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Author for correspondence:
U. E. Vincent
e-mail: u.vincent@lancaster.ac.uk

Vibrational and stochastic resonances in driven nonlinear systems

U. E. Vincent[1,2], P. V. E. McClintock[2], I. A. Khovanov[3], and S. Rajasekar[4],

[1]Department of Physical Sciences, Redeemer's University, P.M.B. 230, Ede, Nigeria
[2]Department of Physics, Lancaster University, Lancaster LA1 4YB, United Kingdom
[3]School of Engineering, University of Warwick, Coventry, CV4 7Al, United Kingdom
[4]School of Physics, Bharathidasan University, Tiruchirappalli – 620 024, Tamilnadu, India.

Nonlinear systems are abundant in nature. Their dynamics has been investigated very extensively, motivated partly by their multidisciplinary applicability, ranging from all branches of physical and mathematical sciences through engineering to the life sciences and medicine. When driven by external forces, nonlinear systems can exhibit a plethora of interesting and important properties – one of the most prominent being that of resonance. In the presence of a second, higher frequency, driving force, whether stochastic or deterministic/periodic, a resonance phenomenon arises that can generally be termed stochastic resonance or vibrational resonance. Operating a system in or out of resonance promises applications in several advanced technologies, such as the creation of novel materials at the nano, micro and macroscales including, but not limited to, materials having photonic band gaps, quantum control of atoms and molecules as well as miniature condensed matter systems. Motivated in part by these potential applications, this 2-part Theme Issue provides a concrete up-to-date overview of vibrational and stochastic resonances in driven nonlinear systems. It assembles state-of-the-art, original contributions on such induced resonances – addressing their analysis, occurrence, and applications from either the theoretical, numerical or experimental perspectives, or through combinations of these.

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1. Introduction

Nonlinearity is an inherent property that determines the behaviours of many natural and technological systems. In addition to nonlinearity, external factors such as driving forces, whether of stochastic/noisy or deterministic/periodic origin are also known to impact on these systems. The action of driving forces on nonlinear systems can induce a plethora of intriguing and potentially useful phenomenon - one of which is *resonance*, a phenomenon in physics originally linked to the matching of a system's natural frequency of vibration to the frequency of an external force driving it, or the matching of two or more frequencies within a system, and ultimately leading to an enhancement of the output signal [1]. Developments in the study of resonance phenomenon have shown that, in most dynamical systems, resonance can also occur in other forms that are not necessarily associated with frequency matching. Thus, the definition of *resonance* has been generalized by relaxing this restriction and including all known processes leading to the enhancement, suppression or optimization of a system's response through the variation/perturbation/modulation of any system property.

Resonance, in its great diversity of forms has been reported in a wide range of systems, ranging from biological to chemical and physical systems, as well as mechanical and engineering systems. For instance, in physical and chemical systems, atomic and molecular spectroscopy is arguably one of the most prominent manifestation of resonance, wherein the photon energy is varied such that it matches the difference in the energies of two levels of species, and consequently leading to the appearance of sharp peaks. At nuclear level, resonance occurs in electron and many particle scattering processes [2]. In all of these chemical and nuclear processes, reactive resonances appear as quasi-bound states or transiently short-lived chemical species, and they can arise from energy wells in vibrationally adiabatic potentials [3]. Reactive resonance is important because it can dramatically enhance the reaction probability. In the cardiovascular system (CVS), resonance can occur involving: (i) the cardiac oscillator at 1 Hz; (ii) respiratory oscillations at 0.2 Hz; (iii) myogenic oscillations at 0.1 Hz; (iv) neurogenic processes and vascular tone (VT) baroreflexes at 0.03 Hz giving rise to a $\approx$ 15-s VT response delay in BP changes; (v) NO-dependent and NO-independent endothelial oscillations near 0.01 Hz that provide an indication of endothelial health on which the immune system depends. Because of being parts of a strongly-coupled system, these oscillations mutually modulate. They are manifested in many different physiological variables e.g. blood pressure and blood flow, and all of them are potentially applicable in clinical sciences [4–7]. In mechanical systems, the occurrence of resonance is sometimes undesirable because it can have a destructive impact on structural integrity leading to mechanical failure. Thus, understanding the frequency regime or parameter space in which resonance occur in mechanical devices is desirable for engineering applications. In some other instances, mechanical resonance can be utilized positively for particular purposes, including, but not limited to (i) increasing the efficiency of machines such as found in resonance drilling machines, (ii) the reduction in the amount of energy required to maintain stable oscillations in vibratory conveyors, (iii) massive energy harvesting which typically can be multiple times greater than excitation energy obtained by means of continuous supplies [8].

When resonance occurs in a nonlinear system it is referred to as nonlinear resonance [1] characterized by the strong dependence of a system's maximum response on the properties of external driving force. The nonlinear resonance plays the fundamental role in developing instability and multistability in oscillatory systems [9]. Typically the nonlinear resonance is considered for a single frequency forcing. Recently it was shown that resonance like phenomena can arise when a system is acted upon by a combination of deterministic harmonic driving forces. In the case of the dual-frequency deterministic forcing the resonance manifest itself as the enhancement and optimization of a system's maximum response at low-frequency (LF) due to the presence of a high-frequency (HF) external driving force. Although the concept of resonances in driven systems originated from physics and mathematics, its application in a diversity of disciplines, such as control engineering, neurosciences, communication, image processing and medicine, formed the motivation for the Theme Issue. Thus, the Theme Issue is partly aimed at creating a platform for cross-fertilization of ideas, enabling experts in widely separated areas to become aware of each other's work.

Noise or random forces, on the one hand, was for a very long time widely perceived as constituting a nuisance to for a system, due to its capabilities to distort signals being processed. This belief was held until 1981 when Benzi et al. [10] showed that noise can indeed produce counterintuitive, yet positive effects on nonlinear systems by boosting weak signals resonantly. These noise-induced or noise-enhanced co-operative effects leading to the amplification of the system's response to a weak periodic driving signal, termed *Stochastic Resonance (SR)*, have since then received huge research attention. The interest in SR is motivated by its numerous natural and technological applications such as in weak signal detection, machine fault detection, image quality filtering and enhancement of energy harvesting [11–14]. Stochastic resonance is in general characterized by the appearance of a skewed bell shaped curve when the signal-to-noise ratio or the amplification factor is plotted against the variation in the noise intensity.

On the other hand, deterministic driving forces such as HF harmonic forces can effectively replace the role played by stochastic actions, inducing an optimal response to an LF signal, thereby also producing a skewed bell shaped response curve in a similarly resonant fashion as found in SR phenomenon. This effect, now known as *Vibrational Resonance* (VR) was proposed twenty years ago by Landa and McClintock [15]. The main features of VR are illustrated in Figure 1, adapted from the review article by Roy-Layinde *et al.* [16] where typical vibrations of a position-dependent mass system driven by dual-frequency forces give rise to the appearance of resonance peaks taking the form of ridge-shaped "hills", stretching along the mass nonlinearity parameter value λ, and spreading across the two parameter planes of interest. In parallel with *SR*, *VR* has been closely investigated, because in engineering applications, the use of harmonics driving forces is more appropriate and easily achieved, in particular for logical control operations. Harmonic driving forces are, in addition, capable of providing the shorter switching times and broader optimal parameter region that are essential for practical improvement of the robustness and response speed of engineering systems [17]. Since VR was originally proposed two decades ago, numerous scientific publications have appeared about the phenomenon in different scientific contexts, scattered across a wide range of journals. Some of the recent developments of VR and SR are fascinating, and include logical, entropic, anti- and ghost-VR/SR. Moreover, the applications of VR have been gaining momentum in diverse disciplines and a wide range of its occurrences and applications relating to communications systems, signal output identification, filtering, optimisation and control, separation and extraction, and a host of advanced and promising technological processes for improved efficiency and operating conditions, have now been reported [1,16–23].

Notably, no journal has so far united and documented this rapidly evolving field of *VR* and *SR* in a topical issue like the present one. The intention, here, is to provide a concrete up-to-date overview of vibrational and stochastic resonances in driven nonlinear systems; by assembling state-of-the-art, original contributions on VR and SR - addressing their analysis, occurrence, and applications from either the theoretical, numerical and experimental perspectives, or through combinations of these. Additionally, examples of novel developments reflecting non -trivial influence of high-frequency excitation on the system response are presented. By drawing together state-of-the-art knowledge on the scientific connection between VR and SR, the issue will provide a reference point and firm basis for future research on, and potential applications of, these important classes of nonlinear resonance thereby, we hope, making a significant impact on the field. Finally, the theme issue is also timely in that it marks 20 years of research on vibrational resonance, allowing us to take stock and to consider what has been achieved and how best to move forward in the direction of practical applications.

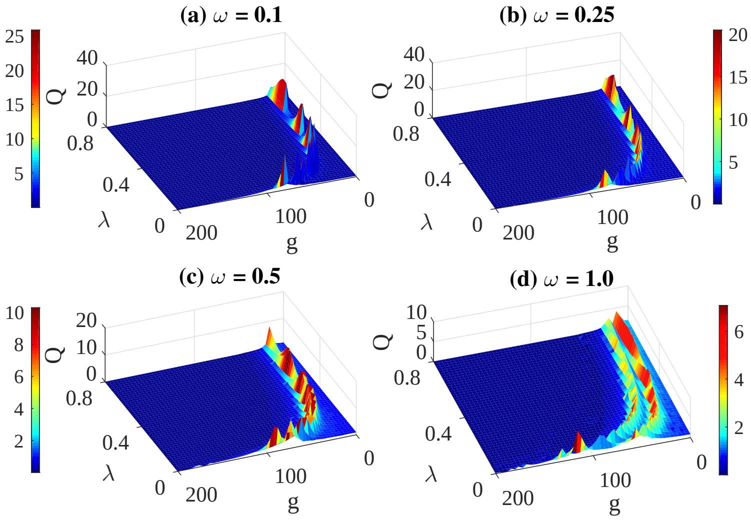

Figure 1. [Color Online] Typical vibrations of a position-dependent mass system driven by dual-frequency forces showing the appearance of resonance peaks taking the form of ridge-shaped "hills", stretching along the mass nonlinearity parameter values λ, and spreading across the two planes. The response of the system was numerically computed as a function of the strength of mass nonlinearity λ and the high-frequency signal amplitude g for different values of the low frequency ω [Roy-Layinde et al. (2021). Phil Trans R. Soc. A 379: 20200227].

2. General Content of the Issue

The issue is presented in two parts and has been organized with papers in three broad domains: Theory, Methods and Analysis; Complex Networks; and Experimental Applications. The contributors are high-level experts in the field and have brought their wealth of experiences to enrich our knowledge in the subject-matter. The theoretical part consists of reviews on fundamental aspects and types of nonlinear resonance induced by driving dynamical systems deterministically and stochastically. It converges on the two most prominent and important types: VR and SR, which are the focus of the issue. The methods employed for both theoretical and numerical analyses are presented in what we hope is a simple, accessible and coherent manner for a diversity of different systems. This is then followed by some discussions of new theoretical approaches and perspectives for analyzing VR, in particular for a generalized nonlinear system with a position-dependent mass, thereby highlighting a new research direction.

In the analysis part, the similarities and differences between stochastic and vibrational resonance phenomena, and their influences on dynamical systems, are discussed based on nonlinear bistable and deformable potential systems, including the effects of parametric driving, nonlinear damping, time-delay and fractional order. Furthermore, some aspects of linear response for trapped quantum particles, radiation-matter interactions and vibrationally isolated solid bodies are presented. The analysis also include expositions on the enhancement of nonlinear response at higher harmonics and at the sub-harmonics of weak fields at sum and difference frequencies, and suppression and stabilization of dynamical systems. Some discussions of linear anti-resonance responses in coupled systems are presented and their implications considered.

Driven nonlinear systems often form important elements of complex networks. The collective dynamics arising from network interactions can impact significantly on the response to the driving forces of an individual system. There are two papers on Complex Networks. One of these reviews the enhancement of weak signals in complex networks due to stochastic and vibrational resonance. The second article considers the effects of three different kinds of autapses in the chaotic response of a single neuron and a network.

Finally, the Experimental Applications part consists of six articles presenting five broad application domains: vibrational machines for bulk material processing, signal processing and bearing fault detection, image processing, logic gate operations, energy harvesting and optical communication systems –– all of which are considered from the perspectives of both VR and SR.

The authorship of these articles is highly diverse in age, gender and seniority from early-career researchers to emeritus professors, broadly distributed geographically across nearly all continents of the world.

Part One consists of the following eleven articles including this introductory paper:

1. Introduction – Vibrational and stochastic resonance in driven nonlinear systems by Vincent *et al.* [24].
2. Vibrational resonances in driven oscillators with position-dependent mass by Roy-Layinde *et al.* [16].
3. Stochastic resonance in deformable potential with time-delayed feedback by Wadop Ngouongo *et al.* [25].
4. Delay-induced resonance suppresses damping-induced unpredictability by Coccolo *et al.* [26].
5. On representing noise by deterministic excitations for interpreting the stochastic resonance phenomenon by Sorokin and Demidov [27].
6. Vibrational resonance in a driven two-level quantum system; linear and nonlinear response by Shibashis and Ray [28].
7. The effect of high-frequency stochastic actions on the low-frequency behavior of dynamic systems by Kremer [29].
8. Adaptive stochastic resonance in bistable system driven by noisy NLFM signal: phenomenon and application by Yang *et al.* [30].
9. Amplification of optical signals in a bistable VCSEL by vibrational resonance by Chizhevsky [23].
10. Construction of logic gates exploiting resonance phenomena in nonlinear systems by Murali *et al.* [22].
11. Study of vibrational resonance in nonlinear signal processing by Pan *et al.* [21]

Thus Part One opens with the paper by Roy-Layinde *et al.* [16], presenting a comprehensive but succinct review of VR in driven oscillators. The paper highlights the wide range of systems and the varieties of potential structures for which VR has been observed and reported: from bistable and quintic oscillators to multi-stable systems, fractional-order systems to systems with deformable potentials, overdamped systems to systems with nonlinear dissipation, delayed dynamical systems to biological and oscillatory networks, to mention but a few examples. Roy-Layinde *et al.* [16] further highlights emergent phenomena associated with VR such as phase-locking modes, ghost-resonances, aperiodic resonances, the appearance of subharmonic and superharmonic resonances, etc. all of which bring fresh perspectives. On the applications of VR, the article identified four broad domains of VR applications: energy-related applications, signal processing, mechanical faults detections and the design of Dual Input Multiple Output (DIMO) logic gates and memory devices. Finally, Roy-Layinde *et al.* [16] propose the extension of the VR phenomenon to systems whose mass are position-dependent, highlighting the significant and potential applications inherent in addressing such problems, both from theoretical and numerical simulation perspectives.

The articles by Wadop Ngouongo *et al.* [25] on "*Stochastic resonance in deformable potential with time-delayed feedback*" and Coccolo *et al.* [26] on "*Delay-induced resonance suppresses damping-induced unpredictability*" consider VR and SR in time-delayed systems, which may arise from memory effects or due to a finite information and energy propagation time as can be found in biological networks. From a mathematical point of view, time-delay may be considered as the difference between the current state of a system and its future state. Wadop Ngouongo *et al.* [25] present fresh insights into the SR phenomenon in nonlinear underdamped deformable systems modelled by the Remoissenet-Peyrard potential which had been treated earlier without time-delay by Wadop Ngouongo *et al.* [31] in the SR setting and by Vincent *et al.* [32] in the VR configuration. The authors [25] show that the resultant effect of time-delayed feedback on the occurrence of SR in the system is the appearance of double resonance peaks due to a reduction in the effective potential depths and its barrier height. SR was in general interpreted from the point of view of the input energy E_{in} associated with the thermodynamic energy arising from Langevin force. The second paper on time-delay systems by Coccolo *et al.* [26] emphasizes the competition between damping and forcing; and shows that damping-induced unpredictability can be controlled by means of delay-induced resonance in time-delayed Duffing oscillator. The control action was accomplished by replacing one of the two periodic forces employed in the vibrational resonance scheme with a time-delay force term.

The article by Sorokin and Demidov [27], "*On representing noise by deterministic excitations for interpreting the stochastic resonance phenomenon*" explicitly analyzed the resonances induced by noise and high-frequency driving forces applied to general linear/nonlinear systems with additive/multiplicative noise. They argue that, counterintuitively, although in the additive driving force case, both stochastic and deterministic driving forces produces increased system responses, the underlying physical mechanisms are unrelated. The paper shows that under the action of the high-frequency deterministic forces, the time-averaged potential collapses to single well rather than a double well, so that oscillation takes place in the single well; whereas in the case of noisy driving, the resonance occurs when the frequency of the low-frequency periodic excitation input, and the "frequency" of the spontaneous jumps of the particle between the potential wells due to the noisy input, are close. In the parametric driving case they show that, by replacing higher noise intensities with high-frequency deterministic driving, the prediction and description of noise-induced instability associated with the increased response of the system is impossible because the high-frequency deterministic driving, which is a requirement for the occurrence of VR, does not capture the low frequency stochastic components. This is the justification for their proposed new approach, consisting of high and low frequency deterministic driving forces [27].

From the physical applications point of view, Shibashis and Ray [28] in their paper "*Vibrational resonance in a driven two-level quantum system; linear and nonlinear response*", consider the response of a two-level quantum system interacting with two deterministic time-periodic electromagnetic fields. By examining the linear, second-harmonic and Stokes sand anti-Stokes Raman responses of the dressed two-level quantum systems whose dynamics are governed by loss-free Bloch equations, the authors opine that quantum optics can offer a stimulating direction for future research on VR. Indeed, quantum VR was also recently reported by Olusola *et al.* [33] in a Tietz-Hua quantum oscillator. These works apparently set the basis for an extension of VR to the quantum regime; and would indeed trigger new research into applications in condensed matter structure where quantum resonance-effects arising from dual-frequency driving forces are prevalent.

Also related to applications, Kremer [29] in his paper titled "*The effect of high-frequency stochastic actions on the low frequency behavior of dynamic systems*" examines and analyses theoretically and experimentally, VR in a mechanical system. In the theoretical setting, the system considered is a nonlinear dynamical system driven by a finite number of independent random processes whose coefficients are dependent on the dynamical variables of the system and slow time. This translates to an experimental model of a vibration machine for bulk material processing undergoing random oscillations of the bulk material mass. Notably, the high-frequency noisy excitation from the bulk material is connected with change in the effective stiffness which, when varied, corresponds to tuning the operational rotation speed, thereby leading to extreme responses of the amplitude.

Yang *et al.* [30] in their paper "*Adaptive stochastic resonance in bistable system driven by noisy NLFM signal: phenomenon and application*", proposed the application of the stochastic resonance phenomenon to signal processing based on a new adaptive technique. The adaptive approach is generally highly promising because it makes possible the dynamical variation of the system parameters during signal transmission in noisy environments. This is particularly important in the smart control of damping due to vibrations, and the control of electro-hydraulic testing equipment. The authors combine empirical mode decomposition (EMD) with an adaptive piecewise re-scaled technique to realize nonlinear frequency modulation (NLFM) signals. Furthermore, they experimentally demonstrate the application of their new scheme to the detection of faults in machine bearings, a frequent problem encountered in mechanical engineering.

The paper on the *Amplification of optical signals in a bistable VCSEL by vibrational resonance* by Chizhevsky [23] reviews the amplification and detection of weak signals in connection with VR and SR. It draws inspirations from previous experimental realizations of VR, and discusses recent results from experimentally realized optical communication systems based on VR. It further demonstrates that VR can be used efficiently to control all-optical polarization switchings in bistable lasers in the development of optical switching devices for communication systems.

Murali *et al.* [22] present an overview of logical stochastic resonance (LSR), in which a noise-driven bistable system induces a logical output response to input signals, in an optimal window of moderate noise intensity. A noise-free configuration is also considered, thus exploring the benefits and cooperation of deterministic harmonic driving forces in logical operations. By driving a bistable system with periodic forces, the authors also demonstrate numerically and experimentally the phenomenon of logical vibrational resonance (LVR) for a broad range of moderate driving frequencies. Furthermore, they show that the interplay of periodic forcing and noise driving leads to the occurrence of logic response analogous to those of LSR.

Finally, Pan *et al.* [21] discuss the application of vibrational resonance to nonlinear signal processing. They report scenarios in which VR can be optimized for the improved performance of nonlinear sensors by optimally tuning the amplitude of high-frequency periodic vibrations injected into the sensor arrays. The method described was shown to be efficient for detecting the probability of weak signals and for the accurate estimation of unknown parameters of a system.

Part Two of the theme issue consists of nine articles, in addition to the introductory preface, highlighting the main content of each article. These nine articles address some specific domains of the theme issue such as complex networks of neurons, entropic stochastic resonance, triple cavity systems as well as applications to image processing and energy harvesting.

3. Summary and Conclusion

The Theme Issue focuses on the rapidly growing scientific subfield of vibrational and stochastic resonances in nonlinear systems driven by dual-frequency forces, the second driving force being of either deterministic or stochastic origin. The disciplines covered are, in general, multiple and interdisciplinary, ranging from physical systems and biological neural networks to engineering applications. It aggregates and reviews earlier works and the state-of-the-art on VR and SR and its various experimental applications in driven two-level quantum systems, a vibration machine for bulk material processing, nonlinear signal processing for the smart control of dampers due to vibrations and electro-hydraulic testing equipments and machine bearing fault diagnosis. It also emphasizes applications to the amplification and detection of weak signals, and the control of all-optical polarization switchings in bistable lasers employed in optical switching devices in communication systems. In particular, VR can be utilized for the improved performance of sensors by optimally tuning the amplitude of high-frequency periodic vibrations injected into the sensor arrays. Special attention is also paid to exploration of the benefits that can be derived from the cooperation between deterministic harmonic and stochastic driving forces for logical operations. The issue also presents and discusss the scientific connection between VR and SR, thereby providing a reference point and firm basis for future research on these important classes of nonlinear resonance, as well as spotlighting future research directions in the subfield.

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