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Equity Market Connectedness across Regimes of Geopolitical Risks: Historical Evidence and Theory

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Abstract

We use a threshold VAR model to capture connectedness of the equity returns of the G7 in a regime-contingent manner as defined by low- and high-geopolitical risks (GPR). We find that connectedness is statistically stronger when GPR is at its higher rather than lower regime, but more importantly, this observation can be associated with threats of geopolitical adverse events, rather than with their actual realization. To explain our empirical observations we employ a model of international trade in assets and international relative asset prices. We introduce uncertainty in the future dividend payments combined with ambiguity aversion of agents to changes in the expected dividends. This allows us to model a geopolitical threat as a shock that affects the level of ambiguity about future dividends. At the same time, a geopolitical act is defined as a shock to the current period endowment of a given country, with limited effects on asset prices and returns. Our obtained results have important portfolio allocation implications for investors.

JEL Classification*: C32, F12, F40, G12, G15*

Keywords*: Geopolitical Risk, Equity Market Connectedness, Threshold VAR, Asset Trade, Multi-Country Macroeconomic Model*

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1 Introduction

The crashes in S&P500 index caused by the 9/11 terrorist attacks in New York and the 2004 and 2005 attacks in Madrid and London, have revealed that GPR is a new type of risk that investors and financial institutions may be facing. Several contributions such as (Pastor and Veronesi, 2012, 2013; Lehkonen and Heimonen, 2015; Kelly et al., 2016; Goel et al., 2017) document that stock markets are not immune to the (geo)political environment.

On the other side, with the increasing global financial integration, the analysis of cross-countries stock return correlations has recently become of central importance to both international finance literature as well as to investment practitioners, especially those involved in global financial markets. Along these lines, recent research highlights the importance of discount rate factors in the time variation of global equity market correlations or comovement, partially driven by changes in the level of risk aversion in financial markets (Miranda-Agrippino and Rey, 2015; Demirer et al., 2018; Rey, 2015; Xu, 2019; Pastor and Veronesi, 2020; Bekaert et al., forthcoming). Geopolitical risks, often cited by central bankers, the financial press, and business investors, lead to significant shifts in the state of the economy, earnings projections as well as in the level of risk aversion and uncertainty (Caldara and Iacoviello, 2019). On these grounds one can hypothesize that GPR can drive stock market comovements.

Motivated by the aforementioned issues, we study the relation between GPR and equity market comovement and we add to the existing literature in two aspects. From an empirical perspective, we analyze, for the first time, the regime-specific role of overall GPR on the connectedness of stock returns of the G7 (Canada, France, Germany, Italy, Japan, the United Kingdom (UK), and the United States (US)). We utilize the GPR index developed by (Caldara and Iacoviello, 2019) which captures "the risk associated with wars, terrorist acts, and tensions between states that affect the normal and peaceful course of international relations". On top of being tractable and auditable, the GPR general index is split in two sub-indices which allows us to distinguish between risks emanating from geopolitical threats

(GPT) and the actual realization of such adverse events, called geopolitical acts (GPA). We use the longest possible span of monthly historical data available for these economies, covering 1924 to 2020, and in the process we would also minimize the possibility of sample selection bias. Besides availability of data, the choice of G7 equity markets is primarily motivated by their importance in the global financial system, with these countries representing nearly two-third of global net wealth, and nearly half of world output. Empirically speaking, we compute the index of stock market connectedness with the method proposed by Diebold and Yilmaz (2009); Diebold and Yılmaz (2014). While they rely on the variance decomposition derived from a linear Vector Autoregressive (VAR) model, we use a threshold VAR (TVAR) model instead, as developed by Alessandri and Mumtaz (2019) which allows us to highlight the varying degree of stock market returns connectedness across endogenously determined phases of the GPR, based on the correlation asymmetry phenomena.

Using this framework, we find that connectedness is statistically stronger when GPR is at its higher rather than lower regime. More importantly, this observation can be associated with risks due to the threat of geopolitical risks rather than the actual realization of such events. While in agreement with studies showing that terrorist attacks have short-lived effects on stock markets with limited spillovers across the borders (see (Chesney et al., 2011; Goel et al., 2017; Balcilar et al., 2018b), it should be emphasized that our results are also closely linked to the way the GPT and GPA indexes are constructed. GPT is obtained through an automated search that identifies articles containing references to geopolitcal, nuclear, war and terrorist threats (or risks or fear or uncertainty) while GPA is associated only to the occurrence of war acts and terrorist acts. Thus, if -preceding or following a terrorist act- there are increasing concerns about future adverse events, these heightened fears are all captured by the GPT rather than GPA.

The second contribution of this paper is providing a theoretical framework to interpret our novel empirical results. Specifically, we start by developing a model of international trade in assets and international relative asset prices in a two country–two period framework. We build upon the model of Martin and Rey (2004) and extend it along two dimensions. First, we introduce uncertainty in the future dividend payments combined with ambiguity aversion of agents to changes in the expected dividends. This allows us to model a geopolitical threat as a shock that affects the level of ambiguity about future dividends. On the other side, a geopolitical act is defined as a shock to the current period endowment of a given country, with limited effects on asset prices and returns. This model is in line with the behaviour of investors who lack confidence about estimation of events, forming expectations using a worst case probability, as in Epstein and Schneider (2010) and Ilut and Schneider (2014), or the threat of adverse events causing agents to reassess macroeconomic tail risks, as in Kozlowski et al. (2019). Finally, we extend the benchmark model of two countries to a model of n countries, and we show that our main empirical results continue to hold in a more general framework as well.

To keep the analysis tractable, our simple theoretical framework discards many aspects of international asset allocation such as inflation and exchange rate risk, cross-country informational deficiencies, and human capital and labor, to name a few. While these factors are definitely important, they may obfuscate the focus of the article.

The focus on GPR in explaining equity market comovements is grounded on the increasing importance that this index has received recently. Notably, as highlighted by (Caldara and Iacoviello, 2019) geopolitical uncertainties are considered a salient risk to the economic outlook in both the World Economic Outlook of the International Monetary Fund as well as in the Economic Bulletin of the European Central Bank. Moreover, in the Gallup survey conducted in 2017, 75 percent of the 1000 investors respondents, expressed concerns about the economic impact of the various military and diplomatic tensions taking place around the world, ranking GPR ahead of political and economic uncertainty. In fact, few recent studies document the existence of a link between GPR and the financial makets empirically (see Bouri et al. (Forthcoming); Baur and Smales (2020) and references therein).

One must point out that there is indeed a vast existing literature that relates terror attacks on stock market movements (see among others Glaser and Weber (2005); Chuliá et al. (2009); Chesney et al. (2011); Goel et al. (2017); Balcilar et al. (2018b)). Unlike these studies which base their conclusions on an event study approach, our analysis uses the GPR index which includes not only terror attacks, but also other forms of geopolitical tensions such as war risks, military threats, and Middle East tensions. Moreover the GPR clearly distinguishes between the heightened threats of adverse events and their actual realization. Hence, these indices allow us to capture GPRs of various forms in a continuous fashion, going beyond the effect of specific events at specific points in time. This allows us to develop more sophisticated empirical models while separating the channel of GPT from the GPA.

At this stage, it must be emphasized that, analyzing correlation or connectedness of equity returns is a critical input not only for asset allocation decisions, but also for risk management and hedging applications. Consequently, there is a large literature on equity market correlations, documenting the presence of a conditional pattern in return correlations with respect to market conditions (see for example, Forbes and Rigobon (2002); Morana and Beltratti (2008); Bekaert et al. (2009); Hou et al. (2011)). The so-called correlation asymmetry phenomenon reported in a number of studies including Longin and Solnik (1995, 2001); Ang and Chen (2002); Ang and Bekaert (2002); Campbell et al. (2002); Goetzmann et al. (2005), among others, refers to the asymmetric pattern in which equity returns tend to be more correlated during bear market regimes as well as during periods of extreme price fluctuations, which could be associated with periods of heightened GPR. Building on this evidence, Krishnan et al. (2009) further documents the presence of a correlation risk premium in returns (after controlling for asset volatility and other risk factors). Hence, understanding the drivers of comovement is not only a topic of interest for effective international diversification strategies, but also has implications for pricing and hedging.

The remainder of the paper is organized as follows: Section 2 outlines the empirical model along with the data, while Section 3 presents the results. Section 4 develops the theoretical framework for 2 and n countries, with Section 5 concluding the paper.

2 Empirical model and data

2.1 Data

The data used in this paper are monthly stock price indices of the G7 countries namely, Canada (S&P TSX 300 Composite Index), France (CAC All-Tradable Index), Germany (CDAX Composite Index), Italy (Banca Commerciale Italiana Index), Japan (Nikkei 225 Index), the UK (FTSE All Share Index), and the US (S&P500 Index). The data on the stock indices are derived from Global Financial Data, and covers the monthly period of 1924:07 to 2020:01.[1] The stock price indices are converted into log returns in percentage, i.e., the first-difference of the natural logarithm of the indices multiplied by 100.

The geopolitical regimes are determined using the indices associated with GPR developed by Caldara and Iacoviello (2019).[2] Caldara and Iacoviello (2019) construct the GPR index by counting the occurrences of words related to geopolitical tensions, derived from automated text searches in the three newspapers for which electronic access to all articles is available from 1899 through ProQuest Historical Newspapers – The New York Times, the Chicago Tribune, and the Washington Post. Then, Caldara and Iacoviello (2019) calculate the index by counting, for each newspaper, the number of articles that contain the search terms for each month starting in 1899. The search identifies articles containing references to six groups of words: Group 1 includes words associated with explicit mentions of geopolitical risk, as well as mentions of military-related tensions involving large regions of the world or US involvement. Group 2 includes words directly related to nuclear tensions. Groups 3 and 4 include mentions related to war threats and terrorist threats, respectively. Finally, Groups 5 and 6 capture press coverage of actual adverse geopolitical events (as opposed to just risks) which can be reasonably expected to lead to increases in geopolitical uncertainty, such as terrorist acts or the beginnings of wars. Understandably, Groups 1 to 4 capture threats of

[1] https://globalfinancialdata.com/.

[2] GPRs data are available for download from: https://sites.google.com/view/dariocaldara/geopolitical-risk?authuser=0.

geopolitical risk, while Groups 5 and 6 encompass actual acts of geopolitical risk.

Since acts and threats could both produce varied impacts on stock market comovements, we conduct our analyses based on global threats and acts separately, as well as the overall GPR. In fact, Caldara and Iacoviello (2019) points out that geopolitical threats predict geopolitical acts (though not the other way round), suggesting that the threat index could contain signals about future geopolitical developments, to the extent that geopolitical threats have a stronger influence than acts on US macroeconomic and financial variables. Note that, the GPR indices are filtered as per Hamilton (2018), which is necessary to allow for the identification of regimes that are above and below a threshold.

2.2 DY connectedness index

We compute the index of financial connectedness with the method proposed by Diebold and Yilmaz (2009), Diebold and Yılmaz (2014) (hereafter DY). DY approach builds connectedness measures from pieces of variance decomposition. In VAR models, variance decomposition indicates how much of the forecast error variance of each variable in the model can be explained by shocks to other variables. For example the H-step forecast error variance $\mathrm{d}^H{}_{i,j}$ is the fraction of variable i's H-step ahead forecast variance due to shocks in variable j. The connectedness index is given by the sum of the off-diagonal entries in the variance decomposition matrix. It should be noticed that contemporaneously (when $H = 0$), this coincides with the sum of the covariances in the variance-covariance matrix.

2.3 Threshold VAR model

Our analysis tests the hypothesis that there is a shift in the behaviour of stock market returns across different phases of geopolitical risk. This makes using a constant parameter model an unsuitable option. We deal with the variation of parameters across regimes by means of a threshold VAR model (TVAR).

Following Alessandri and Mumtaz (2019) in this section we introduce the TVAR model defined as:

$$Y_t = \left[c_1 + \sum_{j=1}^{P} B_{1,j} Y_{t-j} + \Omega_1^{1/2} e_t\right] S_t + \left[c_2 + \sum_{j=1}^{P} B_{2,j} Y_{t-j} + \Omega_2^{1/2} e_t\right] (1 - S_t) \tag{2.1}$$

where

$$S_t = 0 \Leftrightarrow Z_{t-d} \leq Z^* \tag{2.2}$$

The matrix of endogenous variables in the first specification is Y_t and contains stock return data for each of the G7 countries. Given the monthly frequency of data we choose a lag length of 12. The model allows for two regimes determined by the level of the threshold variable Z_{t-d} relative to an unobserved threshold level Z^*. In our analysis the threshold variable is assumed to be the d^{th} lag of the geopolitical index and its two sub-indexes, namely the Threat index (GPT) and Act index (GPA), while the delay d is unknown and estimated in the model.

The threshold variable is assumed to cause the switch across regimes in a deterministic way. The regimes identified by this specification are high and low geopolitical risk.

The high number of the parameters to be estimated favors the choice of Bayesian methods for the estimation strategy. In the spirit of Bańbura et al. (2010) we impose a natural conjugate prior for the parameters via dummy observations:

$$Y_{D,1} = \begin{pmatrix} \frac{diag(\gamma_1 \sigma_1 ... \gamma_N \sigma_N)}{\tau} \\ 0_{N \times (P-1) \times N} \\ .... \\ diag(\sigma_1 .... \sigma_N) \\ .... \\ 0_{1 \times N} \end{pmatrix}; \; X_{D,1} = \begin{pmatrix} \frac{J_P \otimes diag(\sigma_1 .... \sigma_N)}{\tau} & 0_{NP \times 1} \\ 0_{N \times NP} & 0_{N \times 1} \\ .... & \\ 0_{1 \times NP} & c \end{pmatrix} \tag{2.3}$$

where γ_1 to γ_N denotes the prior mean for the coefficients on the first lag, τ is the tightness of the prior on the VAR coefficients and c is the tightness of the prior on the constant terms. The prior means are determined as the OLS estimates of the coefficients of an AR(1) regression estimated for each endogenous variable using a training sample. The σ_i scaling factors are chosen using the standard deviation of the error terms from the preliminary AR(1) regressions. As is standard in the literature, we set the overall tightness parameter τ to 0.1 while the prior on the constant is imposed to 1. Additionally we introduce a prior on the sum of the lagged dependent variables by adding the following dummy observations:

$$Y_{D,2} = \frac{diag(\gamma_1\mu_1....\gamma_N\mu_N)}{\lambda};\ X_{D,2} = \begin{pmatrix} \frac{(1_{1\times P})\otimes diag(\gamma_1\mu_1....\gamma_N\mu_N)}{\lambda} & 0_{N\times 1} \end{pmatrix} \tag{2.4}$$

where μ_i denotes the sample means of the endogenous variables calculated using the training sample. As customary in the literature, the tightness on the sum of coefficients is set to $\lambda = 10\tau$. Given the natural conjugate prior the posterior distribution takes the form:

$$p(B_i \setminus \Sigma_i) \sim \mathcal{N}(B_i^*, \Sigma_i \otimes (X_i^{*'} X_i^*)^{-1}) \tag{2.5}$$

$$p(\Sigma_i \setminus B_i) \sim \mathcal{IW}(S_i^*, T_i^*) \tag{2.6}$$

where

$$B_i^* = (X_i^{*'} X_i^*)^{-1}(X_i^{*'} Y_i^*) \tag{2.7}$$

$$S_i^* = (Y_i^* - X_i^* b_j)'(Y_i^* - X_i^* b_j)$$

for i=1,2 denoting the two regimes ; $Y^* = [Y; Y_{D,1}; Y_{D,2}]$, $X^* = [X; X_{D,1}; X_{D,2}]$ and b_j is the draw of the VAR coefficients B reshaped to be conformable with ${X^*}_i$ while T_i^* is the number of rows of $Y^*{}_i$. We impose a normal prior for $Z^* \sim N(z, 10)$, where z is the 90th percentile of the GPR index. We assume a flat prior for the delay parameter d but we limit its values between 1 and 12.

To simulate the posterior distribution of the unknown parameters we use a Gibbs sampler with a Metropolis - Hastings step (see Appendix A for details).

Generalized identification. Koop et al. (1996) introduce the generalized impulse response functions (hereafter GIRF) that are more appropriate than standard impulse responses in case of non-linear models. GIRFS are defined as follows:

$$GIRF_t^S = E(Y_{t+k} \setminus \Psi_t, Y_{t-1}^S, \mu) - E(Y_{t+k} \setminus \Psi_t, Y_{t-1}^S) \tag{2.8}$$

where Ψ_t denotes all the parameters and hyper-parameters of the model, k is the forecasting horizon under consideration, $S = 0, 1$ denotes the regime and μ is the shock. Equation 2.8 characterizes the GIRF as the difference between two conditional expectations, one in which we condition on the structural shock μ , and one in which we assume the shock to be equal to zero. The estimation of GIRF requires Monte Carlo simulations in which the impulse response for each regime is calculated for all possible values of that specific regime and the average response conditioned on that regime is obtained. We define the shock as:

$$\mu_j^s = \Sigma_s v_j / \sqrt{\Sigma_{s,jj}} \tag{2.9}$$

where s identifies the regime (low or high geopolitical risk), j is the endogenous variable on which we apply the shock while v_j is a $1 \times N$ selection vector with its j^{th} element equal to unity and zeros elsewhere. The GIRF are unique and are not affected by the reordering of the variables. This is an appealing characteristic for our application where the ordering of the variables would be hard to justify[3]. In the generalized framework the shocks are

[3] The Generalized variance decomposition is preferred in most of the DY index applications for its invariance to the ordering of variables.

not necessarily orthogonal. Since our focus is only on the general connectedness index, this should not affect our results.

3 Results

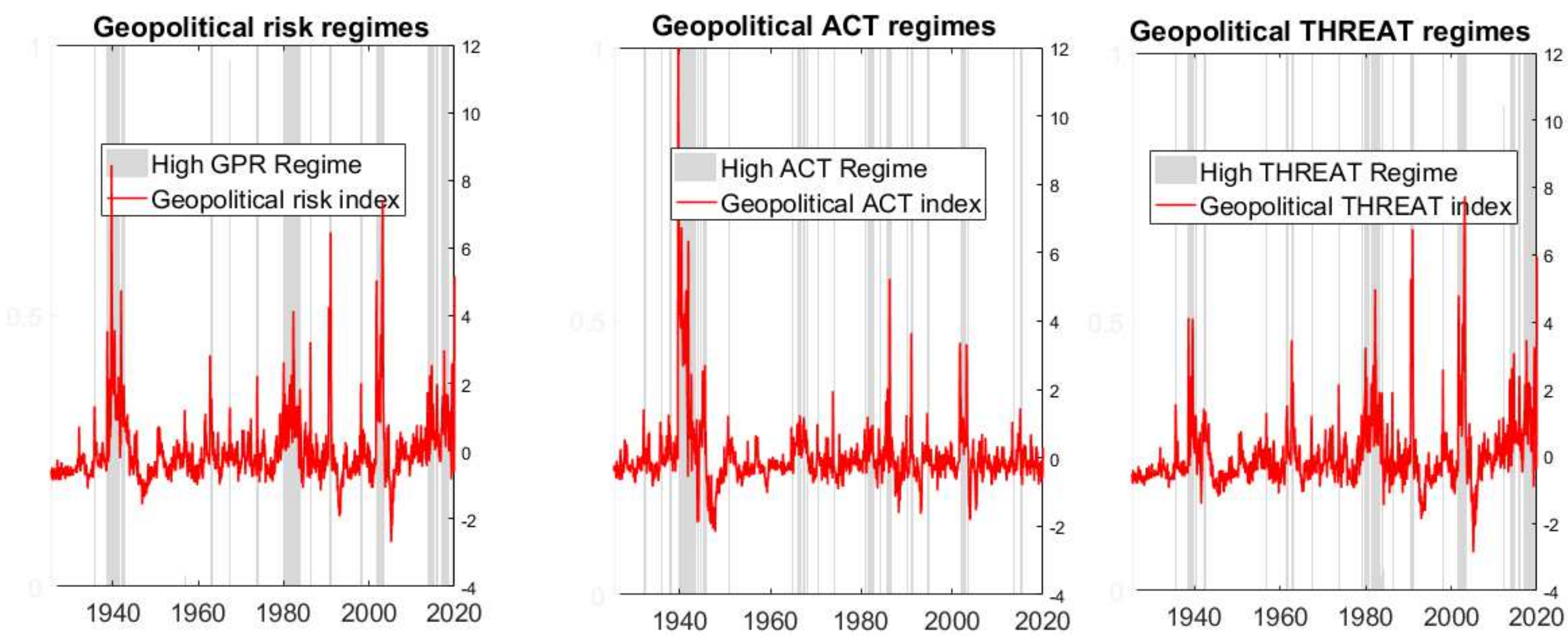
We present the features of the regimes identified by the TVAR model described by equations 2.1-2.2. The regimes are introduced in Figure 3.1. The gray area represents the median estimate of $1-S_t$ which is equal to 1 when the threshold variable Z_{t-d} is below the estimated threshold level Z^*. The threshold variable is given by the GPR index or one of its two sub-indexes, GPT and GPA respectively. We interpret these regimes as high and low global geopolitical risk regimes. The GPR general index raises in the occasion of the World War II (WW2), in early 80s with the intensification of Cold War and Middle East tensions (e.g. the Arab-Israeli conflict and Iran-Iraq war), in early 2000s around the World Trade Center attack and in the last decade. Regarding the two sub-indexes, the high GPA regime is covering most of the WW2, and a few episodes in early 80s and the 2001 crisis. The high GPT regime is less represented during the WW2 but cover a wide period in the early 80s and are the main driver of the general GPR in the last decade characterized by a high level of geopolitical threats.

Figure 3.2 shows the financial connectedness index in high and low GPR regime. Median across saved draws is reported for the 1 year (1Y) and 5 years (5Y) ahead forecasting horizon. The connectedness index is noticeably larger in high GPR regime. This result is valid for the GPT regime while no relevant variation is observed across GPA regime. As described in Caldara and Iacoviello (2019), one possible interpretation of the asymmetric effects of acts and threat regimes is that the act component of GPR leads to a resolution of the uncertainty around a particular set of events. By contrast, during threat regimes asset prices are affected because there is an increase in uncertainty and in the probability of future adverse events.

We attest the statistical significance of our findings as follows. We compute the difference in the connectedness index in high and low GPR regime for each saved draw. We plot the median of this difference together with the 95 high probability density intervals (HPDI). Results reported in figure 3.3 show that, unlike for the act regime, the difference in the financial connectedenss across threat regimes is statistically greater than zero for most of the forecasting horizons. These findings are in line with (Chesney et al., 2011; Goel et al., 2017) who document empirically that actual terror attacks have a short-lived impact on stock markets and with (Brounen and Derwall, 2010) and (Balcilar et al., 2018b) who show that there is limited evidence of any cross border effects of terrorist attacks. Moreover, (Baur and Smales, 2020) employing the same GPT ad GPA sub-indexes as we do, find that safe heaven assets react to geopolitical threats but not to acts. From a theoretical perspective, our results suggest that investors may be forming expectations using a worst case probability scenario, as in Ilut and Schneider (2014), or that the threat of adverse events is causing agents to reassess macroeconomic tail risks, as in Kozlowski et al. (2019). However, we develop a formal theoretical model to provide explanation to our empirical findings along the lines of the above studies.

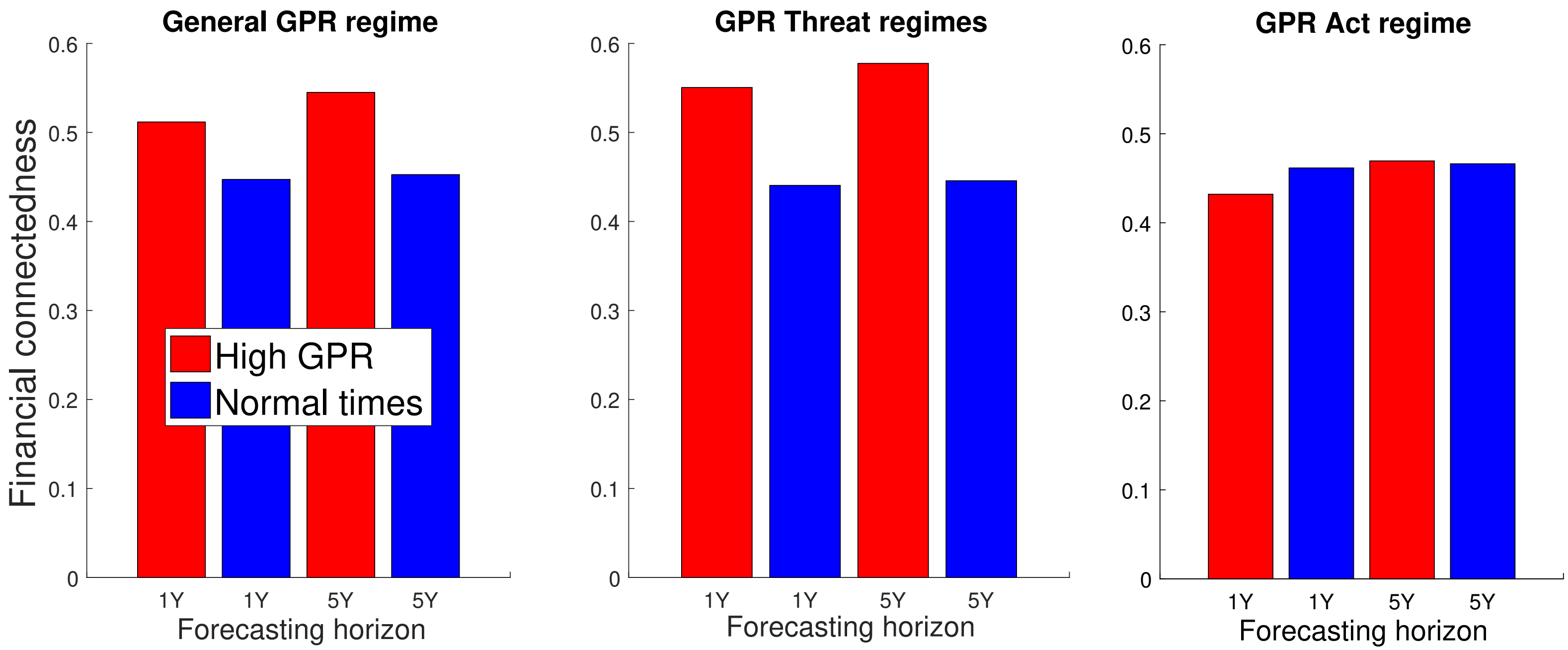
Sensitivity checks. We first check the sensitivity of our results to the prior mean z of the threshold level Z^* choosing z as the 80th percentile of the GPR index rather than the 90th percentile in the benchmark model. Results from this exercise are reported in Figure B.1 in the Appendix B and show that our main findings hold. Finally, we test the robustness of our estimates to the filtering method used for the GPR index, our threshold variable. We replace the benchmark Hamilton's filter with the HP filter. The results from this experiment are reported in Figure B.2 and show that our conclusions are, for all practical purposes, unchanged.

Figure 3.1 – Model-based geopolitical risk regimes



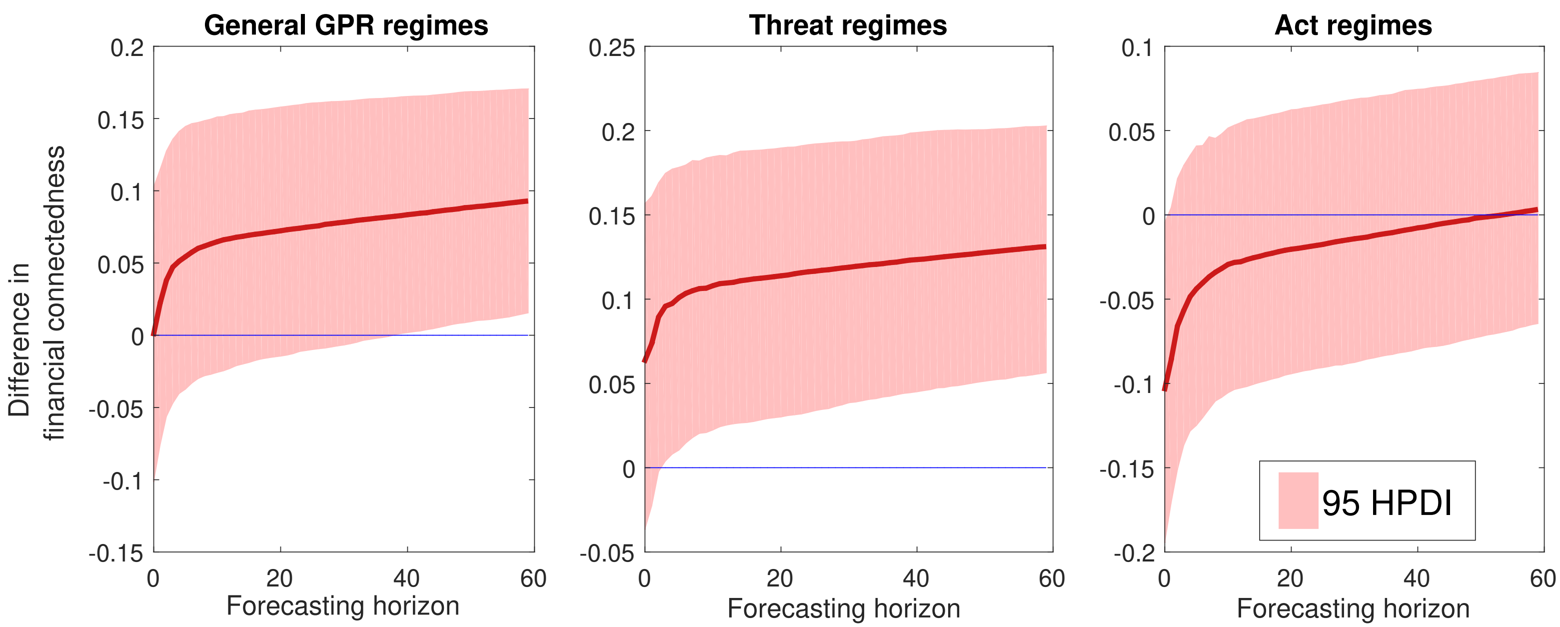
Grey bands represent the high geopolitical risk regimes for the GPR index (left), GPA index (middle) and GPT index (right). Regimes are model-based determined.

Figure 3.2 – Financial connectedness and geopolitical regimes



The figure reproduces the median of the connectedness index in high(red bar) and low (blue bar) GPR regime. The index is reported 1 year ahead (1Y) and 5 years ahead (5Y) for GPR regime (left), GPT regime (middle) and GPA regime (right). Forecasting horizon on the X axis.

Figure 3.3 – Difference in the financial connectedness across high and low GPR regimes.



The figure reports the difference in the connectedness index between high and low GPR regime (left), GPT regime (middle) and GPA regime (right). Red line is the median across saved draws; bands represent 95 HPDIs.

4 Theoretical model

In this section, we lay out a formal model for geopolitical risk to investigate the connectedness of returns and investment decisions among countries following an increase in geopolitical act compared to an increase in geopolitical threat. The model is constructed to be consistent with our main empirical findings - that connectedness of asset returns increases with GPR associated threats while it is unaffected by changes in acts of GPR - thus it helps us interpret them. In addition, the model has an auxiliary result, namely that stock returns as well decline with GPT while it does not react to shocks in GPA which is consistent with Caldara and Iacoviello (2019).

Our basic setup builds on Martin and Rey (2004) who model international trade in assets and international relative asset prices in a two countries - two periods framework. We depart from this baseline setup along two dimensions. First we introduce uncertainty in the future dividend payments combined with ambiguity aversion of agents to changes in the expected dividends. This allows us to model a geopolitical threat as a shock that affects the level of ambiguity about future dividends. And second, a geopolitical act is defined as a shock to the current period endowment of a given country, with limited effects on asset prices and returns. Understandably, the way we define GPT and GPA in the theoretical model is consistent with how these two indices are measured, as discussed in detail above in the data segment of the paper associated with the TVAR model to produce the results of regime-specific connectedness. Finally, for ease of exposition we start by investigating a benchmark of two countries; we then extend the model to include n countries and we show that our main results hold in this framework as well.

4.1 Benchmark model: two countries

We start by examining a benchmark two-period model of two countries A and B with $N_i = \{1, 2, ..., n_i\}$ being the set of n_i agents in country $i \in \{A, B\}$ identical ex ante. For simplicity, we assume that agents are identified by their country A and B. In the first period t, every agent is endowed with y_i ($i \in \{A, B\}$) units of traded goods. Agents are assumed risk averse and each faces the decision of whether to consume, invest in risky projects or trade on the stock market. We define Z_i as the set of z_i fixed-size projects developed in country i, at a cost $f(z_i)$ assuming it is increasing in the number of projects and convex ($f'(.) > 0, f''(.) > 0$). The consumption in period $(t+1)$ consists of the dividends raised for the different projects, which depends on the L exogenous and equally likely states of nature, each occurring with a probability $\frac{1}{L}$. We adopt the assumption of an incomplete market such that $\sum_i z_i < L$.

We depart from Martin and Rey (2004) who assume exogenous dividend, and define the dividend of a project h as $d(1 + \mu_{t+1})$ if state $h \in \{1, ..., L\}$ occurs and is equal to zero otherwise, such that $\mu_t \in [-a_t, -a_t + 2\,|a_t|]$ and $a_{t+1} = \rho a_t + \epsilon_t, \epsilon_t \sim \mathcal{N}(0, \sigma_\epsilon)$. μ_t and a_t represent the prior beliefs and the level of ambiguity respectively.

Furthermore, we assume that agents lack confidence in the estimation of events, and are therefore ambiguity averse. Following Epstein and Schneider (2010) and Ilut and Schneider (2014), ambiguity averse agents evaluate payoffs using a worst case scenario probability selected to minimize the expected utiliy. The minimizing mean chosen will be $\mu_{t+1} = E(-a_{t+1}) = -\rho a_t$. An increase in ambiguity is reflected in the increase of the width of the interval (and vice-versa), making the worst mean worse. In our setting, an increase in the level of ambiguity a_t is associated with the occurance of a geopolitical threat.

Assumption 1 (Ambiguity Aversion) Agents evaluate their payoffs using a worst case probability, and choose μ_t to be $-\rho a_t$.

We denote c_{it} as agent $i's$ consumption in period t, s^i_j as the demand of agent from country j for asset sold by agent from country i and α_i as the proportion of asset developed and sold by agent i, with p_i being the price of that asset. which will increase in the event of a geopolitical threat. International trade of assets involves a transaction cost τ which is assumed to be paid by the buyer and eliminated from the share.

The following assumption is fundamental to our model as it characterises the difference between a geopolitical threat and a geopolitical act.

Assumption 2 (Act vs. Threat) A GPA is a shock to the agent's endowment y_i and results in a reduction of consumption in the current period; whereas a GPT increases ambiguity a_t without affecting future dividends.

Accordingly, the budget constraint for a representative agent of country A (symmetric for agent from B) can be written as follows:

$$c_{At} + f(z_A) + (n_A - 1)p_A s_A^A + n_B(1+\tau)p_B s_A^B = y_A + p_A\alpha_A \tag{4.1}$$

The income of the agent appearing on the right-hand side of equation 4.1 consists of the initial endowment and the revenue from the part sold of her project. The expenditure side of the equation includes consumption at time t, the cost of investment in the projects $f(z_A)$, and the demands for domestic assets $(n_A - 1)p_A s_A^A$ as well as foreign assets $n_B(1+\tau)p_B s_A^B$.

The consumption of the agent from country A in the second period $(t+1)$ involves the dividends of her asset determined by the state of nature that occurs:

$$c_{A(t+1)} = \begin{cases} d(1+\mu_{t+1})s_A^i & \text{if } i \in Z_A \text{occurs} \\ d(1+\mu_{t+1})(1-\tau)s_A^j & \text{if } j \in Z_B \text{occurs} \\ d(1+\mu_{t+1})(1-\alpha_A) & \text{if } A \in Z_A \text{occurs} \\ 0 & \text{otherwise} \end{cases} \tag{4.2}$$

where $(1-\alpha_A)$ is the portion of the asset the agent developed that she keeps to herself.

We consider the following expected utility function of agent in country A (analogously for agent in country B)

$$EU_{At} = c_{At} + \beta E\left(\frac{c_{A(t+1)}^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}}\right) \tag{4.3}$$

such as β is the discount rate and σ is the inverse degree of risk aversion, assuming $\sigma > 1$ as in Martin and Rey (2004).

The agent chooses c_{At}, z_A, s_A^i and α_A that maximise her expected utility EU_{At}.

Substituting for c_{At} and $c_{A(t+1)}$ in EU_{At}, we obtain the following objective function

$$EU_{At} = y_A + \alpha_A p_A - f(z_A) - (n_A - 1)p_A s_A^A - n_B(1+\tau)p_B s_A^B$$

$$+ \frac{D(1-\rho a_t)^{1-\frac{1}{\sigma}}}{(1-\frac{1}{\sigma})}\left((n_A - 1)s_A^{A^{1-\frac{1}{\sigma}}} + (1-\tau)^{1-\frac{1}{\sigma}} n_B s_A^{B^{1-\frac{1}{\sigma}}} + (1-\alpha_A)^{1-\frac{1}{\sigma}}\right) \tag{4.4}$$

where $D \equiv \beta d^{1-\frac{1}{\sigma}}/L$.

Proposition 1. *The solution of the maximisation problem of the agent from country A is given by:*

$$\begin{cases} s_A^A = D^\sigma(1-\rho a_t)^{\sigma-1}p_A^{-\sigma} \\ s_A^B = D^\sigma(1-\rho a_t)^{\sigma-1}(1-\tau)^{\sigma-1}(1+\tau)^{-\sigma}p_B^{-\sigma} \\ \alpha_A = 1 - D^\sigma(1-\rho a_t)^{\sigma-1}p_A^{-\sigma} \end{cases} \tag{4.5}$$

Proof. See Appendix C for the proof of proposition 1 along with all the remaining proofs. □

The equilibrium condition for every stock market is equating sold shares of an asset to the sum of aggregate domestic demand and aggregate foreign demand:

$$\alpha_A = (n_A - 1)s_A^A + (1+\tau)n_B s_B^A \tag{4.6}$$

The equilibrium asset prices are therefore

$$p_A = (1-\rho a_t)^{\frac{\sigma-1}{\sigma}} D(n_A + n_B\phi)^{\frac{1}{\sigma}} \tag{4.7}$$

where $\phi \equiv \left(\frac{1-\tau}{1+\tau}\right)^{\sigma-1}$ such that $\phi < 1$ since $\sigma > 1$, where ϕ depicts the level of financial integration (see Martin and Rey (2004)).

Substituting for the respective prices, equilibrium demands become

$$
\begin{cases}
s_A^A = \frac{1}{n_A+n_B\phi} \\
s_A^B = \frac{(1-\tau)^{\sigma-1}}{(1+\tau)^{\sigma}(n_B+n_A\phi)} \\
\alpha_A = \frac{n_A-1+n_B\phi}{n_A+n_B\phi}
\end{cases}
\tag{4.8}
$$

We follow Martin and Rey (2004)'s definition of returns and expected returns of a representative agent respectively as follows

$$
R_A := \frac{d(1+\mu_{t+1})}{Lp_A}
$$

$$
E(R_A) = \frac{d(1-\rho a_t)}{L(1-\rho a_t)^{\frac{\sigma-1}{\sigma}} D(n_A+n_B\phi)^{\frac{1}{\sigma}}} = \frac{d(1-\rho a_t)^{\frac{1}{\sigma}}}{LD(n_A+n_B\phi)^{\frac{1}{\sigma}}}
$$

Lemma 2. *The covariance of returns between country A and country B is given by*

$$
COV(R_A, R_B) = \frac{d^{\frac{2}{\sigma}}}{\beta^2(n_A+n_B\phi)^{\frac{1}{\sigma}}(n_B+n_A\phi)^{\frac{1}{\sigma}}}\left[\frac{(1-\rho a_t)^2+\sigma_\epsilon^2}{(1-\rho a_t)^{\frac{2\sigma-2}{\sigma}}} - (1-\rho a_t)^{\frac{2}{\sigma}}\right]
$$

Proposition 3. *The covariance between returns of countries A and B increases with ambiguity at a rate decreasing with respect to financial integration and size, while it is unaffected by changes in the initial endowment hence by movements in GPA.*

Next we examine how changes in ambiguity and financial integration impact prices, investment and returns, as well as size effects.

Proposition 4. *(Comparative statics)*

The price of an asset decreases with ambiguity and increases with financial integration. The expected returns decrease both with ambiguity and financial integration.

Remark 5. *Prices, returns of assets and the covariance between returns of countries A and B are not affected by movements in initial endowment, hence by changes in the GPA.*

4.2 n countries

In this section, we examine a model of a set $N = \{1, 2, ..., n\}$ of n countries. There is a set M_i of m_i agents belonging to country $i \in N$ and Z_i is set of z_i projects developed by an agent in country i. The remaining definiftions, notations and assumptions are identical to those introduced in the case of two countries. Furthermore, the detailed derivations of the model can be found in Appendix D.

Proposition 6. *The demand functions solving the agent's maximisation problem are given by:*

$$\begin{cases} s_i^i = D^{\sigma}(1-\rho a_t)^{\sigma-1} p_i^{-\sigma} \\ s_i^j = D^{\sigma}(1-\rho a_t)^{\sigma-1}(1-\tau)^{\sigma-1}(1+\tau)^{-\sigma} p_j^{-\sigma} \\ \alpha_i = 1 - D^{\sigma}(1-\rho a_t)^{\sigma-1} p_i^{-\sigma} \end{cases}$$

We can obtain the equilibrium price and the equilibrium demands as follows:

$$p_i = D(1-\rho a_t)^{1-\frac{1}{\sigma}}(m_i + \phi \sum_{j\in N, j\neq i} m_j)^{\frac{1}{\sigma}}$$

and

$$\begin{cases} s_i^i = \frac{1}{m_i + \phi \sum_{j\in N, j\neq i} m_j} \\ s_i^j = \frac{(1-\tau)^{\sigma-1}}{(1+\tau)^{\sigma}(m_i + \phi \sum_{j\in N, j\neq i} m_j)} \\ \alpha_i = \frac{m_i + \phi \sum_{j\in N, j\neq i} m_j - 1}{m_i + \phi \sum_{j\in N, j\neq i} m_j} \end{cases}$$

The expected returns are written as

$$E(R_i) = \frac{d(1-\rho a_t)}{LD(1-\rho a_t)^{1-\frac{1}{\sigma}}(m_i + \phi \sum_{j\in N, j\neq i} m_j)^{\frac{1}{\sigma}}} = \frac{d(1-\rho a_t)^{\frac{1}{\sigma}}}{LD(m_i + \phi \sum_{j\in N, j\neq i} m_j)^{\frac{1}{\sigma}}}$$

The following corollary extends the comparative statics in Proposition 4 to the n countries framework. Moreover, it should be mentioned that as in the benchmark model, a change in GPA, reflected in the shock to the endowment, does not have an impact on the prices, returns and connectedness in returns.

Corollary 7. *An increase in ambiguity decreases prices, and an increase in financial integration increases prices. Moreover, both ambiguity and financial integration diminish expected returns.*

Next, we start by deriving the covariance across returns in order to inverstigate connectedness.

Lemma 8. *The covariance among returns on every pair of assets from two countries i and j, is given by*

$$COV(R_i, R_j) = \frac{d^{\frac{2}{\sigma}} \left(\frac{(1-\rho a_t)^2+\sigma_\epsilon^2}{(1-\rho a_t)^{2-\frac{2}{\sigma}}} - (1-\rho a_t)^{\frac{2}{\sigma}} \right)}{\beta^2 (m_i + \phi \sum_{k\in N, k\neq i} m_k)^{\frac{1}{\sigma}} (m_j + \phi \sum_{k\in N, k\neq j} m_k)^{\frac{1}{\sigma}}}$$

To measure connectedness in returns across the n countries, we define the Connectedness index I as

$$I \coloneqq \sum_{i=1}^{n} \sum_{j=1, i\neq j}^{n} Cov(R_i, R_j)$$

which can be rewritten as

$$I = \frac{d^{\frac{2}{\sigma}}}{\beta^2} \left(\frac{(1-\rho a_t)^2+\sigma_\epsilon^2}{(1-\rho a_t)^{2-\frac{2}{\sigma}}} - (1-\rho a_t)^{\frac{2}{\sigma}} \right) \sum_{i=1}^{n} \sum_{j=1, i\neq j}^{n} (m_i + \phi \sum_{k\in N, k\neq i} m_k)^{-\frac{1}{\sigma}} (m_j + \phi \sum_{k\in N, k\neq j} m_k)^{-\frac{1}{\sigma}}$$

Theorem 9. *The connectedness of returns among different countries increases with ambiguity.*

4.3 Consistency with the empirical findings.

Theorem 9 suggests that connectedness of returns across countries increases with the GPT risk while it is not affected by changes in GPA. This prediction is in line with our main empirical results. The intuition goes as follows: the increase in the act component of GPR is resolved in the period of occurrence hence it affects the current period endowment but not the expectations of the future, therefore the investment decisions do not change with GPA movements. As mentioned in Caldara and Iacoviello (2019), this lack of reaction of financial markets to act shocks, might as well arise since the act event could trigger a coordinated policy response that ends up giving protection on the worst possible outcomes.

By contrast, threat shocks increase uncertainty and send signals about future adverse events which we capture in our model by a surge in the ambiguity about future dividends. Hence ambiguity averse agents will synchronize in their investments decisions by evaluating payoffs using the worse case scenario probability. This leads to an increase in the financial connectedness as observed in the empirical exercise.

Our model has predictions for the financial first moments as well. Corollary 7 suggests that an increase in the GPT which translates in a raise in the ambiguity of future dividends, depresses stock returns since the ambiguity averse agents will give the highest probability of occurrence to the worse case scenario. For the same reasons expressed above, changes in GPA leave asset prices and returns unaffected. Although beyond the scope of our empirical exercise, these results have been validated by Caldara and Iacoviello (2019).

5 Conclusion

The study of the factors that drive the comovement of global equity markets is not only a topic of interest for effective diversification strategies, but also has implications for pricing and hedging. GPR associated with actual acts and threats are often cited by central bankers, the financial press, and business investors as a determinant of investment decisions, and hence, are believed to affect business cycles and financial markets. In this paper we analyzed the role of such risks in determining connectedness of the G7 equity markets. Using a long span of monthly historical data from 1924 to 2020, and a threshold VAR model to capture regime specific comovements, we showed that connectedness is statistically stronger when GPR is at its higher rather than lower regime, but more importantly, this observation can be associated with risks due to geopolitical threats, rather than actual acts. To explain our empirical observations, we developed a model of international trade in assets and international relative asset prices in a two country–two period framework. In this model, we introduced uncertainty in the future dividend payments combined with ambiguity aversion of agents to changes in the expected dividends, which allowed us to model a geopolitical threat as a shock that affects the level of ambiguity about future dividends. At the same time, a geopolitical act was defined as a shock to the current period endowment of a given country, with limited effects on asset prices and returns. We also highlight that our theoretical explanation continues to hold in a general model of n countries.

From the perspective of investor, our results tend to suggest that in the wake of heightened GPR resulting from threats, diversification benefits will be limited across the G7 countries, though possibility of portfolio allocation should be available during risks resulting from actual acts of war and terror. As part of future research, it would be interesting to extend our work to emerging markets, using emerging economy-specific GPR indices developed by **?**.

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A TVAR model Algorithm

Step 1. Given an initial value for $Z^*$and d, separate the data into two regimes (below and above the threshold).

Step 2. Sample the VAR parameters B_i and Σ_i in each regime i=1,2:

$$H(B_i \setminus \Sigma_{i,}Y_t, Z^*) \sim N(vec(B_i^*), \Sigma_i \otimes (X_i^* X_i^*)^{-1})$$

$$H(\Sigma_i \setminus B_i, Y_t, Z^{*)} \sim \mathcal{IW}(S_i^*, T_i^*)$$

Step 3. Use a MH step to sample Z*and then compute the acceptance probability α.

$$Z^*_{new} = Z^*_{old} + \Phi^{1/2}e, e \sim \mathcal{N}(0,1)$$
$$\alpha = \frac{F(Y \setminus B_i, \Sigma_i, d_i, Z^*_{new})p(Z^*_{new})}{F(Y \setminus B_i, \Sigma_i, d_i, Z^*_{old})p(Z^*_{old})}$$

where $F(Y \setminus B_i, \Sigma_i, Z^*_{new})p(Z^*_{new})$ is the likelihood of the VAR computed as the product of the likelihoods in the two regimes. We choose the scaling factor Φ to ensure that the acceptance rate remains between 20% and 40%.

Step 4. Draw the delay parameter d from the multinomial distribution with probability:

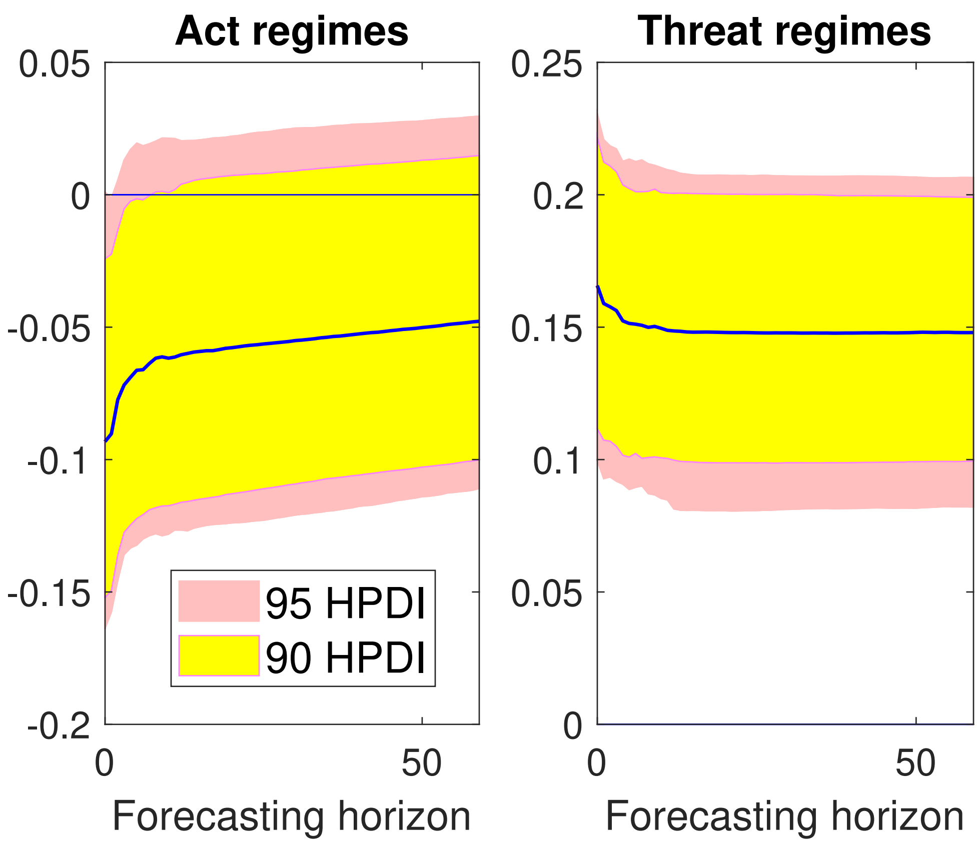
$$\frac{L(Y \setminus d, \Psi)}{\sum_{d=1}^{n} L(Y \setminus d, \Psi)}$$

where L (.) is likelihood function, Ψ denotes all the other parameters and n the maximum value d can take.

Algorithm 1: TVAR model estimation. MH within Gibbs sampling algorithm.

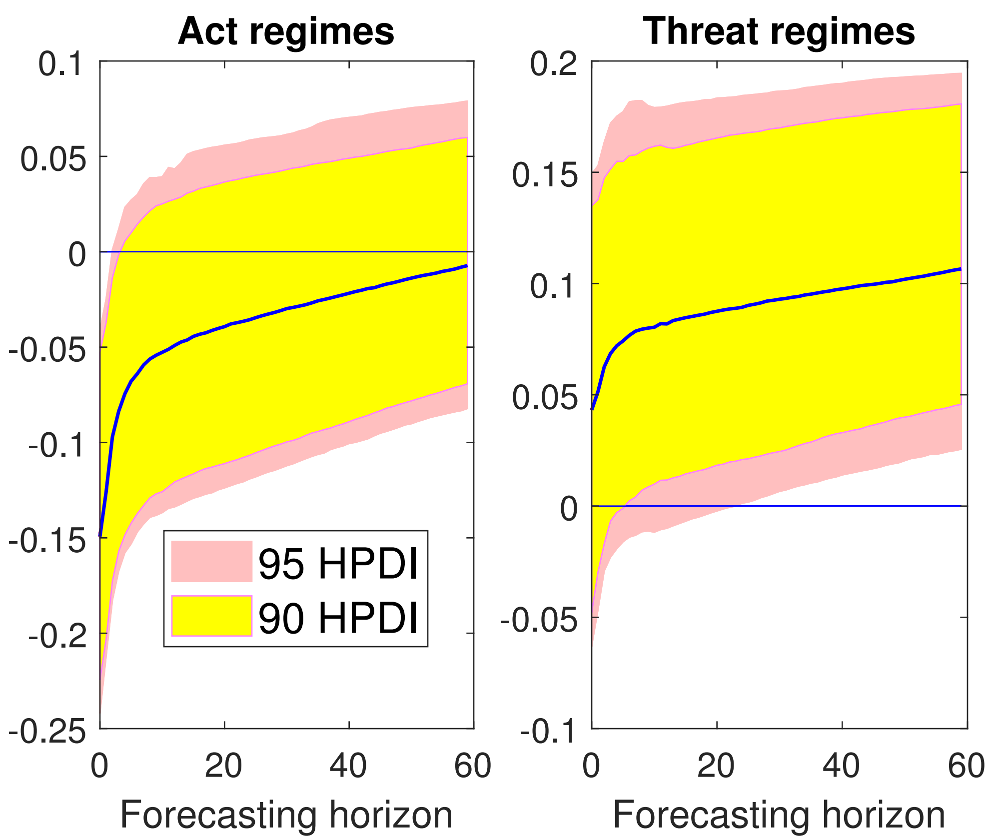
B Sensitivity analysis

Figure B.1 – Sensitivity analysis to the threshold prior mean



The figure reports the difference in the connectedness index between high and low GPA regime (left) and GPT regime (right) with the prior mean for the threshold level set at the 80th percentile. Blue line is the median across saved draws; bands represent 95 and 90 HPDIs.

Figure B.2 – Sensitivity analysis to the filtering method



The figure reports the difference in the connectedness index between high and low GPA regime (left) and GPT regime (right) with the threshold variable HP filtered. Blue line is the median across saved draws; bands represent 95 and 90 HPDIs.

C Proofs

Proof of Proposition 1 The first order conditions are:

$$\frac{\partial EU_{At}}{\partial s_A^A} = 0 \iff \frac{D(1-\rho a_t)^{1-\frac{1}{\sigma}}(n_A-1)}{1-\frac{1}{\sigma}}\left(1-\frac{1}{\sigma}\right) s_A^{A^{-\frac{1}{\sigma}}} = (n_A-1)p_A$$

$$\frac{\partial EU_{At}}{\partial s_A^B} = 0 \iff \frac{D(1-\rho a_t)^{1-\frac{1}{\sigma}}(1-\tau)^{1-\frac{1}{\sigma}} n_B}{1-\frac{1}{\sigma}}\left(1-\frac{1}{\sigma}\right) s_A^{B^{-\frac{1}{\sigma}}} = n_B(1+\tau)p_B$$

$$\frac{\partial EU_{At}}{\partial \alpha_A} = 0 \iff p_A = (1-\frac{1}{\sigma})\frac{D(1-\rho a_t)^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}}(1-\alpha_A)^{-\frac{1}{\sigma}}$$

Proof of Lemma 2

$$
\begin{aligned}
COV(R_A, R_B) &= \\
& E(R_A R_B) - E(R_A)E(R_B) \\
&= E\left[\frac{d^2(1+\mu_{t+1})^2}{L^2 p_A p_B}\right] - \frac{d(1-\rho a_t)^{\frac{1}{\sigma}}}{LD(n_A+n_B\phi)^{\frac{1}{\sigma}}}\frac{d(1-\rho a_t)^{\frac{1}{\sigma}}}{LD(n_B+n_A\phi)^{\frac{1}{\sigma}}} \\
&= \frac{d^2}{L^2 p_A p_B} E\left[1+2\mu_{t+1}+\mu_{t+1}^2\right] - \frac{d^2(1-\rho a_t)^{\frac{2}{\sigma}}}{L^2D^2(n_A+n_B\phi)^{\frac{1}{\sigma}}(n_B+n_A\phi)^{\frac{1}{\sigma}}} \\
&= \frac{d^2}{L^2 p_A p_B}\left(1-2\rho a_t + E(\mu_{t+1}^2)\right) - \frac{d^2(1-\rho a_t)^{\frac{2}{\sigma}}}{L^2D^2(n_A+n_B\phi)^{\frac{1}{\sigma}}(n_B+n_A\phi)^{\frac{1}{\sigma}}} \\
&= \frac{d^2}{L^2 p_A p_B}(1-2\rho a_t + Var(\mu_{t+1}) + (E(\mu_{t+1}))^2) - \frac{d^2(1-\rho a_t)^{\frac{2}{\sigma}}}{L^2D^2(n_A+n_B\phi)^{\frac{1}{\sigma}}(n_B+n_A\phi)^{\frac{1}{\sigma}}} \\
&= \frac{d^2}{L^2 p_A p_B}(1-2\rho a_t + Var(\mu_{t+1}) + \rho^2 a_t^2) - \frac{d^2(1-\rho a_t)^{\frac{2}{\sigma}}}{L^2D^2(n_A+n_B\phi)^{\frac{1}{\sigma}}(n_B+n_A\phi)^{\frac{1}{\sigma}}} \\
&= \frac{d^2}{L^2 p_A p_B}\left[(1-\rho a_t)^2 + \sigma_\epsilon^2\right] - \frac{d^2(1-\rho a_t)^{\frac{2}{\sigma}}}{L^2D^2(n_A+n_B\phi)^{\frac{1}{\sigma}}(n_B+n_A\phi)^{\frac{1}{\sigma}}} \\
&= \frac{d^2}{L^2D^2(n_A+n_B\phi)^{\frac{1}{\sigma}}(n_B+n_A\phi)^{\frac{1}{\sigma}}}\left[\frac{(1-\rho a_t)^2+\sigma_\epsilon^2}{(1-\rho a_t)^{\frac{2\sigma-2}{\sigma}}} - (1-\rho a_t)^{\frac{2}{\sigma}}\right]
\end{aligned}
$$

Recall $D = \frac{\beta d^{1-\frac{1}{\sigma}}}{L}$,substituting

$$COV(R_A, R_B)$$

$$= \frac{d^{\frac{2}{\sigma}}}{\beta^2(n_A+n_B\phi)^{\frac{1}{\sigma}}(n_B+n_A\phi)^{\frac{1}{\sigma}}}\left[\frac{(1-\rho a_t)^2+\sigma_\epsilon^2}{(1-\rho a_t)^{\frac{2\sigma-2}{\sigma}}} - (1-\rho a_t)^{\frac{2}{\sigma}}\right]$$

Proof of Proposition 3

- $$\frac{\partial Cov}{\partial a_t} = \frac{d^{\frac{2}{\sigma}}}{\beta^2(n_A + n_B\phi)^{\frac{1}{\sigma}}(n_B + n_A\phi)^{\frac{1}{\sigma}}}\left[\frac{2\rho(1-\rho a_t)^{\frac{2}{\sigma}-3}(\sigma-1)\sigma_\epsilon^2}{\sigma}\right]$$

  - change with financial integration

$$\frac{\partial}{\partial \phi}\frac{\partial Cov}{\partial a_t}$$

$$= \quad -\frac{2\rho d^{\frac{2}{\sigma}}(1-\rho a_t)^{\frac{2}{\sigma}-3}(\sigma-1)\sigma_\epsilon^2}{\beta^2\sigma^2}\left(\frac{n_B}{(n_A+n_B\phi)^{\frac{1}{\sigma}+1}(n_B+n_A\phi)^{\frac{1}{\sigma}}} + \frac{n_A}{(n_B+n_A\phi)^{\frac{1}{\sigma}+1}(n_A+n_B\phi)^{\frac{1}{\sigma}}}\right) < 0$$

  - change with size

$$\frac{\partial}{\partial n_A}\frac{\partial Cov}{\partial a_t}$$

$$= -\frac{2\rho d^{\frac{2}{\sigma}}(1-\rho a_t)^{\frac{2}{\sigma}-3}(\sigma-1)\sigma_\epsilon^2}{\beta^2\sigma^2}\left(\frac{(n_A+n_B\phi)^{-1} + \phi(n_B+n_A\phi)^{-1}}{(n_A+n_B\phi)^{\frac{1}{\sigma}}(n_B+n_A\phi)^{\frac{1}{\sigma}}}\right) < 0$$

  - change with risk aversion (subsitute for $\phi = \left(\frac{1-\tau}{1+\tau}\right)^{\sigma-1}$)

$$\frac{\partial}{\partial \sigma}\frac{\partial Cov}{\partial a_t}$$

$$= \frac{\partial}{\partial \sigma}\left[\frac{d^{\frac{2}{\sigma}}}{\beta^2(n_A + n_B\left(\frac{1-\tau}{1+\tau}\right)^{\sigma-1})^{\frac{1}{\sigma}}(n_B + n_A\left(\frac{1-\tau}{1+\tau}\right)^{\sigma-1})^{\frac{1}{\sigma}}}\left[\frac{2\rho(1-\rho a_t)^{\frac{2}{\sigma}-3}(\sigma-1)\sigma_\epsilon^2}{\sigma}\right]\right]$$

$$= \frac{2d^{\frac{2}{\sigma}}\sigma_\epsilon^2\rho(1-\rho a_t)^{\frac{2}{\sigma}-3}}{\beta^2\sigma^3}\left(n_B + n_A\left(-1+\frac{2}{1+\tau}\right)^{\sigma-1}\right)^{-\frac{1}{\sigma}}\left(n_A + n_B\left(-1+\frac{2}{1+\tau}\right)^{\sigma-1}\right)$$

$$\times\left[-\sigma(\sigma-1) + \sigma^2 - 2(\sigma - 1\ln d - 2(\sigma-1)\ln(1-\rho a_t) + \right.$$

$$(\sigma-1)\left(-\frac{n_A\sigma(1+\tau)(\frac{2}{1+\tau}-1)^\sigma(\ln(1-\tau)+\ln(\frac{1}{1+\tau})}{n_B - n_B\tau + n_A(1+\tau)(\frac{2}{1+\tau}-1)^\sigma} + \right.$$

$$\left.\ln\left(n_B + n_A(\frac{2}{1+\tau}-1)^{\sigma-1}\right)\right) +$$

$$(\sigma-1)\left(-\frac{n_B\sigma(1+\tau)(\frac{2}{1+\tau}-1)^\sigma(\ln(1-\tau)+\ln(\frac{1}{1+\tau})}{n_A - n_A\tau + n_B(1+\tau)(\frac{2}{1+\tau}-1)^\sigma} + \right.$$

$$\ln\left(n_A + n_B(\frac{2}{1+\tau} - 1)^{\sigma-1}\right)\Bigg)\Bigg]$$

Proof of Proposition (Comparative Statics)

$$\frac{\partial p_A}{\partial a_t} = -D\rho\frac{\sigma-1}{\sigma}\left(\frac{n_A + n_B\phi}{1-\rho a_t}\right)^{\frac{1}{\sigma}} < 0$$

$$\frac{\partial z_A}{\partial a_t} = -\frac{D\rho}{2\lambda}\frac{\sigma-1}{\sigma}\left(\frac{n_A + n_B\phi}{1-\rho a_t}\right)^{\frac{1}{\sigma}} < 0$$

$$\frac{\partial E(R_A)}{\partial a_t} = -\frac{\rho d}{LD(n_A + n_B\phi)^{\frac{1}{\sigma}}}(1-\rho a_t)^{\frac{1}{\sigma}-1} < 0$$

$$\frac{\partial p_A}{\partial \phi} = \frac{Dn_B(1-\rho a_t)^{\frac{\sigma-1}{\sigma}}}{\sigma}(n_A + n_B\phi)^{\frac{1-\sigma}{\sigma}} > 0$$

$$\frac{\partial z_A}{\partial \phi} = \frac{Dn_B(1-\rho a_t)^{\frac{\sigma-1}{\sigma}}}{2\lambda\sigma}(n_A + n_B\phi)^{\frac{1-\sigma}{\sigma}} > 0$$

$$\frac{\partial E(R_A)}{\partial \phi} = -\frac{d(1-\rho a_t)^{\frac{1}{\sigma}}n_B}{LD\sigma}(n_A + n_B\phi)^{\frac{1-\sigma}{\sigma}} < 0$$

Proof of Proposition 6 FOCs:

$$\frac{\partial EU_{it}}{\partial s_i^i} = 0 \iff D(1-\rho a_t)^{1-\frac{1}{\sigma}}(m_i - 1)s_i^{i^{-\frac{1}{\sigma}}} - (m_i - 1)p_i = 0$$

$$\frac{\partial EU_{it}}{\partial s_i^j} = 0 \iff D(1-\rho a_t)^{1-\frac{1}{\sigma}}m_j(1-\tau)^{1-\frac{1}{\sigma}}s_i^{j^{-\frac{1}{\sigma}}} - m_j(1+\tau)p_j = 0$$

$$\frac{\partial EU_{it}}{\partial \alpha_i} = 0 \iff -D(1-\rho a_t)^{1-\frac{1}{\sigma}}(1-\alpha_i)^{-\frac{1}{\sigma}} + p_i = 0$$

Proof of Lemma 8 Covariance of returns of every pair of countries:

$$COV(R_i, R_j) = E(R_iR_j) - E(R_i)E(R_j)$$

$$= E\left(\frac{d^2(1+\mu_{t+1})^2}{L^2p_ip_j}\right) - \frac{d^2(1-\rho a_t)^{\frac{2}{\sigma}}}{L^2D^2(m_i + \phi\sum_{k\in N, k\neq i} m_k)^{\frac{1}{\sigma}}(m_j + \phi\sum_{k\in N, k\neq j} m_k)^{\frac{1}{\sigma}}}$$

$$= \frac{d^2E\left(1+\mu_{t+1}^2+2\mu_{t+1}\right)}{L^2D^2(1-\rho a_t)^{2-\frac{2}{\sigma}}(m_i+\phi\sum_{k\in N,k\neq i}m_k)^{\frac{1}{\sigma}}(m_j+\phi\sum_{k\in N,k\neq j}m_k)^{\frac{1}{\sigma}}} - \frac{d^2(1-\rho a_t)^{\frac{2}{\sigma}}}{L^2D^2(m_i+\phi\sum_{k\in N,k\neq i}m_k)^{\frac{1}{\sigma}}(m_j+\phi\sum_{k\in N,k\neq j}m_k)^{\frac{1}{\sigma}}}$$

$$= \frac{d^2\left(1-2\rho a_t+E(\mu_{t+1}^2)\right)}{L^2D^2(1-\rho a_t)^{2-\frac{2}{\sigma}}(m_i+\phi\sum_{k\in N,k\neq i}m_k)^{\frac{1}{\sigma}}(m_j+\phi\sum_{k\in N,k\neq j}m_k)^{\frac{1}{\sigma}}} - \frac{d^2(1-\rho a_t)^{\frac{2}{\sigma}}}{L^2D^2(m_i+\phi\sum_{k\in N,k\neq i}m_k)^{\frac{1}{\sigma}}(m_j+\phi\sum_{k\in N,k\neq j}m_k)^{\frac{1}{\sigma}}}$$

$$= \frac{d^2\left((1-\rho a_t)^2+\sigma_\epsilon^2\right)}{L^2D^2(1-\rho a_t)^{2-\frac{2}{\sigma}}(m_i+\phi\sum_{k\in N,k\neq i}m_k)^{\frac{1}{\sigma}}(m_j+\phi\sum_{k\in N,k\neq j}m_k)^{\frac{1}{\sigma}}} - \frac{d^2(1-\rho a_t)^{\frac{2}{\sigma}}}{L^2D^2(m_i+\phi\sum_{k\in N,k\neq i}m_k)^{\frac{1}{\sigma}}(m_j+\phi\sum_{k\in N,k\neq j}m_k)^{\frac{1}{\sigma}}}$$

$$= \frac{d^2\left(\frac{(1-\rho a_t)^2+\sigma_\epsilon^2}{(1-\rho a_t)^{2-\frac{2}{\sigma}}}-(1-\rho a_t)^{\frac{2}{\sigma}}\right)}{L^2D^2(m_i+\phi\sum_{k\in N,k\neq i}m_k)^{\frac{1}{\sigma}}(m_j+\phi\sum_{k\in N,k\neq j}m_k)^{\frac{1}{\sigma}}}$$

Recall $D=\frac{\beta d^{1-\frac{1}{\sigma}}}{L}$,substitute.

Proof of Theorem 9

$$\frac{\partial I}{\partial a_t}=\frac{d^{\frac{2}{\sigma}}}{\beta^2}\left[\frac{2\rho(1-\rho a_t)^{\frac{2}{\sigma}-3}(\sigma-1)\sigma_\epsilon^2}{\sigma}\right]\sum(m_i+\phi\sum_{k\in N,k\neq i}m_k)^{-\frac{1}{\sigma}}(m_j+\phi\sum_{k\in N,k\neq j}m_k)^{-\frac{1}{\sigma}}>0$$

D N countries

We write the budget contraint for an agent from country i as

$$c_{it+}f(z_i)+(m_i-1)p_is_i^i+\sum_{j\in N,j\neq i}m_j(1+\tau)p_js_i^j=y_i+p_i\alpha_i \tag{D.1}$$

The consumption in period $(t+1)$ depends on the state of nature and is defined as follows

$$c_{i(t+1)} = \begin{cases} d(1+\mu_{t+1})(1-\alpha_i) & \text{if } i \in Z_i \text{ occurs} \\ d(1+\mu_{t+1})s_i^i & \text{if } k \in Z_i, k \neq i \text{ occurs} \\ d(1+\mu_{t+1})(1-\tau)s_i^j & \text{if } j \in Z_j, j \in N \setminus \{i\} \text{ occurs} \\ 0 & \text{otherwise} \end{cases} \tag{D.2}$$

The agent's objective function is her expected utility

$$EU_{it} = c_{it} + \beta E\left(\frac{c_{i(t+1)}^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}}\right) \tag{D.3}$$

where β is the discount rate, and σ is the inverse degree of risk aversion ($\sigma > 1$). This can be rewritten as follows

$$\begin{aligned} EU_{it} &= y_i + p_i\alpha_i - f(z_i) - (m_i-1)p_i s_i^i - \sum_{j\in N, j\neq i} m_j(1+\tau)p_j s_i^j \\ &+\frac{D(1-\rho a_t)^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}}\left((m_i-1){s_i^i}^{1-\frac{1}{\sigma}} + \sum_{j\in N, j\neq i} m_j(1-\tau)^{1-\frac{1}{\sigma}}{s_i^j}^{1-\frac{1}{\sigma}} + (1-\alpha_i)^{1-\frac{1}{\sigma}}\right) \end{aligned}$$

The equilibrium condition for every $i \in N$ can be writen as:

$$\alpha_i = (m_i-1)s_i^i + \sum_{j\in N, j\neq i}(1+\tau)m_j s_j^i$$

We define the returns on the asset developed by agent from country i as $R_i = \frac{d(1+\mu_{t+1})}{Lp_i}$.