EFFICIENCY IN LOWER AND UPPER SECONDARY EDUCATION AS A NETWORK: IMPLICATIONS FOR THE NORTHERN POWERHOUSE

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ABSTRACT

The efficiency of secondary schools in England is evaluated using a network data envelopment analysis. School-specific efficiency scores at the various nodes of the network are regressed against environmental variables. It is found, inter alia, that schools located in the Northern Powerhouse are less efficient than others at the upper secondary level, though no less efficient at lower secondary level.

Keywords: education, network data envelopment analysis, Northern Powerhouse
JEL Classification: C44, I21

Introduction

In a BBC TV documentary first aired in March 2014, broadcaster Evan Davis argued for the small Yorkshire town of Hebden Bridge to become the second city of the UK.[1] The suggestion was, of course, outrageous. Hebden Bridge has a population of around 4500 people. The point being made by Davis was that Hebden Bridge is the centre of a latent inverted city – one with a green area at the centre and with the major urban areas of Liverpool, Manchester, Leeds and Sheffield around the periphery. Given improved transport links, the cities of England's north could combine to become a globally significant metropolis.

In June 2014, the then Finance Minister George Osborne declared that 'The cities of the north are individually strong, but collectively not strong enough… We need a Northern Powerhouse'.[2] To help build that powerhouse, substantial investment of public funds has been promised to improve the transport infrastructure[3], and major investment has been sought from Chinese investors to support housing and industrial development.[4]

The core idea underpinning the Northern Powerhouse is to be found in the work of the linguist George Zipf (1935). His research established that there is a common distribution that describes the frequency with which the most common word, the second most common word, the third most common word and so on appears in a text. Zipf's law, as it came to be known, was subsequently found to describe many analogous processes, including the distribution of city size within countries. Hence New York has roughly twice the population of Los Angeles, Delhi has about twice the population of Mumbai, and Sao Paolo has about double the population of Rio de Janiero. Spain is an exception to this rule – with Madrid (at around 5 million) and Barcelona (at about 4 million) being much bigger than other cities. The United Kingdom is another exception – London and its surrounding area has a population of around 14 million, but the next biggest cities are Birmingham and Manchester (both close to 3 million). Alongside Manchester, other urban areas in the north include West Yorkshire (2.2 million people), Merseyside (1.4 million) and South Yorkshire (1.3 million). Given the existence of agglomeration economies, a strong case can be made that England's second city is too small. Indeed, the meta-analysis conducted by Melo et al. (2009) cites numerous research papers that point to a central estimate of the elasticity of productivity with respect to city population of around 0.05. Expanding (say) Manchester to 2½ times its current size could lead to a 7.5% increase in its per capita output. Since the North West and Yorkshire and Humberside together account for some 16% of the national gross value added, this means that achieving the hitherto unrealised agglomeration economies in the Northern Powerhouse could permanently add around 1% to the national Gross Domestic Product. Small wonder then that the Northern Powerhouse has captured imaginations and become a key element of the government's national industrial strategy.[5]

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[1] 'OK, so Hebden Bridge isn't our second city. But its ability to attract urban professionals suggests there is a big city struggling to emerge… This city is a long spread out one, a bit like Los Angeles.' https://www.youtube.com/watch?v=EpUNIKB-WaU

[2] https://www.gov.uk/government/speeches/chancellor-we-need-a-northern-powerhouse

[3] http://www.transportforthenorth.com

[4] https://www.gov.uk/government/news/northern-powerhouse-investments-showcased-to-chinese-investors

[5] https://beisgovuk.citizenspace.com/strategy/industrial-strategy/supporting_documents/buildingourindustrialstrategygreenpaper.pdf

That said, there are some difficulties to be overcome. The major provincial cities of Liverpool, Manchester and Leeds lie along a long straight line rather than within a tight circle, so distances within the Northern Powerhouse are considerable. This has motivated the focus on improving transport infrastructure. That is, in itself, hindered somewhat by the presence of the Pennines, a mountain range that separates Manchester from Leeds and Sheffield. Indeed one of the more ambitious infrastructure projects being mooted is a trans-Pennine tunnel.[6] Cultural factors run deep – the rivalry between Manchester and Liverpool is not simply due to the competing claims of football clubs, but goes back at least as far as 1894 when Manchester industrialists opened the Manchester Ship Canal (similar in size to the Panama canal) as a means of bypassing dues raised on exports and imports at Liverpool. A further difficulty – and one that is the focus of the present paper – concerns education and the skills of the workforce. Maps based on 2011 Census data show the Northern Powerhouse to be a coldspot for the percentage of adult population holding a HE qualification.[7] In his 2016 Budget, George Osborne appointed Sir Nick Weller, head of the Dixons Academy Trust, a chain of schools in Bradford, to report on transforming educational standards in the north. Crucial for the growth prospects of a Northern Powerhouse is the ability of employers in the north to attract highly skilled workers

In this paper, I shall examine aspects of the performance of secondary schools throughout England. In doing so, I shall focus on the analysis of efficiency using the method of network data envelopment analysis (NDEA), providing for the first time an examination of schools that explicitly recognises their network structure, whereby the outputs of lower secondary education become an input into upper secondary schooling. I shall show that there are clear economic determinants associated with school performance, but that there is evidence of systematic inefficiency in some (but not all) aspects of schools' performance in the Northern Powerhouse. The remainder of the paper is structured as follows. In the next section, I discuss the NDEA model. I then bring the data to the model, and present results. The paper ends with a concluding section.

Methodology

The seminal work of Farrell (1957), Boles (1971), Førsund and Hjalmarsson (1974), and Charnes et al. (1978) gave rise to a substantial literature on data envelopment analysis. The basic model involves the evaluation of the relative efficiency of each of a number of decision making units (DMU), where each unit converts multiple inputs into multiple outputs. The method is particularly useful in contexts where market prices for the various inputs and outputs are absent, so that there are no natural weights that can be used meaningfully to aggregate the inputs and outputs into totals that can be compared by simple analysis of ratios. It assumes that, for each DMU independently, the weights vector is chosen that maximises that unit's ratio of weighted output to weighted input relative to that of its peers. Hence the linear program

$$\text{Max } h_k = \sum_{r=1}^{s} u_r y_{rk} \text{ s.t. } \sum_{i=1}^{m} v_i x_{ik} = 1, \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \leq 0, u_r \geq 0 \forall r, v_i \geq 0 \forall i$$

is solved for each of the k DMUs, yielding an efficiency score, h_k, for the kth DMU that lies in the unit interval. Here y_{rk} is the rth output of the kth DMU, x_{ik} is the ith input of the kth

[6] https://www.gov.uk/government/news/manchester-to-sheffield-trans-pennine-road-tunnel-routes-shortlisted
[7] http://www.hefce.ac.uk/analysis/coldspots/employment/

DMU and the u and v terms are the weights, choice of which maximises h_k. By allowing each DMU, in effect, to choose its own weights vector, the method allows a heterogeneity of missions across producers. This is at once part of the appeal of DEA, in recognising and celebrating difference between producers, and one of its demerits, since (at least in its simple form) it fails to penalise outliers. In effect, DEA constructs a production frontier for each DMU and calculates as an efficiency score the ratio of the radial distance from the origin, first to the production point of the DMU itself, and second to the frontier.

The basic DEA model has been extended in a variety of ways over the last three decades. A development on which we focus here involves the application of DEA to a system of production that is characterised by a network. This approach has its origins in the work of Färe (1991) and Färe and Grosskopf (1996a, 1996b, 2000), and has been developed further by Tone and Tsutsui (2009) to cast it in a slacks-based framework that allows convenient solution. The technology assumed within the network model is given by

$$\begin{aligned}
&\mathbf{x}^k \geq \Sigma_j \mathbf{x}_j^k \lambda_j^k, \quad \forall k \\
&\mathbf{y}^k \geq \Sigma_j \mathbf{y}_j^k \lambda_j^k, \quad \forall k \\
&\mathbf{z}^{k,h} \geq \Sigma_j \mathbf{x}_j^{k,h} \lambda_j^k, \qquad \forall k,h \text{ as outputs from node k} \\
&\mathbf{z}^{k,h} \geq \Sigma_j \mathbf{x}_j^{k,h} \lambda_j^h, \qquad \forall k,h \text{ as inputs into node h}
\end{aligned} \tag{2}$$

where **x** is the vector of inputs, **y** the vector of outputs and **z** a vector of intermediate products that are outputs from one node of the network and are simultaneously inputs into another node. The λ denote intensity vectors that are specific to the superscripted node. The summations are across all decision-making units. The linking activities are defined so that $\mathbf{Z}^{k,h}\lambda^h = \mathbf{Z}^{k,h}\lambda^k$ where $\mathbf{Z}^{k,h} = (\mathbf{z}_1^{k,h},\ldots, \mathbf{z}_n^{k,h})$. In this way decision-making units are free to decide upon the levels of the intermediate outputs that will be produced.

Subject to this restriction and to (2), the linear program

$$\theta^* = \min_{\lambda^k,\, s^{k-}} \ \Sigma_k \left[1 - \left(\sum_{i=1}^{m_k} s_i^{k-}/x_i^k\right)/m_k\right] \tag{3}$$

is solved for each DMU simultaneously to evaluate the overall efficiency scores θ*. Here m_k represents the number of inputs to the kth node and s is a slack. This overall score can then routinely be broken down into node-specific scores

$$\theta_k = 1 - \left(\sum_{i=1}^{m_k} s_i^{k-}/x_i^k\right)/m_k \quad \forall k \tag{4}$$

By contruction, the overall efficiencies that emerge from a network DEA are in general equal to or lower than those that are obtained from a corresponding ‘black-box’ model. This highlights areas (nodes) where it would be possible to improve – and possibly substantially improve – efficiency even in situations where overall performance appears to be strong.

In Figure 1, a simple network is shown. There are two nodes, labelled GCSE results and A level results. These represent the outcomes of national examinations typically taken by students in England at ages 16 and 18 respectively. The General Certificate of Secondary Education (GCSE) is a series of subject-specific examinations, taken across a wide range of subjects at the end of lower secondary education. Subject to satisfactory performance in these examinations (usually deemed to be 5 or more passes at a minimum grade of C, including mathematics and English), students may – if they choose to do so – progress to upper secondary education, during which they specialise in a relatively small number of subjects that are examined in the Advanced level (A level) examinations. Students that pass two or more A levels are then deemed to be qualified to enter higher education. In the network shown in Figure 1, the number of students in the relevant grade (cohort) and the number of teachers in a school are inputs into the production process; numbers of students achieving satisfactory performance at GCSE is an intermediate output; this forms an input into the second node (A level performance), alongside any students that join from other schools at this juncture.

Data

Data on the variables in this network come from the School Performance Tables published by the UK government.[8] These data concern all schools in England; schools in Wales, Scotland and Northern Ireland operate under distinct systems of education and are not therefore included. The data include information about student numbers, teaching staff, finances, type of school, location, and results achieved at the various key stages of education. Our interest is in secondary schools that provide education over the ages 11-18; not all secondary schools do, since many provide lower secondary education only, with students wishing to progress to study at A level then transferring to colleges or to schools that do provide upper secondary, as well as lower secondary, education. There are a little over 1500 of the latter type of schools.

Results

With so many data points, it would not be helpful to report results for each individual school. It is instructive, however, to look for patterns in the results, and that is the aim of the present section.

The network illustrated in Figure 1 is just one example of the way in which the secondary education system can be modelled. In Figure 2, an alternative view is presented. Here there are more inputs into the system – we divide student numbers into two component parts, namely students who receive free school meals and those who do not. This enables the model to make allowance for the social composition of the student body, since those having free school meals come from poorer backgrounds, since entitlement is based on receipt of various welfare payments. Another additional input used in this model is the total teacher salary bill of the school. This variable is positively related to the number of teachers, of course, but adding it to the model allows for variation in teacher seniority, a proxy for teacher quality. The model shown in Figure 2 also has a richer set of outputs at the first node – in addition to viewing the intermediate output of progression to upper secondary study in the same school as a positive outcome, numbers of students exiting the school to enter further education elsewhere, and (separately) to enter apprenticeship schemes are considered as final outputs at this stage.

In the remainder of this section, we consider results obtained by evaluating efficiency for each of the two networks shown in Figures 1 and 2. Figures 3-8 report the average efficiency

[8] https://www.compare-school-performance.service.gov.uk/

scores obtained by schools within each local authority in England, for each of the two models. Darker areas represent lower levels of efficiency. The magnified square to the right of the main map in each figure shows the pattern across the London boroughs. The basic DEA model appears in Figure 3. This shows regions of relative inefficiency in Tyneside, parts of the Northern Powerhouse, Shropshire, Oxfordshire and Somerset. This pattern is repeated in Figure 4, which shows (for the basic model) efficiencies at node 1 of the network (in effect, measuring schools' efficiency at converting inputs into successful results at GCSE). Of the London boroughs, Enfield, Lewisham appear as inefficient. (Kensington and Chelsea also appears inefficient, and this is something of a surprise as it is located in an affluent area – but there is only one school in the data, and close examination reveals that the software fails to identify an efficiency score for this school at node 1.) In Figure 5, Herefordshire, Tyneside, parts of the Northern Powerhouse, Southwark, and Thurrock (east of London) appear relatively inefficient. The relatively high inefficiency scores observed within the relatively densely populated areas of Tyneside and the Northern Powerhouse pose a challenge for the government's plans to regenerate those areas.

The results for the full model (Figure 6) likewise indicate relatively low levels of efficiency in parts of the Northern Powerhouse, and also in some Midlands towns (Wolverhampton, Coventry) and in Cornwall, Oxfordshire, Bedfordshire, Kent and Sussex. Several London boroughs also fall into this group, including Wandsworth, Kingston, Croydon, Lewisham, Greenwich, Hillingdon, Enfield, Camden and Islington. For the most part, this pattern is explained by the distribution of efficiencies at node 1 (that is, at lower secondary level), as seen in Figure 7. Some areas, particularly in Tyneside and the Northern Powerhouse, and in Shropshire and Herefordshire, suffer relatively low efficiency levels at node 2 (A level) though.

Figures 9-11 show, for the full model, the distribution of efficiencies. Figure 9 shows the efficiencies obtained from a standard DEA model; these are generally high, with a median value of around 0.87, and with a very small number of schools achieving scores below 0.6. The median efficiencies for each of the nodes are lower – 0.65 for the first node and just 0.29 for the second. It is clear from Figure 11 that there is a small number of schools with unusually strong performance; indeed the upper quartile of this distribution is at an efficiency score of just 0.43. While the distribution of node 2 efficiencies across schools has a large range, the distribution of local authority average node 2 efficiencies is somewhat narrower (see Figure 13) and this is why few authorities are shaded red in the map in Figure 8.

Some schools that perform strongly at node 1 in the full model do so because many of their students progress to further education at another institution rather than remaining with the school. These schools tend to achieve low efficiency scores at node 2 because students have not progressed *within* the school.

In future years, our ability to model the education system as a network will be enhanced by the availability of destinations data beyond upper secondary level. In particular, data will be available on students attending university – this measure being disaggregated so that, for example, the numbers from each school attending Russell Group universities will be known.

Figures 14 and 15 report, respectively for the basic and full network DEA models, scatterplots showing the relationship between node 1 and node 2 efficiency scores for each school. It is readily observed that several schools obtain zero efficiency score at node 1 in the basic model. This appears to be due to a computational problem, and so we concentrate our attention on Figure 15 and the results of the full network DEA model. A positive correlation exists between node 1 and node 2 efficiency, but it is clear that many schools are some way from the line of best fit – and in particular there are many schools with low node 2 and high node 1 scores, and also a few schools that score well at node 2 but poorly at node 1. Of those schools that are close to the line of best fit, casual inspection of the data reveals that larger schools tend to achieve relatively high efficiency scores at both nodes, while small schools tend to achieve low scores at both nodes. The group of schools that achieves high node 2 but low node 1 scores is typically large, and characterised by low proportions of students on free school meals; their low node 1 scores may therefore be attributable to the fact that they have favourable inputs.

To test this more formally, we perform, at school level, separate regressions of the standard DEA efficiency scores and the network DEA node 1 and node 2 efficiency scores against a set of explanatory variables. These include the total school roll and the unemployment rate in the local authority in which the school is located (in 2013).[9] We also include a binary variable to indicate whether or not the school is in the Northern Powerhouse.[10] The results are shown in Table 1. It is readily observed that the model has little explanatory power in connection with the node 1 efficiencies, but that it performs considerably better in explaining the node 2 efficiency scores. The absence of a Northern Powerhouse effect at lower secondary level may be due to the diversity of outputs considered at this level; it is known that there is disproportionately high participation in apprenticeships amongst young people in the north of England (Dominguez-Reig and Sellen, 2017). At upper secondary level, it appears that school efficiency increases with school size, and also with the prosperity of the local area (as measured by the inverse of the unemployment rate). Moreover, on average, at upper secondary level, the efficiency score of schools located within the Northern Powerhouse is some 4 percentage points lower than schools elsewhere.

This last finding is of considerable policy importance as government seeks to regenerate lagging regions. Infrastructure investments are likely to provide benefits, but the importance of human capital as a driver of growth is also well known. Unlike physical infrastructure, human capital is mobile; investment in schooling in poorer regions may have a perverse effect if it encourages the ablest to move away. A key task faced by those responsible for the development of the Northern Powerhouse will be to improve the efficiency of upper secondary education while ensuring that students schooled to higher levels remain within, or are attracted to, the region as they enter employment and progress within the labour market.

Conclusion

While the methods of DEA have been widely applied in the area of education, applications of network DEA have been few and far between (Johnes, 2013). In this paper, we have noted the network character of secondary schools in England, where (some of) the output of lower secondary education forms an input into upper secondary education, normally delivered within the same institution.

[9] http://webarchive.nationalarchives.gov.uk/20160105160709/http://www.ons.gov.uk/ons/rel/subnational-labour/regional-labour-market-statistics/december-2013/rft-lm-table-li01-december-2013.xls

[10] No unambiguous definition of the area covered by the Northern Powerhouse exists. For the purpose of this exercise, we deem it to comprise the following local authorities: Bolton; Halton; Knowsley; Liverpool; Manchester; Oldham; Rochdale; Salford; St. Helens; Stockport; Tameside; Trafford; Warrington; Wigan; Bradford; Calderdale; Kirklees; Leeds; Sheffield; and Wakefield.

Investigating the efficiency of secondary schools in England as a network has highlighted the fact that schools are typically performing better (in the sense of achieving higher levels of efficiency) at lower than at upper secondary level. Particularly at upper secondary level, there are clearly factors that are associated with better or worse performance. Once of these is location. Schools located within the Northern Powerhouse, on average, have lower efficiency at upper secondary level (but not at lower secondary level) than those located elsewhere. This is a topic worthy of further investigation, but in the meantime, it would appear that the education and development of highly skilled workers presents a challenge for the north that should not be overlooked by policy makers.

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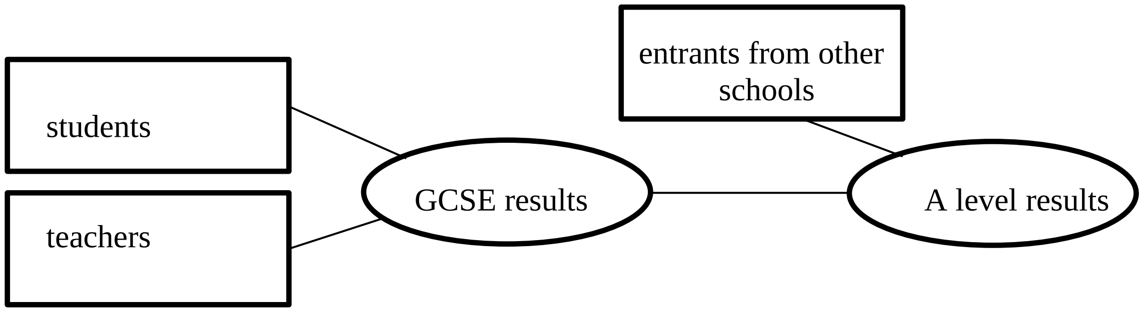

Figure 1 Basic network DEA model

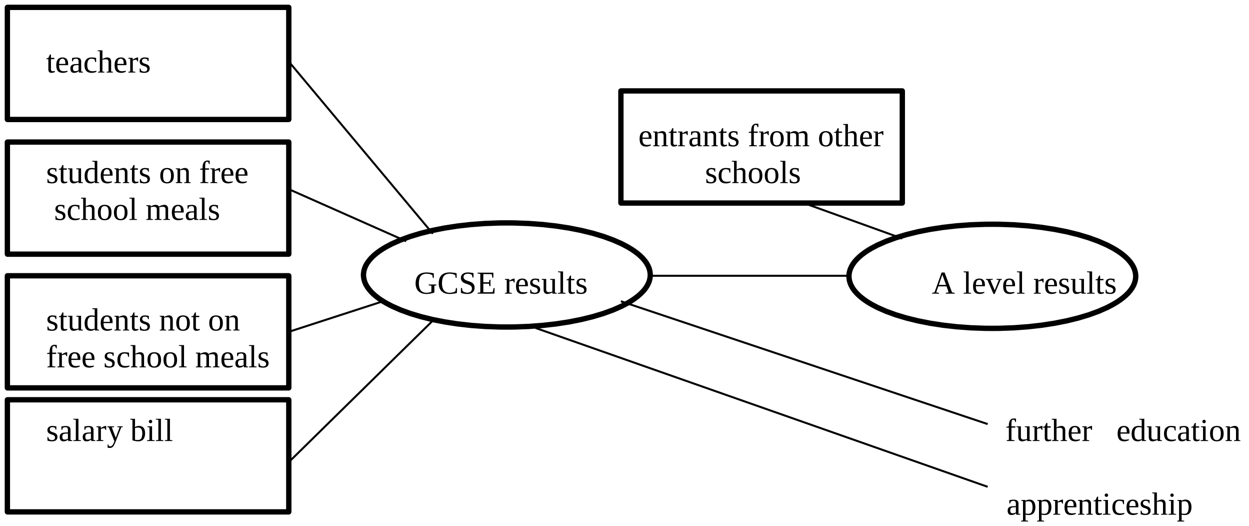

Figure 2 Full network DEA model

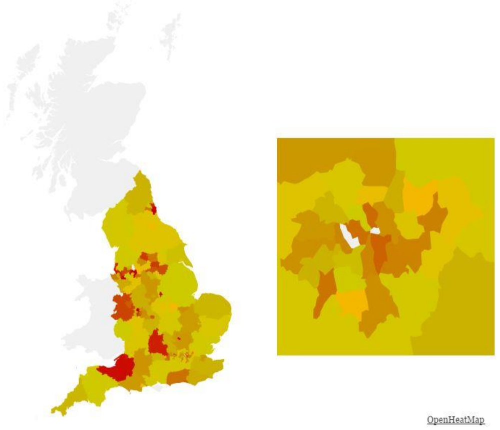

Figure 3 Basic DEA model

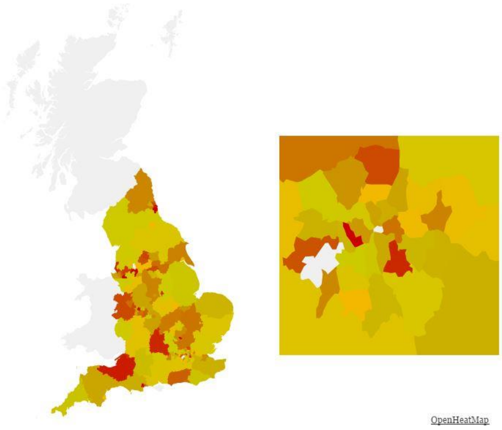

Figure 4 Basic network DEA model – node 1

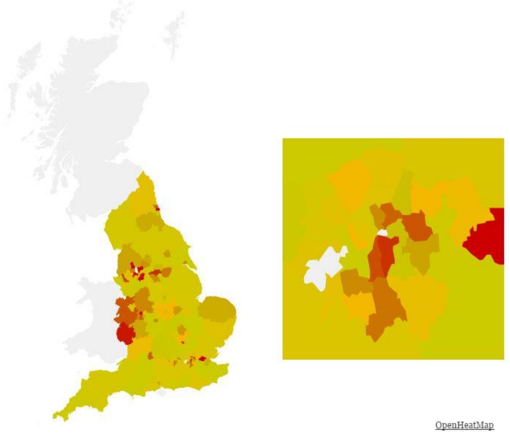

Figure 5 Basic network DEA model – node 2

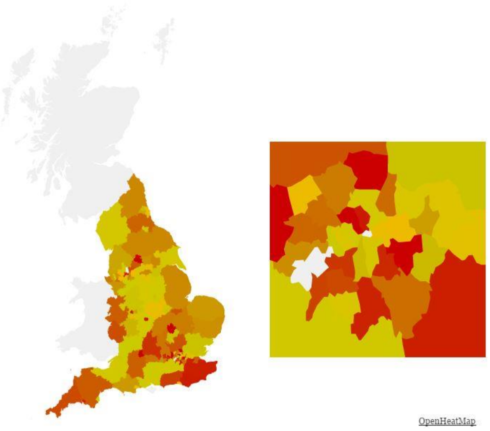

Figure 6 Full DEA model

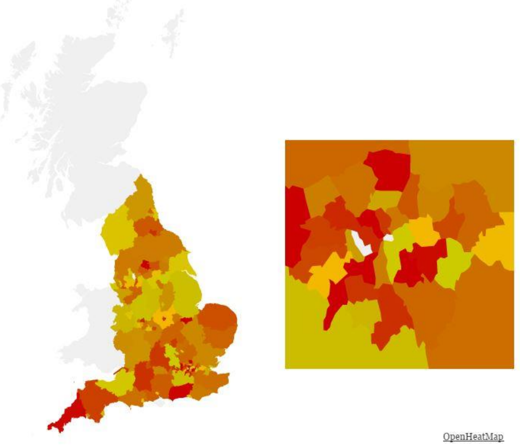

Figure 7 Full network DEA model – node 1

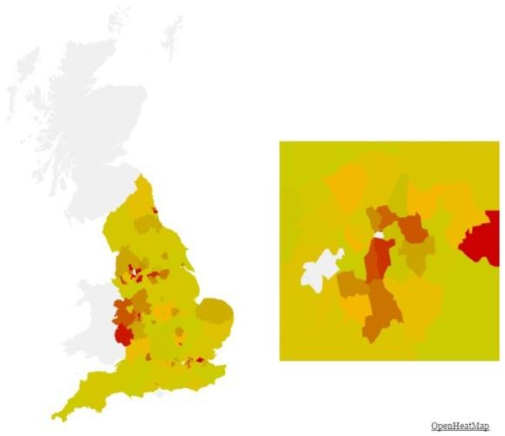

Figure 8 Full network DEA model – node 2

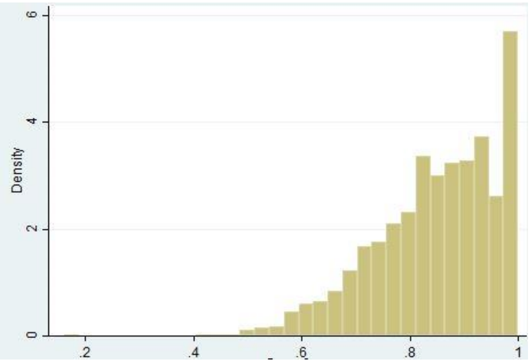

Figure 9 Full DEA model efficiencies

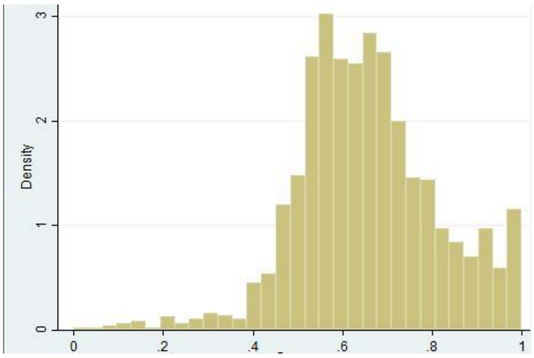

Figure 10 Full network DEA model efficiencies – node 1

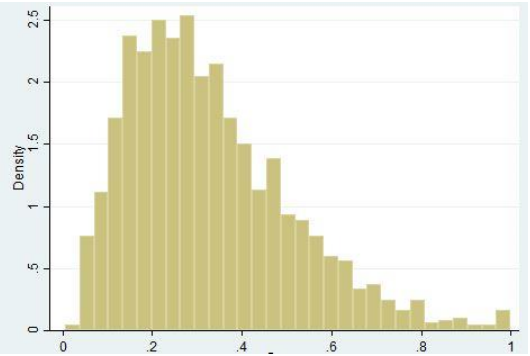

Figure 11 Full network DEA model efficiencies – node 2

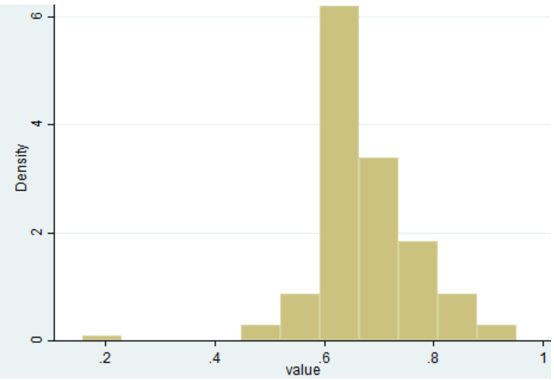

Figure 12 Local authority average efficiencies, full network DEA model, node 1

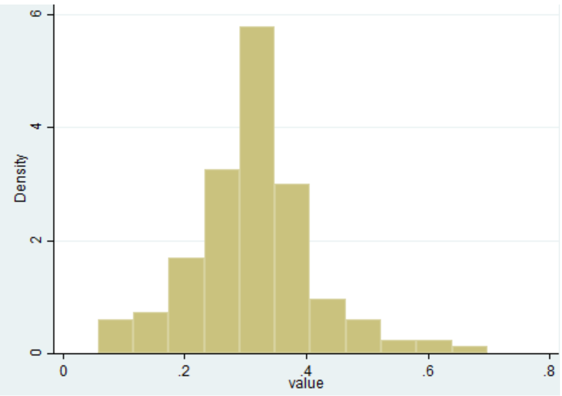

Figure 13 Local authority average efficiencies, full network DEA model, node 2

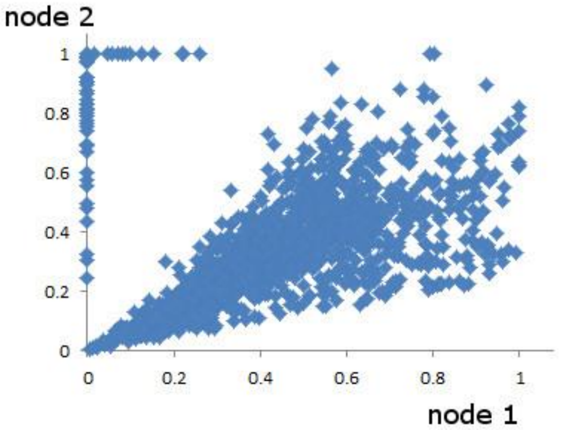

Figure 14 Scatterplot of node 1 v node 2 efficiency scores in basic network DEA model

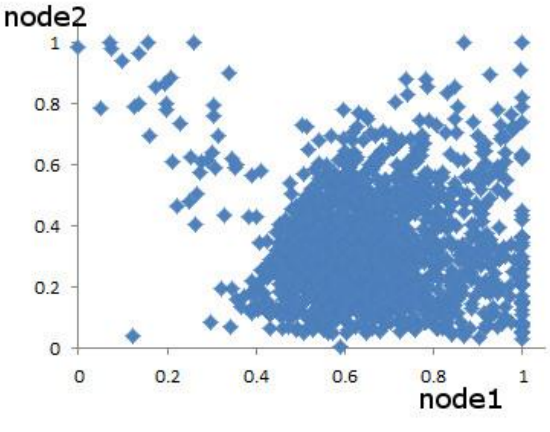

Figure 15 Scatterplot of node 1 v node 2 efficiency scores in full network DEA model

Table 1

Explanatory variable	Node 1	Node 2
School size (total pupils on roll)	-0.0000	0.0002
	(0.31)	(17.51)
Unemployment rate in local authority	-0.0026	-0.0058
	(1.50)	(3.08)
Northern Powerhouse	0.0178	-0.0363
	(1.24)	(2.46)
R^2	0.0023	0.2053
N	1507	1510

Note: t statistics based on robust standard errors in parentheses.