Non-galvanic calibration and operation of a quantum dot thermometer

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A cryogenic quantum dot thermometer is calibrated and operated using only a single non-galvanic gate connection. The thermometer is probed with radio-frequency reflectometry and calibrated by fitting a physical model to the phase of the reflected radio-frequency signal taken at temperatures across a small range. Thermometry of the source and drain reservoirs of the dot is then performed by fitting the calibrated physical model to new phase data. The thermometer can operate at the transition between thermally broadened and lifetime broadened regimes, and outside the temperatures used in calibration. Electron thermometry was performed at temperatures between 3.0K and 1.0K, in both a 1K cryostat and a dilution refrigerator. In principle, the experimental setup enables fast electron temperature readout with a sensitivity of $4.0 \pm 0.3 \text{mK}/\sqrt{\text{Hz}}$, at kelvin temperatures. The non-galvanic calibration process gives a readout of physical parameters, such as the quantum dot lever arm. The demodulator used for reflectometry readout is readily available and very affordable.

I. INTRODUCTION

Electron temperature is a fundamental parameter that can limit the performance of low-temperature experiments and applications. Electron thermometry is an essential tool in understanding the behaviour of low-temperature circuitry [1], for example the processors used in quantum computers or devices used to study exotic electronic phases and materials. Accurate and fast electron temperature readout is also a valuable tool for thermodynamic experiments. Increasingly sensitive quantum circuits require delicate and non-invasive electronic thermometry. Quantum dot (QD) and single-electron transistor (SET) conduction thermometry are well established as a powerful approach to monitor electron temperatures [1–11]. However, these thermometers require the measurement of current through the QD, which can complicate or interfere with other electronic measurements in the experiment. Furthermore, the extra galvanic connections for dot thermometry can be a source of additional noise and parasitic heating. Local charge sensing can be used to probe the occupation of a QD without having to pass a direct current through it [5, 12–15]. This approach still requires galvanic connections to read the charge sensor. Similarly, radio frequency (RF) reflectometry techniques have recently allowed QD thermometry to be performed without measuring current through the QD, which is effective outside the lifetime broadened regime [16, 17]. However, for both charge sensing and reflectometry, connections to the source and drain of the dot are still needed to measure the dot lever arm for calibration.

Here we demonstrate how a completely non-galvanic QD thermometer can be calibrated and used with a single capacitive gate connection, including a calibration of the lever arm with no DC source-drain bias. The QD thermometer has the flexibility to be used on any conducting reservoir and operates in an intermediate regime where the maximum temperature is much lower than the QD charging energy $k_B T \ll E_c$ and the minimum temperature is similar to the tunnel coupling $k_B T \sim \hbar \Gamma$.

II. EXPERIMENTAL DETAILS

The QD device used in our experiment is a silicon-on-insulator trigate accumulation-mode field-effect transistor (FET) with a channel length, width and height of 80nm, 30nm and 10nm respectively, shown in Fig 1a and b. For more details about the device, see reference [18]. At cryogenic temperatures, when a positive sub-threshold DC voltage $V_{tg}$ is applied to the top gate, a localisation potential appears in the Si channel underneath the gate. This allows bound electron states to accumulate, and creates a single QD. Corner dots [19, 20] do not appear in the structure due to the narrow width of the channel. The FET top gate also acts as a plunger gate for the QD via capacitance $C_{tg}$, allowing tuning of the next available energy level in the QD, which is labelled $E_{QD}$, by adjusting $V_{tg}$. The source and drain electrodes are both grounded and act as a single reservoir of electrons with a Fermi level $\mu$ (Fig 1c). Throughout the experiment, the back gate was grounded.

The QD energy level $E_{QD}$ is broadened by tunnel coupling to the reservoir [21, 22]. This is described with a broadened density of states in the form of a normalised Lorentzian, given by:

$$n(E_{QD}, E) = \frac{1}{\pi} \frac{\hbar \Gamma}{(E_{QD} - E)^2 + (\hbar \Gamma)^2}, \quad (1)$$

where $\hbar$ is the reduced Planck’s constant [21]. The tun-
The QD energy level \( E_{QD} \) has both a constant geometric capacitance and a variable tunneling capacitance \( C_t \) as a function of \( E_{QD} \), given by \( C_t = e\alpha / \partial V_{tg} \), with the reflected signal magnitude \(|R|\) and phase \(\phi\) shown in red and blue, respectively. The black dashed lines show the fitted circuit model, which measures a Q-factor of 63. Thermometry is performed on resonance at \( f_0 = 593 \text{ MHz} \), where phase is most responsive.

The derivative of \( f \) with respect to \( V_{tg} \), denoted \( f' \), is

\[
 f'(V_{tg}) = \frac{1}{4k_B T_e \cosh^{-2} \left( -\alpha e(V_{tg} - V_0) \right) / 2k_B T_e } ,
\]  

where \( T_e \) is the electron temperature of the reservoirs, \( V_0 \) is the value of \( V_{tg} \) when \( P_{QD} = 1/2 \), and \( k_B \) is the Boltzmann constant. The \( T_e \) dependence of \( C_t \) via Eq. (6) is the basis for the non-galvanic QD thermometer.

The capacitance \( C_t \) was measured by RF reflectometry using the setup shown in Fig 2a. A resonant circuit was connected to the QD top gate and consisted of a NbTiN-on-quartz spiral inductor \( L = 96 \text{ nH} \), and a coupling capacitor \( C_c = 0.18 \text{ pF} \). The inductor is placed parallel to the measured capacitance to help improve the loaded Q-factor, which helps achieve higher responsiveness to a change in capacitance [29]. Modelling the circuit using the measured RF reflection \(|R|\) and

\[
 P_{QD}(E_{QD}) = \int_{-\infty}^{\infty} f(E)n(E_{QD}, E)\text{d}E . \tag{2} 
\]  

This is the convolution of the two functions, so \( P_{QD} \) becomes:

\[
 P_{QD}(E_{QD}) = (f * n) . \tag{3} 
\]  

Due to the spin degeneracy of the extra electron in the QD, \( P_{QD} = 1/2 \) occurs when \( E_{QD} = \mu + k_B T \ln 2 \) [17, 23]. The QD has both a constant geometric capacitance and a variable ‘tunneling’ capacitance \( C_t \), given by [17, 24–28]:

\[
 C_t = e\alpha / \partial V_{tg} , \tag{4} 
\]  

where \( \alpha = C_{tg} / C_{\Sigma} \) is the top gate lever arm, \( C_{\Sigma} \) is the total QD capacitance, \( \partial V_{tg} = -\partial E_{QD} / \partial V_{tg} \), and \( e \) is the elementary charge. Inserting Eq. (3) into this definition gives us the tunneling capacitance of the QD-reservoir system:

\[
 C_t(V_{tg}) = e\alpha (f * n)' = e\alpha (f' * n) . \tag{5} 
\]
phase \( \phi \) (shown in Fig 2b) gives a resonance frequency of \( f_0 = 593.4 \text{MHz} \) with a Q-factor of 63, a parasitic capacitance of \( C_p = 0.57 \text{pF} \) and a parasitic resistance of \( R_p = 43.2 \text{k}\Omega \). A modelled circuit loss to ground is represented by resistance \( R_p \), which affects the resonance Q-factor [29]. The parasitic capacitance \( C_p \) includes the geometric gate capacitance \( C_{tg} \). The demodulation of \([R]\) and \( \phi \) was performed with an ‘ADL5387’ active quadrature demodulator chip. To allow a DC bias \( V_{tg} \) to be applied to the top gate, a 100pF capacitor was placed after the inductor to avoid DC-shorting the device and to create a good RF ground at the frequency of operation. A 96k\Omega resistor was used to prevent RF signals from escaping via the DC bias line. The resonant frequency depends on total top gate capacitance via \( f_0 = 1/2\pi \sqrt{L(C_c + C_p + C_{tg})} \). At the circuit resonant frequency, \( \phi \approx C_1 \), when \( C_1 \ll C_p + C_c \) [17, 28, 30–32]. This gives a change in reflected signal phase that depends on \( T_e \) according to:

\[
\phi - \phi_0 = Ae\alpha \left(f' + n\right),
\]

where \( A \) is a fitting constant, which tells us the phase change due to a small change in capacitance, and \( \phi_0 \) is the circuit phase offset at the resonant frequency when \( C_1 \approx 0 \). If the constants \( A, \alpha \) and \( \Gamma \) are known, measuring \( \phi - \phi_0 \) as a function of \( V_{tg} - V_0 \) and fitting the model described by Eq. (7) gives a readout of electron temperature \( T_e \). A demonstration of this technique is shown in Fig 3.

III. RESULTS

The QD thermometer was measured in two fridge systems: a cryogen-free 1K cryostat\(^1\) and a cryogen-free dilution refrigerator\(^2\), where it was calibrated and operated. In both systems, the fridge temperature was monitored by a ruthenium oxide resistance fridge thermometer\(^3\) mounted alongside the QD thermometer during data collection. The reading of the ruthenium oxide fridge thermometer is denoted as \( T_f \).

In the 1K cryostat, calibration was done by reading the reflected signal phase across a number of sweeps of \( V_{tg} \) at a set of fridge temperatures \( T_f = 2.0, 2.5, 3.0 \text{K} \). A single least-squares fit using Eq. (7) was performed on the collective phase traces for this set of temperatures (Fig 3). It was assumed that the electron temperature is well thermalised with the fridge temperature at 2K and above, and so during the calibration the condition \( T_e = T_f \) was applied to Eq. (7). For each phase trace, \( \phi_0 \) and \( V_0 \) were individually fitted. The calibration fit produced an estimate for the values \( \alpha = 0.74 \pm 0.02 \), \( \Gamma = 270 \pm 20 \text{ns}^{-1} \) and \( A = 5.13 \pm 0.06 \text{rad pF}^{-1} \). This implies \( \Gamma \gg 2\pi f_0 \), therefore dissipative components were neglected and the cyclic tunnelling was considered adiabatic. With these three constants defined, the thermometer was calibrated and ready for operation.

\(^1\) \( \text{Oxford Instruments 'Io'} \)
\(^2\) \( \text{BlueFors 'LD250'} \)
\(^3\) \( \text{Model ref 'ROTH-GEN' in the 1K cryostat and 'RuO2.RX-102B' in the dilution refrigerator} \)

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![FIG. 3. QD thermometer calibration in 1K cryostat, showing three experimental phase traces taken at fridge temperatures \( T_f = 2.0, 2.5, 3.0 \text{K} \). Each trace shows the change in reflected signal phase \( \phi - \phi_0 \) against top gate voltage \( V_{tg} - V_0 \) around the Coulomb peak of the QD. The solid black lines show the least-square fit of Eq. (7) onto the data, assuming the electron temperature \( T_e \) equals the fridge thermometer readout \( T_f \). This calibration procedure estimates \( \alpha = 0.74 \pm 0.02 \), \( \Gamma = 270 \pm 20 \text{ns}^{-1} \) and \( A = 5.13 \pm 0.06 \text{rad pF}^{-1} \). These three constants are then included within Eq. (7), allowing electron thermometry to be performed, shown here with a \( T_e \) fit to data taken at \( T_f = 1.35 \text{K} \), yielding an electron temperature \( T_e = 1.4 \pm 0.1 \text{K} \).](image)

![FIG. 4. Electron temperature \( T_e \) (orange) and max phase \( \phi_{\text{MAX}} \) (green) readout from QD thermometer in 1K cryostat. Thermometry readout was generated by fitting \( T_e \), via calibrated Eq. (7), to the phase curve observed by sweeping the \( V_{tg} \) over the QD Coulomb peak. The peak phase \( \phi_{\text{MAX}} \) was measured at \( V_{tg} = V_0 \), with no fitting process required. The fridge temperature \( T_f \) was read from a ruthenium oxide fridge thermometer thermally linked to the QD device. The dashed line highlights where \( T_e = T_f \). The dotted line represents the model prediction from calibrated Eq. (7), assuming \( T_e = T_f \).](image)
To use the QD thermometer, \( \phi - \phi_0 \) was measured as a function of \( V_{\text{tg}} - V_0 \) and fitted with the calibrated Eq. (7) to give a readout of electron temperature \( T_e \). A series of thermometry readings were taken at varying fridge temperatures between 3.0 K and 1.3 K. Fridge temperature \( T_f \) was monitored for each QD thermometer reading of \( T_e \) (Fig 4). The QD thermometer agreed with the fridge thermometer across the range of temperatures, even at temperatures below the calibration range. It is worth noting that this works in the intermediate regime where \( k_B T \approx \hbar \Gamma \) because the tunnel broadening is taken into account within the physical model. For quicker electron temperature readout, the QD can be tuned to where \( V_{\text{tg}} = V_0 \) so that \( \phi - \phi_0 = \phi_{\text{MAX}} \), which was directly converted to an electron temperature via Eq. (7), using the previously calibrated values of \( \alpha \), \( \Gamma \) and \( A \). This approach worked effectively, even at fridge temperatures below the calibration data.

Finally, to provide an independent confirmation of the calibration process, the QD source and drain connections were ungrounded to apply a source-drain voltage \( V_{\text{sd}} \) across the QD and measure a charge stability diagram of the device. Using the relationship

\[
\alpha = \frac{1}{m_d - m_s},
\]

where \( m_d = \Delta V_{\text{tg}}^{d} / \Delta V_{\text{sd}}^{d} \) is the gradient along the ‘drain resonance’ side of a Coulomb diamond and \( m_s = \Delta V_{\text{tg}}^{s} / \Delta V_{\text{sd}}^{s} \) is the ‘source resonance’ side, we can see the lever arm \( \alpha = 0.74 \pm 0.02 \) matches well with the Coulomb diamond geometry in Fig 5. This demonstrates that the lever arm of a QD can be obtained using one non-galvanic gate connection and measurements spanning a range of fridge temperatures. The order of magnitude of source-drain current \( |I_{\text{sd}}| \) from the unblocked QD was found to be in the order of \( \sim 10 \text{nA} \). This agrees with the calibration fit of the total tunnel rate constant \( 270 \pm 20 \text{ns}^{-1} \), which equates to a source-drain current of \( e\Gamma = 6.9 \pm 0.5 \text{nA} \) for a single electron transport channel.

To study operation at lower temperatures, the QD thermometer was mounted into a dilution refrigerator. The calibration fit was performed as described above, with phase data taken at 1300 mK and 1600 mK. Within this system, the three calibration constants were found to be \( \alpha = 0.84 \pm 0.03 \), \( \Gamma = 510 \pm 10 \text{ns}^{-1} \), and \( A = 0.75 \pm 0.07 \text{rad pF}^{-1} \). The change in \( A \) is attributed to the differences between the two fridge systems affecting the RF electronics, such as parasitic capacitance \( C_p \) changing due to a different metallic geometry near to the QD chip. Both \( \alpha \) and \( \Gamma \) are sensitive to the shape and position of the QD in the Si channel, which are likely to be different after a thermal cycle of the device.

The electron temperature readout \( T_e \) from the QD ther-
modulator in the dilution refrigerator agreed with the fridge temperature readout $T_f$ above 1 K, despite the fact that $k_B T_f < \hbar \Gamma$ (Fig 6a). Below 1 K there was deviation of $T_e$ away from $T_f$, although the point $T_e = T_f$ remains within one standard deviation of experimental uncertainty (For details on how the error is determined, see section II in the Supplemental Material [33]). The deviation of $T_e$ readout and its increase in uncertainty occurs because the QD energy level is strongly tunnel-broadened and the response of the phase trace to temperature becomes weaker (For details on the influence of tunnel broadening, see section III and FIG. 4 in the Supplemental Material [33]). With a reduced $\Gamma$ the QD thermometer would work at lower temperatures. This can be achieved by adjusting the device design geometry. Checking the charge stability diagram, the predicted lever arm $\alpha = 0.84 \pm 0.03$ matches the Coulomb diamond geometry well (Fig 6b). The dilution refrigerator experiment has an effective white noise phase sensitivity of $1.1 \pm 0.1 \mu \text{rad}/\sqrt{\text{Hz}}$ in this case dominated by the measurement chain. With this equipment and the $\phi_{\text{MAX}}$ measurements demonstrated in Fig 4, the QD thermometer setup could achieve a potential sensitivity of $11 \pm 1 \text{mK}/\sqrt{\text{Hz}}$ at $3.0 \text{K}$, and $4.0 \pm 0.3 \text{mK}/\sqrt{\text{Hz}}$ at $1.3 \text{K}$ (For details on the thermometer sensitivity, see section I in the Supplemental Material [33]).

IV. CONCLUSIONS

We have described the successful calibration and operation of a QD thermometer readout via a single capacitive connection in two separate cryostats, which introduces a new level of simplicity and versatility for measuring electron temperature. The calibration uses limited data to generate a physical model of the QD-reservoir system, which correctly estimates physical parameters such as the QD lever arm. Electron thermometry was successfully performed with the calibrated QD thermometer in a 1.0 K to 3.0 K range. The QD thermometer was also used for faster readout by monitoring the phase when the QD has an occupation probability of 1/2. In this mode of operation, the noise floor of the measurement should allow for a sensitivity of $4.0 \pm 0.3 \text{mK}/\sqrt{\text{Hz}}$ and $11 \pm 1 \text{mK}/\sqrt{\text{Hz}}$, at 1.3 K and 3.0 K respectively. This process worked with the same QD chip in both cryostats. The QD thermometer can operate even in the case where $k_B T_e < \hbar \Gamma$, however with the system and techniques used here, the thermometry uncertainty starts to increase below 1 K due to strong tunnel coupling overriding the temperature dependence. Careful analysis of the thermometry uncertainty reveals the coldest limit of the QD thermometer, when the electron temperature readout confidence boundary increases beyond a usable size. A redesign of the QD device to reduce the tunnel rate would be needed to decrease the uncertainty at lower temperatures. The ability to fully calibrate and operate a non-galvanic electron thermometer with a single RF line simplifies the application of the device substantially. This device provides a versatile, sensitive and effective tool for monitoring electron temperature in nanoelectronic devices at cryogenic temperatures.

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[33] See Supplemental Material at [URL will be inserted by publisher] for extra material on the quantum dot thermometer temperature sensitivity, electron thermometry error, quantum dot energy scales and coulomb peak location temperature dependence.