A Markovian Model for the Analysis of Age of Information in IoT Networks

Qamar Abbas, Syed Ali Hassan, Haris Pervaiz and Qiang Ni

Abstract—**Age of Information (AoI) is a critical metric in status update systems as these systems require the fresh updates. This paper investigates the uplink of an Internet-of-thing (IoT) network where L nodes transmit their information packets to a base station. The effects of the arrival rate of packets at the nodes, the number of nodes in the system, and queue length of each node have been studied by devising a discrete time Markov chain (MC) model. This model helps in predicting the values of AoI and probability of packet drops in such systems. The notion of first-in first-out is used for queuing, which transmits the oldest packet first, resulting in decreasing the overall AoI of the system. The results show that AoI increases with the increase in queue length, number of nodes and arrival rate and we quantify the aforementioned metrics using the MC model. The results found using the MC model are also validated using extensive simulations.**

Index Terms—**Internet-of-thing, age of information, Markov chain, arrival rate, queuing theory.**

I. Introduction

With the explosive growth of Internet-of-thing (IoT) systems, real-time status updates have become a crucial and ubiquitous form of communication. To quantify the information freshness about these remote systems, age of information (AoI) has been recently introduced [1]. AoI is defined as the time elapsed since the generation of the latest successfully received update about the source system. The notion of AoI is different from throughput and delay because system utilization can be maximized by allowing the nodes to send the arrived packets as soon as possible [2]. However, this may result in backlogging of the communication system. Similarly, the delay can be reduced by decreasing the number of updates, but this can result in obsolete packets because of the lack of fresh updates.

AoI has been studied so far as a concept, performance metric and a tool [3]. In real-time monitoring of IoT systems, AoI is crucial to be considered as the stale information can degrade the system performance in such systems. For example, in [4], AoI is critical in agricultural monitoring as the latest information is needed for precision agriculture applications. Precision agriculture along with IoT can be used to enhance productivity of agriculture crops by monitoring soil properties, moisture level, meteorological behavior, etc [5]. Similarly, [6] models the problem of real-time scheduling based on AoI in wireless ad hoc networks by using a scheduling policy to improve the AoI without sacrificing the feasibility optimality. A simplified method to evaluate AoI based on stochastic hybrid system using a finite-state Markov chain is formulated for a multiple source network sharing a single server [7]. The authors proposed closed form AoI expressions for simple queues and the results are compared for different queuing systems but the lossy queuing systems increase the probability of packet drops. [8] uses deadlines to drop the outdated packets in a down-link IoT network updating multiple nodes. AoI is evaluated using a fixed and random deadlines and showed the advantages of respective deadlines in different deadline regimes.

Although recent studies focus on minimizing AoI but the probability of packet drops in such systems is also important to discuss. This letter provides a performance analysis of an uplink IoT system with the help of Markov chains where the number of nodes and the queue length in each node is used to quantify both the AoI and probability of packet drops. Specifically, we assume a first-in first-out (FIFO) queue model where each information packet carries a distinct information about a multi-sensory scenario and each information packet is critical to reach the receiver with minimum AoI. We use a discrete time BER/G/1/Q queuing model where the arrivals occur at a time instant with probability λ following a Bernoulli distribution, and a queue length Q as analyzed in [9] and computes the probability of packet drops and AoI of the network using the state transition matrix of Markov chain. The main contributions of this letter are outlined below.

- We model the AoI and probability of packet drops using a discrete time Markov chain (MC) model and study various attributes of network performance by analyzing the state probability distribution.
- We quantify the effects of number of nodes, queue length and arrival rate on AoI and packet drops using the proposed analytical model.

Qamar Abbas and Syed Ali Hassan are with the School of Electrical Engineering Computer Science (SEECS), National University of Sciences & Technology (NUST), Pakistan. (e-mail: qabbas.phdee17seecs@seecs.edu.pk; ali.hassan@seecs.edu.pk).

Haris Pervaiz and Qiang Ni are with the Department of Computing and Communications Lancaster university, UK (e-mail: h.b.pervaiz@lancaster.ac.uk; q.ni@lancaster.ac.uk).

II. System Model

Consider the uplink of a wireless network consisting of a base-station (BS) and L sensor nodes denoted by the set $\mathbb{L} = \{l_1, ..., l_L\}$, having a queue length, Q, each where each node extracts real-time information from an environmental source. Each node transmits information to the BS in the form of packets, where the inter-arrival times are independent and size of all packets is considered the same. We consider BER/G/1/Q queuing model where the arrival process follows a Bernoulli process with parameter λ.

When a packet arrives at the queue of a sensor, it waits in the queue where each sensor has a queue length of Q. The waiting time of i^{th} node increases as L increases because the total transmission time depends on the number of nodes. Similarly, the waiting time increases as Q increases hence, with the increase in L and Q, the information freshness of each node is also compromised. We assume that every packet is carrying unique information and when a node has Q packets in its queue and another packets arrives, the oldest packet is discarded from the queue to ensure the information freshness. The discarded packets due to the queue overloading are counted as packet drops. We model the AoI and the probability of packet drops, P_{drops}, using Markov chain, which is described in the next section.

III. Proposed Markov Model

At a certain time n, the state of the system can be described as the number of packets waiting for transmission in the queues of each sensor and the decision metric for transmission at the BS. Hence, the state of system at time n, represented as $X(n)$ is given as

$$X(n) = \{\mathbb{S}_1(n), \mathbb{S}_2(n), .., \mathbb{S}_L(n),\ \mathbb{D}_1(n), \mathbb{D}_2(n), .., \mathbb{D}_L(n)\}, \tag{1}$$

where $\mathbb{S}_i(n)$, $i \in \{1, 2, ...L\}$ shows the number of packets waiting in the queue of the i^{th} sensor at time n and its value is given as a Q-tuple indicator function, i.e.,

$$\mathbb{S}_i(n) = \begin{cases} 0 & \text{if the node has no packets in queue} \\ 1 & \text{if the node has one packet in queue} \\ \vdots & \\ Q & \text{if the node has } Q \text{ (maximum) packets in queue.} \end{cases} \tag{2}$$

Similarly the BS decision for the i^{th} node is also a binary indicator function such that for i^{th} node and at time n,

$$\mathbb{D}_i(n) = \begin{cases} 0 & \text{if the node is not transmitting} \\ 1 & \text{if the node is transmitting.} \end{cases} \tag{3}$$

Note that as only one node can transmit at one time, therefore $\mathbb{D}_i(n) = 1$ implies $\sum_{\substack{\forall n \\ f \neq i}} \mathbb{D}_f(n) = 0$, i.e., all other nodes are not transmitting.

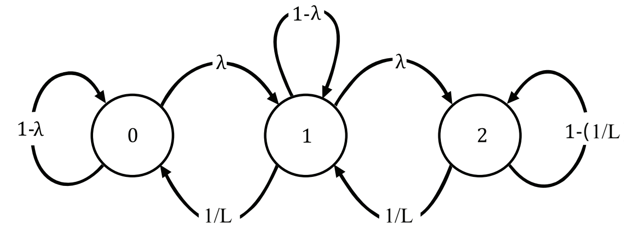

Fig. 1: The state transition diagram of a single node for $Q = 2, L = 2$

The next state of the system only depends on the current state and the arrival probability, λ, on all nodes satisfying the Markovian property. We now divert our attention on the state transition of the i^{th} node. The transition of the i^{th} node to state $\mathbb{S}_i(n+1)$ given that the current state is $\mathbb{S}_i(n)$ depends on $\mathbb{D}_i(n)$ and λ. For instance, if the current state of i^{th} node, i.e., $\mathbb{S}_i(n)$=1 and $\mathbb{D}_i(n)$=1, the next state of the node will be 0 with a unit probability as the only packet in its queue is transmitted. Similarly, if $\mathbb{S}_i(n)$=1 and $\mathbb{D}_i(n)$=0, the next state of the node will be 2 with probability λ (packet arrives) or $\mathbb{S}_i(n+1) = 1$ with probability $1 - \lambda$ as shown in Fig. 1. When a node is in Q state, the oldest packet will be replaced by the new arrived packet which are counted as dropped packets. This implies that, when the current state is Q, on arrival of a new packet, the oldest packet will be dropped until any packet is transmitted to change current state to any state less than Q.

The next state of the i^{th} node given that the node is in state $S_i(n)$ at a time n is given as,

$$\mathbb{S}_i(n+1) = \begin{cases} \mathbb{S}_i(n) & \text{if } \mathbb{D}_i(n) = 0 \text{ and no arrival} \\ \mathbb{S}_i(n) + 1 & \text{if } \mathbb{D}_i(n) = 0 \text{ and arrival} \\ \mathbb{S}_i(n) - 1 & \text{if } \mathbb{D}_i(n) = 1. \end{cases} \tag{4}$$

The total number of states for the Markov chain X given L and Q is denoted as L_S and is given as

$$L_S = L \times (Q+1)^L. \tag{5}$$

Suppose there are two nodes, i.e., $L = 2$ and queue length of each sensor is 2 i.e., $Q = 2$. The BS allocates a time slot for transmission to the two sensors randomly. The probability of transmission, in this case, will be $1/2$. The total number of transition states will be 18 according to (5). The $L_s = 18$ states show all possible number of packets in the queues of each sensor with all possible decisions the BS makes. Suppose both of the sensors have a single packet waiting in their queue, and the first node gets a time slot for transmission. The current state of the system will be $X(n) = 1110$, where the first two digits indicate that both sensors have one packet in their queues and the last two digits show the decision BS makes, respectively. If the BS decides to allow transmission to the first sensor, the next possible states will be $X(n+1) = 0110$ or $X(n+1) = 0101$, given that there is no arrival on both nodes. In this example, the state of the first node becomes 0 as the waiting packet is transmitted and there can be no arrival on the first node in this slot as it is in transmission state. The second node is not transmitting, therefore, there is a probability of arrival λ on the second node. The state of the second sensor will be 2 if there is any packet arrival or will remain to state 1 if there is no arrival. If there is an arrival on the second node, the next possible states are $X(n+2) = 0210$ or $X(n+2) = 0201$. Considering the above discussions, let the first L digits of the system state of $X(n)$ are represented by the vector $\mathbf{a}_n$ such that $\mathbf{a}_n \in \mathbf{Z}^{+(1\times L)}$ where $\mathbf{Z}^{+}$ is the set of positive integers including zero and the last L digits of the system state are represented by the vector $\mathbf{b}_n$. The purpose is to find the transition probability of the system from the state $X(n) = \{\mathbf{a}_n \ \ \mathbf{b}_n\}$ to the state $X(n+1) = \{\mathbf{a}_{n+1} \ \ \mathbf{b}_{n+1}\}$, where we need the values of α, β and γ, which are found using Algorithm 1. In the algorithm, first of all the node which is transmitting is found using the variable *index*. γ counts the number of nodes which has full queue i.e. Q. Similarly, α denotes the number of nodes which has got arrival. β is the number of nodes which remains in same state and have less than Q packets in the queue.

Algorithm 1: Finding the arguments (α, β and γ) of Transition Probability

```
Input: X(n), X(n+1), L, Q
Output: α, β, γ
Initialize: α, β, γ=0 , c̃_n(m) ← 0, m = 1, ..., L
 Extract a_n & b_n from X(n) where, the dimension
 of a_n, b_n is [1 × L]
 Find c_n using c_n = a_n − b_n
 index ← find(c_n ≠ a_n)
 if a_{n+1}(index)==c_n(index) then
   for i = 1 to L do
     if c_n(i) == Q then
       c̃_n(i) ← 0
        γ ← γ + 1
        else if c_n(i) ≤ 0 then
          c̃_n(i) ← 0
       end
       else
          c̃_n(i) ← c_n(i)
       end
     end
   end
   Find Δ ∈ Z^(1×L) using Δ = a_{n+1} - c̃_n
    for j = 1 to L do
     if Δ(j) == 1 then
       α ← α + 1
     end
   end
   Find β = L − α − 1
end
```

For any pair of $X(n)$ and $X(n+1)$, to find the transition probability from $X(n)$ to $X(n+1)$ the following propositions are applied:

Proposition 1.
For the states $X(n)$ and $X(n+1)$, when any element of the vector $\mathbf{\Delta}$ does not belong to {0,1}, $\mathbb{P}_{(X(n),X(n+1))|(\mathbb{D}(n))} = 0$.

Proposition 2.
For the states $X(n)$ and $X(n+1)$, when all elements of the vector $\mathbf{\Delta}$ belong to {0,1}, the transition probability is given as

$$\mathbb{P}_{(X(n),X(n+1))|(\mathbb{D}(n))} = \frac{1}{L}\left(\lambda^{\alpha}(1-\lambda)^{\beta-\gamma}\right). \quad (6)$$

The transition to and from all transient states are incorporated in a transition probability matrix, $\mathbf{P}$, for which a single entry is given by Proposition 1 or 2. Please note that the matrix $\mathbf{P}$ is sparse owing to the fact that many transitions are prohibited given the state space of the system. After the formation of stochastic matrix, using the property of Markov chain, let $\mathbf{v}$ be the eigenvector of $\mathbf{P}$ corresponding to the eigenvalue χ, then,

$$(\mathbf{P} - \chi\mathbf{I})\mathbf{v} = \mathbf{0}. \quad (7)$$

The eigenvector $\mathbf{v}$ after normalization provides the state probability distribution of the system $\boldsymbol{\phi}$.

IV. Age of Information Model

Consider a packet, p, is generated at a node at time $u(n)$, we define AoI of that packet as the waiting time for transmission in the queue. The age of that packet can be written as

$$\psi_p(n) = n - u(n), \quad (8)$$

where n is the current time instant. AoI of a packet is incremented by 1 at next time slot if the packet is not transmitted while it becomes zero if it is transmitted. Since we are considering the age of a packet till its successful transmission, therefore, the a time slot required for transmission is not considered. According to the above discussion, AoI in the next slot of any packet which is oldest packet in a node is written as

$$\psi_p(n+1) = \begin{cases} 0 & \text{if } \mathbb{D}_i(n+1) = 1 \\ \psi_p(n)+1 & \text{if } \mathbb{D}_i(n+1) = 0 \end{cases} \quad (9)$$

following the well known sawtooth pattern introduced in [1].

As we know that the length of vector $\mathbb{S}_i(n)$ shows the number of packets currently waiting in the queue of i^{th} node, we can find the AoI of i^{th} node by summing the ages of all packets currently waiting in its queue. Let the length of vector $\mathbb{S}_i(n)$ at time n be κ and $\psi_p(n)$ be the AoI of the p^{th} packet in i^{th} node then,

$$\text{AoI}_i(n) = \sum_{p=1}^{\kappa} \psi_p(n). \quad (10)$$

TABLE I: Mean Absolute Error

λ	L=3,Q=2	L=3,Q=3	L=4,Q=2	L=5,Q=2
0.1	0.034%	0.040%	0.020%	0.013%
0.3	0.025%	0.038%	0.017%	0.010%
0.5	0.021%	0.037%	0.016%	0.007%
0.7	0.017%	0.023%	0.015%	0.006%
0.9	0.013%	0.015 %	0.012%	0.004%

If the state of the i^{th} node at time n is $\mathbb{S}_i(n)$, then the AoI at time $n+1$ will be summation of AoI in the current time slot, i.e., $\text{AoI}_i(n)$, and the state $\mathbb{S}_i(n)$ with probability $1-\lambda$ when $\mathbb{D}_i(n+1)=0$. Similarly, when $\mathbb{D}_i(n+1)=0$, the AoI at time $t+1$ will be summation of AoI in the current time slot, i.e., $\text{AoI}_i(n)$, and the state $\mathbb{S}_i(n)$+1 with probability λ. If $\mathbb{D}_i(n+1)=1$, the AoI at time $n+1$ will be $\text{AoI}_i(n)$-$\mathbb{S}_i(n)$ with unit probability. The AoI for i^{th} node at BS at time $n+1$ can be expressed as,

$$\text{AoI}_i(n+1) = \begin{cases} \text{AoI}_i(n)+\mathbb{S}_i(n) & \text{if } D_i(n+1)=0 \text{ and no arrival} \\ \text{AoI}_i(n)+\mathbb{S}_i(n)+1 & \text{if } D_i(n+1)=0 \text{ and arrival} \\ \text{AoI}_i(n)-\mathbb{S}_i(n) & \text{if } D_i(n+1)=1 \end{cases} \tag{11}$$

As per our assumption above, the BS schedules transmission of each node unbiasedly and the arrival λ on each node is kept same. Therefore, the AoI at each node will also be same. When we find the AoI at any random node, we can also find AoI of the entire system, $\overline{\text{AoI}}$, at $n+1$ using (11) as

$$\overline{\text{AoI}}(n+1) = \sum_{i=1}^{L} AoI_i(n+1). \tag{12}$$

Similarly, the probability of packet drops (P_{drops}) on each node will also be equal as it is the function of λ and the state of a node. P_{drops} at the i^{th} node can be found using the state distribution vector of the system $\boldsymbol{\phi}$. The probability of packet drops on the i^{th} node can be given as,

$$P_{drops} = \frac{(L-1)\lambda}{L} \times \sum_{j=y}^{L_s} \boldsymbol{\phi}(j) \tag{13}$$

where, $y = \frac{Q(\frac{L_s}{L}+1)+1}{Q+1}$. We consider a Rayleigh fading channel between the nodes and the base station. The outage probability at a given SNR threshold Γ is given as, $P_{out} = 1 - e^{-\Gamma(P_t/\sigma^2)}$. Where, P_t is the transmit power of nodes and σ is noise power spectral density. Therefore the effective AoI with fading can be found as, $AoI_{eff} = \frac{AoI}{1-P_{out}}$. The AoI_{eff} is more realistic as it takes outage in to account.

V. Performance Evaluation and Discussions

This section discusses the performance of the network in terms of AoI and probability of packet drops for different queue lengths and the arrival rates on each node. The results are found analytically using Markov chain and compared with the simulation results.

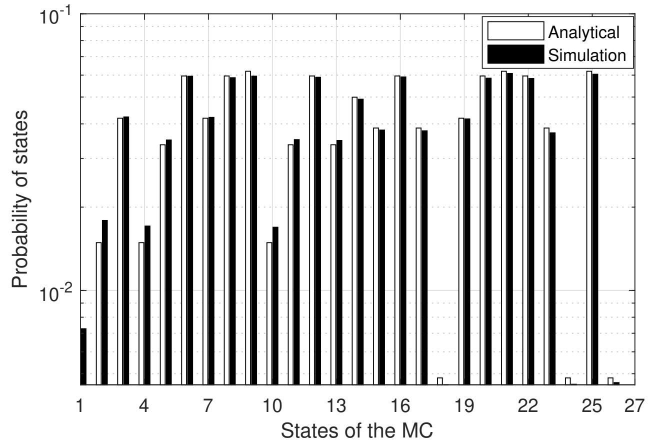

Fig. 2: The state probability distribution of the system for $L=3, Q=3$ and $\lambda=0.7$

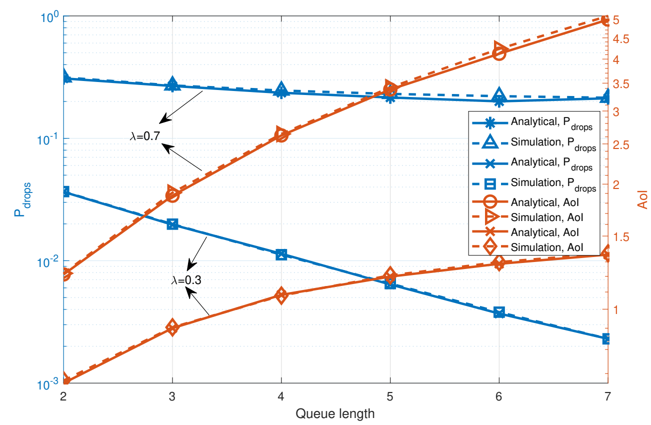

Fig. 3: AoI and P_{drops} trade-off for queue length,Q, and $L=3$

In Fig. 2, the state probability distribution is shown for $L=3$, $Q=3$ and $\lambda=0.5$. The horizontal axis shows the states of the system which has 27 length for the given combination of L and Q calculated using Eq. (5) and vertical axis shows the probability of all states. The hollow bars show the probability distribution using the MC model while the filled bars are numerical simulation results. It can be seen that the proposed analytical model has a close agreement with simulations, thereby validating the MC model. As the arrival rate is 0.5 therefore the graph is not shifted on either side. If the arrival rate is greater than 0.5, the graph shifts to right side because the probability of states representing greater number of packets in the queue increases. Similarly, if the arrival rate is less than 0.5, the graph shifts on the left side as the probability of states representing lower number of packets increases.

The mean absolute error between the analytical and simulation results are computed in Table I for various values of L, Q and λ and it can be seen that the proposed model is valid for a variety of parameter combinations.

Fig. 3 shows the trade-off between AoI and probability of packet drops. The y-axis on left side of the graph shows the probability of packet drops, while y-axis on the right side denotes AoI without introducing fading and queue lengths are plotted on the x-axis. It is depicted that increase in the queue length increases the AoI while it decreases probability of packet drops. The results are shown for arrival rates 0.3 and 0.7 and also validated using simulation result. For a given L, the AoI increases with the increase in Q because with increase in queue length, the stale information packets will reside in the queue for longer time instead of dropping with queue overloading in smaller queue length. Similarly, for a constant queue length, when L increases, the AoI also increases because with the increase in L, the transmission probability of a node decreases. Increase in Q, decreases packet drops because there will be more space for the packets in the queue. It also depicts that the packet drops decrease with the increase in queue length for a given number of nodes.

In Fig. 4, the evolution of AoI with the time steps is shown for different number of nodes and different queue lengths for $\lambda = 0.5$. For each L and Q pair, the results are compared for different SNR thresholds i.e., no outage, $\Gamma = -10dB$ and $\Gamma = -5.2dB$ for operating SNR=0 dB. It is shown that initially with the increase in number of steps, AoI is increasing and after some steps it gets the steady state and becomes constant. The result indicates that AoI increases with increment in queue length and number of nodes. The results show that with the increase in Γ, the outage probability increases which results in increasing AoI. it is also revealed from the graph that the AoI gets its steady state later for larger number of nodes and similarly the steady state is attained later for larger queue length for a given number of nodes. When the number of nodes and/or queue length increases, the number of states increases therefore, the steady state is attained after more time steps.

Similarly, Fig. 5 shows the evolution of AoI with the number of time steps for different arrival rates. The results are simulated for $L = 3$ and $Q = 3$ for different arrivals and outage values at SNR operating at 0 dB. The results indicate that AoI increases with the arrival rate because when arrival rate increases the number of packets in the queue becomes large as compared to the lower arrival rate and thus AoI increases. Likewise, when the Γ is increased, the outage probability is also increased incrementing AoI.

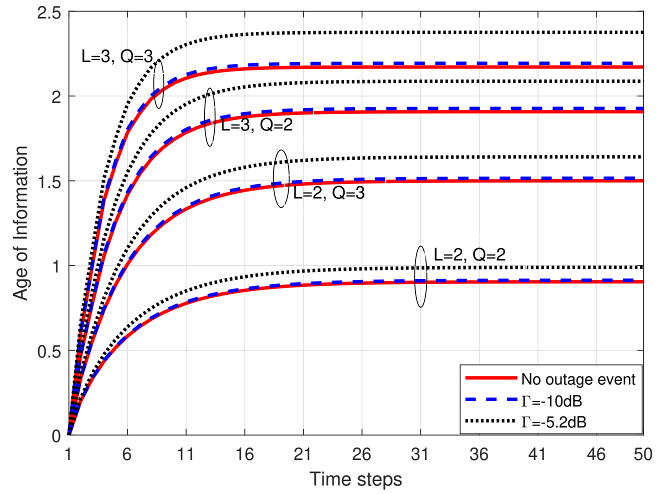

Fig. 4: Evolution of AoI with number of time steps for $\lambda = 0.5$

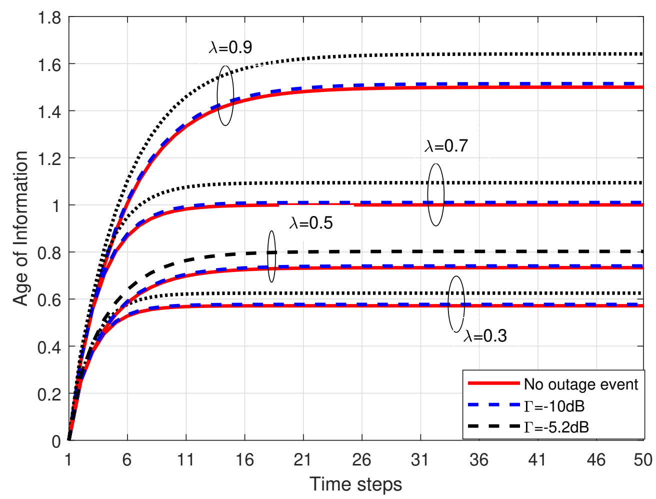

Fig. 5: Evolution of AoI with number of time steps for $L = 3, Q = 3$

VI. Conclusion

This paper investigated the effects of the arrival rate of packets, queue length, and the number of nodes in an up-link IoT system on AoI and probability of packet drops using discrete time Markov chain. We considered first-in first-out queuing system which processes the oldest packet first, thereby, decreasing the overall AoI. The accuracy of the approach is evaluated using mean absolute error between the probability distribution of the proposed MC and simulation results. The results showed that the probability state distribution, AoI and probability of packet drops are function of arrival rate, queue length and number of nodes in the system. The trade-off between AoI and probability of packet drops is also shown and quantified using the proposed analytical model.

References

[1] S. Kaul, M. Gruteser, V. Rai, and J. Kenney, "Minimizing age of information in vehicular networks," in *2011 8th Annual IEEE Communications Society Conference on Sensor, Mesh and Ad Hoc Communications and Networks*, pp. 350–358.

[2] S. Kaul, R. Yates, and M. Gruteser, "Real-time status: How often should one update?" in *2012 Proceedings IEEE INFOCOM*, pp. 2731–2735.

[3] A. Kosta, N. Pappas, and V. Angelakis, "Age of information: A new concept, metric, and tool," *Foundations and Trends in Networking*, vol. 12, no. 3, pp. 162–259, 2017.

[4] Q. Abbas, S. Zeb, S. A. Hassan, R. Mumtaz, and S. A. R. Zaidi, "Joint optimization of age of information and energy efficiency in IoT networks," in *2020 IEEE 91st Vehicular Technology Conference (VTC2020-Spring)*, pp. 1–5.

[5] Shafi, Uferah and Mumtaz, Rafia and García-Nieto, José and Hassan, Syed Ali and Zaidi, Syed Ali Raza and Iqbal, Naveed, "Precision agriculture techniques and practices: From considerations to applications," *Sensors*, vol. 19, no. 17, p. 3796, 2019.

[6] N. Lu, B. Ji, and B. Li, "Age-based scheduling: Improving data freshness for wireless real-time traffic," in *Proceedings of the Eighteenth ACM International Symposium on Mobile Ad Hoc Networking and Computing*, 2018, pp. 191–200.

[7] R. D. Yates and S. K. Kaul, "The age of information: Real-time status updating by multiple sources," *IEEE Transactions on Information Theory*, vol. 65, no. 3, pp. 1807–1827, 2018.

[8] J. Li, Y. Zhou, and H. Chen, "Age of information for multicast transmission with fixed and random deadlines in iot systems," *IEEE Internet of Things Journal*, 2020.

[9] V. Tripathi, R. Talak, and E. Modiano, "Age optimal information gathering and dissemination on graphs," in *IEEE INFOCOM 2019-IEEE Conference on Computer Communications*. IEEE, 2019, pp. 2422–2430.