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Abstract

This is the first paper to provide a comprehensive theoretical analysis of the third and fourth order lottery preferences implied by cumulative prospect theory (CPT). We consider the lottery choices from three alternative reference points: the status quo, the expected payout and the MaxMin. We report a large number of new results given the standard assumptions about probability weighting. We demonstrate, for example, the general result that from the status quo reference point there is no third order reflection effect but there is a fourth order reflection effect. When the average payout or the MaxMin is the reference point, we lose generality but can demonstrate that representative individuals with power value functions can make prudent or imprudent, temperate or intemperate choices depending on the precise magnitude of lottery payoffs. In addition to this, we show that these representative CPT individuals can exhibit some surprising combinations of second with third and fourth order risk attitudes. Throughout the paper, we contrast our theoretical predictions with results reported in the literature and we are able to reconcile some conflicting evidence on higher order risk preferences.

Keywords: cumulative prospect theory, decision making under risk, experiments, higher order preferences, reflection effect

JEL codes: D81, E21

1 Introduction

Over the last three decades theoretical research has demonstrated the important role that higher order preferences, particularly prudence or temperance, play in economic models of risky choice such as savings, auctions, asset pricing and several other (see Trautmann and van de Kuilen (2018) for a review of this literature). Experimental evidence of the 'pure' effect of those higher order risk attitudes was facilitated by the introduction of choice-based definitions of preferences of any order by Eeckhoudt and Schlesinger (2006). This major contribution set out the method for revealing higher order risk preferences employing experimental methods. They demonstrated how the choices between particular lottery pairs could be employed to elicit higher order preferences. Subsequently, a number of experimental studies have employed choices between lotteries to reveal the higher order preferences of experimental subjects (a few prominent examples within the framework of risk include Deck and Schlesinger 2010, 2014, Maier and Rüger, 2012, Noussair et al. 2014, Ebert and Wiesen, 2011, 2014; and Heinrich and Mayrhofer, 2018, and within the framework of ambiguity Baillon et al. 2018).

In this paper, we will focus on the first two higher order preferences, namely, risk apportionment of order 3 and 4, also known as prudence and temperance, respectively.[1]

A key feature of the experimental research reported to date on higher order lottery choices is that researchers have endeavoured to implement their appropriate reference point by experimental procedure and lottery design. This reference point is then assumed in analysis of the responses of the experimental subjects.[2] Maier and Rüger (2012), Bleichrodt and van Bruggen (2018) and Brunette and Jacob (2019) report results where the status quo reference point is assumed. Alternatively, Deck and Schlesinger (2010) and Ebert and Wiesen (2014) assumed that the reference point was the average payoff.

Recent research by Baillon et al. (2019) developed a test for the appropriate reference point employing second order lottery choices and assuming CPT preferences. Two of the reference points

[1] "Prudence" was coined by Kimball (1990) in his analysis of precautionary savings within and expected-utility framework. "Temperance," on the other hand, was first employed by Kimball (1992) in his work on precautionary motives for holding assets.

[2] For example, the experimental design in Maier and Rüger (2012) was set up to try to ensure that the status quo was the reference point. They write: "*We carefully designed our experiment to implement the status quo prior to the second date as the reference point.*"

they considered, the status quo and the MaxMin (the maximum outcome subjects can obtain for sure), were identified as the most frequently employed each by around 30% of their experimental subjects.[3,4] In this paper, we will examine third and fourth order lottery choices as employing these two reference points as well as the average or expected return of the lotteries since, as noted above, the expected payout has been employed in prominent studies on higher order preferences.

Deck and Schlesinger (2010 p.1414, 2012 pp.28-29) conjecture that any higher order lottery choices are in principle consistent with CPT due to the interplay of cumulative weighting of probabilities, the value function's properties over gains and losses, and loss aversion. We examine this conjecture employing a framework based on the method of Yaari (1987) in conjunction with a definition of the representative experimental CPT subject. From the status quo reference point the representative CPT subject is assumed to exhibit solely the form of probability weighting whereby small probabilities are over-weighted and larger ones are under-weighted. From the expected payout or MaxMin reference point, we assume the parametric model of CPT and the range of parameter values reported in the prominent research of Tversky and Kahneman (1992), Deck and Schlesinger (2010), Ebert and Wiesen (2014) and Baillon et al. (2019).

The results of our analysis reveal that from the status quo reference point we can derive the general result that CPT individual will always makes prudent and temperate lottery choices when all payoffs are positive, and prudent and intemperate lottery choices when all lottery payoffs are negative. For mixed lotteries, when the status quo is the reference point, all third and fourth order lottery choices are feasible.

From the expected value reference point, our anaysis reveals that the representative CPT subject exhibits prudent or imprudent, temperate or intemperate lottery choices depending on the precise lottery payoffs employed and their risky choice parameter values. From the MaxMin reference

[3] Kőszegi and Rabin (2006, 2007) (KR) assume the reference point is expectations-based, taken to be the entire distribution of expected outcomes. In choosing between two lotteries, the expected utility of each lottery is evaluated by determining the expected utilities of the difference between each possible outcome and the reference point outcome. Following the method of Baillon et al. (2019), who illustrated the implications of the KR assumption for the expected utility of a second order lottery, we can show that the KR hypothesis implies the individual is indifferent between the prudent and imprudent lotteries. This is demonstrated in the Appendix.

[4] We are not aware of any study on higher-order preferences that has assumed the MaxMin reference point. We also note that Baillon et al. (2019) found little support for expected payout as the reference point.

point, the representative CPT subject always makes a prudent or temperate lottery choice.

These findings have a number of implications for experimental research. Whilst most experimental studies report that the majority third order lottery choice is prudent. However, the experimental evidence does not support a majority fourth order preference. For example, Deck and Schlesinger (2010) reported more intemperate choices than temperate, which contrasted with the reverse finding of Noussair et al. (2014) and Ebert and Wiesen (2014). Trautmann and van de Kuilen (2018) in their review report the ranges of the average proportion of risk averse, prudent and temperate choices observed in experimental research which span the proportions [46,84], [45,96], and [38,87], respectively.[5] Overall, the experimental results reveal that there is a non-negligible proportion of imprudent and intemperate lottery choices which are inconsistent with everywhere-concave expected utility theory (EUT) models. We will provide an explanation of how these experimental findings on higher order risk preferences can be reconciled with cumulative prospect theory (CPT) of Tversky and Kahneman.

The implications of our analysis of third and fourth order lottery choices from the status quo reference point is particularly interesting given the lottery choices reported by Maier and Rüger (2012), Bleichrodt and van Bruggen (2018) and Brunette and Jacob (2019) who all assumed the status quo reference point. Maier and Rüger reported that the average proportion of prudent choices when all lottery payoffs were positive was, for ten lottery choices, 60% with a maximum of 70% and a minimum of 49%. The corresponding figures when all lottery payoffs were losses was, for seven lottery choices, 55%, 63% and 49%, respectively. The proportions of temperate or intemperate choices reported by Maier and Rüger also show wide variations. For the all-gains lotteries the average proportion of temperate choices reported was, for ten lottery choices, 58% with a maximum of 73% and minimum of 43%. The corresponding figures for the seven all-losses lotteries were 54%, 69%, and 40%, respectively.

Bleichrodt and van Bruggen (2018) report that, for twelve all-gain lotteries, the average proportion of prudent choices was 56% and, for twelve all-gain lotteries, the average temperate response was 40.4%. The corresponding figures for twelve of each all-losses lotteries were an average of

[5] Those figures are based on table A.1 in Trautmann and van de Kuilen (2018). The figures in the text correspond to averages in the studies reported in Trautmann and van de Kuilen (2018), but if the range within each of those studies were to be employed instead, the overall range of values would be significantly wider.

39.8% prudent and 69.6% intemperate. Brunette and Jacob (2019) report, for five lottery choices, an average of 2.63 prudent choices in the all-gain domain, and 1.29 in the all-loss domain. The corresponding figures for the five temperate/intemperate choices were 3.42 temperate for all gains and 2.87 for all losses.

Clearly, the results reported in these three experimental studies suggest that there is a significant proportion of lottery choices that are not consistent with the choices that would be made by the representative CPT or EUT subject from the status quo reference point.

Our new result that from the average reference point the CPT representative subjects can exhibit any third or fourth order preference, enables us to help explain the experimental findings reported by Deck and Schlesinger (2010) and Ebert and Wiesen (2014). In these two studies the average reference point was assumed. Whilst both studies reported a majority third order preference of prudence, they differed in the majority fourth order preference. The parameter values of the representative CPT subject reported in Deck and Schlesinger imply an intemperate majority choice employing their lottery payoffs and would also imply an intemperate majority choice employing the lottery payoffs employed in Ebert and Wiesen (2014). Conversely, the parameter values of the representative subject reported in Ebert and Wiesen imply a majority temperate choice employing their lottery payoffs and would also imply a majority temperate choice employing the lottery payoffs employed in Deck and Schlesinger. Both sets of parameters reported in these two studies imply majority prudent choices for the lottery payoffs employed.

The rest of the paper is organised as follows. In section 2, we set out the method of elicitation of third and fourth order preferences as well as the parametric model of CPT employed in our analysis. Sections 3, 4, and 5 present the predictions of CPT for risk apportionment of order 3 and 4 assuming the three different reference points: status quo, average payoff, and MaxMin, respectively, and employing a variety of lottery payoff structures. The final section provides the conclusions.

2 The CPT model and elicitation of higher order preferences

Our analysis of higher order preferences follows the methodology developed by Deck and Schlesinger (2014). Deck and Schlesinger's method derives from Eeckhoudt et al. (2009) and generalises the approach of Eeckhoudt and Schlesinger (2006). The elicitation of higher order preferences is done through the choices of 50-50 lottery pairs. We will write $[z\_y]$ to denote a lottery of equally likely payoffs z and y. Let us denote an individual's endowment $W > 0$, reductions in wealth $k_1 > 0, k_2 > 0$, and zero-mean random variables $\widetilde{\varepsilon}_1 \equiv [-\varepsilon_1\_\varepsilon_1], \widetilde{\varepsilon}_2 \equiv [-\varepsilon_2\_\varepsilon_2]$, whose distributions are assumed to be statistically independent of one another, and $\varepsilon_1, \varepsilon_2 > 0$. The 50-50 lottery $A_n \equiv [W\_W + z + y]$ has more nth-degree risk ($n = 2, 3$ or 4) than lottery $B_n \equiv [W + z\_W + y]$.[6] We will only examine risk attitudes up to order four, so z will take the values of either $-k_1$ or $\widetilde{\varepsilon}_1$, and y will be either $-k_2$ or $\widetilde{\varepsilon}_2$. In the terminology of Deck and Schlesinger (2014), individuals are labelled as "risk apportionate of order n" if they dislike the lottery with more nth-degree risk, i.e., $B_n \succ A_n$. Table 1 presents the classification of risk attitudes and their corresponding lottery pair structure.

Table 1. Risk attitudes and lottery pair structure (B_n, A_n)

Risk preference	B_n	A_n
$n = 2$, Risk aversion		
$z = -k_1, y = -k_2$	$B_2 \equiv [W - k_1\_W - k_2]$	$A_2 \equiv [W\_W - k_1 - k_2]$
$n = 3$, Prudence		
$z = \widetilde{\varepsilon}_1, y = -k_2$	$B_3 \equiv [W + \widetilde{\varepsilon}_1\_W - k_2]$	$A_3 \equiv [W\_W - k_2 + \widetilde{\varepsilon}_1]$
$n = 4$, Temperance		
$z = \widetilde{\varepsilon}_1, y = \widetilde{\varepsilon}_2$	$B_4 \equiv [W + \widetilde{\varepsilon}_1\_W + \widetilde{\varepsilon}_2]$	$A_4 \equiv [W\_W + \widetilde{\varepsilon}_1 + \widetilde{\varepsilon}_2]$

An individual who is prudent has a preference to combine the relatively "good" outcome (endowment W without the loss of k_2) with the "bad" outcome (zero-mean risk $\widetilde{\varepsilon}_1$), as in lottery B_3, instead of combining the two relatively "bad" outcomes together (wealth reduction of k_2 and the zero-mean risk $\widetilde{\varepsilon}_1$), as it is the case in lottery A_3. Likewise, an individual is temperate if she prefers to combine the relatively "good" and "bad" outcomes (B_4) instead of combining the two "bad" outcomes together (zero-mean risks $\widetilde{\varepsilon}_1$ and $\widetilde{\varepsilon}_2$) as it is the case in A_4.

In our analysis, we will initially assume linear value functions. Assuming linearity of the value functions implies that the representation of different risk preferences is done exclusively through

[6] Using Deck and Schlesinger's (2014) notation, the lottery structure to elicit temperance could also be constructed with a combination of $n = 1, m = 3$ (see Deck and Schlesinger, 2014 p.1919). To simplify notation and the analysis below, we only consider the case of $n = 2, m = 2$.

subjective cumulative probability weighting transformations (e.g. Yaari (1987), in his dual theory) from the status quo reference point (except for mixed lotteries) and through subjective cumulative probability weighting and loss aversion from the average or MaxMin reference points.[7] In the case of the status quo reference point, when all lottery payoffs are either gains or losses, this enables us to determine the higher order lottery choices since all that is necessary to determine the preference, given the probability weighting, is the assumption of value functions that are everywhere risk-averse over gains and risk-seeking over losses. However, when we relax the assumption of linear value functions, we can only determine the third or fourth order lottery choice from the average or MaxMin reference points by assuming a particular form of the value functions since the degree of risk-aversion or risk-seeking plays a role in the higher order choice. We therefore employ the parametric models of CPT reported in four prominent studies. We assume the representative agent is defined by the most common parameterization of CPT that includes power value functions, $v(.)$, with reference point r, parameter $\alpha \in [0,1]$, and a loss aversion parameter λ, in conjunction with an inverse-S-shaped weighting function, $w(p)$.[8]

$$v(x) = \begin{cases} (x-r)^{\alpha} & \text{for } x > r \\ -\lambda\,(r-x)^{\alpha} & \text{for } r \geq x \end{cases}$$

$$w^{+}(p) = \frac{p^{\gamma}}{(p^{\gamma}+(1-p)^{\gamma})^{\frac{1}{\gamma}}}$$
$$w^{-}(p) = \frac{p^{\delta}}{\left(p^{\delta}+(1-p)^{\delta}\right)^{\frac{1}{\delta}}}$$

where $w^{+}(p)$ and $w^{-}(p)$ are the probability weighting functions for gains and losses, respectively.[9] In CPT, the probability weighting function does not apply to the probability density

[7] Also in experimental research on second order risk preferences over small lottery payoffs numerous researchers report that linear value functions are a good approximation to the value function (e.g. Fehr-duda et al., 2006; Abdellaoui et al., 2008; Ring et al., 2018; and L'Haridon and Vieider, 2019). It is also of interest to note that, in the only paper that has estimated a CPT model for higher order preferences, Ebert and Wiesen (2014) report a value of the power exponent as 0.97, presumably not statistically different from unity.

[8] The value function $v(x)$ can be generalised if parameter α differs over gains and over losses. We keep this simpler form for the following reasons: (i) it provides a more intuitive understanding of the role played by the degree of risk seeking/risk aversion in the determination of higher order preferences; (ii) many of the prominent studies listed below assume equal power coefficient over gains and losses; and (iii) generalising the results would be relatively straightforward.

[9] Our results will apply in general to any probability weighting function that overweights small probabilities and

function but to the cumulative probability distribution. For gains, $x > r$, the cumulative weighting is applied from the largest to smallest gain, or in the case of losses, from the largest loss to the smallest.

Table 2 lists the four model specifications from Tversky and Kahneman (1992), Deck and Schlesinger (2010), Ebert and Wiesen (2014), and Baillon et al. (2019), which we will refer to from here onwards as TK, DS, EW, and BBS, respectively.[10] The parameters of DS and EW are based on studies on third and fourth order preferences and that of TK and BBS on second order.

Table 2. Four Parameterisations of the CPT Model

	α	λ	γ	δ
Tversky and Kahnemann (1992) (TK)	0.88	2.25	0.61	0.69
Deck and Schlesinger (2010) (DS)	0.88	2.25	0.65	0.65
Ebert and Wiesen (2014) (EW)	0.97	1.53	0.43	0.43
Baillon, Bleichrodt and Spinu (2019) (BBS)	0.48	2.34	0.53	0.53

We employ the range of parameter values to derive the implications for third and fourth order lottery preferences from the three different reference points. We find that the third and fourth order lottery choices of the representative CPT individual will depend on the precise parameter values and lottery payoff structure. This enables us to explain the conflicting experimental results reported.

The next three sections present analytical expressions that define risk apportionment of order 3 and 4. Each section corresponds to a different assumption about the appropriate reference point.

underweights large probabilities. We have decided to illustrate our framework with the function in Tversky and Kahneman (1992) since it is employed in three of the most prominent studies that report parameter values.

[10]Baillon et al. (2019) report a one parameter Prelec weighting function $w(p) = e^{-(-\ln p)^{0.43}}$ with the same parameter over gains and losses. When $\gamma = \delta = 0.53$ the weighting function described in this section is employed to proxy the Prelec function since it makes no important difference to our reported results in our setting.

3 Reference point 1: Status Quo

3.1 Prudence

3.1.1 Domain of gains: Lottery payoffs are all gains

From the status quo reference point the probabilities and payoffs of the prudent, B_3, and imprudent, A_3, lottery pair are given by[11]

$$\begin{aligned} B_3 &: 0.5, (W - k_2)\,; 0.25, (W + \varepsilon_1)\,; 0.25, (W - \varepsilon_1) \\ A_3 &: 0.5, (W)\,; 0.25, (W - k_2 + \varepsilon_1)\,; 0.25, (W - k_2 - \varepsilon_1)\,. \end{aligned}$$

The cumulative weighting of probabilities in CPT implies there are two cases depending on the relative magnitudes of k_2 and ε_1. The first case we consider is $k_2 > \varepsilon_1$. Given this lottery payoff structure, the value of the prospects, $V(B_3)$ and $V(A_3)$, are given by the following expressions

$$\begin{aligned} V(B_3) &= w^+(0.25)(W+\varepsilon_1) + \left(w^+(0.5) - w^+(0.25)\right)(W-\varepsilon_1) + \left(1 - w^+(0.5)\right)(W-k_2) \\ V(A_3) &= w^+(0.5)(W) + \left(w^+(0.75) - w^+(0.5)\right)(W+\varepsilon_1-k_2) + \left(1 - w^+(0.75)\right)(W-\varepsilon_1-k_2). \end{aligned}$$

Risk apportionment of order 3 requires $V(B_3) > V(A_3)$, and this condition is met if

$$\begin{aligned} \left(1 + 2w^+(0.25)\right)\varepsilon_1 &> 2w^+(0.75)\,\varepsilon_1 \\ 1 + 2w^+(0.25) &> 2w^+(0.75) \\ 0.5 + w^+(0.25) &> w^+(0.75). \end{aligned} \tag{1}$$

Expression (1) can also be written in terms of probability distortion as $\widetilde{w}^+(0.25) > \widetilde{w}^+(0.75)$, where $\widetilde{w}^+(p) = w^+(p) - p$. This condition holds for the representative CPT subject who overweights probabilities of 0.25 ($\widetilde{w}^+(0.25) > 0$) and underweights probabilities of 0.75 ($\widetilde{w}^+(0.75) < 0$). The probability weighting assumed for the representative CPT experimental subject therefore implies that, with linear value function, the prudent lottery, B_3, will always be chosen.

[11] A lottery L with payoffs x and y and corresponding probabilities p_x and p_y is represented by $L : p_x, x; p_y, y$.

Introducing risk aversion over gains in the absence of probability distortion also implies a prudent choice. Consequently, in this case, the CPT individual will always exhibit prudence which is not dependent upon a power value function. [12]

The order of the lottery payoff structure is changed for the second case, when $k_2 < \varepsilon_1$. Nevertheless, the condition for the agent to exhibit prudence when $\alpha = 1$ is also (1), and the conclusions are therefore the same.

3.1.2 Domain of Losses: Lottery payoffs are all negative

In this case, the weighting of cumulative probability of payoffs is from largest loss to smallest loss. Since the subjective expected value of the lottery is negative, we will ignore the negative sign. For the payoff structure when $k_2 > \varepsilon_1$, we obtain, assuming a power value function, the value of the prospects is given by[13]

$$\begin{aligned} V(B_3) &= w^-(0.5)\,(W+k_2)^\alpha + \big(w^-(0.75) - w^-(0.5)\big)\,(W+\varepsilon_1)^\alpha + \big(1-w^-(0.75)\big)\,(W-\varepsilon_1)^\alpha \\ V(A_3) &= w^-(0.25)\,(W+k_2+\varepsilon_1)^\alpha + \big(w^-(0.5) - w^-(0.25)\big)\,(W+k_2-\varepsilon_1)^\alpha + \big(1-w^-(0.5)\big)\,(W)^\alpha\,. \end{aligned}$$

Since we are dealing with losses, a prudent choice requires the absolute value of $V(B_3)$ to be less than the absolute value of $V(A_3)$.[14] The condition for risk apportionment of order 3 with linear

[12] In Rank Dependent Utility (RDU) of Quiggin (1982), the ranking of cumulative weighting of probabilities is not the same as in CPT since the weighting is from worst to best outcome. The weighting is therefore different to CPT over gains but the same over losses (see Quiggin (1993, p.57) or Neilson (2001, p3)). Nevertheless, over all-gains and over all-losses the results are the same as in CPT, namely, the agent behaves always as prudent.

[13] The loss aversion parameter λ drops out from the expression because it multiplies both $V(B_3)$ and $V(A_3)$.

[14] To clarify the argument about lottery pair choices in the all-losses case, we provide the following numerical example. Let us consider a linear value function and assume that $W = -8$, $k_2 = -5$, $\widetilde{\varepsilon}_1 \equiv [3\_ -3]$, employing negative values we obtain

$$\begin{aligned} V(B_3) &= w^-(0.5)(-13) + \big(w^-(0.75) - w^-(0.5)\big)\,(-11) + \big(1-w^-(0.75)\big)\,(-5) \\ &= w^-(0.5)(-2) + w^-(0.75)(-6) - 5 \\ V(A_3) &= w^-(0.25)(-16) + \big(w^-(0.5) - w^-(0.25)\big)\,(-10) + \big(1-w^-(0.5)\big)\,(-8) \\ &= w^-(0.25)(-6) + w^-(0.5)(-2) - 8, \end{aligned}$$

so that $V(B_3) > V(A_3)$ if $6w^-(0.25) + 3 > 6w^-(0.75)$, which is equivalent to the expression $\big(2w^-(0.25)+1\big) > 2w^-(0.75)$.

value functions, when $\alpha = 1$, is obtained as $2w^{-}(0.75) < 1 + 2w^{-}(0.25)$. This is the same condition for prudence in the domain of gains, expression (1). Consequently, the cumulative weighting of probabilities implies that a CPT individual with a linear value function would make the prudent lottery choice.

When we introduce risk-seeking value preferences the third derivative of the value function is negative which enhances the imprudent lottery over the prudent lottery but the negative sign implies that the prudent lottery exhibits less negative expected value and is preferred.We illustrate this point in Figure 1 employing the parameterisations of DS and EW described in the previous section. We observe that across all values of α $(0 < \alpha \leq 1)$[15] the absolute value of $V(B_3)$ is below that of $V(A_3)$ and that they only intersect at $\alpha = 0$. The same implications follow for lottery pairs where $k_2 < \varepsilon_1$.

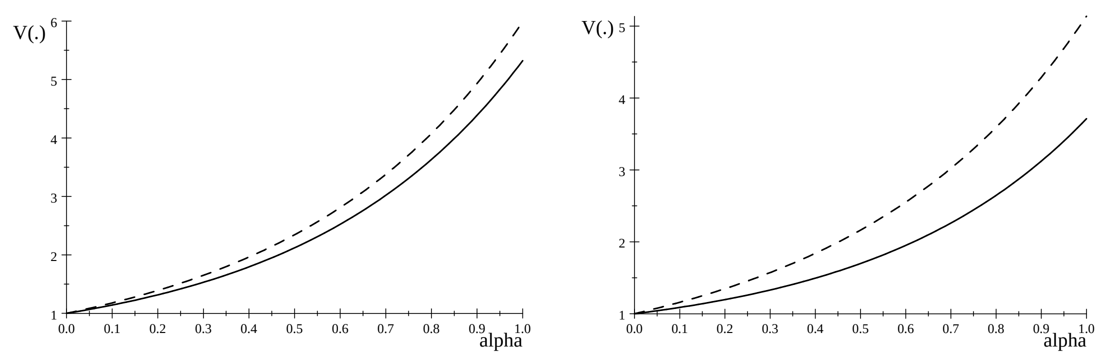

Figure 1. Plots of absolute values of $V(B_3)$ -solid line- and $V(A_3)$ -dash line- for paramaterisatons DS (left) and EW (right) for lottery payoffs $W = -3, k_2 = -6, \varepsilon_1 = 2$

3.1.3 Mixed Domain

In the case where lottery payoffs involve both gains and losses, we find that the representative CPT subject can exhibit prudent or imprudent lottery choices. There are many possible lottery structures in this case dependent on the precise values of the payoffs. We illustrate with a lottery payoff sturucture reported in the experimental literature where $W = 0$ and $k_2 > \varepsilon_1$. In this case, the values of the prospects are the following

[15].Note that $\alpha < 1$ is a necessary condition for risk seeking over losses with a power value function. A smaller α implies a higher degree of risk seeking

$$V(B_3) = w^+(0.25)(\varepsilon_1)^\alpha - \lambda w^-(0.5)(k_2)^\alpha - \lambda\left(w^-(0.75) - w^-(0.5)\right)(\varepsilon_1)^\alpha$$
$$V(A_3) = -\lambda w^-(0.25)\left(\varepsilon_1 + k_2\right)^\alpha - \lambda\left(w^-(0.5) - w^-(0.25)\right)\left(k_2 - \varepsilon_1\right)^\alpha.$$

With linear value functions, the condition for the prudent choice ($B_3 \succ A_3$) is $w^+(0.25) + 2\lambda w^-(0.25) > \lambda w^+(0.75)$; and when $\alpha = 0$, the condition is $w^+(0.25) > \lambda(w^+(0.75) - w^+(0.5))$. The second condition is violated for representative values of probability distortion and loss aversion. Consequently, our representative CPT subject exhibits a switch point from prudent to imprudent.

To illustrate, we employ as an example the lottery payoffs $W = 0, k_2 = 6, \varepsilon_1 = 4$ employed in experimental work by Maier and Rüger (2012). Assuming the parameter values for loss aversion and probability weighting in KT, Figure 2 illustrates that the individual is prudent only if the risk aversion parameter is higher than a specific value, in particular $\alpha > 0.643$.

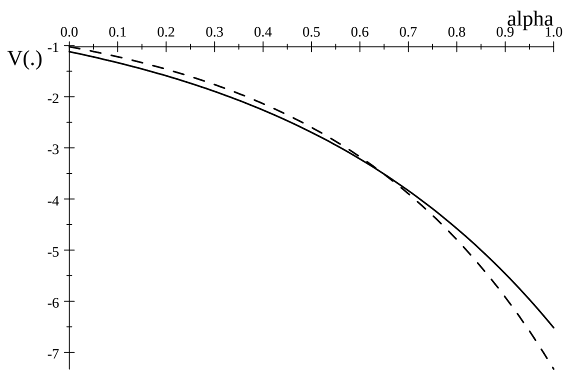

Figure 2. Plots of $V(B_3)$ -solid line- and $V(A_3)$ -dash line- for parameterisation TK and lottery payoffs $W = 0, k_2 = 6, \varepsilon_1 = 4$

3.2 Temperance

3.2.1 Domain of Gains

To analyse risk apportionment of order 4, or temperance, we proceed as in the previous subsection and first consider the case where all payoffs are in the gains domain, that is, $W > \varepsilon_2 + \varepsilon_1$. Assuming that the payoffs of the two independent risks are different[16], $\varepsilon_2 > \varepsilon_1$, the probabilities and outcomes of the two lotteries are the following

[16] When $\varepsilon_2 = \varepsilon_1$, the conclusions of our analysis are not changed but the mathematics is, and they are available upon request.

$$
\begin{aligned}
B_4 \quad &: \quad 0.25, (W+\varepsilon_2)\,; 0.25, (W+\varepsilon_1)\,; 0.25, (W-\varepsilon_1)\,; 0.25, (W-\varepsilon_2) \\
A_4 \quad &: \quad 0.5, (W)\,; 0.125, (W+\varepsilon_2+\varepsilon_1)\,; 0.125, (W+\varepsilon_2-\varepsilon_1)\,; 0.125, (W-\varepsilon_2+\varepsilon_1)\,; 0.125, (W-\varepsilon_2-\varepsilon_1)\,.
\end{aligned}
$$

Employing the cumulative probability weighting process from highest to lowest gain, the value of the prospects assuming power value functions are given by

$$
\begin{aligned}
V(B_4) &= w^+(0.25)\,(W+\varepsilon_2)^\alpha + \left(w^+(0.5) - w^+(0.25)\right)(W+\varepsilon_1)^\alpha + \left(w^+(0.75) - w^+(0.5)\right)(W-\varepsilon_1)^\alpha \\
&\quad + \left(1 - w^+(0.75)\right)(W-\varepsilon_2)^\alpha \\
V(A_4) &= w^+(0.125)\,(W+\varepsilon_2+\varepsilon_1)^\alpha + \left(w^+(0.25) - w^+(0.125)\right)(W+\varepsilon_2-\varepsilon_1)^\alpha + \left(w^+(0.75) - w^+(0.25)\right)(W)^\alpha \\
&\quad + \left(w^+(0.875) - w^+(0.75)\right)(W-\varepsilon_2+\varepsilon_1)^\alpha + \left(1 - w^+(0.875)\right)(W-\varepsilon_2-\varepsilon_1)^\alpha\,.
\end{aligned}
$$

In the case of a linear value function, $\alpha = 1$, the condition for the temperate lottery choice, $V(B_4) > V(A_4)$, is given by

$$
\begin{aligned}
1 + 2w^+(0.5) &> 2w^+(0.125) + 2w^+(0.875) \\
0.5 + w^+(0.5) &> w^+(0.125) + w^+(0.875).
\end{aligned}
$$

This condition can also be written in terms of probability distortion as $\widetilde{w}^+(0.5) > \widetilde{w}^+(0.125) + \widetilde{w}^+(0.875)$. For the standard probability weighting functions that overweight small probabilities, this condition holds. Since all lottery payoffs are gains, introducing risk aversion into the value functions reinforces the effect of the probability weighting so that the representative CPT subject always makes the temperate lottery choice when the status quo is the reference point.

3.2.2 Domain of Losses

When the lottery payoffs are all negative and $\varepsilon_2 > \varepsilon_1$, the value of temperate and intemperate lotteries are given by

$$\begin{aligned} V(B_4) &= w^-(0.25)\,(W+\varepsilon_2)^\alpha + \big(w^-(0.5)-w^-(0.25)\big)\,(W+\varepsilon_1)^\alpha + \big(w^-(0.75)-w^-(0.5)\big)\,(W-\varepsilon_1)^\alpha \\ &\quad + \big(1-w^-(0.75)\big)\,(W-\varepsilon_2)^\alpha \\ V(A_4) &= w^-(0.125)\,(W+\varepsilon_2+\varepsilon_1)^\alpha + \big(w^-(0.25)-w^-(0.125)\big)\,(W+\varepsilon_2-\varepsilon_1)^\alpha + \big(w^-(0.75)-w^-(0.25)\big)\,(W)^\alpha \\ &\quad + \big(w^-(0.875)-w^-(0.75)\big)\,(W-\varepsilon_2+\varepsilon_1)^\alpha + \big(1-w^-(0.875)\big)\,(W-\varepsilon_2-\varepsilon_1)^\alpha\,. \end{aligned}$$

Since the weighting order of probabilities is the same in the domain of gains and losses, the condition for choice of the intemperate lottery over losses when the value functions are linear is the smallest absolute value of the prospects. Consequently, with linear value functions the representative CPT individual exhibits intemperance. Introducing risk-seeking, $\alpha < 1$, favours the intemperate lottery. The representative CPT individual therefore always makes the intemperate lottery choice. Consequently, there is a reflection effect in fourth order preferences when the status quo is the reference point.

3.2.3 Mixed domain

For lottery payoffs that involve both gains and losses there are a number of different lottery payoff structures dependent on whether $\varepsilon_2 + \varepsilon_1 > W$ or $\varepsilon_1 > W$. We found, after analysing these different lottery structures, that the representative CPT subject individual can make either temperate or intemperate lottery choices.

We also note that if $W = 0$ the formal analysis from the status quo reference point is the same as from the expected reference point, and therefore to save space, we present the analysis of that particular case when we assume the average payoff is the reference point.

4 Reference Point 2: Average payoff

Deck and Schlesinger (2010) and Ebert and Wiesen (2014) analyse higher order preferences within a CPT framework assuming the reference point is the expected payoff from the lottery so that the reference points for the order 3 and 4 are respectively $W - 0.5k_2$ and W. This implies that the lotteries now exhibit both gains and losses and therefore fall within the mixed domain. It also implies that the endowment, W, plays no role in determining higher order preferences. We demonstrate that from the average payoff reference point the representative CPT subject, defined by the range of parameters in Table 2, can make prudent or imprudent, temperate or intemperate choices dependent upon the precise lottery payoffs. As a consequence, our representative CPT individuals can exhibit some, perhaps surprising, combinations of second and third or fourth order preferences.

If we determine the certainty equivalent of the chosen third or fourth order lottery choice we must employ the lottery prior to transformation to the average reference point. We find that the certainty equivalents of the untransformed lottery can be lower or higher than the expected value dependent upon the value of the endowment (W). Consequently, the representative CPT subject can exhibit, for example, an imprudent third order preference or an intemperate fourth order preference but be risk averse based on the certainty equivalent of the untransformed lotteries. This also implies that the correlation between second order lottery choices and higher order lottery choices will be lottery specific if the reference point is the average payoff.

Another implication of the analysis is that a researcher can examine, *ex-ante*, the higher order lottery choices for different lottery payoff structures employing a range of representative parameters and determine which payoff structure is the most appropriate for eliciting imprudent or intemperate lottery choices and therefore more likely to reject alternative models such as EUT.

4.1 Temperance

We first consider risk apportionment of order 4 since, as noted above, some prominent studies report that temperate choices are the majority (Noussair et al., 2014; and Ebert and Wiesen, 2014) whilst others the intemperate choice (Deck and Schlesinger, 2010). The lottery pair to elicit temperance are the following, $\varepsilon_2 > \varepsilon_1$

$$\begin{aligned} B_4 &: \quad 0.25, (\varepsilon_2); 0.25, (\varepsilon_1); 0.25, (-\varepsilon_2); 0.25, (-\varepsilon_1) \\ A_4 &: \quad 0.125, (\varepsilon_2 + \varepsilon_1); 0.125, (\varepsilon_2 - \varepsilon_1); 0.125, (-\varepsilon_2 + \varepsilon_1); 0.125, (-\varepsilon_2 - \varepsilon_1). \end{aligned}$$

The value of the prospects for our representative CPT subjects are given by

$$
\begin{aligned}
V(B_4) &= w^+(0.25)(\varepsilon_2)^\alpha + \left(w^+(0.5) - w^+(0.25)\right)(\varepsilon_1)^\alpha - \lambda w^-(0.25)(\varepsilon_2)^\alpha - \lambda\left(w^-(0.5) - w^-(0.25)\right)(\varepsilon_1)^\alpha \\
V(A_4) &= w^+(0.125)\left(\varepsilon_2 + \varepsilon_1\right)^\alpha + \left(w^+(0.25) - w^+(0.125)\right)\left(\varepsilon_2 - \varepsilon_1\right)^\alpha - \lambda w^-(0.125)\left(\varepsilon_2 + \varepsilon_1\right)^\alpha \\
&\quad -\lambda\left(w^-(0.25) - w^-(0.125)\right)\left(\varepsilon_2 - \varepsilon_1\right)^\alpha.
\end{aligned}
$$

If we consider the choice of lottery when the individual has linear value functions so that $\alpha = 1$, we find that the individual will be temperate if

$$\left(w^+(0.5) + 2\lambda w^-(0.125)\right) > 2w^+(0.125) + \lambda w^-(0.5).$$

In this case, we find that the probability weighting and loss aversion parameters play a crucial role in determining the fourth order lottery choice when the value functions are linear. For EW and BB we find their representative subjects will make the temperate choice, whilst the representative subjects of TK and DS would make an intemperate choice. When we set $\alpha = 0$, which is as if assuming infinitely risk aversion over gains and risk seeking over losses, risk apportionment of order 4 holds if

$$w^+(0.5) + \lambda w^-(0.25) > w^+(0.25) + \lambda w^-(0.5).$$

In this case, we find with the representative set of parameters there are no switches in preferences. Consequently, the precise values of the probability distortion and loss aversion coefficient play a crucial role in determining whether, from the average reference point, a temperate or intemperate lottery choice will be made.

Figure 3 illustrates the effect that different parameter combinations have in the agent's decision. We employ lottery payoffs $\varepsilon_1 = 3.5, \varepsilon_2 = 7$ (as in Ebert and Wiesen 2014), and parameter values from the four CPT specifications outlined in Table 2. In two of the cases, TK and DS, the lines for $V(B_4)$ and $V(A_4)$ never cross each other, implying the agent would always exhibit intemperance. In the other two cases, EW and BBS, the two lines cross each other, meaning the power exponent will determine whether the individual makes the temperate or intemperate lottery choice. For example, for the values of α reported in their papers, 0.97 in EW and 0.48 in BBS, the agent would choose the temperate and the intemperate lottery choice, respectively. The overall conclusion from this analysis is that the representative CPT individual can make temperate or intemperate lottery choices.

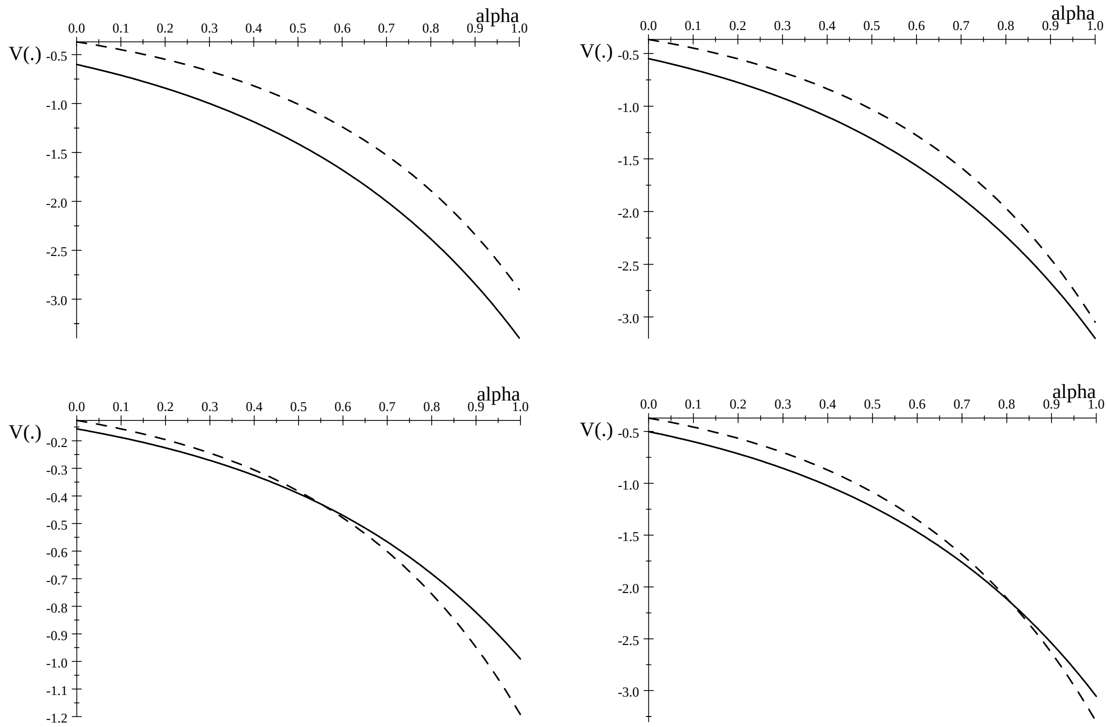

Figure 3 depicts $V(B_4)$ -solid lines- and $V(A_4)$ -dashed lines- for lottery structure $\varepsilon_1 = 3.5, \varepsilon_2 = 7$ and parameter values employed in TK -top left-, DS -top right-, EW -bottom left- and BBS -bottom right-.

To illustrate that the representative individual can exhibit surprising combinations of second and higher order preferences, let consider the lottery payoffs $W = 55, \varepsilon_1 = 5, \varepsilon_2 = 45$. Employing the parameters from TK described in Table 2 we find that $V(A_4) > V(B_4)$, and therefore the agent makes the intemperate choice. The certainty equivalent of the untransformed intemperate lottery choice is 45.93, which is less than 55 (the average payoff), hence the agent is risk averse, and intemperate.

4.2 Prudence

From a reference point of expected return the implied prudent or imprudent lottery choice is given by

$$
\begin{aligned}
B_3 &: \quad 0.5, (-0.5k_2)\,; 0.25, (\varepsilon_1 + 0.5k_2)\,; 0.25, (-\varepsilon_1 + 0.5k_2) \\
A_3 &: \quad 0.5, (0.5k_2); 0.25, (\varepsilon_1 - 0.5k_2); 0.25, (-\varepsilon_1 - 0.5k_2).
\end{aligned}
$$

The relative size of k_2 and ε_1 now determines three possible cases in terms of valuation of the prospects. The first one is when $\varepsilon_1 \geq k_2 > 0.5k_2$. The subjective expected values are the following

$$
\begin{aligned}
V(B_3) &= w^+(0.25)\,(\varepsilon_1 + 0.5k_2)^\alpha - \lambda w^-(0.25)\,(\varepsilon_1 - 0.5k_2)^\alpha - \lambda\left(w^-(0.75) - w^-(0.25)\right)(0.5k_2)^\alpha \\
V(A_3) &= w^+(0.25)\,(\varepsilon_1 - 0.5k_2)^\alpha + \left(w^+(0.75) - w^+(0.25)\right)(0.5k_2)^\alpha - \lambda w^-(0.25)\,(\varepsilon_1 + 0.5k_2)^\alpha .
\end{aligned}
$$

If we consider the choice of lottery when the individual has linear value functions so that $\alpha = 1$, we find that the individual will be prudent if

$$
3\left(w^+(0.25) + \lambda w^-(0.25)\right) > w^+(0.75) + \lambda w^-(0.75).
$$

It is easy to see that this condition holds for all representative CPT parameter values because small probabilities are overweighted while large probabilities are underweighted. Conversely, if we set $\alpha = 0$ which is as if assuming infinitely risk aversion over gains and risk seeking over losses, risk apportionment of order 3 holds if

$$
w^+(0.25) + \lambda w^-(0.25) > w^+(0.75) + \lambda w^-(0.75).
$$

In this case the imprudent choice will be made by the representative CPT subject. Since the representative CPT subject makes the prudent lottery choice with linear value functions but the imprudent choice if sufficiently risk averse over gains or risk seeking over losses with the representative values of loss aversion, this implies that there can be prudent or imprudent lottery choices dependent upon these parameters in experimental research. For instance, in the case of lottery payoffs $\varepsilon_1 = 9, k_2 = 1$, in three out of the four CPT parameterisations in Table 2, namely TK, EW, and DS, the individual would exhibit the prudent lottery choice, whilst in the fourth parameterisation, BBS, the CPT representative agent would be exhibit the imprudent lottery choice.

The following example illustrates how payoff magnitudes can impact on the cross over point that determines different risk attitudes of order 3 employing parameter values reported by TK and

BBS. Let us consider two lottery pair designs, one with $\varepsilon_1 = 9, k_2 = 1$, and another one with $\varepsilon_1 = 25, k_2 = 25$. Figure 4 plots the value of the prospects in each of those two cases. We observe that the level of risk aversion at which agents switch from imprudent to prudent behaviour differs between the two parameterisations and lottery payoff structure.

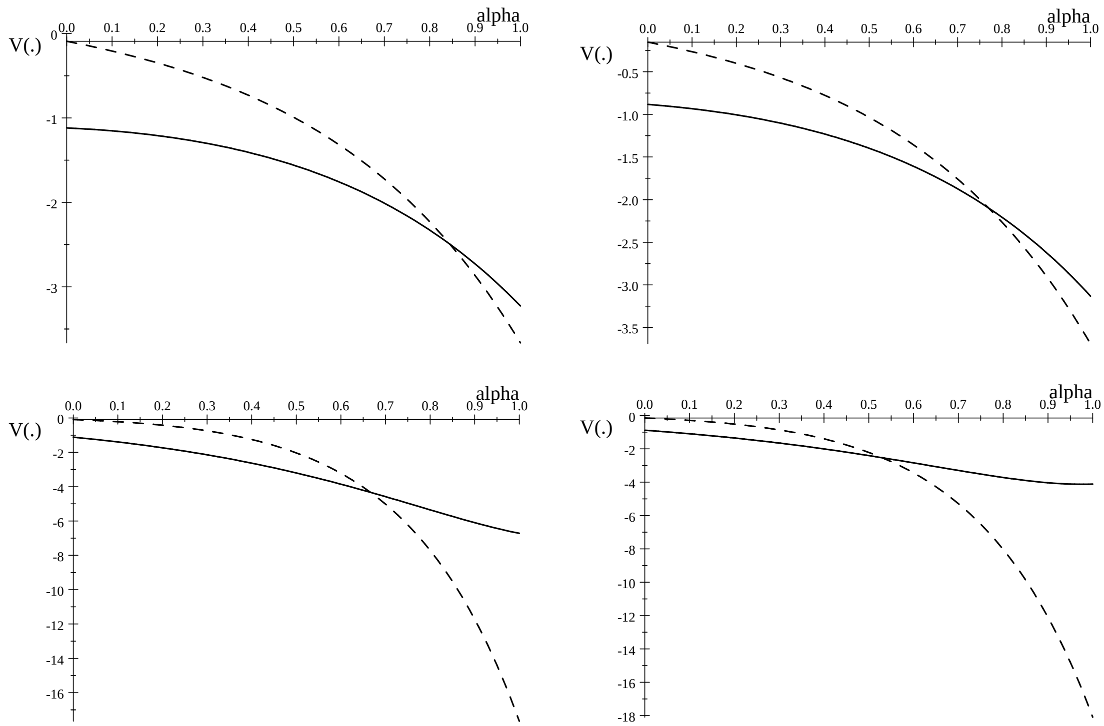

Figure 4 plots $V(B_3)$ -solid line- and $V(A_3)$ -dashed line- for lottery designs $\varepsilon_1 = 9, k_2 = 1$ -top-, and $\varepsilon_1 = 25, k_2 = 25$ -bottom-, and parameters in TK -left- and BBS -right-.

We now demonstrate that the representative CPT individual can make the imprudent lottery choice when exhibiting either risk averse or risk loving second order preferences. We employ one of the examples above where the lottery choice is the imprudent one ($V(A_3) > V(B_3)$). We then compute the certainty equivalent of this lottery choice employing the untransformed lottery A_3, which is the relevant lottery for computing the certainty equivalent of the imprudent lottery choice, $V(A_3)$. With lottery payoffs $\varepsilon_1 = 9, k_2 = 1$, and the EW values of loss aversion and probability distortion, but with a power exponent $\alpha = 0.6$, the imprudent lottery choice, $V(A_3)$, has a certainty equivalent based on lottery A_3 when $W = 10$ of 3.48, a value which is less than the expected value of 9.5. Consequently, a risk-averse CPT subject can make an imprudent lottery choice but be risk averse. However, if the endowment is increased to $W = 100$, the certainty equivalent of the untransformed lottery A_3 is 103.6, and the CPT individual therefore makes an imprudent choice but is risk-seeking.

This analysis demonstrates how imprudent third order lottery choices from the average reference point, which can exceed thirty percent (see Trautmann and van de Kuilen, 2018), may exhibit a low correlation with second order preferences obtained directly from second order lottery choices. The relevant correlation is between the imprudent choices and the certainty equivalent of the untransformed imprudent lottery choice.

Finally, we examine the other two cases that arise from different payoff structure in the lottery design for prudence. In the case where $0.5k_2 > \varepsilon_1$, the subjective expected values are the following

$$\begin{aligned} V(B_3) &= w^+(0.25)\,(\varepsilon_1 + 0.5k_2)^\alpha + \left(w^+(0.5) - w^+(0.25)\right)(0.5k_2 - \varepsilon_1)^\alpha - \lambda w^-(0.5)\,(0.5k_2)^\alpha \\ V(A_3) &= w^+(0.5)\,(0.5k_2)^\alpha - \lambda w^-(0.25)\,(\varepsilon_1 + 0.5k_2)^\alpha - \lambda\left(w^-(0.5) - w^-(0.25)\right)(0.5k_2 - \varepsilon_1)^\alpha. \end{aligned}$$

In this case we find that the representative subject can be prudent or imprudent. The same implication applies to the third case where $k_2 > \varepsilon_1$ but $\varepsilon_1 \geq 0.5k_2$.[17] Overall, the analysis of this section reveals the way in which model parameters and lottery payoff structures can imply different higher order lottery choices for the representative CPT subject.

5 Reference Point 3: MaxMin

Baillon et al. (2019) report that the MaxMin criterion, defined as the maximum outcome that a subject can reach for sure, is, together with the status quo, the most common reference point used in decision under risk. The results in their study of second order risky choices show that MaxMin is employed by 30% of experimental subjects as the reference point.

Employing our framework we find, from the MaxMin reference point, the representative CPT with power value functions and the range of loss aversion reported in the representative studies subject will always choose the prudent and the temperate lotteries.[18] Consequently, the representa-

[17] In this case the associated prospect values are given by

$V(B_3) = w^+(0.25)\,(\varepsilon_1 + 0.5k_2)^\alpha - \lambda w^-(0.5)\,(0.5k_2)^\alpha - \lambda\left(w^-(0.75) - w^-(0.5)\right)(\varepsilon_1 - 0.5k_2)^\alpha$

$V(A_3) = w^+(0.5)\,(0.5k_2)^\alpha + \left(w^+(0.75) - w^+(0.5)\right)(\varepsilon_1 - 0.5k_2)^\alpha - \lambda w^-(0.25)\,(\varepsilon_1 + 0.5k_2)^\alpha.$

[18] In this section, we have omitted the formal derivations to preserve some space, but they are all available upon request.

tive CPT subject from the MaxMin reference point makes the same third and fourth order lottery choices as is the case from the status quo reference point when all lottery payoffs are gains.[19]

6 Conclusions

In this paper, we have investigated the third and fourth order lottery choices of representative agents in CPT from three reference points. An important finding is that a CPT subject will, in common with an EUT subject, always make prudent or temperate lottery choices from the status quo reference point when all lottery payoffs are gains and prudent or intemperate lottery choices when all lottery payoffs are losses. As a consequence, there is no reflection effect for third order lottery choices but there is for fourth order lottery choices from the status quo reference point.

From the average payout reference point, we demonstrated that, dependent upon the precise magnitude of lottery payoffs, all third and fourth order lottery choices can be made by the representative CPT subjects who exhibit the range of parameter values for probability weighting, power value functions and loss aversion reported in prominent studies. Our analysis also revealed that from the average payout reference point there are combinations of second and third or fourth order preferences that have not previously been associated with the representative CPT agent. We also showed that the correlation between second and higher order lottery choices will be lottery specific if the reference point is the average payoff.

Our results have further implications for experimental research that fits parametric models to explain higher order lottery preferences. For instance, our findings shed light on whether the design of an experiment would be informative enough to discriminate between CPT and alternative risky-choice models.

[19] We note that it is possible to find cases where a CPT individual makes the intemperate choice. For instance with $\alpha = 0.93$, and parameters $\lambda = 1.3$, $\gamma = \delta < 0.52$, and lottery design $\varepsilon_2 = 9, \varepsilon_1 = 1$. However, we do not consider this parameterisation as a representative CPT agent because the minimum loss aversion parameter λ considered in Table 2 is 1.53.

Appendix. Kőszegi and Rabin (2006, 2007) reference point

In the Köszegi and Rabin (2006, 2007) approach the reference point is dependent upon the entire distribution of expected outcomes in each lottery. Following Baillon et al. (2019) and equating final wealth with outcome, the expected utility of the difference in lottery payoffs from each reference point are evaluated.

$$\begin{aligned}
U(B_3) &= 0.5\,[0.5U(0) + 0.25U(k_2+\varepsilon_1) + 0.25U(k_2-\varepsilon_1)] + 0.25\,[0.5U(-k_2-\varepsilon_1) + 0.25U(0) + 0.25U(-2\varepsilon_1)] \\
&\quad +0.25\,[0.5U(\varepsilon_1-k_2) + 0.25U(2\varepsilon_1) + 0.25U(0)] \\
&= 0.5\,[0.25U(k_2+\varepsilon_1) + 0.25U(k_2-\varepsilon_1)] + 0.25\,[0.5U(-k_2-\varepsilon_1) - 0.5U(\varepsilon_1)] \\
&\quad +0.25\,[0.5U(\varepsilon_1-k_2) + 0.5U(\varepsilon_1)] \\
U(A_3) &= 0.5\,[0.5U(0) + 0.25U(\varepsilon_1-k_2) + 0.25U(-\varepsilon_1-k_2)] + 0.25\,[0.5U(k_2-\varepsilon_1) + 0.25U(0) + 0.25U(-2\varepsilon_1)] \\
&\quad +0.25\,[0.5U(k_2+\varepsilon_1) + 0.25U(2\varepsilon_1) + 0.25U(0)] \\
&= 0.5\,[0.25U(\varepsilon_1-k_2) + 0.25U(-\varepsilon_1-k_2)] + 0.25\,[0.5U(k_2-\varepsilon_1) - 0.5U(\varepsilon_1)] \\
&\quad +0.25\,[0.5U(k_2+\varepsilon_1) + 0.5U(\varepsilon_1)]\,,
\end{aligned}$$

which implies $U(B_3) = U(A_3)$ and the individual is indifferent between the prudent and imprudent lotteries.

Regarding risk apportionment of order 4, our analysis available upon request shows that the Kőszegi and Rabin approach implies the representative subject will exhibit temperance.

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