

LANCASTER UNIVERSITY

DOCTORAL THESIS

Essays on
Applied Microeconomic Theory

Author:

Konstantinos PROTOPAPPAS

Supervisors:

Prof. Alexander MATROS

Dr David RIETZKE

Dr Orestis TROUMPOUNIS

*A thesis submitted in partial fulfilment of the requirements
for the degree of Doctor of Philosophy
in the*

Department of Economics
Lancaster University Management School



August 2020

Declaration of Authorship

I declare that this thesis, titled *Essays on Applied Microeconomic Theory* and the work presented in it are my own. I confirm that:

- This work was done wholly or mainly while in candidature for a research degree at this University.
- Where any part of this thesis has previously been submitted for a degree or any other qualification at this University or any other institution, this has been clearly stated.
- Where I have consulted the published work of others, this is always clearly attributed.
- Where I have quoted from the work of others, the source is always given. With the exception of such quotations, this thesis is entirely my own work.
- I have acknowledged all main sources of help.
- Where the thesis is based on work done by myself jointly with others, I have made clear exactly what was done by others and what I have contributed myself.

“... πρὸς ἑμαυτὸν δ’ οὖν ἀπιὼν ἐλογιζόμεν ὅτι τούτου μὲν τοῦ ἀνθρώπου ἐγὼ σοφώτερός εἰμι· κινδυνεύει μὲν γὰρ ἡμῶν οὐδέτερος οὐδὲν καλὸν καὶ ἀγαθὸν εἶδέναι, ἀλλ’ οὗτος μὲν οἶεται τι εἶδέναι οὐκ εἰδώς, ἐγὼ δέ, ὥσπερ οὖν οὐκ οἶδα, οὐδὲ οἶομαι· ἔοικα γοῦν τούτου γε σμικρῶ τι ἀὐτῷ τούτῳ σοφώτερος εἶναι, ὅτι ἂ μὴ οἶδα οὐδὲ οἶομαι εἶδέναι.”

Σωκράτης

“... as I went away, I thought to myself, ‘I am wiser than this man; for neither of us really knows anything fine and good, but this man thinks he knows something when he does not, whereas I, as I do not know anything, do not think I do either. I seem, then, in just this little thing to be wiser than this man at any rate, that what I do not know I do not think I know either’.”

Socrates

LANCASTER UNIVERSITY

Abstract

Lancaster University Management School
Department of Economics

Doctor of Philosophy

Essays on Applied Microeconomic Theory

by Konstantinos PROTOPAPPAS

This thesis consists of three chapters which cover multiple fields in applied microeconomic theory, such as behavioural industrial organisation (Chapter 1), contest theory (Chapters 2 and 3), political economy (Chapter 3) and optimal pricing (Chapters 1 and 3).

Particularly, in the first chapter, I study the role of loss-averse consumers' expectations about future consumption in the pricing policy of a monopolistic firm. Firm offers a contract consisting of two units of service and consumer makes two sequential consumption choices: buy first unit or not and if she buys first unit, buy second unit or not. Before signing the contract, consumer forms expectations about consumption of second unit. I, first, study consumer's optimal consumption strategy. Then, I derive firm's optimal pricing strategy for each case of consumer's expectations. Interestingly, if consumer expects to buy second unit with high probability, firm finds it optimal to prevent her from buying it offering a three-part tariff contract, namely, a fixed fee, first unit at a price below marginal cost and second unit at a price above it.

In the second chapter, I study a two-stage contest with two players *ex ante* asymmetric in abilities and a prize awarded at the second stage. At the first stage, players compete with each other and the winner earns a lower effort cost at the second stage. Then, with the second-stage effort costs having been allocated, the players compete again to win the final award. I consider two cases of the first stage: simultaneous and sequential moves by the players and find equilibrium efforts, expected payoffs as well as provide comparative static results. The *ex ante* advantaged player will never be inactive, even if her opponent plays first. Moreover, in equilibrium her equilibrium effort and expected payoff are always higher than her opponent's, regardless of the time she makes effort. I, also, endogenise the timing of the players' efforts and show that both players prefer sequential first stage with the weak player exerting effort first.

Finally, the third chapter is an application of the model of the second chapter to a game with two candidates and two interest groups. Groups offer two kinds of costly contributions

to achieve political influence: a) pre-electoral campaign contributions to their favourite candidates that increase their probability of winning the election, and b) post-electoral lobbying contributions to the winning candidate to affect the implemented policy. Candidates, in turn, are the first to act by strategically choosing the lobbying prices they will charge the groups once in office. I characterise the equilibrium values for lobbying prices, campaign, and lobbying contributions and show that: a) candidates commit to charge a lower lobbying price the group that supports them in the election, justifying the preferential treatment of certain groups once in office, and b) less skilled candidates tend to promise a larger preferential treatment to the group that supports them than more skilled candidates.

Acknowledgements

First, I would like to express my deepest gratitude to my supervisors Alexander Matros, David Rietzke and Orestis Troumpounis for their excellent supervision. I am confident that they are the best supervisors I could have. Alexander, with his experience and deep knowledge on the field, was remarkably helpful showing me the way to overcome any obstacles in my research. His constructive comments and suggestions were crucial in the process of writing this thesis. Dave's research approach and critical thinking have been very helpful to my research so far. I can tell that the discussions we had have made me a better economist. Orestis has been more than a supervisor to me. Not only did he motivate and guide me throughout my studies but, also, his sincere interest in my welfare and future career made my PhD life and job search process much easier than I expected.

I am also grateful to Themis Pavlidis and Maurizio Zanardi for their trust and support over the past few years. My sincere thanks are extended to the academic and administrative staff of the Department of Economics at Lancaster University Management School who contributed to my doctoral studies. Special thanks to Elias Bouacida, Renaud Foucart, Patricia Esteve-Gonzalez, Giorgio Motta and Caren Wareing for the valuable advice and support.

Of course, I could not omit to mention my friends and colleagues who have always been there for me and made my PhD studies an amazing experience. Anwar, Apo, Ari, Gianni, Kosta, Marie, Natasa, Niko, Sakib, Sophie...thank you for everything!

There are no words to express my gratitude and respect to the person who has been my inspiration and motivation since the beginning of this journey. Katerina, thank you for your encouragement and patience. I acknowledge that tolerating me for the last few years has not been an easy-going task.

Finally, I would like to thank my parents and sister for their unconditional support throughout my studies. Needless to say that this thesis would not have been accomplished if it were not for them.

Contents

Declaration of Authorship	iii
Abstract	vii
Acknowledgements	ix
1 Loss Aversion, Expectations and Optimal Pricing	1
1.1 Introduction	1
1.2 Model	6
1.2.1 Timing	6
1.2.2 Consumer's utility	7
1.2.3 Consumer's behaviour and expectations	9
1.2.4 Firm	9
1.3 Results	10
1.3.1 Consumer's strategy	10
Period 2	10
Period 1	12
1.3.2 Firm's strategy	14
1.4 Uniform distribution	17
1.5 Conclusion	19
2 Competition for advantage: A two-stage contest	21
2.1 Introduction	21
2.2 The game	24
2.2.1 Second stage	25
2.2.2 First stage	27
Simultaneous game	27
2.3 Sequential first stage	33
2.4 Endogenous timing	44
2.5 Conclusion	45
3 Optimal Lobbying Pricing	47
3.1 Introduction	47
3.2 The game	50
3.3 Results	53

3.3.1	Lobbying stage	53
3.3.2	Election stage	54
3.3.3	Pricing stage	56
3.4	Asymmetric Candidates	60
3.4.1	Lobbying stage	61
3.4.2	Election stage	61
3.4.3	Pricing stage	64
	Candidates	64
	Interest groups	67
3.5	Conclusion	69
	References	71
	Appendix A	79
	Appendix B	85
	Appendix C	95

List of Figures

1.1	Timing of the game	7
1.2	Expected social surplus as a function of p_1 in the case $q = 1$ ($c = 0.5, \lambda = 2$).	18
1.3	Expected social surplus as a function of p_1 in the case $q = 0$ ($c = 0.5, \lambda = 2$).	19
2.1	Player 1 and player 2's expected payoffs for $0 < a \leq \sqrt{2} - 1$	31
2.2	Player 1 and player 2's expected payoffs for $\sqrt{2} - 1 < a \leq 1$	32
2.3	Player 1 (leader) and player 2's (follower) first-stage effort.	36
2.4	Player 1 (leader) and player 2's (follower) expected payoffs for $\frac{1}{2} < a \leq 1$	38
2.5	Player 1 (leader) and player 2's (follower) expected payoffs for $0 < a \leq \frac{1}{2}$	39
2.6	Player 2 (leader) and player 1's (follower) first-stage effort.	41
2.7	Player 2 (leader) and player 1's (follower) expected payoffs for $2 - \sqrt{2} < a \leq 1$	43
2.8	Player 2 (leader) and player 1's (follower) expected payoffs for $0 < a \leq 2 - \sqrt{2}$	43
3.1	Contributions of groups i and j as a function of $c_{I,i}$ ($c_{J,j} = c_{I,j} = c_{J,i} = 1$).	56
3.2	Equilibrium of the pricing stage game for different values of δ	66
3.3	Equilibrium lobbying prices as a function of δ	67
3.4	Candidates' expected lobbying revenues as a function of δ	67
3.5	Total equilibrium lobbying effort as a function of δ . (e_i^* : total equilibrium lobbying effort if candidate i wins.)	68
3.6	Equilibrium election contributions as a function of δ	69
3.7	Expected payoffs of the groups as a function of δ	69

List of Tables

2.1	Player 1 and player 2's expected payoffs.	44
-----	---	----

*To my friend Giannis,
who did not manage to see this thesis complete.*

Chapter 1

Loss Aversion, Expectations and Optimal Pricing

1.1 Introduction

Experimental evidence shows that not only do consumers assess economic outcomes according to absolute measures but, also, they compare them to relevant reference points. The most remarkable indication of such reference-dependent preferences is loss aversion, in the sense that a loss-averse consumer is more sensitive to losses relative to her reference point than to gains relative to it (Kahneman & Tversky 1979, Kahneman et al. 1990, 1991). Reference-dependent preferences are known to affect consumers' behaviour and firms are aware of it and decide their strategies accordingly (Blinder et al. 1998, Odean 1998, Genesove & Mayer 2001).

Typically, a consumer's reference point in the money she has to pay in order to buy a good could be either her *status quo*, how much she usually spends on this good or the regular price of this good. In this study, instead, we consider a consumer similar to Kőszegi & Rabin (2006) whose reference point is determined by her probabilistic beliefs about future outcomes. However, while Kőszegi & Rabin (2006) consider a loss-averse consumer with reference-dependent preferences facing a one-shot decision problem regarding the purchase of a good, we assume that consumer's reference-dependent preferences arise after she has already consumed the good once and decides whether to consume again. In particular, we

assume that before a consumer buys the good for the first time, she forms expectations about her future behaviour given that she buys now.

To give some intuition, imagine that a customer of a mobile network provider wants to decide whether to use data on her mobile phone in order to watch the first season of a TV show. Consumer knows that if she decides to watch the first season, she will want to watch the second season as well with some probability. If, for example, she expects to watch the second season with certainty, doing so will be considered as a favourable outcome by her, offering her some consumption utility, while not watching it will be perceived as a loss. Nevertheless, although it is an undesirable outcome for her, consumer may also get positive utility (gain-loss utility) even if she ends up not watching the second season. This positive utility may arise in the case where consumer perceives not paying the price to buy data as a gain in her total utility greater than the loss she feels for not watching the show.

After consumer has formed expectations about future consumption, we follow the timing in [Grubb \(2015\)](#) in which a monopolistic firm offers a contract and consumer decides whether to accept it or to choose her outside option which is normalised to zero. If she accepts, in the first period, she learns her valuation of consuming a unit of service and makes a binary choice deciding whether to buy first unit or not. If she does so, in the second period, consumer learns her valuation of consuming an additional unit of service and decides whether to buy second unit or not. Then, consumer makes the payment to the firm, which depends on the number of units purchased and on the prices of the units.

We, first, find consumer's valuation thresholds above which she decides to purchase. In the second period, the optimal threshold depends on the price of the unit consumer considers to buy, the level of loss aversion as well as her *ex ante* expectation about consumption in this period. Actually, if consumer *ex ante* expects to buy second unit with sufficiently high probability, she does buy it even if her valuation of this unit is lower than the price of it. In contrast, if she *ex ante* expects to buy second unit with low probability, she does not buy it even for some levels of valuation greater than the price of it. Regarding optimal first-period threshold, it depends not only on the price of first unit but, also, on the price of the second

unit, the expectation of second-period valuation, the level of loss aversion and consumer's *ex ante* expectation about second-period consumption. This happens because when consumer decides whether to buy first unit, being forward-looking, she also takes into consideration the probability of buying second unit which, in turn, depends on the price of it, her level of loss aversion and the *ex ante* expectation of second-unit consumption.

Proceeding to study the pricing implications of such consumer's preferences, we find that, in equilibrium, depending on consumer's *ex ante* expectation of buying, a profit-maximising firm finds it optimal to deviate from a pricing strategy that involves extraction of consumer surplus through a fixed fee and marginal cost pricing for each unit. Actually, if a consumer *ex ante* expects to buy second unit with sufficiently low probability, firm finds it optimal to distort prices charging for the first unit a price above its marginal cost and a price below it for the second one. It acts so because since consumer's *ex ante* expectation to buy second unit is low, the probability of actually buying it is low as well. Thus, a profit-maximising monopolistic firm finds it optimal to charge a price below marginal cost for the second unit in order to raise the probability that consumer buys it and, hence, increase her perceived surplus. In turn, to maximise expected profit and compensate for the second-period below marginal cost pricing, firm finds it optimal to charge a price above marginal cost for the first unit.

The optimal pricing strategy of the firm becomes even more interesting when consumer's *ex ante* expectation of buying second unit is sufficiently high. In this case, firm finds it optimal to charge a price above marginal cost for the second unit. Firm, knowing that consumer expects to buy second unit with high probability, finds it optimal to increase the price of it so that to decrease the probability of buying it. This way, firm increases consumer's gain-loss utility in money dimension if she does not buy rather than her consumption utility if she buys. In order to induce consumer buy first unit and, therefore, increase the probability that she will experience an increase in her gain-loss utility in period 2, firm finds it optimal to sell first unit at a price below marginal cost. Interestingly, if this marginal cost is zero, which is the case, for instance, in mobile network providers where the cost of an additional

minute or MB of data is zero, firm finds it optimal to offer first unit for free. This particular type of contract consisting of a fixed fee, an allowance of units for which marginal price is zero, and a positive marginal price for subsequent units beyond the allowance is known as three-part tariff and is common in several markets such as mobile phone service, electricity, health care and debit-card transactions.

Standard non-linear pricing models have not been able to explain such pricing schemes with increasing marginal prices. Instead, they predict that prices should be decreasing rather than increasing (Mussa & Rosen 1978, Maskin & Riley 1984). Moreover, consideration of consumer with biased beliefs such as naive quasi-hyperbolic discounting for leisure goods (DellaVigna & Malmendier 2004), myopia (Gabaix & Laibson 2006, Miao 2010) and self-control naivety (Esteban et al. 2007, Heidhues & Koszegi 2010) can explain marginal prices above marginal cost but cannot explain marginal prices below it for low levels of consumption. Also, hyperbolic discounting for investment goods can provide an explanation for marginal pricing below marginal cost but not for marginal pricing above it (DellaVigna & Malmendier 2004).

A number of studies that explain the introduction of three-part tariffs are the following. Grubb (2009) shows that it is optimal for a firm to offer three-part tariff contracts if consumers are overconfident, in the sense that they make two mistakes: overestimate the probability of consuming initial units and underestimate the probability of extremely high usage. Eliaz & Spiegel (2006) study a screening model with dynamically inconsistent preferences who differ in their degree of their ability to predict the change in their future changes. They find that the optimal menu of contracts offered by the principal can be implemented by a menu of three-part tariffs and not by a menu of two-part tariffs. Grubb (2015) considers consumers who are inattentive, in the sense that they do not keep track of their past usage of service. For example, a mobile phone user might not realise that she has exceeded her free allowance of data and, therefore, does not know that surfing the internet is now charged at a high rate. Again, in order for three-part tariff scheme to be optimal for the firm, consumer has to make the two mistakes mentioned above. Finally, in a recent study, Triviza (2019)

considers a consumer who is habit-forming—her current consumption increases her value of future consumption. She shows that if consumer is naive habit-forming, namely, she realises that she is habit-forming only if she has already consumed, three-part tariff is optimal for the firm. Still, for this price scheme to be optimal, consumer is assumed to make one mistake which is underestimation of the likelihood of future consumption.

In all the above cases, the common assumption that gives rise to three-part tariffs is that consumers have a systematic bias in predicting their future preferences, while firms do not. In the first study, consumers underestimate the variance of the future demand while the firm knows the actual one. In the second study, principal knows the agent’s future utility while the agent is partially aware of her changing tastes. In the last two studies, consumers update their planned future behaviour after they have consumed. As a consequence, despite the fact that firms *ex ante* know the “true” expected consumer surplus, they cannot extract it all since consumers believe that their expected surplus is different to the “true” one.

In this study, we show that a three-part tariff pricing scheme can be optimal for a firm even if there is not asymmetric information between the consumer and itself about consumer’s expected utility and consumer stays consistent to her *ex ante* planned consumption strategy. Furthermore, in contrast to the above studies where firm knows consumer’s “true” expected utility but consumer has a misperception about it and learns the “true” one when she has already consumed, in our case, while consumer and firm have different perception about consumer’s expected utility, it is this utility they both take into consideration when they search for optimal consumption and pricing strategy, respectively. Also, unlike in other studies, our consumer does not update her strategy when she has already consumed and has to decide whether to consume more or not. Finally, while the above studies suggest that firm sells the second unit expensively because consumers eventually become willing to pay the high price and acquire it, in this paper, we propose that if firm faces consumers with high expectations of buying second unit, it sells it at a high price to prevent them from buying and, as a consequence, feel their perceived utility increased because of the increase in their gain-loss utility.

There is a growing literature considering interaction between loss-averse consumers and rational firms. For instance, [Heidhues & Kőszegi \(2004\)](#) consider a model with a monopolistic firm and consumers who assess paying high prices as a loss compared to lower possible prices. They show that even when marginal costs are continuously distributed, firm does not change the price it charges for most small cost changes, result that interpret as price stickiness. In a “cover version” of that model, [Spiegler \(2012\)](#) assumes that consumer has a single reference point in price and he shows that expected price charged by firms are lower than in the case without loss-averse consumers, a result also present in [Heidhues & Kőszegi \(2008\)](#) in an oligopolistic setting.¹

The rest of the paper is structured as follows. In [Section 1.2](#), we present the model. In [Section 1.3](#), we solve the model and provide the results regarding consumer’s optimal behaviour and firm’s optimal pricing strategy. In [Section 1.4](#), we graphically illustrate our findings assuming uniform distribution. Finally, [Section 1.5](#) concludes the paper.

1.2 Model

1.2.1 Timing

Following [Grubb \(2015\)](#), we consider a two-period model, with one consumer and one firm. Consumer’s available consumption choices are to buy a unit of service in period 1 and if she does so, to also buy an additional unit of service in period 2.²

Before the game starts, consumer forms expectations about consumption in period 2 which are common knowledge between the firm and the consumer. After these expectations have been formed, consumer and firm learn the distribution of consumer’s valuation of the service and firm offers a contract $\mathbf{p} = \{p_0, p_1, p_2\}$, where p_0 is a fixed fee, p_1 is the price of

¹See also [Popescu & Yaozhong \(2007\)](#), [Karle & Peitz \(2014\)](#) and [Karle & Möller \(2020\)](#) who study the interaction between firms and loss-averse consumers in different settings.

²The main interest of the paper is focused on the case where the decision about purchase in both period 1 and period 2 is involved. Therefore, we omit the case where consumer can buy first unit in period 2. This assumption makes our model less complicated and the results more elegant while not significantly affecting them.

first unit and p_2 the price of second unit of service.³ If consumer accepts the contract, she pays the fixed fee, p_0 . Otherwise, she chooses the outside option which is normalised to zero.

If consumer accepts the contract, the timing of the game is shown in Figure 1.1. Denote by $k_t \in \{0, 1\}$ consumer's choice of buying one unit of service in period t , which is equal to zero if she does not buy and equal to one if she does. In period $t = \{1, 2\}$, consumer learns her valuation of service, v_t . More specifically, at $t = 1$, consumer learns v_1 . Then, she makes a binary purchase decision: either to pay p_1 and purchase a unit of service ($k_1 = 1$) or not ($k_1 = 0$). If she chooses not to buy in period 1, consumer gains zero utility and the game ends. If she chooses to buy, consumer moves to period 2 where she learns the valuation of an additional unit of service, v_2 , and chooses whether to purchase ($k_2 = 1$) or not ($k_2 = 0$).

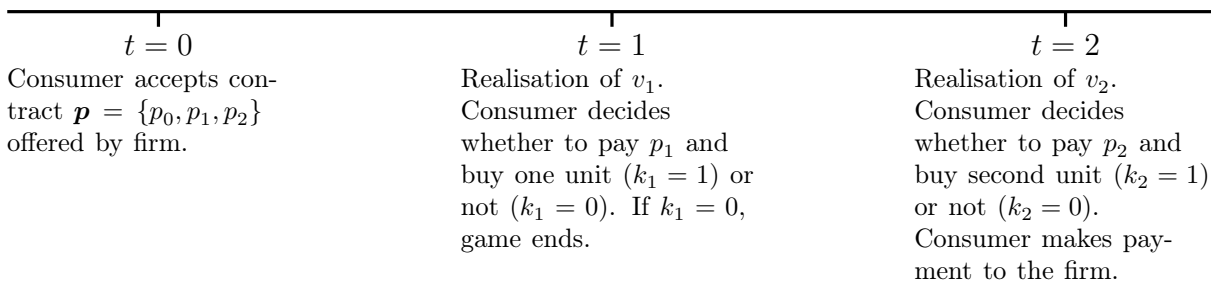


FIGURE 1.1: Timing of the game

1.2.2 Consumer's utility

We assume that consumer has no *ex ante* expectations about consumption of first unit.⁴ However, she does have *ex ante* expectations about consuming second unit of service. For instance, if consumer *ex ante* expects to consume second unit of service, then, doing so is considered by her as a favourable outcome while if she expects not to consume it, in this case, not consuming it is the favourable outcome.

³Since first and second unit can only be bought in period 1 and 2 respectively, we could equivalently consider p_t to be the price of a unit of service in period t .

⁴The main results of the paper can be obtained even if we consider expectations about consumption of first unit as well. However, for simplicity, we assume that consumer behaves as an expected utility maximiser in period 1.

To formalise our model, since consumer has no expectations about consumption of first unit, her utility of consuming first unit is simply the difference between her valuation of the unit in period 1 minus the price of the unit, $v_1 - p_1$, where v_t is the valuation of a unit of service in period t , drawn independently with cumulative distribution function F , with support on $[0, 1]$ and probability density function f .

Now, if consumer purchases in period 1 ($k_1 = 1$) and her *ex ante* expectation was to buy second unit of service with probability q , we derive her period 2 utility from [Kőszegi & Rabin \(2006\)](#). Thus, if the price of second unit is p_2 , consumer's net utility of buying it is

$$v_2 - p_2 + (1 - q)(v_2 - \lambda p_2). \quad (1.1)$$

The term $v_2 - p_2$ is consumption utility from buying. The remaining term is gain-loss utility from comparing buying to not buying. Thus, if, for instance, $q = 1$, i.e., consumer *ex ante* expected to buy second unit with certainty, her utility from buying it is only the consumption utility, $v_2 - p_2$. On the other hand, if $q = 0$, i.e., consumer *ex ante* expected not to buy second unit with certainty, her utility is the consumption utility plus the gain-loss utility which is assessed as a gain in the value of the second unit of service, v_2 , and a foregone gain in the price of it, p_2 , which because of the assumption of loss aversion is multiplied by $\lambda > 1$.

If, now, consumer does not buy second unit, in contrast to a non-loss-averse consumer who earns zero utility, our consumer's net utility is

$$q(p_2 - \lambda v_2). \quad (1.2)$$

Relative to the expectation to buy, not buying is a gain of p_2 and a loss of v_2 . For example, if consumer *ex ante* expects to buy second unit with certainty, not buying it and, therefore, not paying p_2 , is considered as a gain in the money dimension and a loss in the service dimension. If consumer *ex ante* expects not to buy second unit, not doing so results to zero utility for her.

Consumer's total net utility is, then,

$$U = -p_0 + k_1 \{v_1 - p_1 + k_2 [v_2 - p_2 + (1 - q) (v_2 - \lambda p_2)] + (1 - k_2) q (p_2 - \lambda v_2)\} \quad (1.3)$$

1.2.3 Consumer's behaviour and expectations

Consumer's optimal consumption strategy is a function mapping valuations to purchase decisions,

$$\mathbf{k}^* : [0, 1] \longrightarrow \{0, 1\}.$$

Before proceeding, we would like to provide some intuition about consumer's possible expectations and, particularly, the two extreme ones, namely, consumer believes that she will buy second unit with certainty and consumer believes that she will not buy second unit with certainty. In the first case, imagine that a customer of a mobile network provider wants to decide whether to use her data on her mobile phone in order to watch the first season of a TV show. She has no expectations about watching the first season but she knows that if she does so, she will want to watch the second season as well.⁵ In the second case, consider a consumer that buys some minutes on his mobile phone and believes that she will not need to make any other calls when these minutes are over.⁶

1.2.4 Firm

Payment to the firm is a function of prices $\mathbf{p} = (p_0, p_1, p_2)$ and consumer's purchase choices, $\mathbf{k} = (k_1, k_2)$. This is

⁵This type of behaviour might remind the consumer in [Becker & Murphy \(1988\)](#) whose current consumption of the good affects future consumption. However, in this study, we do not model rational addiction, as [Becker & Murphy \(1988\)](#) do, but rather it might be one possible explanation why our consumer has such expectations.

⁶This consumer reminds the one in [Grubb \(2009\)](#) where she underestimates the probability of buying second unit but, eventually, when she consumes first unit, she figures out the true probability of buying it and updates her planned strategy. In our case, when consumer has bought first unit and decides whether to buy the second one or not, she does not modify her planned strategy but, actually, she makes her decision taking into consideration her *ex ante* expectation of consuming it.

$$P(\mathbf{k}, \mathbf{p}) = p_0 + k_1 p_1 + k_1 k_2 p_2,$$

where firm receives the fixed fee, p_0 , regardless of the units sold and the prices of the units sold if any.

If firm's cost per unit of service is $c \in [0, 1]$, expected profit of the firm is its revenue minus cost for each unit sold. Thus, firm's profit function when consumer makes optimal consumption decisions is

$$\Pi = \mathbb{E} [P(\mathbf{k}^*, \mathbf{p}) - c(k_1^* + k_2^*)].$$

1.3 Results

1.3.1 Consumer's strategy

Period 2

Let us go backwards and study consumer's behaviour in period 2. When consumer learns her valuation of the service in period 2 and has to decide whether to buy second unit or not, she will choose to buy if her utility of buying is greater than the utility of not buying. Thus, consumer sets a threshold of second-period valuation above which she buys second unit and below which she prefers not to buy. Setting eq. (1.1) greater than eq. (1.2) yields our first proposition:

Proposition 1.1. *If consumer is loss-averse and her ex ante expectation is to buy second unit of service with probability q , her optimal strategy is to buy it if $v_2 > v_2^*$, where*

$$v_2^* = \frac{[1 + q + (\lambda - 1)q] p_2}{2 + (\lambda - 1)q}.$$

Observing the optimal threshold of buying second unit in Proposition 1.1, we can see the effect of consumer's *ex ante* expectations and loss aversion on her behaviour. Indeed, in contrast to a non-loss-averse consumer who buys second unit if her second-period valuation

is greater than the price of the second unit, a loss-averse consumer sets a lower threshold of valuation above which she decides to buy second unit. In order to avoid the even greater loss in utility she will incur if she does not buy second unit, she may buy it even if her valuation in the period 2 is lower than the price, implying negative payoff for her but still less negative than the payoff she gets if she does not buy. It becomes clear that loss aversion makes consumer biased in favour of buying second unit compared to a non-loss-averse consumer.

Differentiating the valuation threshold in Proposition 1.1 with respect to p_2 and q , we have, respectively,

$$\frac{\partial v_2^*}{\partial p_2} = \frac{1 + q + (\lambda - 1) q}{2 + (\lambda - 1) q} > 0, \quad \frac{\partial v_2^*}{\partial q} = -\frac{(3 + \lambda)(\lambda - 1) p_2}{[2 + (\lambda - 1) q]^2} < 0.$$

For given q , as p_2 increases, v_2^* also goes up. The intuition is straightforward: as the price of second unit increases, consumer decreases the probability of buying it by raising the optimal valuation threshold. On the contrary, for given p_2 , v_2^* is decreasing in q . The greater the *ex ante* probability that consumer believes she will buy second unit, the more prone she is to doing so when she is in period 2.

While the above derivatives are monotonic, differentiating v_2^* with respect to λ and keeping p_2 and q constant, we obtain that

$$\frac{\partial v_2^*}{\partial \lambda} = \frac{2(1 - 2q) p_2}{[2 + (\lambda - 1) q]^2} \begin{cases} > 0 \text{ if } q < \frac{1}{2} \\ < 0 \text{ if } q > \frac{1}{2} \end{cases}.$$

Whether v_2^* is increasing or decreasing in λ depends on the level of q . Specifically, if consumer *ex ante* expects to buy second unit with sufficiently high probability, the more loss averse she becomes, the more she increases the probability of buying it by lowering the optimal valuation threshold. This is because when q is high, the negative effect of λ on consumer's utility is stronger as she does not purchase and, therefore, she increases the probability of

purchasing. On the contrary, if q is low, an increase in λ reduces consumer's utility if she does not buy less than if she does buy and, therefore, she decreases the probability of buying by increasing her optimal valuation threshold.

Period 1

Having determined utilities and optimal thresholds regarding second unit, we can now express consumer's expected utility in the contracting period. Denote by v_1 consumer's valuation of the service in period 1 and by v_1^* the valuation threshold above which consumer decides to buy first unit. Consumer's expected utility in the contracting period is

$$\begin{aligned} \mathbb{E}(U) = -p_0 + \int_{v_1^*}^1 \left\{ v_1 - p_1 + \int_{v_2^*}^1 [v_2 - p_2 + (1 - q)(v_2 - \lambda p_2)] f(v_2) dv_2 + \right. \\ \left. + \int_0^{v_2^*} q(p_2 - \lambda v_2) f(v_2) dv_2 \right\} f(v_1) dv_1. \end{aligned} \quad (1.4)$$

The first term of the sum is the fixed fee consumer pays to the firm if she accepts the contract. The second term of the sum, the long integral, is consumer's expected utility if she buys in both periods.

Consumer's optimal consumption strategy is to purchase in period 1 if her valuation exceeds threshold v_1^* . Hence, consumer's maximisation problem is

$$\max_{v_1^*} U(v_1^*).$$

Maximising with respect to v_1^* and using previous results about second period optimal thresholds, we obtain the following proposition:

Proposition 1.2. *If consumer is loss-averse and her ex ante expectation is to buy second unit of service with probability q , her optimal strategy is to buy first unit if $v_1 > v_1^*$, where*

$$v_1^* = p_1 - qp_2 - (2 - q) \int_{v_2^*}^1 [1 - F(v)] dv + q\lambda \int_0^{v_2^*} [1 - F(v)] dv.$$

Proof. See Appendix A. □

Regarding the period 1 optimal threshold, we observe that it depends not only on p_1 , but also on p_2, λ, q and the expectation of v_2 . When consumer decides whether to purchase in period 1, she takes into consideration the price of first unit and her valuation of the service. Nevertheless, consumer is forward-looking. Therefore, since buying in period 1 creates the probability of buying also in period 2—which in turn creates the probability that the loss aversion effect kicks in—before buying in period 1, consumer also considers the price of the second unit, the *ex ante* expectation of buying second unit, her level of loss aversion and the expectation of her period 2 valuation of service.

In what follows, we provide some comparative statics for the period 1 optimal threshold. First, differentiating the optimal threshold, v_1^* , with respect to the price of the first unit, p_1 , we find that $\partial v_1^*/\partial p_1 = 1 > 0$. The effect of an increase in the price of first unit on the period 1 valuation threshold is clear and the intuition is straightforward: as first unit of service becomes more expensive, consumer decreases the probability of buying in period 1 by increasing the optimal period 1 valuation threshold.

On the contrary, differentiating v_1^* with respect to p_2 , we obtain that

$$\frac{\partial v_1^*}{\partial p_2} = 1 + (\lambda - 1)q - [1 + \lambda - (\lambda - 1)q] F(v_2^*)$$

which yields the following lemma:

Lemma 1.1. *Consumer increases the probability of buying first unit as the price of second unit increases if $1 - F(v_2^*) < \frac{1}{2}$ and $q > \frac{[1 - F(v_2^*)]^{(\lambda+1)}}{F(v_2^*) + [1 - F(v_2^*)]^\lambda}$.*

Consumer decreases her optimal valuation threshold in period 1 as the price of second unit increases if her *ex ante* expectation of buying it, q , is sufficiently high but the actual probability of buying it, $1 - F(v_2^*)$, is sufficiently low. A non-loss-averse consumer, who compares only prices to valuations, would decrease the probability of buying as the price of second unit goes up. However, in the case with a loss-averse consumer, depending on her *ex ante* expectation of buying second unit, doing so might be considered as a loss either in service or

money dimension. As a consequence, consumer might even raise the probability of buying first unit and, therefore, the probability of buying second unit as well, even if second unit becomes more expensive.

For instance, if $q = 1$, i.e., consumer *ex ante* expects to buy second unit with certainty, we have that $\partial v_1^*/\partial p_2 = 1 - 2F\left(\frac{2p_2}{1+\lambda}\right)$, indicating that v_1^* is decreasing in p_2 if $1 - F\left(\frac{2p_2}{1+\lambda}\right) < 1/2$. The intuition is the following: because p_2 is assessed as a loss in money dimension when consumer buys second unit but as a gain when she does not, since now the probability of buying second unit is lower than the probability of not buying it, as p_2 goes up, consumer increases the probability of buying first unit and, therefore, the probability of facing the gain of p_2 in period 2. Nevertheless, if $q = 0$, i.e., consumer *ex ante* expects not to buy second unit, we have that $\partial v_1^*/\partial p_2 = (1 + \lambda) \left[1 - F\left(\frac{(1+\lambda)p_2}{2}\right)\right] > 0$. Now, the two possible scenarios are the following: either consumer buys second unit and incurs a loss in the money dimension or she does not buy it and gains zero utility. It follows that in neither of the two cases a raise in the price of the second unit can be beneficial for the consumer since not only does it increase the probability of not buying and, therefore, getting zero utility but, also, it reduces the utility of buying. Thus, since there is not any potential gain-loss utility induced by an increase in the price of the second unit, if this price increases, consumer decreases the probability of buying first unit by increasing her optimal period 1 valuation threshold.

1.3.2 Firm's strategy

To begin with, we know that the optimal pricing strategy of a monopolistic firm when consumer is non-loss-averse is to set a two-part tariff, namely price equal to marginal cost for each unit and fixed fee equal to the consumer surplus.⁷ Since a non-loss-averse consumer is assumed not to have any expectations or even if she has, she does not take them into account when planning her consumption strategy, firm is able to have the first best profit since it can extract all the consumer surplus through a fixed fee and set marginal cost pricing for each unit sold.⁸

⁷See also Grubb (2015).

⁸For details, see Appendix A.

Now, assume that we have a consumer with the preferences described in the previous section and a rational, profit-maximising monopolistic firm which can observe consumer's preferences and expectations as well as the distribution of consumer's valuations, F . Recall that firm's cost per unit of service is constant between the two periods and equal to $c \in [0, 1]$. Firm's profit for selling unit i is $p_i - c$. Hence, firm's expected total profit is

$$\mathbb{E}(\pi) = p_0 + \int_{v_1^*}^1 \left(p_1 - c + \int_{v_2^*}^1 (p_2 - c) f(v_2) dv_2 \right) f(v_1) dv_1. \quad (1.5)$$

The first term is the fixed fee firm paid if consumer accepts the contract and the second term is the expected profit from selling first unit, which also considers the possibility of selling second unit.

Let us now express firm's profit as $\pi = S - U$, where S is the social surplus produced.⁹ Firm's objective is, then,

$$\max_{\mathbf{p}} \mathbb{E}(\pi) = \mathbb{E}[S(\mathbf{p})] - \mathbb{E}[U(\mathbf{p})] \quad \text{s.t.} \quad \mathbb{E}(U) \geq 0. \quad (1.6)$$

Firm maximises its expected profit which is the difference between expected social and expected consumer surplus subject to the participation constraint. Since we have one monopolistic firm, it maximises its profit setting expected consumer's surplus equal to her reservation level, which is zero. Thus, firm chooses marginal prices, p_1 and p_2 , and adjusts the fixed fee, p_0 , in order to make the participation constraint binding, i.e., $\mathbb{E}(U) = 0$.

Adding eq. (1.4) and eq. (1.5) together and simplifying, we obtain that expected social surplus is

⁹Note that this is the social surplus as perceived by consumer and firm. If one argues that loss aversion is irrational, she will define consumer surplus as $\tilde{U} = -p_0 + k_1[v_1 - p_1 + k_2(v_2 - p_2)]$, where consumer's utility consists only of consumption and not gain-loss utility, and social surplus as $\tilde{S} = \tilde{U} + \pi$.

$$\begin{aligned} \mathbb{E}(S) = \int_{v_1^*}^1 \left\{ v_1 - c + \int_{v_2^*}^1 [v_2 - c + (1 - q)(v_2 - \lambda p_2)] f(v_2) dv_2 + \right. \\ \left. + \int_0^{v_2^*} q(p_2 - \lambda v_2) f(v_2) dv_2 \right\} f(v_1) dv_1. \end{aligned} \quad (1.7)$$

Maximising eq. (1.6) with respect to p_1 and p_2 using eq. (1.7) yields Proposition 1.3 which provides firm's optimal pricing strategy.

Proposition 1.3. *If consumer is loss-averse and her ex ante expectation is to buy second unit of service with probability q , a monopolistic firm's optimal pricing strategy is:*

$$\begin{aligned} \text{If } c > 0 : & \begin{cases} p_1 > c, p_2 < c & \text{if } q < \bar{q} \\ p_1 < c, p_2 > c & \text{if } q > \bar{q} \end{cases} \\ \text{If } c = 0 : & \begin{cases} p_1 > 0, p_2 = 0 & \text{if } q < \bar{q} \\ p_1 = 0, p_2 > 0 & \text{if } q > \bar{q} \end{cases} \end{aligned}$$

where $\bar{q} = \frac{[1-F(v_2^*)]^\lambda}{F(v_2^*)+[1-F(v_2^*)]^\lambda}$ and p_0 is equal to the corresponding perceived consumer surplus.

Proof. See Appendix A. □

When consumer's *ex ante* expectation is to buy second unit with sufficiently low probability, if she buys, her consumption utility and gain-loss utility are greater than her gain-loss utility if she does not buy. Thus, the firm, so as to create as much perceived consumer surplus as possible and extract it through the fixed fee, has incentive to sell second unit at a price even below marginal cost and, therefore, make the purchase of it more possible. In order to compensate for the below marginal cost price of the second unit, firm needs to sell first unit at a price greater than marginal cost. In turn, consumer, being forward-looking, is willing to pay a high price for the first unit and, consequently, create the possibility of gaining the utility from buying second unit as well.

On the other hand, if consumer *ex ante* expects to buy second unit with sufficiently high probability, when she compares buying to not buying, her gain-loss utility is greater in the latter case. Therefore, firm finds it optimal to increase the price of the second unit above marginal cost so that to decrease the probability that consumer actually buys it and, hence, increase her gain-loss utility from not paying a high price.

In order to increase the probability that consumer buys first unit and, therefore, period 2 actually takes place, firm finds optimal to sell first unit at a price below marginal cost. Interestingly, if firm's per unit marginal cost is zero, as in the mobile network providers market where the cost of each MB of internet provided is negligible, and assuming non-negative prices, firm finds it optimal to offer first unit for free. This pricing scheme (three-part tariff) is common in the market of mobile phone service, where the contracts offered include a fixed fee, an allowance of minutes or data for free and high prices for consumption beyond this allowance. However, in contrast to previous literature which suggests that firms sell subsequent units of service expensively because consumers eventually become willing to pay these high prices and acquire them, what we propose is that firms, knowing that consumers expect to overconsume, charge these high prices to prevent them from buying, make their expected perceived consumer surplus greater and absorb this high surplus through fixed fees.

1.4 Uniform distribution

In this section, let us assume that consumer's valuation of service in each period is distributed uniformly in $[0, 1]$, i.e., $f(v) = 1$ and $F(v) = v$. This is a common assumption in the literature which allows us to graphically illustrate the findings of Section 1.3.

In Figure 1.2, fixing p_2 and λ , we graphically compare expected social surpluses as functions of p_1 in the case where $q = 1$. The blue curve is social surplus when firm sets marginal cost pricing for both units. The blue dashed and dotted curves depict expected social surpluses when firm sets price for the second unit above and below marginal cost, respectively. Finally, the light orange curve is expected social surplus when consumer is non-loss-averse

and firm's optimal strategy is to set marginal cost pricing for both units.¹⁰ We observe that the maximum possible expected social surplus is when firm charges second unit above marginal cost. We can also notice that at the point where expected social surplus is at the maximum level, the price of the first unit on the horizontal axis is below marginal cost. Thus, we graphically demonstrate that firm finds it optimal to offer a contract consisting of a three-part tariff. Because consumer expects to buy second unit, by selling it at a high price, firm increases consumer surplus as perceived by her since it induces her not to buy it and, therefore, gain-loss utility from not paying this high price. In order to increase the probability that consumer experiences this gain in her utility, firm charges a low price for first unit to make purchase of it more likely.

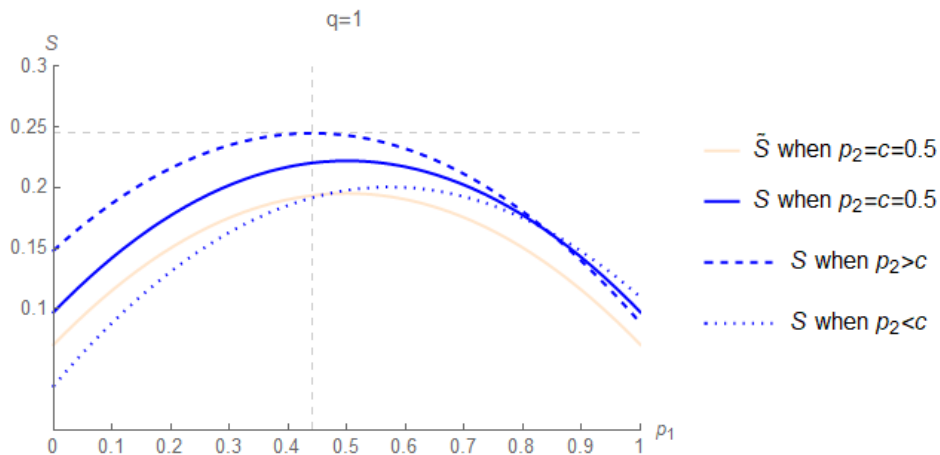


FIGURE 1.2: Expected social surplus as a function of p_1 in the case $q = 1$ ($c = 0.5$, $\lambda = 2$).

Now, Figure 1.3 shows the graph of expected social surpluses in the case where $q = 0$. In contrast to the previous graph, we observe here that if consumer is loss-averse and *ex ante* expects not to buy second unit, expected social surplus is maximised when firm sells second unit at a price below marginal cost. Also, at the point where expected social surplus is maximised, price of the first unit is above marginal cost. Thus, we graphically verify firm's optimal pricing strategy for this case found in Section 1.3. However, differently to the

¹⁰For details on non-loss-averse consumer's strategy and optimal firm's response, see Appendix A.

previous case, we also notice that expected social surplus can be even greater if consumer is non-loss-averse.

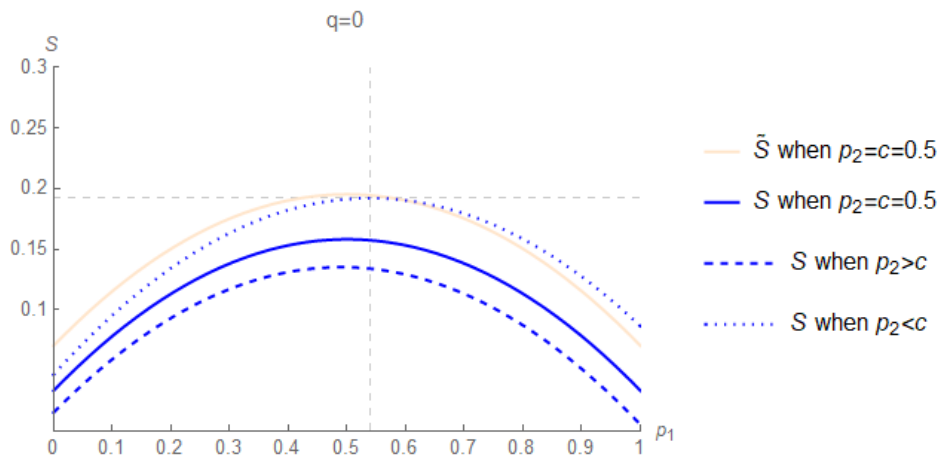


FIGURE 1.3: Expected social surplus as a function of p_1 in the case $q = 0$ ($c = 0.5$, $\lambda = 2$).

1.5 Conclusion

In this study, we have analysed, in a dynamic setting, how a loss-averse consumer’s expectations about future consumption affect the pricing strategy of a rational profit-maximising monopolistic firm. These expectations, combined with consumer’s non-standard preferences, induce firm to deviate from the “first best” two-part tariff pricing, i.e., marginal cost pricing and extraction of all consumer surplus through a fixed fee, which would be the case with a non-loss-averse consumer. However, depending on consumer’s expectations, this distortion in prices might even be beneficial for the firm despite consumer’s misperception of consumer surplus. This occurs because firm can exploit consumer’s expectations and create more consumer surplus as perceived by her and then absorb it through a fixed fee making higher profit than in the case of a non-loss-averse consumer market.

Chapter 2

Competition for advantage: A two-stage contest

2.1 Introduction

There are many situations in which a number of contestants compete with each other exerting costly efforts in order to win a prize. These efforts include, among others, monetary expenditures or time spending, while the prize awarded may include monopoly rents, sports awards or fame. Beginning from the seminal works of [Tullock \(1980\)](#) and [Krueger \(1974\)](#) who study rent-seeking, these competitions have been widely studied by economists, spawning the contest theory literature.¹

While most of the early research focuses on single-stage contests ([Hillman & Riley 1989](#), [Baye et al. 1993](#)), there are cases in which contests take place in more than one stage. In this contests, the efforts submitted at the first stage affect the efforts and, therefore, the payoffs of the subsequent stages.² For instance, consider a pre-election period with two lobbies, each of them supporting a specific candidate. In order to help its favourite candidate win the election and, consequently, come to power, each lobby exerts effort (e.g., campaign

¹For surveys of the literature, see [Nitzan \(1994\)](#), [Corchón \(2007\)](#), [Konrad \(2009\)](#) and [Corchón & Serena \(2017\)](#).

²[Harris & Vickers \(1987\)](#) are the first to study a multi-stage race, in which the player who makes a specific number of advances is the winner of the game. [Konrad & Kovenock \(2009\)](#) extend this study by adding intermediate prizes awarded to the winner of each round. Some other relative studies are [Klumpp & Polborn \(2006\)](#), [Fu & Lu \(2012\)](#) and [Gelder \(2014\)](#).

contributions, endorsements, media campaigns) to persuade the constituency to vote for her. Then, the election takes place and one of the two candidates wins. The lobbies do not have a direct benefit or loss at the end of the election since no policies have been implemented yet. Now, both lobbies have to exercise pressure in order to influence the incumbent's political decisions in favour of their members' welfare, regardless of who the winner of the election is. However, it would be reasonable to assume that, once the winner of the election takes office, the lobby which supported her prior to the election, being politically aligned with her, will be more efficient at influencing her potential policy than the lobby that supported the opposite candidate.

In this paper, we examine a two-stage contest with two players competing to win a single, non-divisible prize. The two players are *ex ante* asymmetric in abilities, meaning that one of them has an *ex ante* cost advantage. At the first stage, there is no award, but the two players compete with each other in order to gain an advantage at the second stage. At the beginning of the second stage, the first-stage winner and loser have been determined. The first-stage winner obtains the advantage, which is expressed as a lower effort cost than her opponent's one at the second stage. Then, with the second-stage effort costs having been allocated according to the outcome of the first stage, the two players compete again with each other to win the final prize.

Regarding the structure of the game, at the second stage, the two players choose efforts simultaneously, while at the first stage, we consider two cases: simultaneous and sequential movement by the players. A model similar to the one of the first case has been analysed by [Clark et al. \(2018\)](#). A difference between their model and the one in this paper is that in the former, the main question addressed is the optimal prize distribution between the two stages, whereas, in our study, the prize is indivisible and awarded at the end of the contest. Also, in [Clark et al. \(2018\)](#), the favourite player's *ex ante* advantage is carried over to the second stage and, thus, affects the outcome of the game. In this paper, the *ex ante* asymmetry affects only the outcome of the first stage and does not influence the second-stage asymmetry.

A justification of the elimination of the first-stage asymmetry at the second stage could be the following. The forms of lobbying at the two stages are different. At the first stage, the interest groups try to affect the voters' behaviour, while at the second stage they attempt to influence the incumbent politician's behaviour. Therefore, we assume that advantage at influencing the constituency at the first-stage does not necessarily affect the advantage at affecting the politician's policy at the second stage.

Finally, [Clark et al. \(2018\)](#) only consider simultaneous moves by the players at the first stage, whereas in this paper, we, also, study the case in which players move sequentially. This case arises when players do not choose how much effort to make simultaneously ([Dixit 1987](#), [Linster 1993](#), [Leininger 1993](#), [Morgan 2003](#), [Serena 2017](#)). For instance, since 1948, in National Presidential Conventions, the parties of the incumbent presidents choose to have their conventions closer to the general election ([Morgan 2003](#)).³ What distinguishes the above studies from ours is that we do not examine a sequential contest independently, but we embody it to the first stage of a two-stage contest. For example, before the election takes place, a lobby might choose a candidate to contribute to before the opposing lobby does. Then, we calculate the equilibrium efforts and expected payoffs of the two settings and provide comparative static results. We compare them with each other, as well as with the equilibrium efforts and expected payoffs of a contest with symmetric players at the first stage. One of our results is that the strong (*ex ante* advantaged) player's total equilibrium effort and expected payoff are higher than the weak (*ex ante* disadvantaged) player's, regardless of whether she makes effort at the same time as her opponent, before or even after her. This result indicates that the level of the *ex ante* advantage is a crucial factor in this contest.

Next, we endogenise the timing of the players' first-stage efforts. In line with previous literature ([Baik & Shogren 1992](#), [Leininger 1993](#), [Morgan 2003](#)), we show that if the two players can choose the time of making efforts by declaring their intention to be the leader or the follower, not only do both players prefer sequential first stage but, also, the weak player always chooses to move first and the strong player to follow. This result establishes that

³See [Morgan \(2003\)](#) for more examples.

sequential equilibrium arises endogenously, implying that the simultaneity of movements is not a valid assumption.

This paper is also related to [Clark & Nilssen \(2013\)](#) who examine learning by doing in a dynamic contest. However, one of their assumptions is that the greater the effort exerted by a player at the first stage, the lower her effort cost at the second stage. In our model, a player has to win at the first stage in order to enjoy a lower effort cost at the second stage, regardless of the amount of effort she made.

Another study that is close to ours is [Beviá & Corchón \(2013\)](#). They, also, study a two-period contest in which the second-stage asymmetry between the players depends on the outcome of the first stage. However, each player's ability depends on the share of the prize earned in the first period.

Our study is, also, linked to the sequential contests literature. [Leininger \(1993\)](#) studies a contest in which two players are allowed to choose the timing of their moves and shows that sequential movement arises endogenously. [Morgan \(2003\)](#) assumes that the players' valuations of the prize are not known until they commit to the timing of their moves. He, also, finds that in equilibrium, the two players choose to make efforts sequentially. [Fu \(2006\)](#) extends the above studies to the asymmetric information case.

The remainder of the paper is organised as follows. In [Section 2.2](#), we present our model and the results of the analysis of the simultaneous first stage. In [Section 2.3](#), we analyse the contest with sequential moves at the first stage. Endogenous timing in the players' choice of effort is introduced in [Section 2.4](#). Finally, some concluding remarks are in [Section 2.5](#).

2.2 The game

Assume a two-stage contest with two players competing for a prize normalised to 1 and awarded at the second stage. At the first stage, both players exert effort to gain an advantage at the second stage. Their cost functions are both linear, but one of the two players, say player 1, has *a priori* a lower effort cost than her opponent. We refer to this player as the strong player and to her opponent, player 2, as the weak player. Submitting efforts x_1 and

x_2 , respectively, player 1's cost function is $c_1(x_1) = ax_1$, where $0 < a \leq 1$, while player 2's cost function is $c_2(x_2) = x_2$. It is clear that player 1's advantage is captured by her lower marginal cost of effort, $a \leq 1$.⁴

At the beginning of the second stage, the winner (player w) and loser (player l) of the first stage have been determined. Now, assuming that player w and player l make efforts y_w and y_l , respectively, player w 's cost function is $c_w(y_w) = c_w y_w$, while player l 's cost function is $c_l(y_l) = c_l y_l$, where $0 < c_w < c_l < 1$ and c_w, c_l are known to the players. Thus, we could say that the award the two players compete for at the first stage is a lower cost of effort at the second stage.

Using the lottery contest success function (CSF) proposed by [Tullock \(1980\)](#) and axiomatised by [Skaperdas \(1996\)](#) and [Clark & Riis \(1998\)](#), we assume that each player's probability of winning at both stages equals the ratio of this player's effort to the sum of efforts. Thus, in general, when player i makes effort e_i , $i = 1, 2$, her probability of winning is

$$p_i(e_1, e_2) = \begin{cases} \frac{e_i}{e_1 + e_2}, & \text{if } \max\{e_1, e_2\} > 0 \\ \frac{1}{2}, & \text{otherwise.} \end{cases}$$

The second case in the above CSF indicates that if both players exert zero effort, they win the prize with equal probability. We use backward induction to determine the sub-game perfect Nash equilibrium.

2.2.1 Second stage

The winner (player w) and loser (player l) of the first stage are known at the second stage. Submitting effort y_i , player i 's probability of winning is $\frac{y_i}{y_w + y_l}$, $i = w, l$.

⁴We use non-strict inequality in order to include also the symmetric case $a = 1$, where the two players' marginal costs are equal.

Player i 's expected payoff is given by the probability of winning at the second stage times the prize minus the second-stage expenditures⁵,

$$\pi_i = \frac{y_i}{y_w + y_l} - c_i y_i, \quad i = w, l. \quad (2.1)$$

Player i chooses how much effort to exert in order to maximise her expected payoff. The first order conditions of the problem are

$$\frac{\partial \pi_i}{\partial y_i} = \frac{y_j}{(y_w + y_l)^2} - c_i = 0, \quad i, j \in \{w, l\}, j \neq i \quad (2.2)$$

and the second order sufficiency conditions are

$$\frac{\partial^2 \pi_i}{\partial y_i^2} = -\frac{2y_j}{(y_w + y_l)^3} < 0 \quad i, j \in \{w, l\}, j \neq i$$

which are satisfied for every y_j .

From eq. (2.2) we can find that, in equilibrium, player i chooses effort

$$y_i^* = \frac{c_j}{(c_l + c_w)^2} \quad i, j \in \{w, l\}, j \neq i. \quad (2.3)$$

Observing the equilibrium efforts, we can verify the result of [Nti \(1999\)](#), i.e., the first-stage winner's equilibrium effort at the second stage, y_w^* , is higher than the first-stage loser's one, y_l^* .

Substituting eq. (2.3) into eq. (2.1), we obtain that player i 's expected payoff is

$$\pi_i^* = \frac{c_j^2}{(c_l + c_w)^2}, \quad i, j \in \{w, l\}, j \neq i \quad (2.4)$$

It is straightforward that since $c_w < c_l$, in equilibrium, player w 's expected payoff is higher than player l 's. In the next section, we proceed with the analysis of the first stage.

⁵The second stage of the contest is common in the literature and previously analysed (see [Clark & Nilssen \(2013\)](#) among others).

2.2.2 First stage

Simultaneous game

We assume that at the first stage of the contest, the two players simultaneously choose their effort x_i , $i = 1, 2$. Recall that player 1 has an exogenous *ex ante* advantage, implying for her effort cost equal to a , where $0 < a \leq 1$. Player 2's effort cost is equal to 1. Player i 's expected payoff is the sum of the probability of winning at the first stage and, thus, earning a payoff π_w at the second stage, plus the probability of losing at the first stage and, thus, earning a payoff π_l at the second stage, minus the first-stage expenditure. Therefore, player i 's expected payoff is

$$\mathbb{E}(\pi_i) = \frac{x_i}{x_1 + x_2} \pi_w^* + \left(1 - \frac{x_i}{x_1 + x_2}\right) \pi_l^* - ax_i, \quad \begin{array}{l} a < 1 \text{ if } i = 1, \\ a = 1 \text{ if } i = 2. \end{array}$$

The first-stage winner and loser's payoffs are given by eq. (2.4). Thus, substituting eq. (2.4) into the above equation and letting $\frac{c_w}{c_l} = c < 1$, we obtain the following expression for the expected payoffs.⁶

$$\mathbb{E}(\pi_i) = \frac{x_i}{x_1 + x_2} \frac{1}{(1+c)^2} + \left(1 - \frac{x_i}{x_1 + x_2}\right) \frac{c^2}{(1+c)^2} - ax_i, \quad \begin{array}{l} a < 1 \text{ if } i = 1, \\ a = 1 \text{ if } i = 2. \end{array} \quad (2.5)$$

Player 1 and player 2 choose how much effort to exert in order to maximise their expected payoffs. The first order conditions of eq. (2.5) are

$$\frac{\partial \mathbb{E}(\pi_i)}{\partial x_i} = -a + \frac{(1-c)x_j}{(1+c)(x_1+x_2)^2} = 0, \quad \begin{array}{l} a < 1 \text{ if } i = 1, \\ a = 1 \text{ if } i = 2 \end{array} \quad j = 1, 2, j \neq i, \quad (2.6)$$

⁶The term c can be interpreted as the level of advantage the two players compete for at the first stage. When c increases, which means that either c_w increases, c_l decreases or both, the potential advantage becomes lower and *vice versa*.

and the second order sufficient conditions for a maximum are

$$\frac{\partial^2 \mathbb{E}(\pi_i)}{\partial x_i^2} = -\frac{2(1-c)x_j}{(1+c)(x_1+x_2)^3} < 0, \quad i, j \in \{1, 2\}, j \neq i$$

which are satisfied for every x_j since $c < 1$.

Solving eq. (2.6) for x_1 and x_2 , and substituting the results into eq. (2.5), we obtain the following proposition:

Proposition 2.1. *When players move simultaneously at the first stage, there is a unique sub-game perfect Nash equilibrium in which player 1 and player 2's first-stage efforts are*

$$x_1^* = \frac{1}{(1+a)^2} \frac{1-c}{1+c} \quad \text{and} \quad x_2^* = \frac{a}{(1+a)^2} \frac{1-c}{1+c}, \quad (2.7)$$

respectively, and their expected payoffs

$$\mathbb{E}^*(\pi_1) = \frac{1}{(1+a)^2} \frac{1+a(2+a)c^2}{(1+c)^2} \quad \text{and} \quad \mathbb{E}^*(\pi_2) = \frac{1}{(1+a)^2} \frac{a^2+(1+2a)c^2}{(1+c)^2}, \quad (2.8)$$

respectively. The strong player's first-stage effort and expected payoff are greater than or equal to the weak player's.

Proof. See Appendix B. □

It is clear that when $c = 1$, players do not have incentive to exert any effort at the first stage since they do not have any advantage to gain at the second stage. Moreover, when $a = 1$, the problem reduces to a symmetric game, where in equilibrium, both players exert effort $\bar{x} = \frac{1}{4} \frac{1-c}{1+c}$ and earn expected payoff $\bar{\mathbb{E}}(\pi) = \frac{1}{4} \frac{1+3c^2}{(1+c)^2}$.

Comparing eq. (2.7) and eq. (2.8) to the equilibrium efforts and payoffs in the symmetric simultaneous game, yields the following proposition:

Proposition 2.2. *When the first stage is asymmetric [symmetric], player 1's [player 2's] first-stage effort and expected payoff are greater than or equal to the ones in the case with symmetric [asymmetric] first stage.*

Proof. See Appendix B. □

Because of her lower marginal cost, player 1's effort is higher in the asymmetric simultaneous game than in the symmetric simultaneous game, resulting to higher expected payoff. Nevertheless, in the asymmetric case, player 2 has a comparative disadvantage, and thus, makes lower effort and has lower expected payoff than in the symmetric case where there are not any advantages.

Conducting comparative static analysis of the equilibrium efforts, we find:

$$\frac{\partial x_1^*}{\partial c} = -\frac{2}{(1+a)^2(1+c)^2} < 0 \quad \text{and} \quad \frac{\partial x_2^*}{\partial c} = -\frac{2a}{(1+a)^2(1+c)^2} < 0.$$

We summarise the above result in the following proposition:

Proposition 2.3. *When players move simultaneously at the first stage, their first-stage equilibrium efforts increase as the level of the second-stage advantage they compete for increases.*

The interpretation of this result is intuitive. If c falls, which means that either the first-stage winner effort cost at the second stage, c_w , decreases or the first-stage loser effort cost at the second stage, c_l , increases, or both, the two players have more incentives to raise their initial effort and, therefore, increase their probability of winning. On the other hand, as c goes up, the players are not highly-motivated to win at the first stage and, therefore, they lessen their effort. Moreover, the result that $\partial x_2^*/\partial c < 0$ does not hold when the first-stage advantage is carried over to the second stage as in [Clark et al. \(2013\)](#) since in that case, the sign of the partial derivative is ambiguous and depends on the level of this advantage.

Regarding how players react when a varies, we have:

$$\frac{\partial x_1^*}{\partial a} = -\frac{2}{(1+a)^3} \frac{1-c}{1+c} < 0 \quad \text{and} \quad \frac{\partial x_2^*}{\partial a} = \frac{1-a}{(1+a)^3} \frac{1-c}{1+c} > 0.$$

Player 1 makes greater [lower] effort as her own marginal cost decreases [increases], while player 2 makes lower [greater] effort as her opponent's marginal cost decreases [increases].

As demonstrated earlier, the equilibrium efforts are monotonic in c and a . Nevertheless, comparative static analysis of the equilibrium expected payoffs is a more complicated task.

In particular, comparative static analysis for player 1's equilibrium expected payoff yields the following:

- For $0 < a \leq \sqrt{2} - 1$,

$$\frac{\partial \mathbb{E}^* (\pi_1)}{\partial c} < 0, \quad \forall c \in (0, 1).$$

- For $\sqrt{2} - 1 < a \leq 1$,

$$\frac{\partial \mathbb{E}^* (\pi_1)}{\partial c} \begin{cases} < 0, & \text{if } c < \frac{1}{a^2+2a} \\ > 0, & \text{if } c > \frac{1}{a^2+2a}. \end{cases}$$

Similarly, for player 2 we have:

$$\frac{\partial \mathbb{E}^* (\pi_2)}{\partial c} \begin{cases} < 0, & \text{if } c < \frac{a^2}{2a+1} \\ > 0, & \text{if } c > \frac{a^2}{2a+1} \end{cases} \quad \forall a \in (0, 1].$$

Finally, differentiating the expected payoffs with respect to a , we find:

$$\frac{\partial \mathbb{E}^* (\pi_1)}{\partial a} = -\frac{2}{(1+a)^3} \frac{1-c}{1+c} < 0$$

and

$$\frac{\partial \mathbb{E}^* (\pi_2)}{\partial a} = \frac{2a}{(1+a)^3} \frac{1-c}{1+c} > 0.$$

The above results are summarised in the following proposition:

Proposition 2.4. *When players make efforts simultaneously at the first stage,*

- if $0 < a \leq \sqrt{2} - 1$, the player 1's expected payoff decreases [increases] as the second-stage advantage decreases [increases].*

- ii) if $\sqrt{2} - 1 < a \leq 1$, player 1's expected payoff decreases [increases] as the second-stage advantage increases if $c > [<] \frac{1}{a^2 + 2a}$.
- iii) player 2's expected payoff decreases [increases] as the second-stage advantage decreases if $c < [>] \frac{a^2}{2a+1}$, $\forall a \in (0, 1]$.
- iv) player 1's [player 2's] expected payoff is increasing [decreasing] in her own [opponent's] *ex ante* advantage.

Let us focus on player 1 and discuss the first two cases.⁷ Proposition 2.4(i) states that when player 1 is sufficiently strong *ex ante* ($a \leq \sqrt{2} - 1$), her expected payoff is increasing in the second-stage advantage (Figure 2.1). This occurs because since her *ex ante* advantage is high, the probability of winning at the first stage and, therefore, gaining the second-stage advantage at the second stage, is high, too. Thus, a raise in the second-stage advantage may only increase her expected payoff.

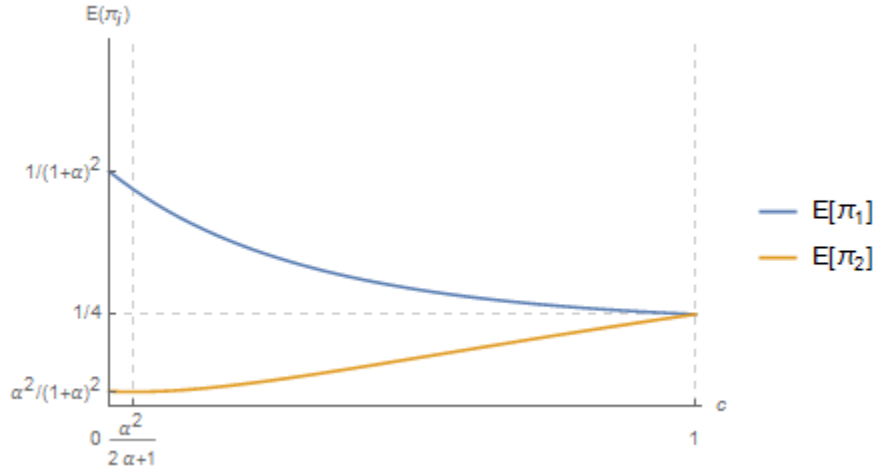


FIGURE 2.1: Player 1 and player 2's expected payoffs for $0 < a \leq \sqrt{2} - 1$.

On the other hand, in Proposition 2.4(ii), we observe that since player 1 is not sufficiently strong ($a > \sqrt{2} - 1$) and, therefore, the probability of winning at the first stage is not as high as in the previous case, the way player 1's expected payoff reacts to changes in the second-stage advantage depends on the level of this advantage. In particular, if the second-stage

⁷The same intuition holds for player 2 in Proposition 2.4(iii).

advantage the two players compete for is sufficiently great or, equivalently, c is sufficiently low, player 1's expected payoff remains being increasing in it. However, if the second-stage advantage is sufficiently low, player 1's expected payoff decreases as this advantage becomes greater (Figure 2.2).

If we consider that the second-stage advantage is the “prize” the two players compete for at the first stage, this part of the proposition implies that player 1's expected payoff may be decreasing in the level of the “prize”. It is interesting that this occurs despite the fact that player 1 still exerts effort greater than or equal to player 2's ($a \leq 1$) and, thus, her probability of winning the “prize” continues to be greater than or equal to her opponent's. Observing the blue curve in Figure 2.2, starting from the point $c = 1$ and moving to the left, a marginal decrease in c reduces player's expected payoff. The intuition is that the increase in effort induced by the decrease in c is not beneficial for the player since even if she wins at the first stage, her second-stage advantage will still be not significant. Nevertheless, when the second-stage advantage exceeds a specific level, despite the greater effort induced, expected payoff increases because of the high potential second-stage advantage if player 1 wins at the first stage.

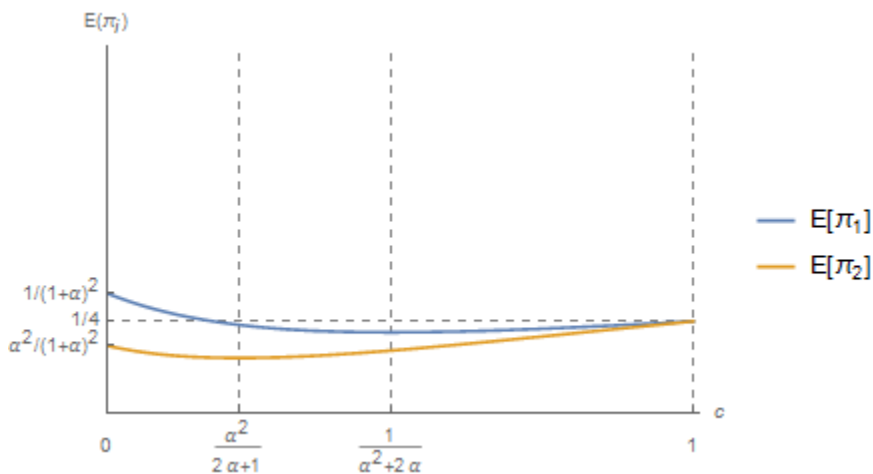


FIGURE 2.2: Player 1 and player 2's expected payoffs for $\sqrt{2} - 1 < a \leq 1$.

Nti (1999) considers two players with different valuations of the prize and shows that a player's expected payoff increases with her own valuation and decreases with the other

player’s valuation. In our model, there are two crucial differences. First, players are asymmetric in their cost function and not in the valuation of the second-stage advantage and second, the “prize” of the first stage is not an actual award but earning it only increases the probability of winning the final prize at the second stage. Because of these reasons, a player’s expected payoff is not monotonic in the second-stage advantage but convex instead.

2.3 Sequential first stage

We, now, extend the model to the case where the two players choose their amount of effort sequentially at the first stage.⁸ Following [Leininger \(1993\)](#), we distinguish two possible cases. In the first case (1-2), the strong player (player 1) chooses first how much effort to exert and the weak player (player 2) chooses effort second knowing player 1’s earlier decision. In the second case (2-1), player 2 makes effort first and player 1 follows.

First case (1-2). Player 1 exerts effort first and player 2 makes her own effort after having observed her rival’s choice. Thus, she solves the same problem as at the simultaneous first stage, namely,

$$\max_{x_2 \geq 0} \left\{ \frac{x_2}{x_1 + x_2} \frac{1}{(1+c)^2} + \frac{x_1}{x_1 + x_2} \frac{c^2}{(1+c)^2} - x_2 \right\}. \quad (2.9)$$

The solution of this optimisation problem is

$$x_2^* = -x_1 + \left(x_1 \frac{1-c}{1+c} \right)^{\frac{1}{2}}.$$

Taking into consideration that $x_2^* \geq 0$, meaning that a player is not able to make negative effort, player 2’s equilibrium strategy as a function of player 1’s effort (player 2’s reaction function) is

⁸Formally, the contest is a three-stage game. However, we will refer to the first two stages as “sequential first stage”.

$$x_2^*(x_1) = \begin{cases} -x_1 + \left(x_1 \frac{1-c}{1+c}\right)^{\frac{1}{2}} & \text{if } x_1 < \frac{1-c}{1+c} \\ 0 & \text{if } x_1 \geq \frac{1-c}{1+c}. \end{cases} \quad (2.10)$$

Player 2 makes a positive effort only if player 1's effort is less than $\frac{1-c}{1+c}$.

When player 1 decides how much effort to exert, she takes into account player 2's reaction function eq. (2.10). Her optimisation problem is, thus,

$$\max_{x_1 \geq 0} \left\{ \frac{x_1}{x_1 + x_2^*(x_1)} \frac{1}{(1+c)^2} + \frac{x_2^*(x_1)}{x_1 + x_2^*(x_1)} \frac{c^2}{(1+c)^2} - ax_1 \right\}$$

and substituting eq. (2.10) into the above expression, we find that when player 1 is the leader of the game, her expected payoff is

$$\mathbb{E}^L(\pi_1) = \begin{cases} \left(x_1 \frac{1-c}{1+c}\right)^{\frac{1}{2}} + \frac{c^2}{(1+c)^2} - ax_1 & \text{if } x_1 < \frac{1-c}{1+c} \\ \frac{1}{(1+c)^2} - ax_1 & \text{if } x_1 \geq \frac{1-c}{1+c}. \end{cases} \quad (2.11)$$

The first-order condition for $x_1 < \frac{1-c}{1+c}$, is

$$-a + \frac{1}{2} \left(\frac{1-c}{x_1(1+c)} \right)^{\frac{1}{2}} = 0$$

and the second order sufficiency condition is

$$-\frac{1}{4} \left(\frac{1-c}{x_1^3(1+c)} \right)^{\frac{1}{2}} < 0, \quad \forall c \in (0, 1).$$

Solving the first order condition for x_1 , we find that player 1 maximises her expected payoff when she chooses effort $x_1^L = \frac{1-c}{4a^2(1+c)}$ and from eq. (2.10) we discover that player 2 responds choosing $x_2^F = \frac{2a-1}{4a^2} \frac{1-c}{1+c}$. For $x_1 \geq \frac{1-c}{1+c}$, we can easily notice that player 1's expected

payoff is maximised when she chooses effort equal to $x_1^L = \frac{1-c}{1+c}$, which implies zero effort for player 2. Substituting the equilibrium first-stage efforts into eq. (2.11) for player 1 and eq. (2.9) for player 2, we obtain their expected payoffs. Since $x_1^L < \frac{1-c}{1+c}$ if and only if $a > \frac{1}{2}$,⁹ the following proposition emerges:

Proposition 2.5. *When at the first stage player 1 moves first and player 2 follows, their equilibrium first-stage efforts are*

$$(x_1^L, x_2^F) = \begin{cases} \left(\frac{1-c}{4a^2(1+c)}, \frac{2a-1}{4a^2} \frac{1-c}{1+c} \right) & \text{if } \frac{1}{2} < a \leq 1 \\ \left(\frac{1-c}{1+c}, 0 \right) & \text{if } 0 < a \leq \frac{1}{2} \end{cases}$$

and their expected payoffs

$$\left(\mathbb{E}^L(\pi_1), \mathbb{E}^F(\pi_2) \right) = \begin{cases} \left(\frac{1+(4a-1)c^2}{4a(1+c)^2}, \frac{(2a-1)^2+(4a-1)c^2}{4a^2(1+c)^2} \right) & \text{if } \frac{1}{2} < a \leq 1 \\ \left(\frac{1-a(1-c^2)}{(1+c)^2}, \frac{c^2}{(1+c)^2} \right) & \text{if } 0 < a \leq \frac{1}{2}. \end{cases}$$

Player 1's first-stage equilibrium effort and expected payoff are always greater than or equal to player 2's.

Proof. See Appendix B. □

Comparative static analysis yields the following:

⁹Solving the inequality, we have:

$$\begin{aligned} x_1^L < \frac{1-c}{1+c} &\implies \frac{1-c}{4a^2(1+c)} < \frac{1-c}{1+c} \\ &\implies a > \frac{1}{2} \end{aligned}$$

and since $a \in (0, 1]$, we have $\frac{1}{2} < a \leq 1$.

- For $\frac{1}{2} < a \leq 1$,

$$\frac{\partial x_1^L}{\partial c} = -\frac{1}{2a^2} \frac{1}{(1+c)^2} < 0 \quad \text{and} \quad \frac{\partial x_2^F}{\partial c} = \frac{1-2a}{2a^2} \frac{1}{(1+c)^2} < 0.$$

- For $0 < a \leq \frac{1}{2}$,

$$\frac{\partial x_1^L}{\partial c} = -\frac{2}{(1+c)^2} < 0 \quad \text{and} \quad \frac{\partial x_2^F}{\partial c} = 0.$$

Also, differentiating the equilibrium efforts with respect to a , we find that, for $\frac{1}{2} < a \leq 1$,

$$\frac{\partial x_1^L}{\partial a} = -\frac{1}{2a^3} \frac{1-c}{1+c} < 0 \quad \text{and} \quad \frac{\partial x_2^F}{\partial a} = \frac{1-a}{2a^3} \frac{1-c}{1+c} > 0,$$

where we observe that player 1's effort is decreasing in a , while player 2's effort increases as her opponent's *ex ante* advantage increases. However, if $0 < a \leq \frac{1}{2}$, player 1's choice of effort does not depend on her *ex ante* advantage since she always chooses to make effort equal to $\frac{1-c}{1+c}$, which induces player 2 not to make any effort. In such a case, it is pointless for player 1 to exert greater effort since her opponent chooses to enter the game only if player 1's effort cost is greater than $\frac{1}{2}$.

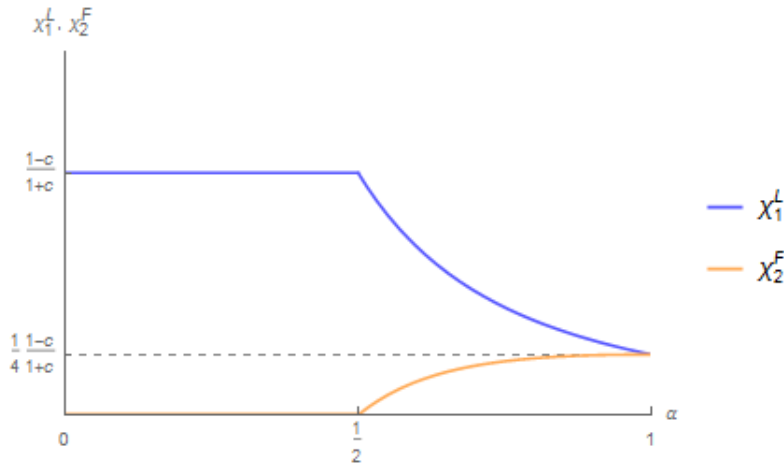


FIGURE 2.3: Player 1 (leader) and player 2's (follower) first-stage effort.

Figure 2.3 illustrates the two players' optimal strategies when player 1 chooses effort first. When $a \leq \frac{1}{2}$, only the strong player submits effort, while when $a > \frac{1}{2}$, her effort declines

and the opponent player's effort increases as a goes up. When a becomes equal to 1, neither player has an *ex ante* advantage and they both choose the symmetric equilibrium effort, $\bar{x} = \frac{1-c}{4(1+c)}$. In the latter case, because of the symmetric setting, sequential moves by the players do not affect the simultaneous equilibrium level of efforts.

We summarise the above results in the following proposition:

Proposition 2.6. *When at the first stage player 1 moves first and player 2 follows,*

- if $\frac{1}{2} < a \leq 1$,
 - i) both players' first-stage equilibrium effort increases as the second-stage advantage increases.
 - ii) player 1's [player 2's] effort increases [decreases] as her own [opponent's] *ex ante* advantage increases.
- if $0 < a \leq \frac{1}{2}$,
 - i) player 1's first-stage equilibrium effort increases as the second-stage advantage increases, while player 2's first-stage effort remains constant and equal to zero.
 - ii) both players' equilibrium efforts remain constant as the *ex ante* advantage varies.

Comparative static analysis for player 1's equilibrium expected payoff yields the following:

- For $\frac{1}{2} < a \leq 1$,

$$\frac{\partial \mathbb{E}^L(\pi_1)}{\partial c} = \frac{(4a-1)c-1}{2a(1+c)^3} \begin{cases} < 0, & \text{if } 0 < c < \frac{1}{4a-1} \\ > 0, & \text{if } \frac{1}{4a-1} < c \leq 1 \end{cases}$$

- For $0 < a \leq \frac{1}{2}$,

$$\frac{\partial \mathbb{E}^L(\pi_1)}{\partial c} = \frac{a(1+c)-1}{2a(1+c)^3} < 0, \quad \forall c \in (0, 1).$$

It is obvious that when her *ex ante* advantage is not sufficiently great ($\frac{1}{2} < a \leq 1$), the way player 1's expected payoff reacts to changes in the second-stage advantage, also, depends on the level of this advantage. When it is sufficiently big ($0 < c < \frac{1}{4a-1}$), player 1's expected payoff increases as this advantage increases (or, equivalently, c decreases). However, when the advantage the two players compete for is not sufficiently great ($\frac{1}{4a-1} < c < 1$), player 1's expected payoff can even be decreasing in this advantage (Figure 2.4).

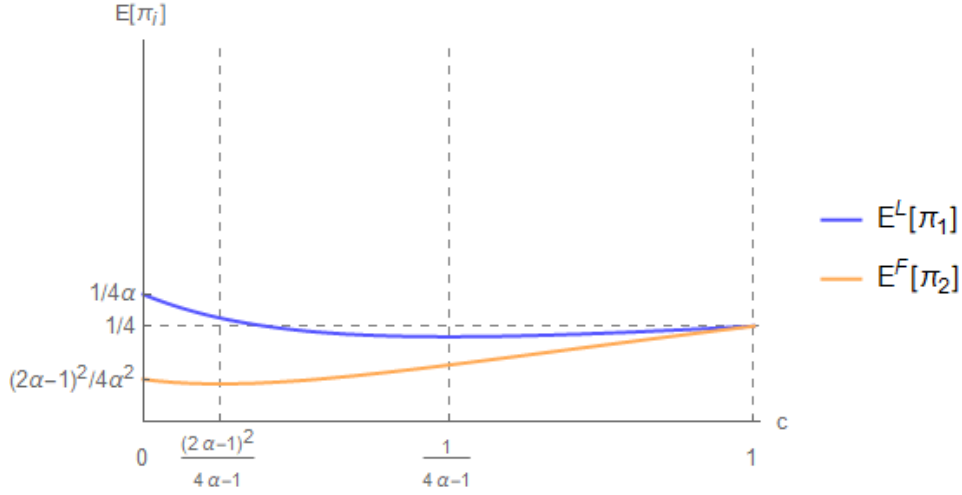


FIGURE 2.4: Player 1 (leader) and player 2's (follower) expected payoffs for $\frac{1}{2} < a \leq 1$.

On the other hand, when player 1's *ex ante* advantage is significant ($0 < a \leq \frac{1}{2}$), the probability that she will gain the second-stage advantage is greater than in the previous case and, thus, an increase in this advantage also increases her expected payoff (Figure 2.5).

Similarly to the analysis above, for player 2 we have:

- For $\frac{1}{2} < a \leq 1$,

$$\frac{\partial \mathbb{E}^F(\pi_2)}{\partial c} = \frac{(4a-1)c - (2a-1)^2}{2a^2(1+c)^3} \begin{cases} < 0, & \text{if } 0 < c < \frac{(2a-1)^2}{4a-1} \\ > 0, & \text{if } \frac{(2a-1)^2}{4a-1} < c < 1 \end{cases}$$

- For $0 < a \leq \frac{1}{2}$,

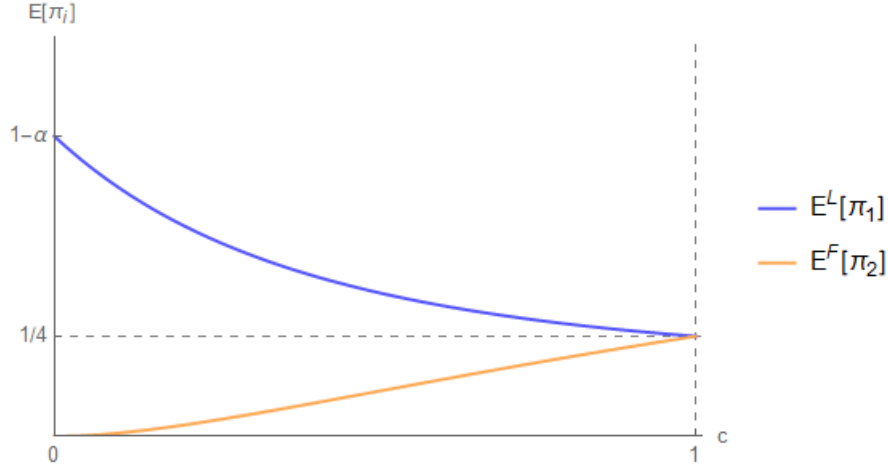


FIGURE 2.5: Player 1 (leader) and player 2's (follower) expected payoffs for $0 < a \leq \frac{1}{2}$.

$$\frac{\partial \mathbb{E}^F(\pi_2)}{\partial c} = \frac{2c}{(1+c)^3} > 0, \quad \forall c \in (0, 1).$$

If player 1 has a great *ex ante* advantage over player 2 then the latter one's expected payoff decreases as the second-stage advantage increases regardless of the level of this advantage. Nevertheless, if her opponent's *ex ante* advantage is not sufficiently great, player 2's expected payoff can be increasing in the second-stage advantage if it is sufficiently important.

Second case (2-1). Now, player 2 chooses first how much effort to make and player 1 follows after having observed this amount of effort. Following the same process as before, player 1's (follower) reaction function is

$$x_1^*(x_2) = \begin{cases} -x_2 + \left(\frac{x_2(1-c)}{a(1+c)} \right)^{\frac{1}{2}} & \text{if } x_2 < \frac{1-c}{a(1+c)} \\ 0 & \text{if } x_2 \geq \frac{1-c}{a(1+c)}. \end{cases}$$

Player 2's (leader) expected payoff is

$$\mathbb{E}^L(\pi_2) = \begin{cases} \left(ax_2 \frac{1-c}{1+c} \right)^{\frac{1}{2}} + \frac{c^2}{(1+c)^2} - x_2 & \text{if } x_2 < \frac{1}{a} \frac{1-c}{1+c} \\ \frac{1}{(1+c)^2} - x_2 & \text{if } x_2 \geq \frac{1}{a} \frac{1-c}{1+c}. \end{cases}$$

The first-order condition for $x_2 < \frac{1}{a} \frac{1-c}{1+c}$ is

$$-1 + \frac{1}{2} \left(\frac{a}{x_2} \frac{1-c}{1+c} \right)^{\frac{1}{2}} = 0$$

and the second order sufficiency condition is

$$-\frac{1}{4} \left(\frac{a}{x_2^3} \frac{1-c}{1+c} \right)^{\frac{1}{2}} < 0, \quad \forall c \in (0, 1) \text{ and } \forall a \in (0, 1].$$

Solving the first order condition, we find that player 2's expected payoff is maximised when she chooses effort $\frac{a}{4} \frac{1-c}{1+c}$ and the corresponding expected payoff is $\frac{a+(4-a)c^2}{4(1+c)^2}$. Equivalently, for $x_2 \geq \frac{1}{a} \frac{1-c}{1+c}$, player 2's optimal effort is $\frac{1}{a} \frac{1-c}{1+c}$, which yields expected payoff equal to $\frac{a-1+c^2}{a(1+c)^2}$.

Comparing player 2's expected payoffs for the various values of x_2 , we discover that $\frac{a+(4-a)c^2}{4(1+c)^2} \geq \frac{(a-1)+c^2}{a(1+c)^2}$, $\forall a \in (0, 1]$, (for proof, see *Appendix B*) meaning that her expected payoff is always greater when she makes effort lower than $\frac{1}{a} \frac{1-c}{1+c}$ regardless of the value of a . Thus, she always chooses effort equal to $\frac{a}{4} \frac{1-c}{1+c}$ and player 1 replies choosing effort equal to $\frac{2-a}{4} \frac{1-c}{1+c}$.

The two players' equilibrium efforts and expected payoffs are summarised in the following proposition:

Proposition 2.7. *When at the first stage player 2 moves first and player 1 follows, their equilibrium first-stage efforts are*

$$x_2^L = \frac{a}{4} \frac{1-c}{1+c} \quad \text{and} \quad x_1^F = \frac{2-a}{4} \frac{1-c}{1+c}$$

and their expected payoffs

$$\mathbb{E}^L(\pi_2) = \frac{a + (4 - a)c^2}{4(1 + c)^2} \quad \text{and} \quad \mathbb{E}^F(\pi_1) = \frac{(2 - a)^2 + (4 - a)ac^2}{4(1 + c)^2},$$

respectively. Player 1's first-stage equilibrium effort and expected payoff are greater than or equal to player 2's.

Proof. See Appendix B. □

From the above proposition, the following corollary emerges:

Corollary 2.1. *When at the first stage player 2 moves first, player 1 will never be inactive.*

As we can see in Figure 2.6, the above lemma brings out a situation different from the “1-2” case, in which the follower may response choosing not to participate at the first stage. Now, the follower, being the strong player, will never prefer to stay inactive at the first stage. In addition, despite the fact that player 2 makes her move first, player 1 will still exert higher first-stage effort, but not as high as her first-stage effort when she moves first.

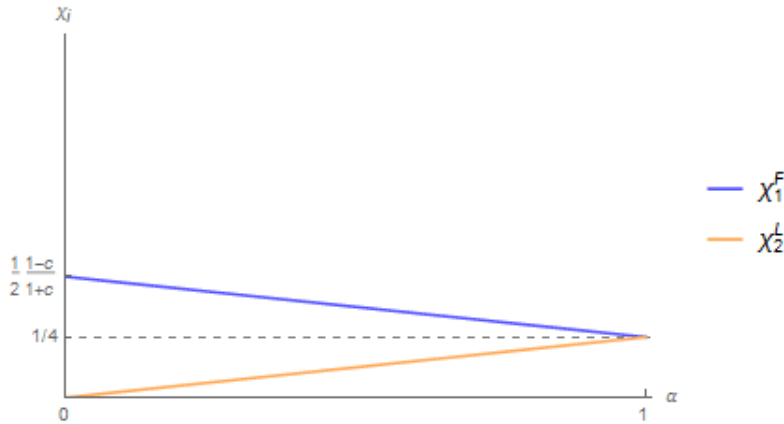


FIGURE 2.6: Player 2 (leader) and player 1's (follower) first-stage effort.

Comparison of the equilibrium efforts and expected payoffs of the simultaneous, “1-2” and “2-1” case provides the following corollary:

Corollary 2.2. *Player 1's first-stage effort and expected payoff are greater than or equal to player 2's regardless of the sequence of their moves.*

The following proposition demonstrates the results of the comparison among the total efforts of the type of contests mentioned above.

Proposition 2.8. *i) When at the first stage player 1 moves first, aggregate effort of the two-stage contest is greater than or equal to aggregate effort when the two players move simultaneously.*

ii) When at the first stage the two players move simultaneously, aggregate effort of the two-stage contest is greater than or equal to aggregate effort when player 2 moves first.

Proof. See Appendix B. □

Comparative static analysis of the equilibrium efforts gives us the following results:

$$\frac{\partial x_2^L}{\partial c} = -\frac{a}{2(1+c)^2} < 0 \quad \text{and} \quad \frac{\partial x_2^L}{\partial a} = \frac{1}{4} \frac{1-c}{1+c} > 0$$

for player 2, and

$$\frac{\partial x_1^F}{\partial c} = -\frac{2-a}{2(1+c)^2} < 0 \quad \text{and} \quad \frac{\partial x_1^F}{\partial a} = -\frac{1}{4} \frac{1-c}{1+c} < 0$$

for player 1. We can note that both players' equilibrium first-stage efforts decrease as the second-stage advantage decreases. Also, as player 1's *ex ante* advantage increases, her effort increases, while her opponent's effort decreases.

Regarding player 2's expected payoff, we have the following comparative static results:

$$\frac{\partial \mathbb{E}^L(\pi_2)}{\partial c} = \frac{(4-a)c - a}{2(1+c)^3} \begin{cases} < 0, & \text{if } c < \frac{a}{4-a} \\ > 0, & \text{if } c > \frac{a}{4-a} \end{cases} \quad \forall a \in (0, 1].$$

Likewise, for player 1 we have

- For $0 < a \leq 2 - \sqrt{2}$,

$$\frac{\partial \mathbb{E}^F(\pi_1)}{\partial c} < 0, \quad \forall c \in (0, 1).$$

- For $2 - \sqrt{2} < a \leq 1$,

$$\frac{\partial \mathbb{E}^F(\pi_1)}{\partial c} = \frac{a(4-a)c - (2-a)^2}{2(1+c)^3} \begin{cases} < 0, & \text{if } c < \frac{(2-a)^2}{a(4-a)} \\ > 0, & \text{if } c > \frac{(2-a)^2}{a(4-a)}. \end{cases}$$

The above analysis of the two players' expected payoffs is illustrated in Figure 2.7 and Figure 2.8.

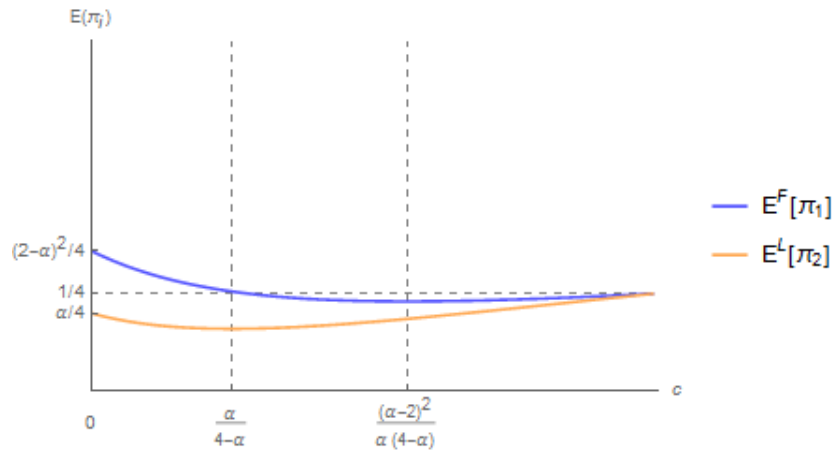


FIGURE 2.7: Player 2 (leader) and player 1's (follower) expected payoffs for $2 - \sqrt{2} < a \leq 1$.

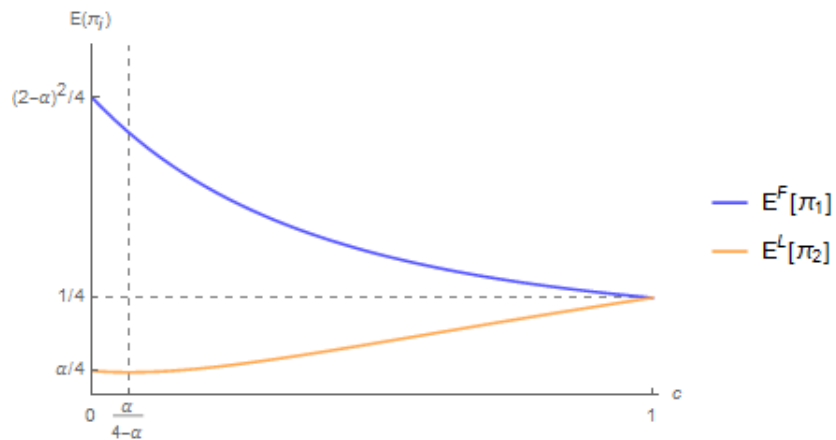


FIGURE 2.8: Player 2 (leader) and player 1's (follower) expected payoffs for $0 < a \leq 2 - \sqrt{2}$.

2.4 Endogenous timing

In this section, we assume that before the contest begins, each player is able to decide whether she wants to move early and, thus, become a leader or late and, thus, become a follower. In order to decide their strategy, the two players have to compare their expected payoff in each situation.

There are four possible scenarios: (L, L), (F, F), in which both players choose to move first and second respectively; (L, F), in which player 1 chooses to play first and player 2 second and (F, L), in which player 2 chooses to play first and player 1 chooses to follow. Table 2.1 includes the expected payoffs in each of the above cases.

Player 1 \ Player 2	L	F
L	$\mathbb{E}^*(\pi_1), \mathbb{E}^*(\pi_2)$	$\mathbb{E}^L(\pi_1), \mathbb{E}^F(\pi_2)$
F	$\mathbb{E}^F(\pi_1), \mathbb{E}^L(\pi_2)$	$\mathbb{E}^*(\pi_1), \mathbb{E}^*(\pi_2)$

TABLE 2.1: Player 1 and player 2's expected payoffs.

Equilibrium analysis of the game yields the following proposition:

Proposition 2.9. *There is a unique sub-game perfect Nash equilibrium, in which player 2 chooses to make effort first and player 1 chooses to follow.*

Proof. See Appendix B. □

The above proposition demonstrates that when at the first stage players compete for a second-stage advantage, the sequence of moves with the weak player leading is the only sub-game perfect Nash equilibrium.

It, also, yields an interesting result for the strong player. The fact that $\mathbb{E}^*(\pi_1) \leq \mathbb{E}^L(\pi_1) \leq \mathbb{E}^F(\pi_1)$ means that besides the fact that player 1 has greater payoff when she moves first than when she moves simultaneously with player 2, she has even greater payoff when she enters the contest after her rival. This fact implies that even if her *ex ante* advantage is

high ($0 < a \leq \frac{1}{2}$), player 1 does not choose to make effort first and, therefore, induce player 2 to abstain from participating at the first stage, but, instead, she always prefers her opponent to participate. On the other hand, because of the fact that $\mathbb{E}^F(\pi_2) \leq \mathbb{E}^*(\pi_2) \leq \mathbb{E}^L(\pi_2)$, player 2 has the greatest possible payoff when she moves first and the lowest when she moves after player 1.

2.5 Conclusion

We have analysed a two-stage contest with two players competing with each other to gain an advantage at the first stage, which will help them win a final prize at the second stage. In the case with simultaneous moves by the two players at the first stage, we find that the stronger player exerts higher first-stage effort and earns greater expected payoff.

When players move sequentially at the first stage, if the favourite player is strong enough, her opponent will choose not to participate at the first stage. She opts to withdraw from the competition for advantage and take part only at the final stage of the contest. On the other hand, regardless of the order of moves, the strong player not only will take part at the first stage but, also, she will exert higher effort and earn greater expected payoff than her rival. Moreover, total effort of the two-stage contest is higher when at the first stage the favourite player moves first. Finally, if players can choose the timing of their first-stage efforts, they both prefer an order in which the weak player plays first and the strong one follows.

What we can derive from this study is that the strong player's *ex ante* advantage is the pivotal factor in this two-stage contest since it makes her front-runner in all cases discussed above. However, the level of her leverage depends on the degree of the *ex ante* heterogeneity in abilities between the two players, as well as on the order of their first-stage moves. A natural extension of the model includes the introduction of incomplete information about the first-stage and second-stage advantages. This task is left for future research.

Chapter 3

Optimal Lobbying Pricing

3.1 Introduction

It is generally admitted that interest groups play an important role in the political process. A significant part of the lobbying literature ([Austen-Smith 1987](#), [Baron 1994](#)) considers interest groups with electoral motives. These groups take the candidates' positions as given and offer them campaign contributions. Contributions are typically spent by candidates on campaign activities that aim at persuading the constituency to vote for them and, thus, affecting their probability of winning. Another strand of the literature assumes that lobbying occurs after the election. Lobbying takes the form of either monetary contributions offered to the incumbent politician ([Krueger 1974](#), [Becker 1983](#), [Lohmann 1995](#), [Bennedsen & Feldmann 2006](#), [Martimort & Semenov 2008](#)) or informational lobbying with exogenous lobbying costs ([Potters & van Winden 1992](#)). That is, the relative literature, typically, considers political influence either before or after the election as separate cases.¹

Nevertheless, political influence is a continuous activity which does not end once the election is concluded. After the election, and regardless of the winner, interest groups need to influence the implemented policies. [Tripathi et al. \(2002\)](#) use data from the 1995 Lobby

¹See, for instance, [Baron \(1994\)](#), [Grossman & Helpman \(1994, 1996\)](#). Two papers that study both combine campaign contributions and lobbying are [Austen-Smith \(1995\)](#) and [Felli & Merlo \(2012\)](#). However, [Austen-Smith \(1995\)](#) “abstracts from details of the electoral process” assuming that the probability that a candidate wins is exogenous and, thus, not affected by the campaign contributions she receives. Moreover, [Felli & Merlo \(2012\)](#) consider a citizen-candidate model of representative democracy and their main question is to what extent lobbying affects equilibrium policy outcomes.

Disclosure Act and find strong connection between campaign expenditures and lobbying. More specifically, “groups that have both a lobbyist and a PAC [...] account for fully 70% of all interest group expenditures and 86% of all PAC contributions”.

In this paper, we consider jointly pre and post-electoral political influence and, endogenously, determine the optimal level of the “access” price politicians charge to the interest groups. Our results show that a candidate commits to charge a lower lobbying price to the group that is going to support her at the election. That is, our results support the idea that contributions may buy “access” for lobbying activities such as “preferential treatment” by the incumbent or amplification of the group’s message (Wright 1990). As put by Wright (1990), our result shows that “representatives may ‘hear you better’ [...] when a contribution precedes lobbying”.

Adding the assumption that candidates are asymmetric in skills, i.e., one candidate is inherently stronger than the other one in the election, we show that again both candidates treat one of the groups preferentially. Nevertheless, such preferential treatment is much more pronounced for the access prices announced by the less skilled candidate. Also, we show that total lobbying activity is more intense if the less skilled candidate wins the election, indicating that a less skilled politician is more prone to political influence once in office. This result is in line with the perspective of “informational lobbying” where interest groups possess valuable information or advice and they strategically provide it to an unaware politician in office.

We model the above situation as a three-stage game with two symmetric interest groups and two candidates. At stage zero (*pricing stage*), the two candidates set the level of lobbying prices they are going to charge the two groups if they are elected. At the first stage (*election stage*), each group offers a monetary contribution to its favourite candidate that is used by her in order to increase her electoral support and, then, the election takes place. At the second stage (*lobbying stage*), the two groups compete exerting lobbying efforts in order to influence the winner’s policy decision.

The first and second stages have the characteristics of a two-stage Tullock contest. Groups compete against each other twice, once before and once after the election takes place. At

the lobbying stage (the second stage of the “contest”), the two groups engage in a contest for a given prize, e.g., their favourite policy being implemented, by exerting costly lobbying efforts.² A unit of lobbying effort can be viewed, for instance, as an hour spent in the incumbent’s office. At the election stage (the first stage of the “contest”), the two groups compete by offering campaign contributions to their favourite candidate so that they affect the electoral outcome.³ That is, like [Austen-Smith \(1987\)](#) and [Baron \(1994\)](#), groups have an electoral motive, meaning that each group takes as given the candidates’ positions and offers a contribution to its favourite candidate in order to increase her probability of winning the election. The “award” the two groups fight for at the election stage is an endogenously determined lower lobbying price at the lobbying stage.

The prices of lobbying efforts charged by the winning candidate are chosen strategically by the two candidates at stage zero (pricing stage). Candidates can choose to charge the two groups any price so as to maximise their expected revenue from lobbying. By announcing the offered lobbying prices, candidates simultaneously affect their probability of being elected and their revenues from lobbying if in office.

The endogenous choice of lobbying prices (stage zero) constitutes our main contribution to the existing literature in contest theory⁴ and, more particularly, works on multi-stage contests ([Gradstein & Konrad 1999](#), [Moldovanu & Sela 2006](#), [Konrad & Kovenock 2009](#), [Fu et al. 2015](#)). [Clark et al. \(2018\)](#) consider a two-stage contest in which the winner of the first stage faces *by assumption* a lower cost of effort at the second stage. Not only does our study provide support to this assumption, but also our results differ significantly due to the different assumed objective functions. In contrast to us, [Clark et al. \(2018\)](#) are concerned about the maximisation of the total effort exerted at the two stages of the contest and they find that the optimal level of second-stage cost offered by an effort-maximising contest designer should be zero. As a result, the two-stage contest reduces to a one-shot contest

²[Besley & Coate \(2001\)](#) model lobbying as a menu-auction and [Felli & Merlo \(2012\)](#) assume that the winner of the election chooses with which groups she will trade off policy favours for transfers.

³We assume that contributions cannot be carried over to the next stage and, thus, the whole amount is spent at the election stage.

⁴For surveys on the contest theory literature, see [Nitzan \(1994\)](#), [Konrad \(2009\)](#), [Balart et al. \(2016\)](#), [Corchón & Serena \(2017\)](#).

since the loser of the first stage does not make any effort at the second stage. The fact that we are concerned about candidates' expected revenue, rather than total effort, makes the two-stage contest meaningful and allows us to determine the optimal revenue-maximising level of the second-stage effort cost. [Clark et al. \(2018\)](#), instead, assume that this cost is out of the designer's control and study optimal prize distribution between the two stages.

In previous work, [Clark & Nilssen \(2013\)](#), also, consider a two-stage contest in which they, exogenously, assume that the greater the effort exerted by a player at the first stage, the lower her effort cost at the second stage. In our model, instead, the levels of effort cost (lobbying prices) are set by the candidates endogenously before the first-stage efforts (contributions) are made.

Another study close to ours is [Beviá & Corchón \(2013\)](#). They study a two-stage contest in which the second-stage asymmetry between the players depends on the outcome of the first stage. In their model, each player's second-stage ability depends on the share of the prize earned in the first period. On the contrary, in our study, first, there is no award for the interest groups at the election stage and, second, only the group that supported the winner of the election benefits at the lobbying stage, regardless of the level of expenditures it made before the election.

The rest of the paper goes as follows. In [Section 3.2](#), we formalise the game. In [Section 3.3](#), we analyse the symmetric game and present the results of the analysis. In [Section 3.4](#), we study the case with candidates of different skills while [Section 3.5](#) concludes the study.

3.2 The game

Consider two symmetric candidates, $\{A, B\}$, and two symmetric interest groups, $\{a, b\}$, where groups a and b support candidates A and B , respectively. The game consists of three stages: the pricing stage, the election stage and the lobbying stage.

At stage zero (pricing stage), the two candidates, having as objective the maximisation of their expected lobbying revenue, choose the level of lobbying prices to charge each group if they win the election. Denote by $c_{I,i} > 0$ the lobbying price set by candidate I for the

group that supported her at the election, group i , and $c_{I,j} > 0$ for the opposite group, group j , where for $I = A$, $i = a$ and $j = b$ and for $I = B$, $i = b$ and $j = a$.

At the first stage (election stage), the two groups choose their levels of contribution to their favourite candidate. The contributions received by the two candidates are spent at this specific stage for campaign purposes and determine their probability of winning the election.⁵ Particularly, the probability that a specific candidate wins is increasing in the contribution received by her and decreasing in the contribution received by her opponent. Let $x_i \geq 0$ denote the contribution made by group i to candidate I and the marginal cost of contribution be the same for both groups and normalised to 1. Since each candidate I is supported by group i , we express the probability that candidate I wins at the election stage as a lottery contest success function first introduced by [Tullock \(1980\)](#), namely

$$q_I = \begin{cases} \frac{x_i}{x_i + x_j}, & \text{if } \max \{x_i, x_j\} > 0 \\ \frac{1}{2}, & \text{otherwise} \end{cases} \quad (3.1)$$

while the probability that she loses, or, equivalently, candidate J , supported by group j , wins is $q_J = 1 - q_I$.⁶

At the final stage (lobbying stage), the two groups lobby the winner of the election. Assume that the winner is candidate I who was supported by group i . Denote the lobbying effort of group i by $y_i \geq 0$. The lobbying price group i is charged is $c_{I,i}$, while the price group j is charged is $c_{I,j}$. Considering linear cost functions for the two groups, the cost function of group i is $C_i(y_i) = c_{I,i}y_i$, while the cost function of group j is $C_j(y_j) = c_{I,j}y_j$. The

⁵We consider impressionable voters, i.e., they are swayed toward one party or the other by the messages they receive during the campaign ([Baron 1994](#), [Grossman & Helpman 1996](#)).

⁶At the election stage, we assume that the probability that a party wins is given by a Tullock contest success function depending only on the efforts of the two groups. [Jia \(2008\)](#) and [Jia et al. \(2013\)](#) show that this CSF is microfounded when voters are impressionable with utility $t^i(x_i, x_j, \theta^i) = x_i \theta^i$, where θ^i is a random variable, independently drawn from an Inverse Exponential Distribution, that measures how much voter i is affected by campaign spending (See also [Balart et al. \(2018\)](#)).

probability that group i wins at the lobbying stage is

$$p_i = \begin{cases} \frac{y_i}{y_i + y_j}, & \text{if } \max\{y_i, y_j\} > 0 \\ \frac{1}{2}, & \text{otherwise} \end{cases}$$

while the the probability that group j wins is $p_j = 1 - p_i$. The winning group of the lobbying stage enjoys the implementation of its favourite policy, the valuation of which is equal for both groups and normalised to 1. Expected payoff of each group is the probability of winning minus its lobbying expenditure. Hence, at the lobbying stage, expected payoffs of groups i and j are, respectively,

$$\pi_i = p_i - c_{I,i}y_i, \quad \text{and} \quad \pi_j = 1 - p_i - c_{I,j}y_j. \quad (3.2)$$

Going backwards, at the election stage, the expected payoff of group i , Π_i , is the following: the probability that its favourite candidate wins times its lobbying-stage payoff if she wins, plus the probability that her favourite candidate loses times its lobbying-stage payoff if she loses (slightly abusing notation we refer to this as π_{-i}), minus its contribution. Making use of eq. (3.1), we have that at the election stage, expected payoff of group i is

$$\begin{aligned} \Pi_i &= q_I \pi_i + (1 - q_I) \pi_{-i} - x_i \\ &= \frac{x_i}{x_i + x_j} \pi_i + \frac{x_j}{x_i + x_j} \pi_{-i} - x_i. \end{aligned} \quad (3.3)$$

At the first stage of the game, the two candidates set the lobbying prices they are going to charge if they win the election. They do so having as objective to maximise their lobbying revenue, where losing the election implies zero payoff. Therefore, at the beginning of the game, candidate I 's expected revenue, r_I , is equal to the probability that she wins the election times her expected lobbying revenue if she wins, namely

$$\begin{aligned}
r_I &= q_I (c_{I,i}y_i + c_{I,j}y_j) \\
&= \frac{x_i}{x_i + x_j} (c_{I,i}y_i + c_{I,j}y_j).
\end{aligned} \tag{3.4}$$

We proceed with the analysis of the game.

3.3 Results

We make use of backward induction to determine the sub-game perfect Nash equilibrium of the game in pure strategies.

3.3.1 Lobbying stage

This stage of the game is an asymmetric Tullock contest previously analysed, among others, by [Nti \(1999\)](#). Assume that the winner of the election is candidate I . Maximising eq. (3.2) with respect to y_i and y_j , respectively, we find that in equilibrium, lobbying efforts of groups i and j are, respectively,

$$y_i^* = \frac{c_{I,j}}{(c_{I,i} + c_{I,j})^2} \quad \text{and} \quad y_j^* = \frac{c_{I,i}}{(c_{I,i} + c_{I,j})^2} \tag{3.5}$$

yielding payoffs

$$\pi_i^* = \frac{c_{I,j}^2}{(c_{I,i} + c_{I,j})^2} \quad \text{and} \quad \pi_j^* = \frac{c_{I,i}^2}{(c_{I,i} + c_{I,j})^2}. \tag{3.6}$$

We observe that if $c_{I,i} \leq c_{I,j}$, then $y_i^* \geq y_j^*$ and $\pi_i^* \geq \pi_j^*$. If a politician charges a lower lobbying price to one group, then this group exerts greater lobbying effort and earns greater payoff than the other one.

3.3.2 Election stage

Group i chooses the level of contribution, x_i , to offer to her favourite candidate, I , in order to maximise its expected payoff. Using eq. (3.3) and eq. (3.6), the maximisation problem of group i is

$$\begin{aligned} \max_{x_i} \Pi_i &= \max_{x_i} \{q_I \pi_i^* + (1 - q_I) \pi_{-i}^* - x_i\} \\ &= \max_{x_i} \left\{ \frac{x_i}{x_i + x_j} \frac{c_{I,j}^2}{(c_{I,i} + c_{I,j})^2} + \left(1 - \frac{x_i}{x_i + x_j}\right) \frac{c_{J,j}^2}{(c_{J,i} + c_{J,j})^2} - x_i \right\}. \end{aligned} \quad (3.7)$$

The equilibrium contributions offered by the two groups are given by the following proposition.

Proposition 3.1. *In the equilibrium of the symmetric election stage game, group i offers contribution*

$$x_i^* = \begin{cases} \frac{(c_{I,j}c_{J,i} - c_{I,i}c_{J,j})[c_{J,i}c_{I,j} + c_{I,i}(2c_{J,i} + c_{J,j})][c_{I,i}c_{J,j} + c_{I,j}(c_{J,i} + 2c_{J,j})]^2}{4(c_{I,i} + c_{I,j})^4(c_{J,i} + c_{J,j})^4}, & \text{if } c_{A,a}c_{B,b} < c_{B,a}c_{A,b} \\ 0, & \text{if } c_{A,a}c_{B,b} \geq c_{B,a}c_{A,b}. \end{cases} \quad (3.8)$$

Proof. The first-order conditions of eq. (3.7) are

$$\frac{\partial \Pi_i}{\partial x_i} = \frac{x_j \left[\frac{c_{I,j}^2}{(c_{I,i} + c_{I,j})^2} - \frac{c_{J,j}^2}{(c_{J,i} + c_{J,j})^2} \right]}{(x_i + x_j)^2} - 1 = 0 \quad (3.9)$$

while the second-order sufficient conditions for a maximum are

$$\frac{\partial^2 \Pi_i}{\partial x_i^2} = \frac{2x_j \left[\frac{c_{J,j}^2}{(c_{J,i} + c_{J,j})^2} - \frac{c_{I,j}^2}{(c_{I,i} + c_{I,j})^2} \right]}{(x_i + x_j)^3} < 0,$$

which are satisfied for $c_{A,a}c_{B,b} < c_{B,a}c_{A,b}$.

Solving eq. (3.9) simultaneously for the two groups, a and b , we obtain the results in the proposition. \square

The first thing we notice in Proposition 3.1 is that either both groups choose not to make any contribution or both groups make positive contributions in the election stage. Groups make zero contributions if the product of the lobbying prices set by their favourite candidates is greater than the product of the lobbying prices set by the opposite candidates. That is, if interest groups are not treated sufficiently well by the candidates and the announced lobbying prices are not favourable, they prefer to remain inactive.

The equilibrium characterisation also offers several interesting comparative statics (for details, see Appendix C). First, the equilibrium effort of group i , x_i^* , is decreasing in the price its favourite candidate has committed to charge to it, $c_{I,i}$, i.e., $\partial x_i^*/\partial c_{I,i} < 0$. This indicates that as a candidate decreases the price she is going to charge to the group that supports her if she wins the election, the group supporting her contributes more in order to increase this candidate's probability of winning and, therefore, the probability that the group will pay this lower price (blue curve in Figure 3.1). Second, the equilibrium effort of group i , x_i^* , is increasing in the price its favourite candidate has committed to charge the opposite group j , $c_{I,j}$, i.e., $\partial x_i^*/\partial c_{I,j} > 0$. This shows that a group increases the contribution to its favourite candidate as she increases the lobbying price she commits to charge to the opposite group if she wins. By increasing its contribution, a group increases the probability that her favourite candidate wins the election and, in consequence, that its favourable policy will be implemented at the lobbying stage since the opposite group will make a low lobbying effort having to pay a high lobbying price.

However, differentiating x_i^* with respect to $c_{J,j}$, we find that the effect of a change in the lobbying price candidate J commits to charge to her favourite group on the contribution received by candidate I is non-monotonic (green curve in Figure 3.1). More specifically, if a candidate commits to charge to the group supporting her a relatively low lobbying price, in case she wins, the payoff of the opposite group at the lobbying stage will be low. As

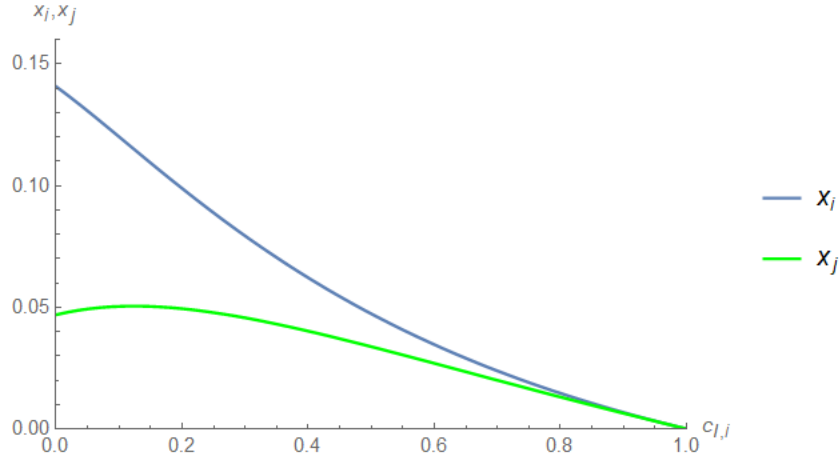


FIGURE 3.1: Contributions of groups i and j as a function of $c_{I,i}$ ($c_{J,j} = c_{I,j} = c_{J,i} = 1$).

this price goes up, the latter group may increase its contribution in order to increase the probability that its favourite candidate wins and, consequently, decrease the probability that it will get a low payoff. If, now, the lobbying price a candidate commits to charge to her favourite group is relatively high, in case she wins, the payoff of the opposite group is going to be high as well. Thus, as this price increases even more, the latter group has an incentive to decrease its contribution to its favourite candidate in order to increase the probability that the opposite candidate wins and, therefore, the probability that it gets a high payoff.⁷

3.3.3 Pricing stage

Candidate I commits to charge to groups i and j prices $c_{I,i}$ and $c_{I,j}$, respectively, that maximise eq. (3.4) taking into consideration eq. (3.5) and eq. (3.8). Candidate I 's maximisation problem is, then,

⁷A similar intuition applies for the effect of a change in $c_{I,j}$ on x_j^* .

$$\begin{aligned}
\max_{c_{I,i}, c_{I,j}} r_I &= \max_{c_{I,i}, c_{I,j}} \{q_I (c_{I,i}y_i^* + c_{I,j}y_j^*)\} \\
&= \max_{c_{I,i}, c_{I,j}} \left\{ \frac{x_i^*}{x_i^* + x_j^*} \left[c_{I,i} \frac{c_{I,j}}{(c_{I,i} + c_{I,j})^2} + c_{I,j} \frac{c_{I,i}}{(c_{I,i} + c_{I,j})^2} \right] \right\} \\
&= \max_{c_{I,i}, c_{I,j}} \left\{ \frac{c_{I,i}c_{I,j} (c_{I,i}c_{J,j} + c_{I,j} (c_{J,i} + 2c_{J,j}))}{(c_{I,i} + c_{I,j})^3 (c_{J,i} + c_{J,j})} \right\}.
\end{aligned} \tag{3.10}$$

Let us, now, explore how candidates' expected revenue changes when the lobbying prices change (for a complete analysis, see Appendix C). First, candidate I 's expected revenue is strictly increasing in the lobbying price candidate J commits to charge to group j , i.e., $\partial r_I / \partial c_{J,j} > 0$. This happens because group j decreases its contribution to her favourite candidate, J , and, therefore, the probability that candidate I wins goes up increasing I 's expected revenue. Moreover, candidate I 's expected revenue is strictly decreasing in the lobbying price candidate J commits to charge to group i , i.e., $\partial r_I / \partial c_{J,i} < 0$. This occurs because I 's expected revenue decreases through her probability of winning the election which is decreasing in the lobbying price J commits to charge to group i .

However, while a candidate's expected revenue is monotonic in the lobbying prices set by the opposite candidate, this is not the case for the lobbying prices set by herself. Candidate I 's expected revenue is decreasing in $c_{I,i}$, i.e., $\partial r_I / \partial c_{I,i} < 0$, if $c_{I,j}$ and $c_{J,j}$ are sufficiently low. This happens because as $c_{I,i}$ goes up, group i decreases its contribution to candidate I and, also, because $c_{I,j}$ is low, group j increases its contribution to its own favourite candidate, J . These two effects lead to a decrease in candidate I 's probability of winning and, thus, to a reduction in her expected revenue. On the other hand, candidate I 's expected revenue is increasing in $c_{I,i}$, i.e., $\partial r_I / \partial c_{I,i} > 0$, if $c_{I,j}$ is sufficiently high and $c_{J,j}$ sufficiently low, or if just $c_{J,j}$ is sufficiently high. Now, while along group i also group j decreases its contribution making the effect in candidate I 's probability of winning ambiguous, it is the positive effect of an increase in $c_{I,i}$ and a high $c_{I,j}$ on her revenue if she wins that makes her expected

revenue increasing in $c_{I,i}$.⁸

The candidates' maximisation problem highlights the main trade-off they face when optimally setting the "access" prices interest groups will have to pay to them if they win the office. On the one hand, as a candidate opts for higher lobbying prices, the higher her lobbying revenue if she wins the election. On the other hand, higher lobbying prices decrease the contribution received at the election stage and, therefore, the probability of winning the election. Balancing these two opposing effects appropriately determines the optimal lobbying prices as characterised in the following proposition:

Proposition 3.2. *There are multiple equilibria in the symmetric pricing stage game in all of which the lobbying prices candidate $I \in \{A, B\}$ commits to charge to the two groups satisfy the ratio $c_I^* \equiv \frac{c_{I,i}^*}{c_{I,j}^*} = \frac{1}{2}(\sqrt{5} - 1)$. Each candidate receives contribution $x^* = \frac{1}{4}(\sqrt{5} - 2)$ and her lobbying revenue is $r^* = \sqrt{5} - 2$. Each group has expected payoff $\Pi^* = \frac{3-\sqrt{5}}{4}$.*

Proof. To solve candidates' maximisation problem, we find the first-order conditions of eq. (3.10):

$$\begin{aligned}\frac{\partial r_I}{\partial c_{I,i}} &= \frac{c_{I,j} (c_{I,j}^2 (c_{J,i} + 2c_{J,j}) - c_{I,i}^2 c_{J,j} - 2c_{I,i} c_{I,j} (c_{J,i} + c_{J,j}))}{(c_{I,i} + c_{I,j})^4 (c_{J,i} + c_{J,j})} = 0 \\ \frac{\partial r_I}{\partial c_{I,j}} &= \frac{c_{I,i} (c_{I,i}^2 c_{J,j} + 2c_{I,i} c_{I,j} (c_{J,i} + c_{J,j}) - c_{I,j}^2 (c_{J,i} + 2c_{J,j}))}{(c_{I,i} + c_{I,j})^4 (c_{J,i} + c_{J,j})} = 0\end{aligned}$$

The above first-order conditions for $c_{I,i}$ and $c_{I,j}$ yields the following reaction functions:

$$\begin{aligned}c_{I,i}(c_{I,j}, c_{J,i}, c_{J,j}) &= \frac{c_{I,j} \left(\sqrt{3c_{J,i}c_{J,j} + c_{J,i}^2 + 3c_{J,j}^2} - c_{J,i} - c_{J,j} \right)}{c_{J,j}} \\ c_{I,j}(c_{I,i}, c_{J,i}, c_{J,j}) &= \frac{c_{I,i} \left(\sqrt{3c_{J,i}c_{J,j} + c_{J,i}^2 + 3c_{J,j}^2} + c_{J,i} + c_{J,j} \right)}{c_{J,i} + 2c_{J,j}}.\end{aligned}$$

⁸A similar intuition applies for the effect of a change in $c_{I,j}$ on r_I .

Solving the above system of equations simultaneously for the two candidates yields infinite solutions which satisfy the equation in the proposition. Substituting these results to eq. (3.8), eq. (3.7) and eq. (3.10), we find the expected contributions candidates receive, the expected payoffs of the two groups and the two candidates' expected lobbying revenues, respectively.

To guarantee that these solutions constitute a Nash equilibrium, we proceed as follows. Assume that candidate J sets $c_J^* = \frac{1}{2}(\sqrt{5} - 1)$. We will check whether candidate I has an incentive to set a lobbying price ratio different from $c_I^* = \frac{1}{2}(\sqrt{5} - 1)$, say $c'_I = k > 0$, and earn revenue greater than $\sqrt{5} - 2$. Substituting $c_{J,j} = \frac{1}{2}(\sqrt{5} - 1)c_{J,i}$ and $c_{I,i} = kc_{I,j}$ to candidate I 's objective function, eq. (3.10), and after some algebra, we obtain:

$$\begin{aligned} \frac{k [(\sqrt{5} - 1)k + 2\sqrt{5}]}{(1 + \sqrt{5})(k + 1)^3} &> \sqrt{5} - 2 \\ \implies k \left[(\sqrt{5} - 1)k + 2\sqrt{5} \right] - (\sqrt{5} - 2) (1 + \sqrt{5})(k + 1)^3 &> 0 \end{aligned}$$

Consider the function $f(k) = k [(\sqrt{5} - 1)k + 2\sqrt{5}] - (\sqrt{5} - 2) (1 + \sqrt{5})(k + 1)^3$, where $k \in (0, \infty)$. Differentiating f , we have $f'(k) = k [3(\sqrt{5} - 3)k + 8\sqrt{5} - 20] + 5\sqrt{5} - 9$ which is equal to zero when $k = \frac{1}{2}(\sqrt{5} - 1)$. The second derivative is $f''(k) = -6(3 - \sqrt{5})k - (20 - 8\sqrt{5}) < 0$, so f is concave, meaning that at $k = \frac{1}{2}(\sqrt{5} - 1)$, f is maximised. At this point, we also have that $f(\frac{1}{2}(\sqrt{5} - 1)) = 0$ which indicates that the function cannot be greater than zero. Thus, candidate I does not have incentive to deviate. \square

Since $c_I^* = \frac{1}{2}(\sqrt{5} - 1) < 1$, the above proposition states that each candidate commits to charge her favourite group a lobbying price lower than the one she commits to charge to the opposite group. This is an interesting result since we obtain endogenously—and not assume *a priori* like the majority of the studies in the literature—that the interest group that supports a candidate will be in an advantageous position after the election if the specific candidate wins.

Proposition 3.2 highlights that a candidate's expected revenue and contributions received do not depend on the absolute values of the lobbying prices she commits to charge but only

on their ratio. This indicates that although a lobbying revenue maximising candidate should set the lobbying price ratio of Proposition 3.2, the absolute values of the prices may depend on her own views about lobbying. Some candidates may prefer to charge high prices and appear interacting very rarely with the groups, while other candidates may prefer to charge low prices and appear interacting more frequently with the groups.

For instance, assume candidate A commits to lobbying prices $c_{A,a} = \frac{1}{2}(\sqrt{5} - 1)$ and $c_{A,b} = 1$ while candidate B commits to lobbying prices $c_{B,b} = 5(\sqrt{5} - 1)$ and $c_{A,b} = 10$. That is, while both pairs of announced prices satisfy the equilibrium ratio in Proposition 3.2, group B charges 10 times more than group A both lobbyists. While either A or B winning will be generating the same revenue, lobbying efforts in the two cases are not equal. In the case candidate A wins, lobbying effort made by groups a and b will be $\frac{1}{2}(3 - \sqrt{5})$ and $\sqrt{5} - 2$, respectively, which yields a total lobbying effort of $\frac{1}{2}(\sqrt{5} - 1)$. If candidate B wins, lobbying effort exerted by groups a and b will be $\frac{1}{10}(\sqrt{5} - 2)$ and $\frac{1}{20}(3 - \sqrt{5})$, respectively, the sum of which is $\frac{1}{20}(\sqrt{5} - 1)$. It is clear that the intensity of lobbying is greater in the first case. Hence, the absolute values of the prices a candidate commits to charge depend on her own views regarding lobbying. The more a candidate enjoys interacting with interest groups, the lower prices she sets. On the contrary, a candidate that does not prefer to undergo lobbying pressure sets higher lobbying prices as long as they satisfy the equilibrium ratio.

3.4 Asymmetric Candidates

In this section, we relax the assumption that the outcome of the election is determined solely by the contributions of the groups and consider that it is also affected by the two candidates' skills, e.g., inherent attributes. We express this asymmetry between the two candidates with the following assumption: in the election stage, if the two candidates exert the same amount of effort, their probabilities of winning are not equal, but the more skilled candidate's probability of winning is greater. The stages of the game remain the same as in the previous section.

3.4.1 Lobbying stage

To simplify analysis, let us assume that whoever candidate wins the election commits to charge to the opposite group lobbying price equal to 1. Hence, at the pricing stage, candidate A and B have to determine only the lobbying price that they are going to charge to their favourite group if they win, c_A and c_B , respectively. From eq. (3.5) and eq. (3.6), substituting $c_{I,j} = 1$, we have that the equilibrium lobbying efforts exerted by group i and j if candidate I , who is supported by group i , wins are:

$$y_i^* = \frac{1}{(c_I + 1)^2} \quad \text{and} \quad y_j^* = \frac{c_I}{(c_I + 1)^2}$$

and the corresponding payoffs

$$\pi_i^* = \frac{1}{(c_I + 1)^2} \quad \text{and} \quad \pi_j^* = \frac{c_I^2}{(c_I + 1)^2}.$$

3.4.2 Election stage

Without loss of generality, let us assume that the more skilled candidate is candidate A , supported by group a . We denote candidate A 's advantage by $\delta > 1$, by which we multiply group a 's contribution to her. Therefore, the maximisation problem of group a in this case is

$$\max_{x_a} \Pi_a = \max_{x_a} \left\{ \frac{\delta x_a}{\delta x_a + x_b} \frac{1}{(c_A + 1)^2} + \left(1 - \frac{\delta x_a}{\delta x_a + x_b} \right) \frac{c_B^2}{(c_B + 1)^2} - x_a \right\}$$

while the maximisation problem of group b is

$$\max_{x_b} \Pi_b = \max_{x_b} \left\{ \frac{x_b}{\delta x_a + x_b} \frac{1}{(c_B + 1)^2} + \left(1 - \frac{x_b}{\delta x_a + x_b} \right) \frac{c_A^2}{(c_A + 1)^2} - x_b \right\}.$$

Maximising the above expressions with respect to x_a and x_b , respectively, yields the following result:

Proposition 3.3. *In the equilibrium of the asymmetric election stage game, groups a and b offer contributions*

$$x_a^* = \frac{\delta (1 - c_A c_B) [(c_A + 2) c_B + 1]^2 [c_A (c_B + 2) + 1]}{(c_A + 1)^2 (c_B + 1)^2 [\delta (c_A + 2) c_B + c_A (c_B + 2) + \delta + 1]^2}$$

and

$$x_b^* = \frac{\delta (1 - c_A c_B) [(c_A + 2) c_B + 1] [c_A (c_B + 2) + 1]^2}{(c_A + 1)^2 (c_B + 1)^2 [\delta (c_A + 2) c_B + c_A (c_B + 2) + \delta + 1]^2},$$

respectively.

Proof. Differentiating the first maximisation problem with respect to x_a , gives us

$$\frac{\delta x_b \left[\frac{1}{(c_A + 1)^2} - \frac{c_B^2}{(c_B + 1)^2} \right]}{(\delta x_a + x_b)^2} - 1 = 0$$

which yields the reaction function of group a

$$x_a(x_b) = \frac{\sqrt{\delta (c_A + 1)^2 x_b (c_B + 1)^2 [1 + 2c_B - c_A (c_A + 2) c_B^2]}}{\delta (c_A + 1)^2 (c_B + 1)^2} - \frac{x_b}{\delta}.$$

Similarly, for group b we have

$$\frac{\delta x_a (1 - c_A c_B) [c_A (c_B + 2) + 1]}{(c_A + 1)^2 (c_B + 1)^2 (\delta x_a + x_b)^2} - 1 = 0$$

which gives us its reaction function

$$x_b(x_a) = \frac{\sqrt{\delta x_a (c_A + 1)^2 (c_B + 1)^2 (1 - c_A c_B) [c_A (c_B + 2) + 1]}}{(c_A + 1)^2 (c_B + 1)^2} - \delta x_a.$$

Equating the two reaction functions to each other, we obtain the equilibrium contribution levels in the proposition.

The second-order sufficient conditions for a maximum in the maximisation problem of group a is

$$\frac{2\delta^2 x_b (c_A c_B - 1) [(c_A + 2) c_B + 1]}{(c_A + 1)^2 (c_B + 1)^2 (\delta x_a + x_b)^3} < 0$$

and of group b

$$\frac{2\delta x_a (c_A c_B - 1) [c_A (c_B + 2) + 1]}{(c_A + 1)^2 (c_B + 1)^2 (\delta x_a + x_b)^3} < 0.$$

We can notice that both are satisfied when $c_A c_B < 1$. □

Comparing the equilibrium contributions of Proposition 3.3 to each other and taking into consideration the above second-order sufficient conditions for a maximum, $c_A c_B < 1$, we obtain the following corollary:

Corollary 3.1.

- i) $x_a^* > x_b^*$ if $c_A < 1$ and $c_A < c_B$*
- ii) $x_b^* > x_a^*$ if ($c_A \leq 1$ and $c_B < c_A$) or $c_A > 1$*

The above corollary provides us with some interesting information. Although candidate A , and therefore group a , have an advantage over the opposing candidate and group, this does not mean that group a always makes greater contribution than group b , as it is common in contests. In fact, contributions also depend on the lobbying prices the two candidates have committed to charge to the two groups if they win. Particularly, contribution by group a is greater than the one by group b only if the lobbying price offered by candidate A is lower than 1 as well as lower than the lobbying price offered by candidate B . Regarding group b , it offers greater contribution if the lobbying price candidate B commits to charge is lower than the lobbying price set by candidate A or if the price candidate A commits to charge is greater than 1.

3.4.3 Pricing stage

Candidates

Adding asymmetry to our model highly complicates the analysis of the first stage of the game. Nevertheless, we are able to show graphically that an equilibrium exists and, using numerical simulations, the equilibrium lobbying prices. Now, in contrast to the previous section, because of the asymmetry in candidates' abilities, their expected revenues are not equal. Thus, we will study each candidate separately.

Regarding candidate A , she wants to set the optimal lobbying price to charge to her favourite group in order to maximise her expected revenue. Therefore, making use of the equilibrium contributions from Proposition 3.3, her maximisation problem is

$$\begin{aligned} \max_{c_A} r_A &= \max_{c_A} \left\{ \frac{\delta x_a^*}{\delta x_a^* + x_b^*} \left[c_A \frac{1}{(1 + c_A)^2} + \frac{c_A}{(1 + c_A)^2} \right] \right\} \\ &= \max_{c_A} \left\{ \frac{2c_A [\delta (c_A + 2) c_B + \delta]}{(c_A + 1)^2 [\delta (c_A + 2) c_B + c_A (c_B + 2) + \delta + 1]} \right\}. \end{aligned}$$

The first-order condition of the above maximisation problem is

$$\frac{\partial r_A}{\partial c_A} = \frac{2(1 - c_A) [\delta (c_A + 2) c_B + \delta]^2 - 2\delta \{c_A^2 (c_B + 2) [(c_A + 3) c_B + 2] + c_A - 2c_B - 1\}}{(c_A + 1)^3 [\delta (c_A + 2) c_B + c_A (c_B + 2) + \delta + 1]^2} = 0.$$

Solving the above equation yields candidate A 's reaction function, $c_A(c_B)$, which is a quite complicating expression. We can guarantee, though, through numerical simulations, that candidate A 's optimal lobbying price is lower than 1 for every c_B . This states that as in the symmetric case, in the asymmetric case as well, candidate A will offer a lower lobbying price to her favourite group than to the opposing one. The second-order sufficient condition for a maximum is also satisfied in the optimal c_A . We can also guarantee that candidate A 's reaction function is increasing in c_B , which indicates that candidate A always increases the lobbying price she commits to as the opposite candidate increases her own (Figure 3.2).

Candidate B 's maximisation problem is

$$\begin{aligned} \max_{c_B} r_B &= \max_{c_B} \left\{ \frac{x_b^*}{\delta x_a^* + x_b^*} \left[c_B \frac{1}{(1+c_B)^2} + \frac{c_B}{(1+c_B)^2} \right] \right\} \\ &= \max_{c_B} \left\{ \frac{2c_B [c_A (c_B + 2) + 1]}{(c_B + 1)^2 \{c_A [(\delta + 1)c_B + 2] + 2\delta c_B + \delta + 1\}} \right\}. \end{aligned}$$

The first-order condition of the above problem is

$$\begin{aligned} &2\{1 + \delta - c_B (1 + \delta + 4\delta c_B) - c_A^2 [-4 + (1 + \delta) c_B^2 (3 + c_B)] \\ \frac{\partial r_B}{\partial c_B} &= \frac{-2c_A [-2 - \delta + c_B + c_B^2 (1 + 4\delta + \delta c_B)]}{(c_B + 1)^3 \{c_A [(\delta + 1) c_B + 2] + 2\delta c_B + \delta + 1\}^2} = 0 \end{aligned}$$

the solution of which yields candidate B 's reaction function. We can assure that, like c_A , c_B cannot be greater than 1 either, meaning that the less skilled candidate also commits to a lower lobbying price to her favourite candidate.

We, also, argue that c_B is not always increasing in c_A , but for large δ it can rather be decreasing. We can observe this in Figure 3.2 focusing on candidate B 's reaction function for any level of δ . The intuition behind it is that if the more skilled candidate is not sufficiently skilled, the opposite candidate finds it optimal to respond with an increase in her lobbying price to an increase in the more skilled candidate's price. However, if the difference in skills is sufficiently large, the less skilled candidate lowers the price as the more skilled one increases it. She does so in order to induce her favourite group contribute more at the election stage and, therefore, assist her to catch up the opponent's high probability of winning because of her skills. Moreover, we can notice that the more skilled the opposite candidate is, the lower lobbying price the less skilled candidate commits to.

Equilibrium. Since the two candidates' reaction functions are quite complicated expressions, the solution of the system of them is complicated as well. In Figure 3.2, we plot the candidates' reaction functions for three different values of δ . It is clear that there is a unique

equilibrium in pure strategies in each case, at the point where the two reaction curves intersect. We can also notice that as δ goes up, the equilibrium moves to the upper left side of the graph indicating that, in equilibrium, the more skilled a candidate is, the greater lobbying price she charges to her favourite group and the lower price the opposite candidate charges to her own favourite group.

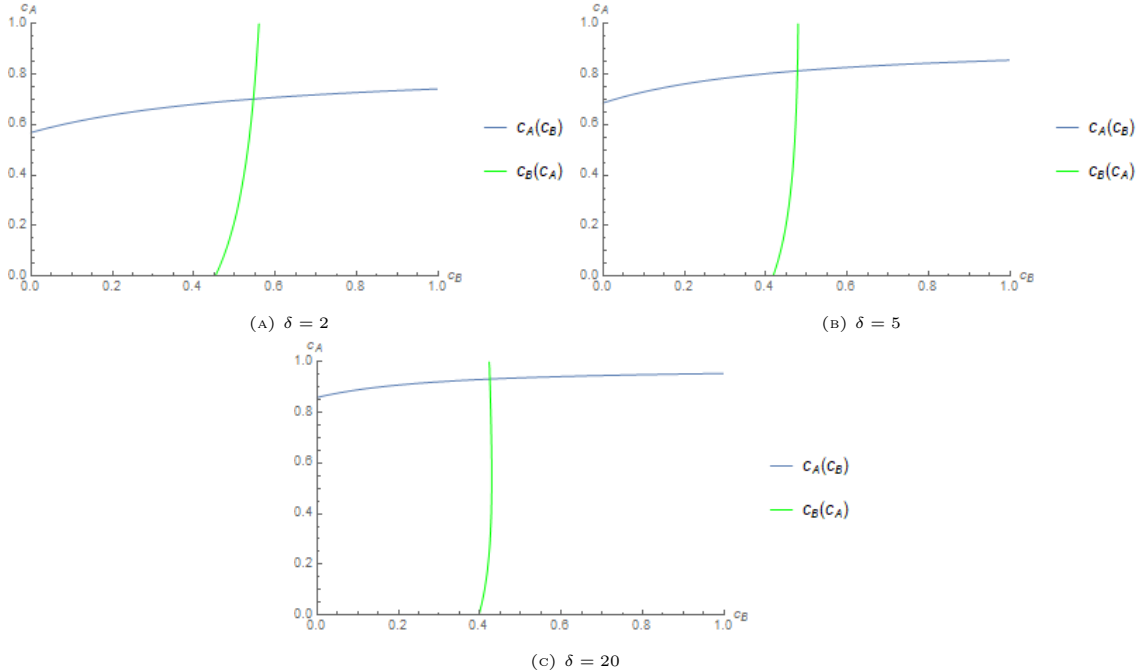


FIGURE 3.2: Equilibrium of the pricing stage game for different values of δ .

This observation can be validated in Figure 3.3 where we provide the graph of the two candidates' equilibrium lobbying prices as a function of δ . It becomes clear that the more skilled candidate A is, the higher lobbying price she affords to commit to in equilibrium. This occurs because in spite of the greater price her favourite group is likely to face at the lobbying stage, it is not willing to decrease its contribution to her since the probability of winning the election becomes greater because of the candidates' skills. On the other hand, as a candidate becomes more skilled, her opponent lowers the lobbying price she commits to charge to her supportive group in order to deter it from being discouraged and decrease its contribution to her.

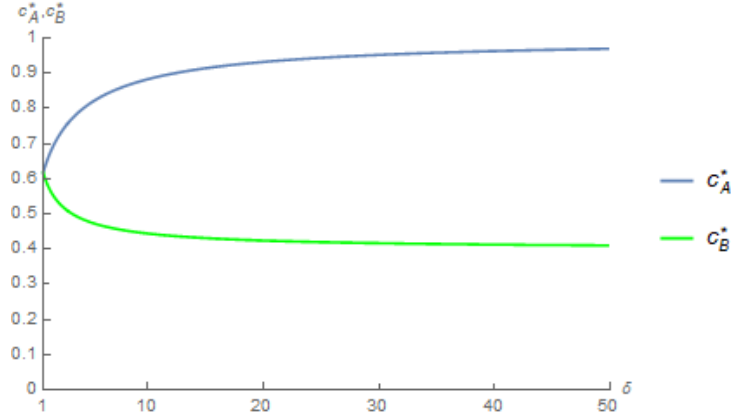


FIGURE 3.3: Equilibrium lobbying prices as a function of δ .

In Figure 3.4, we observe that the more skilled candidate's expected lobbying revenue is greater than her opponent's. Also, starting from equal levels of revenue in the case of symmetric candidates, as a candidate becomes more skilled, her opponent's expected lobbying revenue decreases and goes to zero for high levels of asymmetry.

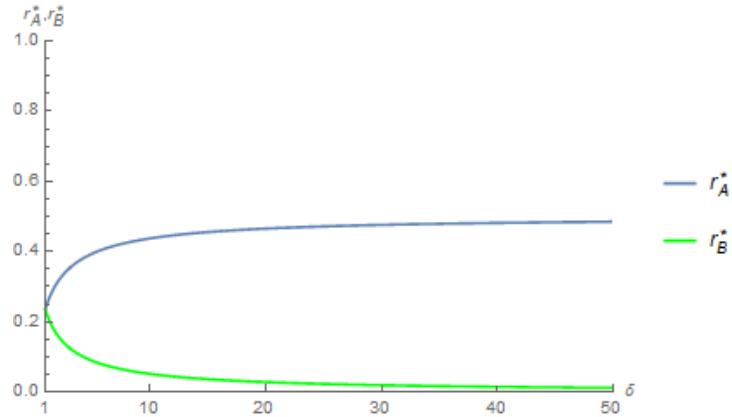


FIGURE 3.4: Candidates' expected lobbying revenues as a function of δ .

Interest groups

In what follows, we provide the analysis results regarding the two groups. In Figure 3.5, we can notice that total lobbying effort is greater if the less skilled candidate wins since the lower prices she has committed to facilitates the lobbying process. Thus, our model predicts that a less skilled politician is more vulnerable to political pressure if she is elected.

Moreover, this result is consistent to the concept of “informational lobbying”, where informed lobbyists attempt to strategically transmit information to uninformed politicians (Calvert 1985, Austen-Smith & Wright 1992, Bannedsen & Feldmann 2002).

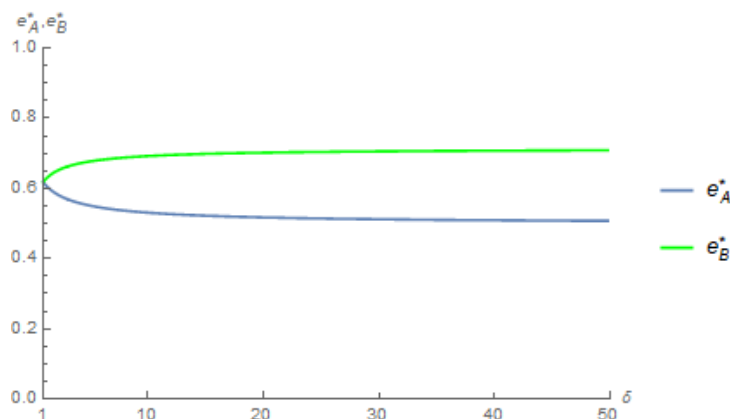


FIGURE 3.5: Total equilibrium lobbying effort as a function of δ . (e_i^* : total equilibrium lobbying effort if candidate i wins.)

In Figure 3.6, we can see that election contribution made by the group supporting the less skilled candidate is slightly greater. This makes sense since offering greater contribution, the group attempts to compensate for the opposite candidate’s advantage. Also, we can observe that as the difference between the two candidates’ skills increases, contributions of both groups tend to zero since the outcome of the election becomes foreseeable and potential contributions by either group will not affect it significantly. The more skilled candidate does not need any contributions since her advantage is already sufficiently strong because of her skills and, thus, group a does not find it necessary to contribute. On the other hand, the opposite candidate is at such an unfavourable position so that group b finds a potential contribution futile.

Finally, Figure 3.7 gives us the expected payoffs of the groups as functions of candidate A ’s skills. Interestingly, we can notice that the group that supports the more skilled candidate has a greater payoff but only slightly. Actually, for higher levels of δ , the expected payoffs of two groups become equal. The intuition is that for low levels of asymmetry, greater expected payoff of group a is attributed to candidate A ’s higher probability of winning and the relatively low lobbying price the group is going to pay. Nevertheless, for higher levels

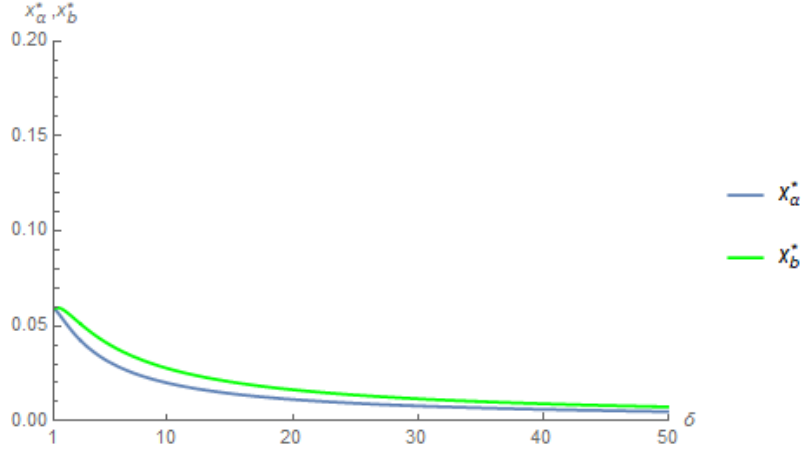


FIGURE 3.6: Equilibrium election contributions as a function of δ .

of asymmetry, despite candidate A 's even greater probability of winning, the lobbying price group a should pay is greater as well and as it approaches the price charged to the opposite group, the payoffs of the two groups become equal.

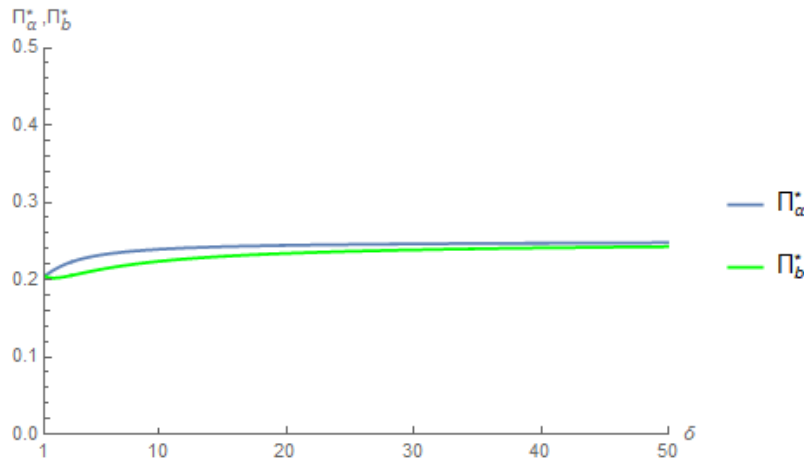


FIGURE 3.7: Expected payoffs of the groups as a function of δ .

3.5 Conclusion

In this study, we have analysed optimal lobbying pricing in a game with two candidates and two interest groups. First, candidates commit to lobbying prices if they win the election. Then, each group makes contribution to its favourite candidate in order to help her win

the election. After the election, the two groups make lobbying efforts in order to affect the incumbent politician's policy.

We obtain, endogenously, that politicians commit to charge lower lobbying prices to the groups that support them than to the opposite groups if they win the election. We also show that the more skilled a politician is in the election, the greater the lobbying price she commits to charge to her favourite group but not greater than the one she commits to charge to the opposite group. These results establish the transaction between politicians and lobbies, and confirm the common view that some specific groups are treated favourably by politicians in power.

References

- Austen-Smith, D. (1987), ‘Interest groups, campaign contributions, and probabilistic voting’, *Public Choice* **54**, 123–139.
- Austen-Smith, D. (1995), ‘Campaign Contributions and Access’, *American Political Science Review* **89**(03), 566–581.
- Austen-Smith, D. & Wright, J. R. (1992), ‘Competitive lobbying for a legislator’s vote’, *Social Choice and Welfare* **9**, 229–257.
- Baik, K. H. & Shogren, J. F. (1992), ‘Strategic Behavior in Contests: Comment’, *American Economic Review* **82**(1), 359–362.
- Balart, P., Casas, A. & Troumpounis, O. (2018), Technological change, campaign spending and Polarization Technological change, campaign spending and polarization.
- Balart, P., Flamand, S. & Troumpounis, O. (2016), ‘Strategic choice of sharing rules in collective contests’, *Social Choice and Welfare* **46**(2), 239–262.
- Baron, D. P. (1994), ‘Electoral Competition with Informed and Uninformed Voters’, *American Political Science Review* **88**(01), 33–47.
- Baye, M. R., Kovenock, D. & de Vries, C. G. (1993), ‘Rigging the Lobbying Process : An Application of the All-Pay Auction’, *American Economic Review* **83**(1), 289–294.
- Becker, G. S. (1983), ‘A Theory of Competition Among Pressure Groups for Political Influence’, *Quarterly Journal of Economics* **98**(3), 371.

- Becker, G. S. & Murphy, K. M. (1988), 'A Theory of Rational Addiction', *Journal of Political Economy* **96**(4), 675–700.
- Bennedsen, M. & Feldmann, S. E. (2002), 'Lobbying Legislatures', *Journal of Political Economy* **110**(4), 919–946.
- Bennedsen, M. & Feldmann, S. E. (2006), 'Informational lobbying and political contributions', *Journal of Public Economics* **90**(4-5), 631–656.
- Besley, T. & Coate, S. (2001), 'Lobbying and Welfare in a Representative Democracy', *Review of Economic Studies* **68**(1), 67–82.
- Beviá, C. & Corchón, L. C. (2013), 'Endogenous strength in conflicts', *International Journal of Industrial Organization* **31**(3), 297–306.
- Blinder, A., Canetti, E. R., Lebow, D. E. & Rudd, J. B. (1998), *Asking About Prices: A New Approach to Understanding Price Stickiness*, Russell Sage Foundation.
- Calvert, R. L. (1985), 'The Value of Biased Information: A Rational Choice Model of Political Advice', *Journal of Politics* **47**(2), 530–555.
- Clark, D. J. & Nilssen, T. (2013), 'Learning by doing in contests', *Public Choice* **156**(1-2), 329–343.
- Clark, D. J., Nilssen, T. & Sand, J. Y. (2013), 'Motivating over time: Dynamic win effects in sequential contests'.
- Clark, D. J., Nilssen, T. & Yngve, J. (2018), 'Dynamic Win Effects in Sequential Contests', (9037).
- Clark, D. J. & Riis, C. (1998), 'Contest Success Functions: An Extension', *Economic Theory* **11**, 201–204.
- Corchón, L. C. (2007), 'The theory of contests: A survey', *Review of Economic Design* **11**(2), 69–100.

- Corchón, L. C. & Serena, M. (2017), Contest theory, *in* L. C. Corchón & M. A. Marini, eds, 'Handbook of game theory and industrial organization', Edward Elgar, Cheltenham, UK.
- DellaVigna, S. & Malmendier, U. (2004), 'Contract Design and Self-Control: Theory and Evidence', *Quarterly Journal of Economics* **119**(2), 353–402.
- Dixit, A. (1987), 'Strategic Behavior in Contests', *American Economic Review* **77**, 891–898.
- Eliasz, K. & Spiegler, R. (2006), 'Contracting with Diversely Naive Agents', *Review of Economic Studies* **73**(3), 689–714.
- Esteban, S., Miyagawa, E. & Shum, M. (2007), 'Nonlinear pricing with self-control preferences', *Journal of Economic Theory* **135**(1), 306–338.
- Felli, L. & Merlo, A. (2012), 'Endogenous Lobbying', *Journal of the European Economic Association* **4**(1), 180–215.
- Fu, Q. (2006), 'Endogenous Timing of Contest with Asymmetric Information', *Public Choice* **129**(12), 1–23.
- Fu, Q. & Lu, J. (2012), 'The optimal multi-stage contest', *Economic Theory* **51**(2), 351–382.
- Fu, Q., Lu, J. & Pan, Y. (2015), 'Team contests with multiple pairwise battles', *American Economic Review* **105**(7), 2120–2140.
- Gabaix, X. & Laibson, D. (2006), 'Shrouded Attributes, Consumer Myopia, and Information Suppression in Competitive Markets', *Quarterly Journal of Economics* **121**(2), 505–540.
- Gelder, A. (2014), 'From Custer to Thermopylae: Last stand behavior in multi-stage contests', *Games and Economic Behavior* **87**, 442–466.
- Genesove, D. & Mayer, C. (2001), 'Loss Aversion and Seller Behavior: Evidence from the Housing Market', *Quarterly Journal of Economics* **116**(4), 1233–1260.
- Gradstein, M. & Konrad, K. A. (1999), 'Orchestrating Rent Seeking Contests', *Economic Journal* **109**(458), 536–545.

- Grossman, G. M. & Helpman, E. (1994), ‘Protection for Sale’, *American Economic Review* **84**(4), 833–850.
- Grossman, G. M. & Helpman, E. (1996), ‘Electoral Competition and Special Interest Politics’, *Review of Economic Studies* **63**(2), 265.
- Grubb, M. D. (2009), ‘Selling to Overconfident Consumers’, *American Economic Review* **99**(5), 1770–1807.
- Grubb, M. D. (2015), ‘Consumer Inattention and Bill-Shock Regulation’, *Review of Economic Studies* **82**(1), 219–257.
- Harris, C. & Vickers, J. (1987), ‘Racing with Uncertainty’, *Review of Economic Studies* **54**(1), 1–21.
- Heidhues, P. & Kőszegi, B. (2004), The impact of consumer loss aversion on pricing.
- Heidhues, P. & Kőszegi, B. (2008), ‘Competition and Price Variation when Consumers Are Loss Averse’, *American Economic Review* **98**(4), 1245–1268.
- Heidhues, P. & Koszegi, B. (2010), ‘Exploiting naïvete about self-control in the credit market’, *American Economic Review* **100**(5), 2279–2303.
- Hillman, A. L. & Riley, J. G. (1989), ‘Politically contestable rents and transfers’, *Economics & Politics* **I**(1), 17–39.
- Jia, H. (2008), ‘A stochastic derivation of the ratio form of contest success functions’, *Public Choice* **135**(3-4), 125–130.
- Jia, H., Skaperdas, S. & Vaidya, S. (2013), ‘Contest functions: Theoretical foundations and issues in estimation’, *International Journal of Industrial Organization* **31**(3), 211–222.
- Kahneman, D., Knetsch, J. L. & Thaler, R. H. (1990), ‘Experimental Tests of the Endowment Effect and the Coase Theorem’, *Journal of Political Economy* **98**(6), 1325–1348.

- Kahneman, D., Knetsch, J. L. & Thaler, R. H. (1991), ‘Anomalies The Endowment Effect, Loss Aversion, and Status Quo Bias’, *Journal of Economic Perspectives* **5**(1), 193–206.
- Kahneman, D. & Tversky, A. (1979), ‘Prospect Theory: An Analysis of Decision under Risk’, *Econometrica* **47**(2), 263–292.
- Karle, H. & Möller, M. (2020), ‘Selling in Advance to Loss Averse Consumers’, *International Economic Review* **61**(1), 441–468.
- Karle, H. & Peitz, M. (2014), ‘Competition under consumer loss aversion’, *The RAND Journal of Economics* **45**(1), 1–31.
- Klumpp, T. & Polborn, M. K. (2006), ‘Primaries and the New Hampshire Effect’, *Journal of Public Economics* **8**(90), 1073–1114.
- Konrad, K. A. (2009), *Strategy and Dynamics in Contests*, Oxford University Press, Oxford, UK.
- Konrad, K. A. & Kovenock, D. (2009), ‘Multi-battle contests’, *Games and Economic Behavior* **66**(1), 256–274.
- Kőszegi, B. & Rabin, M. (2006), ‘A Model of Reference-Dependent Preferences’, *Quarterly Journal of Economics* **121**(4), 1133–1165.
- Krueger, A. O. (1974), ‘The Political Economy Seeking of the Society’, *American Economic Review* **64**(3), 291–303.
- Leininger, W. (1993), ‘More Efficient Rent-Seeking: A Münchhausen Solution’, *Public Choice* **75**, 43–62.
- Linster, B. G. (1993), ‘Stackelberg Rent-Seeking’, *Public Choice* **77**(2), 307–321.
- Lohmann, S. (1995), ‘Information, access, and contributions: A signaling model of lobbying’, *Public Choice* **85**(3-4), 267–284.

- Martimort, D. & Semenov, A. (2008), 'Ideological uncertainty and lobbying competition', *Journal of Public Economics* **92**(3-4), 456–481.
- Maskin, E. & Riley, J. (1984), 'Monopoly with Incomplete Information', *The RAND Journal of Economics* **15**(2), 171–196.
- Miao, C. H. (2010), 'Consumer myopia, standardization and aftermarket monopolization', *European Economic Review* **54**(7), 931–946.
- Moldovanu, B. & Sela, A. (2006), 'Contest architecture', *Journal of Economic Theory* **126**(1), 70–96.
- Morgan, J. (2003), 'Sequential contests', *Public Choice* **116**(1-2), 1–18.
- Mussa, M. & Rosen, S. (1978), 'Monopoly and product quality', *Journal of Economic Theory* **18**(2), 301–317.
- Nitzan, S. (1994), 'Modelling rent-seeking contests', *European Journal of Political Economy* **10**(1), 41–60.
- Nti, K. O. (1999), 'Rent-seeking with asymmetric valuations', *Public Choice* **98**, 415–430.
- Odean, T. (1998), 'Are investors reluctant to realize their losses?', *Journal of Finance* **53**(5), 1775–1798.
- Popescu, I. & Yaozhong, W. (2007), 'Dynamic pricing strategies with reference effects', *Operations Research* **55**(3), 413–429.
- Potters, J. & van Winden, F. (1992), 'Lobbying and asymmetric information', *Public Choice* **74**(3), 269–292.
- Serena, M. (2017), 'Sequential contests revisited', *Public Choice* **173**(1-2), 131–144.
- Skaperdas, S. (1996), 'Contest success functions', *Economic Theory* **7**(2), 283–290.

- Spiegler, R. (2012), 'Monopoly pricing when consumers are antagonized by unexpected price increases: A "cover version" of the Heidhues-Koszegi-Rabin model', *Economic Theory* **51**(3), 695–711.
- Tripathi, M., Ansolabehere, S. & Snyder, J. M. (2002), 'Are PAC Contributions and Lobbying Linked? New Evidence from the 1995 Lobby Disclosure Act', *Business and Politics* **4**(02), 131–155.
- Triviza, E. (2019), Optimal Non-Linear Pricing Scheme when Consumers are Habit Forming.
- Tullock, G. (1980), Efficient Rent Seeking, *in* J. Buchanan, R. Tollison & G. Tullock, eds, 'Toward a Theory of Rent-Seeking Society', College Station: Texas A&M University Press, pp. 97–112.
- Wright, J. R. (1990), 'Contributions, Lobbying, and Committee Voting in the U.S. House or Representatives', *American Political Science Review* **84**(2).

Appendix A

Proof of Proposition 1.2

Differentiating eq. (1.4) with respect to v_1^* , we have

$$\begin{aligned} \frac{\partial U}{\partial v_1^*} = -f(v_1^*) \left\{ v_1^* - p_1 + \int_{v_2^*}^1 \left\{ (2-q)v_2 - [1 - (1-q)\lambda]p_2 \right\} f(v_2) dv_2 + \right. \\ \left. + \int_0^{v_2^*} q(p_2 - \lambda v_2) f(v_2) dv_2 \right\} \end{aligned} \quad (\text{A.1})$$

Setting eq. (A.1) equal to zero, we get

$$\begin{aligned} v_1^* &= p_1 - \int_{v_2^*}^1 \left\{ (2-q)v_2 - [1 - (1-q)\lambda]p_2 \right\} f(v_2) dv_2 + \int_0^{v_2^*} q(p_2 - \lambda v_2) f(v_2) dv_2 \\ &= p_1 + [1 + (1-q)\lambda]p_2 + q - 2 - q\lambda \int_0^{v_2^*} F(v_2) dv_2 + (2-q) \int_{v_2^*}^1 F(v_2) dv_2 \end{aligned}$$

Substituting v_2^* from Proposition 1.1 to the above equation, we obtain

$$\begin{aligned}
v_1^* &= p_1 + [1 + (1 - q)\lambda]p_2 + q - 2 + q\lambda \int_0^{\frac{[1+q+(\lambda-1)q]p_2}{2+(\lambda-1)q}} [1 - F(v)] dv - \\
&\quad - q\lambda \frac{[1 + q + (\lambda - 1)q]p_2}{2 + (\lambda - 1)q} - (2 - q) \int_{\frac{[1+q+(\lambda-1)q]p_2}{2+(\lambda-1)q}}^1 [1 - F(v)] dv + \\
&\quad + 2 - q - (2 - q) \frac{[1 + q + (\lambda - 1)q]p_2}{2 + (\lambda - 1)q} \\
&= p_1 + [1 + (1 - q)\lambda]p_2 + q\lambda \int_0^{v_2^*} [1 - F(v)] dv - \frac{q\lambda p_2 [1 + q + (1 - q)\lambda]}{2 + q(\lambda - 1)} - \\
&\quad - (2 - q) \int_{v_2^*}^1 [1 - F(v)] dv - \frac{(2 - q) [1 + q + (1 - q)\lambda]p_2}{2 + q(\lambda - 1)} \\
&= p_1 + [1 + (1 - q)\lambda]p_2 - [1 + q + (1 - q)\lambda]p_2 + q\lambda \int_0^{v_2^*} [1 - F(v)] dv - \\
&\quad - (2 - q) \int_{v_2^*}^1 [1 - F(v)] dv \\
&= p_1 - qp_2 - (2 - q) \int_{v_2^*}^1 [1 - F(v)] dv + q\lambda \int_0^{v_2^*} [1 - F(v)] dv
\end{aligned}$$

which gives us the result in Proposition 1.2.

Proof of Proposition 1.3

Firm's maximisation problem is

$$\max_{\mathbf{p}} E[\pi] = E[S(\mathbf{p})] - E[U(\mathbf{p})] \quad \text{s.t. } E[U] = 0.$$

The first-order conditions are

$$\begin{aligned}\frac{\partial \pi}{\partial p_1} &= \frac{\partial S}{\partial v_1^*} \frac{\partial v_1^*}{\partial p_1} + \frac{\partial S}{\partial p_1} = 0 \\ \frac{\partial \pi}{\partial p_2} &= \frac{\partial S}{\partial v_1^*} \frac{\partial v_1^*}{\partial p_2} + \frac{\partial S}{\partial p_2} = 0\end{aligned}$$

where S is given by eq. (1.7) and v_1^* by Proposition 1.2. We have, then,

$$\begin{aligned}\frac{\partial S}{\partial v_1^*} &= -f(v_1^*) \left\{ v_1^* - c + \int_{v_2^*}^1 [(2-q)v_2 - (1-q)\lambda p_2 - c] f(v_2) dv_2 + \right. \\ &\quad \left. + \int_0^{v_2^*} q(p_2 - \lambda v_2) f(v_2) dv_2 \right\}\end{aligned}$$

where substituting v_1^* from Proposition 1.2 and simplifying yields

$$\frac{\partial S}{\partial v_1^*} = f(v_1^*) [2c - p_1 - p_2 - F(v_2^*)(c - p_2)].$$

Regarding p_1 , we have that $\partial v_1^*/\partial p_1 = 1$ and $\partial S/\partial p_1 = 0$. Thus, from the FOC, we get

$$\frac{\partial \pi}{\partial p_1} = \frac{\partial S}{\partial v_1^*} = f(v_1^*) [2c - p_1 - p_2 - F(v_2^*)(c - p_2)] = 0. \quad (\text{A.2})$$

Regarding p_2 , we have

$$\begin{aligned}\frac{\partial S}{\partial p_2} &= \int_{v_1^*}^1 \left\{ q \int_0^{v_2^*} f(v_2) dv_2 - (1-q)\lambda \int_{v_2^*}^1 f(v_2) dv_2 + \frac{[1 + \lambda - (\lambda - 1)q] f(v_2^*)(c - p_2)}{2 + (\lambda - 1)q} \right\} \\ &= [1 - F(v_1^*)] \left\{ qF(v_2^*) - (1-q)\lambda [1 - F(v_2^*)] + \frac{[1 + \lambda - (\lambda - 1)q] f(v_2^*)(c - p_2)}{2 + (\lambda - 1)q} \right\}.\end{aligned}$$

Moreover, we have already found in Section 1.3 that

$$\frac{\partial v_1^*}{\partial p_2} = 1 + (\lambda - 1)q - [1 + \lambda - (\lambda - 1)q] F(v_2^*).$$

Substituting in FOC, we obtain

$$\begin{aligned}
\frac{\partial \pi}{\partial p_2} = & f(v_1^*) [2c - p_1 - p_2 - F(v_2^*)(c - p_2)] \times \\
& \times \left\{ 1 + (\lambda - 1)q - [1 + \lambda - (\lambda - 1)q] F(v_2^*) \right\} + \\
& + [1 - F(v_1^*)] \left\{ qF(v_2^*) - (1 - q)\lambda [1 - F(v_2^*)] + \right. \\
& \left. \frac{[1 + \lambda - (\lambda - 1)q] f(v_2^*)(c - p_2)}{2 + (\lambda - 1)q} \right\} = 0.
\end{aligned} \tag{A.3}$$

Solving eq. (A.2) for p_1 and substituting to eq. (A.3), we have

$$\frac{[1 - F(v_1^*)] A}{2 + (\lambda - 1)q} = 0$$

where $A = [2 + (\lambda - 1)q] \{ [q + (1 - q)\lambda] F(v_2^*) - (1 - q)\lambda \} + [1 + \lambda - (\lambda - 1)q] f(v_2^*)(c - p_2)$.

Solving the above for p_2 , gives us

$$p_2 = c + \frac{B}{D}. \tag{A.4}$$

where $B = [2 + (\lambda - 1)q] \{ [q + (1 - q)\lambda] F(v_2^*) - (1 - q)\lambda \}$ and $D = [1 + \lambda - (\lambda - 1)q] f(v_2^*)$.

First, we show that $D > 0$. Assume that the opposite is true, $D < 0$. Then, since $f(v_2^*) > 0$, we have $[1 + \lambda - (\lambda - 1)q] f(v_2^*) < 0 \implies 1 + \lambda - (\lambda - 1)q < 0 \implies 1 + \lambda < (\lambda - 1)q \implies q > \frac{\lambda + 1}{\lambda - 1}$, which cannot be true since $q \leq 1$. Regarding B , we have

$$\begin{aligned}
B > [<] 0 & \implies [2 + (\lambda - 1)q] \left\{ [q + (1 - q)\lambda] F(v_2^*) - (1 - q)\lambda \right\} > [<] 0 \\
& \implies [q + (1 - q)\lambda] F(v_2^*) - (1 - q)\lambda > [<] 0, & \text{since } 2 + (\lambda - 1)q > 0 \\
& \implies q \{ F(v_2^*) + [1 - F(v_2^*)] \lambda \} > [<] [1 - F(v_2^*)] \lambda \\
& \implies q > [<] \frac{[1 - F(v_2^*)] \lambda}{F(v_2^*) + [1 - F(v_2^*)] \lambda} = \bar{q}.
\end{aligned}$$

We have so far that, $p_2 = c + \frac{B}{D}$, where $D > 0$ and $B > [<] 0$ if $q > [<] \bar{q}$. Therefore, $p_2 > [<] c$ if $q > [<] \bar{q}$.

Now, substituting eq. (A.4) to eq. (A.2) and after some algebra, we have

$$p_1 = c + \frac{[1 - F(v_2^*)] [2 + (\lambda - 1) q] \left\{ (1 - q) \lambda - [q + (1 - q) \lambda] F(v_2^*) \right\}}{[1 + \lambda - (\lambda - 1) q] f(v_2^*)}$$

or, equivalently,

$$p_1 = c - [1 - F(v_2^*)] \frac{B}{D}.$$

Since $1 - F(v_2^*) > 0$, $D > 0$ and $B > [\leq] 0$ if $q > [\leq] \bar{q}$, we have that $p_1 < [\geq] c$ if $q > \bar{q}$.

Non-loss-averse consumer

If consumer maximises her expected utility, she does not have any expectations about future consumption or if she does, she does not take them into consideration when she decides whether to buy or not. Thus, consumer buys second unit if her valuation in period 2 is greater than the price of the second unit, i.e., $v_2 > p_2$. Consumer's expected utility at the contracting period is

$$E(\tilde{U}) = -p_0 + \int_{\tilde{v}_1^*}^1 \left[v_1 - p_1 + \int_{p_2}^1 (v_2 - p_2) f(v_2) dv_2 \right] f(v_1) dv_1.$$

Maximising the above expected utility with respect to \tilde{v}_1^* , we get the optimal first-period valuation threshold which is

$$\tilde{v}_1^* = p_1 - \int_{p_2}^1 [1 - F(v)] dv.$$

When consumer decides whether to buy or not first unit, she only takes into consideration the price of it and the probability of buying second unit which, in turn, depends on her expected valuation in period 2 and the price of the second unit.

Regarding firm's strategy, similarly to Section 1.3, firm maximises expected social surplus which is

$$\tilde{S} = \int_{\tilde{v}_1^*}^1 \left[v_1 - c + \int_{p_2}^1 (v_2 - c) f(v_2) dv_2 \right] f(v_1) dv_1.$$

Following a similar process as in Section 1.3, we can find that the above expected social surplus is maximised when $p_1 = p_2 = c$ and p_0 is equal to consumer surplus.

Appendix B

Proof of Proposition 2.1. Dividing the two equilibrium efforts, we have

$$\frac{x_1^*}{x_2^*} = \frac{\frac{1}{(1+a)^2} \frac{1-c}{1+c}}{\frac{a}{(1+a)^2} \frac{1-c}{1+c}} = \frac{1}{a} \implies x_1^* = \frac{1}{a} x_2^*$$

and because $0 < a \leq 1 \implies \frac{1}{a} \geq 1$, we have $x_1^* \geq x_2^*$.

For $0 < a \leq 1$, we have

$$\begin{aligned} \mathbb{E}^*(\pi_1) - \mathbb{E}^*(\pi_2) &= \frac{1}{(1+a)^2} \frac{1+a(2+a)c^2}{(1+c)^2} - \frac{1}{(1+a)^2} \frac{a^2 + (1+2a)c^2}{(1+c)^2} \\ &= \frac{1}{(1+a)^2} \frac{(1-c^2) - a^2(1-c^2)}{(1+c)^2} \\ &= \frac{1}{(1+a)^2} \frac{(1-c)(1+c)(1-a)(1+a)}{(1+c)^2} \geq 0, \quad \forall a \in (0, 1]. \end{aligned}$$

Thus, $\mathbb{E}^*(\pi_1) \geq \mathbb{E}^*(\pi_2)$.

Proof of Proposition 2.2

$$\begin{aligned} x_2^* \leq \bar{x} &\implies \frac{a}{(1+a)^2} \frac{1-c}{1+c} \leq \frac{1}{4} \frac{1-c}{1+c} \\ &\implies 4a \leq (1+a)^2 \\ &\implies 1 - 2a + a^2 \geq 0 \\ &\implies (a-1)^2 \geq 0, \end{aligned}$$

which is true for all a .

Now we show that $\bar{x} \leq x_1^*$.

$$\begin{aligned}
\bar{x} \leq x_1^* &\implies \frac{1}{4} \frac{1-c}{1+c} \leq \frac{1}{(1+a)^2} \frac{1-c}{1+c} \\
&\implies (1+a)^2 \leq 4 \\
&\implies 1+a \leq 2, \quad \text{since } 1+a > 0 \\
&\implies a \leq 1,
\end{aligned}$$

which is true. Thus, $x_2^* \leq \bar{x} \leq x_1^*$.

Now, we show that $\mathbb{E}^*(\pi_2) \leq \bar{\mathbb{E}}(\pi)$.

$$\begin{aligned}
\mathbb{E}^*(\pi_2) \leq \bar{\mathbb{E}}(\pi) &\implies \frac{1}{(1+a)^2} \frac{a^2 + (1+2a)c^2}{(1+c)^2} \leq \frac{1}{4} \frac{1+3c^2}{(1+c)^2} \\
&\implies 3a^2 + c^2 + 2ac^2 - 1 - 2a - 3a^2c^2 \leq 0 \\
&\implies 3a^2(1-c^2) - 2a(1-c^2) - (1-c^2) \geq 0 \\
&\implies (3a^2 - 2a - 1)(1-c^2) \leq 0 \\
&\implies 3(a-1) \left(a + \frac{1}{3}\right) (1-c^2) \leq 0,
\end{aligned}$$

which is true since $a - 1 \leq 0$ for $0 < a \leq 1$.

Finally, we prove that $\bar{\mathbb{E}}(\pi) \leq \mathbb{E}^*(\pi_1)$:

$$\begin{aligned}
\bar{E}(\pi) \leq \mathbb{E}^*(\pi_1) &\implies \frac{1}{4} \frac{1+3c^2}{(1+c)^2} \leq \frac{1}{(1+a)^2} \frac{1+a(2+a)c^2}{(1+c)^2} \\
&\implies 3 - 3c^2 - 2a + 2ac^2 - a^2 + a^2c^2 \geq 0 \\
&\implies 3(1-c^2) - 2a(1-c^2) - a^2(1-c^2) \geq 0 \\
&\implies -(a^2 + 2a - 3)(1-c^2) \geq 0 \\
&\implies -(a-1)(a+3)(1-c^2) \geq 0,
\end{aligned}$$

which is true since $a-1 \leq 0$ for $0 < a \leq 1$. Thus, $\mathbb{E}^*(\pi_2) \leq \bar{E}(\pi) \leq \mathbb{E}^*(\pi_1)$.

In the following, we show that total effort in a contest with asymmetric first stage is higher than or equal to total effort in a contest with symmetric first stage, i.e. $e_{\text{asym}} \geq e_{\text{sym}}$.

We have that

$$\begin{aligned}
e_{\text{asym}} &= y_w^* + y_l^* + x_1^* + x_2^* \\
&= \frac{c_l}{(c_l + c_w)^2} + \frac{c_w}{(c_l + c_w)^2} + \frac{1}{(1+a)^2} \frac{1-c}{1+c} + \frac{a}{(1+a)^2} \frac{1-c}{1+c} \\
&= \frac{1}{c_l + c_w} + \frac{1}{1+a} \frac{1-c}{1+c}
\end{aligned}$$

and

$$\begin{aligned}
e_{\text{sym}} &= y_w^* + y_l^* + 2\bar{x} \\
&= \frac{c_l}{(c_l + c_w)^2} + \frac{c_w}{(c_l + c_w)^2} + 2 \frac{1}{4} \frac{1-c}{1+c} \\
&= \frac{1}{c_l + c_w} + \frac{1}{2} \frac{1-c}{1+c}.
\end{aligned}$$

We have, then,

$$\begin{aligned}
e_{\text{asym}} \geq e_{\text{sym}} &\implies \frac{1}{c_l + c_w} + \frac{1}{1+a} \frac{1-c}{1+c} \geq \frac{1}{c_l + c_w} + \frac{1}{2} \frac{1-c}{1+c} \\
&\implies \frac{1}{1+a} \geq \frac{1}{2} \\
&= a \leq 1,
\end{aligned}$$

which is true. Thus, $e_{\text{asym}} \geq e_{\text{sym}}$.

Proof of Proposition 2.5. For $\frac{1}{2} < a \leq 1$, we have

$$\begin{aligned}
\mathbb{E}^L(\pi_1) - \mathbb{E}^F(\pi_2) &= \frac{1 + (4a-1)c^2}{4a(1+c)^2} - \frac{(2a-1)^2 + (4a-1)c^2}{4a^2(1+c)^2} \\
&= \frac{[a - (2a-1)^2] + [a(4a-1) - (4a-1)]c^2}{4a^2(1+c)^2} \\
&= \frac{(4a^2 - 5a + 1)(c^2 - 1)}{4a^2(1+c)^2} \\
&= \frac{4(a-1)\left(a - \frac{1}{4}\right)(c-1)(c+1)}{4a^2(1+c)^2} \geq 0, \quad \forall a \in \left(\frac{1}{2}, 1\right].
\end{aligned}$$

For $0 < a \leq \frac{1}{2}$, we have

$$\begin{aligned}
\mathbb{E}^L(\pi_1) - \mathbb{E}^F(\pi_2) &= \frac{(1-a) + ac^2}{(1+c)^2} - \frac{c^2}{(1+c)^2} \\
&= \frac{(1-a) - (1-a)c^2}{(1+c)^2} \\
&= \frac{(1-a)(1-c^2)}{(1+c)^2} \\
&= \frac{(1-a)(1-c)}{(1+c)} > 0, \quad \forall a \in \left(0, \frac{1}{2}\right].
\end{aligned}$$

Thus, $\mathbb{E}^L(\pi_1) \geq \mathbb{E}^F(\pi_2)$ for $0 < a \leq 1$.

Proof of Proposition 2.7. For $0 < a \leq 1$, we have

$$\begin{aligned}
\mathbb{E}^L(\pi_2) - \mathbb{E}^F(\pi_1) &= \frac{a + (4-a)c^2}{4(1+c)^2} - \frac{(2-a)^2 + (4-a)ac^2}{4(1+c)^2} \\
&= \frac{(a^2 - 5a + 4)c^2 - (a^2 - 5a + 4)}{4(1+c)^2} \\
&= \frac{(a-4)(a-1)(c-1)(c+1)}{4(1+c)^2} \leq 0, \quad \forall a \in (0, 1].
\end{aligned}$$

Thus, $\mathbb{E}^L(\pi_2) \leq \mathbb{E}^F(\pi_1)$ for $0 < a \leq 1$.

Proof of Proposition 2.8. *i)* We find the level of total effort when at the first stage, players move simultaneously, e_{sim} , and when player 1 moves first, e_{12} .

$$\begin{aligned}
e_{\text{sim}} &= y_w^* + y_l^* + x_1^* + x_2^* \\
&= \frac{c_l}{(c_l + c_w)^2} + \frac{c_w}{(c_l + c_w)^2} + \frac{1}{(1+a)^2} \frac{1-c}{1+c} + \frac{a}{(1+a)^2} \frac{1-c}{1+c} \\
&= \frac{1}{c_l + c_w} + \frac{1}{1+a} \frac{1-c}{1+c}.
\end{aligned}$$

For $\frac{1}{2} < a \leq 1$,

$$\begin{aligned}
e_{12} &= y_w^* + y_l^* + x_1^L + x_2^F \\
&= \frac{c_l}{(c_l + c_w)^2} + \frac{c_w}{(c_l + c_w)^2} + \frac{1}{4a^2} \frac{1-c}{1+c} + \frac{2a-1}{4a^2} \frac{1-c}{1+c} \\
&= \frac{1}{c_l + c_w} + \frac{1}{2a} \frac{1-c}{1+c}.
\end{aligned}$$

Calculating the difference between e_{12} and e_{sim} , we obtain

$$\begin{aligned}
e_{12} - e_{\text{sim}} &= \frac{1}{c_l + c_w} + \frac{1}{2a} \frac{1-c}{1+c} - \frac{1}{c_l + c_w} - \frac{1}{1+a} \frac{1-c}{1+c} \\
&= \left(\frac{1}{2a} - \frac{1}{1+a} \right) \frac{1-c}{1+c} \\
&= \frac{1-a}{2a(1+a)} \frac{1-c}{1+c} \geq 0, \quad \forall a \in \left(\frac{1}{2}, 1 \right].
\end{aligned}$$

For $0 < a \leq \frac{1}{2}$,

$$\begin{aligned}
e_{12} &= y_w^* + y_l^* + x_1^L + x_2^F \\
&= \frac{c_l}{(c_l + c_w)^2} + \frac{c_w}{(c_l + c_w)^2} + \frac{1-c}{1+c} + 0 \\
&= \frac{1}{c_l + c_w} + \frac{1-c}{1+c}.
\end{aligned}$$

The difference between e_{12} and e_{sim} is

$$\begin{aligned}
e_{12} - e_{\text{sim}} &= \frac{1}{c_l + c_w} + \frac{1-c}{1+c} - \frac{1}{c_l + c_w} - \frac{1}{1+a} \frac{1-c}{1+c} \\
&= \frac{(1+a)(1-c) - (1-c)}{(1+a)(1+c)} \\
&= \frac{a(1-c)}{(1+a)(1+c)} > 0, \quad \forall a \in \left(0, \frac{1}{2} \right].
\end{aligned}$$

Thus, $e_{12} \geq e_{\text{sim}}$ for $0 < a \leq 1$.

ii) Now we find how much total effort is exerted when player 2 moves first at the first stage.

$$\begin{aligned}
e_{21} &= y_w^* + y_l^* + x_2^L + x_1^F \\
&= \frac{c_l}{(c_l + c_w)^2} + \frac{c_w}{(c_l + c_w)^2} + \frac{a}{4} \frac{1-c}{1+c} + \frac{2-a}{4} \frac{1-c}{1+c} \\
&= \frac{1}{c_l + c_w} + \frac{1}{2} \frac{1-c}{1+c}.
\end{aligned}$$

Subtracting e_{sim} from e_{21} yields

$$\begin{aligned}
e_{21} - e_{\text{sim}} &= \frac{1}{c_l + c_w} + \frac{1}{2} \frac{1-c}{1+c} - \frac{1}{c_l + c_w} - \frac{1}{1+a} \frac{1-c}{1+c} \\
&= \frac{a-1}{2(1+a)} \frac{1-c}{1+c} \leq 0, \quad \forall a \in (0, 1].
\end{aligned}$$

Thus, $e_{21} \leq e_{\text{sim}}$ for $0 < a \leq 1$.

Proof of Proposition 2.9. First we show that $\mathbb{E}^*(\pi_1) \leq \mathbb{E}^L(\pi_1) \leq \mathbb{E}^F(\pi_1)$, $\forall a \in (0, 1]$.

For $\frac{1}{2} < a \leq 1$, we have

$$\begin{aligned}
\mathbb{E}^L(\pi_1) - \mathbb{E}^F(\pi_1) &= \frac{1 + (4a-1)c^2}{4a(1+c)^2} - \frac{(2-a)^2 + (4-a)ac^2}{4(1+c)^2} \\
&= \frac{(c^2-1)(a^3 - 4a^2 + 4a - 1)}{4a(1+c)^2} \\
&= \frac{(c-1)(c+1)[a(a-2)^2 - 1]}{4a(1+c)^2} \leq 0, \quad \forall a \in \left(\frac{1}{2}, 1\right]
\end{aligned}$$

and

$$\begin{aligned}
\mathbb{E}^L(\pi_1) - \mathbb{E}^*(\pi_1) &= \frac{1 + (4a - 1)c^2}{4a(1+c)^2} - \frac{1}{(1+a)^2} \frac{1 + a(2+a)c^2}{(1+c)^2} \\
&= \frac{(1+a)^2 + (1+a)^2(4a-1)c^2 - 4a - 4a^2(2+a)c^2}{4a(1+a)^2(1+c)^2} \\
&= \frac{(1-c^2)(1-2a+a^2)}{4a(1+c)^2} \\
&= \frac{(1-c)(1+c)(1-a)^2}{4a(1+c)^2} \geq 0, \quad \forall a \in \left(\frac{1}{2}, 1\right].
\end{aligned}$$

For $0 < a \leq \frac{1}{2}$, we have

$$\begin{aligned}
\mathbb{E}^L(\pi_1) - \mathbb{E}^F(\pi_1) &= \frac{(1-a) + ac^2}{(1+c)^2} - \frac{(2-a)^2 + (4-a)ac^2}{4(1+c)^2} \\
&= -\frac{a(1-c^2)}{4(1+c)^2} \\
&= -\frac{a(1-c)(1+c)}{4(1+c)^2} < 0, \quad \forall a \in \left(0, \frac{1}{2}\right]
\end{aligned}$$

and

$$\begin{aligned}
\mathbb{E}^L(\pi_1) - \mathbb{E}^*(\pi_1) &= \frac{(1-a) + ac^2}{(1+c)^2} - \frac{1}{(1+a)^2} \frac{1 + a(2+a)c^2}{(1+c)^2} \\
&= \frac{a - a^2 - a^3 - ac^2 + a^2c^2 + a^3c^2}{(1+a)^2(1+c)^2} \\
&= \frac{a[c^2(a^2 + a - 1) - (a^2 + a - 1)]}{(1+a)^2(1+c)^2} \\
&= \frac{a(c^2 - 1)(a^2 + a - 1)}{(1+a)^2(1+c)^2} \\
&= -\frac{a(1-c)(1+c)(a^2 + a - 1)}{(1+a)^2(1+c)^2} > 0, \quad \forall a \in \left(0, \frac{1}{2}\right]
\end{aligned}$$

since $a^2 + a - 1 < 0$, $\forall a \in (0, \frac{1}{2}]$. Thus, $\mathbb{E}^*(\pi_1) \leq \mathbb{E}^L(\pi_1) \leq \mathbb{E}^F(\pi_1)$ for $0 < a < 1$.

Now, we show that player 2's dominant strategy is to become a leader.

$$\begin{aligned}
\mathbb{E}^L(\pi_2) - \mathbb{E}^*(\pi_2) &= \frac{a + (4-a)c^2}{4(1+c)^2} - \frac{1}{(1+a)^2} \frac{a^2 + (1+2a)c^2}{(1+c)^2} \\
&= \frac{a - 2a^2 + a^3 - ac^2 + 2a^2c^2 - a^3c^2}{4(1+a)^2(1+c)^2} \\
&= \frac{a[(1-2a+a^2) - c^2(1-2a+a^2)]}{4(1+a)^2(1+c)^2} \\
&= \frac{a(1-c)(a-1)^2}{4(1+a)^2(1+c)} \geq 0, \quad \forall a \in (0, 1].
\end{aligned}$$

Also, for $\frac{1}{2} < a \leq 1$, we have

$$\begin{aligned}
\mathbb{E}^F(\pi_2) - \mathbb{E}^*(\pi_2) &= \frac{(2a-1)^2 + (4a-1)c^2}{4a^2(1+c)^2} - \frac{1}{(1+a)^2} \frac{a^2 + (1+2a)c^2}{(1+c)^2} \\
&= \frac{1 - 3a^2 - 2a + 4a^3 - c^2 + 2ac^2 + 3a^2c^2 - 4a^3c^2}{4a^2(1+a)^2(1+c)^2} \\
&= \frac{(1-c^2)(1-3a^2-2a+4a^3)}{4a^2(1+a)^2(1+c)^2} \\
&= -\frac{(1-c)(1+c)(1-a)[4(a+1)-1]}{4(1+a)^2(1+c)^2} \leq 0, \quad \forall a \in \left(\frac{1}{2}, 1\right]
\end{aligned}$$

and for $0 < a \leq \frac{1}{2}$, we have

$$\begin{aligned}
\mathbb{E}^F(\pi_2) - \mathbb{E}^*(\pi_2) &= \frac{c^2}{(1+c)^2} - \frac{1}{(1+a)^2} \frac{a^2 + (1+2a)c^2}{(1+c)^2} \\
&= \frac{-2ac^2 - 3a^2c^2 - a^2}{(1+a)^2(1+c)^2} \\
&= -\frac{a[c^2(2+3a) + a]}{(1+a)^2(1+c)^2} < 0, \quad \forall a \in \left(0, \frac{1}{2}\right].
\end{aligned}$$

Thus, $\mathbb{E}^F(\pi_2) \leq \mathbb{E}^*(\pi_2) \leq \mathbb{E}^L(\pi_2)$ for $0 < a \leq 1$.

Other proofs. We prove that player 2's expected payoff is always higher when she chooses effort $x_2^L < \frac{1-c}{1+c}$. Let $\mathbb{E}_1^L(\pi_2)$ be player 2's expected payoff when she chooses effort $x_2^L < \frac{1-c}{1+c}$, and $\mathbb{E}_2^L(\pi_2)$ his expected payoff when she chooses effort $x_2^L \geq \frac{1-c}{1+c}$. Then, we have

$$\begin{aligned}
\mathbb{E}_1^L(\pi_2) - \mathbb{E}_2^L(\pi_2) &= \frac{a + (4-a)c^2}{4(1+c)^2} - \frac{(a-1) + c^2}{a(1+c)^2} \\
&= \frac{a^2 + (4-a)ac^2 - 4(a-1) - 4c^2}{4a(1+c)^2} \\
&= \frac{(a^2 - 4a + 4)(1-c^2)}{4a(1+c)^2} \\
&= \frac{(a-2)^2(1-c)}{4a(1+c)} > 0, \quad \forall a \in (0, 1],
\end{aligned}$$

Thus, $\mathbb{E}_1^L(\pi_2) > \mathbb{E}_2^L(\pi_2)$, for $0 < a \leq 1$.

Appendix C

Comparative statics of equilibrium contributions

Differentiating x_i^* with respect to $c_{I,i}$, we obtain

$$\begin{aligned} \frac{\partial x_i^*}{\partial c_{I,i}} &= - \frac{c_{I,j} [c_{J,i}^2 c_{I,j} (3c_{I,i} + c_{I,j}) - c_{I,i} c_{J,i} c_{J,j} (c_{I,i} - 3c_{I,j}) + 2c_{I,i} c_{I,j} c_{J,j}^2] [c_{I,i} c_{J,j} + c_{I,j} (c_{J,i} + 2c_{J,j})]}{2 (c_{I,i} + c_{I,j})^5 (c_{J,i} + c_{J,j})^3} \\ &= - \frac{c_{J,i}^3 c_{I,j}^4 + 2c_{J,i}^2 c_{I,j}^4 c_{J,j} + 3c_{I,i} c_{J,i}^3 c_{I,j}^3 + 4c_{I,i} c_{I,j}^3 c_{J,j}^3 + 8c_{I,i} c_{J,i} c_{I,j}^3 c_{J,j}^2 + 10c_{I,i} c_{J,i}^2 c_{I,j}^3 c_{J,j} \\ &\quad + 2c_{I,i}^2 c_{I,j}^2 c_{J,j}^3 + c_{I,i}^2 c_{J,i} c_{I,j}^2 c_{J,j}^2 + 2c_{I,i}^2 c_{J,i}^2 c_{I,j}^2 c_{J,j} - c_{I,i}^3 c_{J,i} c_{I,j} c_{J,j}^2}{2 (c_{I,i} + c_{I,j})^5 (c_{J,i} + c_{J,j})^3} \\ &= - \frac{c_{J,i}^3 c_{I,j}^4 + 2c_{J,i}^2 c_{I,j}^4 c_{J,j} + 3c_{I,i} c_{J,i}^3 c_{I,j}^3 + 4c_{I,i} c_{I,j}^3 c_{J,j}^3 + 8c_{I,i} c_{J,i} c_{I,j}^3 c_{J,j}^2 + 10c_{I,i} c_{J,i}^2 c_{I,j}^3 c_{J,j} \\ &\quad + 2c_{I,i}^2 c_{I,j}^2 c_{J,j}^3 + c_{I,i}^2 c_{J,i} c_{I,j}^2 c_{J,j}^2 + (c_{I,i}^2 c_{I,j} c_{J,i} c_{J,j}) (2c_{I,j} c_{J,i} - c_{I,i} c_{J,j})}{2 (c_{I,i} + c_{I,j})^5 (c_{J,i} + c_{J,j})^3} < 0, \end{aligned}$$

since $c_{I,j} c_{J,i} > c_{I,i} c_{J,j}$ and, therefore, $2c_{I,j} c_{J,i} > c_{I,i} c_{J,j}$.

Differentiating x_i^* with respect to $c_{I,j}$, we have

$$\begin{aligned}
& c_{J,j} [c_{I,i}^2 c_{J,j} (3c_{J,i} + c_{J,j}) - c_{I,i} c_{J,i} c_{I,j} (c_{J,i} - 3c_{J,j}) + \\
\frac{\partial x_i^*}{\partial c_{I,j}} &= - \frac{2c_{J,i} c_{I,j}^2 c_{J,j} [c_{I,i} c_{J,j} + c_{I,j} (c_{J,i} + 2c_{J,j})]}{2(c_{I,i} + c_{I,j})^3 (c_{J,i} + c_{J,j})^5} \\
& c_{I,i}^4 c_{J,i} c_{J,j}^2 - 2c_{I,i}^3 c_{J,i}^2 c_{I,j} c_{J,j} - 2c_{I,i}^3 c_{I,j} c_{J,j}^3 - c_{I,i}^3 c_{J,i} c_{I,j} c_{J,j}^2 \\
& - 3c_{I,i}^2 c_{J,i}^3 c_{I,j}^2 - 4c_{I,i}^2 c_{I,j}^2 c_{J,j}^3 - 8c_{I,i}^2 c_{J,i} c_{I,j}^2 c_{J,j}^2 \\
& - 10c_{I,i}^2 c_{J,i}^2 c_{I,j}^2 c_{J,j} - c_{I,i} c_{J,i}^3 c_{I,j}^3 - 2c_{I,i} c_{J,i}^2 c_{I,j}^3 c_{J,j} \\
& = - \frac{2(c_{I,i} + c_{I,j})^3 (c_{J,i} + c_{J,j})^5}{2(c_{I,i} + c_{I,j})^3 (c_{J,i} + c_{J,j})^5} \\
& c_{I,i}^3 c_{J,i} c_{J,j} (c_{I,i} c_{J,j} - 2c_{J,i} c_{I,j}) - 2c_{I,i}^3 c_{I,j} c_{J,j}^3 - c_{I,i}^3 c_{J,i} c_{I,j} c_{J,j}^2 \\
& - 3c_{I,i}^2 c_{J,i}^3 c_{I,j}^2 - 4c_{I,i}^2 c_{I,j}^2 c_{J,j}^3 - 8c_{I,i}^2 c_{J,i} c_{I,j}^2 c_{J,j}^2 \\
& - 10c_{I,i}^2 c_{J,i}^2 c_{I,j}^2 c_{J,j} - c_{I,i} c_{J,i}^3 c_{I,j}^3 - 2c_{I,i} c_{J,i}^2 c_{I,j}^3 c_{J,j} \\
& = - \frac{2(c_{I,i} + c_{I,j})^3 (c_{J,i} + c_{J,j})^5}{2(c_{I,i} + c_{I,j})^3 (c_{J,i} + c_{J,j})^5} > 0,
\end{aligned}$$

since $c_{I,i} c_{J,j} < c_{I,j} c_{J,i}$ and, therefore, $c_{I,i} c_{J,j} < 2c_{I,j} c_{J,i}$.

Differentiating x_i^* with respect to $c_{J,j}$, we have

$$\begin{aligned}
& c_{J,i} [c_{I,i}^2 c_{J,j} (3c_{J,i} + c_{J,j}) - c_{I,i} c_{J,i} c_{I,j} (c_{J,i} - 3c_{J,j}) \\
\frac{\partial x_i^*}{\partial c_{J,j}} &= - \frac{+ 2c_{J,i} c_{I,j}^2 c_{J,j} [c_{I,i} c_{J,j} + c_{I,j} (c_{J,i} + 2c_{J,j})]}{2(c_{I,i} + c_{I,j})^3 (c_{J,i} + c_{J,j})^5}
\end{aligned}$$

Some manipulation of the above expression yields that

$$\frac{\partial x_i^*}{\partial c_{J,j}} < (>) 0 \quad \text{if } c_{J,j} > (<) \frac{c_{J,i} \left[-3c_{I,i} c_{I,j} + (c_{I,i} + c_{I,j}) \sqrt{4c_{I,i} c_{I,j} + 9c_{I,i}^2 + 4c_{I,j}^2} - 3c_{I,i}^2 - 2c_{I,j}^2 \right]}{2c_{I,i}^2}.$$

Differentiating x_i^* with respect to $c_{J,i}$, we have

$$\frac{\partial x_i^*}{\partial c_{J,i}} = \frac{c_{J,j} [c_{I,i}^2 c_{J,j} (3c_{J,i} + c_{J,j}) - c_{I,i} c_{J,i} c_{I,j} (c_{J,i} - 3c_{J,j}) + 2c_{J,i} c_{I,j}^2 c_{J,j}] [c_{I,i} c_{J,j} + c_{I,j} (c_{J,i} + 2c_{J,j})]}{2 (c_{I,i} + c_{I,j})^3 (c_{J,i} + c_{J,j})^5}$$

which yields that

$$\frac{\partial x_i^*}{\partial c_{J,i}} < (>) 0 \quad \text{if } c_{J,j} < (>) \frac{c_{J,i} \left[-3c_{I,i} c_{I,j} + (c_{I,i} + c_{I,j}) \sqrt{4c_{I,i} c_{I,j} + 9c_{I,i}^2 + 4c_{I,j}^2} - 3c_{I,i}^2 - 2c_{I,j}^2 \right]}{2c_{I,i}^2}.$$

Differentiation of expected revenue

Differentiating eq. (3.10) with respect to $c_{J,j}$, we have

$$\frac{\partial r_I}{\partial c_{J,j}} = \frac{c_{I,i} c_{J,i} c_{I,j}}{(c_{I,i} + c_{I,j})^2 (c_{J,i} + c_{J,j})^2} > 0, \quad \forall c_{I,i}, c_{I,j}, c_{J,j}, c_{J,i}$$

while differentiating with respect to $c_{J,i}$, we get

$$\frac{\partial r_I}{\partial c_{J,i}} = -\frac{c_{I,i} c_{I,j} c_{J,j}}{(c_{I,i} + c_{I,j})^2 (c_{J,i} + c_{J,j})^2} < 0, \quad \forall c_{I,i}, c_{I,j}, c_{J,j}, c_{J,i}.$$

Now, if we differentiate eq. (3.10) with respect to $c_{I,i}$, we obtain

$$\frac{\partial r_I}{\partial c_{I,i}} = \frac{c_{I,j} [-c_{I,i}^2 c_{J,j} - 2c_{I,i} c_{I,j} (c_{J,i} + c_{J,j}) + c_{I,j}^2 (c_{J,i} + 2c_{J,j})]}{(c_{I,i} + c_{I,j})^4 (c_{J,i} + c_{J,j})}.$$

Manipulation of the above equation yields

$$\frac{\partial r_I}{\partial c_{I,i}} \begin{cases} > 0 & \text{if } \left(c_{J,j} \leq \frac{3c_{J,i}}{2} \text{ and } c_{I,j} > \frac{c_{I,i}(\sqrt{3c_{J,i}c_{J,j}+c_{J,i}^2+3c_{J,j}^2+c_{J,i}+c_{J,j}})}{c_{J,i}+2c_{J,j}} \right) \text{ or } c_{J,j} > \frac{3c_{J,i}}{2} \\ < 0 & \text{if } c_{J,j} < \frac{3c_{J,i}}{2} \text{ and } c_{I,j} < \frac{c_{I,i}(\sqrt{3c_{J,i}c_{J,j}+c_{J,i}^2+3c_{J,j}^2+c_{J,i}+c_{J,j}})}{c_{J,i}+2c_{J,j}}. \end{cases}$$

Similarly, differentiating eq. (3.10) with respect to $c_{I,j}$, we have

$$\frac{\partial r_I}{\partial c_{I,j}} = \frac{c_{I,i} [c_{I,i}^2 c_{J,j} + 2c_{I,i} c_{I,j} (c_{J,i} + c_{J,j}) - c_{I,j}^2 (c_{J,i} + 2c_{J,j})]}{(c_{I,i} + c_{I,j})^4 (c_{J,i} + c_{J,j})}$$

and

$$\frac{\partial r_I}{\partial c_{I,j}} \begin{cases} > 0 & \text{if } c_{J,j} < \frac{3c_{J,i}}{2} \text{ and } c_{I,j} < \frac{c_{I,i}(\sqrt{3c_{J,i}c_{J,j}+c_{J,i}^2+3c_{J,j}^2+c_{J,i}+c_{J,j}})}{c_{J,i}+2c_{J,j}} \\ < 0 & \text{if } \left(c_{J,j} \leq \frac{3c_{J,i}}{2} \text{ and } c_{I,j} > \frac{c_{I,i}(\sqrt{3c_{J,i}c_{J,j}+c_{J,i}^2+3c_{J,j}^2+c_{J,i}+c_{J,j}})}{c_{J,i}+2c_{J,j}} \right) \text{ or } c_{J,j} > \frac{3c_{J,i}}{2}. \end{cases}$$