

# Adaptive Closed-Loop Identification and Tracking Control of an Aerial Vehicle with Unknown Inertia Parameters <sup>\*</sup>

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**Abstract:** In this paper, the problem of adaptive closed-loop parameter estimation and tracking control of a six degree of freedom (6-DOF) nonlinear quadrotor unmanned aerial vehicle (UAV) is studied. To manage the complexity of the problem, the system dynamics is decomposed into two subsystems, i.e. translational dynamics and rotational dynamics. A nested control architecture is adopted to develop both adaptive tracking control and parameter estimation. To stabilize the outer loop, a virtual control input is proposed using a proportional-derivative (PD) controller to track the x, y and z positions. The rotational dynamics of UAV contains unknown inertia parameters appearing in the control structure as well as in a nonlinear dynamic term. An adaptive tracking scheme is designed using the certainty equivalence principle to handle both parameter estimation and tracking control in a closed-loop. The idea behind the controller design is to cancel the nonlinear term in the inner loop by estimating the unknown system parameters. The stability of the whole closed-loop system is proved with a rigorous analytical study. Moreover, the performance of the proposed controller is verified with several numerical analyses.

*Keywords:* Grey box identification, 6-DOF quadrotor, certainty equivalence principle, adaptive tracking control, closed-loop identification, unknown inertia parameters.

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## 1. INTRODUCTION

In recent years, the research outcomes in control for quadrotor unmanned aerial vehicle (UAV) have provided significant technological advances. Broad applications of quadrotor are very useful for mission in various hazardous environments such as in the military and nuclear decommissioning (Montazeri et al. (2020)). Quadrotors also have found their way in geographical photography, volcano monitoring, agriculture data collection and so on (Burrell et al. (2018, 2016); Radoglou-Grammatikis et al. (2020); Santamarina-Campos and Segarra-Oña (2018)).

To extend the implementation of quadrotor, autonomous operation and cognition of quadrotors, either as an operation of a single quadrotor or as a collaborative movement of multiple quadrotors in a networked environment as a cyber-physical system is one of the hottest research areas from the viewpoint of control engineers (Um (2019); Montazeri et al. (2020)). Many interesting results have been developed in various scenarios. In general, the main objective is how to design a proper controller for quadrotor and realistic for practical implementation.

The presence of nonlinearities in the attitude dynamics is one of the most essential issues in designing the controller.

At the beginning of the research, a linear controller for a quadrotor is designed by linearizing its dynamics. Under this situation, this technique only can be applied for limited cases. To solve this issue, a proper nonlinear controller is required to maintain the UAV movement in the full nonlinear operational range. Several results have been presented to tackle the tracking control problem, especially for rotational dynamics. Feedback linearization approach is one of the common techniques proposed for trajectory tracking problem in UAVs as developed in Voos (2009); Zhou et al. (2010). This approach only can be implemented if all parameters of quadrotor available for feedback control design.

In various practical implementations, one or some parameters may unavailable for feedback controller. As a result, a more advanced controller is required. Two common methods to handle nonlinear function with uncertain parameters is robust and adaptive control. The idea behind robust control is to dominate the uncertain nonlinear term. Some interesting results using robust control can be seen in Huang and Chen (2004); Lewis et al. (2003); Chen (2015) for single setting and in Chen and Chen (2016); Zhu and Chen (2014) for collaborative settings. Adaptive control is another approach to handle the nonlinearities with unknown parameters. In this approach, the uncertain nonlinearities term is maintained by estimating the unknown parameters. Some results using adaptive control can be found in Narendra and Annaswamy (1989); Anderson et al.

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(2005); Astolfi et al. (2007); Hovakimyan and Cao (2010) for single setting and in Lewis et al. (2013); Imran et al. (2019, 2020) for collaborative settings.

In general, there are two typical scenarios of attitude dynamics with unknown parameters. The first is the unknown parameter that appears in the nonlinear term separated from control input structure. The results under this setting for quadrotor can be found in Nemati and Montazeri (2018b,a) using sliding mode control and in Imran and Montazeri (2020) using adaptive control. The second is the unknown parameter that appears in the control input structure. The control problem under this scenario is more complicated. In this paper, the inertia parameters of quadrotor are unknown for feedback control design. These unknown parameters appear in control input structure as well as in the separate nonlinear term in the attitude dynamics. The main contribution of this paper is to design an adaptive tracking control for a nonlinear quadrotor with uncertain inertia parameters.

The remainder of the paper is organized as follows. In section 2, the system dynamics of quadrotor is presented. Following that, the outer loop position tracking control as well as the proposed adaptive tracking attitude control system is presented in Section 3. To demonstrate the performance of designed controller, several numerical simulations are conducted in Section 4. Finally, the paper is concluded in the last section with some recommendations for future work.

## 2. SYSTEM DYNAMICS OF QUADROTOR

In this section, we present the dynamic model of a 6-DOF quadrotor UAV, including both translational and rotational dynamics. For this purpose, we follow the notations used in Imran and Montazeri (2020). The translational dynamics is expressed as

$$\ddot{\eta}_1 = -gz_e + J_1(\eta_2)z_e \frac{u}{m} - \frac{k_t}{m}\dot{\eta}_1, \quad (1)$$

where  $z_e = [0 \ 0 \ 1]^T$ ,  $g$  is the gravity acceleration,  $u$  is the thrust force,  $k_t$  is translational drag coefficient and  $m$  is the mass of UAV. Vector  $\eta_1 = [x \ y \ z]^T$  represents the position vector consisting of forward, lateral, and vertical motions respectively; Here  $\eta_2 = [\phi \ \theta \ \psi]^T$  represents the orientation vector consist of roll ( $\phi$ ), pitch ( $\theta$ ) and yaw ( $\psi$ ) motions. Matrix  $J_1(\eta_2)$  is a transformation matrix expressed by

$$J_1(\eta_2) = \begin{bmatrix} \cos \theta \cos \psi & \sin \phi \sin \theta \cos \psi - \cos \phi \sin \psi \\ \cos \theta \sin \psi & \sin \phi \sin \theta \sin \psi + \cos \phi \cos \psi \\ -\sin \theta & \sin \phi \cos \theta \\ & \cos \phi \sin \theta \cos \psi + \sin \phi \sin \psi \\ & \cos \phi \sin \theta \sin \psi - \sin \phi \cos \psi \\ & \cos \phi \cos \theta \end{bmatrix}.$$

By assuming  $\cos \phi$  and  $\cos \theta$  to be non-zero

$$J_1^T(\eta_2) = J_1^{-1}(\eta_2). \quad (2)$$

The rotational dynamics can be represented as

$$\dot{\nu}_2 = w_1 f(\nu_2) + w_2 \tau, \quad (3)$$

where

$$w_1 = \text{diag} \left[ \frac{I_y - I_z}{I_x} \quad \frac{I_z - I_x}{I_y} \quad \frac{I_x - I_y}{I_z} \right]$$

$$f(\nu_2) = [qr \ pr \ pq]^T$$

$$w_2 = I_m^{-1}.$$

The vector  $\nu_2 = [p \ q \ r]^T$  in (3) is the angular velocity vectors,  $I_M = \text{diag}[I_x \ I_y \ I_z]$  is the inertia matrix and  $\tau = [\tau_p \ \tau_q \ \tau_r]^T$  is the torque vector acting on the body frame. Here, it is assumed that the inertia parameters  $I_x$ ,  $I_y$  and  $I_z$  are unknown for the feedback control design.

By putting (1) and (3) together, we get the dynamic model of the UAV as an under-actuated systems, in which the four control inputs are used to control the six system states. Both translational and rotational dynamics are highly coupled between the body and inertial frames and represented by

$$\begin{aligned} \dot{\eta}_1 &= J_1(\eta_2)\nu_1 \\ \dot{\eta}_2 &= J_2(\eta_2)\nu_2, \end{aligned} \quad (4)$$

where  $\nu_1 = [u \ v \ w]^T$  is the linear velocity vector,  $\eta_2 = [\phi \ \theta \ \psi]^T$  is the orientation vector consisting of roll ( $\phi$ ), pitch ( $\theta$ ) and yaw ( $\psi$ ) motions and matrix  $J_2(\eta_2)$  is a transformation matrix represented by

$$J_2(\eta_2) = \begin{bmatrix} 1 & \sin \phi \tan \theta & \cos \phi \tan \theta \\ 0 & \cos \phi & \sin \theta \\ 0 & \frac{\cos \phi}{\cos \theta} & \frac{\cos \phi}{\cos \theta} \end{bmatrix}.$$

The thrust force  $u$  is generated by

$$u = \sum_{\iota=1}^4 f_{\iota} = \sum_{\iota=1}^4 k_{\iota} \Omega_{\iota}^2, \quad (5)$$

where  $f_{\iota}$  is upward-lifting force generated by each rotor,  $k_{\iota}$  is a positive constant gain and  $\Omega_{\iota}$  is the angular speed of the rotor  $\iota$ . The thrust force and the torques acting around the body of UAV have the following relationship

$$\begin{bmatrix} u \\ \tau_p \\ \tau_q \\ \tau_r \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ \frac{l\sqrt{(2)}}{2} & -\frac{l\sqrt{(2)}}{2} & -\frac{l\sqrt{(2)}}{2} & \frac{l\sqrt{(2)}}{2} \\ \frac{l\sqrt{(2)}}{2} & \frac{l\sqrt{(2)}}{2} & -\frac{l\sqrt{(2)}}{2} & -\frac{l\sqrt{(2)}}{2} \\ \frac{2}{d} & \frac{2}{-d} & \frac{2}{d} & -\frac{2}{-d} \end{bmatrix} \begin{bmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \end{bmatrix}, \quad (6)$$

where  $l$  is the arm length and  $d$  is the drag factor.

## 3. PROPOSED CONTROL DESIGN

In this section, the proposed controllers for both translational and rotational dynamics are presented. A virtual PD controller is applied to maintain the translational motions of UAV as presented in Section 3.1.

The presence of nonlinearities in the attitude dynamics is an essential issue in designing attitude controller. If all parameters of the dynamics are known for feedback control design, then the control problem can be simplified by applying a full feedback linearization to cancel the nonlinear terms. However, this is not a viable solution since some parameters are either unavailable or the exact values are unknown for the feedback control design. As a result, a full feedback linearization approach does not give rise to a good performance. The problem becomes more

complicated when the unknown parameters appear in the control input structure. In Section 3.2, an adaptive control technique is developed to handle the tracking control of UAV with inertia parameters  $I_x$ ,  $I_y$  and  $I_z$  as unknown values. These unknown parameters appear in the control input structure as well as in the nonlinear dynamic terms.

### 3.1 Translational control design

In this section, the tracking control for translational dynamics is designed following the results presented in Imran and Montazeri (2020). The tracking error of the system can be defined as

$$\tilde{\eta}_1 = \eta_1 - \eta_{1d}, \quad (7)$$

where  $\tilde{\eta}_1$  and  $\eta_{1d}$  are the error vector position and the desired vector position, respectively. The double integrator dynamics of (7) can be written as

$$\ddot{\tilde{\eta}}_1 = -K_D \dot{\tilde{\eta}}_1 - K_P \tilde{\eta}_1. \quad (8)$$

The control gains  $K_P$  and  $K_D$  are selected to be positive definite, as a result Routh-Hurwitz stability criterion for system dynamics (8) is satisfied. The dynamics (7) can be rewritten as follows

$$\ddot{\eta}_1 = \ddot{\eta}_{1d} - K_D(\dot{\eta}_{1d} - \dot{\eta}_1) - K_P(\eta_{1d} - \eta_1). \quad (9)$$

We define a virtual input  $U = \ddot{\eta}_1 = [U_1 \ U_2 \ U_3]^T$ . Then by substituting  $U$  to (1), we have

$$U = -gz_e + J_1(\eta_2)z_e \frac{u}{m} - \frac{k_t}{m}\dot{\eta}_1, \quad (10)$$

or

$$\frac{u}{m}z_e = J_1^{-1}(\eta_2)(U + gz_e + \frac{k_t}{m}\dot{\eta}_1). \quad (11)$$

By expanding (11), we have the following relationships

$$\begin{aligned} (U_1 + \frac{k_t}{m}\dot{x}) \cos \theta \cos \psi + (U_2 + \frac{k_t}{m}\dot{y}) \cos \theta \sin \psi \\ - (U_3 + g + \frac{k_t}{m}\dot{z}) \sin \theta = 0, \end{aligned} \quad (12)$$

$$\begin{aligned} (U_1 + \frac{k_t}{m}\dot{x})(\sin \phi \sin \theta \cos \psi - \cos \phi \sin \psi) \\ + (U_2 + \frac{k_t}{m}\dot{y})(\sin \phi \sin \theta \sin \psi + \cos \phi \cos \psi) \\ + (U_3 + g + \frac{k_t}{m}\dot{z}) \sin \phi \cos \theta = 0, \end{aligned} \quad (13)$$

$$\begin{aligned} (U_1 + \frac{k_t}{m}\dot{x})(\cos \phi \sin \theta \cos \psi + \sin \phi \sin \psi) \\ + (U_2 + \frac{k_t}{m}\dot{y})(\cos \phi \sin \theta \sin \psi - \sin \phi \cos \psi) \\ + (U_3 + g + \frac{k_t}{m}\dot{z}) \cos \phi \cos \theta = \frac{u}{m}. \end{aligned} \quad (14)$$

The fact that  $\cos \theta \neq 0$ . From (12), we can compute  $\theta$  as follows

$$\theta = \arctan \left( \frac{(U_1 + \frac{k_t}{m}\dot{x}) \cos \psi + (U_2 + \frac{k_t}{m}\dot{y}) \sin \psi}{U_3 + g + \frac{k_t}{m}\dot{z}} \right). \quad (15)$$

From (11), we have

$$\begin{aligned} \left( \frac{u}{m}z_e \right)^\top \left( \frac{u}{m}z_e \right) &= \begin{pmatrix} J_1^{-1}(\eta_2)(U + gz_e + \frac{k_t}{m}\dot{\eta}_1) \\ J_1^{-1}(\eta_2)(U + gz_e + \frac{k_t}{m}\dot{\eta}_1) \end{pmatrix}^\top \\ &= \left( U + gz_e + \frac{k_t}{m}\dot{\eta} \right)^\top \left( U + gz_e \right. \\ &\quad \left. + \frac{k_t}{m}\dot{\eta}_1 \right). \end{aligned} \quad (16)$$

Therefore

$$\begin{aligned} \frac{u}{m} &= \left( (U_1 + \frac{k_t}{m}\dot{x})^2 + (U_2 + \frac{k_t}{m}\dot{y})^2 \right. \\ &\quad \left. + (U_3 + g + \frac{k_t}{m}\dot{z})^2 \right)^{1/2} \end{aligned} \quad (17)$$

From (13) and (14), we obtain

$$\frac{u}{m} \sin(\phi) = (U_1 + \frac{k_t}{m}\dot{x}) \sin(\psi) - (U_2 + \frac{k_t}{m}\dot{y}) \cos(\psi) \quad (18)$$

We can compute the  $\phi$  by substituting (17) to (18) as

$$\begin{aligned} \phi &= \arcsin \left( \left( (U_1 + \frac{k_t}{m}\dot{x}) \sin \psi - (U_2 + \frac{k_t}{m}\dot{y}) \cos \psi \right) \right. \\ &\quad \left. \left( (U_1 + \frac{k_t}{m}\dot{x})^2 + (U_2 + \frac{k_t}{m}\dot{y})^2 \right. \right. \\ &\quad \left. \left. + (U_3 + g + \frac{k_t}{m}\dot{z})^2 \right)^{1/2} \right). \end{aligned} \quad (19)$$

By following similar arguments, thus we can generate  $\phi_d$  and  $\theta_d$  as expressed by

$$\begin{aligned} \phi_d &= \arcsin \left( \left( (U_1 + \frac{k_t}{m}\dot{x}_d) \sin \psi_d - (U_2 + \frac{k_t}{m}\dot{y}_d) \right. \right. \\ &\quad \left. \left. \cos \psi_d \right) \left( (U_1 + \frac{k_t}{m}\dot{x}_d)^2 + (U_2 + \frac{k_t}{m}\dot{y}_d)^2 \right. \right. \\ &\quad \left. \left. + (U_3 + g + \frac{k_t}{m}\dot{z}_d)^2 \right)^{1/2} \right) \end{aligned} \quad (20)$$

$$\theta_d = \arctan \left( \frac{(U_1 + \frac{k_t}{m}\dot{x}_d) \cos \psi_d + (U_2 + \frac{k_t}{m}\dot{y}_d) \sin \psi_d}{U_3 + g + \frac{k_t}{m}\dot{z}_d} \right). \quad (21)$$

The total thrust  $u$  is generated from (14), as expressed by

$$\begin{aligned} u &= m \left( (U_1 + \frac{k_t}{m}\dot{x})(\cos \phi \sin \theta \cos \psi + \sin \phi \sin \psi) \right. \\ &\quad \left. + (U_2 + \frac{k_t}{m}\dot{y})(\cos \phi \sin \theta \sin \psi - \sin \phi \cos \psi) \right. \\ &\quad \left. + (U_3 + g + \frac{k_t}{m}\dot{z}) \cos \phi \cos \theta \right). \end{aligned} \quad (22)$$

### 3.2 Attitude control design

In this section, an adaptive scheme for attitude dynamics of UAV with unknown inertia parameters is developed. The main objective of the adaptive controller is to stabilize the closed-loop system so that not only the tracking error is going to zero but also to estimate the inertia parameters of the system simultaneously. By defining the desired trajectory  $\nu_{2d} = [p_d \ q_d \ r_d]^T$  and the tracking error as  $e = \nu_2 - \nu_{2d}$ , the tracking error dynamics can be written as

$$\dot{e} = -\dot{\nu}_{2d} + w_1 f(\nu_2) + w_2 \tau. \quad (23)$$

The tracking controller is deemed to be successful if

$$\lim_{t \rightarrow \infty} e(t) = 0. \quad (24)$$

Before presenting the main results, we define

$$\begin{aligned} E &= \text{diag}(e), \\ F(\nu_2) &= \text{diag}(f(\nu_2)), \\ N_{2_d} &= \text{diag}(\nu_{2_d}). \end{aligned}$$

The main result of the proposed controller for attitude dynamics is presented in Theorem 3.1.

**Theorem 3.1.** Consider the attitude dynamics (3). The objective of the tracking error (24) is achieved by selecting the controller

$$\tau = -\hat{w}_2(\alpha e + \hat{w}_1 f(\nu_2) - \dot{\nu}_{2_d}), \quad (25)$$

with adaptation law

$$\begin{aligned} \dot{\hat{w}}_1 &= \gamma_1 F(\nu_2) E, \\ \dot{\hat{w}}_2 &= \gamma_2 E(\alpha E + \hat{w}_1 F(\nu_2) - N_{2_d}), \end{aligned} \quad (26)$$

where  $\alpha$ ,  $\gamma_1$  and  $\gamma_2$  are some positive constants.

*Proof:* The dynamics error of closed-loop system (3) under control input (25) can be calculated as follows

$$\begin{aligned} \dot{e} &= -\dot{\nu}_{2_d} + w_1 f(\nu_2) - w_2(\hat{w}_2(\alpha e + \hat{w}_1 f(\nu_2) - \dot{\nu}_{2_d})) \\ &= -\dot{\nu}_{2_d} + w_1 f(\nu_2) - w_2(\tilde{w}_2 + w_2^{-1})(\alpha e + \hat{w}_1 f(\nu_2) - \dot{\nu}_{2_d}) \\ &= -\dot{\nu}_{2_d} + w_1 f(\nu_2) - w_2 \tilde{w}_2(\alpha e + \hat{w}_1 f(\nu_2) - \dot{\nu}_{2_d}) \\ &\quad - \alpha e - \hat{w}_1 f(\nu_2) + \dot{\nu}_{2_d} \\ &= -\alpha e - \tilde{w}_1 f(\nu_2) - w_2 \tilde{w}_2(\alpha e + \hat{w}_1 f(\nu_2) - \dot{\nu}_{2_d}), \end{aligned} \quad (27)$$

where  $\tilde{w}_1 = \hat{w}_1 - w_1$  and  $\tilde{w}_2 = \hat{w}_2 - w_2^{-1}$ .

We select the Lyapunov function of dynamics (23) to be

$$V(e, \tilde{w}_1, \tilde{w}_2) = \frac{1}{2} e^\top e + \text{tr}\left(\frac{1}{2\gamma_1} \tilde{w}_1^2 + \frac{1}{2\gamma_2} w_2 \tilde{w}_2^2\right). \quad (28)$$

Direct calculation shows that the time-derivative of  $V(e, \tilde{w}_1, \tilde{w}_2)$  along the closed-loop system (3)+(25)+(26) is

$$\begin{aligned} \dot{V}(e, \tilde{w}_1, \tilde{w}_2) &= e^\top \dot{e} + \text{tr}\left(\frac{1}{\gamma_1} \tilde{w}_1 \dot{\tilde{w}}_1 + \frac{1}{\gamma_2} w_2 \tilde{w}_2 \dot{\tilde{w}}_2\right) \\ &= e^\top \left( -\alpha e - \tilde{w}_1 f(\nu_2) - w_2 \tilde{w}_2 \right. \\ &\quad \left. (\alpha e + \hat{w}_1 f(\nu_2) - \dot{\nu}_{2_d}) \right) + \text{tr}\left(\frac{1}{\gamma_1} \tilde{w}_1 \dot{\tilde{w}}_1 \right. \\ &\quad \left. + \frac{1}{\gamma_2} w_2 \tilde{w}_2 \dot{\tilde{w}}_2 \right) \\ &= -\alpha e^\top e + e^\top \left( -\tilde{w}_1 f(\nu_2) \right. \\ &\quad \left. - w_2 \tilde{w}_2 (\alpha e + \hat{w}_1 f(\nu_2) - \dot{\nu}_{2_d}) \right) \\ &\quad + \text{tr}\left(\frac{1}{\gamma_1} \tilde{w}_1 \dot{\tilde{w}}_1 + \frac{1}{\gamma_2} w_2 \tilde{w}_2 \dot{\tilde{w}}_2 \right) \\ &= -\alpha e^\top e + \text{tr}\left(\frac{1}{\gamma_1} \tilde{w}_1 \dot{\tilde{w}}_1 \right. \\ &\quad \left. + \frac{1}{\gamma_2} w_2 \tilde{w}_2 \dot{\tilde{w}}_2 - \tilde{w}_1 F(\nu_2) E \right. \\ &\quad \left. - w_2 \tilde{w}_2 E(\alpha E + \hat{w}_1 F(\nu_2) - N_{2_d}) \right) \\ &= -\alpha e^\top e. \end{aligned}$$

From (26) and (27), we can see that  $e(t)$ ,  $\tilde{w}_1$  and  $\tilde{w}_2$  are bounded. To show the tracking error  $e$  is driven asymptotically to zero, we calculate the second time-derivative of Lyapunov function  $V(e, \tilde{w}_1, \tilde{w}_2)$  as

$$\ddot{V}(e, \tilde{w}_1, \tilde{w}_2) = -2\alpha e^\top \dot{e}. \quad (29)$$

It shows from (27) that  $e$  is uniformly bounded, and hence  $\ddot{V}(e, \tilde{w}_1, \tilde{w}_2)$  is bounded. This implies that  $\dot{V}(e, \tilde{w}_1, \tilde{w}_2)$  is uniformly continuous. By Barbalat's Lemma, then  $\lim_{t \rightarrow \infty} e(t) = 0$ . Therefore, the proof is completed. ■

## 4. SIMULATION RESULTS

The performance of the proposed approach is evaluated numerically in this section. The parameters of quadrotor UAV used in this simulation is presented in Table 1

Table 1. The parameters of a quadrotor UAV

Parameter name	Notation	Value
Mass	$m$	0.52kg
Gravity acceleration	$g$	9.8m/s <sup>2</sup>
Translational drag coefficient	$k_t$	0.95
Arm length	$l$	0.205m
Drag factor	$d$	7.5e <sup>-7</sup> kg.m <sup>2</sup> g
Inertia of x-axis	$I_x$	0.0069kg.m <sup>2</sup>
Inertia of y-axis	$I_y$	0.0069kg.m <sup>2</sup>
Inertia of z-axis	$I_z$	0.0129kg.m <sup>2</sup>

To maintain the translational motions, the simulation is conducted using a virtual PD controller (8) with  $K_P = K_D = \text{diag}([100 \ 100 \ 100])$ . In another side, the Theorem 3.1 is proposed to maintain the attitude motions. The gains of (25) and (26) are selected to be

$$\alpha = 10000, \gamma_1 = 0.1, \gamma_2 = 0.01.$$

The simulation results for tracking control of UAV for both the translational and rotational motions are illustrated in Figures 1-4. We can verify that all states of UAV can follow the desired trajectories, as concluded in Theorem 3.1. Moreover, we also present the performance of adaptation law estimation error as presented in Figure 5 and 6. The profiles of torque  $\tau$  and UAV motions in three-dimension (3D) can be seen in Figure 7 and 8, respectively.

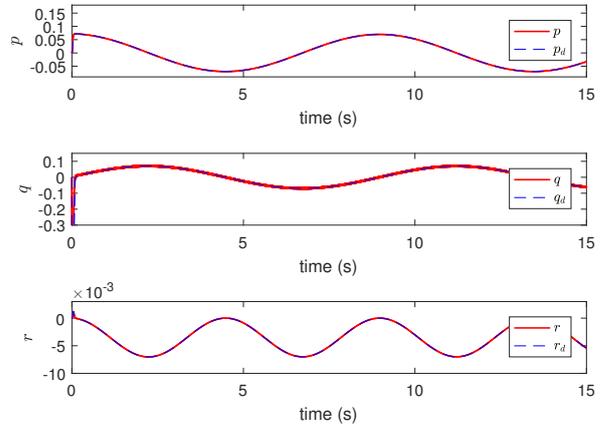


Fig. 1. Profile of  $p$ ,  $q$  and  $r$

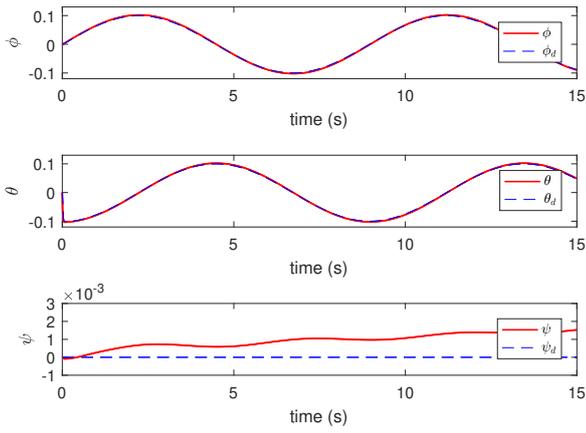


Fig. 2. Profile of  $\phi$ ,  $\theta$  and  $\psi$

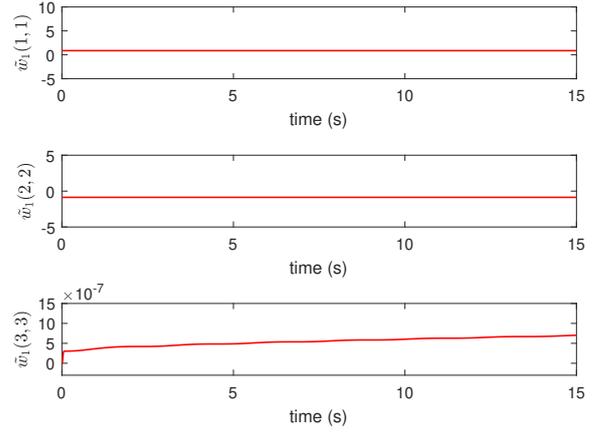


Fig. 5. Profile of estimation error  $\tilde{w}_1$

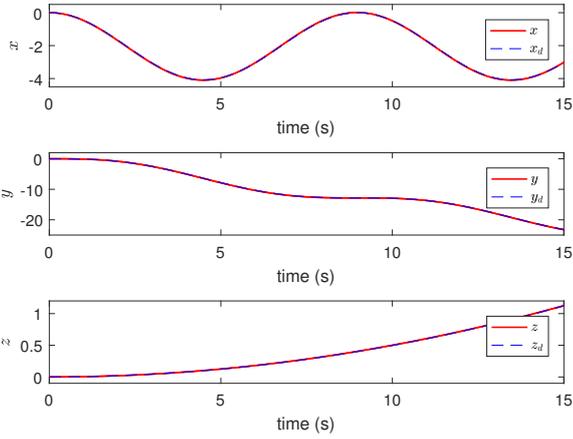


Fig. 3. Profile of  $x$ ,  $y$  and  $z$

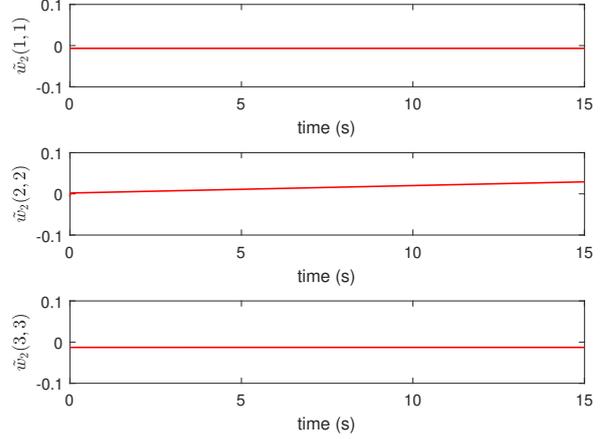


Fig. 6. Profile of estimation error  $\tilde{w}_2$

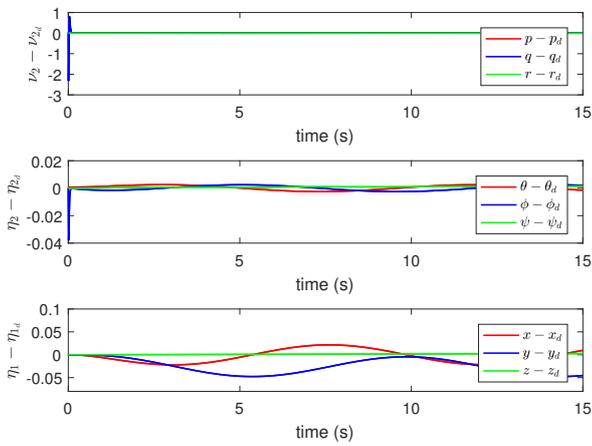


Fig. 4. Profile of tracking error trajectories

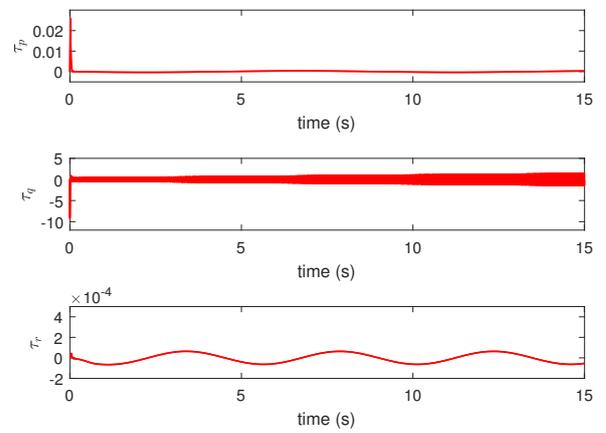


Fig. 7. Profile of  $\tau$

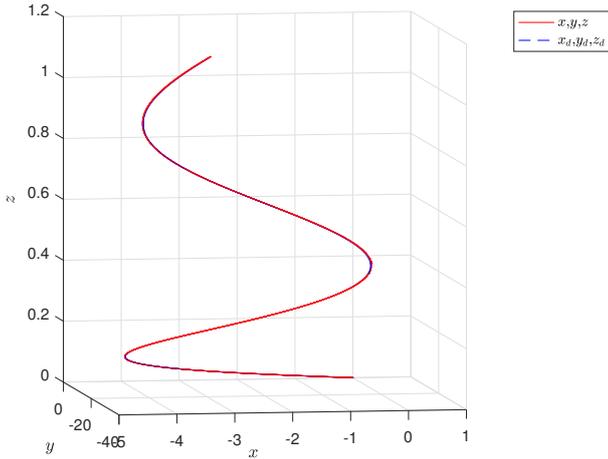


Fig. 8. Profile of  $x$ ,  $y$  and  $z$

## 5. CONCLUSION

We present a fully tracking control for 6-DOF of UAV with unknown inertia. A virtual PD controller is proposed for the tracking position control. Our main contribution is to design an adaptive controller for rotational dynamics with the presence of unknown inertia in the control input structure as well as in the nonlinear function. An adaptive controller is designed to handle the nonlinearities with uncertainties. The effectiveness of the tracking controller is presented in the rigorous proof by applying Barbalat's Lemma. To demonstrate the performance of our approaches, a simulation is conducted for a mini-quadrotor. It will be interesting to apply our scheme for practical implementation in the future work.

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