Optimal Loan Loss Provisions and Welfare*

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Abstract

We study the welfare implications of optimal loan loss provisions in a New Keynesian model featuring endogenous default risk and inflationary credit spreads. A unique link between provisions, credit spreads and inflation can be employed to enhance macroeconomic stability. Optimal provisions are most effective when dealing with cost-push financial shocks inherent in volatile spreads and the zero bound problem of monetary policy. Relaxing provisioning requirements following a recessionary financial disturbance consistently achieves the first-best outcome while nullifying the value of monetary policy under commitment. In contrast, deflationary demand shocks warrant an optimal rise in provisions, which inflate prices yet mildly contract output.

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1 Introduction

In the aftermath of the financial crisis and the Great Recession, the Basel Committee on Banking Supervision together with the International Accounting Standards Board (IASB) have called for the transition from *specific* loan loss provisioning systems (incurred-loss approach) towards a more *dynamic* provisioning regime (expected-loss approach). Until January 2018, the provisioning system in most advanced economies was specific and tied by the International Accounting Standards (IAS) 39, which required banks to set specific provisions related to identified credit losses, such as past due payments (usually 90 days) or other default-like events. Critics of the IAS 39 have argued that setting provisions in an ex-post fashion, after the identification of nonperforming loans, comes too late in the cycle, with the additional provisioning expenses in banks income statements exacerbating the procyclical tendencies of the financial system (see also Jiménez, Ongena, Peydró and Saurina (2017)).

In contrast, according to the new International Financial Reporting Standards (IFRS) 9, which have replaced the IAS 39 as of January 2018, dynamic provisions should be set in a timely manner before credit risk materializes, and allow the financial sector to better absorb losses by drawing upon these provisions in the wake of a negative credit cycle. In this way, provisions are established according to the expected-loss impairment model, potentially curbing procyclicality in the credit markets and providing a more accurate profit and loss account. More specifically, expected provisioning ought to smooth out the evolution of total loan loss provisions, and reduce the need of financial intermediaries to increase costly loan loss reserves during financial downturns. Prior to the IFRS 9, and already back in 2001, the Bank of Spain was one of the first central banks to introduce elements of the expected-loss provisioning system in an effort to dampen excessive credit growth. Under the expected-loss approach, banks must make provisions according to the latent risk over the business cycle, or based on the historical information regarding credit losses for different types of loans. By anticipating better the expected losses lurking in a loan portfolio over some specific horizons, IFRS 9-type provisions aim to promote financial stability.[1]

Despite the recent adjustments in global financial accounting standards and their significance in driving credit cycles, the theoretical literature on the macroeconomic stabilization roles of loan loss provisions and their welfare optimizing behavior is rather limited. Motivated by the recent implementation of more prudential-type provisions, and the limitations of standard monetary policy as a tool for macroeconomic stabilization, as has been evident over more than a decade, in this paper we address the following interrelated questions: *i)* what should be the *optimal* design of loan loss provisions in response to financial market disruptions associated with inflationary and volatile credit spreads; *ii)* what are the macroeconomic and welfare implications of optimally varying provisions against the backdrop of state-contingent liquidity traps?

We answer these questions by providing closed-form analytical and quantitative results in a

[1] In practice, the IFRS 9 expected-loss provisioning system differs from the dynamic "forward-looking" provisioning system that has been implemented in Spain and other Latin American countries. The former approach represents unbiased estimates of loan losses over a specific horizon, while the latter requires banks to make specific and general provisions (based on future losses) according to a formula set by the regulator. However, dynamic provisions include elements of the expected-loss model with both regimes featuring "forward-looking" components intended to reduce financial sector procyclicality. We therefore use the terms dynamic and expected provisioning interchangeably throughout the text despite the differences in actual definitions and implementation. The way we model the dynamic nature of loan loss provisions in this paper is *broadly* more consistent with the IFRS 9 expected-loss approach. At the same time, the focus of this paper is on the *macroeconomic* implications of optimal loan loss provisions rather than the financial intricacies of the various provisioning methods.

small-scale workhorse New Keynesian model à la Woodford (2003). The basic framework is augmented for a supply-side collateral constraint that gives rise to endogenous default risk and inflationary credit spreads. Default risk is determined by the borrowers' leverage, measured in terms of the debt to collateralized output ratio, while the credit spread is shown to be endogenous with respect to risk and costly loan loss provisions. This supply-side financial market friction results in a distorted steady state allocation and in inefficient economic dynamics, both of which call for the implementation of corrective credit policies. Such policies in this paper take the form of state-contingent *regulatory* loan loss provisions that are primarily designed to *smooth* business and credit cycles that result in occasional liquidity traps.[2]

As in Ravenna and Walsh (2006), firms in our model must borrow from a bank in order to finance their labour costs.[3] Therefore, monetary policy, credit risk and loan loss provisions, all of which dictate the loan rate behavior, translate into changes in the firms' marginal costs, price inflation and output through the credit cost channel.[4] Importantly, part of this cost-push conduit is driven by the *provisioning cost channel* relating provisions directly to inflation and the real economic activity. Such unique association has important and novel implications for the design of optimal financial policies aimed at enhancing overall welfare. We argue that optimal provisioning practices are most effective when dealing with inflationary financial shocks that result in high borrowing costs and occasional supply-side-driven liquidity traps. Optimal loan loss provisions can consistently achieve the first-best allocation regardless of whether the nominal interest rate is constrained by the zero lower bound (ZLB) or not. In a demand-side-driven liquidity trap instigated by adverse shocks to the natural rate of interest, prudential regulation policies like the ones proposed by the IFRS 9 bring about a variant of the *paradox of toil* (as popularized by Eggertsson (2010)), wherein otherwise expansionary supply-side measures can paradoxically lead to lower welfare. In our paper, this paradox is highlighted through the provisioning cost channel.

Supporting Gilchrist, Schoenle, Sim and Zakrajšek (2017), adverse financial shocks that raise the lending rate produce inflationary pressures, and result in an output-inflation trade-off. This upshot motivates us to examine optimal financial policies beyond traditional interest rate policy. Following a cost-push credit disturbance, optimal monetary policy under commitment sends the nominal policy rate to its zero bound due to the large inefficient contraction in output that is amplified by the enforcement of IAS 39 specific provisions. We demonstrate that *relaxing* provisioning requirements, even *beyond* the full-smoothing of provisions as proposed by the new IFRS 9 regulation, can be a very effective way to restore complete price and output stability by directly minimizing variations in borrowing costs and eliminating the zero bound problem. In particular, a mildly countercyclical response of loan loss provisions solves the standard output-inflation trade-off

[2] We therefore abstract from micro factors that may influence the individual bank's discretionary provisioning decisions. These factors may include profitability levels, transparency and signaling effects to the financial markets.

[3] The majority of the literature on financial regulation uses credit lines to finance house purchases and investment in physical capital. We instead introduce loans to finance labour costs. This strategy allows us to provide closed-form solutions to the model and to examine the normative properties of loan loss provisions in a tractable textbook New Keynesian model augmented for financial frictions. This modeling viewpoint is also motivated by recent evidence suggesting that variations in working-capital loans following adverse financial shocks can have persistent negative effects on the economic activity (see Fernandez-Corugedo, McMahon, Millard and Rachel (2012) who estimate the cost channel for the U.K economy, and Christiano, Eichenbaum and Trabandt (2015) for the U.S). This result, therefore, warrants the examination of optimal financial policies when firms rely on external finance to support their production activities.

[4] The additional financial frictions present in this model contribute to the standard monetary policy cost channel of Ravenna and Walsh (2006), where the loan rate is simply equal to the risk-free policy rate.

induced by the financial disruption and the credit cost channel. These results hold regardless of how the policy maker implements monetary policy, with loan loss provisions removing the necessity for any costly and time-inconsistent monetary policy commitments. In our model, loan loss provisions stand out as a sole and natural policy reaction against financial shocks inherent in high and volatile borrowing costs that result in occasional supply-side-driven liquidity traps.

Adverse deflationary demand shocks warrant an optimal *increase* in provisions and a prolonged ZLB interest rate policy. Intuitively, raising provisions provides a cost-push effect that lifts the credit spread and increases prices through the credit and provisioning cost channels. We find that higher provisions result in a mild short-run contraction in output, which is optimal when the public authority places a much higher weight on price stability than on output stability in the micro-founded welfare function. At the same time, dynamic and optimal loan loss provisions yield minimal welfare gains relative to the more constrained optimal monetary policy under commitment. These quantitative results prevail despite the qualitatively different impulse response functions. In other words, stabilization following negative shocks to the natural rate of interest is mainly attributed to the optimal and loose monetary policy. From a purely welfare perspective, whether to increase, flatten or lower provisions over the cycle depends largely on what triggers the business cycle and pressingly on the output-inflation dynamics that arise.

Much of the earlier literature on loan loss provisions has empirically examined the role of provisioning practices in shaping the financial cycle (see Laeven and Majnoni (2003), Bikker and Metzemakers (2005) and Jiménez and Saurina (2006), among others). Moreover, in a recent contribution, Jiménez, Ongena, Peydró and Saurina (2017) provide an exhaustive empirical study on the impact of dynamic provisions on the Spanish credit cycle. These authors show that dynamic provisioning smooths credit supply cycles and supports firm performance in bad times. From a theoretical standpoint, we highlight the importance of provisions in also explaining the behavior of real business cycles and inflation, as well as their optimal dynamics and welfare implications.

Our findings also extend the theoretical literature on the effectiveness of loan loss provisioning regimes. Bouvatier and Lepetit (2012a) develop an analytical partial equilibrium model, and show that dynamic provisions, defined by accounting rules to cover for expected losses, can eliminate procyclicality in lending standards induced largely by specific provisions. Agénor and Zilberman (2015) use a standard medium-scale calibrated New Keynesian model and illustrate that dynamic provisions can help mitigate financial and real sector volatility, even more so when implemented together with a credit-augmented monetary policy rule. Unlike these papers, we instead derive analytically and examine the fully optimal behavior of loan loss provisions, as well as quantify the welfare gains from state-contingent optimal provisioning policies following inflationary (deflationary) financial (demand) shocks in a tractable small-scale New Keynesian model. Furthermore, we show that varying provisions alone can completely circumvent adverse financial shocks and importantly liquidity traps generated by such disturbances. This desirable feature of optimal provisions holds regardless of any potential interactions between provisioning rules and monetary policy and/or reserve requirements (as is the case in Agénor and Zilberman (2015) and Agénor and Pereira da Silva (2017), respectively).[5]

In terms of methodology, our paper is related to those of Demirel (2009) and De Fiore and Tristani (2013), who also derive a micro-founded risk premium in a New Keynesian model augmented for the credit cost channel. However, these papers focus solely on optimal monetary policy

[5] In a recent contribution, Rubio and Yao (2020) emphasize the welfare enhancing properties of policy coordination between loan-to-value (LTV) and monetary policy *rules* in a low interest rate environment.

away from liquidity traps, whereas the aim of our paper is to calculate optimal provisioning policies in response to credit and demand shocks, accounting for the ZLB. Correia, De Fiore, Teles and Tristani (2021) show within a classic monetary economy framework that credit subsidies to firms can overcome the zero bound constraint on nominal interest rates following shocks that raise credit spreads. Our paper supports this result in the context of a New Keynesian model where the financial policy is conducted via loan loss provisions that directly affect the financial intermediation pricing decisions. Put differently, provisions in our model affect bank profits by acting similarly to a macroprudential tax / subsidy, which feeds into the bank's optimal loan rate determination.

The rest of the paper is structured as follows: Section 2 describes the model and the market clearing conditions. Section 3 derives the steady state and the short-run equilibrium properties. Section 4 details the parameterization of the model. Section 5 examines the dynamics and welfare implications of state-dependent optimal provisioning policies. Section 6 concludes.

2 The Model

The economy consists of households, a final good (FG) firm, a continuum of monopolistic intermediate good (IG) firms, a perfectly-competitive commercial bank (the bank), and a benevolent public authority. At the beginning of the period and following the realization of aggregate shocks, the bank issues deposits to households in order to meet the IG firms' demand for working-capital loans that enable production. The bank sets the loan rate based on the deposit rate, the finance-risk premium and regulatory loan loss provisions that cover for a fraction of nonperforming loans. In particular, borrowing IG firms face end-of-period idiosyncratic productivity shocks that give rise to credit default risk, and hence to an ex-ante finance premium and required provisions. For the going lending rate, IG firms pay households labour income, compute their marginal costs, and set prices subject to Calvo (1983)-type nominal price rigidities. Using a standard Dixit-Stiglitz (1977) technology, the FG firm combines all intermediate goods to produce a homogeneous final good used only for consumption purposes. Finally, the public authority sets the deposit-policy rate and the required loan loss provisions to loan ratio. We now turn to a more detailed exposition of the economic environment and equilibrium properties.

2.1 Households

The objective of the representative household is to maximize the following expected lifetime utility:

$$U_t = \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t \vartheta_t \left[\frac{C_t^{1-\varsigma}}{1-\varsigma} - \frac{H_t^{1+\gamma}}{1+\gamma} \right], \tag{1}$$

where $\mathbb{E}_t$ is the expectations operator, $H_t = \int_0^1 H_{j,t} dj$ are the total hours supplied to the production sector, and C_t is composite consumption. Moreover, $\beta \in (0,1)$ is the discount factor, ς is the inverse of the intertemporal elasticity of substitution in consumption, and γ is the inverse of the Frisch elasticity of labour supply. The preference shock, also referred to as a discount factor or simply a demand shock, follows the $AR(1)$ process:

$$\vartheta_t = (\vartheta)^{1-\rho_\vartheta} (\vartheta_{t-1})^{\rho_\vartheta} \exp \left(s.d(\alpha^\vartheta) \cdot \alpha_t^\vartheta \right), \tag{2}$$

where ϑ is the steady state value of the shock, ρ_ϑ is the degree of persistence, and α_t^ϑ is a random shock distributed as standard normal with a constant standard deviation given by $s.d(\alpha^\vartheta)$.[6]

Households enter period t with real cash holdings M_t. In return for supplying working hours, households receive their wage bill $W_t H_t$ paid as cash at the start of the period, with W_t representing the real wage. This cash is then used to make real deposits D_t at the bank. The interest rate on deposits, which also represents the main policy rate, is denoted by R_t^D. The households remaining cash balances of $M_t + W_t H_t - D_t$ become available to purchase the aggregate consumption good subject to a cash-in-advance (CIA) constraint, $C_t \leq M_t + W_t H_t - D_t$. Such restriction represents the implicit cost of holding intra-period deposits that yield interest but that cannot be used for transaction services. At the end of the period, households receive a lump-sum liquidity cash transfer from the public authority (T_t), as well as total profits from the production and financial intermediation sectors ($J_t \equiv J_t^P + J_t^{FI}$). The real value of cash carried over to period $t+1$ is:

$$M_{t+1} \frac{P_{t+1}}{P_t} = M_t + W_t H_t - D_t - C_t + R_t^D D_t + J_t + T_t. \tag{3}$$

Taking the real wage (W_t) and the aggregate price level (P_t) as given, the first-order conditions of the household's problem with respect to C_t, D_t, M_{t+1} and H_t can be summarized as:

$$C_t^{-\varsigma} = \beta \mathbb{E}_t \left(C_{t+1}^{-\varsigma} R_t^D \frac{\vartheta_{t+1}}{\vartheta_t} \frac{P_t}{P_{t+1}} \right), \tag{4}$$

$$H_t^\gamma C_t^\varsigma = W_t. \tag{5}$$

The lower bound constraint that satisfies the households no-arbitrage condition between cash-financed consumption and deposits must apply to $R_t^D \geq 1$, which is an equilibrium restriction in a model with a CIA constraint.

2.2 Production

A perfectly-competitive FG firm produces total output Y_t by assembling differentiated intermediate goods $Y_{j,t}$, indexed by $j \in (0,1)$, using a Dixit-Stiglitz (1977) technology, $Y_t = \left[\int_0^1 Y_{j,t}^{(\lambda-1)/\lambda} dj \right]^{\lambda/(\lambda-1)}$, where $\lambda > 0$ is the constant elasticity of substitution between intermediate goods. The corresponding demand for each product j is then given by $Y_{j,t} = (P_{j,t}/P_t)^{-\lambda} Y_t$, with $P_{j,t}$ denoting the price set by intermediate firm j, and $P_t = \left[\int_0^1 P_{j,t}^{1-\lambda} dj \right]^{1/(1-\lambda)}$ representing the aggregate price index of the final good that satisfies $P_t Y_t = \int_0^1 P_{j,t} Y_{j,t} dj$.

Each intermediate good is produced by a monopolistic IG firm using the following linear production function:

$$Y_{j,t} = \varepsilon_{j,t} H_{j,t}, \tag{6}$$

where $H_{j,t}$ is employment demand by firm j, and $\varepsilon_{j,t}$ is an idiosyncratic shock distributed uniformly over the interval $(\underline{\varepsilon}, \bar{\varepsilon})$ with a mean of unity and a constant variance.[7] The firm must partly borrow from the bank in order to pay the households' labour income. Let $L_{j,t}$ be the amount borrowed by

[6] Steady state values are denoted by dropping the time subscript.

[7] This simple distribution helps to achieve clear and transparent expressions for credit risk, loan loss provisions and borrowing costs without loss of generality.

firm j, then the real financing constraint is:

$$L_{j,t} = \kappa W_t H_{j,t}, \tag{7}$$

with $\kappa \in (0,1)$ determining the fraction of wage bill that must be financed at the beginning of the period prior to production.

Financing labour costs bears risk and in case of default the bank expects to seize a fraction χ_t of the firm's output. The term χ_t follows the $AR(1)$ shock process:

$$\chi_t = (\chi)^{1-\rho_\chi} (\chi_{t-1})^{\rho_\chi} \exp\left(s.d\,(\alpha^\chi) \cdot (\alpha_t^\chi)\right), \tag{8}$$

where $\chi \in (0,1)$ is the steady state value of this fraction, ρ_χ is the degree of persistence, and α_t^χ is a random shock with a normal distribution and a constant standard deviation denoted by $s.d\,(\alpha^\chi)$. The collateral shock characterizes the financial (credit) shock in this model, as it directly impacts credit risk at the IG firm level as well as the loan-deposit rate spread as shown below.[8] In the good states of nature, each IG firm pays back the bank principal plus interest on loans. Consequently, default occurs when the expected value of seizable collateralized output $(\chi_t Y_{j,t})$, net of state verification and enforcement costs, is less than the amount that needs to be repaid to the lender at the end of the period, i.e., $\chi_t Y_{j,t} < R_t^L L_{j,t}$, with R_t^L standing for the gross lending rate. Using (6) and (7), the threshold value $(\varepsilon_{j,t}^M)$ below which the borrower defaults is:

$$\varepsilon_{j,t}^M = \frac{\kappa R_t^L W_t}{\chi_t}. \tag{9}$$

Therefore, the cut-off point is related to the aggregate credit shock, borrowing costs and the real wage, and is identical across all firms. However, in our model, the loan rate not only depends on the risk-free rate and the finance premium (as in Agénor and Aizenman (1998)), but also on the credit risk shock and the loan loss provisions to loan ratio.[9] Importantly, notice that for $Y_{j,t} = H_{j,t}$, $\mathbb{E}_t \varepsilon_{j,t} = 1$ and output serving as collateral, the fraction of wage bill that must be financed in advance of production must equal the loan to output ratio (see (6) and (7)). Then, from (9), higher demand for working-capital loans and increasing leverage in 'good times' translate to higher financial risk. Such supply-side default risk results in a distorted steady state and amplified economic dynamics, both of which justify credit interventions in the form of loan loss provisions. Given the uniform properties of $\varepsilon_{j,t}$, the probability of default is:

$$\Phi_t = \int_{\underline{\varepsilon}}^{\varepsilon_t^M} f(\varepsilon_t) d\varepsilon_t = \frac{\varepsilon_t^M - \underline{\varepsilon}}{\bar{\varepsilon} - \underline{\varepsilon}}. \tag{10}$$

The pricing decision during period t takes place in two stages. First, each IG firm minimizes the cost of employing labour, taking its effective costs as given. The idiosyncratic shock is unknown at the time of the labour hiring decision, even though the commercial bank sets the loan rate (derived below) by accounting for *perceived* default in the required risk premium. IG firms therefore internalize the possibility of default once they borrow from the bank at the rate R_t^L. Defining the

[8] Tayler and Zilberman (2016) also introduce a similar type of financial / collateral shock that directly affects credit default risk and the credit spread.

[9] As we solve explicitly for the risk of default using a threshold condition, the supply-side collateral constraint in this model, from which we derive the cut-off point, is always binding.

time-t real profit function of firm j as $J^P_{j,t} = Y_{j,t} - \kappa R^L_t W_t H_{j,t} - (1-\kappa) W_t H_{j,t}$, first-order conditions yield the expected real marginal cost:[10]

$$mc_t = \frac{\left[1 + \kappa \left(R^L_t - 1\right)\right] W_t}{\mathbb{E}_t \varepsilon_{j,t}}. \tag{11}$$

Second, each IG producer chooses the optimal price for its good subject to Calvo (1983)-type nominal rigidities, where a portion of ω firms keep their prices fixed while a portion of $1-\omega$ firms reset prices optimally. Denoting P^*_t as the optimal price set by IG producers who can reset prices at time t, then the standard maximization problem produces the optimal price setting rule:

$$\frac{P^*_t}{P_t} = \left(\frac{\lambda}{\lambda - 1}\right) \frac{\mathbb{E}_t \sum_{s=0}^{\infty} \omega^s \beta^s C^{-\varsigma}_{t+s} mc_{t+s} \left(\frac{P_{t+s}}{P_t}\right)^{\lambda} Y_{t+s}}{\mathbb{E}_t \sum_{s=0}^{\infty} \omega^s \beta^s C^{-\varsigma}_{t+s} \left(\frac{P_{t+s}}{P_t}\right)^{\lambda - 1} Y_{t+s}}, \tag{12}$$

where $pm \equiv \left(\frac{\lambda}{\lambda-1}\right)$ is the price mark-up.

2.3 The Financial Sector

2.3.1 Balance Sheet and Loan Loss Reserves

Consider a perfectly-competitive bank, which raises funds through deposits (D_t) in order to supply working-capital loans (L_t) to IG firms. Moreover, the bank holds government bonds (B_t), a safe asset, yielding a gross return of R^B_t. As the loan portfolio takes into account expected loan losses, loan loss reserves (LLR_t) are subtracted from total loans, consistent with standard practice where reserves are treated as an accounting entry - contra asset (see Walter (1991) and Jiménez, Ongena, Peydró and Saurina (2017)). The bank lends to a continuum of IG firms and therefore its balance sheet in real terms is $L_t - LLR_t + B_t = D_t$, where $L_t \equiv \int_0^1 L_{j,t} dj$ is the aggregate lending to borrowing firms.

The bank must also satisfy regulation in the form of setting loan loss provisions (a flow), which are deducted from current earnings. Loan loss reserves (a stock) are assumed to be invested into a safe asset such that $LLR_t = B_t$. This ensures that loan loss reserves are a liquid asset and available to face losses (as in Bouvatier and Lepetit (2012a), Agénor and Zilberman (2015) and Agénor and Pereira da Silva (2017)). Further, in equilibrium government bonds are issued in zero net supply so we do not need to specify the evolution of loan loss reserves in order to examine the direct effects of loan loss provisioning practices on the bank's intra-period pricing decisions.[11] Using these results,

[10] Note that $\mathbb{E}_t \varepsilon_{j,t}$ is identical across all firms during the pricing decision stage that takes place at the beginning of period t, just *after* the realization of aggregate shocks and *before* the idiosyncratic shocks that occur at the very end of the period. Hence, under symmetry, the subscript j can be dropped from the marginal cost and consequently from the optimal price level derived below.

[11] In practice, *variations* in loan loss reserves are equal to the flow of provisions plus unanticipated charged-off loans (subtracted from earnings) minus charged-off loans (Walter (1991)). Due to the intra-period nature of loans and rational expectations, we do not model charged-off loans given that there is no distinction between the fraction of nonperforming loans and the fraction of charged-off loans (both of which are equal to default risk). To avoid a zero steady state value of provisions and to simplify the analysis, we assume that loan loss reserves are invested in government bonds in each period.

the bank's balance sheet boils down to:

$$L_t = D_t. \tag{13}$$

2.3.2 Loan Loss Provisions and Nonperforming Loans

In practice, regulatory loan loss provisions can be set according to two specifications: *i)* a *specific* provisioning system, where loan loss provisions are driven by contemporaneous nonperforming loans and fit identified loan losses - "point-in-time" losses ; *ii)* a prudential *dynamic* provisioning system that requires the bank to make provisions related to both current risk (specific provisions) *and* expected losses over the whole business cycle - "through-the-cycle" expected losses.

In this model, specific provisions are determined by the rule:

$$LLP_t^s = l_0 \Phi_t L_t, \tag{14}$$

whereas a dynamic prudential expected-loss provisioning rule evolves according to:

$$LLP_t^d = l_0 \Phi^{l_1} \Phi_t^{1-l_1} L_t. \tag{15}$$

In both rules, the term $l_0 > 0$ represents the steady state coverage ratio, measuring the expected average of nonperforming loans which are covered by loan loss provisions. Furthermore, l_1 denotes the degree of provisions smoothing under the dynamic system, with Φ representing the steady state latent risk of default or the long-run fraction of nonperforming loans.[12] Similar to Agénor and Zilberman (2015), the difference between the two provisioning regimes is reflected by the value of l_1.[13] For $0 < l_1 < 1$, provisions are determined by a weighted average of the "point-in-time" current losses and "through-the-cycle" expected losses. Alternatively, rewriting (15) as $LLP_t^d = l_0 \Phi_t \left(\frac{\Phi_t}{\Phi}\right)^{-l_1} L_t$ shows that whenever actual default risk is lower than its normal steady state value, the financial authority requires the bank to raise dynamic provisions in order to avoid excessive lending.[14] Setting $l_1 = 1$ produces the *fully* expected-loss provisioning rule, $LLP_t^d = l_0 \Phi L_t$, implying a *flat* loan loss provisions to loan ratio over the business cycle.

Nonetheless, the focus of this paper is to calculate and study the normative properties of Ramsey *optimal* state-contingent loan loss provisions LLP_t^{opt}, rather than to examine the impact of (sub-)optimal simple rules (which is the case in Agénor and Pereira da Silva (2017)). In the analysis below, we compare the macroeconomic effects of optimal loan loss provisions with the outcomes arising from implementing the rules described in (14) and (15).

Finally, in the long-run both rules (14) and (15) boil down to:

$$LLP = l_0 \Phi L, \tag{16}$$

and therefore obey the "through-the-cycle" expected loss regime. We assume (16) is also the steady

[12] By placing a higher weight on steady state latent risk, provisioning systems are considered, in the banking and Basel terminology, to be more "forward looking" or "dynamic". More precisely, dynamic provisions do not necessarily depend on the statistical prediction of nonperforming loans in period $t+1$. Rules are specified in order to smooth provisions made by the bank over the whole business cycle (see also Bouvatier and Lepetit (2012a)).

[13] We could also have dynamic provisions respond to a measure of the economic activity such as cyclical output, in line with the more general macroprudential approach of financial regulation. However, default and output are endogenous in this model (see equation (33) below), thus making such analysis redundant.

[14] The qualitative results of the paper remain unchanged if we instead consider a dynamic provisioning rule that is linked to deviations of expected default ($\mathbb{E}_t \Phi_{t+1}$) from its' steady state level (Φ).

state level of provisions when examining optimal policy.[15]

2.3.3 Lending Rate Decision

At the beginning of period t, the bank breaks-even from its intermediation activity, such that the expected income from lending to a continuum of IG firms minus the flow expense of loan loss provisions and other loan-related costs for the given period, is equal to the total costs of borrowing deposits from households,

$$\int_{\varepsilon_{j,t}^M}^{\bar{\varepsilon}} \left(R_t^L L_{j,t}\right) f(\varepsilon_{j,t}) d\varepsilon_{j,t} + \int_{\underline{\varepsilon}}^{\varepsilon_{j,t}^M} \left(\chi_t Y_{j,t}\right) f(\varepsilon_{j,t}) d\varepsilon_{j,t} - LLP_t^i - cL_{j,t} = R_t^D D_t, \tag{17}$$

where $f(\varepsilon_{j,t})$ is the probability density function of $\varepsilon_{j,t}$, and $i = (s, d, opt)$ an index for the imposed regulatory provisioning regime. The first element on the left-hand side is the repayment to the bank in the non-default states while the second element is the expected return to the bank in the default states. We assume the bank faces additional costs related to supplying loans given by $cL_{j,t}$, where $c > 0$ is a fixed cost parameter that may include transaction, issuance, administration, monitoring and/or industrial costs in the banking system.[16] These general lending resource-management costs ($cL_{j,t}$) are assumed to be linearly increasing with the volume of loans.

We use equations (6), (7), (9) for $\chi_t \varepsilon_{j,t}^M H_{j,t} = \kappa R_t^L L_{j,t}$, (10), (13), divide by $L_{j,t}$, and apply the characteristics of the uniform distribution in (17) to obtain the loan rate equation:[17]

$$R_t^L = \nu_t \left(R_t^D + \frac{LLP_t^i}{L_t} + c \right), \tag{18}$$

where the term $\nu_t \equiv \left[1 - \frac{(\bar{\varepsilon} - \underline{\varepsilon})}{2\varepsilon_t^M} \Phi_t^2 \right]^{-1} > 1$ is defined as the finance-risk premium, which itself is also a positive function of the lending rate (see (9) and (10)). Conditions (17) and (18) ensure zero bank profits in equilibrium.

The loan rate is affected by various components: *i)* the direct *monetary policy cost channel* associated with changes in the main policy rate; *ii)* the *finance premium channel*, which is related to the fact that the bank expects to receive back only a fraction of its loans and seize collateral in case of default. The bank internalizes the positive risk of default and consequently charges a higher loan rate; *iii)* the *provisioning cost channel* - a higher loan loss provisions to loan ratio lowers the profitability of the bank and thus requires an increase in the loan rate for the break-even condition to be satisfied. This can explain procyclicality in the credit markets generated by specific provisioning systems (see also Bouvatier and Lepetit (2012a, 2012b)). Moreover, through

[15] In line with Angelini, Neri and Panetta (2014) and Rubio and Yao (2020), for example, our initial approach towards loan loss provisions (regulatory and accounting tool) is *positive*. That is, empirically-relevant specific provisions initially exist because government regulation requires it (see IAS 39 or the more recent IFRS 9 in practice). The question we ask therefore is: in the presence of regulatory loan loss provisions, which macroprudential-type optimal policies can achieve the highest welfare and alleviate liquidity traps? Unlike these models, nevertheless, our focus is on fully optimal provisions (rather than optimal simple regulatory bank capital and LTV rules).

[16] Dia and VanHoose (2017) and Dia and Menna (2016) find that banking sector industrial costs have a significant impact on the loan pricing decision in both normal times and in times of distress. The introduction of c does not alter the dynamics of the model, but helps to easily target a more empirically-relevant steady state loan rate.

[17] The subscript j is dropped given that employed labour hours and demand-driven loans are the same for all borrowing firms.

an internal propagation mechanism, the higher borrowing costs amplify the rise in credit risk and produce further procyclicality in financial variables (see equations (9), (10) and (18)). More precisely, increasing provisions may result in higher financial risk, supporting empirical findings by Jin, Kanagaretnam and Lobo (2011).

2.4 Public Authority

The public authority sets the required level of loan loss provisions according to a specific, dynamic or an optimal regime. In addition, the social planner operates under optimal monetary policy commitment, and targets the short-term policy rate that respects the model-implied ZLB constraint,

$$R_t^D \geq 1. \tag{19}$$

In the analysis below, we also examine the macroeconomic effects of a standard Taylor (1993) interest rate rule in this environment. Our choice of optimal monetary commitment used as the policy benchmark is motivated by the extensive implementation of various forward guidance measures by several central banks in recent years, including the Federal Reserve since 2003.[18] Given that optimal commitment falls under the umbrella of more generalized forward guidance policies, such strategy is arguably a good approximation of contemporary monetary practice.

2.5 Market Clearing

The market clearing condition in the production sector requires $Y_t \int_0^1 \left(\frac{P_{j,t}}{P_t}\right)^{-\lambda} dj = H_t \int_0^1 \varepsilon_{j,t} dj$, with $H_t = H_{j,t}$ in a symmetric equilibrium. Using the distribution properties of the idiosyncratic shocks, which satisfy $\int_0^1 \varepsilon_{j,t} dj = 1$ and have a mean of unity, we obtain:

$$Y_t \Delta_t = H_t, \tag{20}$$

with $\Delta_t \equiv \int_0^1 \left(\frac{P_{j,t}}{P_t}\right)^{-\lambda} dj$ defined as the usual price dispersion index. From (11), the real marginal cost faced by IG firms is:

$$mc_t = \left[1 + \kappa \left(R_t^L - 1\right)\right] W_t. \tag{21}$$

The equilibrium condition in the market for loans is derived from (7) and (13) and given by:

$$L_t = \kappa W_t H_t = D_t. \tag{22}$$

In line with the cost channel literature that feature a CIA constraint, we assume the end-of-period lump-sum cash injection from the public authority is $T_t = M_{t+1} \frac{P_{t+1}}{P_t} - M_t$. Following the financial intermediation process, the public authority receives $LLP_t^i + cL_t = J_t^{FI}$, which is paid back to households in a lump-sum fashion. In a symmetric aggregate equilibrium, we substitute all profits from the production sector, total profits from the financial intermediation process, the equilibrium condition in the credit market (22), and the lump-sum cash injection in (3) to obtain the goods market clearing condition:

$$Y_t = C_t. \tag{23}$$

[18] See also Bilbiie (2019) and Campbell, Fisher, Justiniano, and Melosi (2017) for a review.

Finally, substitute (20) and (23) in (5) to retrieve the equilibrium real wage:

$$W_t = \Delta_t^{\gamma} Y_t^{\varsigma+\gamma}. \tag{24}$$

3 Steady State and Aggregate Equilibrium Dynamics

To solve the model, we log-linearize the behavioral equations and market clearing conditions around the non-stochastic, zero inflation ($\pi = 1$) steady state. We start by calculating the long-run equilibrium. The steady state marginal cost is derived from (12) and (21) and is given by:

$$mc = \left[1 + \kappa\left(R^L - 1\right)\right] W = \frac{(\lambda - 1)}{\lambda} \equiv (pm)^{-1}. \tag{25}$$

From (24) and the definition of Δ we have:

$$W = Y^{\varsigma+\gamma}. \tag{26}$$

Plugging (26) in (25) gives the long-run output equation,

$$Y = \left[\frac{(pm)^{-1}}{1 + \kappa\left(R^L - 1\right)}\right]^{\frac{1}{\varsigma+\gamma}}, \tag{27}$$

as well as the steady state loan to GDP ratio after employing (20) and (22),

$$\frac{L}{Y} = \frac{\kappa W H}{Y} = \kappa Y^{\varsigma+\gamma} = \frac{\kappa (pm)^{-1}}{1 + \kappa\left(R^L - 1\right)}. \tag{28}$$

To derive the average values of the financial variables unique to this model, we first substitute (25), (26) and (27) in the long-run version of (9) to obtain:

$$\varepsilon^M = \left[\frac{\kappa R^L}{1 + \kappa\left(R^L - 1\right)}\right] (pm)^{-1} \chi^{-1}. \tag{29}$$

Combining (29) with (10) produces the long-run value of default risk:

$$\Phi = \frac{\left[\frac{\kappa R^L}{1+\kappa\left(R^L-1\right)}\right] (pm)^{-1} \chi^{-1} - \underline{\varepsilon}}{\bar{\varepsilon} - \underline{\varepsilon}}. \tag{30}$$

Finally, substituting (30), $R^D = \beta^{-1}$ (from (4)), and $LLP/L = l_0\Phi$ (from (16)) all in (18) gives the long-run loan rate:

$$R^L = \nu\left(\beta^{-1} + l_0\Phi + c\right), \tag{31}$$

where $\nu \equiv \left[1 - \frac{(\bar{\varepsilon}-\underline{\varepsilon})}{2\varepsilon^M}\Phi^2\right]^{-1}$ is the average risk premium.

In the steady state, the loan loss provisions to loan ratio ($l_0\Phi$) acts similarly to a tax on the banking sector, raising the cost of borrowing and hence reducing output and the loan to GDP ratio. From a normative perspective, the optimal coverage ratio should be negative as it serves as a financial subsidy that lowers R^L and brings output closer to its long-run first-best level ($Y = 1$).

However, as shown below, setting state-contingent provisions to counter various shocks provides an extra degree of freedom to the policy maker in the short-run, and could be a very powerful stabilization tool irrespective of the conduct of monetary policy. To avoid the use of a long-run financial subsidy, and given that our focus is on the *short-run* normative implications of loan loss provisions, in the welfare analysis we assume instead a long-run labour subsidy that eliminates both monopolistic and cost channel distortions in (27).

We now use the steady state relationships derived above and define the log-linear deviations of each variable X_t from its respective steady state level X as $\hat{X}_t$. Log-linearizing equations (9), (10) and (24) yields:

$$\hat{\varepsilon}_t^M = R_t^L + (\varsigma + \gamma)\,\widehat{Y}_t - \hat{\chi}_t, \tag{32}$$

$$\hat{\Phi}_t = \left(\frac{\varepsilon^M}{\varepsilon^M - \underline{\varepsilon}}\right)\left[\hat{R}_t^L + (\varsigma + \gamma)\,\widehat{Y}_t - \hat{\chi}_t\right]. \tag{33}$$

By log-linearizing (18), and using (32) and (33), the loan rate is approximated by:

$$\hat{R}_t^L = \Lambda_1\left[\beta^{-1}\hat{R}_t^D + l_0\Phi\widehat{llp}_t^i\right] + \Lambda_2\left[(\varsigma + \gamma)\,\hat{Y}_t - \hat{\chi}_t\right]. \tag{34}$$

The constants $\Lambda_1 \equiv \frac{\left[2\varepsilon^M(\bar{\varepsilon} - \underline{\varepsilon}) - \left(\varepsilon^M - \underline{\varepsilon}\right)^2\right]}{2\varepsilon^M(\bar{\varepsilon} - \varepsilon^M)R^L} > 0$ and $\Lambda_2 \equiv \frac{\left(\varepsilon^M + \underline{\varepsilon}\right)\left(\varepsilon^M - \underline{\varepsilon}\right)}{2\varepsilon^M(\bar{\varepsilon} - \varepsilon^M)} > 0$ reflect the degree of financial market imperfections given their relation with ε^M and χ, that, in turn, govern the values of the steady state financial variables related to default risk. The term $\widehat{llp}_t^i \equiv \widehat{LLP}_t^i - \widehat{L}_t$ is the log-linearized loan loss provisions to loan ratio determined by the type of provisioning regime in place: specific, dynamic or optimal ($i = s, d, opt$). The log-linearized specific and dynamic provisioning rules are derived from (14) and (15) and given by:

$$\widehat{llp}_t^s = \hat{\Phi}_t, \tag{35}$$

$$\widehat{llp}_t^d = (1 - l_1)\,\hat{\Phi}_t. \tag{36}$$

The log-linearized loan to GDP ratio is obtained from (20), (22) and (24) and evolves according to:

$$\hat{L}_t - \hat{Y}_t = (\varsigma + \gamma)\,\hat{Y}_t. \tag{37}$$

From (33), (34) and (37), credit risk and the loan rate are positive functions of the loan to GDP ratio. Thus, our model provides micro-foundations to why the credit to GDP ratio may be a useful indicator for measuring credit risk inherent in escalating volumes of lending. In normal times, and in the absence of shocks, higher productivity is linked with increasing real marginal costs, higher levels of debt and leverage, and consequently elevated borrowing costs (as proxied by the finance premium). Our approach to modeling financial frictions in a simple and stylized New Keynesian model contributes to Cúrdia and Woodford (2016), for example, who posit a reduced-form nondecreasing relationship between debt and spreads. The analytical tractability of our model is not hampered by the introduction of working-capital loans and the endogenous credit spread.

Finally, using the log-linearized equations derived above, and approximating (4) and (12) allows to express the entire model in terms of inflation and output. Specifically, the New Keynesian model with financial risk and loan loss provisions can be condensed to the following system:

$$\widehat{\pi}_t = \beta\mathbb{E}_t\widehat{\pi}_{t+1} + k_p\left\{(\varsigma + \gamma)\,(1 + \delta\Lambda_2)\,\hat{Y}_t + \delta\left[\Lambda_1\left(\beta^{-1}\hat{R}_t^D + l_0\Phi\widehat{llp}_t^i\right) - \Lambda_2\hat{\chi}_t\right]\right\}, \tag{38}$$

$$\widehat{Y}_t = \mathbb{E}_t \widehat{Y}_{t+1} - \varsigma^{-1} \left(\hat{R}_t^D - \mathbb{E}_t \widehat{\pi}_{t+1} - \hat{r}_t^e \right), \tag{39}$$

with $\delta \equiv \frac{\kappa R^L}{(1+\kappa(R^L-1))}$, $k_p \equiv \frac{(1-\omega)(1-\omega\beta)}{\omega}$, and $\hat{r}_t^e \equiv \hat{\vartheta}_t - \mathbb{E}_t \hat{\vartheta}_{t+1}$ defined as the natural rate of interest that is a function of only the demand shock. The stochastic process for $\hat{R}_t^D, \widehat{llp}_t^i$ and the $AR(1)$ processes for the financial and demand shocks close the model.

Equation (38) is the extended NKPC establishing the short-run AS relation between inflation and output, augmented for the financial shock, $\hat{\chi}_t$, the loan loss provisions to loan ratio, $\widehat{llp}_t^i$, and the degree of financial frictions, Λ_1 and Λ_2.[19] In this setup, both the loan loss provisions to loan ratio and the micro-founded structural financial shock enter as cost-push components in the NKPC, and do not alter the efficient output level.[20] Equation (39) is the Euler equation that determines the AD schedule.

A novel aspect of this model is that the loan rate is driven primarily by provisioning practices as well as the elements of the marginal cost. Thus, loan loss provisions provide an additional channel through which the policy maker can alter borrowing costs and economic activity by directly targeting inflation, without relying on the standard demand and cost channels of monetary policy. This provisioning cost channel operates through the wider credit cost channel linking borrowing costs to inflation and output. To illustrate, higher external financing costs experienced by borrowing firms lead to a rise in inflation, yet also exert downward pressures on the economic activity by shrinking the real wage, labour supply and consequently GDP. Lowering loan loss provisions, in turn, can reduce these economic inefficiencies by directly targeting the lending rate. Compared to Ravenna and Walsh (2006), the loan rate term (the source of the credit cost channel) in (34) is driven largely by the factors determining credit risk ($\hat{\Phi}_t$) and provisioning policies ($\widehat{llp}_t^i$) rather than merely variations in $\hat{R}_t^D$. This general equilibrium framework allows us to study how different accounting systems for loan loss provisions translate into real macroeconomic effects.

4 Parameterization and Solution Strategy

The baseline parameterization used to simulate the model is summarized in Table 1. Parameters that characterize tastes, preferences, price stickiness, elasticities, price mark-ups and technology are all standard in the New Keynesian literature. In what follows, we calibrate the parameters that are unique to this setup in order to approximately match some of the model's steady state financial variables with their U.S. counterparts between the period 2002Q1 to 2020Q3. We consider this particular period to assess the potency of supply-side provisioning policies in a low interest rate environment that has characterized the U.S. economy (and the Eurozone) over the past two decades, and also more recently in light of the Covid-19 economic crisis.

[19] Without the financial friction ($\Lambda_1 = \Lambda_2 = 0$), the cost-push financial shock *disappears* from the model. Indeed, setting $\delta > 0$ together with $\Lambda_1 > 0$ and $\Lambda_2 > 0$ gives rise to the risk-adjusted credit cost channel and the inflationary cost-push shock in this framework.

[20] Without Total Factory Productivity (TFP) shocks, the efficient level of output is set to unity, implying that cyclical output is equal to the output gap.

Table 1: Parameter Values

Parameter	Value	Description
β	0.998	Discount factor
ς	1.00	Inverse of elasticity of intertemporal substitution
γ	0.20	Inverse of the Frisch elasticity of labour supply
pm	1.20	Price mark-up
ω	0.70	Degree of price stickiness
κ	0.515	Fraction of wage bill financed in advance
$\bar{\varepsilon}^F$	1.57	Idiosyncratic productivity shock upper range
$\underline{\varepsilon}^F$	0.43	Idiosyncratic productivity shock lower range
χ	0.99	Average fraction of collateral seized in default states
l_0	0.40	Loan loss provisions coverage ratio
c	0.009	Loan-related management costs
$s.d(\alpha^\chi)$	0.10	Standard deviation - financial shock
ρ_χ	0.90	Degree of persistence - financial shock
$s.d(\alpha^\vartheta)$	0.012	Standard deviation - demand shock
ρ_ϑ	0.70	Degree of persistence - demand shock

From (28)-(31), all steady state financial variables are endogenous with respect to one another and don't have a closed-form solution unless $\kappa = 1$. Hence, this small-scale framework requires a careful parameterization that can achieve a close match to the data of some important variables as explained below, but not an exact one due to the endogenous relationship between Φ, L/Y, R^L and their dependence on R^D.[21]

Jointly setting $\beta = 0.998$, $(\underline{\varepsilon}, \bar{\varepsilon}) = (0.43, 1.57)$, $\chi = 0.99$, $\kappa = 0.515$, $c = 0.009$, and $l_0 = 0.40$, implies a long-run annualized fraction of nonperforming loans of 2.20% (vs. 2.10% in the U.S. data), and a loan loss provisions to loan ratio of 0.88% (vs. around 0.80% in the data).[22] The per annum loan rate for our parameterization is 5.28%, which, together with an annualized policy rate of 0.80%, generates a loan-deposit rate spread value of 4.48%. This borrowing cost spread is comparable with the U.S. data over the sample period wherein the average credit spread between Moody's seasoned BAA corporate bond minus the Federal Funds Rate was roughly 4.40%.[23] Finally, the long-run model-implied credit to GDP ratio is 49.91%, which exceeds the 44.11% average non-financial corporate business loans to GDP ratio in the sample period, but representative of the average ratio over the past two years.[24]

As for the main shocks examined in our paper, we fix the persistence parameters of the financial and the demand shocks to $\rho_\chi = 0.90$ and $\rho_\vartheta = 0.70$, while the standard deviations associated with these shocks are $s.d(\alpha^\chi) = 0.10$ and $s.d(\alpha^\vartheta) = 0.012$, respectively. These numbers are within range

[21] Nevertheless, our results are qualitatively robust to alternative parameterizations.

[22] The median of annualized provisions across 70 large internationally active banks rose from 35 basis points in the second half of 2019 to 105 basis points in the first half of 2020 (see also De Araujo, Cohen and Pogliani (2021)). Taking into account financial distress periods in which provisions tend to substantially increase, the steady state loan loss provisions to loan ratio in our model is consistent with the roughly 20 year long-run average.

[23] Without provisions, we would need to re-calibrate the structural parameters of the model such as χ, κ, c and the range $\left(\varepsilon^F, \bar{\varepsilon}^F\right)$ to bring the cut-off point (ε^M), default risk (Φ) and the loan rate (R^L) to the values currently used in the paper. Having provisions in the baseline framework improves the model's ability to capture more empirically-relevant steady state relationships and interest rate spreads.

[24] All the long-run averages are retrieved from the Federal Reserve Economic Data (FRED) data produced by the St. Louis Fed, as well as from the World Bank database and Bloomberg.

of the calibrated values found in previous studies by Adam and Billi (2006), Christiano, Motto and Rostagno (2014), and Benes and Kumhof (2015). To quantitatively solve the model with an occasionally binding ZLB constraint, we implement the piecewise-linear methodology developed in Guerrieri and Iacoviello (2015), and confirm the results with Holden's (2016) DynareOBC algorithm.

5 Optimal Provisioning and Welfare

In this section, we calculate optimal monetary and financial policies in response to inflationary financial shocks and deflationary demand shocks, and compare the macroeconomic effects of $\widehat{llp}_t^{opt}$ with specific and dynamic provisioning regimes. The policy maker's objective function is given by a second-order approximation of the household's ex-ante expected utility,[25]

$$\mathbb{E}_t \sum_{t=0}^{\infty} \beta^t U_t \approx U - \frac{1}{2} U_C C \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t \left[\left(\frac{\lambda}{\kappa_p} \right) \widehat{\pi}_t^2 + (\varsigma + \gamma) \widehat{Y}_t^2 \right]. \tag{40}$$

We measure the welfare gain of a particular policy i as a fraction of the consumption path under the benchmark case (denoted by I) that must be given up in order to obtain the benefits of welfare associated with policy i; $\mathbb{E}_t \sum_{t=0}^{\infty} \beta^t U_t \left(C_t^i, H_t^i \right) = \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t U_t \left((1-\Lambda) C_t^I, H_t^I \right)$, where Λ is a measure of welfare gain in units of steady state consumption. Given the utility function adopted and with $\varsigma = 1$, the expression for the consumption equivalent (Λ) in percentage terms is:

$$\Lambda = \left\{ 1 - \exp \left[(1-\beta) \left(\mathbb{W}_t^i - \mathbb{W}_t^I \right) \right] \right\} \times 100, \tag{41}$$

with $\mathbb{W}_t^i = \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t U_t \left(C_t^i, H_t^i \right)$ representing the unconditional expectation of lifetime utility under policy i, and $\mathbb{W}_t^I = \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t U_t \left(C_t^I, H_t^I \right)$ the welfare associated with the benchmark policy I. A higher positive Λ implies a larger welfare gain and hence indicates that the policy is more desirable from a welfare perspective.

5.1 Optimal Policy at the ZLB

This section determines the optimal Ramsey plans when the public authority can implement state-contingent loan loss provisions in face of occasional liquidity traps. The benevolent policy maker chooses state-contingent paths for inflation, output, the nominal interest rate and loan loss provisions to maximize its objective function (40) subject to the equilibrium constraints ((38) and (39)), and the ZLB. The associated Lagrangian is:

$$\mathfrak{L}_t = \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t \left\{ \begin{array}{c} -\frac{1}{2} \left[\left(\frac{\lambda}{\kappa_p} \right) \widehat{\pi}_t^2 + (1+\gamma) \widehat{Y}_t^2 \right] - \\ -\hat{\zeta}_{1,t} \left[\widehat{\pi}_t - \beta \mathbb{E}_t \widehat{\pi}_{t+1} - k_p \left[(1+\gamma)(1+\delta\Lambda_2) \hat{Y}_t + \delta \left(\Lambda_1 \left(\beta^{-1} \hat{R}_t^D + \Phi \widehat{llp}_t \right) - \Lambda_2 \hat{\chi}_t \right) \right] \right] - \\ -\hat{\zeta}_{2,t} \left[\hat{Y}_t - \mathbb{E}_t \hat{Y}_{t+1} + \hat{R}_t^D - \mathbb{E}_t \widehat{\pi}_{t+1} - \hat{r}_t^e \right] - \hat{\zeta}_{3,t} \left[-\hat{R}_t^D + \ln \beta \right] \end{array} \right\},$$

[25] See online Appendix A for the full derivation.

where $\hat{R}_t^D \geq \ln \beta$ represents the zero bound restriction expressed in log-deviations. Moreover, $\hat{\zeta}_{1,t}, \hat{\zeta}_{2,t}$ and $\hat{\zeta}_{3,t}$ are the Lagrange multipliers on the model constraints. Under commitment and treating the shocks $\hat{r}_t^e$ and $\hat{\chi}_t$ as given, the resulting first-order conditions read:

$$-\left(\frac{\lambda}{\kappa_p}\right)\widehat{\pi}_t - \hat{\zeta}_{1,t} + \hat{\zeta}_{1,t-1} + \beta^{-1}\hat{\zeta}_{2,t-1} = 0, \tag{42}$$

$$-(1+\gamma)\hat{Y}_t + k_p(1+\gamma)(1+\delta\Lambda_2)\hat{\zeta}_{1,t} - \hat{\zeta}_{2,t} + \beta^{-1}\hat{\zeta}_{2,t-1} = 0, \tag{43}$$

$$k_p\delta\Lambda_1\beta^{-1}\hat{\zeta}_{1,t} - \hat{\zeta}_{2,t} + \hat{\zeta}_{3,t} = 0, \tag{44}$$

$$\hat{\zeta}_{1,t} = 0. \tag{45}$$

The complementary slackness condition is:

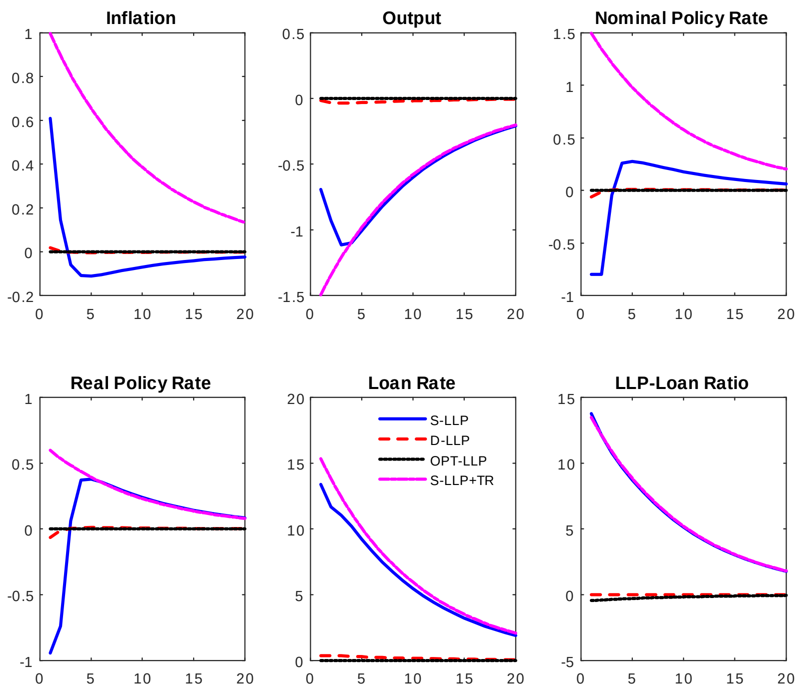
$$\hat{\zeta}_{3,t}\left(-\hat{R}_t^D + \ln\beta\right) = 0, \quad \hat{\zeta}_{3,t} \geq 0, \tag{46}$$

where the initial conditions satisfy $\hat{\zeta}_{1,-1} = \hat{\zeta}_{2,-1} = \hat{\zeta}_{3,-1} = 0$. The optimal state-contingent evolution of the endogenous variables $\left\{\widehat{\pi}_t, \hat{Y}_t, \hat{R}_t^D, \widehat{llp}_t\right\}$ is then characterized by the above conditions together with constraints (38) and (39). Under commitment, optimal policy becomes history-dependent as reflected by the lagged Lagrange multipliers in (42) and (43). These additional state variables reflect "promises" that must be kept from past commitments.

5.1.1 Financial Shocks

Figure 1 displays the dynamic responses of key variables to an adverse financial shock stemming from a one standard deviation negative shock to $\hat{\chi}_t$. We compare between the following regimes: *i)* Optimal Ramsey monetary policy with specific loan loss provisions (labeled 'S-LLP'); *ii)* Optimal Ramsey monetary policy with dynamic loan loss provisions (labeled 'D-LLP'); and *iii)* Joint optimal monetary and provisioning policies under commitment (labeled 'OPT-LLP'). We plot the above scenarios also against the case where the policy maker implements a standard Taylor interest rate rule $\hat{R}_t^D = \max(\ln\beta, \phi_\pi\widehat{\pi}_t)$, with $\phi_\pi = 1.5$, alongside specific provisions (labeled 'S-LLP+TR').

Figure 1: Adverse Financial Shock with various Provisioning and Monetary Policies



Note: Interest rates, inflation rate and the LLP-loan ratio are measured in annualized percentage point deviations from steady state. The rest of the variables are measured in terms of annualized percentage deviations.

A negative financial shock translates into an steep rise in credit risk and consequently in the loan rate through the finance premium effect. The hike in the lending rate, coupled with the rise in risk, raises the marginal cost and increases price inflation. The downward pressure on the real wage instigated by the jump in inflation discourages both labour supply and the demand for consumption goods, leading to a deterioration in output. Thus, financial shocks give rise to a trade-off between inflation and output stabilization in this model.[26] Compared to the situation where provisions are absent, the increase in default risk and the corresponding incurred-loss provisioning policy accelerate the escalation in the lending rate due to the credit intermediation costs involved in raising provisions. In fact, given our parameterization, risk and provisions, which are linked through the rule (35), explain most of the loan rate fluctuations as also seen from (34). The cost-push nature of specific provisions, feeding through the provisioning and credit cost channels, exacerbates the surge in inflation, and intensifies the decline in output and working-capital loans.

As a result of the inefficient contraction in GDP and *in spite* of the rise in inflation, the commitment policy maker *cuts* the nominal policy rate to its ZLB (see 'S-LLP' impulse response functions in Figure 1). The accommodative monetary policy helps to smooth the adjustment of

[26] See also Gilchrist, Schoenle, Sim and Zakrajšek (2017), who show that adverse financial shocks may produce a rise in *aggregate* inflation.

output at the expense of short-lived inflationary pressures. At the same time, such demand-pull inflation is mitigated by the direct monetary policy cost channel in which the fall in $\hat{R}_t^D$ contains part of the initial spike in $\widehat{\pi}_t$. As output starts recovering from period 5, the forward-looking social planner promises to generate mild future deflation, which helps to further alleviate the immediate cost-push repercussions in the first few periods. Committing to future deflation in this case acts a substitute for interest rate cuts. Overall, our model explains how the ZLB policy is an optimal commitment plan also against inflationary supply-side financial shocks that are amplified by the enactment of inefficient specific provisioning policies which were officially in place until January 2018.

The negative effects of specific provisions become even more pronounced when the policy maker implements a standard interest rate rule instead of optimal monetary commitment. With $\phi_\pi > 1$, the higher price level resulting from the intensity of the provisioning cost channel lead to a *hike* in the policy rate. The contractionary monetary policy renders higher borrowing costs, which, in combination, bring about a further increase in inflation and an amplified fall in output. For a specified Taylor rule, the ZLB constraint therefore does *not* bind following a recessionary cost-push disturbance, preventing the apparatus of the otherwise more expansionary optimal monetary commitment stance. The welfare loss from implementing policy 'S-LLP+TR' relative to 'S-LLP' is of -0.09% of permanent consumption.

The responses of the key variables under the 'D-LLP' policy are examined under the assumption of a fully expected-loss provisioning practice, where $l_1 = 1$ and $\widehat{llp}_t^d = 0$. Dynamic provisions that follow a simple latent risk rule deliver near-complete output and inflation stabilization. The relative fall in the lending rate attributed to the weaker provisioning cost channel reduces volatility in nominal, real and financial variables through the credit cost channel, and leads to a welfare gain of 0.08% relative to the 'S-LLP' regime. This result highlights the importance of smoothing the loan loss provisions to loan ratio based on the expected-loss approach, broadly consistent with the IFRS 9 as well as with the Spanish dynamic provisioning system (see also Jiménez and Saurina (2006) and Jiménez, Ongena, Peydró and Saurina (2017)).

It is important to note that despite the evolution of international accounting regulatory standards from specific to expected-loss dynamic provisioning systems over the past few years, banks still have great discretion and flexibility when setting provisions for loan losses. In response to the Covid-19 crisis, banks have significantly revised upwards their loan loss provisions due to the sharp rise in nonperforming loans (see De Araujo, Cohen and Pogliani (2021)). Such bank-specific discretionary policies that don't necessarily follow the historical average dynamic *smoothing* approach of loan loss provisions would also produce a higher credit spread, similar to the case of specific provisions.[27] Either way, the conclusions above hint that incurred-loss provisioning practices or an abrupt rise in provisions in times of financial distress may keep borrowing costs excessively high, and as a result perhaps partly contribute to the "missing deflation" phenomenon.

We now turn to study the optimal dynamics of loan loss provisions. As can be inferred from Figure 1, the welfare maximizing provisioning policy ('OPT-LLP') calls for some *excess* smoothing of loan loss provisions relative to loans. A decline in provisions of around 0.4 percentage points offsets the immediate procyclical impact of risk on loan loss provisions and the loan rate instigated

[27] We leave the analysis of moving from a low to a *permanently* high steady state default risk and provisions for future research. Such long-run changes in expected nonperforming loans may also reflect the recent changes in banks provisioning practices in light of Covid-19. In our model and in line with most of the literature, all variables converge back to their original steady states once the shocks dissipate.

by the specific provisioning system and the financial shock. While dynamic provisions that follow a simple latent historical average risk rule deliver near-complete output and inflation stabilization, a fully optimal rule can entirely eliminate the welfare costs inherent in the inflationary financial shock and the credit market imperfections. Access to optimal loan loss provisions adds the first-order condition $\hat{\zeta}_{1,t} = 0$, which removes the constraint imposed by the AS schedule, and delivers a welfare benefit of 0.11% relative to the 'S-LLP' policy. Using a grid-search, we calculate the optimal dynamic provisions parameter in (36) that mimics the dynamics of optimal provisions, and find that $l_1^{opt} = 1.0358$ achieves $\widehat{\pi}_t = \widehat{Y}_t = 0$ for all t following the financial shock. That is, without restricting the value of l_1, the optimal dynamic provisioning rule involves excess smoothing in the sense that provisions react more than proportionally to steady state deviations in default risk. Optimal policy conducted either via a simple excess smoothing-type of rule or a direct reaction to the size of the financial shock warrants a fall in provisioning requirements and *no* change in the nominal interest rate.

To further highlight the irrelevance of the monetary policy standpoint when provisions are set optimally following financial shocks, consider a situation where the interest rate is pegged, $\hat{R}_t^D = 0$, and no other shocks, $\hat{r}_t^e = 0$. From (38), it is clear there will be a state-contingent adjustment in loan loss provisions,

$$\widehat{llp}_t^{opt} = \frac{\Lambda_2}{\Lambda_1 l_0 \Phi} \hat{\chi}_t, \tag{47}$$

that will satisfy $\widehat{\pi}_t = \mathbb{E}_t \widehat{\pi}_{t+1} = 0$ and $\widehat{Y}_t = \mathbb{E}_t \widehat{Y}_{t+1} = 0$ in (38) and (39). When the policy rate is instead $\hat{R}_t^D = \phi_\pi \widehat{\pi}_t$, we solve the model using the method of undetermined coefficients to obtain closed-form solutions for inflation and output:

$$\widehat{\pi}_t = \Upsilon_\chi \delta \left(\Lambda_1 l_0 \Phi \widehat{llp}_t^i - \Lambda_2 \hat{\chi}_t \right), \tag{48}$$

$$\widehat{Y}_t = -\frac{\left(\phi_\pi - \rho_\chi\right)}{\left(1 - \rho_\chi\right)} \Upsilon_\chi \delta \left(\Lambda_1 l_0 \Phi \widehat{llp}_t^i - \Lambda_2 \hat{\chi}_t \right), \tag{49}$$

where,

$$\Upsilon_\chi \equiv \frac{k_p \left(1 - \rho_\chi\right)}{\left\{ \left(1 - \beta \rho_\chi\right) \left(1 - \rho_\chi\right) + k_p \left[(\varsigma + \gamma)(1 + \delta \Lambda_2) \left(\phi_\pi - \rho_\chi\right) - \left(1 - \rho_\chi\right) \delta \Lambda_1 \beta^{-1} \phi_\pi \right] \right\}} > 0.$$

The optimal first-best policy that yields $\widehat{\pi}_t = \widehat{Y}_t = 0$ for all t in (48) and (49) is *then* still determined by (47), that also corresponds with the dynamics displayed under the 'OPT-LLP' regime in Figure 1. A provisioning system that follows (47), or an excess smoothing simple rule with $\widehat{llp}_t^{d,opt} = -0.0358 \hat{\Phi}_t$, completely counteracts and in fact insulates the real economy from the negative consequences of the cost-push financial shock. Indeed, zero welfare losses are attained irrespective of the value of ϕ_π, and/or any other form of costly and time-inconsistent monetary policy commitments.[28] In this way, loan loss provisions accomplish the goals of monetary policy without escalating the trade-offs between output and inflation that arise from financial shocks and the monetary policy cost channel. Optimal provisions stand out as a natural financial instrument that can deal with the source of credit distortions and volatile spreads, and that can execute the first-best outcome regardless of whether the economy is in a supply-side-driven liquidity trap or

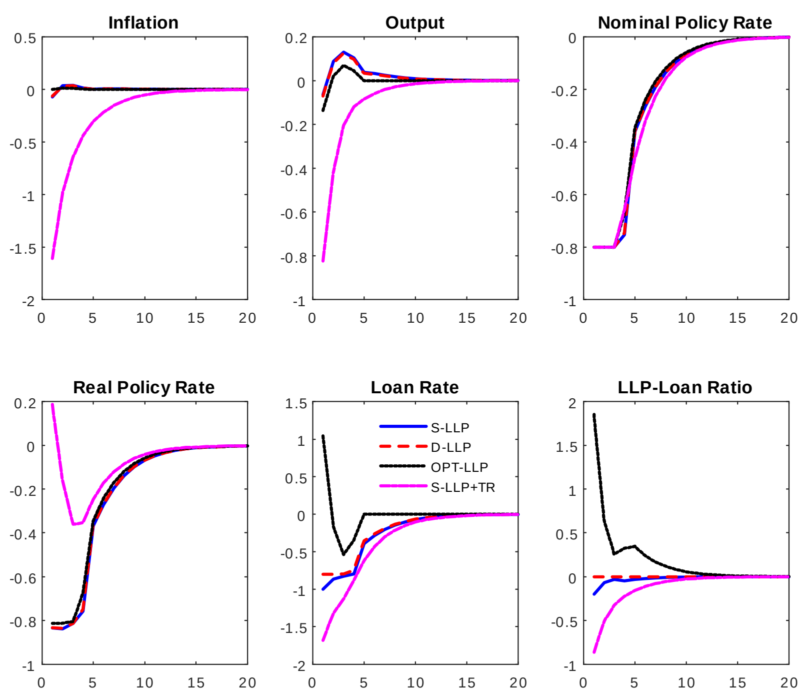
[28] We only require $\phi_\pi > 1$ to ensure determinacy in the case of a Taylor rule.

not.

5.1.2 Demand Shocks

Figure 2 presents the optimal responses of the key variables of the model to a one standard deviation negative demand shock to $\hat{\vartheta}_t$. As before, we compare between the following scenarios: *i)* Optimal Ramsey monetary policy with specific loan loss provisions (labeled 'S-LLP'); *ii)* Optimal Ramsey monetary policy with dynamic loan loss provisions (labeled 'D-LLP'); *iii)* Joint optimal monetary and provisioning policies under commitment (labeled 'OPT-LLP'); and *iv)* A simple Taylor rule $\hat{R}_t^D = \max\left(\ln\beta, \phi_\pi \widehat{\pi}_t\right)$, and specific provisions (labeled 'S-LLP+TR').

Figure 2: Adverse Demand Shock with various Provisioning and Monetary Policies



Note: Interest rates, inflation rate and the LLP-loan ratio are measured in annualized percentage point deviations from steady state. The rest of the variables are measured in terms of annualized percentage deviations.

Starting from the examination of a Taylor rule and specific provisions, a large negative natural rate of interest drives inflation and output below their steady state levels, thereby pushing the nominal policy rate to zero. The policy rate is kept at its ZLB for 3 periods. As the marginal cost declines with $\hat{Y}_t$, the risk premium and the credit spread are lowered upon the impact of the shock. The fall in risk, in turn, reduces the required level of specific provisions such that the deflationary effect of the shock is magnified via the provisioning cost channel.[29] These supply-side-driven deflationary pressures in a ZLB environment lift the real interest rate and deepen the demand-side-driven recession.

With optimal monetary commitment and specific provisions, the public authority extends the ZLB policy to 4 periods, with the intention to dampen the financial-side deflationary outcome stemming from the specific provisioning regime. The expansionary monetary policy limits the fall in aggregate demand and induces a gradual and persistent economic expansion from the third period. The added stimulus to the system generated by the promise to keep $\widehat{\pi}_t$ and $\hat{Y}_t$ positive even after the economy escapes the liquidity trap facilitates better stabilization of these variables in the first few periods through the expectations channel. For the *same* parametrization and moments employed in our framework, one can show that the 4 period duration at the ZLB under monetary commitment is longer than the optimal commitment ZLB time spell in the benchmark New Keynesian model without financial frictions, provisions and a cost channel (e.g. Adam and Billi (2006)). Intuitively, the deflationary by-product induced by the provisioning cost channel justifies a more persistent inflationary output boom in relation to the standard New Keynesian framework. The welfare gain from the 'S-LLP' regime relative to 'S-LLP-TR' system is 0.101%.

Optimal provisioning policy calls for an *increase* in the loan loss provisions to loan ratio in order to counteract the deflationary effects of the adverse demand shock. In this way, higher loan loss provisions serve to increase the loan rate, which via the credit cost channel, mitigate the fall in inflation. On the one hand, the relative rise in $\widehat{\pi}_t$ in a demand-side-driven liquidity trap lowers the real interest rate and raises $\widehat{Y}_t$. On the other hand, the drop in the real wage instigated by rising inflationary pressures reduces the labour supply and provokes a decline in output. In our model, the latter channel dominates such that the inflationary financial policy makes the short-run decline in output slightly more severe compared to the optimal monetary regime with specific provisions. From a normative perspective, the 'OPT-LLP' policy unambiguously minimizes the asymptotic standard deviations in inflation, and provides an overall welfare gain of 0.112% compared to the 'S-LLP-TR' regime, or 0.011% relative to the 'S-LLP' outcome. The improvement in inflation variability enables a more moderated output expansion from the third period, thereby contributing further to overall lifetime welfare.

A provisioning system with $LLP_t^d/L_t = l_0\Phi$ or $\widehat{llp}_t^d = 0$ yields a negligible welfare gain of 0.002% in relation to the optimal monetary policy plan accompanied with an IAS 39 provisioning system ('S-LLP'). Such prudential policy marginally limits the decline in provisions and consequently in prices. Nonetheless, this full-smoothing provisioning regime is still inadequate to overcome the substantial deflationary impact of the shock and the welfare-detrimental effects that arise. Therefore, the interaction of the ZLB and the provisioning cost channel can reverse the implications of a prudential financial policy that in more 'normal' circumstances or following financial shocks would lower the inflation rate and expand economic activity. Relative to the optimal provisioning regime, dynamic provisions contribute to the *paradox of toil* by amplifying the fall in prices and reducing overall

[29] Eggertsson, Juelsrud, Summers and Wold (2019) show that lending rates across advanced economies have been steadily falling since 2011 and in the aftermath of the financial crisis which was initially associated with elevated credit spreads. In that sense, financial shocks in our model resemble more the onset of the financial crisis, whereas preference shocks, which produce procyclical credit spreads, are more consistent with the situation further into the crisis. Moreover, according to recent economic data obtained by the Federal Deposit Insurance Corporation / Haver analytics and the St. Louis Fed's FRED database, the loan loss provisions and loan loss reserves to loan ratios have also significantly declined since 2011, at least until the recent Covid-19 crisis (see also Balasubramanyan and Madias (2015)). Therefore, the relationship between specific provisions, credit spreads and macroeconomic variables is not inconsistent with the data.

welfare through the provisioning cost channel. Table 2 summarizes the above discussion.

Table 2: Standard deviations and welfare gains from optimal LLP and monetary policies at the ZLB

	S-LLP+TR	S-LLP	D-LLP	OPT-LLP
	$s.d\,(\widehat{\pi}_t) = 2.46$ $s.d(\widehat{Y}_t) = 1.21$	$s.d\,(\widehat{\pi}_t) = 0.24$ $s.d(\widehat{Y}_t) = 0.31$	$s.d\,(\widehat{\pi}_t) = 0.22$ $s.d(\widehat{Y}_t) = 0.32$	$s.d\,(\widehat{\pi}_t) = 0.08$ $s.d(\widehat{Y}_t) = 0.34$
Welfare Cost/Gain in %	–	0.101	0.103	0.112

Notes: i) The standard deviations of key variables are represented in annualized rates.
ii) The welfare cost / gain is measured relative to the S-LLP+TR regime.

Unlike financial shocks, enacting optimal provisions in response to deflationary demand shocks yields minimal welfare gains. Most of the stabilization effects are attributed to the accommodative optimal monetary policy. Nevertheless, from a qualitative perspective, optimal financial policy in a demand-side-driven liquidity trap requires an inflationary and a mild contractionary provisioning regime in the first few periods.[30]

Another result worth mentioning despite being quantitatively small is that when optimal provisions are in place, the policy rate leaves the ZLB territory one period earlier compared to the other regimes which involve optimal monetary commitment with either specific or dynamic provisions. Intuitively, the positive impact of the credit and provisioning cost channels on inflation as the economy exits the liquidity trap enables the public authority to raise the interest rate at a quicker pace.

Our state-contingent results show that altering regulatory loan loss provisions can provide meaningful welfare gains following financial shocks and to a lesser extent in response to demand shocks. Implementing accounting / regulatory provisioning practices can be very potent in achieving the primary mandates of central banks, without any reliance on monetary policy or forward guidance-type policy commitments. This result holds particularly when the economy is subject to supply-side financial shocks resulting in occasional liquidity traps. Notwithstanding, in response to demand-side shocks, strictly following the IFRS 9 guidelines and non-contingently smoothing provisions over the business cycle can defeat the purpose of promoting overall macroeconomic stability.

6 Concluding Remarks

This paper sheds new insights on how provisioning practices impact the financial system, real economy and welfare, as well as how they should be set optimally in response to various shocks that provoke the ZLB. During recessions triggered by cost-push financial shocks, our model supports, to a large extent, the recent calls by the Basel committee and the IASB to re-design accounting principles such that banks engage in "through-the-cycle" smoothing of loan loss provisions (which under new regulations can be calculated as Tier-2 bank capital). In practice, the new IFRS 9 accounting framework obliges banks to recognize losses earlier in the credit cycle, with the aim to dampen procyclicality instigated by specific provisions. Beyond the flattening of provisions as put forward by the IFRS 9, we additionally advocate for some excess smoothing of provisions, and

[30] Implementing a sub-optimal standard monetary policy Taylor rule alongside optimal provisions results in a much larger output-inflation trade-off (see online Appendix B).

show that optimal loan loss provisions can in fact eliminate the standard output-inflation trade-off induced by credit disturbances. In other words, loan loss provisions can shield the real economy from the adverse consequences of volatile credit spreads and can completely circumvent supply-side-driven liquidity traps. For deflationary demand shocks, higher provisions generate cost-push pressures that limit price deflation. In the absence of other financial / fiscal policies, monetary policy remains responsible for most of the GDP stabilization effects following demand-driven disturbances.

A natural extension to this model would be to simultaneously account for countercyclical capital requirements and dynamic (or optimal) provisioning systems, and understand how such policy tools interact with one another. In this setup featuring a credit cost channel, both dynamic provisions and countercyclical capital buffers would impact the real economy through their effect on borrowing costs. Assuming that bank capital is more costly than deposits (due to a tax advantage of debt over equity for example), a regulatory rule which relaxes (tightens) equity requirements during bad (good) times can also lead to significant welfare improvements and therefore act as a substitute to loan loss provisions. However, in the presence of an effective dynamic provisioning system, as already implemented in some countries, the countercyclical weight on a Basel III type equity rule would not need to be too aggressive in order to mitigate welfare losses. Put differently, a small adjustment in bank capital requirements, based on the nature of the business cycle and unexpected losses, would suffice to further promote macroeconomic stability when dynamic provisions (covering for expected losses) are set. In this case, bank capital and loan loss provisions would be complementary to one another. We leave the formal analysis of this important issue for future research.

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7 Appendix A - Welfare Function Derivation (not for publication)

The derivation of the loss function as presented in the paper strictly follows Woodford (2003) and Ravenna and Walsh (2006, online appendix). To derive a second-order approximation of the representative utility function, it is first necessary to clarify some additional notation. For any variable $\mathbb{X}_t$, let $\mathbb{X}$ be its steady state value, $\mathbb{X}_t^e$ be its efficient level, $\tilde{\mathbb{X}}_t = \mathbb{X}_t - \mathbb{X}$ be the deviation of $\mathbb{X}_t$ around its steady state, and finally $\hat{\mathbb{X}}_t = \log(\mathbb{X}_t/\mathbb{X})$ be the log-deviation of $\mathbb{X}_t$ around its correspondent steady state. Using a second-order Taylor approximation, the variables $\tilde{\mathbb{X}}_t$ and $\hat{\mathbb{X}}$ can be related using the following equation,

$$\frac{\mathbb{X}_t}{\mathbb{X}} = 1 + \log\left(\frac{\mathbb{X}_t}{\mathbb{X}}\right) + \frac{1}{2}\left[\log\left(\frac{\mathbb{X}_t}{\mathbb{X}}\right)\right]^2 = 1 + \hat{\mathbb{X}}_t + \frac{1}{2}\hat{\mathbb{X}}_t^2. \tag{50}$$

As we can write $\tilde{\mathbb{X}}_t = \mathbb{X}\left(\frac{\mathbb{X}_t}{\mathbb{X}} - 1\right)$, it follows that $\tilde{\mathbb{X}}_t \approx \mathbb{X}\left(\hat{\mathbb{X}}_t + \frac{1}{2}\hat{\mathbb{X}}_t^2\right)$.

Utility is assumed to be separable in consumption and leisure,

$$U_t = \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t \left\{ \frac{\vartheta_t C_t^{1-\varsigma}}{1-\varsigma} - \frac{\vartheta_t H_t^{1+\gamma}}{1+\gamma} \right\}. \tag{51}$$

We start by approximating the utility from consumption. With the steady state value of the discount factor shock (ϑ) equal to 1, the second order expansion for $U(C_t, \vartheta_t)$ yields,

$$\begin{aligned} U(C_t, \vartheta_t) \;\; \approx \;\; & U(C,1) + U_C(C,1)\tilde{C}_t + \frac{1}{2}U_{CC}(C,1)\tilde{C}_t^2 + \\ & U_\vartheta(C,1)\widetilde{\vartheta}_t + \frac{1}{2}U_{\vartheta,\vartheta}\widetilde{\vartheta}_t^2 + U_{C,\vartheta}\widetilde{\vartheta}_t\tilde{C}_t, \end{aligned} \tag{52}$$

which according to our utility function (51) results in,

$$\begin{aligned} U(C_t, \vartheta_t) \;\; \approx \;\; & U(C,1) + U_C(C,1)\tilde{C}_t + \frac{1}{2}U_{CC}(C,1)\tilde{C}_t^2 + \\ & U_\vartheta(C,1)\widetilde{\vartheta}_t + \frac{1}{2}U_{\vartheta,\vartheta}\widetilde{\vartheta}_t^2 + U_{C,\vartheta}\widetilde{\vartheta}_t\tilde{C}_t, \end{aligned}$$

using $\widetilde{\vartheta}_t \approx \widehat{\vartheta}_t$ and $U_{C,\vartheta} = U_C(C,1)$, the above becomes,

$$\begin{aligned} U(C_t, \vartheta_t) \;\; \approx \;\; & U(C,1) + U_C(C,1)C\left(\hat{C}_t + \frac{1}{2}\hat{C}_t^2\right) - \frac{1}{2}\varsigma U_C(C,1)C\left(\hat{C}_t + \frac{1}{2}\hat{C}_t^2\right)^2 + \\ & + U_\vartheta(C,1)\widehat{\vartheta}_t + \frac{1}{2}U_{\vartheta,\vartheta}\widehat{\vartheta}_t^2 + U_C(C,1)C\widehat{\vartheta}_t\left(\hat{C}_t + \frac{1}{2}\hat{C}_t^2\right), \end{aligned}$$

ignoring the terms $\mathbb{X}^i$ for $i > 2$ yields,

$$\begin{aligned} U(C_t, \vartheta_t) \;\; \approx \;\; & U(C,1) + U_C(C,1)C\left[\left(1 + \widehat{\vartheta}_t\right)\hat{C}_t + \frac{1}{2}\left(1-\varsigma\right)\hat{C}_t^2\right] + \\ & + U_\vartheta(C,1)\widehat{\vartheta}_t + \frac{1}{2}U_{\vartheta,\vartheta}\widehat{\vartheta}_t^2, \end{aligned} \tag{53}$$

We next derive an expression for the disutility from labour. The Taylor expansion for $V(H_t, \vartheta_t)$ gives,

$$\begin{aligned} V(H_t, \vartheta_t) &\approx V(H,1) + V_H(H,1)\tilde{H}_t + \frac{1}{2}V_{HH}(H,1)\tilde{H}_t^2 + \\ &+V_{\vartheta}(C,1)\widetilde{\vartheta}_t + \frac{1}{2}V_{\vartheta,\vartheta}\widetilde{\vartheta}_t^2 + V_{C,\vartheta}\widetilde{\vartheta}_t\tilde{H}_t, \end{aligned} \tag{54}$$

where aggregate employment is,

$$\tilde{H}_t = \int_0^1 \tilde{H}_{j,t}dj,$$

and employment at firm j,

$$\tilde{H}_{j,t} \approx H\left[\hat{H}_{j,t} + \frac{1}{2}\hat{H}_{j,t}^2\right].$$

For the purpose of calculating the *ex-ante* loss function, we ignore the effects of the idiosyncratic shock that takes place at the *end of the period* such that $\hat{\varepsilon}_{j,t}^F = 0$. We therefore only examine the ex-ante uniform properties of this shock to calculate welfare. Using this assumption, each firm faces the following technology function,

$$\hat{H}_{j,t} = \hat{Y}_{j,t}.$$

Thus, we can define employment as,

$$\hat{H}_t = H\left[\int_0^1 \hat{Y}_{j,t}dj + \frac{1}{2}\int_0^1 \hat{Y}_{j,t}^2 dj\right]. \tag{55}$$

Substituting (55) into (54) and using $H = Y$ results in,

$$\begin{aligned} V(H_t, \vartheta_t) &\approx V(Y,1) + V_H(Y,1)Y\left[\int_0^1 \hat{Y}_{j,t}dj + \frac{1}{2}\int_0^1 \hat{Y}_{j,t}^2 dj\right] + \\ &+\frac{1}{2}V_{HH}(Y,1)Y^2\left[\int_0^1 \hat{Y}_{j,t}dj\right]^2 + V_{\vartheta}(C,1)\left(\widehat{\vartheta}_t + \frac{1}{2}\widehat{\vartheta}_t^2\right) + \\ &+\frac{1}{2}V_{\vartheta,\vartheta}\left(\widehat{\vartheta}_t + \frac{1}{2}\widehat{\vartheta}_t^2\right)^2 + \\ &+V_{H,\vartheta}Y\left(\widehat{\vartheta}_t + \frac{1}{2}\widehat{\vartheta}_t^2\right)\left[\int_0^1 \hat{Y}_{j,t}dj + \frac{1}{2}\int_0^1 \hat{Y}_{j,t}^2 dj\right]. \end{aligned}$$

ignoring terms of $\mathbb{X}^i$ for $i > 2$ yields,

$$\begin{aligned} V(H_t, \vartheta_t) &\approx V(Y,1) + V_H(Y,1)Y\left[\int_0^1 \hat{Y}_{j,t}dj + \frac{1}{2}\int_0^1 \hat{Y}_{j,t}^2 dj\right] + \\ &+\frac{1}{2}V_{HH}(Y,1)Y^2\left(\int_0^1 \hat{Y}_{j,t}dj\right)^2 + \\ &+V_\vartheta(Y,1)\widehat{\vartheta}_t + \frac{1}{2}V_{\vartheta,\vartheta}\widehat{\vartheta}_t^2 + V_{H,\vartheta}Y\widehat{\vartheta}_t\int_0^1 \hat{Y}_{j,t}dj. \end{aligned} \tag{56}$$

Given the demand function of each firm j, aggregate output is approximated by,

$$\hat{Y}_t = \int_0^1 \hat{Y}_{j,t}dj + \frac{1}{2}\left(\frac{\lambda-1}{\lambda}\right)var_j\hat{Y}_{j,t},$$

hence,

$$\left(\int_0^1 \hat{Y}_{j,t}dj\right)^2 = \left[\hat{Y}_t - \frac{1}{2}\left(\frac{\lambda-1}{\lambda}\right)var_j\hat{Y}_{j,t}\right]^2 \approx \hat{Y}_t^2,$$

and,

$$\int_0^1 \hat{Y}_{j,t}^2 dj = \left(\int_0^1 \hat{Y}_{j,t}dj\right)^2 + var_j\hat{Y}_{j,t}.$$

Therefore,

$$\int_0^1 \hat{Y}_{j,t}^2 dj \approx \widehat{Y}_t^2 + var_j\hat{Y}_{j,t},$$

and,

$$\int_0^1 \hat{Y}_{j,t}dj \approx \hat{Y}_t.$$

Using $V_{H,\vartheta} = V_H(Y,1)$ and the above results, (56) becomes,

$$\begin{aligned} V(H_t, \vartheta_t) &\approx V(Y,1) + V_H(Y,1)Y\left\{\left(1+\widehat{\vartheta}_t\right)\hat{Y}_t + \frac{1}{2}\left(\frac{1}{\lambda}\right)var_j\hat{Y}_{j,t} + \frac{1}{2}(1+\gamma)\hat{Y}_t^2\right\} + \\ &+V_\vartheta(Y,1)\widehat{\vartheta}_t + \frac{1}{2}V_{\vartheta,\vartheta}\widehat{\vartheta}_t^2. \end{aligned} \tag{57}$$

To determine total utility we subtract (57) from (53) to obtain,

$$\begin{aligned}U(C_t,\vartheta_t)-V(H_t,\vartheta_t) &= U(C,1)-V(Y,1)+U_C(C,1)C\left[\left(1+\widehat{\vartheta}_t\right)\hat{C}_t+\frac{1}{2}\left(1-\varsigma\right)\hat{C}_t^2\right]- \\ &-V_H(Y,1)Y\left\{\left(1+\widehat{\vartheta}_t\right)\hat{Y}_t+\frac{1}{2}\left(\frac{1}{\lambda}\right)var_j\hat{Y}_{j,t}+\frac{1}{2}\left(1+\gamma\right)\hat{Y}_t^2\right\} \\ &+\left(U_\vartheta(C,1)-V_\vartheta(Y,1)\right)\widehat{\vartheta}_t+\frac{1}{2}\left(U_{\vartheta,\vartheta}-V_{\vartheta,\vartheta}\right)\widehat{\vartheta}_t^2. \qquad (58)\end{aligned}$$

Note that the steady state labour market equilibrium condition is $\frac{V_H}{U_C}=W=\frac{1}{(pm)(1+\kappa(R^L-1))}$, with $pm\equiv\frac{\lambda}{(\lambda-1)}$ defined as the price mark-up. We define Ξ such that,

$$1-\Xi\equiv\frac{1}{(pm)\left(1+\kappa(R^L-1)\right)}.$$

Then $V_H(H,1)Y$ can be written as $U_C\left(C,1\right)Y(1-\Xi)$. In this way, Ξ acts as a subsidy to labour costs that eliminates all distortions in the steady state equilibrium. As in Ravenna and Walsh (2006), given that Ξ is small, terms such as $(1-\Xi)\widehat{Y}^2$ simply boil down to $\widehat{Y}_t^2$.[31] With these assumptions, we can now rewrite equation (58) as,

$$\begin{aligned}U(C_t,\vartheta_t)-V(H_t,\vartheta_t) &= U(C,1)-V(Y,1)+U_C(C,1)C\left[\left(1+\widehat{\vartheta}_t\right)\hat{C}_t+\frac{1}{2}\left(1-\varsigma\right)\hat{C}_t^2\right]- \\ &-U_C\left(C,1\right)Y(1-\Xi)\left[\begin{array}{c}\left(1+\widehat{\vartheta}_t\right)\hat{Y}_t+ \\ +\frac{1}{2}\left(\frac{1}{\lambda}\right)var_j\hat{Y}_{j,t}+\frac{1}{2}\left(1+\gamma\right)\hat{Y}_t^2\end{array}\right]+ \\ &+\left(U_\vartheta(C,1)-V_\vartheta(Y,1)\right)\widehat{\vartheta}_t+\frac{1}{2}\left(U_{\vartheta,\vartheta}-V_{\vartheta,\vartheta}\right)\widehat{\vartheta}_t^2. \qquad (59)\end{aligned}$$

Using $C=Y$ and collecting terms,

$$\begin{aligned}U(C_t,\vartheta_t)-V(H_t,\vartheta_t) &= U(C,1)-V(Y,1) \\ &+U_C(C,1)Y\left\{\begin{array}{c}\left(1+\widehat{\vartheta}_t\right)\left[\hat{C}_t-(1-\Xi)\hat{Y}_t\right]+ \\ +\frac{1}{2}\left(1-\varsigma\right)\hat{C}_t^2-\frac{1}{2}\left(1+\gamma\right)\hat{Y}_t^2\end{array}\right\} \\ &-\frac{1}{2}U_C(C,1)Y\left(\frac{1}{\lambda}\right)var_j\hat{Y}_{j,t}+ \\ &+\left(U_\vartheta(C,1)-V_\vartheta(Y,1)\right)\widehat{\vartheta}_t+\frac{1}{2}\left(U_{\vartheta,\vartheta}-V_{\vartheta,\vartheta}\right)\widehat{\vartheta}_t^2.\end{aligned}$$

[31] Note that like Ravenna and Walsh (2006), the value of Ξ is increasing with the price markup and the loan rate, which in our model is larger due to the presence of the various financial frictions.

Substituting the log-linear representation of consumption, $\hat{C}_t = \hat{Y}_t$, gives,

$$\begin{aligned} U(C_t,\vartheta_t) - V(H_t,\vartheta_t) &= U(C,1) - V(Y,1) \\ & +U_C(C,1)Y \left\{ \begin{array}{c} \left(1+\widehat{\vartheta}_t\right)\Xi\hat{Y}_t + \\ \frac{1}{2}(1-\varsigma)\hat{Y}_t^2 - \frac{1}{2}(1+\gamma)\hat{Y}_t^2 \end{array} \right\} \\ & -\frac{1}{2}U_C(C,1)Y\left(\frac{1}{\lambda}\right) var_j \hat{Y}_{j,t} + \\ & + \left(U_\vartheta(C,1) - V_\vartheta(Y,1)\right)\widehat{\vartheta}_t + \frac{1}{2}\left(U_{\vartheta,\vartheta} - V_{\vartheta,\vartheta}\right)\widehat{\vartheta}_t^2, \end{aligned}$$

or,

$$\begin{aligned} U(C_t,\vartheta_t) - V(H_t,\vartheta_t) &= U(C,1) - V(Y,1) \\ & +\frac{1}{2}U_C(C,1)Y\left\{\left[(1-\varsigma)-(1+\gamma)\right]\hat{Y}_t^2 + 2\left(1+\widehat{\vartheta}_t\right)\Xi\hat{Y}_t\right\} \\ & -\frac{1}{2}U_C(C,1)Y\left(\frac{1}{\lambda}\right) var_j \hat{Y}_{j,t} + \\ & + \left(U_\vartheta(C,1) - V_\vartheta(Y,1)\right)\widehat{\vartheta}_t + \frac{1}{2}\left(U_{\vartheta,\vartheta} - V_{\vartheta,\vartheta}\right)\widehat{\vartheta}_t^2, \end{aligned}$$

collecting terms,

$$\begin{aligned} U(C_t,\vartheta_t) - V(H_t,\vartheta_t) &= U(C,1) - V(Y,1) \\ & +\frac{1}{2}U_C(C,1)Y\left\{-(\varsigma+\gamma)\left[\hat{Y}_t^2 - 2\frac{\Xi}{(\varsigma+\gamma)}\left(1+\widehat{\vartheta}_t\right)\hat{Y}_t\right]\right\} \\ & -\frac{1}{2}U_C(C,1)Y\left(\frac{1}{\lambda}\right) var_j \hat{Y}_{j,t} + \\ & + \left(U_\vartheta(C,1) - V_\vartheta(Y,1)\right)\widehat{\vartheta}_t + \frac{1}{2}\left(U_{\vartheta,\vartheta} - V_{\vartheta,\vartheta}\right)\widehat{\vartheta}_t^2, \end{aligned}$$

or,

$$\begin{aligned} U(C_t,\vartheta_t) - V(H_t,\vartheta_t) &= U(C,1) - V(Y,1) \\ & +\frac{1}{2}U_C(C,1)Y\left\{-(\varsigma+\gamma)\left[\left(\hat{Y}_t - \frac{\Xi}{(\varsigma+\gamma)}\left(1+\widehat{\vartheta}_t\right)\right)^2 - \left(\frac{\Xi}{-(\varsigma+\gamma)}\right)^2\left(1+\widehat{\vartheta}_t\right)^2\right]\right\} \\ & -\frac{1}{2}U_C(C,1)Y\left(\frac{1}{\lambda}\right) var_j \hat{Y}_{j,t} + \left(U_\vartheta(C,1) - V_\vartheta(Y,1)\right)\widehat{\vartheta}_t + \frac{1}{2}\left(U_{\vartheta,\vartheta} - V_{\vartheta,\vartheta}\right)\widehat{\vartheta}_t^2. \end{aligned}$$

Collecting all terms that are independent of policy stabilization and denoting them as *tip* results in,

$$\begin{aligned} U(C_t,\vartheta_t) - V(H_t,\vartheta_t) &= U(C,1) - V(Y,1) \\ & +\frac{1}{2}U_C(C,1)Y\left\{ \begin{array}{c} -(\varsigma+\gamma)\left[\left(\hat{Y}_t - \frac{\Xi}{(\varsigma+\gamma)}\right)^2 + \left(\frac{\Xi}{(\varsigma+\gamma)}\right)^2\right] \\ -\left(\frac{1}{\lambda}\right) var_j \hat{Y}_{j,t} \end{array} \right\} + tip \end{aligned} \quad (60)$$

Assuming that the term $\frac{\Xi}{(\varsigma+\gamma)}$ is a small constant, the above boils down to,

$$\begin{aligned} U(C_t,\vartheta_t) - V(H_t,\vartheta_t) &= U(C,1) - V(Y,1) \\ &+\frac{1}{2}U_C(C,1)Y\left\{-\left(\varsigma+\gamma\right)\hat{Y}_t^2 - \left(\frac{1}{\lambda}\right)var_j\hat{Y}_{j,t}\right\} + tip. \end{aligned}$$

Given the demand function for each intermediate good, $Y_{j,t} = Y_t\left(\frac{P_{j,t}}{P_t}\right)^{-\lambda}$, we have,

$$\log Y_{j,t} = \log Y_t - \lambda\left(\log P_{j,t} - \log P_t\right),$$

so,

$$var_j \log Y_{j,t} = \lambda^2 var_j \log P_{j,t}.$$

Note the price adjustment mechanism involves a randomly chosen fraction $(1-\omega)$ of all firms acting optimally by adjusting prices in each period. Defining $\Delta_t \equiv var_j \log P_{j,t}$ then Woodford (2003, pp. 694-696) shows that,

$$\Delta_t \approx \omega\Delta_{t-1} + \left(\frac{\omega}{1-\omega}\right)\hat{\pi}_t^2.$$

Assuming Δ_{t-1} is the initial degree of price dispersion, then,

$$\sum_{t=0}^{\infty}\beta_t^t\Delta_t = \left[\frac{\omega}{(1-\omega)(1-\omega\beta)}\right]\sum_{t=0}^{\infty}\beta^t\hat{\pi}_t^2 + tip. \tag{61}$$

Combining (61) with (60), the present discounted value of the representative household welfare is,

$$\mathbb{W}_t \equiv \sum_{t=0}^{\infty}\beta^t U_t \approx U - \sum_{t=0}^{\infty}\beta^t\mathcal{L}_t,$$

where the associated losses from welfare are given by,

$$\sum_{t=0}^{\infty}\beta^t\mathcal{L}_t = \frac{1}{2}U_C(C,1)Y\mathbb{E}_0\sum_{t=0}^{\infty}\beta^t\left[\left(\frac{\lambda}{\kappa_p}\right)\hat{\pi}_t^2 + \left(\varsigma+\gamma\right)\hat{Y}_t^2\right], \tag{62}$$

with $\kappa_p = \frac{(1-\omega)(1-\omega\beta)}{\omega}$. With $\varsigma = 1$ and $Y = C$, (62) boils down to,

$$\sum_{t=0}^{\infty}\beta^t\mathcal{L}_t = \frac{1}{2}\mathbb{E}_0\sum_{t=0}^{\infty}\beta^t\left[\left(\frac{\lambda}{\kappa_p}\right)\hat{\pi}_t^2 + \left(1+\gamma\right)\hat{Y}_t^2\right]. \tag{63}$$

Welfare Measure

In considering loan loss provisions policies, we measure the welfare benefit of a particular optimal policy j as a fraction of the consumption path under the benchmark case (Policy I) that must be given up in order to obtain the benefits of welfare associated with optimal provisioning; $\mathbb{E}_t\sum_{t=0}^{\infty}\beta^t U_t\left(C_t^j, H_t^j\right) = \mathbb{E}_t\sum_{t=0}^{\infty}\beta^t U_t\left((1-\Lambda)C_t^I, H_t^I\right)$, where superscript j refers to Policies II and III and superscript I refers to Policy I. Given the utility function adopted and with $\varsigma = 1$, the expression for Λ in percentage terms is,

$$\Lambda = \left\{1 - \exp\left[(1-\beta)\left(\mathbb{W}_t^j - \mathbb{W}_t^I\right)\right]\right\} \times 100,$$

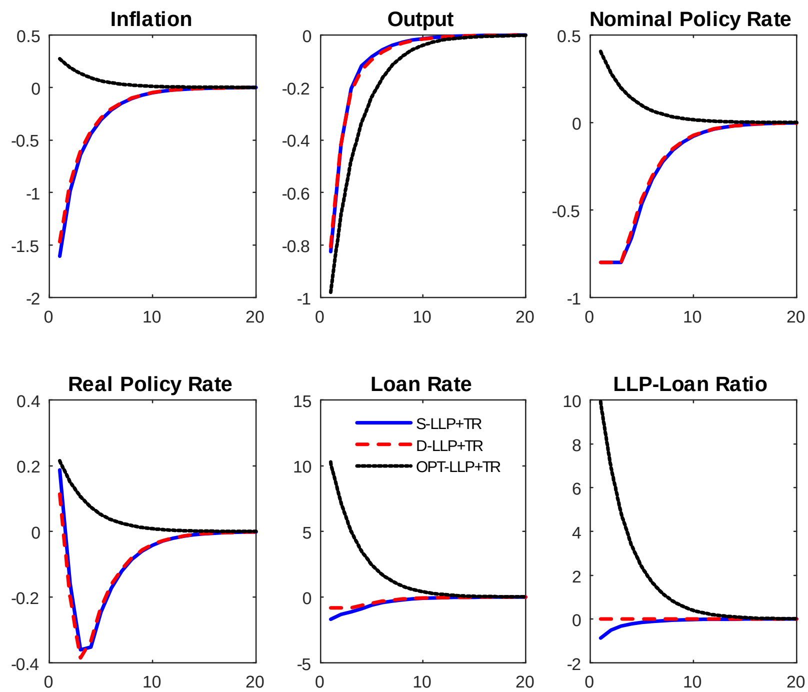
where $\mathbb{W}_t^j = \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t U_t \left(C_t^j, H_t^j\right)$ represents the unconditional expectation of lifetime utility under policy $j = II, III$, and $\mathbb{W}_t^I = \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t U_t \left(C_t^I, H_t^I\right)$ is the welfare associated with the benchmark Policy I. Converting the loss function to the welfare measure gives,

$$\mathbb{W}_t \equiv U - \frac{1}{2}\frac{U_C C}{(1-\beta)}\left[\left(\frac{\lambda}{\kappa_p}\right) var(\hat{\pi}_t) + (1+\gamma)\, var\left(\hat{Y}_t\right)\right].$$

8 Appendix B - Demand Shocks, Provisioning Rules and Taylor (1993) Monetary Policy Rules (not for publication)

To complement the analysis in Section 5.1.2, in Figure A below we use a monetary policy Taylor rule as a benchmark in all impulse response functions and compare the macroeconomic implications of a negative demand shock when the financial policy follows: *i)* specific provisions; *ii)* dynamic provisions; and *iii)* optimal provisions.

Figure A: Performance of Provisioning Rules with a Taylor (1993) Monetary Policy Rule - Demand Shock



Note: Interest rates, inflation rate and the LLP-loan ratio are measured in annualized percentage point deviations from steady state. The rest of the variables are measured in terms of annualized percentage deviations.

The loan loss provisions implications from Figure A are qualitatively similar to the provisioning responses with optimal commitment monetary policy. Specifically, the stabilization gains from implementing dynamic provisions relative to specific provisions are negligible, whereas optimal financial policy calls for a rise in provisions to counteract the deflationary pressures stemming from the adverse demand shock. Such policy results in a larger decline in output but with smaller deviations in prices. The optimal provisioning regime prevents the policy rate from falling to its lower bound, with the rise in $\hat{R}_t^D$ helping to stabilize the inflationary outcome from raising $\widehat{llp}_t^{opt}$. This, however, comes at the cost of a bigger output contraction. As explained in the main text, most of the inflation-output stabilization following a negative demand shock comes from the access to optimal commitment plans that considerably mitigate the aforementioned trade-off.