RELATIVE PRODUCTIVITY AND SEARCH UNEMPLOYMENT IN AN OPEN ECONOMY*

Luisito BERTINELLI† Olivier CARDI‡
University of Luxembourg CREA Lancaster University Management School
Romain RESTOUT§
Université de Lorraine BETA (CNRS UMR 7522)

Abstract

Using a panel of eighteen OECD countries, we find empirically that the long-run effects of higher productivity of tradables relative to non-tradables vary across time, space and stages of the business cycle. More specifically, our evidence reveals that elasticities of the relative wage and relative price of non-tradables with respect to relative productivity of tradables increase over time. Our estimates also show that the fall in the relative wage is more pronounced whilst the appreciation in the relative price is less in countries where labor markets are more regulated and during periods of recession. To rationalize the evidence, we differentiate between labor mobility and hiring costs by developing a two-sector open economy model with search in the labor market and an endogenous sectoral labor force participation decision. While time-declining labor mobility costs can account for the time-increasing effects of a productivity differential, international differences in labor market regulation and variations of hiring costs across the business cycle, respectively, can rationalize the cross-country and state-dependent effects we estimate empirically. Finally, labor market frictions have important implications for sectoral unemployment since labor mobility and hiring costs bias labor demand toward the traded sector which results in a greater decline in unemployment in tradables relative to unemployment in non-tradables following higher relative productivity.

Keywords: Relative productivity of tradables; Search theory; Labor mobility; Labor market institutions; State-dependency; Sectoral price and wage differences; Sectoral unemployment dynamics.

JEL Classification: E24; F16; F32; F41; J64.

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†Correspondence address: University of Luxembourg, CREA, Faculty of Law, Economics and Finance. 162 A, avenue de la Faïencerie, L-1511 Luxembourg. Phone: +352 46 66 44 6620. Fax: +352 46 66 44 6633. E-mail: luisito.bertinelli@uni.lu.

‡Corresponding author: Olivier Cardi. Correspondence address: Lancaster University Management School, Bailrigg, Lancaster LA1 4YX. E-mail: o.cardi@lancaster.ac.uk.

§Corresponding address: Université de Lorraine, Université de Strasbourg, CNRS, BETA, 54000, Nancy, France. Phone: +33 03 54 50 43 72. Fax: +33 03 54 50 43 51. E-mail: romain.restout@univ-lorraine.fr.
1 Introduction

According to the Balassa [1964] and Samuelson [1964] (BS henceforth) effect, higher productivity in tradables relative to non-tradables puts upward pressure on the relative price of non-tradables and appreciates the real exchange rate. Despite the fact that the link between the relative price and relative productivity finds some strong support in the data, estimates at an individual level documented by Canzoneri et al. [1999], Kakkar [2003] and Chong et al. [2012] reveal that this relationship varies greatly across OECD countries. This link also varies across time as estimates by Bergin et al. [2006] indicate that the BS effect has gradually strengthened over time. In this paper, we disentangle labor mobility costs across sectors from hiring costs and show that this distinction is crucial when it comes to explaining the variations of the relative price effects of a productivity differential across time, space and stages of the business cycle.

Our paper contributes to a growing literature which has recently put forward labor market frictions to rationalize the estimated effect of higher relative productivity of tradables on relative prices. To account for the link between sectoral productivity and relative prices as implied by the BS model, Berka et al. [2018] consider shocks to the labor wedge which fuel inflation of tradables. Beyond the fact that Berka et al. [2018] highlight the terms of trade channel while we focus on movements in the relative price of non-tradables, the major difference with our approach is that the previous authors treat shocks to the labor wedge (resulting from unexplained labor market frictions) and shocks to sectoral TFPs separately. We model instead labor mobility costs and allow for search frictions so that hiring costs are endogenously determined by both labor market policies and the state of the economy in the business cycle; such labor market frictions determine the magnitude of the appreciation in the relative price of non-tradables following higher relative productivity.

In this regard, our work is complementary to Cardi and Restout’s [2015] analysis which reveals that labor mobility costs tend to curb inflation of non-tradables. However, by abstracting from search frictions in the labor market, the authors cannot disentangle workers’ mobility costs from hiring costs and thus neither can account for the cross-country dispersion in the relative price effects of higher relative productivity of tradables nor the time-varying effects. Our key contribution is to show that time-declining labor mobility costs can account for the time-increasing effects of a productivity differential we document empirically, while international differences in labor market regulation (LMR henceforth) and variations of hiring costs along the business cycle can rationalize estimated cross-country and state-dependent effects, respectively.¹

¹By abstracting from search frictions, Cardi and Restout [2015] cannot model the effects of labor market institutions or the state of the economy in the business cycle on hiring costs and thus cannot rationalize the cross-country (see Online Appendix F) and/or state-dependent effects. Because time-varying effects can be caused by labor mobility costs and LMR, and since the latter is absent from their analysis and thus cannot be controlled for, Cardi and Restout’s [2015] setup is inappropriate to rationalize the variations over time of the effects of higher relative productivity.
By using a panel of eighteen OECD countries, our estimates reveal that an increase in the relative productivity of tradables lowers significantly non-traded relative to traded wages which is consistent with the presence of labor mobility costs. When estimating elasticities of the relative wage and relative price of non-tradables with respect to relative productivity in rolling sub-samples, we find that the former has increased over time from -0.32 to -0.15, while the appreciation in the relative price appears to be more pronounced. Concomitantly, the magnitude of labor reallocation across sectors following higher relative productivity has almost doubled over the same period which suggests that time-increasing estimated elasticities are driven by time-declining labor mobility costs.

Hiring costs which emerge naturally in an environment with search frictions vary with LMR and across stages of the business cycle. Using a set of indicators to capture the extent of LMR, the decline in the relative wage is found empirically to be more pronounced and the appreciation in the relative price to be less in countries where the unemployment benefit scheme is more generous or the worker bargaining power (measured by the bargaining coverage) is larger. While the relative wage also falls more in countries where legal protection against dismissals is stricter, we find empirically that the relative price appreciates by a larger amount. Furthermore, when we differentiate the effects of a productivity differential according to the state of the economy in the business cycle, our estimates reveal that the decline in the relative wage is more pronounced while the relative price appreciates less during periods of recession.

While matching frictions cause search unemployment, labor mobility costs lead sectoral unemployment to adjust at different rates across sectors. Our estimates show that an increase in the relative productivity of tradables lowers the unemployment rate of tradables more than that of non-tradables and this decline turns out to be less pronounced over time. By affecting hiring costs, search frictions matter as well as we find that the fall in the unemployment differential between tradables and non-tradables is amplified in countries where LMR is higher or during recessions.

In order to account for our evidence, we put forward a variant of a two-sector open economy model with tradables and non-tradables and search in the labor market along with an endogenous labor force participation decision in the lines of Shi and Wen [1999]. Like Alvarez and Shimer [2011], workers cannot switch sectors without going through a spell of search unemployment which gives rise to labor mobility costs. Since the elasticity of labor supply at the extensive margin measures the extent of job search costs, it determines the degree of labor mobility across sectors. Labor mobility costs resulting from an endogenous sectoral labor force participation decision are pivotal to our work since standard search

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\(^2\) We consider an endogenous sectoral labor force participation decision by assuming that representative household members experience disutility from working and searching efforts in each sector. Relocating hours worked from one sector to another is costly as the representative household must incur a searching cost for a job in this sector. In contrast to Matsuyama [1992] who assumes the irreversibility of the career decision, workers can move between sectors, at some cost though.
frictions are not sufficient on their own to account for the decline in the relative wage we estimate empirically. Conversely, hiring costs resulting from search frictions determine the magnitude of the relative wage decline which varies with labor market institutions and across stages of the business cycle.

One key feature of our open economy model with search frictions is its dynamic nature. When workers experience mobility costs, higher relative productivity of tradables leads traded firms to post more job vacancies than non-traded firms in order to encourage workers to shift toward the traded sector. Because search frictions make hiring costly and labor mobility costs amplify recruitment expenditure, higher hirings give rise to a current account deficit along the transitional path. As the country must fulfill the intertemporal solvency condition, net exports must increase in the long-run. Higher demand for tradables mitigates the appreciation in the relative price of non-tradables caused by the increase in traded relative to non-traded output. The rise in net exports also biases labor demand toward the traded sector which drives down non-traded relative to traded wages and generates a greater decline in the unemployment rate of tradables than that of non-tradables, in line with the evidence. The dynamic nature of our setup resulting from search frictions plays a pivotal role since keeping net exports fixed prevents the model from matching the evidence when traded and non-traded goods are complements in consumption. With an elasticity of substitution between traded and non-traded goods smaller than one (as our estimates suggest), higher relative productivity of tradables increases the share of non-tradables. Because labor demand is biased toward the non-traded sector, both the relative wage of non-tradables and the unemployment differential between tradables and non-tradables increase instead of declining.

When we calibrate our model to a representative OECD economy and allow traded and non-traded goods to be complements, our quantitative analysis reveals that the long-run increase in net exports driven by the accelerated hiring process more than offsets the rise in the share of non-tradables. Higher demand for tradables lowers both the relative wage of non-tradables and the unemployment differential between tradables and non-tradables while the appreciation in the relative price is mitigated in line with our estimates. If we shut down search frictions, hiring costs vanish so that net exports remain fixed, thus preventing the model to account for the evidence.

When we control for the variations of LMR over time, we find that time-declining labor mobility costs alone can account for the time-increasing effects of higher relative productivity we document empirically. Intuitively, lower labor mobility costs mitigate the rise in hiring costs resulting from search frictions so that demand for goods and labor turns out to be less biased toward tradables because net exports increase less.

While labor mobility costs create an asymmetry across sectors, search frictions play a crucial role by mitigating or amplifying this asymmetry in sector adjustment. More specif-
ically, search frictions give rise to hiring costs which vary with LMR and across stages of the business cycle. In an economy where unemployment benefits are more generous or the worker bargaining power is higher or during recessions, demand for goods and labor is further biased toward tradables which amplifies the decline in the relative wage of non-tradables and mitigates the relative price appreciation, in line with our evidence. Intuitively, an economy with higher LMR or in recession has more unemployed workers and fewer job vacancies. Because a low labor market tightness makes hiring more profitable, recruiting expenditure increases more following higher relative productivity, thus amplifying the current account deficit and thus the long-run increase in net exports. Our quantitative results also show that the relative price of non-tradables appreciates more while the relative wage declines by a larger amount in countries with stringent employment protection legislation (EPL henceforth) in accordance with our empirical findings. Like Hopenhayn and Rogerson [1993] and Veracierto [2008], the strictness of legal protection against dismissals is modelled as a tax on reducing employment. While higher productivity causes a fall in labor supply due to the positive wealth effect, traded employment increases and non-traded establishments are shrinking since productivity gains are concentrated in the traded sector. Non-traded firms are thus subject to the firing tax which further biases labor demand toward the traded sector. The greater increase in traded relative to non-traded output results in a greater appreciation in the relative price.

To further assess the role of search frictions, we calibrate the model to country-specific data and investigate the implications of labor market institutions for the cross-country dispersion in estimated effects. While the model generates a wide dispersion in the relative wage and the relative price responses across countries, we find quantitatively that it can account for the larger decline in the relative wage and the smaller appreciation in the relative price in countries where labor market regulation is higher.\(^3\) Our cross-country analysis also reveals that a productivity differential of one percent results in a decline in the relative unemployment rate of tradables which appears to be insignificant in countries having more flexible labor markets but ranging between twofold and fourfold of that obtained for a representative OECD country in economies with higher LMR.

The remainder of the paper is organized as follows. In section 2, we document evidence on the long-run effects of higher relative productivity of tradables and contrast these effects across time, space and stages of the business cycle. In section 3, we develop an open economy version of the two-sector model with both imperfect mobility of labor arising from searching efforts and unemployment arising from matching frictions in both sectors. Section 4 derives analytical results to guide our discussion on the role of labor mobility costs and LMR. In section 5, we conduct a quantitative analysis to assess the ability of our model to

\(^3\)When using a measure of LMR which encompasses the three dimensions of labor market institutions, we find that the relative price significantly appreciates less in countries where labor markets are more regulated.
account for the variations of the effects across time, space and stages of the business cycle. Section 6 summarizes our main results and concludes. The Online Appendix provides a description of the dataset along with additional empirical results, and shows robustness checks.\(^4\)

**Related literature.** Our paper is at the cross-roads of three strands of the literature investigating the adjustment of open economies to structural shocks. First, it is closely related to the BS theory which has been renewed by Bergin et al. [2006], Ghironi and Melitz [2005], and Christopoulos et al. [2012]. Whilst the latter paper puts forward financial frictions as an explanation of the cross-country dispersion in the BS effect, the former two papers show that heterogenous productivity among firms and/or entry and exit of firms amplifies the BS effect. Recently, Cardi and Restout [2015] and Berka et al. [2018] have put forward labor market frictions to account for the BS effect found in the data. However, the two aforementioned works abstract from search frictions and thus cannot disentangle labor mobility from hiring costs which prevent the aforementioned works to account for the cross-country and state-dependent effects we document empirically.

Our paper also adds to a fast growing literature which contrasts empirically and theoretically the response of output and unemployment to fiscal or tax shocks across stages of the business cycle, see e.g., Auerbach and Gorodnichenko [2012], Michaillat [2014]. By producing an asymmetry in the size of hiring costs across stages of the business cycle, our model with search frictions allows us to rationalize the state-dependent effects we estimate.

Third, our work is also related to the literature employing a multi-sector model with search frictions in the labor market and emphasizing the key role of the costs of sectoral reallocation in shaping the response of the economy to sector-specific shocks. As in Lilien [1982], labor mobility costs tend to increase search unemployment before labor fully adjusts following asymmetric shocks across sectors. In contrast to Lilien [1982], reduced search for a job caused by the positive wealth effect lowers unemployment in both sectors since we allow for the transition between leisure and labor force. Like Kehoe et al. [2018], we find that the response of sectoral labor is influenced by the elasticity of substitution between traded and non-traded goods together with the cost of sectoral reallocation. In the same vein as Kambourov [2009] and Cosar [2013], we investigate the quantitative implications of labor market policies when workers experience barriers to labor mobility. Beyond the fact that the authors focus on trade shocks, a key dimension of our setup which is absent from that of Kambourov [2009] or Cosar [2013] who assume that trade is balanced, is the dynamics of the net foreign asset position which brings about a change in the composition of the demand of goods and allows our model to generate productivity effects in line with our empirical findings.

\(^4\) A Technical Appendix available upon request from the authors contains all the proofs, derivations of analytical results, and extensions of the baseline model.
2 Empirical Facts

In this section, we explore empirically the effects of higher productivity in tradables relative to non-tradables across time, space and stages of the business cycle. We focus on relative price as well as relative wage effects because the movement in the relative wage reveals the presence of labor market frictions. Since unemployment emerges naturally in an economy with search frictions, we also investigate the effect on the unemployment differential between tradables and non-tradables. We denote the level of the variable in upper case, the logarithm in lower case (except for the unemployment rate which is expressed in percentage point), and the percentage deviation from its initial steady-state by a hat.

2.1 Developing Intuitions about Labor Market Frictions’ Implications

To set the stage for the empirical analysis, we build up intuition about how the theory developed by Balassa [1964] and Samuelson [1964] (BS hereafter) is modified when relaxing the assumption of perfectly competitive labor markets. Like BS we consider an open economy where the terms of trade are fixed and further assume that traded and non-traded goods are produced by using labor only.\(^5\) The introduction of labor market frictions implies that traded relative to non-traded output is no longer perfectly elastic to the relative price of non-tradables which turns out to be affected by demand shifts. While in section 4 we identify two transmission channels through which higher relative productivity tilts demand toward traded or non-traded goods, we restrict below our attention to one channel for clarity purposes. Importantly, this channel depends on the size of labor market frictions which vary along two dimensions, say labor mobility and hiring costs.

**Labor Mobility Costs.** As shall be clear in section 3, workers’ costs of switching sectors are the result of job search costs. Like Alvarez and Shimer [2011], workers experience mobility costs as they have to search for a job before being employed in the other sector. Because searching for a job is time-consuming, such an activity is costly in utility terms. Such utility loss may capture sector-specific human capital, see e.g., Lee and Wolpin [2006], Dix-Carneiro [2014], Kambourov [2009], Ritter [2014], and/or geographical mobility costs, see e.g., Kennan and Walker [2011].\(^6\) Labor mobility costs lead traded firms to post more job vacancies (than non-traded firms) with the purpose to encourage workers to shift their hours worked toward the traded sector. Since the hiring process is costly and labor mobility costs amplify recruitment expenditure, a productivity differential produces a current account deficit.\(^7\) For the intertemporal solvency condition to hold, net exports must increase in the

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\(^5\) In Online Appendix A, we lay out a simple model with search frictions which provides a formal background of the discussion in subsection 2.1.

\(^6\) It is worth mentioning that Artuç et al. [2010], Caliendo et al. [2019] explore the labor reallocation effects between traded/manufacturing and non-traded/service sectors following trade shocks and report large labor mobility costs across sectors. The authors obtain a closed-form structural equation that relates gross flows of workers across sectors to intersectoral wage differentials like in Horvath [2000] who abstract from search frictions.

\(^7\) In a model with search frictions, labor becomes an asset which can be accumulated. Labor accumulation
long-run. Because a greater demand for tradables biases labor demand toward the traded sector, higher relative productivity of tradables lowers the relative wage of non-tradables. Increased demand for tradables also mitigates the appreciation in the relative price of non-tradables caused by higher traded relative to non-traded output. Since prices of non-traded goods are not high enough to even lower relative productivity gains out, traded firms hire more than non-traded firms which results in a larger decline in unemployment in tradables relative to that in non-tradables.

**Hiring Costs.** Hiring costs matter as well in determining the responses of the economy to higher relative productivity of tradables as they vary according to labor market institutions and across stages of the business cycle. Intuitively, in a country with higher LMR or in an economy in recession, there are more unemployed workers and less job vacancies. Thus the labor market tightness is low which makes hiring more profitable since it is easier to fulfill job vacancies. Because the elasticity of hiring is higher, recruiting expenditure increases more following a productivity differential, thus resulting in a greater current account deficit and thus in a larger increase in net exports. Higher demand for tradables causes the relative wage to fall more and the relative price of non-tradables to appreciate less which biases the decline in unemployment toward tradables.

### 2.2 Data Construction

Before empirically exploring the effects of higher relative productivity, we briefly describe the dataset we use and provide details about data construction below as well as in Online Appendix B. Our sample consists of a panel of eighteen OECD countries for eleven 1-digit ISIC-rev.3 industries. To split these eleven industries into traded and non-traded sectors, we follow the classification suggested by De Gregorio et al. [1994] that we updated by following Jensen and Kletzer [2006].

For the relative price and the relative wage, our sample covers the period 1970-2007. We use the EU KLEMS [2011] database which provides domestic currency series of value added in current and constant prices, labor compensation and employment (number of hours worked) for each sector \( j \) (with \( j = T, N \)), permitting the construction of price indices \( p^j \) (in log) which correspond to sectoral value added deflators, sectoral wage rates \( w^j \) (in log), and sectoral measures of productivities \( a^j \) (in log). The relative price of non-tradables at time \( t \) in country \( i \), \( p_{i,t} \), is the log of the ratio of the non-traded value added deflator to the traded value added deflator (i.e., \( p_{i,t} = p_{i,t}^N - p_{i,t}^T \)). The relative wage \( \omega_{i,t} \) is the log of the ratio of the non-traded wage to the traded wage (i.e., \( \omega_{i,t} = w_{i,t}^N - w_{i,t}^T \)). We use sectoral labor productivities \( A^j_{i,t} = Y^j_{i,t}/L^j_{i,t} \) to approximate technical change which are constructed from constant-price series of value added \( Y^j_{i,t} \) and hours worked \( L^j_{i,t} \).

We construct time series for sectoral unemployment rate, \( u^j \), as the ratio of the number leads to recruitment expenditure which produces a current deficit, just like in a model with capital investment or firm entry.
of unemployed workers $U^j$ in sector $j$ to the labor force $F^j \equiv L^j + U^j$ in this sector. Unemployed persons in industry $j$ are those who lost their job in industry $j$. Data was extracted from LABORSTA database (ILO) which provides series for unemployed workers by economic activity for fourteen OECD countries out of eighteen listed in Online Appendix B.\(^8\) The longest available period ranges from 1987 to 2007. On average, our data covers thirteen years per country (see Online Appendix C.3).\(^9\) Then we subtract $u^N$ from $u^T$ to construct the unemployment differential between tradables and non-tradables, i.e., $u^T - u^N$.

### 2.3 Effects of a Productivity Differential across Time

A way to gauge the role of labor mobility costs in determining the adjustment of the economy to a productivity differential is to investigate whether the effects of a change in relative productivity vary over time and explore their relationship with time-varying labor reallocation across sectors caused by higher relative productivity.

**Empirical strategy.** To perform this experiment, we run the regression of the relative wage, $\omega$, the relative price, $p$, and the unemployment differential, $u^T - u^N$, on relative productivity in rolling sub-samples:

$$x_{i,t} = \delta_i + \alpha \cdot \text{productivity differential}_{i,t} + \epsilon_{i,t}, \quad (1)$$

where $x = \omega, p, u^T - u^N$, $\alpha = \beta, \gamma, \sigma$, subscripts $i$ and $t$ denote the country and the year, $\epsilon_{i,t}$ is an i.i.d. error term and country fixed effects are captured by country dummies $\delta_i$. Since $p, \omega$ and the productivity differential (i.e., $a^T - a^N$), display trends, we ran unit root and then cointegration tests. Having verified that these two assumptions are empirically supported, we estimate long-run elasticities for the relative wage, $\beta$, and the relative price, $\gamma$, by using the fully modified OLS (FMOLS) estimator for cointegrated panel proposed by Pedroni \[2000\], \[2001\].\(^{10}\) Since the time horizon is too short to recourse to cointegration techniques for the unemployment rate differential, we explore empirically (1) in variation and estimate the effect of a productivity growth differential on the change in the unemployment differential, $\sigma$, by using a panel fixed effects regression.

Following Wacziarg and Wallack \[2004\], we compute the labor reallocation index in year $t$ for country $i$ denoted by $LR_{i,t}$ by calculating the rate of workers who have shifted from one sector to another over $\tau$ years:

$$LR_{i,t}(\tau) = \frac{\sum_{j=1}^{N} |L_{i,t}^j - L_{i,t-\tau}^j| - \frac{1}{2} \sum_{j=1}^{N} L_{i,t}^j - \frac{1}{2} \sum_{j=1}^{N} L_{i,t-\tau}^j}{0.5 \sum_{j=1}^{N} (L_{i,t}^j + L_{i,t-\tau}^j)}.$$  \(2\)

\(^8\)It is worth mentioning that we started this paper a few years ago and in the meanwhile, the dataset provided by ILO which gives unemployment by economic activity has been removed from the web site and no longer exists.

\(^9\)Whereas we are able to construct time series of sectoral unemployment rates for Korea, data for the unemployment benefit replacement rate, used as a control variable, are not available before 2002 and thus this country is removed from the sample.

\(^{10}\)We alternatively estimate eq. (1) by using the dynamic OLS (DOLS) estimator. Results are almost identical and can be found in Online Appendix E.
where \( \tau = 5 \) and \( L_{j,t}^i \) denotes employment in sector \( j = T, N \). To estimate the effect of higher relative productivity on labor reallocation, we run regression (1) in rolling sub-samples where \( x_{it} = LR_{i,t} \).\(^{11}\)

**Labor mobility costs.** Before turning to time-varying effects, we start with the long-run responses for the whole sample. As shown in column 1 of Table 1, a 1% increase in the relative productivity of tradables lowers \( \omega \) by 0.22%, which reveals the presence of labor mobility costs. Such labor mobility costs curb non-tradable inflation since \( p \) appreciates by 0.64% only (see panel B), i.e., less than the productivity differential of 1%. Furthermore, column 1 of Table 2 reveals that \( u^T \) falls more than \( u^N \).

**Time-varying elasticities and labor reallocation.** Whilst we estimate \( \beta, \gamma, \) and \( \sigma \) in rolling sub-samples, to check results’ robustness, we consider different window lengths.\(^{12}\) As can be seen in the first row of Fig. 1 which reports the elasticity of the relative wage to relative productivity (i.e., \( \beta \)) in the solid black line, the response of \( \omega \) has increased over time (i.e., \( \beta \) becomes less negative). The increase in the response of \( \omega \) over time, especially in the nineties, is associated with more labor reallocation following higher relative productivity, as shown in the dotted black line. The increase in worker mobility across sectors over the nineties echoes the evidence documented by Kambourov and Manovskii [2009] on U.S. data.

The second row of Fig. 1 reveals that as more workers shift from one sector to another, \( p \) appreciates more over time (until the beginning of 2000’s), i.e., \( \gamma \) takes higher values. Focusing on panels 1(a) and 1(c), the magnitude of labor reallocation reaches a peak at the beginning of 2000’s and then tends to be declining. Such a pattern tracks pretty well the fall in \( \gamma \) from 2002 onwards and to a lesser extent the merely declining path of \( \beta \) which starts later, in 2005. Another piece of evidence which corroborates the role of labor reallocation in shaping the labor market adjustment across time is the increase in \( \sigma \) which captures the response of the unemployment differential to a rise in relative productivity, as can be seen in panel 1(e) of Fig. 1.\(^{13}\)

Whilst Fig. 1 reveals that the effects of a productivity differential are increasing over time and are concomitant to time-increasing labor reallocation, larger shifts of labor across sectors can be caused by lower mobility costs or changes in LMR or both.

**LMR across Time.** As can be seen in Fig. 2 which plots three dimensions of LMR over time, their evolution has opposite effects on labor reallocation. On the one hand, the rise in the unemployment benefit replacement rate shown in the dotted blue line together

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\(^{11}\)We are interested in the long-run effects of higher relative productivity on labor reallocation and thus consider \( \tau = 5 \) like Wacziarg and Wallack [2004]. Since the labor reallocation index is stationary, relative productivity is expressed in growth rate.

\(^{12}\)When estimating \( \beta \) and \( \gamma \), we run the same regression as in eq. (1), except that we consider overlapping subperiods of different fixed lengths, i.e., \( T = 20 \) and \( T = 25 \). More specifically, for \( T = 20 \), we estimate eqs. (1) over 1970-1990, 1971-1991, ...,1987-2007, and for \( T = 25 \), over 1970-1995, ..., 1982-2007.

\(^{13}\)When running the regression of the unemployment differential on relative productivity of tradables in growth rate, we add unemployment benefit replacement as a control; due to data availability, we consider one unique window length (i.e., \( T = 12 \)) and exclude BEL, DNK, JPN, USA as the time horizon for sectoral unemployment data taken from ILO is too short for these countries.
with the fall in EPL shown in the dashed red line increases labor reallocation. On the other hand, the collective bargaining coverage shown in the solid black line reaches a peak at the beginning of the eighties and then declines from 72% to 62% which exerts a negative impact on labor reallocation.\textsuperscript{14}

Because the effects on labor reallocation caused by the movements in the LMR indicators somewhat cancel out, changes in LMR cannot be responsible for the sharp increase in labor reallocation which doubles over the nineties. Thus time-increasing effects of higher relative productivity can only be the result of time declining labor mobility costs. As we shall see in subsection 5.3, when we calibrate the model to the data and let labor mobility costs along with the three dimensions of LMR vary across time, numerical results reveal that time-declining labor mobility costs alone can account for time-varying effects of higher relative productivity of tradables.

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\subsection*{2.4 Effects of a Productivity Differential across Countries}

While overall LMR does not vary much across time as its components vary in opposite direction, labor market institutions vary considerably across countries.\textsuperscript{15} In the following, we put forward international differences in LMR to account for the cross-country dispersion in the elasticity of the relative wage, the relative price and the unemployment differential w.r.t. relative productivity.

\textbf{Dimensions of LMR.} We consider three dimensions of LMR. The first aspect is the difficulty of redundancy that we capture through the EPL index provided by the OECD; this index which captures the strictness of legal protection against dismissals for permanent workers has the advantage to be available for all countries of our sample over the period 1985-2007. In order to have a more accurate measure of the difficulty of redundancy, we adjust EPL for regular workers with the share of permanent workers in the economy (see Boeri and Van Ours [2008]). The indicator is denoted by $EPL_{adj}$. The generosity of unemployment benefit systems is measured by using the replacement rate, denoted by $\varrho$. The data we use are taken from the Benefits and Wages database provided by the OECD which calculates the average of the net unemployment benefit for three durations of unemployment (1st year, 2nd and 3rd year, 4th and 5th year). In the empirical literature, the worker bargaining power is commonly captured by the bargaining coverage; we thus use this indicator, denoted by $BargCov$, which gives the proportion of employees covered

\begin{footnotesize} 
\textsuperscript{14}The effects of the three dimensions of LMR on labor reallocation are discussed in section 4. A more generous unemployment benefit scheme and/or a higher bargaining coverage lead to greater labor reallocation by increasing the marginal benefit of job search. On the contrary, as emphasized by Kambourov [2009] a stricter EPL lowers labor reallocation by reducing hiring and thus the marginal benefit of search.

\textsuperscript{15}In Online Appendix B, we plot estimated responses of $\omega$ to a productivity differential against LMR indicators. While the relative wage elasticity displays a wide cross-country dispersion, estimates indicate that $\omega$ falls more in countries where LMR is higher.
\end{footnotesize}
by collective bargaining. Data are taken from the ICTWSS database (Visser [2009]).

Implications of more generous unemployment benefits or higher worker bargaining power. Whilst LMR further biases labor demand toward the traded sector, labor market institutions influence goods and labor market variables through two distinct channels according to the type of LMR. As mentioned in section 2.1, in countries where unemployment benefits are more generous or the worker bargaining power is higher, labor demand in the traded sector is more elastic to productivity gains. In Online Appendix E.7, we provide evidence which supports the transmission channel emphasized in section 2.1. More specifically, we find that countries where the unemployment benefit scheme is more generous or the collective bargaining coverage is higher experience a greater increase in the balance of trade in the long-run following a rise in the relative productivity of tradables. As a result, the traded wage is expected to increase by a larger amount and the unemployment rate of tradables to decline more. In addition, the greater increase in the demand for tradables further mitigates the appreciation in the relative price of non-tradables caused by the productivity differential.

Implications of stricter protection against dismissals. In countries with higher firing costs, we expect the non-traded wage to rise less, the unemployment rate of non-tradables to decrease by a smaller amount and the relative price to appreciate more. The intuition is as follows. Because higher productivity lowers aggregate labor supply through the positive wealth effect while the non-traded sector experiences relatively low productivity gains, the shrinking non-traded establishments are subject to the redundancy cost. As a result, they are less prone to recruit more workers when productivity increases. Labor demand in the non-traded sector is thus less elastic to productivity gains in countries where EPL is more pronounced, which mitigates increases in $w^N$ and the decline in $u^N$. Since traded relative to non-traded output increases more, $p$ appreciates by a larger amount.

Empirical strategy. To empirically explore the implications of LMR for the effects of a productivity differential, we perform a simple split-sample analysis. Hence, for each sub-sample, we run the following regression:

$$x_{i,t}^k = \delta_i^k + \alpha^k \cdot \text{productivity differential}_{i,t}^k + \epsilon_{i,t}^k,$$  \quad k = H, L, \tag{3}

where $x = \omega, p, u^T - u^N$ and the superscript $k = H, L$ means 'High' or 'Low' LMR.\footnote{Because the movements in $p$ can be influenced by changes in the cost of entry in product market triggered by competition-oriented policies, we add country-specific linear time trends when we run the regression (3) for each sub-sample in order to control for these effects.}

\footnote{Online Appendix E.5 provides the values for all LMR indicators. For $\omega$ and $p$, we base the split-sample analysis on the median of the sample for the three dimensions of LMR. In Online Appendix E.5, we show that whether we use the median or the mean sample, our split-sample analysis is robust to the threshold used when we explore the implications of three dimensions of LMR. Due to the small effect of higher relative productivity on the unemployment differential, we base the split-sample analysis on the sample mean instead since it produces the traditional distinction between English-speaking and Continental European economies. More specifically, using the sample mean, IRL, AUS, GBR, JPN, CAN, USA are classified in the group of countries with low LMR while the rest of the countries, AUT, SWE, DNK, FIN, BEL, ESP, DEU, ITA, are classified in the group of countries with high LMR.}
For each sub-sample, we estimate the elasticity $\alpha$ for the relative wage (labelled $\beta^k$), the relative price (labelled $\gamma^k$), and the unemployment differential (labelled $\sigma^k$).\textsuperscript{18} Building on the above discussion, we expect $\beta^H$ and $\sigma^H$, which captures the response of $\omega$ and $u^T - u^N$ to a productivity differential in countries with higher LMR, to be larger (in absolute terms) than $\beta^L$ and $\sigma^L$. Whilst LMR biases labor demand toward the traded sector, regardless of the type of labor market institutions, the three dimensions of LMR must be distinguished for the response of $p$; $p$ should appreciate more in countries with stricter legal protection against dismissals (i.e., $\gamma^H > \gamma^L$) and is expected to increase less in countries with more generous unemployment benefit scheme or a higher bargaining coverage (i.e., $\gamma^H < \gamma^L$).

Relative wage elasticity and LMR. The FMOLS estimates are reported in columns 2-5 of Table 1 for countries with high and low LMR. As the results in panel A of Table 1 show, the decline in $\omega$ is significantly greater for countries with more regulated labor markets, i.e., $|\beta^H| > |\beta^L|$. While countries providing lower unemployment benefits experience a decline in $\omega$ of -0.16% approximately, the second set of countries with generous unemployment benefits experience a fall in $\omega$ of -0.26% (see column 2). Furthermore, as shown in column 3 of Table 1, $\omega$ falls by -0.24% in countries where the worker bargaining power is relatively higher instead of -0.18% in economies with a lower bargaining coverage. A similar pattern emerges when we exploit a third dimension of LMR, namely the strictness of employment protection (see column 4). Since series for EPL are available over 1985-2007, we run again the regression (3) for each sub-sample over this period to be consistent. We find that $\omega$ declines by 0.17% in countries with higher firing costs while $\omega$ declines by only 0.13% in the second set of countries. Because LMR includes three indicators, we have recourse to a principal component analysis in order to have one overall indicator reflecting all the dimensions of labor market institutions. As displayed by column 5 of Table 1, we find that countries with more regulated labor markets experience a larger decline in $\omega$. Finally, we detect a significant difference in the responses of $\omega$ between countries with low and high LMR as shown in the third line of Table 1 which indicates that imposing the restriction $\beta^L = \beta^H$ is strongly rejected at a 1% significance level.

Relative price elasticity and LMR. Turning to the relative price, columns 2 and 3 of panel B in Table 1 show that higher relative productivity of tradables causes an appreciation in $p$ which is significantly smaller in countries with more generous unemployment benefits or a higher bargaining coverage. Conversely, as displayed by column 4 of Table 1, stricter EPL tends to amplify the increase in $p$, in line with our conjecture. However, the difference in the relative price responses caused by EPL between the two sub-samples is not statistically significant. Because $EPL_{adj}$ does not seem to exert substantial effects on $\gamma$, it is thus not surprising to find that the overall LMR index tends to mitigate the appreciation in $p$ as shown in the column 5 of Table 1. As discussed later, this finding is in line with

\textsuperscript{18}To estimate $\sigma$, we consider eq. (3) in variation.
our quantitative results which show that large differences in EPL do not cause marked differences in $\gamma$, the cross-country dispersion in the response of $p$ being mostly driven by differences in unemployment benefit replacement rates.

**Unemployment differential adjustment and LMR.** To explore the implications of LMR for the response of the unemployment rate differential, we split our sample into groups with less and more regulated labor markets by using the mean value of the index which encompasses the three dimensions of LMR.\(^{19}\) Our analysis covers 14 countries out of which 8 are classified as countries with more regulated labor markets. Contrasting estimates of $\sigma^H$ with those of $\sigma^L$ shown in column 3 of Table 2 reveals that a rise in the relative productivity of tradables drives down $u^T$ relative to $u^N$, and more so in countries where LMR is higher. More specifically, the unemployment differential declines by 0.033 and 0.036 ppt in economies with low and high LMR, respectively. Column 4 shows that estimated effects between the two subsamples are more distinct when controlling for the replacement rate and EPL, $u^T - u^N$ declining by 0.032 ppt in countries with low LMR and by 0.041 ppt with high LMR.

### 2.5 Effects of a Productivity Differential across Stages of Business Cycle

While hiring costs vary according to LMR, hiring costs also vary across stages of the business cycle. Since the elasticity of hiring is higher in recessions as a result of a low labor market tightness, we expect $\omega$ and the unemployment differential between tradables and non-tradables to fall more following higher relative productivity and $p$ to appreciate less.

**Empirical Strategy.** In order to contrast the effects of higher relative productivity of tradables in recessions with those during expansions, we have to identify the state of the economy in the business cycle. Following standard practice, we define a recession period as a situation where the output gap declines, i.e., the economy is moving from its peak to trough, and an expansion period as a situation where the output gap increases, i.e., the economy is moving from its trough to peak.\(^{20}\) Expansions (recessions) are periods where the output gap $dy_{it} - d\bar{y}_{it}$ is positive ($dy_{it} - d\bar{y}_{it}$ is negative), with $\bar{y}$ the potential GDP in log. A recession lasts 3.8 years and an expansion 4 years on average. In order to insure that the differences in the effects of a productivity differential are pronounced enough across stages of the business cycle, we consider expansions and recessions which last at least 3 years.\(^{21}\) We alternatively identify periods of expansion and recession by using the unemployment gap, $u_{it} - \bar{u}_{it}$ with $u$ and $\bar{u}$ the actual and natural unemployment rate, respectively. To investigate whether the response of the economy to a productivity

\(^{19}\)Because the effect of an increase in $A^T/A^N$ on the unemployment differential is small since the latter variable is the difference between two sectoral ratios, we find it convenient to base the split-sample analysis on the mean value instead of the median as we obtain more clear-cut results in this case.

\(^{20}\)To compute the output gap, we logged real GDP $Y_{it}$ and estimate its trend, $\bar{y}_{it}$, by applying a Hodrick-Prescott filter with a smoothing parameter of $\lambda = 100$ (as we use annual data).

\(^{21}\)In Online Appendix E.6, we consider all recessions/expansions or alternatively recessions/expansions which last at least 2 years. Our results are robust to business cycle duration.
differential varies across stages of the business cycle, we perform a split-sample analysis and run regression (3) when economies are in recession and next when economies are in expansion. Since unemployment is relatively Low (High) when the economy is in expansion (recession), the estimated elasticity is denoted by the superscript $L$ ($H$).

**Empirical Results.** The FMOLS estimates of $\beta^k$ and $\gamma^k$ ($k = H, L$) are reported in the last two columns of Table 1. In accordance with our hypothesis, a 1% increase in the relative productivity of tradables lowers $\omega$ by 0.29% in recessions and 0.22% only in expansions. Conversely, $p$ appreciates less in recessions than in expansions, i.e., by 0.58% vs. 0.64%, respectively. Turning to the sectoral unemployment effects displayed by the last two columns of Table 2, a productivity differential further lowers $u^T$ relative to $u^N$ when the economy is in recession, $\sigma^H$ being statistically significant when we use the unemployment gap to identify the state of the economy in the business cycle.

### 3 The Framework

The country is small in terms of both world goods and capital markets, and faces a given world interest rate, $r^{*}$. The small open economy is populated by a constant number of identical households and firms that have perfect foresight and live forever. Households decide on labor market participation and consumption while firms decide on hirings. The economy consists of two sectors. One sector produces a traded good denoted by the superscript $T$ that can be exported while the other sector produces a non-traded good denoted by the superscript $N$. Both goods are used for consumption. The traded good is chosen as the numeraire. The labor market, in the tradition of Diamond-Mortensen-Pissarides, consists of a matching process within each sector between the firms who post job vacancies and unemployed workers who search for a job. Time is continuous and indexed by $t$.

#### 3.1 Households

At each instant the representative agent consumes traded goods, $C^T(t)$, and non-traded goods, $C^N(t)$, which are aggregated by a constant elasticity of substitution function:

$$C(t) = \left[ \phi^{\frac{1}{\phi}} (C^T(t))^{\frac{\phi-1}{\phi}} + (1 - \phi)^{\frac{1}{\phi}} (C^N(t))^{\frac{\phi-1}{\phi}} \right]^{\frac{\phi}{\phi-1}},$$

where $0 < \phi < 1$ is the weight of the traded good in the overall consumption bundle and $\phi > 0$ is the intratemporal elasticity of substitution.

The economy that we consider consists of a representative household with a measure one continuum of identical infinitely lived members. At any instant, members in the household

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22The price of the traded good is determined on the world market and exogenously given for the small open economy. Hence, real exchange rate movements are exclusively caused by the long-run adjustment in the relative price of non-tradables. Evidence documented by Burstein et al. [2006] reveals that half of all cyclical real exchange rate variation is accounted for by the relative price of non-traded to traded goods.

23Our paper builds on Heijdra and Ligthart [2009]. Unlike the authors, we consider a two-sector framework where the sectoral elasticity of labor supply at the extensive margin determines the transition across sector labor force and explore the implications of LMR.
derive utility from consumption goods $C(t)$ and experience disutility from working and searching efforts. More precisely, the representative household comprises members who engage in only one of the following activities: working and searching a job in each sector, or enjoying leisure. Assuming that the representative individual is endowed with one unit of time, leisure is defined as $1 - F^T(t) - F^N(t)$, with $F^j(t)$ the labor force in sector $j = T, N$ defined as the sum of units of labor time, and time spent on searching for a job in sector $j$, $U^j(t)$, i.e., $F^j(t) = L^j(t) + U^j(t)$. For later use, we denote by $u^j$ the sectoral unemployment rate defined as $u^j(t) = U^j(t)/F^j(t)$. Unemployed agents are randomly matched with job vacancies according to a matching function described later. Since the timing of a match is random, agents face idiosyncratic risks. To simplify the analysis, we assume that members in the household perfectly insure each other against variations in labor income, see e.g., Merz [1995] and Andolfatto [1996].

The representative household chooses the time path of consumption and labor force to maximize the following objective function:

$$\Upsilon = \int_0^\infty \left\{ \frac{1}{1-\frac{1}{\sigma_C}} C(t)^{1-\frac{1}{\sigma_C}} - \frac{1}{1 + \frac{1}{\sigma_L}} F(t)^{1+\frac{1}{\sigma_L}} \right\} e^{-\beta t} dt,$$

where $\beta > 0$ is the consumer’s subjective time discount rate, $\sigma_C > 0$ the intertemporal elasticity of substitution for consumption, $F(t)$ the aggregate labor force, and $\sigma_L$ the elasticity of labor supply at the extensive margin.

Because the labor force is not constant, we allow for the transition between employment and unemployment, and the transition between leisure and labor force. Since the labor force in sector $j$ is not constant either, we allow for the transition between the traded and the non-traded sector. As in Alvarez and Shimer [2011], a worker in one sector cannot switch to the other sector without going through a spell of search unemployment which generates a labor mobility cost. Because $\sigma_L$ is assumed to be symmetric across sectors and determines the extent of the utility loss from searching a job in sector $j$, the degree of labor mobility increases when $\sigma_L$ takes higher values. The elasticity of labor supply at the extensive margin thus collapses to the elasticity of substitution between $F^T$ and $F^N$ as captured by a CES aggregator:

$$F(t) = \left[ \zeta^T F^T(t)^{\frac{1+\sigma_L}{\sigma_L}} + \zeta^N F^N(t)^{\frac{1+\sigma_L}{\sigma_L}} \right]^{\frac{\sigma_L}{1+\sigma_L}},$$

where $\zeta^j > 0$ parametrizes the disutility from working and searching efforts in sector $j = T, N$. When $\sigma_L = 0$, labor immobility emerges as a special case since workers’ costs of switching sectors are prohibitive. Letting $\sigma_L$ tend towards infinity and setting $\zeta^T = \zeta^N = 1$, eq. (6) collapses to $F(t) = F^T(t) + F^N(t)$ which implies that labor force is perfectly substitutable across sectors. When $\sigma_L$ takes intermediate values (i.e., $0 < \sigma_L < \infty$), traded and non-traded labor force are no longer perfect substitutes. As $\sigma_L$ takes lower values, workers experience greater disutility when shifting. While an endogenous sectoral labor
force participation decision generates imperfect substitutability between sectoral labor force and
and echoes the modelling approach by Cardi and Restout [2015] to generate labor mobility costs, hiring costs emerge naturally in our model with search frictions and are distinct from workers’ switching costs. As we shall see in section 4 and 5, such a distinction is crucial when it comes to explaining time-varying, cross-country and state-dependent effects.\footnote{Instead of considering workers’ heterogeneity and sector-specific human capital like Kambourov [2009] and Cosar [2013], we generate imperfect mobility of labor by assuming that workers must search for a job before shifting from one sector to another. While this modelling strategy amounts to assuming that the worker regains sector-specific human capital and prevents us from investigating distributional issues, it allows us to derive analytical expressions and characterize sectoral unemployment dynamics by using phase diagrams.}

Denoting by $m^j(t)$ the rate at which unemployed agents find jobs and $s^j$ the exogenous rate of job separation, employment in sector $j$ evolves gradually according to:

$$\dot{L}^j(t) = m^j(t) U^j(t) - s^j L^j(t). \quad (7)$$

Households supply $L^j(t)$ units of labor services in sector $j = T, N$ for which they receive the product wage $W^j(t)$. We denote by $A(t)$ the stock of financial wealth held by households which comprises internationally traded bonds, $B(t)$, and shares on domestic firms. Because foreign bonds and domestic shares are perfect substitutes, the stock of financial wealth yields net interest rate earnings $\rho^i A(t)$. Denoting by $T(t)$ the lump-sum taxes, the flow budget constraint is equal to households’ real disposable income less consumption expenditure:

$$\dot{A}(t) = \rho^i A(t) + \sum_j W^j(t) L^j(t) + \sum_j R^j U^j(t) - T(t) - P_C (P(t)) C(t), \quad (8)$$

where $P_C$ is the consumption price index which is increasing in the relative price of non-tradables, $P$, and $R^j$ represents unemployment benefits received by job seekers in sector $j$.

Denoting by $\lambda(t)$ and $\xi^j(t)$ the shadow prices of wealth and finding a job in sector $j$, respectively, the key equations characterizing optimal household behavior are:\footnote{First-order conditions consist of (9a) and (9c) together with $\xi^j (F^j)^{1/\sigma_L} = m^j \xi^j + R^j \lambda$ and $\dot{\xi} = (s^j + \beta) \xi^j - [\lambda W^j - \xi^j (F^j)^{1/\sigma}]$. Denoting by $\xi^j \equiv \xi^j / \lambda$, using (9a) and (9c), we get (9b) and (9d).}

$$C(t) = (P_C(t) \lambda(t))^{-\sigma_C}, \quad (9a)$$

$$F^j(t) = \{ \lambda(t) \left[ m^j (\theta^j(t)) \xi^j(t) + R^j \right] / \xi^j \}^{\sigma_L}, \quad (9b)$$

$$\dot{\lambda}(t) = \lambda(t) (\beta - \rho^i), \quad (9c)$$

$$\dot{\xi}^j(t) = (s^j + \rho^i) \xi^j(t) - \left[ W^j(t) - \frac{\xi^j (F^j(t))^{1/\sigma_L}}{\lambda(t)} \right], \quad (9d)$$

and the appropriate transversality conditions. In order to generate an interior solution, we impose $\beta = \rho^i$; hence, (9c) implies that $\lambda$ must remain constant over time, i.e., $\lambda(t) = \bar{\lambda}$. Eq. (9b) shows that labor market participation increases with the reservation wage $W^j(t)$, which is defined as the sum of the expected value of a job, $m^j(t) \xi^j(t)$, and the unemployment benefit, $R^j$.\footnote{Instead of considering workers’ heterogeneity and sector-specific human capital like Kambourov [2009] and Cosar [2013], we generate imperfect mobility of labor by assuming that workers must search for a job before shifting from one sector to another. While this modelling strategy amounts to assuming that the worker regains sector-specific human capital and prevents us from investigating distributional issues, it allows us to derive analytical expressions and characterize sectoral unemployment dynamics by using phase diagrams.}
Intra-temporal allocation of consumption follows from the following optimal rule:

\[
\left( \frac{1 - \phi}{\phi} \right) \frac{C_T}{C_N} = P^\phi. \tag{10}
\]

An appreciation in the relative price of non-tradables \( P \) lowers expenditure on tradables relative to expenditure on non-tradables (i.e. \( C_T/P^N \)) when \( \phi < 1 \). Applying Shephard’s lemma and denoting by \( \alpha_C \) the share of non-traded goods in consumption expenditure yields expenditure in non-tradables and tradables, i.e., \( PC^N = \alpha_C PC \) and \( C^T = (1 - \alpha_C) PC \).

### 3.2 Firms

Each sector consists of a large number of identical firms which use labor, \( L_j \), as the sole input in a linear technology, \( Y_j = A_j L_j \). Firms post job vacancies \( V_j \) to hire workers and face a cost per job vacancy \( \kappa_j \) which is assumed to be constant. Like Kehoe et al. [2018], the cost per job vacancy is measured in terms of the traded good. In the quantitative analysis, we explore the robustness of our results to this assumption by alternatively considering that the cost per job vacancy is expressed in terms of the non-traded good. Firms pay the wage \( W^j \) decided by the generalized Nash bargaining solution. As producers face a labor cost \( W^j \) per employee and a cost per hiring of \( \kappa^j \), the profit function of the representative firm in sector \( j \) is:

\[
\pi^j(t) = \Xi^j(t)L^j(t) - W^j(t)L^j(t) - \kappa^j(t)V^j(t) - x^j \max \left\{ 0, -\dot{L}^j(t) \right\}, \tag{11}
\]

where \( \Xi^j \) is the marginal revenue product of labor (i.e., \( \Xi^T = A^T \) and \( \Xi^N = PA^N \)).

Following Hopenhayn and Rogerson [1993] who abstract from search frictions, Alvarez and Veracierto [2001], Heijdra and Ligthart [2002], Veracierto [2008], who consider search frictions, the strictness of legal protection against dismissals is captured by a tax on reducing employment denoted by \( x^j \). While firms must make a payment \(-x^j\dot{L}^j(t) > 0\) whenever they decrease their employment level, firms experience simultaneously outflow and inflow of workers. As we shall see below, because the decision of hiring (i.e., the decision to post job vacancies) and employment adjustment choices are distinct, a tax on reducing employment amounts to paying taxes upon job separation, \( x^j s^jL^j \), and receiving hiring subsidies, \( x^j f^jV^j \), at the same time, the former being larger than the latter amount. When we calibrate the firing tax to the data, we restrict attention to the transfer from the firm to the laid-off worker which includes advance notification and severance payments since according to the evidence documented by Garibaldi and Violante [2005], red-tape costs account for a small fraction of the firing tax, i.e., less than 20%.

Denoting by \( f^j \) the rate at which a vacancy is matched with unemployed agents, the

\[26\]As underlined by Garibaldi and Violante [2005] and Bentolila et al. [2012], EPL imposes a firing cost to the firm which has two separate components: a transfer from the firm to the worker to be laid off which includes the requirements to provide the worker with advance notification and severance payments, and red-tape costs which refer to a set of administrative procedures and legal expenditures.
law of motion for labor is given by:

$$\dot{L}^j(t) = f^j(t)V^j(t) - s^j L^j(t). \quad (12)$$

Denoting by $\gamma^j(t)$ the shadow price of employment to the firm, the maximization problem yields the following first-order conditions:

$$\begin{align*}
\gamma^j(t) + x^j \mathbb{I}_{L^j < 0} &= \frac{\kappa^j}{f^j(\theta^j(t))}, \quad (13a) \\
\gamma^j(t) &= \gamma^j(t) \left( r^* + s^j \right) - (\Xi^j(t) - s^j x^j \mathbb{I}_{L^j < 0} - W^j(t)). \quad (13b)
\end{align*}$$

While the firing tax is in effect when $\dot{L}^j(t) < 0$ (as captured by the indicator function), the net employment change is the result of total hirings and total separations; since $V^j$ is a control variable and $L^j$ is a state variable so that hiring decisions and employment adjustment choices are distinct mathematically, $x^j$ is split into a hiring subsidy in eq. (13a) and a tax upon job separation in eq. (13b). Eq. (13a) requires the marginal cost of vacancy, $\kappa^j$, to be equal to the expected marginal benefit of hiring inclusive of the hiring subsidy, $f^j \left( \gamma^j + x^j \mathbb{I}_{L^j < 0} \right)$. Solving (13b) forward and invoking the transversality condition yields:

$$\gamma^j(t) = \int_t^\infty [\Xi^j(\tau) - s^j x^j \mathbb{I}_{L^j < 0} - W^j(\tau)] e^{(r^* + s^j)(t-\tau)} d\tau, \quad (14)$$

where $r^* + s^j$ is the risk-of-job-destruction discount rate. Eq. (14) states that $\gamma^j(t)$ is equal to the present discounted value of the cash flow earned on an additional worker, consisting of the excess of marginal revenue of labor $\Xi^j(t)$ over the wage $W^j(t)$ and the expected firing cost $s^j x^j \mathbb{I}_{L^j < 0}$. Following higher productivity $A^j$, the marginal revenue of labor $\Xi^j(t)$ rises; hence hiring becomes more profitable which induces firms to post job vacancies, but less so in countries with a higher firing cost $x^j$, in line with the evidence documented by Adhvaryu et al. [2013]. Differentiating $\gamma^j(t)L^j(t)$ w.r.t. time, inserting (12) together with (13b), solving and invoking the transversality condition shows that the value of firm’s labor force is equal to the present value of its profit:

$$\gamma^j(t)L^j(t) = \int_t^\infty \pi^j(\tau) e^{-r^*(\tau-t)} d\tau. \quad (15)$$

### 3.3 Matching and Wage Determination

In each sector, there are job-seeking workers $U^j$ and firms with job vacancies $V^j$ which are matched in a random fashion. Assuming a constant returns to scale matching function, the number of labor contracts $M^j$ concluded per job seeker $U^j$ gives the job finding rate $m^j$ which is increasing in the labor market tightness $\theta^j$:

$$m^j(t) = M^j(t)/U^j(t) = X^j \left( V^j(t)/U^j(t) \right)^{\alpha^j_V} = X^j \left( \theta^j(t) \right)^{\alpha^j_V}, \quad (16)$$

where $\alpha^j_V$ represents the elasticity of vacancies in job matches and $X^j$ corresponds to the matching efficiency. The number of matches $M^j(t)$ per job vacancy gives the worker-finding rate for the firm, $f^j(t)$, which is decreasing in $\theta^j(t)$:

$$f^j(t) = M^j(t)/V^j(t) = X^j \left( \theta^j(t) \right)^{\alpha^j_V-1}. \quad (17)$$

18
When a vacancy and a job-seeking worker meet, a rent is created which is equal to 
\[ \xi(t) + \gamma^j(t) + x^j \cdot \mathbb{1}_{Lj<0}, \]
where \( \xi(t) \) is the value of an additional job, \( \gamma^j(t) \) is the value of an additional worker, and \( x^j \) corresponds to the hiring subsidy. The division of the rent between the worker and the firm determined by generalized Nash bargaining leads to the product wage \( W^j \) defined as a weighted sum of the marginal revenue product of labor plus the interest income from the hiring subsidy and the reservation wage:

\[ W^j(t) = \alpha_W^j \left( \Xi^j(t) + r^* x^j \cdot \mathbb{1}_{Lj<0} \right) + \left( 1 - \alpha_W^j \right) W_R^j(t), \]  

(18)

where \( \alpha_W^j \) corresponds to the bargaining power of the worker. Inserting the Nash bargaining solution, i.e., \( \alpha_W^j \left( \gamma^j(t) + x^j \cdot \mathbb{1}_{Lj<0} \right) = \left( 1 - \alpha_W^j \right) \xi^j(t) \), into \( W_R^j(t) = m(t) \xi^j(t) + R^j \) allows us to express the reservation wage in terms of the average hiring cost per job seeker \( k^j \theta^j(t) \), i.e., \( W_R^j(t) = \frac{\alpha_W^j}{1-\alpha_W^j} k^j \theta^j(t) + R^j \). When the firm fires a worker, it must pay to the State \( x^j s^j \) instantaneously while when it hires a new worker, the firm obtains from the State \( x^j \) which is equivalent to \( \int_t^\infty (r^* + s^j) x^j e^{(r^*+s^j)(t-\tau)} d\tau \). Combining the latter result with (14) leads to \( \gamma^j(t) + x^j \cdot \mathbb{1}_{Lj<0} = \int_t^\infty \left( \Xi^j(\tau) + r^* x^j \cdot \mathbb{1}_{Lj<0} - W^j(\tau) \right) e^{(r^*+s^j)(t-\tau)} d\tau \) which explains why \( r^* x^j \) shows up in the surplus from an additional hiring displayed by the first term on the RHS of eq. (18). Intuitively, the interest income from the hiring subsidy deposited at a bank is left available to the firm to pay the firing tax when the worker is laid-off. While the presence of the hiring subsidy slightly increases the surplus from an additional worker by \( r^* x^j \), this term is found quantitatively to be very small so that it has no impact on targeted ratios and thus the initial equilibrium is identical whether EPL is high or low. Conversely, the firing tax \( x^j \), which is in effect when \( \hat{L}^j(t) < 0 \), lowers \( \pi^j(t) \) by \( x^j \hat{L}^j(t) < 0 \) at each instant of time (see eq. (11)). The value of firm’s labor force (see eq. (15)) declines which mitigates the incentives to post job vacancies following a rise in \( A^j \), and all the more so in countries where the tax on reducing employment is higher.

3.4 Government

The final agent in the economy is the government. Unemployment benefits \( R^T U^T + R^N U^N \) are covered by lump-sum taxes \( T \) and the proceeds from the firing tax \( \sum_j x^j \cdot \max \left\{ 0, -\hat{L}^j \right\} \) according to the following balanced budget constraint:

\[ \sum_j x^j \cdot \max \left\{ 0, -\hat{L}^j \right\} + T = \sum_j R^j U^j. \]  

(19)

Like Veracierto [2008], the proceeds are rebated to households as lump sum transfers. As we shall see, because higher productivity generates a positive wealth effect which encourages agents to reduce time devoted to job search, unemployment benefits shown on the RHS of eq. (19) decline. The excess of the proceeds from the firing tax over unemployment benefits is paid to households as lump-sum transfers which square well with our assumption of considering the firing tax as a transfer from the firm to the laid-off worker.
3.5 Market Clearing Conditions

We have to impose the market clearing condition for the non-traded good:

\[ Y^N(t) = C^N(t). \]  

(20)

Using the definition of the stock of financial wealth \( A(t) \equiv B(t) + \sum_j \gamma_j(t)L^j(t) \), differentiating with respect to time, substituting the accumulation equations of labor (7) and financial wealth (8) together with the dynamic equation for the shadow value of an additional worker (13b), using (19) and (20), the current account is:

\[ \dot{B}(t) = r^*B(t) + Y^T(t) - C^T(t) - \kappa^TV^T(t) - \kappa^NV^N(t). \]  

(21)

As shall be clear later, the current account adjustment plays a pivotal role in driving our results. In this regard, it is worth mentioning that our assumption of hiring costs measured in traded good units does not affect our conclusions since a current account deficit aims at covering any excess of domestic absorption over domestic output, regardless of whether expenditure falls on traded or non-traded goods.

3.6 Steady-State

We now describe the steady-state of the economy. Due to the lack of empirical estimates at a sectoral level, we impose \( \alpha_jV_j = \alpha_V \) and \( \alpha_jW_j = \alpha_W \) from now on.

First, setting \( \dot{B} = 0 \) into (21), denoting by \( \upsilon_{NX} \equiv NX/Y^T \) the ratio of net exports to traded output, and using (20) yields the goods market equilibrium:\footnote{Denoting by \( \upsilon_B \equiv r^*/r^T \) the ratio of interest receipts to traded output and \( \upsilon_{V_t} \equiv \gamma^T/V^T \) the ratio of the cost of hiring in sector \( j = T, N \) to traded output, the zero current account equation implies \( \upsilon_B - \upsilon_{V_t} = -\upsilon_{NX} \). While for simplicity purposes, we refer to \( \upsilon_{NX} \) as the ratio of net exports to traded output, it also includes hiring expenditure, i.e., \( NX \equiv Y^T - C^T = NX + \kappa TV^T + \kappa N V^N \) with \( NX \equiv Y^T - C^T - \kappa TV^T - \kappa N V^N \) corresponding to the ‘true’ definition of the trade balance.}

\[ \frac{Y^T(1-\upsilon_{NX})}{Y^N} = \frac{\phi}{1-\phi}, \]  

(22)

where we have inserted the allocation of aggregate consumption expenditure between traded and non-traded goods given by (10). According to (22), following a rise in traded output relative to non-traded output, the relative price of non-tradables, \( P \), must appreciate to clear the goods market and all the more so as the elasticity of substitution \( \phi \) is smaller.

Second, setting \( \dot{\gamma}_j = 0 \) into (13b), using (13a) to eliminate \( \gamma_j \), and inserting \( W_j \) given by (18) leads to the vacancy creation equation which states that the marginal benefit of an additional worker to the firm, i.e., \( (1 - \alpha V_j) s_j + r^* \Psi_j \) where \( \Psi_j = \Xi_j + r^* x^j - W_j \) is the overall surplus created when a job-seeking worker and a firm with a job vacancy conclude a contract, equals the expected costs of recruitment per worker, i.e., \( \kappa^j/f^j \). Inserting (17) and combining hiring decisions for the traded and non-traded sectors give:

\[ \frac{\kappa^T (s^T + r^*) X^N \left( \theta^T / \theta^N \right)^{1-\alpha_V}}{\kappa^N (s^N + r^*) X^T \left( \theta^N / \theta^T \right)^{1-\alpha_V}} = \frac{\Xi^T + r^* x^T - W^T_R}{\Xi^N + r^* x^N - W^N_R}, \]  

(23)
where \( \Xi^T = A^T \) and \( \Xi^N = PA^N \). According to the vacancy creation equation described by (23), higher \( A^T/A^N \) has an expansionary effect on labor demand in the traded sector and thus pushes up \( \theta^T/\theta^N \) as long as \( \phi > 1 \). Conversely, when \( \phi < 1 \), \( P \) appreciates by more than the productivity differential which raises the share of non-tradables and thus biases labor demand toward the non-traded sector. Since our estimates of \( \phi \) reveal that the elasticity is smaller than one for the whole sample, we restrict attention to this case in the following.

Third, setting \( \xi^j = 0 \) into (9d) leads to \( \xi^j = \frac{\sigma W \Psi^j}{\sigma + \phi} \). Rewriting the latter equation by inserting the vacancy creation equation for sector \( j \) to eliminate \( \Psi^j \) gives the expected value of finding a job, i.e., \( m^j \xi^j = \frac{\sigma W}{1 - \sigma W} \kappa^j \theta^j \). Plugging this equation into (9b) leads to the equality between the utility loss from participating in the labor market in sector \( j \), \( \xi^j(F^j)^{1/\phi} / \lambda \), and the marginal benefit from search, \( \frac{\sigma W}{1 - \sigma W} \kappa^j \theta^j + R^j = W^j_R \). Combining the decision of search for the traded and the non-traded sector gives:

\[
\frac{L^T}{L^N} = \frac{m^T m^N + s^N}{m^N m^T + s^T} \left( \frac{W^T_R s^N}{W^N_R s^T} \right)^{\sigma_L},
\]

where we set \( \hat{L}^j = 0 \) into (7) to eliminate \( U^j \). According to (24), a rise in \( \theta^T/\theta^N \) has an expansionary effect on hours worked in the traded sector because more unemployed agents find a job while workers are also encouraged to increase their participation to the labor force in this sector, and all the more so as \( \sigma_L \) takes larger values.

The long-term equilibrium comprise three equations (22)-(24) which can be solved for relative employment, \( L^T / L^N \), the ratio of sectoral labor market tightness, \( \theta^T/\theta^N \), and the relative price, \( P \), as functions of relative productivity, \( A^T/A^N \), and \( \nu_{NX} \). Inserting these solutions into the Nash bargaining wage (18) and \( u^j = \frac{s^j}{s^T + m^j} \) allows us to express the relative wage, \( \Omega = W^N / W^T \) and the unemployment differential \( u^T - u^N \), in terms of \( A^T/A^N \) and \( \nu_{NX} \). This procedure to solve for the steady-state enables us to break down analytically the effects of a productivity differential between tradables and non-tradables into two components as detailed in the next section.\(^{29}\)

### 4 Higher Relative Productivity and Labor Market Frictions

Since the forces which shape the relative wage and relative price responses to an increase in \( A^T/A^N \) determine the behavior of the unemployment rate differential between tradables and non-tradables, we first explore their adjustment. We thus analytically break down the relative wage and relative price effects in two components to shed some light on the trans-

\(^{28}\)Differentiating \( u^j = \frac{s^j}{s^T + m^j} \) w.r.t. the labor market tightness \( \theta^j \) and subtracting \( du^N \) from \( du^T \) leads to \( du^T - du^N = -\alpha_N \left[ u^T (1 - u^T) \theta^T - u^N (1 - u^N) \theta^N \right] \).

\(^{29}\)When solving the steady-state, changes in the net foreign asset position and thus in net exports as reflected by changes in \( \nu_{NX} \) are assumed to be exogenous. Such a procedure allows us to isolate the effects stemming from changes in the trade balance and hiring expenditure. The ratio \( \nu_{NX} \) can be expressed in terms of sectoral productivities by using the intertemporal solvency condition obtained by linearizing (21) and invoking the intertemporal solvency condition.
mission mechanism and investigate the implications of LMR and the state of the economy.\textsuperscript{30} Then we extend this analysis to the unemployment rate differential between tradables and non-tradables. The analytical tractability of our model allows us to characterize the transitional dynamics for sectoral unemployment rates by using phase diagrams.

4.1 Inspecting the Transmission Mechanism

Relative price. Equating demand (22) and supply (23)-(24) of tradables in terms of non-tradables, leads to a relationship between the deviation in percentage of the relative price from its initial steady-state and the productivity differential:\textsuperscript{31}

\[
\hat{p} = \frac{(1 + \Theta^T) \hat{a}^T - (1 + \Theta^N) \hat{a}^N}{(\phi + \Theta^N)} + \frac{\ln (1 - v_{NX})}{(\phi + \Theta^N)},
\]

(25)

where we set

\[
\Theta^j = \Sigma^j [\alpha_V w^j + \sigma_L \chi^j], \quad \Sigma^j = \frac{\Xi^j (s^j + r^*)}{\Psi^j [(1 - \alpha_V) (s^j + r^*) + \alpha_W m^j]},
\]

(26)
in order to write expressions in a compact form: \(\chi^j = \frac{\alpha_W}{1 - \alpha_W} \kappa^j \theta^j / W^j \) stands for the share of the surplus associated with a labor contract in the reservation wage and \(\Sigma^j\) is the elasticity of \(\theta^j\) w.r.t. \(\Xi^j\). The elasticity \(\Theta^j\) of sectoral employment \(L^j\) w.r.t. the marginal revenue of labor \(\Xi^j\) is a measure of the degree of labor mobility across sectors which captures both the size of workers’ mobility costs and the extent of search frictions. In order to facilitate the discussion, we assume that \(\Theta^j \simeq \Theta^j\).\textsuperscript{32} Under this assumption, (25) reduces to:

\[
\hat{p} = \frac{(1 + \Theta)}{(\phi + \Theta)} (\hat{a}^T - \hat{a}^N) + \frac{\ln (1 - v_{NX})}{(\phi + \Theta)},
\]

(27)

where \(\ln (1 - v_{NX}) \simeq -dv_{NX}\) by using a first-order Taylor approximation.

Eq. (27) breaks down the relative price response into two components: a labor market frictions effect and a labor accumulation effect. The first term on the RHS of (27) corresponds to the labor market frictions effect. Through this channel, a productivity differential appreciates \(p\). The reason is that higher relative productivity of tradables raises traded relative to non-traded output so that \(p\) must increase to clear the goods market. Importantly, the size of the relative price appreciation is given by the elasticity \((1 + \Theta) / (\phi + \Theta)\). As long as \(\sigma_L < \infty\), workers experience an intersectoral labor mobility cost so that the term \(\Theta\)

\textsuperscript{30}It compares the steady-state of the model before and after the increase in relative productivity of tradables. Details of derivation can be found in a Technical Appendix.

\textsuperscript{31}Totally differentiating the goods market equilibrium (22) yields: \((\hat{g}^T - \hat{g}^N) = \phi \hat{p} - \ln (1 - v_{NX})\). Using the fact that \(w^j_N = \chi^j \theta^j\) and totally differentiating the vacancy creation equation for sector \(j\) gives the deviation in percentage of the sectoral labor market tightness from its initial steady-state, i.e., \(\theta^j = \Sigma^j \hat{\xi}^j\). Totally differentiating the decision of search equation for sector \(j\) leads to \(\hat{p} = \sigma_L \chi^j + [\alpha_V \eta^j + \sigma_L \chi^j] \theta^j\). Substituting the former into the latter, differentiating the production function to eliminate \(\hat{p}\), and using the fact that \(\chi^j W^j = 2 w^j \eta^j\) at the steady-state, one obtains \(\hat{g}^j = \hat{a}^j + \Theta^j \hat{\xi}^j\) where \(\Theta^j\) is given by (26). The output differential along the labor market equilibrium is thus given by \((\hat{g}^T - \hat{g}^N) = -\Theta^N \hat{p} + (1 + \Theta^T) \hat{a}^T - (1 + \Theta^N) \hat{a}^N\). Combining the goods with the labor market equilibrium leads to (25).

\textsuperscript{32}For the baseline calibration, while labor market parameters are allowed to vary across sectors \(\Theta^T\) and \(\Theta^N\) are very similar if not identical. It is only when the firing costs are important that \(\Theta^T\) and \(\Theta^N\) differ substantially.
takes finite values. In this configuration, \( p \) is jointly determined by technological and demand conditions. If the elasticity \( \phi \) between traded and non-traded goods in consumption is smaller than one, \( p \) must appreciate by more than the productivity differential to clear the goods market.

The second term on the RHS of (27) reveals that higher relative productivity of tradables also impinges on \( p \) by affecting the trade balance expressed as a share of traded output, i.e., \( v_{NX} \). More precisely, through the labor accumulation channel, higher relative productivity of tradables increases \( v_{NX} \) which exerts a negative impact on \( p \) by raising the demand for tradables in the long-run. Intuitively, higher productivity, \( A^j \), raises the shadow value of an additional worker \( \gamma^j \) and thus induces firms in both sectors to hire more. Because job vacancies, \( V^j \), is a jump variable, it overshoots on impact. Since hiring is a costly activity and labor mobility costs amplify the rise in recruitment expenditure, a current account deficit shows up in the short-run to finance the accelerated hiring process. For the country to remain solvent, the deterioration in the net foreign asset position must be offset by a steady-state increase in net exports. The improvement in the trade balance has an expansionary effect on the demand for tradables which drives down \( p \), regardless of the value of the elasticity of substitution, \( \phi \). This result echoes estimates by Lane and Milesi-Ferretti [2004] who find that countries with a larger decline in the net foreign position have more depreciated relative price of non-tradables.

To conclude, as long as the elasticity of labor supply takes finite values (i.e., \( \sigma_L < \infty \)), we will have to determine numerically if the labor accumulation effect more than offsets the labor market frictions effect when \( \phi < 1 \) so that \( \hat{p} < 1\% \) following a 1\% increase in the relative productivity of tradables.

Relative wage. We now explore the long-run response of the relative wage of non-tradables to a productivity differential. To do so, we first totally differentiate the vacancy creation equation that we substitute into the Nash bargaining wage (18) expressed in rate of change relative to the steady-state:\( ^{33} \)

\[
\hat{w}^j = \Omega^j \hat{\Xi}^j, \quad \Omega^j = \frac{\Xi^j}{W^j} \left[ (1 - \alpha_V) (s_j + r^*) + m^j \right] > 0, \tag{28}
\]

where \( \hat{\Xi}^T = \hat{\alpha}^T \) and \( \hat{\Xi}^N = \hat{\rho} + \hat{\alpha}^N \). Calculating \( \hat{\omega} \equiv \hat{w}^N - \hat{w}^T \) by using (28) and substituting (25) yields the deviation in percentage of the relative wage from its initial steady-state:

\[
\hat{\omega} = \left\{ \Omega^N \left[ \frac{(1 + \Theta^T) \hat{\alpha}^T + \left( \frac{\phi - 1}{\phi + \Theta^N} \right) \hat{\alpha}^N}{\phi + \Theta^N} \right] - \Omega^T \hat{\alpha}^T \right\} + \Omega^N \frac{d \ln (1 - v_{NX})}{\phi + \Theta^N}. \tag{29}
\]

\(^{33}\)Totally differentiating (18) gives \( \hat{w}^j = \frac{\alpha_W \Xi^j}{V^j} \hat{\Xi}^j + (1 - \alpha_W) \chi^j W^j \hat{\theta}^j \). Inserting in the above equation the vacancy creation equation expressed in percentage deviation from initial steady-state, i.e., \( \theta^j = \Sigma^j \hat{\Xi}^j \), and using the fact that at the steady-state, \( \chi^j W^j \theta^j = m^j \xi^j = \frac{m^j \alpha_W \Psi^j}{\phi + \Theta^N} \), one obtains (28).
Assuming $\Theta^j \simeq \Theta$ and $\Omega^j \simeq \Omega$ to facilitate the discussion implies that (29) reduces to:
\[
\hat{\omega} = -\Omega \left[ \frac{(\phi - 1)}{\phi + \Theta} \left( \hat{a}^T - \hat{a}^N \right) - \frac{\ln (1 - v_{NX})}{\phi + \Theta} \right].
\] (30)

According to (30), as long as workers experience a utility loss when shifting (i.e., assuming $\sigma_L < \infty$), higher relative productivity of tradables impinges on $\omega$ through two channels.

When keeping fixed $v_{NX}$, (30) reduces to $-\Omega \left( \frac{\phi - 1}{\phi + \Theta} \right) (\hat{a}^T - \hat{a}^N)$. Hence, through the labor market frictions channel, higher relative productivity increases $\omega$ when $\phi < 1$. With an elasticity of substitution $\phi$ smaller than one, a productivity differential raises the share of non-tradables which biases labor demand toward non-tradables and increases $\omega$ in contradiction with our empirical findings. Conversely, as captured by the second term on the RHS of (30), a productivity differential also impinges on $\omega$ through a labor accumulation channel. More specifically, by raising the demand for traded goods, higher net exports bias labor demand toward the traded sector and thus exert a negative impact on $\omega$. Since the long-run change in $\omega$ is the result of two opposite effects when $\phi < 1$, we address this ambiguity numerically later.

**Elasticity of labor supply at the extensive margin.** In our model $\sigma_L$ plays a key role in the determination of adjustment in $\omega$ and $p$. When $\sigma_L = 0$, labor mobility costs are prohibitive so that the labor force is fixed in both sectors. As will be clear later when discussing quantitative results, the absence of labor mobility across sectors reduces the likelihood that our model trustfully replicates our empirical findings. Conversely, when we let $\sigma_L$ tend toward infinity, we have $\Theta^j \rightarrow \infty$ so that workers are no longer subject to switching costs. Applying l’Hôpital’s rule, eq. (25) reduces to $\lim_{\sigma_L \rightarrow \infty} \hat{p} = \left( \frac{\Sigma^T \chi^T}{\Sigma^N \chi^N} \hat{a}^T - \hat{a}^N \right)$. For the baseline calibration, we find that $\lim_{\sigma_L \rightarrow \infty} \hat{p} \simeq \hat{a}^T - \hat{a}^N$ as in the standard BS model. Regarding the relative wage, eq. (29) reduces to $\lim_{\sigma_L \rightarrow \infty} \hat{\omega} = -\left( \Omega^T - \Omega^N \frac{\Sigma^T \chi^T}{\Sigma^N \chi^N} \right) \hat{a}^T$. Such an equality reflects the fact that even if mobility costs are absent, higher relative productivity of tradables may produce different sectoral wage responses because search parameters vary across sectors. However, the quantitative analysis conducted in section 5 reveals that $\chi^T \simeq \chi^N$, $\Sigma^T \simeq \Sigma^N$ and $\Omega^T \simeq \Omega^N$ (as long as firing costs are low); hence, when $\sigma_L \rightarrow \infty$, we have $\hat{\omega} \simeq 0$ so that standard search frictions are insufficient on their own to produce significant long-run movements in $\omega$.

**Search frictions.** While search frictions cannot generate $\hat{\omega} < 0$, labor mobility costs are not sufficient on their own either to account for the evidence. If we shut down search frictions in eqs. (27) and (30), $\Theta$ collapses to $\sigma_L$ and $\Omega$ reduces to 1 while the labor accumulation channel vanishes since labor is no longer an asset that can be accumulated. When $\phi < 1$, only the labor market frictions channel is in effect so that $p$ appreciates by more than 1% and $\omega$ increases, in contradiction with our evidence. As shown in the next section, search frictions also play a critical role by affecting hiring costs.

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34 For the baseline scenario of our quantitative analysis, i.e., when calibrating to a typical OECD economy, $\Omega^T$ and $\Omega^N$ are almost identical.

24
4.2 Implications of LMR

We now explore the ability of our model to account for our empirical findings established in section 2.4. While searching for a job is costly because it is time consuming, in a model with search in the labor market, hiring is also a costly activity. By affecting the marginal benefit of hiring, labor market institutions determine the elasticity of labor demand to productivity gains and thus mitigate or amplify the wage differential caused by labor mobility costs. Because LMR influences the hiring process and the subsequent adjustment of sectoral output to technology shocks, labor market policies also affect the relative price adjustment. Since the transmission mechanism varies according to the type of LMR, we differentiate between the firing cost on the one hand, the generosity of the unemployment benefit scheme and the worker bargaining power on the other.

4.2.1 Higher Firing Tax

In our model, the strictness of legal protection against dismissals is captured by a firing tax denoted by $x^j$ paid to the State by the representative firm in the sector which reduces employment. Productivity gains exert two opposite effects on labor $L^j$. On the one hand, by producing a positive wealth effect, as reflected by a fall in the shadow value of wealth $\lambda$, higher productivity drives down labor supply which exerts a negative impact on employment (see eq. (9b)). On the other hand, by increasing the marginal revenue of labor, a rise in $A^j$ induces firms to recruit more which pushes up employment. Because productivity gains are biased toward the traded sector, hours worked increase in the traded sector while labor in the non-traded sector declines. As non-traded establishments are shrinking, firms must pay a firing cost on reducing employment. Thus, according to (14), higher productivity induces non-traded firms to post more job vacancies but less so as the firing tax is increased because the surplus from hiring rises by a smaller amount. Since hirings in the non-traded sector are relatively less profitable in countries where the firing tax is higher, the labor market tightness $\theta^N$ (and thus $W^N$) increases by a smaller amount.

When $\phi < 1$, higher relative productivity of tradables increases the surplus of hirings in the non-traded sector relative to that in the traded sector. Hence, the ratio of labor market tightness (i.e., $\theta^T / \theta^N$) falls, but less so as the firing cost is higher. Consequently, $\omega$ increases less through the labor market frictions effect. Since non-traded firms tend to recruit less in countries where the firing tax is higher, labor and thus output of non-tradables increases by a smaller amount so that $p$ appreciates more.

A higher firing tax also curbs recruiting expenditure which mitigates the current account deficit and thus the long-run increase in net exports. Hence, the labor accumulation channel is somewhat moderated by the firing tax which mitigates the fall in $\omega$ and $p$.

Analytically, in terms of (29), a higher firing tax (paid by non-traded firms) lowers substantially the term $\Omega^N$ which is the elasticity of $W^N$ to the marginal revenue of labor.
When $\phi < 1$, the term in braces in (29) is positive but smaller as the firing tax $x$ is increased. Regarding the relative price equation (25), a stricter employment legislation against dismissals lowers $\Theta^N$ and thus amplifies the effect of higher $A_T/A_N$ on $p$. Moreover, as mentioned above, in countries where the firing tax is higher, net exports increase less which mitigates the rise in $v_{NX} > 0$ (see the second term in eqs. (25) and (29)). Thus, the firing tax moderates the labor accumulation effect and thus mitigates the negative impact on $p$ and $\omega$.

In sum, the larger appreciation in $p$ along the labor market frictions channel and its smaller depreciation through the labor accumulation channel implies that a higher firing tax unambiguously amplifies the rise in $p$ in line with our evidence. Conversely, the effect of stricter EPL on the response of $\omega$ is ambiguous since it mitigates its rise through the labor market frictions channel and dampens its decline through the labor accumulation channel.

### 4.2.2 Higher Unemployment Benefits or Larger Worker Bargaining Power

In our framework, the generosity of the unemployment benefit scheme is captured by the level of $R^j$; unemployment benefits are assumed to be a fixed proportion $\varrho$ of the wage rate $W^j$, i.e., $R^j = \varrho W^j$. Additionally, a higher worker bargaining power measured empirically by the bargaining coverage is captured by the parameter $\alpha_W$.

In contrast to a firing tax, raising $\varrho$ or $\alpha_W$ leads to a larger long-run rise in net exports and thus amplifies the decline in $\omega$ and mitigates the appreciation in the relative price through the labor accumulation channel. The reason is as follows. In countries where unemployment benefits are more generous or the worker bargaining power is larger, there are more job-seeking workers and less job vacancies, thus resulting in lower labor market tightness $\theta^j$ in both sectors. Consequently, following higher productivity, firms are more willing to recruit additional workers because hiring is more profitable as the probabilities of fulfilling vacancies ($f^j$) are much higher. Hence, the open economy experiences a larger current account deficit along the transitional path which must be matched in the long-run by a greater improvement in the balance of trade. The larger current account adjustment in countries where labor markets are more regulated is in line with the evidence documented by Ju et al. [2014]. By amplifying the rise in net exports and thus the demand for tradables, higher relative productivity of tradables exerts a larger negative impact on $\omega$ and $p$ in countries with a higher $\varrho$ or a larger $\alpha_W$.

While a productivity differential lowers further $\omega$ and $p$ through higher net exports, increased labor mobility tends to mitigate the impact of the trade balance. More precisely, larger values of $\varrho$, by reducing the expected cost of hiring (because the probability $f^j$ is higher), or higher values of $\alpha_W$, by raising the marginal benefit of search, increase the mobility of labor across sectors (captured by $\Theta^j$). Because workers are more willing to search for a job in countries with higher $\alpha_W$ or $\varrho$, larger values of $\Theta^j$ mitigate the negative
impact of increased net exports on $\omega$ and $p$.

Since it is found analytically that the three dimensions of LMR exert opposite effects on the elasticity of $\omega$ and $p$ to a productivity differential, we conduct a quantitative analysis in section 5.

4.3 State-Dependency

Whilst the elasticity of hiring w.r.t. to a productivity differential depends on LMR, it also varies across stages of the business cycle. Because the surplus from hiring depends on the level of labor productivity, a model with search frictions produces an asymmetry in hiring between periods of expansion and recession which allows us to rationalize the state-dependent effects we document empirically.

Intuitively, during a recession, as captured by a low labor productivity $A^j$, there are more unemployment workers and less job vacancies. This can be seen formally by using the vacancy creation equation which equates the cost of hiring, i.e., $\kappa^j/f^j (\theta^j)$, to the marginal benefit of an additional worker to the firm, i.e., $(1 - \alpha W^j) \Psi^j$ where $\Psi^j = \Xi^j + r^* x^j \mathbb{1}_{L^j < 0} - W^j R$ is the overall surplus created by a successful match. As $A^j$ takes smaller values, the surplus $\Psi^j$ gets lower which reduces $\theta^j$. Because it is easier to fulfill job vacancies (i.e., $f^j$ increases), hiring turns out to be more profitable during recessions. Thus, as in an economy with high LMR, an increase in relative productivity of tradables further biases the demand for goods and labor toward tradables through the labor accumulation channel. In the next section, we calibrate the model to quantify the effects of a productivity differential when the economy is in recession and contrast them when the economy is in expansion.

4.4 Effects on Sectoral Unemployment Rates

We now emphasize the implications of labor market frictions for unemployment effects of higher relative productivity. Importantly, our framework is tractable enough to analyze the adjustment of sectoral unemployment in the long- as well as the short-run.

We begin with the long-run effect of $A^T/A^N$ on the unemployment rate differential between tradables and non-tradables. Setting $\dot{L}^j = 0$ into (7) gives us the standard negative relationship between the unemployment rate, $u^j$, and labor market tightness, $\theta^j$:

$$u^j = \frac{s^j}{s^j + m^j (\theta^j)}. \quad (31)$$

The labor market steady-state in sector $j = T, N$ is described by a decision of search- and a vacancy creation-schedule (henceforth labelled $DS^j$ and $VC^j$), respectively:

$$L^j = (1 - u^j) \left( \frac{\lambda W^j_R / \xi^j}{\sigma_L} \right), \quad (32a)$$

$$\frac{\kappa^j}{f^j} = (1 - \alpha W^j) \Psi^j, \quad (32b)$$

Setting $\dot{L}^j = 0$ into (7) and $\dot{\xi}^j = 0$ into (9b) leads to the $DS^j$-schedule in sector $j$. Setting $\gamma^j = 0$ into (13b), and inserting $W^j$ given by (18) leads to the $VC^j$-schedule in sector $j$. 

\textsuperscript{35}Setting $\dot{L}^j = 0$ into (7) and $\dot{\xi}^j = 0$ into (9b) leads to the $DS^j$-schedule in sector $j$. Setting $\gamma^j = 0$ into (13b), and inserting $W^j$ given by (18) leads to the $VC^j$-schedule in sector $j$. 

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where \( W^j \) and \( \Psi^j \) are the reservation wage and overall surplus from an additional job in sector \( j \). Eqs. (31) and (32a) determine the \( DS_j \)-schedule which is downward-sloping in the \((u^j, L^j)\)-space, as shown in Fig. 3. Intuitively, a rise in \( \theta^j \) raises the probability of finding a job and thereby the marginal benefit of search which increases \( L^j \) and lowers \( u^j \). Eqs. (31) and (32b) determine the \( VC_j \)-schedule which is vertical in the traded sector (see Fig. 3(a)) and upward-sloping in the non-traded sector (see Fig. 3(b)).\(^{36}\) Intuitively, a rise in \( L^N \) increases non-traded output and thereby exerts a downward pressure on \( p \); because the marginal benefit of hiring falls, \( \theta^N \) declines, and thus \( u^N \) increases. Since the terms of trade are fixed, a rise in \( L^T \) leaves \( u^T \) unaffected along the \( VCT \)-schedule.

The initial steady-state is at point \( H^T_0 \) in Fig. 3 while the final steady-state is at \( H^T_1 \). A rise in \( A^T/A^N \) produces a positive wealth effect which lowers labor supply and thus shifts the \( DS_j \)-schedule downward in sector \( j \). At the same time, by raising the surplus from hiring, higher \( A^j \) shifts the \( VC_j \)-schedule to the left. As firms recruit more, \( \theta^j \) increases which lowers \( u^j \) in both sectors. Under certain conditions we detail below, the shift of the \( VC_j \)-schedule to the left is larger in the traded sector which results in a greater decline in \( u^T \) than in \( u^N \).

**Steady-State Effects.** Using the fact that \( \hat{\theta}^j = \Sigma^j / \hat{\Xi}^j \) where \( \Sigma^j \) is given by (26), totally differentiating (31) and inserting (27), subtracting \( du^N \) from \( du^T \), the change in the unemployment rate differential between tradables and non-tradables reads as:

\[
du^T - du^N = -\alpha_V u (1 - u) \left[ \left( \frac{1}{\phi + \Theta} \right) \left( \theta^T - \theta^N \right) - \frac{\ln (1 - u_{NX})}{(\phi + \Theta)} \right],
\]

where we assume that search parameters are such that \( \Theta^j \simeq \Theta, u^j \simeq u, \Sigma^j \simeq \Sigma \) to facilitate the discussion. When we let \( \sigma_L \to \infty \), the term \( \Theta \) tends toward infinity as well so that the unemployment rate differential remains unchanged.\(^{37}\) Intuitively, when job search costs are absent, \( p \) appreciates by the same amount as \( \theta^T - \theta^N \) so that the marginal revenue of labor and thus labor market tightness rises evenly across sectors.

As captured by the first term on the RHS of (33), if \( \phi < 1 \), higher \( A^T/A^N \) lowers \( u^T \) less than \( u^N \), i.e., \( du^T - du^N > 0 \), through the labor market frictions channel. The second term on the RHS of (33) reveals that the long-run improvement in the balance of trade drives down the unemployment rate differential, i.e., \( du^T - du^N < 0 \), through the labor accumulation channel. Whilst numerical results discussed in the next section show

\(^{36}\)Totally differentiating (31) and (32a) leads to \( DS_j \)-schedule in the \((u^j, L^j)\)-space, i.e.,

\[
\frac{\partial ln L^j}{\partial ln u^j}(\theta^j = 0) = \frac{\alpha_V u^j}{\alpha_V (1 - u^j)} < 0.
\]

Totally differentiating (31) and (32b) leads to the \( VC_j \)-schedule in the \((u^j, L^j)\)-space,

i.e.,

\[
\frac{\partial ln L^j}{\partial ln u^j}(\theta^j = 0) = \frac{(1 - \alpha_V) \phi^j + \chi_j \gamma_j}{\alpha_V (1 - u^j) \Xi^j \phi^j} > 0 \text{ where } \Xi^j = \partial \xi^j / \partial L^j \leq 0.
\]

\(^{37}\)When we let search parameters vary across sectors and \( \sigma_L \) tend to infinity, the unemployment rate differential reduces to:

\[
\lim_{\sigma_L \to \infty} \left( du^T - du^N \right) = -\alpha_V \left[ u^T \left( 1 - u^T \right) \Sigma^T - u^N \left( 1 - u^N \right) \Sigma^N \right] \theta^T,
\]

where we used the fact that \( \lim_{\sigma_L \to \infty} \dot{p} = \theta^T - \theta^N \). The term in brackets on the RHS of the above equation is merely positive for the baseline calibration.
that the latter channel predominates, LMR should amplify the decline in $u^T$ relative to $u^N$. Intuitively, in countries where the worker bargaining power is higher or unemployment benefits are more generous, net exports and thus the demand for tradables increases more which further raises $\theta^T$ through the labor accumulation channel. In addition, as EPL becomes more stringent, $\theta^N$ increases less though the labor market frictions channel.

**Short-Run Effects.** The dynamic effects of a productivity differential on $u^j$ are depicted in Fig. 3(a) and Fig. 3(b). The stable branch labelled $X^j$ is downward-sloping and flatter than the $DS^j$-schedule. Along the stable transitional path, $L^j$ and $u^j$ vary in opposite direction. Because labor is a state variable, $L^j$ remains unchanged on impact. On the contrary, $U^j$, is a control variable which falls sharply on impact since the positive wealth effect encourages agents to reduce time devoted to job search. Thus both $u^T$ and $u^N$ decrease at time $t = 0$. Graphically, the economy jumps initially at $H^j$. As can be seen in Fig. 3(b), $u^N$ overshoots its new steady-state level and thus declines more on impact than $u^T$. Intuitively, while the positive wealth effect lowers $U^j$ in both sectors, higher $A^T/A^N$ mitigates the decline in $u^T$ by exerting a positive impact on the marginal benefit of search. The adjustment in $L^j$ along the transitional path reverses this outcome though since productivity gains are biased toward the traded sector. As employment builds up in the traded sector, thus lowering $u^T$ along the stable path, the gradual decrease in $L^N$ raises $u^N$. In the long-run, higher $A^T/A^N$ lowers the unemployment rate differential, i.e., $du^T - du^N < 0$, as long as the labor accumulation channel more than offsets the labor market frictions channel.

< Please insert Figure 3 about here >

5 Quantitative Analysis

In this section, we analyze the effects of a labor productivity differential between tradables and non-tradables quantitatively. For this purpose we solve the model numerically. Therefore, first we discuss parameter values before turning to the quantitative analysis.

5.1 Calibration

We first calibrate the model to a representative OECD country and investigate whether the model can account for the evidence we document empirically when one parameter at a time is modified. Later, we move a step further and calibrate the model to country-specific data and explore whether the model can rationalize our empirical findings once we let all parameters of interest vary across countries. To calibrate our model, we estimated a set of parameters so that the initial steady-state is consistent with the key empirical properties of a representative OECD economy. Our sample covers the eighteen OECD economies in

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38 Technically, the assumption $\beta = r^*$ requires the joint determination of the transition and the steady state.
our dataset. Since we calibrate a two-sector model with labor market frictions, we pay particular attention to match the labor market differences between the two sectors. To do so, we carefully estimate a set of sectoral labor market parameters shown in Table 6 in Online Appendix C.3.\(^{39}\) Because we consider an open economy setup with traded and non-traded goods, we calculate the non-tradable content of employment, consumption, and government spending, and the productivity in tradables in terms of non-tradables, for all countries in our sample, as summarized in Table 4 in Online Appendix C.1. Our reference period for the calibration of the non-tradable share given in Table 4 is running from 1990 to 2007 while labor market parameters have been computed over various periods due to data availability. To capture the key properties a typical OECD economy which is chosen as the baseline scenario, we take unweighed average values shown in the last line of Tables 4 and 6. Some of the values of parameters can be taken directly from the data, but others like \(\kappa^T, \kappa^N, X^T, X^N, \zeta^N, \varphi\), together with initial conditions \((B_0, L_0^T, L_0^N)\), need to be endogenously calibrated to fit a set of labor market and non-tradable content features.\(^{40}\)

We choose the model period to be one month and therefore set the world interest rate, \(r^*\), which is equal to the subjective time discount rate, \(\beta\), to 0.4\%.

We start with the values of the labor market parameters which are chosen so as to match a typical OECD economy. We set the matching efficiency in the traded (non-traded) sector \(X^T (X^N)\) so as to target a monthly job finding rate \(m^T (m^N)\) of 17.4\% (17.0\%). In accordance with estimates shown in the last line of column 6 (column 8) of Table 6, the job destruction rate \(s^T (s^N)\) in the traded (non-traded) sector is set to 1.48\% (1.54\%), which together with the job finding rate \(m^T (m^N)\) leads to an unemployment rate \(u^T (u^N)\) of 7.9\% (8.3\%). We obtain an overall unemployment rate \(u\) of 8.1\%. We choose the recruiting cost \(\kappa^T\) and \(\kappa^N\), respectively, to target the labor market tightness \(\theta^T = 0.24\) and \(\theta^N = 0.34\) displayed by the last line of columns 10 and 11 of Table 6. The share of recruiting costs in GDP is 2.3\%.

Unemployment benefit replacement rates and the firing cost shown in the latter two columns of Table 6 correspond to averages over 1980-2007 (except Korea: 2001-2007) and 1980-2005, respectively. The unemployment benefits replacement rate, \(\varrho\), has been set to 52.4\%. To calibrate the firing cost, we take data from FRDB-IMF Labor Institutions Database [2010]; we add the advance notice and the severance payment which are averages after 4 and 20 years of employment. Since the advance notice and the severance payment are both expressed in monthly salary equivalents, we have \(x^j = \tau W^j\) with \(\tau \geq 0\). For the

\(^{39}\)To calibrate the labor market for the traded and the non-traded sector, we need to estimate the job finding and the job destruction rate for each sector. To do so, we apply the methodology developed by Shimer [2012].

\(^{40}\)As detailed in Online Appendix I, the steady-state can be reduced to seven equations which jointly determine \(\theta^T, \theta^N, m^T, m^N, L^T/L^N\) (and thus \(L^N/L\)), \(P\) (and thus \(\alpha_C\)), \(B\) (and thus \(\upsilon_{NX}\)). Among the 20 parameters that the model contains, 14 have empirical counterparts while the remaining 6 parameters, i.e., \(\kappa^T, \kappa^N, X^T, X^N, \zeta^N, \varphi\), together with initial conditions \((B_0, L_0^T, L_0^N)\) must be set in order to match \(\theta^T, \theta^N, m^T, m^N, L^N/L, \alpha_C, \upsilon_{NX}\).
baseline calibration, we set the firing tax \( \tau \) to 4.2. We model firing costs as a tax that firms have to pay to the State when their employment levels decline, i.e., if \( \dot{L} < 0 \). As mentioned previously, because traded employment monotonically increases while the non-traded sector reduces continuously employment following a productivity differential, only the non-traded sector is subject to the firing tax, i.e., \( x^N > 0 \) and \( x^T = 0 \).

Using U.S. data, Barnichon [2012] reports an elasticity of the matching function with respect to unemployed workers of about 0.6, an estimate which lies in the middle of the plausible range reported by Petrongolo and Pissarides [2001]. Hence, we set \( 1 - \alpha_V \) to 0.6. As it is common in the literature, we impose the Hosios condition, and set the worker bargaining power \( \alpha_W \) to 0.6 in the baseline scenario.

Next, we turn to the elasticity of labor supply at the extensive margin which is assumed to be symmetric across sectors. Using data from the PSID, Fiorito and Zanella [2012] find that aggregate time-series results deliver an extensive margin elasticity in the range of 0.8-1.4, which is substantially larger than the corresponding estimate (i.e., 0.2-0.3) reported by Chetty et al. [2011]. We choose a value for \( \sigma_L \) of 0.6 which is halfway between these two sets of findings in our baseline setting but conduct a sensitivity analysis with respect to this parameter.41 Furthermore, in order to target a non-tradable content of labor of 66% which corresponds to the 18 OECD countries’ unweighted average shown in the last line of Table 4, we normalize \( \zeta^T \) to 1 and choose a value for \( \zeta^N \) that parametrizes the disutility from working and searching for a job in the non-traded sector, of 0.18 (see eq. (5)).

We now turn to the calibration of consumption-side parameters. Building on our panel data estimations, we set the elasticity of substitution between between traded and non-traded goods to 0.8.42 The weight of consumption in non-tradables \( 1 - \phi \) is set to 0.44 to target a non-tradable content in total consumption expenditure (i.e., \( \alpha_C \)) of 42%, in line with the our estimates for the whole sample shown in the last line of Table 4. The intertemporal elasticity of substitution for consumption \( \sigma_C \) is set to 1.

For calibration purposes, we introduce government spending on traded and non-traded goods in the setup. We set \( G^N \) and \( G^T \) so as to yield a non-tradable share of government spending of 90%, and government spending as a share of GDP of 20%.43 We assume that, in the initial steady-state, net exports are nil and thus choose initial conditions \( (B_0, L^T_0, L^N_0) \) in order to target \( v_{NX} = 0 \).

Because we find empirically that the stage of the business cycle matters in determining the effects of a higher relative productivity of tradables, we also calibrate the model to data

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41Blundell, Bozio and Laroque [2011] estimate an elasticity of labor supply at the extensive margin of 0.34 for women and 0.25 for men on U.K. data. Using Japanese data, Kuroda and Yamamoto [2008] report a Frisch elasticity on the extensive margin which falls in the range of 0.6 to 0.8 for both sexes. Mustre-del-Río [2015] finds a value of 0.71 for the responsiveness of labor at the extensive margin which varies between 0.18 for men and 1.46 for women.

42In Online Appendix C.2, we describe the empirical strategy to estimate \( \phi \). Last column of Table 4 reports estimates of \( \phi \) for each country and the whole sample (equal to 0.8).

43Eq. (19) can be rewritten as follows: \( \sum_j x^j \cdot \max \left \{ 0, -\dot{L}^j \right \} + T = (R^T U^T + R^N U^N) + G^T + PG^N \)
when the economy is in recession or in expansion. We proceed as follows. We calculate the output gap for each country in our sample over 1970-2007 (except Japan: 1974-2007). Then we multiply the average duration of a cycle by the average output gap for each country to calculate the cumulated output loss in recession or output gain in expansion; we consider the situation of an economy in the middle of the cycle and thus the cumulated output gap is halved. Next, we map the cumulated output gain or loss into an unemployment gap by using estimates of Okun’s law provided by Ball et al. [2017] for each country in our sample. The rise in unemployment relative to trend following a 1 ppt increase in the output gap is 0.42 on average. Using this value, we find that the cumulated increase in the unemployment rate relative to trend after about 2 years is 1.4 ppt in recessions whilst its cumulated decline is 1.2 ppt in expansions. Since the trend unemployment rate is 8.1%, we choose initial values for sectoral labor productivity, $A^j$, so that the unemployment rate of a representative OECD economy is 9.5% in recessions and 6.9% in expansions. We modify sectoral labor productivity so that the ratio $A^T/A^N$ is unchanged at 1.28.

We consider a permanent increase in the productivity index $A^j$ of both sectors biased towards the traded sector so that the labor productivity differential between tradables and non-tradables, i.e., $\hat{a}^T - \hat{a}^N$, is 1%. While in our baseline calibration we set $\sigma_L = 0.6$, $\alpha_W = 0.6$, $\varrho = 0.524$, $\tau = 4.2$, we conduct a sensitivity analysis with respect to these four parameters by setting alternatively: $\sigma_L$ to $\infty$, 0, 0.16 and 1.22, $\alpha_W$ to 0.9, $\varrho$ to 0.782, and $\tau$ to 13.\textsuperscript{44} Finally, in the last two columns of Table 3, we compare the results for an economy in recession with those obtained when the economy is in expansion.

5.2 Results

We now assess the ability of the model to account for our empirical findings according to which a 1% permanent increase in $A^T/A^N$ lowers the relative wage (by 0.22%), appreciates the relative price (by 0.64%), and lowers $u^T$ relative to $u^N$ (by 0.034 ppt). We also investigate the implications of the three dimensions of LMR for the effects of higher $A^T/A^N$ and contrast the effects across stages of the business cycle.

The responses of $\omega$, $p$, and the unemployment differential computed numerically are summarized in Table 3. Panels A and B of Table 3 report the long-run changes for $\omega$ and $p$, respectively, expressed as a percentage while panel C gives the change in unemployment differential in percentage point of the labor force. For comparison purposes, column 1 summarizes our empirical evidence for the whole sample. The numbers reported in the first line of each panel give the (overall) responses of these variables to $\hat{a}^T - \hat{a}^N = 1%$.

\textbf{Mapping theoretical results into empirical estimates.} Before discussing quan-

\textsuperscript{44}We let $\sigma_L$ vary between 0.16 and 1.22 as these values are those which allow the model to replicate the increase in $\beta$ from -0.32 to -0.15 as shown in Figure 1(a). When conducting the sensitivity analysis, we raise $\varrho$ from 52.4% to 78.2% and $\tau$ from 4.2 to 13, which correspond to the highest value in our sample of countries for the replacement rate and the firing cost, respectively.
tative results, we relate our analytical results to the elasticity of $z = \omega, p, u^T - u^N$ with respect to the productivity differential, i.e., $\beta, \gamma, \sigma$, which are estimated empirically (see eq. (1)). When search frictions are similar across sectors, the long-run responses of $p, \omega, u^T - u^N$ reduce to (27), (30), (33), respectively. In this configuration, there exists a direct mapping between analytical expressions of $\frac{dz}{\sigma^2 - \sigma}$, and empirical estimates of $\gamma, \beta, \sigma$, respectively. In contrast, when search frictions vary across sectors, we have to correct for the inherent discrepancy between theoretical and empirical values. This discrepancy originates from sector-varying $\Theta_j$ and $\Omega_j$ which makes the theoretical elasticity of $z$ w.r.t. $A^T/A^N$ different. To map the deviation in percentage of $z$ from its initial steady-state into elasticity estimated empirically, we need to adjust numerically computed values with a term that captures the extent to which search frictions vary across sectors. Once the discrepancy is accounted for, we are able to relate $\gamma, \beta$, and $\sigma$ estimated empirically to their analytical counterpart. Whilst Online Appendix D shows analytical expressions adjusted with the bias originating from sector-varying search frictions, the last line of each panel of Table 3 displays the size of the bias which remains low if not insignificant.

**No mobility costs.** In our model, labor market frictions vary along two dimensions. If we abstract from hiring costs, i.e., if we set $\kappa^j = 0$, and shut down labor mobility costs, i.e., if we let $\sigma_L \rightarrow \infty$, the model reduces to the standard BS model without unemployment. In this situation, a productivity differential of 1% appreciates $p$ by 1% while $\dot{\omega} = 0$. In column 3 of Table 3, we consider the responses when we shut down labor mobility costs, i.e., setting $\sigma_L \rightarrow \infty$, while still considering search frictions. As it clearly stands out, standard search frictions are not sufficient on their own to account for the evidence.

**With mobility costs.** Numerical results summarized in column 2 show that when calibrating to a typical OECD economy, a model with labor market frictions can produce a decline in $\omega$, a less than proportional increase in $p$, and a fall in $u^T$ which is more pronounced than $u^N$, as found in the data. To shed light on the transmission mechanism of higher relative productivity in a model with labor market frictions, we numerically break down the responses into two components: a labor market frictions channel stemming from the change in the share of non-tradables and a labor accumulation channel triggered by the accelerated hiring process which increases the share of tradables in the long-run.

As shown in the second line of panels A and B, a 1% increase in the relative productivity of tradables raises $\omega$ by 0.11% and appreciates $p$ by 1.15% through the labor market frictions effect. Because inflation in non-tradables more than offsets the productivity differential, $u^T - u^N$ increases. Intuitively, when $\phi < 1$, traded and non-traded goods are complements in consumption so that $p$ appreciates by more than the productivity differential. As a result, higher relative productivity of tradables raises the share of non-tradables into expenditure which biases labor demand toward the non-traded sector, thus resulting in an increase in $\omega$ and $u^T - u^N$. 

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As shown in the third line of panels A, B, C, the labor accumulation effect counteracts the labor market frictions effect. More specifically, higher $A^T/A^N$ also raises net exports which has an expansionary effect on hiring in the traded sector, thus driving down $\omega$ by 0.34% and lowering $u^T$ relative to $u^N$ by 0.023 ppt. Higher demand for tradables also depreciates $p$ by 0.36%. Importantly, the labor accumulation effect more than offsets the labor market frictions effect so that $\omega$ declines by 0.22%, $u^T$ relative to $u^N$ falls by 0.011 ppt, whilst $p$ appreciates by 0.78%, as summarized in the first line of panels A, C, and B, respectively.

Our model with search in the labor market and an endogenous sectoral labor force participation sheds light on three sets of factors influencing the mobility of labor across sectors and thus the responses of $\omega$, $p$, and the unemployment differential to higher relative productivity of tradables: the workers’ mobility cost reflected by a utility loss when increasing the search intensity for a job in one sector (as captured by $\sigma_L$), labor market institutions (captured by $\alpha_W$, $\varrho$, $\tau$), and the state of the economy in the business cycle (as captured by the initial value of $A^j$).

**Role of labor supply at the extensive margin.** As we move from column 4 to column 6, the elasticity of labor supply at the extensive margin $\sigma_L$ is raised from zero to 1.22. Column 4 of panels A, B, and C of Table 3 shows numerical results if labor is totally immobile across sectors as captured by setting $\sigma_L = 0$. In this configuration, the labor force is fixed in both sectors because the mobility cost is prohibitive. Since the decision of search is inelastic to the sectoral wage, $\omega$ falls by 0.48% instead of 0.22% in the baseline scenario. Hence, such a polar case tends to substantially overstate the decline in $\omega$ and thus confirms the pivotal role of an endogenous labor force participation decision. As shown, in columns 5 and 6 of panels A and B of Table 3, raising $\sigma_L$ from 0.16 to 1.22 lowers the utility loss induced by the shift from one sector to another which in turn moderates the decline in $\omega$ from -0.32 to -0.15, respectively, these values corresponding to the lowest and highest elasticity of the relative wage w.r.t. relative productivity estimated empirically in rolling sub-samples (see Figure 1(a)). As the mobility cost is lowered, $p$ appreciates more because demand is less biased toward traded goods which mitigates the decline in the unemployment differential. Because Fiorito and Zanella [2012] find larger values than 0.6 for $\sigma_L$, in Online Appendix H we re-calibrate our model by choosing $\sigma_L = 1$ or $\sigma_L = 3$. We find that our conclusions remain unchanged although the decline in $\omega$ is less pronounced and the appreciation in $p$ is amplified.

**Implications of higher unemployment benefits and worker bargaining power.** Scenarios summarized in columns 7 and 8 of Table 3 show that, in line with our evidence, raising the worker bargaining power $\alpha_W$ or the unemployment benefit replacement rate $\varrho$ mitigates the appreciation in $p$ from 0.78% to 0.74% and 0.70%, respectively, as shown in the first line of panel B, because the demand for traded goods increases more which further
depreciates $p$ through the labor accumulation channel. By tilting labor demand toward the traded sector, LMR also amplifies the decline in $\omega$ from 0.22% to 0.26% and 0.29%, respectively, in accordance with our evidence. Finally, the first line of panel C reveals that the decline in $u^T - u^N$ is twofold when $\alpha_W$ is higher and fourfold when $\varrho$ takes higher values, respectively.

**Implications of higher firing costs.** Column 9 of Table 3 gives results when the firing cost, $\tau$, is about three times larger than in the baseline scenario. In accordance with our empirical findings, raising $\tau$ drives down further both $\omega$ and the unemployment differential. Intuitively the firing cost curbs the expansionary effect of higher productivity gains on hiring by non-traded firms and thus further biases labor demand toward the traded sector. Moreover, as shown in the first line of panel B, countries with stringent legal protection against dismissals also experience a larger appreciation in $p$ which squares well with our estimates. Quantitatively, we may notice that the differences produced by increasing the firing cost are quantitatively small, in accordance with our evidence.

**Implications of the state in the business cycle.** In columns 10 and 11, we compare the responses of a representative OECD economy with high and low unemployment. As can be seen in panel A and C, an increase in relative productivity lowers $\omega$ and $u^T - u^N$ more because the elasticity of hiring to a productivity differential is amplified when the economy is in recession. Panel B also shows that non-tradable inflation is reduced from 0.79% in expansion to 0.76% in recession.

**Robustness to the definition of hiring costs.** In the baseline model, we assume that the cost per job vacancy is expressed in terms of the traded good (column 2). In column 12 of Table 3, we investigate the robustness of our results to alternatively assuming that the cost per job vacancy paid by traded and non-traded firms is expressed in terms of the non-traded good. Since the cost per job vacancy, i.e., $P(t)\kappa^j$, depends on the relative price of non-tradables, an appreciation in $p$ increases the hiring cost. Contrasting numerical results shown in column 2 with those in column 12 reveals that our results are robust to the definition of hiring costs although $p$ appreciates more while $\omega$ declines less because the labor accumulation channel is mitigated. Intuitively, while higher relative productivity of tradables leads both sectors to recruit more, the rise in the cost per job vacancy brought about by the appreciation in $p$ mitigates hirings and thus recruitment expenditure. Since the current account deficit is lower along the transitional path, net exports increase by a smaller amount in the long-run so that demand is less biased toward tradables. Because productivity gains which favor hirings are concentrated in the traded sector, traded firms are disproportionately affected by the rise in the hiring cost, thus resulting in a moderate decline in the unemployment differential between tradables and non-tradables (see panel C). In Online Appendix G, we also explore the robustness of our results by alternatively considering that the cost per job vacancy in the non-traded (traded) sector is expressed in
terms of the non-traded (traded) good and find that all of our conclusions in the main text hold.

5.3 Time-Varying Effects of Higher Relative Productivity

According to the evidence documented in subsection 2.3, the effects of a productivity differential increase over time. While our evidence points the role of time-declining labor mobility costs, we further explore this hypothesis below.

To perform this exercise, we calibrate the model to a representative OECD economy except that we let the three dimensions of LMR vary over time and consider a decline in labor mobility costs in order to account for the rise in the elasticity of the relative wage, $\beta$, we estimate empirically. More specifically, we set the working bargaining power, $\alpha_W$, to the bargaining coverage by using time series from the ICTWSS database (Visser [2009]). To calibrate the firing cost, we take data from FRDB-IMF Labor Institutions Database [2010] and set the firing tax $\tau$ in monthly salary equivalents. We also let the unemployment benefit replacement rate (i.e., $\varrho$) vary across time, see subsection 2.4 for details about data.

While the values of parameters which captured the extent of LMR are taken from the data, $\sigma_L$ is set to replicate the elasticity of the relative wage to a productivity differential (i.e., $\beta$) we estimate empirically for each year over the period 1990-2007 which is displayed by the solid blue line with circles in Fig. 4(a). Next we contrast empirical estimates (shown by the blue line) with model’s predictions when we let both LMR and $\sigma_L$ vary across time, as shown by the black line with triangles, and alternatively when we shut down LMR and increase $\sigma_L$, as shown in the red line. Because the black and the red lines can be merely distinguished, time-declining labor costs alone caused by the rise in $\sigma_L$ from 0.16 to 1.22 can account for time-increasing effects of a productivity differential. Regarding the relative price (see Fig. 4(b)), whilst the model is able to produce the rise in $\gamma$ over the nineties, it somewhat misses the fall in $\gamma$ starting from 2003. Finally, as can be seen in Fig. 4(c), time-increasing $\sigma_L$ can account for the rise in the elasticity $\sigma$ of the unemployment differential w.r.t. relative productivity.

---

45 We exclude a few countries because data were missing which leaves us with 12 countries. The collective bargaining coverage averages 60.7%, which corresponds roughly to the value we set for $\alpha_W$ when we calibrate the model to a representative OECD economy; the bargaining coverage declines from 65% in 1990 to about 57% in 2007.

46 Since time series stop in 2005 for all countries in our sample, we set $\tau$ in 2006 and 2007 to its 2005 value.

47 Parameters $\alpha_W$, $\varrho$, and $\tau$ are set to their average values, i.e., 0.6, 0.52, 4.2, respectively, to plot the red line.

48 The decline in $\gamma$ in 2000’s could be attributed to pro-competitive policies implemented by European countries which have targeted especially non-traded industries.
5.4 Cross-Country Effects and LMR

We now move a step further by calibrating our model to country-specific data and assess the ability of our model to account for the negative relationship between the size of the effects of a productivity differential and LMR. We use the same baseline calibration for each economy, except for the elasticity of substitution $\phi$ between traded and non-traded goods, and labor market parameters which are allowed to vary across countries. More specifically, $\phi$ is set in accordance with its estimates shown in the last column of Table 4. The parameters which capture the degree of LMR such as the firing cost, $\tau$, and the replacement rate, $\varrho$, are set to their values shown in the latter two columns of Table 6. The matching efficiency $X^j$ in sector $j = T, N$ is set to target the job finding rate $m^j$ shown by columns 5 and 7 of Table 6. The job destruction rate in sector $j$, $s^j$, is set in accordance with its value reported in columns 6 and 8 of Table 6. The costs per job vacancy $\kappa^T$ and $\kappa^N$ are chosen to target the aggregate labor market tightness $\theta$ shown in column 13 and the ratio of sectoral labor market tightness $\theta^T/\theta^N$ obtained by dividing column 10 by column 11.49

In Figure 5, we plot numerically computed elasticity of $\omega$, $p$ and $u^T - u^N$, respectively, against our measure of LMR which encompasses the extent of the worker bargaining power, the generosity of the unemployment benefit scheme and the strictness of legal protection against dismissals. Despite the wide dispersion in the responses of $\omega$ and $p$, the trend line in Figure 5(a) and 5(b) reveals that the $\omega$ falls more and $p$ appreciates less in countries with more regulated labor markets. An additional major implication of our two-sector model with search frictions is that higher relative productivity leads to a decline in $u^T - u^N$. As can be seen in Figure 5(c), the decline in $u^T$ relative to $u^N$ remains insignificant in English-speaking countries where LMR is low but remains substantial in Belgium, Denmark, France, Germany, Spain where $du^T - du^N$ varies from $-0.061$ to $-0.022$ ppt.

6 Conclusion

The literature exploring the long-run effects of a productivity differential between tradables and non-tradables on the relative price of non-tradables commonly assumes frictionless labor markets. In this paper, we differentiate between labor mobility and hiring costs to account for the effects of higher productivity of tradables relative to non-tradables which appear to vary across time, space and stages of the business cycle. Our first set of evidence suggests the presence of time-declining labor mobility costs which mitigate the sectoral wage differential and amplify the appreciation in the relative price over time. Our second set of evidence

49 Ideally, the recruiting cost $\kappa^j$ would be set in order to target $\theta^j$; however, the series for job vacancies by economic activity are available for a maximum of seven years and for a limited number of countries. On the contrary, the OECD provides data for job openings (for the whole economy) over the period 1980-2007 allowing us to calculate the labor market tightness, i.e., $\theta = V/U$, for several countries that we target along with the ratio $\theta^T/\theta^N$ by choosing $\kappa^T$ and $\kappa^N$. 

37
evidence reveals that hiring costs matter in determining the variations of the effects across countries and across stages of the business cycle. More specifically, we find that the relative wage of non-tradables falls more and the relative price of non-tradables appreciates less in countries where LMR is higher. When we differentiate the effects between recessions and expansions, we find empirically that the decline in the relative wage is more pronounced during a recession while the increase in the relative price is less. Since unemployment emerges naturally in an economy with search frictions, we also investigate empirically the effect on the unemployment differential between tradables and non-tradables. Our third set of evidence shows that the decline in the unemployment differential caused by higher relative productivity turns out to be less pronounced over time, and appears to be more pronounced in countries where labor markets are more regulated or during recessions.

To account for the evidence, we develop a two-sector open economy model with search in the labor market and an endogenous sectoral labor force participation decision. As in Alvarez and Shimer [2011], workers cannot reallocate hours worked from one sector to another without searching for a job in this sector. Because such an activity is costly in utility terms, workers experience a switching cost which varies with the elasticity of labor supply at the extensive margin. We find analytically that two sets of parameters play a pivotal role in the determination of the relative wage and relative price responses to higher productivity in tradables relative to non-tradables: i) preference parameters such as the elasticity of labor supply at the extensive margin and the elasticity of substitution in consumption between tradables and non-tradables, ii) parameters capturing labor market institutions such as the firing tax, the unemployment benefit replacement rate and the worker bargaining power.

Our quantitative analysis indicates that, regardless of the value of the elasticity of substitution between tradables and non-tradables, when the elasticity of labor at the extensive margin falls within the range of values documented by the empirical literature, our model can account for the fall in the relative wage and the greater decline in the unemployment rate of tradables relative to that of non-tradables along with the less than proportional appreciation in the relative price. On the contrary, the situations of total immobility or perfect mobility of labor across sectors that emerge as special cases cannot account for the evidence. When we control for the variations of LMR across time, we find that time-declining labor mobility costs are responsible for the time-increasing effects of higher relative productivity we document empirically.

While labor mobility costs create an asymmetry in the sector adjustment, hiring costs which emerge naturally in a model with search frictions play a key role in amplifying or mitigating this asymmetry. In line with the evidence aforementioned, our numerical results show that the variations of hiring costs across stages of the business cycle can account for state-dependent effects of a productivity differential. When we let labor market policies vary across countries, we find that international differences in LMR can rationalize cross-
country effects of a productivity differential.

References


Figure 1: Plot of Estimates of $\beta$, $\gamma$, and $\sigma$ in Rolling Sub-Samples against Intersectoral Labor Reallocation Notes: We estimate $\beta$, $\gamma$, $\sigma$, and the effect of higher relative productivity on labor reallocation across sectors by running regression (1) in rolling sub-samples. The first row of Figure 1 plots FMOLS estimates for the response of the relative wage to a rise in the relative productivity of tradables (shown in the solid black line) against the intersectoral labor reallocation caused by higher relative productivity (shown in the dotted black line). The first two panels in the second row of Figure 1 plot FMOLS estimates for the response of the relative price to a rise in the relative productivity of tradables (shown in the solid black line) against the intersectoral labor reallocation caused by higher relative productivity (shown in the dotted black line). Sample: 18 OECD countries, 1970-2007. Figure 1(e) plots the estimated response of the unemployment rate differential to higher relative productivity (shown in the dotted black line) against the intersectoral labor reallocation caused by higher relative productivity (shown in the solid black line). Sample: 10 OECD countries, 1987-2007.

Table 1: Panel Cointegration FMOLS Estimates of $\beta$ and $\gamma$ for the Whole and Sub-Samples

<table>
<thead>
<tr>
<th></th>
<th>Whole Sample</th>
<th>Split-Sample: LMR</th>
<th>Split-Sample: Business Cycle</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\varphi$</td>
<td>$\gamma$</td>
<td>$\sigma$</td>
</tr>
<tr>
<td>A. Relative Wage</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\beta^H$</td>
<td>-0.72**</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\beta^L$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$t(\hat{\beta}^L - \hat{\beta}^H) \mid (\hat{\gamma} = 0)$</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>B. Relative Price</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\gamma^H$</td>
<td>0.636**</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\gamma^L$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$t(\hat{\gamma}^L - \hat{\gamma}^H) \mid (\hat{\gamma} = 1)$</td>
<td>0.000</td>
<td>0.000</td>
<td>0.987</td>
</tr>
<tr>
<td>Observations</td>
<td>68</td>
<td>642</td>
<td>642</td>
</tr>
<tr>
<td>Countries</td>
<td>18</td>
<td>17</td>
<td>17</td>
</tr>
<tr>
<td>mean LMR/unemp gap (high)</td>
<td>-</td>
<td>0.609</td>
<td>0.864</td>
</tr>
<tr>
<td>mean LMR/unemp gap (low)</td>
<td>-</td>
<td>0.391</td>
<td>0.491</td>
</tr>
</tbody>
</table>

Notes: $^a$, $^b$, and $^c$ denote significance at 1%, 5%, and 10% levels. Heteroskedasticity and autocorrelation consistent standard errors are reported in parentheses. Column 1 shows estimates for the full sample by running regression shown in eq. (1). The third row of panel A and B (i.e., $t(\hat{\beta} = 0$ and $t(\hat{\gamma} = 1$) reports the p-value of the test of $H_0: \beta = 0$ and $H_0: \gamma = 1$, respectively. In the last six columns, we report estimates when we explore empirically eq. (3). From column 2 to 5, we investigate whether LMR influences the responses of the relative wage, $\beta$, and the relative price, $\gamma$, to a productivity differential. We split the sample of 18 OECD countries into two sub-samples, based on the median of the sample, and run regression (3) for the group of countries with high and low LMR; $\beta^H$ ($\beta^L$) and $\gamma^H$ ($\gamma^L$) capture the responses of the relative wage and the relative price, respectively, in countries with high (low) LMR. The last two columns show the responses of $\omega$ and $p$ across stages of the business cycle. The state of the economy is measured by means of the output gap (column 6) or alternatively by using the unemployment gap (column 7); $\beta^H$ and $\gamma^H$ ($\beta^L$ and $\gamma^L$) refers to the responses of the relative wage and the relative price, respectively, when the economy displays high (low) unemployment, i.e. during a recession (expansion). The third row of panel A and B (i.e., $t(\hat{\beta}^H = \hat{\beta}^L$, $t(\hat{\gamma}^L = \hat{\gamma}^H)$) reports the p-value of the test of $H_0: \beta^H = \beta^L$ and $\gamma^L = \gamma^H$, respectively. $'p'$ is the unemployment benefits replacement rate, 'EPLadj' the strictness of employment protection against dismissals adjusted with the share of permanent workers, 'BargCov' is the bargaining coverage, 'LMR' refers to the LMR index obtained by using a principal component analysis.
Figure 2: Labor Market Indicators over Time

Notes: Figure 2 plots three indicators. The solid black line shows the OECD countries’ average of the collective bargaining coverage which gives the proportion of employees covered by collective bargaining. Data are taken from the ICTWSS database (Visser [2009]). We use a linear interpolation to replace missing data between two dates. The dotted blue line shows the OECD countries’ average of the unemployment benefit replacement rate. Data are taken from the Benefits and Wages database provided by the OECD which calculates the average of the net unemployment benefit for three durations of unemployment (1st year, 2nd and 3rd year, 4th and 5th year). Because data for Korea are not available before 2002 for the bargaining coverage and 2001 for the replacement rate, we exclude this country from the sample to calculate the mean of these two indicators. The dashed red line shows the OECD countries’ average of the employment protection legislation index for regular workers adjusted with the share of permanent workers in the economy. Source: OECD.

Table 2: Panel OLS Estimates of $\sigma$ for the Whole and Sub-Samples

<table>
<thead>
<tr>
<th></th>
<th>Whole sample</th>
<th>Split-Sample: LMR</th>
<th>Split-Sample: Business Cycle</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>No control</td>
<td>With controls</td>
<td>No control</td>
</tr>
<tr>
<td>$\sigma^H$</td>
<td>$-0.034^a$ (0.013)</td>
<td>$-0.037^a$ (0.014)</td>
<td>$-0.036^c$ (0.020)</td>
</tr>
<tr>
<td>$\sigma^L$</td>
<td>$-0.033^c$ (0.018)</td>
<td>$-0.032^c$ (0.019)</td>
<td>$-0.034^c$ (0.019)</td>
</tr>
<tr>
<td>Obs.</td>
<td>164</td>
<td>164</td>
<td>164</td>
</tr>
<tr>
<td>Countries</td>
<td>14</td>
<td>14</td>
<td>14</td>
</tr>
</tbody>
</table>

Notes: all regressions include country fixed effects. $^a$, $^b$ and $^c$ denote significance at 1%, 5% and 10% levels. Heteroskedasticity and autocorrelation consistent standard errors are reported in parentheses. The table reports panel OLS estimates of eq. (1) expressed in variations, i.e., we run the regression of the change in unemployment differential between tradables and non tradables on the productivity growth differential. The first two columns show results for the full sample with no controls and two controls (i.e., EPL$\text{adj}$ the strictness of employment protection against dismissals adjusted with the share of permanent workers, and $\varrho$ the unemployment benefits replacement rate), respectively. In the last four columns, we report estimates when we explore empirically eq. (3) in variation. In columns 3 and 4, we split the sample of 14 OECD countries into two sub-samples on the basis of the mean sample of the labor market regulation (LMR) index obtained by using a principal component analysis. Coefficient $\sigma^H$ ($\sigma^L$) captures the effect of a 1% increase in the relative productivity of tradables on the unemployment rate differential between tradables and non tradables in countries with high (low) labor market regulation.

The last two columns show the responses of the unemployment differential following higher relative productivity across stages of the business cycle. The state of the economy is measured by means of the output gap (column 5) or alternatively by using the unemployment gap (column 6); $\sigma^H$ ($\sigma^L$) refers to the response of the unemployment differential when the economy displays high (low) unemployment, i.e. during a recession (expansion).
Figure 3: Phase Diagrams and Dynamics for Sectoral Unemployment Rates.

(a) Traded sector: \((u^T, L^T)\)-space
(b) Non traded sector: \((u^N, L^N)\)-space

Figure 4: Theoretical vs. Empirical Estimates of Time-Varying Elasticity \(\beta, \gamma,\) and \(\sigma\). Notes: \(\beta\) and \(\gamma\) are the elasticities of the relative wage and relative price w.r.t. relative productivity; empirical estimates correspond to FMOLS estimates for the responses of the relative wage and relative price in rolling sub-samples with a window length \(T = 20\) shown in the first column of Figure 1; \(\sigma\) is the change in the unemployment differential following a productivity growth differential of 1% estimated in rolling sub-samples with window length \(T = 12\). Responses of the relative wage, relative price and relative unemployment estimated empirically are shown in the blue line. Responses computed numerically are shown in the black and the red line. We use the same calibration as for a representative OECD economy and choose a value for the elasticity of labor supply at the extensive margin for each year, \(\sigma_L\), in order to replicate the empirically estimated value of \(\beta\) shown in the blue line of Figure 4(a). Whilst in the black line, we let the three LMR indicators, including the unemployment benefit replacement rate, \(\tau\), the firing tax, \(x\), and the worker bargaining power measured by collective bargaining coverage, \(\alpha_W\), vary across time, in the red line, we compute numerically the elasticities by keeping LMR constant over time in order to give a sense of its consequences.

Figure 5: Cross-Country Relationship between Simulated Responses to Higher Relative Productivity and LMR. Notes: Horizontal axes display the LMR index obtained by using a principal component analysis which encompasses the three dimensions of labor market institutions. Vertical axes in the top panels report simulated long-run responses of the relative wage, relative price, and unemployment differential to higher relative productivity from the baseline model with search frictions and an endogenous labor force participation decision.
Table 3: Decomposition of Long-Term Responses to Higher Productivity in Tradables Relative to Non Tradables

<table>
<thead>
<tr>
<th>Data</th>
<th>OECD</th>
<th>Labor force</th>
<th>Barg. power</th>
<th>Replac. rate</th>
<th>Firing</th>
<th>Rec</th>
<th>Exp</th>
<th>Hiring</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
<td>(4)</td>
<td>(5)</td>
<td>(6)</td>
<td>(7)</td>
<td>(8)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(u = 8.1%)</td>
<td>(σ_L → ∞)</td>
<td>(σ_L = 0)</td>
<td>(σ_L = 0.16)</td>
<td>(σ_L = 1.22)</td>
<td>(α_W = 0.9)</td>
<td>(φ = 0.782)</td>
<td>(τ = 13)</td>
</tr>
</tbody>
</table>

A. Relative Wage

| Relative wage, $\hat{\omega}$ | -0.22 | -0.22 | 0.00 | -0.48 | -0.32 | -0.15 | -0.26 | -0.29 | -0.23 | -0.24 | -0.21 | -0.14 |
| Labor market frictions | 0.11 | 0.00 | 0.18 | 0.18 | 0.07 | 0.11 | 0.09 | 0.07 | 0.11 | 0.11 | 0.11 | 0.11 |
| Labor accumulation | -0.34 | 0.00 | -0.68 | -0.50 | -0.23 | -0.37 | -0.39 | -0.33 | -0.36 | -0.33 | -0.27 | -0.27 |
| Bias | -0.01 | 0.00 | -0.02 | -0.00 | -0.01 | -0.01 | -0.02 | -0.03 | -0.01 | -0.01 | -0.01 | -0.01 |

B. Relative Price

| Relative price, $\hat{\rho}$ | 0.64 | 0.78 | 1.02 | 0.51 | 0.68 | 0.85 | 0.74 | 0.70 | 0.80 | 0.76 | 0.79 | 0.86 |
| Labor market frictions | 1.15 | 1.04 | 1.22 | 1.22 | 1.11 | 1.14 | 1.13 | 1.18 | 1.15 | 1.15 | 1.15 | 1.15 |
| Labor accumulation | -0.36 | 0.00 | -0.71 | -0.53 | -0.24 | -0.39 | -0.42 | -0.36 | -0.38 | -0.35 | -0.29 | -0.29 |
| Bias | 0.01 | 0.02 | -0.00 | 0.01 | 0.01 | 0.00 | 0.00 | 0.02 | 0.00 | 0.01 | 0.01 | 0.01 |

C. Unemployment diff.

| Unemp. diff., $da_T^N - da_N^N$ | -0.034 | -0.011 | 0.004 | -0.028 | -0.025 | 0.000 | -0.027 | -0.045 | -0.012 | -0.014 | -0.008 | -0.008 |
| Labor market frictions | 0.014 | 0.008 | 0.021 | -0.004 | 0.029 | 0.021 | 0.035 | 0.011 | 0.016 | 0.012 | 0.012 | 0.012 |
| Labor accumulation | -0.023 | 0.000 | -0.046 | -0.028 | -0.017 | -0.045 | -0.073 | -0.021 | -0.027 | -0.019 | -0.017 | -0.017 |
| Bias | 0.003 | 0.004 | 0.003 | -0.007 | 0.011 | 0.003 | 0.006 | 0.001 | 0.003 | 0.002 | 0.003 | 0.003 |

Notes: Effect of a 1% increase in the productivity of tradables relative to non-tradables. Panels A, B, C show the deviation in percentage relative to steady-state for the relative wage, $\omega \equiv w_T - w_N$, the relative price of non-tradables, $p \equiv p_N - p_T$, and the unemployment differential between tradables and non-tradables, $u_T - u_N$, and break down changes in a labor market frictions effect (keeping net exports fixed), and a labor accumulation effect (triggered by the long-run adjustment in net exports); 'unemp. diff.' mean unemployment differential in panel C. The fourth line in each panel 'Bias' shows the magnitude of the bias caused by search frictions varying across sectors which must be subtracted from the sum of the second and the third line of Table 3. The first line shows the numerically computed response to a productivity differential once the inherent discrepancy between theoretical and empirical values caused by search frictions varying across sectors has been corrected. Parameter $\phi$ is the elasticity of substitution in consumption between tradables and non tradables; $\sigma_L$ is the elasticity of labor supply at the extensive margin; $\alpha_W$ corresponds to the worker bargaining power; $\rho$ is the unemployment benefits replacement rate; $\tau$ is the firing tax expressed in monthly salary equivalents. In our baseline calibration we set $\phi = 0.8$, $\sigma_L = 0.6$, $\alpha_W = 0.6$, $\rho = 0.524$, $\tau = 4.2$. Columns 10 and 11 show the effects when the economy is in recession (labelled 'Rec') and in expansion (labelled 'Exp'). We calibrate the model by reducing sectoral labor productivity $A^f$ so that the ratio $A_T^N / A_N^N = 1.28$ is kept unchanged and the aggregate unemployment rate is 9.5% and 6.9% in a recession and in an expansion, respectively. The last column shows model’s predictions when the cost per job vacancy paid by traded and non-traded firms is expressed in terms of the non-traded good. 'NT HC' means non-traded hiring costs.
A Simple Model with Labor Market Frictions

To set the stage for the empirical analysis, in the main text we revisit the theory developed by Balassa [1964] and Samuelson [1964] (BS hereafter) by relaxing the assumption of perfectly competitive labor markets in order to build up intuition regarding the implications of labor market frictions. We lay out below a simple model with search frictions which provides a formal background of the discussion in subsection 2.1. We denote below the percentage deviation from initial steady-state by a hat.

As it is commonly assumed, the country is small in terms of both world goods and capital markets, and thus faces an exogenous international price for the traded good normalized to unity. Each sector produces $Y_j$ by using labor, $L_j$, according to a linear technology, $Y_j = A^j L_j$, where $A^j$ represents the labor productivity index.

Because firms face a cost by maintaining job vacancies, they receive a surplus equal to the marginal revenue of labor $\Xi_j$ less the product wage $W_j$. Symmetrically, so as to compensate for the cost of searching for a job, unemployed workers receive a surplus equal to $W_j$ less the reservation wage $W_j^R$. We denote by $\Psi_j$ the overall surplus created when a job-seeking worker and a firm with a job vacancy conclude a contract:

$$\Psi_j = \Xi_j - W_j^R \left( \theta^j \right),$$

where $\Xi_j = A_j^T$, $\Xi_j^N = P A_j^N$ with $P$ corresponding to the relative price of non tradables, and we denote by $\theta^j$ the labor market tightness in sector $j$, defined as the ratio of job vacancies to unemployed workers; when firms post more job vacancies, $\theta^j$ rises which raises the reservation wage, i.e., $\hat{w}_j^R = \chi_j^j \hat{\theta}^j$ where $0 < \chi_j^j < 1$ represents the share of the surplus associated with a labor contract in the marginal benefit of search.

The product wage $W_j$ paid to the worker in sector $j$ is equal to the reservation wage plus a share $\alpha_{W_j}$ of the overall surplus:

$$W_j = W_j^R \left( \theta^j \right) + \alpha_{W_j} \Psi_j,$$

where the worker bargaining power $\alpha_{W_j}$ is assumed to be symmetric across sectors. Denoting the relative wage by $\Omega \equiv W_j^N/W_j^T$ and differentiating (35) leads to the sectoral wage differential:

$$\dot{\omega} \equiv \dot{w}_j^N - \dot{\bar{w}}^T = -\chi_j W_j^R \left( \dot{\theta}^j - \dot{\bar{\theta}}^N \right) - \frac{\alpha_{W_j} \dot{\Psi}_j}{W_j^T} \left( \dot{\Psi}_j - \dot{\Psi}_j^N \right),$$

where we assume that initially $W_j^T \simeq W_j^N$ and $\chi_j W_j^R \simeq \chi_j W_j^N$ and $\Psi_j \simeq \Psi$ to ease the interpretation. In a model abstracting from labor market frictions, as the standard BS model, searching for a job is a costless activity so that $\Psi$ and $\chi$ are nil; hence sectoral wages rise at the same speed. Conversely, in a model with labor market frictions, a productivity differential between tradables and non tradables may lower $\omega$. The reason is as follows. First, as captured by the first term on the RHS of (36), higher $A_j^T/A_j^N$ induces traded firms to recruit more than non traded firms; because agents experience a utility loss when increasing the search intensity for a job in the traded sector, traded firms must increase wages to attract workers as reflected by the rise in the ratio $\theta^T/\theta^N$. Moreover, as shown by the second term on the RHS of (36), by raising $\Psi_j^T/\Psi_j^N$, a productivity differential between tradables and non tradables lowers $\omega$; intuitively, higher $A_j^T/A_j^N$ increases the surplus from an additional job in the traded sector relative to the non traded sector, $\Psi_j^T/\Psi_j^N$, the worker obtaining a share equal to $\alpha_{W_j}$.

Denoting the job destruction rate by $s^j$ and the job finding rate by $m^j$, and using the fact that at the steady-state, the flow of unemployed workers who find a job is equalized with the flow of employed workers who lose their job, the unemployment rate $u^j$ in sector $j$ reads as $u^j = \frac{s^j}{s^j + m^j(\theta^j)}$. Totally differentiating $u^j$ and denoting the elasticity of vacancies in job matches by $\alpha_V$, allows us to express the unemployment rate differential between tradables and non tradables in terms of the differential in sectoral labor market tightness:

$$du^T - du^N = -\alpha_V u (1 - u) \left( \dot{\theta}^T - \dot{\theta}^N \right),$$

where we assume that at the initial steady-state, search parameters are such that $u^j \simeq u$. According to (37), higher $A_j^T/A_j^N$ results in a decline in $u^T$ relative to $u^N$ by raising the ratio $\theta^T/\theta^N$ as traded firms recruit more than non traded firms.

When a labor contract is concluded with a worker, the representative firm in sector $j$ receives the marginal revenue of labor $\Xi_j^j$ which must cover the recruiting cost plus the dividend per worker equivalent to $(1 - \alpha_{W_j}) \Psi_j$ and the wage rate paid to the worker:

$$\Xi_j^j = \left( 1 - \alpha_{W_j} \right) \Psi_j + W_j^j.$$

Differentiating (38) and subtracting $\Xi^T_j$ from $\Xi^N_j$ leads to:

$$\dot{\bar{\psi}} = (\bar{a}^T - \bar{a}^N) + \frac{W_j}{\Xi_j^j} (\bar{w}_j^N - \bar{\bar{w}}^T) - \frac{(1 - \alpha_{W_j}) \Psi_j}{\Xi_j^j} \left( \dot{\Psi}_j - \dot{\Psi}_j^N \right),$$

46
where we assume that initially $\Xi^j \simeq \Xi$, $\Psi^j \simeq \Psi$, and $W^j \simeq W$. According to (39), when abstracting from labor market frictions, as the BS model, the surplus $\Psi$ is nil while sectoral wages increase at the same speed so that $p$ must appreciate by the same amount as $\tilde{a}^T - \tilde{a}^N$. Conversely, in a model with labor market frictions, as captured by the second term on the RHS of (39), $\omega$ falls because traded firms have to pay higher wages to compensate for the workers’ mobility costs. Moreover, as shown by the third term on the RHS of (39), since traded firms recruit more than non traded firms, the hiring cost must be covered by an increase in $\Psi^T/\Psi^N$, the firm obtaining a share equal to $1 - \alpha_W$. Thus, by lowering $\omega$ and increasing the hiring cost in the traded sector relative to that in the non traded sector, a productivity differential of 1% appreciates $p$ by less than 1%.

The relative wage and relative price equations described by (36) and (39), respectively, allow us to explain why labor market frictions imply that sectoral wages may no longer rise at the same speed and the elasticity of the relative price w.r.t. the productivity differential may be smaller than one. However, such conclusions are established by abstracting from the goods market equilibrium which matters as long as labor is not perfectly mobile across sectors. In section 4, we show that the full steady-state can be solved for the relative price and the relative wage, i.e., $P = P^N/P^T = P(A^T, A^N)$ and $\Omega \equiv W^N/W^T = \Omega(A^T, A^N)$. Because all variables display trends, our empirical strategy consists in estimating the cointegrating relationships with relative productivity.

In the main text, we also explore empirically whether higher $A^T/A^N$ leads to $du^T - du^N < 0$. Whilst the standard BS model abstracting from labor market frictions cannot address unemployment issues, standard search frictions are not sufficient on their own to lower the unemployment rate differential following a rise in $A^T/A^N$. More specifically, for higher relative productivity to result in a decline in $u^T$ relative to $u^N$, as shown in eq. (37), traded firms must recruit more than non traded firms. For this to happen, the appreciation in the $p$ must be less than the productivity differential otherwise non traded firms are able to exactly offset lower productivity gains by setting higher prices. As discussed above, the relative price appreciates less than proportionately if workers experience mobility costs.

B Data for Empirical Analysis

Country Coverage: Our sample consists of a panel of 18 OECD countries: Australia (AUS), Austria (AUT), Belgium (BEL), Canada (CAN), Germany (DEU), Denmark (DNK), Spain (ESP), Finland (FIN), France (FRA), the United Kingdom (GBR), Ireland (IRL), Italy (ITA), Japan (JPN), Korea (KOR), the Netherlands (NLD), Norway (NOR), Sweden (SWE), and the United States (USA).


Sources: We use the EU KLEMS [2011] database (the March 2011 data release) for all countries of our sample with the exceptions of Canada and Norway. For these two countries, sectoral data are taken from the Structural Analysis (STAN) database provided by the OECD [2011]. Both the EU KLEMS and STAN databases provide annual data at the ISIC-rev.3 1-digit level for eleven industries.

The eleven industries are split into tradables and non tradables sectors. To do so, we adopt the classification proposed by De Gregorio et al. [1994]. Following Jensen and Kletzer [2006], we have updated this classification by treating ”Financial Intermediation” as a traded industry. We construct traded and non traded sectors as follows (EU KLEMS codes are given in parentheses):

- **Traded Sector**: ”Agriculture, Hunting, Forestry and Fishing” (A-B), ”Mining and Quarrying” (C), ”Total Manufacturing” (D), ”Transport, Storage and Communication” (I) and ”Financial Intermediation” (J).

- **Non Traded Sector**: ”Electricity, Gas and Water Supply” (E), ”Construction” (F), ”Wholesale and Retail Trade” (G), ”Hotels and Restaurants” (H), ”Real Estate, Renting and Business Services” (K) and ”Community Social and Personal Services” (L-Q).

Once industries have been classified as traded or non traded, for any macroeconomic variable $X$, its sectoral counterpart $X^j$ for $j = T, N$ is constructed by adding the $X_k$ of all sub-industries $k$ classified in sector $j = T, N$ as follows $X^j = \sum_{k \in j} X_k$. In the following, we provide details on data construction (mnemonics are in parentheses):

- **Relative wage of non tradables**, $\Omega$, is calculated as the ratio of the nominal wage in the non traded sector $W^N$ to the nominal wage in the traded sector $W^T$, i.e., $\Omega = W^N/W^T$. The sectoral nominal wage $W^j$ for sector $j = T, N$ is calculated by dividing labor compensation in sector $j$ (LAB) by total hours worked by persons engaged (H_EMP) in that sector.

- **Relative price of non tradables**, $P$, corresponds to the ratio of the value added deflator of non traded goods $P^N$ to the value added deflator of traded goods $P^T$, i.e., $P = P^N/P^T$. The value added deflator $P^j$ for sector $j = T, N$ is calculated by dividing value added at current
Relative productivity of tradables

The generosity of the unemployment benefit scheme is commonly captured by the

The construction of employment protection legislation

The worker bargaining power is measured by the

The trend lines in Figures 6(a), 6(b), 6(c) show that the estimated responses of the relative wage and our three measures of LMR are positively related across countries. We also we have recourse to a principal component analysis to construct an indicator that gives a more accurate measure of the degree of LMR. Figure 6(d) displays the traditional distinction between English-speaking and Continental European economies, labor markets being much less regulated in the former than the latter countries. Importantly, the trend line is upward sloping, thus suggesting that higher productivity in tradables relative to non tradables lowers the relative wage more in countries where LMR is more pronounced.

In order to identify the state of the economy across the business cycle, we use alternatively the output or the unemployment gap:

Because time series for the unemployment benefit replacement rate and bargaining coverage are available only from the beginning of the 2000’s for Korea and thus are too short, we exclude this country from Figures 6(b) and 6(c).
Figure 6: Labor Market Regulation and The Relative Wage Response to Higher Productivity of Tradables relative to Non Tradables

Notes: Figure 6 plots fully modified OLS estimates of relative wage responses to a labor productivity differential against indicators of labor market regulation. Horizontal axis displays the FMOLS estimates for each country which are taken from Table 14. For easier reading, we show the absolute value of the change in the relative wage (i.e., $|\beta_i|$). Firing cost is captured by the employment protection legislation index adjusted with the share of permanent workers in the economy (source: OECD); the generosity of unemployment benefit scheme is measured by the average of net unemployment benefit replacement rates for three duration of unemployment (source: OECD); the worker bargaining power is measured by the bargaining coverage (source: Visser [2009]); in Figure 6(d), we have recourse to a principal component analysis in order to have one overall indicator encompassing the three dimensions of labor market regulation.
• **Output gap** is computed as the deviation of output from trend, i.e. $y_{it} - ar{y}_{it}$ where $y_{it}$ and $\bar{y}_{it}$ denote the log of actual and potential real GDP, respectively. Log potential GDP $\bar{y}$ is obtained by applying a Hodrick-Prescott filter with a smoothing parameter of $\lambda = 100$ (as we use annual data) to the series $y$. Gross domestic product is the real gross domestic product (GDPV). Recessions are periods where $dy_{it} - d\bar{y}_{it} < 0$. Source: OECD Economic Outlook Database. Data coverage: 1970-2007 for all countries.

• **Unemployment gap** is computed as $u_{it} - \bar{u}_{it}$ where $u_{it}$ and $\bar{u}_{it}$ is the actual and natural unemployment rate, respectively. The natural unemployment rate $\bar{u}$ is obtained by applying a Hodrick-Prescott filter with a smoothing parameter of $\lambda = 100$ to the series $u$. The actual unemployment rate is defined as unemployment in percentage of the civilian labour force.

Following Ramey and Zubairy [2017], we define a period of recession as $u_{it} - \bar{u}_{it} > 0$. Source: OECD Population and Labour Force Database. Data coverage: 1970-2007 for all countries (except for NOR: 1972-2007).

### C Data for Calibration

#### C.1 Non Tradable Share

Table 4 shows the non-tradable content of labor, consumption, government spending, and gives the share of government spending on the traded and non traded goods in the sectoral output. The second to last column of Table 4 also shows the ratio of traded real labor productivity to the non traded real labor productivity, $A^T/A^N$. Our sample consists of 18 OECD countries mentioned in section B, including 12 European countries plus Australia, Canada, Korea, Japan, Norway, the United-States. Our reference period for the calibration corresponds to the period 1990-2007. The choice of this period has been dictated by data availability.

To calculate the non tradable share of employment we split the eleven industries into traded and non traded sectors by adopting the classification proposed by De Gregorio et al. [1994] and updated by Jensen and Kletzer [2006] (Source: EU KLEMS [2011]). The non-tradable share of labor, shown in column 1 of Table 4 averages to 66%.

To split consumption expenditure (at current prices) into consumption in traded and non traded goods, we made use of the Classification of Individual Consumption by Purpose (COICOP) published by the United Nations (Source: United Nations [2011]). Among the twelve items, the following ones are treated as consumption in traded goods: "Food and Non-Alcoholic Beverages", "Alcoholic Beverages Tobacco and Narcotics", "Clothing and Footwear", "Furnishings, Household Equipment", "Transport", "Miscellaneous Goods and Services". The remaining items are treated as consumption in non traded goods: "Housing, Water, Electricity, Gas and Fuels", "Health", "Communication", "Education", "Restaurants and Hotels". Because the item "Recreation and Culture" is somewhat problematic, we decided to consider it as both tradable (50%) and non tradable (50%) with equal shares. Data coverage: 1990-2007 for AUS, AUT, CAN, DNK, FIN, FRA, GBR, ITA, JPN, KOR, NLD, NOR, and USA, 1991-2007 for DEU, 1993-2007 for SWE, 1995-2007 for BEL and ESP and 1996-2007 for IRL. Note that the non-tradable share of consumption shown in column 2 of Table 4 averages to 42%.

Sectoral government expenditure data (at current prices) were obtained from the Government Finance Statistics Yearbook (Source: IMF [2011]) and the OECD General Government Accounts database (Source: OECD [2012b]). Adopting Morshed and Turnovsky’s [2004] methodology, the following four items were treated as traded: "Fuel and Energy", "Agriculture, Forestry, Fishing, and Hunting", "Mining, Manufacturing, and Construction", "Transport and Communications". Items treated as non traded are: "Government Public Services", "Defense", "Public Order and Safety", "Education", "Health", "Social Security and Welfare", "Environment Protection", "Housing and Community Amenities", "Recreation Cultural and Community Affairs". Data coverage: 1990-2007 for BEL, DNK, FIN, GBR, IRL, ITA, JPN, NOR and USA, 1990-2006 for CAN, 1991-2007 for DEU, 1995-2007 for SWE, 1996-2007 for IRL. Note that the non-tradable share of government expenditure shown in column 3 of Table 4 averages to 90%. While government spending as a share in GDP is shown in column 4, the proportion of government spending on the traded and non traded good (i.e., $G^T/Y^T$ and $G^N/Y^N$) are shown in columns 5 and 6 of Table 4. They average 5% and 29%, respectively. In column 4, government spending is government final consumption expenditure at current prices and the GDP is the gross domestic product at current prices. Source: OECD Economic Outlook Database. Data coverage: 1990-2007 for all countries.

The second to last column of Table 4 displays the ratio of labor productivity of tradables relative to non tradables ($A^T/A^N$) averaged over the period 1990-2007 for all countries. Source: the EU KLEMS [2011] and STAN database. As shown in column 7, the traded sector is in average 28 percent more productive than the non traded sector.
Table 4: Data to Calibrate the Two-Sector Model (1990-2007)

<table>
<thead>
<tr>
<th>Countries</th>
<th>Non tradable Share</th>
<th>G/Y</th>
<th>G'/Y'</th>
<th>G'/Y''</th>
<th>A'/A''</th>
<th>Elasticity</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Labor Consumption</td>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
<td>(4)</td>
<td>(5)</td>
</tr>
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<td>AUS</td>
<td>0.68</td>
<td>0.43</td>
<td>n.a.</td>
<td>n.a.</td>
<td>0.18</td>
<td>n.a.</td>
</tr>
<tr>
<td>AUT</td>
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<td>0.42</td>
<td>0.90</td>
<td>0.19</td>
<td>0.05</td>
<td>0.27</td>
</tr>
<tr>
<td>BEL</td>
<td>0.68</td>
<td>0.42</td>
<td>0.91</td>
<td>0.22</td>
<td>0.06</td>
<td>0.30</td>
</tr>
<tr>
<td>CAN</td>
<td>0.69</td>
<td>0.43</td>
<td>0.91</td>
<td>0.20</td>
<td>0.05</td>
<td>0.30</td>
</tr>
<tr>
<td>DEU</td>
<td>0.65</td>
<td>0.40</td>
<td>0.91</td>
<td>0.19</td>
<td>0.05</td>
<td>0.27</td>
</tr>
<tr>
<td>DK</td>
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<td>0.94</td>
<td>0.26</td>
<td>0.05</td>
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<tr>
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<td>0.88</td>
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<td>0.24</td>
</tr>
<tr>
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<td>0.89</td>
<td>0.22</td>
<td>0.06</td>
<td>0.34</td>
</tr>
<tr>
<td>FRA</td>
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<td>0.23</td>
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<td>0.31</td>
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<tr>
<td>GBR</td>
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<td>0.40</td>
<td>0.93</td>
<td>0.20</td>
<td>0.04</td>
<td>0.29</td>
</tr>
<tr>
<td>IRL</td>
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<td>0.17</td>
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<td>0.28</td>
</tr>
<tr>
<td>ITA</td>
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<td>0.91</td>
<td>0.19</td>
<td>0.05</td>
<td>0.27</td>
</tr>
<tr>
<td>JPN</td>
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<td>0.86</td>
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<td>KOR</td>
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</tr>
<tr>
<td>NLD</td>
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<td>0.23</td>
<td>0.07</td>
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<tr>
<td>NOR</td>
<td>0.66</td>
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<td>0.88</td>
<td>0.21</td>
<td>0.06</td>
<td>0.34</td>
</tr>
<tr>
<td>SWE</td>
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<td>0.92</td>
<td>0.27</td>
<td>0.06</td>
<td>0.39</td>
</tr>
<tr>
<td>USA</td>
<td>0.73</td>
<td>0.51</td>
<td>0.90</td>
<td>0.16</td>
<td>0.05</td>
<td>0.20</td>
</tr>
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<td>EU-12</td>
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</tr>
<tr>
<td>OECD</td>
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<td>0.90</td>
<td>0.20</td>
<td>0.05</td>
<td>0.29</td>
</tr>
</tbody>
</table>

C.2 Elasticity of Substitution in consumption ($\phi$)

To estimate the elasticity of substitution in consumption $\phi$ between traded and non traded goods, we first derive a testable equation by inserting the optimal rule for intra-temporal allocation of consumption (10) into the goods market equilibrium which gives \[
\frac{C_T}{C_N} \equiv \frac{Y_T - NX}{Y_T - EN} - \frac{E_T}{E_N} \frac{NX}{EN} - \frac{E_T}{EN}\frac{NX}{E_N}
\] where $NX \equiv B - r^*B$ is net exports, $E_T \equiv G_T + I_T + F$ (with $F \equiv \kappa^T V_T + \kappa^N V_N$) and $EN \equiv G^N + I^N$; note that we include investment in order to be consistent with accounting identities. Inserting the optimal rule for intra-temporal allocation of consumption (10) into the goods market equilibrium, and denoting the ratio of $E_T$ to traded value added adjusted with net exports at current prices by $v_{ET} = \frac{P^T Y_T}{P^T Y_N}$, and the ratio of $EN$ to non traded value added at current prices by $v_{E_N} = \frac{P^N E^N}{P^N Y_N}$, the goods market equilibrium can be written as follows \[
\frac{Y_T - NX}{Y_N} = \frac{Y_T - NX}{Y_N} - \frac{E_T}{E_N}\frac{NX}{EN} - \frac{E_T}{EN}\frac{NX}{E_N}
\] Isolating \[(Y_T - NX)/Y_N \text{ taking logarithm yields } \ln \left( \frac{Y_T - NX}{Y_N} \right) = \alpha + \phi \ln P \text{ where } \alpha \equiv \ln \left( 1 + \frac{\gamma_T}{\gamma_N} \right) + \ln \left( \frac{\gamma_T}{\gamma_N} \right).\] Adding an error term $\mu$, we estimate $\phi$ by running the regression of the (logged) output of tradables adjusted with net exports at constant prices in terms of output of non tradables on the (logged) relative price of non tradables:

\[
\ln \left( \frac{Y_T - NX}{Y_N} \right)_{t,t} = f_t + f_t + \alpha_t + \phi_t \ln P_{t,t} + \mu_{t,t}, \tag{40}
\]

where $f_t$ and $f_t$ are the country fixed effects and time dummies, respectively. Because the term $\alpha$ is composed of ratios which may display a trend over time, we add country-specific linear trends, as captured by $\alpha_t$. Instead of using time series for sectoral value added, we can alternatively make use of series for sectoral labor compensation. Multiplying both sides of \[
\frac{(Y_T - NX)(1 - v_{ET})}{Y_N(1 - v_{E_N})} = \left( \frac{\gamma_T}{\gamma_N} \right) \frac{P^T}{P^N}
\] and then by $\frac{P^T}{P^N}$, with $\rho = \frac{W^T L^T}{P^T Y_T}$, denoting by $\gamma_T = \left( W^T L^T - \rho^T P^T NX \right)$ (with $PT = \frac{W^T L^T}{P^T Y_T}$) and $\gamma_N = W^N L_N$, and taking logarithm yields $\ln \left( \frac{\gamma_T}{\gamma_N} \right) = \eta + (\phi - 1) \ln P$ where $\eta$ is a term composed of both preference (i.e., $\phi$) and production (i.e., $\rho$) parameters, and the (logged) ratio of $E_T$ ($EN$) to $W^T L^T - \rho^T P^T NX$ ($W^N L_N$). We thus estimate $\phi$ by exploring alternatively the following empirical relationship:

\[
\ln \left( \frac{\gamma_T}{\gamma_N} \right)_{t,t} = g_t + g_t + \eta_t + \delta_t \ln P_{t,t} + \zeta_{t,t}, \tag{41}
\]

where $\delta_t = (\phi_t - 1); g_t$ and $g_t$ are the country fixed effects and time dummies, respectively; we add country-specific trends, as captured by $\eta_t$, because $\eta$ is composed of ratios that may display a trend over time.
Table 5: Estimates of the Elasticity of Substitution in Consumption between Tradables and Non Tradables (φ)

<table>
<thead>
<tr>
<th>Country</th>
<th>φ^{DOLS}</th>
<th>φ^{FMOLS}</th>
<th>φ^{DOLS}</th>
<th>φ^{FMOLS}</th>
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<tr>
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<td>eq. (40)</td>
<td>eq. (40)</td>
<td>eq. (41)</td>
<td>eq. (41)</td>
</tr>
<tr>
<td>AUS</td>
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<td>0.295</td>
<td>0.011</td>
<td>0.375</td>
</tr>
<tr>
<td>AUT</td>
<td>0.574</td>
<td>1.019</td>
<td>0.910</td>
<td>1.414</td>
</tr>
<tr>
<td>BEL</td>
<td>0.268</td>
<td>0.034</td>
<td>0.393</td>
<td>0.749</td>
</tr>
<tr>
<td>CAN</td>
<td>0.306</td>
<td>0.439</td>
<td>0.332</td>
<td>0.569</td>
</tr>
<tr>
<td>DEU</td>
<td>0.976</td>
<td>1.126</td>
<td>1.190</td>
<td>1.363</td>
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<tr>
<td>DNK</td>
<td>1.243</td>
<td>1.925</td>
<td>1.698</td>
<td>1.320</td>
</tr>
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<td>0.527</td>
<td>0.782</td>
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<td>0.355</td>
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<td>1.556</td>
<td>1.043</td>
<td>2.061</td>
<td>1.412</td>
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<tr>
<td>FRA</td>
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<td>0.896</td>
<td>1.169</td>
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<tr>
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<td>0.321</td>
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<td>0.427</td>
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<td>0.832</td>
<td>0.713</td>
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</tr>
<tr>
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<td>EU-12</td>
<td>0.590</td>
<td>0.599</td>
<td>0.890</td>
<td>0.832</td>
</tr>
<tr>
<td>Whole sample</td>
<td>0.586</td>
<td>0.800</td>
<td>0.853</td>
<td>0.934</td>
</tr>
</tbody>
</table>

Notes: Data coverage: 1970-2007 (except Japan: 1974-2007). All regressions include country fixed effects, time dummies and country specific trends. a, b and c denote significance at 1%, 5% and 10% levels. Heteroskedasticity and autocorrelation consistent t-statistics are reported in parentheses.

Time series for sectoral value added at constant prices, labor compensation, and the relative price of non tradables are taken from EU KLEMS [2011] (see section B). Net exports correspond to the external balance of goods and services at current prices taken from OECD Economic Outlook Database. To construct time series for net exports at constant prices NX, data are deflated by the traded value added deflator of traded goods (i.e., P^T).

Since the LHS term of (40) and (41) and the relative price of non tradables as well display trends, we ran unit root and then cointegration tests. Having verified that these two assumptions are empirically supported, we estimate the cointegrating relationships by using DOLS and FMOLS estimators for cointegrated panel proposed by Pedroni [2000], [2001]. DOLS and FMOLS estimates are reported in Table 5, considering alternatively eq. (40) or eq. (41). Estimates of φ are reported in the last column of Table 4. As a reference model, we consider FMOLS estimates when exploring the empirical relationship (40); running regression (40) gives an estimate for the whole sample of 0.800 which is close to the value documented by Mendoza [1995] who reports an estimate of 0.74. As shown in Table 5, the estimated value of φ for Belgium is statistically significant only when exploring the empirical relationship (41) for this economy; in the last column of Table 4, we set φ to 0.749 for Belgium. Because estimates for Italy are negative by using alternatively eq. (40) or eq. (41), the estimate of φ for this country is left blank in the last column of Table 4 and φ is set to our panel data estimation for EU-12, i.e., 0.599, when calibrating the model for each country.

C.3 Labor Market Variables

We now describe the data employed to calibrate the model, focusing on labor market variables. To begin with, EU-10 refers to the following ten European countries: Austria, Belgium, Germany,
<table>
<thead>
<tr>
<th>Country</th>
<th>Period</th>
<th>(u_j^T)</th>
<th>(u_j^N)</th>
<th>(u)</th>
<th>(m_j^T)</th>
<th>(m_j^N)</th>
<th>(s_j^N)</th>
<th>Period</th>
<th>(\theta_j^T)</th>
<th>(\theta_j^N)</th>
<th>Period</th>
<th>(\rho)</th>
<th>(\tau)</th>
</tr>
</thead>
<tbody>
<tr>
<td>AUS</td>
<td>95-07</td>
<td>0.072</td>
<td>0.062</td>
<td>0.065</td>
<td>0.304</td>
<td>0.0236</td>
<td>0.278</td>
<td>04-05</td>
<td>0.17</td>
<td>0.27</td>
<td>80-07</td>
<td>0.13</td>
<td>52.4</td>
</tr>
<tr>
<td>AUT</td>
<td>94-07</td>
<td>0.037</td>
<td>0.044</td>
<td>0.042</td>
<td>0.126</td>
<td>0.0048</td>
<td>0.123</td>
<td>005</td>
<td>0.17</td>
<td>0.27</td>
<td>80-07</td>
<td>0.18</td>
<td>52.8</td>
</tr>
<tr>
<td>BEL</td>
<td>01-07</td>
<td>0.077</td>
<td>0.079</td>
<td>0.078</td>
<td>0.067</td>
<td>0.0056</td>
<td>0.064</td>
<td>005</td>
<td>0.05</td>
<td>0.05</td>
<td>82-03</td>
<td>0.05</td>
<td>65.2</td>
</tr>
<tr>
<td>CAN</td>
<td>95-07</td>
<td>0.101</td>
<td>0.091</td>
<td>0.094</td>
<td>0.067</td>
<td>0.0075</td>
<td>0.062</td>
<td>006-07</td>
<td>0.21</td>
<td>0.40</td>
<td>80-07</td>
<td>0.09</td>
<td>60.6</td>
</tr>
<tr>
<td>DEU</td>
<td>94-04</td>
<td>0.064</td>
<td>0.061</td>
<td>0.062</td>
<td>0.245</td>
<td>0.0167</td>
<td>0.247</td>
<td>016-01</td>
<td>0.21</td>
<td>0.21</td>
<td>81-07</td>
<td>0.09</td>
<td>65.1</td>
</tr>
<tr>
<td>DNK</td>
<td>92-07</td>
<td>0.147</td>
<td>0.161</td>
<td>0.156</td>
<td>0.097</td>
<td>0.0167</td>
<td>0.094</td>
<td>018-04</td>
<td>0.03</td>
<td>0.03</td>
<td>80-04</td>
<td>0.03</td>
<td>47.2</td>
</tr>
<tr>
<td>ESP</td>
<td>91-07</td>
<td>0.087</td>
<td>0.118</td>
<td>0.107</td>
<td>0.137</td>
<td>0.0130</td>
<td>0.0180</td>
<td>02-07</td>
<td>0.21</td>
<td>0.21</td>
<td>81-07</td>
<td>0.09</td>
<td>65.1</td>
</tr>
<tr>
<td>FIN</td>
<td>75-04</td>
<td>n.a.</td>
<td>n.a.</td>
<td>0.081</td>
<td>0.067</td>
<td>0.0059</td>
<td>0.067</td>
<td>009</td>
<td>0.30</td>
<td>0.48</td>
<td>80-07</td>
<td>0.24</td>
<td>58.3</td>
</tr>
<tr>
<td>GBR</td>
<td>88-07</td>
<td>0.073</td>
<td>0.066</td>
<td>0.068</td>
<td>0.163</td>
<td>0.0129</td>
<td>0.161</td>
<td>01-07</td>
<td>0.30</td>
<td>0.48</td>
<td>80-07</td>
<td>0.24</td>
<td>58.2</td>
</tr>
<tr>
<td>IRL</td>
<td>86-97</td>
<td>0.130</td>
<td>0.154</td>
<td>0.144</td>
<td>0.048</td>
<td>0.0071</td>
<td>0.045</td>
<td>008</td>
<td>0.30</td>
<td>0.48</td>
<td>80-07</td>
<td>0.24</td>
<td>58.2</td>
</tr>
<tr>
<td>ITA</td>
<td>93-07</td>
<td>0.094</td>
<td>0.098</td>
<td>0.097</td>
<td>0.062</td>
<td>0.0065</td>
<td>0.059</td>
<td>006</td>
<td>0.30</td>
<td>0.48</td>
<td>80-07</td>
<td>0.24</td>
<td>58.2</td>
</tr>
<tr>
<td>JPN</td>
<td>03-07</td>
<td>0.033</td>
<td>0.033</td>
<td>0.033</td>
<td>0.171</td>
<td>0.0058</td>
<td>0.165</td>
<td>005</td>
<td>0.27</td>
<td>0.27</td>
<td>80-07</td>
<td>0.27</td>
<td>50.1</td>
</tr>
<tr>
<td>KOR</td>
<td>92-07</td>
<td>0.027</td>
<td>0.041</td>
<td>0.035</td>
<td>0.262</td>
<td>0.0072</td>
<td>0.262</td>
<td>011-07</td>
<td>0.30</td>
<td>0.48</td>
<td>80-07</td>
<td>0.18</td>
<td>53.5</td>
</tr>
<tr>
<td>NLD</td>
<td>83-04</td>
<td>n.a.</td>
<td>n.a.</td>
<td>0.064</td>
<td>0.047</td>
<td>0.0032</td>
<td>0.047</td>
<td>003-02</td>
<td>0.30</td>
<td>0.48</td>
<td>80-07</td>
<td>0.19</td>
<td>54.9</td>
</tr>
<tr>
<td>NOR</td>
<td>83-04</td>
<td>n.a.</td>
<td>n.a.</td>
<td>0.045</td>
<td>0.035</td>
<td>0.0143</td>
<td>0.035</td>
<td>014</td>
<td>0.43</td>
<td>0.65</td>
<td>80-07</td>
<td>0.59</td>
<td>26.1</td>
</tr>
<tr>
<td>SWE</td>
<td>95-07</td>
<td>0.056</td>
<td>0.060</td>
<td>0.059</td>
<td>0.233</td>
<td>0.0138</td>
<td>0.231</td>
<td>014</td>
<td>0.17</td>
<td>0.17</td>
<td>82-07</td>
<td>0.19</td>
<td>54.9</td>
</tr>
<tr>
<td>USA</td>
<td>03-07</td>
<td>0.048</td>
<td>0.053</td>
<td>0.052</td>
<td>0.444</td>
<td>0.0224</td>
<td>0.440</td>
<td>024</td>
<td>0.43</td>
<td>0.65</td>
<td>01-07</td>
<td>0.59</td>
<td>26.1</td>
</tr>
<tr>
<td>Average EU-12</td>
<td>0.087</td>
<td>0.093</td>
<td>0.091</td>
<td>0.124</td>
<td>0.018</td>
<td>0.122</td>
<td>0.0125</td>
<td>01-07</td>
<td>0.21</td>
<td>0.30</td>
<td>0.12</td>
<td>55.9</td>
<td>4.3</td>
</tr>
<tr>
<td>Average OECD</td>
<td>0.079</td>
<td>0.083</td>
<td>0.081</td>
<td>0.174</td>
<td>0.018</td>
<td>0.170</td>
<td>0.0154</td>
<td>01-07</td>
<td>0.24</td>
<td>0.34</td>
<td>0.15</td>
<td>52.4</td>
<td>4.2</td>
</tr>
</tbody>
</table>

Notes: Regarding sectoral unemployment rates, job finding and separation rates for DNK, the period 1994-2004 has to be read 1994-1998 and 2002-2004; \(u_j\) is the sectoral unemployment rate (source: ILO); \(m_j^T\) and \(s_j^N\) are the monthly job finding and job destruction rates in sector \(j = T, N\), respectively (source: ILO); the monthly job destruction rate has been estimated by adopting the methodology developed by Shimer [2012] except for FRA, NLD, NOR and KOR; \(\theta_j\) is the labor market tightness in sector \(j\) (source: Eurostat for European countries, Labour Market Statistics from the Office for National Statistics for the U.K., Bureau of Labor Statistics for the U.S.); \(\rho\) is the average net unemployment benefit replacement rate over the period 1980-2007 (source: OECD Benefits and Wages Database); \(\tau\) (with \(x = \tau W\)) is the firing cost expressed in monthly salary equivalents and is averaged over the period 1980-2005 (source: Fondazione De Benedetti).
Denmark, Spain, Finland, the United Kingdom, Ireland, Italy, Sweden; EU-12 includes EU-10 along with France and the Netherlands.

We construct the following labor market variables:

- **Sectoral unemployment rate** denoted by \( u^j \) (\( j = T, N \)) is the number of unemployed workers \( U^j \) in sector \( j \) as a share of the labor force \( F^j \equiv L^j + U^j \) in this sector. Unemployed persons in industry \( j \) are those who lost their job in industry \( j \) according to BLS definition. LABORSTA database from ILO provides series for unemployed workers by economic activity for fifteen OECD countries out of eighteen in our sample. The longest available period ranges from 1987 to 2007. On average, our data covers 12.8 years per country. Series cover 18 sectors, according to ISIC Rev.3.1 classification. To construct \( L^j \) and \( U^j \) for \( j = T, N \), we map the classification used previously to compute series for sectoral wages, prices and real labor productivity indexes (see section B) into the 1-digit ISIC-rev.3 classification. The mapping was clear for all industries except for "Not classifiable by economic activity” (1-digit ISIC-Rev.3, code: X) when constructing \( L^j \) and \( U^j \), and, "Unemployed seeking their first job” to identify \( U^j \). These two categories have been split between tradables and non tradables according to the shares of total unemployment (excluding the two categories) between tradables and non tradables by year and country. In a few rare cases, the sum of sectoral unemployment provided by ILO did not correspond to total unemployment. These differences were usually due to missing data for some industries in the sectoral databases. In these cases, we added these differences in level, keeping however the share of each sector constant. In Table 7 we provide an overview of the classifications used to construct traded and non traded sectors. Once industries have been classified as traded or non traded, series for unemployed and employed workers are constructed by adding unemployed and employed workers of all sub-industries \( k \) in sector \( j = T, N \) in the form \( U^j = \sum_{k \in j} U_k \) and \( L^j = \sum_{k \in j} L_k \). Data coverage: AUS (1995-2007), AUT (1994-2007), BEL (2001-2007), CAN (1987-2007), DEU (1995-2007), DNK (1994-1998 and 2002-2004), ESP (1992-2007), FIN (1995-2007), GBR (1988-2007), IRL (1986-1997), ITA (1993-2007), JPN (2003-2007), KOR (1992-2007), SWE (1995-2007) and USA (2003-2007). Data for unemployed workers by economic activity are not available for FRA, NLD and NOR.

- **Sectoral labor market tightness** denoted by \( \theta^j \) (\( j = T, N \)) is calculated as the ratio of job vacancies in sector \( j \) (\( V^j \)) to the number of unemployed workers in that sector (\( U^j \)). To construct \( \theta^j \), we collect information on job vacancies and unemployed workers by economic activity. Sources for \( V^j \): Job Openings and Labor Turnover Survey (JOLTS) provided by the Bureau of Labor Statistics (BLS) for USA, Eurostat database (NACE 1-digit) for a range of European Countries, Labour Market Statistics from the Office for National Statistics for the UK. Sources for \( U^j \): Current Population Survey (CPS) published by the BLS for USA and LABORSTA (ILO) for European Countries.\(^{51}\) As shown in Table 7, the level of detail in the definition of traded and non traded sectors differs across databases in two dimensions. First, the number of items to split disaggregated data varies across nomenclatures from a low eleven categories in the Eurostat database to a high of eighteen items in the LABORSTA database. Second, the definitions of items are not harmonized across the different sets of data. To generate sectoral variables in a consistent and uniform way, series on disaggregated data for vacancies and unemployed workers are added up to form traded and non traded sectors following, as close as possible, the classification we used for value added, hours worked and labor compensation. Once industries have been classified as traded or non traded, series for employment vacancies (unemployed workers resp.) are constructed by adding job openings (unemployed workers resp.) of all sub-industries \( k \) in sector \( j = T, N \) in the form \( V^j = \sum_{k \in j} V_k \) (\( U^j = \sum_{k \in j} U_k \) resp.). Data coverage for \( V^j \) and \( U^j \): AUT (2004-2005), DEU (2006-2007), FIN (2002-2007), GBR (2001-2007), SWE (2005-2007) and USA (2001-2007).


- **Job finding rate** denoted by \( m^j \) (\( j = T, N \)) is computed at a sectoral level by adopting the methodology proposed by Shimer [2012]. As Shimer [2012], we ignore movements in and out of the overall labor force. Since we compute the job finding rate for the traded and the non traded sector, we have to further assume that labor force is fixed at a sectoral level, i.e., we ignore reallocation of labor across sectors. More details on the model and the derivation

\(^{51}\)The JOLTS and CPS databases provide (not seasonally adjusted) monthly data on vacancies and unemployed workers. We convert monthly data series into annual data series by summing the twelve monthly data points.
of the results below can be found in the Technical Appendix. The monthly job finding rate \( m^{j,<1}(t) \) for sector \( j \) at time \( t \) is computed as follows:

\[
m^{j,<1}(t) = -\ln \left(1 - M^{j,<1}(t)\right), \tag{42}
\]

where \( t \) indexes months and the probability of finding a job \( M^{j,<1} \) within one month is given by

\[
M^{j,<1}(t) = 1 - \frac{(1 - \alpha^{j,<1}(t))U^j(t)}{U^j(t-1)}, \tag{43}
\]

with \( \alpha^{j,<1} = \frac{U^{j,<1}(t)}{U^j(t)} \) the share of unemployment less than one month (\( U^{j,<1}(t) \)) among total monthly unemployment (\( U^j(t) \)) in sector \( j \). Source: LABORSTA database from ILO for data on employment and unemployment at the sectoral level, and, OECD for unemployment by duration.

- **Job destruction rate** denoted by \( s^j \) \( (j = T, N) \) is estimated by solving this equation:

\[
U^j(t) = \psi^j(t) \frac{s^j(t)}{s^j(t) + m^{j,<1}(t)} \left(U^j(t) + L^j(t)\right) + \left(1 - \psi^j(t)\right)U^j(t-1), \tag{44}
\]

where \( \psi^j \) is the monthly rate of convergence to the long-run sectoral unemployment rate:

\[
\psi^j(t) = 1 - e^{-\left(s^j(t) + m^{j,<1}(t)\right)} \tag{45}
\]

When estimating \( s^j \) by using (44), the unemployment rate has not necessarily reached its long-run equilibrium. Since we calibrate the model so that the initial steady state is consistent with the empirical properties of each OECD economy, we have computed values for \( s^j \) which are consistent with the steady-state sectoral unemployment rate \( u^j = \frac{s^j}{s^j + m^j} \) where \( u^j \) is the actual value taken from the data and \( m^j \) is computed by using (42). Reassuringly, average values for job destruction rates obtained from eq. (44) are close to those derived from the long-run equilibrium of the unemployment rate. More details can be found in the Technical Appendix.

- **Unemployment benefit net replacement rate** denoted by \( \varrho \) is shown in column 14 of Table 6 and is defined in section B. Replacement rates are averaged over 1980-2007 for all countries except Korea (2001-2007). Average EU-12 unemployment benefit replacement rate shown in Table 6 is the unweighted average of twelve EU members’ replacement rates. Source: OECD, Benefits and Wages Database.

- **Firing cost** denoted by \( \tau \) is shown in the last column of Table 6 is a measure of the strictness of legal protection against dismissals captured by the firing tax \( x = \tau \cdot W \) in our model; it is calculated as the sum of the average advance notice and average severance payment after 4 and 20 years of employment. \( \tau \) is expressed in monthly salary equivalents and is averaged over the period 1980-2005. Source: Fondazione de Benedetti.

Series of employment and unemployment by economic activity provided by ILO are not available for France, the Netherlands, Norway; while such data is available for Korea, unemployment by duration provided by the OECD is not available and thus prevents the estimation of the monthly job finding and job destruction rates. For these four countries, we proceeded as follows:


- **Unemployment rate** denoted by \( u \) is is the number of unemployed people as a percentage of the labor force. Coverage: FRA (1975-2004), the NLD (1983-2004), NOR (1983-2004). Source: OECD, LFS database.

- **Monthly job separation rate** denoted by \( s \) is computed so as to be consistent with the steady-state unemployment rate given by \( u = \frac{s}{s + m} \).

### D Mapping Theoretical Results into Elasticities Estimated Empirically

To map the deviation in percentage of \( p \) and \( \omega \) from their initial steady-state into elasticities estimated empirically, we need to adjust numerically computed values with a term that captures the extent to which search frictions vary across sectors. Once the discrepancy is accounted for, we are
### Table 7: Sectoral Classifications for Labor Market Variables

<table>
<thead>
<tr>
<th>Sector</th>
<th>EU KLEMS/STAN</th>
<th>LABORSTA Employment</th>
<th>LABORSTA Unemployment</th>
<th>JOLTS (BLS)</th>
<th>CPS (BLS)</th>
<th>EUROSTAT</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tradables</td>
<td>Agriculture, Hunting, Forestry and Fishing (A)</td>
<td>Agriculture, Hunting, Forestry and Fishing (A)</td>
<td>Agriculture, Hunting, Forestry and Fishing (A)</td>
<td>Mining and logging</td>
<td>Agriculture and fishing</td>
<td>mining and logging</td>
</tr>
<tr>
<td>Tradables</td>
<td>Mining and Quarrying (C)</td>
<td>Manufacturing (D)</td>
<td>Manufacturing (D)</td>
<td>Manufacturing (C)</td>
<td>Manufacturing</td>
<td>Manufacturing</td>
</tr>
<tr>
<td>Tradables</td>
<td>Transport and Storage and Communication (I)</td>
<td>Financial Intermediation (J)</td>
<td>Financial Intermediation (J)</td>
<td>Information</td>
<td>Financial activities</td>
<td>Financial Intermediation</td>
</tr>
<tr>
<td>Non-Tradables</td>
<td>Electricity, Gas and Water Supply (E)</td>
<td>Construction (F)</td>
<td>Wholesale and Retail Trade (G)</td>
<td>Construction (F)</td>
<td>Wholesale trade</td>
<td>Construction</td>
</tr>
<tr>
<td>Non-Tradables</td>
<td>Hotels and Restaurants (H)</td>
<td>Real Estate, Renting and Business Activities (K)</td>
<td>Compulsory Social Security (L)</td>
<td>Leisure and hospitality</td>
<td>Leisure and hospitality</td>
<td>Leisure and hospitality</td>
</tr>
<tr>
<td>Non-Tradables</td>
<td>Community Social and Personal Services (LtQ)</td>
<td>Education (M)</td>
<td>Education and health</td>
<td>Education and health</td>
<td>Other services</td>
<td>Other services</td>
</tr>
<tr>
<td>Unclassified</td>
<td>Self-employed, unincorporated and unpaid family workers</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
able to relate $\gamma$ and $\beta$ estimated empirically to their analytical counterpart which we denote by $\gamma^{\text{predict}}$ and $\beta^{\text{predict}}$, respectively.\footnote{The correction term for $\rho$ and $\omega$ is $\left(\frac{1 + \Theta_T^T}{\phi + \Theta_N^T}\right) \left[ 1 - \left(\frac{1 + \Theta_T^T}{1 + \Theta_N^T}\right) \tilde{\omega} \right]$ and \[ \left(\frac{\Omega^T - \Omega_N^T \left(\frac{1 + \Theta_T^T}{\phi + \Theta_N^T}\right)}{\phi + \Theta_N^T} \right) \left(\frac{1 + \Theta_T^T}{1 + \Theta_N^T}\right) \tilde{\rho} \right] \tilde{N}, \text{ respectively. It is worth mentioning that the magnitude of the bias originating from sector-varying search frictions is quantitatively low.}}

\begin{align}
\gamma^{\text{predict}} &= \left(\frac{1 + \Theta_T^T}{\phi + \Theta_N^T}\right) \left(1 + \frac{\Omega^T - \Omega_N^T \left(\frac{1 + \Theta_T^T}{\phi + \Theta_N^T}\right)}{\phi + \Theta_N^T} \right) \frac{\ln(1 - \nu_{NX}^N)}{\Phi^T - \Phi_N^T}, \quad (46a) \\
\beta^{\text{predict}} &= -\left[ \Omega^T - \Omega_N^T \left(\frac{1 + \Theta_T^T}{\phi + \Theta_N^T}\right) \right] + \left(\frac{\Omega^T - \Omega_N^T \left(\frac{1 + \Theta_T^T}{\phi + \Theta_N^T}\right)}{\phi + \Theta_N^T} \right) \frac{\ln(1 - \nu_{NX}^N)}{\Phi^T - \Phi_N^T}. \quad (46b)
\end{align}

where the second term on the RHS of (46a) and (46b) captures the negative impact on $p$ and $\omega$ of the long-run adjustment in net exports caused by rise in $A_T^T/A_N^N$. More details can be found in Technical Appendix J.

The numerical counterpart of $\sigma$ which corresponds to the estimated effect of higher relative productivity on the unemployment rate differential adjusted with the bias originating from sector-varying search frictions, i.e.,

\[ \sigma^{\text{predict}} = -\alpha_V \Delta^T + \alpha_V u^N \left(1 - u^N\right) \frac{\Sigma^N}{\phi + \Theta_N^T} \frac{\ln(1 - \nu_{NX}^N)}{\Phi^T - \Phi_N^T}, \quad (47) \]

where $\Delta^T = \left[u^T \left(1 - u^T\right) + \Omega^T - \Omega_N^T \left(1 - u_N^N\right) \left(1 - \frac{1 + \Theta_T^T}{\phi + \Theta_N^T}\right) \right]\left(\frac{1 + \Theta_T^T}{1 + \Theta_N^T}\right) \tilde{\omega} \tilde{N}$.

\section*{E More Empirical Results and Robustness Checks}

\subsection*{E.1 A First Glance at the Data}

We begin by examining the data for the 18 OECD economies over the period 1970-2007. Figure 7 plots the average relative price growth against the average relative wage growth which have been scaled (i.e., divided) by the average productivity growth differential between tradables and non tradables. Quantitatively, the BS model predicts that a productivity differential between tradables and non tradables of 1% leaves unaffected the relative wage of non tradables and appreciates the relative price of non tradables by 1%. Hence, according to the BS model, all countries should be positioned at point BS along the X-axis with coordinates (1,0). However, we find that all countries are positioned to the south-west of point BS. Quantitatively, we find that a productivity differential between tradables and non tradables by 1% is associated with a fall in the relative wage which varies between -0.02% for Belgium and -0.41% for Denmark. Regarding the relative price, we find that its increase of prices in traded industries such as 'Mining and Quarrying' (which accounts for about one fourth of GDP) over 1995-2007.

The data seem to challenge the conventional wisdom that labor mobility would gradually eliminate wage differences across sectors. If it were the case, the ratio of the non traded wage to the traded wage would remain unchanged. However, we observe that the relative wage tends to fall. Moreover, because non traded wages increase by a smaller amount than if labor were perfectly mobile, the relative price of non tradables appreciates by a smaller amount than suggested by the standard BS model. To confirm these findings, in the following, we have recourse to panel data unit root tests and cointegration methods.

\subsection*{E.2 Panel Unit Root Tests}

We test for the presence of unit roots in the logged relative wage $\omega$ (i.e., $w^N - w^T$) and in the difference between the (log) relative price $p$ (i.e., $p^T - p^N$) and the (log) relative productivities (i.e., $a_T - a_N^N$). If the wage equalization hypothesis was right, sectoral wages would increase at the same speed so that the relative wage of non tradables would be stationary. As a result, the non tradable unit labor cost would rise by the same amount as the productivity differential. Hence, the difference between the (logged) relative price and the (logged) relative productivity should be stationary as well.

We consider five panel unit root tests among those most commonly used in the literature: i) Levin, Lin and Chu’s [2002] test based on a homogenous alternative assumption, ii) a t-ratio type test statistic by Breitung [2000] for testing a panel unit root based on alternative detrending methods, iii) Im, Pesaran and Shin’s [2003] test that allows for a heterogeneous alternative, iv) Fisher type test by Maddala and Wu [1999], and v) Hadri [2000] who proposes a test of the null of stationarity against the alternative of a unit root in the panel data. Results are summarized in Table 8. Although
Figure 7: The Relative Price and the Relative Wage Growth. **Notes:** Figure 7 plots the annual average growth of the relative price of non tradables and the relative wage of non tradables, both scaled by the average productivity growth differential between tradables and non tradables, for each country of our sample over 1970-2007.

Table 8: Panel Unit Root Tests (p-values) for the relative wage and the relative price

<table>
<thead>
<tr>
<th>Test</th>
<th>Stat</th>
<th>Variables</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>(\omega)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(p)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(a^T - a^N)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(p - (a^T - a^N))</td>
</tr>
<tr>
<td>Levin et al. [2002]</td>
<td>t-stat</td>
<td>0.075</td>
</tr>
<tr>
<td>Breitung [2000]</td>
<td>t-stat</td>
<td>0.273</td>
</tr>
<tr>
<td>Im et al. [2003]</td>
<td>W-stat</td>
<td>0.558</td>
</tr>
<tr>
<td>Maddala and Wu [1999]</td>
<td>ADF</td>
<td>0.329</td>
</tr>
<tr>
<td></td>
<td>PP</td>
<td>0.289</td>
</tr>
<tr>
<td>Hadri [2000]</td>
<td>Z-\mu</td>
<td>0.000</td>
</tr>
</tbody>
</table>

Notes: For all tests, except for Hadri [2000], the null of a unit root is not rejected if p-value \(\geq 0.05\) at a 5% significance level. For Hadri [2000], the null of stationarity is rejected if p-value \(\leq 0.05\) at a 5% significance level. ADF and PP are the Maddala and Wu’s [1999] \(P\) test based on Augmented Dickey-Fuller and Phillips-Perron p-values respectively.

The time span of data is relatively short, we also ran these five panel unit root tests for sectoral unemployment rates along with the unemployment rate differential. Results are displayed in Table 10.

As shown in the first column Table 8, all panel unit root tests, reveal that the relative wage variable is non-stationary at a 5% significance level. This finding suggests that labor market frictions prevent wage equalization across sectors in the long run. Regarding the relative price of non tradables and the productivity of tradables relative to productivity of non tradables, these variables are found to be non-stationary. As shown in the last column, the difference between the relative price of non tradables and the relative productivity is integrated of order one which implies that the productivity differential is not fully reflected in the non tradable unit labor cost and thus the relative price. As can be seen in the first two columns of Table 10, sectoral unemployment rates are stationary, except for Hadri’s [2000] test.

The common feature of first generation tests is the restriction that all cross-sections are independent. We also consider some second generation unit root tests that allow cross-unit dependencies. We consider the tests developed by: i) Bai and Ng [2002] based on a dynamic factor model, ii) Choi [2001] based on an error-component model, iii) Pesaran [2007] based on a dynamic factor model and iv) Chang [2002] who proposes the instrumental variable nonlinear test. The results of second generation unit root tests are shown in Table 9.

In all cases, except for the Choi [2001] and Pesaran’s [2007] tests applied to \(\omega\) and \(p - (a^T - a^N)\), we fail to reject the presence of a unit root in the relative price, the relative wage, the productivity differential, and the difference \(p - (a^T - a^N)\), when cross-unit dependencies are taken into account.
Table 9: Panel Unit Root Tests (second generation) for the relative wage and the relative price

<table>
<thead>
<tr>
<th>Test</th>
<th>Stat</th>
<th>Variables</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bai and Ng [2002]</td>
<td>$Z_{d_e}$</td>
<td>$0.267$ 0.151 0.038 0.530</td>
</tr>
<tr>
<td></td>
<td>$P_{d_e}$</td>
<td>$0.251$ 0.150 0.050 0.498</td>
</tr>
<tr>
<td>Choi [2001]</td>
<td>$P_m$</td>
<td>$0.000$ 0.988 0.992 0.407</td>
</tr>
<tr>
<td></td>
<td>$Z$</td>
<td>$0.053$ 1.000 1.000 0.653</td>
</tr>
<tr>
<td></td>
<td>$L^*$</td>
<td>$0.047$ 1.000 1.000 0.662</td>
</tr>
<tr>
<td>Pesaran [2007]</td>
<td>$CIPS$</td>
<td>$0.010$ 0.320 0.450 0.015</td>
</tr>
<tr>
<td></td>
<td>$CIPS^*$</td>
<td>$0.010$ 0.320 0.450 0.015</td>
</tr>
<tr>
<td>Chang [2002]</td>
<td>$S_N$</td>
<td>$1.000$ 1.000 1.000 1.000</td>
</tr>
</tbody>
</table>

Notes: For all tests, the null of a unit root is not rejected if p-value $\geq 0.05$ at a 5% significance level. $r$ is the estimated number of common factors. For the idiosyncratic components, $P_{d_e}$ is a Fisher’s type statistic based on p-values of the individual ADF tests. Under $H_0$, $P_{d_e}$ has a $\chi^2$ distribution. $Z_{d_e}$ is the standardized Choi’s type statistic. Under $H_0$, $Z_{d_e}$ has a $N(0,1)$ distribution. For the idiosyncratic components, the estimated number of independent stochastic trends in the common factors is reported. The first estimated value is derived from the filtered test $MQ_c$ and the second one is derived from the corrected test $MQ_f$. The $P_m$ test is a modified Fisher’s inverse chi-square test. The $Z$ test is an inverse normal test. The $L^*$ test is a modified logit test. All these three statistics have a standard normal distribution under $H_0$. $CIPS$ is the mean of individual Cross sectionally ADF statistics (CADF). $CIPS^*$ denotes the mean of truncated individual CADF statistics. The $S_N$ statistic corresponds to the average of individual non-linear IV t-ratio statistics. It has a $N(0,1)$ distribution under $H_0$. Corresponding p-values are in parentheses.

Table 10: Panel Unit Root Tests (p-values) for sectoral unemployment rates

<table>
<thead>
<tr>
<th>Test</th>
<th>Stat</th>
<th>Variables</th>
</tr>
</thead>
<tbody>
<tr>
<td>Levin et al. [2002]</td>
<td>t-stat</td>
<td>$0.000$ 0.000 0.000</td>
</tr>
<tr>
<td>Breitung [2000]</td>
<td>t-stat</td>
<td>$0.049$ 0.045 0.000</td>
</tr>
<tr>
<td>Im et al. [2003]</td>
<td>W-stat</td>
<td>$0.000$ 0.003 0.000</td>
</tr>
<tr>
<td>Maddala and Wu [1999]</td>
<td>ADF</td>
<td>$0.000$ 0.003 0.000</td>
</tr>
<tr>
<td></td>
<td>PP</td>
<td>$0.000$ 0.000 0.000</td>
</tr>
<tr>
<td>Hadri [2000]</td>
<td>$Z_{d_e}$-stat</td>
<td>$0.074$ 0.051 0.013</td>
</tr>
</tbody>
</table>

Notes: For all tests, except for Hadri [2000], the null of a unit root is not rejected if p-value $\geq 0.05$ at a 5% significance level. For Hadri [2000], the null of stationarity is rejected if p-value $\leq 0.05$ at a 5% significance level. ADF and PP are the Maddala and Wu’s [1999] $P$ test based on Augmented Dickey-Fuller and Phillips-Perron p-values respectively.
Table 11: Panel cointegration tests results (p-values)

<table>
<thead>
<tr>
<th></th>
<th>wage equation</th>
<th>price equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Panel tests</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Non-parametric ν</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>Non-parametric ρ</td>
<td>0.012</td>
<td>0.003</td>
</tr>
<tr>
<td>Non-parametric t</td>
<td>0.004</td>
<td>0.002</td>
</tr>
<tr>
<td>Parametric t</td>
<td>0.046</td>
<td>0.000</td>
</tr>
<tr>
<td>Group-mean tests</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Non-parametric ρ</td>
<td>0.388</td>
<td>0.449</td>
</tr>
<tr>
<td>Non-parametric t</td>
<td>0.167</td>
<td>0.220</td>
</tr>
<tr>
<td>Parametric t</td>
<td>0.016</td>
<td>0.001</td>
</tr>
</tbody>
</table>

Notes: The null hypothesis of no cointegration is rejected if the p-value is below 0.05 (0.10 resp.) at 5% (10% resp.) significance level.

E.3 Cointegration Tests and Alternative Cointegration Estimates

To begin with, we report the results of parametric and non parametric cointegration tests developed by Pedroni ([1999]), ([2004]). We regress the (log) relative wage \( ω \) and the (log) relative price \( p \) on the (log) relative productivity, respectively:

\[
ω_{i,t} = δ_i + β \cdot (a_{i,t}^T - a_{i,t}^N) + v_{i,t}, \tag{48a}
\]

\[
p_{i,t} = α_i + γ \cdot (a_{i,t}^T - a_{i,t}^N) + u_{i,t}, \tag{48b}
\]

where \( i \) and \( t \) index country and time and \( v_{i,t} \) and \( u_{i,t} \) are i.i.d. error terms. Country fixed effects are captured by country dummies \( δ_i \) and \( α_i \).

Cointegration tests are based on the estimated residuals of equations (48a) and (48b). Table 11 reports the tests of the null hypothesis of no cointegration. All Panel tests reject the null hypothesis of no cointegration between \( p \) and \( a^T - a^N \) at the 1% significance level while three Panel tests reject the null hypothesis of no cointegration between \( ω \) and \( a^T - a^N \) at the 5% significance level. Group-mean parametric t-test confirms cointegration between \( p \) and the labor productivity differential and between \( ω \) and \( a^T - a^N \) at 5% and 1% significance level, respectively, while group-mean non parametric t-tests are somewhat less pervasive. Pedroni [2004] explores finite sample performances of the seven statistics. The results reveal that group-mean parametric t-test is more powerful than other tests in finite samples. By and large, panel cointegration tests provide evidence in favor of cointegration between the relative price and relative productivity, and between the relative wage and relative productivity.

As robustness checks, we compare our group-mean FMOLS estimates and group-mean DOLS estimates with one lag (\( q = 1 \)), with alternative estimators. First, we consider the group-mean DOLS estimator with 2 lags (\( q = 2 \)) and 3 lags (\( q = 3 \)). Second, we estimate cointegration relationships (48a) and (48b) using the panel DOLS estimator (Mark and Sul [2003]). We also use alternative econometric techniques to estimate cointegrating relationships (3): the dynamic fixed effects estimator (DFE), the mean group estimator (MG, Pesaran and Smith [1995]), the pooled mean group estimator (PMG, Pesaran et al. [1999]). All results are displayed in Table 12 and show that estimates of \( β \) and \( γ \) are close to those shown in Table 1 of the paper, except for the dynamic fixed effects estimator which suggests a fall in \( ω \) of 0.1% instead of 0.2%.

E.4 Estimating the Effects of Higher Relative Productivity

Kakkar [2003], Cardi and Restout [2015] estimate empirically the effects of higher productivity of tradables relative to non tradables by using cointegration techniques. Whist Kakkar [2003] focuses exclusively on the relative price effects of a productivity differential, Cardi and Restout [2015] also investigate empirically the long-run response of the relative wage. Like Cardi and Restout [2015], we estimate the relative price and relative wage effects but it differs along several dimensions. First, we measure technological change with sectoral labor productivity instead of sectoral TFP in order to be consistent with the model developed in section 3 where we abstract from physical capital accumulation. Second, our dataset includes eighteen OECD countries instead of fourteen. We provide below estimates for the whole sample and for each OECD country. Third, we are interested in the main text in the variations of the effects of a productivity differential across time, space and stages of the business cycle. Fourth, we analyze the effects of higher relative productivity on the unemployment differential.
Table 12: Alternative Cointegration Estimates of $\beta$ and $\gamma$

<table>
<thead>
<tr>
<th></th>
<th>Relative wage eq. (48)</th>
<th>Relative price eq. (48a)</th>
</tr>
</thead>
<tbody>
<tr>
<td>DOLS ($q = 2$)</td>
<td>$-0.223^{a}$ 0.000</td>
<td>$0.658^{a}$ (77.95)</td>
</tr>
<tr>
<td></td>
<td>$(-27.69)$</td>
<td></td>
</tr>
<tr>
<td>DOLS ($q = 3$)</td>
<td>$-0.220^{a}$ 0.000</td>
<td>$0.673^{a}$ (79.22)</td>
</tr>
<tr>
<td></td>
<td>$(-26.77)$</td>
<td></td>
</tr>
<tr>
<td>DOLS ($q = 4$)</td>
<td>$-0.218^{a}$ 0.000</td>
<td>$0.678^{a}$ (84.96)</td>
</tr>
<tr>
<td></td>
<td>$(-26.51)$</td>
<td></td>
</tr>
<tr>
<td>DFE</td>
<td>$-0.105^{a}$ 0.006</td>
<td>$0.697^{a}$ (13.35)</td>
</tr>
<tr>
<td></td>
<td>$(-2.51)$</td>
<td></td>
</tr>
<tr>
<td>MG</td>
<td>$-0.145^{a}$ 0.000</td>
<td>$0.608^{a}$ (17.25)</td>
</tr>
<tr>
<td></td>
<td>$(-7.63)$</td>
<td></td>
</tr>
<tr>
<td>PMG</td>
<td>$-0.164^{a}$ 0.000</td>
<td>$0.668^{a}$ (31.03)</td>
</tr>
<tr>
<td></td>
<td>$(-10.59)$</td>
<td></td>
</tr>
<tr>
<td>Panel DOLS ($q = 1$)</td>
<td>$-0.214^{a}$ 0.000</td>
<td>$0.621^{a}$ (22.39)</td>
</tr>
<tr>
<td></td>
<td>$(-6.32)$</td>
<td></td>
</tr>
<tr>
<td>Panel DOLS ($q = 2$)</td>
<td>$-0.216^{a}$ 0.000</td>
<td>$0.620^{a}$ (22.62)</td>
</tr>
<tr>
<td></td>
<td>$(-6.85)$</td>
<td></td>
</tr>
<tr>
<td>Panel DOLS ($q = 3$)</td>
<td>$-0.213^{a}$ 0.000</td>
<td>$0.624^{a}$ (23.88)</td>
</tr>
<tr>
<td></td>
<td>$(-6.42)$</td>
<td></td>
</tr>
</tbody>
</table>

Notes: All regressions include country fixed effects. Heteroskedasticity and autocorrelation consistent t-statistics are reported in parentheses. * denotes significance at 1% level. The columns $t(\beta = 0)$ and $t(\gamma = 1)$ report the p-value of the test of $H_0 : \beta = 0$ and $H_0 : \gamma = 1$ respectively.

Since $p$, $\omega$ and $a^T - a^N$ display trends, we ran unit root and then cointegration tests. Having verified that these two assumptions are empirically supported, we estimate the cointegrating relationships by using fully modified OLS (FMOLS) and dynamic OLS (DOLS) procedures for the cointegrated panel proposed by Pedroni [2000], [2001]. Both estimators give similar results and coefficients $\beta$ and $\gamma$ of the cointegrating relationships are significant at 1%. In Table 13, we report results for DOLS estimator. Two major results emerge. First, estimates reported in the first column of Table 13 reveal that a productivity differential between tradables and non tradables by 1% lowers the relative wage by about 0.22% and appreciates the relative price by 0.64%. Second, as shown in the second line of panel A and B in Table 13, the predictions of the model abstracting from labor market frictions are strongly rejected: the slope of the cointegrating vector $\beta$ ($\gamma$) is statistically significantly different from zero (one).

We now assess if our conclusion for the whole sample also holds for each country. To do so we run again the regression of relative wage and relative price on relative productivity by letting $\beta$ and $\gamma$ vary across countries. Table 14 shows DOLS and FMOLS estimates for the eighteen countries of our sample. The first result that emerges is that the responses display a wide dispersion across countries. The second result is that despite these large cross-country variations, higher productivity in tradables relative to non tradables significantly lowers $\omega$ in all countries while $p$ rises less than the productivity differential.

Because long-run movements in both the relative wage and relative price reveal the presence of labor market frictions, we also run the regression of the change in the unemployment rate differential between tradables and non tradables on the relative productivity of tradables in growth rate:\footnote{Since time series for the unemployment rate differential do not display a unit root process, we express labor productivity in growth rate. Moreover, on average, the time horizon is too short to recourse to cointegration techniques.}

$$
   du_{it}^T - du_{it}^N = \eta_i + \sigma \cdot (\tilde{a}_{it}^T - \tilde{a}_{it}^N) + z_{it},
$$

(49)

where $\eta_i$ are the country fixed effects and $z_{it}$ are i.i.d. error terms. As can be seen in the first line of Table 17, a 1% increase in the relative productivity of tradables lowers the unemployment rate in the traded relative to the non traded sector by 0.034 percentage point. Columns 2 to 4 reveal that our result is robust to the inclusion of control variables for labor market regulation and thus sectoral unemployment rates adjust unevenly in all specifications.\footnote{In the second (third) column of Table 17, we include employment protection legislation adjusted with the share of permanent workers (unemployment benefit replacement rate) since these variables are available for a yearly basis. The fourth column shows that results are unchanged when we add two control variables.}

### E.5 Split-Sample Analysis

In this subsection, we provide more details about the split-sample analysis we perform in the main text in order to differentiate the effects of a productivity differential according to the degree of labor
Table 13: Panel Cointegration DOLS Estimates of $\beta$ and $\gamma$ for Sub-Samples when the Split is Based on Sample Mean

<table>
<thead>
<tr>
<th>A. Relative Wage</th>
<th>Sub-Samples</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\varrho$</td>
</tr>
<tr>
<td>$\beta$</td>
<td>$-0.223^a$</td>
</tr>
<tr>
<td>$t(\hat{\beta}) = 0$</td>
<td>0.000</td>
</tr>
<tr>
<td>$\beta^H$</td>
<td>$-0.261^a$</td>
</tr>
<tr>
<td>$\beta^L$</td>
<td>$-0.158^a$</td>
</tr>
<tr>
<td>$t(\hat{\beta}^L = \hat{\beta}^H)$</td>
<td>0.000</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>B. Relative Price</th>
<th>Sub-Samples</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\gamma$</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>$0.646^a$</td>
</tr>
<tr>
<td>$t(\hat{\gamma}) = 1$</td>
<td>0.000</td>
</tr>
<tr>
<td>$\gamma^H$</td>
<td>$0.791^a$</td>
</tr>
<tr>
<td>$\gamma^L$</td>
<td>$1.123^a$</td>
</tr>
<tr>
<td>$t(\hat{\gamma}^L = \hat{\gamma}^H)$</td>
<td>0.000</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Observations</td>
<td>680</td>
<td>642</td>
<td>642</td>
<td>414</td>
<td>390</td>
</tr>
<tr>
<td>Countries</td>
<td>18</td>
<td>17</td>
<td>17</td>
<td>18</td>
<td>17</td>
</tr>
<tr>
<td>mean LMR (high)</td>
<td>$-0.609$</td>
<td>0.864</td>
<td>2.280</td>
<td>1.376</td>
<td></td>
</tr>
<tr>
<td>mean LMR (low)</td>
<td>$-0.391$</td>
<td>0.491</td>
<td>1.296</td>
<td>-0.512</td>
<td></td>
</tr>
</tbody>
</table>

Notes: $^a$, $^b$ and $^c$ denote significance at 1%, 5% and 10% levels. Heteroskedasticity and autocorrelation consistent standard errors are reported in parentheses. The rows $t(\hat{\beta}) = 0$ and $t(\hat{\gamma}) = 1$ report the p-value of the test of $H_0: \beta = 0$ and $H_0: \gamma = 1$ respectively. To investigate whether LMR influences the responses of the relative wage, $\beta$, and the relative price, $\gamma$, to a productivity differential, we split the sample of 18 OECD countries into two subsamples by using the sample mean and run the regressions (48a)-(48b) for the high and low-labor market regulation countries. $\beta^H$ ($\beta^L$) and $\gamma^H$ ($\gamma^L$) capture the responses of the relative wage and the relative price, respectively, in countries with high (low) labor market regulation. The row $t(\hat{\beta}^L = \hat{\beta}^H)$ ($t(\hat{\gamma}^L = \hat{\gamma}^H)$) reports the p-value of the test of $H_0: \beta^L = \beta^H$ ($\gamma^L = \gamma^H$). '$\varrho$' is the unemployment benefits replacement rate, 'EPL$_{adj}$' the strictness of employment protection against dismissals adjusted with the share of permanent workers, 'BargCov' the bargaining coverage and 'LMR' the labor market regulation index obtained by using a principal component analysis.
Table 14: Panel Cointegration Estimates of $\beta_i$ and $\gamma_i$ for Each Country (eqs. (48a)-(48b))

<table>
<thead>
<tr>
<th>Country</th>
<th>$\beta_i^\text{DOLS}$</th>
<th>$\beta_i^\text{MOLS}$</th>
<th>$\hat{\gamma}_i^\text{DOLS}$</th>
<th>$\hat{\gamma}_i^\text{MOLS}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>AUS</td>
<td>$-0.047$</td>
<td>$-0.062^a$</td>
<td>$0.567^a$</td>
<td>$0.559^a$</td>
</tr>
<tr>
<td></td>
<td>($-1.51$)</td>
<td>($-2.19$)</td>
<td>($10.95$)</td>
<td>($10.88$)</td>
</tr>
<tr>
<td>AUT</td>
<td>$-0.220^a$</td>
<td>$-0.231^a$</td>
<td>$0.687^a$</td>
<td>$0.689^a$</td>
</tr>
<tr>
<td></td>
<td>($-12.62$)</td>
<td>($-19.95$)</td>
<td>($20.14$)</td>
<td>($21.89$)</td>
</tr>
<tr>
<td>BEL</td>
<td>$-0.150^a$</td>
<td>$-0.135^a$</td>
<td>$0.732^a$</td>
<td>$0.740^a$</td>
</tr>
<tr>
<td></td>
<td>($-6.36$)</td>
<td>($-5.74$)</td>
<td>($17.49$)</td>
<td>($17.52$)</td>
</tr>
<tr>
<td>CAN</td>
<td>$-0.298^a$</td>
<td>$-0.299^a$</td>
<td>$0.549^a$</td>
<td>$0.524^a$</td>
</tr>
<tr>
<td></td>
<td>($-6.11$)</td>
<td>($-7.19$)</td>
<td>($4.95$)</td>
<td>($5.10$)</td>
</tr>
<tr>
<td>DEU</td>
<td>$-0.502^a$</td>
<td>$-0.493^a$</td>
<td>$0.532^a$</td>
<td>$0.517^a$</td>
</tr>
<tr>
<td></td>
<td>($-20.69$)</td>
<td>($-22.96$)</td>
<td>($9.76$)</td>
<td>($10.70$)</td>
</tr>
<tr>
<td>DNK</td>
<td>$-0.366^a$</td>
<td>$-0.355^a$</td>
<td>$0.361^a$</td>
<td>$0.357^a$</td>
</tr>
<tr>
<td></td>
<td>($-4.96$)</td>
<td>($-5.86$)</td>
<td>($9.51$)</td>
<td>($12.64$)</td>
</tr>
<tr>
<td>ESP</td>
<td>$-0.231^a$</td>
<td>$-0.236^a$</td>
<td>$0.689^a$</td>
<td>$0.709^a$</td>
</tr>
<tr>
<td></td>
<td>($-8.30$)</td>
<td>($-11.10$)</td>
<td>($19.14$)</td>
<td>($21.50$)</td>
</tr>
<tr>
<td>FIN</td>
<td>$-0.197^a$</td>
<td>$-0.193^a$</td>
<td>$0.645^a$</td>
<td>$0.628^a$</td>
</tr>
<tr>
<td></td>
<td>($-11.14$)</td>
<td>($-12.99$)</td>
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<td>($23.02$)</td>
</tr>
<tr>
<td>FRA</td>
<td>$-0.396^a$</td>
<td>$-0.395^a$</td>
<td>$0.787^a$</td>
<td>$0.790^a$</td>
</tr>
<tr>
<td></td>
<td>($-6.56$)</td>
<td>($-7.00$)</td>
<td>($29.79$)</td>
<td>($31.01$)</td>
</tr>
<tr>
<td>GBR</td>
<td>$-0.152^b$</td>
<td>$-0.161^a$</td>
<td>$0.842^a$</td>
<td>$0.810^a$</td>
</tr>
<tr>
<td></td>
<td>($-2.35$)</td>
<td>($-2.94$)</td>
<td>($6.63$)</td>
<td>($7.41$)</td>
</tr>
<tr>
<td>IRL</td>
<td>$-0.187^a$</td>
<td>$-0.193^a$</td>
<td>$0.554^a$</td>
<td>$0.562^a$</td>
</tr>
<tr>
<td></td>
<td>($-3.64$)</td>
<td>($-4.20$)</td>
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<td>($19.20$)</td>
</tr>
<tr>
<td>ITA</td>
<td>$-0.265^a$</td>
<td>$-0.282^a$</td>
<td>$0.761^a$</td>
<td>$0.727^a$</td>
</tr>
<tr>
<td></td>
<td>($-10.04$)</td>
<td>($-11.74$)</td>
<td>($23.91$)</td>
<td>($24.34$)</td>
</tr>
<tr>
<td>JPN</td>
<td>$-0.161^a$</td>
<td>$-0.157^a$</td>
<td>$0.879^a$</td>
<td>$0.898^a$</td>
</tr>
<tr>
<td></td>
<td>($-8.05$)</td>
<td>($-9.29$)</td>
<td>($42.50$)</td>
<td>($41.06$)</td>
</tr>
<tr>
<td>KOR</td>
<td>$-0.403^a$</td>
<td>$-0.393^a$</td>
<td>$0.529^a$</td>
<td>$0.532^a$</td>
</tr>
<tr>
<td></td>
<td>($-10.77$)</td>
<td>($-12.53$)</td>
<td>($40.46$)</td>
<td>($45.56$)</td>
</tr>
<tr>
<td>NLD</td>
<td>$-0.331^a$</td>
<td>$-0.307^a$</td>
<td>$0.724^a$</td>
<td>$0.731^a$</td>
</tr>
<tr>
<td></td>
<td>($-5.90$)</td>
<td>($-5.82$)</td>
<td>($15.95$)</td>
<td>($18.04$)</td>
</tr>
<tr>
<td>NOR</td>
<td>$-0.071^a$</td>
<td>$-0.081^a$</td>
<td>$0.094$</td>
<td>$0.034$</td>
</tr>
<tr>
<td></td>
<td>($-5.84$)</td>
<td>($-6.17$)</td>
<td>($0.75$)</td>
<td>($0.29$)</td>
</tr>
<tr>
<td>SWE</td>
<td>$-0.020$</td>
<td>$-0.009$</td>
<td>$0.908^a$</td>
<td>$0.882^a$</td>
</tr>
<tr>
<td></td>
<td>($-0.66$)</td>
<td>($-0.52$)</td>
<td>($11.23$)</td>
<td>($18.13$)</td>
</tr>
<tr>
<td>USA</td>
<td>$-0.017$</td>
<td>$-0.033$</td>
<td>$0.784^a$</td>
<td>$0.765^a$</td>
</tr>
<tr>
<td></td>
<td>($-0.69$)</td>
<td>($-1.47$)</td>
<td>($23.50$)</td>
<td>($24.80$)</td>
</tr>
<tr>
<td>EU-12</td>
<td>$-0.252^a$</td>
<td>$-0.249^a$</td>
<td>$0.685^a$</td>
<td>$0.679^a$</td>
</tr>
<tr>
<td></td>
<td>($-26.89$)</td>
<td>($-30.24$)</td>
<td>($58.20$)</td>
<td>($64.78$)</td>
</tr>
</tbody>
</table>

All sample  $-0.223^a$  $-0.223^a$  $0.646^a$  $0.636^a$
$(-29.72)$  $(-33.85)$  $(76.54)$  $(83.04)$

Notes: Heteroskedasticity and autocorrelation consistent t-statistics are reported in parentheses. $^a$, $^b$ and $^c$ denote significance at 1%, 5% and 10% levels.
E.5.1 Relative Wage and Relative Price Effects of Higher Relative Productivity of Tradables: Implications of Labor Market Regulation

To empirically explore the implications of labor market regulation for the effects of a productivity differential between tradables and non tradables, we apply cointegration techniques and perform a simple split-sample analysis. We consider three indicators which capture the extent of regulation on labor markets: the unemployment benefit replacement rate, the collective bargaining coverage, and the employment protection legislation index. We also have recourse to a principal component analysis to construct an indicator that gives a more accurate measure of the degree of labor market regulation. Source and data construction are detailed in section A. We take the median to split the sample of 18 countries in 9 countries with high and 9 with low labor market regulation. Table 15 shows values of each labor market indicator for each country. For each indicator, countries are ranked in decreasing order.

We first compare the relative wage behavior of 9 countries with high and 9 economies with low labor market regulation by running the regression of the relative wage on relative productivity for each sub-sample:

$$\omega_{i,t} = \delta + \beta^H (a_{it}^T - a_{it}^N) + v_{i,t}, \quad c = H, L,$$

(50)

where $\beta^H$ ($\beta^L$) captures the response of the relative wage to a productivity differential in countries with higher (lower) labor market regulation.

We adopt a similar approach for the relative price. Because the movements of non tradables can be influenced by changes in the cost of entry in product market triggered by competition-oriented policies, we add country-specific linear time trends when we run the regression for each sub-sample in order to control for these effects:

$$p_{i,t} = \delta + \alpha_i + \gamma^H (a_{it}^T - a_{it}^N) + u_{i,t}, \quad c = H, L,$$

(51)

where $\gamma^H$ ($\gamma^L$) captures the response of the relative price to a productivity differential in countries where the index that captures the extent of labor market regulation is above (below) the median. Because the movements in $p$ can be influenced by changes in the cost of entry in product market triggered by competition-oriented policies, we add country-specific linear time trends, $\alpha_i$, when we run the regression (51) for each sub-sample in order to control for these effects.

Building on our model’s predictions, we expect the relative wage to decline more (i.e., $|\beta^H|$ is expected to take higher values) and the relative price to appreciate less (i.e., $|\gamma^H|$ is expected to take lower values) in countries where the unemployment benefit scheme is more generous (i.e., $\text{BargCov}$ is higher). While we expect the relative wage to decline more in countries with strictness legislation against dismissals (i.e., $\text{EP L_adj}$ takes higher values), the relative price should appreciate by a larger amount. While estimates shown in Table 13 corroborate all of our hypothesis related to the implications of labor market regulation for the relative wage and relative price effects of a productivity differential, Table 16 shows results when we base the split-sample analysis on sample mean for the three dimensions of labor market regulation. Reassuringly, all of our conclusions hold when we base the split of the sample of 18 OECD countries on sample mean. In a nutshell, our results are robust to the threshold used to perform the split-sample analysis.

E.5.2 Effect on Unemployment Rate Differential of Higher Relative Productivity of Tradables: Implications of Labor Market Regulation

One prediction of the two-sector model with search frictions developed in the paper is that a productivity differential between tradables and non tradables lowers the unemployment rate in both the traded and non traded sector, the decline of the former being larger than that of the latter. When we investigate the implications of labor market regulation, our model also predicts that the decline in the unemployment rate differential between tradables and non tradables following higher relative productivity of tradables is more pronounced in countries where labor markets are more regulated. To test these predictions, we proceed in two stages.

Firstly, indexing countries and time by $i$ and $t$ respectively, we explore the following relationship empirically:

$$du_{it}^T - du_{it}^N = \eta_i + \sigma_i (a_{it}^T - a_{it}^N) + \lambda_i LMR_{it} + z_{i,t},$$

(52)

where $\eta_i$ are the country fixed effects and $z_{i,t}$ are i.i.d. error terms. The dependent variable is the difference between the change in the unemployment rate in the traded sector and the change in the unemployment rate in the non traded sector (so that the unemployment rate differential is expressed in percentage point); we construct the productivity differential by taking growth rates in order to remove the time trend, i.e., $a_{it}^T - a_{it}^N$, since $a_{it}^T - a_{it}^N$ displays a unit root process, see section E.2.
### Table 15: Split-Sample Analysis: Labor Market Indicators

<table>
<thead>
<tr>
<th>Collective Bargaining Coverage</th>
<th>Unemployment Benefit Replacement Rate</th>
<th>Employment Protection Legislation</th>
<th>Labor Market Regulation</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
<td>(4)</td>
</tr>
<tr>
<td>AUT</td>
<td>97.10</td>
<td>DNK</td>
<td>72.21</td>
</tr>
<tr>
<td>BEL</td>
<td>94.22</td>
<td>BEL</td>
<td>66.86</td>
</tr>
<tr>
<td>SWE</td>
<td>89.08</td>
<td>NLD</td>
<td>66.70</td>
</tr>
<tr>
<td>FIN</td>
<td>86.07</td>
<td>GBR</td>
<td>63.04</td>
</tr>
<tr>
<td>FRA</td>
<td>85.38</td>
<td>DEU</td>
<td>61.39</td>
</tr>
<tr>
<td>NLD</td>
<td>84.50</td>
<td>FIN</td>
<td>59.33</td>
</tr>
<tr>
<td>ITA</td>
<td>83.26</td>
<td>IRL</td>
<td>53.65</td>
</tr>
<tr>
<td>ESP</td>
<td>82.45</td>
<td>DK</td>
<td>53.60</td>
</tr>
<tr>
<td>AUS</td>
<td>75.51</td>
<td>JPN</td>
<td>51.24</td>
</tr>
<tr>
<td>DEU</td>
<td>69.38</td>
<td>ESP</td>
<td>47.18</td>
</tr>
<tr>
<td>IRL</td>
<td>57.58</td>
<td>FR</td>
<td>47.18</td>
</tr>
<tr>
<td>NOR</td>
<td>44.83</td>
<td>NLD</td>
<td>43.18</td>
</tr>
<tr>
<td>CAN</td>
<td>35.75</td>
<td>CAN</td>
<td>41.34</td>
</tr>
<tr>
<td>GBR</td>
<td>24.15</td>
<td>USA</td>
<td>25.72</td>
</tr>
<tr>
<td>JPN</td>
<td>15.50</td>
<td>KOR</td>
<td>7.68</td>
</tr>
</tbody>
</table>

Mean 65.60 Mean 49.91 Mean 1.79 Mean 0.40


### Table 16: Panel Cointegration Estimates of $\beta$ and $\gamma$ for Sub-Samples

<table>
<thead>
<tr>
<th>LMR</th>
<th>$\beta$</th>
<th>BargCov</th>
<th>EPL$^{adj}$</th>
<th>LMR</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>DOLS</td>
<td>FMOLS</td>
<td>DOLS</td>
<td>FMOLS</td>
</tr>
<tr>
<td>A. Relative Wage</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\beta^H$</td>
<td>$-0.261^a$</td>
<td>$-0.255^a$</td>
<td>$-0.233^a$</td>
<td>$-0.168^a$</td>
</tr>
<tr>
<td></td>
<td>(−23.04)</td>
<td>(−25.65)</td>
<td>(−27.28)</td>
<td>(−30.59)</td>
</tr>
<tr>
<td>$\beta^L$</td>
<td>$-0.158^a$</td>
<td>$-0.166^a$</td>
<td>$-0.163^a$</td>
<td>$-0.116^a$</td>
</tr>
<tr>
<td></td>
<td>(−16.34)</td>
<td>(−19.14)</td>
<td>(−9.32)</td>
<td>(−11.23)</td>
</tr>
<tr>
<td>$t(\hat{\beta}_L = \hat{\beta}_H)$</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>B. Relative Price</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\gamma^H$</td>
<td>0.791$^a$</td>
<td>0.776$^a$</td>
<td>0.754$^a$</td>
<td>0.713$^a$</td>
</tr>
<tr>
<td></td>
<td>(6.37)</td>
<td>(7.15)</td>
<td>(10.19)</td>
<td>(10.90)</td>
</tr>
<tr>
<td>$\gamma^L$</td>
<td>1.123$^a$</td>
<td>1.037$^a$</td>
<td>1.410$^b$</td>
<td>1.346$^b$</td>
</tr>
<tr>
<td></td>
<td>(12.81)</td>
<td>(13.60)</td>
<td>(8.96)</td>
<td>(9.92)</td>
</tr>
<tr>
<td>$t(\hat{\gamma}_L = \hat{\gamma}_H)$</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>Countries</td>
<td>17</td>
<td>17</td>
<td>18</td>
<td>17</td>
</tr>
<tr>
<td>Observations</td>
<td>642</td>
<td>642</td>
<td>414</td>
<td>390</td>
</tr>
<tr>
<td>mean LMR (high)</td>
<td>0.609</td>
<td>0.823</td>
<td>2.221</td>
<td>1.280</td>
</tr>
<tr>
<td>mean LMR (low)</td>
<td>0.391</td>
<td>0.365</td>
<td>1.108</td>
<td>-0.964</td>
</tr>
</tbody>
</table>

Notes: $^a$ and $^b$ denote significance at 1% and 5% levels. To investigate whether labor market regulation influences the responses of the relative wage, $\beta$, and the relative price, $\gamma$, to a productivity differential, we split the sample of 18 OECD countries into two subsamples and run the regressions (50)-(51) for the high and low-labor market regulation countries. $\beta^H$ ($\beta^L$) and $\gamma^H$ ($\gamma^L$) capture the responses of the relative wage and the relative price, respectively, in countries with high (low) labor market regulation. The row $t(\hat{\beta} = \bar{\beta}^H)$ ($t(\hat{\gamma}_L = \bar{\gamma}^H)$) reports the p-value of the test of $H_0: \beta = \bar{\beta}^H$ ($\gamma = \bar{\gamma}^H$). 'EPL$^{adj}$' the strictness of employment protection against dismissals adjusted with the share of permanent workers, BargCov the bargaining coverage and 'LMR' the labor market regulation index obtained by using a principal component analysis.

Since sectoral unemployment rates can be directly affected by labor market regulation, we add a control $LMR_{it}$, which varies over time. Because bargaining coverage is available on a yearly basis for four countries only, whilst data availability is erratic for the rest of the countries, we do not include this
indicator in our analysis. On the contrary, the adjusted employment protection legislation index, \(ELP_{adj}\), and the unemployment benefit replacement rate, \(\varrho\), are available on a yearly basis since 1985, except Korea.

Turning to the implications of labor market regulation, we perform a split-sample analysis on the basis of the labor market regulation index (LMR) obtained by running a principal component analysis. The number of observations of the sub-sample of countries with high (low) labor market regulation is 94 (70). We estimate the regression (53) for countries with high or low LMR, without (column 1) or with one (columns 2 and 3) or two (column 4) labor market control variables: \(\sigma^H (\sigma^L)\) captures the response of the unemployment rate differential between tradables and non tradables, respectively, in countries with high (low) labor market regulation. \(EPL_{adj}\) is the strictness of employment protection against dismissals adjusted with the share of permanent workers, \(\varrho\) is the unemployment benefits replacement rate.

### E.6 State-Dependency Effects of Higher Relative Productivity of Tradables

In the main text, we differentiate the effect of higher relative productivity of tradables relative to non tradables on the relative wage, the relative price of non tradables and the relative unemployment rate of tradables across stages of the business cycle. We provide below more details about data construction and consider recessions/expansions of lower durations whilst in the main text, we restrict attention to recessions and expansions which last at least three years.

**Identifying Recession and Expansion Periods.** In order to contrast the effects of technology shocks biased toward the traded sector in expansions with those in recessions, we have to identify the state of the economy across the business cycle. Following standard practice (see Riera-Crichton, Vegh, and Vuletin [2015] for instance), we define a recession period as a situation where the output gap declines, i.e., the economy is moving from its peak to trough, and an expansion...
period as a situation where the output gap increases, i.e., the economy is moving from its trough to peak. Denoting real GDP in country $i$ at time $t$ by $Y_{it}$ and logged real GDP with low case letters, $y_{it}$, applying a Hodrick-Prescott filter with a smoothing parameter of $\lambda = 100$ (as we use annual data), we obtain a measure of the output gap which allows us to identify expansions and recessions:

\[
\begin{align*}
\text{Expansions:} & \quad dy_{it} - d\tilde{y}_{it} > 0, \\
\text{Recessions:} & \quad dy_{it} - d\tilde{y}_{it} < 0,
\end{align*}
\]

where $\tilde{y}_{it}$ is the potential GDP at time $t$ in country $i$.

This measure of the state of the economy can be criticized on the grounds that recession periods are not necessarily periods of high unemployment, see Ramey and Zubairy [2017]. We alternatively identify expansion and recession periods by calculating the difference between actual unemployment (as a share of the labor force), denoted by $u_{it}$, and the natural rate of unemployment, $\bar{u}_{it}$:

\[
\begin{align*}
\text{Expansions:} & \quad u_{it} - \bar{u}_{it} < 0, \\
\text{Recessions:} & \quad u_{it} - \bar{u}_{it} > 0,
\end{align*}
\]

where $\bar{u}_{it}$ is obtained by applying a Hodrick-Prescott filter with a smoothing parameter of $\lambda = 100$ to the time series of unemployment rate, $u_{it}$. Definition (55) implies that unemployment increases in recessions and declines in expansions.

**Descriptive Statistics.** Before turning to estimates, it is useful to look at the descriptive statistics related to expansions and recessions summarized in Table 18 for the output gap dummy and in Table 19 for the unemployment rate gap dummy. In each table, columns (1) and (2) ((3) and (4)) resp.) give for each of the 18 OECD countries of our panel the percentage of time spent in expansion and recession states which last at least two (three resp.) consecutive years. Using the same thresholds of durations, columns (5) to (8) gives the average duration of episodes of expansion and recession. For all columns, the last line shows the 18 OECD countries’ average. Some features deserve some comments.

First, data indicate that for both measures of the state of the business cycle the time spent in expansions which last at least two years amounts to 52% on average (see column (1) of Tables 19 and 20). Notably, our estimates are well in line with that reported in Riera-Crichton, Vegh, and Vuletin [2015] who document that industrial countries spend, on average, 50% of the time in an expansionary regime. Moreover, the number of years identified as recessions in higher with the unemployment rate gap dummy (45%) than that when using the output gap dummy (35%). This discrepancy between our two measures of the state of the business cycle may reflect the high degree of persistence of the unemployment rate during bad times.\(^5\) Second, when we restrict attention to episodes of recessions and expansions which last at least three consecutive years, a typical OECD economy spends on average 42% of the time in expansion for our two measures of the state of the economy. The corresponding figures for persistent recessions measured with the output gap and the unemployment rate gap are 23 and 48 percent, respectively. Once again, time spent in bad times is found to be higher when we use the unemployment rate gap to identify the state of the economy in the business cycle. Third, columns (5) and (6) of Table 18 reveal that a typical business cycle identified with the output gap dummy has a duration of 6.4 years including an expansion of 3.4 years and a recession of 3 years. When using the unemployment rate gap dummy, the duration of the cycle is higher with a cross-country average of 9.2 years characterized by an expansion of 4.3 years and a recession of 4.9 years (see Table 19). Obviously, dropping short expansions and recessions of one or two years, increases the average duration of each state of the economy as shown in columns (7) and (8) of both tables.

**Empirical Strategy.** Once we have identified periods of expansion and recession for each OECD country, we conduct a split-sample analysis to assess the role of the state of economy for the transmission of higher relative productivity of tradables. Hence, we compare the elasticity of variable $x = \omega, p, u^{T} - u^{N}$ for periods of expansion with the elasticity for periods of recession. We run the regression for each sub-sample:

\[
x^{s,x}_{it} = \delta^{x}_{s} + \alpha^{x} \cdot \text{productivity differential}_{s,t}^{x} + \epsilon^{x}_{s,t},
\]

\(^{55}\)The figures in columns (1) and (2) (along with (3) and (4)) do not sum up to 1 because by considering only expansions and recessions which last at least two or three consecutive years, we drop all observations corresponding to expansions or recessions of smaller duration.

\(^{56}\)Another explanation relies to the fact that recession periods identified with the output gap are not necessarily periods in which actual unemployment is rising but lower than its trend level, and hence is not an indicator of a state of slack according to the unemployment rate gap dummy.
### Table 18: State of the Business Cycle: Output Gap Measured by (54)

<table>
<thead>
<tr>
<th>Fraction of Time in Exp. (L) or Rec. (H)</th>
<th>Duration (years) of Exp. (L) or Rec. (H)</th>
</tr>
</thead>
<tbody>
<tr>
<td>L ≥ 2y</td>
<td>H ≥ 2y</td>
</tr>
<tr>
<td>(1)</td>
<td>(2)</td>
</tr>
<tr>
<td>AUS</td>
<td>0.297</td>
</tr>
<tr>
<td>AUT</td>
<td>0.514</td>
</tr>
<tr>
<td>BEL</td>
<td>0.514</td>
</tr>
<tr>
<td>CAN</td>
<td>0.514</td>
</tr>
<tr>
<td>DEU</td>
<td>0.541</td>
</tr>
<tr>
<td>DNK</td>
<td>0.541</td>
</tr>
<tr>
<td>ESP</td>
<td>0.541</td>
</tr>
<tr>
<td>FIN</td>
<td>0.622</td>
</tr>
<tr>
<td>FRA</td>
<td>0.595</td>
</tr>
<tr>
<td>GBR</td>
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</tr>
<tr>
<td>IRL</td>
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</tr>
<tr>
<td>ITA</td>
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<tr>
<td>JPN</td>
<td>0.471</td>
</tr>
<tr>
<td>KOR</td>
<td>0.459</td>
</tr>
<tr>
<td>NLD</td>
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</tr>
<tr>
<td>NOR</td>
<td>0.568</td>
</tr>
<tr>
<td>SWE</td>
<td>0.541</td>
</tr>
<tr>
<td>USA</td>
<td>0.649</td>
</tr>
<tr>
<td>Panel</td>
<td>0.525</td>
</tr>
</tbody>
</table>

Notes: L refers to low unemployment (i.e., the economy is in expansion) and H to high unemployment (i.e., the economy is in recession); y is the contraction for years.

### Table 19: State of the Business Cycle: Unemployment Gap Measured by (55)

<table>
<thead>
<tr>
<th>Fraction of Time in Exp. (L) or Rec. (H)</th>
<th>Duration (years) of Exp. (L) or Rec. (H)</th>
</tr>
</thead>
<tbody>
<tr>
<td>L ≥ 2y</td>
<td>H ≥ 2y</td>
</tr>
<tr>
<td>(1)</td>
<td>(2)</td>
</tr>
<tr>
<td>AUS</td>
<td>0.447</td>
</tr>
<tr>
<td>AUT</td>
<td>0.500</td>
</tr>
<tr>
<td>BEL</td>
<td>0.500</td>
</tr>
<tr>
<td>CAN</td>
<td>0.447</td>
</tr>
<tr>
<td>DEU</td>
<td>0.474</td>
</tr>
<tr>
<td>DNK</td>
<td>0.500</td>
</tr>
<tr>
<td>ESP</td>
<td>0.579</td>
</tr>
<tr>
<td>FIN</td>
<td>0.505</td>
</tr>
<tr>
<td>FRA</td>
<td>0.447</td>
</tr>
<tr>
<td>GBR</td>
<td>0.579</td>
</tr>
<tr>
<td>IRL</td>
<td>0.526</td>
</tr>
<tr>
<td>ITA</td>
<td>0.474</td>
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<tr>
<td>JPN</td>
<td>0.526</td>
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<tr>
<td>KOR</td>
<td>0.632</td>
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<td>NLD</td>
<td>0.474</td>
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<td>NOR</td>
<td>0.500</td>
</tr>
<tr>
<td>SWE</td>
<td>0.526</td>
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<tr>
<td>USA</td>
<td>0.526</td>
</tr>
<tr>
<td>Panel</td>
<td>0.512</td>
</tr>
</tbody>
</table>

Notes: L refers to low unemployment (i.e., the economy is in expansion) and H to high unemployment (i.e., the economy is in recession); y is the contraction for years.
where the superscript $s = H, L$ refers to the High unemployment (i.e., recession) and Low unemployment (i.e., expansion). For each sub-sample $s$, we estimate the elasticity $\alpha$ for the relative wage (labelled $\beta^s$), the relative price (labelled $\gamma^s$), and the unemployment differential (labelled $\sigma^s$). For the relative price and the relative wage of nontradables, the labor productivity differential is expressed in level, i.e. $(\bar{a}^T - a^N)$ as we employ cointegration techniques to tackle the presence of unit roots in $p, \omega$ and $(\bar{a}^T - a^N)$. Instead, when estimating the change in the unemployment differential for each sub-sample, $\sigma^s$, the productivity differential is expressed in growth rate as $(\bar{a}^T - a^N)$ and we use a panel fixed effects regression.

Table 20 presents the estimated elasticity $\gamma^s$ and $\beta^s$ for $s = H, L$ from regression (56) in which the dependent variable is either relative wage or the relative price of nontradables. In columns (1) and (2), the state of the business cycle is identified with the sign of the output gap, and to allow for the possibility of state-dependence that might arise only in more persistent recessions or, alternatively, during extreme booms, we consider 2 different durations: expansions and recessions which last 2 years or more (column (1) labelled ≥ 2y) and 3 years or more (column (2) labelled ≥ 3y). The same exercise is repeated in columns (3) and (4) with our second measure of the state of the economy, i.e. the unemployment rate gap.

We refer column (2) as the baseline scenario. For the relative wage, the estimated coefficients $\beta^H$ and $\beta^L$ of -0.289 and -0.215 are highly significant. For the relative price equation, the corresponding estimated coefficients are $\gamma^H = 0.581$ and $\gamma^L = 0.638$ and are significantly different from zero too. In line with model’s predictions, the relative wage falls more ($\beta^H < \beta^L < 0$) while the relative price appreciates less in recessions than in expansions ($0 < \gamma^H < \gamma^L$). Remarkably, the difference in the estimated coefficient for $\omega$ is statistically significant, as shown in the line $t(\beta^L = \beta^H)$ the slope of the cointegrating vector in expansion $\beta^H$ is statistically different (at the 4% level) from the estimated coefficient in recession $\beta^L$. However, the difference $(\gamma^H - \gamma^L)$ is not statistically significant.

Next, our main conclusions are robust to the variable used to measure the state of the economy. Whether we identify 3-year expansions and recessions with output gap (column (2)) or unemployment rate gap (column (4)), the estimates remain highly significant. For the relative wage equation, one can see some indication that the estimated coefficient in recession increases from $\beta^H = -0.289$ with the output gap to $\beta^H = -0.242$ with the unemployment rate gap. Regarding the relative price of nontradables, using an alternative measure of the state of the business cycle does not affect the results as the estimated coefficients ($\gamma^H = 0.631$ and $\gamma^L = 0.630$) are both significantly different from zero but the hypothesis $\gamma^H = \gamma^L$ can not be rejected at conventional level.

Finally, the duration of regimes does not seem to drive the results. Specifically, when contrasting our estimates in columns (2) and (4) for the baseline scenarios with those shown in columns (1) and (3) respectively, for the alternative duration of expansions and recessions, our main conclusions hold: i) the estimated coefficients $\gamma^s$ and $\beta^s$ for $s = H, L$ are all statistically different from zero, iii) these estimates are close to their corresponding baseline values displayed in columns (2) and (4), and, iii) in all these runs one can verify that $\gamma^L > \gamma^H > 0$ and $0 > \beta^L > \beta^H$.

In Table 21, we present our estimated elasticity $\sigma^s$ for $s = H, L$ from regression (56) applied to the unemployment differential. In all regressions, we find that $|\sigma^H| > |\sigma^L|$. This result confirms our theoretical model which implies that the unemployment differential between tradables and non tradables falls more in recessions than in expansions. Note that when we use the output gap dummy (unemployment rate resp.) to identify periods of recession and expansion, the coefficient $\sigma^H$ is significant at least at the 10% level when considering the unemployment gap to identify the state of the economy in the business cycle. A possible explanation for this lack of significance is that our dataset covers only 11 countries split into two sub-samples which reduces significantly the number of observations.

E.7 Trade Balance Adjustment and Labor Market Regulation

In the main text, we show that the effects of higher relative productivity of tradables can be broken down into a labor market frictions effect and a labor accumulation effect. For the baseline calibration, we find numerically that the labor accumulation effect more than offsets the labor market frictions effect so that the relative price of non-tradables appreciates less than proportionately (i.e., by a lower amount than the productivity differential), the relative wage of non-tradables and the unemployment differential between tradables and non-tradables decline, in line with our estimates. Intuitively, in an open economy model where workers experience mobility costs, higher relative productivity of tradables leads traded firms to post more job vacancies than non-traded firms in order to encourage workers to shift toward the traded sector. Because the hiring process is costly and labor mobility costs increases hiring expenditure, a current account deficit shows up which must be offset by a long-run rise in net exports. By amplifying the long-run increase in net exports, LMR biases labor

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57Because DOLS and FMOLS estimates are very similar, for clarity purposes, Table 20 shows FMOLS estimates only.
Table 20: Panel FMOLS Estimates of $\beta$ and $\gamma$ for Expansions and Recessions

<table>
<thead>
<tr>
<th>Duration (years) of Exp. (L) and Rec. (H)</th>
<th>Output Gap</th>
<th>Unempl. Gap</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\geq 2y$</td>
<td>$\geq 3y$</td>
</tr>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
</tr>
<tr>
<td>A. Relative Wage</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\beta^H$</td>
<td>$-0.235^a$</td>
<td>$-0.289^a$</td>
</tr>
<tr>
<td></td>
<td>(0.011)</td>
<td>(0.028)</td>
</tr>
<tr>
<td>$\beta^L$</td>
<td>$-0.206^a$</td>
<td>$-0.215^a$</td>
</tr>
<tr>
<td></td>
<td>(0.008)</td>
<td>(0.008)</td>
</tr>
<tr>
<td>$t(\hat{\beta}^H = \hat{\beta}^L)$</td>
<td>0.110</td>
<td>0.039</td>
</tr>
</tbody>
</table>

| B. Relative Price                         |            |             |
| $\gamma^H$                                | 0.651$^a$  | 0.581$^a$   |
|                                          | (0.016)    | (0.075)     |
| $\gamma^L$                                | 0.651$^a$  | 0.638$^a$   |
|                                          | (0.012)    | (0.013)     |
| $t(\hat{\gamma}^H = \hat{\gamma}^L)$    | 1.000      | 0.522       |

Notes: $^a$, $^b$ and $^c$ denote significance at 1%, 5% and 10% levels. Heteroskedasticity and autocorrelation consistent standard errors are reported in parentheses. $\beta^H$ and $\gamma^H$ ($\beta^L$ and $\gamma^L$) refer to the responses of the relative wage and the relative price, respectively, when unemployment is high (low), i.e., when the economy is in recession (expansion). The row $t(\hat{\beta}^H = \hat{\beta}^L)$ ($t(\hat{\gamma}^H = \hat{\gamma}^L)$) reports the p-value of the test of $H_0 : \beta^H = \beta^L$ ($\gamma^H = \gamma^L$).

Table 21: Panel OLS Estimates of $\sigma$ for Expansions and Recessions

<table>
<thead>
<tr>
<th>Duration (years) of Exp. (L) and Rec. (H)</th>
<th>Output Gap</th>
<th>Unempl. Gap</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\geq 2y$</td>
<td>$\geq 3y$</td>
</tr>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
</tr>
<tr>
<td>$\sigma^H$</td>
<td>$-0.038$</td>
<td>$-0.052$</td>
</tr>
<tr>
<td></td>
<td>(0.041)</td>
<td>(0.055)</td>
</tr>
<tr>
<td>$\sigma^L$</td>
<td>$-0.035^c$</td>
<td>$-0.034^c$</td>
</tr>
<tr>
<td></td>
<td>(0.018)</td>
<td>(0.019)</td>
</tr>
<tr>
<td>$t(\hat{\sigma}^H = \hat{\sigma}^L)$</td>
<td>0.960</td>
<td>0.810</td>
</tr>
</tbody>
</table>

| Observations                             | 138        | 109        |
| Countries                                | 11         | 11         |

Notes: $^a$, $^b$ and $^c$ denote significance at 1%, 5% and 10% levels. Heteroskedasticity and autocorrelation consistent standard errors are reported in parentheses. $\sigma^H$ ($\sigma^L$) refer to the responses of the unemployment differential when unemployment is high (low), i.e. the economy is in recession (expansion). The row $t(\hat{\sigma}^H = \hat{\sigma}^L)$ reports the p-value of the test of $H_0 : \sigma^H = \sigma^L$. We exclude Belgium, Japan, and the United States from the sample since for these countries, the time horizon is too short and they display either one recession or one expansion but not both.

Demand toward the traded sector. More specifically, in an economy where labor markets are more regulated, there are more job seekers and less job vacancies and thus the labor market tightness is lower which makes hiring more profitable. Firms recruit more workers which amplifies the current account deficit and further increases net exports in the long-run, thus resulting in a higher demand for traded goods. In line with our hypothesis, we find numerically that $\omega$ and the unemployment differential decline more in countries where labor markets are more regulated and the relative price appreciates less. As emphasized in section 4.2, only the generosity of the unemployment benefit scheme and the worker bargaining power influence the strength of the labor accumulation channel while EPL operates through the labor market frictions channel.

Since the labor accumulation effect plays a key role in reconciling the theory with our empirical findings, we provide below some evidence which supports the labor accumulation channel. More specifically, we run the regression in panel data of the balance of trade (in percentage of GDP) on
Figure 8 plots OLS estimates of trade balance responses to a labor productivity growth differential against indicators of labor market regulation. Vertical axis plots panel OLS estimates of $\beta_1$ obtained by running regression (57) for one country at a time. Horizontal axis displays the labor market regulation index. The generosity of unemployment benefit scheme is measured by the average of net unemployment benefit replacement rates for three duration of unemployment (source: OECD); the worker bargaining power is measured by the bargaining coverage (source: Visser [2009]); in Figure 8(c), we have recourse to a principal component analysis in order to have one overall indicator encompassing the two dimensions of labor market regulation mentioned above. Sample: 18 OECD countries, 1970-2007, annual data.

The productivity growth differential:

$$nx_{i,t} = \delta + \beta_1 \cdot (\hat{a}_t^T - \hat{a}_t^N)_{i,t} + \beta_2 \cdot (\hat{a}_t^T - \hat{a}_t^N)_{i,t} \cdot \text{LMR}_{i,t} + \epsilon_{i,t},$$

where $nx_{i,t} = NX_{i,t}/Y_{i,t}$ is the ratio of the trade balance to GDP, $(\hat{a}_t^T - \hat{a}_t^N)_{i,t}$ is the productivity growth differential, and $\text{LMR}_{i,t}$ is the labor market regulation index obtained by using a principal component analysis. Net exports correspond to the external balance of goods and services at current prices taken from OECD Economic Outlook Database. Because time series for net exports as a percentage of GDP are stationary and the ratio of productivity of tradables to non-tradables is non-stationary, we estimate $\beta_1$ and $\beta_2$ by using a panel fixed effects regression where the productivity differential is expressed in growth rates so that the LHS and the RHS are both stationary. It is worth mentioning coefficients estimated by running the regression (57) capture the long-run effect of a productivity growth differential.

According to our model’s predictions, a rise in the relative productivity of tradables improves the balance of trade in the long-run, and all the more so in countries where LMR is higher. It is worth mentioning that our model predicts an improvement in the balance of trade in the long-run for the whole sample but the balance of trade adjustment displays a wide dispersion across countries. More specifically, countries where LMR is low such as the U.S., and/or countries with low values of the elasticity of substitution between traded and non-traded goods, such as Canada or the U.K., experience a decline in net exports in the long-run. As a first pass on the implications of LMR, we estimate $\beta_1$ by exploring empirically eq. (57) for one country at a time; for each country, we run the regression of the balance of trade in percentage of GDP on the productivity growth differential. In Fig. 8, we plot $\beta_1$ against two indicators of labor market policies, namely the generosity of the unemployment benefit scheme and the collective bargaining coverage. In line with our model’s predictions, most of the countries (i.e., two-third) experience a rise in the balance of trade following a rise in productivity differential between tradables and non-tradables. Importantly, Fig. 8(a) and 8(b) show that there exists a positive cross-country relationship between the long-run improvement in the balance of trade and LMR, the latter being captured by the generosity of the unemployment benefit scheme and collective bargaining coverage, respectively. In Fig. 8(c), we plot $\beta_1$ against the LMR indicator which encompasses the two dimensions of labor market institutions, say the generosity of the unemployment benefit scheme and collective bargaining coverage. Since time series for the unemployment benefit replacement rate and the collective bargaining coverage are only available after 2000 for Korea, data availability prevents us to construct a consistent LMR indicator by using a principal component analysis and thus we exclude this country from our analysis in Fig. 8(c). In line with our model’s predictions, countries where labor markets are more regulated experience a greater improvement in the balance of trade.

We now explore empirically equation (57). Our sample excludes Korea since a LMR indicator which encompasses the two dimensions of the labor market cannot be constructed due to data availability for this country. The first column of Table 22 shows that a productivity growth differential between tradables and non-tradables increases the balance of trade in the long-run. The second column of Table 22 reveals that the increase in the balance of trade is larger in countries where LMR is higher, in line with our model’s predictions.
F Labor Market Frictions and Cross-Country Effects of Higher Relative Productivity

In this section, we document some evidence indicating that the labor market frictions index constructed by Cardi and Restout [2015] cannot account for the cross-country dispersion in the effects of higher relative productivity when we let this measure vary between countries.

Cardi and Restout [2015] find empirically that a 1% permanent increase in the relative productivity of tradables leads to an appreciation in the relative price of non-tradables which is smaller than 1% and lowers the relative wage of non-tradables. To rationalize the evidence, they develop an open economy version of the neoclassical model with tradables and non-tradables and assume that agents experience labor mobility costs when shifting hours worked from one sector to another. Their quantitative analysis reveals that the model can account for the relative price and relative wage responses to higher relative productivity of tradables as long as workers experience mobility costs.

To calibrate the model to the data, the authors estimate empirically the degree of labor mobility across sectors which plays a pivotal role in the quantitative analysis. To measure the degree of labor mobility, Cardi and Restout [2015] draw on Horvath [2000] and estimate the elasticity of labor supply across sectors for each of the fourteen OECD countries of their sample. Their estimates and those by Cardi and Restout [2015] is 0.98.

Table 22: Panel Estimates of Regression (57)

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\beta_1)</td>
<td>0.168^b (0.073)</td>
<td>0.163^b (0.074)</td>
</tr>
<tr>
<td>(\beta_2)</td>
<td></td>
<td>0.004^b (0.002)</td>
</tr>
<tr>
<td>Observations</td>
<td>626</td>
<td>626</td>
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<tr>
<td>Countries</td>
<td>17</td>
<td>17</td>
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<tr>
<td>Fixed Effects</td>
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<td>Yes</td>
</tr>
</tbody>
</table>

Notes: ^a, ^b and ^c denote significance at 1%, 5% and 10% levels. Heteroskedasticity and autocorrelation consistent standard errors are reported in parentheses.

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<td>Fixed Effects</td>
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<td>Yes</td>
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<td>17</td>
<td>17</td>
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<td>Fixed Effects</td>
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To calibrate the model to the data, the authors estimate empirically the degree of labor mobility across sectors which plays a pivotal role in the quantitative analysis. To measure the degree of labor mobility, Cardi and Restout [2015] draw on Horvath [2000] and estimate the elasticity of labor supply across sectors for each of the fourteen OECD countries of their sample. Their estimates and those by Cardi and Restout [2015] is 0.98.
Table 23: Estimates of the Elasticity of Labor Supply across Sectors ($\epsilon$)

<table>
<thead>
<tr>
<th>Country</th>
<th>$\hat{\epsilon}_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>AUS</td>
<td>0.635*</td>
</tr>
<tr>
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<tr>
<td>BEL</td>
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<tr>
<td>CAN</td>
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<td>DNK</td>
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<tr>
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Countries 18  
Observations 1326  
Data coverage 1971-2007  
Country fixed effects yes  
Time dummies yes  
Time trend no

Notes: *, + and * denote significance at 1%, 5% and 10% levels. Heteroskedasticity and autocorrelation consistent t-statistics are reported in parentheses.
country. Since estimates of $\epsilon$ are not statistically significant for the Netherlands and Norway, we remove these two countries from the scatter-plots shown in the second row. Inspection of the trend line in Fig. 9 reveals that international differences in the elasticity of labor supply across sectors cannot account for the cross-country dispersion in the relative wage and relative price responses to a productivity differential.

The first reason to this is that search frictions create a wedge between the marginal product of labor and the marginal rate of substitution between consumption and leisure which prevents from estimating consistently labor mobility costs by adopting the methodology of Horvath [2000]. The second reason is that the degree of labor mobility (i.e., captured by $\epsilon$ in Cardi and Restout [2015]) encompasses both labor mobility costs and LMR in our model with imperfectly competitive labor markets. In our setup with search frictions in the labor market, the degree of labor mobility across sectors is measured by the elasticity of sectoral employment w.r.t. the marginal revenue product of labor, denoted by $\Theta^j$. This term is a function of labor mobility costs and LMR. In a model with perfectly competitive labor markets such as that considered by Cardi and Restout [2015], $\Theta^j$ collapses to $\epsilon$ which captures the extent of labor mobility costs; when $\epsilon$ takes larger values, labor mobility costs are lower. In the present paper where we consider both labor mobility costs and search frictions, $\Theta^j$ is increasing in $\sigma_L$, the worker bargaining power, $\alpha_W$, and the unemployment benefit replacement rate, $\varrho$. When we shut down search frictions, $\Theta^j$ collapses to $\sigma_L$. Conversely, in a model with search frictions, $\Theta^j$ is a function of labor mobility costs and hiring costs, the latter being influenced by LMR or the stage of the economy in the business cycle. According to our model’s predictions, $\Theta^j$ will take larger values as labor mobility costs are lower and LMR, captured by the worker bargaining power and/or the unemployment benefit replacement rate, $\varrho$. Since a fall in labor mobility costs and a rise in LMR have opposite effects on the relative price and relative wage effects, the empirical strategy proposed by Cardi and Restout [2015] cannot account for the cross-country dispersion in the relative wage and relative price responses as it stands out from 9. In Online Appendix B, we document some evidence indicating that the cross-country dispersion in the relative wage responses is driven by international differences in LMR since by using the three dimensions of LMR, we detect a positive cross-country relationship between the magnitude of the decline in the relative wage following higher relative productivity and the extent of LMR.

Figure 9: FMOLS Estimates for Relative Price and Relative Wage Responses to Higher Relative Productivity against Elasticity of Labor Supply across Sectors. Notes: Horizontal axes display countries’ estimates of the elasticity of labor supply across sectors, $\epsilon$. Vertical axis display FMOLS estimates of the relative price and relative wage responses, denoted by $\gamma$ and $\beta$, to a 1% permanent increase in the relative productivity of tradables. The first row of Fig. 9 shows panel data estimations by Cardi and Restout [2015]; sample: 14 OECD countries, 1970-2007. The second row shows our panel data estimations of $\epsilon$ and FMOLS estimates of $\beta$ and $\gamma$; sample: 18 OECD countries, 1970-2007.
G Robustness To Alternative Definitions of Hiring Costs

In the main text, both traded and non-traded firms face a cost of posting job vacancies and we assume that hiring costs paid by traded and non-traded firms are expressed in terms of traded good. This assumption amounts to considering that each firm produces a final good by renting labor services from a competitive human resource arm and these employment agencies are treated as tradables. As a result, hiring costs show up in the current account equation but do not appear in the market clearing condition for non-tradables.

In this section, we conduct a robustness check with respect to the assumption that recruiting costs are measured in traded good units. We consider a first extension where we assume that recruiting costs paid by non-traded firms are non-tradables and hiring costs paid by traded firms are tradables. We also consider a second extension where hiring costs are expressed in non-traded good units. Both the the labor market frictions and the labor accumulation channels exert similar effects as in the baseline model on the relative price, the relative wage and the unemployment differential. When hiring costs paid by non-traded firms are non-tradables and hiring costs paid by traded firms are tradables, all of the results found in the paper hold. While we discuss the numerical results below, section L and section M of the Technical Appendix emphasize the main changes with respect to the baseline model when we relax the assumption of hiring costs measured in terms of the traded good, and detail the steps to solve the model together with the analytical decomposition of steady-state changes in the relative price, the relative wage and the unemployment differential.

We detect some differences quantitatively since the labor market frictions channel is amplified and the labor accumulation channel is mitigated by assuming that hiring costs are expressed in terms of the non-traded good. First, when hiring costs are expressed in terms of the non-traded good, i.e., $P(t)\kappa N$, the appreciation in the relative price of non-tradables, $P(t)$, increases the cost per job vacancy. Because the non-traded reservation wage is a function of the cost per job vacancy, i.e., $W_{Nt} = \frac{1}{1-\omega} P(t)\kappa N \theta^N(t) + R^N$, the appreciation in the relative price of non-tradables further increases $W_{Nt}$ which magnifies the rise in the relative wage of non-tradables $\omega$ through the labor market frictions channel. Because recruiting costs increase more than in the baseline model as a result of the appreciation in the relative price of non-tradables, it leads non-traded firms to moderate their hiring. Because hiring increases less, non-traded output increases by a smaller amount which amplifies the excess supply of traded goods relative to non-traded goods and thus magnifies the appreciation in the relative price of non-tradables through the labor market frictions channel. While any excess of expenditure over total output must be covered by a current account deficit regardless of their tradedness, recruiting expenditure increases less when recruiting costs are both traded and non-traded which mitigates the current account deficit in the short-run. Because net exports rise by a smaller amount in the long-run for the intertemporal solvency condition to hold, the labor accumulation channel is mitigated.

When hiring costs paid by traded and non-traded firms are both expressed in non-traded good units, we reach the same conclusion except for an increase the unemployment benefit replacement rate. We show in section 4 that a rise in the replacement rate exerts two opposite effects. All else being equal, countries where unemployment benefits are more generous experience a larger current account deficit in the short-run and thus a greater increase in net exports in the long-run which depreciates the relative price of non-tradables and amplifies the decline in both the relative wage and the unemployment differential. While net exports increase more following a rise in the replacement rate like in the baseline case, more generous unemployment benefits result in a greater mobility of labor across sectors which mitigates the labor accumulation channel instead of amplifying it. When hiring costs are expressed in non-traded units, the latter effect dominates because enhanced labor mobility more than offsets the larger increase in net exports.

G.1 Traded vs. Non-Trade Hiring Costs

To assess the robustness of our results to the definition of the cost per job vacancy, we simulate the model laid out in section L of the Technical Appendix and contrast the results with those obtained in the baseline model where hiring costs are expressed in terms of the traded good. To ease the comparison of both models, we keep the calibration discussed in section I unchanged, except for $\kappa N$ which must be increased from 0.575 to 0.835 in order to target a non-traded labor market tightness of 0.34. The hiring costs supported by non-traded firms, i.e., $P \kappa N V N$, account for 1.3% of GDP and hiring costs supported by traded firms, i.e., $\kappa^T V^T$, account for 1.1% of GDP. Column 1 of Table 24 shows our FMOLS estimates for comparison purposes. Columns 2-11 show numerical results when hiring costs paid by non-traded firms are expressed in non-traded good units and hiring costs paid by traded firms are expressed in traded good units.

Main results. Like in the baseline case, we correct for the bias caused by search frictions which vary across sectors (see the last line of each panel). Numerical results show that the bias caused by search frictions is very small and identical to that in the baseline case where hiring costs
are measured in terms of the traded good. All of the conclusions reached in the main text hold when hiring costs are measured in both traded and non-traded good units. More specifically, a 1% permanent increase in the productivity of tradables relative to non-tradables:

- appreciates the relative price by less than the productivity differential (0.85%, see panel B of column 2 of Table 24);
- lowers the relative wage (by 0.17%, see panel A of column 2 of Table 24);
- lowers the unemployment differential between tradables and non-tradables (by 0.034%, see panel C of column 2 of Table 24);
- appreciates \( p \) less and further lowers both \( \omega \) and the unemployment differential in countries where the worker bargaining power is higher (see column 7), or unemployment benefits are more generous (see column 8), or in recession (see column 10);
- appreciates \( p \) more and leads to a larger decline in both \( \omega \) and the unemployment differential in countries where EPL is stricter (see column 9);
- appreciates \( p \) more and lowers less both \( \omega \) and the unemployment differential as labor mobility costs fall (i.e., we move from column 4 to column 6).

Importantly, the predictions of a model assuming search frictions but abstracting from labor mobility costs (see column 2 of Table 24) do not fit the data as such a model generates an appreciation in the relative price which is larger than the productivity differential and increases the relative wage. However the model is able to produce a fall in the unemployment differential.

**Quantitative differences.** Quantitatively, we detect some differences which are moderate however. For comparison purposes, let us recall the results for a representative OECD economy in the baseline case. As shown in column 2 of Table 3 where hiring costs are expressed in traded good units, a 1% permanent increase in the relative productivity of tradables generates an appreciation in the relative price by 0.78% and a decline in the relative wage by 0.22%, and leads the unemployment differential to fall by 0.011%. As can be seen in column 2 of Table 24, \( p \) appreciates more (0.85% instead of 0.78%), the relative wage falls less (-0.17% instead of -0.22%) and the unemployment differential falls more (by -0.034% instead of -0.011%). Thus the ability of a model where hiring costs are both traded and non-traded to account for our estimates is lower for the relative price and the relative wage but higher for the unemployment differential.

Intuitively, when hiring costs paid by non-traded firms are expressed in terms of the non-traded good, the appreciation in the relative price through the labor market frictions channel leads to a smaller increase in job vacancies posted by non-traded firms since the appreciation in the relative price increases the cost of hiring. Because non-traded labor increases less, the excess supply of traded goods is larger which amplifies the appreciation in the relative price (1.19% instead of 1.15% in the baseline scenario). Because the rise in hiring costs caused by the appreciation in the relative price amplifies the rise in the reservation wage, the non-traded wage increases more relative to the traded wage (i.e., by 0.14% instead of 0.11%). Since non-traded firms post less job vacancies as a result of higher recruiting costs, the unemployment rate of tradables falls more than the unemployment rate of non-tradables. While both the relative price and the relative wage increase more through the labor market frictions channel, the labor accumulation channel exerts a smaller negative impact on \( p \) (-0.34% instead of -0.36%) and \( \omega \) (-0.32% instead of -0.34%). The reason is that hiring expenditure increases by a smaller amount than in the baseline case which mitigates the current account deficit and thus the rise in net exports in the long-run. Because the labor market frictions are larger and the labor accumulation channel smaller, assuming that hiring costs paid by non-traded firms are expressed in terms of the non-traded good results in a larger appreciation in \( p \) and a smaller decline in \( \omega \).

In columns 7 and 8 of Table 24, we assume that hiring costs are both tradables and non-tradables and investigate the effects of a rise in the relative productivity of tradables in countries where the worker bargaining power is higher (\( \alpha_W \) is set to 0.9) and the unemployment benefit scheme is more generous (\( \varrho \) is set to 0.78%), respectively. In line with the results obtained when assuming that hiring costs are measured in terms of the traded good, increasing the worker bargaining power or the unemployment benefit replacement rate amplifies the labor accumulation channel which results in a larger decline in the relative wage and a smaller appreciation in the relative price. However, the differences are less pronounced than if hiring costs were measured in traded good units. The reason is twofold. First, in contrast to the baseline model where the labor market frictions channel exerts a slightly smaller positive impact on \( p \) and \( \omega \) when \( \alpha_W \) is increased, both the relative wage and the relative price of non-tradables appreciate more (as can be seen in the second row of panel A and B) as we move from column 2 to column 7. The explanation lies in the fact that when recruiting costs paid by non-traded firms are expressed in terms of the non-traded good, the appreciation in \( p \) raises significantly the non-traded reservation wage which more than offsets the negative impact of increased labor mobility (caused by an increase in \( \alpha_W \)) on \( \omega \). Second, while an increase in
LMR magnifies the labor accumulation effect as a result of a larger current account deficit in the short-run, this channel is somewhat mitigated however because the rise in the cost per job vacancy caused by the appreciation in $p$ leads non-traded firms to moderate hiring which mitigates recruiting expenditure and thus the current account deficit compared with the baseline model.

In column 9 of Table 24, we assume that hiring costs are both tradables and non-tradables and investigate the effects of a rise in the relative productivity of tradables in countries where the firing tax is higher ($\tau$ is set to 13). In accordance with the results discussed in the main text, increasing the firing tax mitigates the labor market frictions channel for the relative wage which results in a larger decline in the relative wage and amplifies the labor market friction channel for the relative price which results in a larger appreciation in the relative price of non-tradables.

In columns 10 and 11 of Table 24, we contrast the effects of a 1% permanent increase in the relative productivity of tradables during a recession period with the effects when the economy is in expansion. In line with the results highlighted in the main text, when the economy is in recession, a permanent increase in the relative productivity of tradables leads to a greater current account deficit in the short-run followed by a large rise in net exports in the long-run which amplifies the labor accumulation channel. Consequently, the relative wage and the unemployment differential fall more while the relative price appreciates less.

Finally, in columns 3-6 of Table 24, we explore the role of labor mobility costs by letting $\sigma_L$ vary between infinity and zero. When we move from column 4 to column 6, $\sigma_L$ takes larger values which result in higher labor mobility across sectors. In accordance with the conclusions established in the main text, both the relative wage and the unemployment differential decline less whilst the relative price appreciates more. In column 3, we shut down labor mobility costs by letting $\sigma_L$ tend toward infinity. Like in the baseline model, a model abstracting from labor mobility costs cannot account for the effects of a productivity differential we estimate empirically, except for the unemployment differential. As mentioned above, in a model where hiring costs paid by non-traded firms are expressed in terms of the non-traded good, the appreciation in the relative price of non-tradables increases the cost per job vacancy which leads non-traded firms to mitigate their hiring. Because the non-traded labor market tightness increases significantly less, the unemployment rate of tradables falls more than the unemployment rate of non-tradables along the labor market frictions channel, in contrast to the baseline scenario.

G.2 Traded vs. Non-Traded Hiring Costs: Time-Varying and Cross-Country Effects

In this subsection, we explore the ability of the model where hiring costs are both expressed in traded and non-traded good units to account for the time-varying and cross-country effects we document empirically in section 2.

Time-varying effects. In Fig. 10, we calibrate the model to a representative OECD economy, as described in section 5.1 and choose a value for the elasticity of labor supply at the extensive margin, $\sigma_L$, so as to replicate the estimated relative wage response, i.e., $\beta$, to a productivity differential in rolling subsamples. The blue line with circles shows empirical results in rolling subsamples with a window length of twenty years. The black line with triangles shows model’s predictions when we let both $\sigma_L$ and LMR vary over time. LMR encompasses three dimensions: the worker’s bargaining power $\alpha_W$ (captured by collective bargaining coverage), the generosity of the unemployment benefit scheme $\varrho$ (as captured by the unemployment benefit replacement rate), and the strictness of legal protection against dismissals (as captured by the EPL index taken from the OECD adjusted with the share of permanent workers in the economy). The dashed red line shows results when we let $\sigma_L$ vary over time and keep LMR unchanged.

We obtain the same results as in the main text. The rise in the relative wage response (i.e., $\beta$ becomes less negative) is associated with a fall in labor mobility costs over time. Time-declining labor mobility costs mitigate the appreciation in the relative price of non-tradables over time. As mentioned in the main text, the model misses the declining appreciation in $p$ at the beginning of the 2000’s however. Importantly, the variation of LMR over time does not influence the relative price and relative wage effects over time since the black line and the red line cannot be differentiated. While for the baseline model where hiring costs are expressed in terms of the traded good, time-varying LMR does not make any difference (see the dashed red line and the black line with triangles in Fig. 4(c)), time-varying LMR improves substantially the fit of the model to the data as it stands out from Fig. 10(c).
Table 24: Decomposition of Long-Term Responses to Higher Relative Productivity of Tradables when Hiring Costs are both Traded and Non-Traded Costs

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<td>A</td>
<td>ω = 0.88%</td>
<td>p = 0.67%</td>
<td>uT - uN = 9.5%</td>
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<tr>
<td>B</td>
<td>ω = 0.82%</td>
<td>p = 0.71%</td>
<td>uT - uN = 6.95%</td>
</tr>
<tr>
<td>C</td>
<td>ω = 0.80%</td>
<td>p = 0.70%</td>
<td>uT - uN = 6.90%</td>
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</table>

Note: Effect of a 1% increase in the productivity of tradables relative to non tradables. Panels A, B, C show the deviation in percentage relative to steady-state for the relative wage, $\omega = w_N - w_T$, the relative price of non tradables, $p = p_N - p_T$, and the unemployment differential between tradables and non tradables, $u_T - u_N$, and break down changes in a labor market frictions effect (keeping net exports fixed), and a labor accumulation effect (triggered by the long-run adjustment in net exports). The fourth line in each panel shows the magnitude of the bias caused by search frictions varying across sectors; the first line shows the numerically computed response to a productivity differential once the inherent discrepancy between theoretical and empirical values has been corrected. Parameter $\phi$ is the elasticity of substitution in consumption between tradables and non tradables; $\sigma_L$ is the elasticity of labor supply at the extensive margin; $\alpha_W$ corresponds to the worker bargaining power; $\varrho$ is the unemployment benefits replacement rate; $\tau$ is the firing tax expressed in monthly salary equivalents. In our baseline calibration we set $\phi = 0.8$, $\sigma_L = 0.6$, $\alpha_W = 0.6$, $\varrho = 0.6$, $\omega = 0.6$, $\omega = 0.6$, $\omega = 0.6$. The last two columns of the table shows the effects when the economy is in recession (labelled 'Rec') and in expansion (labelled 'Exp'). We calibrate the model by reducing sectoral labor productivity $A_j$ so that the unemployment benefits replacement rate is kept unchanged and the aggregate unemployment rate is 9.5% in a recession and 6.9% in an expansion, respectively.
In this section, we relax the assumption that hiring costs are tradable and consider that recruiting costs paid by non-traded as well as traded firms are expressed in terms of the non-traded good. Section M of the Technical Appendix details the steps to solve the model and to decompose analytically the steady-state changes.

To assess the implications of relaxing the assumption that employment agencies are tradables only, we simulate the model laid out in section M of the Technical Appendix and contrast the results with those obtained in the baseline model where hiring costs are expressed in terms of the traded good. To ease the comparison of both models, we keep the calibration detailed in section I unchanged, except for $\kappa^N$ which must be increased from 0.575 to 0.835 in order to target a non-traded labor market tightness of 0.34 while...
Figure 11: Cross-Country Relationship between Simulated Responses to Higher Relative Productivity and LMR when Hiring Costs are both Traded and Non-Traded. Notes: Horizontal axes display the LMR index obtained by using a principal component analysis which encompasses the three dimensions of labor market institutions. Vertical axes in the top panels report simulated long-run responses of the relative wage, relative price, and unemployment differential to higher relative productivity from the baseline model with search frictions and an endogenous labor force participation decision.

$\kappa^T$ must be increased from 1.482 to 1.995 to target $\theta^T = 0.24$. All remaining parameters remain unchanged and all ratios are identical to those targeted for the OECD representative economy. The hiring costs supported by non-traded firms, i.e., $P\kappa^N V^N$, account for 1.4% of GDP and hiring costs supported by traded firms, i.e., $P\kappa^T V^T$, account for 1.5% of GDP. Columns 3-10 of Table 25 show numerical results when hiring costs are expressed in terms of the non-traded good. Column 1 of Table 25 shows our estimates for comparison purposes. Column 2 of Table 25 shows numerical results for the baseline model in the main text when hiring costs are expressed in terms of traded good.

Like in the main text, we correct for the bias caused by search frictions which vary across sectors. As can be seen in the last line of each panel, the effect is very small. By and large, our results are robust to alternatively defining the cost per job vacancy in terms of the non-traded good. More specifically, in line with our evidence, as shown in column 3, the relative price appreciates by less than 1%, the relative wage declines and the unemployment rate of tradables falls more than the unemployment rate of non-tradables. When we contrast the responses of $p$, $\omega$, and $u^T - u^N$ to a 1% increase in the relative productivity of tradables between column 2 and column 3, we detect some differences however. More specifically, considering that hiring costs are expressed in terms of the non-traded good (column 3), the labor accumulation channel is mitigated as a result of a lower current account deficit. The reason is that the appreciation in $p$ now increases both hiring costs in the traded and the non-traded sector which mitigates the rise in recruiting expenditure and thus the labor accumulation channel. The labor market frictions channel is slightly amplified for the relative wage since the appreciation in $p$ amplifies the rise in the non-traded reservation wage which is only caused by an increase in $\theta^N$ in the baseline model.

In columns 4 and 5 of Table 25, we consider a fall in labor mobility costs, as captured by a rise in $\sigma_L$. In line with the results in the main text, a decrease in labor mobility costs amplifies the appreciation in $p$ and mitigates the decline in both $\omega$ and the unemployment differential.

In columns 6-7 of Table 25, we estimate numerically the effects of a 1% permanent increase in the relative productivity of tradables in countries where the worker bargaining power is higher or the replacement rate is larger. It is worth mentioning that we increase $\alpha_W$ from 0.6 to 0.75 instead of 0.9 because the latter value is too high to ensure saddle-path stability. To ensure the existence of the convergence along a saddle-path, we cannot increase $\alpha_W$ above 0.75 when hiring costs are non-tradables. In line with the evidence, the relative price appreciates less, and both the relative wage and the unemployment differential fall more in countries where the collective bargaining coverage is higher because the labor accumulation channel is amplified. However, as shown in column 7, we do not reach this result for the relative price and the relative wage in countries where the replacement rate is higher ($\varphi$ is set to 0.78%). The reason is that increased labor mobility caused by a more generous unemployment benefit scheme mitigates the labor market frictions channel substantially. In addition, the labor accumulation channel is mitigated instead of being
amplified when we move from column 3 to column 7. As mentioned above, when hiring costs are measured in terms of the non-traded good, the effect of a productivity differential on the reservation wage is amplified by the appreciation in the relative price of non-tradables. Hence, the combination of this effect and a more generous unemployment benefit scheme provides high incentive to shift labor across sectors and higher labor mobility softens the rise in the relative wage through the labor market frictions channel and the fall in the relative wage through the labor accumulation channel. Because labor is more mobile across sectors, the excess demand for traded goods is lower which results in a mitigated labor accumulation channel for the relative price as well.

In column 8 of Table 25, we investigate the effects of a 1% increase in the relative productivity of tradables in countries where legal protection against dismissals is stricter (τ is set to 13). In accordance with the results discussed in the main text, increasing the firing tax mitigates the labor market frictions channel for the relative wage which results in a larger decline in the relative wage and amplifies the labor market friction channel for the relative price which results in a larger appreciation in the relative price of non-tradables.

In columns 9-10 of Table 25, we contrast the effects of a 1% permanent increase in the relative productivity of tradables during a recession period with the effects when the economy is in expansion. In line with the results highlighted in the main text, when the economy is in recession, a permanent increase in the relative productivity of tradables leads to a greater current account deficit in the short-run followed by a larger rise in net exports in the long-run which amplifies the labor accumulation channel. Henceforth, the relative wage and the unemployment differential fall more while the relative price appreciates less.

To conclude, except for an economy where the unemployment benefits replacement rate is higher, all the conclusions reached in the main text hold.

H Robustness Check: Elasticity of Labor Supply at the Extensive Margin

In this section, we review the literature estimating the value of the elasticity of labor supply at the extensive margin and we conduct a sensitivity analysis with respect to this parameter.

Empirical literature. Fiorito and Zanella [2012] use micro data from the PSID to construct a panel of individuals over the period 1968-1997. Aggregate time series are obtained by aggregating these individuals each year. They find that aggregate time-series results deliver an extensive margin elasticity in the range of 0.8-1.4, which is substantially larger than the corresponding estimate (i.e., 0.2-0.3) reported by Chetty, Friedman, Manoli, and Weber [2011]. A value of 0.6 is halfway between these two sets of findings. This value is close to the estimates of the Frisch elasticity at the extensive margin on U.S. data documented by Mustre-del-Río [2015] who reports a value of 0.71. Mui and Schoef er [2019] show that the labor supply curve will be isoelastic if the reservation wedge distribution is power-law-like. In Mui and Schoefer [2019], the Frisch elasticity of labor supply at the extensive margin collapses to the shape parameter of the power law distribution. The authors find an elasticity of labor supply at the extensive margin ranging from a high of 3 to a low of 0.5 depending on whether the wedge perturbations are small or large. Blundell, Bozio and Laroque [2011] report an elasticity of labor supply at the extensive margin of 0.34 for women and 0.25 for men on U.K. data.

Another strand of the literature develops RBC models and uses information on cyclical variations of labor market variables to make inference about the elasticity of labor supply at the extensive margin. While we consider an endogenous labor force participation decision by assuming that representative household members experience disutility from working and searching efforts, Haefke and Reiter [2011] consider a pool of workers with different productivity so that only the most productive agents devote time to market activities (rather than to home activities). Haefke and Reiter [2011] find that an aggregate labor supply elasticity along the extensive margin of around 0.3 for men and 0.5 for women can replicate the variability of unemployment and participation, and the negative correlation

58 The reservation wedge is the hypothetical percent shift in an individual’s potential labor earnings required to render her indifferent between employment and nonemployment.
Table 25: Decomposition of Long-Term Responses to Higher Relative Productivity of Tradables: Non-Traded Hiring Costs

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<td>(5)</td>
<td>(6)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(σ_L = 0.16)</td>
<td>(σ_L = 1.22)</td>
<td>(α_W = 0.75)</td>
<td>(ρ = 0.782)</td>
<td>(τ = 13)</td>
<td>(u = 9.5%)</td>
</tr>
<tr>
<td><strong>A. Relative Wage</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Relative wage, $\bar{\omega}$</td>
<td>-0.22</td>
<td>-0.143</td>
<td>-0.092</td>
<td>-0.171</td>
<td>-0.126</td>
<td>-0.153</td>
<td>-0.147</td>
<td>-0.140</td>
</tr>
<tr>
<td>Labor market frictions effect</td>
<td>0.112</td>
<td>0.114</td>
<td>0.078</td>
<td>0.111</td>
<td>0.005</td>
<td>0.074</td>
<td>0.112</td>
<td>0.115</td>
</tr>
<tr>
<td>Labor accumulation effect</td>
<td>-0.341</td>
<td>-0.268</td>
<td>-0.181</td>
<td>-0.294</td>
<td>-0.237</td>
<td>-0.257</td>
<td>-0.271</td>
<td>-0.266</td>
</tr>
<tr>
<td>Bias caused by sector differences</td>
<td>-0.011</td>
<td>-0.011</td>
<td>-0.011</td>
<td>-0.012</td>
<td>-0.016</td>
<td>-0.030</td>
<td>-0.012</td>
<td>-0.011</td>
</tr>
<tr>
<td><strong>B. Relative Price</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Relative price, $\bar{p}$</td>
<td>0.64</td>
<td>0.801</td>
<td>0.915</td>
<td>0.833</td>
<td>0.876</td>
<td>0.880</td>
<td>0.856</td>
<td>0.864</td>
</tr>
<tr>
<td>Labor market frictions effect</td>
<td>1.148</td>
<td>1.151</td>
<td>1.118</td>
<td>1.148</td>
<td>1.137</td>
<td>1.184</td>
<td>1.150</td>
<td>1.152</td>
</tr>
<tr>
<td>Labor accumulation effect</td>
<td>-0.360</td>
<td>-0.285</td>
<td>-0.196</td>
<td>-0.311</td>
<td>-0.259</td>
<td>-0.283</td>
<td>-0.289</td>
<td>-0.283</td>
</tr>
<tr>
<td>Bias caused by sector differences</td>
<td>0.005</td>
<td>0.005</td>
<td>0.007</td>
<td>0.004</td>
<td>0.002</td>
<td>0.021</td>
<td>0.005</td>
<td>0.005</td>
</tr>
<tr>
<td><strong>C. Unemployment Differential</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Unemployment differential, $du_T - du_N$</td>
<td>-0.034</td>
<td>-0.008</td>
<td>0.00</td>
<td>-0.011</td>
<td>-0.014</td>
<td>-0.008</td>
<td>-0.009</td>
<td>-0.006</td>
</tr>
<tr>
<td>Labor market frictions effect</td>
<td>0.014</td>
<td>0.012</td>
<td>0.021</td>
<td>0.016</td>
<td>0.034</td>
<td>0.009</td>
<td>0.014</td>
<td>0.011</td>
</tr>
<tr>
<td>Labor accumulation effect</td>
<td>-0.023</td>
<td>-0.017</td>
<td>-0.011</td>
<td>-0.024</td>
<td>-0.041</td>
<td>-0.016</td>
<td>-0.020</td>
<td>-0.014</td>
</tr>
<tr>
<td>Bias caused by sector differences</td>
<td>0.003</td>
<td>0.003</td>
<td>0.010</td>
<td>0.003</td>
<td>0.007</td>
<td>0.001</td>
<td>0.003</td>
<td>0.003</td>
</tr>
</tbody>
</table>

Notes: Effect of a 1% increase in the productivity of tradables relative to non tradables. Panels A, B, C show the deviation in percentage relative to steady-state for the relative wage, $\omega = w^N - w^T$, the relative price of non tradables, $p = p^N - p^T$, and the unemployment differential between tradables and non tradables, $u^T - u^N$, and break down changes in a labor market frictions effect (keeping net exports fixed), and a labor accumulation effect (triggered by the long-run adjustment in net exports). The fourth line in each panel shows the magnitude of the bias caused by search frictions varying across sectors; the first line shows the numerically computed response to a productivity differential once the inherent discrepancy between theoretical and empirical values has been corrected. Parameter $\phi$ is the elasticity of substitution in consumption between tradables and non tradables; $\sigma_L$ is the elasticity of labor supply at the extensive margin; $\alpha_W$ corresponds to the worker bargaining power; $\rho$ is the unemployment benefits replacement rate; $\tau$ is the firing tax expressed in monthly salary equivalents. In our baseline calibration we set $\phi = 0.8$, $\sigma_L = 0.6$, $\alpha_W = 0.6$, $\rho = 0.524$, $\tau = 4.2$. The last two columns of the table shows the effects when the economy is in recession (labelled ‘Rec’) and in expansion (labelled ‘Exp’). We calibrate the model by reducing sectoral labor productivity $A^j$ so that the ratio $A^T/A^N = 1.28$ is kept unchanged and the aggregate unemployment rate is 9.5% and 6.9% in a recession and in an expansion, respectively.
of unemployment and GDP they document empirically.

**Labor mobility at the extensive margin vs. intensive margin.** In our model, we consider an endogenous sectoral labor force participation decision. We assume that the Frisch elasticity of labor supply at the extensive margin which determines the sectoral labor force participation decision. Since this parameter is symmetric across sectors and because a worker has to go through a spell of search unemployment to find a job in the other sector, the Frisch elasticity of labor supply at the extensive margin captures the extent of sector-specific skills and thus the magnitude of labor mobility costs. When we detail the calibration of $\sigma_L$ in the paper, we restrict attention to the empirical literature estimating the Frisch elasticity of labor supply at the extensive margin. The elasticity of labor supply at the extensive margin also measures the degree of labor force mobility across sectors.

In our model, we consider a representative household setup where the familiar isoelastic intensive-margin MacCurdy [1981] preferences is extended to the extensive margin:

$$
\Phi(t) \equiv \frac{C(t) \frac{1}{\sigma_C} - F(t) \frac{1}{1 + \frac{1}{\sigma_L}}}{1 - \frac{1}{\sigma_C}} - F(t) \frac{1}{1 + \frac{1}{\sigma_L}},
$$

where $F(t)$ is the aggregate labor force and $\sigma_L$ is the Frisch elasticity of labor supply at the extensive margin (i.e., keeping the marginal utility of wealth constant). We assume that labor force in the traded and the non-traded sectors are imperfect substitutes and aggregated by means of a CES function:

$$
F(t) = \left[ \frac{\zeta_T}{1 + \frac{1}{\sigma_L}} F^T(t) \frac{1 + \sigma_L}{1 + \frac{1}{\sigma_L}} + \frac{\zeta_N}{1 + \frac{1}{\sigma_L}} F^N(t) \frac{1 + \sigma_L}{1 + \frac{1}{\sigma_L}} \right]^{\frac{1}{1 + \sigma_L}},
$$

where $\sigma_L$ is the elasticity of substitution between traded and non-traded labor force. This parameter thus captures the extent of workers’ mobility costs across sectors since when $\sigma_L$ takes lower values, the disutility from searching for a job in sector $j$ gets larger. It shares some common features with the specification of Cardi and Restout [2015] (borrowed from Horvath [2000]) who allow for imperfect mobility of labor across sectors by considering that sectoral hours worked are imperfect substitutes in a similar fashion as (60). In contrast, the authors abstract from search frictions and thus cannot disentangle labor mobility costs from hiring costs while this distinction is key to reproducing time-varying, cross-country, and state-dependent effects.

**Estimates of labor mobility costs.** Estimates of $\epsilon$ by using the methodology pioneered by Horvath [2000] who abstract from search unemployment may give a sense of the magnitude of labor mobility costs as captured by $\sigma_L$ in the present model. Adopting the methodology of Horvath [2000] who abstracts from search frictions to estimate the elasticity of labor supply across sectors, when we consider the eighteen countries of our sample and impose $\delta_i = \delta$ into eq. (58), we find a value of 0.527 as shown in the last line of Table 23. This estimated value is close to the value of 0.6 we choose for the elasticity of sectoral labor force participation across sectors. While this value is informative, we have to be cautious in two respects. First, estimates of $\epsilon$ refer to labor mobility costs and not to labor force mobility because unemployment is absent. Second, in a model with search frictions, the marginal product of labor no longer collapses to the wage rate and labor market institutions influence the response of hours worked to a change in the relative share of value added paid to workers in sector $j$. In other words, $\delta_i$ might reflect the size of labor mobility costs together with LMR.

**Extensive vs. intensive margin.** In our model, we allow for labor supply at the extensive margin while hours worked are supplied inelastically as they are determined by search frictions. These frictions are crucial for our analysis since the degree of labor mobility no longer collapses to the elasticity of labor supply at the extensive margin but instead to $\Theta^j$ which depends on $\sigma_L$ as well as labor market institutions. In the lines of Shi and Wen [1999], Heer and Schubert [2012], Heijdra and Ligthart [2002], [2009], we consider a family which comprises a large number of members. The overall household has a fixed time endowment which is normalized to unity for convenience so that leisure $l$ is defined as $l = 1 - F^T - F^N$. Search effort of an unemployed household member and worked hours are
supplied inelastically while we allow for an endogenous sectoral labor force participation decision. More precisely, to determine his/her labor force participation decision, the household member equates the marginal cost with the marginal benefit of entering the labor force:

\[
\frac{\zeta^j((F^j(t))^{1/\sigma_L}) \lambda}{\lambda} = m^j(\theta^j(t)) \xi^j(t) + R^j.
\]

As shown in the LHS term, labor supply is elastic at the extensive margin. Then, depending on the job destruction rate, \(s^j\), and the job finding rate, \(m^j(\theta^j)\), labor force is split between working time and job search. Along the transitional dynamics, using the fact that \(U^j(t) = F^j(t) - L^j(t)\), agents supply working time \(L^j(t)\) according to the following accumulation equation \(\dot{L}^j(t) = m^j(t)U^j(t) - s^j\dot{L}^j(t)\) = \(m^j(t)F^j(t) - (m^j(t) + s^j)L^j(t)\), where \(F^j(t)\) is the labor force in sector \(j\). The flows of workers in and out of employment are equal to each other in any symmetric equilibrium, i.e., \(m^j(t)U^j(t) = f^j(t)V^j(t)\). Hence eq. \(\dot{L}^j(t) = m^j(t)U^j(t) - s^j\dot{L}^j(t)\) and eq. \(\dot{L}^j(t) = f^j(t)V^j(t) - s^j\dot{L}^j(t)\) indicate that the demand for labor indeed equates the supply.

**Robustness check with respect to the elasticity of labor supply at the extensive margin.** In the main text, we choose a value of 0.6 for \(\sigma_L\). Because Fiorito and Zanella [2012] find larger values than 0.6 for the elasticity of labor supply at the extensive margin, in Table 26 we calibrate our model by choosing \(\sigma_L = 1\) and we re-estimate all scenarios considered in the main text. While column 1 reports our FMOLS estimates and column 3 shows numerical results when \(\sigma_L\) is set to 1, we show the baseline scenario with \(\sigma_L = 0.6\) in column 2 for comparison purposes. Since Peterman [2016] finds estimates for the macro Frisch elasticity of labor supply close to 3-4 (which includes both the intensive and extensive margin), we explore the effects of a 1% permanent increase in the relative productivity of tradables when \(\sigma_L = 3\). Because the elasticity of labor supply at the extensive margin slightly modifies the value of labor market tightness in the non-traded sector at the initial steady-state, we set \(\kappa^N = 0.461\) and \(\kappa^N = 0.294\) when \(\sigma_L = 1\) and \(\sigma_L = 3\), respectively, instead of \(\kappa^N = 0.575\) in the baseline scenario, to target \(\theta^N = 0.34\).

Contrasting results in the baseline scenario with those when \(\sigma_L = 1\), we find that our results are unchanged although the decline in \(\omega\) is less pronounced and the appreciation in \(p\) is amplified. The reason is that \(\sigma_L\) measures both the elasticity of labor supply at the extensive margin and the extent of workers’ costs of switching sectors so that higher values of \(\sigma_L\) lead to a greater mobility. In columns 4-8, we re-estimate the effects of a 1% permanent increase in the relative productivity of tradables when LMR is higher or when the economy is in recession (‘Rec’) or in expansion (‘Exp’). All of the conclusions reached in the main text hold. In column 9, we set \(\sigma_L = 3\). Like in the baseline scenario, we find that the relative price appreciates by less than the productivity differential while both the relative wage and the unemployment differential fall. Because labor mobility costs are much lower than in the baseline case, the model imposing \(\sigma_L = 3\) understates the decline in the relative wage and overstates the appreciation in the relative price we document empirically.

## I Calibration Procedure

In this section, we provide more details about the calibration to a representative OECD economy and to data from 18 OECD countries whose source and construction are detailed in section C.

### I.1 Initial Steady-State

Assuming that the elasticity of labor supply at the extensive margin (\(\sigma_L^j\)), the elasticity of vacancies in job matches (\(\alpha_V^j\)), and the worker bargaining power (\(\alpha_W^j\)) are symmetric across sectors, i.e., \(\sigma_L^j = \sigma_L\), \(\alpha_V^j = \alpha_V\), and \(\alpha_W^j = \alpha_W\), and normalizing to 1 the parameters \(\zeta^T\) and \(A^N\) that correspond to the disutility from working and searching for a job in the traded sector and the productivity of labor in the non-traded sector, respectively, the calibration reduces to 20 parameters: \(r^*, \beta, \sigma_G, \sigma_L, \phi, \varphi, \zeta^N, \omega_G = \frac{G}{F} \omega_{GN} = \frac{F^G N}{F}, A^T, s^T, s^N, 1 - \alpha_V, \alpha_W, \kappa^N, X^T, X^N, x^N, \theta, \) and initial conditions \(B_0, L^0_N, L^0_T\).
Table 26: Decomposition of Long-Term Responses to Higher Productivity in Tradables Relative to Non Tradables: Robustness Check w.r.t. $\sigma_L$

<table>
<thead>
<tr>
<th>Data</th>
<th>OECD</th>
<th>LS</th>
<th>Barg. power</th>
<th>Replac. rate</th>
<th>Firing</th>
<th>Rec</th>
<th>Exp</th>
<th>High LS</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
<td>(4)</td>
<td>(5)</td>
<td>(6)</td>
<td>(7)</td>
<td>(8)</td>
</tr>
<tr>
<td>$\sigma_L = 0.6$</td>
<td>$\sigma_L = 1$</td>
<td>(w = 0.9)</td>
<td>(p = 0.782)</td>
<td>(r = 13)</td>
<td>(u = 9.5%)</td>
<td>(u = 6.9%)</td>
<td>($\sigma_L = 3$)</td>
<td></td>
</tr>
</tbody>
</table>

### A. Relative Wage

- Relative wage, $\omega$
  - $-0.22$ | $-0.22$ | $-0.16$ | $-0.19$ | $-0.21$ | $-0.16$ | $-0.17$ | $-0.15$ | $-0.07$
- Labor market frictions effect
  - $0.11$ | $0.09$ | $0.09$ | $0.08$ | $0.06$ | $0.09$ | $0.09$ | $0.04$
- Labor accumulation effect
  - $-0.34$ | $-0.26$ | $-0.29$ | $-0.30$ | $-0.25$ | $-0.27$ | $-0.25$ | $-0.12$
- Bias caused by sector differences
  - $-0.01$ | $-0.01$ | $-0.01$ | $-0.01$ | $-0.02$ | $-0.01$ | $-0.01$ | $0.00$

### B. Relative Price

- Relative price, $\bar{p}$
  - $0.64$ | $0.78$ | $0.84$ | $0.81$ | $0.79$ | $0.87$ | $0.83$ | $0.85$ | $0.94$
- Labor market frictions effect
  - $1.15$ | $1.13$ | $1.12$ | $1.12$ | $1.17$ | $1.12$ | $1.13$ | $1.08$
- Labor accumulation effect
  - $-0.36$ | $-0.27$ | $-0.30$ | $-0.33$ | $-0.27$ | $-0.29$ | $-0.26$ | $-0.13$
- Bias caused by sector differences
  - $0.01$ | $0.01$ | $0.01$ | $0.01$ | $0.03$ | $0.01$ | $0.01$ | $0.01$

### C. Unemployment Differential

- Unemployment differential, $du_T - du_N$
  - $-0.034$ | $-0.011$ | $-0.007$ | $-0.018$ | $-0.030$ | $-0.007$ | $-0.009$ | $-0.005$ | $-0.001$
- Labor market frictions effect
  - $0.014$ | $0.013$ | $0.019$ | $0.032$ | $0.010$ | $0.014$ | $0.011$ | $0.010$
- Labor accumulation effect
  - $-0.023$ | $-0.017$ | $-0.034$ | $-0.056$ | $-0.016$ | $-0.021$ | $-0.014$ | $-0.008$
- Bias caused by sector differences
  - $0.003$ | $0.003$ | $0.003$ | $0.006$ | $0.001$ | $0.003$ | $0.002$ | $0.003$

Notes: Effect of a 1% increase in the productivity of tradables relative to non tradables. Panels A, B, C show the deviation in percentage relative to steady-state for the relative wage, $\omega \equiv w_N - w_T$, the relative price of non tradables, $\bar{p} \equiv p_N - p_T$, and the unemployment differential between tradables and non tradables, $u_T - u_N$, and break down changes in a labor market frictions effect (keeping net exports fixed), and a labor accumulation effect (triggered by the long-run adjustment in net exports). The fourth line in each panel shows the magnitude of the bias caused by search frictions varying across sectors; the first line shows the numerically computed response to a productivity differential once the inherent discrepancy between theoretical and empirical values has been corrected. Parameter $\phi$ is the elasticity of substitution in consumption between tradables and non tradables; $\sigma_L$ is the elasticity of labor supply at the extensive margin; $\alpha_W$ corresponds to the worker bargaining power; $\varrho$ is the unemployment benefits replacement rate; $\tau$ is the firing tax expressed in monthly salary equivalents. In column 2, we consider our baseline calibration where we set $\phi = 0.8$, $\sigma_L = 0.6$, $\alpha_W = 0.6$, $\varrho = 0.524$, $\tau = 4.2$. In column 3, we set $\sigma_L = 1$ (LS’ means labor supply) while in column 9, we set $\sigma_L = 3$. 


Since we focus on the long-run equilibrium, the tilde is suppressed for the purposes of clarity. The steady-state of the open economy comprises 14 equations:

\[ C = (PC\check{\lambda})^{-\sigma_C}, \quad (61a) \]
\[ U^T = \frac{s^T L^T}{m^T}, \quad (61b) \]
\[ U^N = \frac{s^N L^N}{m^N}, \quad (61c) \]
\[ m^T = X^T (\theta^T)^{\alpha_V}, \quad (61d) \]
\[ m^N = X^N (\theta^N)^{\alpha_V}, \quad (61e) \]
\[ L^T = \frac{m^T}{s^T + \lambda^T} \left[ \frac{\lambda W_R^T}{\sigma_L} \right]^{\sigma_L}, \quad (61f) \]
\[ L^N = \frac{m^N}{s^N + \lambda^N} \left[ \frac{\lambda W_R^N}{\sigma_L} \right]^{\sigma_L}, \quad (61g) \]
\[ \frac{\kappa^T}{f^T} = \frac{(1 - \alpha_W) \Psi^T}{s^T + r^*}, \quad \Psi^T = A^T - W_R^T, \quad (61h) \]
\[ \frac{\kappa^N}{f^N} = \frac{(1 - \alpha_W) \Psi^N}{s^N + r^*}, \quad \Psi^N = PA^N + r^* x^N - W_R^N, \quad (61i) \]
\[ V^T = \theta^T U^T, \quad (61j) \]
\[ V^N = \theta^N U^N, \quad (61k) \]
\[ A^N L^N = C^N + G^N, \quad (61l) \]
\[ r^* B + A^T L^T = C^T + G^T + \kappa^T \theta^T U^T + \kappa^N \theta^N U^N, \quad (61m) \]

and the intertemporal solvency condition
\[ B - B_0 = \Phi^T (L^T - L_0^T) + \Phi^N (L^N - L_0^N), \quad (61n) \]

where the system jointly determines \(C, U^T, U^N, m^T, m^N, L^T, L^N, \theta^T, \theta^N, V^T, V^N, P, B, \check{\lambda}.\)

Some of the values of parameters can be taken directly from data, but others need to be endogenously calibrated to fit a set of an average OECD economy features. Among the 20 parameters, 6 parameters, i.e., \(\kappa^T, \kappa^N, X^T, X^N, \zeta^N, \varphi,\) together with initial conditions \((B_0, L_0^T, L_0^N)\) must be set in order to match key properties of a typical OECD economy. More precisely, the parameters \(\kappa^T, \kappa^N, X^T, X^N, \zeta^N, \varphi,\) together with the set of initial conditions are set to target \(\theta^T, \theta^N, m^T, m^N, L^N / L, \alpha_C, \upsilon_{NG},\) Denoting by \(\upsilon_{GN}\) the ratio of government spending in non tradables, \(G^N,\) to the non traded output, \(Y^N,\) the steady-state can be reduced to the following seven equations:

\[ \frac{\kappa^T}{f^T} = \frac{(1 - \alpha_W) \Psi^T}{s^T + r^*}, \quad (62a) \]
\[ \frac{\kappa^N}{f^N} = \frac{(1 - \alpha_W) \Psi^N}{s^N + r^*}, \quad (62b) \]
\[ m^T = X^T (\theta^T)^{\alpha_V}, \quad (62c) \]
\[ m^N = X^N (\theta^N)^{\alpha_V}, \quad (62d) \]
\[ \frac{A^T L^T (1 - \upsilon_{NX})}{A^N L^N (1 - \upsilon_{GN})} = \frac{\varphi - P^\Phi}{1 - \varphi}, \quad (62e) \]
\[ \frac{L^T}{L^N} = \frac{m^T m^N + s^N}{m^N m^T + s^T} \left( \frac{W_R^T \zeta^N}{W_R^N} \right)^{\sigma_L}, \quad (62f) \]
\[ B - B_0 = \Phi^T (L^T - L_0^T) + \Phi^N (L^N - L_0^N), \quad (62g) \]

which jointly determine \(\theta^T, \theta^N, m^T, m^N, L^T / L^N, P, B.\) The ratio \(L^T / L^N\) implicitly determines \(L^N / L:\)

\[ \frac{L^N}{L} = \frac{L^N}{L^T + L^N} = \frac{1}{L^T + 1}. \quad (63) \]
The relative price of non tradables $P$ implicitly determines the non tradable content of consumption expenditure:

$$\alpha_C = \frac{(1 - \varphi) P^{1 - \phi}}{\varphi + (1 - \varphi) P^{1 - \phi}}.$$  \hfill (64)

The net foreign asset position $B$ implicitly determines $v_{NX} = \frac{NX}{\nu_T}$ with $NX = Y^T - C^T - GT$ and $-v_{NX} = v_B - v_{VT} - v_{VN}$ with $v_B = \frac{\nu_B}{\nu_T}$, $v_{VT} = \frac{\nu_{VT}}{\nu_T}$ and $v_{VN} = \frac{\nu_{VN}}{\nu_T}$. To see it, multiply both sides of eq. (62g) by $\frac{\nu_T}{\nu_T}$:

$$v_B = v_{B0} + r^* \Phi_T \left( \frac{1}{A^T} - v_{L^T_0} \right) + r^* \Phi_N \left( \frac{L^N}{A^T L^T} - v_{L^N_0} \right),$$ \hfill (65)

where $v_{B0} = \frac{r^* B_0}{\nu_T}$, $v_{L^T_0} = \frac{L^T_0}{\nu_T}$, $v_{L^N_0} = \frac{L^N_0}{\nu_T}$. Since we have

$$v_{VT} = \frac{\kappa^T T^T S^T}{A^T m^T},$$ \hfill (66a)

$$v_{VN} = \frac{\kappa^N N^N S^N L^N}{A^T m^N L^T},$$ \hfill (66b)

where we used the fact that $V^j = \theta^j U^j$ and $U^j = \frac{\nu_T L^j}{m^j}$ at the steady-state; according to (66) the ratios $v_{VT} = \frac{\nu_{VT}}{\nu_T}$ and $v_{VN} = \frac{\nu_{VN}}{\nu_T}$ are pinned down by $\theta^T$, $\theta^N$, $m^T$, $m^N$, $L^N/L^T$ which are endogenously determined by system (62). Eqs (65) and (66) determine the ratio of net exports to traded output (i.e., $v_{NX}$):

$$v_B - v_{VT} - v_{VN} = -v_{NX}. \hfill (67)$$

In order to finish the proof that system (62) can be solved for $\theta^T$, $\theta^N$, $m^T$, $m^N$, $L^T/L^N$, $P$, $B$, we have to determine analytical expressions of $W^T_R$, $W^N_R$, $\Psi^T$, $\Psi^N$. The reservation wage in sector $j$, $W^j_R$, is defined as the sum of the expected value of a job $m^j \xi^j = \frac{\alpha_W}{1 - \alpha_W} \kappa^j \theta^j$ and the unemployment benefit $R^j = \varrho W^j$. The Nash bargaining wage in sector $j$, $W^j$, can be rewritten as follows:

$$W^j = \alpha_W (\Xi^j + r^* x^j) + (1 - \alpha_W) \left( \frac{\alpha_W}{1 - \alpha_W} \kappa^j \theta^j + \varrho W^j \right),$$ \hfill (68)

Plugging (68) into the definition of the reservation wage in sector $j$, we have:

$$W^j_R = \frac{\alpha_W}{1 - \alpha_W} \kappa^j \theta^j + \varrho W^j,$$

$$= \frac{\alpha_W}{1 - \alpha_W} \kappa^j \theta^j + \varrho \frac{\alpha_W (\Xi^j + r^* x^j + \kappa^j \theta^j)}{1 - (1 - \alpha_W) \varrho}. \hfill (69)$$

Since $\Xi^T = A^T$ and $\Xi^N = PA^N$, the reservation wage in the traded sector, $W^T_R$, is a function of $\theta^T$, while the reservation wage in the non traded sector, $W^N_R$, is a function of $\theta^N$ and $P$. Since $\Psi^j = \Xi^j - W^j_R$, the overall surplus from an additional job in the traded sector, $\Psi^T$, is a function of $\theta^T$, while the overall surplus from an additional job in the non traded sector, $\Psi^N$, is a function of $\theta^N$ and $P$.

To begin with, labor market parameters of the traded sector, i.e., the matching efficiency $X^T$ and the recruiting cost $K^T$, can be set to target the monthly job finding rate $m^T$ and the labor market tightness $\theta^T$. To show it more formally, we first compute the share of the overall surplus from an additional worker obtained by the firm, $(1 - \alpha_W) \Psi^T$, which is equal to the excess of labor productivity over the Nash bargaining wage, $A^T - W^T$; inserting (68), one obtains:

$$(1 - \alpha_W) \Psi^T = A^T - \alpha_W \left( A^T + \kappa^T \theta^T \right) - \frac{\alpha_W (A^T + \kappa^T \theta^T)}{1 - (1 - \alpha_W) \varrho},$$

$$= \frac{(1 - \alpha_W) (1 - \varrho) A^T - \alpha_W \kappa^T \theta^T}{1 - (1 - \alpha_W) \varrho}. \hfill (70)$$
Plugging (70) into (62a) and using the fact that \( f^T = \frac{m^T}{\vartheta} \) allows us to rewrite the vacancy-creation equation in the traded sector as follows:

\[
\frac{\kappa^T \theta^T}{m^T} (s^T + r^*) = \frac{(1 - \alpha_W) (1 - \vartheta) A^T - \alpha_W \kappa^T \theta^T}{1 - (1 - \alpha_W) \vartheta}. \tag{71}
\]

Equations (62c) and (71) form a separate subsystem which jointly determine \( \theta^T \) and \( m^T \); parameters \( \kappa^T \) and \( X^T \) are set in order to target \( \theta^T \) and \( m^T \) shown in Table 6. It is worthwhile mentioning that while theoretically \( \kappa^T \) and \( X^T \) jointly determine \( \theta^T \) and \( m^T \), we find numerically that \( \theta^T \) is mostly affected by \( \kappa^T \) while \( m^T \) is mostly determined by \( X^T \).

The remaining equations (62b), (62d)-(62g) form a separate subsystem which jointly determine \( m^N \), \( \theta^N \), \( P \), \( L^T/L^N \), and \( v_{NX} \):

\[
\frac{\kappa^N \theta^N}{m^N} (s^N + r^*) = \frac{(1 - \alpha_W) (1 - \vartheta) (PA^N + r^* x^N) - \alpha_W \kappa^N \theta^N}{1 - (1 - \alpha_W) \vartheta}, \tag{72a}
\]

\[
m^N = X^N (\theta^N)^{\alpha_V}, \tag{72b}
\]

\[
\frac{A^T L^T (1 - v_{NX})}{A^N L^N (1 - v_{GN})} = \frac{\varphi}{1 - \varphi} P^\phi, \tag{72c}
\]

\[
L^T = m^T m^N + s^T \left( \frac{W_R^T}{W_R^N} \right)^{\sigma_L}, \tag{72d}
\]

\[
v_{NX} = -(v_B - v_{VT} - v_{VN}), \tag{72e}
\]

where \( v_B, v_{VT}, v_{VN} \) are given by eqs. (65), (67), (68), respectively; to rewrite (62b) as (72a), we used the fact that \( (1 - \alpha_W) \Psi^N = \frac{(1 - \alpha_W) (1 - \vartheta) (PA^N + r^* x^N) - \alpha_W \kappa^N \theta^N}{1 - (1 - \alpha_W) \vartheta} \). Remembering that \( P \) determines \( \alpha_C \) and \( LT/LN \) determines \( LN/L \), parameters \( \kappa^N \), \( X^N \), \( \varphi \), \( \zeta^N \) and initial conditions \( (B_0, L_0^T, L_0^N) \) are set in order to target \( \theta^N \) and \( m^N \) (see columns 11 and 7 in Table 6), \( \alpha_C \) and \( LN/L \) (see columns 2 and 1 in Table 4), \( v_{NX} \approx 0 \) as we assume that at the initial steady-state, the balance of trade is nil. While theoretically the four parameters and initial conditions are endogenously determined to target \( \theta^N, m^N, \alpha_C, \frac{LN}{L} \) and \( v_{NX} \), we find numerically that \( \theta^N \) is mostly affected by \( \kappa^N \), \( m^T \) by \( X^N \), \( \alpha_C \) by \( \varphi \), \( \frac{LN}{L} \) by \( \zeta^N \), and \( v_{NX} \) by initial conditions.

### I.2 Calibration to a Representative OECD Economy

In order to assess the ability of our model to account for the evidence, we proceed in two steps. We first calibrate the model to a representative OECD country and investigate whether the model can account for the evidence we document empirically in section 2 when one parameter at a time is modified. In the next subsection, we calibrate the model to country specific data and explore whether the model can rationalize our empirical findings once we let all parameters vary across countries.

This subsection provides more details about how we calibrate the model to match the key empirical properties of a representative OECD economy. Our reference period for the calibration of the non tradable share given in Table 4 is running from 1990 to 2007 while labor market parameters have been computed over various periods. Due to the availability of data, we were able to estimate sectoral unemployment rates for 10 European countries and 5 OECD economies as ILO does not provide series for sectoral employment and unemployment for France, the Netherlands, and Norway at a sectoral level. Regarding Korea, while ILO provides data necessary for the computation of sectoral unemployment rates, the OECD does not provide unemployment by duration for this country which prevents the computation of job finding and job destruction rates. Data for the labor markets are described in Table 6.\(^{59}\)

We first describe the parameters that are taken directly from the data; we start with the preference parameters shown in panel A of Table 27:

\(^{59}\)For sectoral unemployment rates, and monthly job finding and job destruction rates, we take the EU-10 unweighted average due to data availability.
• One period in the model is a month.
• The world interest rate, \( r^\star \), equal to the subjective time discount rate, \( \beta \), is set to 0.4%.
• We assume that utility for consumption is logarithmic and thus set the intertemporal elasticity of substitution for consumption, \( \sigma_c \), to 1.
• We set the elasticity of substitution (in consumption) between traded and non traded goods to 0.8 in the baseline calibration.\(^{60}\)
• Next, we turn to the elasticity of labor supply at the extensive margin which is assumed to be symmetric across sectors. We choose \( \sigma_L \) to be 0.6 in our baseline setting but conduct a sensitivity analysis with respect to this parameter. See Online Appendix H for a review of the literature estimating the Frisch elasticity of labor supply at the extensive margin.

Next, we describe the calibration of the non-tradable content of consumption expenditure, employment, government spending displayed by panel B:

• The weight of consumption in non tradables \( 1 - \varphi \) is set to 0.44 to target a non-tradable content in total consumption expenditure (i.e. \( \alpha_c \)) of 42%, in line with the average of our estimates shown in the last line of Table 4.

• In order to target a non tradable content of labor of 66% which corresponds to the 18 OECD countries’ unweighted average shown in the last line of Table 4, we set \( \zeta_N \) to 0.18 (see eq. (5)) while \( \zeta_T \) has been normalized to 1.

• Government spending as a percentage of GDP is set to 20% and we set the non tradable content of government expenditure, i.e., \( \omega_{GN} = \frac{PGN}{G} \), to 90%.\(^{61}\)

• We assume that traded firms are 28 percent more productive than non traded firms in line with our estimates; we thus normalize \( A_N \) to 1 and set \( A_T \) to 1.28;

We describe below the choice of parameters characterizing the labor markets of a typical OECD economy in panel C:

• In line with our estimates shown in the last line of Table 6, we set the rates of separation in the traded (i.e., \( s^T \)) and the non traded (i.e., \( s^N \)) sector to 1.48% and 1.54% respectively.

• We set \( 1 - \alpha_V \) to 0.6 in line with the estimates documented by Barnichon [2012] who reports an elasticity of the matching function with respect to unemployed workers of about 0.6.

• As it is common in the literature, we impose the Hosios [1990] condition, and set the worker bargaining power \( \alpha_W \) to 0.6 in the baseline scenario.

• To target the labor market tightness for a representative OECD economy in the traded sector, \( \theta_T = 0.24 \), and in the non traded sector, \( \theta_N = 0.34 \), we set the recruiting cost to \( \kappa_T = 1.482 \) and \( \kappa_N = 0.575 \) in the traded and the non traded sector respectively.

• When calibrating to a representative OECD economy, we set the matching efficiency in the traded (non traded) sector \( X^T \) (\( X^N \)) to 0.307 (0.262) to target a monthly job finding rate \( m^T \) (\( m^N \)) of 17.4% (17.0%). A job destruction rate in the traded (non traded) sector \( s^T \) (\( s^N \)) of 1.48% (1.54%) together with a monthly job finding rate of 17.4% (17.0%) leads to an unemployment rate \( u^T \) (\( u^N \)) of 7.9% (8.3%) in the traded (non traded) sector.

\(^{60}\)Last column of Table 4 reports estimates for the elasticity of substitution \( \phi \) between traded and non traded goods. For the whole sample, we find empirically an elasticity of 0.8.

\(^{61}\)The market clearing condition for the traded good and the non traded good at the steady-state are \( r^\star B + Y_T = C_T + G_T + \kappa_T V_T + \kappa_N V_N \) and \( Y_N = C_N + G_N \), respectively.
Finally, we present the parameters that capture the labor market institutions shown in panel D:

- Since the advance notice and the severance payment are both expressed in monthly salary equivalents, we have $x_j = \tau W_j$ with $\tau \geq 0$. Values of $\tau$ are shown in the last column of Table 6. For the baseline calibration, we set the firing tax $\tau$ to 4.2. When conducting the sensitivity analysis, we set $\tau$ to 13 which corresponds to the highest value for the firing cost.

- Assuming that unemployment benefits are a fixed proportion of the wage rate, i.e., $R_j = \varrho W_j$, with $\varrho$ the replacement rate, we choose a value for $\varrho$ of 52.4%, in line with our estimates shown in Table 6. When conducting the sensitivity analysis, we set $\varrho$ to 78.2% which corresponds to the highest value for the unemployment benefit replacement rate.

Finally, we choose values for $B_0$, $L_0^T$, $L_0^N$ for the ratio of net exports to traded output to be nil at the initial steady-state, i.e., $v_{NX} \approx 0$.

I.3 Calibration to Each OECD Economy

In a second stage, we move a step further and compare the predicted values with estimates for each country and the whole sample as well. The initial steady-state of each OECD economy is described by the system (62) that comprises seven equations. To calibrate our model to each OECD economy in our sample, we use the same baseline calibration for each country, except for the elasticity of substitution $\phi$ between traded and non-traded goods, and labor market parameters which are allowed to vary across economies. More specifically, the elasticity of substitution $\phi$ between traded and non-traded goods is set in accordance with its estimates shown in the last column of Table 4.\textsuperscript{62} The parameters which capture the degree of labor market regulation such as the firing cost $x$, and the replacement rate $\varrho$ are set to their values shown in the last two columns of Table 6. The matching efficiency $\lambda_j$ in sector $j$ is set to target the job finding rate $m_j$ summarized in columns 5 and 7 of Table 6. The job destruction rate $s_j$ is set in accordance to its value reported in columns 6 and 8 of Table 6. Ideally, the recruiting cost $\kappa^j$ would be set in order to target $\theta^j$; however, the series for job vacancies by economic activity are available for a maximum of seven years and for a limited number of countries. On the contrary, the OECD provides data for job openings (for the whole economy) over the period 1980-2007 allowing us to calculate the labor market tightness, i.e., $\theta = V/U$, for several countries that we target along with the ratio $\theta^T/\theta^N$ by choosing $\kappa^T$ and $\kappa^N$. Thus, when calibrating the model to each OECD economy, the costs per job vacancy $\kappa^T$ and $\kappa^N$ are chosen to target the aggregate labor market tightness $\theta$ shown in column 13 and the ratio of sectoral labor market tightness $\theta^T/\theta^N$ obtained by dividing column 10 by column 11.

When data for sectoral labor market tightness are not available, we target the average value $\theta^T/\theta^N$ for EU-12 if the country is a member of the European Union, the average value for the US for English-speaking countries (excluding European economies), and average value for the OECD otherwise. When data for job openings are not available at an aggregate level, we first calibrate the model to EU-12 (US, OECD), in particular choosing $\kappa^T$ and $\kappa^N$ to target an aggregate labor market tightness $\theta$ of 0.12 (0.59, 0.18) and a ratio $\theta^T/\theta^N$ of 0.75 (0.66, 0.77); then, we set $\kappa^T$ and $\kappa^N$ chosen for EU-12 if the country is a member of the European Union, chosen for the US for Canada, and chosen for the OECD otherwise. Finally, because labor market parameters cannot be calculated at a sectoral level for France, the Netherlands and Norway, we assume that the job destruction rate $s$ and the matching efficiency $X$ are identical across sectors and are chosen in accordance with estimates shown in column 6 (or alternatively in column 8) of Table 6 for the former and to target $m_j$ shown in column 5 (or alternatively in column 7) of Table 6 for the latter.

Table 28 gives a sense of the correction term in columns 3 and 6 and compares $\hat{\omega}'$ with $\omega'^j$, and $\hat{p}'$ with $p'^j$.

\textsuperscript{62}We also choose the weight of consumption in non tradables $1 - \varphi$ to target a non-tradable content in total consumption expenditure (i.e., $a_C$) for each country in line with our estimates shown in column 2 of Table 4.
<table>
<thead>
<tr>
<th>Definition</th>
<th>Value</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Period of time</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$a^T - a^N$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1% month</td>
<td>1%</td>
<td>standard</td>
</tr>
<tr>
<td><strong>A. Preferences</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Subjective time discount rate, $\beta$</td>
<td>0.4%</td>
<td>equal to the world interest rate</td>
</tr>
<tr>
<td>Intertemporal elasticity of substitution for consumption, $\sigma_C$</td>
<td>1</td>
<td>standard</td>
</tr>
<tr>
<td>Elasticity of labor supply at the extensive margin, $\sigma_L$</td>
<td>0.6</td>
<td>Fiorito and Zanella [2012]</td>
</tr>
<tr>
<td>Elasticity of substitution, $\phi$</td>
<td>0.8</td>
<td>our estimates (KLEMS [2011], OECD Economic Outlook)</td>
</tr>
<tr>
<td><strong>B. Non Tradable Share</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Weight of consumption in non traded goods, $1 - \varphi$</td>
<td>0.44</td>
<td>set to target $\alpha_C = 42%$ (United Nations [2011])</td>
</tr>
<tr>
<td>Disutility from working and searching for a job, $\zeta$</td>
<td>0.18</td>
<td>set to target $L^N/L = 66%$ (KLEMS [2011])</td>
</tr>
<tr>
<td>Non Tradable content of government expenditure, $\omega_{GN}$</td>
<td>0.90</td>
<td>our estimates (OECD [2012b], IMF [2011])</td>
</tr>
<tr>
<td>Labor productivity index for the traded sector, $A^T$</td>
<td>1.28</td>
<td>our estimates (KLEMS [2011])</td>
</tr>
<tr>
<td><strong>C. Labor Market</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Job destruction rate in sector $j = T$, $s^T$</td>
<td>1.48%</td>
<td>our estimates (ILO)</td>
</tr>
<tr>
<td>Job destruction rate in sector $j = N$, $s^N$</td>
<td>1.54%</td>
<td>our estimates (ILO)</td>
</tr>
<tr>
<td>Elasticity of matches w.r.t. $U$, $1 - \alpha_V$</td>
<td>0.6</td>
<td>Barnichon [2012]</td>
</tr>
<tr>
<td>Worker bargaining power, $\alpha_W$</td>
<td>0.6</td>
<td>Hoes [1990] condition</td>
</tr>
<tr>
<td>Hiring cost in sector $j = T$, $\kappa^T$</td>
<td>1.482</td>
<td>set to target $\theta^T$</td>
</tr>
<tr>
<td>Hiring cost in sector $j = N$, $\kappa^N$</td>
<td>0.575</td>
<td>set to target $\theta^N$</td>
</tr>
<tr>
<td>Matching efficiency in sector $j = T$, $X^T$</td>
<td>0.307</td>
<td>set to target $m^T$</td>
</tr>
<tr>
<td>Matching efficiency in sector $j = N$, $X^N$</td>
<td>0.262</td>
<td>set to target $m^N$</td>
</tr>
<tr>
<td><strong>D. Labor Market Institutions</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Firing cost, $x^N$</td>
<td>4.2</td>
<td>our estimates (Fondazione De Benedetti)</td>
</tr>
<tr>
<td>Replacement rate, $\varphi$</td>
<td>52.4%</td>
<td>our estimates (OECD, Benefits and Wages Database)</td>
</tr>
</tbody>
</table>
Table 28: Comparison of Computed Numerically Responses Before and After Bias Correction

<table>
<thead>
<tr>
<th>Country</th>
<th>Relative wage response</th>
<th>Relative price response</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
</tr>
<tr>
<td></td>
<td>$\hat{\omega}$</td>
<td>$\hat{\omega}'$</td>
</tr>
<tr>
<td>AUS</td>
<td>0.179</td>
<td>0.172</td>
</tr>
<tr>
<td>AUT</td>
<td>-0.337</td>
<td>-0.318</td>
</tr>
<tr>
<td>BEL</td>
<td>-0.294</td>
<td>-0.281</td>
</tr>
<tr>
<td>CAN</td>
<td>0.009</td>
<td>0.015</td>
</tr>
<tr>
<td>DEU</td>
<td>-0.423</td>
<td>-0.420</td>
</tr>
<tr>
<td>DNK</td>
<td>-0.527</td>
<td>-0.515</td>
</tr>
<tr>
<td>ESP</td>
<td>-0.286</td>
<td>-0.261</td>
</tr>
<tr>
<td>FIN</td>
<td>-0.384</td>
<td>-0.355</td>
</tr>
<tr>
<td>FRA</td>
<td>-0.355</td>
<td>-0.346</td>
</tr>
<tr>
<td>GBR</td>
<td>-0.049</td>
<td>-0.050</td>
</tr>
<tr>
<td>IRL</td>
<td>-0.171</td>
<td>-0.148</td>
</tr>
<tr>
<td>ITA</td>
<td>-0.272</td>
<td>-0.266</td>
</tr>
<tr>
<td>JPN</td>
<td>-0.152</td>
<td>-0.145</td>
</tr>
<tr>
<td>KOR</td>
<td>-0.685</td>
<td>-0.640</td>
</tr>
<tr>
<td>NLD</td>
<td>-0.286</td>
<td>-0.280</td>
</tr>
<tr>
<td>NOR</td>
<td>-0.292</td>
<td>-0.286</td>
</tr>
<tr>
<td>SWE</td>
<td>0.134</td>
<td>0.144</td>
</tr>
<tr>
<td>USA</td>
<td>-0.037</td>
<td>-0.035</td>
</tr>
<tr>
<td>EU-12</td>
<td>-0.160</td>
<td>-0.149</td>
</tr>
<tr>
<td>Whole sample</td>
<td>-0.229</td>
<td>-0.218</td>
</tr>
</tbody>
</table>

Notes: $\hat{p}$ and $\hat{\omega}$ correspond to deviations in percentage of the relative price and the relative wage from their initial steady-state which are computed numerically following a productivity differential of 1%; we denote by $\hat{p}'$ and $\hat{\omega}'$ the steady-state changes in the relative price and relative wage computed numerically once their values have been adjusted with the bias originating from the presence of search frictions which vary across sectors. Columns 3 and 6 show that magnitude of bias for the relative wage and the relative price which must be subtracted from $\hat{p}$ and $\hat{\omega}$ in order to make elasticities computed numerically directly comparable with $\beta$ and $\gamma$ which are estimated empirically.
I.4 Calibration of the Model according to the State in the Business Cycle

In the main text, we explore quantitatively the magnitude of the effects of higher relative productivity of tradables according to the state of the economy in the business cycle. To calibrate to the model to the data, we proceed as follows. We calculate the output gap for each country in our sample over 1970-2007 (except for Japan: 1974-2007) by applying a Hodrick-Prescott filter with a smoothing parameter of $\lambda = 100$ to logged real GDP $y_{it}$. Expansions (recessions) are periods where $dy_{it} - d\bar{y}_{it} > 0$ ($dy_{it} - d\bar{y}_{it} < 0$) where potential GDP (in log) is $\bar{y}_{it}$. As shown in the first two columns of Table 29, the average duration of a recession 3.8 years and 4 years for an expansion.

Then we multiply the average duration of a cycle by the average output gap for each country to calculate the cumulated output loss in recession or output gain in expansion; we consider the situation of an economy in the middle of the cycle and thus the cumulated output gap the outcome is halved:

$$\sum_{t=1}^{(T)^s} \frac{y_{it} - \bar{y}_{it}}{2}$$

where $T$ is the average duration of a cycle and $s = H$ for recessions ($H$ means High unemployment) and $s = L$ for expansions ($L$ means low unemployment). Columns 3 and 4 of Table 29 show the cumulated output gap when the economy is in the middle of the cycle. On average, the cumulated output loss relative to trend is 3.2 ppt of GDP during recessions whilst the output gain is 2.7 ppt of GDP in expansions.

Next, we need to translate the cumulated output gain or loss in unemployment gap by using estimates of Okun’s Law documented by Ball et al. [2017] (see Table 1) for each country in our sample. Column 5 of Table 29 shows the Okun coefficient which measures the short-run responsiveness of unemployment relative to output fluctuations. The rise in unemployment relative to trend following a 1 ppt increase in the output gap is 0.42 on average. Multiplying the halved cumulated output gap by the Okun coefficient gives the cumulated increase in unemployment relative to trend when the economy is in expansion or in recession. As displayed by columns 6 and 7 of Table 29, the cumulated unemployment gap after about 2 years in recessions is 1.4 ppt whilst its cumulated decline is 1.2 ppt after about 2 years in expansion. When we calibrate the model to a representative OECD economy, we choose labor market parameters in order to generate an unemployment rate of 8.1%. To explore the implications of the state of the economy in the business cycle, we choose initial values for sectoral labor productivity, $A_j^T$, so that the unemployment rate of a representative OECD economy is 9.5% (i.e., $\bar{u} + 1.4 = 8.1 + 1.4 = 9.5\%$ of the labor force) in recessions and 6.9% (i.e., $\bar{u} - 1.2 = 8.1 - 1.2 = 6.9\%$ of the labor force) in expansions. Moreover, we modify sectoral labor productivity so that the ratio $A^T/A^N$ is unchanged at 1.28.
Table 29: Estimates of Cumulated Unemployment relative to Trend in the Middle of the Cycle

<table>
<thead>
<tr>
<th>Country</th>
<th>Duration cycle</th>
<th>Output gap</th>
<th>Coef Okun's Law</th>
<th>Unemployment gap</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>rec. ($T^H$)</td>
<td>exp. ($T^L$)</td>
<td>$\sum_{t=1}^{T^H} (\frac{y_{it} - \bar{y}<em>{it}}{2})</em>{rec}$</td>
<td>$\sum_{t=1}^{T^L} (\frac{y_{it} - \bar{y}<em>{it}}{2})</em>{exp}$</td>
</tr>
<tr>
<td>---------</td>
<td>----------------</td>
<td>------------</td>
<td>-----------------</td>
<td>-----------------</td>
</tr>
<tr>
<td>AUS</td>
<td>3.00</td>
<td>3.50</td>
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<td>JPN</td>
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<tr>
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<td>OECD</td>
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References


RELATIVE PRODUCTIVITY AND SEARCH UNEMPLOYMENT IN AN OPEN ECONOMY

TECHNICAL APPENDIX
NOT MEANT FOR PUBLICATION

Luisito BERTINELLI, Olivier CARDI, and Romain RESTOUT

- Section A presents the source and construction of the data used in the empirical and quantitative analysis, and provides summary statistics as well.
- Sections B-D give more details on the model. Section B develops an open economy version of the neoclassical model with search frictions and sectoral endogenous labor supply, and derives first-order conditions. Section C presents the matching process and derives the Nash bargaining wage. Section D sets out the approach taken to solve the model, analyzes equilibrium dynamics, and provides formal solutions.
- Section E characterizes the initial steady-state and the transitional paths by using phase diagrams.
- In section F, we describe the graphical framework which allows us to characterize initial steady-state values for the relative wage and the relative price.
- In section G, we decompose analytically the steady-state changes in the relative wage and the relative price following higher relative productivity of tradables.
- In section H, we analyze graphically the long-term adjustment in the relative price and the relative wage following a productivity shock biased toward the traded sector and investigate the implications of labor market regulation.
- In section I, we break down the change in the unemployment rate differential into labor market frictions and labor accumulation effects.
- In section J, we detail the steps of derivation of the effects of a productivity differential once we have corrected for the bias caused by search frictions which vary across sectors.
- In section K, we explore the case of total immobility (i.e., \( \sigma_L = 0 \)) as well as perfect mobility (i.e., \( \sigma_L \to \infty \)) in order to highlight the role of the elasticity of the labor supply at the extensive margin.
- In section L, we relax the assumption that the cost per job vacancy is expressed in terms of the traded good and consider instead that recruiting costs paid by non-traded firms are non-tradables and hiring costs paid by traded firms are tradables. In section M, we alternatively consider that the hiring costs are expressed in terms of the non-traded good. These two sections detail the steps to solve the model and decompose analytically the steady-state changes.
A Data Description

In this section, we present a complete description of our dataset. First, we provide details on the data sources and variables construction used in the empirical analysis and to calibrate the model. Then, we describe the empirical strategy implemented to estimate a parameter involved in our quantitative analysis, i.e., the elasticity of substitution in consumption between traded and non traded goods $\phi$.

A.1 Data for Empirical Analysis: Source and Construction

**Coverage:** Our sample consists of a panel of 18 countries: Australia (AUS), Austria (AUT), Belgium (BEL), Canada (CAN), Germany (DEU), Denmark (DNK), Spain (ESP), Finland (FIN), France (FRA), the United Kingdom (GBR), Ireland (IRL), Italy (ITA), Japan (JPN), Korea (KOR), the Netherlands (NLD), Norway (NOR), Sweden (SWE), and the United States (USA).

**Period Coverage:** The period is running from 1970 to 2007, with the exception of Japan (1974-2007) for which the starting date differs due to sectoral data availability. The choice of countries is restricted by the availability of sufficiently detailed data on sectoral variables over a long time horizon.

**Sources:** We use the EU KLEMS [2011] database (the March 2011 data release) for all countries of our sample with the exceptions of Canada and Norway. For these two countries, sectoral data are taken from the Structural Analysis (STAN) database provided by the OECD [2011]. Both the EU KLEMS and STAN databases provide annual data at the ISIC-rev.3 1-digit level for eleven industries.

The eleven 1-digit ISIC-rev.3 industries are split into tradables and non tradables sectors. To do so, we adopt the classification proposed by De Gregorio et al. [1994] who treat an industry as traded when it exports at least 10% of its output. Following Jensen and Kletzer [2006], we have updated the classification suggested by De Gregorio et al. [1994] by treating “Financial Intermediation” as a traded industry. Jensen and Kletzer [2006] use the geographic concentration of service activities within the United States to identify which service activities are traded domestically. The authors classify activities that are traded domestically as potentially traded internationally. The idea is that when a good or a service is traded, the production of the activity is concentrated in a particular region to take advantage of economies of scale in production.

Jensen and Kletzer [2006] use the two-digit NAICS (North American Industrial Classification System) to identify tradable and non tradable sectors. We map their classification into the NACE-ISIC-rev.3 used by the EU KLEMS database. The mapping was clear for all sectors except for ”Real Estate, Renting and Business Services”. According to the EU KLEMS classification, the industry labelled ”Real Estate, Renting and Business Services” is an aggregate of five sub-industries: ”Real estate activities” (NACE code: 70), ”Renting of Machinery and Equipment” (71), ”Computer and Related Activities” (72), ”Research and Development” (73) and ”Other Business Activities” (74). While Jensen and Kletzer [2006] find that industries 70 and 71 can be classified as tradable, they do not provide information for industries 72, 73 and 74. We decided to classify ”Real Estate, Renting and Business Services” as non tradable.

**Traded Sector** comprises the following industries: Agriculture, Hunting, Forestry and Fishing; Mining and Quarrying; Total Manufacturing; Transport, Storage and Communication; and Financial Intermediation.

**Non Traded Sector** comprises the following industries: Electricity, Gas and Water Supply; Construction; Wholesale and Retail Trade; Hotels and Restaurants; Real Estate, Renting and Business Services; and Community Social and Personal Services.

Relevant to our work, the EU KLEMS and STAN database provides series, for each industry and year, on value added at current and constant prices, permitting the derivation of sectoral deflators of value added, as well as details on labor compensation and employment data, allowing the construction of sectoral wage rates. We describe below the construction for the data employed in section 2 (mnemonics are given in parentheses):

- Sectoral value-added deflator $P_{it}^j$ for $j = T, N$: value added at current prices (VA) over
value added at constant prices (VA_QI) in sector \( j \). Source: EU KLEMS database. The relative price of non tradables \( P_t \) corresponds to the ratio of the value added deflator of non traded goods to the value added deflator of traded goods: \( P_t = P_t^N / P_t^T \).

- Sectoral labor \( L_j^t \) for \( j = T, N \): total hours worked by persons engaged (H_EMP) in sector \( j \). Source: EU KLEMS database.

- Sectoral nominal wage \( W_j^i \) for \( j = T, N \): labor compensation in sector \( j \) (LAB) over total hours worked by persons engaged (H_EMP) in that sector. Source: EU KLEMS database. The relative wage, \( \Omega_t \) is calculated as the ratio of the nominal wage in the non traded sector to the nominal wage in the traded sector: \( \Omega_t = W_t^N / W_t^T \).

- Sectoral labor productivity \( A_j^t \) for \( j = T, N \): value-added at constant prices in sector \( j \) (VA_QI) over total hours worked by persons engaged (H_EMP) in that sector. Source: EU KLEMS database. The relative productivity of tradables is the ratio of traded \( A_t^T \) to non traded labor productivity \( A_t^N \).

- The construction of sectoral unemployment rates, \( u_j^t \), is detailed below in section A.2.

Because data source and construction are heterogenous across variables as a result of different nomenclatures, Table 30 provides a summary of the classification adopted to split value added and its demand components, hours worked, labor compensation, unemployed workers as well into tradables and non tradables.

Summary statistics of the data used in the empirical analysis are displayed in Table 31. As shown in columns 1, 2, 4, all countries of our sample experience higher productivity gains in tradables relative to non tradables, an appreciation in the relative price of non tradables (except for Norway) and a decline in the ratio of the non traded wage relative to the traded wage. Moreover, for the vast majority of the countries (11 over the 15 providing data on sectoral unemployment), the average of the unemployment differential \( du_t^T - du_t^N \) is negative (see column 3).

To empirically assess the role of labor market institutions in the determination of the relative wage response to higher productivity in tradables relative to non tradables, we use three indicators aimed at capturing the stringency of labor market regulation. We detail below the construction and the source of these three indicators:

- The strictness of legal protection against dismissals for permanent workers is measured by the employment protection legislation index, \( EPL_{i,t} \) in country \( i \) at time \( t \), provided by OECD. Source for \( EPL_{i,t} \): OECD Labour Market Statistics database. Data coverage: 1985-2007 (1990-2007 for KOR). This index can be misleading since regulation was eased for temporary contracts (in Spain) while the regulation for workers with permanent contracts hardly changed. To have a more accurate measure of legal protection against dismissals, we construct a new index denoted by \( EPL_{i,t}^\text{adj} \) in country \( i \) at time \( t \) by adjusting \( EPL_{i,t} \) for regular workers with the share \( \text{share \ perm}_{i,t} \) of permanent workers in the economy, i.e., \( EPL_{i,t}^\text{adj} = EPL_{i,t} \times \text{share \ perm}_{i,t} \). Source for share \( \text{perm}_{i,t} \): OECD Labour Market Statistics database. Data coverage: 1985-2007 (1990-2007 for KOR).

- The generosity of the unemployment benefit scheme, \( g_{i,t} \) in country \( i \) at time \( t \), is commonly captured by the unemployment benefit replacement rate. It is worthwhile noticing that the unemployment benefit rates are very similar across counties when considering short-term unemployment (less than one year) but display considerable heterogeneity for long-term unemployment. To have a more accurate measure of the generosity of the unemployment benefit scheme, we calculate \( g \) as the average of the net unemployment benefit (including social assistance and housing benefit) replacement rates (for two earnings levels and three family situations) for three durations of unemployment (1 year, 2&3 years, 4&5 years). Source: OECD, Benefits and Wages Database. Data coverage: 2001-2007. In order to have longer time series, we calculated \( g \) over the period running from 1970 to 2000, by using the growth rate of the
Table 30: Construction of Sectoral Variables and Data Sources

<table>
<thead>
<tr>
<th>Variable</th>
<th>Countries covered</th>
<th>Period</th>
<th>Construction and aggregation</th>
<th>Database</th>
</tr>
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<tr>
<td>Value added $Y^T$ &amp; $Y^N$ (constant prices)</td>
<td>BEL, DEU, DNK, ESP, FIN, FRA, GBR, IRL, ITA, JPN (74-07), KOR, NLD, SWE, USA</td>
<td>1970-2007</td>
<td>$T$: Agriculture, Mining, Manufacturing, Transport, Finance Intermediation $N$: Electricity, Construction, Trade, Hotels, Real Estate, Personal Services</td>
<td>EU KLEMS</td>
</tr>
<tr>
<td>Value added $P^TY^T$ &amp; $P^NY^N$ (current prices)</td>
<td>BEL, DEU, DNK, ESP, FIN, FRA, GBR, IRL, ITA, JPN (74-07), KOR, NLD, SWE, USA</td>
<td>1970-2007</td>
<td>$T$: Agriculture, Mining, Manufacturing, Transport, Finance Intermediation $N$: Electricity, Construction, Trade, Hotels, Real Estate, Personal Services</td>
<td>EU KLEMS</td>
</tr>
<tr>
<td>Labor $L^T$ &amp; $L^N$ (total hours worked by persons engaged)</td>
<td>BEL, DEU, DNK, ESP, FIN, FRA, GBR, IRL, ITA, JPN (74-07), KOR, NLD, SWE, USA</td>
<td>1970-2007</td>
<td>$T$: Agriculture, Mining, Manufacturing, Transport, Finance Intermediation $N$: Electricity, Construction, Trade, Hotels, Real Estate, Personal Services</td>
<td>EU KLEMS</td>
</tr>
<tr>
<td>Labor compensation $LAB^T$ &amp; $LAB^N$ (current prices)</td>
<td>BEL, DEU, DNK, ESP, FIN, FRA, GBR, IRL, ITA, JPN (74-07), KOR, NLD, SWE, USA</td>
<td>1970-2007</td>
<td>$T$: Agriculture, Mining, Manufacturing, Transport, Finance Intermediation $N$: Electricity, Construction, Trade, Hotels, Real Estate, Personal Services</td>
<td>EU KLEMS</td>
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<tr>
<td>Consumption $C^T$ &amp; $C^N$ (constant prices)</td>
<td>BEL (95-07), DEU (91-07), DNK, ESP (95-07), FIN (75-07), FRA, ITA, GBR (90-07), IRL (96-07) JPN (80-07), KOR, NLD (80-07), SWE (93-07), USA</td>
<td>1990-2007</td>
<td>$T$: Food, Beverages, Clothing, Furnishings, Transport, Recreation, Other $N$: Housing, Health, Communication, Education, Restaurants, Recreation (Recreation is defined as 50% tradable and 50% non tradable)</td>
<td>COICOP</td>
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<tr>
<td>Trade balance $NX$ (constant prices)</td>
<td>BEL, DEU, DNK, ESP, FIN, FRA, GBR, IRL, ITA, JPN, KOR, NLD, SWE, USA</td>
<td>1970-2007</td>
<td>External balance of goods and services at current prices (source: OCDE) over price of traded goods ($P^T$)</td>
<td>authors’ calculations</td>
</tr>
<tr>
<td>Price $P^T$ &amp; $P^N$ (value added deflator)</td>
<td>BEL, DEU, DNK, ESP, FIN, FRA, GBR, IRL, ITA, JPN (74-07), KOR, NLD, SWE, USA</td>
<td>1970-2007</td>
<td>Value added at current prices ($P^TY^T$) over value added at constant prices ($Y^T$)</td>
<td>authors’ calculations</td>
</tr>
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<td>Relative Price $P$ (index 1993=100)</td>
<td>BEL, DEU, DNK, ESP, FIN, FRA, GBR, IRL, ITA, JPN (74-07), KOR, NLD, SWE, USA</td>
<td>1970-2007</td>
<td>Value added deflator of non traded goods ($P^N$) over value added deflator of traded goods ($P^T$)</td>
<td>authors’ calculations</td>
</tr>
<tr>
<td>Wage $W^T$ &amp; $W^N$ (nominal and per hour)</td>
<td>BEL, DEU, DNK, ESP, FIN, FRA, GBR, IRL, ITA, JPN (74-07), KOR, NLD, SWE, USA</td>
<td>1970-2007</td>
<td>Labor compensation ($LAB^T$) over total hours worked by hired persons ($L^T$)</td>
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<td>Relative Wage $\Omega$ (index 1993=100)</td>
<td>BEL, DEU, DNK, ESP, FIN, FRA, GBR, IRL, ITA, JPN (74-07), KOR, NLD, SWE, USA</td>
<td>1970-2007</td>
<td>Nominal wage in non tradables ($W^N$) over nominal wage in tradables ($W^T$)</td>
<td>authors’ calculations</td>
</tr>
<tr>
<td>Unemployment rate $u^T$ &amp; $u^N$</td>
<td>AUS (95-07), AUT (94-07), BEL (01-07), CAN (87-07), DEU (95-07), DNK (94-98 &amp; 02-04), ESP (92-07), FIN (95-07), GBR (88-07), IRL (96-07), ITA (93-07), JPN (93-07), KOR (92-07), SWE (95-07), USA (03-07)</td>
<td>Unemployed workers $U^T$ as a share of the labor force $F^T \equiv L^T + U^T$</td>
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<td>Sectoral Productivity $A^T$ &amp; $A^N$ (index 1995=100)</td>
<td>BEL, DEU, DNK, ESP, FIN, FRA, GBR, IRL, ITA, JPN (74-07), KOR, NLD, SWE, USA</td>
<td>1970-2007</td>
<td>Measured by labor productivity $A^T = Y^T / L^T$</td>
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<tr>
<td>Relative Productivity (index 1995=100)</td>
<td>BEL, DEU, DNK, ESP, FIN, FRA, GBR, IRL, ITA, JPN (74-07), KOR, NLD, SWE, USA</td>
<td>1970-2007</td>
<td>Computed as the ratio $A^T/A^N$</td>
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</table>
The historic OECD measure of benefit entitlements which is defined as the average of the gross unemployment benefit replacement rates for two earnings levels, three family situations and three durations of unemployment. Source: OECD, Benefits and Wages Database. Data coverage: 1970-2001 for all countries while data are unavailable for Korea.


Summary statistics of the labor market regulation indicators used in the empirical analysis are displayed in the last three columns of Table 31.

The construction, together with descriptive statistics, sources and data coverage, of the two measures we used to identify the state of the economy across the business cycle is detailed below in subsection E.6.

Table 31: Summary Statistics per Country

<table>
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<tr>
<th>Countries</th>
<th>Variables</th>
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<td>SWE</td>
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<tr>
<td>USA</td>
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</tr>
<tr>
<td>Average</td>
<td>1.79</td>
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</table>

A.2 Calibration of the Labor Market

To calibrate the labor market for the traded and the non traded sector, we need to estimate the sectoral unemployment rate, the job finding and the job destruction rate for each sector, and the sectoral labor market tightness. We provide below the source and construction of the data.

A.2.1 Source and Construction of Data

In this subsection, we first describe the data employed to calibrate some key features of OECD labor markets. Then, we present the dataset we use to estimate a set of sectoral search unemployment parameters. Summary statistics for the key indicators of the labor market are displayed in Table 32.

- **Sectoral unemployment rate**, $u^j$, is the number of unemployed workers $U^j$ in sector $j = T, N$ as a share of the labor force $L^j + U^j$ in this sector. LABORSTA database from the International Labour Organization (ILO) provides annual data for unemployed and employed workers at the 1-digit ISIC-rev.3 level. To construct $L^j$ and $U^j$ for $j = T, N$, we map the classification used previously to compute series for sectoral wages, prices and real labor productivity indexes (see section A.1) into the 1-digit ISIC-rev.3 classification used by the LABORSTA database. The mapping was clear for all industries except for “Not classifiable by economic activity” (1-digit ISIC-Rev.3 code: X) when constructing $L^j$ and $U^j$, and, “Unemployed seeking their first job” to identify $U^j$. These two categories have been split between tradables and non tradables according to the shares of total unemployment (excluding the two sectors) between tradables and non tradables by year and country. In a few rare cases, the sum of sectoral employment provided by ILO did not correspond to total unemployment. These differences were usually due to missing data for some industries in the sectoral databases. In these cases, we added these differences in level, keeping however the share of each sector constant. In Table 32 we provide an overview of the classifications used to construct traded and non traded sectors variables. Once industries have been classified as traded or non traded, series for unemployed and employed workers are constructed by adding unemployed and employed workers of all sub-industries $k$ in sector $j = T, N$ in the form $U^j = \sum_{k \in j} U_k$ and $L^j = \sum_{k \in j} L_k$. Data coverage: AUS (1995-2007), AUT (1994-2007), BEL (2001-2007), CAN (1987-2007), DEU (1995-2007), DNK (1994-1998 and 2002-2004), ESP (1992-2007), FIN (1995-2007), GBR (1988-2007), IRL (1986-1997), ITA (1993-2007), JPN (2003-2007), KOR (1992-2007), SWE (1995-2007) and USA (2003-2007). Data for unemployed workers by economic activity are not available for FRA, NLD and NOR.

- **Labor market tightness**, $\theta^j$ for $j = T, N$, is calculated as the ratio of employment vacancies in sector $j$ ($V^j$) to the number of unemployed workers in that sector ($U^j$). To construct the variables $\theta^j$, we collect information on job vacancies and unemployed workers by economic activity. Sources for $V^j$: Job Openings and Labor Turnover Survey (JOLTS) provided by the Bureau of Labor Statistics (BLS) for USA and Eurostat database (NACE 1-digit) for a range of European Countries, Labour Market Statistics from the Office for National Statistics for the UK. Sources for $U^j$: Current Population Survey (CPS) published by the BLS for USA and LABORSTA (ILO) for European Countries.63 As shown in Table 32, the level of detail in the definition of traded and non traded sectors differs across databases in two dimensions. First, the number of items to split disaggregated data varies across nomenclatures from a low eleven categories in the Eurostat database to a high of eighteen items in the LABORSTA database. Second, the definitions of items are not harmonized across the different sets of data. To generate sectoral variables in a consistent and uniform way, series on disaggregated data for vacancies and unemployed workers are added up to

63 The JOLTS and CPS databases provide (not seasonally adjusted) monthly data on vacancies and unemployed workers. We convert monthly data series into an annual data series by summing the twelve monthly data points.
form traded and non traded sectors following, as close as possible, the classification we used for value added, hours worked and labor compensation. Once industries have been classified as traded or non traded, series for employment vacancies (unemployed workers resp.) are constructed by adding job openings (unemployed workers resp.) of all sub-industries $k$ in sector $j = T, N$ in the form $V^j = \sum_{k \in j} V_k$ ($U^j = \sum_{k \in j} U_k$ resp.). Data coverage for $V^j$ and $U^j$: AUT (2004-2005), DEU (2006-2007), FIN (2002-2007), GBR (2001-2007), SWE (2005-2007) and USA (2001-2007).

For reason of space, Table 32 does not provide the classification between tradables and non tradables for job vacancies for the United Kingdom. The classification is detailed below. The Office for National Statistics provides series for the UK that cover 19 sectors, according to SIC 2007 classification. Sectors have been aggregated into tradables (Financial and insurance activities; Information and communication; Manufacturing; Mining and quarrying; Transport and storage) and non tradables (Accommodation and food service activities; Administrative and support service activities; Arts, entertainment and recreation; Construction; Education; Electricity, gas, steam and air conditioning supply; Human health and social work activities; Other service activities; Public administration and defense; Compulsory social security; Real estate activities; Water supply, sewerage, waste and remediation activities; Wholesale and retail trade; repair of motor vehicles and motor cycles).

### A.2.2 The Methodology

In this section, we present the approach we adopted to measure the job finding and employment exit rates by using readily accessible data. We apply the methodology developed by Shimer [2012] who assumes that the labor force is fixed. Applying the same logic to our two-sector model, we need to impose that the labor force $F^j$ is fixed at a sectoral level. The implication of such an assumption is twofold. First, we explicitly assume that there are no movements into and out of the labor force at an aggregate level. Second, we assume that there are no movements between the traded and the non traded sectors. Reassuringly, Shimer [2012] shows that a two-state model where workers simply transit between employment and unemployment does a good job of capturing unemployment fluctuations. Because the reallocation of labor across sectors is relatively low, the second assumption should not substantially affect the results. In particular, Shimer [2012] finds that the job finding rate to worker averaged 0.44 over the post-war period for the U.S., while our own estimates indicate that the job finding rate averages about 0.40 from 2003 to 2007.

The presentation below borrows heavily from Elsby, Hobijn, and Sahin [2013]. We assume that during period $t$, all unemployed workers find a job according to a Poisson process with arrival rate $m^j(t) = -\ln (1 - M^j(t))$ and all employed workers lose their job according to a Poisson process with arrival rate $\psi^j(t) = -\ln (1 - S^j(t))$. We refer to $m^j(t)$ and $s^j(t)$ as the job finding and job destruction rates in sector $j$ and to $M^j(t)$ and $S^j(t)$ as the corresponding probabilities.

The evolution over time of the unemployed workers, which we denote by $U^j(t)$, can be written as:

$$\dot{U}^j(t) = s^j(t)L^j(t) - m^j(t)U^j(t),$$  \hspace{1cm} (74)

where $L^j(t)$ is employment in sector $j$; the evolution over time of the unemployed workers can be written alternatively by using the fact that $L^j(t) = F^j - U^j(t)$:

$$\dot{U}^j(t) = s^j(t)\left(F^j - U^j(t)\right) - m^j(t)U^j(t),$$  \hspace{1cm} (75)

where $s^j(t)$ is the monthly rate of inflow into unemployment, $m^j(t)$ is the monthly outflow rate from unemployment, and $t$ indexes months.

Collecting terms, assuming that the job destruction rate and the job finding rate are constant within years and solving eq. (75), pre-multiplying by $e^{-(m+s)t}$, and integrating over the time interval $[t-12, t]$, leads to the temporal path for unemployed workers:

$$U^j(t) = \psi^j(t)\tilde{U}^j(t)F^j(t) + (1 - \psi(t))U^j(t-12);$$  \hspace{1cm} (76)
<table>
<thead>
<tr>
<th>Sector</th>
<th>EU KLEMS/STAN</th>
<th>LABORSTA Employment</th>
<th>LABORSTA Unemployment</th>
<th>JOLTS (BLS)</th>
<th>CPS (BLS)</th>
<th>EUROSTAT</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tradables</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Agriculture, Hunting, Forestry and Fishing</td>
<td>Agriculture, Hunting, Forestry and Fishing (A-B)</td>
<td>Agriculture, Hunting, Forestry (A) Fishing (B)</td>
<td>Agriculture</td>
<td>Mining and logging</td>
<td>Agriculture and fishing</td>
<td></td>
</tr>
<tr>
<td>Mining and Quarrying</td>
<td>Mining and Quarrying (C)</td>
<td>Mining and Quarrying (C)</td>
<td>Mining and Quarrying (C)</td>
<td>Manufacturing</td>
<td>Mining and quarrying</td>
<td></td>
</tr>
<tr>
<td>Total Manufacturing</td>
<td>Manufacturing (D)</td>
<td>Manufacturing (D)</td>
<td>Manufacturing (D)</td>
<td>Transportation</td>
<td>Manufacturing</td>
<td></td>
</tr>
<tr>
<td>Transport and Storage and Communication</td>
<td>Transport, Storage and Communications (I)</td>
<td>Transport, Storage and Communications (I)</td>
<td>Transport, Storage and Communications (I)</td>
<td>Information</td>
<td>Transportation and utilities</td>
<td></td>
</tr>
<tr>
<td>Financial Intermediation</td>
<td>Financial Intermediation (J)</td>
<td>Financial Intermediation (J)</td>
<td>Financial Intermediation (J)</td>
<td>Finance and insurance</td>
<td>Information</td>
<td></td>
</tr>
<tr>
<td>Non Tradables</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Electricity, Gas and Water Supply (E)</td>
<td>Electricity, Gas and Water Supply (E)</td>
<td>Electricity, Gas and Water Supply (E)</td>
<td>Electricity, Gas and Water Supply (E)</td>
<td>Construction</td>
<td>Construction</td>
<td></td>
</tr>
<tr>
<td>Construction (F)</td>
<td>Construction (F)</td>
<td>Construction (F)</td>
<td>Construction (F)</td>
<td>Wholesale trade</td>
<td>Wholesale and retail trade</td>
<td></td>
</tr>
<tr>
<td>Wholesale and Retail Trade (G)</td>
<td>Wholesale and Retail Trade (G)</td>
<td>Wholesale and Retail Trade (G)</td>
<td>Wholesale and Retail Trade (G)</td>
<td>Retail trade</td>
<td>Wholesale retail trade</td>
<td></td>
</tr>
<tr>
<td>Hotels and Restaurants (H)</td>
<td>Hotels and Restaurants (H)</td>
<td>Hotels and Restaurants (H)</td>
<td>Hotels and Restaurants (H)</td>
<td>Real estate and rental</td>
<td>Hotels and restaurants</td>
<td></td>
</tr>
<tr>
<td>Real Estate, Renting and Business Activities</td>
<td>Real Estate, Renting and Business Activities (K)</td>
<td>Real Estate, Renting and Business Activities (K)</td>
<td>Real Estate, Renting and Business Activities (K)</td>
<td>Business services</td>
<td>Business services</td>
<td></td>
</tr>
<tr>
<td>Personal Services (LtQ)</td>
<td>Other Community, Social and Personal Service Activities (O) Households with Employed Persons (P) Extra-Territorial Organizations and Bodies (Q) Not classifiable by economic activity (X)</td>
<td>Other Community, Social and Personal Service Activities (O) Households with Employed Persons (P) Extra-Territorial Organizations and Bodies (Q) Not classifiable by economic activity (X)</td>
<td>Other Community, Social and Personal Service Activities (O) Households with Employed Persons (P) Extra-Territorial Organizations and Bodies (Q) Not classifiable by economic activity (X)</td>
<td>Leisure and hospitality</td>
<td>Leisure and hospitality</td>
<td></td>
</tr>
<tr>
<td>Unclassified</td>
<td>Other services</td>
<td>Other services</td>
<td>Other services</td>
<td>Other services</td>
<td>Other services</td>
<td></td>
</tr>
</tbody>
</table>

Table 32: Summary of Sectoral Classifications
where $\tilde{\alpha}^j$ is the long-run unemployment rate in sector $j$:

$$\tilde{\alpha}^j(t) = \frac{s^j(t)}{s^j(t) + m^j(t)}, \quad (77)$$

and $\psi^j$ is the annual rate of convergence to the long-run sectoral unemployment rate:

$$\psi^j(t) = 1 - e^{-(s^j(t) + m^j(t))12}. \quad (78)$$

To infer the monthly outflow probability $M^j(t)$ and then the monthly job finding rate $m^j(t)$, we follow Shimer [2012] and write the dynamic equations of sectoral unemployment and sectoral short term unemployment, i.e.,

$$\dot{U}^j(t + d) = s^j(t)L^j(t) - m^j(t)U^j(t), \quad (79a)$$

$$\dot{U}^{j,<d}(t + d) = s^j(t)L^j(t) - m^j(t)U^{j,<d}(t), \quad (79b)$$

where $U^{j,<d}(t + d)$ denotes short-term unemployment, i.e., the stock of unemployed workers who are employed at some time $\tau \in [t, t + d]$ but lose their job and thus are unemployed at time $t + d$; hence, by construction, $U^{j,<d}(t) = 0$ since all short-term unemployed workers were employed at time $t$. Combining (79a) and (79b) to eliminate $s^j(t)L^j(t)$ leads to a dynamic equation relating changes of unemployment to changes of short-term unemployment:

$$\dot{U}^j(t + d) = \dot{U}^{j,<d}(t + d) - m^j(t)\left(U^j(t) - U^{j,<d}(t)\right). \quad (80)$$

Solving eq. (80) above by integrating over $[t - d, t]$, and using the fact that at time $t$, short-term unemployment is such that $U^{j,<d}(t) = 0$, leads to:

$$U^j(t + d) = U^{j,<d}(t + d) + e^{-m^j(t)d}U^j(t).$$

Inserting $e^{-m^j(t)d}(t + d) = (1 - M^{j,<d}(t))$ where $M^{j,<d}$ is the probability that an unemployed worker exits unemployment within $d$ months, one obtains:

$$U^j(t + d) - U^j(t) = U^{j,<d}(t + d) - M^{j,<d}(t)U^j(t). \quad (81)$$

Eq. (81) states that the change of unemployment in sector $j$ is equal to the inflows into unemployment $U^{j,<d}(t + d)$ of workers who were employed at time $t$ but are unemployed at time $t + d$ less the number of unemployed workers who find a job $M^{j,<d}(t)U^j(t)$. Solving (81) for $M^{j,<d}(t)$, it is possible to write the probability that an unemployed worker exits unemployment within $d$ months as

$$M^{j,<d}(t) = 1 - \left[\frac{U^j(t + d) - U^{j,<d}(t + d)}{U^j(t)}\right]. \quad (82)$$

The probability of finding a job within $d$ months given by eq. (82) can be mapped as the monthly job finding rate for unemployment duration $d = 1, 3, 6, 12$:

$$m^{j,<d}(d) = -\frac{1}{d} \ln \left(1 - M^{j,<d}(d)\right). \quad (83)$$

To estimate the monthly job finding rate, we use the duration of unemployment lower than one month. In this configuration, the probability of finding a job can be rewritten as follows:

$$M^{j,<1}(t) = 1 - \left[\frac{U^j(t) - U^{j,<1}(t)}{U^j(t - 1)}\right]$$

or alternatively

$$1 - M^{j,<1}(t) = \frac{U^j(t) - U^{j,<1}(t)}{U^j(t - 1)}. \quad (84)$$

Since $U^j(t - 1)$ corresponds to monthly unemployment, we have to convert annual data on a monthly basis:

$$U^j(t - 1) = \left(U^j(t - 12)\right)^{1/12} \left(U^j(t)\right)^{11/12}. \quad (85)$$
We estimate the job destruction rate can be estimated by solving the following equation:

\[ m^{j<1}(t) = - \ln \left( U^j(t) - U^{j<1}(t) \right) + \ln \left( U^j(t - 1) \right), \tag{86} \]

where the construction of \( U^j(t - 1) \) is given by eq. (85) while the same logic applies to \( U^j(t) \).

Since series for unemployment by duration are expressed in percentage, we define \( \alpha^{j<1}(t) \) the share of unemployment less than one month among total unemployment as follows:

\[ \alpha^{j<1}(t) = \frac{U^{j<1}(t)}{U^j(t)}. \tag{87} \]

Because the share of short-term unemployment is not available by economic activity, we assume that \( \alpha^{j<1}(t) \) is identical across sectors:

\[ \alpha^{j<1}(t) = \alpha^{T<1}(t) = \alpha^{N<1}(t). \tag{88} \]

The job destruction rate can be estimated by solving this equation:

\[ U^j(t) = \psi^j(t) \frac{s^j(t)}{s^j(t) + m^{j<1}(t)} (U^j(t) + L^j(t)) + (1 - \psi^j(t)) U^j(t - 1), \tag{89} \]

where \( \psi^j \) is the monthly rate of convergence to the long-run sectoral unemployment rate:

\[ \psi^j(t) = 1 - e^{-\left(s^j(t) + m^{j<1}(t)\right)}. \tag{90} \]

### A.2.3 Computation of the job finding rate and the job separation rate at a sectoral level

To estimate the monthly job finding rate, \( m^{j<1} \), and the job destruction rate, \( s^j \), for \( j = T, N \), we proceed as follows:

- We estimate \( \alpha^{<1}(t) = \alpha^{j<1}(t) = \frac{U^{<1}(t)}{U(t)} \) where \( U^{<1}(t) \) is unemployment of duration less than one month.
- Using the fact that \( U^{j<1}(t) = \alpha^{<1}(t) U^j(t) \), the probability of finding a job is

\[ M^{j<1}(t) = 1 - \left[ \frac{1 - \alpha^{<1}(t) U^j(t)}{U^j(t - 1)} \right], \tag{91} \]

where \( U^j(t - 1) \) corresponds to monthly unemployment which is calculated as follows

\[ U^j(t - 1) = \left( U^j(t - 12) \right)^{1/12} \left( U^j(t) \right)^{11/12} \]

by using annual data.

- The monthly job finding rate is:

\[ m^{j<1}(t) = - \ln \left( 1 - M^{j<1}(t) \right) \tag{92} \]

- The job destruction rate can be estimated by solving the following equation:

\[ U^j(t) = \psi^j(t) \frac{s^j(t)}{s^j(t) + m^{j<1}(t)} (U^j(t) + L^j(t)) + (1 - \psi(t)) U^j(t - 1), \tag{93} \]

where \( \psi^j \) is the monthly rate of convergence to the long-run sectoral unemployment rate:

\[ \psi^j(t) = 1 - e^{-\left(s^j(t) + m^{j<1}(t)\right)}. \tag{94} \]

To compute \( m^{j<1} \) and \( s^j \), we need series for unemployment by economic activity in order to construct \( U^j \), and unemployment less than one month in order to estimate \( \alpha^{<1}(t) \). For unemployment at the sectoral level, data are taken from ILOSTAT database (ILO) while unemployment less than one month is provided by OECD which gives unemployment by duration. Data coverage: AUS (1995-2007), AUT (1994-2007), BEL (2001-2007), CAN
Table 33: Comparison of Actual Values with Calculated Values for the Sectoral Unemployment Rates

<table>
<thead>
<tr>
<th>Country</th>
<th>Actual</th>
<th>Calculated</th>
<th>Error</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$u^*$</td>
<td>$u^N$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$u^{*}$</td>
<td>$u^N$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$u^* - u^N$</td>
<td>$u^N - u^{*}$</td>
<td></td>
</tr>
<tr>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
<td>(4)</td>
</tr>
<tr>
<td>(5)</td>
<td>(6)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>AUS</td>
<td>0.072</td>
<td>0.062</td>
<td>-0.012-0.004</td>
</tr>
<tr>
<td>AUT</td>
<td>0.037</td>
<td>0.044</td>
<td>0.0010.007</td>
</tr>
<tr>
<td>BEL</td>
<td>0.077</td>
<td>0.079</td>
<td>0.0020.001</td>
</tr>
<tr>
<td>CAN</td>
<td>0.082</td>
<td>0.084</td>
<td>-0.004-0.002</td>
</tr>
<tr>
<td>DEU</td>
<td>0.101</td>
<td>0.099</td>
<td>0.001-0.003</td>
</tr>
<tr>
<td>DNK</td>
<td>0.064</td>
<td>0.061</td>
<td>-0.0030.001</td>
</tr>
<tr>
<td>ESP</td>
<td>0.147</td>
<td>0.161</td>
<td>0.0060.001</td>
</tr>
<tr>
<td>FIN</td>
<td>0.087</td>
<td>0.118</td>
<td>-0.0010.001</td>
</tr>
<tr>
<td>GBR</td>
<td>0.073</td>
<td>0.066</td>
<td>0.002-0.002</td>
</tr>
<tr>
<td>IRL</td>
<td>0.130</td>
<td>0.154</td>
<td>-0.0020.010</td>
</tr>
<tr>
<td>ITA</td>
<td>0.094</td>
<td>0.098</td>
<td>-0.0100.001</td>
</tr>
<tr>
<td>JPN</td>
<td>0.033</td>
<td>0.033</td>
<td>0.0090.008</td>
</tr>
<tr>
<td>SWE</td>
<td>0.056</td>
<td>0.060</td>
<td>0.0130.015</td>
</tr>
<tr>
<td>USA</td>
<td>0.048</td>
<td>0.053</td>
<td>0.0010.001</td>
</tr>
</tbody>
</table>

(1987-2007), DEU (1995-2007), DNK (1994-1998 and 2002-2004), ESP (1992-2007), FIN (1991-2007), GBR (1988-2007), IRL (1986-1997), ITA (1993-2007), JPN (2003-2007), SWE (1995-2007) and USA (2003-2007). Because we calibrate the model so that the initial steady state is consistent with the empirical properties of each OECD economy while the series for the sectoral job separation rates are computed when the economy is out of the steady-state, we need to compute values for $s^j$ which are consistent with the steady-state sectoral unemployment rate $\tilde{u}^j = \frac{s^j}{s^j + m^j}$, given the computed value for $m^j$. The two first columns in Table 33 show the actual values for the sectoral unemployment rates while columns 3 and 4 give the values for steady-state sectoral unemployment rates computed by using its long-run equilibrium $\tilde{u}^j = \frac{s^j}{s^j + m^j}$ where the job finding rate $m^j$ is taken from columns 5 and 7 of Table 6 and the job destruction rate has been computed by solving eq. (93). The two last columns of Table 33 show the difference between the actual and the predicted value. Reassuringly, because computed values for $m^j$ and $s^j$ by using (92) and (93) are averaged over a long enough time horizon so that the unemployment rate should have reached its long-run value, actual and predicted values are close in most of the cases, except for Sweden, Australia and Italy (for $u^T$), and Ireland (for $u^N$). The values for sectoral job destruction rates shown in columns 6 and 8 of Table 6 are thus calculated by using the long-run equilibrium expression for the sectoral unemployment rate, i.e.,

$$s^j = \frac{m^j u^j}{1 - u^j},$$

where $u^j$ is taken from columns 2 and 3 of Table 6 and $m^j$ is taken from columns 5 and 7 of Table 6. Computed values for $s^j$ using (95) are shown in columns 6 and 8 of Table 6.

For France, Korea, the Netherlands, and Norway, data are not available to compute the job finding and the job separation rate. We proceed as follows to get estimates of $m$ and $s$ when calibrating the model for each economy:

1. Because data for unemployment by economic activity are not available for FRA, NLD, and NOR, estimates for the job finding rate $m = m^j$ are taken from Hobijn and Sahin [2009]. Note that estimates are not available at a sectoral level so that we have to assume that the job finding rate is identical across sectors, i.e., $m^j = m$. Building on estimates by Hobijn and Sahin [2009], we set $m = 6.7\%$ for France (1975-2004), $m = 4.7\%$ for the Netherlands (1983-2004), and $m = 30.5\%$ for Norway (1983-2004). To compute the job separation rate, we use the steady-state expression for the unemployment rate $u = \frac{1}{s + m}$, where the unemployment rate is averaged over the appropriate period, i.e., 1975-2004 for France, 1983-2004 for the Netherlands and 1983-2004 for Norway. Series for harmonized unemployment rates are taken from Labor Force Survey, OECD.
While we can construct series for unemployment by economic activity for Korea, series for unemployment by duration is not provided by the OECD for this economy. We thus average the job finding rates taken from Chang et al. [2004] over 1993-1994, i.e., $m = 26.2\%$ and compute the job destruction rate by using the steady-expression for the unemployment rate $u^j = \frac{sl}{st+sm}$ where $u^j$ is the sectoral unemployment rate calculated by using the LABORSTA database from ILO.

### A.3 Elasticity of substitution in consumption ($\phi$): Empirical Strategy

When including physical capital investment and denoting recruiting costs by $F \equiv \kappa^T V^T + \kappa^N V^N$, according to the goods market equilibrium, we have:

$$
\frac{Y^T - NX - IT - G^T - F}{Y^N - IN - GN} = \frac{C^T}{C^N},
$$

(96)

where we used the fact that $\dot{B} = r^{s} B = NX$ with $B$ the net foreign asset position and $NX$ net exports. Inserting the optimal rule for intra-temporal allocation of consumption (10), i.e., $\frac{C^T}{C^N} = \left(\frac{\varphi}{1-m}\right) P^{\rho}$, into (96) leads to

$$
\frac{Y^T - NX - IT - G^T - F}{Y^N - IN - GN} = \left(\frac{\varphi}{1-\varphi}\right) P^{\rho}.
$$

(97)

According to the market clearing condition, we could alternatively use data for consumption or for sectoral value added along with times series for its demand components to estimate $\phi$. Unfortunately, classifications for valued added by industry and for consumption by items are different (because nomenclatures are different) and thus it is most likely that $C^T$ differs from $Y^T - NX - G^T - IT - F$, and $C^N$ from $Y^N - G^N - IN$ as well. Because time series for traded and non traded consumption display a short time horizon for half countries of our sample while data for sectoral value added and net exports are available for the 18 OECD countries of our sample over the period running from 1970 to 2007 (except for Japan: 1974-2007), we find appropriate to estimate $\phi$ by computing $Y^T - NX - E^T$ and $Y^N - EN$ where $E^T \equiv G^T + I^T + F$ and $E^N \equiv G^N + I^N$. Yet, a difficulty shows up because the classification adopted to split government spending and investment expenditure into traded and non traded items is different from that adopted to break down value added into traded and non traded components. Moreover, the time horizon is short at a disaggregated level for most of the countries, especially for time series of $G^I$. To overcome these difficulties, we proceed as follows. Denoting the ratio of $E^T \equiv G^T + I^T + F$ to traded value added adjusted with net exports at current prices by $v_{ET} = \frac{PT^T}{PT^T - PX^T}$, and denoting the ratio of $E^N \equiv G^N + I^N$ to non traded value added at current prices by $v_{EN} = \frac{PN^N}{PN^N - PX^N}$, the goods market equilibrium (97) can be rewritten as follows:

$$
\frac{(PT^T - PX^T) (1 - v_{ET})}{P^N Y^N (1 - v_{EN})} = \left(\frac{\varphi}{1-\varphi}\right) P^{\rho-1},
$$

or alternatively

$$
\frac{(Y^T - NX) (1 - v_{ET})}{Y^N (1 - v_{EN})} = \left(\frac{\varphi}{1-\varphi}\right) P^{\rho}.
$$

(98)

Setting

$$
\alpha \equiv \ln \frac{(1 - v_{EN})}{(1 - v_{ET})} + \ln \left(\frac{\varphi}{1-\varphi}\right),
$$

(99)

and taking logarithm, eq. (98) can be rewritten as follows:

$$
\ln \left(\frac{Y^T - NX}{Y^N}\right) = \alpha + \phi \ln P.
$$

(100)

Indexing time by $t$ and countries by $i$, and adding an error term $\mu$, we estimate $\phi$ by exploring the following empirical relationship:

$$
\ln \left(\frac{Y^T - NX}{Y^N}\right)_{i,t} = f_i + f_t + \alpha t + \phi_i \ln P_{i,t} + \mu_{i,t},
$$

(101)
where \( f_i \) captures the country fixed effects, \( f_t \) are time dummies, and \( \mu_{i,t} \) are the i.i.d. error terms. Because the term (99) is composed of ratios which may display a trend over time, we add country-specific trends, as captured by \( \alpha_{i,t} \). Eq. (101) corresponds to eq. (40) in Online Appendix C.2.

Instead of using time series for sectoral value added, we can alternatively make use of series for sectoral labor compensation. Multiplying both sides by \( \frac{P_T}{P_T N} \) and then by \( \frac{\rho_T}{\rho_T} \) with \( \rho^j = \frac{W^j L^j}{P^j Y^j} \) the sectoral labor income share, eq. (98) can be rewritten as follows

\[
\ln \left( \frac{W^T L^T - \rho^T P^T N X}{W^N L^N} \right) = \eta + (\phi - 1) \ln P. \tag{102}
\]

where

\[
\eta \equiv \ln \left( \frac{1 - \nu E^N}{1 - \nu E^T} \right) + \ln \left( \phi \frac{\rho^T}{\rho^N} \right). \tag{103}
\]

Indexing time by \( t \) and countries by \( i \), and adding an error term \( \mu_{i,t} \), we estimate \( \phi \) by exploring the following empirical relationship:

\[
\ln \left( \frac{\gamma_T}{\gamma^N} \right)_{i,t} = g_i + g_t + \eta_{i,t} + \delta_{i,t} \ln P_{i,t} + \zeta_{i,t}, \tag{104}
\]

where \( \delta_{i} = (\phi_{i} - 1) \); \( g_t \) are time dummies which capture common macroeconomic shocks. Because \( \eta_{i} \) is composed of preference parameters (i.e., \( \varphi \)), and (logged) ratios which may display trend over time, we introduce country fixed effects \( g_{i,t} \), and add country-specific trends, as captured by \( \eta_{i,t} \). Once we have estimated \( \delta_{i,t} \), we can compute \( \hat{\phi}_{i} = \hat{\delta}_{i} + 1 \) where a hat refers to point estimate in this context. Eq. (104) corresponds to eq. (41) in the text.

**B Two-Sector Open Economy with Search Frictions**

In this section, we determine the first-order conditions and next we conduct an analysis of equilibrium dynamics.

**B.1 Households**

The representative household chooses the time path of consumption and labor force to maximize the following objective function:

\[
\Upsilon = \int_{0}^{\infty} \left\{ \frac{1}{1 - \frac{1}{\sigma_C} C(t)^{1 - \frac{1}{\sigma_C}} - \frac{1}{1 + \frac{1}{\sigma_L}} F(t)^{1 + \frac{1}{\sigma_L}}} \right\} e^{-\beta t} dt, \tag{105}
\]

where \( \beta > 0 \) is the consumer’s subjective time discount rate, \( \sigma_C > 0 \) is the intertemporal elasticity of substitution for consumption; \( \sigma_L \) is the elasticity of labor supply at the extensive margin which is symmetric across sectors. We assume that labor force in the traded and the non-traded sectors are imperfect substitutes and aggregated by means of a CES function:

\[
F(t) = \left[ \zeta^T F^T(t)^{\frac{1 + \sigma_L}{\sigma_L}} + \zeta^N F^N(t)^{\frac{1 + \sigma_L}{\sigma_L}} \right]^{\frac{\sigma_L}{1 + \sigma_L}}, \tag{106}
\]

where \( \zeta^j > 0 \) parametrizes the disutility from working and searching efforts in sector \( j = T, N \), and \( \sigma_L \) is the elasticity of substitution between traded and non-traded labor force which captures the extent of workers’ moving costs.

Inserting (106) into (105) and denoting the disutility function from working and searching efforts by \( \nu^j \left( L^j(t) + U^j(t) \right) = -\frac{\zeta^j}{1 + \frac{1}{\sigma_L}} F^j(t)^{\frac{1 + \sigma_L}{\sigma_L}} \), the instantaneous utility reads as follows:

\[
\Phi(t) \equiv \frac{1}{1 - \frac{1}{\sigma_C}} C(t)^{1 - \frac{1}{\sigma_C}} + \sum_j \nu^j \left( L^j(t) + U^j(t) \right). \tag{107}
\]
We drop the time index below when it causes no confusion. The current-value Hamiltonian for the representative household’s optimization problem is:

\[
\mathcal{H}^H = \Phi + \lambda \left[ r^* A + W^T L^T + W^N L^N + R^T U^T + R^N U^N - P_C C - T \right] \\
+ \xi^{T,j} \left[ m^T U^T - s^T L^T \right] + \xi^{N,j} \left[ m^N U^N - s^N L^N \right],
\]

where \( A, L^j (j = T, N) \) are state variables; \( \lambda, \xi^j (\text{with } j = T, N) \) are the corresponding co-state variables; \( C \) and \( U^j \) are the control variables.

Assuming that the representative agent takes \( m \) as given, first-order conditions for households are:

\[
\begin{align*}
C &= (P_C \lambda)^{-\sigma_C}, \\
-v_F^T (L^T + U^T) &= m_T \xi^{T,j} + R^T \lambda, \\
-v_F^N (L^N + U^N) &= m_N \xi^{N,j} + R^N \lambda,
\end{align*}
\]

where \( \xi^{j} \) (with \( j = T, N \)) is the utility value of the marginal job and \( \lambda \) the marginal utility of wealth.

Since \( \xi^{j} \) represents the utility value from an additional job and \( \hat{\lambda} \) corresponds to the marginal utility of wealth, the pecuniary value of the marginal job is \( \xi^{j} (\tau) \equiv \frac{\xi^{j} (\tau)}{\lambda} \) for \( \tau \in [t, \infty) \). Using this definition, we can rewrite (109d) as follows:

\[
\dot{\xi}^j = (s^j + r^* \xi^j) - \left( W^j + \frac{v_F^j}{\lambda} \right).
\]

Abstracting from search costs implies that the marginal rate of substitution between labor and consumption, \( -\frac{v_F^j}{\lambda} \), has to be equal to the wage rate \( W^j \). In this case, the shadow price of employment \( \xi^j \) is null. As long as agents face search costs, the real wage rate must exceed the disutility from entering the labor force \( -\frac{v_F^j}{\lambda} \). Since the quantity \( -\frac{v_F^j}{\lambda} \) can be viewed as being the worker’s reservation wage, we will refer to \( W^j + \frac{v_F^j}{\lambda} \) as the worker’s surplus (by keeping in mind that \( v_F^j < 0 \)).

Solving (110) forward and using the transversality condition \( \lim_{t \to \infty} \xi^j L^j \exp (-r^* t) = 0 \), we get:

\[
\xi^j (t) = \int_t^\infty \left[ W^j (\tau) - W^j_R (\tau) \right] e^{(s^j + r^*) (t - \tau)} d\tau,
\]

where \( W^j_R \) is the reservation wage given by

\[
W^j_R = -\frac{v_F^j}{\lambda} = m^j \left( \theta^j \right) \xi^j + R^j.
\]

Differentiating \( \xi^j (t) L^j (t) \) w. r. t. time and substituting the law of motion for employment \( \dot{L}^j (t) \) (7) and the dynamic optimality condition (110) yields:

\[
\frac{d}{dt} (\xi^j L^j) = r^* \xi^j L^j - \left( W^j + \frac{v_F^j}{\lambda} \right) \frac{d}{dt} \xi^j L^j - \left( W^j L^j + \frac{v_F^j}{\lambda} \xi^j \left( m^j U^j - s^j L^j \right) \right),
\]

\[
= r^* \xi^j L^j - \left[ \left( W^j + \frac{v_F^j}{\lambda} \right) \frac{d}{dt} \xi^j L^j - \xi^j m^j U^j \right],
\]

\[
= r^* \xi^j L^j - \left( W^j L^j + R^j U^j + \frac{v_F^j}{\lambda} F^j \right),
\]

\[14\]
where \( F^j \equiv L^j + U^j \) is the labor force and we have inserted eqs. (109b)-(109c), i.e., we used the fact that \( m^j \xi^j = -\frac{\nu_F^j}{\lambda} - R^j \). Solving forward, making use of the transversality condition, we get:

\[
\xi^j(t)L^j(t) = \int_t^\infty \left[ (W^j L^j + R^j U^j) + \frac{\nu_F^j}{\lambda} F^j \right] e^{-\rho(t-s)} ds.
\]

(113)

Differentiating \( \frac{\nu_F^j(U^j + L^j)}{\lambda} = m^j (\theta^j) \xi^j + R^j \) w.r.t. time and inserting (110), we can derive the dynamic equation for job seekers in sector \( j \):

\[
-\frac{\nu_F^j}{\lambda} \dot{U}^j = m^j (\theta^j) \xi^j + \alpha_V^j m^j (\theta^j) \xi^j \frac{\dot{\theta}^j}{\theta^j} + \frac{\nu_F^j}{\lambda} \dot{L}^j,
\]

\[
= \left[ (s^j + r^*) + \alpha_V^j \frac{\dot{\theta}^j}{\theta^j} \right] m^j (\theta^j) \xi^j - m^j (\theta^j) \left( W^j + \frac{\nu_F^j}{\lambda} \right) + \frac{\nu_F^j}{\lambda} \dot{L}^j.
\]

where we used the fact that \( \frac{\nu_F^j}{m^j} \frac{\dot{\theta}^j}{\theta^j} = \alpha_V^j \). Substituting \( m^j \xi^j = -\frac{\nu_F^j}{\lambda} - R^j \), we get:

\[
\frac{\nu_F^j}{\lambda} \dot{U}^j = \left( \frac{\nu_F^j}{\lambda} + R^j \right) \left[ (s^j + r^*) + \alpha_V^j \frac{\dot{\theta}^j}{\theta^j} \right] m^j (\theta^j) \xi^j - \left( W^j + \frac{\nu_F^j}{\lambda} \right) \dot{L}^j.
\]

(114)

**B.2 Firms**

We consider a traded sector which produces a good denoted by the superscript \( T \) that can be exported or consumed domestically. We also consider a non traded sector which produces a good denoted by the superscript \( N \) that can be consumed only domestically. Each sector consists of a large number of identical firms. Both the traded and non-traded sectors use labor, \( L^T \) and \( L^N \), according to constant returns to scale production functions:

\[
Y^T = A^T L^T, \quad \text{and} \quad Y^N = A^N L^N.
\]

(115)

Firms post job vacancies \( V^j \) to hire workers and face a cost per job vacancy \( \kappa^j \) which is assumed to be constant and measured in terms of the traded good. Firms pay the wage \( W^j \) decided by the generalized Nash bargaining solution. We also consider that firms must pay a firing tax \( x^j \) per job loss which captures the extent of employment protection legislation (see e.g., Heijdra and Ligthart [2002], Veracierto [2008]).

As producers face a labor cost \( W^j \) per employee, a cost per hiring of \( \kappa^j \), the profit function of the representative firm in the traded sector is:

\[
\pi^T = A^T L^T - W^T L^T - \kappa^T V^T - x^T \max \left\{ 0, -\dot{L}^T \right\},
\]

(116)

where \( x^T \) is a firing tax in the traded sector when \( \dot{L}^T < 0 \) otherwise \( x^T = 0 \).

Symmetrically, denoting by \( P \) the price of non traded goods in terms of traded goods, the profit function of the representative firm in the non traded sector is:

\[
\pi^N = PA^N L^N - W^N L^N - \kappa^N V^N - x^N \max \left\{ 0, -\dot{L}^N \right\},
\]

(117)

where \( x^N \) is a firing tax in the non traded sector when \( \dot{L}^N < 0 \) otherwise \( x^N = 0 \).

Denoting by \( f^j \) the rate at which a vacancy is matched with unemployed agents, the law of motion for labor is given by:

\[
\dot{L}^j = f^j (\theta^j) - s^j L^j,
\]

(118)

where \( f^j V^j \) represents the flow of job vacancies which are fulfilled; note that \( f^j \) decreases with labor tightness \( \theta^j \).

The current-value Hamiltonian for the sector \( j \)'s representative firm optimization problem is:

\[
H^j = \Xi^j L^j - W^j L^j - \kappa^j V^j + (\gamma^j + x^j) \left( f^j V^j - s^j L^j \right),
\]

(119)
where $\Xi^j$ is the marginal revenue of labor with $\Xi^T \equiv A^T$ and $\Xi^N \equiv PA^N$ and $\gamma^j$ is the co-state variable associated to the law of motion equation (118).

First-order conditions can be written as follows:

$$\gamma^j + x^j \mathbb{1}_{L^j < 0} = \frac{\kappa^j}{f^j(\theta^j)},$$

$$\gamma^j = \gamma^j (\tau^* + s^j) - (\Xi^j - s^j x^j) \mathbb{1}_{L^j < 0} - W^j,$$  \hspace{1cm} (120a)

$$\gamma^j = \gamma^j (\tau^* + s^j) - (\Xi^j - s^j x^j) \mathbb{1}_{L^j < 0} - W^j,$$  \hspace{1cm} (120b)

where $\gamma^j$ represents the pecuniary value of an additional job to the representative firm of sector $j = T, N$. This can be seen more formally by solving (120b) forward and using the appropriate transversality condition. This yields:

$$\gamma^j(t) = \int_t^\infty \left[ (\Xi^j(\tau) - W^j(\tau) - s^j x^j \mathbb{1}_{L^j < 0} ) e^{(s^j + \tau^*)(\tau-t)} d\tau. \right]$$

We drop the indicator function below when it causes no confusion.

Differentiating $\gamma^j(t) L^j(t)$ w.r.t. time and inserting the law of motion for employment $\dot{L}^j(t)$ together with the dynamic optimality condition (120b), we obtain:

$$\frac{d}{dt} (\gamma^j L^j) = \gamma^j L^j + \gamma^j \dot{L}^j = \gamma^j (\tau^* + s^j) L^j + x^j s^j L^j - (\Xi^j - W^j) L^j + \gamma^j (f^j V^j - s^j L^j),$$

$$= r^* \gamma^j L^j - [\Xi^j L^j - W^j L^j - \gamma^j f^j V^j - x^j s^j L^j] = r^* \gamma^j L^j - \pi^j,$$

where we used the fact that $\gamma^j = \kappa^j / f^j - x^j$, $\pi^j = \Xi^j L^j - W^j L^j + x^j \dot{L}^j - \kappa^j V^j$ and $\dot{L}^j = f^j \theta^j - s^j L^j$. Using the first-order condition (120a) and solving forward, making use of the transversality condition, we get:

$$\gamma^j(t) L^j(t) = \int_t^\infty \left[ (\Xi^j L^j - W^j L^j - \kappa^j V^j - x^j \max \{0, -L^j\} ) e^{-r^*(\tau-t)} d\tau, \right]$$

$$= \int_t^\infty \pi^j e^{-r^*(\tau-t)} d\tau. \hspace{1cm} (122)$$

Eq. (122) corresponds to eq. (15) in the main text.

C Matching and Wage Determination

In each sector, there are job-seeking workers $U^j$ and firms with job vacancies $V^j$ which are matched in a random fashion. Assuming a constant returns to scale matching function, the number of labor contracts $M^j$ concluded per job seeker $U^j$ gives the job finding rate $m^j$ which is increasing in the labor market tightness $\theta^j$:

$$m^j = \frac{M^j}{U^j} = X^j \left( \frac{V^j}{U^j} \right)^{\alpha^j_V} = X^j (\theta^j)^{\alpha^j_V}, \hspace{1cm} \alpha^j_V \in (0, 1),$$

where $\alpha^j_V$ represents the elasticity of vacancies in job matches and $X^j$ corresponds to the matching efficiency.\(^{64}\) The number of matches $M^j$ per job vacancy gives the worker-finding rate for the firm:

$$f^j = \frac{M^j}{V^j} = X^j (\theta^j)^{\alpha^j_V-1}. \hspace{1cm} (124)$$

Eq. (124) shows that the instantaneous probability of the firm finding a worker is higher the lower the labor market tightness $\theta^j$.

The representative firm of sector $j$ posts job vacancies in order to hire workers. We assume that the wage rate is derived from a bargaining between the firm and the worker. The wage rate $W^j$ is set so as to maximize the following expression:

$$W^j(t) = \arg\max \mathcal{H}_W = \arg\max (\xi^j(t))^{\alpha^j_W} (\gamma^j(t) + x^j) ^{1-\alpha_W}, \hspace{1cm} 0 \leq \alpha^j_W \leq 1, \hspace{1cm} (125)$$

\(^{64}\)Note that the flows of workers in and out of employment are equal to each other in any symmetric equilibrium, i.e., $m^U^j = f^j V^j$. Hence equations $\dot{L}^j = f^j V^j - s^j L^j$ and $\dot{L}^j = m^U^j - s^j L^j$ indicate that the demand for labor indeed equals the supply.
where $\alpha_W^j$ and $1 - \alpha_W^j$ correspond to the bargaining power of the worker and the firm, respectively. The first-order condition determining the current wage, $w(t)$ writes as follows:

$$\frac{\partial \mathcal{H}_W^j}{\partial W_j(t)} = \frac{\alpha_W^j \mathcal{H}_W^j}{\xi^j(t)} \frac{\partial \xi^j(t)}{\partial W_j(t)} + \frac{1 - \alpha_W^j}{\gamma^j(t) + x^j} \mathcal{H}_W^j \frac{\partial \gamma^j(t)}{\partial W_j(t)} = 0. \quad (126)$$

Differentiating (111) and (121) w.r.t. the wage rate $W_j$, we get: $\frac{\partial \xi^j(t)}{\partial W_j(t)} = 1$ and $\frac{\partial \gamma^j(t)}{\partial W_j(t)} = -1$; inserting these into (126):

$$\alpha_W^j (\gamma^j(t) + x^j) = \left(1 - \alpha_W^j\right) \xi^j(t). \quad (127)$$

By differentiating (127) w.r.t. time, inserting the dynamic equations for $\xi^j$ given by (110) and for $\gamma^j$ given by (120b), bearing in mind that $\gamma^j + x^j = \frac{1 - \alpha_W^j}{\alpha_W^j} \xi^j$ (see eq. (127)), rearranging terms, leads to the wage rate:

$$W_j = \alpha_W^j (\Xi^j + r^* x^j) + \left(1 - \alpha_W^j\right) W_{Rj}^j. \quad (128)$$

where $W_{Rj}^j = -v_F^j/\bar{\lambda}$ represents the reservation wage.

An alternative expression for the reservation wage $W_{Rj}^j$ which is equal to $-v_F^j/\bar{\lambda} = m^j (\theta^j) \xi^j + R^j$ can be derived as follows. Eliminating $\xi^j$ from (112) by making use of (127), i.e., $\xi^j = \frac{\alpha_W^j}{1 - \alpha_W^j} (\gamma^j + x^j)$, inserting (120a), i.e., $\gamma^j + x^j = \kappa^j / f^j$, and using the fact that $m^j / f^j = \theta^j$, the reservation wage can be rewritten as follows:

$$W^j = m^j (\theta^j) \xi^j + R^j,$$

$$= m^j \frac{\alpha_W^j}{1 - \alpha_W^j} \frac{\kappa^j}{f^j} + R^j,$$

$$= \frac{\alpha_W^j}{1 - \alpha_W^j} \kappa^j \theta^j + R^j. \quad (129)$$

D Solving the Model

D.1 Short-Run Static Solutions

In this subsection, we compute short-run static solutions for consumption and the relative price of non tradables. Static efficiency condition (109a) can be solved for consumption which of course must hold at any point of time:

$$C = C (\bar{\lambda}, P), \quad (130)$$

with

$$C_{\bar{\lambda}} = \frac{\partial C}{\partial \bar{\lambda}} = -\sigma_C \frac{C}{\bar{\lambda}} < 0, \quad (131a)$$

$$C_P = -\sigma_C \frac{C}{P} < 0, \quad (131b)$$

where $\sigma_C$ corresponds to the intertemporal elasticity of substitution for consumption.

Denoting by $\phi$ the intratemporal elasticity of substitution between the tradable and the non tradable good and inserting short-run solution for consumption (109a) into intratemporal allocations between non tradable and tradable goods, i.e., $C^N = P'_C C$ and $C^T = [P^C - PP'_C] C$, allows us to solve for $C^T$ and $C^N$:

$$C^T = C^T (\bar{\lambda}, P), \quad C^N = C^N (\bar{\lambda}, P), \quad (132)$$
where the partial derivatives are:

\[ C^T_\lambda = -\sigma_C \frac{C^T}{\lambda} < 0, \]  
(133a)

\[ C^T_P = \alpha_C \frac{C^T}{P} (\phi - \sigma_C) \leq 0, \]  
(133b)

\[ C^N_\lambda = -\sigma_C \frac{C^N}{\lambda} < 0, \]  
(133c)

\[ C^N_P = -\frac{C^N}{P} \left[(1 - \alpha_C) \phi + \alpha_C \sigma_C\right] < 0, \]  
(133d)

where we use the fact that \(-\frac{P^*_P P}{P} = \phi (1 - \alpha_C) > 0\) and \(P^*_PC = C^N\).

Inserting the short-run static solution for consumption in non tradables \(C^N (\tilde{\lambda}, P)\) given by (132) into the market clearing condition for non tradables (20) allows us to solve for the relative price of non tradables:

\[ P = P \left(L^N, \tilde{\lambda}, A^N\right), \]  
(134)

where

\[ P_{LN} = \frac{\partial P}{\partial LN} = \frac{A^N}{C^P_P} < 0, \]  
(135a)

\[ P_\lambda = \frac{\partial P}{\partial \lambda} = -\frac{C^N}{C^P_P} < 0, \]  
(135b)

\[ P_{AN} = \frac{\partial P}{\partial A^N} = \frac{L^N}{C^P_P} < 0. \]  
(135c)

Inserting (135) into (132), the short-run static solutions for \(C^T\) and \(C^N\) become:

\[ C^T = C^T \left(L^N, \tilde{\lambda}, A^N\right), \quad C^N = C^N \left(L^N, \tilde{\lambda}, A^N\right), \]  
(136)

where the partial derivatives are:

\[ \frac{\dot{C}^T}{\tilde{\lambda}} = -\frac{\sigma_C \phi}{[(1 - \alpha_C) \phi + \alpha_C \sigma_C]} < 0, \]  
(137a)

\[ \frac{\dot{C}^T}{L^N} = \frac{C^T}{A^N} = -\frac{(\phi - \sigma_C)}{[(1 - \alpha_C) \phi + \alpha_C \sigma_C]} \frac{\omega_N}{\omega_C} \leq 0, \]  
(137b)

\[ \frac{\dot{C}^N}{\tilde{\lambda}} = -\sigma_C + \sigma_C = 0, \]  
(137c)

\[ \frac{\dot{C}^N}{L^N} = \frac{\dot{C}^N}{A^N} = \frac{\omega_N}{\omega_C} > 0. \]  
(137d)

We denote by a hat the rate of change of the variable and rewrite \(\frac{C^N}{A^N L^N} = \frac{P_{CN}}{P_{CN}} + \frac{P_{PC}}{P_{PC}} + \frac{Y}{P_{AT}} L^N = \frac{\omega_N}{\omega_C} \) with \(\alpha_C\) the non tradable content of consumption expenditure, \(\omega_C\) the GDP share of consumption expenditure and \(\omega_N\) the non tradable content of GDP.

### D.2 Derivation of the Dynamic Equation of the Current Account

Using the fact that \(A \equiv B + \gamma^T L^T + \gamma^N L^N\), differentiating with respect to time, noticing that \((\gamma^L j) = r^* \gamma^L j - \pi^j\), the accumulation equation of traded bonds is given by:

\[ \dot{B} = A - \gamma^T L^T - \gamma^T L^\prime j - \gamma^N L^N - \gamma^N L^N, \]
\[ = r^* \left(A - \gamma^T L^T - \gamma^N L^N\right) + \pi^T + \pi^N + W^T L^T + W^N L^N + R^T U^T + R^N U^N - T - P_C C. \]

Remembering that \(\pi^j = \Xi^j - W^j L^j - \kappa^j V^j - x^j \cdot \max\{0, -\dot{L}^j\}\), inserting the market clearing condition for the non traded good (20) and the balanced government budget (19), the current account equation reduces to:

\[ \dot{B}(t) = r^* B(t) + A^T L^T(t) - C^T(t) - G^T - \kappa^T V^T(t) - \kappa^N V^N(t). \]  
(138)
D.3 Equilibrium Dynamics and Formal Solutions

D.3.1 Dynamic System

Differentiating (120a) w. r. t. time, using (120b) yields

\[
\frac{\dot{\theta}^j}{\theta^j} = \frac{1}{1 - \alpha^j_V} \frac{\gamma^j + x^j}{\gamma^j + x^j}.
\]

Eliminating \( \gamma^j + x^j \) by using (120a), leads to the dynamic equation for labor market tightness \( \theta^j \):

\[
\dot{\theta}^j(t) = \frac{\theta^j(t)}{1 - \alpha^j_V} \left\{ (s^j + r^*) - \frac{f^j(\theta^j(t))}{\kappa^j} \left[ (\Xi^j + r^*x^j) - W^j \right] \right\}.
\]

Setting the overall surplus from an additional job in sector \( j \):

\[
\Psi^j(t) = (\Xi^j(t) + r^*x^j) + \frac{v^j_F(t)}{\lambda},
\]

Inserting the Nash bargaining wage \( W^j \) given by (128) into \( [(\Xi^j + r^*x^j) - W^j] \), the dynamic equation for labor market tightness \( \theta^j \) can be rewritten as follows:

\[
\dot{\theta}^j(t) = \frac{\theta^j(t)}{1 - \alpha^j_V} \left\{ (s^j + r^*) - \frac{f^j(\theta^j(t))}{\kappa^j} \left[ 1 - \alpha^j_W \right] \Psi^j(t) \right\}.
\]

The overall surplus from an additional job in the traded and the non traded sector, respectively, is given by:

\[
\Psi^T = (A^T + r^*x^T) + \frac{v^T_F}{\lambda}, \quad \Psi^N = [P(L^N, \bar{\lambda}, A^N) A^N + r^*x^N] + \frac{v^N_F}{\lambda},
\]

where the short-run static solution for the relative price of non tradables (134) has been inserted into the overall surplus from a match into the non traded sector. Partial derivatives are given by:

\[
\begin{align*}
\Psi^T_{LT} &= \Psi^T_{UT} = \frac{v^T_F}{\lambda} < 0, \quad (142a) \\
\Psi^N_{LN} &= P_L A^N + \frac{v^N_F}{\lambda} < 0, \quad (142b) \\
\Psi^N_{UN} &= \frac{v^N_F}{\lambda} < 0, \quad (142c) \\
\Psi^N_{AN} &= P_A A^N + P = \frac{A^N L^N}{C^N_P} + P, \\
&= \frac{A^N L^N}{C^N_P} \left\{ 1 - [(1 - \alpha_C) \phi + \alpha_C \sigma_C] \frac{\alpha_C \omega_C}{\omega_N} \right\} < 0, \quad (142d) \\
\Psi^N_{\lambda} &= P_\lambda A^N - \frac{v^N_F}{(\lambda)^2}, \\
&= -1 \frac{1}{\lambda} \left\{ \frac{\sigma_C P A^N}{[(1 - \alpha_C) \phi + \alpha_C \sigma_C]} + \frac{v^N_F}{\lambda} \right\} < 0, \quad (142e)
\end{align*}
\]

where \( P_L < 0, C^N_P < 0 \), and we use the fact that \( \frac{G^N}{A^N L^N} = \frac{PC^N C^N}{A^N C^N} \frac{Y}{P_A A^N L^N} = \frac{\alpha_C \omega_C}{\omega_N} \).

The adjustment of the open economy towards the steady-state is described by a dynamic system which comprises six equations. We consider that the utility function is additively separable in the disutility received by working and searching in the two sectors. Such a specification makes it impossible to switch from one sector to another instantaneously without going through a spell of search unemployment, as in Alvarez and Shimer [2011]. Because workers must search for a job to switch from one sector to another, i.e., cannot
relocate hours worked from one sector to another instantaneously, the dynamic system is block recursive. The first (second) dynamic system consists of the law of motion of employment in the traded (non traded) sector described by (7), the dynamic equations for labor tightness and job seekers given by (140) and (114), respectively. We denote the steady-state value with a tilde.

**Traded Sector**

Linearizing the accumulation equation for traded labor (7) by setting \( j = T \) and the dynamic equations for labor market tightness (140) and job seekers (114) in the traded sector, we get in matrix form:

\[
\begin{pmatrix}
\dot{L}^T, \dot{\theta}^T, \dot{U}^T
\end{pmatrix}^T = J^T \begin{pmatrix}
L^T(t) - \bar{L}^T, \theta^T(t) - \bar{\theta}^T, U^T(t) - \bar{U}^T
\end{pmatrix}^T
\]

where \( J^T \) is given by

\[
J^T \equiv \begin{pmatrix}
-s^T & (m^T)\dot{U}^T & m^T \left( \frac{\partial \theta^*}{\partial t} \right) \\
\frac{1-\alpha_W^T}{1-\alpha_V^T} \alpha^T & (s^T + r^*) & -\frac{1-\alpha_W^T}{1-\alpha_V^T} \alpha^T v_{EF}^T \\
2s^T + r^* & \frac{1-\alpha_W^T}{1-\alpha_V^T} \bar{m}^T - (m^T)\dot{U}^T & (s^T + r^*) - \bar{m}^T + \frac{\alpha_V^T}{1-\alpha_V^T} \bar{m}^T
\end{pmatrix},
\]

and where we used the fact that:

\[
\begin{align*}
\tilde{f}(1 - \alpha_W^T) \tilde{\Psi}^T &= \kappa^T, \\
v_{EF}^T + R^T &= -\bar{m}^T \tilde{\xi}^T = -\frac{\bar{m}^T}{s^T + r^*} \tilde{\Psi}^T, \\
1 + \frac{\alpha_V^T}{1-\alpha_V^T} \tilde{f}(1 - \alpha_W^T) \tilde{\Psi}^T &= \frac{1}{1-\alpha_V^T}.
\end{align*}
\]

The trace denoted by \( \text{Tr} \) of the linearized \( 3 \times 3 \) matrix (144) is given by:

\[
\text{Tr}J^T = (s^T + r^*) + r^* - \frac{\bar{m}^T}{1-\alpha_V^T} \left[ \alpha_W^T - \left( 1 - \alpha_V^T \right) \right].
\]

The determinant denoted by \( \text{Det} \) of the linearized \( 3 \times 3 \) matrix (144) is unambiguously negative:

\[
\text{Det}J^T = - (s^T + r^*) (s^T + \bar{m}^T) \left[ (s^T + r^*) + \frac{\alpha_W^T}{1-\alpha_V^T} \bar{m}^T \right] < 0.
\]

Assuming that the Hosios condition holds, i.e., setting \( \alpha_W^T = 1 - \alpha_V^T \), the trace reduces to:

\[
\text{Tr}J^T = (s^T + r^*) + r^*,
\]

while the determinant is given by:

\[
\text{Det}J^T = - (s^T + r^*) (s^T + r^* + \bar{m}^T) (s^T + \bar{m}^T) < 0.
\]

From now on, for clarity purpose, we impose the Hosios condition in order to avoid unnecessary complications. We relax this assumption when analyzing steady-state effects and conducting a quantitative exploration of the effects of higher productivity of tradables relative to non tradables. Note that all conclusions related to the analysis of equilibrium dynamics hold whether the Hosios conditions is imposed or not.

Denoting by \( \nu_i^T \) the eigenvalue in the traded sector, the characteristic equation for the matrix \( J \) (144) of the linearized system writes as follows:

\[
(s^T + r^* - \nu_i^T) \left\{ (\nu_i^T)^2 - r^*_i \nu_i^T + \frac{\text{Det}J^T}{s^T + r^*} \right\} = 0.
\]

The characteristic roots obtained from the characteristic polynomial of degree two can be written as follows:

\[
\nu_i^T \equiv \frac{1}{2} \left\{ r^* \pm \sqrt{(r^*)^2 - 4 \frac{\text{Det}J^T}{s^T + r^*}} \right\} \geq 0, \quad i = 1, 2.
\]
We denote by \( \nu_1^T < 0 \) and \( \nu_2^T > 0 \) the stable and unstable eigenvalues respectively which satisfy:

\[
\nu_1^T < 0 < r^* < \nu_2^T. \tag{151}
\]

Let \( \nu_3^T \) be the second unstable characteristic root which writes as:

\[
\nu_3^T = s^T + r^* > 0. \tag{152}
\]

Since the system features one state variable, \( L^T \), and one negative eigenvalue, two jump variables, \( \theta^T \) and \( U^T \), and two positive eigenvalues, the equilibrium yields a unique one-dimensional saddle-path. Inserting (145) and (146) into (150), the stable and unstable eigenvalues reduce to:

\[
\nu_1^T = -(s^T + \tilde{m}^T), \quad \nu_2^T = (s^T + r^* + \tilde{m}^T). \tag{153}
\]

### Non Traded Sector

Linearizing the accumulation equation for non traded labor (7) by setting \( j = N \) and the dynamic equations for labor market tightness (140) and job seekers (114) in the non traded sector, we get in matrix form:

\[
\begin{pmatrix}
\tilde{L}^N, \tilde{\theta}^N, \tilde{U}^N
\end{pmatrix} = J^N \begin{pmatrix}
L^N(t) - \tilde{L}^N, \theta^N(t) - \tilde{\theta}^N, U^N(t) - \tilde{U}^N
\end{pmatrix}, \tag{154}
\]

where \( J^N \) is given by

\[
J^N = \begin{pmatrix}
-s^N & (m^N)' \tilde{U}^N & m^N (\tilde{\theta}^N) \\
\frac{1-\alpha_N^N}{1-\alpha_V^N} \tilde{m}^N \left( P_{LN} A^N + \frac{v^N_F}{\lambda} \right) & (s^N + r^*) & -\frac{1-\alpha_N^N}{1-\alpha_V^N} \tilde{m}^N \frac{v^N_F}{\lambda} \\
(2s^N + r^*) + \frac{\alpha_N^N \tilde{m}^N}{1-\alpha_V^N} \left( P_{LN} A^N \tilde{\lambda} \right. & 
\left. \frac{1}{v^N_F} + 1 \right) & - (m^N)' \tilde{U}^N (s^N + r^*) - \tilde{m}^N \frac{\alpha_N^N}{1-\alpha_V^N} \tilde{m}^N
\end{pmatrix}, \tag{155}
\]

and where we used the fact that:

\[
\frac{\tilde{f}^N (1 - \alpha_N^N) \tilde{\Psi}^N}{s^N + r^*} = \kappa^N, \quad \frac{v^N}{\lambda} + R^N = -\tilde{m}^N \tilde{\xi}^N = -\tilde{m}^N \alpha_W^N \tilde{\Psi}^N, \quad 1 + \frac{\alpha_N^N}{1-\alpha_V^N} \frac{\tilde{f}^N (1 - \alpha_N^N) \tilde{\Psi}^N}{\kappa^N (s^N + r^*)} = \frac{1}{1 - \alpha_V^N}. \tag{156}
\]

The trace denoted by \( \text{Tr} \) of the linearized \( 3 \times 3 \) matrix (155) is given by:

\[
\text{Tr} J^N = (s^N + r^*) + r^* + \tilde{m}^N \frac{\alpha_N^N}{1-\alpha_V^N} \left[ \alpha_W^N - (1 - \alpha_V^N) \right]. \tag{157}
\]

The determinant denoted by \( \text{Det} \) of the linearized \( 3 \times 3 \) matrix (155) is unambiguously negative:

\[
\text{Det} J^N = -\left( s^N + r^* \right) \left\{ \left( s^N + \tilde{m}^N \right) \left[ (s^N + r^*) + \frac{\alpha_W^N}{1-\alpha_V^N} \tilde{m}^N \right] \right\} + \left( \frac{1-\alpha_N^N}{1-\alpha_V^N} \tilde{m}^N P_{LN} A^N \tilde{m}^N \frac{\alpha_N^N}{1-\alpha_V^N} \tilde{\xi}^N \tilde{\lambda} \tilde{\Psi}^N \frac{v^N_F}{\lambda} \right) < 0, \tag{158}
\]

where \( P_{LN} < 0 \).

Assuming that the Hosios condition holds, i.e., setting \( \alpha_W^N = 1 - \alpha_V^N \), the trace reduces to:

\[
\text{Tr} J^N = (s^N + r^*) + r^*, \tag{159}
\]

while the determinant is given by:

\[
\text{Det} J^N = -\left( s^N + r^* \right)^2 \left( s^N + \tilde{m}^N \right) \left\{ \frac{\left( s^N + r^* + \tilde{m}^N \right)}{s^N + r^*} - \frac{P_{LN} \tilde{L}^N}{P} \frac{\tilde{P} A^N}{\left( 1 - \alpha_N^N \right) \tilde{\Psi}^N} \left( \tilde{\lambda} \tilde{\Psi}^N \sigma_L + \alpha_V^N \tilde{\omega}^N \right) \right\} < 0, \tag{160}
\]

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where we have rewritten the last term as follows:

\[
\frac{1 - \alpha^N_W \bar{m}^N}{1 - \alpha^N_W} P_{LN} A N \bar{m}^N \frac{\alpha^N_W}{\bar{m}^N} (\frac{\alpha^N_W}{\bar{m}^N})^N \bar{\theta}^N \nu^N - \alpha V \bar{U}^N = 1 - \alpha^N_W \bar{m}^N \frac{\alpha^N_W}{\bar{m}^N} P_{LN} A N \bar{m}^N \frac{\alpha^N_W}{\bar{m}^N} \bar{\theta}^N \nu^N - \alpha V \bar{U}^N,
\]

and where we used the fact that \(\frac{\alpha^N_W}{\bar{m}^N} = -\chi^N \nu^N \), \(\bar{\theta}^N = \bar{\theta}^N / \bar{\theta}^N\), and \(\nu^N = \nu^N / \nu^N\) to get the second line, \(\tilde{f}^N(1 - \alpha_W) = (s^N + r^s)^N \), \(\bar{m}^N \bar{U}^N = s^N \bar{L}^N\), and \(\bar{U}^N / \bar{P}^N = \bar{U}^N / \bar{P}^N = \bar{U}^N / \bar{P}^N\) to get the third line, \(\tilde{u}^N = \frac{s^N}{s^N + \nu^N}\), multiplying the numerator and the denominator by \(\tilde{W}^N\) and rearranging terms to get the last line.

We impose the Hosios condition in order to avoid unnecessary complications. Denoting by \(\nu^N\) the eigenvalue, the characteristic equation for the matrix \(J\) (155) of the linearized system writes as follows:

\[
(s^N + r^s - \nu^N_i) \left\{ (\nu^N_i)^2 - r^s \nu^N_i + \frac{\text{Det}J^N_N}{s^N + r^s} \right\} = 0.
\]

The characteristic roots obtained from the characteristic polynomial of degree two write as follows:

\[
\nu^N_i = \frac{1}{2} \left\{ r^s \pm \sqrt{(r^s)^2 - 4 \frac{\text{Det}J^N_N}{s^N + r^s}} \right\} \geq 0, \quad i = 1, 2.
\]

We denote by \(\nu^N_1 < 0\) and \(\nu^N_2 > 0\) the stable and unstable eigenvalues respectively which satisfy:

\[
\nu^N_1 < 0 < r^s < \nu^N_2.
\]

As it will become useful later, \(\nu^N_1 (r^s - \nu^N_1) = \frac{\text{Det}J^N_N}{s^N + r^s}\) which can be rewritten as follows

\[
\frac{\text{Det}J^N_N}{s^N + r^s} = -\left(s^N + r^s\right) \left(s^N + \bar{m}^N\right) \left\{ \frac{(s^N + r^s + \bar{m}^N)}{(s^N + r^s)} + \frac{\omega^N}{\alpha^N \omega^N [1 - (1 + \alpha^N) \phi + \alpha^N \sigma^N]} \right\} < 0.
\]

where we used the fact that \(\frac{\alpha^N \omega^N}{\omega^N} = \frac{\alpha^N \omega^N}{\omega^N}\) and \(P_{LN} = \frac{A^N}{C^N} < 0\).

Let \(\nu^N_2\) be the second unstable characteristic root which writes as:

\[
\nu^N_2 = s^N + r^s > 0.
\]

Since the system features one state variable, \(L^N\), and one negative eigenvalue, two jump variables, \(\theta^N\) and \(U^N\), and two positive eigenvalues, the equilibrium yields a unique one-dimensional saddle-path.

### D.4 Formal Solutions for \(\dot{\theta}^T(t)\) and \(\dot{U}^T(t)\)

Setting the constant \(D^T_2 = 0\) to insure a converging adjustment for all macroeconomic aggregates, the stable paths are given by:

\[
L^T(t) - \bar{L}^T = D^T_1 e^{\nu^T_1 t},
\]

\[
\theta^T(t) - \bar{\theta}^T = \omega^T_2 D^T_1 e^{\nu^T_1 t},
\]

\[
U^T(t) - \bar{U}^T = \omega^T_3 D^T_1 e^{\nu^T_1 t},
\]

(166a)

(166b)

(166c)
where \( D_1^T = L_0^T - \tilde{L}^T \), and elements \( \omega_{21}^T \) and \( \omega_{31}^T \) of the eigenvector (associated with the stable eigenvalue \( \nu_1^T \)) are given by:

\[
\omega_{21}^T = \frac{1 - \alpha_{\bar{W}} \bar{m} \nu_1^T v_{\bar{F}}}{\alpha_{\bar{V}} \bar{m}} \left( \tilde{m}^T + s^T + \nu_1^T \right) = 0, \quad (167a)
\]

\[
\omega_{31}^T = \left( \frac{s^T + \nu_1^T}{\bar{m}} \right) - \left( \frac{(m^T)'}{\bar{m}} \right) \tilde{U}^T \omega_{21}^T \leq 0. \quad (167b)
\]

We have normalized \( \omega_{31}^T \) to unity. Inserting \( \nu_1^T = s^T + \tilde{m}^T \) (see (153)) into (167a) and (167b), eigenvectors reduce to:

\[
\omega_{21}^T = 0, \quad \omega_{31}^T = -1. \quad (168)
\]

From (168), the dynamics for labor market tightness \( \theta^T \) degenerate while job seekers are negatively correlated with employment along a stable transitional path.

### D.5 Formal Solutions for \( \theta^N(t) \) and \( U^N(t) \)

Setting the constant \( D_2^N = 0 \) to insure a converging adjustment for all macroeconomic aggregates, the stable paths are given by:

\[
\begin{align*}
L^N(t) - \tilde{L}^N &= D_1^N e^{\nu_1^N t}, \\
\theta^N(t) - \tilde{\theta}^N &= \omega_{21}^N D_1^N e^{\nu_1^N t}, \\
U^N(t) - \tilde{U}^N &= \omega_{31}^N D_1^N e^{\nu_1^N t},
\end{align*} \quad (169a, b, c)
\]

where \( D_1^N = L_0^N - \tilde{L}^N \), and elements \( \omega_{21}^N \) and \( \omega_{31}^N \) of the eigenvector (associated with the stable eigenvalue \( \nu_1^N \)) are given by:

\[
\begin{align*}
\omega_{21}^N &= \frac{1 - \alpha_{\bar{W}} \bar{m}^N}{\alpha_{\bar{V}} \bar{m}} \left[ \bar{m}^N \left( P_{LN} A^N + v_{\bar{F}}^N \right) + \left( s^N + \nu_1^N \right) v_{\bar{F}}^N \right] \leq 0, \quad (170a)
\omega_{31}^N &= \left( \frac{s^N + \nu_1^N}{\bar{m}^N} \right) - \left( \frac{(m^N)' \tilde{U}^N}{\bar{m}} \right) \omega_{21}^N \leq 0. \quad (170b)
\end{align*}
\]

We have normalized \( \omega_{11}^N \) to unity. The signs of (170a) and (170b) will be determined later.

### D.6 Formal Solution for the Stock of Foreign Bonds \( B(t) \)

Substituting first the short-run static solutions for consumption in tradables given by (136), and using the fact that \( V^T = U^T \theta^T \), the accumulation equation for traded bonds (138) can be written as follows:

\[
\dot{B}(t) = r^* B(t) + A^T L^T(t) - C^T (L^N(t), \bar{\lambda}, A^N) - G^T - \kappa T \theta(t) U^T(t) - \kappa^N \theta^N(t) U^N(t). \quad (171)
\]

Linearizing (171) in the neighborhood of the steady-state and inserting stable solutions given by (166) and (169) yields:

\[
\dot{B}(t) = r^* \left( B(t) - \tilde{B} \right) + A^T \left( L^T(t) - \tilde{L}^T \right) + A^N \left( L^N(t) - \tilde{L}^N \right), \quad (172)
\]

where we set:

\[
\begin{align*}
A^T &= A^T - \kappa^T \tilde{U}^T \omega_{21}^T - \kappa^T \tilde{\theta}^T \omega_{31}^T = A^T + \kappa T \tilde{\theta}^T > 0, \\
A^N &= -C_{LN}^T - \kappa N \tilde{U}^N \omega_{21}^N - \kappa N \tilde{\theta}^N \omega_{31}^N,
\end{align*} \quad (173a, b)
\]

\[
\begin{align*}
&= -C_{LN}^T - \kappa N \tilde{U}^N (1 - \alpha_{\bar{V}}^N) \omega_{21}^N - \kappa N \tilde{\theta}^N \left( s^N + \nu_1^N \right) / \bar{m}^N > 0, \quad (173b)
\end{align*}
\]

where we have inserted (170b) and used the fact that \( (m^N)' \theta^N / m = \alpha_{\bar{V}} \) to get (173b); note that \( C_{LN}^T \approx 0 \) because our estimates of \( \phi \) average about 1 while we set \( \sigma_C \) to one. The
sign of (173b) follows from the fact that $\omega_{21}^N < 0$ (see (201)) and $s^N + \nu_1^N < 0$; the latter result stems from the fact that $\nu_1^T = -(s^T + \bar{m}^T)$; because we have the following set of inequalities $\text{Det}_JN^{N+1} < \text{Det}_JN^T < 0$, $\nu_1^N < -(s^N + \bar{m}) < 0$ and thereby $s^N + \nu_1^N < 0$.

Solving the differential equation (172) yields:

$$B(t) = \tilde{B} + \left( B_0 - \tilde{B} \right) - \frac{\lambda^T D_1^T}{\nu_1^T - r^*} - \frac{\lambda^T D_1^N}{\nu_1^N - r^*} e^{r^* t} + \frac{\lambda^T D_1^T}{\nu_1^T - r^*} e^{r^* t} + \frac{\lambda^T D_1^N}{\nu_1^N - r^*} e^{r^* t}. \quad (174)$$

Invoking the transversality condition for intertemporal solvency, and using the fact that $D_1^T = L_0^T - \tilde{L}^T$ and $D_1^N = L_0^N - \tilde{L}^N$, we obtain the linearized version of the nation’s intertemporal budget constraint:

$$\tilde{B} - B_0 = \Phi^T \left( \tilde{L}^T - L_0^T \right) + \Phi^T \left( \tilde{L}^N - L_0^N \right), \quad (175)$$

where we set

$$\Phi^T = \frac{\lambda^T}{\nu_1^T - r^*} = -\left( \frac{A^T + \kappa^T \theta^T}{(s^T + \bar{m}^T + r^*)} \right) < 0, \quad \Phi^N = \frac{\lambda^N}{\nu_1^N - r^*} < 0. \quad (176)$$

Equation (176) can be solved for the stock of foreign bonds:

$$\tilde{B} = B \left( \tilde{L}^T, \tilde{L}^N \right), \quad B_{L^T} = \Phi^T < 0, \quad B_{L^N} = \Phi^N < 0. \quad (177)$$

For the national intertemporal solvency to hold, the terms in brackets of equation (174) must be zero so that the stable solution for net foreign assets finally reduces to:

$$B(t) - \tilde{B} = \Phi^T \left( L^T(t) - \tilde{L}^T \right) + \Phi^N \left( L^N(t) - \tilde{L}^N \right). \quad (178)$$

### E Graphical Apparatus

Before turning to the decomposition of steady-state effects, we investigate graphically the long-run effects of a productivity differential.

#### E.1 Steady-State

Using (129), the steady-state of the open economy is described by the following set of equations:

$$\tilde{C} = [P_C (\tilde{P}) \lambda]^{-\sigma_C}, \quad (179a)$$

$$s^T \tilde{L}^T = m^T (\bar{\theta}^T) \tilde{U}^T, \quad (179b)$$

$$s^N \tilde{L}^N = m^N (\bar{\theta}^N) \tilde{U}^N, \quad (179c)$$

$$\left( \tilde{L}^T + \tilde{U}^T \right) = \left[ \tilde{\lambda} \left( \frac{\alpha_W^T}{1 - \alpha_W^T} \kappa^T \tilde{\theta}^T + R^T \right) \right]^{\sigma_L}, \quad (179d)$$

$$\left( \tilde{L}^N + \tilde{U}^N \right) = \left[ \tilde{\lambda} \left( \frac{\alpha_W^N}{1 - \alpha_W^N} \kappa^N \tilde{\theta}^N + R^N \right) \right]^{\sigma_L}, \quad (179e)$$

$$\frac{\kappa^T}{f^T (\bar{\theta}^T)} = \frac{(1 - \alpha_W^T) \tilde{\psi}^T}{s^T + r^*}, \quad (179f)$$

$$\frac{\kappa^N}{f^N (\bar{\theta}^N)} = \frac{(1 - \alpha_W^N) \tilde{\psi}^N}{s^N + r^*}, \quad (179g)$$

$$A^N \tilde{L}^N = \tilde{C}^N, \quad (179h)$$

$$s^* \tilde{B} + A^T \tilde{L}^T - \tilde{C}^T - \kappa^T \bar{\theta}^T \tilde{U}^T - \kappa^N \bar{\theta}^N \tilde{U}^N, \quad (179i)$$

and the intertemporal solvency condition

$$\tilde{B} - B_0 = \Phi^T \left( \tilde{L}^T - L_0^T \right) + \Phi^T \left( \tilde{L}^N - L_0^N \right), \quad (179j)$$
where \( C^N = P'C'C \) and \( C^T = (1 - \alpha_C) P'C \) and we used the fact that \( V^j = U^j \theta^j \). The steady-state equilibrium defined by ten equations jointly determines \( C, L^T, L^N, \bar{U}^T, \bar{U}^N, \tilde{\theta}^T, \tilde{\theta}^N, \bar{P}, \bar{B}, \bar{\lambda} \).

**E.2 Isoclines and Stable Path in the \((\theta^T, L^T)\)-space**

The labor market in the traded sector can be summarized graphically by Figure 12(a) that traces out two schedules in the \((\theta^T, L^T)\)-space. More precisely, eliminating \( \bar{U}^T \) from eq. (179b) by using (179b), i.e., \( \bar{U}^T = \frac{\bar{x}^T}{\bar{m}^T} \), the system which comprises eqs. (179b), (179d) and (179f) can be reduced to two equations:

\[
\begin{align*}
\dot{L}^T &= \frac{\bar{m}^T}{\bar{m}^T + \bar{m}^T} \left[ \tilde{\lambda} \left( \frac{\alpha^T_W}{1 - \alpha^T_W} \kappa^T \tilde{\theta}^T + R^T \right) \right]^\sigma^L, \\
\kappa^T \dot{f}^T(\tilde{\theta}^T) &= \frac{(1 - \alpha^T_W)}{(\bar{m}^T + \bar{m}^T)} \tilde{\Psi},
\end{align*}
\]

where \( \bar{m}^T = m^T \left( \tilde{\theta}^T \right) \) and \( \dot{f}^T = f^T \left( \tilde{\theta}^T \right) \); using the fact the reservation wage \( W^T_R = -\frac{v^T}{\bar{x}} \) is equal to \( \left( \frac{\alpha^T_W}{1 - \alpha^T_W} \kappa^T \tilde{\theta}^T + R^T \right) \) (see eq. (129)), the overall surplus from hiring in the traded sector is given by:

\[
\tilde{\Psi} \equiv \left( A^T + r^* x^T \right) - \left( \frac{\alpha^T_V}{1 - \alpha^T_W} \kappa^T \tilde{\theta}^T + R^T \right).
\]

Totally differentiating eq. (180a) yields

\[
\dot{\tilde{L}}^T = \sigma_L \ddot{\lambda} + \left[ \alpha^T_V \ddot{u}^T + \sigma_L \ddot{x}^T \right] \tilde{\theta}^T,
\]

where \( \ddot{u}^T = \frac{\bar{x}^T}{\bar{m}^T + \bar{m}^T} \) and \( 0 < \ddot{x}^T = \frac{\sigma^T_W}{1 - \sigma^T_W} \bar{x}^T \tilde{\theta}^T \) \( \bar{m}^T < 1 \). The slope of the \( \ddot{L}^T = 0 \) schedule in the \((\theta^T, L^T)\)-space writes as:

\[
\left. \frac{\ddot{\tilde{L}}^T}{\tilde{\theta}^T} \right|_{\ddot{L}^T=0} = \left[ \alpha^T_V \ddot{u}^T + \sigma_L \ddot{x}^T \right] > 0.
\]

Hence the decision of search (henceforth labelled DST) schedule is upward sloping in the \((\theta^T, L^T)\)-space. According to (182), a fall in the marginal utility of wealth \( \dot{\lambda} \) shifts downward the DST-schedule.

Totally differentiating eq. (180b) yields

\[
\dot{\tilde{\theta}}^T \left[ \left( 1 - \alpha^T_V \right) \ddot{\Psi}^T + \ddot{x}^T \bar{W}_R^T \right] = A^T \tilde{a}^T,
\]

where we used (179f) and the fact that \(- (f^T)^T \dot{\theta}^T / f^T = (1 - \alpha^T_V) \). The slope of the \( \ddot{\theta}^T = 0 \) schedule in the \((\theta^T, L^T)\)-space can be written as:

\[
\left. \frac{\ddot{\tilde{L}}^T}{\tilde{\theta}^T} \right|_{\ddot{\theta}^T=0} = +\infty.
\]

Hence the vacancy creation (henceforth labelled VCT) schedule is a vertical line in the \((\theta^T, L^T)\)-space. According to (184), a rise in labor productivity in the traded sector \( A^T \) shifts to the right the VCT-schedule.

Having determined the patterns of isoclines in the \((\theta^T, L^T)\)-space, we now analyze the slope of the stable path. To determine the pattern of the stable path, we have to estimate:

\[
\frac{L^T(t) - L^T(t)}{\dot{\theta}^T(t) - \bar{\theta}^T} = \frac{1}{\omega_{21}^T L^T},
\]

which can be estimated using eqs. (186)(185).
Using the fact that \( \omega_{21}^T = 0 \) (see (168)), the slope of the stable branch labelled \( SS^T \) in the \((\theta, L)\)-space rewrites as:

\[
\left. \frac{ \dot{\tilde{L}}^T }{ \dot{\tilde{\theta}}^T } \right|_{SS^T} = +\infty.
\]  

(187)

According to (187), the stable branch coincides with the \( VCT \)-schedule (see Figure 12(a)) as the dynamics for \( \theta^T \) degenerate.

**E.3 Isoclines and Stable Path in the \((\theta^N, L^N)\)-space**

The labor market in the non traded sector can be summarized graphically by Figure 12(b) that traces out two schedules in the \((\theta^N, L^N)\)-space. More precisely, eliminating \( \tilde{U}^N \) from eq. (179e) by using (179c), i.e., \( \tilde{U}^N = s^N \tilde{m}^N \), and inserting the short-run static solution for the relative price of non tradables given by (134) implies that the system which comprises eqs. (179c), (179e), (179g), and (179h) can be reduced to two equations:

\[
\tilde{L}^N = \tilde{m}^N \left[ \tilde{\lambda} \left( \frac{\alpha_W}{1 - \alpha_W} \tilde{\theta}^N + R^N \right) \right]^{\sigma_L},
\]  

(188a)

\[
\frac{\kappa^N}{f^N \left( \tilde{\theta}^N \right)} = \left( \frac{1 - \alpha_N}{s^N + r^*} \right) \tilde{\Psi}^N,
\]  

(188b)

where \( \tilde{m}^N = m^N \left( \tilde{\theta}^N \right) \) and \( \tilde{f}^N = f^N \left( \tilde{\theta}^N \right) \); using the fact the reservation wage \( W^N_R = -\frac{\nu_N}{\lambda} \) is equal to \( \left( \frac{\alpha_N}{1 - \alpha_W} \kappa^N \tilde{\theta}^N + R^N \right) \) (see eq. (129)), the overall surplus from hiring in the non
traded sector is given by:

\[ \tilde{\Psi}^N = \left( P \left( \lambda, L^N, A^N \right) A^N + r^* x^N \right) - \left( \frac{\alpha_N^N}{1 - \alpha_N^N} \kappa^N \tilde{\vartheta}^N + R^N \right). \]  

(189)

Totally differentiating eq. (188a) yields

\[ \hat{L}^N = \sigma_L \hat{\lambda} + \left[ \alpha_V^N \hat{\vartheta}^N + \sigma_L \hat{\lambda} \right] \hat{\theta}^N, \]  

(190)

where \( \hat{\vartheta}^N = \frac{s^N}{m_i^N + \tilde{m}_i^N} \) and \( 0 < \hat{\lambda}^N = \frac{\alpha_N^N}{1 - \alpha_N^N} \kappa^N \tilde{\vartheta}^N W_R < 1 \). The slope of the \( \hat{L}^N = 0 \) schedule in the \((\theta^N, L^N)\)-space writes as:

\[ \frac{\hat{L}^N}{\hat{\theta}^N} \bigg|_{\hat{L}^N = 0} = \left[ \alpha_V^N \hat{\vartheta}^N + \sigma_L \hat{\lambda} \hat{\lambda}^N \right] > 0. \]  

(191)

Hence the decision of search (henceforth labelled \( DSN \)) schedule is upward-sloping in the \((\theta^N, L^N)\)-space. According to (190), a fall in the marginal utility of wealth \( \lambda \) shifts downward the \( DSN \)-schedule.

Totally differentiating eq. (188b) yields

\[ \hat{\theta}^N \left[ (1 - \alpha_V^N) \tilde{\Psi}^N + \hat{\lambda}^NW_R \right] = - \frac{P A^N \left\{ \omega_N \hat{L}^N + \sigma_C \omega_C \omega_C \hat{\lambda} + \left[ \omega_N - \omega_C \alpha_C \left( (1 - \alpha_C) \phi + \alpha_C \sigma_C \right) \right] \hat{\vartheta}^N + \right\}}{\alpha_C \omega_C \left( (1 - \alpha_C) \phi + \alpha_C \sigma_C \right)}, \]  

(192)

where we used (179g) and the fact that \(- (f^N)^t \theta^N / f^N = (1 - \alpha_V^N)\). The slope of the \( \hat{\theta}^N = 0 \) schedule in the \((\theta^N, L^N)\)-space is:

\[ \frac{\hat{\theta}^N}{\hat{\theta}^N} \bigg|_{\hat{\theta}^N = 0} = - \frac{\left( (1 - \alpha_V^N) \tilde{\Psi}^N + \hat{\lambda}^N W_R \right)}{PA^N} \frac{\alpha_C \omega_C \left( (1 - \alpha_C) \phi + \alpha_C \sigma_C \right)}{\omega_N} < 0. \]  

(193)

Hence the vacancy creation (henceforth labelled \( VCN \)) schedule is downward-sloping in the \((\theta^N, L^N)\)-space. According to (193), since \( \left[ \omega_N - \omega_C \alpha_C \left( (1 - \alpha_C) \phi + \alpha_C \sigma_C \right) \right] \geq 0 \), a rise in labor productivity in the non traded sector \( A^N \) may shift to the left or to the right the \( VCN \)-schedule depending on whether \( \phi \) takes high or low values; it is worthwhile mentioning that higher productivity in tradables relative to non tradables shifts to the right the \( VCN \)-schedule by appreciating the relative price and thus by raising the marginal revenue of labor in the non traded sector, i.e., by increasing \( \Xi^N \equiv PA^N \). Moreover, a fall in the marginal utility of wealth \( \lambda \) shifts to the right the \( VCN \)-schedule by appreciating the relative price of non tradables.

Having determined the patterns of isoclines in the \((\theta^N, L^N)\)-space, we now analyze the slope of the stable path. To do so, we use the third line of the Jacobian matrix (155) to rewrite the element \( \omega_{21}^N \) of the eigenvector:

\[ \omega_{21}^N = \left( 2s^N + r^* \right) \left( s^N + r^* - \nu_i^N \right) \left( \frac{s^N + \nu_i^N}{\tilde{m}_i^N} \right) + \tilde{m}_i^N \left( \frac{P_{LN}}{m_i^N} \frac{A^N}{e^N} + 1 \right). \]  

(194)

The first two terms in the numerator of (194) can be rewritten as follows:

\[ (2s^N + r^*) \left( s^N + r^* - \nu_i^N \right) \left( \frac{s^N + \nu_i^N}{s^N} \right) = s^N + \left( s^N + r^* \right) \left( s^N + \tilde{m}_i^N \right) + \nu_i^N \left( r^* - \nu_i^N \right), \]  

(195)

where \( \nu_i^N \left( r^* - \nu_i^N \right) \) is equal to the determinant of the Jacobian matrix (155) given by (160). To determine the pattern of the stable path in the \((\theta^N, L^N)\)-space, we have to estimate:

\[ \frac{L^N (1 - \tilde{L}^N)}{\theta^N (1 - \tilde{\theta}^N)} = \frac{1}{\omega_{21}^N \tilde{L}^N}. \]  

(196)
Inserting (164) into (196), the slope of the stable branch labelled $S^N S^N$ in the $(\theta^N, L^N)$-space can be rewritten as follows:

$$\frac{\dot{L}^N}{\dot{\theta}^N}_{S^NS^N} = \frac{1}{\omega^N_{21}} \frac{\dot{\theta}^N}{L^N} = - \frac{s^N + \bar{m}^N + r^* - \nu^N_1}{(s^N + r^*)} \left( 1 - \alpha_N^V \right) \frac{\bar{\Psi}^N}{PA^N} \frac{\alpha_C \omega_C \left[ (1 - \alpha_C) \phi + \alpha_C \sigma_C \right]}{\omega_N} < 0,$$

(197)

where we denote by $\bar{\theta}$ the hat the rate of change relative to initial steady-state. According to (197), the stable branch $S^NS^N$ is downward-sloping in the $(\theta^N, L^N)$-space.

To get (197), we proceed as follows. We first have rewritten the numerator of eigenvector $\omega^N_{21}$ given by (194) (set $i = 1$) by using (195) and by inserting $\frac{\lambda_{21} F}{s^N + r^*}$ (which is equal to $\nu^N_1 (r^* - \nu^N_1)$) given by (164):

$$\bar{s}^N + \frac{s^N + \bar{m}^N - (s^N + r^* + \bar{m}^N)}{\bar{m}^N} \left( \frac{P_{LN} A^N \bar{\lambda}}{\bar{v}_P} + 1 \right)$$

$$\begin{equation}
\quad - \frac{\omega_N \bar{\lambda}}{\alpha_C \omega_C \left[ (1 - \alpha_C) \phi + \alpha_C \sigma_C \right]} \left( \frac{s^N + r^*}{s^N + m^N} \right) \frac{\bar{\chi}^N s_{CL} + \alpha_N^V \bar{u}^N}{(1 - \alpha_N^V)} \rho_N^N \rho_{\bar{m}^N}, \quad (198)
\end{equation}$$

$$\begin{equation}
= - \frac{\omega_N \bar{\lambda}}{\alpha_C \omega_C \left[ (1 - \alpha_C) \phi + \alpha_C \sigma_C \right]} \left( \frac{s^N + r^*}{s^N + m^N} \right) \frac{\alpha_N^V \bar{u}^N}{(1 - \alpha_N^V)} \rho_N^N \rho_{\bar{m}^N}. \quad (199)
\end{equation}$$

To get the last line, we computed the following term $\bar{m}^N \left( \frac{P_{LN} A^N \bar{\lambda}}{\bar{v}_P} + 1 \right)$ as follows:

$$\begin{equation}
\bar{m}^N \left[ \frac{P_{LN} A^N \bar{\lambda}}{\bar{v}_P} + 1 \right] = \bar{m}^N \left[ \frac{P_{LN} \bar{L}^N \bar{P}_{A^N}}{P} \frac{\bar{\lambda}}{\bar{v}_P} + 1 \right],
\end{equation}$$

(200)

where we used the fact that $\frac{\bar{v}_P^N}{\bar{v}_P} = \sigma_L$ to get the first line, $\bar{L}^N = \frac{\bar{m}^N}{s^N + \bar{m}^N}$ and $\frac{P_{LN} \bar{L}^N}{P} = \frac{\omega_N \bar{\lambda}}{\alpha_C \omega_C \left[ (1 - \alpha_C) \phi + \alpha_C \sigma_C \right]}$ to get the second line, $\bar{m}^N \bar{S}^N = \bar{m}^N \frac{\alpha_N^V \bar{\psi}^N}{s^N + r^*} = - \bar{\chi}^N \frac{\rho_N^N}{\bar{\chi}}$ to get (200). Inserting (200) into (198), rearranging terms, we get (199).

Inserting first (200), and multiplying $\omega^N_{21}$ (setting setting $i = 1$ into (194)) by $\bar{L}^N / \bar{\theta}^N$, we get:

$$\begin{equation}
\omega^N_{21} \frac{\bar{L}^N}{\bar{\theta}^N} = - \frac{\omega_N \bar{\lambda}}{\alpha_C \omega_C \left[ (1 - \alpha_C) \phi + \alpha_C \sigma_C \right]} \left( \frac{s^N + r^*}{s^N + \bar{m}^N} \right) \frac{\bar{\psi}^N}{\bar{\chi}^N W_R^N} \bar{\psi}^N \left( \frac{s^N + \bar{m}^N + r^*}{s^N + r^*} \right) \frac{\bar{L}^N}{F^N}, \quad (201)
\end{equation}$$

where we used the fact that $(m^N)' \theta^N / m^N = \alpha_N^V$ and $\bar{u}^N = \bar{U}^N / \bar{F}^N$ to get the first line, $\frac{\bar{L}^N}{F^N} = \frac{\bar{m}^N}{s^N + \bar{m}^N}$ to get (201).

Because both the VCN-schedule and the stable branch $S^N S^N$ are downward sloping, we have now to determine whether the stable branch $S^N S^N$ is steeper or flatter than the VCN-schedule. To do so, we compute the following term which shows up in eq. (193):

$$\begin{equation}
(1 - \alpha_N^V) \bar{\psi}^N + \bar{\chi}^N W_R^N = \left( 1 - \alpha_N^V \right) \bar{\psi}^N \left( \frac{s^N + \bar{m}^N + r^*}{s^N + r^*} \right), \quad (202)
\end{equation}$$

where we used the fact that $\bar{\chi}^N W_R^N = \bar{m}^N \bar{\psi}^N = \bar{m}^N \left( \frac{s^N}{s^N + r^*} \right) \bar{\psi}^N$. Since $\left( \frac{s^N + \bar{m}^N + r^* - \nu^N_1}{s^N + r^*} \right) > \left( \frac{s^N + \bar{m}^N + r^*}{s^N + r^*} \right)$, inspection of (193) and (197) implies that the $S^N S^N$-schedule is steeper than the VCN-schedule (see Figure 12(b)).
We turn now to the transitional adjustment along the stable path in the \((L^N, U^N)\)-space by making use of (170b):

\[
U^N(t) - \dot{U}^N = \omega_{31}^N \left(L^N(t) - \dot{L}^N\right),
\]

(203)

where \(\omega_{31}^N\) is given by eq. (170b). To sign the slope of the transitional path in the \((L^N, U^N)\)-space, we use the third line of the Jacobian matrix (155) to rewrite the element \(\omega_{21}^N\) of the eigenvector:

\[
\omega_{21}^N = \frac{(2s^N + r^*) + (s^N + r^* - \nu_1^N)}{(m^N)^2} \frac{(s^N + m^N + r^* - \nu_1^N)}{\Psi_{L^N}},
\]

(204)

where \(\Psi_{L^N}\) and \(\Psi_{U^N}\) and the partial derivatives (evaluated at the steady-state) of the overall surplus from an additional job \(\Psi^N\) in the non traded sector:

\[
\Psi_{L^N} = \partial \Psi^N \frac{\partial}{\partial L^N} = P_L A^N + \frac{v_{FE}^N}{\lambda} < 0, \quad (205a)
\]

\[
\Psi_{U^N} = \partial \Psi^N \frac{\partial}{\partial U^N} = \frac{v_{FE}^N}{\lambda} < 0. \quad (205b)
\]

Inserting (204) into (170b) allows to rewrite \(\omega_{31}^N\) as follows:

\[
\omega_{31}^N = \left(s^N + \nu_1^N\right) - \frac{(m^N)^2 \dot{U}^N}{m^N} \omega_{21}^N,
\]

\[
= \frac{s^N + \nu_1^N}{m^N} - \frac{(2s^N + r^*) + (s^N + r^* - \nu_1^N)}{\Psi_{L^N}} \frac{(s^N + m^N + r^* - \nu_1^N)}{\Psi_{U^N}},
\]

\[
= \frac{s^N + \nu_1^N - (2s^N + r^*)}{\Psi_{L^N}} \frac{(s^N + m^N + r^* - \nu_1^N)}{\Psi_{U^N}},
\]

\[
= - \frac{s^N + r^* - \nu_1^N + \Psi_{L^N}}{\Psi_{U^N}} < 0,
\]

(206)

where \(\nu_1^N < 0\) is the stable root for the non traded labor market. Since according to (205), \(\Psi_{L^N} < 0\) and \(\Psi_{U^N} < 0\), we have \(\omega_{31}^N < 0\). Hence, as employment declines in the non traded sector, job seekers increase in this sector.

### E.4 Isocones and Stable Path in the \((u^T, L^T)\)-space

One can alternatively analyze the transitional adjustment in the \((u^T, L^T)\)-space. To do so, we first determine the slopes of the isoclines \(\dot{L}^T = 0\) and \(\dot{u}^T = 0\) in the \((u^T, L^T)\)-space. Hence, we first determine the relationship between labor market tightness and the unemployment rate by using the definition of the latter, i.e. \(\ddot{u}^T = \frac{s^T + m^T}{s^T + m^T} \dot{u}^T\). To alleviate the notation, we assume:

\[
\alpha_V = \alpha_L^T, \quad \sigma_L = \sigma_L^T.
\]

(207)

Totally differentiating the equation that describes the steady-state level of the unemployment rate, we have:

\[
\ddot{u}^T = \frac{-1}{\alpha_V} \left(s^T + \dot{m}^T\right) \ddot{u}^T.
\]

(208)

The slope of the \(\dot{L}^T = 0\) schedule in the \((u^T, L^T)\)-space writes as:

\[
\frac{\dot{L}^T}{\ddot{u}^T} \bigg|_{L^T=0} = - \left[\alpha_V \ddot{u}^T + \sigma_L \dot{L}^T\right] \frac{1}{\alpha_V \left(s^T + \dot{m}^T\right)} < 0.
\]

(209)

Hence the DST-schedule is downward-sloping in the \((u^T, L^T)\)-space, as displayed in Figure 13(a).
Using eq. (184) together with eq. (208), we have:
\[
- \frac{1}{\alpha_V} \left( \frac{s^T + \bar{m}^T}{\bar{m}^T} \right) \frac{\partial \tilde{u}^T}{\partial \tilde{u}} \left[ (1 - \alpha_V T) \tilde{W} + \tilde{Z} \tilde{W}_R \right] = A^T \tilde{u}^T.
\]
The slope of the \( \tilde{u}^T = 0 \) schedule in the \((u^T, L^T)\)-space thus reads as:
\[
\frac{\tilde{L}^T}{\tilde{u}^T} \bigg|_{\tilde{u}^T = 0} = +\infty \quad (210)
\]

As a result, the \( VCT \)-schedule is a vertical line in the \((u^T, L^T)\)-space, as displayed in Figure 13(a).

Having determined that the patterns of isoclines, we turn now to the transitional adjustment along the stable path labelled \( XX^T \). We begin by linearizing \( u^j(t) = \frac{s^j}{\bar{m}^j} \) in the neighborhood of the steady-state which leads to:
\[
\frac{L^j(t) - L^j_{\text{ss}}}{u^j(t) - u^j_{\text{ss}}} \bigg|_{XX^T} = -\bar{F}^j \frac{\tilde{u}^j}{L^j} \left( \frac{1}{\tilde{L}^j} \right) \frac{1}{(1 - \tilde{u}^j) \omega_{31}^j - \tilde{u}^j},
\]
\[
= \frac{s^j}{\bar{m}^j} \left( \frac{1}{(1 - \tilde{u}^j) \omega_{31}^j - \tilde{u}^j} \right), \quad (212)
\]
where we used the fact that:
\[
\bar{F}^j \tilde{u}^j \frac{L^j}{L^j} = \frac{U^j}{L^j},
\]
\[
= \frac{s^j}{\bar{m}^j} \frac{s^j + \bar{m}^j}{s^j + \bar{m}^j} \tilde{u}^j = \frac{s^j}{\bar{m}^j},
\]
since \( \tilde{U}^j / \tilde{L}^j = \frac{s^j}{\bar{m}^j} \).

Focusing on the traded sector, inserting the stable path (see section D.4) for job seekers, i.e., \( U^T(t) - \tilde{U}^T = \omega_{31}^j D^j e^{\nu T} \) with \( \omega_{31}^j = -1 \) (see eq. (168)), the stable path \( XX^T \) shown in Figure 13(a) is described by:
\[
\frac{\tilde{L}^T(t)}{\tilde{u}^T(t)} \bigg|_{XX^T} = -\frac{s^T + \bar{m}^T}{\bar{m}^T} \tilde{u}^T < 0,
\]
\[
= -\frac{\tilde{u}^T}{1 - \tilde{u}^T} \quad (213)
\]

Eq. (214) reveals that in countries where the unemployment benefit scheme is more generous (i.e., \( q \) takes higher values) or worker bargaining power is greater (i.e., \( \alpha_W \) takes higher values), the stable path becomes steeper since labor market tightness is initially low and thus the unemployment rate \( u^T \) is high.

We now demonstrate that the slope of the eigenvector (214) in the \((u^T, L^T)\)-space is larger (i.e., less negative) than the slope of the \( DST \)-schedule described by eq. (209):
\[
0 > -\frac{s^T + \bar{m}^T}{\bar{m}^T} \tilde{u}^T > -\left[ \alpha_V \tilde{u}^T + \sigma_L \tilde{X}^T \right] \frac{1}{\alpha_V} \left( \frac{s^T + \bar{m}^T}{\bar{m}^T} \right),
\]
\[
0 > -\sigma_L \tilde{X}^T \left( \frac{s^T + \bar{m}^T}{\alpha_V} \right), \quad (215)
\]
Since the term on the RHS of inequality is unambiguously negative, the stable branch which corresponds to the $X X^T$-schedule is flatter than the $DST$-schedule.

The adjustment of labor and unemployment rate in the traded sector is depicted in Figure 3(a). Following an increase in productivity of tradables relative to non tradables, the decision of search-schedule shifts (slightly) to the left as a result of the positive wealth effect (captured by a decline in $\bar{\lambda}$, see eq. (182)); at the same time, the vacancy creation-schedule which is vertical also shifts to the left (see eq. (184)) as a result of the rise in $A^T$ which encourages firms to post more job vacancies; as a result, $\bar{\theta}^T$ increases which raises the probability of finding a job and thus lowers unemployment. The unemployment rate declines on impact. Along the stable path, $u^T$ falls while employment builds up.

E.5 Isoclines and Stable Path in the $(u^N, L^N)$-space

The steady-state level of the non traded sector is described by:

$$\tilde{u}^N = \frac{s^N}{s^N + \overline{m}^N \left( \bar{\theta}^N \right)}$$  \hspace{1cm} (216)

Totally differentiating eq. (216) leads to:

$$\hat{\tilde{\theta}}^N = - \frac{1}{\sigma_V} \left( \frac{s^N + \overline{m}^N}{\overline{m}^N} \right) \tilde{u}^N.$$  \hspace{1cm} (217)

The slope of the $\dot{L}^N = 0$ schedule in the $(u^N, L^N)$-space reads as:

$$\frac{\dot{L}^N}{\dot{u}^N} = - \left[ \alpha_V \tilde{u}^N + \sigma_{LX}^N \right] \frac{1}{\alpha_V} \left( \frac{s^N + \overline{m}^N}{\overline{m}^N} \right) < 0.$$  \hspace{1cm} (218)
Hence the $DSN$-schedule is downward-sloping in the $(u^N, L^N)$-space, as displayed in Figure 13(b).

Inserting first (134) and totally differentiating eq. (188b) leads to:

$$
\left[ (1 - \alpha V) \tilde{\psi}^N + \chi^N \tilde{W}_R^N \right] \tilde{\theta}^N = P_A A^N d\lambda + P_{LN} A^N dL^N + \left( P_{AN} A^N + \tilde{P} \right) dA^N,
$$

(219)

where $P_{LN} < 0$.

Inserting eq. (217) into eq. (219) gives us the slope of of the $\tilde{\theta}^N = 0$ schedule in the $(u^N, L^N)$-space:

$$
\frac{\dot{L}_N}{\dot{u}_N} \bigg|_{\dot{u}_N = 0} = - \left[ (1 - \alpha V) \tilde{\psi}^N + \chi^N \tilde{W}_R^N \right] \alpha V P_{LN} A^N L^N (s^N + \tilde{m}^N) > 0.
$$

(220)

where the positive sign of eq. (220) follows from eq. (136) indicating that $P_{LN} < 0$. As a result, the $VCN$-schedule is an upward-sloping line in the $(u^N, L^N)$-space, as displayed in Figure 13(b).

Having determined the patterns of isoclines, we turn now to the transitional adjustment along the stable path labelled $XX^N$ by making use of (212):

$$
\left. \frac{L_N(t)-\tilde{L}_N}{L^N} \right|_{u^N} = \frac{1}{\tilde{u}_N} \frac{L^N}{L^N} \left| \frac{(1 - \tilde{u}_N) \omega_{31}^N - \tilde{u}_N}{(1 - \tilde{u}_N) \omega_{31}^N - \tilde{u}_N} \right|.
$$

(221)

As will be useful, we first determine the expression of eigenvector $\omega_{31}^N$ by inserting eq. (194) into (170b):

$$
\omega_{31}^N = \frac{\left( s^N + r^* - \nu_1^N \right) + \tilde{m}^N \left( P_{LN} A^N \frac{\lambda}{v_{FF}} + 1 \right)}{(s^N + \tilde{m}^N + r^* - \nu_1^N)}.
$$

(222)

Then, we use (222) to derive an expression for $(1 - \tilde{u}_N) \omega_{31}^N - \tilde{u}_N$:

$$
(1 - \tilde{u}_N) \omega_{31}^N - \tilde{u}_N = - \frac{(s^N + \tilde{m}^N + r^* - \nu_1^N) + (1 - \tilde{u}_N) \tilde{m}^N P_{LN} A^N \frac{\lambda}{v_{FF}}}{(s^N + \tilde{m}^N + r^* - \nu_1^N)}.
$$

(223)

Inserting (223) into eq. (221) gives us the slope of the stable path $XX^N$ in the $(u^N, L^N)$-space:

$$
\left. \frac{\dot{L}_N(t)}{\dot{u}_N(t)} \right|_{XX^N} = \frac{- s^N \left( s^N + \tilde{m}^N + r^* - \nu_1^N \right)}{\tilde{m}^N \left( s^N + \tilde{m}^N + r^* - \nu_1^N \right) + (1 - \tilde{u}_N) \tilde{m}^N P_{LN} A^N \frac{\lambda}{v_{FF}}} < 0.
$$

(224)

Since $v_{FF}^N < 0$ and $P_{LN} < 0$, the stable branch $XX^N$ is downward-sloping in the $(u^N, L^N)$-space.

We now demonstrate that the slope of the stable branch (224) in the $(u^N, L^N)$-space is larger (i.e., less negative) than the slope of the $DSN$-schedule described by eq. (218):

$$
0 > \frac{s^N \left( s^N + \tilde{m}^N + r^* - \nu_1^N \right)}{\tilde{m}^N \left( s^N + \tilde{m}^N + r^* - \nu_1^N \right) + (1 - \tilde{u}_N) \tilde{m}^N P_{LN} A^N \frac{\lambda}{v_{FF}}} > - \frac{1}{\alpha V} \left( s^N + \tilde{m}^N \right),
$$

$$
\left( s^N + \tilde{m}^N + r^* - \nu_1^N \right) \alpha V \tilde{u}_N < \alpha V \tilde{u}_N + \sigma L \tilde{x}^N \left[ \left( s^N + \tilde{m}^N + r^* - \nu_1^N \right) + \left(1 - \tilde{u}_N \right) \tilde{m}^N P_{LN} A^N \frac{\lambda}{v_{FF}} \right] \tilde{u}_N < \sigma L \tilde{x}^N \left( s^N + \tilde{m}^N + r^* - \nu_1^N \right) + \alpha V \tilde{u}_N - \sigma L \tilde{x}^N \left(1 - \tilde{u}_N \right) \tilde{m}^N P_{LN} A^N \frac{\lambda}{v_{FF}}.
$$

(225)

Since the term on the RHS of inequality is unambiguously positive, the stable branch which corresponds to the $XX^N$-schedule is flatter than the $DSN$-schedule, as can be seen in Figure 13(b).

The adjustment of labor and unemployment rate in the non traded sector is depicted in Figure 3(b). Following an increase in productivity of tradables relative to non tradables, the
decision of search-schedule shifts to the left as a result of the positive wealth effect (captured by a decline in $\bar{\lambda}$); at the same time, the vacancy creation-schedule which is upward-sloping also shifts to the left (see eq. (219)) as a result of the rise in $A^N$ which encourages firms to post more job vacancies. More specifically, a rise in $A^N$ has an ambiguous effect on $PA_N$. Assuming $\sigma_C = \phi = 1$, $A^N$ has no impact whilst the positive wealth effect stimulates consumption in non tradables and thus appreciates the relative price of non tradables which increases the surplus from an additional job. Consequently, $\theta^N$ increases which raises the probability of finding a job and thus lowers $u^N$ in the long-run. The unemployment rate declines significantly on impact and overshoots its new steady-state level. Along the stable path, $u^N$ increases while employment declines. Intuitively, as $L^N$ falls along $XX^N$, the relative price appreciates which induces non traded firms to post more job vacancies. The rise in the labor market tightness $\theta^N$ leads agents to search for a job and thus increases the number of job seekers. The decline in employment $L^N$ triggered by the positive wealth effect and the rise in the number of job seekers $U^N$ produces an increase in $u^N$ along the stable path.

F Solving Graphically for the Steady-State

The steady-state can be described by considering alternatively the goods market or the labor market. Due to the lack of empirical estimates at a sectoral level, and to avoid unnecessary complications, we impose $\alpha^T_L = \alpha^V$, $\alpha^T_W = \alpha_W$ from now on.

F.1 Steady-State

We first show that the steady-state of the economy consisting of six equations which can be solved for sectoral employment and labor market tightness, i.e., $L^j = L^j (A^T, A^N)$ and $\theta^j = \theta^j (A^T, A^N)$ with $j = T, N$, the stock of foreign assets, $B = B (A^T, A^N)$, and the shadow value of wealth, $\bar{\lambda}$.

First, setting $\dot{\theta}^j = 0$ into eq. (140), we obtain the vacancy creation equation (which holds for the traded sector and non traded sector):

$$\frac{\kappa^j}{f^j (\theta^j)} = \frac{(1 - \alpha_W) \tilde{\Psi}^j}{s^j + r^*}, \quad \tilde{\Psi}^j \equiv (\Xi^j + r^*x^j) - \tilde{W}_R^j, \quad j = T, N, \quad (226)$$

where $\Xi^N = P(.)A^N$ with $P(.)$ given by eq. (134). The LHS term of eq. (226) represents the expected marginal cost of recruiting in sector $j = T, N$. The RHS term represents the marginal benefit of an additional worker which is equal to the share, received by the firm, of the rent created by the encounter between a vacancy and a job-seeking worker. A rise in labor productivity raises the surplus from hiring $\tilde{\Psi}^j$; as a result, firms post more job vacancies which increases the labor market tightness $\theta^j$.

Second, setting $\dot{\xi}^j = 0$ into eq. (110) and using the fact that $W^j - W^T_R = \alpha_W \tilde{\Psi}^j$ leads to $\xi^j = \frac{\alpha_W \tilde{\Psi}^j}{s^j + r^*}$. Rewriting the latter equation by inserting the vacancy creation equation (226) for sector $j$ to eliminate $\tilde{\Psi}^j$ gives the expected value of finding a job, i.e., $m^j \xi^j = \frac{\alpha_W}{1 - \alpha_W} \kappa^j \theta^j$. Plugging this equation into (9b) leads to the equality between the utility loss from participating the labor market in sector $j$ and the marginal benefit from search, i.e.,

$$\frac{\xi^j (F^j)^{\frac{1}{\alpha_W}}}{m^j} = \frac{\alpha_W}{1 - \alpha_W} \kappa^j \theta^j + R^j.$$

Setting $\dot{L}^j = 0$ into eq. (7) to eliminate $U^j$ so that $F^j = \left(\frac{s^j + m^j}{m^j}\right)L^j$, the decision of search equation reads as (which holds for the traded sector and non traded sector):

$$L^j = \left[\frac{m^j}{m^j + s^j} \left(\frac{\bar{\lambda}}{\tilde{\xi}^j} \left(\frac{\alpha_W}{1 - \alpha_W} \kappa^j \theta^j + R^j\right)\right)^{\sigma^L}\right]^{\sigma^L}, \quad j = T, N, \quad (227)$$

where $\left(\frac{\alpha_W}{1 - \alpha_W} \kappa^j \theta^j + R^j\right)$ corresponds to the reservation wage, $W^T_R$, reflecting the marginal benefit from search. According to (227), a higher labor market tightness increases labor $L^j$ by raising the job-finding rate for the worker and thus the employment rate $\frac{m^j}{m^j + s^j}$.
Moreover, for given $\tilde{\lambda}$, the rise in the reservation wage $\frac{\alpha w}{1 - \alpha w} \kappa^j \theta^j + R^j$ induces agents to supply more labor.

Third, setting $\tilde{B} = 0$ into eq. (21), we obtain the market clearing condition for the traded good:

$$r^* B + A^T L^T - C^T - \kappa^T U^T \theta^T - \kappa^N U^N \theta^N = 0,$$

(228)

where $C^T = C^T (L^N, \tilde{\lambda}, A^N)$.

The system which comprises eqs. (226)-(228) can be solved for the steady-state sectoral labor market tightness and employment, and traded bonds. All these variables can be expressed in terms of the labor productivity index $A^j$ and the marginal utility of wealth, i.e., $\theta^T = \theta^T (A^T)$, $L^T = L^T (\tilde{\lambda}, A^T)$, $\theta^N = \theta^N (\tilde{\lambda}, A^N)$, $L^N = L^N (\lambda, A^N)$, and $B = B (\tilde{\lambda}, A^T, A^N)$. Inserting first $B = B (\tilde{\lambda}, A^T, A^N)$, and $L^j = L^j (\tilde{\lambda}, A^N)$, the intertemporal solvency condition (175) can be solved for the equilibrium value of the marginal utility of wealth:

$$\tilde{\lambda} = \lambda (A^T, A^N).$$

(229)

Setting first $\tilde{L}^j = 0$ into (7), inserting $L^j = L^j (\tilde{\lambda}, A^j)$, one can solve for $U^j$; then the relationship $V^j = \theta^j U^j$ can be solved for the steady-state job vacancy in sector $j$. Using the fact that $C^T = C^T (L^N, \tilde{\lambda}, A^N)$, inserting $L^N (\lambda, A^N)$ and using the fact that $Y^T = A^T L^T$ with $L^T = L^T (\tilde{\lambda}, A^T)$, allows us to solve for ratio $v_{NX} = \frac{Y^T - C^T}{Y^T}$:

$$v_{NX} = v_{NX} (A^T, A^N),$$

(230)

where we have eliminated $\tilde{\lambda}$ by using (229).

### F.2 The Goods Market: Graphical Apparatus

To build intuition about steady-state changes, we investigate graphically the long-run effects of a rise in $A^T/A^N$. To do so, it is convenient to rewrite the steady-state as follows:

$$\frac{C^T}{C^N} = \frac{\varphi}{1 - \varphi} B^\varphi,$$

(231a)

$$\frac{L^T}{L^N} = \frac{m^T}{m^N} \left( \frac{s^N}{s^N + m^N} \right) \left[ \frac{\lambda W^R_L / \zeta^T}{\zeta^T} \right]^{\sigma_L},$$

(231b)

$$\frac{\kappa_T}{\kappa_N} = \frac{(1 - \alpha W)^T}{(1 - \alpha W)^N},$$

(231c)

$$\frac{f^T (\theta^T)}{f^N (\theta^N)} = \frac{(s^T + r^*)}{(s^N + r^*)},$$

(231d)

$$\frac{Y^T (1 + v_B - v_V^T - v_V^N)}{Y^N} = \frac{C^T}{C^N}.$$

(231e)

We denote by $v_B \equiv \frac{r^* B}{Y^T}$ the ratio of interest receipts to traded output, by $v_V^j \equiv \frac{\nu^j V^j}{Y^T}$ the share of hiring cost in sector $j = T, N$ in traded output. Remembering that $Y^T = A^T L^T$ and $Y^N = A^N L^N$, the system (231) can be solved for $C^T/C^N, L^T/L^N, \theta^T, \theta^N$, and $P$, as functions of $A^T, A^N, (1 + v_B - v_V^T - v_V^N)$. Inserting these functions into $Y^N = C^N$ (see eq. (179h)), and $B - B_0 = \Phi^T (L^T - L^T_0) + \Phi^N (L^N - L^N_0)$ (see eq. (179j)), the system can be solved for $B$ and $\tilde{\lambda}$ as functions of $A^T$ and $A^N$. Hence, when solving the system (231), we assume that the stock of foreign bonds and the marginal utility of wealth are exogenous which allows us to separate intratemporal reallocation effects triggered by the change in the share of tradables from the dynamic (or intertemporal) effects stemming from the accelerated hiring process that increases the demand for tradables in the long-run.

When focusing on the goods market, the equilibrium can be characterized by two schedules in the $(y^T - y_N, p)$-space where we denote the logarithm in lower case. The steady state is summarized graphically in Figure 14(b).

Denoting by $v_{NX} \equiv NX/Y^T$ the ratio of net exports to traded output, with $v_{NX} \equiv -(v_B - v_V^T - v_V^N)$, and inserting (231a) into the market clearing condition (231e) leads to

$$\frac{Y^T}{Y^N} = \frac{\varphi}{1 - \varphi (1 - v_{NX})} P^\varphi.$$

(232)
Eq. (232) corresponds to eq. (22) in the text. Totally differentiating (232) and denoting the percentage deviation from its initial steady-state by a hat yields the goods market equilibrium-schedule (GME henceforth):

$$\left(\hat{y}^T - \hat{y}^N\right)^{GME} = \phi \hat{p} - d \ln \left(1 - v_{NX}\right).$$  \hspace{1cm} (233)

According to (233), the GME-schedule is upward-sloping in the \((y^T - y^N, p)\)-space with a slope equal to \(1/\phi\). Following a rise in traded output relative to non traded output, the relative price of non tradables must appreciate to clear the goods market, and all the more so as the elasticity of substitution \(\phi\) is smaller. The 45° dotted line allows us to consider two cases. When \(\phi > 1 \,(\phi < 1)\), the GME-schedule is flatter (steeper) than the 45° dotted line.

We now characterize the labor market equilibrium. Totally differentiating (226) gives the deviation in percentage of the sectoral labor market tightness from its initial steady-state, i.e., \(\hat{\beta}^j = \frac{\Xi^j}{\left(1 - \alpha^j_v\right)\Psi^j + \chi^j W^j_R} \hat{\beta}^j\). Totally differentiating (227) gives the deviation in percentage of sectoral labor from its initial steady-state, i.e., \(\hat{\beta}^j = \sigma^j_L \hat{\lambda} + \left[\alpha^j_v \hat{\beta}^j + \sigma^j_L \chi^j\right] \hat{\beta}^j\). Substituting the former into the latter, differentiating the production function \(y^j = \Lambda^j L^j\) to eliminate \(\hat{\lambda}\), and using the fact that \(\chi^j W^j_R = \frac{\alpha^j_L \Psi^j}{\chi^j \Psi^j + \chi^j W^j_R}\) at the steady-state, one obtains the labor market equilibrium (LME henceforth) schedule:

$$\left(\hat{y}^T - \hat{y}^N\right)^{LME} = -\Theta^N \hat{p} + (1 + \Theta^T) \hat{a}^T - (1 + \Theta^N) \hat{a}^N,$$  \hspace{1cm} (234)

where we set

$$\Theta^j = \frac{\Xi^j \left(s^j + r^\ast\right) \left[\alpha^j_v \hat{\beta}^j + \sigma^j_L \chi^j\right]}{\Psi^j \left(1 - \alpha^j_v\right) \left(s^j + r^\ast\right) + \alpha^j_L m^j},$$  \hspace{1cm} (235)

in order to write formal solutions in a compact form. As depicted in Figure 14(b), the LME-schedule is downward-sloping in the \((y^T - y^N, p)\)-space with a slope equal to \(-1/\Theta^N\) (see eq. (234)). An appreciation in the relative price of non tradables raises the surplus from hiring which induces non traded firms to post more job vacancies. By raising the expected value of a job, the consecutive rise in the labor market tightness induces agents to increase the search intensity for a job in the non traded sector but less so as the elasticity of labor supply \(\sigma^j_L\) is lower. More precisely, lower values of \(\sigma^j_L\) indicate that workers experience a larger switching cost from one sector to another; in this configuration, the term \(\Theta^j\) is smaller so that the LME-schedule is steeper. Conversely, when we let \(\sigma^j_L\) tend toward infinity, the case of perfect mobility of labor across sectors is obtained; in this configuration, the LME-schedule becomes a horizontal line.
F.3 The Labor Market: Graphical Apparatus

When focusing on the labor market, the model can be summarized graphically by two schedules in the \((i^T - i^N, \ln \left( \frac{g^T}{g^N} \right))\)-space, as shown in Figure 14(a).

As will be useful later, we first solve for the relative price of non tradables by using the goods market clearing condition (232). Using production functions, i.e., \(Y^j = A^jL^j\), solving (232) for the relative price yields:

\[
P = \left[ \frac{1 - \varphi}{1 - \varphi} \right] (1 - v_{NX}) \left( \frac{A^T}{A^N} \right) \left( \frac{L^T}{L^N} \right)^{\frac{1}{2}}. \tag{236}
\]

Applying the implicit function theorem, we have:

\[
P = P \left( \left( \frac{L^T}{L^N} \right), (1 - v_{NX}), \left( \frac{A^T}{A^N} \right) \right), \tag{237}
\]

where

\[
\hat{p} = \frac{1}{\hat{\phi}} \left[ d \ln \left( \frac{L^T}{L^N} \right) + d \ln \left( \frac{A^T}{A^N} \right) + d \ln (1 - v_{NX}) \right]. \tag{238}
\]

F.3.1 The Decision of Search Schedule in the \((i^T - i^N, \ln \left( \frac{g^T}{g^N} \right))\)-space

Imposing \(\sigma^j = \sigma_L\) into (231b), which implies that the marginal utility of wealth does not impinge relative labor supply, the decision of search equation reduces to:

\[
\frac{L^T}{L^N} = \frac{m^T m^N + s^N}{m^T + s^T} \left( \frac{W^T_R \epsilon^N_s}{W^N_R \epsilon^T_s} \right)^{\sigma_L}, \tag{239}
\]

where \(W^j_R = \frac{\alpha^j}{1 - \alpha^j} \kappa^j \tilde{\theta}^j + R^j\) is the reservation wage. Eq. (239) corresponds to eq. (24) in the text. Taking logarithm and differentiating eq. (236) yields:

\[
\frac{i^T - i^N}{\ln \left( \frac{g^T}{g^N} \right)} = \frac{1}{\alpha_U + \sigma_L \chi} \left( \frac{i^T}{i^N} \right). \tag{240}
\]

Inspection of (241) reveals that the DS-schedule:

- is upward-sloping in the \((i^T - i^N, \ln \left( \frac{g^T}{g^N} \right))\)-space;

- is steeper as the workers are more reluctant to shift hours worked across sectors (i.e., the elasticity of labor supply \(\sigma_L\) is smaller), the unemployment benefit scheme is more generous or the worker bargaining power \(\alpha_U\) is lower (because higher unemployment benefits \(R\) or a lower worker bargaining power both reduce the share of the surplus associated with a labor contract in the marginal benefit of search \(\chi\)).

F.3.2 The Vacancy-Creation Schedule in the \((i^T - i^N, \ln \left( \frac{g^T}{g^N} \right))\)-space

Dividing (231c) by (231d) and using (124) leads to the vacancy creation equation:

\[
\frac{\kappa^T (s^T + r^*) X_N}{\kappa^N (s^N + r^*) X_T} \left( \frac{\theta^T}{\theta^N} \right)^{1 - \alpha_U} = \frac{\Xi^T + r^* \theta^T - W^T_R}{\Xi^N + r^* \theta^N - W^N_R}. \tag{242}
\]
where \( \frac{\Xi^T + r^T x^T - W_R^T}{\Xi^N + r^N x^N - W_R^N} = \Psi^T \Psi^N \). Eq. (242) corresponds to eq. (23) in the text. Totally differentiating (242) by sing the fact that the change in overall surplus \( \Psi^j \) in percentage is given by

\[
\hat{\Psi}^j = \frac{\Xi^j \hat{\xi}^j - \chi^j W_R^j \hat{\theta}^j}{\Psi^j},
\]

yields:

\[
\left( \hat{\theta}^T - \hat{\theta}^N \right) \bigg|^{VC} = \frac{\Xi^T \hat{\alpha}^T - \Xi^N (\hat{p} + \hat{\alpha}^N)}{(1 - \alpha_V) \Psi^T + \chi^T W_R^T} - \frac{\Xi^N (\hat{p} + \hat{\alpha}^N)}{(1 - \alpha_V) \Psi^N + \chi^N W_R^N}.
\]

Eliminating the relative price by using (238), collecting terms, assuming that initially \( \Xi^j \sim \Xi, \Psi^j \sim \Psi, W_R^j \sim W_R, \chi^j \sim \chi \), eq. (244) can be rewritten as follows:

\[
\left( \hat{\theta}^T - \hat{\theta}^N \right) \bigg|^{VC} = -\frac{\Xi}{\phi [(1 - \alpha_V) \Psi + \chi W_R]} \left( \hat{i}^T - \hat{i}^N \right) + \frac{\Xi [(\phi - 1) (\hat{a}^T - \hat{\alpha}^N) - d \ln (1 - \nu_{NX})]}{\phi [(1 - \alpha_V) \Psi + \chi W_R]}.
\]

Inspection of (245) reveals that the VC-schedule:

- is downward-sloping in the \((\hat{i}^T - \hat{i}^N, \ln (\hat{a}^T / \hat{\alpha}^N))\)-space with a slope equal to \(- \frac{\Xi}{\phi [(1 - \alpha_V) \Psi + \chi W_R]}\);
- is steeper as the elasticity of substitution between traded and non traded goods \( \phi \) is smaller or the worker bargaining power is lower (because it reduces \( \chi W_R \));
- shifts to the right following higher productivity of tradables relative to non tradables (i.e., \( \hat{a}^T - \hat{\alpha}^N > 0 \)) as long as \( \phi > 1 \) or when the country experiences a higher steady-state trade balance surplus, i.e., if \(-d \ln (1 - \nu_{NX}) \approx d \nu_{NX} > 0\);

G Long-Run Relative Price and Relative Wage Effects of Higher Relative Productivity of Tradables

This section analyzes analytically the consequences on the relative wage and the relative price of an increase in relative sectoral productivity \( A^T / A^N \). It compares the steady-state of the model before and after the productivity shock biased towards the traded sector. To shed some light on the transmission mechanism, we analytically break down the relative wage and relative price effects in two components: a labor market frictions effect and a labor accumulation effect.

Equating demand for tradables in terms of non tradables given by eq. (233) and supply (234) yields

\[
\left( \hat{y}^T - \hat{y}^N \right) = \phi \hat{\rho} - d \ln (1 - \nu_{NX}),
\]

\[
= -\Theta^N \hat{\rho} + (1 + \Theta^T) \hat{a}^T - (1 + \Theta^N) \hat{\alpha}^N.
\]

Collecting terms leads to the deviation in percentage of the relative price from its initial steady-state:

\[
\hat{\rho} = \frac{(1 + \Theta^T) \hat{a}^T - (1 + \Theta^N) \hat{\alpha}^N}{(\phi + \Theta^N)} + \frac{d \ln (1 - \nu_{NX})}{(\phi + \Theta^N)}.
\]

Eq. (246) corresponds to eq. (25) in the text. It is worthwhile noticing that \( \hat{\rho} \) given by eq. (246) is determined by the system which comprises the goods market equilibrium (232), the decision of search equation (239), and the vacancy creation equation (242). This implies that \( P = P (A^T, A^N, \nu_{NX}) \). Invoking the intertemporal solvency condition (175) allows us to solve for \( \nu_{NX} = v_{NX} (A^T, A^N) \).
To determine the long-run adjustment in the relative wage, \( \Omega \equiv W^N/W^T \), we first derive the deviation in percentage of the sectoral wage. To do so, we totally differentiate the vacancy creation equation for sector \( j \) given by eq. (226):

\[
\hat{\theta}^j = \frac{\Xi^j}{(1 - \alpha_V) \Psi^j + \chi^j W^R_R} \hat{\Xi}^j. \tag{247}
\]

We repeat the Nash bargaining wage given by eq. (18) for convenience by imposing \( \alpha^j_W = \alpha_W \):

\[
W^j = \alpha_W \left( \Xi^j + r^* x^j \right) + (1 - \alpha_W) W^R_R. \tag{248}
\]

Totally differentiating (248) and plugging the change in the labor market tightness leads to:

\[
\hat{w}^j = \frac{\alpha_W \Xi^j}{W^j} \hat{\Xi}^j + \frac{\alpha_W (1 - \alpha_W) \psi^j + \chi^j W^R_R}{W^j} \hat{\theta}^j,
\]

\[
= \Xi^j \left[ \frac{\alpha_W (1 - \alpha_V) \Psi^j + \chi^j W^R_R}{W^j (1 - \alpha_V) \Psi^j + \chi^j W^R_R} \right]. \tag{249}
\]

Using the fact that at the steady-state, we have \( \chi^j W^R_R = m^j \xi^j = \frac{m^j \alpha_W \psi^j}{s^j + r^*} \), eq. (249) can be rewritten as follows:

\[
\hat{w}^j = \Xi^j \left[ \frac{\alpha_W (1 - \alpha_V) \Psi^j + \frac{m^j \alpha_W \psi^j}{s^j + r^*}}{W^j} \right].
\]

\[
= \Xi^j \frac{\alpha_W (1 - \alpha_V) (s^j + r^*) + m^j}{W^j [(1 - \alpha_V) (s^j + r^*) + \alpha_W m^j]} \hat{\Xi}^j. \tag{250}
\]

Eq. (250) corresponds to eq. (28) in the text. In order to write formal solutions in a compact form, we set:

\[
\Omega^j \equiv \Xi^j \frac{\alpha_W (1 - \alpha_V) (s^j + r^*) + m^j}{W^j [(1 - \alpha_V) (s^j + r^*) + \alpha_W m^j]}. \tag{251}
\]

Using the fact that \( \hat{\Xi}^N = \hat{p} + \hat{a}^N \) and \( \hat{\Xi}^T = \hat{a}^T \), subtracting \( \hat{w}^N \) from \( \hat{w}^N \) by combining (250) and (251) and inserting (246) leads to the deviation in percentage of the relative wage:

\[
\hat{\omega} = \hat{w}^N - \hat{w}^T,
\]

\[
= \Omega^N (\hat{p} + \hat{a}^N) - \Omega^T \hat{a}^T,
\]

\[
= \left\{ \Omega^N \left[ \frac{(1 + \Theta^T) \hat{a}^T + (\phi - 1) \hat{a}^N}{(\phi + \Theta^N)} \right] - \Omega^T \hat{a}^T \right\} - \Omega^N \frac{d u_{N X}}{\phi + \Theta^N}. \tag{252}
\]

Eq. (252) corresponds to eq. (29) in the text.

H Analyzing Graphically the Long-Run Effects of Higher Relative Productivity

This section analyzes graphically the consequences on the relative wage and the relative price of an increase in the relative productivity of tradables, \( A^T/A^N \), by breaking down the relative wage and relative price effects into a labor market frictions effect and a labor accumulation effect.
H.1 Effects of Higher Productivity in Tradables Relative to Non Tradables

In order to facilitate the discussion, we assume that $\Theta^j \simeq \Theta$. Under this assumption, eq. (246) reduces to:

$$
\hat{p} = \frac{(1 + \Theta)(\hat{a}^T - \hat{a}^N)}{(\phi + \Theta)} + \frac{d\ln(1 - v_{NX})}{(\phi + \Theta)},
$$

(253)

where $d\ln(1 - v_{NX}) \simeq -dv_{NX}$ by using a first-order Taylor approximation.

Eq. (253) breaks down the relative price response into two components: a labor market frictions effect and an accumulation effect. The first term on the RHS of eq. (253) corresponds to the labor market frictions effect. When we let $\sigma_L$ tend toward infinity, we have $\lim_{\sigma_L \to \infty} \frac{1}{\phi + \Theta} = 1$; in this configuration, a productivity differential between tradables and non tradables by 1% appreciates the relative price by 1% as well, in line with the prediction of the standard BS model. Graphically, as shown in Figure 15(a), the $LME$-schedule is a horizontal line because the allocation of the labor force across sectors is perfectly elastic to the ratio of sectoral reservation wages. A productivity shock biased toward the traded sector shifts higher the $LME$-schedule which results in a relative price appreciation, from $p_0$ to $p_{BS}^*$, i.e., by the same amount as the productivity differential. The $LME$-schedule intercepts the $45^\circ$ line at point $BS^*$.

As long as $\sigma_L < \infty$, workers experience a mobility cost when moving from one sector to another; hence, the term $\Theta$ takes finite values while graphically, the $LME$-schedule is downward sloping in the $(y^T - y^N, p)$-space. Graphically, higher productivity in tradables relative to non tradables shifts to the right the $LME$-schedule from $LME_0$ to $LME_1$; this shift corresponds to the labor market frictions effect. If $\phi > 1$, the $GME$-schedule is flatter than the $45^\circ$ line so that the intersection is at $G'$; since $\hat{p} > p_{BS}$, the relative price appreciates by less than the productivity differential between tradables and non tradables, in line with our empirical findings. Conversely, if $\phi < 1$, the relative price must appreciate more than proportionately (i.e., by more than 1%) following higher productivity of tradables relative to non tradables (by 1 percentage point). In this configuration, the $GME$-schedule is steeper that the $45^\circ$ line so that the $LME_1$-schedule intercepts the $GME$-schedule at a point which lies to the north west of $BS^*$. Hence, through the labor market frictions channel, a productivity differential between tradables and non tradables by 1% appreciates the relative price of non tradables by less (more) than 1% if traded and non traded goods are substitutes (complements).

The second term on the RHS of eq. (253) reveals that a productivity differential between tradables and non tradables also impinges on the relative price of non tradables by affecting net exports and hiring expenditure expressed as a share of traded output, as summarized by $dv_{NX}$. The combined effect of the improvement in the trade balance and permanently increased hiring expenditure has an expansionary effect on the demand for tradables which drives down the relative price of non tradables, as captured by $dv_{NX} > 0$. In terms of Figure 15(a), the labor accumulation channel shifts the $GME$-schedule to the right, regardless of the value of the elasticity of substitution between traded and non traded goods. It is worthwhile noticing that a change in $v_{NX}$ no longer impinges on the relative price $p$ and thus the labor accumulation channel vanishes when we let $\sigma_L$ tend toward infinity, i.e., if agents are not subject to switching costs from one sector to another. Formally, we have $\lim_{\sigma_L \to \infty} \frac{1}{\phi + \Theta} = 0$. In this case, the $GME_1$-schedule intercepts the $LME_1$-schedule at $BS_1$. Unlike, when $\sigma_L < \infty$, the intercept is at $G_1$ if $\phi > 1$.

We turn to the relative response. To facilitate the discussion, we assume that $\Theta^j \simeq \Theta$ and $\Omega^j \simeq \Omega$ so that eq. (252) reduces to:

$$
\tilde{\omega} = -\Omega \left[ \frac{(\phi - 1)}{\phi + \Theta} (\hat{a}^T - \hat{a}^N) + \frac{dv_{NX}}{\phi + \Theta} \right].
$$

(254)

Through the labor market frictions channel, captured by the first term in brackets in the RHS of eq. (254), higher productivity growth in tradables relative to non tradables lowers the relative wage $\omega$ only if $\phi > 1$. In terms of Figure 15(b), technological change biased toward the traded sector shifts to the right the $VC$-schedule from $VC_0$ to $VC'$. Unlike,
with an elasticity $\phi$ smaller than one, the $VC$-schedule would shift to the left because the share of non tradables rises which has an expansionary effect on recruitment in the non traded sector.

As captured by the second term on the RHS of eq. (254), a productivity differential between tradables and non tradables also impinges on the relative wage through a labor accumulation channel. Graphically, as depicted in Figure 15(b), higher productivity in tradables relative to non tradables shifts further to the right the $VC$-schedule from $VC'$ to $VC_1$. Hence, while $\omega$ unambiguously declines if the elasticity of substitution is larger than one, when $\phi < 1$, the relative wage response to a productivity differential is ambiguous. In the latter case, a productivity differential between tradables and non tradables drives down $\omega$ through the labor accumulation channel while it increases the relative wage through the labor market frictions channel.

### H.2 Implications of Labor Market Institutions

In this subsection, we analyze graphically the implications of labor markets institutions for the relative wage response to technological change biased toward the traded sector. In our framework, the strictness of legal protection against dismissals is captured by a firing tax denoted by $x^j$ paid to the State by the representative firm in the sector which reduces employment. The generosity of the unemployment benefit scheme is captured by the level of $R^j$: unemployment benefits are assumed to be a fixed proportion $\varrho$ of the wage rate $W^j$, i.e., $R^j = \varrho W^j$. Additionally, a higher worker bargaining power measured empirically by the bargaining coverage is captured by the parameter $\alpha_W$. Because the transmission mechanism varies according the type of labor market institution, we differentiate between the firing cost on the one hand, the generosity of the unemployment benefit scheme and the worker bargaining power on the other.

The implications of a higher firing tax is depicted in Figure 16(a) where we assume an elasticity between traded and non traded goods in consumption $\phi$ larger than one. In this configuration, as mentioned previously, technological change biased toward the traded sector shifts to the right the $VC$-schedule. As highlighted in Figure 16(a), higher productivity in tradables relative to non tradables shifts further to the right the $VC$-schedule from $VC'$ to $VC''$, thus resulting in a larger increase in $\theta^T/\theta^N$ because hiring in the non traded sector which decumulates employment is limited by the firing tax. Consequently, the relative wage $\omega$ declines more, in line with our empirical findings, through a stronger labor market frictions effect. However, a higher firing tax also moderates the decline in the relative wage since net exports increase less. Intuitively, as recruiting expenditure are curbed by the firing tax, the productivity differential leads to a smaller current account deficit, thus moderating the necessary trade balance improvement.
In contrast to a firing tax, raising the unemployment benefit replacement rate or the worker bargaining power leads to a larger long-run rise in net exports and thus amplifies the decline in the relative wage through the labor accumulation channel. The implication of a higher replacement rate $\varrho$ or a larger worker bargaining power $\alpha_W$ is depicted in Figure 16(b) where we consider an elasticity of substitution $\phi$ larger than one. Figure 16(b) shows that technological change biased toward the traded sector shifts further to the right the $VC$-schedule from $VC_1$ to $VC_2$ in countries where the replacement rate $\varrho$ is higher or the worker bargaining power $\alpha_W$ larger. As mentioned above, the larger increase in net exports amplifies the expansionary effect on hiring in the traded sector which pushes up further the ratio of labor market tightness $\theta^T/\theta^N$. Hence, the relative wage of non tradables falls more through a stronger labor accumulation effect. Raising $\varrho$ or $\alpha_W$ also modifies the labor market frictions channel by increasing the mobility of labor across sectors.\footnote{In countries with a higher worker bargaining power $\alpha_W$, firms are willing to recruit more (because it is relatively less costly due to a higher probability to fill a job vacancy) while workers are less reluctant to move from one sector to another (since they receive a larger share $\chi$ of the surplus associated with a labor contract in the marginal benefit of search). In economies with a more generous unemployment benefit scheme, while workers are more reluctant to move from one sector to another (because $\chi$ falls), the vacancy creation is more elastic to technological change. Since the latter effect predominates, the labor mobility rises.} Because we find numerically that raising $\varrho$ or $\alpha_W$ merely modifies the relative wage response to a productivity differential between tradables and non tradables through the labor market frictions channel, we restrict our attention to the labor accumulation channel in Figure 15(b).

In this section, we investigate the effects of higher productivity in tradables relative to non tradables on the unemployment rate differential. To alleviate the notation, we drop the superscript $\hat{x}$ to denote steady-state values since we focus on steady-state changes.

To write analytical expression in a compact form, it is useful to set:

$$\Sigma^j = \frac{\Xi^j}{(1 - \alpha_V) \Psi^j + \chi^j W^j_R}. \quad (255)$$

which implies (see eq (184) for the traded sector and eq. (192) for the non traded sector):

$$\hat{\theta}^j = \Sigma^j \Xi^j. \quad (256)$$
Differentiating the definition of the steady-state level for the sectoral unemployment rate described by:

\[ u^j = \frac{s^j}{s^j + m^j (\theta^j)} \quad (257) \]

one obtains the standard negative relationship between \( u^j \) and the labor market tightness in sector \( j \):

\[ \dot{u}^j = -\alpha_V \frac{m^j}{s^j + m^j} \hat{\theta}^j. \quad (258) \]

Using the fact that \( \hat{\xi}^T = \hat{a}^T \) and \( \hat{\xi}^N = \hat{p} + \hat{a}^N \), subtracting \( \hat{a}^N \) from \( \hat{a}^T \) by using (256) and (258), one obtains:

\begin{align*}
\hat{a}^T - \hat{a}^N &= -\alpha_V \left\{ \frac{m^T}{s^T + m^T} \Sigma^T \hat{a}^T - \frac{m^N}{s^N + m^N} \Sigma^N \left( \hat{p} + \hat{a}^N \right) \right\}, \\
&= -\alpha_V \left\{ \left[ \frac{m^T}{s^T + m^T} \Sigma^T - \frac{m^N}{s^N + m^N} \Sigma^N \left( \frac{1 + \Theta^T}{\phi + \Theta^N} \right) \right] \hat{a}^T - \frac{m^N}{s^N + m^N} \Sigma^N \left( \frac{\phi - 1}{\phi + \Theta^N} \right) \hat{a}^N \right\} + \alpha_V m^N \frac{\Sigma^N \ln (1 - u_{NX})}{(\phi + \Theta^N)}. \quad (259)
\end{align*}

where we have inserted the decomposition of the steady-state change of the relative price of non tradables given by eq. (246) to determine the percentage change in the labor market tightness in the non traded sector:

\[ \hat{\theta}^N = \Sigma^N \left( \hat{p} + \hat{a}^N \right), \]

\[ \Sigma^N \left( \frac{1 + \Theta^T}{\phi + \Theta^N} \right) \hat{a}^T + \Sigma^N \left[ 1 - \frac{1 + \Theta^N}{\phi + \Theta^N} \right] \hat{a}^N + \Sigma^N \frac{\ln (1 - u_{NX})}{(\phi + \Theta^N)}. \]

Using the fact that at the steady-state, \( \frac{m^j}{s^j + m^j} = (1 - u^j) \), eq. (259) can be rewritten as follows:

\begin{align*}
\hat{a}^T - \hat{a}^N &= -\alpha_V \left\{ \left[ (1 - u^T) \Sigma^T - (1 - u^N) \Sigma^N \left( \frac{1 + \Theta^T}{\phi + \Theta^N} \right) \right] \hat{a}^T - (1 - u^N) \Sigma^N \left( \frac{\phi - 1}{\phi + \Theta^N} \right) \hat{a}^N \right\} + \alpha_V (1 - u^N) \frac{\Sigma^N \ln (1 - u_{NX})}{(\phi + \Theta^N)}. \quad (261)
\end{align*}

To facilitate the discussion of the effect of a productivity differential on the unemployment rate in the traded relative to the non traded sector, we assume that at the initial steady-state, we have \( \Theta^j \simeq \Theta \), \( u^j \simeq u \), \( \Sigma^j \simeq \Sigma \), and we multiply both sides of eq. (261) by \( u \) in order to express the unemployment differential in percentage point so that eq. (261) reduces to:

\[ du^T - du^N = -\alpha_V u (1 - u) \Sigma \left[ \left( \frac{\phi - 1}{\phi + \Theta} \right) (\hat{a}^T - \hat{a}^N) - \frac{\ln (1 - u_{NX})}{(\phi + \Theta)} \right]. \quad (262) \]

Eq. (262) corresponds to equation (33) in the main text. Eq. (262) breaks down the response of the unemployment differential to a productivity differential into two components: a labor market frictions effect and a labor accumulation effect. The first term on the RHS of (262) corresponds to the labor market frictions effect. Through this channel, higher productivity gains in tradables relative to non tradables lower or increase the unemployment rate in the traded sector relative to the non traded sector depending on whether the elasticity of substitution between tradables and non tradables \( \phi \) is smaller or higher than one. If \( \phi < 1 \), as our evidence suggest, a productivity differential between tradables and non tradables appreciates the relative price of non tradables more than proportionately. Because the share of non tradables increases, non traded firms recruit more which result in a larger decline in \( u^N \) relative to \( u^T \). The second term on the RHS corresponds to the labor accumulation effect. Through this channel, the long-run increase in net exports.
raises the demand for tradables and thus encourages firms to recruit more. When \( \phi < 1 \),
the labor market frictions effect and the labor accumulation effect have conflicting effects
on the unemployment differential between tradables and non tradables. If the labor accumu-
lation effect predominates, a productivity differential lowers the unemployment rate in
the traded sector by a larger amount than that in the non traded sector. When \( \phi > 1 \),
higher productivity in tradables relative to non tradables unambiguously drives down the
unemployment differential between tradables and non tradables.

\[ \textbf{J Correction of the Bias to map Theoretical results into Elasticities Estimated Empirically} \]

In this section, we compute the bias originating from search frictions varying across sectors
which must be accounted for in order to map theoretical results for the responses to a
productivity differential into elasticities estimated empirically.

The long-run change of the relative price (25) can be rewritten as follows:

\[
\hat{p} = \frac{(1 + \Theta^T) \hat{a}^T - (1 + \Theta^N) \hat{a}^N}{\phi + \Theta^N} + \frac{d \ln (1 - u_{NX})}{\phi + \Theta^N},
\]

\[
= \left( \frac{1 + \Theta^T}{\phi + \Theta^N} \right) \left\{ (\hat{a}^T - \hat{a}^N) + \hat{a}^N \left[ 1 - \frac{(1 + \Theta^N)}{1 + \Theta^T} \right] \right\} + \frac{d \ln (1 - u_{NX})}{\phi + \Theta^N}. \tag{263}
\]

Because empirically we consider a productivity differential \( \hat{a}^T - \hat{a}^N \), to make our estimates
comparable with our numerical results, we have to adjust the long-run change in the relative
price computed numerically with the following term:

\[
\text{bias} \hat{p} = \left( \frac{1 + \Theta^T}{\phi + \Theta^N} \right) \left[ 1 - \frac{(1 + \Theta^N)}{1 + \Theta^T} \right] \hat{a}^N. \tag{264}
\]

Subtracting (264) from (263) leads to:

\[
\hat{p}' = \hat{p} - \text{bias} \hat{p}, \tag{265}
\]

\[
= \left( \frac{1 + \Theta^T}{\phi + \Theta^N} \right) (\hat{a}^T - \hat{a}^N) + \frac{d \ln (1 - u_{NX})}{\phi + \Theta^N}, \tag{266}
\]

where we denote by \( \hat{p}' \) the value of \( \hat{p} \) which has been adjusted with the bias originating from
the presence of search frictions which vary across sectors and thus make the elasticity \( \Theta^j \) of
sectoral employment \( L^j \) w.r.t. the marginal revenue of labor, \( \Xi^j \), slightly different between
sectors. Once the value of \( \hat{p} \) has been adjusted with, we can map the deviation in percentage
of the relative price of non tradables from its initial steady-state derived analytically into
the elasticity of the relative price, \( \gamma \), estimated empirically:

\[
\gamma = \frac{\hat{p}'}{\hat{a}^T - \hat{a}^N},
\]

\[
= \left( \frac{1 + \Theta^T}{\phi + \Theta^N} \right) + \frac{1}{\phi + \Theta^N} \frac{d \ln (1 - u_{NX})}{\hat{a}^T - \hat{a}^N}. \tag{267}
\]

**Eq. (267) corresponds to eq. (46a).** The first term on the RHS of eq. (267) corresponds
to the effect of a productivity differential \( \hat{a}^T - \hat{a}^N \) of 1% on the relative price keeping net
exports fixed while the second term captures the impact of the long-run adjustment in net
exports caused by rise in productivity of tradables relative to non tradables of 1%.

The same logic applies to the relative wage. The long-run reaction of the relative wage
described by (179j) can be rewritten as follows:

\[
\hat{\omega} = - \left\{ \left[ \Omega^T - \Omega^N \left( \frac{1 + \Theta^T}{\phi + \Theta^N} \right) \right] \hat{a}^T - \left[ \Omega^N - \Omega^N \left( \frac{1 + \Theta^N}{\phi + \Theta^N} \right) \right] \hat{a}^N \right\} + \Omega^N \frac{d \ln (1 - u_{NX})}{\phi + \Theta^N},
\]

\[
= - \left[ \Omega^T - \Omega^N \left( \frac{1 + \Theta^T}{\phi + \Theta^N} \right) \right] \left\{ (\hat{a}^T - \hat{a}^N) + \left\{ 1 - \frac{\Omega^N - \Omega^N \left( \frac{1 + \Theta^N}{\phi + \Theta^N} \right)}{\Omega^T - \Omega^N \left( \frac{1 + \Theta^T}{\phi + \Theta^N} \right)} \right\} \hat{a}^N \right\}
\]

\[
+ \Omega^N \frac{d \ln (1 - u_{NX})}{\phi + \Theta^N}. \tag{268}
\]
We have to adjust the long-run change in the relative wage computed numerically with the following term:

$$\text{bias } \hat{\omega} = - \left[ \Omega^T - \Omega^N \left( \frac{1 + \Theta^T}{\phi + \Theta^N} \right) \right] \left\{ 1 - \frac{\Omega^N - \Omega^N \left( \frac{1 + \Theta^N}{\phi + \Theta^N} \right)}{\Omega^T - \Omega^N \left( \frac{1 + \Theta^N}{\phi + \Theta^N} \right)} \right\} \hat{a}^N. \quad (269)$$

Subtracting (269) from (268) leads to:

$$\hat{\omega}' = \hat{\omega} - \text{bias } \hat{\omega}, \quad (270)$$

$$= - \left[ \Omega^T - \Omega^N \left( \frac{1 + \Theta^T}{\phi + \Theta^N} \right) \right] (\hat{a}^T - \hat{a}^N) + \frac{\Omega^N \frac{d \ln (1 - v_{NX})}{\hat{a}^T - \hat{a}^N}}{\phi + \Theta^N}, \quad (271)$$

where we denote by $\hat{\omega}'$ the value of $\hat{\omega}$ which has been adjusted with the bias originating from the presence of search frictions which vary across sectors and thus make $\Theta^j$ along with $\Omega^j$ slightly different between sectors. Once the value of $\hat{\omega}$ has been adjusted with, we can map the deviation in percentage of the relative wage from its initial steady-state derived analytically into the elasticity of the relative wage, $\beta$, estimated empirically:

$$\beta = \frac{\hat{\omega}'}{\hat{a}^T - \hat{a}^N},$$

$$\beta = - \left[ \Omega^T - \Omega^N \left( \frac{1 + \Theta^T}{\phi + \Theta^N} \right) \right] + \frac{\Omega^N \frac{d \ln (1 - v_{NX})}{\hat{a}^T - \hat{a}^N}}{\phi + \Theta^N}. \quad (272)$$

Eq. (272) corresponds to eq. (46b). The first term on the RHS of eq. (272) corresponds to the effect of a productivity differential $\hat{a}^T - \hat{a}^N$ of 1% on the relative wage keeping net exports fixed while the second term captures the impact of the long-run adjustment in net exports caused by rise in productivity of tradables relative to non tradables of 1%. It is worthwhile mentioning that the rise in net exports exerts a negative impact on both $\beta'$ and $\hat{\omega}'$ and thus the term $\frac{d \ln (1 - v_{NX})}{\hat{a}^T - \hat{a}^N}$ which shows up in eqs. (267) and (272) is negative. The numerical computation of the unemployment rate differential is subject to the same bias the relative price and the relative wage. The long-run reaction of the unemployment differential between tradables and non tradables described by (261) can be rewritten as follows:

$$du^T - du^N = -\alpha_V \left\{ u^T (1 - u^T) \Sigma^T - u^N (1 - u^N) \Sigma^N \left( \frac{1 + \Theta^T}{\phi + \Theta^N} \right) \right\} \hat{a}^T$$

$$- u^N (1 - u^N) \Sigma^N \left( \frac{\phi - 1}{\phi + \Theta^N} \right) \hat{a}^N + \alpha_V u^N (1 - u^N) \Sigma^N \frac{d \ln (1 - v_{NX})}{\phi + \Theta^N},$$

$$= -\alpha_V \Delta^T \hat{a}^T - \hat{a}^N + \hat{a}^N \left[ 1 - \frac{u^N (1 - u^N) \Sigma^N \left( \frac{\phi - 1}{\Delta^T} \right)}{\phi + \Theta^N} \right]$$

$$+ \alpha_V u^N (1 - u^N) \Sigma^N \frac{d \ln (1 - v_{NX})}{\phi + \Theta^N}, \quad (273)$$

where we set

$$\Delta^T = \left[ u^T (1 - u^T) \Sigma^T - u^N (1 - u^N) \Sigma^N \left( \frac{1 + \Theta^T}{\phi + \Theta^N} \right) \right]. \quad (274)$$

We have to adjust the long-run change in the relative wage computed numerically with the following term:

$$\text{bias } (du^T - du^N) = -\alpha_V \Delta^T \hat{a}^N \left[ 1 - \frac{u^N (1 - u^N) \Sigma^N \left( \frac{\phi - 1}{\Delta^T} \right)}{\phi + \Theta^N} \right]. \quad (275)$$

Subtracting (275) from (273) leads to:

$$(du^T - du^N)' = (du^T - du^N) - \text{bias } (du^T - du^N), \quad (276)$$

$$= -\alpha_V \Delta^T (\hat{a}^T - \hat{a}^N) + \alpha_V u^N (1 - u^N) \Sigma^N \frac{d \ln (1 - v_{NX})}{\phi + \Theta^N}, \quad (277)$$

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where we denote by \((du^T - du^N)\)' the value of \(du^T - du^N\) which has been adjusted with the bias originating from the presence of search frictions which vary across sectors and thus make \(\Theta^j\) along with \(\Sigma^j\) slightly different between sectors. Once the value of \(du^T - du^N\) has been adjusted with, we can map the unemployment rate differential derived analytically into its response, \(\sigma\), estimated empirically:

\[
\sigma = \frac{(du^T - du^N)'}{\bar{a}^T - \bar{a}^N} = -\alpha_V \Delta^T + \alpha_V u^N \left(1 - u^N\right) \frac{\Sigma^N}{\phi + \Theta^N} \frac{d \ln (1 - u^N)}{dT - \bar{a}^N}. \tag{278}
\]

Eq. (278) corresponds to eq. (47). Eq. (278) is used to compute numerically the response of the unemployment rate differential to higher relative productivity of tradables by 1%, as reported in Table 3. When we abstract from labor mobility costs and let \(\sigma_L\) tend toward infinity, the unemployment rate differential reduces to eq. (326). In this case, changes in \(u^T\) relative to \(u^N\) are only driven by differences in search frictions between sectors.

K The Role of Endogenous Sectoral Labor Force Participation Decision

In this section, we look at a special case of the model for which the sectoral labor force is inelastic, i.e., \(\sigma_L = 0\) (reflecting the situation of labor immobility across sectors), in order to highlight the role of an endogenous sectoral labor force participation decision in driving the long-run effects of a productivity differential between tradables and non tradables. Then, we analyze the implications of \(\sigma_L \to \infty\) (reflecting the situation of perfect mobility of labor across sectors).

K.1 Equilibrium Dynamics when \(\sigma_L = 0\)

To begin with, we determine the dynamic system. Denoting by \(\bar{W}_R^j\) the reservation wage in sector \(j\), the first-order conditions for the traded and the non traded sector described by eqs. (109b)-(109c) respectively, implies that \(F^j \equiv L^j + U^j = (\bar{L} \bar{W}_R^j/\xi^j)^{\sigma_L}\) with \(W^j_R \equiv R^j + m^j (\theta^j) \xi^j\). Using the fact that \(U^j = (\bar{L} \bar{W}_R^j/\xi^j)^{\sigma_L} - L^j\), the dynamic equation for employment (7) can be rewritten as follows:

\[
\dot{L}^j = m^j (\theta^j) \left(\bar{L} \bar{W}_R^j/\xi^j\right)^{\sigma_L} - \left[s^j + m^j (\theta^j)\right] L^j. \tag{279}
\]

Assuming that labor force is fixed, i.e., setting \(\sigma_L = 0\), then the equation above reads as:

\[
\dot{L}^j = m^j (\theta^j) - \left[s^j + m^j (\theta^j)\right] L^j. \tag{279}
\]

Imposing \(\alpha_W = \alpha_W^j = \alpha_W^j\) and using the fact that \(m^j (\theta^j) \xi^j = \frac{\alpha_W}{1 - \alpha_W} \kappa^j \theta^j\) together with \(\frac{\bar{V}_R^j}{\lambda} = W^j_R\) and \(W^j_R \equiv R^j + m^j (\theta^j) \xi^j\), the Nash bargaining wage can be rewritten as follows:

\[
W^j = \alpha_W (\bar{Z}^j + r^j x^j) - (1 - \alpha_W) \frac{\bar{V}_R^j}{\lambda},
\]

\[
= \alpha_W (\bar{Z}^j + r^j x^j + \kappa^j \theta^j) + (1 - \alpha_W) R^j. \tag{280}
\]

We now determine the dynamic equation for the labor market tightness. Plugging (280) into (140) yields:

\[
\dot{\theta}^j(t) = \frac{\theta^j(t)}{(1 - \alpha_V^j)} \left\{ (s^j + r^j) - \frac{f^j (\theta^j(t))}{\kappa^j} \left[ (\bar{Z}^j + r^j x^j) - W^j \right] \right\},
\]

\[
= \frac{\theta^j(t)}{(1 - \alpha_V^j)} \left\{ (s^j + r^j) - \frac{f^j (\theta^j(t))}{\kappa^j} (1 - \alpha_W) \Psi^j \right\}, \tag{281}
\]

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where the overall surplus from an additional job $\Psi^j$ is:

$$\Psi^j = \Xi^j + r^j x^j - \frac{\alpha_W}{1 - \alpha_W} \kappa^j \theta^j - R^j,$$  \hspace{1cm} (282)

with $\Xi^T = A^T$ and $\Xi^N = PA^N$.

**Traded Sector**

Linearizing the accumulation equation for labor (279) and the dynamic equation for labor market tightness (281) in the traded sector, we get in matrix form:

$$
\begin{pmatrix}
\hat{L}^T, \hat{\theta}^T
\end{pmatrix}^T = J^T \begin{pmatrix}
L^T(t) - \bar{L}^T, \theta^T(t) - \bar{\theta}^T
\end{pmatrix}^T
$$

(283)

where $J^T$ is given by

$$
J^T = \begin{pmatrix}
-(s^T + \tilde{m}^T) & (\tilde{m}^T)' \left(1 - \bar{L}^T\right) \\
0 & \left([s^T + r^*] + \tilde{m}^T \frac{\alpha_W}{1 - \alpha_V}\right)
\end{pmatrix},
$$

(284)

with $\tilde{m}^T = m^T \left(\hat{\theta}\right)$.

The trace denoted by $\text{Tr}$ of the linearized $2 \times 2$ matrix (283) is given by:

$$
\text{Tr}.J^T = r^* + \frac{\tilde{m}^T}{1 - \alpha_V} \left[\alpha_W - (1 - \alpha_V)\right].
$$

(285)

The determinant denoted by $\text{Det}$ of the linearized $2 \times 2$ matrix (144) is unambiguously negative:

$$
\text{Det}.J^T = - (s^T + \tilde{m}^T) \left([s^T + r^*] + \frac{\alpha_W}{1 - \alpha_V} \tilde{m}^T\right) < 0.
$$

(286)

From now on, for clarity purpose, we impose the Hosios condition in order to avoid unnecessary complications:

$$
\alpha_W = (1 - \alpha_V).
$$

(287)

Denoting by $\nu^T$ the eigenvalue, the characteristic equation for the matrix $J$ (284) of the linearized system writes as follows:

$$
\left(\nu^T_i\right)^2 - r^* \nu^T_i + \text{Det}.J^T = 0.
$$

(288)

The characteristic roots obtained from the characteristic polynomial of degree two can be written as follows:

$$
\nu^T_i \equiv \frac{1}{2} \left\{r^* \pm \sqrt{(r^*)^2 - 4 \text{Det}.J^T}\right\} \geq 0, \quad i = 1, 2,
$$

$$
\equiv \frac{1}{2} \left\{r^* \pm \sqrt{(r^*)^2 + 4 \left(s^T + \tilde{m}^T\right)^2 + 4r^* \left(s^T + \tilde{m}^T\right)}\right\},
$$

$$
= \frac{1}{2} \left\{r^* \pm \left[r^* + 2 \left(s^T + \tilde{m}^T\right)\right]\right\},
$$

(289)

where we used the fact that $\text{Det}.J^T = - (s^T + \tilde{m}^T) \left(s^T + r^* + \tilde{m}^T\right)$.

We denote by $\nu^T_1 < 0$ and $\nu^T_2 > 0$ the stable and unstable eigenvalues respectively which satisfy:

$$
\nu^T_1 = - (s^T + \tilde{m}^T) < 0 < r^* < \nu^T_2 = \left(s^T + r^* + \tilde{m}^T\right).
$$

(290)

**Non Traded Sector**

Linearizing the accumulation equation for non traded labor (279) by setting $j = N$ and the dynamic equation for labor market tightness (281) in the non traded sector by inserting first the solution for the relative price of non tradables (134), i.e., $P = P \left(L^N, \bar{\lambda}, A^N\right)$, we get in matrix form:

$$
\begin{pmatrix}
\hat{L}^N, \hat{\theta}^N
\end{pmatrix}^T = J^N \begin{pmatrix}
L^N(t) - \bar{L}^N, \theta^N(t) - \bar{\theta}^N
\end{pmatrix}^T
$$

(291)
where $J^N$ is given by
\[
J^N = \begin{pmatrix}
-(s^N + \bar{m}^N) & (m^N)'(1 - \bar{L}^N) \\
-1 - \alpha_W \bar{m}^N \tilde{P}_{LN} A^N & \left((s^N + r^*) + \bar{m}^N \alpha_w \frac{1}{1 - \alpha_V}\right)
\end{pmatrix},
\] (292)
with $P_{LN} = \frac{\partial P}{\partial LN} = \frac{A^N}{C^P} < 0$.

The trace is:
\[
\text{Tr}J^N = r^* + \frac{\bar{m}^N}{1 - \alpha_V} [\alpha_W - (1 - \alpha_V)].
\] (293)

The determinant denoted by Det of the linearized $2 \times 2$ matrix (292) is unambiguously negative:
\[
\text{Det}J^N = -(s^N + \bar{m}^N) \left[(s^N + r^*) + \frac{\alpha_W \bar{m}^N}{1 - \alpha_V} \right] + \frac{1 - \alpha_W \bar{m}^N}{1 - \alpha_V} \tilde{P}_{LN} A^N \left((m^N)'(1 - \bar{L}^N) \right) < 0.
\] (294)

Assuming that the Hosios condition (287) holds, the determinant (294) can be rewritten as follows:
\[
\text{Det}J^N = -(s^N + \bar{m}^N) (s^N + r^*) \left[\frac{(s^N + r^*) \bar{m}^N}{s^N + r^*} - \frac{1 - \alpha_W \bar{m}^N \tilde{P}_{LN} A^N m^N}{1 - \alpha_V} \left((s^N + r^*) \right) \left((s^N + \bar{m}^N) \right)\right],
\] (295)
where we computed the following term:
\[
\frac{1 - \alpha_W \bar{m}^N \tilde{P}_{LN} A^N m^N}{1 - \alpha_V \left((s^N + r^*) \right) \left((s^N + \bar{m}^N) \right)},
\]
\[
= \left(1 - \alpha_W\right) \frac{\bar{m}^N m^N \bar{m}^N \tilde{U}^N}{\bar{m}^N \tilde{U}^N} \tilde{P}_{LN} A^N \left((s^N + r^*) \right),
\]
\[
= \alpha_V \frac{\bar{m}^N \tilde{U}^N}{\bar{m}^N \tilde{U}^N} \tilde{P}_{LN} A^N \left((s^N + r^*) \right),
\]
\[
= \left(\frac{\alpha_V}{1 - \alpha_V}\right) \frac{\tilde{U}^N \tilde{P}_{LN} \tilde{L}^N}{\tilde{U}^N \tilde{P}_{LN} \tilde{L}^N} \tilde{P}_{LN} A^N.
\] (296)

To get (296), we used the fact that $\frac{1 - \alpha_W}{\alpha_V} \tilde{L}^N = 1 - \tilde{L}^N = \tilde{U}^N$, $\tilde{m}^N \tilde{U}^N = s^N \tilde{L}^N$, and $\tilde{u}^N = s^N \tilde{L}^N$.

We denote by $\nu_1^N < 0$ and $\nu_2^N > 0$ the stable and unstable eigenvalues respectively which satisfy:
\[
\nu_1^N < 0 < r^* < \nu_2^N.
\] (297)

**K.2 Formal Solutions for $L^T(t)$ and $\theta^T(t)$**

The stable paths for the labor market in the traded sector are given by:
\[
L^T(t) - \tilde{L}^T = D^T_1 e^{r^T t},
\] (298a)
\[
\theta^T(t) - \bar{\theta}^T = \omega^T_2 D^T_1 e^{r^T t},
\] (298b)
where $D^T_1 = L^T_0 - \tilde{L}^T$, and element $\omega^T_2$ of the eigenvector (associated with the stable eigenvalue $\nu^T_1$) is given by:
\[
\omega^T_2 = \frac{(s^T + \bar{m}^T + \nu^T_1)}{m^T \bar{m}^T (1 - \tilde{L}^T)} = 0.
\] (299)

where we used the fact that $\nu^T_1 = - (s^T + \bar{m}^T)$ (see eq. (290)). From (298a), the dynamics for labor market tightness $\theta^T$ degenerate.
K.3 Formal Solutions for $L^N(t)$ and $\theta^N(t)$

The stable paths for the labor market in the non traded sector are given by:

$$L^N(t) - \bar{L}^N = D_1^N e^{\nu_1^N t}, \tag{300a}$$
$$\theta^N(t) - \bar{\theta}^N = \omega_2^N D_1^N e^{\nu_1^N t}, \tag{300b}$$

where $D_1^N = L_0^N - \bar{L}^N$, and element $\omega_2^N$ of the eigenvector (associated with the stable eigenvalue $\nu_1^N$) is given by:

$$\omega_2^N = \frac{(s^N + \bar{m}^N + \nu_1^N)}{m^t N (1 - \bar{L}^N)},$$

where we have inserted (170b) and used the fact that

$$\frac{1 - \alpha_{\nu}}{1 - \alpha_C} \frac{m^N}{m^N} P_{LN} A^N = \frac{1 - \alpha_{\nu}}{1 - \alpha_C} \frac{m^N}{m^N} \bar{m}^N - \nu_1^N < 0. \tag{301}$$

K.4 Formal Solution for the Stock of Foreign Bonds $B(t)$

Substituting first the short-run static solutions for consumption in tradables given by (136), and using the fact that $V^j = U^j \theta^j$, the accumulation equation for traded bonds (138) can be written as follows:

$$B(t) = r^* B(t) + A^T L^T(t) - C^T (L^N(t), \bar{\lambda}, A^N) - \kappa^T \theta^N(t) (1 - L^N(t)) - \kappa^N \theta^N(t) (1 - L^N(t)), \tag{302}$$

where we used the fact that $U^j = 1 - L^j$ when $\sigma_L = 0$.

Linearizing (302) in the neighborhood of the steady-state and inserting stable solutions given by (298) and (300) yields:

$$\dot{B}(t) = r^* (B(t) - \bar{B}) + A^T (L^T(t) - \bar{L}^T) + \Lambda^N (L^N(t) - \bar{L}^N), \tag{303}$$

where we set:

$$\Lambda^T = A^T + \kappa^T \bar{\theta}^T - \kappa^T (1 - \bar{L}^T) \omega_{21}^T > 0, \tag{304a}$$
$$\Lambda^N = -C_{LN}^T - \kappa^N \bar{\lambda}^N \omega_{21}^N - \kappa^N \bar{\theta}^N \omega_{21}^N = -C_{LN}^N + \kappa^N \bar{\theta}^N \left[ 1 - \frac{(s^N + \bar{m}^N + \nu_1^N)}{\alpha_V \bar{m}^N} \right] > 0, \tag{304b}$$

where we have inserted (170b) and used the fact that $(m^N)'/\theta^N/m^N = \alpha_V$ to get (304b); note that $C_{LN}^N \approx 0$ as long as $\phi \approx \sigma_C$ in line with evidence for a typical OECD economy. The sign of (304b) follows from the fact that $\omega_{21}^N < 0$ (see (301)).

Solving the differential equation (303) yields:

$$B(t) = \dot{B} + \left( B_0 - \bar{B} \right) - \frac{\Lambda^T D_1^T}{\nu_1^T - r^*} \left( \frac{A^T + \kappa^T \bar{\theta}^T}{s^T + \bar{m}^T + r^*} \right) \nu_1^T \nu_1^T e^{\nu_1^T t} + \frac{\Lambda^N D_1^N}{\nu_1^N - r^*} e^{\nu_1^N t}. \tag{305}$$

Invoking the transversality condition for intertemporal solvency, and using the fact that $D_1^T = L_0^T - \bar{L}^T$ and $D_1^N = L_0^N - \bar{L}^N$, we obtain the linearized version of the nation’s intertemporal budget constraint:

$$\dot{B} - B_0 = \Phi^T (L^T - L_0^T) + \Phi^N (L^N - L_0^N), \tag{306}$$

where we set

$$\Phi^T = \frac{\Lambda^T}{\nu_1^T - r^*} = - \frac{A^T + \kappa^T \bar{\theta}^T}{s^T + \bar{m}^T + r^*} < 0, \quad \Phi^N = \frac{\Lambda^N}{\nu_1^N - r^*} < 0. \tag{307}$$

Equation (307) can be solved for the stock of foreign bonds:

$$\dot{B} = B \left( L^T - L_0^T \right), \quad B_LT = \Phi^T < 0, \quad B_LN = \Phi^N < 0. \tag{308}$$

For the national intertemporal solvency to hold, the terms in brackets of equation (305) must be zero so that the stable solution for net foreign assets finally reduces to:

$$B(t) - \bar{B} = \Phi^T (L^T(t) - L_0^T) + \Phi^N (L^N(t) - L_0^N). \tag{309}$$
K.5 Solving Graphically for the Steady-State

We investigate graphically the long-run effects of a rise in the the ratio of sectoral productivity. Assuming \( \alpha^j_W = \alpha^j_W \) and setting \( \sigma_L = 0 \), the steady-state (231) reduces to the following system which comprises five equations:

\[
\begin{align*}
\tilde{C}_T & = \frac{\varphi}{1 - \varphi} \tilde{p}^\phi, \\
\tilde{L}_T \tilde{L}_N & = \tilde{m}_T (s^N + \tilde{m}^N) \zeta^N, \\
\kappa_T \left( \hat{\theta}^T \right) & = \frac{(1 - \alpha^T_W) \bar{\Psi}^T}{(s^T + r^*)}, \\
\kappa_N \left( \hat{\theta}^N \right) & = \frac{(1 - \alpha^N_W) \bar{\Psi}^N}{(s^N + r^*)}, \\
\tilde{Y}_T \left( 1 - v_{NX} \right) \tilde{Y}_N & = \tilde{C}_T \tilde{C}_N,
\end{align*}
\]

where \( v_{NX} = v_B - v^T_V - v^N_V \).

Goods Market

Because we restrict ourselves to the analysis of the long-run effects, the tilde is suppressed for the purposes of clarity. To characterize the steady-state, we focus on the goods market which can be summarized graphically by two schedules in the \((y^T - y^N, p)\)-space, where we denote the logarithm of variables with lower-case letters.

The goods market equilibrium (GME)-schedule that we repeat for convenience is identical to (233):

\[
\begin{align*}
\tilde{y}^T \left( \varphi \tilde{p} - d \ln (1 - \varphi \tilde{v}_{NX}) \right) = \Phi.
\end{align*}
\]

The GME-schedule is upward-sloping in the \((y^T - y^N, p)\)-space and the slope of the GME-schedule is equal to \(1/\varphi\).

The labor market equilibrium (LME)-schedule that we repeat for convenience is identical to (241),

\[
\begin{align*}
\hat{y}^T \left( -\Theta^T \tilde{p} + (1 + \Theta^T) \hat{a}^T - (1 + \Theta^N) \hat{a}^N \right),
\end{align*}
\]

except for the elasticity \( \Theta^j \) of employment to the marginal revenue of labor which reduces to:

\[
\begin{align*}
\Theta^T & \equiv \frac{A^T \alpha^T_U u^T}{\left(1 - \alpha^T_v\right) \Psi^T + \bar{\chi}^T W^P} > 0, \\
\Theta^N & \equiv \frac{P A^N \alpha^N_U u^N}{\left(1 - \alpha^N_v\right) \Psi^N + \bar{\chi}^N W^P} > 0.
\end{align*}
\]

The LME-schedule is downward-sloping in the \((y^T - y^N, p)\)-space and the slope of the LME-schedule is equal to \(-1/\Theta^T\). When \( \sigma_L = 0 \), \( \Theta^j \) is smaller so that the LME-schedule is steeper.

Labor Market

Imposing \( \sigma_L = 0 \) into eq. (231b), the decision of search (DS)-schedule reduces to:

\[
\begin{align*}
\tilde{L}_T \tilde{L}_N & = \tilde{m}_T m^N + s^N \zeta^N, \\
\tilde{Y}_T \left( 1 - \nu_{NX} \right) \tilde{Y}_N & = \tilde{C}_T \tilde{C}_N
\end{align*}
\]

Taking logarithm and differentiating eq. (314) yields:

\[
\begin{align*}
\tilde{l}^T - \tilde{l}^N & = \alpha^T u^T \hat{\theta}^T - \alpha^N u^N \hat{\theta}^N.
\end{align*}
\]
Assuming that the labor markets display similar features across sectors, i.e., \( w^j \simeq u \), eq. (315) reduces to:

\[
\left. \left( \tilde{\theta}^T - \tilde{\theta}^N \right) \right| = \left. \frac{1}{\alpha V} \left( \tilde{i}^T - \tilde{i}^N \right) \right|_{\sigma_L = 0} = \left( T^N - i^N \right).
\]

The DS-schedule is upward-sloping in the \((T^N - i^N, \ln(\tilde{\theta}^T / \tilde{\theta}^N))\)-space. Comparing (316) with (241), it is straightforward to show that the DS-schedule becomes steeper when \( \sigma_L = 0 \). The VC-schedule is downward-sloping and identical to (245).

**K.6 Effects of Higher Relative Productivity of Tradable**

When \( \sigma_L = 0 \)

Equating demand for tradables in terms of non tradables given by eq. (311) and supply (312) yields the deviation in percentage of the relative price from its initial steady-state (246). When assuming \( \Theta^j \simeq \Theta' \), eq. (246) reduces to:

\[
\hat{p} = \frac{(1 + \Theta') \left( \hat{a}^T - \hat{a}^N \right)}{\left( \phi + \Theta' \right)} + \frac{\ln (1 - \nu_{NX})}{\left( \phi + \Theta' \right)},
\]

where

\[
\Theta' \equiv \frac{\Xi \alpha V u}{\left(1 - \alpha V \right) + \chi W_R} < \Theta \equiv \frac{\Xi \left[ \alpha V u + \sigma L \chi \right]}{\left(1 - \alpha V \right) + \chi W_R},
\]

with \( \Theta \) given by (235). Assuming \( \sigma_L = 0 \) lowers the elasticity \( \Theta \) of sectoral employment w.r.t. marginal revenue of labor. Intuitively, increased productivity induce firms to post more job vacancies which raises the labor market tightness and thus the probability of finding a job. When \( \sigma_L > 0 \), higher \( \theta^j \) increases \( L^j \) through two channels: i) by triggering an outflow from unemployment, and ii) by inducing agents to increase the search intensity for a job. Because the latter effect vanishes if \( \sigma_L = 0 \), employment becomes less responsive to productivity gains, as captured by a lower \( \Theta \), i.e., \( \Theta' < \Theta \) (see inequality (320)). Since \( \Theta' < \Theta \), comparing eq. (317) with eq. (27) shows that when setting \( \sigma_L = 0 \), the labor market friction effect captured by the first term on the RHS of eq. (317) is moderated or amplified depending on whether \( \phi \) is larger or smaller than one. In the former case, traded output increases less so that the relative price of non tradables must appreciate by a smaller amount to clear the goods market. If \( \phi < 1 \), a productivity differential between tradables and non tradables raises the share of non tradables and thus has an expansionary effect on labor demand in the non traded sector. When \( \sigma_L = 0 \), as detailed below, firms must increase wages by a larger amount. To compensate for the higher unit labor cost, non traded firms set higher prices so that \( p \) increases more. Irrespective of whether \( \phi \) is larger or smaller than one, a productivity differential between tradables and non tradables exerts a larger negative impact on \( p \) when \( \sigma_L = 0 \) through the labor accumulation effect. The reason is that following higher net exports, because the reallocation of labor across sectors is absent, traded output increases less which in turn triggers a greater excess of demand for tradables, thus leading to a larger depreciation in the relative price of non tradables (i.e., a larger decline in \( p \)).

Equating labor supply (316) with labor demand (245) while assuming \( \Theta^j \simeq \Theta \) and \( \Omega \simeq \Omega \) leads to the deviation in percentage of the relative wage from its initial steady-state:

\[
\hat{\omega} = -\frac{\Omega}{\phi + \Theta} \left[ (\phi - 1) \left( \hat{a}^T - \hat{a}^N \right) + \nu_{NX} \right].
\]

Eq. (319) shows that assuming a fixed labor force by setting \( \sigma_L = 0 \) amplifies both the labor market frictions effect (captured by the first term on the RHS of eq. (319)) and the labor market accumulation effect (captured by the second term on the RHS of eq. (319)). Intuitively, higher productivity shifts the VC-schedule along a steeper DS-schedule, thus resulting in larger changes in the ratio \( \theta^T / \theta^N \) and in the relative wage \( \hat{\omega} \). As discussed in section 5.2, across all scenarios, even if the labor market frictions effect raises the relative wage (when setting \( \phi < 1 \)), the labor market accumulation effect predominates. Setting \( \sigma_L = 0 \) amplifies the negative impact of the labor accumulation effect on the relative wage by such an amount that the model cannot account quantitatively for the size of decline in the relative wage (i.e., tends to overstate the decline in \( \omega \)) found in the data.
K.7 Effects of Higher Relative Productivity of Tradables When $\sigma_L \to \infty$

In this subsection, we investigate the relative price and relative wage effects of higher productivity of tradables relative to non tradables when we let $\sigma_L$ tend toward infinity. In this configuration, the case of perfect mobility of labor emerges.

As mentioned in section F, the steady-state can be characterized graphically by considering alternatively the goods market or the labor market. When we let $\sigma_L$ tend toward infinity, eq. (235) implies that $\Theta^j$, which captures the elasticity of sectoral employment w.r.t. the marginal revenue product of labor, tends toward infinity. Inspection of (233) and (234) indicates that when $\sigma_L \to \infty$, the slope of the $GME$-schedule (equal to $1/\phi$) is unaffected while the $LME$-schedule (whose slope is equal to $1/\Theta^N$) becomes a horizontal line. Letting $\sigma_L$ tend toward infinity into (246) and applying l'Hôpital’s rule leads to the steady-state change in the relative wage driven by standard search frictions alone:

$$
\lim_{\sigma_L \to \infty} \hat{p} = \lim_{\sigma_L \to \infty} \frac{1 + \Theta^T}{\phi + \Theta^N \hat{a}^T - 1 + \Theta^N \phi} = \frac{\chi^T \Sigma^T}{\chi^N \Sigma^N} \hat{a}^T - \hat{a}^N. \tag{320}
$$

Applying l'Hôpital’s rule to the relative price effect once the bias has been controlled for as described by eq. (266) leads to:

$$
\lim_{\sigma_L \to \infty} \hat{p}' = \lim_{\sigma_L \to \infty} \frac{1 + \Theta^T}{\phi + \Theta^N} (\hat{a}^T - \hat{a}^N),
\quad = \frac{\chi^T \Sigma^T}{\chi^N \Sigma^N} (\hat{a}^T - \hat{a}^N). \tag{321}
$$

where we used the fact that $\lim_{\sigma_L \to \infty} \Theta^N = \infty$, $\frac{\partial \hat{a}^j}{\partial \sigma_T} = \Sigma^j \chi^j$.

According to our quantitative analysis, while labor market parameters are allowed to vary across sectors, the term in front of $\hat{a}^T$ is close to one for the baseline calibration. As a result, a 1 percentage point increase in the productivity differential between tradables and non tradables appreciates the relative price of non tradables by 1% approximately. Assuming that $\Theta^j \approx \Theta$ and applying l’Hôpital’s rule, the rate of change of the relative price described by eq. (27) reduces to:

$$
\lim_{\sigma_L \to \infty} \hat{p} = \hat{a}^T - \hat{a}^N. \tag{322}
$$

Consequently, a model with labor market frictions reaches the same conclusion as the standard neoclassical model with a competitive labor market as long as the elasticity of labor supply at the extensive margin tends toward infinity.

Inspection of (241) and (245) indicates that when $\sigma_L \to \infty$, the $DS$-schedule (whose slope is equal to $-\frac{1}{\alpha^L + \sigma_L}$) becomes a horizontal line while the $V'C$-schedule (whose slope is equal to $-\frac{\hat{a}^T}{\alpha(1-\alpha^L)\hat{a}^T + \chi^N}$) is unaffected. Letting $\sigma_L$ tend toward infinity into (252) and applying l’Hôpital’s rule leads to the steady-state change in the relative wage driven by standard search frictions alone:

$$
\lim_{\sigma_L \to \infty} \hat{\omega} = \lim_{\sigma_L \to \infty} -\Omega^T \hat{a}^T + \Omega^N \hat{a}^N + \Omega^N \left[ \frac{1 + \Theta^T}{\phi + \Theta^N \hat{a}^T - 1 + \Theta^N \phi} \right],
\quad = -\left[ \Omega^T - \Omega^N \chi^T \Sigma^T \right] \hat{a}^T. \tag{332}
$$

Applying l’Hôpital’s rule to the relative wage effect once the bias has been controlled for as described by eq. (271) leads to:

$$
\lim_{\sigma_L \to \infty} \hat{\omega}' = \lim_{\sigma_L \to \infty} -\left[ \Omega^T - \Omega^N \frac{1 + \Theta^T}{1 + \Theta^N} \right] (\hat{a}^T - \hat{a}^N),
\quad = -\left[ \Omega^T - \Omega^N \chi^T \Sigma^T \right] \hat{a}^T - \hat{a}^N. \tag{324}
$$
Assuming that $\Theta^j \simeq \Theta$ and applying l'Hôpital’s rule, the rate of change of the relative wage described by eq. (30) reduces to:

$$\lim_{\sigma_L \to \infty} \dot{\omega} = (\Omega^N - \Omega^T) \dot{a}^T,$$

(325)

where $\Omega^j$ captures the elasticity of the sectoral wage w.r.t the marginal revenue of labor; according to (325), the effect of higher productivity in tradables relative to non tradables on the relative wage is proportional to $\Omega^N - \Omega^T$. More precisely, when we let $\sigma_L \to \infty$, while the ratio of labor market tightness remains unaffected if $\Theta^j \simeq \Theta$, technological change biased toward the traded sector may influence the relative wage as long as the elasticity of sectoral wage w.r.t. the marginal revenue of labor $\Omega^j$ varies across sectors. For our benchmark parametrization, we have $\Omega^j \simeq \Omega$ so that the relative wage is (almost) unaffected by a productivity differential.

Applying l’Hôpital’s rule to the unemployment differential once the bias has been controlled for as described by eq. (277) leads to:

$$\lim_{\sigma_L \to \infty} (du^T - du^N)' = \lim_{\sigma_L \to \infty} -\alpha_V \left[ u^T (1 - u^T) \Sigma^T - u^N (1 - u^N) \Sigma^N \left( \frac{1 + \Theta^T}{\phi + \Theta^N} \right) \right] (\dot{a}^T - \dot{a}^N),$$

$$= -\alpha_V \Sigma^T \left[ u^T (1 - u^T) - u^N (1 - u^N) \frac{\chi^T}{\chi^N} \right] (\dot{a}^T - \dot{a}^N).$$

(326)

When search frictions do not differ across sectors, then $\lim_{\sigma_L \to \infty} (du^T - du^N)' = 0$.

In conclusion, a model with search frictions reaches the same conclusions as the standard neoclassical model with a competitive labor market as long as the elasticity of labor supply at the extensive margin tends toward infinity.

### L Non-Traded and Traded Hiring Costs

In the main text, both traded and non-traded firms pay a cost per job vacancy expressed in terms of the traded good. In this section, we relax this assumption and consider that recruiting costs paid by non-traded firms are expressed in terms of the non-traded good and hiring costs paid by traded firms are expressed in terms of the traded good. We emphasize below the main changes regarding the baseline model and we detail the steps to solve the model.

#### L.1 Main Changes to the Setup

The profit function for traded firms is identical to (117). Each sector consists of a large number of identical firms which use labor, $L^j$, as the sole input in a linear technology, $Y^j = A^j L^j$. Firms post job vacancies $V^j$ to hire workers and face a cost per job vacancy $\kappa^j$ which is assumed to be constant. Denoting by $P$ the price of non traded goods in terms of traded goods, the profit function of the representative firm in the non traded sector is:

$$\pi^N(t) = P(t) A^N L^N(t) - W^N(t)L^N(t) - P(t) \kappa^N V^N(t) - x^N \max \left\{ 0, -\dot{L}^N(t) \right\},$$

(327)

where we assume that the cost per job vacancy is measured in terms of the non-traded good.

First-order conditions for the traded sector are unchanged while for the non-traded sector, they can be rewritten as follows:

$$\gamma^N(t) + x^N = \frac{P(t) \kappa^N}{\bar{f}^N(\bar{\theta}^N(t))},$$

(328a)

$$\dot{\gamma}^N(t) = \gamma^N(t) (r^* + s^N) - (\Xi^N(t) - x^N s^N - W^N(t)), $$

(328b)

where $\gamma^N$ represents the pecuniary value of an additional job to the representative firm of sector $N$. Differentiating (328a) w.r.t. time leads to:

$$\frac{\dot{\bar{\theta}}^N(t)}{\bar{\theta}^N(t)} = \left[ \frac{\dot{\gamma}^N(t) + x^N}{\gamma^N(t) + x^N} \frac{\dot{P}(t)}{P(t)} \right] \frac{1}{1 - \alpha_V}.$$

(329)
Inserting eq. (328b) into (329) leads to the dynamic equation for the labor market tightness in the non-traded sector:

\[ \dot{\theta}^N(t) = \frac{\theta^N(t)}{(1 - \alpha_N^V)} \left\{ (s^N + r^*) - \frac{f^N(\theta^N(t)) (1 - \alpha_N^N)}{P(t)\kappa^N} \Psi^N(t) - \frac{\dot{P}(t)}{P(t)} \right\}. \]  

(330)

where the overall surplus \( \Psi^N(t) \) is

\[ \Psi^N(t) = P(t)\Lambda^N + r^*x^N + \frac{v^N}{\lambda}. \]

(331)
The market clearing condition for the non-traded sector now reads as follows:

\[ A^N L^N(t) = C^N (\bar{\lambda}, P(t)) + \kappa^N V^N(t). \]

(332)

Solving (332) for the relative price of non-tradables leads to:

\[ P(t) = P \left( L^N(t), V^N(t), \bar{\lambda}, A^N \right). \]

(333)

Using the fact that \( V^j(t) = U^j(t)\theta^j(t), \) differentiating (333) w.r.t. time, i.e., \( \dot{P}(t) = P_L N L^N(t) + P_{VN} \left( \theta^N \dot{U}^N(t) + U^N \dot{\theta}^N(t) \right), \) the dynamic equation for the relative price of non-tradables, i.e., \( \dot{P}(t)/P(t), \) reads as follows:

\[ \frac{\dot{P}(t)}{P(t)} = \frac{A^N}{C_P^N P} \dot{L}^N(t) - \frac{\kappa^N V^N \theta^N(t)}{C_P^N P} - \frac{\kappa^N \theta^N(t)}{C_P^N P} \dot{U}^N(t). \]

(334)

Eliminating \( \dot{P}/P \) from (330) by inserting (334) leads to the dynamic equation of the non-traded labor market tightness:

\[ \dot{\theta}^N(t) = \frac{\theta^N(t)}{(1 - \alpha_N^V)} \left\{ (s^N + r^*) - \frac{f^N(\theta^N(t)) (1 - \alpha_N^N)}{P(t)\kappa^N} \Psi^N(t) - \frac{\dot{P}(t)}{P(t)} \right\} \]

(335)

By assuming that hiring costs are expressed in terms of the non-traded good, the solution method becomes more complex since \( L^N(t) \) and \( \dot{U}^N(t) \) now show up in the equation (see the previous eq. (140) when hiring costs are expressed in terms of the traded good).

As shall be useful below to write the dynamics in a compact form, we set:

\[ a_1^N = \left(1 - \alpha_N^V\right) \left[ 1 - \frac{a_4^N}{1 - \alpha_N^V} \right], \]

(336a)

\[ a_2^N = \frac{A^N + \kappa^N \theta^N}{C_P^N P}, \]

(336b)

\[ a_3^N = \frac{\kappa^N \theta^N}{C_P^N P} \bar{\lambda}, \]

(336c)

\[ a_4^N = \frac{\kappa^N V^N}{C_P^N P} + \frac{\kappa^N \theta^N}{C_P^N P} \bar{\lambda} \left( \frac{v^N}{\lambda} + R^N \right) \alpha_N^V. \]

(336d)

Inserting the dynamic equation for job seekers (114) into (335) and making use of (336), leads to the dynamic equation for the non-traded labor market tightness:

\[ \dot{\theta}^N(t) = \frac{\theta^N(t)}{a_1^N} \left\{ (r^* + s^N) - \frac{f^N(\theta^N(t)) (1 - \alpha_N^N)}{P(t)\kappa^N} \Psi^N(t) - a_2^N \dot{L}^N(t) \right\} \]

(337)

\[ + a_3^N \left( \frac{v^N}{\lambda} + R^N \right) \left( s^N + r^* \right) + m^N (\theta^N(t) \alpha_N^W \Psi^N(t) \right\} \}. \]
L.2 Equilibrium Dynamics

Linearizing the accumulation equation for non traded labor (7) by setting \( j = N \) and the dynamic equations for labor market tightness (337) and job seekers (114) in the non-traded sector, we get:

\[
\begin{pmatrix}
L^N, \theta^N, \bar{U}^N
\end{pmatrix}^T = J^N \left( L^N(t) - \bar{L}^N, \theta^N(t) - \bar{\theta}^N, U^N(t) - \bar{U}^N \right)^T,
\]

(338)

where \( J^N \) is the Jacobian matrix described by:

\[
J^N = \begin{pmatrix}
-s^N & m^N & m^N \\
x_{21}^N & x_{22}^N & x_{23}^N \\
x_{31}^N & x_{32}^N & x_{33}^N
\end{pmatrix},
\]

(339)

where we computed the following linearized terms:

\[
x_{21}^N = \frac{\theta^N}{a_1^N} v_{FF} \left\{ \left( f^N \left( 1 - \alpha_W^N \right) \frac{1}{P^N} + a_3^N (s^N + r^*) + m^N \alpha_W^N \right) \right\}
\]

(340a)

\[
x_{22}^N = \frac{\theta^N}{a_1^N} \left\{ \left( 1 - \alpha_V^N \right) \frac{(s^N + r^*)}{\theta^N} + a_3^N \alpha_V^N f^N \alpha_W^N \Psi^N \right\}
\]

(340b)

\[
x_{23}^N = \frac{\theta^N}{a_1^N} v_{FF} \left\{ \left( f^N \left( 1 - \alpha_W^N \right) \frac{1}{P^N} + a_3^N (s^N + r^*) + m^N \alpha_W^N \right) \right\}
\]

(340c)

and

\[
x_{31}^N = (2s^N + r^*) + m^N \alpha_W^N + \bar{\lambda} m^N \alpha_W^N A^N P_{LN} + \bar{\lambda} \frac{v^N}{v_{FF}} \left( \frac{v^N}{\lambda} + R^N \right) \frac{\alpha_V^N}{\theta^N} x_{21}^N
\]

(341a)

\[
a_{32}^N = \bar{\lambda} \frac{\alpha_V^N f^N \alpha_W^N \Psi^N}{v_{FF}^N} - (m^N) U^N + \bar{\lambda} \frac{v^N}{v_{FF}^N} m^N \alpha_W^N A^N P_{VN} U^N
\]

(341b)

\[
a_{33}^N = (s^N + r^*) + m^N \alpha_W^N - m^N + \bar{\lambda} \frac{v^N}{v_{FF}^N} m^N \alpha_W^N A^N P_{VN} \theta^N
\]

(341c)

where we used the fact that \( f' \theta / f = - (1 - \alpha_V) \), \( m' \theta / m = \alpha_V \), \( f = m / \theta \), and set

\[
x_2^N = - \left( 1 - \alpha_V \right) \left( \frac{P A^N}{\Psi^N} - 1 \right) + a_3^N m^N \alpha_W^N A^N,
\]

(342)

to write the linearized system in a compact form.

Setting the constant \( D_2^N = 0 \) associated with the unstable eigenvalue \( \nu_2^N \) to insure a converging adjustment for all macroeconomic aggregates, the stable paths are given by:

\[
L^N(t) - \bar{L}^N = D_1^N e^{\nu_1^N t},
\]

(343a)

\[
\theta^N(t) - \bar{\theta}^N = \omega_2^N D_1^N e^{\nu_2^N t},
\]

(343b)

\[
U^N(t) - \bar{U}^N = \omega_3^N D_1^N e^{\nu_3^N t}.
\]

(343c)
Using the fact that \( A \equiv B + \gamma T L^T + \gamma N L^N \), differentiating with respect to time, noticing that \( (\gamma J L) = r^* \gamma J L - \pi J \), the accumulation equation of traded bonds is given by:

\[
\dot{B} = A - \gamma T L^T - \gamma T L^N - \gamma N L^N,
\]

\[
= r^* (A - \gamma T L^T - \gamma N L^N) + \pi T + \pi N + W^T L^T + W^N L^N + R^T U^T + R^N U^N - T - P_C C.
\]

Remembering that \( \pi_j = \Xi_j - W^j L^T - P \kappa V_j - \pi^j \cdot \max \left\{ 0, -\lambda^j \right\} \), inserting the market clearing condition for the non-traded good (332) and the balanced government budget (19), the current account equation reduces to:

\[
\dot{B}(t) = r^* B(t) + A^T L^T(t) - C^T(t) - G^T - \kappa T V^T(t).
\]  

(344)

Substituting first the short-run static solution for \( P \) (333) into the static solution for consumption in tradables given by (136), and using the fact that \( V^j = U^j \theta^j \), the accumulation equation for traded bonds (344) can be written as follows:

\[
\dot{B}(t) = r^* B(t) + A^T L^T(t) - C^T \left( L^N(t), \theta^N(t), U^N(t), \lambda, \Lambda \right) - G^T - \kappa T \theta^T(t) U^T(t).
\]

(345)

Linearizing (345) in the neighborhood of the steady-state and inserting stable solutions given by (166) and (343) yields:

\[
\dot{B}(t) = r^* \left( B(t) - \bar{B} \right) + \Lambda_T \left( L^T(t) - \bar{L}^T \right) + \Lambda^N \left( L^N(t) - \bar{L}^N \right),
\]

(346)

where we set:

\[
\Lambda_T = A_T - \kappa T \bar{U}^T \omega^T t_1 - \kappa T \bar{\theta}^T \omega^T t_1,
\]

(347a)

\[
\Lambda^N = - \left[ C_{LN} + C_{\theta N} \omega^N t_1 + C_{U N} \omega^N \right].
\]

(347b)

L.3 Decomposition of Steady-State Changes

Assuming \( \alpha^j W = \alpha W \) and \( \alpha^j V = \alpha V \), the steady-state reads as follows:

\[
\frac{C^T}{C^N} = \frac{\varphi}{1 - \varphi},
\]

(348a)

\[
\frac{L^T}{L^N} = \frac{m_T}{m_N} \left( \frac{s^N + m_N}{s^N + m_T} \right) \left[ \frac{\lambda W^T_R / \zeta^T}{\lambda W^N_R / \zeta^N} \right]^{\sigma^T},
\]

(348b)

\[
\frac{\kappa^T}{\kappa^N} = \frac{(1 - \alpha W) \Psi^T}{(s^T + r^*)},
\]

(348c)

\[
\frac{P^N}{P^N} = \frac{(1 - \alpha W) \Psi^N}{(s^N + r^*)},
\]

(348d)

\[
\frac{Y^T (1 + \nu_B - v^T)}{Y^N (1 - v^N)} = \frac{C^T}{C^N}.
\]

(348e)

where

\[
W^N_R = \frac{\alpha W}{1 - \alpha W} P^N \theta^N + R^N.
\]

(349)

Assuming that non-traded firms use labor services from non-traded employment agencies modifies eq. (348d), (348e) and (349).

Inserting first (349) into the total surplus from an additional job, i.e., \( \Psi^j = \Xi^j + r^* \lambda^j - W^j_R \), and totally differentiating (348d) leads to the steady-state rate of change in the sectoral labor market tightness:

\[
\dot{\theta}^N = \Sigma^N \bar{a}^N + \bar{p} \left[ \Sigma^N - \Sigma^N \right],
\]

(350)

where we set:

\[
\Sigma^j = \frac{\Xi^j}{\left( 1 - \alpha V \right) \Psi^j + \chi^j W^j_R},
\]

(351a)

\[
\Sigma^N \left[ \frac{\Xi^N}{\left( 1 - \alpha V \right) \Psi^N + \chi^j W^N_R} \left( \frac{\Psi^N + \chi^j W^N_R}{\Xi^N} \right) \right].
\]

(351b)
Denoting by
\[ \Theta^j = \Sigma^j \left[ \alpha_V u^j + \sigma_L \chi^j \right], \quad (352a) \]
\[ \Theta^{Nj} = \Sigma^{Nj} \left[ \alpha_V u^N + \sigma_L \chi^N \right], \quad (352b) \]
differentiating (348b) and inserting (350) leads to labor supply:
\[
\begin{align*}
\hat{\theta}^T \hat{a}^N &= \left[ \alpha_V u^T + \sigma_L \chi^T \right] \hat{\theta}^T - \left[ \alpha_V u^N + \sigma_L \chi^N \right] \hat{\theta}^N - \hat{\rho} \sigma_L \chi^N, \\
&= \Theta^T \hat{a}^T - \Theta^{N} \hat{a}^N - \hat{\rho} \left[ \left( \Theta^N - \Theta^{N, j} \right) + \sigma_L \chi^N \right]. \\
&= \Theta^T \hat{a}^T - \Theta^{N} \hat{a}^N - \hat{\rho} \left[ \Theta^N - \Theta^{N, j} \right] - \hat{\rho} \sigma_L \chi^N. \\
&= \Theta^T \hat{a}^T - \Theta^{N} \hat{a}^N - \hat{\rho} \sigma_L \chi^N. \\
\end{align*}
\]
Using the fact that \( \hat{\theta}^j = \hat{y}^j - \hat{a}^j \) differentiating (348a), inserting (353) and solving for the steady-state change in the relative price of non-tradables leads to:
\[
\hat{p} = \left( 1 + \Theta^T \right) \hat{a}^T - \left( 1 + \Theta^{N} \right) \hat{a}^N + \frac{d \ln \left[ \frac{1 - v_{NX}}{1 - v_{\hat{p}^*}} \right]}{\Delta}, \quad (354)
\]
where we set
\[
\Delta = \phi + \left( \Theta^N - \Theta^{N, j} \right) + \sigma_L \chi^N. \quad (355)
\]
Inserting (349) into the Nash bargaining wage, i.e., \( w^j = \alpha_W \left( \Xi^j + r^j x^j \right) + \left( 1 - \alpha_W \right) W^j_R \), differentiating and substituting the steady-state in the labor market tightness (350) leads to:
\[
\hat{w}^N - \hat{w}^T = \Omega^N \hat{a}^N - \Omega^T \hat{a}^T + \hat{\rho} \left( \Omega^N - \Omega^{N, j} \right), \quad (356)
\]
where we set:
\[
\Omega^j = \frac{\Xi^j}{w^j} \left( 1 - \alpha_V \right) \Psi^j + \chi^j W^j_R, \quad (357a)
\]
\[
\Omega^{N, j} = \frac{1 - \alpha_W}{w^N} \left[ \left( 1 - \alpha_V \right) \Psi^N + \chi^N W^N_R \right]. \quad (357b)
\]
Plugging (354) into (356) and collecting terms leads to the steady-state change in the relative wage of non-tradables:
\[
\begin{align*}
\hat{w} &= - \left\{ \Omega^T - \left( \Omega^N - \Omega^{N, j} \right) \left( 1 + \Theta^T \right) \right\} \hat{a}^T + \left\{ \Omega^N - \left( \Omega^N - \Omega^{N, j} \right) \left( 1 + \Theta^N \right) \right\} \hat{a}^N \\
&+ \left( \Omega^N - \Omega^{N, j} \right) \frac{d \ln \left[ \frac{1 - v_{NX}}{1 - v_{\hat{p}^*}} \right]}{\Delta}. \\
\end{align*}
\]
Differentiating the sectoral unemployment rate described by eq. (257) leads to:
\[
du^T - \frac{du^N}{\Delta} = -\alpha_V \left[ u^T \left( 1 - u^T \right) \hat{\theta}^T - u^N \left( 1 - u^N \right) \hat{\theta}^N \right]. \quad (359)
\]
Inserting (354) into (350), using the fact that \( \hat{\theta}^T = \Sigma^T \hat{a}^T \), substituting the outcome into (359), and collecting terms leads to the unemployment differential between tradables and non-tradables:
\[
\begin{align*}
\frac{du^T}{\Delta} - \frac{du^N}{\Delta} &= -\alpha_V \left\{ u^T \left( 1 - u^T \right) \Sigma^T - u^N \left( 1 - u^N \right) \left[ \frac{\left( \Sigma^N - \Sigma^{N, j} \right) \left( 1 + \Theta^T \right)}{\Delta} \right] \right\} \hat{a}^T \\
&+ \alpha_V \left\{ u^N \left( 1 - u^N \right) \left[ \Sigma^N - \left( \Sigma^N - \Sigma^{N, j} \right) \left( 1 + \Theta^N \right) \right] \right\} \hat{a}^N \\
&+ \alpha_V u^N \left( 1 - u^N \right) \left[ \Sigma^N - \Sigma^{N, j} \right] \frac{d \ln \left[ \frac{1 - v_{NX}}{1 - v_{\hat{p}^*}} \right]}{\Delta}. \quad (360)
\end{align*}
\]
L.4 Correcting for the Bias

We now compute the bias originating from search frictions varying across sectors which must be accounted for in order to map theoretical results for the responses to a productivity differential into elasticities estimated empirically.

Because empirically we consider a productivity differential $\hat{a}^T - \hat{a}^N$, to make our estimates comparable with our numerical results, we have to adjust the long-run change in the relative price computed numerically with the following term:

$$\text{bias } \hat{p} = \frac{1}{\Delta} \left[ (1 + \Theta^T) - (1 + \Theta^N) \right] \hat{a}^N. \quad (361)$$

Subtracting (361) from (354) leads to the 'unbiased' relative price response to a productivity differential:

$$\hat{p}' = \hat{p} - \text{bias } \hat{p}. \quad (362)$$

The same logic applies to the relative wage. The long-run reaction of the relative wage described by (358) must be corrected for the bias which reads as follows:

$$\text{bias } \hat{\omega} = \left\{ \left[ \Omega^T - (\Omega^N - \Omega^N\rho) \left( \frac{1 + \Theta^T}{\Delta} \right) \right] - \left[ \Omega^N - (\Omega^N - \Omega^N\rho) \left( \frac{1 + \Theta^N}{\Delta} \right) \right] \right\} \hat{a}^N. \quad (363)$$

Once the value of $\hat{\omega}$ has been adjusted with, we can map the deviation in percentage of the relative wage from its initial steady-state derived analytically into the elasticity of the relative wage, $\beta$, estimated empirically:

$$\hat{\omega}' = \hat{\omega} - \text{bias } \hat{\omega}. \quad (364)$$

The numerical computation of the unemployment rate differential is subject to the same bias as the relative price and the relative wage. We have to adjust the long-run change in the unemployment differential computed numerically with the following term:

$$\text{bias } (du^T - du^N) = \alpha_V \left\{ u^N \left(1 - u^N\right) \left[ \Sigma^N - \left(\Sigma^N + \Sigma^N\rho \left( \Theta^N - \Theta^T \right) / \Delta \right) \right] - u^T \left(1 - u^T\right) \Sigma^T \right\} \hat{a}^N. \quad (365)$$

The long-run reaction of the unemployment differential between tradables and non tradables described by (360) must be corrected for the bias (365):

$$(du^T - du^N)' = du^T - du^N - \text{bias } (du^T - du^N) \quad (366)$$

L.5 Steady-State Changes when $\sigma_L \to \infty$

Once the bias (361) caused by search frictions is controlled for, the decomposition of the steady-state change in the relative price reads:

$$\hat{p}' = \frac{(1 + \Theta^T)}{\Delta} (\hat{a}^T - \hat{a}^N) + \frac{d \ln \left( \frac{(1 - v_{NX})}{(1 - v_N^T)} \right)}{\Delta}. \quad (367)$$

Letting $\sigma_L$ tend toward infinity and applying l'Hôpital rule leads to:

$$\lim_{\sigma_L \to \infty} \hat{p}' = \lim_{\sigma_L \to \infty} \frac{(1 + \Theta^T)}{\Delta} (\hat{a}^T - \hat{a}^N),$$

$$= \frac{\chi^T}{\chi^N} \left( \Sigma^T / (\Sigma^N - \Sigma^N\rho + 1) \right) (\hat{a}^T - \hat{a}^N), \quad (368)$$

where we used the fact that $\lim_{\sigma_L \to \infty} \Delta = \infty$, $\frac{\partial \Delta}{\partial \sigma_L} = \chi^N (\Sigma^N - \Sigma^N\rho + 1)$ and $\frac{\partial \Theta^j}{\partial \sigma_L} = \Sigma^j \chi^j$.

Once the bias (363) caused by search frictions is controlled for, the decomposition of the steady-state change in the relative wage reads:

$$\hat{\omega}' = - \left\{ \Omega^T - \left( \Omega^N - \Omega^N\rho \right) \left( \frac{1 + \Theta^T}{\Delta} \right) \right\} (\hat{a}^T - \hat{a}^N)$$

$$+ \frac{\left( \Omega^N - \Omega^N\rho \right)}{\Delta} d \ln \left( \frac{(1 - v_{NX})}{(1 - v_N^T)} \right). \quad (369)$$

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Letting $\sigma_L$ tend toward infinity and applying l'Hôpital rule leads to:

$$\lim_{\sigma_L \to \infty} \hat{\omega}' = \lim_{\sigma_L \to \infty} \left( \Omega^T - \frac{(\Omega^N - \Omega^N,') (1 + \Theta^T)}{\Delta} \right) \left( \hat{a}^T - \hat{a}^N \right),$$

$$= - \left( \Omega^T - \frac{(\Omega^N - \Omega^N,') \Sigma^T \chi^T}{\Sigma^N - \Sigma^N, + 1 \chi^N} \right) (\hat{a}^T - \hat{a}^N).$$

(370)

Once the bias (365) caused by search frictions is controlled for, the decomposition of the steady-state change in the unemployment rate of tradables relative to the unemployment rate of non-tradables reads:

$$(du_T - du_N)' = -\alpha V \left( u_T^T (1 - u_T^T) \Sigma^T - u_N^T (1 - u_N^T) \right) \left( \frac{(\Sigma^N - \Sigma^N,') (1 + \Theta^T)}{\Delta} \right) (\hat{a}^T - \hat{a}^N)$$

$$+ \alpha_V u_N^T (1 - u_N^T) \left( \frac{(\Sigma^N - \Sigma^N,') \Sigma^T \chi^T}{\Sigma^N - \Sigma^N, + 1 \chi^N} \right) d \ln \left( \frac{(1 - v_{NT})}{(1 - v_{NT})} \right).$$

(371)

M Non-Traded Hiring Costs

In this section, we relax the assumption that hiring costs are tradables and consider that recruiting costs paid by non-traded as well as traded firms are expressed in terms of the non-traded good.

M.1 Market Clearing Condition

Assuming that both traded and non-traded hiring costs are expressed in non-traded units, the market clearing condition now reads as follows:

$$A^N L^N = C^N (\bar{\lambda}, P) + \kappa^N V^N + \kappa^T V^T,$$

(373)

where we have inserted the short-run static solution for consumption in non tradables $C^N (\bar{\lambda}, P)$ given by (132). Totally differentiating allows us to solve for the relative price of non tradables:

$$P(t) = P \left( L^N(t), V^N(t), V^T(t), \bar{\lambda}, A^N \right),$$

(374)

where

$$P_{L^N} = \frac{\partial P}{\partial L^N} = \frac{A^N}{C^N_P} < 0,$$

(375a)

$$P_{V^N} = \frac{\partial P}{\partial V^N} = -\frac{\kappa^N}{C^N_P} > 0,$$

(375b)

$$P_{V^T} = \frac{\partial P}{\partial V^T} = -\frac{\kappa^T}{C^N_P} > 0,$$

(375c)

$$P_{\bar{\lambda}} = \frac{\partial P}{\partial \bar{\lambda}} = -\frac{C^N_{\bar{\lambda}}}{C^N_P} < 0,$$

(375d)

$$P_{A^N} = \frac{\partial P}{\partial A^N} = \frac{A^N}{C^N_P} < 0.$$
M.2 Firms’ Decisions

Firms pay the wage $W^j$ decided by the generalized Nash bargaining solution. As producers face a labor cost $W^j$ per employee and a cost per hiring of $\kappa^j$, the profit function of the representative firm in sector $j$ is:

$$\pi^j = \Xi^j L^j - W^j L^j - P\kappa^j V^j - x^j \cdot \max\left\{0, -\dot{L}^j\right\}, \quad (376)$$

where $\Xi^j$ is the marginal revenue of labor (i.e., $\Xi^T = A^T$ and $\Xi^N = PA$); $x^j$ is a firing tax paid to the government when layoffs are higher than hirings, i.e., if $\dot{L}^j < 0$. Denoting by $f^j$ the rate at which a vacancy is matched with unemployed agents, the law of motion for labor is given by:

$$\dot{L}^j = f^j V^j - s^j L^j. \quad (377)$$

The current-value Hamiltonian for the sector $j$’s representative firm optimization problem is:

$$\mathcal{H}^j = \Xi^j L^j - W^j L^j - P\kappa^j V^j + \left(\gamma^j + x^j\right) \left(f^j V^j - s^j L^j\right), \quad (378)$$

where $\Xi^j$ is the marginal revenue of labor with $\Xi^T \equiv A^T$ and $\Xi^N \equiv PA$ and $\gamma^j$ is the co-state variable associated to the labor motion equation (118). Denoting by $\gamma^j$ the shadow price of employment to the firm, the maximization problem yields the following first-order conditions:

$$\gamma^j(t) + x^j = \frac{P(t)\kappa^j}{f^j(\theta^j(t))}, \quad (379a)$$

$$\dot{\gamma}^j = \gamma^j \left(r^* + s^j\right) - \left(\Xi^j - x^j s^j - W^j\right). \quad (379b)$$

Noting that (379b) can be rewritten as follows $\gamma^j(t) = \left(\gamma^j(t) + x^j\right) \left(r^* + s^j\right) - \left(\Xi^j(t) + r^* x^j - W^j(t)\right)$ and differentiating (379a) w.r.t. time leads to:

$$\frac{\dot{P}(t)}{P(t)} + \left(1 - \alpha_V^T\right) \frac{\dot{\theta}^T(t)}{\theta^T(t)} = \frac{\dot{\gamma}^j(t)}{\gamma^j(t) + x^j},$$

$$= \left(r^* + s^j\right) - \frac{f^j(t) \left(1 - \alpha_V^N\right)}{P(t)\kappa^j} \Psi^j(t), \quad (380)$$

where we set

$$\Psi^j(t) = \Xi^j(t) + r^* x^j + \frac{\kappa^j}{\lambda}. \quad (381)$$

Because hiring costs are expressed in non-traded units, (380) implies the following relationship between traded and non-traded labor market tightness dynamics:

$$\frac{\dot{\theta}^T(t)}{\theta^T(t)} = \frac{1}{1 - \alpha_V^T} \left\{ \left(r^* + s^T\right) - \frac{f^T(t) \left(1 - \alpha_V^N\right)}{P(t)\kappa^T} \Psi^T(t) \right\} - \left(\Psi^N(t) - \frac{f^N(t) \left(1 - \alpha_W^N\right)}{P(t)\kappa^N} \Psi^N(t) \right) + \left(1 - \alpha_N^N\right) \frac{\dot{\theta}^N(t)}{\theta^N(t)}, \quad (382)$$

where the dynamic equation for the non-traded labor market tightness is described by:

$$\frac{\dot{\theta}^T(t)}{\theta^T(t)} = \frac{1}{1 - \alpha_V^T} \left\{ \left(r^* + s^T\right) - \frac{f^N(t) \left(1 - \alpha_W^N\right)}{P(t)\kappa^N} \Psi^N(t) \right\} + \frac{\dot{P}(t)}{P(t)} \left(1 - \alpha_V^N\right) \frac{\dot{\theta}^N(t)}{\theta^N(t)} = \left(r^* + s^N\right) - \frac{f^N(t) \left(1 - \alpha_W^N\right)}{P(t)\kappa^N} \Psi^N(t). \quad (383)$$

Using the fact that $V^j(t) = U^j(t)\theta^j(t)$, differentiating (374) w.r.t. time, i.e., $P(t) = P_{LN} \bar{L}^N(t) + P_{VN} \left(\theta^N U^N(t) + U^N \dot{\theta}^N(t)\right) + P_{VT} \left(\theta^T U^T(t) + U^T \dot{\theta}^T(t)\right)$, the dynamic equation for the relative price of non-tradables, i.e., $\frac{P(t)}{P(t)}$, reads as follows:

$$\frac{\dot{P}(t)}{P(t)} = \frac{AN}{C_B^P} \bar{L}^N(t) - \frac{\kappa^N V^N \dot{\theta}^N(t)}{C_B^P P} \theta^N(t) - \frac{\kappa^N \theta^N}{C_B^P P} \dot{U}^N(t) - \frac{\kappa^T V^T \dot{\theta}^T(t)}{C_B^P P} \theta^T(t) - \frac{\kappa^T \theta^T}{C_B^P P} \dot{U}^T(t). \quad (384)$$
As shall be useful below to write the dynamics in a compact form, we set:

\[ a_1^N = (1 - a_{4V}^N) \left[ 1 - \frac{a_{3N}^N}{1 - a_{4V}^N} \right. \left. - \frac{a_{4T}^N}{1 - a_{4T}^N} \right], \]  

\[ a_2^N = \frac{AN + \kappa N \theta N}{CP_P P}, \]  

\[ a_2^T = \frac{\kappa T \theta T}{CP_P P}, \]  

\[ a_3^N = \frac{\kappa N \theta N}{CP_P P v_{FF}^N}, \]  

\[ a_3^T = \frac{\kappa T \theta T}{CP_P P v_{FF}^T}, \]  

\[ a_4^N = \frac{\kappa N V^N}{CP_P P} + \frac{\kappa N \theta N}{CP_P P v_{FF}^N} \left( \frac{v_F^N}{\lambda} + R^N \right) \alpha_V^N, \]  

\[ a_4^T = \frac{\kappa T V^T}{CP_P P} + \frac{\kappa T \theta T}{CP_P P v_{FF}^T} \left( \frac{v_F^T}{\lambda} + R^T \right) \alpha_V^T. \]  

Plugging the dynamic equation for the relative price of non-tradables (384), next inserting the dynamic equation for job seekers (114) in the traded and non-traded sector into (383), and eliminating the dynamic equation for the traded labor market tightness by making use of (382) leads to the dynamics for the non-traded labor market tightness:

\[
\dot{\theta}^N(t) = \frac{\theta^N(t)}{a_{1N}^N} \left\{ \left( 1 - \frac{a_{4T}^N}{1 - a_{4V}^N} \right) \left[ (r^* + s^N) - \frac{f^N(\theta^N(t)) (1 - \alpha_W^N) \Psi^N(t)}{P(t) \kappa_N} \right] \\
+ \frac{a_{4T}^N}{1 - a_{4V}^N} \left[ (r^* + s^T) - \frac{f^T(\theta^T(t)) (1 - \alpha_W^T) \Psi^T(t)}{P(t) \kappa_T} \right] \\
+ a_{3N}^N \left( \frac{v_F^N}{\lambda} + R^N \right) (s^N + r^*) + m^N(\theta^N(t)) \alpha_W^N \Psi^N(t) \\
+ a_{3T}^N \left( \frac{v_F^T}{\lambda} + R^T \right) (s^T + r^*) + m^T(\theta^T(t)) \alpha_W^T \Psi^T(t) \right\} - a_2^N \dot{L}^N(t) - a_2^T \dot{L}^T(t). \]  

M.3 Equilibrium Dynamics

The adjustment of the open economy towards the steady-state is described by a dynamic system which comprises six equations. When assuming that hiring costs are non-tradables, the dynamics within each sector cannot be analyzed separately because the relative price dynamics imposes a connection between the two labor markets.

The first dynamic system consists of the law of motion of employment in the non-traded and traded sector described by (7), the dynamic equation for non-traded labor tightness described by eq. (386), the dynamic equation for traded labor market tightness described by eq. (382), and the dynamic equation for job seekers in both sectors given by (114), respectively. We drop the time index to denote the steady-state value. Before linearizing, we recall that \( W^j + \frac{v_F^j}{\lambda} = \alpha_W^j \Psi^j \) and \( \Xi^j + r^j \sigma^j - W^j = \left( 1 - \alpha_W^j \right) \Psi^j \).

Linearizing the accumulation equation for labor in sector \( j = N, T \), the dynamic equation for labor market tightness in sector \( j = N, T \), and the dynamic equations for job seekers in both sectors, we get in matrix form:

\[
\begin{bmatrix}
\dot{L}^N(t), \dot{\theta}^N(t), \dot{U}^N(t), \dot{L}^T(t), \dot{\theta}^T(t), \dot{U}^T(t)
\end{bmatrix}' = J (L^N(t) - L^N, \theta^N(t) - \theta^N, U^N(t) - U^N, L^T(t) - L^T, \theta^T(t) - \theta^T, U^T(t) - U^T)' \]  

\[ \text{Eq. 387} \]
where \( J \) is given by

\[
J \equiv \begin{pmatrix}
-s^N & (m^N)'U^N & m^N & 0 & 0 & 0 \\
x_{21} & x_{22} & x_{23} & x_{24} & x_{25} & x_{26} \\
x_{31} & x_{32} & x_{33} & x_{34} & x_{35} & x_{36} \\
0 & 0 & 0 & -s^T & (m^N)'U^T & m^T \\
x_{51} & x_{52} & x_{53} & x_{54} & x_{55} & x_{56} \\
x_{61} & x_{62} & x_{63} & x_{64} & x_{65} & x_{66}
\end{pmatrix},
\] (388)

where we computed the following linearized terms:

\[
x_{21} = \frac{\theta v}{a_{11}} \left\{ -\left(1 - \frac{a_4^T}{1 - \alpha_1 V} \right) \frac{f^N (1 - \alpha W)}{P \alpha W} + a_3^N \left[ (s^N + r^*) + m^N \alpha W \right] \right\} \\
+ \frac{\theta}{a_{11}} \left( x_2^N P_{LN} + a_2^N s^N \right), \tag{389a}
\]

\[
x_{22} = \frac{\theta v}{a_{11}} \left[ \left(1 - \frac{a_4^T}{1 - \alpha_1 V} \right) \frac{1 - \alpha W}{\theta} \left( s^N + r^* \right) + a_3^N \alpha_V f^N \alpha W \Psi \right] \\
+ \frac{\theta}{a_{11}} \left( x_2^N P_{VN} a W - a_2^N m^N \right), \tag{389b}
\]

\[
x_{23} = \frac{\theta v}{a_{11}} \left\{ -\left(1 - \frac{a_4^T}{1 - \alpha_1 V} \right) \frac{f^N (1 - \alpha W)}{P \alpha W} + a_3^N \left[ (s^N + r^*) + m^N \alpha W \right] \right\} \\
+ \frac{\theta}{a_{11}} \left( x_2^N P_{VN} \theta - a_2^N m^N \right), \tag{389c}
\]

\[
x_{24} = \frac{\theta v}{a_{11}} \left\{ -\left(1 - \frac{a_4^T}{1 - \alpha_1 V} \right) \frac{f^T (1 - \alpha W)}{P \alpha W} + a_3^T \left[ (s^T + r^*) + m^T \alpha W \right] \right\} \\
+ \frac{\theta}{a_{11}} a_2 \alpha V \Psi, \tag{389d}
\]

\[
x_{25} = \frac{\theta V}{a_{11}} \left[ \frac{a_4^T}{1 - \alpha_1 V} \frac{(1 - \alpha T) (s^T + r^*)}{\theta T} \right] \\
+ \frac{\theta}{a_{11}} \left( x_2^N P_T^T U^T - a_2^T (m^T)'U^T \right), \tag{389e}
\]

\[
x_{26} = \frac{\theta v}{a_{11}} \left\{ -\left(1 - \frac{a_4^T}{1 - \alpha_1 V} \right) \frac{f^T (1 - \alpha W)}{P \alpha W} + a_3^T \left[ (s^T + r^*) + m^T \alpha W \right] \right\} \\
+ \frac{\theta}{a_{11}} \left( x_2^N P_{VT} \theta - a_2^T m^T \right), \tag{389f}
\]
and
\[ x_{31} = (2s^N + r^*) + m^N \alpha_W + \frac{\lambda}{v_{FF}} m^N \alpha_W A^N P_{L_N} - \frac{\lambda}{v_{FF}} m^N \alpha_W \Psi^N \alpha_V \gamma^N x_{21}, \quad (390a) \]
\[ x_{32} = \frac{\lambda}{v_{FF}} m^N \alpha_W \Psi^N \alpha_V - (m^N)^T U^N + \frac{\lambda}{v_{FF}} m^N \alpha_W A^N P_{V_N} U^N - \frac{\lambda}{v_{FF}} m^N \alpha_W \Psi^N \alpha_V \gamma^N x_{22}, \quad (390b) \]
\[ x_{33} = (s^N + r^*) + m^N \alpha_W - m^N + \frac{\lambda}{v_{FF}} m^N \alpha_W A^N P_{V_N} \theta^N - \frac{\lambda}{v_{FF}} m^N \alpha_W \Psi^N \alpha_V \gamma^N x_{23}, \quad (390c) \]
\[ x_{34} = \frac{\lambda}{v_{FF}} m^N \alpha_W \Psi^N \alpha_V - \frac{\lambda}{v_{FF}} m^N \alpha_W \Psi^N \alpha_V \theta^N x_{24}, \quad (390d) \]
\[ x_{35} = \frac{\lambda}{v_{FF}} m^N \alpha_W A^N P_{V_T} U^T - \frac{\lambda}{v_{FF}} m^N \alpha_W \Psi^N \alpha_V \gamma^N x_{25}, \quad (390e) \]
\[ x_{36} = \frac{\lambda}{v_{FF}} m^N \alpha_W A^N P_{V_T} \theta^T - \frac{\lambda}{v_{FF}} m^N \alpha_W \Psi^N \alpha_V \theta^N x_{26}, \quad (390f) \]
and
\[ x_{51} = \frac{\theta^T}{1 - \alpha^T} \left[ \frac{f^N (1 - \alpha^N_W)}{\lambda} + x_5^N P_{L_N} + \frac{(1 - \alpha^N_V)}{\theta^N} \right] x_{21}, \quad (391a) \]
\[ x_{52} = \frac{\theta^T}{1 - \alpha^T} \left[ -(1 - \alpha^N_V) \left( \frac{s^N + r^*}{\theta^N} \right) + x_5^N P_{V_N} U^N + \frac{(1 - \alpha^N_V)}{\theta^N} \right] x_{22}, \quad (391b) \]
\[ x_{53} = \frac{\theta^T}{1 - \alpha^T} \left[ f^N \left( \frac{1 - \alpha^T_W}{\lambda} \right) + x_5^N P_{V_N} \theta^N + \frac{(1 - \alpha^N_V)}{\theta^N} \right] x_{23}, \quad (391c) \]
\[ x_{54} = \frac{\theta^T}{1 - \alpha^T} \left[ -f^T \left( \frac{1 - \alpha^T_W}{\lambda} \right) \frac{v_{FF}^T}{\lambda} + \frac{(1 - \alpha^N_V)}{\theta^N} \right] x_{24}, \quad (391d) \]
\[ x_{55} = \frac{\theta^T}{1 - \alpha^T} \left[ (1 - \alpha^T_V) \left( \frac{s^T + r^*}{\theta^T} \right) + x_5^N P_{V_T} U^T + \frac{(1 - \alpha^N_V)}{\theta^N} \right] x_{25}, \quad (391e) \]
\[ x_{56} = \frac{\theta^T}{1 - \alpha^T} \left[ -f^T \left( \frac{1 - \alpha^T_W}{\lambda} \right) \frac{v_{FF}^T}{\lambda} + x_5^N P_{V_T} \theta^T + \frac{(1 - \alpha^N_V)}{\theta^N} \right] x_{26}, \quad (391f) \]
and
\[ x_{61} = \frac{\lambda}{v_{FF}} \left( \frac{v_{F}^T}{\lambda} + R^r \right) \frac{\alpha_T^T}{\theta^T} x_{51}, \quad (392a) \]
\[ x_{62} = \frac{\lambda}{v_{FF}} \left( \frac{v_{T}^T}{\lambda} + R^r \right) \frac{\alpha_T^T}{\theta^T} x_{52}, \quad (392b) \]
\[ x_{63} = \frac{\lambda}{v_{FF}} \left( \frac{v_{F}^T}{\lambda} + R^t \right) \frac{\alpha_T^T}{\theta^T} x_{53}, \quad (392c) \]
\[ x_{64} = (2s^T + r^*) + \alpha_T^T m^T + \frac{\lambda}{v_{FF}} \left( \frac{v_{F}^T}{\lambda} + R^T \right) \frac{\alpha_T^T}{\theta^T} x_{54}, \quad (392d) \]
\[ x_{65} = \frac{\lambda}{v_{FF}} \alpha_T f^T \alpha_T \Psi^T \Psi^T - (m^T)^T U^T + \frac{\lambda}{v_{FF}} \left( \frac{v_{F}^T}{\lambda} + R^T \right) \frac{\alpha_T^T}{\theta^T} x_{55}, \quad (392e) \]
\[ x_{66} = (s^T + r^*) + \alpha_T^T m^T - m^T + \frac{\lambda}{v_{FF}} \left( \frac{v_{F}^T}{\lambda} + R^T \right) \frac{\alpha_T^T}{\theta^T} x_{56}, \quad (392f) \]
where we used the fact that $f'\theta/f = -(1 - \alpha_V)$, $m'\theta/m = \alpha_V$, $f = m/\theta$, and set some
expressions to write the linearized system in a compact form:

\[
\begin{align*}
    x_2^N &= - \left(1 - \frac{a_4^T}{1 - a_4^T}\right) \left(\frac{s^N + r^*}{P}\right) \left(\frac{PA^N}{\Psi^N} - 1\right) + \frac{a_4^T}{1 - a_4^T} \left(\frac{s^T + r^*}{P}\right) \\
    x_5^N &= \frac{f^N (1 - \alpha_N^N) \alpha_N^N}{P_{KN}} \left(\frac{s^T + r^*}{P} - \frac{s^N + r^*}{P}\right).
\end{align*}
\]

(393a)

(393b)

Denoting by \(\omega_k^i\) the \(k\)th element of eigenvector \(\omega^i\) related to eigenvalue \(\nu_i\), the general solution that characterizes the adjustment toward the new steady-state can be written as follows: \(V(t) - V = \sum_{i=1}^{6} \omega^i D_i e^{\nu^i t}\) where \(V\) is the vector of state and control variables. Since the dynamic system comprises two state variables, it must have two negative eigenvalues for the long-run equilibrium to be saddle-path. We denote by \(\nu_1 < \nu_2 < 0\) the two negative eigenvalues. Formal solutions read as follows:

\[
\begin{align*}
    L^N(t) - L^N &= D_1 e^{\nu^1 t} + D_2 e^{\nu^2 t}, \\
    \theta^N(t) - \theta^N &= \omega_1^1 D_1 e^{\nu^1 t} + \omega_2^2 D_2 e^{\nu^2 t}, \\
    U^N(t) - U^N &= \omega_1^1 D_1 e^{\nu^1 t} + \omega_2^2 D_2 e^{\nu^2 t}, \\
    L^T(t) - L^T &= \omega_1^1 D_1 e^{\nu^1 t} + \omega_2^2 D_2 e^{\nu^2 t}, \\
    \theta^T(t) - \theta^T &= \omega_1^1 D_1 e^{\nu^1 t} + \omega_2^2 D_2 e^{\nu^2 t}, \\
    U^T(t) - U^T &= \omega_1^1 D_1 e^{\nu^1 t} + \omega_2^2 D_2 e^{\nu^2 t}.
\end{align*}
\]

(394a)

(394b)

(394c)

(394d)

(394e)

(394f)

Using initial conditions, i.e., \(L^N(0) = L_0^N\) and \(L^T(0) = L_0^T\), setting \(t = 0\) into (394a) and (394d) leads to a system of two equations \(D_1 + D_2 = -dL^N\) and \(\omega_1^1 + \omega_2^2 = -dL^T\) that can be solved for the two arbitrary constants:

\[
\begin{align*}
    D_1 &= \frac{dL^T - \omega_2^2 dL^N}{\omega_2^2 - \omega_1^1}, \\
    D_2 &= \frac{\omega_1^1 dL^N - dL^T}{\omega_2^2 - \omega_1^1}.
\end{align*}
\]

(395a)

(395b)

M.4 Formal Solution for the Stock of Foreign Bonds \(B(t)\)

Substituting first the short-run static solution (136) for consumption in tradables, and inserting the solution (374) for the relative price of non-tradables, i.e., \(P(t) = P (L^N(t), V^N(t), V^T(t), \tilde{\lambda}, A^N)\), the accumulation equation for traded bonds reads as follows:

\[
\dot{B}(t) = r^* B(t) + A^T L^T(t) - C^T (P(\cdot), \tilde{\lambda}) - G^T.
\]

(396)

Linearizing (396) in the neighborhood of the steady-state and inserting stable solutions given by (394) yields:

\[
\dot{B}(t) = r^* \left(B(t) - \tilde{B}\right) + \Lambda_1 D_1 e^{\nu^1 t} + \Lambda_2 D_2 e^{\nu^2 t},
\]

(397)

where we set:

\[
\begin{align*}
    \Lambda_1 &= A^T \omega_1^1 - C^T \left[P_{tN} + P_{vN} (U^N \omega_1^2 + \theta^N \omega_1^3)] + P_{vT} (U^T \omega_5^1 + \theta^T \omega_5^2)\right], \\
    \Lambda_2 &= A^T \omega_2^2 - C^T \left[P_{tN} + P_{vN} (U^N \omega_2^2 + \theta^N \omega_2^3)] + P_{vT} (U^T \omega_5^2 + \theta^T \omega_5^3)\right].
\end{align*}
\]

(398a)

(398b)

Solving the differential equation (397) yields:

\[
B(t) = \tilde{B} + \left[\left(B_0 - \tilde{B}\right) - \frac{\Lambda_1 D_1}{\nu_1 - r^*} - \frac{\Lambda_2 D_2}{\nu_2 - r^*}\right] e^{r^* t} + \frac{\Lambda_1 D_1}{\nu_1 - r^*} e^{\nu^1 t} + \frac{\Lambda_2 D_2}{\nu_2 - r^*} e^{\nu^2 t}.
\]

(399)
Invoking the transversality condition for traded bonds and inserting (395) leads to the intertemporal solvency condition:

\[ B - B_0 = \Phi^T dL^T + \Phi^N dL^N, \]  

(400)

where we set

\[ \Phi^T = \frac{1}{\omega_1^2 - \omega_2^2} \left[ \frac{\Lambda_2}{\nu_2 - r^*} - \frac{\Lambda_1}{\nu_1 - r^*} \right], \]  

(401a)

\[ \Phi^N = \frac{1}{\omega_1^2 - \omega_2^2} \left[ \frac{\Lambda_1 \omega_2^2}{\nu_2 - r^*} - \frac{\Lambda_2}{\nu_1 - r^*} \right], \]  

(401b)

For the national intertemporal solvency to hold, the term in brackets of eq. (399) must be zero so that the stable solution for the net foreign asset position reduces to:

\[ B(t) - \tilde{B} = \Phi_1 D_1 e^{\nu_1 t} + \Phi_2 D_2 e^{\nu_2 t} \]  

(402)

where

\[ \Phi_1 = \frac{\Lambda_1}{\nu_1 - r^*}, \quad \Phi_2 = \frac{\Lambda_2}{\nu_2 - r^*}. \]  

(403)

M.5 Decomposition of Steady-State Changes

Assuming \( \alpha^j_W = \alpha_W \) and \( \alpha^j_V = \alpha_V \), the steady-state reads as follows:

\[ \frac{C^T}{C^N} = \frac{\phi}{1 - \phi}, \]  

(404a)

\[ \frac{L^T}{L^N} = \frac{m^T}{m^N} \left( s^N + m^N \right) \left[ \frac{\lambda W^T_R / \zeta^T}{\lambda W^N_R / \zeta^N} \right]^{\sigma_L}, \]  

(404b)

\[ \frac{P^T}{P^N} = \frac{(1 - \alpha_W) \Psi^T}{(s^T + r^*)}, \]  

(404c)

\[ \frac{P^N}{P^N} = \frac{(1 - \alpha_W) \Psi^N}{(s^N + r^*)}, \]  

(404d)

\[ \frac{Y^T}{Y^N} \frac{(1 + \nu_B)}{(1 + \nu_V)} = \frac{C^T}{C^N}, \]  

(404e)

where we set

\[ W^j_R = \frac{\alpha_W}{1 - \alpha_W} P^j \theta^j + R^j, \]  

(405)

and \( \nu_B = \frac{r^* B}{1 + \nu_B}, \nu_V^j = \frac{\nu^j \nu^j}{\nu^N}. \) Assuming that recruiting costs are non-tradables instead of tradables modifies eqs. (404c)-(404d), (404e) and (405).

Inserting first (405) into the total surplus from an additional job, i.e., \( \Psi^j = \Xi^j + r^* x^j - W^j_R \), and totally differentiating (404c)-(404d) leads to the steady-state rate of change in the sectoral labor market tightness:

\[ \dot{\theta}^N = \Sigma^N \dot{a}^N + \rho \left[ \Sigma^N - \Sigma^N \right], \]  

(406a)

\[ \dot{\theta}^T = \Sigma^T \dot{a}^T - \rho \Sigma^T \dot{a} \]  

(406b)

where we set:

\[ \Sigma^j = \frac{\Xi^j}{(1 - \alpha_V) \Psi^j + \chi^j W^j_R}, \]  

(407a)

\[ \Sigma^{j,t} = \frac{\Xi^j}{(1 - \alpha_V) \Psi^j + \chi^j W^j_R} \left( \frac{\Psi^j + \chi^j W^j_R}{\Xi^j} \right). \]  

(407b)

Denoting by

\[ \Theta^j = \Sigma^j \left[ \alpha_V w^j + \sigma_L \chi^j \right], \]  

(408a)

\[ \Theta^{j,t} = \Sigma^{j,t} \left[ \alpha_V w^j + \sigma_L \chi^j \right], \]  

(408b)
differentiating (404b) and inserting (406a)-(406b) leads to labor supply:

\[ \hat{I} - \hat{N} = \left[ \alpha v u^T + \sigma_L \chi^T \right] \hat{\theta}^T - \left[ \alpha v u^N + \sigma_L \chi^N \right] \hat{\theta}^N + \hat{p} \sigma_L \left( \chi^T - \chi^N \right), \]

\[ = \Theta^T \hat{a}^T - \Theta^T \hat{a}^N - \hat{p} \left[ \Theta^N + \left( \Theta^T - \Theta^N \right) + \sigma_L \left( \chi^N - \chi^T \right) \right]. \tag{409} \]

Using the fact that \( \hat{I} = \hat{y}^T - \hat{a}^T \) differentiating (404a), inserting (409) and solving for the steady-state change in the relative price of non-tradables leads to:

\[ \hat{p} = \frac{\left( 1 + \Theta^T \right) \hat{a}^T - \left( 1 + \Theta^N \right) \hat{a}^N}{\left[ \phi + \Theta^N + \left( \Theta^N - \Theta^N \right) + \sigma_L \left( \chi^N - \chi^T \right) \right]} + \frac{d \ln \left[ \frac{(1-u_{NX})}{(1-u_{NV} - u_{VT}^2)} \right]}{d \ln \left[ \frac{(1-u_{NX})}{(1-u_{NV} - u_{VT}^2)} \right]} \tag{410} \]

Inserting (405) into the Nash bargaining wage, i.e.,\( w^j = \alpha w \left( \Xi^j + r^* x^j \right) + (1 - \alpha w) W^j_R \), differentiating and substituting the steady-state in the labor market tightness (406) leads to:

\[ \hat{w}^N - \hat{w}^T = \Omega^N \hat{a}^N - \Omega^T \hat{a}^T + \hat{p} \left[ \Omega^N + \left( \Omega^T - \Omega^N \right) \right], \tag{411} \]

where we set:

\[ \Omega^j = \frac{\Xi^j}{w^j} = \frac{\Omega^N \left( 1 - \alpha v \right) \Psi^j + \chi^j W^j_R}{(1 - \alpha v) \Psi^j + \chi^j W^j_R}, \tag{412a} \]

\[ \Omega^{j'} = \frac{\Omega^N \left( 1 - \alpha v \right) \Psi^j + \chi^j W^j_R}{w^j \left( 1 - \alpha v \right) \Psi^j + \chi^j W^j_R}. \tag{412b} \]

Plugging (410) into (411) and collecting terms leads to the steady-state change in the relative wage of non-tradables:

\[
\hat{\omega} = - \left( \Omega^T - \frac{\left[ \Omega^N + \left( \Omega^T - \Omega^N \right) \right] \left( 1 + \Theta^T \right)}{\left[ \phi + \Theta^N + \left( \Theta^N - \Theta^N \right) + \sigma_L \left( \chi^N - \chi^T \right) \right]} \right) \hat{a}^T
+ \left( \Omega^N - \frac{\left[ \Omega^N + \left( \Omega^T - \Omega^N \right) \right] \left( 1 + \Theta^N \right)}{\left[ \phi + \Theta^N + \left( \Theta^N - \Theta^N \right) + \sigma_L \left( \chi^N - \chi^T \right) \right]} \right) \hat{a}^N
+ \frac{\left[ \Omega^N + \left( \Omega^T - \Omega^N \right) \right] \left( 1 + \Theta^N \right)}{\left[ \phi + \Theta^N + \left( \Theta^N - \Theta^N \right) + \sigma_L \left( \chi^N - \chi^T \right) \right]} \frac{d \ln \left[ \frac{(1-u_{NX})}{(1-u_{NV} - u_{VT}^2)} \right]}{d \ln \left[ \frac{(1-u_{NX})}{(1-u_{NV} - u_{VT}^2)} \right]}, \tag{413} \]

Differentiating the sectoral unemployment rate described by eq. (257) leads to:

\[ du^T - du^N = -\alpha v \left[ u^T \left( 1 - u^T \right) \hat{\theta}^T - u^N \left( 1 - u^N \right) \hat{\theta}^N \right]. \tag{414} \]

Inserting (410) into (406a)-(406b), substituting the outcome into (414), and collecting terms leads to the unemployment differential between tradables and non-tradables:

\[
du^T - du^N = -\alpha v \left[ u^T \left( 1 - u^T \right) \left[ \Sigma^T - \frac{\Sigma^T \left( 1 + \Theta^T \right)}{\Delta} \right] - u^N \left( 1 - u^N \right) \left[ \frac{\left( \Sigma^N - \Sigma^N \right) \left( 1 + \Theta^T \right)}{\Delta} \right] \right] \hat{a}^T
+ \alpha v \left[ u^N \left( 1 - u^N \right) \sum^N - \frac{\Sigma^N \left( 1 + \Theta^N \right)}{\Delta} \right] - u^T \left( 1 - u^T \right) \frac{\Sigma^T \left( 1 + \Theta^N \right)}{\Delta} \hat{a}^N
- \alpha v \left[ u^T \left( 1 - u^T \right) \sum^T - u^N \left( 1 - u^N \right) \sum^N \right] \frac{d \ln \left[ \frac{(1-u_{NX})}{(1-u_{NV} - u_{VT}^2)} \right]}{d \ln \left[ \frac{(1-u_{NX})}{(1-u_{NV} - u_{VT}^2)} \right]}, \tag{415} \]

where we set

\[ \Delta = \phi + \Theta^N + \left( \Theta^T - \Theta^N \right) + \sigma_L \left( \chi^N - \chi^T \right). \tag{416} \]
M.6 Correcting for the Bias

We now compute the bias originating from search frictions varying across sectors which must be accounted for in order to map theoretical results for the responses to a productivity differential into elasticities estimated empirically.

Because empirically we consider a productivity differential $\hat{a}_T - \hat{a}_N$, to make our estimates comparable with our numerical results, we have to adjust the long-run change in the relative price computed numerically with the following term:

$$\text{bias } \hat{p} = \frac{1}{\Delta} \left[ (1 + \Theta^T) - (1 + \Theta^N) \right] \hat{a}_N. \quad (417)$$

Subtracting (417) from (410) leads to the 'unbiased' relative price response to a productivity differential:

$$\hat{p}' = \hat{p} - \text{bias } \hat{p}. \quad (418)$$

The same logic applies to the relative wage. The long-run reaction of the relative wage described by (413) must be adjusted with the bias which reads as follows:

$$\text{bias } \hat{\omega} = -\left\{ \Omega^T - \left[ \Omega^N + \left( \Omega_{T,T'} - \Omega_{N,T'} \right) \right] \frac{(1 + \Theta^T)}{\Delta} \right\} \hat{a}_N + \left\{ \Omega^N - \left[ \Omega^N + \left( \Omega_{T,N'} - \Omega_{N,N'} \right) \right] \frac{(1 + \Theta^N)}{\Delta} \right\} \hat{a}_N. \quad (419)$$

Once the value of $\hat{\omega}$ has been adjusted with, we can map the deviation in percentage of the relative wage from its initial steady-state derived analytically into the elasticity of the relative wage, $\beta$, estimated empirically:

$$\hat{\omega}' = \hat{\omega} - \text{bias } \hat{\omega}. \quad (420)$$

The numerical computation of the unemployment rate differential is subject to the same bias as the relative price and the relative wage. We have to adjust the long-run change in the relative wage computed numerically with the following term:

$$\text{bias } (du_T - du_N) = -\alpha_V \left\{ u_T^T (1 - u_T^T) \left[ \Sigma_T - \frac{\Sigma_{T,T'} (1 + \Theta^T)}{\Delta} \right] ight\} \hat{a}_N - u_N (1 - u_N) \left[ \frac{(\Sigma_N - \Sigma_{N,N'}) (1 + \Theta^N)}{\Delta} \right] \hat{a}_N + \alpha_V \left\{ u_N (1 - u_N) \left[ \Sigma_N - \frac{(\Sigma_N - \Sigma_{N,N'}) (1 + \Theta^N)}{\Delta} \frac{(1 + \Theta^N)}{\Delta} \right] \right\} \hat{a}_N \quad (421)$$

The long-run reaction of the unemployment differential between tradables and non tradables described by (415) must be adjusted with the bias:

$$(du_T - du_N)' = du_T - du_N - \text{bias } (du_T - du_N) \quad (422)$$

References


