Teacher Educator Perceptions of Mathematical Knowledge for Teaching – A Phenomenographic Study

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This thesis was completed as part of the Doctoral programme in Educational Research.

No sections of the thesis have been published, or submitted for higher degree elsewhere and the thesis is a result of sole research.

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Declaration:
This thesis results entirely from my own work and has not been offered previously for any other degree or diploma.

Signature:

Abstract

Teacher education is a complex policy and practice landscape and teacher educator roles are changing. This thesis investigates variation in how teacher educators in England account for their practices within initial teacher education, in the context of secondary mathematics. It sets out a view of their experiences of teacher education in relation to different ways of accounting for relations between knowledge about *teaching and learning mathematics* and *knowledge of mathematics.* Outcomes are based on interviews with sixteen participants, including both school-based and university-based teacher educators. Teacher educator accounts comprised four, hierarchically inclusive descriptions of learning in teacher education: (1) understanding teaching and learning in mathematics (2) linking teaching and learning with the subject discipline of mathematics (3) integrating teaching and learning with the subject discipline of mathematics (4) reconceptualisation of teaching and learning with mathematics. Accounts of teacher education are structured as expanding opportunities for student teachers to engage critically with the process of reconceptualising mathematical knowledge into 'school maths' knowledge. The most developed accounts position teacher education as drawing on opportunities for student teachers to experience interplay between mathematics and school mathematics through advanced and multiple mathematics perspectives. This interplay is a site of learning in teacher education. Participants hold a range of teacher educator roles, including school-based and university-based roles. Teacher educators in school-led roles were less able to draw on this interplay in practice and so this thesis is a further and timely contribution to the research about university-based teacher educators**.** This thesis contributes a *curriculum forms* perspective that focuses on teacher educator positions on the role of the curriculum in developing new teachers.

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List of abbreviations

CK	Content Knowledge
CCK	Common content knowledge
DfE	Department for Education
HCK	Horizon content knowledge
ITE	Initial Teacher Education
ITT	Initial Teacher Training
KCS	Knowledge of content and students
KCT	Knowledge of content and teaching
MKfT	Mathematical Knowledge for Teaching
ORF	Official reconceptualising field
PCK	Pedagogical Content Knowledge
PGCE	Postgraduate Certificate in Education
PRF	Pedagogical reconceptualising field
PTICK	Pedagogical technology integration knowledge
QTS	Qualified Teacher Status
SCK	Specialized content knowledge
SMK	Subject matter knowledge
TPACK	Technological pedagogical and content knowledge
TPCK	Technological pedagogical knowledge
T&L	Teaching and Learning

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Chapter 1: Introduction

1.1 Context

In this chapter I outline the context for this doctoral research study. I set out the rationale for the study and develop the aims of the research, leading to a statement of the research questions. Following an outline of the methodological approach to the study and an outline of the organisation of the thesis, this section sets out an overview of the originality and contribution offered by this thesis.

I designed the study to explore the perspectives of teacher educators on their teaching practices. Specifically, the practices of teacher educators when developing subject-specific knowledge for teaching in student teachers who are studying to enter the teaching profession as new teachers. The aim was to understand how teacher educators describe negotiated meaning across their working contexts, as reflected in their accounts of practice. The study is set within the context of Initial Teacher Education (ITE) in England, an area that has undergone a significant sector change within the last decade. The study focuses on teacher educators who are working on programmes designed to prepare students teachers to teach mathematics in the secondary school age phase.

The study adopts a curriculum approach to exploring teacher education. This means that the teacher educators' accounts of their practice are examined in the context of forms of knowledge that shape school subjects. In particular, teacher educator accounts reflect perspectives on how what is taught and learned in schools is shaped by different influences on school curriculum forms of knowledge. These influences include forms of subject disciplinary knowledge and academic knowledge. As such, the term 'curriculum forms' will be adopted to describe taking such a curriculum knowledge lens on the outcomes of the study. As background to the curriculum context in England, a national curriculum (Department for Education, 2014) sets out expectations of entitlement for pupils. Due to structural changes made to the organisation of schools, this only applies to particular types of settings, as schools such as academies are not required to provide this statutory curriculum. In Chapter 2, I set out the ways in which the curriculum experienced by pupils in schools is also driven and shaped by examination specifications and accountability regimes. This is a site of tension and debate in education. It is timely to examine teacher educators' accounts of teaching practice. Debate over what is taught and learned in school coincides with changes made to teacher education itself, including a shift to school-led models and, in some cases, to generic models of teacher education in England, leaving less time for subject teaching (Brown et al., 2016). The role of the teacher educator is shifting in response to these changes, and new roles are emerging from this landscape, including school-based and hybrid models of employment for teacher educators, alongside more established higher education roles (White et al., 2015). A *curriculum forms* approach is introduced as a lens on the practices of teacher educators to illuminate the ways in which they draw on different curriculum knowledge and its influence on the curriculum that learners experience in school. At the heart of this analysis are teacher educator positions on the role of the curriculum in developing new teachers.

The initial teacher education (ITE) system in England is recognized as being one of the most complex and fragmented in the world (George and Maguire, 2018) and as such, teacher education is a complex activity (Douglas, 2017). This is set in a broader context whereby education policy and practices are seen as a reflection of a global tendency to position education as a key factor in driving forward economic agendas. The emergence of global education policy approaches are arguably aligned to public sector reform themes, such as quasi-marketisation, and quality and competition agendas (Ball, 2013). Not withstanding this context, this plays out through the education system in England in different ways to that of the rest of the UK and so this study focuses on teacher educators in English schools contexts.

1.2 Rationale

Consideration of the ways in which curriculum perspectives influence the practices of teacher educators is an important contribution to ongoing debates within teacher education. Teacher education has, for many years, wrestled with the challenges of reconciling theory and practice, and the relations between these, both in a practical and a theoretical sense (Ellis, 2010; Jackson and Burch, 2016). This is a particular challenge for teacher educators due to their positioning both inside and outside of higher education (Maguire, 2000). These tensions are felt even more acutely as teacher educators work through the implications of the shift to school-based teacher education in recent years (Hodson et al., 2011; Douglas, 2017).

Established debates in teacher education include those around approaches to teaching and learning, such as the integration of theory and practice as well as *who* teaches, e.g. through debates around teacher identity (Murray, 2008a; Izadinia, 2014; O'Brien and Furlong, 2015). The key debates in relation to the shift to school-based ITE have included the changing ways that university-based teacher educators relate to their subject (Brown, 2016), the role of research/academic accreditation (Cochran-Smith, 2012), the theory-practice interface, professional learning (Jackson and Burch, 2016; Jackson and Burch, 2018) and changing relations between universities and schools (George and Maguire, 2018). A curriculum focused approach contributes a fresh perspective to this area.

The importance of curriculum within the context of teacher education is that it constitutes simultaneously an aim, and a context in the context of developing new teachers for the profession. It therefore, both drives, and shapes, student teacher learning as novice teachers need to become prepared, and able, to teach the school curriculum (Department for Education., 2012). At the same time, this professional learning takes place in a school subject context, a space shaped by a subject discipline having been recontextualised into the school curriculum for the purpose of teaching and learning in the classroom (Bernstein, 2000). Examining learning in a school subject context is relevant to understanding learning in teacher education as it is a form of *knowledge in practice* (Ellis, 2009). A curriculum approach, in the context of this study means that I pay attention to the opportunities for learning in teacher education presented by learning within, and from, the different ways that disciplinary knowledge is transformed into school subject knowledge. As such a lens will be used on the outcomes of the study that aims to illuminate (or make visible), the influences on the way that teacher educators draw on curriculum knowledge. This approach enables a situated exploration of ways that Bernstein's reconceptualisation process plays out in practice in the context of teacher education. It also provides a way to explore teacher educator positions on the role of the curriculum in developing new teachers, bringing a fresh perspective here as it is linked to the context of shifting roles of the teacher educator. Taking such a lens on the findings will be referred to as a *curriculum forms* approach. As such, I will explore a link between mathematical knowledge and mathematics teaching and learning knowledge and use this as a theoretical lens on the practices of teacher education.

From a broader curriculum perspective, tensions around what is taught, learned and assessed in schools mean that changes to the school curriculum impact on teacher education. This impacts on teacher education firstly in relation to the aims of preparing new teachers for the school context, and secondly in relation to the way that the government drives school improvement through policy levers that include teacher education policy. Teacher educators work at the boundary of intersecting systems of education such as the school system and higher education. Understanding how teacher educators reconcile these positions within their accounts of practice is hence of value. Within this study I will consider some of the changing ways in which university and school-based teacher educators position the school curriculum in their practices of developing new teachers.

The study I therefore explore variation across teacher educators in order to develop an understanding of how these contextual conditions may or may not be reflected in accounts of practices. Due to the way in which the school curriculum is partitioned into subjects, and also due to the different ways in which different disciplinary subjects are classified and framed (Bernstein, 2000), I focus on a single subject case, namely mathematics. This provides the opportunity to explore mathematical knowledge for teaching in depth, whilst also reflecting on the broader implications of outcomes for teacher education.

1.3 Research questions

The overall research aim is therefore to determine the range of variation in the ways that ‘mathematical knowledge for teaching’ is described amongst teacher educators currently working in the field of mathematics ITE in England. Furthermore, the study aims to consider the relationships between the features of this variation, roles of teacher educators and teacher educators’ positioning of the school curriculum when developing new teachers.

The research questions, identified from the gaps in the existing research, are as follows:

1. **What is the range of variation in the accounts of the practices of mathematics teacher educators?**
2. **What are the qualitative differences between these accounts?**
3. **What positions on the school curriculum are taken up in the accounts in relation to mathematical knowledge for teaching in developing new teachers?**

1.4 Methodology

The notion that different characteristics of the phenomena, namely the purpose of teacher education, in the context of mathematics education, are discerned at different times led to a phenomenographic approach to this study. Phenomenography as applied to this research design focuses on what is argued to be a finite range of ways of experiencing a phenomenon. It draws on participants' accounts of their experiences rather than abstractions or espoused practices. By drawing together individual descriptions of participants' accounts into a single overview for analysis, I develop a hierarchical overview of descriptions of teacher educators' practices with particular focus on mathematical knowledge for teaching. I then examine the outcome space with respect to teacher educator positions on the mathematics curriculum, in order to enable a further development of relations between teacher educators and their practices.

1.5 Originality and contribution

There is a growing research base in relation to teacher education and mathematical knowledge for teaching. The previous research on university-based teacher educators is being increasingly expanded through the consideration of emerging school-based roles, although studies mainly focus on either school-based or university-based teacher educators rather than a collective. There is limited research based on the perceptions of teacher educators as reflected in accounts of teacher educators in England in relation to these areas.   As such, this research constitutes an original contribution by exploring the relationships between the purposes of teacher education and mathematical knowledge for teaching, as reflected in the accounts of teacher educators. This study specifically provides an original contribution based on a curriculum lens in relation to the practices of teacher educators and, as such, provides a fresh contribution to ongoing debates within teacher education.

1.6 Organisation of the thesis

Within this chapter I frame the study and provide an overview of key features. In Chapter 2 I discuss theoretical positions on the purpose of teacher education, roles of teacher educators, mathematical knowledge for teaching and the school curriculum context. I rigorously explore the methodological position, along with how this frames the study and the key implications of adopting a phenomenographic approach in Chapter 3. In Chapter 4 I provide an overview of the outcomes of the study and present the results as a phenomenographic outcome space. I explore the outcome space in depth in Chapter 5, with particular focus on the structure of the space, and how this is applied to the teacher education context. Particular reference is made to '*curriculum thinking'* (Deng, 2018), a further layer of elaboration of the disciplinary subject involved when a teacher recontextualises an institutional curriculum into classroom practice. A *curriculum forms* approach is developed as a lens on the practices of teacher educators to illuminate ways in which they draw on different curricula that influence the curriculum that learners experience in school. At the heart of this analysis are teacher educator positions on the role of the curriculum in developing new teachers.

Chapter 6 develops this further with consideration of the issues raised for school subjects beyond the mathematics curriculum, leading to consideration of the implications of the study for teacher education. Finally, in chapter 6 I go on to provide a revisit of the overall aims of the study, and an opportunity to reflect on the approach to the study.

1.7 Summary

Within this section I have set out the rationale and context for the study. I have argued that developing an overview of the range of variation reflected in the accounts of teacher educators contributes to developing further understanding about mathematical knowledge for teaching. Crucially, understanding which aspects of the phenomena are discerned by teacher educators and how this may relate to the context within which they operate has the potential to contribute new understandings to the discussion about practices within initial teacher education. In the following chapter I will explore theoretical and contextual background to the work of teacher educators, developing links with curriculum thinking.

Chapter 2: Teacher Education: An Overview

2.1 Introduction

In this chapter, I set out a contextual and theoretical background to the work of teacher educators. I identify links between issues in teacher education and those of the school curriculum, developing this into a focus on a specific school subject: mathematics. Mathematical knowledge for teaching is considered in light of the schools and teacher education context. By identifying areas for exploration, I develop the lines of argument that lead to the research questions, providing a frame for the research study. The argument develops thinking about policy and practice in teacher education and relations with changing roles in teacher education and the school curriculum.

In particular, within this chapter, I set out an argument that there are links between positions on knowledge within the school subject curriculum and how these are recognised and drawn on within the practices of teacher educators as reflected in their accounts:

(i) *Who*, *where* and *how* teachers are inducted into the profession is the subject of much debate, leading to a fragmented policy and practice landscape for teacher education. In particular, the role of the teacher educator is changing, and there are gaps in the previous research around the collective analysis of individual experiences of teacher educators based across established and emerging roles including those based in schools.

(ii) The policy and practice landscape means that teacher educators therefore experience teacher education differently at different times and in different contexts, including individuals experiencing multiple experiences relating to their context as well as their experiences changing over time . This difference in experience could be attributed to the way in which it is possible for teacher education to be seen as having a subset of characteristics out of the set of all possible characteristics.

(iii) Similarly, subject knowledge for teaching, and specifically, mathematical knowledge for teaching, also has the potential to be experienced differently by teacher educators at different times and in different contexts. Similarly, this difference in experience can be attributed to potential for mathematical knowledge for teaching to be seen as having different characteristics from all of the possible characteristics.

(iv) Mathematical knowledge for teaching and its relation to the experiences of teacher educators is timely to explore in the light of current changing landscapes in teacher education. Exploring mathematical knowledge for teaching by developing an overview of the accounts of teacher educators constitutes an original contribution to teacher education research.

(v) School subjects influence the context of teacher education. A curriculum lens offers a fresh perspective on teacher education. In particular, an original perspective would be gained from a study into how university-based and school-based teacher educators engage with different underpinning curricula as sites of learning in their accounts of practices in initial teacher education.

Within the following sections, I therefore set out a conceptual and theoretical background, developing these lines of argument and leading to a set of research questions. Within this section I establish that the research questions prompt an original contribution to knowledge. I also demonstrate how the questions have been developed, and how they relate to previous research.

I will proceed by setting out how teacher education is positioned and experienced depending on the particular context. Arguments that this positioning affects the role of the teacher educator, who undertakes these roles, and their practices are developed. The increasing variation in the locations of teacher educator roles means that this positioning is worthy of closer examination. I examine a single subject, mathematics, in order to enable a focus on experience in practice. I also examine mathematical knowledge for teaching, and how this has developed in a mathematical context building on Shulman's (1986) notion of pedagogical content knowledge (PCK). These developments incorporate specific forms of mathematics that are particular to the purpose of teaching in a mathematics classroom. Links are drawn between these areas of previous research and dimensions such as curricular thinking. I conclude the section by summarising a rationale for exploring the perceptions of teacher educators of mathematics for teaching, in relation to the process of developing new teachers.

2.2 The landscape of teacher education

There are numerous policy and practice features of teacher education that shape the roles of, and influence the experiences of, teacher educators working in the sector. As such, these features can be described as a landscape. This section sets out the policy and practice landscape of teacher education. Through consideration of key issues, I identify the ways in which *where* and *how* teachers are inducted into the profession, which is a subject of debate. With this section I introduce these key issues in teacher education, including contestations about relations between theory and practice, the pedagogy of teacher education, and the role of research. The purpose of this section is to highlight what is argued to be a fragmented policy and practice landscape.

Teacher education is a space of deliberation and tension, both internationally (Grimmett et al., 2009; Cochran-Smith, 2012; Furlong, 2013; Ellis and McNicholl, 2015) and in the English context (Whitty, 2013; Whitty, 2014). These contestations have historically centred around two main areas of debate in teacher education. The first area relates to structural issues, often manifesting in *who/where* questions relating to where the responsibility and resources should be focused for inducting new teachers into the profession. The second area of debate is about the induction and training of new teachers. This manifests in *how* questions about pedagogy in teacher education, including the interplay between theory and practice, the role of research, and a pedagogy of teacher education

Within England, government policy is currently framed around Initial Teacher Training (ITT) models rather than Initial Teacher Education (ITE). This thesis will refer to ITE as this is a broader approach to teacher education than a narrow ‘training’ focus, although nomenclature of ITT and trainees will be used where these relate to government policy. The distinction reflects more than just a nuance of wording; rather it conveys a marked shift to associated apprentice models of teacher training (McNamara and Murray, 2013). The government vision for ITT was articulated in the paper *The Importance of Teaching: The Schools White Paper (Department for Education., 2010),* which framed learning to teach as a practical skill for development, reinforced in subsequent policy papers (Department for Education, 2015; Department for Education., 2016). This positioning of the process of learning to teach as a craft, rather than as an academic endeavor was championed by Michael Gove, the Secretary of State for Education:

> "*The evidence shows the best teacher training is led by teachers; that the skills which define great teaching - managing behaviour, constructing compelling narratives, asking the right questions, setting appropriate tasks - are best learnt from great teachers; that the classroom is the best place for teachers to learn as well as to teach […][School-based ITE], gives aspiring teachers the opportunity to work in a great school from day one, just like student medics in hospitals - learning from more experienced colleagues and immediately putting their new skills into practice.*" (Gove, 2013)

The shift to school-led routes into teaching was driven forward through a series of policy levers such as shifts in resource funding and persistent promotion of school-led routes over university-based routes by the Department for Education. This diversion of funding from university providers to school-led routes was argued by Jackson and Burch (2016) to be one of the most far-reaching changes to ITE since the introduction of minimum required time in schools as part of the teacher training process (Department for Education, 1992). These changes impact on teacher education to the extent that they are seen to be changing what it means to enter the profession and what it means to be a teacher (Brown, 2018).

The changing nature of teacher education is influenced by global and national drivers. Initial Teacher Education in England is currently operated as a quasi-market, meaning that there are aspects of market forces underpinning the government approach to the allocation, rationing and regulation of the resource allocated to training places for ITE (Ball, 2013). Within this market for educational services, there are contracts required for training places between the government, universities, and schools. However, this is a restricted internal market, as only English universities and schools accredited by the Department for Education (DfE) are able to participate. Additionally, and again in contrast to a conventional market, within the ITE quasi-market places can be rationed accordingly to a series of institutional metrics, and they are not stable from year to year. Providers not meeting baseline metrics are required to exit the market. ITE is delivered in partnerships, often between universities and schools, but the term 'partnership' is often ill-defined and arguably masks power imbalances, such as those arising from financial tensions (Ball, 2013). Universities are increasingly positioned as 'providers' and schools positioned simultaneously as partners, customers, and providers. As part of the shifts in influence on what is valued in the process of becoming a new teacher, the nature of teaching itself is contested in England, and the perception of the value of research in relation to teacher education has reduced over time (Beauchamp et al., 2015). Beauchamp points out that, in England, in contrast to other parts of the UK, there is a shift as teacher education moves increasingly away from universities and towards a growing set of school-based providers in line with a policy direction of *'(re)-return to the practical'*. The current policy context direction is that of a shift of resources and decision-making from the university to the school sector, with the result that the long-term involvement of universities in teacher education is uncertain for some (Furlong, 2013).

Furthermore, from a curriculum perspective, tensions around what is taught, learned and assessed in schools means that changes to the school curriculum also impact on teacher education. Teacher curriculum development has not been a focus for many schools in recent years, due to schools' focus on assessment outcomes (Spielman, 2018). This impacts teacher education, including the development of new teachers, in two key ways: firstly due to the aims to prepare new teachers for the school context and secondly due to the way in which the government drives school improvement through policy levers, that include teacher education policy. Furthermore, it is known that there are relations between student teacher orientations towards the curriculum and how teachers develop as they prepare to teach. Twiselton (2004), for example, in her work with student teachers teaching literacy in primary schools, identified three orientations, namely '*task manager*', '*curriculum deliverer*' and '*concept/skill builders*'. Additionally, shifts in what is taught and learned in schools, as well as the diversion of funding from universities engaged in teacher education, has led to increasingly to generic models of teacher education in England, with less time for subject teaching (Brown et al., 2016).

Teacher education has, for many years, wrestled with the challenges of reconciling theory and practice and the relationships between these, both in a practical and a theoretical sense (Ellis, 2010; Jackson and Burch, 2016). This is a particular challenge for teacher educators, due to their positioning both inside and outside higher education (Maguire, 2000). These tensions are felt even more acutely as teacher educators work through the implications of the shift to school-based teacher education in recent years (Hodson et al., 2011). Within this context, the complexity of teacher education is sometimes subject to reductive models, at its worst reducing the process down to a series of 'tips for teachers'. Loughran encapsulates a more nuanced approach stating that:

> *"Teaching teaching is about thoughtfully engaging with practice beyond the technical; it is about using the cauldron of practice to expose pedagogy (especially one's own) to scrutiny."* (Loughran, 2014: p275)

Loughran goes on to suggest that it is through shared and purposeful collaborative inquiry approaches that the sophisticated pedagogical raising can be made explicit and examined as part of the process of developing new teachers.

Within this section I have outlined some of the key debates in teacher education around who inducts new teachers and where and how they are inducted. These debates influence the purpose of teacher education and how this is positioned and experienced in a particular context. These experiences directly affects the role of the teacher educator who undertakes these roles, where they are located and the activity that they undertake. I will explore this further in the next section.

2.3 The role of teacher educators

Within this section I will examine key issues in relation to new and emerging roles in teacher education, and how these are being approached in current research. The policy and practice landscape means that teacher educators therefore experience teacher education differently at different times and in different contexts. I argue that the role of the teacher educator is changing, and there are gaps in the current research around the collective experiences of teacher educators.

As the context and purpose of teacher education undergoes change, the roles of those in the sector have also diversified. The term ‘teacher educator’ is frequently used in the research to mean university-based teacher educators (Cochran-Smith, 2003; Murray, 2008a; Grimmett et al., 2009; Boyd and Harris, 2010; Field, 2012; Izadinia, 2014; Douglas, 2017; Ping et al., 2018; Yamin-Ali, 2018). Less prevalent are studies that examine the roles of school-based teacher educators, such as the van Velzen and Volman (2009), study of the activities of school-based teacher educators, and White’s (2014) study of the perceptions of school-based teacher educators in relation to their teacher/teacher educator roles. The role of a ‘teacher educator’ can be defined generally as that of an education professional involved in the education of teachers (Loughran, 2014). It comprises teacher educators employed by schools, universities, or both (Zeichner, 2010; White et al., 2015). This includes emerging school-based teacher educator roles, where teachers working in schools support the professional learning of student teachers. This would include, for example, a school lead for a school-based programme, delivered in partnership with a university. School-based educators in such roles typically support the development of student teachers through mentoring. Additionally, they undertake subject or professional studies teaching to groups of student teachers, and coordinate aspects of the ITE programme across a school or groups of schools. There is a fuzzy boundary around school-based teacher and teacher educator roles, with some professionals in these roles either reporting that school leaders do not recognise these roles, (for example through status or time allocation), or they do not recognise their role themselves. For this reason, these roles can be described as being roles of ‘hidden professionals’ (Livingston, 2014). There is currently no professional designation via accreditation or professional designation for the role of a teacher educator in England and, as such, it is an ill-defined group (Murray, 2008c; White et al., 2015). An alternative way to frame the role is based on the notion that the work is focused on both the subject (or discipline) alongside the knowledge of how to teach that subject (Murray, 2008b; Murray, 2008c). Although Murray’s (re)-conceptualisation was focused on teacher educators based in Higher Education, this is nevertheless useful.

The diversification of roles presents challenges to teacher educators in relation to the identities they construct. Research in the area of teacher education frequently focuses on identity (Murray, 2008a; Izadinia, 2014;

O'Brien and Furlong, 2015). Identity appears in the teacher education research in relation to themes such as transitions (Amott, 2018), the interplay of identity with external or contextual influences (Grimmett et al., 2009; Dinkelman and Todd, 2011), and tensions between professional and research aspects of the role (Hökkä et al., 2012). Field (2012) explores changes in the this 'hard-won' new professional identity when developing understanding of pedagogy of teacher education, and the tensions involved in the shift from expert teacher to novice teacher educator. Aspects of these tensions are particular to teacher education, as teacher educators not only teach about teaching, but are expected to role model teaching as they do so. This is different, for example, from the tensions that may be identified in other professional areas (Boyd and Harris, 2010; Field, 2012). Transition from professional practice (school teaching roles) into university-based teacher educator academic roles involves thinking about reconciling these identity shifts (Maguire, 2002; Murray, 2008a; Boyd and Harris, 2010). Boyd and Harris (2010) highlight that this transition can often involve '*rapid immersion'* into work, with particular emphasis on teaching" (p20). Meanwhile, teacher educator identities continue to build "*in context, in practice and over time*" (Dinkelman and Todd, 2011: p314). This suggests that length of time in the role of teacher educator is a consideration for any exploration of features of accounts of practice. Similarly, identity shifts may also occur during the transition from the role of a teacher to school-based teacher educator (White, 2014).

Other aspects of recent research relating to teacher educator roles includes the teaching of particular aspects of teacher development such as diversity (Han, 2016), and relations between competence and amount of experience as a teacher educator (Hollins et al., 2014). There is evidence that professional learning of teacher educators is in different forms, and includes informal workplace learning (Bouckaert and Kools, 2018; Ping et al., 2018). Previous research explores informal learning along with on-going professional development for teacher educators (e.g. Montenegro Maggio, 2016; Meeus et al., 2018) and other types of professional learning, including self-study research (Vanassche and Kelchtermans, 2015). Factors shaping the professional development experiences of teacher educators include challenges linked to transitioning from practice, the nature of teacher education, and the value of researching the practices of teacher educators (Loughran, 2014). Almost this entire research base is focused on university-based teacher educators. It is known that the working context of teachers has an influence on the way that their expertise develops (Berliner, 2001). I argue that this extends to ITE expertise for school-based teacher educators roles.

The role of the teacher educator in England has diversified in recent years due to the introduction of school-based training routes as part of a shift from universities to schools (Brown, 2015; Jackson and Burch, 2016; Douglas, 2017). As noted, the majority of studies focus on university-based teacher educators; however, in line with the changing context of teacher education in England, some recent studies focus particularly on the roles of those involved with school-based teacher education (van Velzen and Volman, 2009; Livingston, 2014; White, 2014; White et al., 2015). Brown et al. (2015) looked at experiences of university-based teacher educators along with school-based mentors and student teachers, finding increasing fragmentation in school-based models of ITE. Emerging recent research has focused on emerging and hybrid school-based teacher educator roles. The key debates in relation to the shift to school-based ITE have included the changing ways that university-based teacher educators relate to their subject (Brown, 2016), the role of research/academic accreditation (Cochran-Smith, 2012), the theory-practice interface, professional learning (Jackson and Burch, 2016; Jackson and Burch, 2018), and changing relations between universities and schools (George and Maguire, 2018).   These debates rarely focus on relations between school–based teacher educators' roles and the school curriculum. A curriculum lens on the accounts of teacher educators therefore contributes a fresh perspective to this area.

Changing roles in teacher education have been linked to beliefs about the process of developing beginning teachers, and changes to the way that theory and practice underpin pedagogy in ITE (Brown et al., 2015) . There is, however, a gap in the previous research that looks holistically or collectively at school-based and university-based teacher educators. This may be due to the way that school and university-based teacher educators as groups experience different expectations in terms of their qualifications, roles and, expectations from their employing institutions (White et al., 2015). Both groups, however, contribute to the student teacher experience of the 'what' and the 'how' of teaching on a PGCE programme. Understanding the collective perspectives of these groups, analysed collectively, furthers the understanding of the student teacher experience. This provides a rationale for a focus on participants drawn from both university-based and school-based roles, with a view to understanding their experiences as a collective.

There are a plethora of routes into teaching and so it is necessary to consider a focus for the study. The initial teacher education (ITE) system in England is recognized as being one of the most diverse in the world (McKinsey and company, 2007). Due to the levers of the quasi-market intervention by government in the area of Initial Teacher Education (Ball, 2013), there are differences in the ways that university-led and school-led programmes go about recruiting new entrants to teaching. These differences are mainly due to contextual constraints of schools (Davies et al., 2016). Davies et al also identified evidence which suggests that school-led recruitment practices may tend to emphasise short-term and school-specific needs. There is variability in the content for ITE nationally, with a diverse range of programmes for the development of subject knowledge. For the present study, participants are engaged with programmes underpinned by Postgraduate Certificate in Education (PGCE) programmes to enable a focus on the programme areas where school-based and university-based roles usually interact in the process of inducting new teachers into the profession.

In summary, to develop a meaningful understanding of the accounts of teacher educators in this diverse landscape, within this research study I focus on teacher educators based in both school and university contexts. This thesis contributes to the current gap in relation to understanding school and university-based teacher educators as a collective. Whilst a comparison of conceptions of teacher education of school-based and university-based teacher educators would contribute to existing research, I argue instead that these roles are fluid and often overlapping, and a more nuanced form of understanding is required. This suggests that a focus on the variation *within* a field of teacher education would build effectively on existing research, rather than comparisons *between* groups or sub-fields. The opportunities offered by considering influences on the school curriculum as a site for learning in teacher education has not been explored to date and will be developed in Section 2.6. Within the next section (2.3) I briefly considers teacher educator roles in the mathematics context.

2.4 The role of teacher educators: mathematics focus

A focus on teacher educators who develop teachers to teach mathematics in secondary schools raises some particular issues for consideration. Within the previous research relating specifically to teacher educator roles in mathematics, Beswick and Goos (2018) take a systematic overview and identify gaps around the beliefs underpinning the practice of mathematics teacher educators. They argue that increasing accountability is a reason to be focused on the knowledge of beginning teachers pointing out an inevitability that the context of increasing focus on quality in teacher education leads to increased scrutiny of the practices of teacher educators. Other research focusing on teacher educators in mathematics includes a distinction between stable and contextual dependent beliefs of teacher educators (Rott and Leuders, 2016), situated perspectives (Bednarz and Proulx, 2017), and the mathematical knowledge of mathematics teacher educators (Masingila et al., 2017). Given the previously identified shifts in models of teacher education in relation to subject/generic balance it is therefore relevant to consider teacher beliefs when focusing on a mathematics teacher education.

In relation to mathematics, it is recognised that a student teacher's prior beliefs about the nature of mathematics and experiences of learning mathematics will influence the ways that they teach the subject (Beswick, 2012). It has been identified for some time now that the development of teacher identity in teacher education involves paying attention to existing beliefs and how these are reconciled with developing professional dimensions and the educational context of becoming a teacher (Beijaard et al., 2004; Beijaard and Meijer, 2017). Whilst beliefs are just one aspect of a range of possible affective dimensions such as emotion, disposition and attitude, beliefs are said to be stable and internally held, and so more resistant to change overtime (Leavy and Hourigan, 2018). The beliefs of student teachers are known to influence their actions and decision making (Pajares, 1992).

The boundary between knowledge and beliefs can be challenging to define. Pajares' focuses on the link between teacher beliefs and practice, suggesting that, for educational research, it is useful to focus on teacher "*beliefs about …".* This allows for consideration of context and enables a focus to be on beliefs about, for instance, learning/resources, learners and so on. Dissonance between types of beliefs does mean, however, that not all beliefs will appear in a teacher's practice (Schoenfeld, 2002).

The perceptions of *teachers*, as reflected in their accounts of practice are explored within the research (Bell, 2015; Belt and Belt, 2017; Anthony et al., 2018). Although the accounts of *teacher educators* have been used to explore foci such as their role in curriculum development (Bouckaert and Kools, 2018), or perceptions of their professional work with HE students (Shagrir, 2015), these have rarely been explored in relation to mathematical knowledge for teaching or the role of teacher education. As such, this constitutes an important contribution of this thesis.

Within the following section I will further develop the rationale for exploring teaching and learning of subject knowledge with teacher educators.

2.5 Subject knowledge for teaching

Developing subject knowledge in beginning teachers may be experienced differently by teacher educators at different times and in different contexts. The process of becoming a teacher involves the development of forms of teacher knowledge for the novice teacher. Influential models for conceptualising forms of knowledge for teaching include the separation of 'knowing how' from 'knowing that' (Ryle, 1949) and Ellis's (2009) more recent distinction between 'teacher knowledge' and 'teacher thinking'. Schulman (1986) is renowned for the notion of Pedagogical Content Knowledge (PCK), with its focus on the intersection between subject matter knowledge and knowledge about teaching and learning. This was one of the seven categories of teacher knowledge proposed by Shulman and colleagues (Shulman, 1986; Shulman, 1987; Grossman et al., 1989):

- general pedagogical knowledge;
- knowledge of learners characteristics;
- knowledge of educational context;
- knowledge of educational purpose and values;
- content knowledge;
- curriculum knowledge;
- pedagogical content knowledge.

Shulman acknowledged that the distinction between knowledge and pedagogy is not a centuries old tradition, but rather a more recent one with Shulman's model being a key influence on the way that knowledge for teaching has subsequently developed. PCK is often referred to in the context of *transformation* of subject content matter into subject knowledge for teaching. This is of particular relevance to student teachers, as this implies a process of transformation of their prior knowledge into different forms for teaching. Exploring how teacher educators account for this transformation in their practice has the potential to contribute to this area.

There are a number of critiques of Shulman's conceptualisation of PCK (Depaepe et al., 2013), including a lack of theoretical basis (Ball et al., 2008) with a charge that PCK is '*thinly developed'* (Ball et al., 2008: p389). As a consequence, the lack of description of pedagogical subject knowledge has been a barrier to translating this into the design of opportunities for teacher education (Schneider, 2015). Combined with a lack of *signature pedagogy* (Shulman, 2015) or clear articulation of a pedagogy of teacher education, this suggests that, although a long-established notion, there is still work to do to realise the potential of PCK to contribute to the development of teacher education. Other limitations of Shulman's original conceptualisation of PCK include little attention to: emotion and affect, pedagogy in action, broader societal and cultural context, outcomes (Shulman, 2015), and an uncritical orientation towards the teacher as a transmitter of knowledge (Meredith, 1995).

It is useful to consider Fennema and Franke (1992) situated model for mathematics teaching in relation to these critiques. Their model comprised four elements of mathematics for teaching: knowledge of the content, knowledge of pedagogy, knowledge of students' cognition, and teachers' beliefs. There are similarities with Shulman's model, including the notion that understanding how learners think and learn is crucial to mathematics teaching. Learners' conceptions are considered to be an element in their own right, rather than being part of the pedagogical knowledge as in Shulman's model. What is significant here is that Fennema and Franke's development focuses on knowledge development within context.

Affective dimensions have long been explored in relation to mathematics teacher education, including teacher beliefs (Pajares, 1992; Cady et al., 2006; Beswick, 2012; Hudson et al., 2014; Leavy and Hourigan, 2018), emotional aspects (Cross and Hong, 2009; Nichols et al., 2017), and dispositions (Golding, 2017; Meschede et al., 2017). Research on PCK has been further advanced to integrate affective dimensions (Gess-Newsome, 1999; Magnusson et al., 1999; Kuntze, 2012; Meschede et al., 2017). In the context of mathematics education, *specialised content knowledge* (Ball et al., 2008) has been develop to understand the work of teaching mathematics (Ball, 2017). This has been further explored in relation to student teacher beliefs (Swars et al., 2018).

Subject knowledge for teaching has also developed since its inception (within the context of initial teacher education (van Driel and Berry, 2017)). With regards to developments within ITE, van Driel and Berry's overview identified studies that examined relationships between PCK and the following aspects of ITE: academic coursework, combinations of academic coursework and school-based activity, and mentors and supervisors. They recommend that it would be beneficial for the sector if researchers were to examine the role of school-based mentors and university-based supervisors in supporting the developments of PCK.

(Banks, 2008) takes the view that supporting teachers to engage explicitly with their professional knowledge development is linked to a teacher's' 'personal subject construct'. Within the context of technology Banks draws on a model of teacher knowledge that attempts to synthesise 'school knowledge', 'subject knowledge' and 'pedagogical knowledge'. Whilst, as Ellis (2007) identifies, this model does not capture situated aspects of professional knowledge development (such as social), this nevertheless highlights knowledge development as participation in contrast to 'acquisition' models of learning.

Additionally, (Deng, 2018) suggested that that the underlying assumption that a teacher transforms subject matter knowledge into pedagogical forms does not sufficiently take into account the teachers role in the transformation of institutional curriculum into the content of the classroom curriculum. Furthermore, this is problematic in relation to pedagogic decision choices by teachers. Indeed, (Pollard, 2005) argues that:

> "*The specific nature and form of subject knowledge, skills and other learning challenges is of great importance. Enhancing learning outcomes depends on successfully managing the tensions between preserving the perceived integrity of forms of knowledge and constructively engaging learners*." (Pollard, 2005: p101)

Building on this, I argue that, in the ITE context, the assumption that a teacher transforms subject matter knowledge into pedagogical forms does not take into account the influence on this process by other knowledge formations. The potential results of such formations include disjoints/tensions between the school curriculum, the subject of mathematics, local curricula practices of schools and the influence of the official practices, such as those reflected in government requirements on the curriculum as it is experienced in schools. This supports the notion that a curriculum lens on teacher education can offer a fresh perspective. Furthermore, research based on the accounts of emerging roles, such as school-based teacher educators alongside established roles such as university-based teacher educators, therefore builds on this and constitutes a further contribution to this gap in the research base.

In summary, whilst there is a considerable body of knowledge that has developed since Shulman's foundational work on subject knowledge, this predominantly focuses on teacher knowledge; there has been less of a focus to date on developing this work in relation to the work of teacher educators. Development within teacher education has been taken up particularly in the contexts of mathematics and science (Hasweh, 1987; Gess-Newsome, 1999; Ma, 1999), although studies have also been undertaken in the context of physical education (Loughran and Berry, 2005; Ball et al., 2008; Berry et al., 2015; Schneider, 2015; van Driel and Berry, 2017; Hayden and Baird, 2018), English as a foreign language (Kim et al., 2018), geography (König et al., 2017), technology (Arenas-Martija et al., 2017), history (Koh and Chai, 2016), and outdoor education (Monte - Sano, 2011). Mutton et al. (2011) identified that novice teachers draw on subject knowledge particularly at the start of their trajectories of learning to lesson plan. Research on subject knowledge based on the way that a teacher transforms subject matter knowledge into pedagogical forms has not taken into account the influence on this process of influencing curricula, such as the disciplinary subject, curriculum and is worthy of exploration.

Having established a gap in the previous research in relation to a curriculum approach to teacher education, I take up the development of PCK within mathematics education in section 2.6 to argue that mathematical knowledge for teaching has the potential to be experienced differently by teacher educators depending on their context.

2.6 Subject knowledge for mathematics teaching

Mathematical knowledge for teaching has a range of characteristics and that may be discerned at different times and in different contexts. Shulman's work on pedagogic content knowledge (PCK) for teaching has been further developed over time to include subject areas, some more than others. Key ideas have been adopted and adapted into mathematics education as documented by Depaepe et al. (2013). They responded to critiques of Shulman's model about the assumptions about teachers as transmitters of knowledge levelled with a situated model for mathematics teaching. Their model comprised four elements of mathematics for teaching:

- knowledge of the content,
- knowledge of pedagogy,
- knowledge of students' cognition, and
- teachers' beliefs.

There are similarities with Shulman's model, including the notion that understanding how pupils think and learn is crucial to mathematics teaching. Knowledge of students' conceptions is considered to be an element in its own right instead of being subsumed within the pedagogical knowledge as in Shulman's model. In common with Fennema and Franke, other researchers have developed Shulman's PCK into the context of mathematics education. Ideas about pedagogical knowledge have developed into specific conceptualisations such as 'mathematical knowledge for teaching' (Fennema and Franke, 1992) and the 'knowledge quartet', (Rowland et al., 2003; Petrou and Goulding, 2011), amongst others. The knowledge quartet is a framework, which can be used to identify how subject matter knowledge (SMK) and PCK are played out in the classroom. This consists of the four dimensions of 'Foundation' (knowledge, beliefs and understanding), 'Transformation' (knowledge in action, e.g. planning), 'Connection' (decision-making) and 'Contingency' (the classroom events that cannot be pre-empted or planned for).

Mathematical practices for mathematics teachers are a form of knowledge in action (Ball and Bass, 2000). Building on the idea that the forms of mathematics a teacher draws on to teach mathematics are different from the ways in which a mathematician would know and do mathematics, Ball et al.

(2004) identified mathematical practices associated with 'knowing mathematics for teaching', and compare with, for example, thinking in mathematics. This develops Shulman's model by distinguishing between PCK and SMK. 'Mathematics for teaching' has emerged as a distinct form of mathematical knowledge and discourse (Ball et al., 2004; Adler and Davis, 2006), focusing on knowledge *about* mathematics (Ball, 1990), in contrast to knowledge *of* mathematics. Whilst progress has been made there is still work to do to understand the relational nature of this work and to further unpack the work of teaching itself (Ball, 2017).

Adler and Davis (2006) draw on the works of Ball and Bass to highlight how there are key structural and epistemological differences between the knowledge domains of mathematics and teaching, meaning that developing the notion of 'mathematics for teaching' is non-trivial. By convention, mathematicians working within the subject field draw on the rules of the subject to create compact and logical explanations; a piece of mathematics that is tightly packed and efficient is usually considered to be elegant. Gauss (1777-1855), an influential German mathematician, was even nicknamed by his peers as 'the fox' (Stewart, 2007): a reference to the way in which his published proofs were swept clean of the path that had brought him to his finished product in the same way that a fox's tail brushes its path so that no trace is left. This contrasts with the mathematics that a teacher draws upon in the mathematics classroom, as often this takes a different form. The mathematics involves 'unpacking' mathematics, elaborating on mathematical reasoning for the purpose of explaining, reasoning, and using misconceptions as a pedagogical opportunity. This form of unpacked, elaborated mathematics is a specific form of mathematics, found in the context of mathematics classrooms and, for the specific purposes of teaching mathematics and teaching others to teach mathematics. There are therefore differences between the kinds of mathematics that are drawn upon to both *do* and *teach* mathematics. Teachers who have previously been successful learners of mathematics need to develop this sense of underlying structure in order to make these connections and unpack mathematics effectively for learners (Ma, 1999; Bair and Rich, 2011)

Adler and Davis (2006) identify a further key tension about whether to foreground mathematics or pedagogy when considering subject knowledge for teaching. They note that generic pedagogy and subject research approaches have developed in parallel and there are sometimes tensions between these. Although PCK has permeated scholarship in teacher education this is variable (Ball et al, 2008). Recent research has focused on the kinds of mathematics that teachers need to know meaning that there has been less of a focus on how mathematics is drawn on for teaching, including understanding for is involved in this for the teacher (Ball, 2017).The focus of this study, in line with a focus on the work of teaching, is on ‘knowledge for teaching’ rather than on ‘knowledge for teachers’. This means that, rather than an identity-based approach, the emphasis is on knowledge required to undertake the activities involved when teaching mathematics, thus taking a situated and contextual perspective. This allows for considerations such as the interplay between the school curriculum and assessment, and their impact on forms of knowledge for teaching to be critically examined within this study.

In contrast to Shulman’s approach, Ball and Bass (2008) integrate pedagogical content knowledge (PCK) and content knowledge (CK) into a single domain of knowledge, that of mathematical knowledge for teaching (MKfT) with this approach being developed over a decade (Ma, 1999; Bair and Rich, 2011). This model is relevant to this research study, as it elaborates on ways that knowledge for teaching interacts with knowledge about students and knowledge about curriculum. These are further composed of categories of knowledge, as illustrated in the following figure:

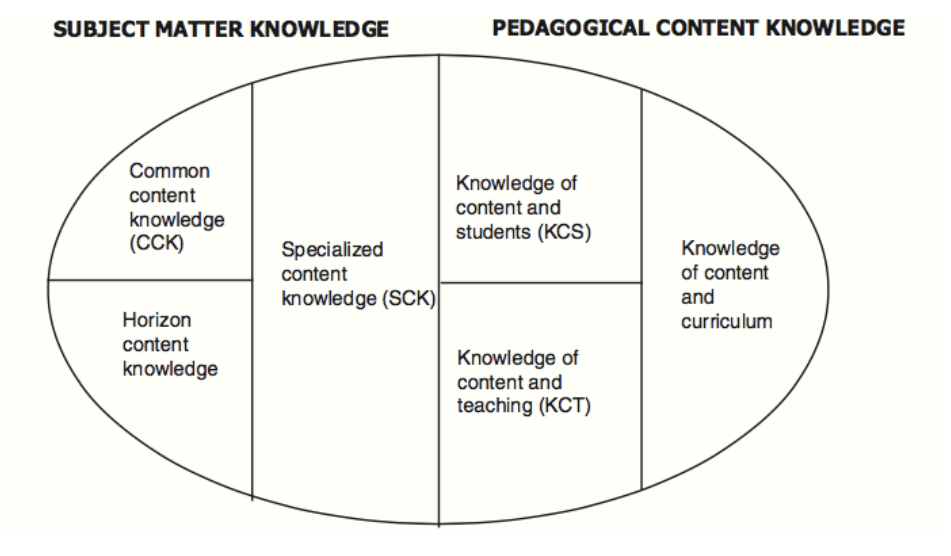

Figure 2.1 Domains of Mathematical Knowledge for Teaching (Ball, 1990; Ball and Bass, 2000; Ball and Bass, 2003; Ball et al., 2004; Ball et al., 2008)

Content knowledge (CK) is broken into three categories. The first, *common content knowledge* (CCK) constitutes the mathematics knowledge and skills that are not necessarily used in teaching, but that all teachers need to draw upon. The second, *specialized content knowledge*[1] *(SCK)*, is the mathematics knowledge and skills that are particular to teaching mathematics, often in the form of unpacking or decompressing mathematical ideas. They are unlikely to appear anywhere other than mathematics classrooms. The third category consists of *horizon content knowledge (HCK)*, which involves knowledge about how mathematical concepts develop over the mathematics curriculum.

Alongside this, pedagogical content knowledge (PCK) is also partitioned into three categories. The first is *knowledge of content and students (KCS)*, that is, the intersection of knowing about pupils and the mathematics itself. This particularly comes into play in relation to learning design, e.g. through anticipating responses to choice of examples or the knowledge of typical misconceptions. The second is *knowledge of content and teaching (KCT)*, which is about a combination of knowing about teaching and knowing about mathematics. This may involve, for example, thinking about why examples are sequenced in particular ways to support leaners development or thinking about the most appropriate representation to use at a particular time. KCT is about an intersection between mathematical understanding and issues that impact upon learning in the classroom. The final category within PCK is *knowledge of content and curriculum*. This means that Ball et al.'s model incorporates curriculum knowledge within the domain of MKfT: something that Shulman conceptualised separately with a separate category of curriculum knowledge.

Alternative models may include further knowledge-based elements such as that of non-local mathematics (Wasserman, 2018), and deep knowledge for the planning for contingent events in learning mathematics (Hurst, 2017). Situated models, such as that developed from cross profession use of mathematics by Bednarz and Proulx (2017) focus on teachers' mathematics within the professional setting of the classroom. Other formats include lines of investigation of the knowledge that teachers need to support pupils with learning through technology with technological pedagogical knowledge (TPCK), technological pedagogical and content knowledge (TPACK), and

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Spelling consistent with Ball and Bass (2008)[1] pedagogical technology integration knowledge (PTICK) (Mishra and Koehler, 2006).

The influential notion of mathematical knoweldge for teaching (MKfT) (Ball et al., 2008) has been developed within mathematics teacher education. Ball and colleagues stated that their model of MKfT is not aligned with particular views of pedagogy or approach. Since then, the previous research contains further directions of development: teacher beliefs of interactional practice (Parker and Adler, 2014), teachers relation with institutional curricula and curriculum planning (Deng, 2018), anthropological approaches (Huillet, 2009), and mathematics professional knowledge in use (Carrillo-Yañez et al., 2018). Parker and Adler (2014) draw on Bernstein to examine how content is drawn from the domains of mathematics and teacher education into mathematics teacher education programmes. Their work focused on the analysis of interactional practice. They argue that, although the practices of the discipline and field *appear* integrated, 'what is constituted as knowledge across them is substantively different' (p217); thus, the outcomes for learners will be distributed in different ways. Research has been undertaken into the types of knowledge that a teacher educator will draw on when they teach PCK to student (Chick and Beswick, 2018) and through inquiry-based approaches (Chick and Beswick, 2018). Ball's work has been further developed and (Hoover et al., 2016) identifies two areas for further progress, based around communicative aspects of teaching as well as curriculum equity and access issues. Mathematical knowledge for teaching has rarely been developed by analysing teacher educators' perceptions as reflected in their accounts and, as such, this research constitutes a contribution to this area.

In summary, in the context of mathematics, Ball and Bass's (2008) influential notion of mathematical knowledge for teaching (MKfT) has developed in a number of ways within teacher education. This has been applied in a range of teacher education contexts. Whilst there is emerging research to suggest that PCK is prioritised in different ways according to the roles of those in teacher education (Brown, 2015), there is potential for exploration of the links between MKfT and teacher educators across contexts. Mathematical knowledge for teaching has rarely been developed by analysing teacher educators' perceptions as reflected in their accounts and, as such, this research constitutes an important contribution to this area. This thesis explores how university- and school-based teacher educators' accounts reflect their engagement with different underpinning curricula, as sites of learning for teacher education.

Having identified gaps in the previous research in relation to the accounts of teacher educators in a range of roles and their perceptions of mathematical knowledge for teaching, in the following section I will develop considerations relating to the school curriculum context.

2.7 School curriculum

In this section I will set out the interaction between school context and curriculum. I argues that a curriculum focus whereby attention is paid to the context and forms of knowledge that shape school subjects can offer teacher education a fresh perspective. I argue that teacher educator accounts reflect perspectives on how what is taught and learned in schools is shaped by different influences on school curriculum forms of knowledge, such as subject disciplinary knowledge or academic knowledge. Teacher educators' positions on curriculum in their practice when developing new teachers therefore both reflects and influences the way that challenges in teacher education about who educates new teachers, and how this takes place, might play out in practice.

In order to examine knowledge for teaching in relation to school mathematics it is useful to frame the subject context for this research given its influence on approaches to curriculum. There is much debate surrounding curriculum knowledge in schools, often focused on a debate about finding a balance between knowledge and skills. Curriculum perspectives are examined in the previous research in the context of social justice (e.g.Young, 2008; Morgan, 2015). The idea that developing critical thinking about disciplinary knowledge structure and relationships between school subjects has the potential to be a site of learning for student teachers is worthy of exploration.

There are broader agendas that impact directly both on the work of teachers through the ways in which teachers are inducted into the profession and through the school curriculum itself. The education system in England reflects both global and local agendas of public service reform, increasingly modelling the activity of schooling on those of business organisations (Whitty, 2010). This means that schools, in common with other public services, are subject to a series of narrow accountability measures and systems. These permeate the system in forms such as league tables, performance management, and key performance indicators, and are reinforced through inspection and other accountability regimes. In relation to teacher induction, the practices of new teachers are steered by the requirements of a set of ‘Teacher Standards’ (Department for Education., 2012). Positioned as professional standards for teachers, these set out minimum requirements within a set of competencies for new teachers, including a minimum level of competency in relation to subject knowledge for teaching. Within a familiarly contrary, tightly controlled-deregulation frame, teachers must achieve these standards in order to achieve qualified teacher status (and hence be provided with the pay and working conditions of a qualified teacher). That said, since 2012 it has no longer been a requirement for teachers in academy status schools to hold Qualified Teacher Status (QTS) (Department for Education, 2012b; Department for Education, 2012a). Nowhere within the regulation is there any requirement for teachers to hold and gain any academic qualification beyond holding a first degree to enter teacher education. This is a symptom of a marked shift to a deprofessionalised workforce in a period where regulation of teacher autonomy is profoundly changing the work of teachers (Gewirtz and Cribb, 2009).

This school context impacts directly on the school in school subjects. The school curriculum is taken here to encompass the full range of provision and experiences offered to pupils (Pollard, 2014). This includes the curriculum intentions as set out in official sources, as well as tacit learning considerations, and the curriculum as experienced by pupils. The official school curriculum is shaped by prescriptive curriculum, assessment and accountability factors. Banks (2008) asserts that school knowledge, in the context of the technology curriculum, ‘*is a function of the schooling process’* (pp.225). Whilst Banks argues that this would be the case even away from the influences such as a national curriculum, it is necessary to consider the subject and school context when reflecting on the strength of these influences within the current schooling context. For authority maintained schools, there is a statutory national curriculum, setting out a form of entitlement for pupils. For other types of school such as academies and free schools the notion of the national curriculum entitlement is advisory. The curriculum offered in all types of schools is further shaped by the accountability regime. This regime incentivises schools to offer particular curriculum formulations, such as the creation of the notion of an 'English baccalaureate' (Department for Education, 2017). Known as 'EBacc', this is meaningless at the level of analysis of an individual pupil as it does not constitute a qualification, nor is it even a curriculum entitlement for pupils; instead, it is a '*performance measure'* for schools (Department for Education, 2017). The EBacc sets out a prescriptive expectation of how schools should compile their curriculum to include mathematics, English, science, a modern foreign language, and either history or geography. Analysis of outcomes shows that there has been a rise in the number of pupils taking these subjects. However attainment has not risen in line with the increase, perhaps highlighting a tension between the government's agenda to privilege particular types of knowledge and the need for pupils to achieve particular grades for progression to further education or employment. (Armitage and Lau, 2018). Many schools in England have adapted their curriculum to align to the policy levers, usually by increasing the curriculum time allocated to subjects with greater accountability such as mathematics and English. This is often at the expense of curriculum time for music, drama and art (Brown et al., 2016). Mathematics is privileged within this model, raising the accountability stakes for this subject. Ball (2013) posits that the resulting pressure of comparison and ranking means that the attentions of schools are therefore focused on the contents of the assessment requirements, in a form of enactment of '*official knowledge'.* At the level of the individual teacher, this is a situation that is negotiated via Ball's (2011) 'paradox of enactment' whereby teachers creatively negotiate with their discipline and with others in a process that may be, at times, uncomfortable, at times pragmatic and, crucially, "*very firmly embedded in the prevailing policies discourses*" (Ball et al., 2011: p265).

The school context therefore impacts on approaches to curriculum in school subjects. This manifests in the previous research in the debate surrounding curriculum knowledge in schools. This tends to focuses on debate concerning balance of knowledge skills or social justice perspectives, rather than on developing critical thinking about knowledge structures and relationships between the school subject and the discipline in relation to the practices of teacher education. Shulman and Shulman (2004) raise the notion of 'curricular capital' with the teacher as the user and interpreter of the curriculum. Their conception links both individual teachers and communities as they work is contextual, cautioning that …"*the policy world is both the sustainer and the executer of the innovations in teaching and learning that occupy our attention*." (p. 267). A *curriculum forms* approach is posited here, whereby the different curriculum knowledge sources and formations that come into play when influencing the curriculum that learners experience in school are used as a lens on the accounts of teacher educators. I argue for an approach whereby teacher educators' engagement with *curriculum forms* in school, as a site of learning for student teachers, is therefore a new angle explored by this thesis.

In the next section I will examine the school subject of mathematics in particular.

2.8 Experiencing school mathematics curriculum

School mathematics has a special position in the wider school curriculum. Broadly, education is positioned as an influencing factor in economic success, and policy is increasingly created and situated within a frame of globalisation (Ball, 2013). Economic interpretations of globalisation are in evidence in English education policy, with the effect of suppressing and efforts to contest or resist this positioning (Maguire, 2002). Furthermore, these interpretations may take the form of localised interpretation of the global or '*glocalisation'* (Ritzer, 2011) (Ball et al., 2011) whereby globalisation is '*nuanced according to locality'* (Marginson and van Der Wende, 2007). In relation to teacher education, this means that the way in which groups of schools induct new teachers into the profession will vary according to the interplay between the local and global factors. Global and national accountability agendas mean that what is taught and learned has shifted towards what Ball (2013) characterises as a '*collection curriculum'* (p112), defined by '*tight boundaries, the authoritative specification of contents and of the sequencing and pacing of knowledge'*. In Bernstein's (2000) terms, this represents a strong classification and framing of the school curriculum.

In the case of mathematics, this strong classification and framing impacts on the teaching of mathematics in the school classroom (Wahlström et al., 2018). I argue that this has an impact on the accounts of teacher educators especially concerning their practices as teacher educators as the curriculum constraints and affordances of schools impacts their practice at the teacher educator/student teacher interface (Brown et al., 2016). Teacher educator accounts will be linked to the need to reconcile the accountabilities of teacher educator roles as well as the reconciliation of different ways to position the purpose of school mathematics.

School mathematics, as experienced in the school classroom, is very different from the mathematical practices of professional mathematicians (Beswick, 2012). Key to this research is the idea that mathematics, as a subject discipline, is transformed into mathematics for teaching and learning in the classroom (Bernstein, 2000). The term 'school maths' is used within this research to signify the mathematics practices experienced by learners within schools. It is useful to draw on Bernstein's notion of a pedagogic device and its analysis of what is involved in the transformation of mathematics knowledge from where it is produced into its form within school classrooms. Bernstein distinguishes between an official reconceptualising field (ORF), created and controlled by state and officialdom, and a pedagogic reconceptualising field (PRF), consisting of the practices of educators, teachers, ITE, educational research. The ORF will influence and shape 'school maths' both in a tightly prescribed mandatory sense (e.g. pupils in England have a statutory entitlement to access the national curriculum) as well as via influence over practices linked to accountability regimes (for example, as identified to happen via the influence of examination boards over classrooms (Puttick, 2015)).

'School maths' is further shaped by the PRF and teachers' understanding and practices. It is a representation of mathematics within a school context, involving a "*particular mode of activity referenced to the performance of certain substitute skills and procedures*" (Brown, 2015: p75). Some scholars develop this further, drawing attention to the multiple layers of recontextualisation involved for a teacher who may conceptualise an institutional curriculum into classroom practice, a further layer of elaboration of the disciplinary subject that requires a form of '*curriculum thinking'* (Deng, 2018). In the context of this research, this is the recontextualisation that takes place at the level of the teacher to recontextualise the school mathematics curriculum into 'school maths', which is experienced by pupils in the school classroom. On a global level, school mathematics is influenced by a broad narrative that mathematics achievement is key to supplying an educated workforce who can contribute to the global and national economy (Brown et al., 2013). Brown argues that, since 'school maths' is a result of the packaging of the subject for the accountability regime (primarily for exams) to which teachers are held to account, there is a consequence of restricting access to learning of mathematics in schools. This is further complicated in England, where the make up of ‘school maths’ is particularly influenced by culturally pervasive beliefs that success in mathematics is a signifier of general intelligence, and that that you can either do mathematics or you cannot (Boaler, 2009).

‘School maths’ is, therefore, a representation of disciplinary mathematics in the school context. The process of representation is not a neutral process and there are consequences about accessibility and inclusion for learners. Conditions with the current school context in England such as the positioning of mathematics within schools means that issues of inclusion in mathematics are of relevance to this research. The nature of the subject itself may also influence these practices. It is worth distinguishing here between the ‘subject discipline and the ‘discipline of the school subject’ as they are distinct. In the case of school mathematics, for example, there are charges that the school subject is more about constructing standards closely aligned to the ‘administration of children’ rather than to mathematical knowledge (Popkewitz, 2004). It has been claimed that the format of assessments, such as exam board specifications, ultimately drive the ways in which teachers present information to pupils, leading to teachers choosing mathematics that the system legitimises over meaningful mathematical learning (Wilson, 2007). Furthermore, whilst the mathematics that a teacher needs to know for teaching is related to the school curriculum, the relationship itself is still unclear (Hoover et al., 2016). Hoover et al (2016) draw attention to the issue that teacher knowledge does not simply map to school curriculum knowledge. raises a current gap in understanding how to organise subject learning for developing teachers that may further benefit the sector. Moreover, within a school-based model, there is emerging evidence that subject progression for beginning teachers is increasingly influenced by regulatory frameworks and policies, rather than the nature or shape of the subject (Brown, 2015). I argue that teacher educator accounts provide insight into the relationship of teacher educators’ responses to the policy context, including the potential to reflect perspectives on how what is taught and learned in schools is shaped by different influences on school curriculum forms of knowledge, such as subject disciplinary knowledge or academic knowledge.

In summary, I have established a gap in the research in relation to how university and school-based teacher educators position different curricula as sites of learning for teacher education. The way in which school mathematics plays out in schools as influenced by different curricula suggests that mathematics presents opportunities as a focus in order to explore a curriculum lens on teacher education. Examining ways that teacher educators engage with relations between the discipline of mathematics and the discipline of the school subject of mathematics therefore provides a specific example of a curriculum angle on learning in teacher education.

2.9 Taking forward teacher education and mathematical knowledge for teaching: a curriculum lens

School subjects influence the context and practices of teacher education. A curriculum lens offers a fresh perspective on teacher education. In particular, I have argued that an original perspective would be gained from a study into how university- and school-based teacher educators engage with different curriculum forms as sites of learning in their accounts of practices in initial teacher education.

Within the above sections I have identified key issues and tensions, many related to global and national agendas, within teacher education. Broadly speaking these focus on structural issues, with tensions about who provides new teacher induction and where and secondly on how this plays out in practice. These are set within the English schools context for the purpose of this study and, whilst many of the issues raised may be familiar to teacher educators in an international context, these do play out in England in particular ways.

I will now focus this study on mathematics subject knowledge development. As such, the specificity of mathematics (Fried, 2011; Jorgensen et al., 2013; Brown et al., 2015) is acknowledged, with its distinctive position in the school curriculum, specific language patterns and clearly defined, hierarchical structures. In the terms of Bernstein (2000), mathematics is a singular, with a very strong grammar, meaning that the practices and discourse are constrained and specialised. Teacher education, moreover, is a form of 'double embedding' of the discourse of mathematics within the discourse of mathematics teacher education (Ensor, 2004). The pedagogic discourses of teacher education, which includes mathematics teacher education is often dependent on context, and indeed may take the form of tacit knowledge in teacher educators and teachers. The approach of this study is to consider particularly how the different curricula of mathematics, ‘school maths’, and indeed those found in teacher education itself, come to play out within the accounts of practices of teacher educators. This research has the potential to contribute to broader discussions about subject knowledge and pedagogy in teaching other school subjects by identifying learning for teacher education across subject disciplines. I address these broader implications in chapter 5.

2.10 Conclusion

In summary, drawing together the key points from this overview of teacher education and mathematical knowledge for teaching, I argue that teacher educators experience teacher education differently at different times and that this is related to context. In particular, the role of the teacher educator is changing and there are gaps in the previous research around the collective experiences of teacher educators based across established and emerging roles, including those based in schools. Mathematical knowledge for teaching and its relation to the experiences of teacher educators is timely to explore in the light of current changing landscapes in teacher education. A curriculum approach, where consideration is given to teacher educators’ positions towards the different influences on how schools subjects play out in the classroom therefore offers a fresh perspective. In particular, exploring how university-based and school-based teacher educators draw on different formations of school subjects (such as the disciplinary subject or the school subject) in their accounts of practice constitutes an original perspective. Exploring the range of such variation provides a perspective on the practices of teacher educators, as reflected in their accounts. Teacher educator positions on mathematical knowledge for teaching will reflect positions on the influencing curricula and these are explored, hence taking a curriculum lens on accounts..

Within this study I therefore explore the perceptions by teacher educators of mathematics for teaching, in relation to teacher education, with the following research questions:

- **What is the range of variation of accounts of the practices of mathematics teacher educators?**
- **What are the qualitative differences between these accounts?**
- **What positions on the school curriculum are taken up in the accounts in relation to mathematical knowledge for teaching in developing new teachers?**

Within the following chapter I set out how the study has been designed to develop answers to the above research questions and develops a rationale for a phenomenographic approach to the research.

Chapter 3: Methodology and Research Methods

3.1 Framing the experiences of teacher educators

Within this study I aim to outline the ways that teacher educators account for *knowledge of teaching and learning mathematics* and *knowledge of mathematics,* and the relations with the developing new teachers in teacher education. I also examine the variation in their accounts. I will proceed in this chapter with an outline of the epistemological assumptions underpinning the research study. Drawing on theoretical context for the methodological approach, I set out the ways in which the study was rigorously designed in order to develop answers to the research questions identified in section 2.10.

3.1.1 Why take a phenomenographic approach to exploring accounts of teacher educators?

Phenomenography is applied to the study design, data generation and analysis of the accounts of teacher educators within this study. The approach is often associated with studies looking at student learning in Higher Education (Tight, 2016); here I apply this to teacher educator accounts of teaching and learning. The research design draws on the idea that different characteristics of a phenomena, here teacher education in the context of mathematics education, are discerned at different times. Within this study, there is a finite range of ways that teacher educators experience the phenomenon of mathematical knowledge for teaching. I take a phenomenographic approach as it draws on participants' accounts of their experiences rather than abstractions or espoused practices. As teacher educator perceptions of mathematics teacher education vary in the ways that the phenomena is experienced, it is possible to discern different aspects of the phenomena. This focus on experience, linked to contextual discernment, suggests that this approach is suited to researching the range of variation of accounts of the process of becoming a mathematics teacher amongst teacher educators. Furthermore, this methodology is underpinned by the idea that a category of description has a meaning (or referential) aspects as well as a structural aspect (Marton and Pong, 2005). This makes it a suitable vehicle for exploring the accounts of subject knowledge for the teaching of mathematics foregrounded in these positions.

3.1.2 Theoretical frame for phenomenographic approach

This study is based on the assumption that there are different ways of experiencing and learning about mathematical knowledge for teaching.

Epistemologically speaking, within this study, exploring meaning for the teacher educators starts with description, on the basis that "*relations have to be relations of something*" (Svensson, 2006:p 167). Furthermore, the different ways of experiencing mathematical knowledge for teaching, as reflected in the description, are logically related to each other (Marton and Booth, 1997). Developing this further, it is therefore assumed that categories of description summarise common clusters of meanings that take on additional significance when considering how they relate to one another.

I therefore take a second order perspective approach to this research, meaning that its basis is the experience of the phenomena of mathematical knowledge for teaching, as described by others (Trigwell, 2006). In particular, this is based on descriptions reflecting relations between thoughts and external reality that vary in its nature. In relation to teacher educators and the practices of teacher education in the context of secondary mathematics this means that descriptions are dependent on the relationship between the teacher educator and the purpose of teacher education in the context of secondary mathematics. The relationship between thought and the external reality of teacher education will vary in nature. Reality for teacher educators therefore consists of different entities; taken collectively, these form a reality of ways understanding and knowing. Descriptions are used for accessing these realities in order to generate the individual accounts of teacher educators, accessing the realities of participants via language and description. Furthermore, categories of description of teacher educator descriptions are developed in order to summarise common clusters of meaning concerning mathematics teacher education. Whilst categories of description stand on their own, attention is given to the significance of how they relate to one another, and what the implications may be for teacher education.

3.2 Data generation

Phenomenography has been taken as an *approach* to the design of the study (Marton and Booth, 1997); as such it has been taken as a framework for design, methods and analysis to enable the application to the experiences of teacher educators. The phenomenographic approach underpins this research design as different characteristics of the phenomena, and the purpose of teacher education in the context of mathematics education, are discerned at different times, and phenomenography focuses on a finite range of ways of experiencing a phenomenon. The research design draws on participants' accounts of their experiences rather than on abstractions or espoused practices, as this enabled a focus on describing variation in the ways that the phenomena is experienced (Marton and Pong, 2005), as I have argued to be the case with teacher educator perceptions of mathematics teacher education.

The descriptions of design and analysis found in Åkerlind (2005a) had a strong influence on the research design process, as Åkerlind's commentary on a case study example articulated how the author had negotiated practical decision making through the research design process with the phenomenographic method. Within the remainder of chapter 3 I outline the approach to data generation and analysis, reflect on the rationale for this approach, and make explicit the lessons learned through this process.

3.2.1 Interview participants

I generated data via sixteen semi-structured, face-to-face interviews. Interviews were used as these provide an opportunity for the participants to articulate experiences. Through semi-structured questioning, participants had the opportunity to unpack key moments and terms. Interviews were robustly designed and had the overarching aim of surfacing meaning for the participant, with a focus on their intentions and purpose (Åkerlind et al., 2005). My rationale for interviewing a total of sixteen teacher educators is that there needed to be sufficient variation in accounts for a systematic phemonenographic analysis. I continued the interviewing until a saturation of themes had been reached. This means that, during the process of iteration of the data categories, a steady state was reached where no new categories of description were required.

The following table sets out the relevant characteristics of the sample of participants, presented in the order they were interviewed:

Ref	School or HE Based	Years in teacher ed.	Teacher Educator Role Details	Solely teacher educator or hybrid role
P1	HEI	25+	HEI tutor, Consultant	Hybrid
P2	School	10+	HEI Tutor: Core and School led PGCE programmes	Hybrid
P3	HEI	25+	HEI incl. Core Tutor	Full
P4	School	<2	Subject Mentor with school-based responsibility incl. Core and School-led PGCE programmes	Hybrid
P5	HEI	25+	HEI Tutor incl. Core and School led PGCE programmes	Full
P6	School	10+	Subject Mentor with school-based responsibility incl. Core and School-led PGCE programmes and CPD maths education	Hybrid
P7	HEI	10+	HEI Tutor incl. Core and School led PGCE programmes	Full
P8	HEI	3+	HEI Tutor incl. Core and School led PGCE programmes and subject knowledge programmes	Full
P9	HEI	15+	HEI Tutor incl. Core and School led PG/UG programmes and subject knowledge programmes	Full
P10	HEI	15+	HEI Tutor incl. Core and School led PGCE programmes	Full
P11	School	5	Subject Mentor with school-based responsibility incl. Core and School-led PGCE programmes & CPD	Hybrid
P12	HEI	10+	HEI Tutor incl. Core and School led PGCE programmes	Full
P13	HEI	5+	HEI Tutor incl. Core and School led PGCE programmes	Full
P14	School	10+	HEI Tutor incl. Core and School led PGCE programmes	Hybrid
P15	HEI	20+	HEI Tutor incl. Core and School led PGCE programmes	Full
P16	School	<1	School-based subject lead with school-based responsibility for Core and School-led PGCE programmes and CPD maths education	Hybrid

Table 3.1 List of characteristics of participant sample for the study

I identified participants to maximise variation within the sample (Marton, 2004). Drawing on the overview of teacher educators' characteristics in section 2.2, this included ensuring that participant characteristics included a range of genders, lengths and variety of experiences as well as a range of settings for initial teacher education (ITE). The sample purposively included university-based teacher educators and school-based teacher educators. This was in line with a broad definition of teacher educators as that of education professionals involved in the education of teachers (Cochran-Smith, 2003; Loughran, 2014) and reflected roles within current ITE landscapes in England as discussed in section 2.2. For the purpose of this study, the sampling strategy included school-based teacher educators who were undertaking lead mentor roles in schools with whole-school responsibility for aspects of teacher education, these were noted in 'hybrid' roles in table 3.1 on the previous page, as I have used the term hybrid to denote participants whose role was not fully or solely that of teacher educator. My sampling strategy excluded mentors whose responsibility was limited to responsibility for a single trainee within a subject department. It is important to state here that subject mentor roles are critically important to the development of trainee teachers; these roles are highly valued and valuable. Indeed, mentoring is the focus of research into the development of student teacher professional knowledge (Mena et al., 2017; Jaspers et al., 2018), and for practices of support beyond induction (Bressman et al., 2018; Jaspers et al., 2018). These roles however were not the focus of this study, as mentoring is only one aspect of the broader set of practices of a teacher educator, and so would limit the range of experiences dealt with in this study.

The sample was furthermore constrained to teacher educators working with student teachers on a Secondary Postgraduate Certificate in Education (PGCE) programmes. The rationale for this is that the current ITE context in England includes a number of diverse routes into teaching (House of Commons Education Committee, 2015). All routes for teaching in schools in England lead to the professional award of Qualified Teacher Status (QTS). Many of these routes are also underpinned by Higher Education provision; usually at Master's level with the majority of student teachers in England following Higher Education underpinned routes. This resulting plethora of routes is a feature of a 'perfect storm' turbulent policy context (House of Commons Education Committee, 2015: p3) as set out in section 2.2.4.

Sampling from the PGCE underpinned sector ensured that the parameters for this study represented the breadth of the sector, whilst keeping within the approaches of the main routes into teaching at this time.

I identified participants who met the parameters for inclusion and contacted from within professional and subject networks. The rationale was that this enabled potential participants who met the required characteristics to be approached for inclusion. This was supplemented by a snowball sampling approach (Browne, 2005) whereby participants suggested further participants from within their professional networks. This was due to the focus on researching accounts of mathematical knowledge or teaching amongst teacher educators, meaning that there was a relatively small, finite group of geographically dispersed potential participants and so this direct approach would be needed to access them. The relevant characteristics of the participants needed to maximise variation were logged and tracked to ensure that the participants were identified in line with a phenomenographic approach and the outlined sampling strategy. This minimised what can be seen as a risk of comprising the sample in some way with this type of network sampling where participants are approached (Savin-Baden and Howell Major, 2013).

I addressed practical issues; I provided participants with information sheets in advance of the interviews. This aligned with an ethical and open approach (British Educational Research Association [BERA], 2018). It also provided the participants with the opportunity to reflect on the purpose of the interview beforehand meaning that accounts could flow from their advance thinking. Additionally, a separate permission sheet was also sent in advance so that when this was shared at the start of the interview and a signature was requested, participants had already seen the form that they would be signing. This adjustment from the pilot interview (where form filling at the start of the interview was a barrier to the flow of the discussion), was how I balanced over-informing and under-informing, a challenge for the principle of informed consent (Kvale, 1996).

3.2.2 Conduct of the interviews

I conducted interviews either face to face or virtually, usually via Skype, depending on the location and preferences of the participant. This flexibility was important for ensuring that travel was not a barrier to including in the sample teacher educators with a range of characteristics in the sample. Interviewing via Skype not only enabled me to access participants at a distance, but also enabled participants to have the opportunity to come forward for interview without having to travel (Lacono et al., 2016). Care was taken to set participants at ease prior to and during the interview, so as to have due regard for the wellbeing of participants, and to ensure that the conditions were conducive for the participants to describe their perspectives freely (Kvale, 1996). I scheduled interviews at a time convenient to the participant to ensure that this was a time most conducive to discussing their practice in an unhurried and reflective manner. For face to face interviews these were also at a location convenient to the participant and, where possible, I provided a drink or brought cake to establish a mutual connection and build rapport. This connection is recognised by others as essential for the successful conduct of a qualitative interview (Schostak, 2006). At the start of each interview, I asked participants whether there were questions about permissions and recording and the format was outlined. Virtual interviews were similar and I took the lead from participants about the form of technology with which they felt more familiar, recognising that familiarity with the technology would potentially impact on the outcomes of the interview (Deakin and Wakefield, 2014). Giving participants the choice meant, for example that the participants who suggested using Skype were already familiar with Skype within their role and so appeared comfortable with its use for an interview. For all interviews, it was important for me to listen carefully to participants, and demonstrate this active listening through body language or verbal cues. This enabled the focus to be maintained on the meaning of the phenomena for the participants. For Skype interviews in particular, this active listening countered the risk that rapport could be lost due to me only having a head and shoulders view, meaning that full body language cues could be lost (Bayles, 2012).

Participants were teacher educators and so I made an assumption that, to some extent, participants were familiar with the notion of reflection on practice. It was important however to ensure a 'safe' environment for the interview, and to use contextual questions to set the scene in the interview before focusing on 'unpacking' the meaning of specific aspects of mathematical knowledge for teaching. This was because there was a risk that if participants found it challenging to articulate a rationale for approaches to teaching and learning mathematics they may feel professionally diminished or uncomfortable. I mitigated this risk both by the establishing of a comfortable environment for the interview, and through prompts that held participants in a safe space through strategies such as acknowledging the challenging nature of the questioning, and being prepared to move on during the interview if appropriate.

3.2.3 Design of the interview to explore meaning for the participants

The interview questions included contextual questions, for the purpose of scene setting, and primary questions, based on the research focus, similar to the approach by Åkerlind (2005a). I adopted this approach as contextual questions aim to both set the scene and to establish a conversation-like approach, again to ensure that the interview was conducive to the teacher educator participants feeling able to respond openly. Primary questions were either open, prompting for specific examples or probing the meaning of *mathematical knowledge for teaching* for the participants. This provided a way of exploring the meaning of the phenomena of mathematics for teaching for the participants (Marton and Booth, 1997). In order to have a discussion about mathematics for teaching, I prompted thinking about 'subject knowledge' in the content of participants' accounts of their practice working with student teachers. My approach was to not make assumptions about meanings of words, and phrases and instead to be open to possibilities. This is consistent with the guidance from Ashworth, which indicates that, for the interview to begin, there needs to be at least a form of superficially shared understanding of the object of research in order to guide the interview but that this understanding needed, for me as researcher, to be held tentatively in order to enter the interviewee's '*lifeworld'* (Ashworth and Lucas, 2000: p299). I prompted interview participants for specific scenarios and examples through the interview, prompting them to recall specific examples; the rationale was that this supported the focus on prompting for meaning and intentions rather than focusing only on words and phraseology (Bowden and Green, 2005).

3.2.4 Pilot interviews

I treated pilot interviews as a key site of learning, recognising that at the start of the process this was an opportunity to make refinements should these be necessary (Howitt, 2011). Close attention to lessons learned and subsequent adjustments to the interview design contributed to the rigorous approach to the research design. Pilot interviews were undertaken, transcribed, scrutinised and, finally, critiqued with my supervisor. This enabled the pilot process to be a rich site of lessons to be learned. Taking an open and explicit approach, I have detailed these lessons learned here.

These lessons included the removal of a question at the start that prompted for biographical information and revision of the script to reflect a more focused approach. My rationale was that scrutiny of the transcripts showed that this had the unintended consequence of eliciting narrative biographical information, albeit interesting, but not aligned to the to the research questions. Furthermore, the narrative approach prevented the starting point from being specific accounts of practice. Ensuring that the script prompted for specific examples provided a purposeful start to the interview, before building on these experiences through the questioning. During the pilot interviews participants were firstly asked a contextual question and, unfortunately, participants provided generalities and discussion of 'espoused' (Eraut, 1994) or idealised practice rather than examples. I removed this question in the next round and revised the script to reflect a more focused approach prompting for specific examples more purposefully at the start of the interview and then building on these experiences through the questioning. Furthermore, scrutiny of the transcripts from the pilot interviews highlighted that there were accounts of generalised descriptions of specifics. Participants referred to 'what I would do there is' or 'what I would usually do is' as a verbal indicator of espoused theory rather than a specific incident or moment in time. Phenomenography draws on the idea that individuals are 'bearers of fragments of different ways of experiencing that phenomenon' (Marton and Booth, 1997) and aims to describe the set of possible ways of experiencing. This means that adapting the interviewing to ensure that the specific 'fragments' were generated through the interviews was necessary. I achieved this by amending the interview questioning to ensure that specific critical incidents were prompted for, and ensuring that the discussion focused on understanding what the participant meant or intended or understood by aspects of mathematical knowledge for teaching.

I gave priority, particularly in the initial pilot interview stage, to keeping my interview skills under review and being prepared to adapt practice. Ashworth and Lucas (2000) refer, for example, to 'stylistic traits' that tend to curtail description. In my immediate post interview reflections I had noted that:

"*I had to constantly curb my natural style which is to reflect, paraphrase and sense make with the participants. Instead I had to actively focus on identifying key phrases for clarification (e.g. in interview 1 I realised that the participant was focusing on a specific definition of accessibility in mathematics and this needed picking up and clarifying) - why had I chosen to focus on what I did – what did I miss, choose to miss or down right ignore? I also had to focus on teasing out specific examples.*" (Extract from post-interview reflective notes)

This highlighted that this type of interview required a pinpointing into specifics and this was different to my more familiar style, which opens up conversation to generalities. Reflecting on this and developing techniques along with self awareness was part of the process of pilot interviewing: something encouraged by Ashworth and Lucas (2000), who reminded me that this type of interview is demanding and is a learnt skill.

I reviewed the transcripts to compare the questions actually asked within the pilot interviews in comparison to the transcripts. In line with the approach suggested by Ashworth and Lucas (2000), I took care to consider whether there was over-domination or under-developed follow through questioning. I identified missed opportunities to follow up specific meanings. I identified ways of amending the questioning technique with greater emphasis on follow up questions such as 'why did you do it like that?', 'What were you trying to achieve?', and 'What do you mean by xxx?' Through this process, the script was adapted to improve the alignment with the phenomenographic method.

Finally, I reflected on the pilot interviews in relation to the need to develop an attitude based on "understanding the meaning of the phenomena *for the interviewee*" (Ashworth and Lucas, 2000; Åkerlind, 2005a: p108). I found this to be a useful guiding prompt. Åkerlind also relates the reconciliation of this open attitude with the role of the interviewer to make decisions about what to follow up, inevitably directing the interview. This suggested a framework, based on Åkerlind's approach that was adopted, to probe further when verbal or non-verbal communication suggested that:

- There was a sense that the underlying intentions of the interviewee had not yet been fully expressed.

- The interviewees' comments appeared to be particularly meaning-laden *for them.*
- There is the opportunity to further explore by asking how new ideas link or contrast with ideas expressed earlier in the interview.

In summary, having reviewed the pilot interview process, I made a number of improvements to the research design and execution as outlined above. These changes underscore the importance of reflecting on pilot interviews as well as underscoring the centrality of a carefully designed interview script. This process ensured that the series of interviews progressed having addressed the identified issues, and with a robustly designed interview process.

3.3 Phenomenographic data analysis

Having outlined the process of data generation, I set out the approach taken to the data analysis and the rationale in this section. This was treated as a dynamic and iterative process undertaken over a six to eight month period. This allowed the on-going development and iteration of the categories of description to take place before these stabilised. Åkerlind's (2005a) account of a phenomenographic data analysis process was a major influence on the approach taken in this study, due to its comprehensive and explicit description. The pilot interviews were included although some sections, such as where participants provided career history, were less relevant to the focus of the study.

3.3.1 Interview and transcription

All of the interviews were audio-recorded as this enabled me to concentrate on the direction of the interview without needing to remember what was being said and also so that I could return to the original responses for later analysis (Kvale, 1996). Even though the phenomenographic interview is generally non-directed, as a researcher, I was a participant in the interview process, through active listening and the need to engage with the participant. I was therefore not in a position to recall the conversation in depth for the record at a later date. This role as researcher in the interview is consistent with a non-directed approach to phenomenographic research, with an exception when "*the interviewer 'leads' the interviewee to focus on some predetermined content in a particular context*" (Walsh, 2000, p 19). My role was to ensure that the interview focused upon the descriptions of subject knowledge as arising out of the discussion of particular experiences. To do this well required high levels of interviewer alertness to ensure that the participants were redirected back to this focus if necessary.

I adopted a mixed approach to transcribing the audio recordings. This was because this provided a balance between needing to be familiar with the interviews, and, pragmatic considerations. Transcribing the pilot interviews and several subsequent transcripts helped me to familiarise myself with the interviews. A transcriber was used for the remaining two-thirds of the transcripts as undertaking a part-time doctoral programme alongside a full time academic role meant making pragmatic decisions about how to target the use of my time. Whether I transcribed the transcripts or the transcriber, the interviews were transcribed close to verbatim. Non-verbal utterances, emotional nuance and sentence fillers such as ‘you know’ were not captured unless they were thought to contribute to the meaning of the discussion. Transcripts were read against the recording, crosschecking against the audio file. Through this process any minor linguistic errors were corrected. The aim was to focus on checking that meanings were correctly captured. Rereading the transcripts against the transcripts was part of the process of becoming very familiar with this data to support the analysis stage.

3.3.2 Phenomenographic analysis

I started analysis when there were a total of twelve transcripts, and I continued to work on the analysis whilst the final interviews were undertaken. This brought the total to sixteen interviews with the subsequent transcripts being integrated as they became available. Interviews were stopped at sixteen as the analysis stabilised and no new findings emerged that suggested refinement to the categories. Adopting an open-minded approach, the additional transcripts were drawn on to strengthen the description of the categories of description and the relationship between them, including boundaries and to identify any gaps.

Although familiar with the transcripts, having interviewed and then carried out accuracy checks, I read these all again, multiple times, with a focus on understanding the intentions and meaning of the teacher educators, rather than just on the face value of words (Bowden and Green, 2005).

I used iteration, or the repeated critical examination of categories against the data, to underpin the approach to analysis in this phenomenographic study (Walsh, 2000). With this came both practical as well as philosophical considerations to take into account. Drawing on part and whole level analysis and adopting the pragmatic approach described, meant that this placed the process towards the construction end of any construction-discovery continuum. Through this process, at all stages, I continued to focus on the research questions with an emphasis on the question: *what is this transcript / quotation telling me about the way that the participant understands mathematics for teaching*? This was in order to mitigate against any potential unintended consequence of favouring my own conceptions and experience due to my familiarity with the topic (Walsh, 2000). Given that the purpose of the research was to uncover and examine considerations other than my own, I argue that openness to conceptions other than my own are woven into the fabric of this research design through guiding attitude, rubric questioning, and the techniques described above.

I therefore adopted a systematic approach to data analysis. A blend of part and whole analysis levels was used, despite the lack of research literature outlining specific descriptions of what researchers actually do when analysing at this stage of phenomenographic research (Åkerlind, 2005b). The approach involved a series of close readings for meaning. I carried out these readings in an iterative manner. I read each script as a whole for meaning. This is not to say that I lost sight of the need to treat the full data set collectively, but focusing on the each script as a unit of reading enabled me to read the sections within their context. This led to an initial identification of organising categories and dimensions, with the role of mathematics content knowledge for teaching being organised under five initial headings. These headings, initial themes from the analysis and my familiarity with the data led to a list of possible dimensions for systematic exploration. These came out of the initial themes and aspects noted through the process to support with responding to the initial research questions. I read each script systematically for meaning about the role of the teacher educator, the role of the teacher, the role of the pupils, the role of mathematics subject knowledge, the role of misconceptions, the role of context, and the role of curriculum. Other aspects of interest in each script that did not come under this heading were also noted. This was organised into a grid and, as throughout this process discussed with my supervisor, inviting critical interrogation. An extract from the organising grid is provided below as an example (again in the interests of openness).

Role of the Teacher educator	Role of the Teacher	Role of the pupils	Role of mathematics subject knowledge	Role of Misconception s	Role of Context	what is taught and learned? (Curriculum?)
To support trainees to become aware of connections in maths. To support trainees to be able to bring parts of mathematics together. To support the development of understanding of the complexity of learning . maths. To develop alternative perspectives on maths, beyond trainees' own initial school experiences.	To be a current learner of mathematics. To understand where a pupil is mathematically and to use this to help them to make subsequent progress. To make connections in maths explicit to pupils. To capture childrens' interest in an authentic mathematical way, eg by providing mathematical problems as context for learning. To challenge pupils.	Pupils make sense of the maths. They are invited to make sense of problems and offer suggestions and solutions. Pupils contribute and join in (not empty vessels). Pupils making sense of maths and linking maths to other areas of maths to solve . Pupils make choices. Pupils work with peers.	Not just about being able to do it yourself. To understand what it means to learn mathematics. It is not even possible to learn all the maths you need. Trainees are able to look things up and critically eventuate and link to own knowledge. Mathematics is reconceptualised into MfT Know how maths extends and develops To know which maths needs explicit teaching and which will arise in context and be able to deal with this. .	Making mistakes and learning from them as part of the classroom culture.	'Real life' maths may not be real life. Context as the mathematical context of the starting point. It illustrates the need for a method or to learn something. History of maths also provides a context for learning maths that supports the development of understanding of subject structure.	Overall curriculum aims as a guide. Beginning teachers need to know about more than what is on the curriculum to develop multiple and advanced perspectives.

Figure 3.1 Extract from data analysis grid to show how initial themes were noted for an exemplar participant

The following stage consisted of further iteration and refinement of this grid including further systematic reading of the transcripts. My readings always focused on the intention of the speaker, leading to a further iteration of the above. The next step was to focus on each of the columns above in turn (instead of on each script as a whole), looking, for each dimension and for variation across the group of transcripts. At this stage, I shifted from reading for meaning within each script to considering each dimension in turn, looking at the collective experiences and perceptions of participants and shifting for aspects of variation. What were the ways in which, for example, the ways that mathematical knowledge for teaching were accounted for across the transcripts? The rationale for this approach was to ensure that the process could take into account the intentions of the participants as reflected in the accounts, and to be open to the meaning of the data. Through this process, working systematically and considering the columns, I identified that there was variation in relation to the ways that mathematics subject knowledge such as mathematical knowledge for teaching was reflected in the accounts. There were differences in how mathematics subject knowledge, pedagogical subject knowledge, and subject matter knowledge were prioritised and foregrounded in the accounts.

I then returned to the individual script: they were read for meaning and intention, this time for the purpose of understanding the role of subject knowledge in the process of becoming a teacher. Each transcript was marked up; transcripts were paper based allowing for ease of reading, mark-up and organisation at this stage. Not only were tentative categories of knowledge (e.g. teaching and learning knowledge, subject matter knowledge) noted, but also interaction between these. At this stage variations emerged, and it is fair to say that the (systematic but messy!) notes made on postits for each category of description represent a true ‘Aha’ moment in the process (Mason, 1982). Again, I provide a photograph here within the thesis for interest and openness.

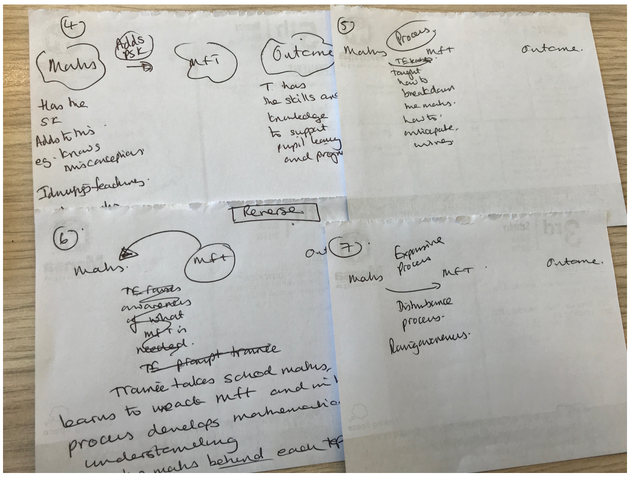

Figure 3.2 Photograph of the notes made about interactions of types of mathematical knowledge as reflected in accounts and captured during the data analysis stage.

Returning to the individual accounts, I constructed a draft summary of categories and considered individual transcripts against this framework. As Åkerlind (2005b) outlined, consideration of individual transcripts, including borderline cases were helpful here and supported the refinement of the categories. Further iterations of the draft framework of categories emerged with each successive rereading of the transcripts against the categories through this iterative process. I noted aspects of variation that were embedded in the intentions of the process for teacher educators where they appeared, and used these to refine the ways in which the categories were distinguished from one another. This was required in order to ensure that the differences between the different categories were more than nuances in words, but instead about different elements that distinguish the categories from each other. This enabled the categories of description to reflect *distinct* ways of experiencing mathematical knowledge for teaching (Marton and Booth, 1997).

An initial diagram was sketched out to demonstrate how the elements fit together for each category of descripton. At this stage in the development of the categories, there were five initial categories of description as reflected in the below notes made at the time:

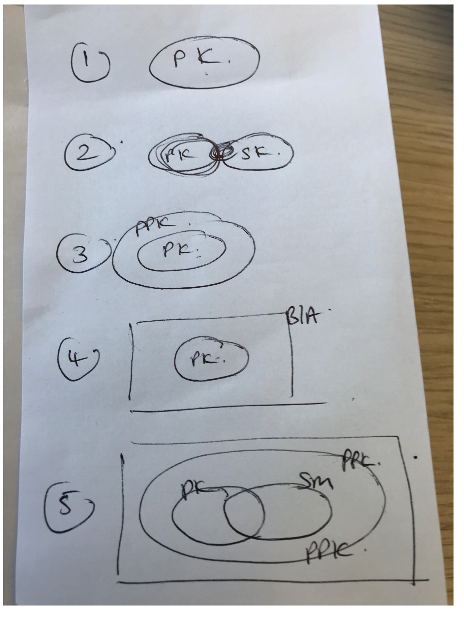

Figure 3.3: Photograph of diagrams reflecting interactions of types of mathematical knowledge as reflected in accounts and captured during the data analysis stage.

I further refined the categories by examining how conceptions were positioned, identifying which elements of knowledge for teaching were foregrounded in each conception. This dynamic process continued reading transcripts as parts, wholes and within the context of treating the data collectively. The categories of description were refined dynamically through this process; definitions were tested again and again until changes reduced before eventual stability, in line with the approach of Marton (1986).

3.4 Issues in phenomenographic methods

I explore a number of issues particular to the type of data analysis in the remainder of this chapter in the context of this study. In particular, in the next sections I discuss ethical considerations, accounts of phenomena, researcher prior knowledge, validity and reliability.

3.4.1 Ethical considerations

The study was undertaken with the relevant institutional ethical approval. Ethical considerations were developed in line with the institutional expectations from the very start of the planning process, in line with the recommendations of developmental phenomenography approaches that recommend that the phenomenographic approach is integrated into planning from beginning to end (Bowden and Walsh, 2000). These provided helpful guiding principles for the conduct of the research. Ethical approval was both required for the study and also considered to be good practice. Key issues for this study included consideration of professional discomfort of participants due to the challenging nature of the questions and anonymity of the university/school setting. The potential for participant discomfort was minimised through the interview design set out in section 3.2.3. The anonymity of the setting was preserved as the transcripts were anonymised and the appropriate record keeping precautions such as password protected participant records were kept.

3.4.2 Accounting for accounts

Within this study, the term ‘accounts’ is used for the descriptions of participants’ experiences, in line with the approaches taken by Åkerlind (2005b), Ashwin et al. (2016) and (Ashwin, 2006). During the interview process, participants were prompted to articulate their reflections on their experiences through the spoken word. The reason was that, through the research process, the study was not about the relationship between the participants and teacher education practices but instead was focused on the collective relationship between participants’ individual accounts and their experience of teacher education in the context of mathematics. This study is therefore based on related assumptions about how individuals make sense of the world and their experiences and these are acknowledged.

The approach to reading transcripts was that sections were always considered within the context of the whole script, and there was interaction between section and whole script reading to make sense of the utterances of the individual within the collective. This rationale is the foregrounding of the assumption that underpins this phenomenographic approach to this study, whereby participants’ experiences are accessible through language and furthermore that these experiences are related in some way to the words spoken by individuals during a phenomenographic interview (Säljö, 1997). There are multiple reasons for responding in a particular way, or not at all, including what Säljö terms a 'communicative obligation' in response to a prompt. Taking this to a logical conclusion he goes on to point out that:

"*Thus, the basic observation of phenomenography "that whatever phenomenon we encounter we experience it in a limited number of qualitatively different ways" (p. 174) can just as easily be accounted for by the fact that there is a limited number of ways of talking about a phenomenon that is perceived as relevant in a particular situation."* (Säljö, 1997: 178)

Therefore sections were always considered within the context of the whole script, with interaction at the data analysis stage between section and whole script reading to make sense of the account of an individual within the collective.

It is useful to highlight here that the study was not designed to compare groups of participants across characteristics such as gender, school/university- based teacher educators or length of service. Some participant groups may be better able to access and articulate accounts of their experiences than others. This reinforces the point that designing the study to make comparisons between these groups is not in line with phenomenographic design principles. Instead, it was the variation in the accounts of experience that was the focus, along with and the *relationship* of the teacher educators within the context of teacher education in mathematics. In line with this approach, the texts were chunked into larger paragraphs rather than into single sentences/phrases at the analysis stage. At the analysis stage of this project, text fragments were not analysed in small sections using close scrutiny approaches such as critical-discourse analysis, which is arguably where articulacy would come to the forefront. Instead, transcripts were read for meaning of the teacher educators, with cross-referencing taking place between the individual sections of text and the overall script, to ensure that this was set in context.

Finally, the term 'finite' is used, rather than 'limited' as a way of describing the outcome space for this study as being based on a specific number of individual ways of experiencing teacher education as reflected within the collectively analysed accounts. Phenomenography is usually underpinned by the notion that there are ‘limited’ ways in which a group can account for a phenomenon. Like Hallett (2014) I found the use of ‘limited’ to be problematic, with the implication that there is something, as yet unexplored, doing the limiting. Instead, the term ‘finite’ is used within this study as it better reflects the way that the study was designed to surface meaning across the range of the participants in order to learn about teacher education and to contribute new perspectives.

3.4.3 Researcher prior knowledge

In the sections above I have explicitly set out the data analysis process. The reason is the approach taken to my own prior knowledge of teacher education and how this is brought to bear on the study, including the data analysis stage. The approach taken is to maintain awareness that this may have an influence and adopt an attitude that acknowledges this and continues an awareness of this through the process. A log was kept of the decisions and approaches adopted through the iterative and interpretative process of summarising the collective data and making meaning as the categories emerge from this process in order to facilitate this awareness. As a teacher educator with a mathematics education background, my view is that it is not possible to claim that I am fully aware of the entirety of my prior knowledge and biases to be able to set these aside completely in line with the convention that, at analysis stage, a phenomenological researcher’s prior knowledge of the research object is set aside or bracketed.

So, for this reason, I have reflected on and acknowledge my own background beyond my involvement in this project as a researcher, and I have made this explicit here. To date, I have taken up different roles through my professional career that have impacted upon my professional identity formation. This professional identity has formed as a process of becoming familiar with the practices of the roles, but also as I transitioned between roles or experienced the need to reconcile roles at different times. For example, I experienced a significant transition from school teacher to university tutor that required considerable reflection and openness to examination of practice and subsequent identity shifts. These roles have included: mathematician, teacher, school-teacher of mathematics, researcher, teacher educator, university-teacher of mathematics, university-teacher of mathematics education, tutor, leader, manager and mentor. Related identity shifts (and developments) have involved perspectives on the relationship between support and challenge for new teachers and moving from paradigms of rational deduction and reasoning to areas where knowledge is contestable, and the relationship between these ways of viewing the world. This means that understanding the relationship of teacher educators with the context of teacher education policy and practice is of particular interest. I reflect that professional life has often involved boundary brokering between one, or more, of the listed roles requiring the movement between different practices, discourses and modes of thinking. These experiences have shaped my identity, involving the taking on and reconciling of assumptions and beliefs.

3.4.4 Validity and reliability

Validity is protected through the process underpinning this study by making choices and interpretations explicit and open and drawing on strategies for assurance of validity within phenomenographic research. The strategy I took within this phenomenographic study comes through the transparency of the process, at all stages and including design and data analysis, drawing on (Cope, 2004) to design the study to incorporate the following verification strategies.

My own preconceived understandings of the phenomenon were bracketed to a point, as highlighted in 3.4.3, and my background where I have continued to reflect on contestability of knowledge in relation to teaching and learning mathematics has been acknowledged. These may, however, still come through in the choice of interview questions, choices within the interview process about what to follow up, and interpretative elements of the analysis; hence these have been reflected upon within this chapter and made explicit. I have also made explicit the sampling strategy, the data generation strategy, and the ways in which I have endeavoured to approach data analysis: being open to possibilities rather than imposing an existing structure. Descriptions have been provided of seeking feedback, from my supervisor and from other researchers through the process. Finally, the results are presented in such a way that they can be scrutinised. The anonymised numbering for each script has been used to present the results in Chapter 4, along with supporting quotations from participants. These are in line with the recommendations of Cope (2004).

Sense checking was a further key aspect of the process of making and refining meaning from the accounts. This was carried out via thorough discussion with my supervisor and peers and inviting critical feedback. I also presented the emerging outcomes at conferences and engaged in critical discussion about the emerging findings. In this way, the perspectives of others were sought and drawn on to ensure that the outcomes have '*communicative validity'* (Åkerlind, 2005b: p.124). As part of this approach to the data analysis process, I not only shared the categories with my supervisor, but also shared the draft interview transcripts along with the pilot interview transcripts and made the full data set available, adopting an open approach through the full process. The rationale was that, as Marton (1986) argues, although it is not necessary for other researchers, with access to the data to reproduce the categories of description replicability is about other researchers recognise the categories through the categories of description identified by the original researcher. The outcome space presented is, in effect, a relation between myself as researcher and the data (Marton and Booth, 1997). This means that this is not the only possible outcome space from this data, and that these are not the only categories that could be developed from this data. Rather, through this commentary, I have aimed to present a series of categories that others would *recognise* from the data set, and I have tested this out with my supervisor and other researchers.

Use of my supervision sessions to discuss the emerging and iterating categories, with my supervisor who had access to my data, enabled an ongoing sense of this communicability to be built into my research process. This was based on a form of 'interjudge reliability' (Sandbergh, 2006): a practice of cross-checking where other researchers, with access to the same data, provide a sense of the extent to which they recognise the categories of description. Maintaining the above described forms of 'interpretative awareness' (Sandbergh, 2006: p209) by having an openness to holding back preconceived theories, through these critical discussions with others, means that the data outcomes described in chapter 4 are a description of the accounts of the participants, rather than an explanation of why the descriptions are as they are.

3.5 Conclusion

In summary, I have applied a phenomenographic approach to describing recollections of experience, making it appropriate as an approach to understand the ways that teacher educators account for their practices of developing mathematical knowledge for teaching in the context of teaching student teachers to become mathematics teachers. Considerations of validity and reliability are reflected in the scale and scope of claims and conclusions of Chapter 3. Taking an explicit and rigorous approach to, being aware of, and accounting for, decisions taken through design, the interview and analysis stage enables this research project to acknowledge assumptions of the approach taken, and yet constitute a useful and authentic contribution to the sector.

Chapter 4: Outcomes

4.1: Introduction to section

Within Chapter 3 I provided an overview of the methodology and approach to this study.

Within this chapter I respond to the first two of the following three research questions:

1. **What is the range of variation of accounts of the practices of mathematics teacher educators?**
2. **What are the qualitative differences between these accounts?**
3. **What positions on the school curriculum are taken up in the accounts in relation to mathematical knowledge for teaching in developing new teachers?**

In accordance with the methodological approaches to data generation and analysis laid out in Chapter 3, I analysed teacher educators' accounts of their practices as mathematics teacher educators phenomenographically. The categories of description and the way in which they are related to form an outcome space (Åkerlind, 2005a) is now presented in this chapter.

An outcome space constitutes an logical and empirical overview of the categories of description (Åkerlind, 2005a) and it is useful to firstly set out some underpinning assumptions here in relation to this presentation of the outcome space and data. The framework for the outcome space presented is of the form of a nested hierarchy. The categories presented, however, are not intended to represent the perceptions of individual teacher educators and not to classify individuals nor individual accounts. Teacher educators were prompted to reflect upon past examples of practice and were encouraged to choose recent examples. These cannot be taken as an indication of either the entirety of their practice nor do they signify an indication of the way that they see their future practices developing. Indeed in some cases individual descriptions reflected different positions at different stages of the interview. This is in line with the reflection by (Marton and Pong, 2005) that it is entirely possible for individuals to provide different types of accounts when the prompt questions are different. Where descriptions of practices have been included in the less inclusive levels of the hierarchy this does not reflect a value judgement on the practices of the individual teacher educator. Indeed, teacher educators' accounts often contained elements of the different levels of description and indicated that they moved between categories within different contexts.

4.2: Variation in accounts of becoming a mathematics teacher: Structural and referential aspects

All of the teacher educators' accounts of practices linked the process of supporting student teachers to become mathematics teachers to the development of student teachers' knowledge of teaching and learning mathematics. Taken collectively, their individual accounts demonstrated variation in ways of describing (experiencing) this process.

The phenomenographic analysis of the descriptions of the teacher educators' accounts reflected qualitative differences in structure between knowledge about *teaching and learning mathematics* and knowledge *of mathematics*. These structural aspects of the outcome space described qualitatively different ways in which the teacher educators foregrounded types of mathematical knowledge in relation to the development of student teachers. This shifted from having knowledge about *teaching and learning mathematics* as a focus through to *knowledge of mathematics* being the focus at different points within the framework. The four categories of description identified in the outcome space are laid out in the following table:

Category	Name	Description of the practices of teacher educators as reflected in the accounts of teacher educators:
1	Teaching and Learning	Student teachers add knowledge about teaching and learning of mathematics to their knowledge of mathematics. They become able to deliver the school mathematics curriculum.
2	Linking 'teaching and learning' with the Subject	Student teachers learn how to add knowledge about teaching and learning of mathematics to their knowledge of mathematics. They know how to unpack school mathematics topics and further develop their own understanding of mathematics They are able to flexibly adapt to changing curriculum content over time.
3	Integrating 'teaching and learning' and Subject	Student teachers' knowledge of mathematics expands into new dimensions of knowledge about T&L mathematics. They develop their own beliefs about learning mathematics. The includes disruption of previously held beliefs. They are able to adapt their teaching to open up possibilities of learning mathematics to learners.
4	Reconceptualise of 'teaching and learning' & Subject	Student teachers reconceptualise knowledge of mathematics into knowledge about teaching and learning of mathematics. There is a change in their understanding of what mathematics is. They have multiple and advanced perspectives on mathematics that informs their teaching and learning of mathematics.

Table 4.1: Outcome Space summarising categories of description of teacher educators' accounts of their practices in mathematics teacher education

These four qualitatively different ways of describing the teacher educators' accounts of the process of becoming a mathematics teacher were further developed by considering structural and referential aspects, as summarised in a phenomenographic outcome space as follows:

	Referential Aspects	
Structural Aspects	Understand	Reconceptualise
Knowledge of mathematics teaching and learning	1	
Relations between mathematics 'teaching and learning' and mathematics knowledge	2	
Synthesis of mathematics 'teaching and learning' and mathematics knowledge	3	4

Table 4.2: Outcome Space Summarising Referential and Structural Aspects of the Teacher Educator accounts of their practice in mathematics teacher education

This outcome space is an attempt to summarise and present the categories of description and the relationships between them. The referential aspects relate to the meaning of teacher educator practices and the structural aspect highlights what is the focus of each category. By, its very nature a summary is a model that may overlook nuances. For example, some accounts, or sections of accounts, were not easily categorised into one category of description or another or overlapped categories. The outcome space is a useful diagram however as it provides an overview of the entirety of the participants' accounts, providing an overview and lens into, these accounts and the underpinning structures. It particularly illustrates the interplay between types of knowledge within the different categories of description and how they are accounted for in preparation for teaching the school mathematics curriculum.

A qualitative difference between the accounts in category of description one and the accounts in categories of description two to four was the foregrounding of *teaching and learning mathematics* in the first category whereas in two to four the *knowledge of mathematics* (as a subject discipline) comes into consideration, with the interaction between these becoming more sophisticated through the categories of description.

Category of description one represented a form of competency level whereby the student teacher acquired the necessary pedagogic (teaching and learning) knowledge to teach and learn mathematics whereas within the descriptions within category of description two onwards the outcomes relate to the student teacher developing new mathematical understanding of mathematics. It is acknowledged that any boundaries drawn between a competency level of being able to use knowledge of mathematics teaching and learning and being able to use to develop new understanding of mathematics teaching and learning is contested as these could be considered to be on a continuum. It is useful to make the distinction between base line competency and further developed application. This distinction is particularly relevant to the context of teacher education. In England, for example, there is a common set of competency based Teacher Standards that trainee teachers must meet as a baseline to become qualified, these same standards are then used currently as the basis for continuing professional development.

Furthermore, the phenomenographic analysis of the teacher educators' accounts reflected qualitative differences in referential aspects about how these types of mathematical knowledge were engaged with or used as a tool within each category of description. Within the first three categories of description, there was an emphasis on the types of knowledge and how they relate to each other. Within the fourth and final category of description, not only would the student teacher understand the interplay, there is also a reconceptualisation for the student teacher, resulting in a change for the student teacher in the form of new learning. This is exemplified in section 4.3.4.

I will now set out the four qualitatively different ways of describing the teacher educators' perception of the practices of becoming a mathematics teacher under four categories of description. Where features of these different ways are identified, these are referenced to the relevant and supporting extracts. Quotations are used within the following sections to exemplify and illustrate features of each category of description.

4.3 Categories of description

4.3.1 Category of description 1: Teaching and learning or 'delivering'

Within this category of description the teacher educators' accounts were consistent with the notion that mathematics teacher education was a process of adding knowledge about *teaching and learning of mathematics* to the student teacher's *knowledge of mathematics*.

Within this category of description, the accounts of teacher educators identified aspects of knowledge about teaching and learning mathematics deemed to be relevant to add to the students' knowledge of mathematics. The emphasis was on the knowledge about *teaching and learning mathematics* that the student teacher needed to acquire. The purpose of this process was that the student teacher then had the necessary pedagogic knowledge to teach and learn mathematics.

Within this category of description, the *knowledge of teaching and learning mathematics* was foregrounded, as an object, process, and outcome. In the following extract, misconceptions for pupils are an example of an aspect of knowledge about teaching and learning within mathematics for the student teacher to acquire. The teacher educator references not just the student teacher's development but also the development of their own *knowledge of mathematics* as part of this process.

> *"Knowing what the misconceptions could be. That surprised me – how many ways there are in which they could be wrong, surprised me frequently. There are multiple and varied ways in which they could get things wrong and I am getting better at predicting what they are now." (P4:18)*[2]

In this example, misconceptions were a type of knowledge about teaching and learning to be acquired by the student teacher. Misconceptions were one example of the types of knowledge to be acquired (P2:2, P5:13, P10:32 , P13:8). Other types of knowledge provided by teacher educators to the student teachers included suggestions for resources, usually for teaching topics on the school mathematics curriculum, (P9:25, P10:4, P10:22, P11:5, P13:8, P16:12, P16:17) and more broadly, suggested approaches to teaching mathematical topics found on the school curriculum (P8:5, P10:4, P10:19,

[2] The notation (PX:Y) is used to reference transcript quotation sources where X denotes the participant number and Y the relevant section of the transcript.

P10:20, P11:18, P14:22 P15:20). The following extract illustrates the significance of identifying resources to the lesson planning process:

> *"Being given a resource was a lot easier – certainly planning around a given resource is a lot easier. So I did offer and said I could find something to get her started but she already had some ideas and she had done some research herself and had …she said 'I have got something that might be appropriate' and so she went away and found it herself. I was pleasantly surprised as I remembered that not knowing where to find resources was one of the most challenging parts of planning lessons when I first got started, when I was a student." (P4: 11)*

The resource had formed a starting point for planning to teach an aspect of the school mathematics curriculum. There were challenges with resources:

> *"You need to have some idea of what could go wrong, the mistakes they may make. And then the practical stuff, finding a resource that they would want to do, whether they will actually do it in some cases, that is the key." (P4:26)*

Within accounts aligned to this category of description, the teacher educators recounted suggesting ideas to student teachers and prompting their thinking as a way of supporting their learning about teaching and learning mathematics. The outcome of this process was that the student teacher had the necessary (pedagogic) knowledge to teach and learn mathematics (e.g. P9:38). This positioned the student teacher as gathering this necessary knowledge, over and above their pre-existing mathematical knowledge, aligned to an overall purpose of delivering the school mathematics curriculum.

Within accounts aligned to this category of description, *knowledge of teaching and learning mathematics* was foregrounded. Examples of this knowledge included being able to make mathematical ideas explicit (P5:9, P5:12, P7:9, P8:1, P9:9, P11:2, P12:29, P16:6), having real life examples of mathematics (P9:25, P11:16, P15:20), knowing common misconceptions in a topic (P5:13, P10:32, P13:8), and knowing the pre-requisites for a topic (P5:24, P6:18, P10:24, P12:27, P16:7).

Unpacking mathematics in this category of description was consistent with the idea of making implicit ideas explicit:

> *"I mean for somebody who has done a maths degree I think knowing, you know negative numbers inside out don't you. I do think it's subject*

knowledge; it's about how do you get a child to understand what is really happening with the numbers because it's easy to say 'well you've got two negative numbers in the middle, what happens there?' It's easy to do that but it's harder to explain the effects of it and having an example to give them so in the child's mind, they get what's going on." (P11:8)

Making mathematical ideas explicit can mean revisiting tacit understanding about mathematical topics. In the following example, the knowledge of teaching and learning mathematics was about the teaching and learning of angle but also about forms of knowing in mathematics. In this case the form of knowing was the mathematical idea of proof and justification:

"I noticed with people who have always found maths very easy at school, is they will often accept a fact like an angle on a straight line adds up to 180 degrees and for them it's so natural, if one angle's 30, the other angle would be 150 degrees and it's something that doesn't need explanation.

It's something they just regard as being obvious and they find great difficulty when they're in the classroom of understanding why pupils, some pupils, will find that not obvious and very difficult to understand and how you actually break that down." (P5: 9)

Explicit unpacking of mathematics involved thinking about sequencing and progression of mathematical concepts and hence explanations:

"It is for them to be able to say – I am going from this line to this line – is there anyone who doesn't get it? Then they can explain it and give an explanation and break it down, make it more concrete. You can work on each step by step until you build the chain of reasoning that gets you from the starting point to the conclusion. Making that concrete and illustrating it with examples usually helps." (P4: 22)

This progression was linked explicitly in the next extract to planning:

"Planning is crucial for all of the subject knowledge, subject knowledge and forward planning. We have, on our lesson planning form, we have a very important box which the trainees have to think about and put some detail in which is called pre-lesson understanding and one of the most common reasons when I see lessons falling apart, when I see my trainees teach, is that they've assumed people will have knowledge that they don't have and so we've built in this idea of a pre-lesson understanding of saying 'what do pupils need to know before the lesson begins in order to access the lesson and how am I going to check they have this knowledge before I start?'"(P5: 24)

The notion that student teachers need a minimum, or baseline knowledge of mathematics and that this was necessary prior to building knowledge about teaching and learning came through in participants' account (P5:15, P5:27, P6:5, P7:2, P9:8, P10:46). In the following extract, a teacher educator gave examples of baseline process knowledge as part of this necessary knowledge for student teachers with mathematical thinking or more relational kinds of knowledge potentially being foregrounded later:

> *"You obviously need the subject knowledge behind you but more important is that you need to be able to see what the key essence of the mathematics is and to be able to identify what – how to go through it. I really want to get my trainees working on relational mathematics – that's one of the key words we have in mathematics education – relational understanding of how things fit together rather than just procedural where it's just learn these rules and if you use those rules you get the right answers in return. That will get them through a test but doesn't actually make them think mathematically and I see a key difference between the two." (P9:9)*

In summary, within this category of description, knowledge of teaching and learning mathematics was foregrounded as an object to acquire and as the focus of a learning process. The next section builds on this idea and outlines the similarities and differences between this category of description and the next.

4.3.2 Category of description 2: Integrating teaching and learning with knowledge of mathematics or 'linking':

Within this category, teacher educators' descriptions were consistent with the notion that teacher education is about the student teacher learning how to add knowledge about *teaching and learning of mathematics* to their *knowledge of mathematics* . Teacher education is about learning how to unpack school mathematics topics and further develop student teachers' own understanding of mathematics.

There was a structural shift between the teacher educator accounts aligned to the category of description 2 when compared with those aligned to the category of description 1. In the previous category it was the knowledge about *teaching and learning mathematics* that was foregrounded in accounts. Within the category of description 2, knowledge *about mathematics* was foregrounded in the accounts with the links between knowledge about mathematics and teaching and learning mathematics providing opportunities for learning as the student teacher develops an understanding of these links.

Within this category of description, teacher educators' accounts of their practice were consistent with student teachers learning *how to add knowledge* about teaching and learning of mathematics to their knowledge of mathematics. Teacher educators' accounts within this category of description described their practice in terms of modelling the process of breaking down mathematics, drawing attention to teaching and learning aspects. The outcome of this process was that the student teachers were able to go through the process of breaking down mathematics in order to plan for learning. Within this category of description, student teachers were positioned as developing their understanding of how to learn new knowledge about teaching and learning mathematics. Not only was the purpose to be able to teach the school curriculum, but they were also learning a process that would enable them to adapt their practice for future changes to the school curriculum - a form of future proofing.

A qualitative difference between this and the previous categories of description was that the *knowledge of mathematics* was foregrounded in this process, as an object that supports this process. Learning within teacher education takes place by learning knowledge of mathematics and knowledge of teaching and learning mathematics, and also through developing an understanding about the links between these.

Accounts of teacher educators consistent with this category of description referred to modelling the process of breaking down mathematics (P5:13, P7:7, P9:11, P9:19, P10:22, P12:16, P13:11, P14:13, P16:11). This took different forms; for example in the following extracts the student teachers experienced this modelling through the use of microteaching exercises:

> *"Every trainee is given a mathematical task to explain and then they have to focus on how do they break down that thing which is something they would do naturally; something they would do, you know, without really thinking about what could be difficult about it. So the first thing they have to do is they have to stand in front of the group and they have to explain that process and then we give kind of group feedback so as a group they're all given opportunities to give suggestions and the critical appraisal of how successful*

it was and what they could do differently and I contribute to that as well and then that also feeds into what 1s are likely to arise, any ambiguity of language which might have been used that could have caused a misconception so that is the activity which is specifically aimed at this, this particular issue of breaking down a process which is natural to the teacher, in the stages that would be understood by a learner." (P5, 12)

Or this came through the resources for planning designed by teacher educators, such as prompt sheets for planning:

"We have, on our lesson planning form, we have a very important box which the trainees have to think about and put some detail in which is called pre-lesson understanding […] we've built in this idea of a pre-lesson understanding of saying 'what do pupils need to know before the lesson begins in order to access the lesson and how am I going to check they have this knowledge before I start?' So all trainees have to answer those two questions before they plan." (P5: 24)

Alternatively, this took the form of modelling the unpacking of a mathematical concept as a framework for developing lesson planning:

"What I'm trying to be able to do with these student trainees is to be able to look at an objective that we've got over a series of lessons and the first question I want them to ask is where do I want to take the children? Where do you want to take the children with this objective? So let's say it's averages – do you want them to go to grouped averages or do you want it to be ungrouped? Or do you want it to just be simple means from raw data if that makes sense? OK once you've pinned where you want them to go, then you can take it to well what's the hardest task you want your children to have and then rewind well where are they now? And then you can plot steps in-between. It's trying to get that sort of philosophy then to transfer to their planning." (P16: 11)

As in the previous category of description, the teacher educator continued to offer resources and prompt thinking through suggestions of ideas, resources and approaches. The qualitative difference, however, was that learning comes through a focus on the 'unpacking' of mathematics topics taught in schools with a resulting development of new mathematical understanding for the student teacher (P2:11, P5:19, P6:7, P6.9, P10:9, P11:16, P12:8, P14:2).

The teacher educator in the next extract illustrated the focus on 'unpacking' and its link with the development of new mathematical understandings in a discussion about unitary method and multipliers:

*"[The student teacher] was a business man and he knew **how** to do the questions but not the mathematics **behind** the questions if you see what I mean? And so I was sort of explaining to him how some students would automatically go for the unitary method which works in some instances but not in others and gave him questions where it wouldn't have worked or where it was harder to do." (P6,7)*

"I've got young teachers who came and sat in for the further maths class and I taught them the higher level by being as part of a class because I think there is knowledge at that end and we don't have 6th form here so they can't also, that idea of thinking beyond the top end, so there's just teaching them and on the other side you are trying to teach them how to break it down and their own understanding […] you're not teaching them any mathematics as such, because they know their maths. You're teaching them the structure and the concepts behind so that they can then explain it." (P6,12)

As in the previous category of description the outcome of this process was that the student teacher then had the necessary knowledge to teach and learn mathematics. The links, however, between *knowledge of mathematics* and knowledge about teaching *and learning mathematics* meant that not only could the student teacher teach the current school mathematics curriculum but they would also be able to adapt to changes when the school curriculum changed in future.

Within this conception, it was through the focus on unpacking topics found in school mathematics that the development of the student teacher's own mathematical knowledge increased as in the following illustrative extract:

"The classic thing is the area of a circle. You don't need to do 20,000 areas of circles because once you've found one measurement on a circle, whether it's the area or the radius or the diameter, you can find every other so it's helping them understand that so by making them write their own questions, I think that forces them to think about the mathematics behind whatever topic it is that they're doing." (P6: 27)

"I think the last example, when we looked at 18 times five and the different ways of doing it and hearing them discuss that with each other and learn by talking to each other I think was quite powerful and I'm trying to represent it using 2D or 3D but I think they were learning that each way was valid and they were appreciating that they were having things shared with them mathematically rather than just the answer because they could all do the

calculation but actually it was a push for some of them I think to articulate a reason." (14:3)

The focus within this category of description was on the process of developing knowledge about teaching and learning as a key aspect of learning how to add knowledge of teaching and learning to the knowledge of the subject matter. This is a way to develop an understanding of the links between teaching and learning and the subject matter of mathematics. Mastery of this process was seen as a skill that underpinned planning for teaching and learning in the classroom by student teachers.

"I want them to have the skill to be able to create questions themselves so that they understand the maths that goes on and what questions are next." (P16: 15)

Although it was acknowledged that modelling could be seen as taking the form of "*genuinely trying to teach the lesson as if [the student teachers] are children*" (P14: 13), in most instances, it was instead described as being about the teacher educator drawing attention to the relevant aspects of their practice. This took different forms, incorporating elements of descriptions to reflective elements, e.g.:

"Justifying the choices that we make during the lesson; commenting on, if I have taught that lesson to children for real if you like, commenting on their reactions, their misconceptions and things that came up and genuinely how I might do it differently next time so that's another important aspect of it isn't it?" (P14: 15)

To summarise, the accounts consistent with the category of description 2 foregrounded *knowledge of mathematics*. The emphasis within this category was on a process of learning to learn so that the student teacher was subsequently able to replicate this process across further areas of the school mathematics curriculum. Through this process, the student teacher developed an understanding of the links between their *knowledge of teaching and learning mathematics* and their *knowledge of mathematics.* This would support student teachers to flexibly adapt to changing curriculum content over time. The next category of description further built on this.

4.3.3 Category of description 3: Synthesis of teaching and learning with knowledge of mathematics or 'disrupting':

Within this category of description the teacher educators' accounts were consistent with the notion that mathematics teacher education was about a synthesis of student teachers' knowledge of mathematics with knowledge about mathematics leading to new knowledge about teaching and learning mathematics. They develop their beliefs about learning mathematics.

As per the previous category of description, the accounts contained descriptions of teacher educators modelling teaching and learning practices to student teachers, usually with the mathematics itself. The process of engaging with teaching and learning in the subject was made explicit by teacher educators, both to support learning and also as a skill to be learned. A qualitative difference is that, within this category of description, there was additionally an emphasis on perspectives that may disrupt and challenge trainees' beliefs about teaching and learning mathematics. The outcome of the process was that there was a change in the student teachers' beliefs and attitudes towards teaching and learning mathematics. There were references to the student teachers subsequently adopting an expansive approach to teaching mathematics. This was about, for example, adopting an open-minded approach, underpinned with an understanding of mathematical possibilities and being able to open up the possibilities of understanding mathematics to all pupils (P1:13, P7:30, P7:33, P8:5).

The *knowledge of mathematics* was foregrounded in these accounts, as an object that supports this developmental process. The student teacher required both the skills to imagine the mathematical possibilities as well as to understand the mathematical underpinnings for the different possibilities presented by mathematical objects such as tasks, problems and mathematical prompts (P1: 14, P7: 35, P8:5, P10:19, P10:25.). They learned to teach and learn mathematics in such a way that opens up possibilities to pupils, without making assumptions about either the limits of mathematical possibilities (P8:3, P13:15, P15:8, P15:10) nor limiting notions of pupils ability (P10:14, P10:25, P10:42).

Accounts aligned to this category of description also often involve 'modelling' processes whereby the teacher educator makes their thinking and purpose explicit but also based around this shift to an expansive approach to teaching and learning, supported by mathematical understanding.

> *"I might say, how I make materials and then how I present a task. I might be modelling, that I can unpack that for them, either at the time or later and say 'well I can make these choices; this is what I was aiming to do with this activity'. To try and help them think about how you would do that in a classroom and how you might do that with different groups of learners and then they will also do that for some tasks as well." (P10: 31)*

Within this category of description, the teacher educators offered perspectives and experiences that disrupt and challenge beliefs about teaching and learning mathematics (P3:10, P7:5, P10:11, P12: 26, P13:7, P15:8). Teacher educators' accounts demonstrated awareness that this is a potentially disruptive and emotional process for student teachers (P7:4, P8:17, P8:21, P8:23, P12:14, P13:5). In the following teacher educator account this disturbance was planned in as a part of the learning process:

> *"I want them to be **disturbed** quite early on and challenged so that they can be the most talented mathematician in the world and they suddenly realise they're **troubled** because I'm asking them to make sense of how a different representation might be used in the classroom. …they're **troubled** because they've thought well the only way to learn to add fractions is to be shown and to practice and then it worked for me so why won't it work for these individuals?" (P7:4)*

Teacher educators offered support for the emotional aspects of this process through a variety of means. This included setting up collaborative planning activity (P7:26) and extended group tasks (P15:8). These affective dimensions were also learning points in themselves. One example of an extended group task is of student teachers being encouraged to explicitly record and discuss these struggles, presenting the struggles alongside the mathematical solutions for a group task:

> *"Part of the task was to record their struggle and actually talk about it so they did a presentation and not only did they present the problem and a solution – because none of the solutions were perfect or really polished, but they had to talk about the struggle and the difficulties they found and the strategies they used to solve these significant problems [….] they did a presentation and*

actually, the joy they got out of doing these was just beautiful and they came in and they did this presentation and they said 'I didn't think of anything else for six weeks […] for some of them it was a significant experience that really changed the way they think about mathematics." (P15: 8-10)

Teacher educators described instances where they were aware of the troubling nature of this work for student teachers. They had considered safeguards to control the impact and extent of this on student teacher wellbeing and professional confidence. For example, in the extract below the teacher educator had considered the kind of school placement environment that would be supportive to experimentation and development:

"They need to have time to develop [fundamentals] so we tend to go very, very deeply in terms of taking early algebraic generalisation. They try it out in school in a kind of ***safe, small group environment*** *and that is a taste of the sort of depth we want you to strive for, across the whole curriculum. " (P7:16)*

One account reflected instances where student teachers sought placements or employment in schools environments where beliefs will not be challenged. This served as an interesting example of a form of self-preservation (P7:34).

The descriptions aligned to this category of description reflected the idea that the process involved the expansion of student teachers' mathematics knowledge into new dimensions. In most accounts, this was a positive experience, however, in some cases there were examples of the opposite happening with the expansion of teachers' mathematical knowledge being reductive and resulting in a contraction. The potential reasons for this are unpacked in Chapter 5, but it should be acknowledged here that, in some circumstances, teacher educators perceived that a student teacher's mathematics experiences on school placement resulted in a contraction of their mathematics knowledge to an entrenched position.

To summarise, the accounts consistent with the category of description 3 foregrounded *knowledge of mathematics* with a focus on synthesising *knowledge of mathematics* with *knowledge about learning and teaching mathematics* . There was an emotional and values based dimension to this category of description, where there were fundamental changes to student teachers' beliefs and attitudes towards the teaching and learning of mathematics. The purpose of teacher education was that the student teacher positioned mathematics teaching as a fundamentally expansive activity, taking in possibilities for all learners. The next, and final, category of description built further on this.

4.3.4 Category of description 4: Reconceptualising knowledge of mathematics into knowledge about teaching and learning of mathematics or 'reconceptualising'.

Within this category of description, the teacher educators' accounts were consistent with the notion that mathematics teacher education was about reconceptualising knowledge of mathematics into knowledge about the teaching and learning of mathematics. There is a change in the student teachers' understanding of what mathematics is.

As in the previous category of description, the accounts aligned to this category positioned teacher educators as offering mathematical tasks and experiences for the purpose of facilitating change in student teachers' understanding of what mathematics is. As in the previous category of description, this included the notion of challenging the student teacher to think in different ways about the subject, including their beliefs about the nature of the subject. The qualitative differences were in the purpose of teacher education outcomes, whereby the student teacher developed multiple and advanced perspective on mathematics and the process of learning mathematics (P3:4, P12:29, P13:25, P15:5). Not only was there a synthesis between the forms of mathematical knowledge, but there was also a fundamental shift and reconceptualization of knowledge. Furthermore, the process of *learning mathematics* was a key part of the process of becoming a mathematics teacher, both individually and with others. This was a category of description consistent with there being a social dimension to learning mathematics as something actively learned with oneself and others (P3:38, P10:20, P12:39, P15:4). As part of the learning process within initial teacher education, the student teacher belonged to, and engaged with, a community of active mathematical learners (P3:26, P12:39, P13:25). The learning of mathematics was positioned as reflexive within the accounts consistent with this category of description and metacognitive aspects were present in accounts:

"Essentially, being able to do it yourself is not enough […] [it is about] exposing connections and offering chances to bring together mathematics at the same time." (P3:7)

Not only was this a reflexive process, but the intentions described in the teacher educators accounts reflected a multi-layered nature of this learning, as illustrated by the following extract:

"They've done all sorts of things from making Origami sundials to measuring trees to, all sorts of different things with a kind of ***dual thing –*** *they're looking, they're engaging with the mathematics; they're evaluating and sometimes it's about that sort of level and sometimes it's more structural. We'll be looking at how mathematics connects so I have a teaching session on geometry next week and one of the things we'll do is we'll look at the connections within mathematics and some of the big ideas within geometry." (P10:22)*

This notion of multi-layered intentions also came through in accounts of teacher educators' practices:

"I'm challenging them to think in a different way about learning the subject. […] developing them as subject specialists because it's about the conversation about how we teach our subject …it's about being understanding what that means in the process of learning mathematics.[…] how the subject develops so the ability to get down to the underpinning principles which inform the more ***advanced mathematics done most recently****." (P12:28)*

"Probably the most important thing is that people have the opportunity to do mathematics themselves, to work on mathematics with other people, and to talk about it. To not be afraid to get it right but to explore mathematics and enjoy that experience […] to realise that mathematics can be enjoyable, has aesthetic qualities… to appreciate that everybody does things differently […] and that this can be a really rich source of learning and appreciation." (P3:40)

Teacher educators raised tensions between learning mathematics for the purpose of developing an advanced perspective and for preparing to teach the school mathematics curriculum:

"Sometimes people will say 'its not on the curriculum so I don't need to do it' and I say if you restrict what you do with beginning teachers to what's on the curriculum you are going to miss quite a lot of important points as having a different, advanced perspective, is different to just 'how do you do this with the kids?" (P3:20)

This issue of the tensions between the curriculum for student teacher learning and the school mathematics curriculum are explored in greater depth within Chapter 5.

Finally, in previous categories of description, it was identified that teacher educator accounts supported the notion that a minimum level of subject knowledge was required before the development of knowledge about teaching and learning could take place. This notion continued to permeate this nested model with the following extract providing an example of how this would develop to a multiple and advanced perspective:

> *"I think personal subject knowledge is the start but it's only the start. I think it's still a about a great deal more. You* ***must*** *have your own subject knowledge, at a high level but actually you have something else that's the basis that you the build on. It's the specialist kind of knowledge that means you've not just communicated the mathematics, but you've enabled others to engage and learn it for themselves and move on with their own mathematics." (P10:46).*

The knowledge of mathematics was foregrounded in this category of description as a process, object, and outcome. Within this category of description, the mathematics was a tool, the development of advanced and new perspectives was an explicit outcome in its own right, and there was a strong social and transformative dimension to the learning.

4.4 Conclusion

In summary, within this chapter I have responded to the research questions by outlining the range of variation of accounts of the process of becoming a mathematics teacher amongst teacher educators. This can be summarized in the following diagram:

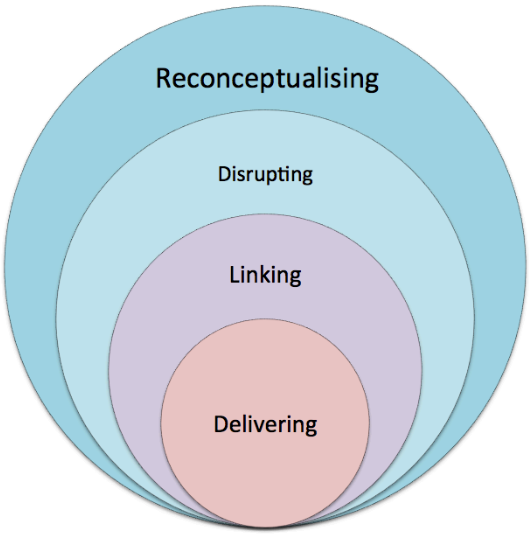

Figure 4.1: A summary model for the teacher educators' accounts of their practices within mathematics teacher education

Within this chapter, I have also outlined the qualitative differences between these categories of description, including highlighting issues at the boundaries between categories. The positions of subject discipline *knowledge of mathematics* and knowledge of *teaching and learning in mathematics* and how these are foregrounded in these positions vary across the categories of description. Accounts at the less developed categories of description foregrounded student teachers' knowledge of teaching and learning mathematics. Moving through to the outer edges of the diagram, representing the more inclusive categories of description, there was an increasing focus on knowledge of mathematics- at its most developed form the knowledge of mathematics took the form of a process, object, and outcome.

Within the following chapter I will discuss the above identified structures and themes in relation to key research. This includes discussion of relations between the purpose of teacher education and mathematical knowledge for teaching. In addition, I make links between teacher education and the ways that teacher educators may draw on curriculum thinking about school subjects. Finally, I explore implications for teacher education within the remainder of the chapter.

Chapter 5: Discussion of Outcomes

5.1 Introduction

This study examines teacher educators' accounts of their practices in mathematics teacher education. I established the range of variation of accounts of practices in teacher education amongst mathematics teacher educators in Chapter 4, in response to the following research questions:

- **What is the range of variation of accounts of the practices of mathematics teacher educators?**
- **What are the qualitative differences between these accounts?**

In this chapter, I consider this variation in relation to the following question concerning the outcomes of the study:

- **What positions on the school curriculum are taken up in the accounts in relation to mathematical knowledge for teaching in developing new teachers?**

I begin the chapter by discussing the outcomes presented in chapter 4 and, where appropriate, making links with the previous research introduced in chapter 2. From this discussion, conclusions are drawn in relation to the context of teacher educator roles and a *curriculum forms* approach is advanced as a lens on these positions on the school curriculum.. A lens enables curriculum aspects of the outcomes to be closely examined. Following this, within chapter 6 I develop implications for policy and practice within initial teacher education.

5.1 Relations between the purpose of teacher education and mathematical knowledge for teaching

Four qualitatively different categories of description of the practices of teacher educators were identified in chapter 4. This discussion now examines the relations between the accounts of teacher education and approaches to subject knowledge learning or mathematical knowledge for teaching. The categories of description comprise four inclusive and hierarchical categories. At the least inclusive level, the category labelled 'delivering', the purpose of teacher education is to develop an understanding of knowledge concerning *teaching and learning mathematics*. The second category of description, 'linking' subsequently incorporates an understanding of the links between *teaching and learning mathematics* with *knowledge of mathematics*. Within the third category of description, 'disrupting' there is a focus on understanding the synthesis of relations between *teaching and learning mathematics* with *knowledge of mathematics*. At the most inclusive level of 'reconceptualising' teacher education represented a reconceptualisation resulting from the synthesis of *teaching and learning mathematics* and *knowledge of mathematics*.

The variation in accounts is such that the more sophisticated descriptions of learning in mathematics education draw on increasingly interdependent relationships between types of knowledge in mathematics. I argue that these increasingly interdependent relations between types of knowledge in mathematics constitute sites of learning in ITE in the accounts of teacher educators with these sites providing opportunity for the integration of new forms of mathematical knowledge for the student teacher. Furthermore, this discussion explores how different forms of curricula, such as formal and informal influences on the knowledge involved in learning 'school maths', 'mathematics' and 'knowledge about teaching and learning mathematics' experienced by student teachers during ITE relate to these types of knowledge, thus taking a curriculum lens on teacher education.

The range of variation reflected in the categories of description differed across the following elements:

- Scope of mathematics curriculum including:
  - Positioning towards the school mathematics curriculum
  - Engagement with mathematics 'around and beyond' the school mathematics curriculum
  - Position/relevance of real life mathematics
- Approaches to teaching and learning
- The role of affective dimensions in the process of becoming a mathematics teacher
- Student teachers as learners of mathematics

I examine the variation in accounts, including across these elements of perception, in this chapter and link this to existing research literature. Furthermore, I argue that these expanding categories of description constitute expanding opportunities for student teachers to engage critically with the process of reconceptualising mathematical knowledge into school mathematics knowledge (‘school maths’). I have developed a *curriculum forms* approach. This approach considers the different curricula and types of knowledge that come into play when influencing the curriculum that pupils experience in school which also subsequently shapes the learning of student teachers in ITE. Curriculum forms is therefore a lens on the practices of teacher educators that illuminates the ways in which they could be considered to be influenced by, and draw on, the different curricula that shape the learners experience of the subject in school. Disjoints between these curricula are identified as opportunities for learning. Within the least developed category of description, the purpose of becoming a mathematics teacher is to become inducted in the regulated and controlled practices of teaching ‘school maths’ a regulated and controlled practice in itself. Within the least developed category of description, there is the least critical engagement with the process of moving mathematics from its original position as a subject discipline into the site of ‘school maths’, and the constraints and affordances that are associated with this. I argue that the expanding categories of description represent a loosening of the boundaries between the practices of ‘school mathematics’ and ‘mathematics’. There is a relationship between engagement with ‘school maths’ and mathematics, and learning in teacher education. Moving through the categories means that the student teacher experiences increasing amounts of interplay between mathematics and school mathematics, with this interplay itself being a site of learning in teacher education. Within the more inclusive categories, this represents teacher education as a space to critically engage with the content and structures of ‘school maths’ itself. It is proposed that the most inclusive category of description of ‘reconceptualisation of the synthesis of teaching and learning and mathematics knowledge’ is therefore a developed position whereby teacher education provides a space for the student teacher to critically engage with the process by which mathematics becomes ‘school maths’. This is done through a lens of advanced and multiple mathematical perspectives. Within the next section I explore the overall structure of these relations.

5.1.1 Structural and referential aspects of the relationships between the purpose of teacher education and mathematical knowledge for teaching

The outcome space set out in table 4.2 shows how the variations in the interplay between *teaching and learning mathematics* and *knowledge of mathematics,* as well as the pedagogical intentions of the teacher educators, as reflected in their accounts, are distributed across the outcome space. This section discusses structural and referential aspects of this outcome space. I start the section by highlighting aspects of a basic orientation to teacher education and then examining how these structural and referential aspects shift as orientations become increasingly more complex.

At a basic, or least inclusive, level, the school mathematics curriculum is experienced as induction into the practices of teaching 'school maths'. School mathematics provides both a context and the parameters for the learning experience of the student teacher, with school mathematics topics serving as a launch point for learning. This occurs, for example, through activities based on starting to plan to teach aspects of the school curriculum through resource gathering.

There is a focus on *teaching and learning mathematics* and, as identified in 5.1.1, the emphasis is on the outcome that the student teacher is able to perform the practices of the school mathematics curriculum, and work with the related texts and skills. Within this category of description the student teacher is therefore positioned as a '*curriculum deliverer'* where success is aligned to successful delivery of the school curriculum over time (Twiselton, 2004), although the emphasis is placed on learning from practice.

Student teacher learning, at this least inclusive level, is for the purpose of being able to perform the practices of the school mathematics curriculum, and the related texts and skills. This reflects a performance mode of pedagogic practice (Bernstein, 2000). There is little reference to the learning of mathematics beyond that deemed to be directly relevant to the school curriculum. The scope of the school mathematics curriculum is predominantly influenced by the demands and requirements of the official recontextualising field (ORF) (Bernstein, 2000). This is evidenced by the frequent references to examination specifications (or schemes of work based on exam specs/national curriculum) as a main influence over the mathematics selected as a focus for learning activity for student teachers. There are echoes of Pais' (2013) reflection that public official discourse about mathematics often articulates its value via the gaining of knowledge and competence for pupils, rather than in relation to its socio-political or socioeconomic value. Real life mathematical examples feature as a motivator, e.g. through the choice of relevant exposition context or to demonstrate use value. At this level, however, there is little acknowledgement that mathematical ideas contextualised in ‘real-life’ can be problematic for children to access due to barriers relating to culture and dispositions (Jorgensen et al., 2013).

Student teachers are positioned as learning to understand barriers and enablers to successful curriculum delivery. Student teachers developing their understanding of pupils’ misconceptions were described in the accounts. The understanding of these is underpinned by the development of knowledge at the intersection of subject content and thinking about pupil learning or in the terms of Ball et al. (2008), concept of *knowledge of content and students (KCS).*

School maths topics usually act as a stimulus for learning, either theoretically or within the context of a school-based activity such as planning a sequence of lessons. The emphasis is therefore predominantly on students teachers’ pedagogical content knowledge (Shulman, 1986). Specifically, and in the terms of Ball and Bass’ (2008) model of mathematical knowledge for teaching, the student teacher is learning about *knowledge of content and students* (KCS) as they consider how pupils may respond to particular mathematical activities. The student teacher is also learning about *knowledge of content and teaching* (KCT), as they learn about how mathematical ideas will play out in teaching practice.

Within the least inclusive descriptions, there is very little reference to explicit engagement with *knowledge of content and curriculum* in either a mathematical or a critical sense by the student teacher. The student teachers’ engagement with the school mathematics curriculum within these accounts is in the sense that they learn its requirements and that this shapes their own learning and the discursive framing (Brown, 2018) of the mathematics learned. The ways that discourses of the official recontextualising field (Bernstein, 2000) shape learning is something that the student teachers are largely unaware of, rather than there being any sense of learning about curriculum thinking (Deng, 2018) within this category of description. However, in line with the finding of Brown (2018), students were aware of the “*policies and associated apparatus validating their practice*”

(Brown, 2018: p80). Indeed, learning was often driven by mathematical topics chosen from schemes of work or discrete learning objectives from formal curriculum materials. Where teacher education is positioned as its most basic level, there is an assumption that student teachers have sufficient pre-existing subject matter knowledge. At this level, ‘sufficient’ means that the student teacher has *common content knowledge* (CCK), that is, the mathematical knowledge and skills that are not necessarily used in teaching, but that all teachers needed to be able to draw upon. In addition, they need to be able to access the next steps in their own learning of mathematics.

As awareness is increasingly in evidence across the outcome space in accounts of the way that ‘school maths’ can be unpacked, there is a referential shift. Not only is teacher education about student teachers adding knowledge about teaching and learning to their knowledge of mathematics, there is also an emphasis on learning how to do this. The second category of description was qualitatively different as this constituted accounts that foregrounded *knowledge of mathematics*. This means that this category of description encompasses the notion that teacher education is about student teachers learning about how to add knowledge about *teaching and learning mathematics* to their *knowledge of mathematics*.

Moving across the outcome space, teacher education therefore becomes increasingly about preparing student teachers to adapt flexibly to the school curriculum with this being a body of knowledge that changes over time, via an evolving selection of skills, discourses and practices from disciplinary mathematics knowledge (Lerman and Adler, 2016). The scope of the school mathematics curriculum within this orientation is still predominantly influenced by the demands and requirements of the official recontextualising field (ORF) (Bernstein, 2000). For example, the mathematics to be ‘unpacked’ is often selected from textbooks or consists of topics from schemes of work or exam specifications. Teacher education becomes increasingly about student teachers learning how to add knowledge. A shift to developing mathematical understanding, however, means that the activities of the pedagogic reconceptualising field (PRF) come to bear on the practices of teacher education. There is a shift to teacher education being a form of future proofing via explicit skill developments, rather than the purpose of teacher education just being to acquire knowledge. Through explicit engagement with these forms of thinking, opportunities are opened up for student teachers to start to engage critically with the process of reconceptualising mathematical knowledge into school mathematics knowledge.

Additionally, teaching and learning practices in teacher education centre on making student teachers aware of the reasoning and rationale behind pedagogical decisions. This is achieved through modelling by teacher educators. This is a non-trivial practice reflecting the multi-layered nature of teacher education, and requiring skilled practice by teacher educators (Loughran and Berry, 2005). There were different ways of making this thinking about teaching and learning explicit and these are reflected within the accounts in line with the finding that there is not a shared agreement about what modelling in teacher education consists of (Boyd and Harris, 2010). For example, some teacher educators referred to role playing learning and teaching. For most, however, this took the form of making metacognition more visible (White, 2011). This modelling highlighted teaching and learning practices that combined knowing about teaching and knowing about mathematics or *content and teaching (KCT)* (Ball et al., 2008). For example, by modelling the process of creating questions and examples 'from scratch' or first principles in order to introduce a new mathematical concept to pupils. There is emphasis on the mathematical content that is specific to the mathematics classroom: distinctive knowledge needed to be able to teach mathematics with understanding (Bair and Rich, 2011). Student teachers learn to re-examine mathematical ideas that they may have drawn on successfully and efficiently in the past. Examples cited by participants include, proficient rubrics for problem solving and tacit application of properties of triangles. They learn to question and break down these ideas, learning a form of unpacking of mathematics, developing their *specialised content knowledge (SCK)* as a specific domain of subject matter knowledge (Ball et al., 2008). Student teacher learning is focused on other forms of SCK; they learn to plan explanations of topic areas that are mathematically accurate, drawing on specialised language and terminology. Their SCK is applied when they plan effective explanations and sequences of learning and, crucially, they are able to respond to pupil questioning with confidence and in a mathematically appropriate way. Student teachers learn how to draw on a range of representations in mathematics (e.g. symbolic, algebraic, graphical) and are able to make connections and use these fluidly and with accuracy. The ubiquity of *specialised content knowledge (SCK)* within accounts from category of description two onwards implies that this is a threshold concept

(Meyer and Land, 2005). SCK not only encapsulates a distinctive form of mathematics knowledge for teaching, but also a particular form of learning for teacher education. The implications of this for teacher education could be further explored, as this raises the question of whether or not effective teacher education is possible without learning sufficient SCK are being explored further.

Teacher education is positioned as increasing levels of synthesis between *mathematics teaching and learning knowledge* and *knowledge of mathematics* across the outcome space. This synthesis is increasingly taking place where these forms of knowledge are reconciled. This process may involve reconciliations of conflicts and tensions, in the process developing new insights.

As teacher educators' accounts increasingly discerned teacher education as being about synthesis of types of knowledge, affective aspects of learning increasingly came into play. Dissonance between personally held prior beliefs about teaching and learning, practices experienced within the school setting, and the experiences of learning mathematics within teacher education were drawn upon by teacher educators as a stimulus for change in the beliefs and attitudes towards learning and teaching mathematics as part of the process of teacher education. This is similar to the approach taken by Leavy and Hourigan (2018). Teacher educator accounts drawing on affective aspects of learning positioned the teacher education process as being about understanding that the student teachers' prior experiences of learning mathematics have been based on a limited range of representations, and there is a need to make sense of mathematics from an increasing range of perspectives. Teacher education is orientated as an epistemological change taking place, challenging student teachers' understanding of the nature of mathematics. This is no longer a construct to be grasped (or not) by the learner, but instead, something that can be grasped by learners that is dependent on the mathematical experience offered by the teacher. Teacher education is therefore about student teachers struggling with mathematics and, as a result, developing their understanding of how others learn the subject.

Accounts of practices of teacher educators drawing on epistemological change position teacher education as drawing on disruption and challenging beliefs about teaching and learning as a form of learning. In relation to the school mathematics curriculum, this means that not only is the school mathematics curriculum a changing entity over time, but that teacher education is positioned as an opportunity to engage with issues of access to the school mathematics curriculum. This happens through developing student teachers' understanding of beliefs about the nature of mathematics and mathematical pedagogy, as it is known that these will impact the way that they teach mathematics (Beswick, 2012). A critical stance is taken towards 'real-life' mathematics, with teacher educators distinguishing between mathematical scenarios that provide authentic motivation for learning from the use of real life maths in authentic contexts. This is similar to the questions raised about tensions in the school mathematics classroom arising from a (mis-)alignment between the kinds of mathematical practices linked to school mathematics, real life mathematics and professional mathematics (Boaler, 2009), although the issue in this case is that of inauthentic positioning (a 'dressing up' of mathematics) rather than the tensions between the types of mathematical practices.

Teacher educators described planning both support and challenge for student teachers going through this process. The foregrounding of these attitudinal factors and inclusion of sense making practices highlights the disruptive nature of learning within this category of description. Teacher education is therefore a learning process that generates a form of cognitive dissonance (Festinger, 1962) between pre-existing beliefs and developing beliefs about teaching and learning mathematics with emerging theory and practice. This can be a troubling process for student teachers involving integrating practice in placement settings along with that of other settings such as university; it is a process of reconciling theory with practice. In some cases these were not reconciled and instead the context of the school setting overwhelmed this process to the extent that the student teachers' mathematics knowledge contracted to align with the conditions presented by the practice setting, as a way to resolve this cognitive dissonance for the student teacher (Beswick, 2012). This is similar to the finding of Schoenfeld (2002) which found that a dissonance between types of beliefs, including those influenced by external pressures, means that some beliefs may not be apparent in a teacher's practice.

There is a shift in the horizon of possibilities for the student teacher from the constraints of the school curriculum to a greater focus on disciplinary parameters, similar to the changing constraints outlined by Twiselton (2004) on the shift between curriculum deliverers and concept builders. Learning mathematics is part of this process and provides a framework for the overall learning experience of student teachers. This more integrative learning approach draws on elements of mathematical knowledge for teaching, as well as a more critical orientation towards *knowledge of content and curriculum* (KCC) (Ball et al., 2008), as student teachers learn to make mathematical as well as pedagogical judgements about the scope and suitability of curricula and materials.

Where student teacher learning is positioned as drawing on increasing mathematical as well as pedagogical knowledge, this therefore positions teacher education as being about developing ways to open up possibilities of understanding mathematics to learners. Student teachers' understanding of misconceptions, for example, is no longer just about why pupils may have particular misconception as a barrier to learning, but also about how these become active teaching opportunities to be proactively designed into planning. In Bass's terms, the *knowledge of content and students (KCS)* is supplemented by *knowledge of content and teaching (KCT),* reflecting an increasingly multilayered positioning towards teaching and learning.

This comprises an increasingly inclusive orientation towards teaching and learning. Knowledge of mathematics develops the student teachers understanding of multiple representations and other mathematical tools for supporting the accessibility of the subject to learners. There is a rejection of the idea that mathematics is a subject that is only accessible to those with a natural innate ability and intelligence and a shift to a more open mindset towards teaching and learning the subject as advocated by Boaler (2016).

An emphasis on drawing increasingly on mathematical rationale alongside pedagogical reasoning for learning in teacher education therefore represents a significant referential shift where reconceptualisation takes place. This means that teacher educators accounts describe student teachers not only drawing on the knowledge types but also changing either themselves or their practices as a result of this synthesis.

At its most sophisticated, teacher education was positioned as 'reconceptualisation of the synthesis of teaching and learning and mathematics knowledge'. This represents a developed position whereby teacher education provides a space for the student teacher to critically engage with the process by which mathematics becomes 'school maths', through a lens of advanced and multiple mathematical perspectives. Student teachers' engage in curriculum thinking, and this is enriched by their knowledge of mathematics as advocated by Deng (2018). Teacher educators' accounts at the most inclusive referred to multi-layered intentions for learning, where the mathematical context acted as a context for the development of the student teacher, as did critical engagement with the school mathematical curriculum.

At its most sophisticated, an advanced and multiple perspective on mathematics was emphasised, unconstrained by the school curriculum. In Bernstein's terms, an advanced perspective on mathematics involves providing student teachers with a tool for critically relating to both the ORF and the PRF, and the tensions between them as part of a process of reconceptualisation of mathematics into school mathematics.

This reconceptualisation constitutes a change in the student teachers' understanding of learning and teaching mathematics, and this comes through engagement with the subject matter. Developing an advanced perspective in this context means that student teachers have a sense of the structure of the subject, and ways of thinking within the subject, contributing firm foundations across domains of mathematical knowledge for teaching (Ball et al., 2008). Teaching and learning in this context also mean that the student teacher's advanced understanding of mathematics supports their understanding of the way that mathematical ideas will play out across the mathematics curriculum; a form of *horizon content knowledge* (Ball and Bass, 2003). This focus on advanced understanding of the subject means that teaching and learning in this context enables student teachers to critically engage with the requirements of mathematics in school.

Within the most inclusive orientations, the learning of mathematics not directly related to the mathematics of the school mathematics curriculum, or '*nonlocal mathematics'* (Wasserman, 2018), presented key learning opportunities. Non-local mathematics is similar to the *specialised content knowledge* (SCK) of Ball's model (Figure 2.1), but differs as SCK relates still to the aspects of mathematics particular to teaching. Nonlocal maths is instead the learning of mathematics not directly related to the school curriculum. Within the accounts of the teacher educators within this study, nonlocal mathematics was used as a vehicle for student teachers to develop their thinking about how to approach the school mathematics curriculum with a mathematical eye, that is, in a mathematically appropriate and coherent way. This was described in two main ways. The first was that the learning of nonlocal maths supported student teachers to develop skills of pre-empting mathematical topics to be taught in the future or 'foreshadowing' (Wasserman, 2018). 'For example, one participant taught student teachers to work with alternative number bases which provided insight into the teaching of place value and working with base 10, enabling student teachers to have insight into choice of examples. The second, known as '*abridging'* (Wasserman, 2018), was the related although opposite pedagogical strategy of simplifying mathematical complexity for example through the choice of example or explanation. Mathematical learning within this category of description is about a sophisticated interaction between a student teacher's advanced perspectives in mathematics and critical perspectives regarding what is taught and learned in schools. This therefore represents a fundamental shift, with the student teacher reconceptualising their understanding of what mathematics is.

The tensions that arise for student teachers going through this transformative process are profound for their identity development, and support for this can be planned into teacher education (Peterman, 2017). Study of related areas such as the history of mathematics were positioned in accounts in this category of description as providing a 'big picture' view of the subject, as well as for reflecting the idea that mathematics is the product of a process of creative human endeavours. This broad view supported the development of open-minded and inclusive approaches to teaching and learning mathematics for student teachers, as espoused by Paolucci (2015). Teaching and learning within this context focuses on what were referred to by one participant as the 'big ideas' in mathematics. A well-developed sense of proportional reasoning as a mathematical concept, for example, would then underpin approaches to teaching and learning of fractions, decimals and percentages. Through this emphasis on the big ideas of mathematics teacher education provides a space for the student teacher to critically engage with the process by which mathematics becomes 'school maths' through a lens of advanced and multiple mathematical perspectives.

There is a change in the student teachers' understanding of what mathematics is. This means that not only do beliefs about the nature of mathematics and the learning of mathematics come into play but also their own relationship and orientation towards the subject are foregrounded. There is recognition that this is a complex process, and teacher educators plan strategies to support the uneven development trajectory of student teachers that results from different starting points and prior beliefs (Beswick, 2012). Teacher education is about student teachers actively engaging with mathematics and having a relationship with mathematics – for example by enjoying mathematics or appreciating its aesthetic qualities.

This overview of structural and relational aspects of the outcome space raises a number of broader questions. If the argument that the school mathematics curriculum is a dynamic and changing representation of mathematics and that the process of reconceptualisation at a systemic and individual level is not a neutral one is accepted, then this analysis has implications for the teacher education sector. What is the role of teacher education in providing a space for curriculum criticality as part of the process of becoming a teacher? What is the role of teacher education in providing a space for multi-layered learning about learning and teaching? At the point at which beliefs and dispositions come into consideration, there are well documented tensions for the student teacher to reconcile this shifting epistemology with practice. How can ITE provide the intellectual and physical space for this process to take place? How can ITE support student teachers to become aware of tensions between cultures, to facilitate and restrict aspects of settings, and to learn from their responses to these?

In summary, within the above section I have set out the ways that the accounts of teacher educators varied, with most inclusive descriptions encapsulating a developed position whereby teacher education provides a space for the student teacher to critically engage with the process by which mathematics becomes 'school maths' through a lens of advanced and multiple mathematical perspectives. In the next section I develop a curriculum perspective on exploring the relationship between these aspects of variation and the purpose of teacher education.

5.1.2 Relationships between the purpose of teacher education and mathematical knowledge for teaching: A curriculum forms approach

In Section 5.1 I have set out the range of variation reflected in the categories of description across the following elements:

- Scope of mathematics curriculum including:
    - Positioning towards the school mathematics curriculum
    - Engagement with mathematics 'around and beyond' the school mathematics curriculum
    - Position/relevance of real life mathematics
- Approaches to teaching and learning
- The role of affective dimensions in the process of becoming a mathematics teacher
- Student teachers as learners of mathematics

In this section I further develop a *curriculum forms* approach to make sense of these ideas and draw together implications for teacher education. This approach explicitly considers how teacher educator accounts reflected that learning from, within, and about the forms that a subject takes within a school curriculum to constitute a learning experience for a student teacher.

The following table illustrates how different dimensions of description of the above elements come into play across the four categories, linked to the theoretical discussion. The elements introduced were Bernstein's (2000) conceptions of the curriculum reconceptualising field, affective dimensions and mathematical knowledge for teaching (Ball, 2004). This illustrates the difference between the categories of description and so is presented as it is useful for describing how the developed categories, and indeed the boundaries between categories, link to these significant aspects of the literature. This is an inclusive model and so each category of description incorporates the elements from previous categories.

Category	Curriculum Recontextualising Field (Bernstein)	Affective Dimensions	Mathematical Knowledge for Teaching (Ball)
1 Delivering	Official	None	Pedagogical, mainly Content and Students (KCS) and Content and Teaching (KCT). Assumes Common Content Knowledge (KCC)
2 Linking	Pedagogical	None	Specialized Content Knowledge (SCK)
3 Disrupting	Pedagogical	Change in orientations towards learning and teaching	Knowledge of Content and Curriculum (KCC)
4 Reconcept ualising	Pedagogical	Change in orientations towards being a learner of mathematics	Horizon Content Knowledge (HCK) Nonlocal knowledge Advanced Perspectives

Table 5.1: Summary of key features of relations for each category of description

The following summary diagram is presented to illustrate how this variation is expressed across the outcome space as a whole:

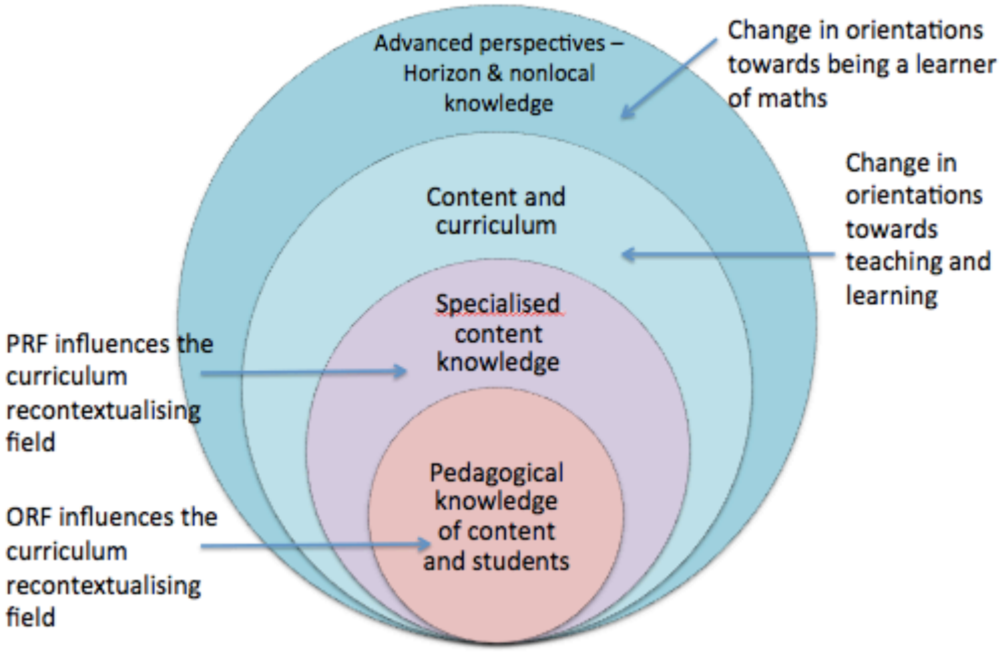

Figure 5.1: A diagram representing how key features of relations vary across the outcome space

From this, the following can be summarised about relations between the purpose of teacher education and mathematical knowledge for teaching

- At the most inclusive level, a *curriculum forms* lens suggests that teacher education is about learning influenced by different knowledge forms of mathematics. That is, learning within, from, and about, the recontextualisation of mathematics and its relationship with 'school maths' through a lens of advanced and multiple mathematical perspectives.

- Less sophisticated orientations reflected a technical and craft-based orientation towards teacher education. Criticality of official policy and practice around the mathematics curriculum in schools comes increasingly part of the learning process within more inclusive categories. Categories become increasingly orientated towards a critically-informed, higher-education underpinned approach to the process of learning in ITE.

- Teacher education increasingly provides opportunities for student teachers to develop and reflect on beliefs, recognising how challenging it is to reconcile what may be deeply held beliefs about teaching and learning and the nature of mathematics.

- There is variation in the way that teacher educators describe the provision of facilitating conditions (e.g. time, space, pedagogical scaffolding) for student teachers to engage critically with the school mathematics curriculum.

- There is variation in the way that teacher educator accounts foreground different dimensions of subject matter knowledge and pedagogical content knowledge. Specialized content knowledge (Ball et al., 2004), in particular, acts as a threshold concept, coming into play when teacher education is positioned as being about more than learning to deliver the 'school maths' curriculum.

Teacher educator accounts describe teacher education, at its most advanced, as therefore a reconceptualisation of forms of mathematics knowledge. This is a developed position whereby teacher education provides a space for student teachers to learn within and around as well as critically engage with the *curriculum influences on* mathematics, such as 'school maths', and the process by which mathematics becomes 'school maths' through a lens of advanced and multiple mathematical perspectives.

Traditional perspectives on ITE suggest that the critical thinking and research skills embedded in Higher Education are a way to conceptualise this model. These links have been identified in section 5.1. Indeed, the teacher education learning space consists of both physical space away from the school setting as well as tools and practices to do this, such as skills for engaging with research-informed approaches. Instead, I suggest that a *curriculum forms* approach provides an alternative approach to exploring features of this model and identifying implications for teacher education. *Curriculum forms* within teacher education is about the opportunities for learning within and from the different ways that a subject appears within the different curricula that inform and influence teacher education. This thesis identifies that there is variation in how this takes place in practice, with different curriculum influences being foregrounded in different ways across the categories of description, as outlined in Chapter 4. It is useful therefore to make this explicit here in relation to the elements identified across the categories summarised in table 5.1. Less inclusive accounts collectively reflected a dominance of the official recontextualising field, and the dominant curriculum for student teacher learning is that of 'school maths'. 'School maths' is the driver for what is learned within teacher education and how it is learned, therefore influencing the curriculum and pedagogy of ITE. At a basic level, teacher education is about delivering the school mathematics curriculum. Furthermore, a characterisation of 'school maths' within this orientation is that it is fixed and not contestable. This means that, where there are tensions between 'school maths' and other forms of mathematics, these are resolved with reference to the dominant format of 'schools maths'.

Moving on to orientations underpinned by learning about how to learn teaching and learning knowledge, there is a change in this positioning, reflected in in the greater influence from the pedagogical reconceptualising field and the threshold concept of specialised content knowledge within this category of description. A *curriculum forms* reflection on this position is that mathematics in school is recognised to be changing over time within this category of description. Different types of knowledge about teaching and learning in mathematics are brought into the learning process of teacher education as necessary tools for student teachers to develop their preparedness to respond to changes over time. By focusing on forms of knowledge about teaching and learning, the student teacher is prepared for not only current forms of school curricula but is also able to respond over time to developments in the curriculum. This is a form of future proofing.

Where teacher education is positioned in the outcome space as expansion into new dimensions of knowledge about teaching and learning, affective dimensions are brought into play within the process of teacher education. The accounts of teacher educators demonstrated how this results in a change in orientation towards teaching and learning mathematics, often with notable shifts in beliefs and attitudes and with an added focus on knowledge of content and the curriculum. Taking a *curriculum forms* perspective whereby curriculum influences and origins are considered, I propose that these shifts are related to orientations towards the school mathematics curriculum. Not only is the school mathematics curriculum changing over time, but it is also seen as being contestable within this and the subsequent category of description. For student teachers, this is often a significant change; their previous experiences of learning mathematics - as pupils, student teachers and lesson observers - have tended to be shaped and strongly influenced by ‘school maths’. For student teachers as learners of mathematics, including both previous study of mathematics at degree level and mathematical learning as part of a teacher education programme, there has additionally been a stronger subject disciplinary curriculum influence. Teacher educator accounts suggest that they are aware of the dissonance between these curriculum influences and plan pedagogic strategies accordingly to challenge and support student teachers with reconciling these forms, recognising that transformational learning takes place as part of this process.

Finally, at its most inclusive aspects of the outcome space, teacher education was about gaining increasingly sophisticated mathematical knowledge. This knowledge has arguably less direct relevance to the school mathematics curriculum, but significant relevance to the learning that takes place within ITE. A *curriculum forms* perspective is that ‘school maths’ and disciplinary mathematics are both contestable and both change over time. Learning within, through, and from these is a reconciliation of different types of knowledge aligned to three areas: ‘school maths’, mathematics, and finally knowledge of teaching and learning mathematics.

The table on the following page summarises how a *curriculum forms* lens provides a view on learning in teacher education in the context of mathematics:

	Curriculum Recontextualising Field	Affective Dimensions	Mathematical Knowledge for Teaching	Curriculum Forms Lens on Learning in Teacher Education
1	Official	None	Pedagogical, mainly Content and Students (KCS) and Content and Teaching (KCT). Assumes Common Content Knowledge (KCC)	School maths shapes the learning. School maths is fixed and not contestable. Teacher education is about delivering the school mathematics curriculum
2	Pedagogical	None	Specialized Content Knowledge (SCK)	School maths changes over time and so forms of knowledge about T&L in maths are brought into learning.
3	Pedagogical	Change in orientations towards learning and teaching	Knowledge of Content and Curriculum (KCC)	School maths is contestable and so other forms of knowledge, the discipline of maths, are brought into learning.
4	Pedagogical	Change in orientations towards being a learner of mathematics	Horizon Content Knowledge (HCK) Nonlocal knowledge (Wasserman, 2018) Advanced Perspectives	School maths and discipline of maths are both contestable and both change over time. Learning is a reconciliation of different types of knowledge in relation to school maths, maths and T&L Maths

Table 5.2: Summary of key features of relations for each category of descriptions with a curriculum forms view of learning in teacher education in the context of mathematics

Again, a diagram is presented to illustrate how this variation is expressed across the outcome space as a whole:

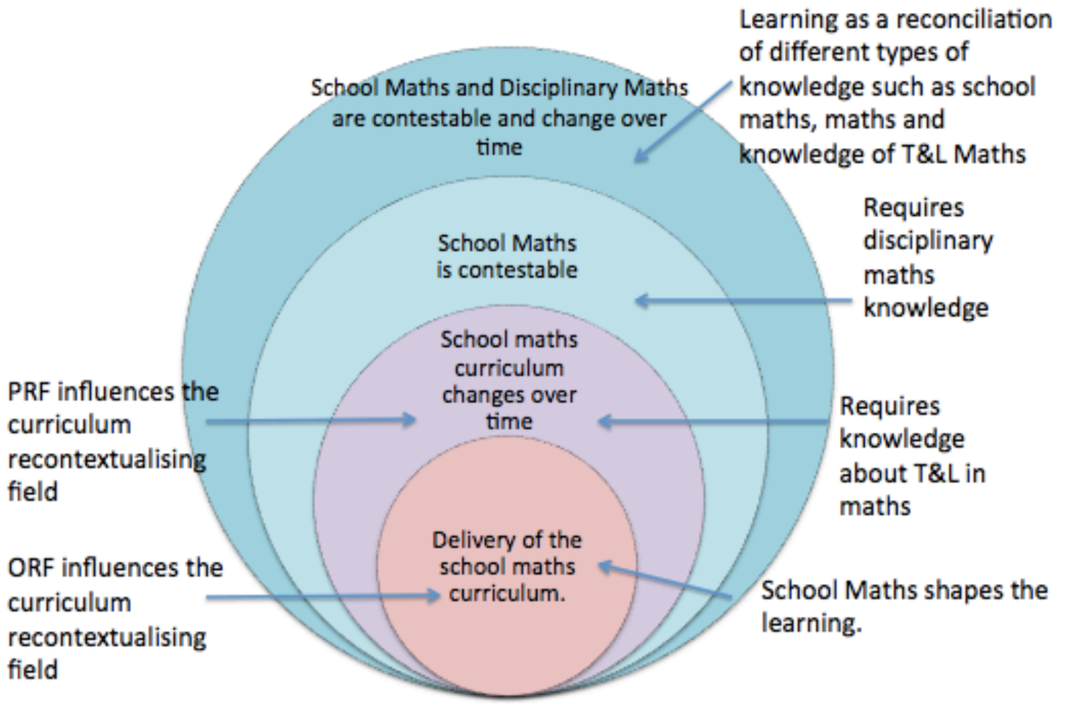

Figure 5.2 *Diagram to illustrate how a curriculum forms view of learning in teacher education in the context of mathematics is expressed across the outcome space*

I have therefore developed the idea of *curriculum forms* in the context of this study as a lens (view of learning) on the purpose of teacher education and learning within this context, taking into account the tensions raised by disjoints between different forms of subject curriculum that shape and frame ITE. Furthermore, these disjoints and differences constitute a transformative place where learning can take place, as this space highlights key ways of thinking and reasoning with a school subject in a way that supports student teacher learning. A *curriculum forms* lens is neither theoretical nor practical, and so is not constrained by the dichotomies about theory and practice that are prevalent within teacher education. The tools that teacher educators drew on in their accounts of teacher education were both theoretical and practical and this approach enables us to make sense of teacher education away from dichotomies that, by their nature, have no resolution. Instead, I posit that this approach enables a way of thinking about teacher education that transcends debates based on dichotomies within the sector.

I advance further discussion about the extension of the lens of *curriculum forms* beyond mathematics in section 6.1.

5.1.3 Summary: the purpose of teacher education and mathematical knowledge for teaching: A curriculum forms approach

In summary, I have explored the relationship between the accounts of teacher education and approaches to subject knowledge learning or mathematical knowledge for teaching in section 5.1. This includes the ways in which Bernstein's notion of the curriculum recontextualising field, affective dimensions of learning, and types of mathematical knowledge for teaching are reflected in teacher education in the mathematics context. A lens of *curriculum forms* has been developed as an original way to draw these elements together and highlight sites of learning within initial teacher education. These learning opportunities concern the pedagogy of teacher education and as such are one of the two main areas of debate in teacher education introduced in section 2.1. The second area of debate related to who/where questions about the location of teacher education and the roles of those engaged in it. A feature of the current context of teacher education, and a design feature of this research is to recognise that teacher educators may be based in a range of settings, including school-based and university-based roles. In the next section I discuss the influence of these contexts on the findings in Chapter 4, and develops the conditions needed for learning to happen. This leads to a further discussion concerning the implications for the teacher education sector in section 6.

5.2 Relationships between the purpose of teacher education and teacher educator contexts

This research examines the individual accounts of school-based and university-based teacher educators, on the basis that this provides a useful insight into sector-wide practices within the current context of ITE in England. Chapter 2 set out that both teacher education and mathematics for teaching have the potential to be experienced differently by teacher educators at different times and in different contexts. The accounts implied relations between the **context of the teacher educator** and their accounts of **subject learning.** As indicated in section 4.1, individual teacher educators' accounts did contain elements of the different levels of description and moved between categories of description within different contexts. It was notable however that, if transcripts were assigned broadly to a single category of description, school-based teacher educator accounts were broadly aligned to category one and two while most university-based teacher educator accounts were aligned to categories of description three and four. This was not the case for any of the other characteristics. Overall, teacher educators with longer lengths of service were HEI based and those with shorter lengths of service were school-based. There was some correspondence, but overall, it was not necessarily the case that accounts from those with the longer lengths of service (particularly those greater than ten years) aligned to the most inclusive categories. In terms of location (school-based or university-based), however, not only were the whole transcripts aligned in the way described above, there were also aspects of the accounts of teacher educators that suggest relationships between descriptions of the purpose of teacher education and their positioning within their role.

Examining this finding more closely, it could be argued that school-based roles provide less opportunity for critical engagement within teacher education. Debate in ITE often frames discussions about teacher educator roles and the location of training in terms of the interplay between theory and practice, and the role of research and pedagogy of teacher education, (including the professional development of tutors and the role of research). This study instead advances a *curriculum forms* approach to considering features of school-based teacher education roles and the close positioning of teacher educators in these roles to the official discourses of the school system. In particular, where teacher educators are situated closer to the school mathematics curriculum, this drives, shapes and constrains the types of learning within ITE. Arguably, in school-based roles, the proximity to forms of pseudo-mathematics within the curriculum inhibits the opportunity to integrate some types of mathematical knowledge. Section 5.2.1 and 5.2.2 explore these relationships through a discussion of features of school-based and university based teacher education respectively. These sections consider how the context of teacher educators impacts the sites of learning that their location enables them to access.

5.2.1 School-based teacher education and the purpose of teacher education

The alignment of school-based teacher educator accounts with the least inclusive categories of description may appear to be a counter-intuitive finding given the significant and necessary contribution to teacher education from those in school-based roles resulting from the ‘close to practice’ nature of the work of the school-based teacher educators. The close-to-practice nature of these roles means that the pedagogic practices associated with the roles often prioritise a practice-based position when reconciling the theory-practice demands of a PGCE programme. Within school-based roles, the practice component is often prioritised over and above the theoretical considerations (Brown et al., 2015). It is worth recalling here, however, that many PGCE programmes are taught by a combination of school and university-based teacher educators, there will be a range of trajectories of development (for example for university-based teacher educators who have moved from school) and an individual teacher educators account may indicate different categories at different times, and so student teachers’ experience of teacher education will be shaped by teacher educators with a range of orientations towards theory and practice. This means that this is a complex situation for student teachers who will likely experience a range of orientations towards the theory-practice debate and will need to reconcile these positions, as they make sense of their own practice. This is thus a different way of examining this alignment, drawing on the idea of *curriculum forms*.

It is known that there is variation in the way that subject and pedagogical knowledge are conceptualised amongst teacher educators, and this is influenced by their location (Brown et al., 2015). Within this study, school-based teacher educators’ accounts tended to focus on pedagogical content knowledge, mainly consisting of *knowledge of content and students* (KCS) and *knowledge of content and teaching* (KCT). Teacher educators assumed that student teachers had a necessary minimum knowledge of *common content knowledge* (CCK), or, where they did not, this was usually remedied through a traditional, audit-do-review style approach. Where school-based teacher educator accounts developed this further, teacher education was usually about developing *specialized content knowledge* (SCK), which I argued earlier in this chapter to be a threshold concept for mathematics teacher education.

An interpretation of this is that school-based teacher educators are strongly influenced by the policies, practices and frameworks of the schools where they are based, and that this frames their pedagogical practices within teacher education. The study outcomes reflected the idea that, for school mathematics, there is a strong framing of 'school maths' within the school mathematics curriculum and that this shaping by official policies and discourses is powerfully held by school-based teacher educators. It could be argued that the demands of these regulatory policies and official frameworks are particularly experienced in the context of school mathematics due to its use value (Pais, 2013). It could also be argued that it is influenced by elitist constructions of the subject's role in society (Boaler, 2016). After all, school mathematics has an almost distinctive position of significant influence on the outcomes and opportunities for individual pupils in its gatekeeper role (Douglas and Attewell, 2017). There is an associated level of accountability for schools, encompassing the punitive consequences of poor performance in mathematics as reflected in accountability measures leading to loss of reputation and status (Perryman et al., 2011). Schemes of work and examination specifications were particularly prevalent as points of reference in school-based teacher educators' accounts, demonstrating the significant influence of examination boards over practices relating to the recontextualisation of knowledge in schools in England (as illustrated by Puttick (2015) and Speilman (2018)). Indeed, when introducing education inspection framework changes in England, Ofsted's Chief Inspector cited the regulatory body's own research, stating that:

> "*The curriculum is not the timetable. Nor is it what we think might be on the exam. We all have to ask ourselves how we have created a situation where second-guessing the test can trump the pursuit of real, deep knowledge and understanding of subjects*." (Spielman, 2018).

These arguments can be developed further by considering them through the lens offered by *curriculum forms*. There are two qualitative differences between categories of description three and two that are useful to consider in the context of school-based teacher education. The first (as illustrated by table 5.1) is that the most inclusive accounts describe teacher education as involving change in affective dimensions. As discussed, this often consists of changes in beliefs about the nature of mathematics and learners. Accounts aligned to the category of description 1 and 2 instead often described student teachers as being inducted into practices of school mathematics, leading to a shaping of their attitudes and beliefs to align with the prevailing ‘school maths’ curriculum. Within this context, where teacher education is about becoming close to practice and understanding it so that this can be replicated, a sense that it is desirable to critique the practices did not permeate the accounts aligned to categories of description 1 and 2. Examples were provided in category of description 1 and 2 accounts of student teachers being taught about ways to deliver schemes of work, supported by class teachers, heads of department, and mentors. Accounts of school-based teacher educators did not, however, contain examples where critical discussion or thinking took place around the policy driving these curriculum practices.

The second qualitative difference between category of description three and two was in the conceptualisation of pedagogical subject knowledge where there were differences between those of school and university-based teacher educators in line with the findings of Brown et al. (2015). The qualitative difference in category of description three was the inclusion of more integrative learning approaches, drawing on elements of mathematical knowledge for teaching, as well as a more critical orientation towards *knowledge of content and curriculum* (KCC) (Ball et al., 2008). Student teachers learn to make mathematical as well as pedagogical judgements about the scope and suitability of curricula and materials. I have argued that school-based teacher educators are subject to regulatory practices and discourses that shape their practice and hence their positioning towards the purpose of teacher education. As a consequence, the strong framing of school mathematics is an example of a regulatory influence on the forms of mathematical activity that can take place (Smith et al., 2013). Consideration of *curriculum forms* suggests that the closeness of school-based teacher educators to the discourses and practices of the school mathematics curriculum presents barriers to the inclusion of the mathematical aspects of the judgements about the scope and suitability of curricula and materials. This appears to restrict forms of engagement with knowledge *of content and curriculum* (KCC) within a school-based context.

Similarly, school-based teacher educator accounts did not include any descriptions of teacher education including the broader forms of mathematics such as ‘*nonlocal mathematics’* (Wasserman, 2018), ‘*horizon content knowledge’* (Ball et al., 2008) or prioritisation of advanced perspectives on the subject. The *curriculum form* approach of this thesis suggests that within the categories of description where these are prevalent, these advanced forms of mathematics are used as a tool, prising apart the different curriculum forms of mathematics, and indeed for reconciling these and blending these back together in knowledge forms that are meaningful for the student teacher in their role as a mathematics teacher. It is surmised, however, that a strong framing of the school mathematics curriculum coupled with the prioritising of ‘relevance’ to school practices within school-based roles means that the circumstances of school-based teacher educators do not provide a motivation or the permitting circumstances to discern these as being a part of the process of teacher education.

In summary, within the school-based context of teacher education, a *curriculum forms* approach suggests that there is a not only a tight framing of the mathematics curriculum, strongly framed by the official recontextualising field (Bernstein, 2000), but that there is little space for levers (such as other forms of mathematics) for prising apart the different forms of curriculum that come into play. This impacts on the ways that teacher educators based in school account for the process of learning. Distance from the school context provides greater affordance for the loosening of this frame at expanding categories of description. Whilst the ability of teacher educators to critically engage with mathematical compromises prevalent in ‘school maths’ is a manifestation of curriculum power and control in pedagogical relations, what is notable here is the way in which that the context constrains or enables the lens of *curriculum forms* to make these explicit and to learn from them.

5.2.2 University-based teacher education and the purpose of teacher education

In addition to the relations described in 5.2.1, there are features specific to university-based tutor roles that influence the relations between university-based roles and teacher education. It is known that there is diversity across ITE programmes internationally in relation to their approach to research in teacher education, and how this is employed as a tool for critical engagement with theory and practice (Flores, 2017). Within the breadth of ITE programmes, however, university-based roles usually provide facilitating conditions, tools and methods for critical engagement due to their position within a Higher Education context. This includes access to research communities, for both teacher educators and student teachers. Engagement with research agendas is usually a continuing professional development expectation of ITE tutors in England. Research-informed approaches to teaching and learning were raised in accounts much more often by university-based teacher educators than school-based teacher educators in the study. Arguably, research skills, access to research literature and evidence based approaches all have the potential to facilitate critical approaches to the school mathematics curriculum.

A further dimension of teacher education, involves the integration of what are often seen as competing discourses in new ways (Zeichner, 2010), such as teacher educators working across departments with groups of mentors and student teachers. Whilst recognising that this is also the case for school-based teacher educators, however, the groups of teachers and mentors they work with are likely to be situated within an alliance of schools where there is some convergence of localized culture and practices. Teacher educators in university-based roles tend to have access to a broader range of school-based practices as part of their roles, bringing a wider perspective to the school mathematics curriculum.

These combinations of research skills, access to research, and broader integration of competing discourses are tools within the accounts of university-teacher educators. The *curriculum forms* approach of this thesis is to suggest that what is key here is that these tools are brought to bear on the possibilities of learning from, with, and through the different forms of mathematics represented in the curricula, shaping the student teacher experience of 'schools maths', 'mathematics', and knowledge about teaching and learning mathematics'. The context of university-based teacher educators provides more access to both the tools with which to do this and the permitting conditions such as providing a learning space for this to take place.

5.2.3 Brief summary of relations between the purpose of teacher education and teacher educator contexts

In summary, teacher educators' definitions of the teacher educator role are growing in diversity, with some researchers choosing to distinguish between professionals working in Higher Education and professionals working in school settings (Douglas, 2017). Within this study, participant teacher educators were based in school or university settings but had in common their contribution to teaching on a Postgraduate Certificate in Education (PGCE) programme, distinguishing this from those who teach on programmes only offering a professional qualification of Qualified Teacher Status (QTS). These parameters mean that the participant sample included school-based and university-based teacher educators. The research findings indicate that there were notable differences, with the university-based teacher educators' accounts being broadly aligned to categories of description three and four in contrast to those in school-based teacher educator roles, whose accounts were usually aligned with the least inclusive categories of description, one and two. A *curriculum forms* lens on the interpretation is offered here that suggests that university-based teacher educator roles provide greater affordances in the forms of tools, methods and spaces for critical engagement with subject and curricula within the context of teacher education than roles based in school settings. Suggestions are made concerning the influence and impact of regulatory regimes within the school setting, and the manifestations of curriculum power and control in pedagogical relations. I have argued that there are barriers to the permitting conditions for the prising apart of school mathematics for critical examination within the process of teacher education. This research project took place in the context of mathematics education, and section 6 explores whether these findings are particular to mathematics or whether or not they apply across other curriculum disciplinary areas.

5.3 Conclusion

In summary, within this research I have explored the positions on the purpose of teacher education taken up in the accounts of teacher educators in relation to mathematical knowledge for teaching. I have argued that the purpose of teacher education is about interplay between *knowledge of teaching and learning* mathematics and *knowledge of mathematics,* with this interplay as a site of learning. Teacher education is therefore about a recontextualisation of mathematics as 'school maths' through a lens of advanced and multiple mathematical perspectives. Taking a *curriculum forms* approach, shape and content of these curricula, including disjoints between these and their contestability, provide opportunities for learning that are taken up by teacher educators to differing degrees depending on their context. In particular, the context influences whether curriculum forms are made explicit, contested, and reconciled before shaping teaching and learning.

Furthermore, this research has considered constraints and affordances relating to school-based and university-based teacher education roles. I argue that these have a notable impact on the positioning of the accounts on teacher education in relation to mathematical knowledge for teaching. To develop mathematical knowledge for teaching, student teachers need to have access to learning experiences that enable the synthesis of mathematical knowledge and knowledge of learning and teaching. To do this, they benefit from proximity to both close-to-practice school mathematics alongside the opportunity to reconceptualise their knowledge by working with educators whose practices enable them to take a critically reflective distance (physical and abstract) from the school mathematics context. In some literature, this distance is positioned as an inhibitor (theory-practice divide) or as an enabler (in the case of university programmes providing space away from the classroom for a safe space for reflection). The findings here contribute a different perspective, framing this distance as about being away from an inhibiting 'pull' of the black hole of 'school mathematics' as shaped by the school context. This *curriculum forms* approach is linked to, but qualitatively different from, the arguments emerging from other research that suggests that student teachers need time away from hegemonic school settings and instead position university as a facilitating space for critical thinking. Within this study I instead take the position that different settings appear to provide different types of opportunity for student teachers to engage critically with the process of reconceptualising mathematical knowledge into school mathematics knowledge ('school maths'), and to develop different knowledge bases concerning the teaching and learning mathematics through this process.

Within Chapter 6 I discuss broader implications of this research for Initial Teacher Education and summarises key implications of this study.

Chapter 6: Conclusions

6.1 Introduction

Within this chapter I proceed with a consideration of the findings of this study in the context of subjects other than mathematics and consider implications for teacher educators. I also summarise key aspects of the study, including originality and contribution to knowledge, reflecting on the approaches taken and signposting ways forward.

My overall research aim was to determine the range of variation of the ways that 'mathematical knowledge for teaching' is described amongst teacher educators currently working in the field of mathematics ITE in England. This contribution to knowledge was achieved and a hierarchical outcome space was developed to represent the range of variation. Teacher educators' individual accounts of practices linked the process of supporting student teachers to become mathematics teachers to the development of student teachers' knowledge of teaching and learning mathematics. When analysed collectively, their accounts demonstrated variation in ways of describing, or experiencing, this process. The study sets out four qualitatively different ways of describing the teacher educators' perception of the process of becoming a mathematics teacher under four categories of description in Chapter 4. These ranged across qualitatively different ways in which the teacher educators foregrounded types of mathematical knowledge in relation to the development of student teachers. There was a shift from having knowledge about *teaching and learning mathematics* as a focus through to *knowledge of mathematics* being the focus at different points within the framework.

Furthermore, the study aimed to address gaps in the research base by considering relations between the features of this variation, roles of teacher educators and the context of teacher education. I took a curriculum approach to exploring these features and, specifically, I developed, and drew on, an original *curriculum forms* lens which illuminated the way that different curricula come to bear on the subject experienced in the school subject classroom. The spaces and disjoints between and around these curricula were explored as a site for learning, and the study explored how teacher educators account for this learning when working with developing teachers. The accounts implied relations between the context of the teacher educator and their accounts of subject learning. I have drawn attention to the notable differences in the ways that this plays out for school-based and university-based teacher educators and have explored this from this curriculum perspective. Taking this curriculum perspective enabled the study to connections to be made between the accounts of practices and the curriculum of the school subject as it plays out in the context of the school setting.

6.2 Teacher education and curriculum thinking: Beyond mathematics

This study focuses on teacher educators' accounts of practice with student teachers within a specific subject of the secondary school curriculum. A *curriculum forms* lens has been taken to explore the relations reflected within these accounts. It is useful to now consider whether there are broader implications for other areas of the school curriculum and more widely for the teacher education sector.

I have argued that the demands of regulatory policies and official frameworks are notably experienced in the context of school mathematics in England due to a potent combination of factors within the subject area. These factors, introduced in chapter 5, include the use value of the subject, societal perceptions of the value of mathematics, the role of the subject as a gatekeeper, particularly for opportunities for work and further study, and the subject's substantial influence over accountability measures for schools. Whilst other school subjects may experience some of these factors, or indeed others, with the potency of effect dependent on the subject, this will come to bear in different ways. It is still appropriate though to consider whether or not aspects of the findings can be applied to school subjects other than mathematics. To this end, categories of description have been shared with teacher educators within other secondary subject areas and other phases such as primary education, with discussion facilitated during conference sessions. Preliminary feedback suggested that the findings are recognisable to the teacher education sector beyond teacher educators working with student teachers undertaking mathematics secondary PGCE programmes.

Within this thesis I have drawn on the idea that the disciplinary subject of mathematics is reconceptualised through its transformation to a school curriculum subject and considered how this relates to the accounts of teacher educators. In Bernstein's terms, mathematics disciplinary knowledge is strongly classified and framed and this provides a specific frame for this study. Within other subjects this classification and framing may be different and furthermore there is variation in forms of reconceptualisation across different subject disciplines (Ashwin, 2009). This means that there may be differences when applying this approach to other subjects which should be considered. Moreover, in the context of higher education, the recontextualisation of the practices of different disciplines, whilst influenced by a range of local, national and global factors, is subject to local institutional context (Ashwin, 2009: p103). This can be considered both in higher education underpinned teacher education, and potentially applied to this discussion of school subjects. This reinforces the idea that there is potential for a range of variation of recontextualisation of subject disciplinary knowledge into school curriculum knowledge, experienced in different ways across school subjects. I have suggested that the *curriculum forms* approach taken to school mathematics could be a tool for reflecting on teacher education practices across a range of subject disciplines, albeit with careful consideration of the specificity of mathematics. The following figure has been developed to frame these considerations and to suggest how a curriculum forms approach may look in the context of subjects other than mathematics. There are questions to be explored about subject specific knowledge for teaching and whether there are threshold concepts in other subjects that would support or further develop this model. What would specialised content knowledge look like in religious studies, for example, or in English literature? What is the role of non-local knowledge in a subject that is less strongly classified and framed than mathematics? Is it the strong classification and framing of mathematics that means that non-local or horizon content knowledge is so significant in mathematics, or would this concept be equally as significant within geography for example?

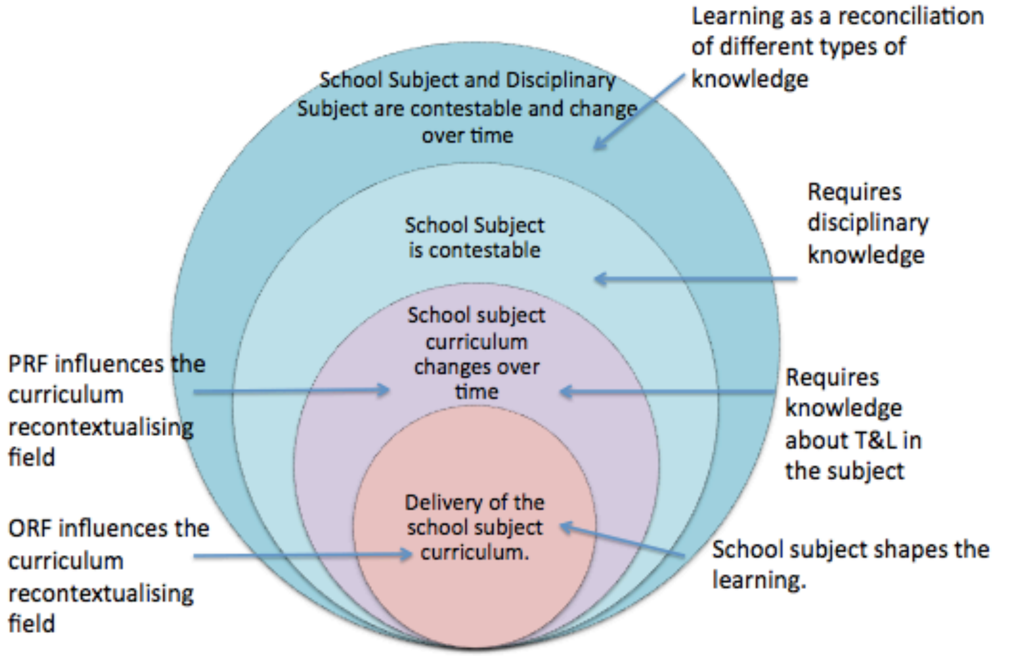

Figure 6.1: A diagram to illustrate how a curriculum forms view of learning in teacher education in the context of a school subject may be expressed across the outcome space

A feature of the variation across the mathematics outcome space that raises questions relates to the way that changes in orientations towards teaching and learning took on such significance within the more inclusive categories of description. Is this the case for other school subjects or is this particularly profound for school mathematics? Would attitudinal change constitute such a significant element of profound change in the accounts of history educators for example? The answers to the above questions are not answered here, and indeed may result in related but different categories of description should the research be replicated with other subject areas, and are therefore worthy of further exploration in future research. I reason, however, that these do not change the broader argument of this thesis, that is, that a curriculum forms approach to analysis and discussion of the accounts of teacher educators has the potential to contribute to the advancement of thinking about the practices of teacher education due to its potential to highlight and illuminate these issues.

6.3 Reflections on the research approach

I brought a phenomenographic approach to bear on the overall aims of this study. This was a useful approach to explore the practices of teacher educators, specifically through their accounts. Teacher educators' practices are underpinned by complex, often implicitly and deeply held interpretations of the multifaceted factors that influence their practices. These interpretations are influenced by context, values, experiences and the interpretation of these- particularly the landscape of hyper-change that currently characterises teacher education was a non-trivial matter. A phenomenographic approach was successful in that its rigorous method provided a framework for this interpretation.

A personal reflection is that the close examination of my own underpinning assumptions, through the study in order to focus on the meaning of the phenomena *for the participants* was also a learning opportunity for me. By setting aside my own assumptions and adopting an awareness based approach, this enabled me to listen to the reflections of others in an authentic way. Not only does this study offer learning to the sector alongside, the process of undertaking this has provided rich learning opportunities to me.

6.4 Originality and contribution

This thesis provides a distinctive contribution, based on curriculum perspectives, in relation to the practices of teacher educators. As such, it is a fresh contribution to ongoing debates within teacher education. Specifically, it has advanced thinking about the ways in which different settings provide different opportunities for student teachers to engage critically with the process of reconceptualising disciplinary knowledge into knowledge for the teaching of school subjects and how this is reflected in the accounts of practices of teacher educators.

This thesis contributes to a gap in the research into the perceptions of teacher educators, as reflected in individual accounts of teacher educators in England. Furthermore, it is unusual to consider individual accounts as a collective consisting of school-based and university-based teacher educators. As such, this research constitutes an original contribution by exploring the relations between teacher education and mathematical knowledge for teaching as reflected in the accounts of teacher educators, where teacher educators are located in school-based or university-based contexts. The study is the first of its kind to develop a phenomenographic outcome space for teacher educators with regards to their accounts of mathematical knowledge for teaching. Additionally, links between the relations reflected in these descriptions and the context of teacher educators have been set out here.

The contribution of this study to the knowledge base is to advance knowledge about the changing role of the teacher educator through analysis and interpretation of the experiences of teacher educators taken collectively across established and emerging roles, including those based in schools. The study contributes a finding that location of roles (university-led or school-led) influences teacher educator positions on the opportunities for learning presented by curriculum matters. Awareness of knowledge influences on curriculum presented more explicit learning opportunities within teacher education for those in roles with some distance from practice, whereas close to practice roles appeared to inhibit these opportunities.

In particular, the thesis advances the research base about mathematical knowledge for teaching by developing a *curriculum forms* perspective on mathematics education whereby the recontextualisation of disciplinary knowledge and its relationship with 'school maths' is examined. This is presented as a phenomenographic overview of the accounts of teacher educators in this area. In the context of mathematics education this links aspects of subject and pedagogical knowledge and orientations to learning with, from and about, the recontextualisation of mathematics and its relationship with 'school maths'. It advances an argument that at its most sophisticated level, this is developed through a lens of advanced and multiple mathematical perspectives.

This thesis therefore presents a curriculum-focused approach, which constitutes a fresh perspective on teacher education. In particular, the study advances original knowledge through a study into how university-based and school-based teacher educators engage with different curriculum influences and the way in which these may be considered a sites of learning in their accounts of practices in initial teacher education.

The thesis advances thinking about the way in which different settings provide different opportunities for student teachers to engage critically with the process of reconceptualising disciplinary knowledge into knowledge for the teaching of school subjects knowledge by identifying how space away from the school setting provides greater opportunity to engage critically with the official recontextualising field, doing so by drawing on increasingly sophisticated forms of knowledge in the subject area. There were notable differences between the accounts of teacher educators in university- and school-based roles. In particular university-based roles provided greater opportunity to explicitly develop orientations towards being a learner in the subject. I employ a *curriculum forms* lens to argue that distance from the immediacy of policy and practice in schools provides a space to both make explicit and learn from the recontextualisation of mathematics and its relationship with 'school maths'. This study therefore contributes to the development of understanding in the sector about how these roles embed in their context and similarities and differences in their contribution to the development of teachers.

6.5 Teacher education and curriculum thinking: Implications for teacher educators

I have employed a curriculum thinking approach to offer a distinctive lens on the practices of teacher education within this study. As such, this work highlights that there is value for teacher educators to be aware of the way that different forms of curricula come to bear both on the school curriculum and on the process of teacher education. The context for the study included consideration of subject knowledge for teaching and its different forms as represented in accounts. The current Teacher Standards in England, for example, sets out a requirement that student teachers "*have a secure knowledge of the relevant subject(s) and curriculum areas, foster and maintain pupils' interest in the subject, and address misunderstandings*" (Department for Education., 2012: p11). This arguably sets out a level of expectation aligned to the dimensions within the category of description one. This is perhaps unsurprising, as the teacher standards set out a minimum level of expectation for Qualified Teacher Status. However, consideration of more sophisticated ways to conceptualise subject knowledge that take into consideration both the complexity of make up of subject knowledge and curriculum forms leads to questions for development. After all, while the teacher standards in isolation define professional knowledge at the point of minimum competency, it is through engagement with higher education that potential for truly transformational learning of the type that Ashwin (2014) argues to be a characteristic of Higher Education. As such, taking a transformational approach to teacher education means considering questions of the vision for teacher education in relation to the development of student teachers' subject knowledge. Should (and how should) teacher education provide the opportunities to develop teacher subject knowledge beyond those articulated in professional teacher competency benchmarks such as the Teacher Standards? What is the role of teacher education in making visible the influences on school subject knowledge? How can student teachers be best prepared for teaching within diverse institutional contexts?

Whilst it appears that there may be implications for the learning of student teachers, this work was focused on the accounts of teacher educators. As such, it does not suggest a model for learning in teacher education; rather the suggested models represent a contribution to the development of understanding about the practices of teacher educators. Additionally, due to the noted relations between the location of the teacher educator roles and the structural aspects of the accounts, it also provides a way of understanding how institutional location comes to bear on accounts of practices across a range of school-based and university-based roles. What this study highlights is that there is value for teacher educators to be aware of the way that different forms of curricula come to bear both on the school curriculum and on the process of teacher education. As such, this offers an alternative approach to dichotomy-based struggles within the sector such as theory-practice, skills-knowledge and other such debates. A *Curriculum forms* lens has brought a different perspective to teacher education for this study. It has value both as a sense making approach to the complex business of ITE, but also in making visible aspects of what we teach new entrants to the profession, and how we teach them. Within this study the lens has been a useful way to explore how the differences in the way that teacher educators account for their practice can be linked to their setting and experience. This can be through scrutiny of underpinning structures and assumptions. Teaching can never be value or assumption free; teacher education therefore has a critical role in not only making this visible but in harnessing the rich learning possibilities of the spaces where these are present.

Within the remainder of this chapter I summarise key aspects of the study, reflect on the approaches taken and signpost ways forward.

6.6 Limitations of the study

This is a phenomenographic study and, as such, is not intended to present generalisation of outcomes, nor draw conclusions about the phenomena under examination (Marton, 1986). The phenomenographic study instead aims to illuminate the ways in which participants perceive the phenomena. As such, the outcome space and subsequent discussion for this study together present an overview of the ways in which teacher educators account for subject knowledge for teaching and the relations with aspects of teacher education such as purpose and location.

The framework for the outcome space presented is of the form of an inclusive hierarchy summarising the collective analysis of individual descriptions. The categories of description presented, are therefore not intended to represent the perceptions of individual teacher educators, and not to classify individuals nor individual accounts. Teacher educators were prompted to reflect upon past examples of practice and were encouraged to choose recent examples. These were not taken as an indication of either the entirety of their practice nor do they signify an indication of aspirations of idealised future practices developing.

This collective and not individual approach to analysis means that, where descriptions of practices were included about the least inclusive levels of the hierarchy, this does not indicate a value judgement about the practices of the individual teacher educator. As noted, teacher educators' accounts often contained elements of the different levels of description and indicated that they moved between categories of description within different contexts.

There is a temporal aspect to this study as the study was undertaken at a particular time. As such, the outcome space represents a collective of meaning at this time and as interpreted through the frameworks of the research base presented here and by myself as the researcher. As such, this is relational and interpretative work and I have set out the checks, balances, and safeguards to ensure that there is a form of communicative validity (Åkerlind, 2005b) of these outcomes.

In the next section I further identify related areas of practice that could be further developed for future research.

6.7 Areas for future research

Within this thesis I have advanced a *curriculum forms* lens on initial teacher education through consideration of the accounts of teacher educators working with student teachers in the context of secondary mathematics. Areas for future research are suggested here, falling into three categories of description- the first relating to how this approach extends beyond mathematics and the second about wider issues within teacher education. Thirdly, suggestions are made regarding future research into implications of location of teacher educator roles. Finally, research areas are identified that arose during the study but were not able to be explored, as they did not align to the constraints of time and focus required for this particular study.

Firstly, future research could explore subject specific knowledge for teaching beyond mathematics with a *curriculum forms* lens. Within the mathematics context, specialized content knowledge for teaching was a threshold concept leading to the more inclusive descriptions. How could threshold concepts in other subjects support or further develop this model? What would specialised content knowledge look like in subjects other than mathematics? What is the role of non-local knowledge in a subject that is less strongly classified and framed than mathematics? Is it the strong classification and framing of mathematics that means that non-local or horizon content knowledge is so significant in mathematics, or would this concept be equally as significant within a different subject. For this study the tight framing of what is taught and learned in mathematics through 'school maths' provided a way to interpret the findings, but how would this play out within other subjects and across the school curriculum as a whole? As has been identified in the study limitations, there are debates in all subjects and a *curriculum forms* approach may offer a lens for exploring these.

More broadly for teacher education, future research could explore how teacher education could provide opportunities to develop teacher subject knowledge, beyond those articulated in professional teacher competency benchmarks such as the Teacher Standards. What is the role of teacher education in making visible the influences on school subject knowledge? How can student teachers be best prepared for teaching within diverse institutional contexts? A feature of the study outcomes was the way that changes in orientations towards teaching and learning took on such significance within the more inclusive categories of description. It would be useful to explore whether this is the case for other school subjects, or whether is this is particularly profound for school mathematics? Would attitudinal change constitute such a significant element of profound change in the accounts of history educators for example?

Thirdly, the qualitative differences between the accounts of university-based teacher educators and school-based teacher educators suggests that there is much to explore in relation to these emerging roles and the curriculum. A *curriculum forms* approach offers the opportunity to explore how teacher education draws on different curricula from a critical perspective in order to make visible the assumptions of this process and maximise the learning opportunities therein.

Finally, it is useful to set out further areas of interest that arose in the study that I was not able to pursue due to time or study design. In the mathematical context, the ubiquity of the role of mathematical misconceptions in accounts suggests that it may be useful to explore teacher educator understanding of these in order to probe how these relate to the curriculum. More broadly, this study focused on the accounts of teacher educators. I would like to explore some of the themes from this study with student teachers and understand their perception, particularly with reference to how they reconcile the different positions afforded by school-based and university-based contexts. Finally, I would like to further explore institutional context and the link with curriculum forms- in what way does institutional context provide conditions for the critical engagement with curriculum forms, by staff, pupils, and student teachers? These are a small number of the myriad of potential future questions for exploration raised by this study.

6.8 Concluding comments

In conclusion, this study has explored the perspectives of teacher educators through their accounts of practice. Through consideration of the variation in their accounts, the study has explored ways in which mathematical knowledge for teaching comes into play across the contexts of teacher educator roles. This study has taken a curriculum approach to exploring the features of the findings. The study has surfaced ideas relating to how what we teach and how we teach cannot be context and value free, and how disjoints between potential curricula provide a space for learning as they provide new ways to consider key issues in subject teaching.

On a personal note, to conclude this study, teaching and learning are joyful, creative, and human endeavors. Teachers and teacher educators have the opportunity to draw on deep knowledge of a subject, strong understanding of learning itself and build relationships to work with others to enable them to learn. Curriculum frameworks shape this work, and whether constraining or enabling, they have the potential to have a profound influence on how the learning unfolds and, as such, have been an illuminating lens for this study into aspects of teacher education.

Appendix 1: Interview Schedule

Interview Overview

Introductory Outline:
For all interviews:
Introductions and outline of the purpose/aims of the project. Outline the use of data. Discuss confidentiality/anonymity.
Outline use of recording and transcription.
Check whether there are any questions re the above.

Interview Prompts:

Firstly, can you tell me about a recent time/incident when you helped somebody to become a mathematics teacher?

Can you describe a particular recent incident -
Where you worked with student teachers to develop their understanding of *mathematics* ?

(Follow up prompts:
- Why did you do that? Why did you do it that way?
- What were you hoping to achieve?
- What do you mean by x?)

Based on your experiences so far what does it mean to you to develop mathematics for teaching in student teachers? (i.e. what do you DO as a teacher educator to develop mathematics for teaching in student teachers?, what are you trying to achieve?).

Can you give me a concrete example of something you have done to make mathematics accessible or developing conceptual understanding when working with student teacher.
- Why did you do that? Why did you do it that way?
- What were you hoping to achieve?
- What do you mean by x?

How do you judge whether you have been successful in developing conceptual understanding to student teachers? (prompt for an example).

Can you tell me what developing conceptual understanding means to you as a teacher educator when working with student teachers?

How do you judge whether you have been successful in developing conceptual understanding to student teachers? (prompt for an example).

Can you tell me what making mathematics accessible means to you as a teacher educator when working with student teachers?

Rounding up….at the start I asked what does it mean to you to develop mathematics for teaching in student teachers. Since then you have had the opportunity to reflect upon specific approaches and episodes. Having been through this process please could you reflect on this and then summarise what mathematics for teaching means to you.

Before we finish do you have anything to add that we haven’t mentioned?

Thank you

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