

Notes: The selected unrestricted error-correction model by the Schwarz (1978) information criterion (SIC) has the following lag order: (6, 0, 2, 6, 0, 6, 4). For brevity, we do not report the 1st, 2nd and 9th decile that produce statistically insignificant speed of adjustments as well as the 1st quartile for the same reason. Statistical inference for the momentum parameter (i.e. the aggregate past changes of ΔPB based on the 6 autoregressive parameters of ΔPB_t) is based on standard distributions, while that of the lagged regressors in levels and that of the error-correction term (ECT) is based on the simulated critical values we generate in this paper. The long-run coefficients (second line of each row) are calculated via the delta method as the coefficient of the lagged-once variable in levels from the unrestricted error-correction model (first line) over the absolute value of the coefficient of the error-correction term. The R^2 for NARDL is 95.4%, and the pseudo R^2 for QNARDLs at 30th, 40th, 50th, 60th, 70th, 75th and 80th percentile are 58.9%, 51.6%, 45.2%, 42.5%, 45.5%, 49.7% and 55.2%, respectively. We round to the 3rd decimal. ^{ga} denotes grey area at 10%, * denotes statistical significance at 1%, ** denotes statistical significance at 2.5%, *** denotes statistical significance at 5% and **** denotes statistical significance at 10%.

Employing the F_{yx}-tests for the 1st stage of the BTP for the NARDL model (see Table 3), we find *F*-statistics (for Cases VIII to XI), which are greater than 24. Thus, we reject the null of no co-integration at every conventional statistical significance level. For the 2nd stage of the BTP the statistic of t_y is -4.431 and thus, it lies in the grey area at 2.5% and 5%, while at 10% we reject the null of no co-integration. To test whether there is a degenerate case of co-integration, we employ the F_x-tests, whose values (for Cases VIII to XI) are also greater than 24.[32] So, the long-run coefficients of 3DM shown in Table 3 (4th row, NARDL) are valid. Moving to the quantile environment (specifically at the 50th percentile), we find *F*-statistics for the F_{yx}-tests that are greater than 8 and for the F_x-tests that are greater than 7. Also, the *t*-statistic for the t_y test is -4.053, which lies in the grey area at 1% significance level however, it is greater than the upper bound at 2.5% significance level. So, we find strong evidence of co-integration using the least absolute deviation (LAD) regression as with the OLS estimator whether or not we include the intercept and the trend terms in the equilibrium relationship, and we do not fall in the degenerate case of co-integration.[33]

Regarding the tests of the lagged regressors, we can see their role at determining the significance of the long-run multipliers. The lagged positive decomposition of *ZSCORE* at the 6th decile is found to be significant at 2.5% and the associated long-run effect to *PB* is significant at 1%. Thus, a degenerate case exists here at 1% significance level. Something similar happens at the median regression as well. One other example of a degenerate case is at the 4th (7th) decile with the negative decomposition of *DPR*. The lagged regressor is found to be significant at 2.5% and the corresponding long-run multiplier at 1%. However, this might

[32] Doing the same for the 4 alternative specifications (Table 9 in the Online Appendix), we find qualitatively the same results.

[33] Repeating the BTP for the rest of the quantiles depicted in Table 3, we find qualitatively the same results except for the estimation at the 4th decile, where the t_y-statistic lies in the grey area at 10% significance level. The critical values for the BTP in our application for the OLS and LAD regressions are in the Online Appendix (Table 1).

be explained by the fact that the speed of adjustment, at the 7th decile is statistically significant at 2.5% and for the 4th decile lies in the grey area at 10% significance level. On the other hand, we find two other cases of degenerate co-integration, where the statistical significance weakens when we move from the lagged regressor to the long-run effect. At the upper quartile and at the 8th decile the lagged "positive" *DPR* is statistically significant at 10% and 5%, respectively. The corresponding long-run multiplier at the 75th percentile is found to cause no effect on *PB*, while at the 80th quantile is significant only at 10%. In both cases, the speed of adjustment is significant at lower significance levels.

From Table 3 we can see that the speed of adjustment is different depending on the quantile of the conditional *PB* distribution. So, a time-varying pattern exists. CKS argue that a plausible explanation for quantile-varying, or time-varying, co-integration is that the underlying relationship may vary over time due to heterogeneous shocks arising at each point of time (see Xiao, 2009) and if this is the case then, the quantile setup is the best estimation strategy because quantile coefficients can be seen as random coefficients (see Koenker and Xiao, 2006). Alternatively, we find location, or quantile, asymmetries for the speed of adjustments of the conditional distribution of *PB*. However, we notice that tail-quantile co-integration is absent and maybe this explains a great part why the linear OLS-based ARDL, given the existence of non-linearities, produces an explosive relationship.[34]

We can confirm the time- or quantile-varying speed of adjustments by testing whether the coefficients of ECT are equal at decile and quartile level. The Wald test (W_y-test) produces a statistic valued at 26.935 for the 9 deciles and 16.256 for the 3 quartiles, which are greater than the critical values of the upper bounds (see Table 1 in the Online Appendix, Panel C1 and C2).

[34] This finding gives more credit to our claims stated earlier in the text that results in works and papers based on ARDL or NARDL or QARDL should be reconsidered with QNARDL modelling. Also, researchers and practitioners in future projects should try both linear and non-linear modelling within both the mean and quantile environment.

So, we reject the null hypothesis of interdecile and interquartile equality of ECTs and, thus, the speeds of adjustment differ. Similarly, we reject the null hypothesis of interdecile and interquartile equality of the lagged regressors in levels (W_x-test), and the lagged regressors in levels and the ECT (W_{yx}-test). The values for the interdecile tests of the lagged regressors, and the lagged regressors and the lagged PB ratio, respectively, are 197.077 and 226.299. For the interquartile tests the corresponding values are 129.598 and 145.193. In all cases, they exceed the critical values of the upper bounds (see Table 1 in the Online Appendix, Panel C1 and C2).[35]

The absolute value of $\left(1+\phi_y\right)$ at the upper percentiles exceeds these of the rest percentiles of interest by a specific threshold. Specifically, we find that the absolute deviation of SOA from unity, at the 8th decile is greater by at least 0.13 than the respective values at 3rd, 4th, 5th, 6th and 7th deciles.[36] So, considering a threshold up to 13% (or 0.13) we find strong evidence of this kind of asymmetry in the convergence rate to equilibrium. The SOA at the 80th percentile denotes a slower convergence rate than the rest of the significant SOAs (30th, 40th, 50th, 60th and 70th percentiles).[37] Alternatively, we find that during periods, where *PB* of the market index is high relative to its fundamentals, convergence to equilibrium is faster than during periods of low *PB* relative to its fundamentals. In other words, fundamentalist traders expect their strategy to be lucrative over a longer horizon during periods of increased financial stress than during periods of stock market buoyancy. From Table 3, we can see that there is a

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[35] However, when we use the general-to-specific approach we see that inference can be misleading if the standard critical values of the Wald test are used. Section 3 of the Online Appendix provides more details.

[36] We use absolute deviations from unity to take into consideration that SOAs close to unity denote a faster convergence rate than SOAs close to 0 or 2, given that we are interested for values that lie in the interval (0, 2). For example, a coefficient of the ECT with value of -0.2 is larger than another one with -1.3 however, -0.2 implies a much slower convergence rate than -1.3 (0.8 > 0.3).

[37] A coefficient of the ECT greater than unity, in absolute values, implies a damped equilibrium path.

co-integrating relationship with the NARDL model, while when we move to the quantile environment a long-run equilibrium relationship exists for the 30th to 80th percentile of the conditional distribution of *PB*.

In our application, when we look at the QNARDL results, we observe in the long run a partial asymmetry of *PB* to shocks in the z-score variable and to shocks in dividend payout ratio (an exemption is at the 8th decile for *DPR*), and a fully asymmetric behavior of *PB* when expected growth of net income changes. Specifically for the median regression, the *p*-value of the Wald test of the equality of the long-run coefficients of *ZSCORE* is 2.6% indicating that at 5% and 10% levels of statistical significance there is evidence of long-run asymmetry.[38] The corresponding *p*-values for *DPR* and *GROWTH* are 1.7% and 3.7%, and reject the hypothesis of long-run symmetry at 5% % statistical level of significance.[39]

When *ZSCORE* increases, i.e. risk decreases, *PB* increases, whereas it does not respond when then there is an increase of risk (*ZSCORE* decreases). On the other hand, decreases of *DPR* increase *PB*, while *PB* is unresponsive to positive shocks of *DPR* (an exemption is at the 8th decile). This might seem strange because when *DPR* decreases, the firms return a smaller part of their profits to their shareholders. However, this is not the case if investors see it as an increase of the expected future growth. So, a *DPR* decrease can also be translated as high expectations for future growth.[40] In confirmation of our claim, the long-run multiplier of expected growth is positive and statistically significant. Moreover, increases in *GROWTH* increase *PB* more than an equal decrease does.

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[38] We employ the delta method for the comparison of the long-run coefficients so that we can infer whether there is an asymmetric behavior. GS and SYG imply that they use the delta method for the long-run symmetry test.

[39] Results remain the same qualitatively at 5% and 10% significance levels for the rest of the quantile regressions shown in Table 3 except for *DPR* at the 8th decile, where a 10% significance level is required.

[40] When *DPR* decreases, the retention ratio increases and thus, expected growth will be larger. See Equation (11) in the "Data description" in Section 3.

Regarding the NARDL and the long-run estimates, we find evidence in favor of a fully asymmetric behavior of *PB* for the *ZSCORE*, dividend payout ratio and expected growth. *PB* increases no matter what happens to risk however, it is more sensitive to risk decreases (*ZSCORE* increases) than to risk increases (*ZSCORE* decreases). Performing a Wald test for the equality of the long-run coefficients of Z*SCORE* in the positive and negative regime, we get a 0% *p*-value, which implies that the "positive" coefficient is greater than the "negative" coefficient at any conventional level of statistical significance.

This is an interesting phenomenon that could be interpreted through an interaction between "Type A" investors and "Type B" investors at the global stock market level. When risk decreases (*ZSCORE* rises), "Type B" investors buy and "Type A" investors sell. If the effect of "Type B" investors is greater than that of "Type A" investors, then *PB* rises. When risk increases (*ZSCORE* deteriorates) "Type A" investors buy and "Type B" investors sell, and if the effect of "Type B" investors dominates that of "Type A" investors then *PB* increases. To sum up, greater risk in the international stock markets seems to be associated with buying activity by "Type A" investors dominating selling activity by "Type B" investors, while lower risk with exactly the opposite effect. However, this phenomenon disappears when we move to the quantile framework. Specifically, when risk decreases (*ZSCORE* rises) the positive effect of "Type B" investors on *PB* dominates. When risk increases (*ZSCORE* falls) on the other hand, the positive effect from "Type A" investors is counterbalanced by the negative effect of "Type B" investors and thus, *PB* does not respond to reductions of the z-score variable.

According to NARDL estimates, *PB* decreases (increases) when *DPR* increases (decreases) with stronger sensitivity to decreases of *DPR*. The *p*-value from the Wald test of the equality of the two long-run coefficients is 0.9% indicating that the coefficient of the negative regime is greater than that of the positive regime at 1% statistical level. Increases in *DPR* can be seen as lower future expected growth and investors penalize increased dividend policies. On the other hand, lower dividends are rewarded for the sake of greater future expected growth. When we move to the quantile framework we witness that *PB* is insensitive to *DPR* increases. Investors value equally their gain from dividends and the loss from the firms' lower expected growth. The only exception is at the 8^{th} decile, where the long-run multiplier of "positive" *DPR* is close to the corresponding coefficient with the NARDL model.

Finally, there is also evidence in favor of full asymmetry for the expected growth of net income in NARDL (the same type exists in the quantile framework). Specifically, the *p*-value of the Wald test of the equality of the long-run coefficients is 0% implying that the positive effect stemmed from higher growth expectations is greater in absolute terms than the negative effect stemmed from lower growth expectations of net income of the S&P Global Index firms. *PB* responds more to *GROWTH* increases than to equal-size decreases. Therefore, the asymmetric behavior of *PB* with respect to the long-run multipliers changes substantially when we depart from OLS to the quantile framework.[41]

When we calculate the long-run marginal effects allowing for a one-standard-deviation change in the long-run coefficients, we find that *GROWTH* dominates *ZSCORE* and *DPR*, and plays the biggest role in determining fundamental values of *PB*. When we use the standard deviations of the variables after the decomposition of SYG, *ZSCORE* captures 10.7%, *DPR* 20.6% and *GROWTH* 68.7% of the variability in *PB*. Performing the same task with the QNARDL model we find similar results. For robustness, we also use the standard deviations of the variables prior the decomposition of SYG and the corresponding numbers for the NARDL model are 6.4%, 39.5% and 54.1% (with QNARDL we also find similar results).

Next, we examine evidence on momentum versus contrarian trading. These are two different forms of chartist strategies. Based on the autoregressive coefficients of our models

[41] With NARDL we observe a fully asymmetric behavior of *PB* for *ZSCORE*, *DPR* and *GROWTH*. With QNARDL we observe partial asymmetry of *PB* to *ZSCORE* and to *DPR* (an exemption is at the 8^{th} decile for *DPR*), and full asymmetry of *PB* to *GROWTH*.

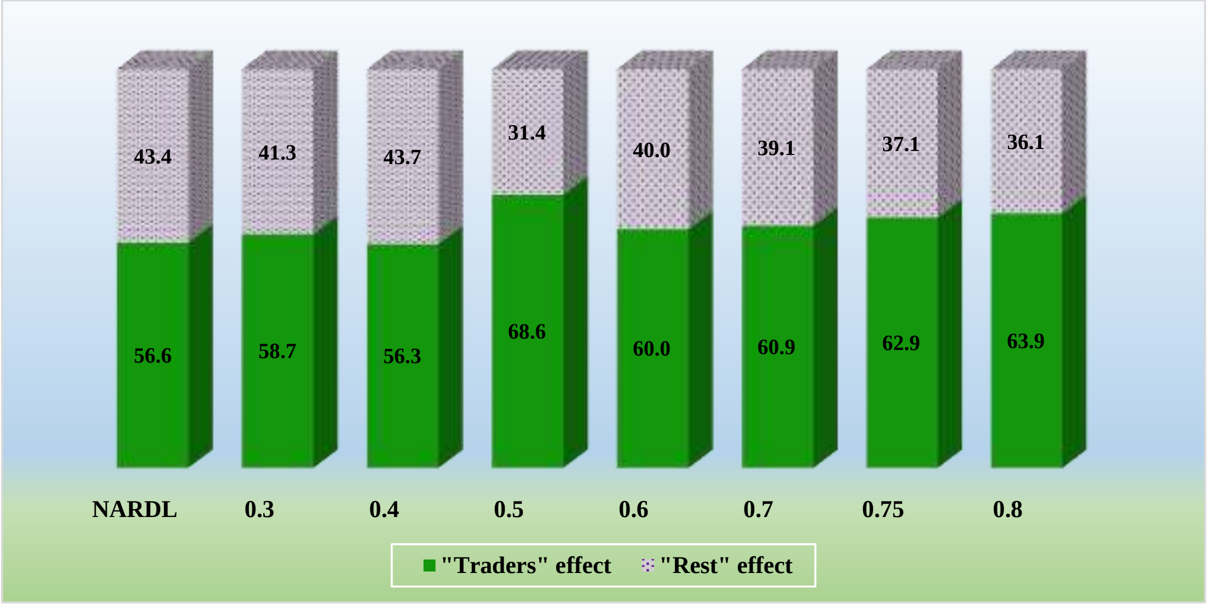
we find strong evidence of momentum, i.e. positive-feedback, trading. Testing whether the sum of the 6 autoregressive coefficients of ΔPB is statistically insignificant we get a 0% *p*-value. The aggregate past changes of ΔPB are statistically significant at any conventional significant level both at the NARDL and QNARDL models (see Equations 12 and 13).

The momentum effect is reinforced in higher quantiles in the QNARDL setup, i.e. during periods with increased market stress. In general, we find heterogeneous momentum trading across the quantiles. Along the transition from the "boom" to the "bust" part of the economic cycle investors seem to rely more on trading based on technical analysis. This type of investors follow "past winners" and get rid of "past losers". Thus, we find evidence of time-varying, quantile-dependent, momentum effect, which becomes more intense during periods characterized by *PB* undervaluation.

We have seen that market participants of the S&P Global Index use both momentum trading strategies (aggregate past *PB* changes exert a positive and statistically significant effect on current *PB*) and fundamental trading strategies (there is co-integration in our model). In Figure 3 we can see that at least 56% of the explained variability of *PB* is due to the presence of traders (both chartists and fundamentalists). Alternatively, about 5% of the total components of the covariance matrix associated with models of Equations (12) and (13) capture more than the half of the variation in *PB*. Figure 4 presents a decomposition of the traders' effect into the fundamentalist and chartist components. Specifically, we observe that at least 69% of the total traders' effect is due to fundamentalists, who predominate the chartists during all periods with verified co-integrating relationships (30th to 80th percentile).[42]

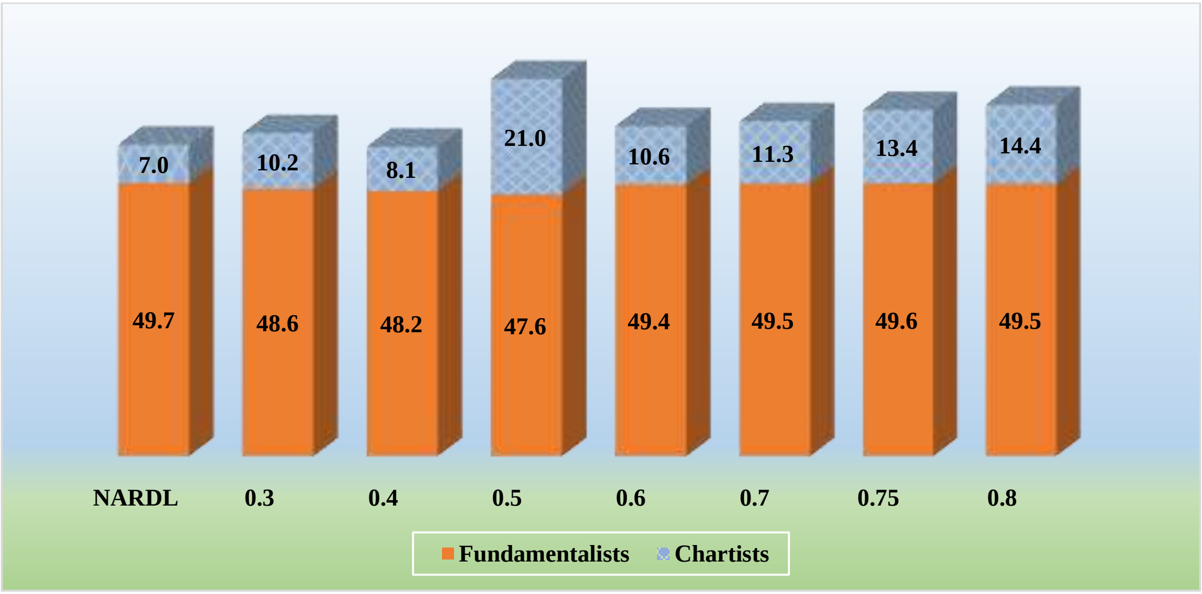
[42] 69% corresponds to the 5th decile and is the minimum value. The greatest values are met in the NARDL model, 88%, and at the 4th decile of QNARDL model, 86%.

Figure 3: Sources of explained variation of the Price-to-Book ratio (in %)



Notes: Numbers shown in Figure 3 are based on the estimates of the models presented in Table 3. "Traders" effect incorporates the covariance effect of the autoregressive variables (36 components), and the covariance effect of the error-correction term (ECT) and the lagged regressors in levels (49 components). "Rest" effect involves the remaining 1,596 components of the covariance matrix. The number of total components of the covariance matrix is 1681, 41^2, where 41 is the number of total variables in the right-hand side. 0.3, 0.4, 0.5, 0.6, 0.7, 0.75 and 0.8 refer to the user-specified quantiles with the Quantile Non-linear Autoregressive Distributed Lag model (QNARDL). We round to the 3rd decimal. Values are in percentages, e.g. 58.7 means 0.587 or 58.7%.

Figure 4: Decomposition of the traders' effect on Price-to-Book valuation



Notes: Figure 4 shows the decomposition of the traders' effect on market valuation (from Figure 3) to two components: the fundamentalist component and the chartist component. 0.3, 0.4, 0.5, 0.6, 0.7, 0.75 and 0.8 refer to the user-specified quantiles with the Quantile Non-linear Autoregressive Distributed Lag model (QNARDL). We round to the 3rd decimal. Values are in percentages, e.g. 48.2 means 0.482 or 48.2%.

4. Concluding remarks

Pesaran, Shin and Smith (2001, PSS) is one of the most influential papers in time-series analysis with many applications in economics and finance. In this paper, we provide useful and important extensions of the bounds testing procedure of PSS in two modelling setups. First, in the ARDL models in unrestricted error-correction form of Pesaran and Shin (1999) in the OLS framework. Second, in the QARDL models in unrestricted error-correction form, of Cho, Kim and Shin (2015) under the conditional quantile framework of Koenker and Bassett (1978).

To demonstrate the new critical values and tests for the bounds testing procedure, we employ an empirical application and we use a variation of the 3D model (3DM) of equity valuation of Bertsatos and Sakellaris (2016) allowing for non-linear responses in the spirit of Shin, Yu and Greenwood-Nimmo (2014). We find that misspecifying an asymmetric relationship as a linear relationship can be misleading in terms of findings. Specifically, we find evidence of no co-integration when we restrict ourselves to linear modelling either with OLS or quantile estimators. More precisely, the long-run equations are explosive. However, once the linearity restriction is lifted and we allow for asymmetric, or non-linear, responses in the ARDL and QARDL models, i.e. when we use NARDL and QNARDL models, the conclusions differ considerably. We discover that there are asymmetric long-run relationships using NARDL and, in the case of quantile environment employing QNARDL, our results show further evidence of time-varying co-integration across quantiles of the conditional distribution of the PB ratio.

These results point to certain caveats on the conventional practice of linear ARDL modelling. Different forms of asymmetry (e.g. time-varying relationships, threshold-oriented non-linear responses) may lead to misleading results under this practice.

To conclude, we discuss some caveats and offer suggestions for future extensions. On the methodological part of the paper, it should be kept in mind that the standard BTP of PSS and the extensions that we present here may yield spurious results when more than one co-integrating relationships exist. Also, this estimation strategy is invalid when there is a variable of second-order integration, $I(2)$, or under the presence of serially correlated errors. Moreover, the critical values we provide with our codes for different sample sizes are based on asymptotic arguments that ignore terms such as lagged differences. This may have limitations in finite samples, in particular when dealing with "short" time-series in an empirical application and when there is a rich lag structure in the estimated ARDL model. Regarding further work, a bounds-testing framework could be expanded to panel data, or a BTP could be developed incorporating a mixture of $I(2)$, $I(1)$ and $I(0)$ regressors to cover projects with "stocks", where the "flows" could be $I(1)$ and $I(0)$ variables.

On the empirical application in the paper, one could link several global events to the PB ratio of the S&P Global Index, or perform extra diagnostic tests (normality tests, homoscedasticity tests, etc). Additionally, one could devise more sophisticated tests of the "chartists and fundamentalists" traders' effects, search for optimal thresholds in the construction of the asymmetric variables (see Greenwood-Nimmo et al., 2012) together with the selection of the lag structure, as well as provide asymmetric dynamic multipliers with their bootstrapped confidence intervals, or emphasize on the short-run coefficients. Finally, we could also engage in detecting possible differences between conditional and unconditional quantile regressions in our study. This is something that Borah and Basu (2013) do in the cross-sectional level for a large US data about medication adherence.

These suggested extensions are beyond the scope of this paper. The main goal of this paper is to present extensions of the BTP of PSS and to demonstrate them with emphasis on the existence of (time-varying) co-integrating relationships either with linear or asymmetric relationships.

5. Declarations

5.1 Funding

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5.2 Data availability

The raw data of the empirical application of this study were downloaded from Datastream. Access restrictions apply to these data, which were downloaded and used under a license to the Laboratory of Financial Applications of Athens University of Economics and Business.

5.3 Computer codes availability

We have created 3 codes in EViews (9th edition) that generate critical values for different sample sizes for tests in the following environments: (i) OLS estimator with an extended BTP of PSS (ARDL of Pesaran and Shin 1999; and PSS), (ii) CQREGs in the spirit of Koenker and Bassett (1978) with a proposed BTP a la PSS (QARDL of CKS), where the user selects the desired percentile for the conditional quantile regression and (iii) CQREGs in the spirit of Koenker and Bassett (1978) with a proposed interpercentile BTP a la PSS. These codes are available on the website of the journal as supplementary information, together with a Readme file that explains their use. The user enters the effective number of observations, i.e. the number of observations after the estimation process.

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