The Simplification, Solution and Estimation of a Small Open DSGE Model:Evidence from the UK and Canada

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November 23, 2019

This dissertation is submitted for the degree of Doctor of Philosophy

Departments of Economics

Dedication

This thesis is dedicated to my parents who encouraged me to pursue the goal of my dreams.

Declaration

This thesis has not been submitted in support of an application for another degree at this or any other university. It is the result of my own work and includes nothing that is the outcome of work done in collaboration except where specifically indicated. Many of the ideas in this thesis were the product of discussion with my supervisor David Peel and Alina Spiru.

The thesis is being submitted in partial fulfillment of the requirements for the degree of:......**PhD**......

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Abstract

This thesis makes three main contributions to the literature on Dynamic Stochastic General Equilibrium (DSGE) models. The first contribution is to bridge the gap between a theoretical small open DSGE model provided by Gali and Monacelli and an empirical model developed by Lubik and Schorfheide, as no previous studies have shown their relationship explicitly. Since all the models suffer from the misspecification problem to some extent, the second contribution is to apply two methodologies including DSGE-VAR approach and indirect inference to study the effect of the possibly misspecified equation of the change rate of terms of trade. The third contribution is to search for the model with the best data fitting in two stages of model comparisons. The thesis assumes that the parameters of the simplified DSGE model are constant at the first stage, and based on the constant parameter models with the best performance on data fitting, it assumes a subset of the parameters including exogenous shock variances and policy parameters follow two independent Markov-switching Markov chains at the second stage.

The empirical results are quite different for the UK and Canada within the sample period covering 1992: Q4 – 2008: Q4. The UK data supports that the movement of the nominal exchange rate should not enter into the monetary policy reaction function. Also, the data supports that it is possible for the UK to experience the two kinds of structural changes, including the economic environment and the behaviours of policymakers simultaneously. Comparatively, Canadian data is in favour of the movement of the nominal exchange rate in the policy function. Moreover, the data supports that it is less likely for Canada to experience two kinds of structural changes simultaneously.

Acknowledgements

It is necessary to thank many people who help me to finish the thesis. First, my thanks go to my supervisor David Peel and Alina Spiru. Professor David suggests me to start my research from Gali and Monacelli's' model and motivates me to carry on the research of model comparisons. Alina is very patient to help me to clarify my research questions and the relevant research methodologies.

Second, I want to appreciate Giorgio Motta and Efthymios Pavlidis. Giorgio give me the first DSGE lectures since I joined Lancaster University since 2014, and he also provides much additional help through my research. Efthymios used to be my PhD director, and he always can offer me much help when something does not go well.

I also thank Professor Xu Wenli from ANHUI University and PhD Nan Yi in Harbin Institute of Technology in China, Professor Hirokuni Iiboshi from Tokyo Metropolitan University in Japan. Professor Xu recommends me to use the tool RISE to study the Markov-Switching DSGE models, and Nan Yi provides much help when I have some coding problems with Matlab. Professor Iiboshi is so kind that he offers me his code resources to solve a standard Markov-Switching New Keynesian model numerically.

Lastly, I give my deepest thanks to my parents, who never get bored with me and always share precious moments and memories with me.

Contents

General Introduction **1**

Previous literature on the development of DSGE models 1

The Solutions to the DSGE Models 4

The Bayesian Estimation of DSGE Models 6

The Structure of the Thesis 10

1 The Simplification and Solution of the Model **11**

1.1 Introduction 11

1.2 A Small Open DSGE Model 12

1.2.1 Types of Model Variables 12

1.2.2 Gali and Monacelli's framework 12

1.2.3 From Gali to Lubik 31

1.3 Solutions to the Model 35

1.3.1 Blanchard and Kahn's Methodology 38

1.3.2 Schur Decomposition 40

1.3.3 Numerical Solutions 42

1.4 Conclusion 48

2 Data Sample and Estimation Methodology **50**

2.1 Introduction 50

2.2 Data and Model Variables 51

2.2.1 Observable and Collected Data for the UK 52

2.2.2 Observable and Collected Data for Canada 56

2.2.3 Measurement Equations 59

2.3 Bayesian Estimation of DSGE model 61
2.3.1 Likelihood function 63
2.3.2 Choice of Priors 64
2.3.3 MCMC Approximation of Bayesian Posteriors 67
2.4 The Analysis of the Estimation Results 70
2.4.1 Estimation Results for the UK 72
2.4.2 Estimation Results for Canada 79
2.5 Check for the Model Specification 86
2.5.1 Model Evaluation by DSGE-VAR 87
2.5.2 Model Evaluation by Indirect Inference 92
2.6 Conclusion 95

3 Model Comparison One: Constant Parameters Estimation 96

3.1 Introduction 96
3.2 Model Comparison of Bayesian Methodology 100
3.3 Bayesian Estimation of the Updated DSGE Models 102
3.3.1 Group One: No Nominal Exchange Depreciation 105
3.3.2 Group Two: No Nominal Exchange Depreciation and Change of Output Deviation 117
3.3.3 Group Three: Nominal Exchange Depreciation and Change of Output Deviation 129
3.3.4 An Overall Remark of the Model Comparison at the First Stage 141
3.4 Conclusion 144

4 Model Comparison Two: Markov-Switching Parameters Estimation 146

4.1 Introduction 146
4.2 Markov-Switching DSGE Models 148
4.3 Estimated Markov Switching DSGE Models for the UK 153
4.3.1 UK: the Model One with Switching Variances 155

4.3.2 UK: the Model Two with the Switching Taylor Rule 166
4.3.3 UK: the Model Three with Switching Variances and Switching Policy Parameters 177
4.3.4 The Model Comparison and Data Analysis for the UK . . . 195
4.4 Estimated Markov Switching DSGE Models for Canada 202
4.4.1 Canada: the Model One with Switching Variances 204
4.4.2 Canada: the Model Two with the Switching Taylor Rule . . 215
4.4.3 Canada: the Model Three with Switching Variances and Switching Policy Parameters 226
4.4.4 Model Comparison and Data Analysis for Canada 244
4.5 Conclusion 250

5 General Conclusion 252
5.1 Main Findings 252
5.2 Limitations and Directions of Further Research 255

Bibliography 257

Appendix 266
Structure of the DSGE Model 266
Measurement Equations of the Observed Data 267

List of Tables

1.1 Calibration 42

1.2 Numerical Solutions for the Calibrated Parameters 46

2.1 Prior Distributions for the UK and Canada 66

2.2 Parameter Estimation Results for the UK and Canada 71

2.3 Numerical Solutions for the UK (Benchmark Model) 77

2.4 Numerical Solutions for Canada (Benchmark Model) 84

2.5 The Calibration of DSGE Prior Weights λ 90

2.6 Power of the Wald Test at 5% significance level 94

3.1 Constant Parameter Estimation Results (Control Group) 104

3.2 Constant Parameter Estimation Results (Group One) 106

3.3 Numerical Solutions for the UK in the Group One 110

3.4 Numerical Solutions for Canada in the Group One 115

3.5 Constant Parameter Estimation Results (Group Two) 118

3.6 Numerical Solutions for the UK in the Group Two 122

3.7 Numerical Solutions for Canada in the Group Two 127

3.8 Constant Parameter Estimation Results (Group Three) 130

3.9 Numerical Solutions for the UK in the Group Three 134

3.10 Numerical Solutions for Canada in the Group Three 139

3.11 Numerical Results of the Posterior Odds Ratio 143

3.12 The Rank of the Data Fitting Performance of Each Group 143

4.1 Parameter Estimation Results of the Benchmark Models 152

4.2 Prior Distributions of the Structural Parameters for the UK 154

4.3 Model One with Markov-Switching Variances(UK) 156
4.4 The Numerical Solution to Regime 1 of the Model One for the UK . 160
4.5 The Numerical Solution to Regime 2 of the Model One for the UK . 161
4.6 Variance Decomposition of the Model One for the UK 165
4.7 Model Two with Markov-Switching Policy Parameters(UK) 167
4.8 The Numerical Solution to Regime 1 of the Model Two for the UK 171
4.9 The Numerical Solution to Regime 2 of the Model Two for the UK 172
4.10 Variance decomposition of the Model Two for the UK 176
4.11 Model Three with 2 Markov Chains(UK) 179
4.12 The Numerical Solution to Regime 1 of the Model Three for the UK 186
4.13 The Numerical Solution to Regime 2 of the Model Three for the UK 187
4.14 The Numerical Solution to Regime 3 of the Model Three for the UK 188
4.15 The Numerical Solution to Regime 4 of the Model Three for the UK 189
4.16 Variance Decomposition of the Model Three for the UK 194
4.17 Log Marginal Data Densities and Ranks of the Models for the UK . 198
4.18 Prior Distributions of the Structural Parameters for Canada 203
4.19 Model One with Markov-Switching Variances(Canada) 205
4.20 The Numerical Solution to Regime 1 of the Model One for Canada 209
4.21 The Numerical Solution to regime 2 of the Model One for Canada . 210
4.22 Variance Decomposition of the Model One for Canada 214
4.23 Model Two with Markov-Switching Policy Parameters(Canada) . . 216
4.24 The Numerical Solution to Regime 1 of the Model Two for Canada 220
4.25 The Numerical Solution to Regime 2 of the Model Two for Canada 221
4.26 Variance Decomposition of the Model Two for Canada 225
4.27 Model Three with 2 Markov Chains(Canada) 228
4.28 The Numerical Solution to Regime 1 of the Model Three for Canada 235
4.29 The Numerical Solution to Regime 2 of the Model Three for Canada 236
4.30 The Numerical Solution to Regime 3 of the Model Three for Canada 237
4.31 The Numerical Solution to Regime 4 of the Model Three for Canada 238
4.32 Variance Decomposition of the Model Three for Canada 243

4.33 Log Marginal Data Densities and Ranks of the Models for Canada . 246

List of Figures

1.1 Impulse response functions for the Calibrated parameters 47

2.1 Data of UK 55

2.2 Data of Canada 58

2.3 Impulse response functions for the benchmark model in the UK . . 78

2.4 Impulse response functions for the benchmark model in Canada . . 85

2.5 Calibrations of DSGE Prior Weights 91

3.1 Impulse response functions for Group One in the UK 111

3.2 Impulse response functions for Group One in Canada 116

3.3 Impulse response functions for Group Two in the UK 123

3.4 Impulse response functions for Group Two in Canada 128

3.5 Impulse response functions for Group Three in the UK 135

3.6 Impulse response functions for Group Three in Canada 140

4.1 Impulse responses,UK(Markov-switching Model One) 162

4.2 Impulse responses,UK(Markov-switching Model Two) 173

4.3 Impulse responses,UK(Markov-switching Variances of Model Three) 190

4.4 Impulse responses,UK(Markov-switching Policy Parameters of Model Three) 191

4.5 Smoothed Probability of High Volatility from the Model One with switching variances 199

4.6 Smoothed Probability of More Strict Inflation Targeting from the Model Two with Switching Policy Parameters 200

4.7 Historical Decomposition Using Model One for the UK 201

4.8 Impulse responses,Canada(Model One) 211
4.9 Impulse responses,Canada(Model Two) 222
4.10 Impulse responses,Canada(Switching Variances of Model Three) . . 239
4.11 Impulse responses,Canada(Switching Taylor Rules of Model Three . 240
4.12 Smoothed probability of high volatility from model one 247
4.13 Smoothed probability of strict inflation from model two 248
4.14 Historical Decomposition Using Model One for Canada 249

General Introduction

The main objective of the thesis is to answer the question that how to improve the data fitting for a given DSGE model based on different specifications of monetary policy function and some subsets of parameter space following Markov-Switching chains. The thesis will prefer the UK and Canada as two sample countries within the sample period 1992:Q4-2008:Q4. There are mainly three reasons motivating the preference. First, the DSGE model adopted by the thesis is appropriate for small open economies, which includes features that the UK and Canada hold to some extent. Second, the UK and Canada share something in common within the chosen sample period. They both announce that they adopt inflation targeting monetary policy after the early 1990s. This thesis does not consider the periods of the zero lower bound after 2008 when the conventional monetary policy does not work. Third, the UK and Canadian monetary policy also exhibit some different features within the chosen sample period. There is a debate about whether the movement of the nominal exchange rate should enter into the design of the monetary policy. Overall, the thesis will study the two sample countries within the sample period at the same time and exhibit their similar and different features, thereby looking for the model offering the best data fitting.

Previous literature on the development of DSGE models

The Dynamic Stochastic General Equilibrium (DSGE) approach can provide microfoundations of households and firms to economic models and inform the policymakers about the impact and feedback of their conduct policy in an economy with assumptions of the real business cycle or New Keynesian nominal rigidity, which plays an increasingly key role in the modern macroeconomic analysis. Ramsey (1927[73],1928[74]) initially mature the framework of this approach. Subsequently, Cass (1965[13]) and Koopmans(1965[54]) carry on developing this methodology. The Lucas critique (Lucas, 1976[61]) and the urgent need to set up micro-founded macroeconomic models lead to a revolution in macroeconomics in the 1980s. It is Kydland and Prescott (1982[55]) who set up the vital work in modern macroeconomic analysis, which is famous for the starting point of the Real Business Cycle (RBC) theory. In the combination between the developed economic theory and the major events in the 20st, the size of DSGE model becomes larger and the structure of it becomes more complicated, because DSGE models gradually incorporate a large number of New Keynesian features(Rotemberg,1982[76];Blanchard and Kiyotaki,1987[9]; Rotemberg and Woodford,1997[77]; Woodford,2011[93];Smet and Wouters, 2003[82],2007[83]). At the meantime, instead of studying the policy and the economy isolated from the world, Some individuals including Clarida,Gali and Gertler (2001[20];2002[21]),Gertler, Gilchrist and Natalucci(2007[38])and Monacelli (2005[66]) start extending the DSGE framework to the context of an open economy in the presence of the trade sector and the currency values in their models. Gali and Monacelli (2005[37]) develop a critical theoretical DSGE model which describe a small open economy which is allowed to trade with the rest of the world and hardly impose a significant impact on the global economy. This model with calibrated parameters can be regarded as the seminal one to guide the policy makers to design the optimal monetary policies. Unexpectedly, as pointed by Diebold (1998[27]), the large scale and the complicated structure of the models may prevent us from measuring the estimates of parameters efficiently and consistently. Some individuals then simplify Gali and Monacelli's framework, and among them, Lubik and Schorfheide (2007[60])'s simplification is applied for practical purpose in terms of constant parameter model (Zheng and Guo,2013[94].) and Markov-Switching DSGE model (Chen and MacDonald,2012[16]). However, few of the literature, including Lubik and Schorfheide, clearly explain the relationship between the theoretical model and the empirical one in terms of the model variables and the parameters. To use their model directly may confuse others because there is not a clear transition from the micro-foundations to the simplified model. Accordingly, the lack of transparency motivates the thesis to set up a bridge between the theoretical model and the simplified one. Chapter 1 of the thesis replicates the derivation process of the empirical model and stands out the meaning of the variables and parameters in the simplified model, thereby identifying whether there are any changes from the theoretical model based on some other different assumptions.

The Solutions to the DSGE Models

The solution to a DSGE model is approximately a VAR model with restricted coefficients deriving from the structural parameters. Blanchard and Kahn (1980[8]) develop a solution method which provides an important condition for the existence and uniqueness of the solutions to the system. That is, the number of explosive eigenvalues should be exactly equal to the number of jump variables with the expectation operators. Sims (2002[80]) proposes a solution method, which is a bit different from Blanchard and Kahn's work. Although they still decouple the system of models into explosive and nonexplosive portions, the expected errors instead of the expectation operators appear in the system. Besides, the technique of QZ or Schur decomposition is applied in their methodology to overcome the singularity of the matrix coefficients. Klein (2000[51]) develops a method which is a hybrid of those of Blanchard and Kahn (1980[8]) and Sims (2001[80]). Like Blanchard and Kahn's method, he distinguishes the predetermined and jump variables of the system. Also, the QZ technique is applied to solve the system in the absence of expectation errors. Furthermore, Uhlig (1995[89]) develops an undetermined coefficient approach, which is very different from the other methodologies. Rather than focusing on solving the system with different portions, he tries to uncover the relationships among the parameters and solve them once for all. Moreover, Svensson and Williams (2005[85]), Farmer et al(2011[32]), Cho (2014[17]) introduce Markov switching to the parameters of the system. They solve the Markov-Switching DSGE model through solving a constant parameters model with the structural parameters and the transition probabilities. They have proved that their solutions are unique and stable. In a word, Blanchard and Kahn's approach is the fundamental one for this thesis to solve the simplified DSGE model, which also incorporates the techniques of QZ decomposition as does in Sims and Klein's work.

It is often difficult to solve the model by hand when the number of the equations and variables are many even for an already simplified system. Thus, it is very convenient to solve the complicated DSGE models with numerical algorithms packaged in some powerful software such like Matlab(Judd, 1998[46]; Miranda and Fackler, 2004[65]; Woodford and Philips, 2011[92]; Brandimarte,2013[10]) and a newly invented software Julia (Caraiani,2018[12]). In this thesis, Dynare (Adjemian,2011[1]) is the primary tool to solve the constant parameter DSGE models and Rise (Maih,2015[62])) is a very efficient tool to solve the Markov-Switching DSGE models. These tools are both attachments of the Matlab.

The Bayesian Estimation of DSGE Models

The thesis will estimate the DSGE models with the Bayesian methodology and evaluate their performances of data fitting with the posterior likelihood ratio test. The combination between a prior probability distribution and the maximum likelihood function of the data in the Bayesian approach can construct a posterior probability distribution which provides a full statistical characterisation of the observed data. Geweke (1999[39]) and Robert and Casella (2013[75]) provide a numerical approach to calculate the integration of the posterior probability densities in the Bayesian framework. Bauwens et al. (2000[5]) offer an application of Bayesian methods to study a wide range of dynamic reduced-form models. Moreover, Koop (2003[53]) writes a textbook which incorporates a general overview of Bayesian statistical methods with further details regarding computational issues.

It is not very common to estimate DSGE models with Bayesian methodology until Smet and Wouters (2003[82], 2007[83]) use this approach to estimate a closed DSGE model for the US economy with data covering the period 1966: Q1-2004: Q4. After that, Adolfson et al. (2007[2]) extend and estimate the model developed by Christiano (2005[18]) for the Eurozone with data covering the period 1970: Q1-2002: Q4. An and Schorfheide (2007[3]) estimate the DSGE model developed by Woodford (2011[93]) with artificial data and evaluate the model with posterior odds ratio and DSGE-VAR approach. In addition to the developed countries, Gabriel et al. (2010[33]) estimate a DSGE model in the presence of financial frictions with Indian data covering period 1980: Q1-2006: Q4. Zheng and Guo (2013[94]) estimate the DSGE model developed by Lubik and Schorfheide(2007[60]) with China data covering the period 1992: Q1-2011: Q4. There are also some useful textbooks regarding the Bayesian estimation of DSGE models (Dejong and Dave,2011[23]; Hashimzade and Thornton,2013[41]; Herbst and Schorfheide, 2015[42]). It is exceedingly beneficial to use such textbooks to address the technical problems occurring in the procedures of estimation.

Lubik and Schorfheide (2007[60]) offer the primary motivation for my research. They estimate a small open DSGE model, which is a simplified version of Gali and Monacelli's model, with the data of Canada, Britain, New Zealand and Australia covering the period 1983: Q1-2002: Q4. They consider two versions of the model with different specification of Taylor principle rules and conclude that it offers the best data fitting for the UK and Canada when it considers the movement of the nominal exchange rate in the policy reaction function. However, the empirical results for the UK are not approved by the Bank of England in their release. The sample period they choose includes at least one structural change when the UK is no longer a member of the Exchange Rate Mechanism (ERM) since 1992. The thesis will adopt a new sample size of the UK and Canada covering the period 1992: Q4-2008: Q4, which excludes the period when the UK is a member of ERM and extends the period until the nominal interest rates experience zero lower bound in the recent financial crisis. Chapter 2 will estimate the original Lubik and Schorfheide's model with the new sample size and reserve the results as a control group in the next chapter. Moreover, it will borrow two methodologies developed from Le et al. (2013[56]) and Del Nergo et al. (2007[26]) to identify the misspecification problems of the small open DSGE model considering whether there is a unit process of the rate change of the terms of trade or not.

Taylor rule policy reaction function (1993[86]) is the equation with different specifications in the estimation. Taylor originally includes the inflation rate and the output gap in the policy reaction function. Gradually, its structure has been changed a lot in examining the coefficients of monetary policy reaction functions in different countries (Judd and Rudebusch, 1998 [45]; Clarida, Gali and Gertler,2000[19]; Nelson,2003[67]; Taylor, 2001[87]). In addition to the nominal exchange rate depreciation in the wake of the relationship between the exchange rate and monetary policy, Walsh (2003)[90] demonstrate that the rate change of output gap should play a significant role in the design of monetary policy and regard it as a speed-limit type of Taylor rule. More specifically, Peel et al. (2004)[70] examine this type of monetary policy with the US data covering the period 1982: Q1-2003: Q1. This type of rule is also applied by Smet and Wouter (2007) in their DSGE model. Overwhelmingly, the policy reaction function incorporates four kinds of specifications based on a combination between the existence of the nominal exchange rate depreciation and the rate change of output. Among them, the original one in Lubik and Schorfheide's model is regarded as the control group while others comprise of the treatment groups. Chapter 3 of the thesis compares the four groups and finds the model with the best data fitting in using the Bayesian likelihood approach. The model with the best fitting performance will be the benchmark model for the next chapter.

Some economies inevitably have experienced structural changes in the past decades. For instance, Nelson(2003[67]) offers guidance of the regime changes of UK monetary policy covering the period 1972-1997, from the period of floating exchange rate to the period of independence of the Bank of England. VARs model is a convenient methodology to study the regime shifts (Catelnuovo and Surico ,2005[14];Benati,2009[6]). Since Davig and Leeper (2006[22]) and Farmer et al. (2011[32]) can provide a unique solution to the Markov-Switching rational expectation models, it motivates individuals to identify the regime shifts in different countries based on diversified versions of DSGE models. Benati and Surico (2009[6]) estimate a Markov-Switching New Keynesian model with US data covering the period of the Great Moderation. Liu and Mumtaz (2011[57]) initially examine the UK data with a small open Markov-Switching DSGE model developed from Justiniano and Preston (2010[48]) covering 1970: Q1-2009: Q1. Chen and MacDonald(2012[16]) estimate a small open DSGE model developed by Lubki and Schorfheid (2007[60]) covering the period 1975:Q1-2010:Q2. Here is one thing to mention, the models developed from Justiniano and Preston (2010[48]) and Lubki and Schorfheid (2007[60]) are both the simplified version of Gali and Monacelli (2005[37])'s framework. Chapter 4 will introduce two similar kinds of Markov chains borrowed from Chen and MacDonald(2012[16]) to estimate the small open

DSGE model which offer the best data fitting in the previous chapter. Chapter 4 tries to find the Markov-switching DSGE model with the best data fitting, and it mainly has three aspects different from the previous literature. First, the chosen sample size is shorter and covers the period 1992: Q4-2008: Q4. The chosen period excludes the potential impacts of the fixed exchange rate regime and the zero lower bound on the monetary policy reaction function. Second, chapter 4 borrows the specification of the monetary policy reaction function offering the best data fitting for each country from the constant parameter estimation in chapter 3. At last, chapter 4 only considers two kinds of structural changes, including the variance of the shocks and the policy parameters.

The Structure of the Thesis

There are five chapters in this thesis. Chapter 1 will derive the core parts of the theoretical small open DSGE model proposed by Gali and Monacelli. It then uncovers the simplification process from the theoretical model to the empirical one provided by Lubik and Schorfheide. At the end of chapter one, it will exhibit a classical method combined with Dynare to solve the model numerically. Chapter 2 will describe the data sample and then introduces the Bayesian methodology to estimate the model for the UK and Canada separately. Moreover, it provides two alternative methodologies linking VARs to DSGE models, thereby checking the model identification regarding the equation of the change rate of the terms of trade. Chapter 3 will regard the empirical results from chapter 2 as the control group and have another three treatment groups based on different specifications of the monetary policy reaction function. It will search for the model with the best performance of data fitting for each of the two countries. Chapter 4 will regard the model with the best performance of data fitting in chapter 3 as the benchmark model and introduce two types of Markov-Switching parameters in the small open DSGE model. It will carry on searching for the model with the best performance of data fitting within the same sample period and tries to answer whether there is a significant improvement of data fitting between the constant parameter model and the Markov-Switching DSGE model. Chapter 5 summarises the main findings of the thesis and offers some implications of the current research with further possible research directions.

Chapter 1

The Simplification and Solution of the Model

1.1 Introduction

The thesis will borrow the small open DSGE model from Lubik and Schorfheide (2007)[60]'s research. This log-linear model is a simplified version of the model developed by Gali and Monacelli (2005)[37]. There are two economies in this model. One is the home country, and the other one is the rest of the world. The DSGE model comprises of a forward-looking IS equation, a forward-looking Philip curve, an exchange rate equation derived based on the law of one price and a Taylor type monetary policy rule and four exogenous stationary processes. In the log-linear model, the log differences of economic variables express the percentage deviations of such variables concerning their stable states. There are two sections in this chapter. The first section will show the derivation process of the log-linear model. It introduces the types of model variables, replicates the derivation of the model based on some essential assumptions and draws attention to the simplification process from the theoretical model to the simplified one. The second section will display the solution methodology to the model. It presents two general ways to solve a given DSGE model and then offers the numerical solutions and the calculated impulse responses functions based on a set of calibrated parameters.

1.2 A Small Open DSGE Model

This section will present the main components of a small open DSGE model derived by Gali and Monacelli(2005 [37]). After that, it will uncover the process of how Lubik and Schorfheide (2007[60]) simplify the model from the previous work. The simplified version of the model has been used in the empirical research to study the behaviour of central banks in different countries(Zheng and Guo (2013[94]), Chen and Macdonald(2012[16])). However, few of them, even Lubik and Schorfheide themselves, has explicitly revealed the process to generate the simplified model from Gali and Monacelli's work. By uncovering the connections between the two versions of models, this section is helpful to understand the assumptions and the limitations of the model better.

1.2.1 Types of Model Variables

It is essential to introduce the types of model variables adopted in the DSGE model in this section. For instance, if M_t is an arbitrary type of economic variable, then m_t is the log value of the economic variable: $m_t = logM_t$. $\tilde{m}_t$ is the log difference of the economic variable: $\tilde{m}_t = m_t - m = logM_t - logM = \frac{M_t - M}{M}$,where M is the steady state of the economic variable. Lubik and Schorfheide opt for $\tilde{m}_t$ in their DSGE model and the steady state $\tilde{m}$ is zero, which implies that the economic variables M_t will converge to their stationary states M in the equilibrium level.

1.2.2 Gali and Monacelli's framework

Gali and Monacelli's framework incorporates about eight segments which are relevant to the following simplifying process. The first segment discusses the intertemporal optimal condition of the household. The second one brings in the definition of terms of trade given the steady state of the purchasing power parity. The third one assumes there is an uncovered interest parity between any two countries in the bond market. The fourth one shows a retailer firm in the market of perfect competition and the fifth one demonstrates wholesale firms in the market of monopolistic competition. The sixth one exhibits the optimal pricing strategy of the wholesale firms when the price is sticky according to the Calvo rule (1983[11]). The seventh segment displays the relationship between the consumption and output in the given small open economy. The final one obtains the potential output and natural rate of the interest rate. The equations and the dependent parameters generated from such eight segments will play critical roles in the simplifying process.

Households

The representative household optimises the following utility function:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi} \right), \tag{1.1}$$

where N_t is hours of labour, C_t is a composite consumption index, β is the intertemporal discount factor, σ is the relative risk aversion coefficient and φ is the marginal disutility with respect to labour supply. In addition, The composite consumption index C_t can be defined as:

$$C_t = \left[(1-\alpha)^{1/\eta} (C_{H,t})^{(\frac{\eta-1}{\eta})} + \alpha^{1/\eta} (C_{F,t})^{(\frac{\eta-1}{\eta})} \right]^{\frac{\eta}{\eta-1}}, \tag{1.2}$$

where $C_{H,t}$ is an index of consumption of domestic produced goods, $C_{F,t}$ is an index of imported goods, η measures the elasticity of substitution between such two kinds of goods, and α is an index of openness. The equation defining $C_{H,t}$ is:

$$C_{H,t} = \left(\int_0^1 C_{H,t}(j)^{\frac{\xi}{\varepsilon-1}} dj \right)^{\frac{\xi}{\xi-1}}, \tag{1.3}$$

where $C_{H,t}(j)$ represents the consumption of home product of good j at time t, and $j \in [0,1]$ denotes that the small economy produces a continuum of differentiated goods in the unit interval. ε is the elasticity of substitution between these many infinitely different goods. Likewise, the equation defining $C_{F,t}$ is:

$$C_{F,t} = \left(\int_0^1 C_{i,t}^{\frac{\gamma}{\gamma-1}} di \right)^{\frac{\gamma}{\gamma-1}}, \tag{1.4}$$

where $C_{i,t}$ is the consumption of goods imported from a specified country i at time t,and γ stands for the elasticity of substitution between different foreign countries in the world. Moreover, $C_{i,t}$ consists of a continuum of differentiated goods in the unit interval:

$$C_{i,t} = \left(\int_0^1 C_{i,t}(j)^{\frac{\xi}{\varepsilon - 1}} dj \right)^{\frac{\xi}{\xi - 1}}, \tag{1.5}$$

where $C_{i,t}(j)$ represents the consumption of product of good j imported from a foreign country i at time t.

It assumes that the total consumption from goods market and bond market cannot exceed the revenue in each period. Thus, the household's period budget can be written as

$$\int_0^1 P_{H,t}(j) C_{H,t}(j) dj + \int_0^1 \int_0^1 P_{i,t}(j) C_{i,t}(j) dj di + E_t[Q_{t,t+1} D_{t+1}] \leq D_t + W_t N_t + T_t, \tag{1.6}$$

where $P_{H,t}(j)$ is the price of home product of good j, $P_{i,t}(j)$ is the price of good j imported from country i. $Q_{t,t+1}$ is the stochastic discount factor for the nominal payoffs D_{t+1} in the period $t+1$ of the bond held at the end of period t. W_t is the nominal wage and T_t is the lump sum transfers (positive) or taxes (negative).

The next task is to take i,j,H and F off from the budget constraint. First, maximise equation (1.3) given the constraint:

$$\int_0^1 P_{H,t}(j) C_{H,t}(j) dj = P_{H,t} C_{H,t}. \tag{1.7}$$

The maximisation procedures arrive at the demand function of the domestic good j.

$$C_{H,t}(j) = \left(\frac{P_{H,t}(j)}{P_{H,t}} \right)^{-\xi} C_{H,t}. \tag{1.8}$$

Substitute the above equation in the equation (1.7) yields the domestic price index:

$$P_{H,t} = (\int_0^1 P_{H,t}(j)^{1-\xi} dj)^{\frac{1}{1-\xi}}. \tag{1.9}$$

Second, maximise equation (1.5) given the constraint:

$$\int_0^1 P_{i,t}(j) C_{i,t}(j) dj = P_{i,t} C_{i,t}. \tag{1.10}$$

The demand function of the imported good j from a foreign country i is

$$C_{i,t}(j) = \left(\frac{P_{i,t}(j)}{P_{i,t}}\right)^{-\xi} C_{i,t}. \tag{1.11}$$

Likewise, the price index(expressed in home currency) for imported goods from country i can be written in the following way:

$$P_{i,t} = (\int_0^1 P_{i,t}(j)^{1-\xi} dj)^{\frac{1}{1-\xi}}. \tag{1.12}$$

By far, it is ready to take j away from the period budget constraint (1.6). Third, maximise equation (1.4) given the constraint

$$\int_0^1 P_{i,t} C_{i,t} di = P_{F,t} C_{F,t}. \tag{1.13}$$

The demand function of the imported good for a given country i is

$$C_{i,t} = \left(\frac{P_{i,t}}{P_{F,t}}\right)^{-\gamma} C_{F,t}. \tag{1.14}$$

The price index (expressed in home currency) for imported goods is

$$P_{F,t} = (\int_0^1 P_{i,t}^{1-\gamma} di)^{\frac{1}{1-\gamma}}. \tag{1.15}$$

Now it is ready to take i away from the period budget constraint (1.6). Finally,maximise equation (1.2) given the constraint:

$$P_{H,t} C_{H,t} + P_{F,t} C_{F,t} = P_t C_t. \tag{1.16}$$

The demand function of the domestic good $C_{H,t}$ is given by

$$C_{H,t} = (1-\alpha)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t. \tag{1.17}$$

The demand function of the imported good $C_{F,t}$ is given by

$$C_{F,t} = \alpha \left(\frac{P_{F,t}}{P_t}\right)^{-\eta} C_t. \tag{1.18}$$

The consumer price index is written in the following way:

$$P_t = [(1-\alpha)(P_{H,t})^{1-\eta} + \alpha(P_{F,t})^{1-\eta}]^{\frac{1}{1-\eta}}. \tag{1.19}$$

After taking H and F away, the period budget constraint (1.6) becomes:

$$P_t C_t + E_t[Q_{t,t+1} D_{t+1}] \leq D_t + W_t N_t + T_t. \tag{1.20}$$

Turning back to the maximisation of the representative household's utility (1.1) given the above budget constraint leads to the intertemporal optimal condition.

$$\beta R_t E_t \left[(\frac{C_{t+1}}{C_t})^{-\sigma} (\frac{P_t}{P_{t+1}}) \right] = 1. \tag{1.21}$$

where $R_t = \frac{1}{E_t Q_{t,t+1}}$ is the gross return on a risk free one period discount bond paying off one unit of home currency at time $t + 1$.Rewrite the intertemporal condition in the log-linear form:

$$\tilde{c_t} = E_t \tilde{c_{t+1}} - \frac{1}{\sigma}(r_t - E_t \pi_{t+1} - \rho), \tag{1.22}$$

where $r_t = R_t - 1$ is the net interest rate, $\pi_t = log(P_t) - log(P_{t-1})$ is CPI inflation, and $\rho = \frac{1}{\beta} - 1$ is the time discount rate.

The derivation process of the household section is the very key component in the framework of a DSGE model. In addition to Gali and Monacelli (2005[37]), the famous book written by Gali(2015[35]) and Walsh (2017[91]) offer a very specified explanation and illustration of the optimal conditions of households in a small open economy. More specifically,the latter one also includes a two-country model developed by Obstfeld and Rogoff (1995[68];1996[69]). The two-country model is useful when someone is interested in examining the impact of national development on the international economy. The thesis is consistent with Gali(2015[35])'s framework, which assumes the domestic economy is tiny and have little impacts on the world economy.

Purchasing Power Parity and Law of One Price

Gali and Monacelli (2005[37]) define the effective terms of trade S_t as the following equation:

$$S_t = \frac{P_{F,t}}{P_{H,t}} = \left(\int_0^1 S_{i,t}^{1-\gamma} di\right)^{\frac{1}{1-\gamma}}, \tag{1.23}$$

where $S_{i,t} = \frac{P_{i,t}}{P_{H,t}}$ is the bilateral terms of trade between the home economy and a foreign country i. In addition, the price of country i's goods $P_{i,t}$ is expressed in the domestic currency. Given the assumptions that the steady state satisfying the purchasing power parity $S = 1$ and the elasticity of substitution between different countries $\gamma = 1$, rewrite the formula above in the log-linear from:

$$\tilde{s_t} = (p_{F,t} - p_F) - (p_{H,t} - p_H) = p_{F,t} - p_{H,t} = \int_0^1 \tilde{s_{i,t}} di. \tag{1.24}$$

Analogically, under the assumption of purchasing power parity and the elasticity of substitution between home product and imported goods $\eta = 1$, rewrite the CPI price formula (1.19) in the log-linear expression:

$$p_t = (1-\alpha)p_{H,t} + \alpha p_{F,t} = p_{H,t} + \alpha \tilde{s}_t. \tag{1.25}$$

Take one period backward of the above equation and subtract it from the above equation yield the following relationship between the CPI inflation and domestic inflation.

$$\pi_t = \pi_{H,t} + \alpha \triangle \tilde{s}_t, \tag{1.26}$$

where the domestic inflation $\pi_{H,t} = p_{H,t} - p_{H,t-1}$ is defined as the rate of change in the index of domestic goods prices.

Gali and Monacelli (2005[37]) also assume the law of one price holds for goods j at all times.

$$P_{i,t}(j) = \varepsilon_{i,t} P_{i,t}^i(j), \tag{1.27}$$

where j represents the good variety, i represents the countries,$\varepsilon_{i,t}$ is the bilateral nominal exchange rate which dimension is $\frac{home_{currency}}{foreign_{currency}}$ and $P_{i,t}^i(j)$ is the price of the same type of good j expressed in the currency of country i. $P_{i,t}$ can be rewritten as the following equation:

$$P_{i,t} = \varepsilon_{i,t} P_{i,t}^{i} = \varepsilon_{i,t} \left(\int_0^1 P_{i,t}^{i}(j)^{1-\xi} dj \right)^{\frac{1}{1-\xi}} \tag{1.28}$$

Substitute the above equation in the equation (1.15) and rewrite $P_{F,t}$ in the following equation:

$$P_{F,t} = \left[\int_0^1 \left(\varepsilon_{i,t} \left(\int_0^1 P_{i,t}^{i}(j)^{1-\xi} dj \right)^{\frac{1}{1-\xi}} \right)^{1-\gamma} di \right]^{\frac{1}{1-\gamma}}. \tag{1.29}$$

Rewrite the above equation in the log-linear form under the assumptions $\gamma = 1$ yields the equation below:

$$p_{F,t} = \int_0^1 (e_{i,t} + p_{i,t}^{i}) di = e_t + p_t^{*}, \tag{1.30}$$

where $e_t = \int_0^1 e_{i,t} di$ is the log nominal effective exchange rate, $p_{i,t}^{i} = \int_0^1 p_{i,t}^{i}(j) dj$ is the log domestic price index for country i expressed in its own currency, and $p_t^{*} = \int_0^1 p_{i,t}^{i} di$ is the log world price index. Combining this formula with the equation (1.24), the log difference of terms of trade is rewritten as the following equation:

$$\tilde{s}_t = \tilde{e}_t + p_t^{*} - p_{H,t}. \tag{1.31}$$

Take one period backward of the above equation and subtract it from the above equation generates an important relationship between CPI inflation and the rate change of nominal exchange rate in the equation below:

$$\pi_t = \pi_t^{*} + \triangle \tilde{e}_t - (1-\alpha) \triangle \tilde{s}_t. \tag{1.32}$$

The definition of terms of trade in Gali and Monacelli (2005[37])'s framework is a bit different from the practical side. Chamberlin and Yueh (2006[15]) define the terms of trade as the ratio of export to import prices, which is just the opposite to the definition of Gali and Monacelli (2005[37]). Generally, an exchange rate depreciation will increase the import price and weaken the terms of trade. The currency depreciation is good for the export sector while bad for the import sector. From the viewpoints of the import sector, the weakened terms of trade will deteriorate the trade balance if the country does not expand the quantity of its export. The exchange rate depreciation $\triangle\tilde{e}_t$ and the change rate of the terms of trade $\triangle\tilde{s}_t$ should move in the opposite direction, and the simplified model will correct the equation above for the practical purpose.

Uncover Interest Parity and Terms of Trade

The intertemporal optimal condition for the representative household in any other country i is written in the following equation:

$$\beta E_t \left[(\frac{C^i_{t+1}}{C^i_t})^{-\sigma} (\frac{P^i_t}{P^i_{t+1}}) \right] = Q^i_{t,t+1}. \tag{1.33}$$

Under the assumption of law of one price and complete security markets, there are no arbitrage opportunities in the bond market:

$$Q_{t,t+1} = Q^i_{t,t+1} \frac{\varepsilon^i_t}{\varepsilon^i_{t+1}}, \tag{1.34}$$

where it implies a potential investor is able to buy the domestic bonds and the foreign bonds at the same discounted current price expressed in the same currency. The combination between equation (1.33) and (1.34) arrives at an uncovered interest rate parity:

$$R_t = R^i_t \frac{\varepsilon^i_{t+1}}{\varepsilon^i_t} = \frac{1}{\varepsilon^i_t} R^i_t \varepsilon^i_{t+1}, \tag{1.35}$$

where it implies the profit of one unit of the domestic currency invested in the domestic bond market is same as the profit of one unit of the domestic currency invested in the foreign bond market. $R^i_t = \frac{1}{Q^i_{t,t+1}}$ is the foreign gross return of one period bond and the dimension of ε^i_t is $\frac{foreign_{currency}}{home_{currency}}$. Combining the equation (1.33) with equation (1.21), it yields the equation below:

$$\frac{C_{t+1}}{C_t} = (\frac{\frac{P^i_t \varepsilon^i_t}{P_t}}{\frac{P^i_{t+1}\varepsilon^i_{t+1}}{P_{t+1}}})^{-\sigma} \frac{C^i_{t+1}}{C^i_t} = (\frac{Q_{i,t}}{Q_{i,t+1}})^{-\sigma} \frac{C^i_{t+1}}{C^i_t}, \tag{1.36}$$

where $Q_{i,t} = \frac{P^i_t \varepsilon^i_t}{P_t}$ is defined as the bilateral real exchange rate for the currency in the country i. It is important to extract an important relationship between the domestic and home consumption from the above equation:

$$C_t = v C^i_t Q^{\frac{1}{\sigma}}_{i,t}, \tag{1.37}$$

where v is a arbitrary constant and can be canceled off when log-linearize the above equation around the steady state $C = C^i = C^*$ under the assumption of the purchasing power parity $Q_i = S_i = 1$:

$$\tilde{c}_t = \tilde{c^*_t} + \frac{1}{\sigma} \tilde{q}_t, \tag{1.38}$$

where $c_t^* = \int_0^1 c_t^i di$ is the index of world consumption and $q_t = \int_0^1 q_{i,t} di = \int_0^1 (e_{i,t} + p_t^i - p_t)$ is the log effective real exchange rate. Combining equation (1.25) and (1.31),the log difference of the real effective exchange rate is expressed by:

$$\tilde{q_t} = \tilde{e_t} + p_t^* - p_t = \tilde{s_t} + p_{H,t} - p_t = (1-\alpha)\tilde{s_t}. \tag{1.39}$$

Substitute the real exchange rate in the equation (1.38), it yields a relationship between the home consumption and the world consumption:

$$\tilde{c_t} = \tilde{c_t^*} + (\frac{1-\alpha}{\sigma})\tilde{s_t}. \tag{1.40}$$

In addition, log-linearize the equation (1.35) will yield a relationship between the home net interest rate and the world net interest rate r_t^*:

$$r_t - r_t^* = E_t \triangle \tilde{e_{t+1}}. \tag{1.41}$$

Combining the above equation with the equation (1.32) yields the relationship between the interest rate and terms of trade:

$$\tilde{s_t} = (r_t^* - \pi_{t+1}^*) + (r_t - E_t \pi_{H,t+1}) + \tilde{s_{t+1}}. \tag{1.42}$$

Uncovered interest rate parity assumes that the bond investors will hold the bond with the highest expected return. However, the assumption of the uncovered interest rate parity is too strong for two reasons (Blanchard,2013[7]). On the one hand, it ignores the transaction cost. For instance, entering in and exists from the UK bond market requires three contracts with different transaction costs. On the other hand, it ignores risk. The forward exchange rate in a year is unknown for a foreigner to decide to buy the domestic bond now. The foreign investor may feel reluctant to hold the domestic bond because the risk of the volatility of the exchange rate is not covered compared to a risk-less arbitrage condition. Having said this, however, the interest parity condition is still a good first approximation of the reality due to the capital movements among the developed and wealthy countries in the world (Dornbusch et al.,2003[30]; Blanchard,2013[7]).

Retailers(Final Goods Firm)

The optimisation problems are a bit complicated in the home production sector, which require the inclusion of two sections: firms called retailers which produce the final goods in the market of perfect competition and firms called wholesalers which produce the intermediate goods in the market of monopolistic competition. The wholesale firms will produce many infinitely differentiate intermediate goods and sell them to the retailer firms with a flexible or sticky price. The retailer firms then aggregate the intermediate goods into the same type of final goods and then sell them with perfect competition.

The objective of the retailer is to maximise the profit in the below equation:

$$P_{H,t}Y_t - \int_0^1 P_{H,t}(j)Y_t(j)dj, \tag{1.43}$$

with the constraint below implying that the elasticity of substitution of the many infinitely intermediate goods is ξ:

$$Y_t = \left(\int_0^1 Y_t(j)^{\frac{\xi-1}{\xi}} \right)^{\frac{\xi}{\xi-1}}. \tag{1.44}$$

Accordingly, the optimisation procedures arrive at the demand function for the wholesale good j:

$$Y_t(j) = \left(\frac{P_{H,t}(j)}{P_{H,t}} \right)^{-\xi} Y_t. \tag{1.45}$$

Substitute the above equation and the equation (1.9) in equation (1.43), the maximum profit of the retailer firms is zero, which proves that there is perfect competition in the market of the retailer firms.

Wholesale Firms with flexible prices strategy

Given the flexible price strategy, the objective of the wholesale firms is to maximize the profit:

$$P_{H,t}(j)Y_t(j) - (1 - \tau_1)W_tN_t(j) \tag{1.46}$$

where τ_1 is the subsidy of employment. The objective is constrained by the equation (1.45) and the production function defined as the below equation:

$$Y_t(j) = A_tN_t(j). \tag{1.47}$$

The optimal price of wholesale good j is written in the following equation:

$$P_{H,t}(j) = \frac{\xi}{\xi - 1}MC_t^n, \tag{1.48}$$

where $MC_t^n = (1 - \tau_1)\frac{W_t}{A_t}$ is the nominal marginal cost. In addition, the marginal cost is independent of j and identical for all wholesale firms. Rewrite the real marginal cost $MC_t = \frac{MC_t^n}{P_{H,t}}$ in the log-linear form:

$$mc_t = log(1 - \tau_1) + w_t - p_{H,t} - a_t. \tag{1.49}$$

The steady state of log real marginal cost $mc = log(\frac{\xi-1}{\xi})$ can be derived from the equation (1.48).

Wholesale Firms with sticky prices strategy

Calvo (1983[11]) assumes the wholesale firms has a θ probability of keeping the price of its good fixed in the following periods and a $1-\theta$ probability of optimally redefining its price. The objective of the wholesale firm to maximize the profit is defined in the below equation:

$$\sum_{k=0}^{\infty} \theta^k E_t[Q_{t,t+k}(Y_{t+k}(P_{H,t}^{*} - MC_{t+k}^{n}))], \tag{1.50}$$

subjecting to the constraint obtained from equation (1.45):

$$Y_{t+k}(j) = (\frac{P_{H,t}^{*}}{P_{H,t+k}})^{-\varepsilon} Y_{t+k} \tag{1.51}$$

The discount factor $Q_{t,t+k} = \beta^k (\frac{C_{t+k}}{C_t})^{-\sigma} (\frac{P_t}{P_{t+k}})$ is derived from the optimal intertemporal condition (1.21). Under the assumptions $\sigma = 1$,$\eta = 1$ and $\gamma = 1$, it arrives at the optimal redefining price $P_{H,t}^{*}$:

$$P_{H,t}^{*} = \frac{\xi}{\xi - 1} \frac{X_{1,t}}{X_{2,t}}, \tag{1.52}$$

where $X_{1,t} = MC_t + \theta\beta E_t X_{1,t+1}$ and $X_{2,t} = P_{H,t}^{-1} + \theta\beta E_t X_{2,t+1}$. The optimal resetting price is identical for all the wholesale firms. In addition, the above equation can be represented by the domestic inflation instead of the domestic price index:

$$\frac{P_{H,t}^{*}}{P_{H,t-1}} = 1 + \pi_{H,t}^{*} = \frac{\xi}{\xi - 1} \frac{1}{P_{H,t-1}} \frac{X_{1,t}}{X_{2,t}} = \frac{\xi}{\xi - 1} (1 + \pi_{H,t}) \frac{X_{1,t}}{X_{2,t}^{*}}, \tag{1.53}$$

where $X_{2,t}^{*} = 1 + \beta\theta(1 + \pi_{H,t+1})^{-1} E_t X_{2,t+1}$. Log-linearize the equation (1.53) around the zero steady inflation rate yields:

$$\pi_{H,t}^{*} = \pi_{H,t} + \tilde{x_{1,t}} - \tilde{x_{2,t}^{*}}, \tag{1.54}$$

where $\tilde{x_{1,t}} = (1 - \theta\beta)\tilde{mc_t} + \theta\beta E_t \tilde{x_{1,t+1}}$ and $\tilde{x_{2,t}^{*}} = -\theta\beta E_t \pi_{H,t+1} + \theta\beta E_t \tilde{x_{2,t+1}^{*}}$. Also, $\tilde{mc_t} = mc_t - mc$ is the log difference of the real marginal cost with respect to its stationary state. Given the price stickness, the domestic price index function (1.9) can be rewritten as follows:

$$P_{H,t}^{1-\xi} = \int_0^{\theta} P_{H,t-1}^{1-\xi}(j)dj + \int_{\theta}^{1} {P_{H,t}^{*}}^{1-\xi} dj = \theta P_{H,t-1}^{1-\xi} + (1 - \theta){P_{H,t}^{*}}^{1-\xi}. \tag{1.55}$$

The above equation can be rewritten in terms of the inflation by dividing $P_{t-1}^{1-\xi}$ from both sides:

$$(1+\pi_{H,t})^{1-\xi} = \theta + (1-\theta)(1+\pi_{H,t}^{*})^{1-\xi} \tag{1.56}$$

Log-linearize the function above yields the relationship between the optimal inflation and the domestic inflation:

$$\pi_{H,t} = (1-\theta)\pi_{H,t}^{*} \tag{1.57}$$

Substitute the above equation in the equation (1.54) yields the domestic Philips curve:

$$\pi_{H,t} = \beta E_t \pi_{H,t+1} + \kappa \tilde{mc}_t, \tag{1.58}$$

where $\kappa = \frac{(1-\theta)(1-\theta\beta)}{\theta}$.

Monopolistic competition and the sticky prices are two central assumptions for the supply side in the DSGE model considering the features of the New Keynesian. The prices of goods are normally higher than the marginal cost, which invalidates the assumption of the perfect competition in the real business cycle model. For instance, Hall(1998[40]) finds evidence of a higher price than the marginal cost in the US economy. Dixit and Stiglitz(1997[28]) mathematically approximate the central idea of the imperfect competition, and after that, most of the DSGE models borrow their ideas to assume there is a continues of differentiated goods locating in the interval from zero to one. More specifically, Torres(2015[88]) offers an excellent and fundamental book to cover the final goods production sector (retailers) in the perfect competition market and the production of the intermediate goods sector (wholesale firms) in the monopolistic competition market. The price stickiness reflects that it is not free for the firms to adjust their market prices given a new equilibrium of the demand and supply for the quantities of their products. Normally, the DSGE literature borrows the pricing methodology proposed by Calvo(1983[11])to add the feature of the staggering price, thereby matching the data better given the derived Philips curve(Gali et al. ,2001[36]; Gali,2002[34]). In addition to Gali(2015[35])and Walsh(2017[91]), McCandless(2008[63]) and Junior (2018[47]) also provide the details to derive the New Keynesian Philip curves.

Consumption and Output

The equilibrium in the goods market for the small open economy requires:

$$Y_t(j) = C_{H,t}(j) + \int_0^1 C^i_{H,t}(j)di, \tag{1.59}$$

where $C^i_{H,t}(j)$ represents foreign country i's consumption for the good j produced in the home country. The simplification of the above equation needs several steps. First, rewrite $C_{H,t}(j)$ in the from of C_t through the equations (1.8) and (1.17):

$$C_{H,t}(j) = (\frac{P_{H,t}(j)}{P_{H,t}})^{-\xi} C_{H,t} = (\frac{P_{H,t}(j)}{P_{H,t}})^{-\xi}(1-\alpha)(\frac{P_{H,t}}{P_t})^{-\eta} C_t. \tag{1.60}$$

Second, rewrite $C^i_{H,t}(j)$ in the form of $C^i_{H,t}$ through equation(1.8) :

$$C^i_{H,t}(j) = (\frac{P_{H,t}(j)}{P_{H,t}})^{-\xi} C^i_{H,t}, \tag{1.61}$$

where it implies the foreign country i prefers to consume good j from the home economy given the price of it $P_{H,t}(j)$ in relation to the whole price index $P_{H,t}$. Next, rewrite $C^i_{H,t}$ in the form of $C^i_{F,t}$ through equation (1.14):

$$C^i_{H,t} = (\frac{P_{H,t}}{P^i_{F,t}\varepsilon_{i,t}})^{-\gamma} C^i_{F,t}. \tag{1.62}$$

The home economy export its good to an foreign country i and the foreign country options to consume the quantity of goods from the home economy $C^i_{H,t}$ among other countries in the world given the export price $P_{H,t}$ comparing with the imported price index $P^i_{F,t}$ for the foreign country in terms of the same currency. Last, rewrite $C^i_{F,t}$ in the form of C^i_t through equation (1.18):

$$C^i_{F,t} = \alpha(\frac{P^i_{F,t}}{P^i_t})^{-\eta} C^i_t. \tag{1.63}$$

The foreign country make a choice on the quantity of imported goods given the imported price index $P^i_{F,t}$ comparing to its own price index P^i_t. Gali and Monacelli assume the preferences are symmetric for consumers, implying the ξ,γ and η are identical across different countries. Combing equations from (1.61) to (1.63), it yields a relationship between the consumption of a certain type of good produced in the home economy $C^i_{H,t}(j)$ and the foreign consumption index C^i_t:

$$C^i_{H,t}(j) = (\frac{P_{H,t}(j)}{P_{H,t}})^{-\xi}(\frac{P_{H,t}}{P^i_{F,t}\varepsilon_{i,t}})^{-\gamma}\alpha(\frac{P^i_{F,t}}{P^i_t})^{-\eta} C^i_t. \tag{1.64}$$

Above all, substituting the equation (1.60) and (1.64) into the equilibrium condition (1.59) generate the following equation:

$$Y_t(j) = (\frac{P_{H,t}(j)}{P_{H,t}})^{-\xi} \left[(1-\alpha)(\frac{P_{H,t}}{P_t})^{-\eta} C_t + \alpha \int_0^1 (\frac{P_{H,t}}{P^i_{F,t}\varepsilon_{i,t}})^{-\gamma} (\frac{P^i_{F,t}}{P^i_t})^{-\eta} C^i_t di \right]. \quad (1.65)$$

Plugging the above equation to the aggregate domestic output (1.44) yields:

$$Y_t = (1-\alpha)(\frac{P_{H,t}}{P_t})^{-\eta} C_t + \alpha \int_0^1 (\frac{P_{H,t}}{P^i_{F,t}\varepsilon_{i,t}})^{-\gamma} (\frac{P^i_{F,t}}{P^i_t})^{-\eta} C^i_t di. \quad (1.66)$$

Notice $(\frac{P^i_{F,t}}{P^i_t})^{-\eta} = (\frac{P_{H,t}}{P_t})^{-\eta}(\frac{P^i_{F,t}\varepsilon_{i,t}}{P_{H,t}})^{-\eta}(\frac{P_t}{\varepsilon_{i,t}P^i_t})^{-\eta} = (\frac{P_{H,t}}{P_t})^{-\eta}(\frac{P^i_{F,t}\varepsilon_{i,t}}{P_{H,t}})^{-\eta}(\frac{1}{Q_{i,t}})^{-\eta}$. Accordingly, the above equation is simplified as:

$$Y_t = (\frac{P_{H,t}}{P_t})^{-\eta} \left[(1-\alpha) C_t + \alpha \int_0^1 (\frac{\varepsilon_{i,t}P^i_{F,t}}{P_{H,t}})^{\gamma-\eta} Q_{i,t} C^i_t di \right]. \quad (1.67)$$

Introducing the definition of terms of trade through equation (1.23), it yields $\frac{\varepsilon_{i,t}P^i_{F,t}}{P_{H,t}} = \frac{P^i_{F,t}}{P^i_{H,t}}\frac{\varepsilon_{i,t}P^i_{H,t}}{P_{H,t}} = S^i_t S_{i,t}$ and then substituting the terms of trade and equation (1.37) into the above equation:

$$Y_t = (\frac{P_{H,t}}{P_t})^{-\eta} C_t \left[(1-\alpha) + \alpha \int_0^1 (S^i_t S_{i,t})^{\gamma-\eta} Q_{i,t}^{\eta-\frac{1}{\sigma}} di \right]. \quad (1.68)$$

Deriving the first order log-linear approximation to the above equation around the steady state with the assumption of purchasing power parity yields:

$$\tilde{y}_t = \tilde{c}_t + \alpha\gamma\tilde{s}_t + \alpha(\eta - \frac{1}{\sigma})\tilde{q}_t = \tilde{c}_t + \frac{\alpha\omega}{\sigma}\tilde{s}_t, \quad (1.69)$$

where $\omega = \sigma\gamma + (1-\alpha)(\sigma\eta - 1)$. The equation above will hold for all countries,implying that:

$$\tilde{y^i_t} = \tilde{c^i_t} + \frac{\alpha\omega}{\sigma}\tilde{s^i_t}. \quad (1.70)$$

Given the assumption $\int_0^1 s^i_t di = 0$, it can generate a world equilibrium condition of the good market by aggregating the output over all countries:

$$\tilde{y^*_t} = \int_0^1 \tilde{y^i_t} di = \int_0^1 \tilde{c^i_t} di = \tilde{c^*_t}. \quad (1.71)$$

Combing equation (1.40), (1.69) and (1.71) generates the relationship between the log difference of domestic output and the log difference of the world output:

$$\tilde{y}_t = \tilde{y^*_t} + \frac{1}{\sigma_\alpha}\tilde{s}_t, \quad (1.72)$$

where $\sigma_\alpha = \frac{\sigma}{1-\alpha+\alpha\omega}$. Finally, combing the Euler equation (1.22) with (1.69) and (1.72) is able to generate the new IS curve in terms of output instead of consumption:

$$\tilde{y_t} = E_t \tilde{y_{t+1}} - \frac{1}{\sigma_\alpha}(r_t - E_t \pi_{H,t+1} - \rho) + \alpha\Theta E_t \triangle \tilde{y^*_{t+1}}, \tag{1.73}$$

where $\Theta = \omega - 1$.

Marginal Cost, Potential Output and Natural Rate of Interest

In this subsection, it will generate a relationship between the marginal cost independent of the price stickness and then introduce the natural rate of output and interest rate.

In addition to the intertemporal optimal Euler condition (1.21), the optimization household utility also yields that the marginal substitution rate of consumption and leisure equating to the real wage price:

$$\frac{C_t^{\sigma}}{N_t^{-\varphi}} = \frac{W_t}{P_t}. \tag{1.74}$$

Rewriting the above formulas in the log form:

$$\sigma c_t + \varphi n_t = w_t - p_t. \tag{1.75}$$

Also, Integrating the production function (1.47) over the domain of $j \in [0, 1]$ combing with the demand equation (1.45):

$$\int_0^1 Y_t(j)dj = \int_0^1 (\frac{P_t(j)}{P_t})^{-\xi} djY_t = A_t \int_0^1 N_t(j)dj = A_t N_t, \tag{1.76}$$

where it assumes $N_t = \int_0^1 N_t(j)dj$ is the total labor supply for the home economy. Taking the log form of the both sides yields the equation below:

$$y_t = a_t + n_t, \tag{1.77}$$

where it assumes the price dispersion $(\frac{P_t(j)}{P_t})^{-\xi}$ is a constant number of one in the first order approximation. Given the equations (1.72),(1.75) and (1.77), rewriting the marginal cost equation in (1.49) as follows:

$$mc_t = -\nu + (\sigma_\alpha + \varphi)y_t + (\sigma - \sigma_\alpha)y_t^* - (1 + \varphi)a_t, \tag{1.78}$$

where $\nu = log(1 - \tau_1)$. The natural rate of output is defined as the output of firms when the price is flexibly determined each period. Thus, substituting the steady log marginal cost $mc = -\mu = log(\frac{1-\xi}{\xi})$ derived from equation (1.48)in the above equation yields the natural rate of output:

$$y_{t,n} = \Omega + \Gamma a_t + \alpha\Psi y_t^*, \tag{1.79}$$

where $\Omega = \frac{\nu - \mu}{\sigma_\alpha + \varphi}$,$\Gamma = \frac{1+\varphi}{\sigma_\alpha + \varphi}$ and $\Psi = -\frac{\sigma_\alpha \Theta}{\sigma_\alpha + \varphi}$. Also, the subtraction of the steady real marginal cost from the equation (1.78)generates the log deviation of real marginal cost:

$$\tilde{mc_t} = (\sigma_\alpha + \varphi)(x_t), \tag{1.80}$$

where $x_t = y_t - y_{t,n}$ is defined as output gap. Noticing that $x_t = \tilde{x_t}$ because y_t and $y_{t,n}$ share the same steady state of log real output, thus $x_t = (y_t - y) - (y_{t,n} - y) = \tilde{y_t} - \tilde{y_{t,n}} = \tilde{x_t}$. Rewriting the domestic Philips curve (1.58) regarding to the output gap instead of real marginal cost:

$$\pi_{H,t} = \beta E_t \pi_{H,t+1} + \kappa_\alpha x_t = \beta E_t \pi_{H,t+1} + \kappa_\alpha \tilde{x_t}, \tag{1.81}$$

where $\kappa_\alpha = \kappa(\sigma_\alpha + \varphi)$. The natural rate of interest is the real interest rate equating the output and natural output. Thus, adding $\tilde{y_{t+1,n}} - \tilde{y_{t,n}}$ to the both sides of the IS curve (1.73) generates the equation below:

$$\tilde{x_t} = E_t \tilde{x_{t+1}} - \frac{1}{\sigma_\alpha}(\tilde{r_t} - E_t \pi_{H,t+1} - \tilde{r_{t,n}}), \tag{1.82}$$

where $\tilde{x_t} = \tilde{y_t} - \tilde{y_{t,n}}$ is the deviation of output gap from its steady state,$\tilde{r_t} = r_t - \rho$ is the deviation of nominal interest rate from its zero inflation steady state ρ and $\tilde{r_{t,n}} = r_{t,n} - \rho$ is the deviation of the natural rate of interest rate from the same zero inflation steady interest rate ρ. The natural rate of interest rate is defined as:

$$r_{t,n} = \rho + \sigma_\alpha \Gamma E_t \triangle a_{t+1} + \alpha \sigma_\alpha (\Theta + \Psi) E_t \triangle y^*_{t+1}. \tag{1.83}$$

1.2.3 From Gali to Lubik

Lubik and Schorfheide (2007[60]) simplify Gali and Monacelli (2005[37])'s model in several ways. In addition to the initial assumptions including purchasing power parity, the law of one price, uncovered interest parity and $\eta = \gamma = 1$, Lubik and Schorfheide (2007[60]) detrend the real economic variables by the non-stationary technology process A_t. They set the marginal substitution between labour and leisure φ to zero. The risk aversion σ is no longer assumed to be one in the simplified model. Moreover, the definition of terms of trade q_1 in the simplified model is the reciprocal of that in Gali and Monacelli (2005[37])'s model. Thus the signs of the terms of trade in the simplified log-linear DSGE model are all opposite to those in the previous section.

IS curve

Lubik and Schorfheide (2007[60]) detrend the aggregate real output domestically and abroad in Gali's model with the a non-stationary technology process $A_t = A_{t-1} + z_t$ following $YY_t = \frac{Y_t}{A_t} = N_t$,so the natural rate of the detrended output in the equation (79) is writing in another way with $y_t = yy_t + a_t$ and $y_t^* = yy_t^* + a_t$:

$$\tilde{yy}_{t,n} = \alpha\Psi\tilde{yy}_t^* = -\alpha\Theta\tilde{yy}_t^*. \tag{1.84}$$

where $\Psi = -\frac{\sigma_\alpha\Theta}{\sigma_\alpha+\varphi} = -\Theta$ when Lubik assumes $\varphi = 0$. The natural rate of interest rate is still same but it replace a_t with z_t which is defined as the rate of change in the technological process:

$$E_t z_{t+1} = E_t a_{t+1} - a_t, \tag{1.85}$$

thus the natural rate of interest (1.83) is rewritten as:

$$r_{t,n} = \rho + \sigma_\alpha\Gamma E_t z_{t+1} + \alpha\sigma_\alpha(\Theta + \Psi)E_t\triangle yy_{t+1}^* = \rho + E_t z_{t+1}, \tag{1.86}$$

where $\Gamma = \frac{1+\varphi}{\sigma_\alpha+\varphi} = \frac{1}{\sigma_\alpha}$.Substituting the new form of natural rate of output and interest rate into the canonical IS equation (1.82) yields:

$$\tilde{yy}_t = E_t\tilde{yy}_{t+1} - \frac{1}{\sigma_\alpha}(\tilde{r}_t - E_t\pi_{t+1} - E_t z_{t+1}) - \frac{\alpha}{\sigma_\alpha}E_t\triangle\tilde{s}_{t+1} + \alpha\Theta E_t\triangle\tilde{yy}_{t+1}^*, \tag{1.87}$$

where $\sigma_\alpha = \frac{\sigma}{1-\alpha+\alpha\omega}$ and $\Theta = \omega - 1 = \sigma\gamma + (1-\alpha)(\sigma\eta - 1) - 1$. It implies that $\tilde{yy_t} = (y_t - a_t) - (y - a_t) = y_t - y = \tilde{y}_t$ and $\tilde{yy}_t^* = \tilde{y}_t^*$. It defines $\tau = \frac{1}{\sigma}$ as the inverse of risk aversion and substitute τ and the assumptions $\eta = \gamma = 1$ into the parameters,which yields

$$\sigma_\alpha = \frac{1}{\tau + \alpha(1-\tau)(2-\alpha)} = \frac{1}{\tau + \lambda} \tag{1.88}$$

and

$$\Theta = \frac{(1-\tau)(2-\alpha)}{\tau} = \frac{\lambda}{\alpha\tau}, \tag{1.89}$$

where $\lambda = \alpha(1-\tau)(2-\alpha)$. The definition of terms of trade in the simplified model is Given by $Q_t^* = \frac{P_{H,t}}{P_{F,t}}$, which is the reciprocal of that in the theoretical model.$P_{F,t}$ is still the import price while $P_{H,t}$ is regarded as the exported price assuming the law of one price always hold in the goods market. Thus, the log form of the terms of trade is defined as $q_t^* = -s_t$. Substituting the new parameters and the new terms of trade into that IS curve yields:

$$\tilde{yy_t} = E_t\tilde{yy}_{t+1} - (\tau+\lambda)(\tilde{r}_t - E_t\pi_{t+1} - E_t z_{t+1}) + \alpha(\tau+\lambda)E_t\triangle\tilde{q}^*_{t+1} + \frac{\lambda}{\tau}E_t\triangle\tilde{yy}^*_{t+1}, \tag{1.90}$$

where the change of world output is defined as:

$$\triangle\tilde{yy}_t^* = \tilde{yy}_t^* - \tilde{yy}_{t-1}^*. \tag{1.91}$$

Philips curve,Exchange rate and Terms of Trade

Lubik and Schorfheide (2007[60]) rewrites the Philips curve (81) in terms of the CPI inflation rate with the aid of equation (1.26) as follows:

$$\pi_t = \beta E_t \pi_{t+1} + \alpha\beta E_t \triangle \tilde{q}^*_{t+1} - \alpha \triangle \tilde{q}^*_t + \frac{\kappa}{\tau+\lambda}\tilde{x}_t, \tag{1.92}$$

where $\kappa_\alpha = \kappa(\sigma_\alpha + \varphi) = \frac{\kappa}{\tau+\lambda}$. The output gap does not change when the aggregate output is detrendend by the non-stationary technology process due to the fact $x_t = y_t - y_{t,n} = (y_t - a_t) - (y_{t,n} - a_t) = yy_t - yy_{t,n}$ and so does the log deviation of output gap $\tilde{x}_t = x_t$. The simplified model directly borrows the exchange rate equation (1.32) and changes the sign of the terms of trade from negative to positive as follows:

$$\pi_t = \pi^*_t + \triangle \tilde{e}_t + (1-\alpha)\triangle \tilde{q}^*_t. \tag{1.93}$$

The first difference of equation (1.72) can lead to the change rate of the terms of trade endogenously:

$$\triangle \tilde{q}^*_t = \sigma_\alpha(\triangle \widetilde{yy}^*_t - \triangle \widetilde{yy}_t) = \frac{1}{\tau+\lambda}(\triangle \widetilde{yy}^*_t - \triangle \widetilde{yy}_t). \tag{1.94}$$

However, Lubik and Schorfheide (2007[60]) replace the equation above with an exogenous process which will be mentioned later. They suggest that the replacement will yield a higher possibility of data matching in the empirical analysis.

Monetary Policy and Exogenous Shock Process

Lubik and Schorfheide (2007[60]) sets the nominal interest rate in response to movements in CPI inflation, output, the nominal exchange rate depreciation with a smoothing term:

$$\tilde{r_t} = \rho_R \tilde{r_{t-1}} + (1-\rho_R)[\phi_\pi \pi_t + \phi_y \tilde{y y_t} + \phi_{\triangle e} \triangle \tilde{e_t}] + \xi_t^R, \xi_t^R \sim NID(0, \sigma_R^2), \quad (1.95)$$

where ϕ_π,ϕ_y and $\phi_{\triangle e}$ are policy coefficients. ρ_R is the smoothing term and ξ_R is an exogenous policy shock.σ_R is the standard deviation of the monetary policy shock. Lubic and Schorfheide also introduce four stationary processes for the exogenous variables z_t,$\triangle q_t$,y_t^* and π_t^* in the model:

$$z_t = \rho_z z_{t-1} + \xi_t^z, \xi_t^z \sim NID(0, \sigma_z^2), \quad (1.96)$$

$$\triangle \tilde{q_t^*} = \rho_q \triangle \tilde{q_{t-1}^*} + \xi_t^q, \xi_t^q \sim NID(0, \sigma_q^2), \quad (1.97)$$

$$\tilde{y y_t^*} = \rho_{y^*} \tilde{y y_{t-1}^*} + \xi_t^{y_t^*}, \xi_t^{y_t^*} \sim NID(0, \sigma_{y^*}^2), \quad (1.98)$$

$$\pi_t^* = \rho_{\pi^*} \pi_{t-1}^* + \xi_t^{\pi_t^*}, \xi_t^{\pi_t^*} \sim NID(0, \sigma_{\pi^*}^2), \quad (1.99)$$

where ρ_z,ρ_q,ρ_{y^*} and ρ_{π^*} are autoregressive coefficients of the AR(1) processes, respectively. ξ_t^z, ξ_t^q,$\xi_t^{y_t^*}$ and $\xi_t^{\pi_t^*}$ are innovations of the four AR(1) processes. σ_z, σ_q, σ_{y^*} and σ_{π^*} are the standard deviation of the corresponding shocks, respectively. z_t is the change rate of the technology process. $\triangle \tilde{q_t^*}$ is the change rate of the terms of trade following an exogenous process instead of the endogenous process (1.93). $\tilde{y y_t^*}$ is the log difference of the detrended world output from its steady state. π_t^* is the world inflation.

1.3 Solutions to the Model

I can solve the model comprising of the equations above using the method developed by Blanchard and Kahn (1980[8]),Klein (2000[51]) or Sims (2002[80]). The Lubik and Schorfheide's model comprise of 10 equations including the IS curve (1.90),natural rate of output (1.84),change of world output(1.91), Philips curve (1.92),law of one price (1.93),monetary policy reaction function (1.95) and 4 AR one processes from (96) to (99). The vector of endogenous variables of the system is $\mathbf{x}_t = [\tilde{yy}_{t,n}, \triangle\tilde{e}_t, \tilde{r}_t, z_t, \tilde{yy}^*_t, \pi^*_t, \triangle\tilde{q}_t, \tilde{yy}_t, \pi_t, \triangle\tilde{yy}^*_t]'$. Among the vector, $\tilde{yy}_{t,n}$ and $\triangle\tilde{e}_t$ are static variables which are just appear at period of time t. $\tilde{r}_t$,z_t,$\tilde{yy}^*_t$ and π^*_t are backward looking variables which are appear at period of time $t-1$ and t. $\triangle\tilde{q}_t$ is the mixed variable which appears at period of time $t-1$, t and $t+1$. $\tilde{yy}_t$,π_t and $\triangle\tilde{yy}^*_t$ are forward looking variables which appear at time t and $t+1$. When handling the solution of a dynamic system, it is often replace the static variables with other endogenous variables so the new vector is $\mathbf{X}_t = [\tilde{r}_t, z_t, \tilde{yy}^*_t, \pi^*_t, \triangle\tilde{q}_t, \tilde{yy}_t, \pi_t, \triangle\tilde{yy}^*_t]'$. The original system of equations becomes

$$\mathbf{B}[\mathbf{X_{1,t+1}}, \mathbf{X_{2,t+1}}]' = \mathbf{A}[\mathbf{X_{1,t}}, \mathbf{X_{2,t}}]' + \mathbf{G}\xi_{\mathbf{t+1}}, \tag{1.100}$$

where $\mathbf{X_{1,t}}$ is the vector of backward looking variables $[\tilde{r}_t, z_t, \tilde{yy}^*_t, \pi^*_t]'$ and $\mathbf{X_{2,t}}$ is the vector of mixed and forward looking variables$[\triangle\tilde{q}_t, \tilde{yy}_t, \pi_t, \triangle\tilde{yy}^*_t]'$. In fact, the mixed variable $\triangle\tilde{q}_t$ should both enters into the vector of backward looking and forward looking vectors. However, it is convenient to illustrate the existence and uniqueness of the solution under the current specification. $\xi_{\mathbf{t}}$ is the vector of exogenous shocks $[\xi_t^R, \xi_t^z, \xi_t^q, \xi_t^{y_t^*}, \xi_t^{\pi_t^*}]'$. **B** is a parameter matrix with size $8 * 8$:

$$\mathbf{B} = \begin{pmatrix} 1 & 0 & 0 & (1-\rho_R)\phi_{\triangle e} & (1-\rho_R)()1-\alpha)\phi_{\triangle e} & -(1-\rho_R)\phi_y & -(1-\rho_R)(\phi_\pi + \phi_{\triangle e}) & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & \tau+\lambda & \frac{\lambda}{\tau} \\ 0 & 0 & 0 & 0 & \alpha\beta & 0 & \beta & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 \end{pmatrix}, \tag{1.101}$$

and **A** is a matrix with size $8 * 8$:

$$\mathbf{A} = \begin{pmatrix} \rho_R & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \rho_z & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \rho_{y^*} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \rho_{\pi^*} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \rho_q & 0 & 0 & 0 \\ \tau+\lambda & -\rho_z(\tau+\lambda) & 0 & 0 & -\alpha(\tau+\lambda)\rho_q & 1 & 0 & 0 \\ 0 & 0 & -\frac{\kappa\alpha\Theta}{\tau+\lambda} & 0 & \alpha & -\frac{\kappa}{\tau+\lambda} & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}, \tag{1.102}$$

and **G** is a matrix with size $8 * 5$:

$$\mathbf{G} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}. \tag{1.103}$$

If the matrix **B** invertible, it is appropriate for the Blanchard and Kahn's methodology to solve the dynamic system. If the matrix is not invertible, it is appropriate for the methodology of matrix decomposition to solve the system. The goal of the solution is to find the transition function:

$$\mathbf{X_{1,t}} = \mathbf{P}\mathbf{X_{1,t-1}} + \mathbf{Q}\xi_{\mathbf{t}}, \tag{1.104}$$

and the policy function:

$$\mathbf{X_{2,t}} = \mathbf{R}\mathbf{X_{1,t-1}} + \mathbf{S}\xi_{\mathbf{t}}. \tag{1.105}$$

After that, it can obtain the static variables $y\tilde{y}_{t,n}$ and $\triangle\tilde{e}_t$ through the equations (1.84) and (1.93) respectively.

1.3.1 Blanchard and Kahn's Methodology

If the inverse of the matrix $\mathbf{B}$ exists,the dynamic system (1.100) becomes

$$\begin{pmatrix} X_{1,t+1} \\ X_{2,t+1} \end{pmatrix} = \mathbf{B^{-1}A} \begin{pmatrix} X_{1,t} \\ X_{2,t} \end{pmatrix} + \mathbf{B^{-1}G}\xi_{\mathbf{t+1}}. \tag{1.106}$$

Rewrite $B^{-1}A = \Gamma\Lambda\Gamma^{-1}$.$\Gamma$ is the eigenvector matrix of the matrix $B^{-1}A$ and Λ is the eigenvalue matrix. In addition, the eigenvalues follows an increasing order and the accordingly eigenvectors are also rearranged at the same time. The above system is rewritten as:

$$\mathbf{Z_{t+1}} = \mathbf{\Lambda Z_t} + \mathbf{\Gamma^{-1}B^{-1}G}\xi_{\mathbf{t+1}}, \tag{1.107}$$

where $\mathbf{Z_t} = \mathbf{\Gamma^{-1}X_t}$. Dividing the eigenvalue matrix in 2 groups based on whether the absolute value of eigenvalue is smaller or bigger than 1.

$$\begin{pmatrix} Z_{1,t+1} \\ Z_{2,t+1} \end{pmatrix} = \begin{pmatrix} \Lambda_1 & 0 \\ 0 & \Lambda_2 \end{pmatrix} \begin{pmatrix} Z_{1,t} \\ Z_{2,t} \end{pmatrix} + \mathbf{\Gamma^{-1}B^{-1}G}\xi_{\mathbf{t+1}}, \tag{1.108}$$

where Λ_1 is a submatrix with size $Q * Q$ and the absolute values of eigenvalues in it are smaller than 1, while Λ_2 is a submatrix with size $O * O$ and the absolute values of eigenvalues in it are equal or bigger than 1. If the system is stationary, $Z_{2,t} = 0$ otherwise it explode after infinity time due to the matrix Λ_2. $Z_{2,t}$ is also a matrix with the size $O * O$. To obtain the X back, rewriting the definition of Z_t:

$$\begin{pmatrix} Z_{1,t} \\ Z_{2,t} \end{pmatrix} = \Gamma^{-1}X_t = \begin{pmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{pmatrix} \begin{pmatrix} X_{1,t} \\ X_{2,t} \end{pmatrix}, \tag{1.109}$$

where the sizes of the sub-matrices in the Γ^{-1} is $[(Q * m), (Q * n); (O * m), (O * n)]$ and m and n are the number of variables in the vector of $X_{1,t}$ and $X_{2,t}$ respectively. According to the stationary condition $Z_{2,t} = 0$,part of the above system becomes:

$$Z_{2,t} = G_{21}X_{1,t} + G_{22}X_{2,t} = 0. \tag{1.110}$$

The above equation determines the policy functions of the dynamic system, capturing the relationship between the jump variables and the the predetermined variables. The Blanchard and Kahn methodology clarifies three conditions of the solutions to the dynamic system. First, if the number of explosive eigenvalues O is larger than the number of the jump variables n, the system has no solutions. Second, if $O < n$,the system has free variables and thus have many infinitely solutions. Third, if $O = n$, there is one and only one solution for this dynamic system. Overall, if the third condition holds, the unique solution derived from the above equation is:

$$X_{2,t} = -G_{22}^{-1} G_{21} X_{1,t}. \tag{1.111}$$

Substitute the unique solution in the rest part of the definition of Z_t (1.109),

$$Z_{1,t} = G_{11} X_{1,t} + G_{12} X_{2,t}, \tag{1.112}$$

And then substitute it back to the dynamic system (1.108):

$$G_{11} X_{1,t+1} + G_{12} X_{2,t+1} = \Lambda_1 (G_{11} X_{1,t} + G_{12} X_{2,t}) + E \xi_{t+1}, \tag{1.113}$$

where E is submatrix of $\Gamma^{-1} B^{-1} G$ with the size $m * 6$. Substitute the unique solution to the above equation and then simplify it to obtain the transition function:

$$X_{1,t} = R X_{1,t-1} + S \xi_t, \tag{1.114}$$

where $R = \left(-G_{11} G_{22}^{-1} G_{21} + G_{12}\right)^{-1} \Lambda_1 (-G_{11} G_{22}^{-1} G_{21} + G_{12})$ and

$$S = \left(-G_{11} G_{22}^{-1} G_{21} + G_{12}\right)^{-1} E.$$

Substitute the transition function to the unique solution to finish deriving the policy function:

$$X_{2,t} = P X_{1,t-1} + Q \xi_t, \tag{1.115}$$

where $P = -G_{22}^{-1} G_{21} R$ and $Q = -G_{22}^{-1} G_{21} S$.

1.3.2 Schur Decomposition

If the inverse of the matrix **B** does not exist,the dynamic system (1.100) becomes

$$QTZ^T \begin{pmatrix} X_{1,t+1} \\ X_{2,t+1} \end{pmatrix} = QSZ^T \begin{pmatrix} X_{1,t} \\ X_{2,t} \end{pmatrix} + G\xi_{t+1}, \tag{1.116}$$

where $B = QTZ^T$ and $A = QSZ^T$. The matrix decomposition is called general Schur decomposition or QZ decomposition. Q and Z are orthogonal unitary matrices and T and S are upper triangular matrices.Following a similar strategy in Blanchard's methodology, rearrange the elements of the upper triangular matrices along the diagonal line based on the increasing order of $\frac{S_{ii}}{T_{jj}}$. Actually, the elements across the diagonal line are also the eigenvalues of the upper triangular matrices. After the simplification, the above system becomes

$$\begin{aligned} &\begin{pmatrix} T_{11} & T_{12} \\ 0 & T_{22} \end{pmatrix} \begin{pmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{pmatrix} \begin{pmatrix} X_{1,t+1} \\ X_{2,t+1} \end{pmatrix} \\ = &\begin{pmatrix} S_{11} & S_{12} \\ 0 & S_{22} \end{pmatrix} \begin{pmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{pmatrix} \begin{pmatrix} X_{1,t} \\ X_{2,t} \end{pmatrix} + Q^T G\xi_{t+1}. \end{aligned} \tag{1.117}$$

To extract the lower part of the dynamic system yields

$$T_{22}(Z_{21}X_{1,t+1} + Z_{22}X_{2,t+1}) = S_{22}(Z_{21}X_{1,t} + Z_{22}X_{2,t}). \tag{1.118}$$

The absolute values of eigenvalues in the sub matrix $T_{22}^{-1}S_{22}$ are not smaller than one. To keep the above equation from generating an explosive path,the condition below is necessary:

$$Z_{21}X_{1,t} + Z_{22}X_{2,t} = 0. \tag{1.119}$$

For this condition to hold, the forward looking variables must be equal to

$$X_{2,t} = -Z_{22}^{-1}Z_{21}X_{1,t}. \tag{1.120}$$

Substitute the above equation in the upper section of the dynamic system yields the transition function:

$$X_{1,t} = RX_{1,t-1} + S\xi_t, \tag{1.121}$$

where $R = (-T_{11}Z_{11}Z_{22}^{-1}Z_{21} + T_{12}Z_{12})^{-1}(-S_{11}Z_{11}Z_{22}^{-1}Z_{21} + S_{12}Z_{12})$. E is the submatrix of Q^TG with size $m{*}6$, so $S = (-T_{11}Z_{11}Z_{22}^{-1}Z_{21} + T_{12}Z_{12})^{-1}E$. Substituting the transition function to the stable condition (1.120) yields the policy function:

$$X_{2,t} = PX_{1,t-1} + Q\xi_t, \tag{1.122}$$

where $P = -Z_{22}^{-1}Z_{21}R$ and $Q = -Z_{22}^{-1}Z_{21}S$.

1.3.3 Numerical Solutions

This section adopts the above methodology to compute the policy and transition functions numerically and then report the impulse response functions of four endogenous variables including $y\tilde{y}_t$,π_t,$\tilde{r_t}$ and $\triangle\tilde{e_t}$. It borrows the parameters values from the table below:

Table 1.1: Calibration

Parameter Names	Parameters Values
τ	0.5
κ	0.3
ϕ_π	1.5
ϕ_y	0.125
$\phi_{\triangle e}$	0.125
ρ_R	0.5
ρ_z	0.2
ρ_q	0.4
ρ_{π^*}	0.8
ρ_{y^*}	0.9
α	0.2
$r^{(A)}$	3.35
π^A	1.92
γ^Q	0.62
σ_R	0.5
σ_z	1.5
σ_{y^*}	1.5
σ_{π^*}	0.55
σ_q	1.5

Note: The parameter values are borrowed from the prior means of parameters within the UK sample in the next chapter.

The parameters are independent of each other. Following Lubik and Schorfheide's specification, a parameter called annual real natural interest percentage rate r^A take the place of the parameter β based on the relationships $\rho = \frac{1}{\beta} - 1$ and $r^A = 100 * 4 * \rho$. The former relationship is present in the previous section. The second relationship generates from two steps. The first step is to transform the quarterly real natural interest rate ρ to the annual real natural interest rate by $4*\rho$. The second step is to report the percentage rate directly without the symbol of percentage,e.g., from 3.35% to 3.35. Thus, the relationship between the parameter r^A and β is $\beta = e^{-\frac{r^A}{400}}$. Table 1.2 reports that the numerical solution to the DSGE model. According to the table, the transition function is calculated as:

$$\begin{pmatrix} \tilde{r}_t \\ z_t \\ \tilde{yy}^*_t \\ \pi^*_t \\ \triangle\tilde{q}_t \end{pmatrix} = \begin{pmatrix} 0.30 & 0.01 & 0.06 & -0.70 & -0.33 \\ 0 & 0.2 & 0 & 0 & 0 \\ 0 & 0 & 0.9 & 0 & 0 \\ 0 & 0 & 0 & 0.8 & 0 \\ 0 & 0 & 0 & 0 & 0.4 \end{pmatrix} \begin{pmatrix} \tilde{r_{t-1}} \\ z_{t-1} \\ \tilde{yy}^*_{t-1} \\ \pi^*_{t-1} \\ \triangle\tilde{q_{t-1}} \end{pmatrix}$$
$$+ \begin{pmatrix} 0.60 & 0.05 & -0.06 & 0.03 & -0.04 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} \xi^R_t \\ \xi^z_t \\ \xi^q_t \\ \xi^{y^*_t}_t \\ \xi^{\pi^*_t}_t \end{pmatrix}, \quad (1.123)$$

and the policy function is calculated as:

$$\begin{pmatrix} \tilde{yy}_t \\ \pi_t \\ \triangle \tilde{yy}_t^* \end{pmatrix} = \begin{pmatrix} -0.35 & 0.02 & -0.30 & 0.06 & 0.08 \\ -0.22 & 0.01 & 0.06 & 0.10 & -0.01 \\ 0 & 0 & -0.10 & 0 & 0 \end{pmatrix} \begin{pmatrix} \tilde{r_{t-1}} \\ z_{t-1} \\ \tilde{yy_{t-1}^*} \\ \pi_{t-1}^* \\ \triangle \tilde{q_{t-1}} \end{pmatrix}$$

$$+ \begin{pmatrix} -0.71 & 0.11 & 0.20 & -0.33 & 0.08 \\ -0.44 & 0.05 & -0.03 & 0.06 & 0.12 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} \xi_t^R \\ \xi_t^z \\ \xi_t^q \\ \xi_t^{y_t^*} \\ \xi_t^{\pi_t^*} \end{pmatrix}, \quad (1.124)$$

and the function for the static variables is

$$\begin{pmatrix} \tilde{yy}_{t,n} \\ \triangle \tilde{e}_t \end{pmatrix} = \begin{pmatrix} 0 & 0 & -0.32 & 0 & 0 \\ -0.22 & 0.01 & 0.06 & -0.70 & -0.33 \end{pmatrix} \begin{pmatrix} \tilde{r_{t-1}} \\ z_{t-1} \\ \tilde{yy_{t-1}^*} \\ \pi_{t-1}^* \\ \triangle \tilde{q_{t-1}} \end{pmatrix}$$

$$+ \begin{pmatrix} 0 & 0 & 0 & -0.36 & 0 \\ -0.44 & 0.05 & -0.83 & 0.06 & -0.88 \end{pmatrix} \begin{pmatrix} \xi_t^R \\ \xi_t^z \\ \xi_t^q \\ \xi_t^{y_t^*} \\ \xi_t^{\pi_t^*} \end{pmatrix}. \quad (1.125)$$

Table 1.2 provides numerical information to compute the impulse response func-

tions for the four endogenous variables including $\tilde{yy_t}$,π_t,$\tilde{r_t}$ and $\triangle\tilde{e_t}$:

$$\begin{pmatrix} \tilde{yy_t} \\ \pi_t \\ \tilde{r_t} \\ \triangle\tilde{e_t} \end{pmatrix} = \begin{pmatrix} -0.35 & 0.02 & -0.30 & 0.06 & 0.08 \\ -0.22 & 0.01 & 0.06 & 0.10 & -0.01 \\ 0.30 & 0.01 & 0.06 & -0.70 & -0.33 \\ -0.22 & 0.01 & 0.06 & -0.70 & -0.33 \end{pmatrix} \begin{pmatrix} r_{\tilde{t-1}} \\ z_{t-1} \\ yy^{\tilde{*}}_{t-1} \\ \pi^*_{t-1} \\ \triangle q_{\tilde{t-1}} \end{pmatrix}$$

$$+ \begin{pmatrix} -0.71 & 0.11 & 0.20 & -0.33 & 0.08 \\ -0.44 & 0.05 & -0.03 & 0.06 & 0.12 \\ 0.60 & 0.05 & -0.06 & 0.03 & -0.04 \\ -0.44 & 0.05 & -0.83 & 0.06 & -0.88 \end{pmatrix} \begin{pmatrix} \xi_t^R \\ \xi_t^z \\ \xi_t^q \\ \xi_t^{y_t^*} \\ \xi_t^{\pi_t^*} \end{pmatrix}. \quad (1.126)$$

The coefficients of the second matrix in the above equation determine the impulse response function for a given size of the structural shock. The impulse response function measures the temporal deviations of economic variables concerning the steady states. Figure 1.1 depicts the calculated impulse response function of the four endogenous variables to the structural shock with the size of one unit of standard deviation offered in table 1.1. For instance, holding everything else constant, a unit of standard deviation of monetary policy shock $\sigma_R = 0.5$ will exert a $-0.71 * \sigma_R = -0.355$ impact on the real output, a $-0.44 * \sigma_R = -0.22$ impact on inflation,a $0.60 * \sigma_R = 0.3$ impact on the nominal interest rate, a $-0.44 * \sigma_R = -0.22$ impact on the change rate of nominal exchange rate.

Table 1.2: Numerical Solutions for the Calibrated Parameters

		static		backward looking				mixed	forward looking		
		$\tilde{yy}_{t,n}$	$\triangle\tilde{e}_t$	$\tilde{r}_t$	z_t	$\tilde{yy}^*_t$	π^*_t	$\triangle\tilde{q}_t$	$\tilde{yy}_t$	π_t	$\triangle\tilde{yy}^*_t$
endogenous	$\tilde{r}_{t-1}$		-0.22	0.30					-0.35	- 0.22	
	z_{t-1}		0.01	0.01	0.20				0.02	0.01	
	$\tilde{yy}^*_{t-1}$	-0.32	0.06	0.03		0.90			-0.30	0.06	-0.10
	π^*_{t-1}		-0.70	0.03			0.80		0.06	0.10	
	$\triangle\tilde{q}_{t-1}$		-0.33	-0.03				0.40	0.08	-0.01	
exogenous	ξ^R_t		-0.44	0.60					-0.71	-0.44	
	ξ^z_t		0.05	0.05	1.00				0.11	0.05	
	ξ^q_t		-0.83	-0.06				1.00	0.20	-0.03	
	$\xi^{y^*_t}_t$	-0.36	0.06	0.03		1.00			-0.33	0.06	1.00
	$\xi^{\pi^*_t}_t$		-0.88	0.04			1.00		0.08	0.12	

Note: This table reports the numerical values of the policy and transition functions.

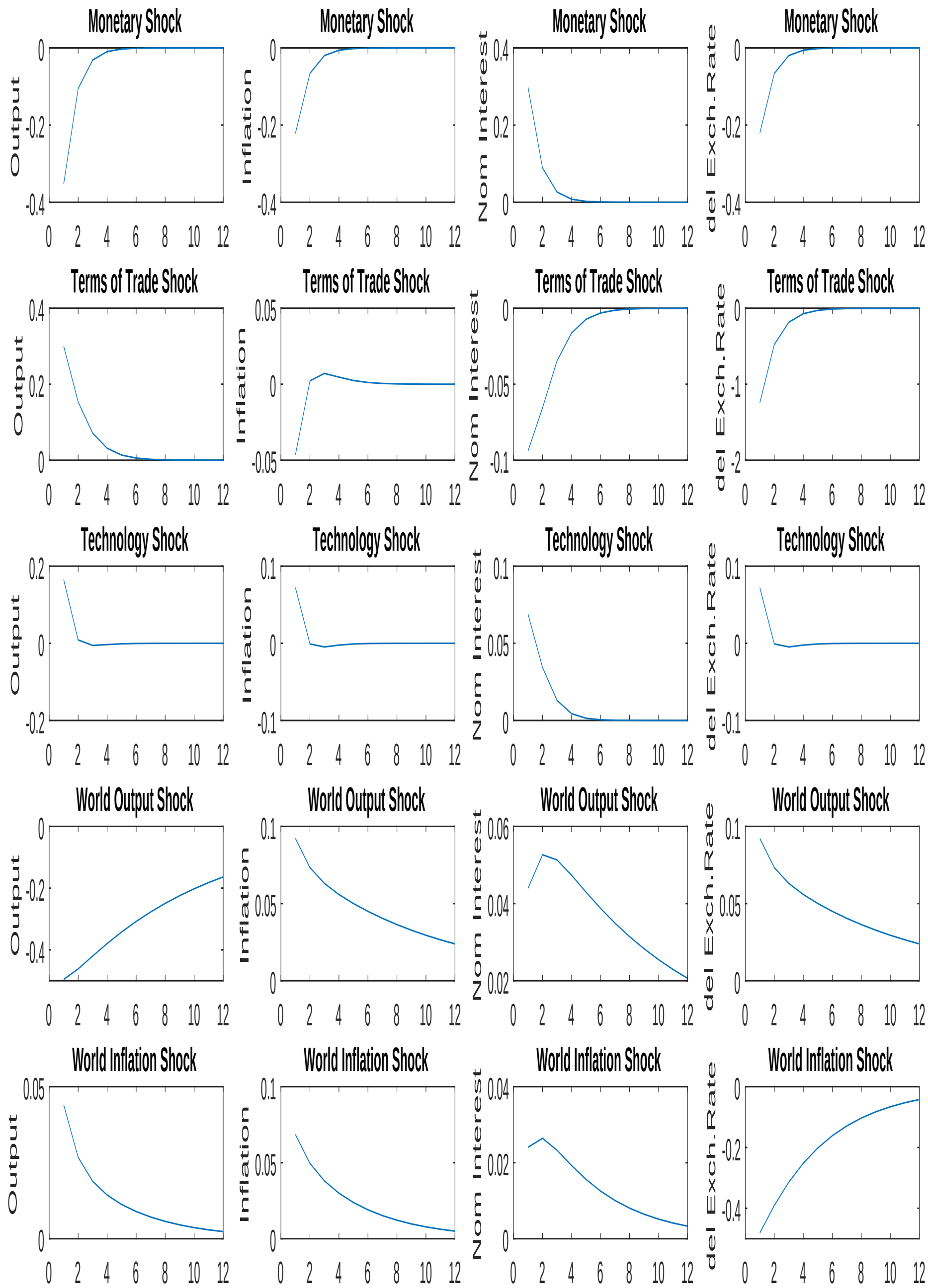

Figure 1.1: Impulse response functions. Note:The figure depicts the impulse response function of real output, inflation rate, nominal interest rate and depreciation exchange rate to one unit structural shock.

1.4 Conclusion

This chapter initially replicates the procedure of Gali and Monacelli(2005[37])'s work to derive a small open DSGE model. In the small open economy, the domestic demand for goods comprises of domestically produced goods and imported goods from many other countries. It assumes that the elasticity of substitution between domestic produced and imported goods is 1. Also, the elasticity of substitution of goods imported from different countries is 1, too. Moreover, when deriving the optimal pricing strategy in the presence of the sticky price, it assumes that the risk aversion is 1. Besides, the uncovered interest rate parity and the law of one price guarantee that there is no arbitrage opportunity between domestic and foreign markets. At the steady state of the economy, the purchasing power parity holds, and the inflation rate is zero. Due to the zero steady inflation rate, the steady nominal interest rate is also the steady real interest rate.

This chapter then tries to uncover the simplification process from Gali and Monacelli's framework to Lubik and Schorfheide(2007[60])'s model. There are mainly four changes in the simplified model. First, it detrends the real output by a non-stationary technology process. Second, the definition of terms of trade is opposite to the theoretical framework. The imported price is the nominator in the theoretical model while it is the denominator in the simplified model. Third, the change rate of terms of trade follows an exogenous stationery process in the simplified model, which will enhance the data fitting of the model. Finally, it ignores the labour supply and relax the assumption that the risk aversion is 1 in deriving the Philips curve. Overall, the log-linearization of the DSGE model comprises of model variables as the percentage deviations with respect to the steady-state values. In the long run, all the economic variables will converge back to their steady state.

This chapter also offers a general introduction to solve the proposed DSGE model. Nowadays, the burden of the computational task is mostly relieved by the advancement of computers. Blanchard and Kahn (1980[8])'s methodology and Schur decomposition are two general methodologies considered in Dynare software, which is one popular tool to solve DSGE models. The core idea of the two methodologies is to guarantee the number of explosive eigenvalues equating to the number of forward-looking variables. The matrix decomposition obtains the number of explosive eigenvalues, and the model assumption determines the number of forward-looking variables. Under this condition, there is one and only one solution to the dynamic system. Typically, the solution comprises a transition function and a policy function. Table 1.2 provides the numerical solutions given a set of calibrated parameters and also includes quantitative information relevant to the impulse response functions depicted in figure 1.1.

Chapter 2

Data Sample and Estimation Methodology

2.1 Introduction

The second chapter estimates the parameters of the small open DSGE model proposed by Lubik and Schorfheide (2007[60]) with data from the UK and Canada covering the period 1992: Q4 -2008: Q4. This chapter aims to introduce a general methodology to estimate the simplified DSGE model, which will yield a benchmark result for the next chapter. There are mainly five sections in this chapter. The first section will introduce the data sample for the UK and Canada and then connect them to the model variables through measurement equations. The second section will illustrate how to run a structural estimation of the DSGE model. The third section will demonstrate, explain and then simulate the empirical results. The fourth section will check for the model specifications with two popular methodologies, including the indirect inference and the DSGE-VAR. The final section concludes.

2.2 Data and Model Variables

This section will introduce the data sample comprising of nominal interest rate, inflation, change of output, change of the nominal exchange rate(foreign currency/home currency) and change of terms of trade for the UK and Canada throughout 1992: Q4 -2008: Q4. All the data are seasonally adjusted and at annually or quarterly frequencies. Apart from the change of terms of trade, it can collect all the time series data from Federal Reserve Economic Data. The change of terms of trade is available from IECONOMICS. The main reason for choosing that sample period 1992: Q4-2008: Q4 is to exclude the periods of non-inflation targeting and the zero lower bounds. The thesis focuses on the inflation targeting regime and also contains the burst of the most recent financial crisis. More specifically, Britain and Canada both announce inflation targeting regimes since 1992, and more or less they all experience a zero lower bound after 2008. After introducing the data, the section then constructs the measurement equations to connect the data to the model variables.

2.2.1 Observable and Collected Data for the UK

There are five types of observable data. It will operate five simple transformations to arrive at the observable data from the originally collected data based on their definitions and frequencies.

Nominal Interest Rate

The observable nominal interest rate r_t^{obs} is the overnight inter-bank rate. The frequencies of the observable interest rate and the collected data are both annuals. Also, it quotes collected data as the percentage rate without the symbol %. Thus, the observable data directly equates to the collected data in terms of the definition and the frequency.

$$r_t^{obs} = InterbankRate_t. \tag{2.1}$$

The averaged observable nominal interest rate is 5.27 throughout 1992: Q4 -2008: Q4 in the UK. The observable nominal interest rate reaches the highest percentage rate of 7.41 in the first quarter of 1998 after the independence of Bank of England, and the lowest percentage rate of 1.65 in the fourth quarter of 2008 preceding the zero lower bound regime.

Inflation Rate

The observable inflation rate π_t^{obs} is the log difference of the CPI, scaled by 400. The frequency of the observable inflation rate is annual, while the frequency of the collected CPI price is quarter. To equate the observable variable and the collected data, it initially calculates the annualised CPI inflation by multiplying 4 of the log difference of the quarterly CPI price. Moreover, it multiplies 100 to remove the symbol %. Thus, it obtains the observable annual inflation rate from the collected quarterly CPI as follows:

$$\pi_t^{obs} = 400 \times LN(\frac{CPI_{real,t}}{CPI_{real,t-1}}). \tag{2.2}$$

The averaged observable inflation rate is 1.92 over the data sample. The observable inflation rate arrives at the highest percentage of 5.73 in the third quarter of 2008 and falls to 0.49 in the next quarter. The lowest observable inflation rate is -0.50 in the first quarter of 2001.

Change of Real Output

The observable rate change of real output $\triangle y_t^{obs}$ is the log difference of the collected real GDP, multiplied by 100. The positive observable variable indicts an output growth in the next period while the negative one implies an output decline in the next period. The frequencies of the observable change rate of the real output and the collected data are both quarters, so it just needs to multiply 100 to remove the symbol %. It obtains the change of the real output from the collected real GDP as follows:

$$\triangle y_t^{obs} = 100 \times LN(\frac{GDP_{real,t}}{GDP_{real,t-1}}). \tag{2.3}$$

The observable change of the real output is 0.62 on average. The observable output grows at the fastest percentage rate of 1.74 in the third quarter of 1999, while declines with the largest percentage rate of 2.2 in the final quarter of 2008 at the wake of the most recent financial crisis.

Change of Nominal Exchange Rate

The observable rate change of nominal exchange rate $\triangle e_t^{obs}$ is the log difference of the nominal effective exchange rate index, multiples by 100. The positive observable variable suggests the nominal exchange rate appreciate in the next period while the negative one suggests the nominal exchange rate depreciate in the next period. The frequencies of the observable data and the nominal effective exchange rate are both quarters, so it just needs to multiply 100 to remove the symbol % away. It calculates the observable change rate of the nominal exchange rate from the collected data as follows:

$$\triangle e_t^{obs} = 100 \times LN(\frac{E_{nominal,t}}{E_{nominal,t-1}}). \tag{2.4}$$

The averaged observable rate change of the nominal exchange rate is -0.25, which implies the currency in the UK depreciates 0.25 on average across the full sample period. The observable nominal exchange rate appreciates with the highest percentage rate of 6.26 in the fourth quarter of 1996 and depreciates with the largest percentage rate of 12.28 in the final quarter of 1992 in the wake of the currency crisis. In the most financial crisis, the nominal exchange rate keeps on depreciating, and the depreciation rate is 6.66 in the fourth quarter of 2008.

Change of Terms of Trade

The observable rate change of the terms of trade $\triangle q_t^{obs}$ is the log difference of the collected terms of trade, defined as the relative prices of exports in terms of imports multiplied by 100. The positive observable variable signals that the price of the exported goods increase faster than the price of the imported goods, which may reflect increasingly higher demand for the locally produced goods and a trade surplus with a fixed amount of the exports and imports. On the contrary, the negative one signals that the price of the exported goods increases slower than the price of the imported goods, which may reflect gradually lower demand for the locally produced goods and a trade deficit with a fixed amount of exports and imports. The frequencies of the observable data and the collected data are both quarters, so it just needs to multiply 100 to remove the symbol %. It calculated the observable rate change of the terms of trade from the collected data as follows:

$$\triangle q_t^{obs} = 100 \times LN(\frac{TOT_t}{TOT_{t-1}}). \tag{2.5}$$

The observable rate change of the terms of trade is -0.04 on average. That is to say, the export price of goods from the UK is four percentage rate lower on average than the import price across the sample. The export price is higher than the import price with the highest percentage rate of 4.55 in the first quarter of 1993, while lower with the largest percentage rate of 4.38 in the final quarter of 1992 after the speculation attack on the currency. The export price index keeps decreasing compared to the import price index in the most recent financial crisis and finally becomes 4.32 % lower than the import price in the final quarter of 2008.

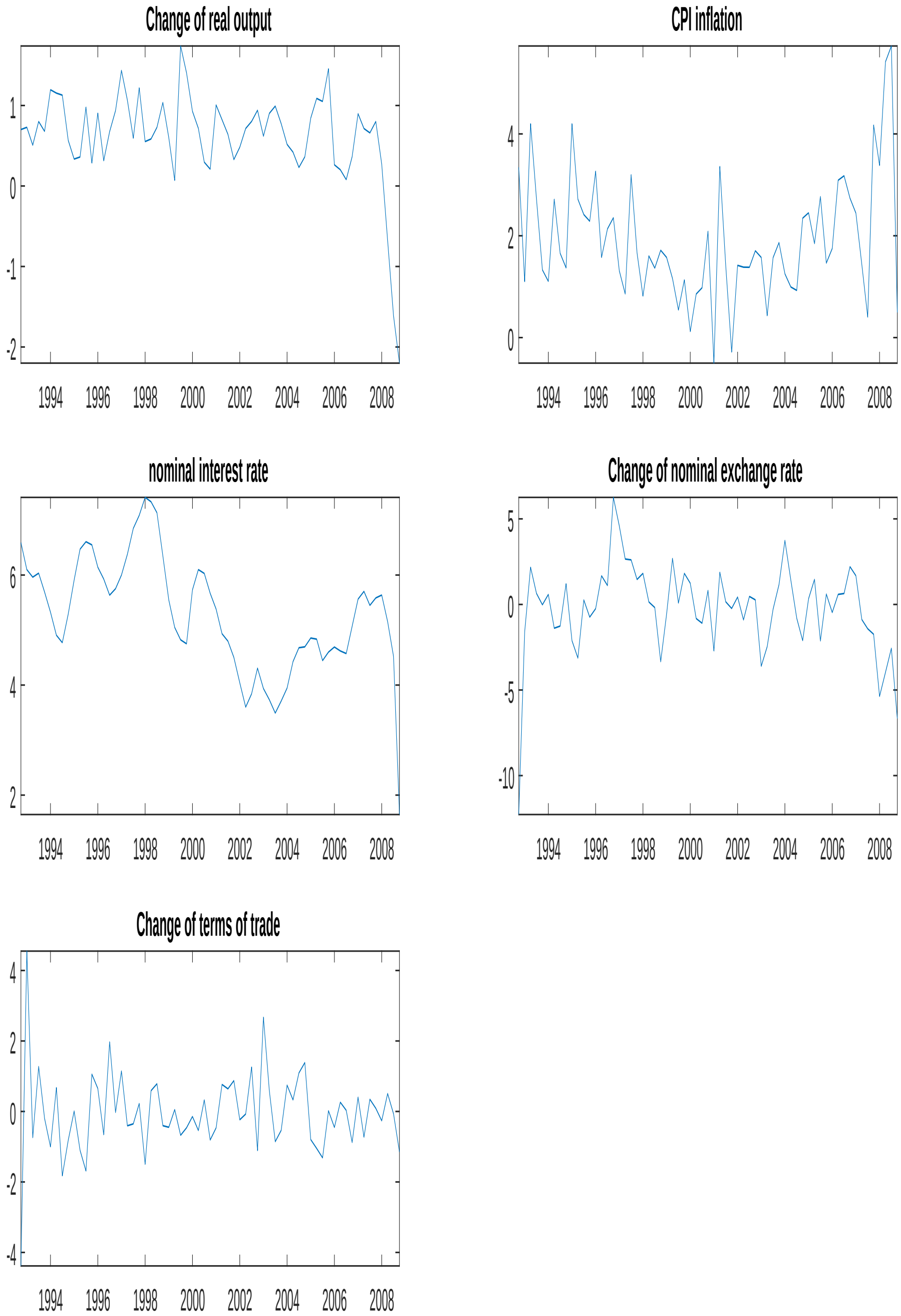

Figure 2.1: Data of UK. Note:The figure depicts the observable variables including the rate change of the quarterly output, the annualised inflation rate, the annualised nominal interest rate, the rate change of the quarterly nominal exchange rate, and the rate change of the quarterly terms of trade covering the period 1992: Q4-2008: Q4 in the UK.

2.2.2 Observable and Collected Data for Canada

It also operates the five same transformations to arrive at the observable data for Canada from the originally collected data based on their definitions and frequencies.

Nominal Interest Rate

The averaged observable nominal interest rate is 4.08 over the data sample in Canada. The highest observable nominal interest rate is 8.03 in the first quarter of 1995, while the lowest is 1.44 in the final quarter of 2008.

Inflation Rate

The averaged observable inflation rate is 1.62. The highest observable inflation rate is 4.54 in the first quarter of 2003 while the lowest is -5.53 in the first quarter of 1994. In the most recent financial crisis, the inflation rate drops from 3.65 in the third quarter of 2008 to -3.45 in the final quarter.

Change of Real Output

The observable rate change of the real output is 0.73 on average. The observable output grows at the highest percentage rate of 1.81 in the first quarter of 1991 and declines with the most significant percentage rate of 1.16 in the final quarter of 2008.

Change of Nominal Exchange Rate

The observable rate change of the nominal exchange rate is -0.01 on average. That is to say, the currency for Canada depreciate only one percentage rate on average across the sample. The observable nominal exchange rate appreciates most with the percentage rate of 6.44 in the second quarter of 2003, while depreciates most with the percentage rate of 12.42 in the final quarter of 2008.

Change of Terms of Trade

The observable rate change of the terms of trade is 0.21 on average. In other words, the average export price is twenty-one percentage higher than the average import price across the sample. The export price is higher than the import price with the most massive percentage rate of 3.76 in the final quarter of 2005 while lower with the most substantial percentage rate of 10.37 in the final quarter of 2008.

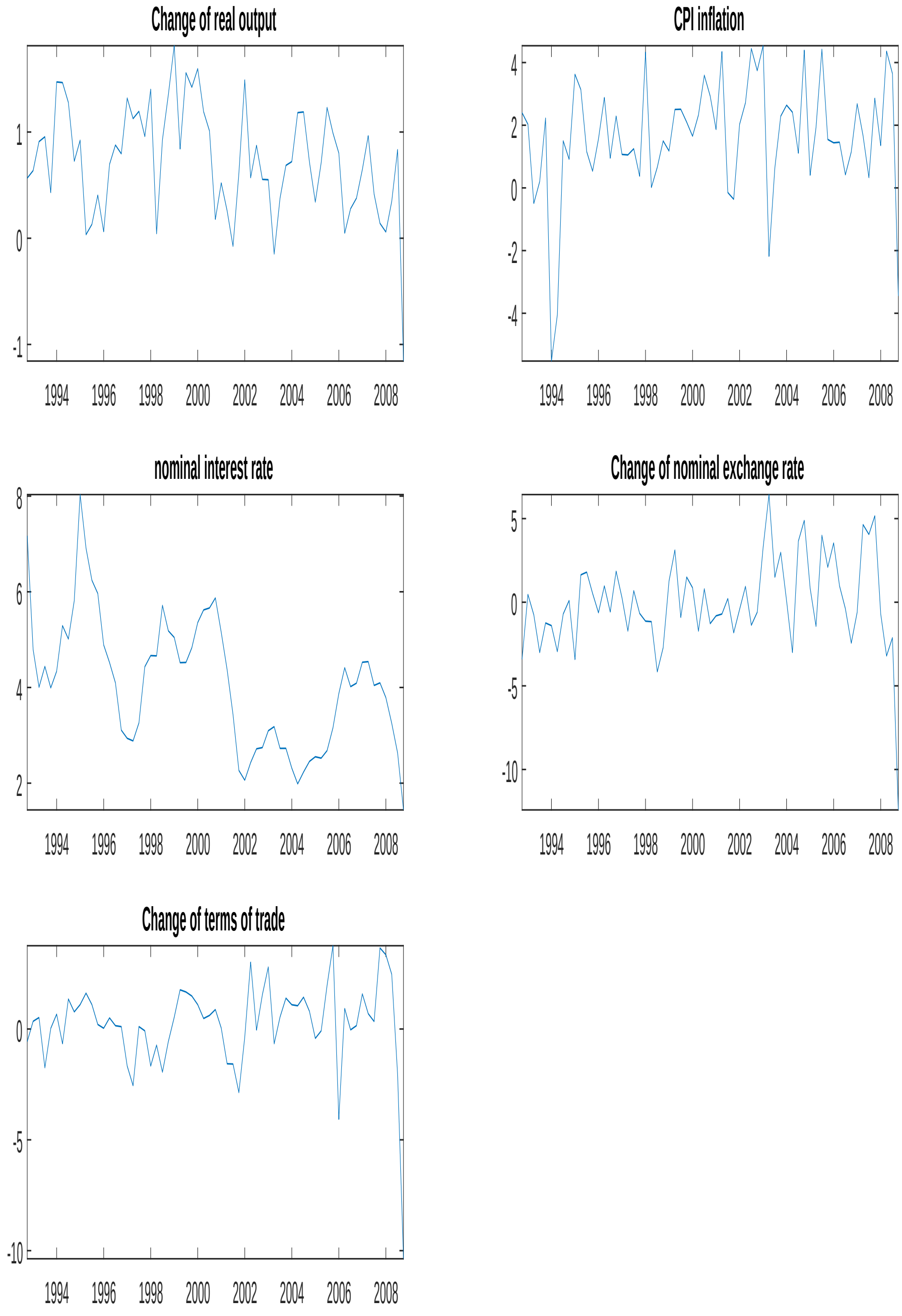

Figure 2.2: Data of Canada. Note:The figure depicts the observable variables including the rate change of the quarterly output, the annualised inflation rate, the annualised nominal interest rate, the rate change of the quarterly nominal exchange rate, and the rate change of the quarterly terms of trade covering the period 1992: Q4-2008: Q4 in Canada.

2.2.3 Measurement Equations

After introducing the five kinds of observable variables for each country, it is then necessary to describe five measurement equations to connect them to the model variables as follows:

$$\begin{bmatrix} r_t^{obs} \\ \pi_t^{obs} \\ \triangle y_t^{obs} \\ \triangle e_t^{obs} \\ \triangle q_t^{obs} \end{bmatrix} = \begin{bmatrix} r^A + \pi^A + 4\tilde{r}_t \\ \pi^A + 4\pi_t \\ \gamma^Q + \triangle y\tilde{y}_t \\ -\triangle \tilde{e}_t \\ \triangle \tilde{q_t^*} \end{bmatrix}, \tag{2.6}$$

where r^A is the steady-state real interest rate, π^A is the observable inflation rate on average and γ^Q is the observable rate change of the real output on average. The measurement equations exhibit a one to one mapping from the observable variables to the model variables. According to the discussion in the previous chapter, the steady-states of the model variables are all zero while the observable variables contain the non-zero means in the data sample. After considering the non-zero means, the observable variables equate the model variables based on the definitions and the adjusted frequencies.

The first measurement equation represents that the observable annual nominal interest rate equates the sum of the steady real interest rate, the average observable inflation rate and the model variable $\tilde{r}_t$ with a multiplication of four. The frequencies of r_t^{obs}, r^A, and π^A are annuals while the frequency of the model variable is quarter, so it multiplies 4 to transfer the frequency of $\tilde{r}_t$ from quarter to annual.

The second measurement equation represents that the observable annual inflation rate equates the averaged inflation rate plus the model variable with a multiplication of four. Likewise, the frequencies of π_t^{obs} and π^A are annuals while the frequency of π_t is quarter, so it multiplies 4 to transfer the frequency of π_t from quarter to annual.

The third measurement equation represents that the observable rate change of the real output equates the averaged rate change of the real output plus the difference of model variable $y\tilde{y}_t$. The frequencies of $\triangle y_t^{obs}$, γ^Q and $y\tilde{y}_t$ are all quarters, so the frequency of each variable in this measurement equation is consistent with each other.

The fourth and fifth measurement equations are similar. On the one hand, the observable values of the rate change of the nominal exchange rate and the terms of trade are minimal on average and thus can be assumed to be zero in the measurement equations. On the other hand, the frequencies of the observable variables and the model variables are all quarters, so there is no need to transform the frequency in the two measurement equations. Consequently, it is natural to connect the observable variables to the model variables directly for the last two measurement equations. However, it should address the unit of the exchange rate in the fourth equation, where denotes the dimension of the observable nominal exchange rate as $\frac{ForeignCurrency}{HomeCurrency}$, while denotes the dimension of the model variable as $\frac{HomeCurrency}{ForeignCurrency}$. Thus, the difference in the dimension of the nominal exchange rate imposes a perfectly negative correlated relationship between the observable variable and the model variable in the log-linear form.

2.3 Bayesian Estimation of DSGE model

In the previous chapter,I have shown that if there is one and only one solution, the solution comprising a transition equation, a policy equation and a static equation to the small open DSGE model can be transformed to a vector auto-regressive representation of the model variables $\mathbf{x}_t$:

$$\mathbf{x}_t = \mathbf{\Phi_1}(\mathbf{\Theta})\mathbf{x_{t-1}} + \mathbf{\Phi}_\xi(\mathbf{\Theta})\xi_\mathbf{t}, \tag{2.7}$$

where $\mathbf{x} = [y\tilde{y}_{t,n}, \triangle\tilde{e}_t, \tilde{r}_t, z_t, \tilde{y y_t^*}, \pi_t^*, \triangle\tilde{q}_t, y\tilde{y}_t, \pi_t, \triangle \tilde{y y_t^*}]'$, $\xi_\mathbf{t} = [\xi_\mathbf{t}^\mathbf{R}, \xi_\mathbf{t}^\mathbf{z}, \xi_\mathbf{t}^\mathbf{q}, \xi_\mathbf{t}^{\mathbf{y_t^*}}, \xi_\mathbf{t}^{\pi_\mathbf{t}^*}]'$, the coefficient matrices $\mathbf{\Phi_1}(\mathbf{\Theta})$ and $\mathbf{\Phi}_\xi(\mathbf{\Theta})$ are functions of the structural parameters $\mathbf{\Theta}$ of the DSGE model. In addition, I also introduce measurement equations (2.6) to connect the observable variables to the model variables. The parameters r^A, π^A and γ^Q are added to the original parameter space. The parameter space Θ now becomes $[\tau, \kappa, \alpha, \phi_\pi, \phi_y, \phi_{\triangle e}$,
$\rho_R, \rho_z, \rho_q, \rho_{\pi^*}, \rho_{y^*}, \sigma_R, \sigma_z, \sigma_{y^*}, \sigma_{\pi^*}, \sigma_q, r^A, \pi^A, \gamma^Q]'$. Moreover, the measurement equation can be rewritten as the following equation:

$$d_t = \Psi_0(\Theta) + \Psi_1(\Theta)t + \Psi_2(\Theta)\mathbf{x_t} + \mathbf{u_t}, \tag{2.8}$$

where d_t is the vector of the observable variables $[r_t^{obs}, \pi_t^{obs}, \triangle y_t^{obs}, \triangle e_t^{obs}, \triangle q_t^{obs}]'$ and u_t is the vector of measurement errors.

$$u_t \sim iidN(0, \Sigma_u) \tag{2.9}$$

Equation (2.7) and (2.8) provide a state-space representation of the DSGE model which offers a joint density for the observable and model variables:

$$p(D_{1:T}, X_{1:T}|\Theta) = \prod_{t=1}^{T} p(d_t, x_t|D_{1:t-1}, X_{1:t-1}, \Theta) = \prod_{t=1}^{T} p(d_t|x_t, \Theta)p(x_t|x_{t-1}, \Theta), \tag{2.10}$$

where $D_{1:T} = d_1, d_2, ..., d_T$ and $X_{1:T} = x_1, x_2, ..., x_T$. In addition, $p(x_t|x_{t-1}, \Theta)$ represents the state transition equation and $p(d_t|x_t, \Theta)$ represents the measurement equation. Here we are at the beginning of the Bayesian inference, which is a method comprising of the likelihood function $p(D_{1:T}|\Theta)$ and the prior distribution of the relevant parameters $p(\Theta)$:

$$p(\Theta|D_{1:t}) = \frac{p(\Theta)p(D_{1:T}|\Theta)}{p(D_{1:T})}, \tag{2.11}$$

where $p(D_{1:t})$ is defined as the marginal likelihood:

$$p(D_{1:t}) = \int p(D_{1:t}|\Theta)p(\Theta)d\Theta. \tag{2.12}$$

2.3.1 Likelihood function

The current goal in this section is to construct the likelihood function $p(D_{1:T}|\Theta)$ from the joint density in the equation above, which implies the model variables $X_{1:T}$ have to be integrated out. The likelihood function is shown in the below equation:

$$p(D_{1:T}|\Theta) = \prod_{t=1}^{T} p(d_t|D_{1:t-1}, \Theta). \tag{2.13}$$

A generic filter is applied to generate the densities $p(d_t|D_{1:t-1}, \Theta)$.There are mainly four steps in one iteration for this filter. First, at time 0 it assumes the initial state $p(x_0|D_{1:0}, \Theta) = p(x_0|\Theta)$. Second, it forecast the model variables in the next period through the transition equation given the initial state:

$$p(x_1|D_{1:0}, \Theta) = \int p(x_1|x_0, D_{1:0}, \Theta)p(x_0|D_{1:0}, \Theta)dx_0. \tag{2.14}$$

Next, it forecast the observable variables in the next period through the measurement equation given the predicted model variables $p(x_1|D_{1:0}, \Theta)$:

$$p(d_1|D_{1:0}, \Theta) = \int p(d_1|x_1, D_{1:0}, \Theta)p(x_1|D_{1:0}, \Theta)dx_1. \tag{2.15}$$

Finally, the model variables in the next period can be updated with Bayesian theorem when the observable variables in the next period are available:

$$p(x_1|D_{1:1}, \Theta) = p(x_1|d_1, D_{1:0}, \Theta) = \frac{p(d_1|x_1, D_{1:0}, \Theta)p(x_1|D_{1:0}, \Theta)}{p(d_1|D_{1:0}, \Theta)}. \tag{2.16}$$

Repeat the procedures above and from the iteration $t-1$ it generates the conditional distribution $p(x_{t-1}|D_{1:t-1}, \Theta)$. The transition equation at time t is shown below:

$$p(x_t|D_{1:t-1}, \Theta) = \int p(x_t|x_{t-1}, D_{1:t-1}, \Theta)p(x_{t-1}|D_{1:t-1}, \Theta)dx_{t-1}. \tag{2.17}$$

The measurement equation is shown below:

$$p(d_t|D_{1:t-1}, \Theta) = \int p(d_t|x_t, D_{1:t-1}, \Theta)p(x_t|D_{1:t-1}, \Theta)dx_t. \tag{2.18}$$

Substitute the above equation in each iteration into the equation 10 will lead to the desired likelihood function. The model variables at time t can also be updated with Bayesian theorem in the following equation when d_t is available:

$$p(x_t|D_{1:t}, \Theta) = p(x_t|d_t, D_{1:t-1}, \Theta) = \frac{p(d_t|x_t, D_{1:t-1}, \Theta)p(x_t|D_{1:t-1}, \Theta)}{p(d_t|D_{1:t-1}, \Theta)}. \tag{2.19}$$

2.3.2 Choice of Priors

The prior distributions of the parameters $p(\Theta)$ are also essential to generate an accurate and reliable estimation results in the framework of Bayes analysis. Table 2.1 provides information about the prior distributions for the UK and Canada separately. The priors setting is broadly consistent with the previous literature such as Rotemberg and Woodford (1998[77]) and Lubik and Schorfheide (2007[60]). In order to guarantee the consistency of the estimation results, it adopts a different number of initial values to prove that different optimisation routines can converge to the same values given the same prior distribution before conducting Bayesian analysis.

Monetary Policy Parameters

The priors for monetary policy parameters are commonly associated with general Taylor rule for both two countries. The prior mean coefficient of inflation rate ϕ_π is 1.5. Following Smet and Wouter (2007[83])'s research, the prior mean coefficients of output ϕ_y is set at 0.125. The prior mean of the exchange coefficient $\phi_{\triangle e}$ is also 0.125. Moreover, the prior for the interest rate smoothing parameter ρ_R follows a beta distribution with mean 0.5 and standard deviation 0.25.

Other Structural Parameters

The priors for the other structural parameters are commonly consistent with Lubik and Schorfheide (2007[60]) except for the slope coefficient of Philips curve κ, which is assumed to be centred around 0.3 instead of 0.5 suggested by Rotemberg and Woodford (1998[77]) and Chen and MacDonald (2011[16]). The intertemporal elasticity of substitution τ centres at 0.5 for both of the two countries, which indicates the relative risk aversion in each country is $\tau^{-1} = 2$. The import share α centries around 0.2 with a standard deviation of 0.2. The prior means of annualised real interest rate $r^{(A)}$, annualised inflation rate $\pi^{(A)}$ and output growth $\gamma^{(A)}$ are set to be roughly consistent with my data sample for the two different countries. Moreover, the annualized real interest rate $r^{(A)}$ is linked to the subjective discount factor $\beta = (1 + \frac{r^{(A)}}{400})^{-1}$. $r^{(A)}$ instead of β will enter into the estimation of the DSGE. The prior mean of real interest rate $r^(A)$ is 3.35% in the UK and 2.47% in Canada over the sample period. The prior of annualised inflation rate $\pi^{(A)}$ centres at 1.92% in the UK and 1.62% in Canada. The prior of the growth rate $\gamma^{(Q)}$ centries around 0.62% in the UK and 0.73% in Canada.

The prior distributions of the parameters in AR(1) processes are majorly consistent with Lubik and Schorfheide (2007[60]). Except for the change rate of the technology process, the prior distributions of the other stationary processes are identical for the UK and Canada. The prior means of the autoregression parameters for the change rate of the terms trade process ρ_q, the world inflation process $\rho_{\pi*}$, and the world output process ρ_{y*} centre at 0.4, 0.8 and 0.9 with corresponding standard deviations 0.2, 0.1 and 0.05, respectively. The priors of the change rate of the technology process ρ_z in the UK and Canada both centre at 0.2, but with higher standard deviations in the UK. Besides, the prior of the innovation shocks to the change rate of the technology process σ_z centres at 1.5 in the UK while only 1 in Canada. The priors of the innovation shocks to the other AR(1) processes σ_R,σ_{y^*}, σ_{π^*} and σ_q are identical in the UK and Canada.

Table 2.1: Prior Distributions for the UK and Canada

Parameters	Domain	Density	UK		Canada	
			Para(1)	Para(2)	Para(1)	Para(2)
τ	[0,1)	Beta	0.5	0.2		
κ	R^+	Gamma	0.3	0.2		
ϕ_π	R^+	Gamma	1.5	0.5		
ϕ_y	R^+	Gamma	0.125	0.05		
$\phi_{\triangle e}$	R^+	Gamma	0.125	0.05		
ρ_R	[0,1)	Beta	0.5	0.25		
ρ_z	[0,1)	Beta	0.2	0.1	0.2	0.05
ρ_q	[0,1)	Beta	0.4	0.2		
ρ_{π^*}	[0,1)	Beta	0.8	0.1		
ρ_{y^*}	[0,1)	Beta	0.9	0.05		
α	[0,1)	Beta	0.2	0.05		
$r^{(A)}$	R^+	Normal	3.35	1	2.47	1
$\pi^{(A)}$	R^+	Normal	1.92	1	1.62	1
$\gamma^{(Q)}$	R^+	Gamma	0.62	0.2	0.73	0.2
σ_R	R^+	Inverse Gamma	0.5	4		
σ_z	R^+	Inverse Gamma	1.5	4	1	4
σ_{y^*}	R^+	Inverse Gamma	1.5	4		
σ_{π^*}	R^+	Inverse Gamma	0.55	4		
σ_q	R^+	Inverse Gamma	1.5	4		

Note: Para(1) and Para(2) are the means and the standard deviations for the relevant distributions.

2.3.3 MCMC Approximation of Bayesian Posteriors

In this section, the goal is to compute the posterior distributions $p(\Theta|D_{1:t})$, which is a difficult task when there are too many dimensions for integration problems. Thus, it introduces a numerical algorithm called Markov Chain Monte Carlo (MCMC) method, including two steps to construct the Bayesian posterior distribution by forming a Markov chain whose invariant distribution approximately equates the posterior distribution. First, it replaces the posterior densities with one proposed density which follows Markov chains. Next, it collects the sample points from the proposed density and adopts the Monte Carlo method to calculate the moments of the parameters.

Ljungqvist and Sargent (2018[58]) offered a clear way to illustrate the way to construct such a proposed probability density following Markov chains. The Markov chain definitely converges to an invariant distribution $\pi(\Theta)$. The invariant distribution equals the posterior one:

$$\pi(\Theta) = p(\Theta|D_{1:t}). \tag{2.20}$$

It defines the Markov chain as a numerical algorithm called Metropolis-Hastings. The target density $p(\Theta|D_{1:t})$ and the proposal density $q(z|\Theta, D_{1:t})$ are two important components in this algorithm. Here z in the proposal density is just a dummy variable for parameter Θ.

First, Draw $\Theta_0 = \Theta_{ML}$, $j = 0$. Normally, a common choice of the initial density is related to the maximum likelihood estimator.
Second, for $j \geq 0$, draw Θ^* from the proposal density $q(\Theta^*|\Theta_j, D_{1:t})$. Θ^* is a candidate for the draw Θ_{j+1}. It is often common to adjust the asymptotic distribution associate with the maximum likelihood estimator $\Theta \sim N(\Theta_{ML}, \Sigma_\Theta)$ to construct the proposal density:

$$q(\Theta^*|\Theta_j, D_{1:t}) = N(\Theta_j, c\Sigma_\Theta), \tag{2.21}$$

where c is a scale parameter to give the acceptance rate between 0.2 and 0.4 at last and Σ_Θ is the inverse of Hessian Matrix V:

$$V = \frac{\partial^2 logp(D_{1:t}|\Theta)}{\partial\Theta\partial\Theta'}|_{\Theta_{ML}}. \quad (2.22)$$

Third, decide whether to accept the candidate by computing the probability of acceptance:

$$r = \frac{p(\Theta^*|D_{1:t})}{p(\Theta_j|D_{1:t})} = \frac{k(\Theta^*|D_{1:t})}{k(\Theta_j|D_{1:t})}, \quad (2.23)$$

where $k(\Theta_j|D_{1:t})$ is the kernel and defined by:

$$logk(\Theta|D_{1:t}) = logL(D_{1:t}|\Theta) + logp(\Theta). \quad (2.24)$$

Consequently, accept $\Theta_{j+1} = \Theta^*$ with the probability min (r,1). Otherwise, $\Theta_{j+1} = \Theta_j$. The Metropolis-Hastings algorithms defines the transition density $\Pi(\Theta, \Theta^*)$ of a Markov Chain mapping Θ_j into Θ_{j+1}:

$$\Pi(\Theta, \Theta^*) = Prob(\Theta_{j+1} = \Theta^*|\Theta_j = \Theta)). \quad (2.25)$$

The invariant distribution of the chain is the posterior:

$$p(\Theta|D_{1:t}) = \int \Pi(\Theta, \Theta^*)p(\Theta|D_{1:t})d\Theta. \quad (2.26)$$

To calculate the moments of the parameter space, Monte Carlo methodology will be applied as follows:

$$E[f(\Theta)] = \frac{1}{N-M}\Sigma_{j=M+1}^{N} f(\Theta_j), \quad (2.27)$$

where N is the total number of draws while M is the number of the initial draws discarded from samples. The above equation can offer a fast and convenient way to compute the posterior mean and variance.

Overall, it follows Sim's optimization routine Csmiwel to maximize the log-likelihood function $logp(\Theta_j|D_{1:t})$ numerically and obtain the posterior mode Θ_{ML}. Secondly, it calculate the inverse Hessian matrix Σ_Θ at the posterior mode to generate the covariance matrix Σ_Θ of the approximate multi-normal distribution $\Theta \sim N(\Theta_{ML}, \Sigma_\Theta)$, which is a benchmark of the proposed density $q(z|\Theta, D_{1:t})$. Thirdly, it applies the Metropolis-Hastings algorithm to generate $N = 200,000$ draws from the posterior distribution and the first $M = 10,000$ draws are burned. Meanwhile, it adjusts the scale of c to have an acceptance rate between 0.2 and 0.4. Finally, it calculate the posterior means of the selected draws by the Monte Carlo method.

2.4 The Analysis of the Estimation Results

Table 2.2 reports the Bayesian estimation results for the UK and Canada together. It will explain the economic intuition of the estimates in each country, and then combine the estimation results with the policy and transition functions computed in the previous chapter to offer the numerical simulation. Although chapter 2 updates the value of the parameters with the real data, the model is still not fully capable of depicting how the real economy runs, because an economic model is just a simplification of the rather complicated realities and cannot be immune to the model misspecification problems. Chapter 2 just sets up the foundations of the bridge linking the simplified model to the real data. The thesis will start struggling to find better connections between them in the next two chapters. Having said this, however, this chapter still provides a benchmark result of the estimation, which has some hints about the different policy specifications and economic structures.

Table 2.2: Parameter Estimation Results for the UK and Canada

	Prior		Posterior			
	UK	Canada	UK		Canada	
Parameters	Mean	Mean	Mean	90%interval	Mean	90%interval
τ	0.500		0.258	[0.124,0.381]	0.283	[0.155,0.410]
κ	0.300		0.461	[0.194,0.726]	0.839	[0.484,1.186]
ϕ_π	1.500		2.589	[1.801,3.351]	2.139	[1.439,2.799]
ϕ_y	0.125		0.121	[0.048,0.191]	0.066	[0.028,0.103]
$\phi_{\triangle e}$	0.125		0.060	[0.025,0.095]	0.128	[0.065,0.187]
ρ_R	0.500		0.813	[0.749,0.878]	0.760	[0.675,0.843]
ρ_z	0.200		0.478	[0.339,0.609]	0.367	[0.263,0.469]
ρ_q	0.400		0.105	[0.010,0.197]	0.544	[0.424,0.661]
ρ_{π^*}	0.800		0.598	[0.441,0.749]	0.449	[0.312,0.580]
ρ_{y^*}	0.900		0.940	[0.899,0.985]	0.955	[0.923,0.988]
α	0.200		0.117	[0.071,0.164]	0.142	[0.086,0.199]
$r^{(A)}$	3.350	2.470	3.127	[2.677,3.565]	2.379	[1.847,2.918]
$\pi^{(A)}$	1.920	1.620	2.263	[1.703,2.838]	1.839	[1.266,2.420]
$\gamma^{(Q)}$	0.620	0.730	0.668	[0.615,0.722]	0.725	[0.670,0.783]
σ_R	0.500		0.194	[0.150,0.239]	0.299	[0.227,0.371]
σ_z	1.500	1.000	1.309	[0.521,2.104]	1.939	[0.871,2.962]
σ_{y^*}	1.500		1.003	[0.422,1.609]	0.809	[0.364,1.231]
σ_{π^*}	0.550		2.510	[2.141,2.860]	2.377	[2.028,2.722]
σ_q	1.500		1.215	[1.027,1.383]	1.921	[1.641,2.192]

Note:The table reports the parameter estimation results for the UK and Canada.

2.4.1 Estimation Results for the UK

Estimates of the Monetary Policy Parameters

ϕ_π is the coefficient of the inflation deviation. The prior mean of this parameter is 1.5, while the posterior mean is 2.589. If the actual inflation is one percentage rate higher than the target inflation, the nominal interest raises by almost 259 base points. ϕ_y is the coefficient of the detrended output deviation. The prior mean of this parameter (0.125) is not very different from the posterior one (0.121). If the real output is one percentage rate higher than its potential value, the nominal interest rate increases by 12.1 basis points. $\phi_{\triangle e}$ is the coefficient of nominal exchange rate depreciation. The prior mean of the parameter (0.125) is much bigger than the posterior one (0.06). If the domestic currency depreciates one percentage rate, the nominal interest rate increases by six basis points. ρ_R is the smoothing term of the nominal interest rate. The posterior mean is 0.813, which is much bigger than the prior one (0.5). Overall, the posterior mean of monetary policy parameters updated from the data sample supports there is a persistent and robust anti-inflationary policy action in the UK. Meanwhile, the movement of the nominal exchange rate only has minimal impact on the policy decision.

Estimates of the Other Structural Parameters

τ is the elasticity of intertemporal substitution and also be the reciprocal of relative risk aversion. The posterior mean of this parameter is 0.258, which is smaller than its prior mean of 0.5. If the real interest rate rise by one percentage rate, the consumption increases by approximately 26 basis points. The elasticity of intertemporal substitution reflects the net impact of the real interest rate on consumption plan. κ, the slope of the Philips curve, is a decreasing function of the price stickiness. The posterior mean of the slope is 0.461. Compared to its prior mean of 0.3, the data support that there is less price stickiness. α is the import share, which reflects the openness of one country. The posterior mean of this parameter is 0.117, which is smaller than the prior mean of 0.2. r^A is the annualised steady-state real interest rate. The posterior mean is 3.127, which is not very different from the prior mean 3.35. $\pi^{(A)}$ is the average of the annualised observable inflation rate. The posterior mean is 2.263, which is bigger than the prior mean of 1.92. $\gamma^{(Q)}$ is the average of the change of quarterly real output. The posterior mean is 0.668, which is not very different from the prior mean of 0.62.

For the AR (1) process of the rate change of terms of trade, ρ_q is the coefficient of the change of terms of trade. The posterior mean of this parameter is 0.105, which is quite smaller than the prior mean 0.4. σ_q is The standard deviation of the shock to this process. The posterior mean of this parameter is 1.215, which is smaller than the prior mean 1.5. For the AR (1) process of the change rate of technology, ρ_z is the coefficient of the change rate of technology. The posterior mean of this parameter is 0.478, which is bigger than the prior mean 0.2. σ_z is the standard deviation of the shock to this process. The posterior mean of this parameter is 1.309, which is smaller than the prior mean 1.5. For the AR (1) process of the world inflation process,ρ_{π^*} is the coefficient of world inflation. The posterior mean of this parameter is 0.598, which is smaller than the prior mean 0.8. σ_{π^*} is the standard deviation of the shock to this process. The posterior mean is 2.510, which is much bigger than its prior mean of 0.55. For the AR (1) process of the world output deviation, ρ_{y^*} is the coefficient of the world output deviation. The posterior mean of the parameter is 0.940, which is more significant than its prior mean of 0.9. σ_{y^*} is the standard deviation of the shock to this process. The posterior mean is 1.003, which is smaller than the prior mean of 1.5. In addition to the volatility in the above four AR(1) processes, the standard deviation of the shock to the monetary policy is σ_R. The posterior mean is 0.194, which is smaller than the prior mean of 0.5.

Numerical Solutions and Simulation Results for the UK

Table 2.3 reports the numerical solution for the UK. According to the table, it computes the transition function as follows:

$$\begin{pmatrix} \tilde{r}_t \\ z_t \\ \tilde{yy}_t^* \\ \pi_t^* \\ \triangle\tilde{q}_t \end{pmatrix} = \begin{pmatrix} 0.382 & 0.055 & 0.020 & 0.001 & -0.003 \\ 0 & 0.478 & 0 & 0 & 0 \\ 0 & 0 & 0.940 & 0 & 0 \\ 0 & 0 & 0 & 0.598 & 0 \\ 0 & 0 & 0 & 0 & 0.105 \end{pmatrix} \begin{pmatrix} \tilde{r}_{t-1} \\ z_{t-1} \\ \tilde{yy}_{t-1}^* \\ \pi_{t-1}^* \\ \triangle\tilde{q}_{t-1} \end{pmatrix}$$
$$+ \begin{pmatrix} 0.470 & 0.115 & -0.028 & 0.021 & 0.002 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} \xi_t^R \\ \xi_t^z \\ \xi_t^q \\ \xi_t^{y_t^*} \\ \xi_t^{\pi_t^*} \end{pmatrix}, \quad (2.28)$$

and the policy function is calculated as:

$$\begin{pmatrix} \tilde{yy}_t \\ \pi_t \\ \triangle\tilde{yy}_t^* \end{pmatrix} = \begin{pmatrix} -0.481 & 0.093 & -0.576 & 0.006 & 0.005 \\ -0.848 & 0.106 & 0.067 & 0.015 & -0.004 \\ 0 & 0 & -0.060 & 0 & 0 \end{pmatrix} \begin{pmatrix} \tilde{r}_{t-1} \\ z_{t-1} \\ \tilde{yy}_{t-1}^* \\ \pi_{t-1}^* \\ \triangle\tilde{q}_{t-1} \end{pmatrix}$$
$$+ \begin{pmatrix} -0.592 & 0.195 & 0.043 & -0.613 & 0.011 \\ -1.043 & 0.222 & -0.038 & 0.071 & 0.026 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} \xi_t^R \\ \xi_t^z \\ \xi_t^q \\ \xi_t^{y_t^*} \\ \xi_t^{\pi_t^*} \end{pmatrix}, \quad (2.29)$$

and the function for the static variables is

$$\begin{pmatrix} \tilde{yy}_{t,n} \\ \triangle\tilde{e}_t \end{pmatrix} = \begin{pmatrix} 0 & 0 & -0.596 & 0 & 0 \\ -0.848 & 0.106 & 0.067 & -0.583 & -0.097 \end{pmatrix} \begin{pmatrix} \tilde{r_{t-1}} \\ z_{t-1} \\ \tilde{yy^*_{t-1}} \\ \pi^*_{t-1} \\ \triangle\tilde{q_{t-1}} \end{pmatrix}$$

$$+ \begin{pmatrix} 0 & 0 & 0 & -0.634 & 0 \\ -1.043 & 0.222 & -0.921 & 0.071 & -0.974 \end{pmatrix} \begin{pmatrix} \xi^R_t \\ \xi^z_t \\ \xi^q_t \\ \xi^{y^*_t}_t \\ \xi^{\pi^*_t}_t \end{pmatrix}. \quad (2.30)$$

Table 2.3 also incorporates the numerical information to compute the impulse response functions for the four endogenous variables including $\tilde{yy}_t$,π_t,$\tilde{r}_t$ and $\triangle\tilde{e}_t$:

$$\begin{pmatrix} \tilde{yy}_t \\ \pi_t \\ \tilde{r}_t \\ \triangle\tilde{e}_t \end{pmatrix} = \begin{pmatrix} -0.481 & 0.093 & -0.576 & 0.006 & 0.005 \\ -0.848 & 0.106 & 0.067 & 0.015 & -0.004 \\ 0.382 & 0.055 & 0.020 & 0.001 & -0.003 \\ -0.848 & 0.106 & 0.067 & -0.583 & -0.097 \end{pmatrix} \begin{pmatrix} \tilde{r_{t-1}} \\ z_{t-1} \\ \tilde{yy^*_{t-1}} \\ \pi^*_{t-1} \\ \triangle\tilde{q_{t-1}} \end{pmatrix}$$

$$+ \begin{pmatrix} -0.592 & 0.195 & 0.043 & -0.613 & 0.011 \\ -1.043 & 0.222 & -0.038 & 0.071 & 0.026 \\ 0.470 & 0.115 & -0.028 & 0.021 & 0.002 \\ -1.043 & 0.222 & -0.921 & 0.071 & -0.974 \end{pmatrix} \begin{pmatrix} \xi^R_t \\ \xi^z_t \\ \xi^q_t \\ \xi^{y^*_t}_t \\ \xi^{\pi^*_t}_t \end{pmatrix}. \quad (2.31)$$

The coefficients of the second matrix in the above equation determine the impulse response function for a given size of the structural shock. The impulse response function measures the temporal deviations of economic variables concerning the steady states. Figure 2.3 depicts the calculated impulse response function of the four endogenous variables to the structural shock with the size of one unit of standard deviation offered in table 2.2.

Holding everything else constant, a unit of standard deviation of monetary policy shock $\sigma_R = 0.194$ will exert a $-0.592 * \sigma_R = -0.115$ impact on the real output deviation, a $-1.043 * \sigma_R = -0.202$ impact on inflation,a $0.47 * \sigma_R = 0.091$ impact on the nominal interest rate deviation, a $-1.043 * \sigma_R = -0.202$ impact on the nominal exchange rate depreciation.

Holding everything else constant, a unit of standard deviation of the shock to the change rate of terms of trade $\sigma_q = 1.215$ will exert a $0.043 * \sigma_q = 0.052$ impact on the real output deviation, a $-0.038 * \sigma_q = -0.046$ impact on inflation, a $-0.028 * \sigma_q = -0.034$ impact on the nominal interest rate deviation, a $-0.921 * \sigma_q = -1.119$ impact on the nominal exchange rate depreciation.

Holding everything else constant, a unit of standard deviation of the shock to the change rate of technology $\sigma_z = 1.309$ will exert a $0.195 * \sigma_z = 0.255$ impact on the real output deviation, a $0.222 * \sigma_z = 0.291$ impact on inflation, a $0.115 * \sigma_z = 0.150$ impact on the nominal interest rate deviation, a $0.222 * \sigma_z = 0.291$ impact on the nominal exchange rate depreciation.

Holding everything else constant, a unit of standard deviation of the shock to the world output deviation $\sigma_{y^*} = 1.003$ will exert a $-0.613 * \sigma_{y^*} = -0.615$ impact on the real output deviation, a $0.071 * \sigma_{y^*} = 0.071$ impact on inflation, a $0.021 * \sigma_{y^*} = 0.021$ impact on the nominal interest rate deviation, a $0.071 * \sigma_{y^*} = 0.071$ impact on the nominal exchange rate depreciation.

Holding everything else constant, a unit of standard deviation of the shock to the world inflation $\sigma_{\pi^*} = 2.510$ will exert a $0.011 * \sigma_{\pi^*} = 0.027$ impact on the real output deviation, a $0.026 * \sigma_{\pi^*} = 0.064$ impact on inflation,a $0.002 * \sigma_{\pi^*} = 0.004$ impact on the nominal interest rate deviation, a $-0.974 * \sigma_{\pi^*} = -2.446$ impact on the nominal exchange rate depreciation.

Table 2.3: Numerical Solutions for the UK (Benchmark Model)

		static		backward looking				mixed	forward looking		
		$\tilde{yy}_{t,n}$	$\triangle\tilde{e}_t$	$\tilde{r}_t$	z_t	$\tilde{yy}^*_t$	π^*_t	$\triangle\tilde{q}_t$	$\tilde{yy}_t$	π_t	$\triangle\tilde{yy}^*_t$
endogenous variables	$\tilde{r}_{t-1}$		-0.848	0.382					-0.481	-0.848	
	z_{t-1}		0.106	0.055	0.478				0.093	0.106	
	$\tilde{yy}^*_{t-1}$	-0.596	0.067	0.020		0.940			-0.576	0.067	-0.060
	π^*_{t-1}		-0.583	0.001			0.598		0.006	0.015	
	$\triangle\tilde{q}_{t-1}$		-0.097	-0.003				0.105	0.005	-0.004	
exogenous variables	ξ^R_t		-1.043	0.470					-0.592	-1.043	
	ξ^z_t		0.222	0.115	1.000				0.195	0.222	
	ξ^q_t		-0.921	-0.028				1.000	0.043	-0.038	
	$\xi^{y^*_t}_t$	-0.634	0.071	0.021		1.000			-0.613	0.071	1.000
	$\xi^{\pi^*_t}_t$		-0.974	0.002			1.000		0.011	0.026	

Note: This table reports the numerical values of the policy and transition functions for the UK.

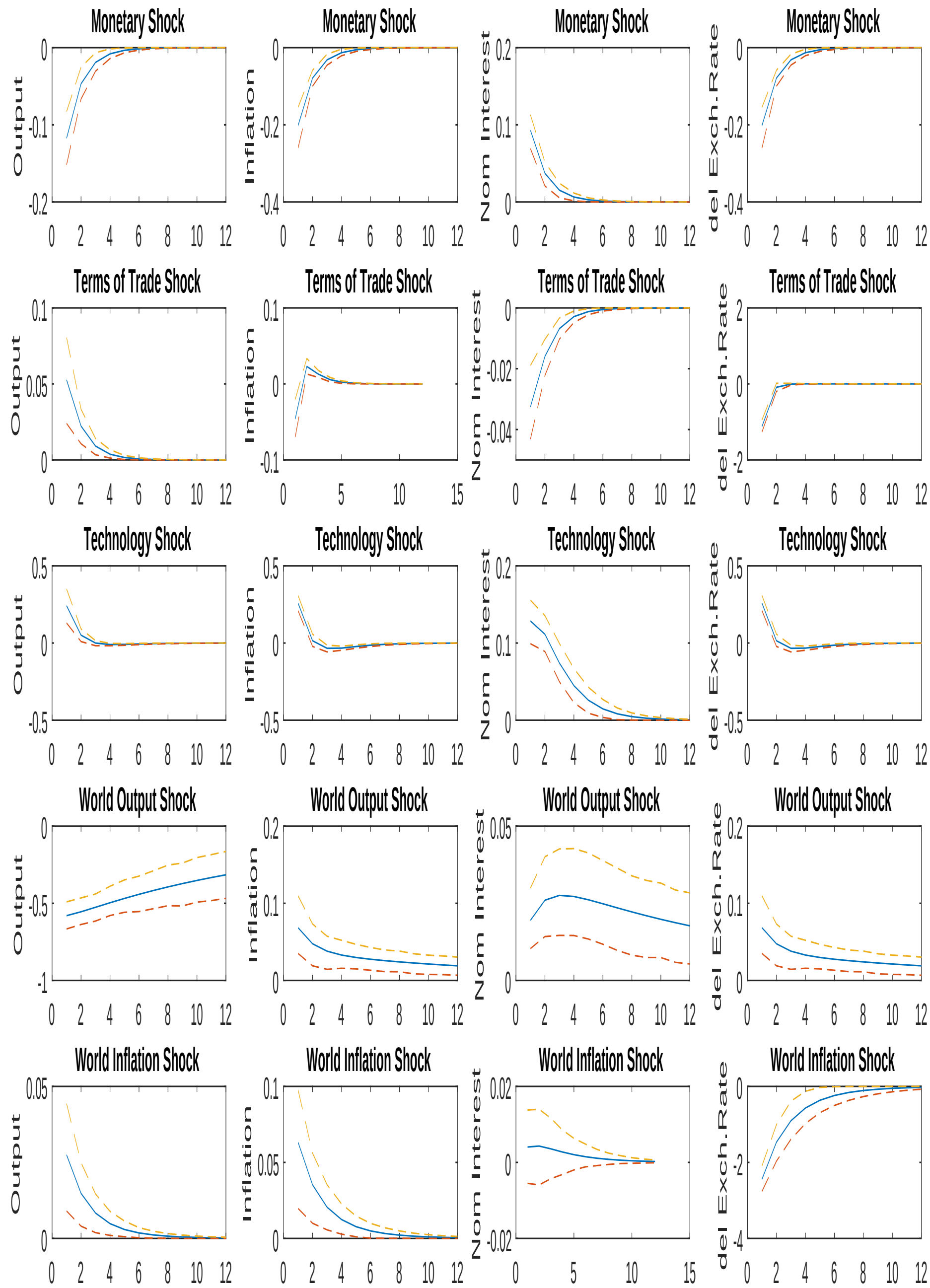

Figure 2.3: Impulse response functions in the UK. Note:The figure depicts the impulse response function of real output, inflation rate, nominal interest rate and depreciation exchange rate in the UK to one unit structural shock.

2.4.2 Estimation Results for Canada

Estimates of the Monetary Policy Parameters

The posterior mean of inflation deviation coefficient ϕ_π is 2.139, which is much higher than the prior mean. If the actual inflation is one percentage rate higher than the target inflation, the nominal interest rate raises by almost 214 base points. The posterior mean of output deviation coefficient ϕ_y is 0.066, which is only a half of the prior mean. If the real output is one percentage rate higher than its potential value, the nominal interest rate increases by about seven basis points. The posterior mean of depreciation rate coefficient $\phi_{\triangle e}$ is 0.128. There is no significant difference between the posterior and the prior mean of this parameter. If the currency depreciates by one percentage rate, the nominal interest rate increases by approximately 13 basis points. The posterior mean of the interest rate smoothing parameter ρ_R is 0.76, which is bigger than the prior mean. Overall, the posterior mean of monetary policy parameters updated from the data sample implies a persistent and strong anti-inflationary policy action in Canada. Also, the movement of the nominal exchange rate plays a crucial part in the Canadian policy decision compared to the case in the UK.

Estimates of the Other Structural Parameters

The posterior mean of the elasticity of intertemporal substitution τ is 0.283, which is smaller than the prior mean. If the real interest rate rise by one percentage rate, the consumption increases by approximately 28 basis points. The posterior mean of the Philips curve κ is 0.839, which is much higher than the prior. The data support that there is less price stickiness compared to the prior assumption. The posterior mean of import share α is 0.142, which is smaller than the prior mean. The posterior mean of the annualised steady-state real interest rate r^A is 2.379, which is not very different from the prior mean 2.470. The posterior mean of the average of annualised observable inflation rate $\pi^{(A)}$ is 1.8379, which is bigger than the prior mean 1.62. The posterior mean of the average of the change of quarterly real output $\gamma^{(Q)}$ is 0.725, which is not very different from the prior mean 0.73.

For the AR (1) process of the rate change of terms of trade, the posterior mean of the coefficient of the change of terms of trade ρ_q is 0.544, which is bigger than the prior mean. The posterior mean of the standard deviation of the shock σ_q to this process is 1.921, which is bigger than the prior mean. For the AR (1) process of the change rate of technology, the posterior mean of the coefficient of the change rate of technology ρ_z is 0.367, which is bigger than the prior mean. The posterior mean of the standard deviation of the shock σ_z to this process is 1.939, which is quite higher than the prior mean of 1. For the AR (1) process of the world inflation process, the posterior mean of the coefficient of world inflation ρ_{π^*} is 0.449, which is smaller than the prior mean. The posterior mean of the standard deviation of the shock σ_{π^*} to this process is 2.377, which is higher than the prior mean. For the AR (1) process of the world output deviation, the posterior mean of the coefficient of the world output deviation ρ_{y^*} is 0.955, which is bigger than the prior mean. The posterior mean of the standard deviation of the shock σ_{y^*} to this process is 0.809, which is smaller than the prior mean. In addition to the volatility in the above four stationary processes, the posterior mean of the standard deviation of the shock to the monetary policy σ_R is 0.299, which is smaller than the prior mean.

Numerical Solutions and Simulation Results for Canada

Table 2.4 reports the numerical solution for Canada. According to the table, it calculates the transition function as follows:

$$\begin{pmatrix} \tilde{r}_t \\ z_t \\ \tilde{yy}_t^* \\ \pi_t^* \\ \triangle\tilde{q}_t \end{pmatrix} = \begin{pmatrix} 0.284 & 0.039 & 0.019 & 0.001 & 0.012 \\ 0 & 0.367 & 0 & 0 & 0 \\ 0 & 0 & 0.9550 & 0 & 0 \\ 0 & 0 & 0 & 0.449 & 0 \\ 0 & 0 & 0 & 0 & 0.544 \end{pmatrix} \begin{pmatrix} \tilde{r_{t-1}} \\ z_{t-1} \\ \tilde{yy_{t-1}^*} \\ \pi_{t-1}^* \\ \triangle\tilde{q_{t-1}} \end{pmatrix}$$
$$+ \begin{pmatrix} 0.374 & 0.106 & 0.022 & 0.020 & 0.002 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} \xi_t^R \\ \xi_t^z \\ \xi_t^q \\ \xi_t^{y_t^*} \\ \xi_t^{\pi_t^*} \end{pmatrix}, \quad (2.32)$$

and the policy function is calculated as:

$$\begin{pmatrix} \tilde{yy}_t \\ \pi_t \\ \triangle\tilde{yy}_t^* \end{pmatrix} = \begin{pmatrix} -0.349 & 0.044 & -0.627 & 0.009 & 0.038 \\ -0.865 & 0.070 & 0.054 & 0.026 & 0.047 \\ 0 & 0 & -0.045 & 0 & 0 \end{pmatrix} \begin{pmatrix} \tilde{r_{t-1}} \\ z_{t-1} \\ \tilde{yy_{t-1}^*} \\ \pi_{t-1}^* \\ \triangle\tilde{q_{t-1}} \end{pmatrix}$$
$$+ \begin{pmatrix} -0.460 & 0.120 & 0.070 & -0.657 & 0.019 \\ -1.138 & 0.192 & 0.087 & 0.056 & 0.059 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} \xi_t^R \\ \xi_t^z \\ \xi_t^q \\ \xi_t^{y_t^*} \\ \xi_t^{\pi_t^*} \end{pmatrix}, \quad (2.33)$$

and the function for the static variables is

$$
\begin{pmatrix} y\tilde{y}_{t,n} \\ \triangle\tilde{e}_t \end{pmatrix} = \begin{pmatrix} 0 & 0 & -0.638 & 0 & 0 \\ -0.865 & 0.070 & 0.054 & -0.423 & -0.420 \end{pmatrix} \begin{pmatrix} \widetilde{r_{t-1}} \\ z_{t-1} \\ \widetilde{yy^*_{t-1}} \\ \pi^*_{t-1} \\ \triangle \widetilde{q_{t-1}} \end{pmatrix}
$$

$$
+ \begin{pmatrix} 0 & 0 & 0 & -0.668 & 0 \\ -1.138 & 0.192 & -0.771 & 0.056 & -0.941 \end{pmatrix} \begin{pmatrix} \xi^R_t \\ \xi^z_t \\ \xi^q_t \\ \xi^{y^*_t}_t \\ \xi^{\pi^*_t}_t \end{pmatrix}. \quad (2.34)
$$

Table 2.4 includes the numerical information to compute the impulse response functions of the four endogenous variables including $y\tilde{y}_t$,π_t,$\tilde{r}_t$ and $\triangle\tilde{e}_t$:

$$
\begin{pmatrix} y\tilde{y}_t \\ \pi_t \\ \tilde{r}_t \\ \triangle\tilde{e}_t \end{pmatrix} = \begin{pmatrix} -0.349 & 0.044 & -0.627 & 0.009 & 0.038 \\ -0.865 & 0.070 & 0.054 & 0.026 & 0.047 \\ 0.284 & 0.039 & 0.019 & 0.001 & 0.012 \\ -0.865 & 0.070 & 0.054 & -0.423 & -0.420 \end{pmatrix} \begin{pmatrix} \widetilde{r_{t-1}} \\ z_{t-1} \\ \widetilde{yy^*_{t-1}} \\ \pi^*_{t-1} \\ \triangle \widetilde{q_{t-1}} \end{pmatrix}
$$

$$
+ \begin{pmatrix} -0.460 & 0.120 & 0.070 & -0.657 & 0.019 \\ -1.138 & 0.192 & 0.087 & 0.056 & 0.059 \\ 0.374 & 0.106 & 0.022 & 0.020 & 0.002 \\ -1.138 & 0.192 & -0.771 & 0.056 & -0.941 \end{pmatrix} \begin{pmatrix} \xi^R_t \\ \xi^z_t \\ \xi^q_t \\ \xi^{y^*_t}_t \\ \xi^{\pi^*_t}_t \end{pmatrix}. \quad (2.35)
$$

Figure 2.4 depicts the calculated impulse response function of the four endogenous variables to the structural shock with the size of one unit of standard deviation offered in table 2.4.

Holding everything else constant, a unit of standard deviation of monetary policy shock $\sigma_R = 0.299$ will exert a $-0.46 * \sigma_R = -0.137$ impact on the real output deviation, a $-1.138 * \sigma_R = -0.340$ impact on inflation,a $0.374 * \sigma_R = 0.112$ impact on the nominal interest rate deviation, a $-1.138 * \sigma_R = -0.340$ impact on the nominal exchange rate depreciation.

Holding everything else constant, a unit of standard deviation of the shock to the change rate of terms of trade $\sigma_q = 1.921$ will exert a $0.070 * \sigma_q = 0.134$ impact on the real output deviation, a $0.087 * \sigma_q = 0.167$ impact on inflation,a $0.022 * \sigma_q = 0.042$ impact on the nominal interest rate deviation, a $-0.771 * \sigma_q = -1.482$ impact on the nominal exchange rate depreciation.

Holding everything else constant, a unit of standard deviation of the shock to the change rate of technology $\sigma_z = 1.939$ will exert a $0.120 * \sigma_z = 0.233$ impact on the real output deviation, a $0.192 * \sigma_z = 0.372$ impact on inflation, a $0.106 * \sigma_z = 0.206$ impact on the nominal interest rate deviation, a $0.192 * \sigma_z = 0.372$ impact on the nominal exchange rate depreciation.

Holding everything else constant, a unit of standard deviation of the shock to the world output deviation $\sigma_{y^*} = 0.809$ will exert a $-0.657 * \sigma_{y^*} = -0.532$ impact on the real output deviation, a $0.056 * \sigma_{y^*} = 0.046$ impact on inflation, a $0.020 * \sigma_{y^*} = 0.016$ impact on the nominal interest rate deviation, a $0.056 * \sigma_{y^*} = 0.046$ impact on the nominal exchange rate depreciation.

Holding everything else constant, a unit of standard deviation of the shock to the world inflation $\sigma_{\pi^*} = 2.377$ will exert a $0.019 * \sigma_{\pi^*} = 0.045$ impact on the real output deviation, a $0.059 * \sigma_{\pi^*} = 0.140$ impact on inflation,a $0.002 * \sigma_{\pi^*} = 0.004$ impact on the nominal interest rate deviation, a $-0.941 * \sigma_{\pi^*} = -2.237$ impact on the nominal exchange rate depreciation.

Table 2.4: Numerical Solutions for Canada (Benchmark Model)

		static		backward looking				mixed	forward looking		
		$\tilde{yy}_{t,n}$	$\triangle\tilde{e}_t$	$\tilde{r}_t$	z_t	$\tilde{yy}^*_t$	π^*_t	$\triangle\tilde{q}_t$	$\tilde{yy}_t$	π_t	$\triangle\tilde{yy}^*_t$
endogenous variables	$\tilde{r}_{t-1}$		-0.865	0.284					-0.349	-0.865	
	z_{t-1}		0.070	0.039	0.367				0.044	0.070	
	$\tilde{yy}^*_{t-1}$	-0.638	0.054	0.019		0.955			-0.627	0.054	-0.045
	π^*_{t-1}		-0.423	0.001			0.449		0.009	0.026	
	$\triangle\tilde{q}_{t-1}$		-0.420	0.012				0.544	0.038	0.047	
exogenous variables	ξ^R_t		-1.138	0.374					-0.460	-1.138	
	ξ^z_t		0.192	0.106	1.000				0.120	0.192	
	ξ^q_t		-0.771	0.022				1.000	0.070	0.087	
	$\xi^{y^*_t}_t$	-0.668	0.056	0.020		1.000			-0.657	0.056	1.000
	$\xi^{\pi^*_t}_t$		-0.941	0.002			1.000		0.019	0.059	

Note: This table reports the numerical values of the policy and transition functions for Canada.

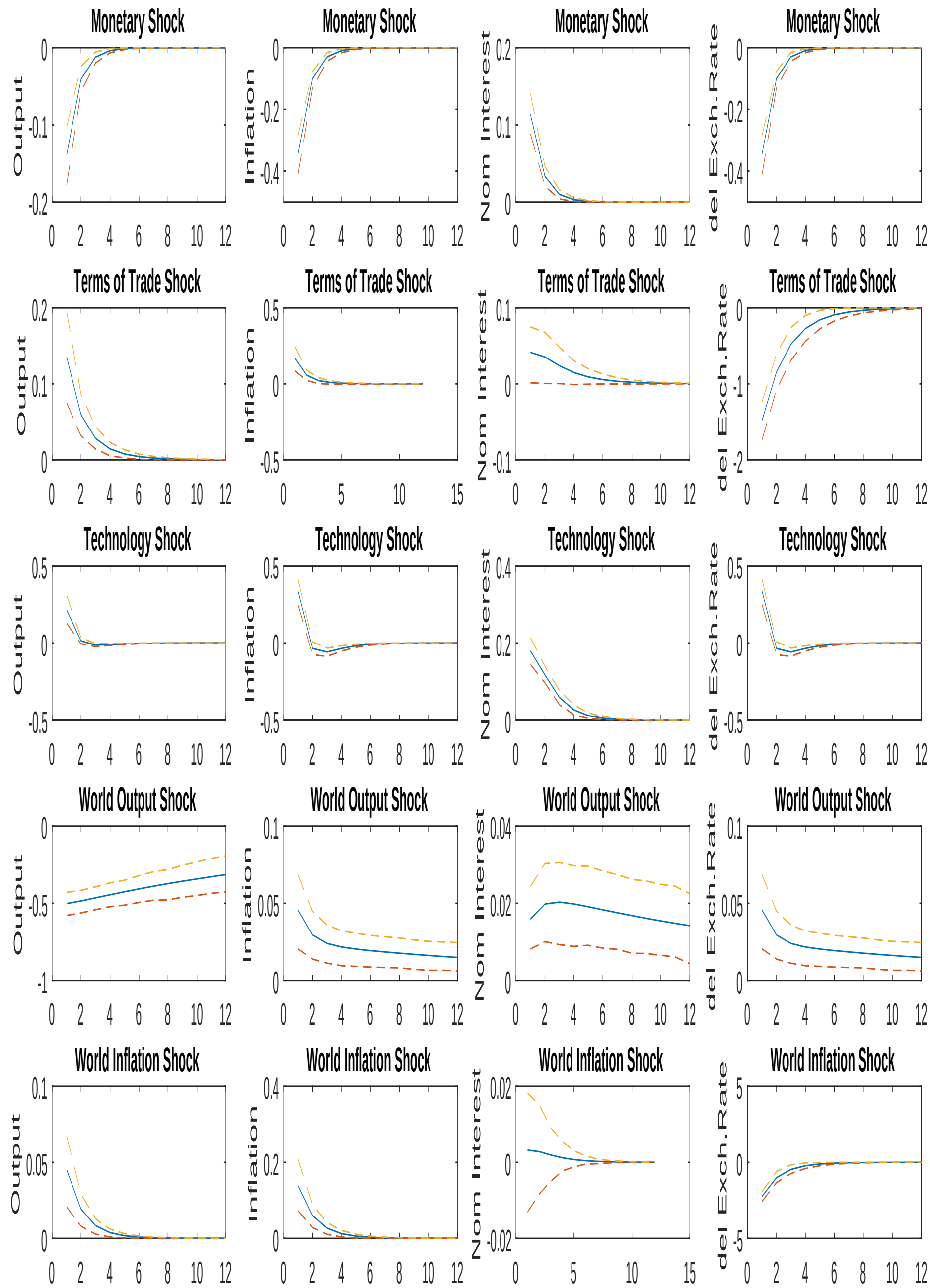

Figure 2.4: Impulse response functions in Canada. Note:The figure depicts the impulse response function of real output, inflation rate, nominal interest rate and depreciation exchange rate in Canada to one unit structural shock.

2.5 Check for the Model Specification

The DSGE models are actually as same as VAR models with restricted parameters. The restrictions of the parameters will impose a negative impact on the fitting of data when they are wrongly specified. In this section, I will apply two popular methodologies to compare the DSGE model with two specifications of the same equation. In the previous chapter, it has learned that the change rate of terms of trade is represented as follows:

$$\triangle \tilde{q_t^*} = \sigma_\alpha(\triangle \tilde{y y_t^*} - \triangle y\tilde{y}_t) = \frac{1}{\tau+\lambda}(\triangle \tilde{y y_t^*} - \triangle y\tilde{y}_t). \tag{2.36}$$

Lubik and Schorfheid suggest this equation should be replaced by a AR(1) exogenous process:

$$\triangle \tilde{q_t^*} = \rho_q \triangle \tilde{q_{t-1}^*} + \xi_t^q, \xi_t^q \sim NID(0, \sigma_q^2), \tag{2.37}$$

which can lead to a better data fitting of the empirical study. Having said this, however, there is no empirical evidence in favour of their suggestions. Due to this, it is better to find a way to compare the DSGE model with equation 2.36 and the model with equation 2.37 in terms of data fitting. Moreover, this section denotes the DSGE model with equation 2.36 as LS_{tot} and the model with equation 2.37 as LS.

2.5.1 Model Evaluation by DSGE-VAR

DSGE models have strong micro foundations and seem to provide a good insight into the business cycle theory. However, all the models are just simplification of the true world and inevitably suffer from the misspecification in different dimensions. The misspecifications can impose a potentially negative impact on the data-fitting performance of DSGE models. Estimating DSGE models is very similar to estimate a vector autoregression (VAR) model with cross-equation restrictions. The unrestricted vector VAR models introduced by Sims (1980[79]) are quite efficient in studying the true data generating process, which sometimes guides the theory through the estimated relationship among the data sets. A Bayesian Var model is then used to desire a better combination between the prior knowledge and the information included in the data (Koop and Korobilis, 2009[52]). Smet and Wouters (2003[82]) also introduce the Bayesian approach in the estimation of the DSGE model.

Del Nergo et al. (2006[25];2007[26]) provides the DSGE-VAR approach to combine the VAR and DSGE, which allows a certain degree of deviations from the cross-equation restrictions, thereby avoiding some potential model misspecifications. They invent a hyper-parameter λ, which is used to gauge the portion of restrictions arising from the DSGE models imposing on the priors of VAR models. This hyper-parameters is also called the weight of the DSGE prior of the VAR model, which represents the ratio of dummy over actual observables. $\lambda \geq \frac{k+n}{T}$, where k is the number of estimated parameters, n is the total number of observables, and T is the actual number of observations.

In summary, if the structural parameters restrict the priors of VARs model completely, then λ converges to infinity. Here is just the case of the pure DSGE models. If the structural parameters no longer restrict the priors of VARs model, then λ converges to zero. Here is the case of the pure VAR models.Del Negro and Schorfheide(2004[24]) propose that an estimated λ through the real data can evaluate how severe a possible misspecification problem can exist in a DSGE. If the estimated λ is very high, the priors of the VARS will concentrate on the model restrictions, and it reflects that the misspecification is not a serious problem for the estimated DSGE model. If the estimated λ is very low, the priors of the VARs will deviate from the model specifications, and it reflects that the misspecification is very significant for the estimated DSGE model. Due to these, the calibration and estimation of the hyperparameter λ can clarify which version of the simplified model suffer less from the misspecification problems.

The first panel of table 2.5 offers the calibration of λ from 0.4 to infinity for the UK. 0.4 is the minimum DSGE prior-weights imposing on the VAR model in Dynare. The log-marginal densities for both of the two models then carry on increasing until the prior weights reach 1 for LS and 0.75 for LS_{tot}, which implies the model restriction exerts a positive impact on the performance of data fitting. After that, the log marginal data density falls as the DSGE prior weights increases and converges to infinity, which implies the model misspecifications dominates now and affect the performance of data fitting negatively. When it enforces the restriction completely, the log marginal data densities for the two models are both very low. Thus, these two models inevitably suffer from model misspecification problems. The first panel of figure 2.5 shows that there is an inverse **U** relationship between the prior weights and the log-marginal density for each of the two models. Moreover, the log-marginal data density calculated from LS is always higher than LS_{tot} at the same DSGE prior weights. Also, the estimated λ for the UK from the LS model is 1.09, while the estimated hyperparameter from the LS_{tot} is 0.85. Overall, the LS_{tot} suffers more from the model misspecification problem in the case of the UK.

The second panel of table 2.5 offers the calibration of λ from 0.4 to infinity for Canada. Similarly, the log marginal data densities for both of the two models initially increases until the DSGE prior weight reaches 1. After that, there is a negative relationship between the calibrated λ and the log marginal data density. When it enforces the model restrictions completely for the two models, the log marginal data densities are very low. The second panel of figure 2.5 shows that there is an inverse **U** relationship between the prior weights and the log-marginal density for each of the two models. Moreover, the log marginal data density calculated from LS is still always higher than LS_{tot} at the same DSGE prior weights. In addition, the estimated λ for Canada from the LS model is 1.16 while from the LS_{tot} is 0.94. The LS_{tot} suffers more from the model misspecification problem in the case of Canada.

The empirical evidence from the UK and Canada are in favour of Lubik and Schefheide' suggestions. That is to say, although the two models are both exposed to a certain level of model misspecification, LS_{tot} suffers more, and the introduction of an exogenous stationary process can relieve the degree of this misspecification to some extent.

Table 2.5: The Calibration of DSGE Prior Weights λ

λ	0.4	0.5	0.75	1	1.25	1.5	2	5	infnity
UK									
LS	-482.36	-465.39	-453.85	-452.00	-452.57	-454.04	-457.96	-471.99	-522.53
LS_{tot}	-497.82	-483.14	-475.98	-477.07	-479.24	-481.49	-485.55	-500.70	-544.69
Canada									
LS	-577.75	-560.44	-548.49	-546.15	-546.42	-547.48	-550.23	-561.19	-586.18
LS_{tot}	-593.23	-577.98	-569.44	-569.18	-570.72	-572.67	-576.41	-588.16	-615.70

Note:The table reports the calibration of the DSGE prior weights ranging from 0.4 to infinity for the UK and Canada,separately.

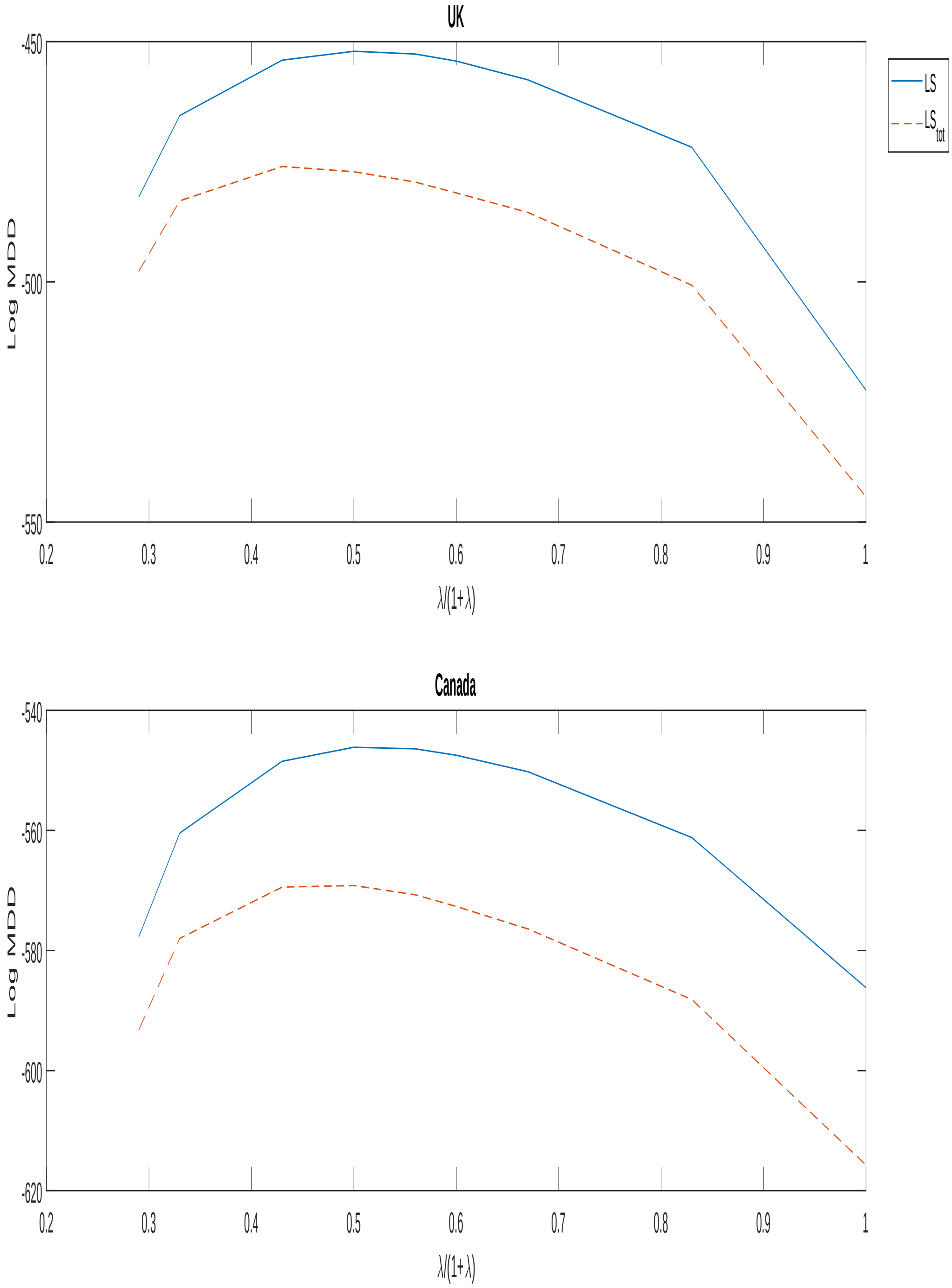

Figure 2.5: Calibrations of DSGE Prior Weights. Note:The figure depicts the calibrated DSGE prior weights and the corresponding log marginal data from two models for the UK and Canada.

2.5.2 Model Evaluation by Indirect Inference

Le et al. (2016[56]) provide a clear explanation of the application of Indirect inference methodology in the model evaluation. This method aims to compare the parameters of the auxiliary model estimated on the simulated data with the parameters of the same model estimated on the actual data. A Wald test measures the difference between the coefficients estimated on the actual data and those on the simulated data. It is often natural to choose VARs model as the auxiliary model.

$$W = (\beta^a - \overline{\beta})'\Omega^{-1}(\beta^a - \overline{\beta}), \tag{2.38}$$

where $\overline{\beta} = \frac{1}{N}\sum_{i=1}^{N}\beta^i$ and $\Omega = \frac{1}{N}\sum_{i=1}^{N}(\beta^i - \overline{\beta})((\beta^i - \overline{\beta})'$. β^a is the VAR estimates on the actual data and β^i is the VAR estimates on the simulated data. N is the total number of simulations. Overall, the Wald statistic measures the distance between the actual VAR parameters β^a and the average of the simulated VAR parameters $\overline{\beta}$.

The implement of the Wald test requires the DSGE model in a standard form as follows:

$$A_0 E_t x_{t+1} = A_1 x_t + \xi_t \tag{2.39}$$

and

$$\xi_t = D\xi_{t-1} + E\varepsilon_t, \tag{2.40}$$

where x_t is the vector of endogenous variables including $[y\tilde{y}_{t,n}, \triangle\tilde{e}_t, \tilde{r}_t, z_t, \widetilde{yy}_t^*, \pi_t^*, \triangle\tilde{q}_t, y\tilde{y}_t,$ $\pi_t, \triangle\widetilde{yy}_t^*]$. ξ_t is the vector of exogenous variables and also called the model residuals including $[\xi_t^R, \xi_t^z, \xi_t^q, \xi_t^{y_t^*}, \xi_t^{\pi_t^*}]$. ε_t is the vector of the innovations. Equation 2.40 contains all the auxiliary models estimated on the actual and the simulated data. There are mainly three steps to calculate the Wald test. First, calculate the model residuals and innovations based on the estimation of the actual data. Second, simulate the data by bootstrapping the innovations. Third, estimate the auxiliary model using the N samples of simulated data and calculate the Wald statistic with equation 2.38. Normally it is convenient to transform the Wald statistic into a t-statistic as follows:

$$T = 1.645\frac{\sqrt{2W^{a}} - \sqrt{2k-1}}{\sqrt{2W^{0.95}} - \sqrt{2k-1}}, \tag{2.41}$$

where W^a is the Wald statistic on the actual data and $W^{0.95}$ is the Wald statistic for the 95th percentile of the simulated data. k is the number of parameters in the AR(1) model. If $W^a = W^{0.95}$, the t statistic is 1.645.

It is appropriate to use Monte Carlo experiments to examine the power of the indirect inference Wald test. It initially creates 1000 samples for LS and LS_{tot} with a sample size of 200. It then bootstraps the innovations 500 times to create the distributions of the Wald statistic across the samples. After that, It generates the falseness by introducing an increasingly positive and negative degree of misspecification for the parameters, alternatively. Likewise, it bootstraps the innovations 500 times again to create the distributions of the Wald statistic with the false parameters. The incorporation of the misspecified parameters can tell us how efficient the Wald test is to use the simulated data to reject a false model at a certain degree. It yields the rejection power as below:

$$power = \frac{\#(T > 1.645)}{\#(samples)}, \tag{2.42}$$

where $\#(T > 1.645)$ is the number of the samples rejecting the model at the significance level of 5% and $\#(samples)$ is the total number of samples which is 1000. In this way, it can tell us how many times the test rejects the model with 95% confidence. Table 2.6 reports the rejection power of the test under different degrees of the falseness. The size of the test is 6.30% and 5.70% for each model when there are no mistakes in the parameters, which are both above 5%. The two models are both rejected at 5% significance level in the Monte Carlo experiment. Moreover, the power of the test reaches nearly 100% when the parameters are only 3% inaccurate for LS and 5% inaccurate for LS_{tot}. Here is another thing to mention in the relationship between the two models. Minford et al. (2019 [64]) test a part of a DSGE model by indirect inference. They augment a subset of the DSGE model with unrestricted VAR equations, which forms a new limited information model. Likewise, the LS is a limited information model and LS_{tot} is a full information

Table 2.6: Power of the Wald Test at 5% significance level

Falseness	0%	1%	3%	5%	7%	10%	15%	20%
LS	6.30%	20.70%	99.30%	100.00%	100.00%	100.00%	100.00%	100.00%
LS_tot	5.70%	14.20%	81.40%	99.40%	100.00%	100.00%	100.00%	100.00%

Note:The table reports the rejection rates under different degrees of parameter falsification. It adds falseness to the parameters by +/- x% alternation. LS denotes the DSGE model with the equation (2.37) and LS_{tot} denotes the DSGE model with the equation (2.36).

model which incorporates the restricted equation of the change rate of terms of trade. Table 2.6 implies that the power of the limited information subset test is much stronger than full information subset tests. Overall, the Indirect Inference Wald test is a little too conservative for both of the two models, while the LS is more intolerant to the degree of falseness ranging from 0% to 3%.

Although the Monte Carlo test shows that the rejection rate of a correct LS is a bit higher than LS_{tot}, it is still too early to judge LS suffers more model misspecification. The implementation of the indirect inference Wald test requires a canonical form of the DSGE model, which assumes the model residuals directly follow the AR processes in equation 2.40. However, the model residuals in LS and LS_{tot} are assumed to be white noise following an identical and independent normal distribution. Instead, the variables in LS and LS_{tot} following AR processes are endogenous backwards-looking variables, which are not the model residuals required by the indirect inference test. The current stage of the implementation of the indirect inference methodology requires changing the assumptions of the shocks to the model LS and LS_{tot}, which may change the original assumptions of the model residuals. It needs some techniques to update the indirect methodology to fit DSGE models with model residuals equating white noise, which is beyond the scope of this thesis.

2.6 Conclusion

Chapter 2 estimates a small open DSGE model developed by Lubik and Schorfheide (2007[60]) using Bayesian methodology with the data collected from the UK and Canada covering period 1992: Q4 to 2008: Q4. This chapter offers benchmark results of Bayesian estimation, which sets up the foundation for the model comparison in the next chapter. In addition to the estimation of the benchmark model, chapter 2 also introduces two methodologies regarding VAR models to check for the problem of model misspecification proposed by Lubik and Schorfheide (2007[60]). The DSGE-VAR approach supports their suggestions of improving data fitting by replacing the restricted equation regarding the change rate of terms of trade of a simple AR(1) process. The indirect inference approach is a bit conservative and rejects both of the two in terms of data fitting. However, the latter approach cannot fit the simplified model very well in the assumptions of the model residuals and innovations at the current stage.

As we can see from the table 2.2, although the posterior mean of the monetary policy coefficients support that UK and Canada both employ a specific type of inflation targeting policy, the weights on output deviation and nominal exchange rate depreciation are quite different between the two countries. More specifically, the nominal exchange rate depreciation plays a more crucial role in the monetary policy reaction function in Canada than in the UK, if the model can fit the data very well. However, it is still too early to evaluate the performance of data fitting in only one model. Although it is impossible to spot the entirely correct model for a specified economy, one still can measure the marginal improvement of the model compared to its benchmark form step by step. The posterior odds ratio embodied in Bayesian techniques is a natural way to compare the performance of different models in terms of data fitting. Next chapter will adjust the monetary policy reaction function several times to compare the updated DSGE models with the benchmark one in this chapter.

Chapter 3

Model Comparison One: Constant Parameters Estimation

3.1 Introduction

Chapter 3 will offer model comparisons at the first stage for one control group and three treatment groups based on different specifications of monetary policy. It will regard the original DSGE model in the previous chapter as the control group and apply the same Bayesian methodology to estimate the updated DSGE models in each of the treatment group. The goal of this chapter is to find the model with the best performance of data fitting for the UK and Canada given the calculated posterior odds ratio.

One resource of the specifications of monetary policy comes from the Taylor rule suggested by Lubik and Schorfheide (2007)[60] in the control group. As mentioned in the earlier chapter, the deviation of nominal interest rate from its steady real interest rate responds to the deviations of inflation, real output and the nominal exchange rate depreciation from their corresponding steady states. More specifically, inflation, real output and the change rate of the nominal exchange rate are assumed to be the important factors for the central bank to make the policy decision.

The other source of the specifications should come from the real world, which is not covered by the original model. For instance, the estimation of the DSGE model with the same monetary policy reaction function may not reflect the difference between the Bank of England and the Central Bank of Canada. For instance, the Central bank of England gets independence since 1997, while the central bank of Canda gets independence since 1992. Besides, the relationship between the monetary policy and the nominal exchange rate is connected closely for Canada, since it can benefit from controlling the volatility of the exchange rate to trade with foreign countries such like America.

The central bank of England, through the monetary policy committee, announces that its main task is to meet the inflation target 2%. Apart from fighting the possible high inflation rate, it also intends to stabilise the output. However, the UK stop committing to a particular exchange rate since the exchange crisis happened in 1992-1993 make the UK leave the European Monetary System (Blanchard,2013 [7]). Nelson(2003)[67] offers an explicit introduction of the monetary policy in the UK, which examines the applications of Taylor rules from 1972 to 1997. He does not consider the exchange rate in the monetary policy reaction function after 1992.

The central bank of Canada also announces that its primary objective is to provide a low and steady inflation rate, which targets 2%. The stabilisation of output is also another important factor in the policy decision. Moreover, as David Dodge, the former governor of the central bank of Canada (2001-2008) said, the exchange rate movements are crucial in the determination of the Canadian monetary policy (Dodge,2005,[29]). Besides, Eichenbaum (2017,[31]) points out that the nominal exchange rate movements instead of the real exchange rate movements should enter into the monetary policy. One study shows that a 1-percentage depreciation in the Canadian dollar leads to a 0.2% rise in the interest rate on average(Ball,2011[4]). Having said this, however, it is necessary to distinguish the exchange rate from the movements of the exchange rate in the policy specifications. The central bank of Canada does not set a target for the exchange rate, but it instead should consider the movements of the exchange rate when the market shocks that change the exchange rate also has a substantial impact on the Canadian economy (Ragan,2005 [71])).

Apart from discussing whether the central bank should incorporate the movements of the exchange rate, some individuals also suggest that it can be helpful to consider the movements of output. Walsh (2003)[90] demonstrate that the change of output gap should play a significant role in the design of monetary policy and call it a speed-limit type of Taylor rule. Likewise, Ragon (2006, [72]) demonstrates that another goal for the Canadian central bank is to offer the stability of the change rate of output.

Based on the discussion above, there will be three treatment groups. The first treatment group will remove the movement of the nominal exchange rates (nominal exchange rate depreciation) away from the monetary policy reaction function. The second treatment group will incorporate the change rate of real output in the policy reaction function but remove the nominal exchange rate depreciation away. The third treatment group will both incorporate the nominal exchange rate depreciation and the rate change of real output. From the estimation of and the comparisons among the control and treatment groups, it can reveal something hidden in the real data. On the one hand, it can tell us which of the adjusted monetary policy can enhance the performance of data fitting most significantly. On the other hand, it can tell us the difference of the monetary policy decisions between the UK and Canada.

Section 3.2 will introduce a methodology to calculate the posterior odds ratio, which is the major way to compare the models using the Bayesian techniques. Section 3.3 will estimate and simulate the DSGE models in each of the three treatment groups. It then compares their estimation results with the control group to find the model with the bests data fitting. Section 3.4 concludes.

3.2 Model Comparison of Bayesian Methodology

This section will introduce a general method to compare the posterior model probabilities of DSGE models with different specifications of monetary policy reaction function in the context of Bayesian estimation.

Chapter 2 offers a way to calculate the marginal data density $p(D_{1:t})$:

$$p(D_{1:t}) = \int p(D_{1:t}|\Theta)p(\Theta)d\Theta. \tag{3.1}$$

In addition, the marginal data density above is an abbreviation for the marginal density associated with DSGE models M_i:

$$p(D_{1:t}|M_i) = \int p(D_{1:t}|\Theta_i, M_i)p(\Theta_i|M_i)d\Theta_i. \tag{3.2}$$

Having known the marginal data density associate with a a given DSGE model, the posterior model probability $\gamma_{i,t}$ updated by the data sample $D_{1:t}$ are calculated by:

$$\gamma_{i,t} = \frac{\gamma_{i,0}p(D_{1:t}|M_i)}{\sum_{i=1}^{I}\gamma_{i,0}p(D_{1:t}|M_i)}, \tag{3.3}$$

where $\gamma_{i,0}$ is the prior model probability and I is the total number of the compared DSGE models.

To compare the performance of different DSGE models in terms of data fitting, it needs to compute the posterior odds ratios $\gamma_{i,j}$:

$$\gamma_{i,j} = \frac{\gamma_{i,0}p(D_{1:t}|M_i)}{\gamma_{j,0}p(D_{1:t}|M_j)}. \tag{3.4}$$

where the factor $\frac{\gamma_{i,0}}{\gamma_{j,0}}$ is called prior odds ratio in favor of M_i, which is generally assumed to be one when we are indifferent to the model specifications. The factor $\frac{p(D_{1:t}|M_i)}{p(D_{1:t}|M_j)}$ is the Bayes factor which denotes the sample evidence in favor of M_i. Moreover, the marginal data density $p(D_{1:t}|M_i)$ is usually represented in the natural log form:

$$lnp(D_{1:t}|M_i) = \sum_{t=1}^{T} ln \int p(d_t|\Theta_i, D_{1:t-1}, M_i)p(\Theta_i|D_{1:t-1}, M_i)d\Theta_i. \tag{3.5}$$

As mentioned above, after the prior odds ratio is assumed to be one, the posterior odds ratio $\gamma_{i,j}$ is updated as:

$$\gamma_{i,j} = \frac{e^{lnp(D_{1:t}|M_i)}}{e^{lnp(D_{1:t}|M_j)}} = e^{(lnp(D_{1:t}|M_i) - lnp(D_{1:t}|M_j))} \tag{3.6}$$

According to Kass and Raftery (1995 [50]), if the posterior odds ratio is between 1 and 3, there is no significant difference between the DSGE model M_i and M_j.If the value is bigger than 3, there is a positive evidence in favor of model M_i. Likewise, if the value is smaller than $\frac{1}{3}$, there is a positive evidence in favor of model M_j. The calculation of the posterior odds ratio leads to a specific type of monetary policy reaction function, which improves the performance of data fitting the most significantly. Aside from enhancing the performance of the model in terms of data fitting, it is helpful to identify whether the simplified DSGE model can capture the distinctive behaviours of central banks in the UK and Canada to some extent.

3.3 Bayesian Estimation of the Updated DSGE Models

This section compares the DSGE models with different specifications of monetary policy reaction function. Table 3.1 reports the estimation results of the original DSGE model used in the previous chapter. The original DSGE model is regarded as the control group. The log marginal density of the control group for the UK is -522.528 and for Canada is -586.176. As mentioned earlier, the control group defines the monetary policy reaction function as the following equation:

$$\tilde{r}_t = \rho_R \tilde{r_{t-1}} + (1-\rho_R)[\phi_\pi \pi_t + \phi_y \tilde{yy_t} + \phi_{\triangle e} \triangle \tilde{e}_t] + \xi_t^R, \xi_t^R \sim NID(0, \sigma_R^2), \quad (3.7)$$

where it assumes the deviation of the nominal interest rate from its steady real interest rate depends on the deviations of inflation rate, real output and nominal exchange rate depreciation from their corresponding steady states. There are three treatment groups totally for each country. The first treatment group assumes that the monetary policy reaction function ignores the nominal exchange rate depreciation:

$$\tilde{r}_t = \rho_R \tilde{r_{t-1}} + (1-\rho_R)[\phi_\pi \pi_t + \phi_y \tilde{yy_t}] + \xi_t^R, \xi_t^R \sim NID(0, \sigma_R^2). \quad (3.8)$$

The second treatment group assumes that the monetary policy reaction function ignores the nominal exchange rate depreciation while incorporate the change rate of the real output:

$$\tilde{r}_t = \rho_R \tilde{r_{t-1}} + (1-\rho_R)[\phi_\pi \pi_t + \phi_y \tilde{yy_t}] + \phi_{\triangle y}(\tilde{yy_t} - \tilde{yy_{t-1}}) + \xi_t^R, \xi_t^R \sim NID(0, \sigma_R^2). \quad (3.9)$$

The third treatment group assumes the monetary policy reaction function incorporate the nominal exchange rate depreciation and the change rate of the real output:

$$\begin{aligned} \tilde{r}_t = \rho_R \tilde{r_{t-1}} + (1-\rho_R)[\phi_\pi \pi_t + \phi_y \tilde{yy_t} + \phi_{\triangle e} \triangle \tilde{e}_t] + \phi_{\triangle y}(\tilde{yy_t} - \tilde{yy_{t-1}}) + \xi_t^R, \\ \xi_t^R \sim NID(0, \sigma_R^2). \end{aligned} \quad (3.10)$$

The following subsections initially report the estimation results of the three treatment groups and then calculate the relative posterior odds ratio given the log marginal data density of the control group. Moreover, those subsections incorporate solutions and simulations of each treatment group. Subsequently, the model comparison leads to finding the model with the best performance of data fitting for the UK and Canada.

Table 3.1: Constant Parameter Estimation Results (Control Group)

	Prior		Posterior			
	UK	Canada	UK		Canada	
Parameters	Mean	Mean	Mean	90%interval	Mean	90%interval
τ	0.500		0.258	[0.124,0.381]	0.283	[0.155,0.410]
κ	0.300		0.461	[0.194,0.726]	0.839	[0.484,1.186]
ϕ_π	1.500		2.589	[1.801,3.351]	2.139	[1.439,2.799]
ϕ_y	0.125		0.121	[0.048,0.191]	0.066	[0.028,0.103]
$\phi_{\triangle e}$	0.125		0.060	[0.025,0.095]	0.128	[0.065,0.187]
ρ_R	0.500		0.813	[0.749,0.878]	0.760	[0.675,0.843]
ρ_z	0.200		0.478	[0.339,0.609]	0.367	[0.263,0.469]
ρ_q	0.400		0.105	[0.010,0.197]	0.544	[0.424,0.661]
ρ_{π^*}	0.800		0.598	[0.441,0.749]	0.449	[0.312,0.580]
ρ_{y^*}	0.900		0.940	[0.899,0.985]	0.955	[0.923,0.988]
α	0.200		0.117	[0.071,0.164]	0.142	[0.086,0.199]
$r^{(A)}$	3.350	2.470	3.127	[2.677,3.565]	2.379	[1.847,2.918]
$\pi^{(A)}$	1.920	1.620	2.263	[1.703,2.838]	1.839	[1.266,2.420]
$\gamma^{(Q)}$	0.620	0.730	0.668	[0.615,0.722]	0.725	[0.670,0.783]
σ_R	0.500		0.194	[0.150,0.239]	0.299	[0.227,0.371]
σ_z	1.500	1.000	1.309	[0.521,2.104]	1.939	[0.871,2.962]
σ_{y^*}	1.500		1.003	[0.422,1.609]	0.809	[0.364,1.231]
σ_{π^*}	0.550		2.510	[2.141,2.860]	2.377	[2.028,2.722]
σ_q	1.500		1.215	[1.027,1.383]	1.921	[1.641,2.192]
Log MDD			-522.528		-586.176	

Note:The table reports the parameter estimation results of UK and Canada considering the original monetary policy rule.

3.3.1 Group One: No Nominal Exchange Depreciation

Table 3.2 reports the estimation results of the first treatment group. The prior distributions for the treatment group are identical to the control group, except that there is no prior setting for the coefficient of the nominal exchange depreciation. The posterior distributions of the first treatment group are not very different from those of the control group.

The log marginal data density of the first treatment group in the UK is -518.651. Thus, the posterior odds ratio is:

$$\gamma_{1,0}^{UK} = e^{(lnp(D_{1:t}^{UK}|M_1) - lnp(D_{1:t}^{UK}|M_0))}, \tag{3.11}$$

where M_0 represents the DSGE model in the control group and M_1 represents the DSGE model in the first treatment group. The numerical results of the posterior odds ratio in the UK, $\gamma_{1,0}^{UK}$, is 48.279, which is much bigger than 3. This ratio supports that the DSGE model in the first treatment group fit the UK data much better than it in the control group. More specifically, the UK data is in favour of the monetary policy reaction function without the movement of the nominal exchange rate.

The log marginal data density of the first treatment group in Canada is -590.343. Likewise, the posterior odds ratio is:

$$\gamma_{1,0}^{Canada} = e^{lnp(D_{1:t}^{Canada}|M_1) - lnp(D_{1:t}^{Canada}|M_0)}. \tag{3.12}$$

The numerical results of the posterior odds ratio in Canada, $\gamma_{1,0}^{Canada}$, is 0.015, which is smaller than $\frac{1}{3}$. This ratio supports that the DSGE model in the first treatment group fit Canada data worse than it in the control group. More specifically, the data from Canada is in favour of the policy reaction function with nominal exchange rate depreciation.

Table 3.2: Constant Parameter Estimation Results (Group One)

	Prior		Posterior			
	UK	Canada	UK		Canada	
Parameters	Mean	Mean	Mean	90%interval	Mean	90%interval
τ	0.500		0.258	[0.126,0.381]	0.308	[0.176,0.439]
κ	0.300		0.455	[0.188,0.715]	0.923	[0.572,1.274]
ϕ_π	1.500		2.388	[1.635,3.076]	2.198	[1.473,2.876]
ϕ_y	0.125		0.108	[0.039,0.170]	0.060	[0.023,0.096]
ρ_R	0.500		0.795	[0.728,0.866]	0.760	[0.676,0.844]
ρ_z	0.200		0.476	[0.332,0.619]	0.373	[0.273,0.468]
ρ_q	0.400		0.097	[0.010,0.185]	0.587	[0.464,0.710]
ρ_{π^*}	0.800		0.604	[0.448,0.778]	0.517	[0.361,0.687]
ρ_{y^*}	0.900		0.937	[0.892,0.983]	0.953	[0.920,0.987]
α	0.200		0.120	[0.069,0.167]	0.156	[0.099,0.210]
$r^{(A)}$	3.350	2.470	3.163	[2.737,3.606]	2.384	[1.847,2.935]
$\pi^{(A)}$	1.920	1.620	2.231	[1.711,2.794]	1.815	[1.250,2.366]
$\gamma^{(Q)}$	0.620	0.730	0.670	[0.616,0.722]	0.729	[0.674,0.784]
σ_R	0.500		0.185	[0.144,0.227]	0.311	[0.236,0.384]
σ_z	1.500	1.000	1.335	[0.508,2.171]	1.775	[0.891,2.673]
σ_{y^*}	1.500		1.005	[0.428,1.617]	0.838	[0.383,1.315]
σ_{π^*}	0.550		2.504	[2.132,2.853]	2.414	[2.051,2.769]
σ_q	1.500		1.216	[1.035,1.394]	1.925	[1.648,2.198]
Log MDD			-518.651		-590.343	

Note:The table reports the parameter estimation results of the UK and Canada considering the monetary policy without the nominal exchange rate depreciation.

Numerical Solution and Simulation Results for the UK in the Group One

Table 3.3 reports the numerical solutions for the UK in the first treatment group. The table yields the transition function as follows:

$$\begin{pmatrix} \tilde{r}_t \\ z_t \\ \tilde{yy}_t^* \\ \pi_t^* \\ \triangle\tilde{q}_t \end{pmatrix} = \begin{pmatrix} 0.381 & 0.055 & 0.021 & 0 & -0.002 \\ 0 & 0.476 & 0 & 0 & 0 \\ 0 & 0 & 0.937 & 0 & 0 \\ 0 & 0 & 0 & 0.604 & 0 \\ 0 & 0 & 0 & 0 & 0.097 \end{pmatrix} \begin{pmatrix} \tilde{r_{t-1}} \\ z_{t-1} \\ \tilde{yy_{t-1}^*} \\ \pi_{t-1}^* \\ \triangle\tilde{q_{t-1}} \end{pmatrix}$$
$$+ \begin{pmatrix} 0.480 & 0.115 & -0.025 & 0.022 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} \xi_t^R \\ \xi_t^z \\ \xi_t^q \\ \xi_t^{y_t^*} \\ \xi_t^{\pi_t^*} \end{pmatrix}, \quad (3.13)$$

and it computes the policy function as follows:

$$\begin{pmatrix} \tilde{yy}_t \\ \pi_t \\ \triangle\tilde{yy_t^*} \end{pmatrix} = \begin{pmatrix} -0.478 & 0.095 & -0.585 & 0 & 0.004 \\ -0.823 & 0.107 & 0.069 & 0 & -0.005 \\ 0 & 0 & -0.063 & 0 & 0 \end{pmatrix} \begin{pmatrix} \tilde{r_{t-1}} \\ z_{t-1} \\ \tilde{yy_{t-1}^*} \\ \pi_{t-1}^* \\ \triangle\tilde{q_{t-1}} \end{pmatrix}$$
$$+ \begin{pmatrix} -0.601 & 0.199 & 0.038 & -0.625 & 0 \\ -1.035 & 0.225 & -0.053 & 0.074 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} \xi_t^R \\ \xi_t^z \\ \xi_t^q \\ \xi_t^{y_t^*} \\ \xi_t^{\pi_t^*} \end{pmatrix}, \quad (3.14)$$

and the function for the static variables is

$$\begin{pmatrix} \tilde{yy}_{t,n} \\ \triangle\tilde{e}_t \end{pmatrix} = \begin{pmatrix} 0 & 0 & -0.606 & 0 & 0 \\ -0.823 & 0.107 & 0.069 & -0.604 & -0.090 \end{pmatrix} \begin{pmatrix} \tilde{r}_{t-1} \\ z_{t-1} \\ \tilde{yy}^*_{t-1} \\ \pi^*_{t-1} \\ \triangle\tilde{q}_{t-1} \end{pmatrix}$$

$$+ \begin{pmatrix} 0 & 0 & 0 & -0.647 & 0 \\ -1.035 & 0.225 & -0.933 & 0.074 & -1.000 \end{pmatrix} \begin{pmatrix} \xi^R_t \\ \xi^z_t \\ \xi^q_t \\ \xi^{y^*_t}_t \\ \xi^{\pi^*_t}_t \end{pmatrix}. \quad (3.15)$$

Table 3.3 also incorporates information to compute the impulse response functions for the four endogenous variables including $\tilde{yy}_t$,π_t,$\tilde{r}_t$ and $\triangle\tilde{e}_t$:

$$\begin{pmatrix} \tilde{yy}_t \\ \pi_t \\ \tilde{r}_t \\ \triangle\tilde{e}_t \end{pmatrix} = \begin{pmatrix} -0.478 & 0.095 & -0.585 & 0 & 0.004 \\ -0.823 & 0.107 & 0.069 & 0 & -0.005 \\ 0.381 & 0.055 & 0.021 & 0 & -0.002 \\ -0.823 & 0.107 & 0.069 & -0.604 & -0.090 \end{pmatrix} \begin{pmatrix} \tilde{r}_{t-1} \\ z_{t-1} \\ \tilde{yy}^*_{t-1} \\ \pi^*_{t-1} \\ \triangle\tilde{q}_{t-1} \end{pmatrix}$$

$$+ \begin{pmatrix} -0.601 & 0.199 & 0.038 & -0.625 & 0 \\ -1.035 & 0.225 & -0.053 & 0.074 & 0 \\ 0.480 & 0.115 & -0.025 & 0.022 & 0 \\ -1.035 & 0.225 & -0.933 & 0.074 & -1.000 \end{pmatrix} \begin{pmatrix} \xi^R_t \\ \xi^z_t \\ \xi^q_t \\ \xi^{y^*_t}_t \\ \xi^{\pi^*_t}_t \end{pmatrix}. \quad (3.16)$$

The coefficients of the second matrix in the above equation determine the impulse response function for a given size of the structural shock. The impulse response function measures the temporal deviations of economic variables from their stable states. Figure 3.1 depicts the calculated impulse response function of the four endogenous variables to the structural shock with the size of one unit of standard deviation offered in table 3.2.

Holding everything else constant, a unit of the standard deviation of the monetary policy shock $\sigma_R = 0.185$ will exert a $-0.601 * \sigma_R = -0.111$ impact on the real output deviation, a $-1.035 * \sigma_R = -0.192$ impact on the inflation, a $0.48 * \sigma_R = 0.089$ impact on the nominal interest rate deviation, a $-1.035 * \sigma_R = -0.192$ impact on the nominal exchange rate depreciation.

Holding everything else constant, a unit of the standard deviation of the shock to the change rate of terms of trade $\sigma_q = 1.216$ will exert a $0.038 * \sigma_q = 0.046$ impact on the real output deviation, a $-0.053 * \sigma_q = -0.064$ impact on the inflation, a $-0.025 * \sigma_q = -0.030$ impact on the nominal interest rate deviation, a $-0.993 * \sigma_q = -1.134$ impact on the nominal exchange rate depreciation.

Holding everything else constant, a unit of the standard deviation of the shock to the change rate of the technology $\sigma_z = 1.335$ will exert a $0.199 * \sigma_z = 0.265$ impact on the real output deviation, a $0.225 * \sigma_z = 0.301$ impact on the inflation, a $0.115 * \sigma_z = 0.153$ impact on the nominal interest rate deviation, a $0.225 * \sigma_z = 0.301$ impact on the nominal exchange rate depreciation.

Holding everything else constant, a unit of the standard deviation of the shock to the world output deviation $\sigma_{y^*} = 1.005$ will exert a $-0.625 * \sigma_{y^*} = -0.628$ impact on the real output deviation, a $0.074 * \sigma_{y^*} = 0.075$ impact on the inflation, a $0.022 * \sigma_{y^*} = 0.023$ impact on the nominal interest rate deviation, a $0.074 * \sigma_{y^*} = 0.075$ impact on the nominal exchange rate depreciation.

Holding everything else constant, a unit of the standard deviation of the shock to the world inflation $\sigma_{\pi^*} = 2.504$ will exert no impact on the real output deviation, the inflation and the nominal interest rate. It only exerts $-1 * \sigma_{\pi^*} = -2.504$ impact on the nominal exchange rate depreciation.

Table 3.3: Numerical Solutions for the UK in the Group One

		static		backward looking				mixed	forward looking		
		$\tilde{yy}_{t,n}$	$\triangle\tilde{e}_t$	$\tilde{r}_t$	z_t	$\tilde{yy}^*_t$	π^*_t	$\triangle\tilde{q}_t$	$\tilde{yy}_t$	π_t	$\triangle\tilde{yy}^*_t$
endogenous variables	$\tilde{r}_{t-1}$		-0.823	0.381					-0.478	-0.823	
	z_{t-1}		0.107	0.055	0.476				0.095	0.107	
	$\tilde{yy}^*_{t-1}$	-0.606	0.069	0.021		0.937			-0.585	0.069	-0.063
	π^*_{t-1}		-0.604				0.604				
	$\triangle\tilde{q}_{t-1}$		-0.090	-0.002				0.097	0.004	-0.005	
exogenous variables	ξ^R_t		-1.035	0.480					-0.601	-1.035	
	ξ^z_t		0.225	0.115	1.000				0.199	0.225	
	ξ^q_t		-0.933	-0.025				1.000	0.038	-0.053	
	$\xi^{y^*_t}_t$	-0.647	0.074	0.022		1.000			-0.625	0.074	1.000
	$\xi^{\pi^*_t}_t$		-1.000				1.000				

Note: This table reports the numerical solutions of the policy and transition functions for the UK in the Group One.

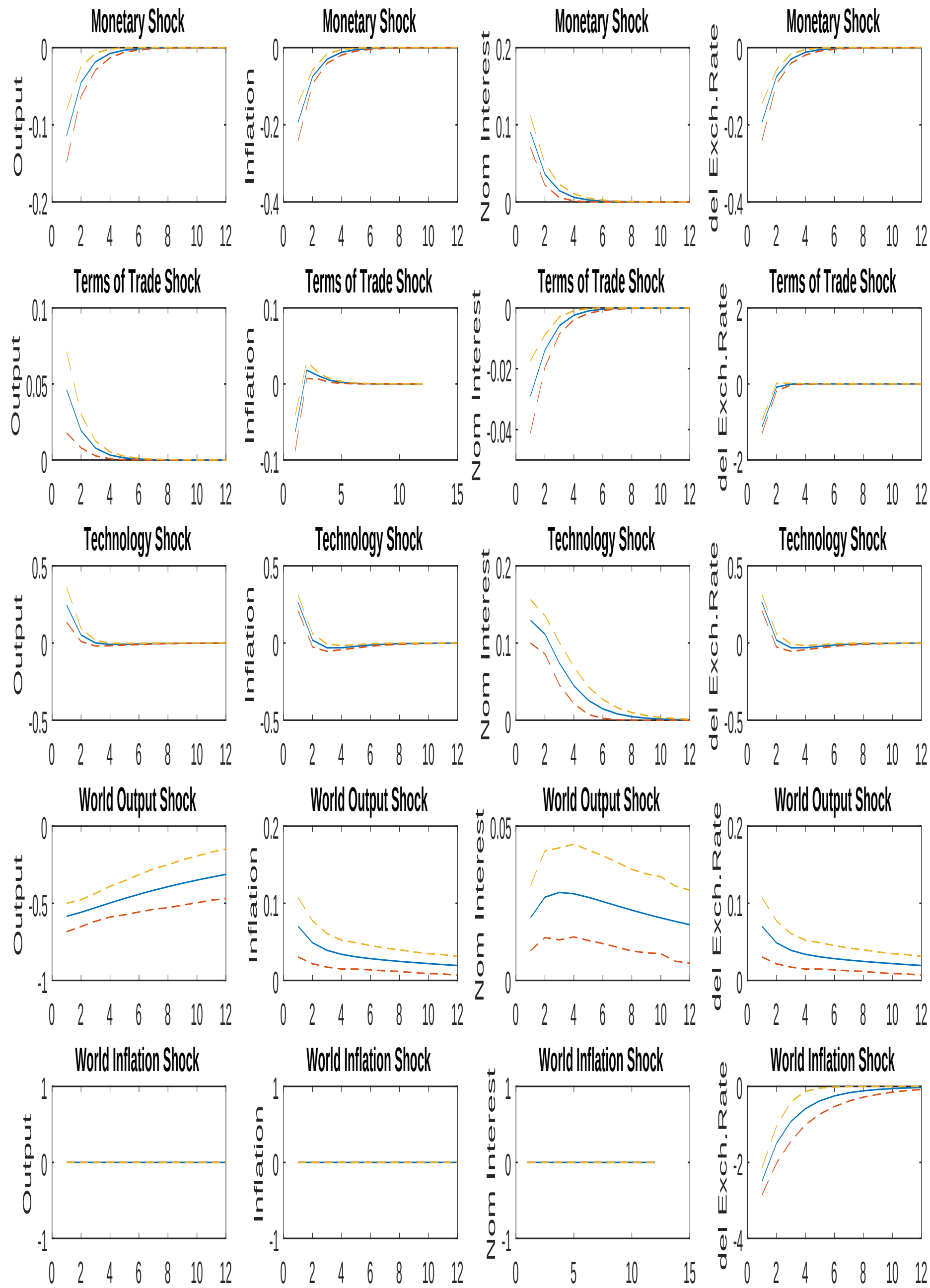

Figure 3.1: Impulse response functions for Group One in the UK. Note:The figure depicts the impulse response functions of the real output, the inflation rate, the nominal interest rate and the depreciation exchange rate in the UK to one unit of the structural shocks.

Numerical Solution and Simulation Results for Canada in the Group One

Table 3.4 reports that the numerical solution for Canada in the first treatment group. According to the table, the computed transition function is:

$$\begin{pmatrix} \tilde{r}_t \\ z_t \\ \tilde{yy}_t^* \\ \pi_t^* \\ \triangle\tilde{q}_t \end{pmatrix} = \begin{pmatrix} 0.273 & 0.042 & 0.018 & 0 & 0.016 \\ 0 & 0.373 & 0 & 0 & 0 \\ 0 & 0 & 0.953 & 0 & 0 \\ 0 & 0 & 0 & 0.517 & 0 \\ 0 & 0 & 0 & 0 & 0.587 \end{pmatrix} \begin{pmatrix} \tilde{r_{t-1}} \\ z_{t-1} \\ \tilde{yy}_{t-1}^* \\ \pi_{t-1}^* \\ \triangle \tilde{q_{t-1}} \end{pmatrix}$$
$$+ \begin{pmatrix} 0.360 & 0.111 & 0.027 & 0.019 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} \xi_t^R \\ \xi_t^z \\ \xi_t^q \\ \xi_t^{y_t^*} \\ \xi_t^{\pi_t^*} \end{pmatrix}, \tag{3.17}$$

and the calculated policy function is:

$$\begin{pmatrix} \tilde{yy}_t \\ \pi_t \\ \triangle\tilde{yy}_t^* \end{pmatrix} = \begin{pmatrix} -0.365 & 0.047 & -0.606 & 0 & 0.036 \\ -0.912 & 0.077 & 0.050 & 0 & 0.029 \\ 0 & 0 & -0.047 & 0 & 0 \end{pmatrix} \begin{pmatrix} \tilde{r_{t-1}} \\ z_{t-1} \\ \tilde{yy}_{t-1}^* \\ \pi_{t-1}^* \\ \triangle \tilde{q_{t-1}} \end{pmatrix}$$
$$+ \begin{pmatrix} -0.480 & 0.127 & 0.061 & -0.635 & 0 \\ -1.200 & 0.208 & 0.050 & 0.053 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} \xi_t^R \\ \xi_t^z \\ \xi_t^q \\ \xi_t^{y_t^*} \\ \xi_t^{\pi_t^*} \end{pmatrix}, \tag{3.18}$$

and the function for the static variables is:

$$\begin{pmatrix} \tilde{yy}_{t,n} \\ \triangle\tilde{e}_t \end{pmatrix} = \begin{pmatrix} 0 & 0 & -0.616 & 0 & 0 \\ -0.912 & 0.077 & 0.050 & -0.517 & -0.466 \end{pmatrix} \begin{pmatrix} \tilde{r_{t-1}} \\ z_{t-1} \\ \tilde{yy^*_{t-1}} \\ \pi^*_{t-1} \\ \triangle\tilde{q_{t-1}} \end{pmatrix}$$

$$+ \begin{pmatrix} 0 & 0 & 0 & -0.646 & 0 \\ -1.200 & 0.208 & -0.794 & 0.053 & -1.000 \end{pmatrix} \begin{pmatrix} \xi^R_t \\ \xi^z_t \\ \xi^q_t \\ \xi^{y^*_t}_t \\ \xi^{\pi^*_t}_t \end{pmatrix}. \quad (3.19)$$

Likewise,table 3.4 provides the information to compute the impulse response functions of the four endogenous variables including $\tilde{yy}_t$,π_t,$\tilde{r}_t$ and $\triangle\tilde{e}_t$:

$$\begin{pmatrix} \tilde{yy}_t \\ \pi_t \\ \tilde{r}_t \\ \triangle\tilde{e}_t \end{pmatrix} = \begin{pmatrix} -0.365 & 0.047 & -0.606 & 0 & 0.036 \\ -0.912 & 0.077 & 0.050 & 0 & 0.029 \\ 0.273 & 0.042 & 0.018 & 0 & 0.016 \\ -0.912 & 0.077 & 0.050 & -0.517 & -0.466 \end{pmatrix} \begin{pmatrix} \tilde{r_{t-1}} \\ z_{t-1} \\ \tilde{yy^*_{t-1}} \\ \pi^*_{t-1} \\ \triangle\tilde{q_{t-1}} \end{pmatrix}$$

$$+ \begin{pmatrix} -0.480 & 0.127 & 0.061 & -0.635 & 0 \\ -1.200 & 0.208 & 0.050 & 0.053 & 0 \\ 0.360 & 0.111 & 0.027 & 0.019 & 0 \\ -1.200 & 0.208 & -0.794 & 0.053 & -1.000 \end{pmatrix} \begin{pmatrix} \xi^R_t \\ \xi^z_t \\ \xi^q_t \\ \xi^{y^*_t}_t \\ \xi^{\pi^*_t}_t \end{pmatrix}. \quad (3.20)$$

Figure 3.2 depicts the calculated impulse response function of the four endogenous variables to the structural shock with the size of one unit of the standard deviation offered in the table 3.2.

Holding everything else constant, a unit of the standard deviation of the monetary policy shock $\sigma_R = 0.311$ will exert a $-0.48 * \sigma_R = -0.149$ impact on the real output deviation, a $-1.2*\sigma_R = -0.373$ impact on the inflation,a $0.36*\sigma_R = 0.112$ impact on the nominal interest rate deviation, a $-1.2*\sigma_R = -0.373$ impact on the nominal exchange rate depreciation.

Holding everything else constant, a unit of the standard deviation of the shock to the change rate of the terms of trade $\sigma_q = 1.925$ will exert a $0.061*\sigma_q = 0.117$ impact on the real output deviation, a $0.05*\sigma_q = 0.096$ impact on the inflation,a $0.027*\sigma_q = 0.052$ impact on the nominal interest rate deviation, a $-0.794*\sigma_q = -1.529$ impact on the nominal exchange rate depreciation.

Holding everything else constant, a unit of the standard deviation of the shock to the change rate of the technology $\sigma_z = 1.775$ will exert a $0.127*\sigma_z = 0.226$ impact on the real output deviation, a $0.208*\sigma_z = 0.369$ impact on the inflation,a $0.111*\sigma_z = 0.198$ impact on the nominal interest rate deviation, a $0.208*\sigma_z = 0.369$ impact on the nominal exchange rate depreciation.

Holding everything else constant, a unit of the standard deviation of the shock to the world output deviation $\sigma_{y^*} = 0.838$ will exert a $-0.635*\sigma_{y^*} = -0.532$ impact on the real output deviation, a $0.053*\sigma_{y^*} = 0.044$ impact on the inflation,a $0.019*\sigma_{y^*} = 0.016$ impact on the nominal interest rate deviation, a $0.053*\sigma_{y^*} = 0.044$ impact on the nominal exchange rate depreciation.

Holding everything else constant, a unit of the standard deviation of the shock to the world inflation $\sigma_{\pi^*} = 2.414$ will only exert a $-1*\sigma_{\pi^*} = -2.414$ impact on the nominal exchange rate depreciation.

Table 3.4: Numerical Solutions for Canada in the Group One

		static		backward looking				mixed	forward looking		
		$\tilde{yy}_{t,n}$	$\triangle\tilde{e}_t$	$\tilde{r}_t$	z_t	$\tilde{yy}^*_t$	π^*_t	$\triangle\tilde{q}_t$	$\tilde{yy}_t$	π_t	$\triangle\tilde{yy}^*_t$
endogenous variables	$\tilde{r}_{t-1}$		-0.912	0.273					-0.365	-0.912	
	z_{t-1}		0.077	0.042	0.373				0.047	0.077	
	$\tilde{yy}^*_{t-1}$	-0.616	0.050	0.018		0.953			-0.606	0.050	-0.047
	π^*_{t-1}		-0.517				0.517				
	$\triangle\tilde{q}_{t-1}$		-0.466	0.016				0.587	0.036	0.029	
exogenous variables	ξ^R_t		-1.200	0.360					-0.480	-1.200	
	ξ^z_t		0.208	0.111	1.000				0.127	0.208	
	ξ^q_t		-0.794	0.027				1.000	0.061	0.050	
	$\xi^{y^*_t}_t$	-0.646	0.053	0.019		1.000			-0.635	0.053	1.000
	$\xi^{\pi^*_t}_t$		-1.000				1.000				

Note: This table reports the numerical solutions of the policy and transition functions for Canada in the Group One.

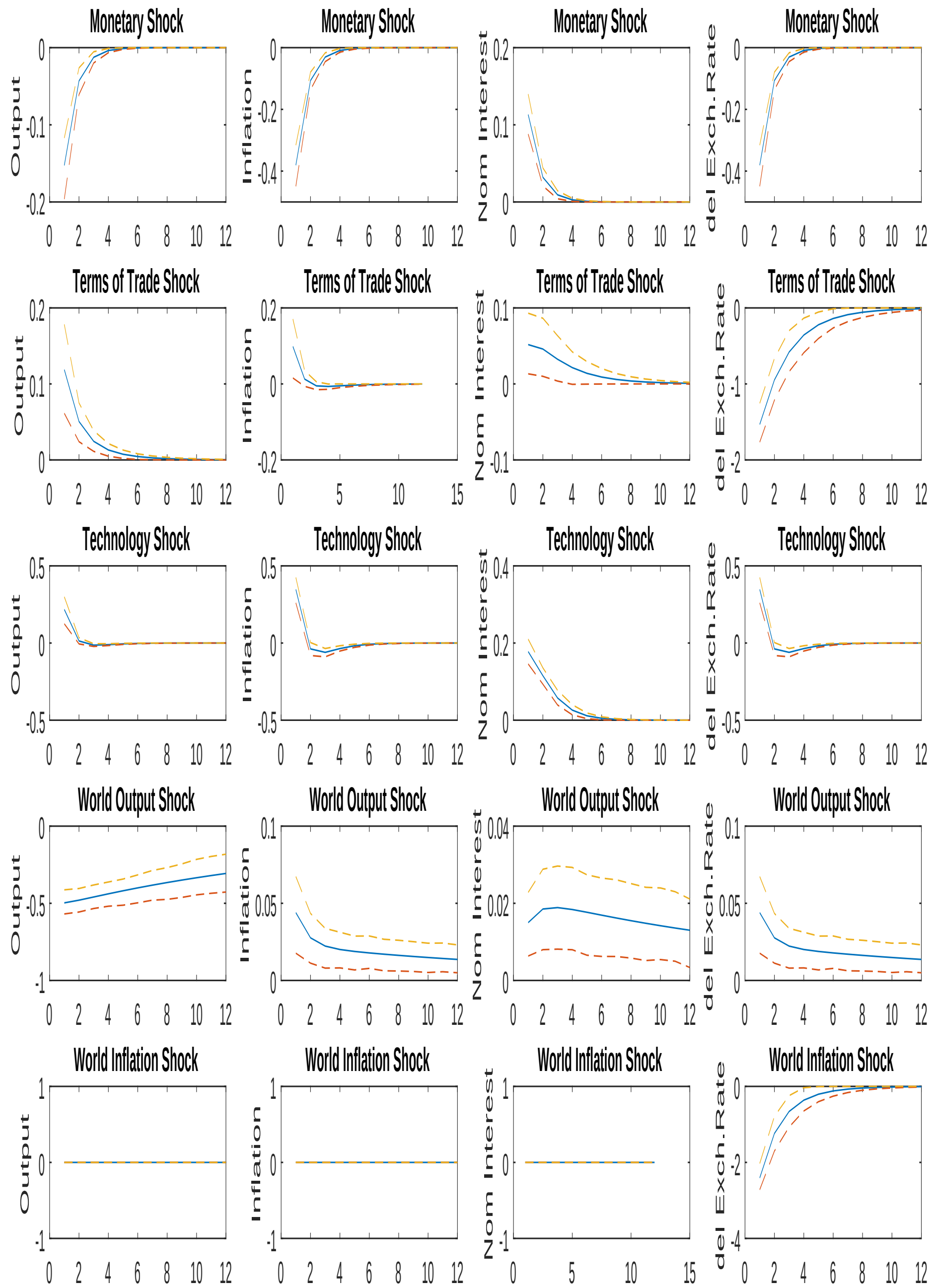

Figure 3.2: Impulse response functions for Group One in Canada. Note:The figure depicts the impulse response function of the real output,the inflation rate, the nominal interest rate and the depreciation exchange rate in Canada to one unit of the structural shocks.

3.3.2 Group Two: No Nominal Exchange Depreciation and Change of Output Deviation

Table 3.5 reports the estimation results of the second treatment group. The prior distributions for the treatment group are identical to the first treatment group, except there is one more prior setting for the coefficient of the change rate of the real output $\phi_{\triangle y}$, which centres at 0.125 with a gamma distribution. The posterior distributions between the second treatment group and the control group are not very different.

The log marginal data density of the second treatment group in the UK is -501.064. Thus, the posterior odds ratio is:

$$\gamma_{2,0}^{UK} = e^{(lnp(D_{1:t}^{UK}|M_2)-lnp(D_{1:t}^{UK}|M_0))}, \tag{3.21}$$

where M_2 represents the DSGE model in the second treatment group. The numerical results of the posterior odds ratio in the UK, $\gamma_{2,0}^{UK}$, is almost 1.22 billion, which is a huge number. This ratio supports that the DSGE model in the second treatment group fit the UK data much better than it in the control group. More specifically, the UK data is significantly in favour of the monetary policy reaction function without nominal exchange rate depreciation and with the change rate of real output.

The log marginal data density of the second treatment group in Canada is -585.955. Likewise, the posterior odds ratio is:

$$\gamma_{2,0}^{Canada} = e^{lnp(D_{1:t}^{Canada}|M_2)-lnp(D_{1:t}^{Canada}|M_0)}. \tag{3.22}$$

The numerical results of the posterior odds ratio in Canada, $\gamma_{2,0}^{Canada}$, is 1.247, which is just between $\frac{1}{3}$ and 3. This odds ratio supports that there is no major difference between the second treatment group and the control group in the performance of data fitting. More specifically, to put more weights on the stabilisation of the rate change of the output cannot offset the loss fitting arising from the ignorance of the movement of the nominal exchange rates for Canada.

Table 3.5: Constant Parameter Estimation Results (Group Two)

	Prior		Posterior			
	UK	Canada	UK		Canada	
Parameters	Mean	Mean	Mean	90%interval	Mean	90%interval
τ	0.500		0.308	[0.151,0.451]	0.306	[0.173,0.437]
κ	0.300		0.366	[0.174,0.547]	0.735	[0.416,1.042]
ϕ_π	1.500		2.249	[1.516,2.946]	2.020	[1.314,2.736]
ϕ_y	0.125		0.097	[0.043,0.147]	0.069	[0.028,0.109]
$\phi_{\triangle y}$	0.125		0.183	[0.133,0.234]	0.157	[0.081,0.233]
ρ_R	0.500		0.812	[0.749,0.875]	0.762	[0.677,0.853]
ρ_z	0.200		0.536	[0.411,0.663]	0.385	[0.284,0.487]
ρ_q	0.400		0.103	[0.010,0.193]	0.579	[0.452,0.717]
ρ_{π^*}	0.800		0.604	[0.448,0.775]	0.513	[0.353,0.667]
ρ_{y^*}	0.900		0.949	[0.916,0.986]	0.952	[0.918,0.988]
α	0.200		0.128	[0.082,0.175]	0.153	[0.098,0.204]
$r^{(A)}$	3.350	2.470	3.127	[2.674,3.594]	2.353	[1.780,2.898]
$\pi^{(A)}$	1.920	1.620	2.182	[1.633,2.753]	1.798	[1.202,2.394]
$\gamma^{(Q)}$	0.620	0.730	0.660	[0.595,0.722]	0.741	[0.674,0.810]
σ_R	0.500		0.149	[0.120,0.178]	0.299	[0.223,0.370]
σ_z	1.500	1.000	0.914	[0.433,1.409]	1.673	[0.814,2.561]
σ_{y^*}	1.500		1.262	[0.463,2.112]	0.859	[0.381,1.340]
σ_{π^*}	0.550		2.507	[2.132,2.858]	2.413	[2.040,2.765]
σ_q	1.500		1.212	[1.033,1.393]	1.920	[1.636,2.191]
Log MDD			-501.064		-585.955	

Note:The table reports the parameter estimation results of UK and Canada considering the monetary policy without the nominal exchange rate depreciation and with the change of the output deviation.

Numerical Solution and Simulation Results for the UK in the Group Two

Table 3.6 reports the numerical solutions for the UK in the second treatment group. According to the table, the transition function is:

$$
\begin{pmatrix} \tilde{r}_t \\ z_t \\ \tilde{yy}^*_t \\ \pi^*_t \\ \tilde{yy}_t \\ \triangle\tilde{q}_t \end{pmatrix}
=
\begin{pmatrix}
0.315 & 0.091 & -0.029 & 0 & -0.071 & -0.002 \\
0 & 0.536 & 0 & 0 & 0 & 0 \\
0 & 0 & 0.949 & 0 & 0 & 0 \\
0 & 0 & 0 & 0.604 & 0 & 0 \\
-0.621 & 0.151 & -0.427 & 0 & 0.140 & 0.005 \\
0 & 0 & 0 & 0 & 0 & 0.103
\end{pmatrix}
\begin{pmatrix} \tilde{r}_{t-1} \\ z_{t-1} \\ \tilde{yy}^*_{t-1} \\ \pi^*_{t-1} \\ \tilde{yy}_{t-1} \\ \triangle\tilde{q}_{t-1} \end{pmatrix}
+
\begin{pmatrix}
0.388 & 0.171 & -0.018 & -0.030 & 0 \\
0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 1 \\
-0.766 & 0.282 & 0.045 & -0.450 & 0 \\
0 & 0 & 1 & 0 & 0
\end{pmatrix}
\begin{pmatrix} \xi^R_t \\ \xi^z_t \\ \xi^q_t \\ \xi^{y^*_t}_t \\ \xi^{\pi^*_t}_t \end{pmatrix}, \quad (3.23)
$$

and the policy function is calculated as:

$$
\begin{pmatrix} \pi_t \\ \triangle\tilde{yy}^*_t \end{pmatrix}
=
\begin{pmatrix}
-0.876 & 0.144 & 0.135 & 0 & 0.198 & -0.006 \\
0 & 0 & -0.051 & 0 & 0 & 0
\end{pmatrix}
\begin{pmatrix} \tilde{r}_{t-1} \\ z_{t-1} \\ \tilde{yy}^*_{t-1} \\ \pi^*_{t-1} \\ \tilde{yy}_{t-1} \\ \triangle\tilde{q}_{t-1} \end{pmatrix}
+
\begin{pmatrix}
-1.079 & 0.268 & -0.063 & 0.142 & 0 \\
0 & 0 & 0 & 1 & 0
\end{pmatrix}
\begin{pmatrix} \xi^R_t \\ \xi^z_t \\ \xi^q_t \\ \xi^{y^*_t}_t \\ \xi^{\pi^*_t}_t \end{pmatrix}, \quad (3.24)
$$

and the function for the static variables is:

$$\begin{pmatrix} \tilde{yy}_{t,n} \\ \triangle\tilde{e}_t \end{pmatrix} = \begin{pmatrix} 0 & 0 & -0.513 & 0 & 0 & 0 \\ -0.876 & 0.144 & 0.135 & -0.604 & 0.198 & -0.096 \end{pmatrix} \begin{pmatrix} \tilde{r}_{t-1} \\ z_{t-1} \\ \tilde{yy}^*_{t-1} \\ \pi^*_{t-1} \\ \tilde{yy}_{t-1} \\ \triangle\tilde{q}_{t-1} \end{pmatrix}$$

$$+ \begin{pmatrix} 0 & 0 & 0 & -0.540 & 0 \\ -1.079 & 0.268 & -0.934 & 0.142 & -1.000 \end{pmatrix} \begin{pmatrix} \xi^R_t \\ \xi^z_t \\ \xi^q_t \\ \xi^{y^*_t}_t \\ \xi^{\pi^*_t}_t \end{pmatrix}. \quad (3.25)$$

Table 3.6 also computes the impulse response functions of the four endogenous variables including $\tilde{yy}_t$,π_t,$\tilde{r}_t$ and $\triangle\tilde{e}_t$:

$$\begin{pmatrix} \tilde{yy}_t \\ \pi_t \\ \tilde{r}_t \\ \triangle\tilde{e}_t \end{pmatrix} = \begin{pmatrix} -0.621 & 0.151 & -0.427 & 0 & 0.140 & 0.005 \\ -0.876 & 0.144 & 0.135 & 0 & 0.198 & -0.006 \\ 0.315 & 0.091 & -0.029 & 0 & -0.071 & -0.002 \\ -0.876 & 0.144 & 0.135 & -0.604 & 0.198 & -0.096 \end{pmatrix} \begin{pmatrix} \tilde{r}_{t-1} \\ z_{t-1} \\ \tilde{yy}^*_{t-1} \\ \pi^*_{t-1} \\ \tilde{yy}_{t-1} \\ \triangle\tilde{q}_{t-1} \end{pmatrix}$$

$$+ \begin{pmatrix} -0.766 & 0.282 & 0.045 & -0.450 & 0 \\ -1.079 & 0.268 & -0.063 & 0.142 & 0 \\ 0.388 & 0.171 & -0.018 & -0.030 & 0 \\ -1.079 & 0.268 & -0.934 & 0.142 & -1.000 \end{pmatrix} \begin{pmatrix} \xi^R_t \\ \xi^z_t \\ \xi^q_t \\ \xi^{y^*_t}_t \\ \xi^{\pi^*_t}_t \end{pmatrix}. \quad (3.26)$$

Figure 3.3 depicts the calculated impulse response functions of the four endogenous variables to the structural shock with the size of one unit of the standard deviation offered in the table 3.5.

Holding everything else constant, a unit of the standard deviation of the monetary policy shock $\sigma_R = 0.149$ will exert a $-0.766 * \sigma_R = -0.114$ impact on the real output deviation, a $-1.079 * \sigma_R = -0.161$ impact on the inflation,a $0.388 * \sigma_R = 0.058$ impact on the nominal interest rate deviation, a $-1.079 * \sigma_R = -0.161$ impact on the nominal exchange rate depreciation.

Holding everything else constant, a unit of the standard deviation of the shock to the change rate of the terms of trade $\sigma_q = 1.212$ will exert a $0.045 * \sigma_q = 0.054$ impact on the real output deviation, a $-0.063 * \sigma_q = -0.076$ impact on the inflation,a $-0.018 * \sigma_q = -0.021$ impact on the nominal interest rate deviation, a $-0.934 * \sigma_q = -1.133$ impact on the nominal exchange rate depreciation.

Holding everything else constant, a unit of the standard deviation of the shock to the change rate of the technology $\sigma_z = 0.914$ will exert a $0.282 * \sigma_z = 0.258$ impact on the real output deviation, a $0.268 * \sigma_z = 0.245$ impact on the inflation,a $0.171 * \sigma_z = 0.156$ impact on the nominal interest rate deviation, a $0.268 * \sigma_z = 0.245$ impact on the nominal exchange rate depreciation.

Holding everything else constant, a unit of the standard deviation of the shock to the world output deviation $\sigma_{y^*} = 1.262$ will exert a $-0.450 * \sigma_{y^*} = -0.568$ impact on the real output deviation, a $0.142 * \sigma_{y^*} = 0.179$ impact on the inflation,a $-0.03 * \sigma_{y^*} = -0.038$ impact on the nominal interest rate deviation, a $0.142 * \sigma_{y^*} = 0.179$ impact on the nominal exchange rate depreciation.

Holding everything else constant, a unit of the standard deviation of the shock to the world inflation $\sigma_{\pi^*} = 2.507$ will only exert a $-1 * \sigma_{\pi^*} = -2.507$ impact on the nominal exchange rate depreciation.

Table 3.6: Numerical Solutions for the UK in the Group Two

		static		backward looking				mixed		forward looking	
		$\tilde{yy}_{t,n}$	$\triangle\tilde{e}_t$	$\tilde{r}_t$	z_t	$\tilde{yy}^*_t$	π^*_t	$\triangle\tilde{q}_t$	$\tilde{yy}_t$	π_t	$\triangle\tilde{yy}^*_t$
endogenous variables	$\tilde{r}_{t-1}$		-0.876	0.315					-0.621	-0.876	
	z_{t-1}		0.144	0.091	0.536				0.151	0.144	
	$\tilde{yy}^*_{t-1}$	-0.513	0.135	-0.029		0.949			-0.427	0.135	-0.051
	π^*_{t-1}		-0.604				0.604				
	$\tilde{yy}_{t-1}$		0.198	-0.071					0.140	0.198	
	$\triangle\tilde{q}_{t-1}$		-0.096	-0.002				0.103	0.005	-0.006	
exogenous variables	ξ^R_t		-1.079	0.388					-0.766	-1.079	
	ξ^z_t		0.268	0.171	1.000				0.282	0.268	
	ξ^q_t		-0.934	-0.018				1.000	0.045	-0.063	
	$\xi^{y^*_t}_t$	-0.540	0.142	-0.030		1.000			-0.450	0.142	1.000
	$\xi^{\pi^*_t}_t$		-1.000				1.000				

Note: This table reports the numerical solutions of the policy and transition functions for the UK in the Group Two.

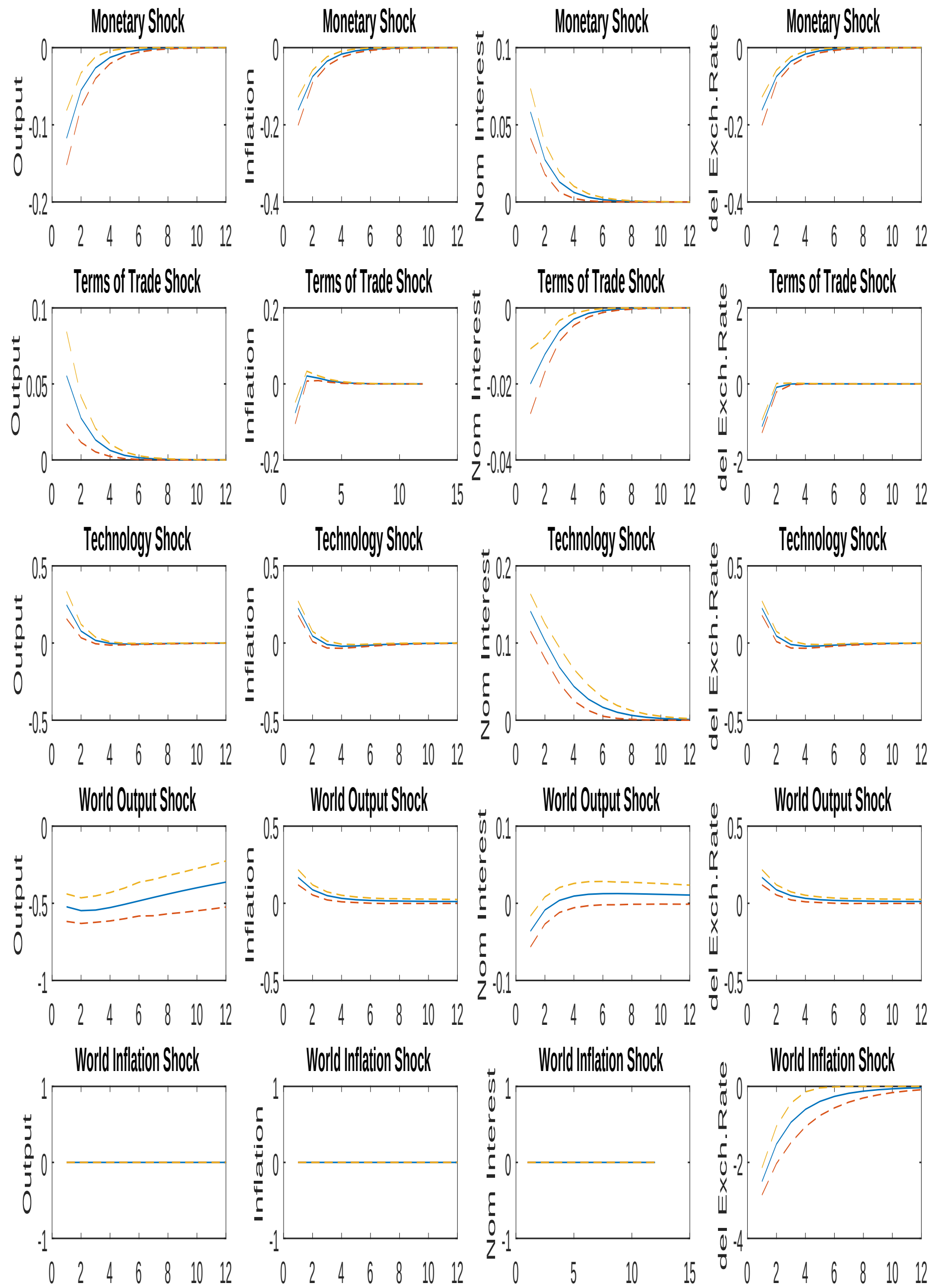

Figure 3.3: Impulse response functions for Group Two in the UK. Note:The figure depicts the impulse response function of the real output,the inflation rate,the nominal interest rate and the depreciation exchange rate in the UK to one unit of the structural shocks.

Numerical Solution and Simulation Results for Canada in the Group Two

Table 3.7 reports the numerical solutions for Canada in the second treatment group. According to the table, the computed transition function is:

$$\begin{pmatrix} \tilde{r}_t \\ z_t \\ \tilde{yy}_t^* \\ \pi_t^* \\ \tilde{yy}_t \\ \triangle\tilde{q}_t \end{pmatrix} = \begin{pmatrix} 0.248 & 0.047 & -0.020 & 0 & -0.051 & 0.017 \\ 0 & 0.385 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.952 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.513 & 0 & 0 \\ -0.419 & 0.056 & -0.547 & 0 & 0.086 & 0.037 \\ 0 & 0 & 0 & 0 & 0 & 0.579 \end{pmatrix} \begin{pmatrix} \tilde{r_{t-1}} \\ z_{t-1} \\ \tilde{yy}_{t-1}^* \\ \pi_{t-1}^* \\ \tilde{yy_{t-1}} \\ \triangle\tilde{q_{t-1}} \end{pmatrix}$$

$$+ \begin{pmatrix} 0.326 & 0.123 & 0.029 & -0.021 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ -0.549 & 0.144 & 0.064 & -0.574 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} \xi_t^R \\ \xi_t^z \\ \xi_t^q \\ \xi_t^{y_t^*} \\ \xi_t^{\pi_t^*} \end{pmatrix}, \quad (3.27)$$

and the policy function is:

$$\begin{pmatrix} \pi_t \\ \triangle\tilde{yy}_t^* \end{pmatrix} = \begin{pmatrix} -0.918 & 0.079 & 0.155 & 0 & 0.189 & 0.021 \\ 0 & 0 & -0.048 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \tilde{r_{t-1}} \\ z_{t-1} \\ \tilde{yy}_{t-1}^* \\ \pi_{t-1}^* \\ \tilde{yy_{t-1}} \\ \triangle\tilde{q_{t-1}} \end{pmatrix}$$

$$+ \begin{pmatrix} -1.205 & 0.204 & 0.036 & 0.163 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} \xi_t^R \\ \xi_t^z \\ \xi_t^q \\ \xi_t^{y_t^*} \\ \xi_t^{\pi_t^*} \end{pmatrix}, \quad (3.28)$$

and the function for the static variables is

$$\begin{pmatrix} \tilde{yy}_{t,n} \\ \triangle\tilde{e}_t \end{pmatrix} = \begin{pmatrix} 0 & 0 & -0.610 & 0 & 0 & 0 \\ -0.918 & 0.079 & 0.155 & -0.513 & 0.189 & -0.469 \end{pmatrix} \begin{pmatrix} \tilde{r}_{t-1} \\ z_{t-1} \\ \tilde{yy}^*_{t-1} \\ \pi^*_{t-1} \\ \tilde{yy}_{t-1} \\ \triangle\tilde{q}_{t-1} \end{pmatrix}$$

$$+ \begin{pmatrix} 0 & 0 & 0 & -0.641 & 0 \\ -1.205 & 0.204 & -0.811 & 0.163 & -1.000 \end{pmatrix} \begin{pmatrix} \xi^R_t \\ \xi^z_t \\ \xi^q_t \\ \xi^{y^*_t}_t \\ \xi^{\pi^*_t}_t \end{pmatrix}. \quad (3.29)$$

Table 3.7 also provides information to compute the impulse response functions of the four endogenous variables including $\tilde{yy}_t$,π_t,$\tilde{r}_t$ and $\triangle\tilde{e}_t$:

$$\begin{pmatrix} \tilde{yy}_t \\ \pi_t \\ \tilde{r}_t \\ \triangle\tilde{e}_t \end{pmatrix} = \begin{pmatrix} -0.419 & 0.056 & -0.547 & 0 & 0.086 & 0.037 \\ -0.918 & 0.079 & 0.155 & 0 & 0.189 & 0.021 \\ 0.248 & 0.047 & -0.020 & 0 & -0.051 & 0.017 \\ -0.918 & 0.079 & 0.155 & -0.513 & 0.189 & -0.469 \end{pmatrix} \begin{pmatrix} \tilde{r}_{t-1} \\ z_{t-1} \\ \tilde{yy}^*_{t-1} \\ \pi^*_{t-1} \\ \tilde{yy}_{t-1} \\ \triangle\tilde{q}_{t-1} \end{pmatrix}$$

$$+ \begin{pmatrix} -0.549 & 0.144 & 0.064 & -0.574 & 0 \\ -1.205 & 0.204 & 0.036 & 0.163 & 0 \\ 0.326 & 0.123 & 0.029 & -0.021 & 0 \\ -1.205 & 0.204 & -0.811 & 0.163 & -1.000 \end{pmatrix} \begin{pmatrix} \xi^R_t \\ \xi^z_t \\ \xi^q_t \\ \xi^{y^*_t}_t \\ \xi^{\pi^*_t}_t \end{pmatrix}. \quad (3.30)$$

Figure 3.4 depicts the calculated impulse response function of the four endogenous variables to the structural shock with the size of one unit of the standard deviation offered in the table 3.5.

Holding everything else constant, a unit of the standard deviation of the monetary policy shock $\sigma_R = 0.299$ will exert a $-0.549 * \sigma_R = -0.164$ impact on the real output deviation, a $-1.205 * \sigma_R = -0.360$ impact on the inflation,a $0.326 * \sigma_R = 0.097$ impact on the nominal interest rate deviation, a $-1.205 * \sigma_R = -0.360$ impact on the nominal exchange rate depreciation.

Holding everything else constant, a unit of the standard deviation of the shock to the change rate of the terms of trade $\sigma_q = 1.920$ will exert a $0.064 * \sigma_q = 0.124$ impact on the real output deviation, a $0.036 * \sigma_q = 0.070$ impact on the inflation,a $0.029 * \sigma_q = 0.055$ impact on the nominal interest rate deviation, a $-0.811 * \sigma_q = -1.556$ impact on the nominal exchange rate depreciation.

Holding everything else constant, a unit of the standard deviation of the shock to the change rate of the technology $\sigma_z = 1.673$ will exert a $0.144 * \sigma_z = 0.241$ impact on the real output deviation, a $0.204 * \sigma_z = 0.342$ impact on the inflation,a $0.123 * \sigma_z = 0.206$ impact on the nominal interest rate deviation, a $0.204 * \sigma_z = 0.342$ impact on the nominal exchange rate depreciation.

Holding everything else constant, a unit of the standard deviation of the shock to the world output deviation $\sigma_{y^*} = 0.859$ will exert a $-0.574 * \sigma_{y^*} = -0.493$ impact on the real output deviation, a $0.163 * \sigma_{y^*} = 0.140$ impact on the inflation,a $-0.021 * \sigma_{y^*} = -0.018$ impact on the nominal interest rate deviation, a $0.163 * \sigma_{y^*} = 0.140$ impact on the nominal exchange rate depreciation.

Holding everything else constant, a unit of the standard deviation of the shock to the world inflation $\sigma_{\pi^*} = 2.413$ only exert a $-1 * \sigma_{\pi^*} = -2.413$ impact on the nominal exchange rate depreciation.

Table 3.7: Numerical Solutions for Canada in the Group Two

		static		backward looking				mixed		forward looking	
		$\tilde{yy}_{t,n}$	$\triangle\tilde{e}_t$	$\tilde{r}_t$	z_t	$\tilde{yy}^*_t$	π^*_t	$\triangle\tilde{q}_t$	$\tilde{yy}_t$	π_t	$\triangle\tilde{yy}^*_t$
endogenous variables	$\tilde{r}_{t-1}$		-0.918	0.248					-0.419	-0.918	
	z_{t-1}		0.079	0.047	0.385				0.056	0.079	
	$\tilde{yy}^*_{t-1}$	-0.610	0.155	-0.020		0.952			-0.547	0.155	-0.048
	π^*_{t-1}		-0.513				0.513				
	$\tilde{yy}_{t-1}$		0.189	-0.051					0.086	0.189	
	$\triangle\tilde{q}_{t-1}$		-0.469	0.017				0.579	0.037	0.021	
exogenous variables	ξ^R_t		-1.205	0.326					-0.549	-1.205	
	ξ^z_t		0.204	0.123	1.000				0.144	0.204	
	ξ^q_t		-0.811	0.029				1.000	0.064	0.036	
	$\xi^{y^*_t}_t$	-0.641	0.163	-0.021		1.000			-0.574	0.163	1.000
	$\xi^{\pi^*_t}_t$		-1.000				1.000				

Note: This table reports the numerical solutions of the policy and transition functions for Canada in the Group Two.

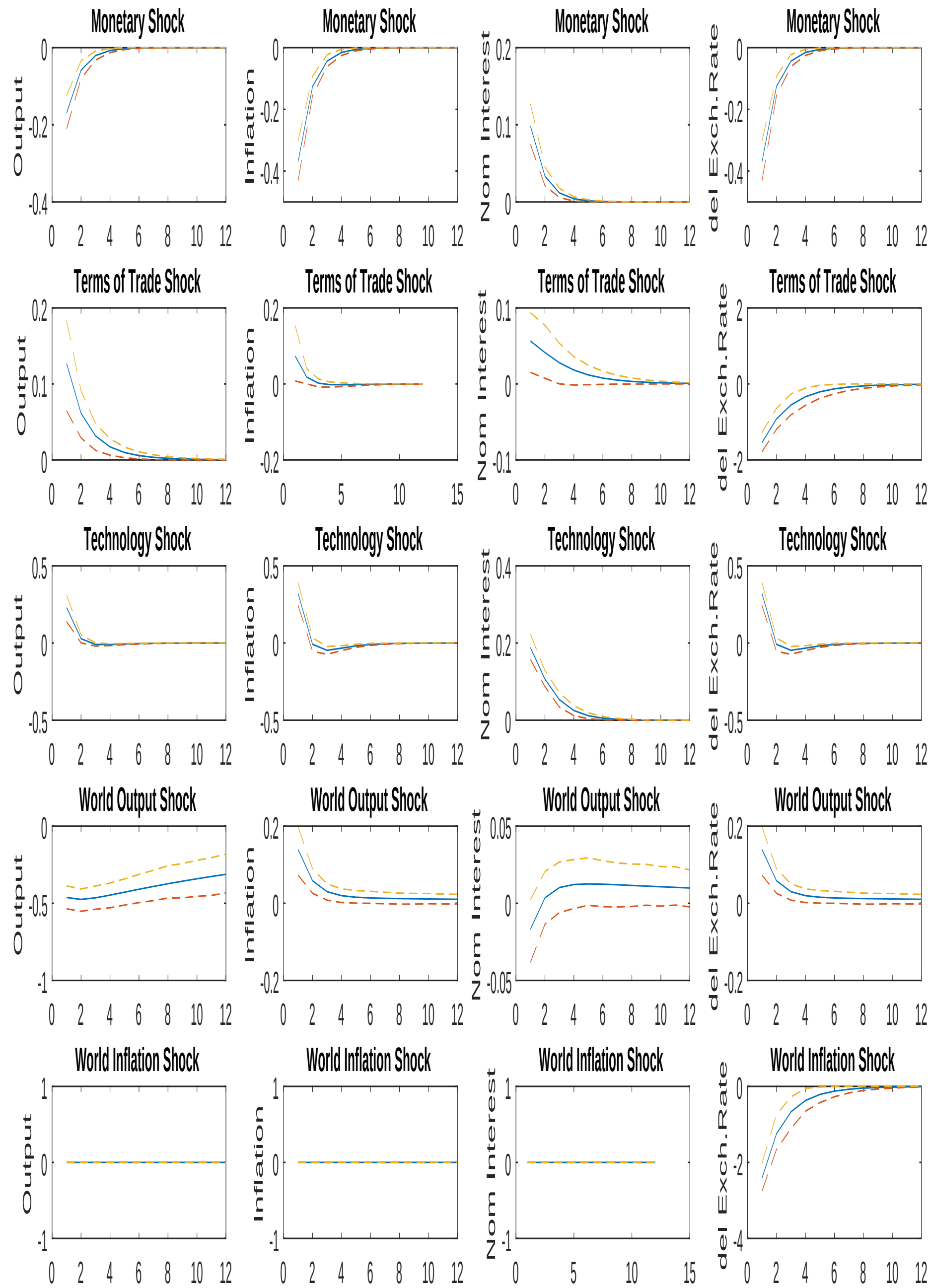

Figure 3.4: Impulse response functions for Group Two in Canada. Note:The figure depicts the impulse response function of the real output,the inflation rate,the nominal interest rate and the depreciation exchange rate in Canada to one unit of the structural shocks.

3.3.3 Group Three: Nominal Exchange Depreciation and Change of Output Deviation

Table 3.8 reports the estimation results of the third treatment group. The prior distributions for the treatment group are identical to the first control group, except there is one more prior setting for the coefficient of the change rate of real output $\phi_{\triangle y}$, which centres at 0.125 with a gamma distribution. The posterior distributions of the third treatment group are still not very different from those of control groups for their common shared parameters.

The log marginal data density of the third treatment group in the UK is -503.067. Thus, the posterior odds ratio is:

$$\gamma_{3,0}^{UK} = e^{(lnp(D_{1:t}^{UK}|M_3) - lnp(D_{1:t}^{UK}|M_0))}, \tag{3.31}$$

where M_3 represents the DSGE model in the third treatment group. The numerical results of the posterior odds ratio in the UK, $\gamma_{3,0}^{UK}$, is almost 0.283 billion, which is very large. This ratio supports that the DSGE model in the third treatment group fit the UK data much better than it in the control group. More specifically, to put more weights on the stabilisation of the change rate of the output in the policy decision fits the UK data much better.

The log marginal data density of the third treatment group in Canada is -582.077. Likewise, the posterior odds ratio is:

$$\gamma_{3,0}^{Canada} = e^{lnp(D_{1:t}^{Canada}|M_3) - lnp(D_{1:t}^{Canada}|M_0)}. \tag{3.32}$$

The numerical results of the posterior odds ratio in Canada, $\gamma_{3,0}^{Canada}$, is 60.280, which is much bigger than 3. This ratio supports that there the DSGE model in the third treatment group fit the data from Canada much better. More specifically, the Canadian data is in favour of the policy reaction function with the nominal exchange rate depreciation and the rate change of the real output.

Table 3.8: Constant Parameter Estimation Results (Group Three)

	Prior		Posterior			
	UK	Canada	UK		Canada	
Parameters	Mean	Mean	Mean	90%interval	Mean	90%interval
τ	0.500		0.338	[0.165,0.518]	0.282	[0.161,0.406]
κ	0.300		0.348	[0.166,0.520]	0.669	[0.372,0.953]
ϕ_π	1.500		2.412	[1.626,3.147]	1.945	[1.304,2.567]
ϕ_y	0.125		0.112	[0.054,0.170]	0.078	[0.034,0.121]
$\phi_{\triangle y}$	0.125		0.200	[0.143,0.258]	0.156	[0.080,0.230]
$\phi_{\triangle e}$	0.125		0.068	[0.029,0.105]	0.128	[0.067,0.188]
ρ_R	0.500		0.827	[0.768,0.886]	0.765	[0.687,0.849]
ρ_z	0.200		0.530	[0.395,0.659]	0.387	[0.290,0.486]
ρ_q	0.400		0.107	[0.010,0.202]	0.538	[0.422,0.658]
ρ_{π^*}	0.800		0.590	[0.441,0.740]	0.441	[0.310,0.570]
ρ_{y^*}	0.900		0.952	[0.918,0.987]	0.955	[0.923,0.987]
α	0.200		0.124	[0.077,0.170]	0.140	[0.085,0.187]
$r^{(A)}$	3.350	2.470	3.106	[2.619,3.584]	2.335	[1.790,2.899]
$\pi^{(A)}$	1.920	1.620	2.188	[1.615,2.800]	1.835	[1.202,2.442]
$\gamma^{(Q)}$	0.620	0.730	0.657	[0.592,0.721]	0.732	[0.668,0.795]
σ_R	0.500		0.155	[0.124,0.185]	0.284	[0.214,0.349]
σ_z	1.500	1.000	0.953	[0.417,1.469]	1.712	[0.850,2.567]
σ_{y^*}	1.500		1.590	[0.453,2.964]	0.833	[0.390,1.281]
σ_{π^*}	0.550		2.499	[2.134,2.863]	2.366	[2.016,2.713]
σ_q	1.500		1.215	[1.032,1.392]	1.916	[1.647,2.187]
Log MDD			-503.067		-582.077	

Note:The table reports the parameter estimation results of UK and Canada considering the monetary policy with the nominal exchange rate depreciation and the change of the output deviation.

Numerical Solution and Simulation Results for the UK in the Group Three

Table 3.9 reports the numerical solution for the UK in the third treatment group. According to the table, the transition function is:

$$\begin{pmatrix} \tilde{r}_t \\ z_t \\ \tilde{yy}^*_t \\ \pi^*_t \\ \tilde{yy}_t \\ \triangle\tilde{q}_t \end{pmatrix} = \begin{pmatrix} 0.308 & 0.089 & -0.025 & 0.002 & -0.074 & -0.002 \\ 0 & 0.530 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.952 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.590 & 0 & 0 \\ -0.661 & 0.151 & -0.351 & 0.009 & 0.160 & 0.006 \\ 0 & 0 & 0 & 0 & 0 & 0.107 \end{pmatrix} \begin{pmatrix} \tilde{r_{t-1}} \\ z_{t-1} \\ \tilde{yy^*_{t-1}} \\ \pi^*_{t-1} \\ \tilde{yy_{t-1}} \\ \triangle\tilde{q_{t-1}} \end{pmatrix}$$

$$+ \begin{pmatrix} 0.372 & 0.168 & -0.019 & -0.027 & 0.004 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ -0.800 & 0.286 & 0.055 & -0.369 & 0.016 \\ 0 & 0 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} \xi^R_t \\ \xi^z_t \\ \xi^q_t \\ \xi^{y^*_t}_t \\ \xi^{\pi^*_t}_t \end{pmatrix}, \quad (3.33)$$

and the calculated policy function is:

$$\begin{pmatrix} \pi_t \\ \triangle\tilde{yy}^*_t \end{pmatrix} = \begin{pmatrix} -0.872 & 0.130 & 0.120 & 0.016 & 0.211 & -0.005 \\ 0 & 0 & -0.048 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \tilde{r_{t-1}} \\ z_{t-1} \\ \tilde{yy^*_{t-1}} \\ \pi^*_{t-1} \\ \tilde{yy_{t-1}} \\ \triangle\tilde{q_{t-1}} \end{pmatrix}$$

$$+ \begin{pmatrix} -1.055 & 0.246 & -0.049 & 0.127 & 0.028 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} \xi^R_t \\ \xi^z_t \\ \xi^q_t \\ \xi^{y^*_t}_t \\ \xi^{\pi^*_t}_t \end{pmatrix}, \quad (3.34)$$

and the function for the static variables is

$$\begin{pmatrix} y\tilde{y}_{t,n} \\ \triangle\tilde{e}_t \end{pmatrix} = \begin{pmatrix} 0 & 0 & -0.434 & 0 & 0 & 0 \\ -0.872 & 0.130 & 0.120 & -0.574 & 0.211 & -0.099 \end{pmatrix} \begin{pmatrix} \tilde{r_{t-1}} \\ z_{t-1} \\ \tilde{yy^*_{t-1}} \\ \pi^*_{t-1} \\ \tilde{yy_{t-1}} \\ \triangle\tilde{q_{t-1}} \end{pmatrix}$$

$$+ \begin{pmatrix} 0 & 0 & 0 & -0.456 & 0 \\ -1.055 & 0.246 & -0.925 & 0.127 & -0.972 \end{pmatrix} \begin{pmatrix} \xi^R_t \\ \xi^z_t \\ \xi^q_t \\ \xi^{y^*_t}_t \\ \xi^{\pi^*_t}_t \end{pmatrix}. \quad (3.35)$$

Table 3.9 also provides information to compute the impulse response functions of the four endogenous variables including $y\tilde{y}_t$,π_t,$\tilde{r}_t$ and $\triangle\tilde{e}_t$:

$$\begin{pmatrix} y\tilde{y}_t \\ \pi_t \\ \tilde{r}_t \\ \triangle\tilde{e}_t \end{pmatrix} = \begin{pmatrix} -0.661 & 0.151 & -0.351 & 0.009 & 0.160 & 0.006 \\ -0.872 & 0.130 & 0.120 & 0.016 & 0.211 & -0.005 \\ 0.308 & 0.089 & -0.025 & 0.002 & -0.074 & -0.002 \\ -0.872 & 0.130 & 0.120 & -0.574 & 0.211 & -0.099 \end{pmatrix} \begin{pmatrix} \tilde{r_{t-1}} \\ z_{t-1} \\ \tilde{yy^*_{t-1}} \\ \pi^*_{t-1} \\ \tilde{yy_{t-1}} \\ \triangle\tilde{q_{t-1}} \end{pmatrix}$$

$$+ \begin{pmatrix} -0.800 & 0.286 & 0.055 & -0.369 & 0.016 \\ -1.055 & 0.246 & -0.049 & 0.127 & 0.028 \\ 0.372 & 0.168 & -0.019 & -0.027 & 0.004 \\ -1.055 & 0.246 & -0.925 & 0.127 & -0.972 \end{pmatrix} \begin{pmatrix} \xi^R_t \\ \xi^z_t \\ \xi^q_t \\ \xi^{y^*_t}_t \\ \xi^{\pi^*_t}_t \end{pmatrix}. \quad (3.36)$$

Figure 3.5 depicts the calculated impulse response function of the four endogenous variables to the structural shock with the size of one unit of standard deviation offered in the table 3.8.

Holding everything else constant, a unit of the standard deviation of the monetary policy shock $\sigma_R = 0.155$ will exert a $-0.800 * \sigma_R = -0.124$ impact on the real output deviation, a $-1.055 * \sigma_R = -0.164$ impact on the inflation,a $0.372 * \sigma_R = 0.058$ impact on the nominal interest rate deviation, a $-1.055 * \sigma_R = -0.164$ impact on the nominal exchange rate depreciation.

Holding everything else constant, a unit of the standard deviation of the shock to the change rate of the terms of trade $\sigma_q = 1.215$ will exert a $0.055 * \sigma_q = 0.067$ impact on the real output deviation, a $-0.049 * \sigma_q = -0.006$ impact on the inflation,a $-0.019 * \sigma_q = -0.023$ impact on the nominal interest rate deviation, a $-0.925 * \sigma_q = -1.124$ impact on the nominal exchange rate depreciation.

Holding everything else constant, a unit of the standard deviation of the shock to the change rate of the technology $\sigma_z = 0.953$ will exert a $0.286 * \sigma_z = 0.272$ impact on the real output deviation, a $0.246 * \sigma_z = 0.234$ impact on the inflation,a $0.168 * \sigma_z = 0.160$ impact on the nominal interest rate deviation, a $0.246 * \sigma_z = 0.234$ impact on the nominal exchange rate depreciation.

Holding everything else constant, a unit of the standard deviation of the shock to the world output deviation $\sigma_{y^*} = 1.590$ will exert a $-0.369 * \sigma_{y^*} = -0.587$ impact on the real output deviation, a $0.127 * \sigma_{y^*} = 0.201$ impact on the inflation,a $-0.027 * \sigma_{y^*} = -0.042$ impact on the nominal interest rate deviation, a $0.127 * \sigma_{y^*} = 0.2011$ impact on the nominal exchange rate depreciation.

Holding everything else constant, a unit of the standard deviation of the shock to the world inflation $\sigma_{\pi^*} = 2.499$ will exert a $0.016 * \sigma_{\pi^*} = 0.040$ impact on the real output deviation, a $0.028 * \sigma_{\pi^*} = 0.069$ impact on the inflation,a $0.004 * \sigma_{\pi^*} = 0.009$ impact on the nominal interest rate deviation, a $-0.972 * \sigma_{\pi^*} = -2.430$ impact on the nominal exchange rate depreciation.

Table 3.9: Numerical Solutions for the UK in the Group Three

		static		backward looking				mixed		forward looking	
		$y\tilde{y}_{t,n}$	$\triangle\tilde{e}_t$	$\tilde{r}_t$	z_t	$\tilde{yy}_t^*$	π_t^*	$\triangle\tilde{q}_t$	$y\tilde{y}_t$	π_t	$\triangle\tilde{yy}_t^*$
endogenous variables	$\tilde{r_{t-1}}$		-0.872	0.308					-0.661	-0.872	
	z_{t-1}		0.130	0.089	0.530				0.151	0.130	
	$\tilde{yy}_{t-1}^*$	-0.434	0.120	-0.025		0.952			-0.351	0.120	-0.048
	π_{t-1}^*		-0.574	0.002			0.590		0.009	0.016	
	$y\tilde{y}_{t-1}$		0.211	-0.074					0.160	0.211	
	$\triangle\tilde{q_{t-1}}$		-0.099	-0.002				0.107	0.006	-0.005	
exogenous variables	ξ_t^R		-1.055	0.372					-0.800	-1.055	
	ξ_t^z		0.246	0.168	1.000				0.286	0.246	
	ξ_t^q		-0.925	-0.019				1.000	0.055	-0.049	
	$\xi_t^{y_t^*}$	-0.456	0.127	-0.027		1.000			-0.369	0.127	1.000
	$\xi_t^{\pi_t^*}$		-0.972	0.004			1.000		0.016	0.028	

Note: This table reports the numerical solutions of the policy and transition functions for the UK in the Group Three.

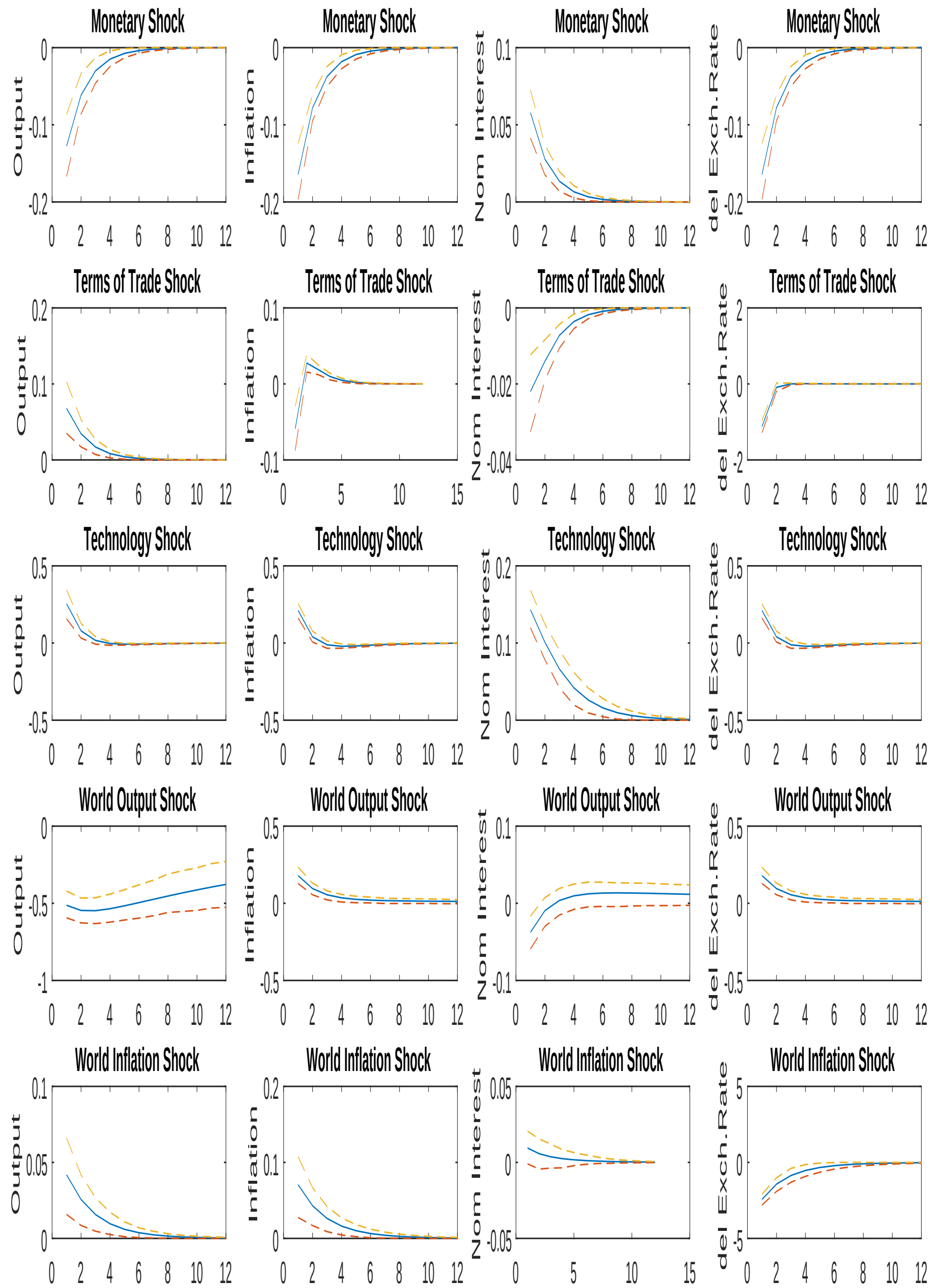

Figure 3.5: Impulse response functions for Group Three in the UK. Note:The figure depicts the impulse response function of the real output,the inflation rate,the nominal interest rate and the depreciation exchange rate in the UK to one unit of the structural shocks.

Numerical Solution and Simulation Results for Canada in the Group Three

Table 3.10 reports the numerical solution for Canada in the third treatment group. According to the table, the transition function is:

$$\begin{pmatrix} \tilde{r}_t \\ z_t \\ \tilde{yy}_t^* \\ \pi_t^* \\ \tilde{yy}_t \\ \triangle\tilde{q}_t \end{pmatrix} = \begin{pmatrix} 0.262 & 0.046 & -0.019 & 0.001 & -0.054 & 0.013 \\ 0 & 0.387 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.955 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.441 & 0 & 0 \\ -0.406 & 0.054 & -0.568 & 0.010 & 0.083 & 0.041 \\ 0 & 0 & 0 & 0 & 0 & 0.107 \end{pmatrix} \begin{pmatrix} \tilde{r_{t-1}} \\ z_{t-1} \\ \tilde{yy_{t-1}^*} \\ \pi_{t-1}^* \\ \tilde{yy_{t-1}} \\ \triangle \tilde{q_{t-1}} \end{pmatrix}$$
$$+ \begin{pmatrix} 0.343 & 0.120 & 0.025 & -0.020 & 0.003 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ -0.531 & 0.140 & 0.076 & -0.594 & 0.022 \\ 0 & 0 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} \xi_t^R \\ \xi_t^z \\ \xi_t^q \\ \xi_t^{y_t^*} \\ \xi_t^{\pi_t^*} \end{pmatrix}, \quad (3.37)$$

and the computed policy function is:

$$\begin{pmatrix} \pi_t \\ \triangle\tilde{yy}_t^* \end{pmatrix} = \begin{pmatrix} -0.883 & 0.076 & 0.163 & 0.026 & 0.181 & 0.041 \\ 0 & 0 & -0.045 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \tilde{r_{t-1}} \\ z_{t-1} \\ \tilde{yy_{t-1}^*} \\ \pi_{t-1}^* \\ \tilde{yy_{t-1}} \\ \triangle \tilde{q_{t-1}} \end{pmatrix}$$
$$+ \begin{pmatrix} -1.155 & 0.195 & 0.077 & 0.171 & 0.060 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} \xi_t^R \\ \xi_t^z \\ \xi_t^q \\ \xi_t^{y_t^*} \\ \xi_t^{\pi_t^*} \end{pmatrix}, \quad (3.38)$$

and the function for the static variables is

$$\begin{pmatrix} \tilde{yy}_{t,n} \\ \triangle\tilde{e}_t \end{pmatrix} = \begin{pmatrix} 0 & 0 & -0.633 & 0 & 0 & 0 \\ -0.883 & 0.076 & 0.163 & -0.414 & 0.181 & -0.421 \end{pmatrix} \begin{pmatrix} \tilde{r}_{t-1} \\ z_{t-1} \\ \tilde{yy}^*_{t-1} \\ \pi^*_{t-1} \\ \tilde{yy}_{t-1} \\ \triangle\tilde{q}_{t-1} \end{pmatrix}$$

$$+ \begin{pmatrix} 0 & 0 & 0 & -0.663 & 0 \\ -1.155 & 0.195 & -0.783 & 0.171 & -0.940 \end{pmatrix} \begin{pmatrix} \xi^R_t \\ \xi^z_t \\ \xi^q_t \\ \xi^{y^*_t}_t \\ \xi^{\pi^*_t}_t \end{pmatrix}. \quad (3.39)$$

Table 3.10 also computes the impulse response functions of the four endogenous variables including $\tilde{yy}_t$,π_t,$\tilde{r}_t$ and $\triangle\tilde{e}_t$:

$$\begin{pmatrix} \tilde{yy}_t \\ \pi_t \\ \tilde{r}_t \\ \triangle\tilde{e}_t \end{pmatrix} = \begin{pmatrix} -0.406 & 0.054 & -0.568 & 0.010 & 0.083 & 0.041 \\ -0.883 & 0.076 & 0.163 & 0.026 & 0.181 & 0.041 \\ 0.262 & 0.046 & -0.019 & 0.001 & -0.054 & 0.013 \\ -0.883 & 0.076 & 0.163 & -0.414 & 0.181 & -0.421 \end{pmatrix} \begin{pmatrix} \tilde{r}_{t-1} \\ z_{t-1} \\ \tilde{yy}^*_{t-1} \\ \pi^*_{t-1} \\ \tilde{yy}_{t-1} \\ \triangle\tilde{q}_{t-1} \end{pmatrix}$$

$$+ \begin{pmatrix} -0.531 & 0.140 & 0.076 & -0.594 & 0.022 \\ -1.155 & 0.195 & 0.077 & 0.171 & 0.060 \\ 0.343 & 0.120 & 0.025 & -0.020 & 0.003 \\ -1.155 & 0.195 & -0.783 & 0.171 & -0.940 \end{pmatrix} \begin{pmatrix} \xi^R_t \\ \xi^z_t \\ \xi^q_t \\ \xi^{y^*_t}_t \\ \xi^{\pi^*_t}_t \end{pmatrix}. \quad (3.40)$$

Figure 3.6 depicts the calculated impulse response function of the four endogenous variables to the structural shock with the size of one unit of standard deviation offered in the table 3.8.

Holding everything else constant, a unit of the standard deviation of the monetary policy shock $\sigma_R = 0.284$ will exert a $-0.531 * \sigma_R = -0.151$ impact on the real output deviation, a $-1.155 * \sigma_R = -0.328$ impact on the inflation,a $0.343 * \sigma_R = 0.097$ impact on the nominal interest rate deviation, a $-1.155 * \sigma_R = -0.328$ impact on the nominal exchange rate depreciation.

Holding everything else constant, a unit of the standard deviation of the shock to the change rate of the terms of trade $\sigma_q = 1.916$ will exert a $0.076 * \sigma_q = 0.146$ impact on the real output deviation, a $0.077 * \sigma_q = 0.147$ impact on the inflation,a $0.025 * \sigma_q = 0.048$ impact on the nominal interest rate deviation, a $-0.783 * \sigma_q = -1.501$ impact on the nominal exchange rate depreciation.

Holding everything else constant, a unit of the standard deviation of the shock to the change rate of the technology $\sigma_z = 1.712$ will exert a $0.140 * \sigma_z = 0.240$ impact on the real output deviation, a $0.195 * \sigma_z = 0.334$ impact on the inflation,a $0.120 * \sigma_z = 0.205$ impact on the nominal interest rate deviation, a $0.195 * \sigma_z = 0.334$ impact on the nominal exchange rate depreciation.

Holding everything else constant, a unit of the standard deviation of the shock to the world output deviation $\sigma_{y^*} = 0.833$ will exert a $-0.594 * \sigma_{y^*} = -0.495$ impact on the real output deviation, a $0.171 * \sigma_{y^*} = 0.142$ impact on the inflation,a $-0.020 * \sigma_{y^*} = -0.017$ impact on the nominal interest rate deviation, a $0.171 * \sigma_{y^*} = 0.142$ impact on the nominal exchange rate depreciation.

Holding everything else constant, a unit of the standard deviation of the shock to the world inflation $\sigma_{\pi^*} = 2.366$ will exert a $0.022 * \sigma_{\pi^*} = 0.053$ impact on the real output deviation, a $0.060 * \sigma_{\pi^*} = 0.141$ impact on the inflation,a $0.003 * \sigma_{\pi^*} = 0.007$ impact on the nominal interest rate deviation, a $-0.940 * \sigma_{\pi^*} = -2.225$ impact on the nominal exchange rate depreciation.

Table 3.10: Numerical Solutions for Canada in the Group Three

		static		backward looking				mixed		forward looking	
		$\tilde{yy}_{t,n}$	$\triangle\tilde{e}_t$	$\tilde{r}_t$	z_t	$\tilde{yy}^*_t$	π^*_t	$\triangle\tilde{q}_t$	$\tilde{yy}_t$	π_t	$\triangle\tilde{yy}^*_t$
endogenous variables	$\tilde{r}_{t-1}$		-0.883	0.262					-0.406	-0.883	
	z_{t-1}		0.076	0.046	0.387				0.054	0.076	
	$\tilde{yy}^*_{t-1}$	-0.633	0.163	-0.019		0.955			-0.568	0.163	-0.045
	π^*_{t-1}		-0.414	0.001			0.441		0.010	0.026	
	$\tilde{yy}_{t-1}$		0.181	-0.054					0.083	0.181	
	$\triangle\tilde{q}_{t-1}$		-0.421	0.013				0.538	0.041	0.041	
exogenous variables	ξ^R_t		-1.155	0.343					-0.531	-1.155	
	ξ^z_t		0.195	0.120	1.000				0.140	0.195	
	ξ^q_t		-0.783	0.025				1.000	0.076	0.077	
	$\xi^{y^*_t}_t$	-0.663	0.171	-0.020		1.000			-0.594	0.171	1.000
	$\xi^{\pi^*_t}_t$		-0.940	0.003			1.000		0.022	0.060	

Note: This table reports the numerical solutions of the policy and transition functions for Canada in the Group Three.

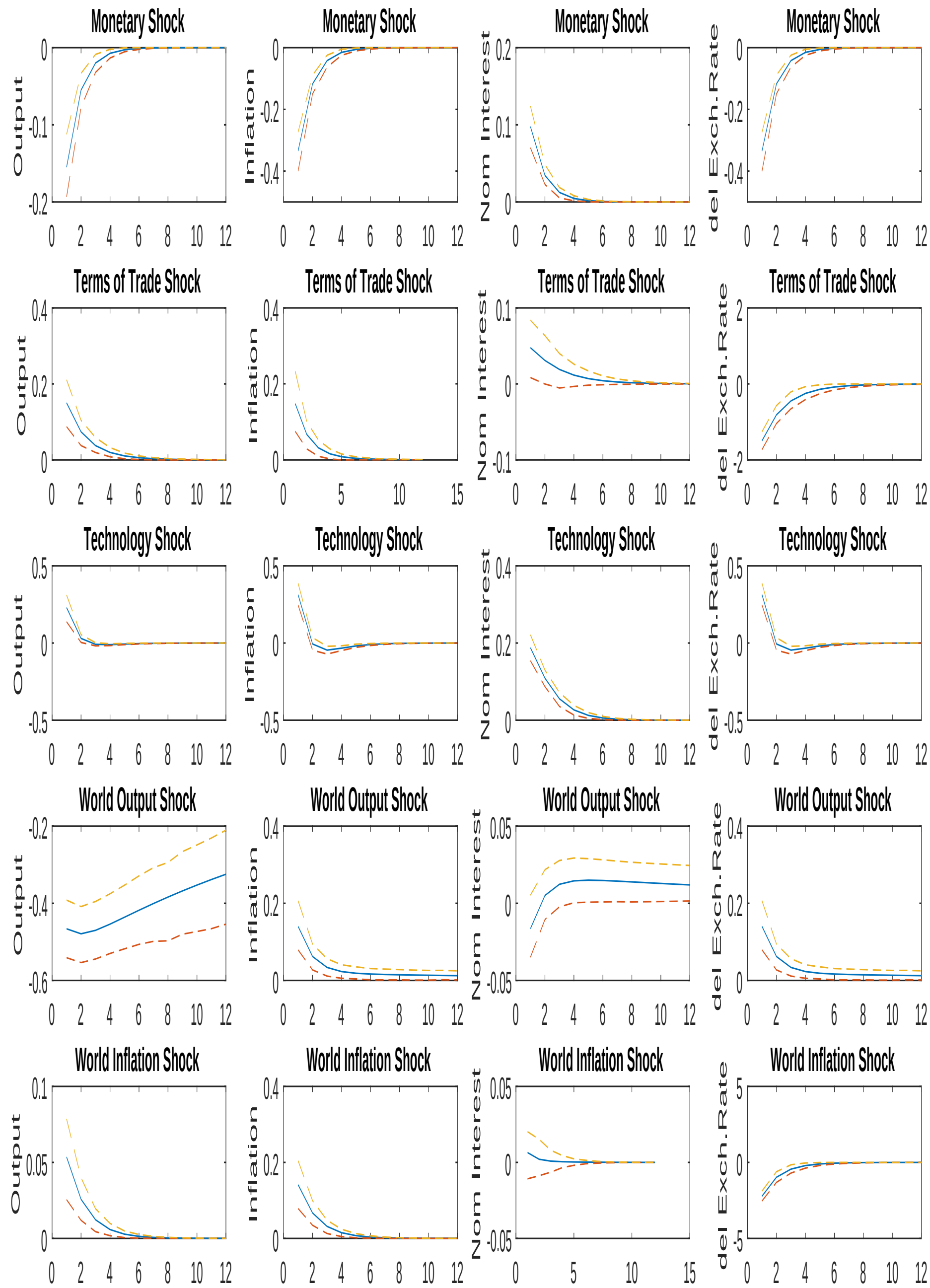

Figure 3.6: Impulse response functions for Group Three in Canada. Note:The figure depicts the impulse response function of the real output,the inflation rate,the nominal interest rate and the depreciation exchange rate in Canada to one unit of the structural shocks.

3.3.4 An Overall Remark of the Model Comparison at the First Stage

Table 3.11 reports the numerical results of the posterior odds ratios generated from the estimation results of the control group and the three treatment groups.

From the first three columns in the row of the UK, it is apparent to see all the three treatment groups perform better than the control group in terms of data fitting. Notably, the second treatment group enhance the data performance most among the three treatment groups. The posterior ratios imply that when it considers no exchange rate depreciation or put the rate change of real output into consideration, the relevant DSGE model will fit the UK data much better. From the next two columns in the same row, it is also apparent to see the second and the third treatment model outperform the first treatment model in terms of data fitting. Likewise, the second treatment model still enhances the performance of data fitting much better than the third treatment group. The comparison shows that considering the rate change of the real output in the monetary policy reaction function can furthermore enhance the data fitting for the UK. From the last column in the row of the UK, the second treatment group outperform the third one in terms of data fitting. Thus, the second treatment group is the best one among all the groups for the UK. More specifically, the ignorance of the movement of the nominal exchange rate and the incorporation of the rate change of the output in the policy decision can best fit the data collected from the UK.

From the first three columns in the row of Canada, it is apparent to see that only the third treatment group outperform the control one. There is a reduction in the performance of data fitting when ignoring the nominal exchange rate depreciation for the first treatment group. Also, there is no enhancement of data fitting when incorporating the change rate of real output but ignoring the nominal exchange rate depreciation for the second treatment group. Only when put them together in the third treatment group, the DSGE model will fit the data much better. From the next two columns in the row of Canada, it shows that the treatment groups with the rate change of the real output outperform the first treatment group which considers no nominal exchange rate depreciation nor the rate change of real output in terms of data fitting. From the last column in the row of Canada, it is easy to find the third treatment group perform much better than the second group. Thus, the third treatment group is the best one among all the groups in Canada. That is to say, the incorporation of the rate change of the real output and the nominal exchange rate depreciation in the policy function can guarantee the simplified DSGE model to provides the best fitting for the Canadian data.

Table 3.12 ranks the performance of data fitting of all the groups for each country. For the UK, all the treatment group perform better than the control group, and the second treatment group performs best to fit the data in the UK. For Canada, only the third treatment group outperforms the control group and performs best. There is no significant difference between the second treatment group and the control group in terms of data fitting and the first treatment group fit the data worst in Canada.

Table 3.11: Numerical Results of the Posterior Odds Ratio

	$\gamma_{1,0}$	$\gamma_{2,0}$	$\gamma_{3,0}$	$\gamma_{2,1}$	$\gamma_{3,1}$	$\gamma_{3,2}$
UK	48.279	11.222E+09	2.830E+08	2.532E+07	5.862E+06	0.232
Canda	0.015	1.247	60.280	80.479	3889.360	48.327

Note: This table reports the numerical values of the posterior odds ratio among the control group and all the three treatment groups.

Table 3.12: The Rank of the Data Fitting Performance of Each Group

	UK	Canada
Control Group	4	2
Treatment Group One	3	3
Treatment Group Two	1	2
Treatment Group Three	2	1

Note: This table reports the ranks of the data fitting performance of each group for each country.

3.4 Conclusion

Chapter 3 offers model comparisons at the first stage to denote the specification of the monetary policy reaction function, which brings the best performance of data fitting. It initially introduces the methodology of calculating the posterior odds ratio. It then estimates the DSGE models in the three treatment groups for each country and computes their posterior odds ratios to carry on the model comparisons.

For the UK, the simplified DSGE model best fit the data considering the rate change of the real output and ignoring the nominal exchange rate depreciation in the policy function. This finding is evident in contrast with Lubik and Schorfheide (2007)'s research, which covers a period that Britain remains in the ERM before 1992. When applying the rule with the best data fitting, Figure 3.3 depicts that the central bank possibly encounters trade-offs in the policy decision. First, a booming world output shock impedes the domestic output, and the central bank cuts the rates to lean against the wind at the expense of the stabilisation of the inflation rates. Also, an increased price of the exports goods arising from the higher demand from the foreign market motivates the domestic output and appreciates the currency value. The central bank instead cut the policy rate to lean against the wind of the disinflation at the expense of the stabilisation of the output.

For Canada, the simplified DSGE model best fit the data considering the rate change of the real output and the nominal exchange rate depreciation in the policy function together. This finding is consistent with Lubik and Schorfheide (2007)'s research and the bank reports in Canada. When applying the rule with the best data fitting, Figure 3.6 depicts that the central bank possibly encounters different situations from the case in the UK. First, the increased export price not only push the domestic output upward but also raise the domestic price level. In the case of the UK, CPI inflation decreases due to the result that the falling price of the imported good because of the currency appreciation outweighs the increasing price of the exported goods. In the case of Canada, CPI inflation increases due to the result that the falling price of the imported good because of the currency appreciation cannot offset the increasing price of the exported goods. Thus, the central bank in Canada tights the monetary policy to lean against the wind from the output and the inflation rate together at the expense of the stabilisation of the movements of the nominal exchange rates. Also, unlike the case in the UK, the world inflation shock can affect the domestic output and CPI inflation in Canada. Thus, the central bank tights the monetary policy at the expense of the stabilisation of the movements of the nominal exchange rate in facing up with the international inflationary pressure.

The next chapter will move to the second stage of the model comparison and prefers the simplified DSGE models with the best data fitting at the first stage as the benchmark models. It continues studying whether introducing some types of structural changes, including the environmental and managerial aspects, can furtherly lead to an improvement in the data fitting.

Chapter 4

Model Comparison Two: Markov-Switching Parameters Estimation

4.1 Introduction

Chapter 4 carries on the model comparison at the second stage to identify whether introducing Markov-switching parameters can improve the performance of data fitting in comparison with the constant parameter DSGE model with the best fitting in chapter 3. Chapter 3 has already discovered that the model which fits the UK data best does not consider the nominal exchange rate depreciation in the policy function while the model which fits the Canadian data best should put the movements of exchange rates into consideration.

Markov-Switching DSGE models are more appropriate than constant parameter DSGE models to analyse the dynamic macroeconomic variables when the selected period potentially includes some kinds of structural changes. Chapter 4 will put forward three types of Markov-Switching models. The first model examines the structural breaks in the variance of exogenous shocks (Stock and Watson, 2003[84]; Sims and Zha, 2006[81]; Justiniano and Primiceri, 2008[49]). The second model considers the structural breaks in the parameters of the policy functions(Clarida, Gali and Gertler,2000[19]; Lubik and Schorfheide, 2004[59]; Davig and Leeper, 2007[22]). The third model explores the two types of structural breaks together (Liu and Mumtaz, 2011[57]; Chen and MacDonald, 2011[16]). After estimating all the three kinds of DSGE models, chapter 4 checks out the Markov-Switching DSGE model with the best performance of data fitting for the UK and Canada. Apart from the model comparison, chapter 4 provides data analysis for the UK and Canada base on the best data fitting model, which explicitly presents the contribution of each of the structural shocks to the dynamic macro-economic variables within the sample period.

Chapter 4 proceeds as follows. Section 2 brings in the methodology put forward by Farmer et al. (2011[32]) to solve and estimate the DSGE model with Markov-switching parameters. Section 3 solves and estimates the mentioned three kinds of Markov-switching DSGE models with the data collected from the UK. It provides the model comparison at the second stage and offers a detailed data analysis for the UK within the sample period. Likewise, section 4 repeats the procedures in the previous section to offer the model comparison and data analysis for Canada within the same sample period. Section 5 concludes.

4.2 Markov-Switching DSGE Models

Chapter 2 have shown how a DSGE model can be represented by the structured equations below:

$$\mathbf{B}(\mathbf{\Theta})\mathbf{x_{t+1}} = \mathbf{A}(\mathbf{\Theta})\mathbf{x_t} + \mathbf{G}(\mathbf{\Theta})\xi_{\mathbf{t+1}}, \tag{4.1}$$

where $\mathbf{x}_t = [y\tilde{y}_{t,n}, \triangle\tilde{e}_t, \tilde{r}_t, z_t, \tilde{yy}^*_t, \pi^*_t, \triangle\tilde{q}_t, y\tilde{y}_t, \pi_t, \triangle\tilde{yy}^*_t]'$, $\xi_{\mathbf{t}} = [\xi^{\mathbf{R}}_{\mathbf{t}}, \xi^{\mathbf{z}}_{\mathbf{t}}, \xi^{\mathbf{q}}_{\mathbf{t}}, \xi^{\mathbf{y}^*_\mathbf{t}}_{\mathbf{t}}, \xi^{\pi^*_\mathbf{t}}_{\mathbf{t}}]'$. B, A and G are the functions of the structural parameters Θ. The parameter space Θ includes $[\tau, \kappa, \alpha, \phi_\pi, \phi_y, \phi_{\triangle e}, \rho_R, \rho_z, \rho_q, \rho_{\pi^*}, \rho_{y^*}, \sigma_R, \sigma_z, \sigma_{y^*}, \sigma_{\pi^*}, \sigma_q]'$. The Markov-switching DSGE model allows its subset of the parameter space to shift between two regimes. Thus, the above structured form can be rewritten as the following equation:

$$\mathbf{B}(\mathbf{\Theta_{s_t}})\mathbf{x_{t+1}} = \mathbf{A}(\mathbf{\Theta_{s_t}})\mathbf{x_t} + \mathbf{G}(\mathbf{\Theta_{s_t}})\xi_{\mathbf{t+1}}, \tag{4.2}$$

where S_t denotes the unobserved state variables which assumes some structural parameters follow a two state Markov process with the following transition probabilities:

$$Prob[S_t = 2|S_t = 1] = p_{12}, Prob[S_t = 1|S_t = 2] = p_{21}. \tag{4.3}$$

There are three kinds of the Markov-switching DSGE models, two types of structural breaks (Markov chains) and two states (regimes) for each break in chapter 4. The first model permits the regime shifts in the standard deviations of the exogenous shocks including $[\sigma_R, \sigma_z, \sigma_{y^*}, \sigma_{\pi^*}, \sigma_q]$. The construction of the first model is motivated by the idea to capture whether there is good luck (low volatility) or bad luck (high volatility) in the researched economy within the sample period. The first model defines the transition probability of the first type of Markov chain as:

$$Prob[S_t = 2|S_t = 1] = Q_{12}, Prob[S_t = 1|S_t = 2] = Q_{21}. \tag{4.4}$$

The second Markov-switching DSGE model enables the regime shifts in the monetary policy parameters including $[\phi_\pi, \phi_y, \phi_{\triangle e}, \phi_{\triangle y}, \rho_R]$. The construction of the second model is to whether there is a potential change in the monetary policy within the sample period. The second model defines the transition probability of the second type of Markov chain as:

$$Prob[S_t = 2|S_t = 1] = P_{12}, Prob[S_t = 1|S_t = 2] = P_{21}. \tag{4.5}$$

The third model allows the two types of regime shifts to happen together. The construction of the third model investigates whether the two independent Markov chains can exist simultaneously in a researched economy within the sample period. The third model still denotes Q and P as the transition probability for each Markov chain.

Farmer et al. (2008[32]) updates equation 4.2 to a model with time invariant parameters:

$$\mathbf{B}^{*}\mathbf{x_{t+1}} = \mathbf{A}^{*}\mathbf{x_t} + \mathbf{G}^{*}\xi_{\mathbf{t+1}}, \tag{4.6}$$

where B^*, A^* and G^* are functions of the structural parameters and the transition probabilities. Farmer et al. (2008) defines that there is a minimal state variable solutions both satisfying the equation 4.2 and 4.6. When the solution is unique and stable, Farmer et al. (2008) rewrites the above equation 4.6 as follows:

$$\mathbf{x}_t = \mathbf{\Phi_1}(\mathbf{\Theta_{S_t}})\mathbf{x_{t-1}} + \mathbf{\Phi}_{\xi}(\mathbf{\Theta_{S_t}})\xi_{\mathbf{t}}, \xi_{\mathbf{t}} \sim \mathbf{iidN}(\mathbf{0}, \mathbf{\Sigma_{S_t}}). \tag{4.7}$$

Chapter 4 uses the same measurement equations introduced in chapter 2 to connect the observable variables to the model variables. The original parameter sapce expands to include the parameters r^A, π^A and γ^Q. Furthermore, the combination between the measurement equations and equation 4.7 can lead to the following equation:

$$d_t = \Psi_0(\Theta_{S_t}) + \Psi_1(\Theta_{S_t})t + \Psi_2(\Theta_{S_t})\mathbf{x_t} + \mathbf{u_t}, \tag{4.8}$$

where d_t is the vector of the observable variables $[r_t^{obs}, \pi_t^{obs}, \triangle y_t^{obs}, \triangle e_t^{obs}, \triangle q_t^{obs}]'$ and u_t is the vector of measurement errors.

$$u_t \sim iidN(0, \Sigma_u) \tag{4.9}$$

Equation 4.7 and 4.8 provide a state-space representation of the DSGE model which offers a joint density for the observable and model variables:

$$p(D_{1:T}, X_{1:T}|\Theta_{S_t}) = \prod_{t=1}^{T} p(d_t, x_t|D_{1:t-1}, X_{1:t-1}, \Theta_{S_t}) = \prod_{t=1}^{T} p(d_t|x_t, \Theta)p(x_t|x_{t-1}, \Theta_{S_t}), \tag{4.10}$$

where $D_{1:T} = d_1, d_2, ..., d_T$ and $X_{1:T} = x_1, x_2, ..., x_T$. The equation above brings in the beginning of the Bayesian inference, which is a method comprising of the likelihood function $p(D_{1:T}|\Theta_{S_t})$ and the prior distribution of the relevant parameters $p(\Theta_{S_t})$:

$$p(\Theta_{S_t}|D_{1:t}) = \frac{p(\Theta_{S_t})p(D_{1:T}|\Theta_{S_t})}{p(D_{1:T})}, \tag{4.11}$$

where $p(D_{1:t})$ is defined as the marginal likelihood:

$$p(D_{1:t}) = \int p(D_{1:t}|\Theta_{S_t})p(\Theta_{S_t})d\Theta_{S_t}. \tag{4.12}$$

Moreover, Θ_{S_t} actually incorporates the structural parameter Θ, the transition probability ϕ^*, and the latent state variables $S_{1:t}$.Thus, the posterior distribution in the equation 4.11 can be rewritten as the following equation:

$$p(\Theta_{S_t}|D_{1:t}) = p(\Theta, \phi^*, S_{1:t}|D_{1:t}) = \frac{p(\Theta, \phi^*)p(S_{1:t}|\phi^*)p(D_{1:T}|\Theta, \phi^*, S_{1:t})}{p(D_{1:T})}, \tag{4.13}$$

where $p(\Theta, \phi^*)$ are the prior distributions for the structural parameters Θ and the transition probability ϕ^*, $p(S_{1:t}|\phi^*)$ are the prior distributions for the latent state variables and $p(D_{1:T}|\Theta, \phi^*, S_{1:t})$ is the likelihood function. Moreover, the marginal data density $p(D_{1:t})$ can be updated as followed:

$$p(D_{1:t}) = \int p(D_{1:T}|\Theta, \phi^*, S_{1:t}))p(\Theta, \phi^*)p(S_{1:t}|\phi^*)d(\Theta, \phi^*, S_{1:t}). \tag{4.14}$$

The goal is to compute the moments of the posterior distributions $p(\Theta, \phi^*, S_{1:t}|D_{1:t})$. As mentioned in chapter 2, I implement the Markov Chain Monte Carlo (MCMC) method to draw the Bayesian posterior distribution approximately. First, I use Sim's optimization routine Csmiwel to maximize the log-likelihood function numerically and arrive at the posterior mode $\Theta_{S_t}^{ML}$. Second, I calculate the inverse Hessian matrix $\Sigma_{\Theta_{S_t}}$ at the posterior mode to generate the covariance matrix of the approximate multi-normal distribution $\Theta_{S_t} \sim N(\Theta_{S_t}^{ML}, \Sigma_{\Theta_{S_t}})$, which is a benchmark of the proposed density $q(z|\Theta_{S_t}, D_{1:t})$. Third, I apply the Metropolis-Hastings algorithm to generate $N = 200,000$ draws from the posterior distribution and the first $M = 10,000$ draws are burned. Meanwhile, adjust the jump scale to have a acceptance rate between 0.2 and 0.4. Finally, I calculate the posterior means of the selected draws by Monte Carlo method. All the algorithms of the solution and estimation above can be incorporated within a very efficient and flexible toolbox named **RISE** invented by professor Junior Maih (Maih,2015[62]).

Table 4.1 reports the estimation results of the constant parameter DSGE models with the best data fitting for the UK and Canada. Chapter 4 regards these models as the benchmark models at the second stage of model comparison. Section 3 and section 4 solve and estimate the three kinds of Markov-switching DSGE models for the UK and Canada, respectively. In comparison with the constant parameter DSGE models, the following two sections examine whether there is an improvement of the data fitting when introducing a specific type of Markov-switching parameters. Apart from the model comparison, these two sections also report the variance decompositions for each type of the regime in the three Markov-switching models and provide historical decompositions of the time series data with the best data fitting DSGE model.

Table 4.1: Parameter Estimation Results of the Benchmark Models

	Prior		Posterior			
	UK	Canada	UK		Canada	
Parameters	Mean	Mean	Mean	90%interval	Mean	90%interval
τ	0.500		0.308	[0.151,0.451]	0.282	[0.161,0.406]
κ	0.300		0.366	[0.174,0.547]	0.669	[0.372,0.953]
ϕ_π	1.500		2.249	[1.516,2.946]	1.945	[1.304,2.567]
ϕ_y	0.125		0.097	[0.043,0.147]	0.078	[0.034,0.121]
$\phi_{\triangle y}$	0.125		0.183	[0.133,0.234]	0.156	[0.080,0.230]
$\phi_{\triangle e}$		0.125			0.128	[0.067,0.188]
ρ_R	0.500		0.812	[0.749,0.875]	0.765	[0.687,0.849]
ρ_z	0.200		0.536	[0.411,0.663]	0.387	[0.290,0.486]
ρ_q	0.400		0.103	[0.010,0.193]	0.538	[0.422,0.658]
ρ_{π^*}	0.800		0.604	[0.448,0.775]	0.441	[0.310,0.570]
ρ_{y^*}	0.900		0.949	[0.916,0.986]	0.955	[0.923,0.987]
α	0.200		0.128	[0.082,0.175]	0.140	[0.085,0.187]
$r^{(A)}$	3.350	2.470	3.127	[2.674,3.594]	2.335	[1.790,2.899]
$\pi^{(A)}$	1.920	1.620	2.182	[1.633,2.753]	1.835	[1.202,2.442]
$\gamma^{(Q)}$	0.620	0.730	0.660	[0.595,0.722]	0.732	[0.668,0.795]
σ_R	0.500		0.149	[0.120,0.178]	0.284	[0.214,0.349]
σ_z	1.500	1.000	0.914	[0.433,1.409]	1.712	[0.850,2.567]
σ_{y^*}	1.500		1.262	[0.463,2.112]	0.833	[0.390,1.281]
σ_{π^*}	0.550		2.507	[2.132,2.858]	2.366	[2.016,2.713]
σ_q	1.500		1.212	[1.033,1.393]	1.916	[1.647,2.187]
Log MDD			-501.064		-582.077	

Note:The table reports the constant parameter estimation results of the UK and Canada best fit the data in chapter 3.

4.3 Estimated Markov Switching DSGE Models for the UK

This section estimates three kinds of Markov-Switching DSGE models for the UK, which includes four components. The first component provides the estimated results, numerical solutions and variance decomposition of the model one with the switching variances. The second component provides the estimated results, numerical solutions and variance decomposition of the model two with the switching monetary policy parameters. The third component provides the estimated results, numerical solutions and variance decomposition of the model three with the switching variances and the switching policy parameters. The final component presents an overall model comparison and yields a general analysis of the UK data based on the best fitting model.

Table 4.2 presents the prior distributions of the structural parameters for the UK prepared for the estimation of the Markov-switching parameters. The structural parameters in chapter 3 and chapter 4 share the same prior distributions. As mentioned before, there are two types of independent Markov chains, and each chain includes two regimes. The first Markov chain Q controls the shifts of the standard deviations across the two regimes representing low and high volatilities, respectively. The prior means of the transition probability in the first chain, $Q_{1,2}$ and $Q_{2,1}$, are both 0.1. The second Markov chain P controls the shifts of the policy parameters across the two regimes representing more strict and less strict inflation targeting behaviours correspondingly. The prior means of the transition probability in the second chain, $P_{1,2}$ and $P_{2,1}$, are both 0.1. Overall, it is helpful to notice that the standard deviations of the exogenous shocks are permitted to shift across low volatility and high volatility in the model one and the model three, while the policy parameters are allowed to shift across more strict inflation targeting and less strict inflation targeting in the model two and the model three.

Table 4.2: Prior Distributions of the Structural Parameters for the UK

Parameters	Domain	Density	Para(1)	Para(2)	Model Spec
τ	[0,1)	Beta	0.5	0.2	
κ	R$\hat{+}$	Gamma	0.3	0.2	
ϕ_π	R$\hat{+}$	Gamma	1.5	0.5	Model 2 & 3
ϕ_y	R$\hat{+}$	Gamma	0.125	0.05	Model 2 & 3
$\phi_{\triangle y}$	R$\hat{+}$	Gamma	0.125	0.05	Model 2 & 3
ρ_R	[0,1)	Beta	0.5	0.25	Model 2 & 3
ρ_z	[0,1)	Beta	0.2	0.1	
ρ_q	[0,1)	Beta	0.4	0.2	
ρ_{π^*}	[0,1)	Beta	0.8	0.1	
ρ_{y^*}	[0,1)	Beta	0.9	0.05	
α	[0,1)	Beta	0.2	0.05	
$r^{(A)}$	R$\hat{+}$	Normal	3.35	1	
$\pi^{(A)}$	R$\hat{+}$	Normal	1.92	1	
$\gamma^{(A)}$	R$\hat{+}$	Gamma	0.62	0.2	
σ_R	R$\hat{+}$	Inverse Gamma	0.5	4	Model 1 & 3
σ_z	R$\hat{+}$	Inverse Gamma	1.5	4	Model 1 & 3
σ_{y^*}	R$\hat{+}$	Inverse Gamma	1.5	4	Model 1 & 3
σ_{π^*}	R$\hat{+}$	Inverse Gamma	0.55	4	Model 1 & 3
σ_q	R$\hat{+}$	Inverse Gamma	1.5	4	Model 1 & 3
P_{12}	[0,1)	Beta	0.1	0.05	Model 2 & 3
P_{21}	[0,1)	Beta	0.1	0.05	Model 2 & 3
Q_{12}	[0,1)	Beta	0.1	0.05	Model 1 & 3
Q_{21}	[0,1)	Beta	0.1	0.05	Model 1 & 3

Note: The table introduces the prior distributions of the structural parameters for the UK.Para(1) and Para(2) are the means and the standard deviations for the relevant distributions,respectively.

4.3.1 UK: the Model One with Switching Variances

Table 4.3 produces the estimation results of the model one for the UK. This model allows the vector of the standard deviations of exogenous shocks $[\sigma_R, \sigma_z, \sigma_{y^*}, \sigma_{\pi^*}, \sigma_q]$ to shift between two regimes. The posterior mean of the transition probability from regime 1 representing low volatility to regime 2 representing high volatility Q_{12} is 0.064, while the transition probability from regime 2 to the regime 1 Q_{21} is 0.090. The asymmetric transition reflects that it is more likely to hold good luck than to take bad luck persistently in the UK within the sample period.

More specifically, regime 1 stands for the low level of volatilities with the posterior means being $[0.141, 0.326, 0.710, 2.288, 0.927]$ compared to $[0.213, 1.653, 2.569, 4.125$ $, 3.506]$ in regime 2. The differences between the two vectors are significant. The standard deviation of the rate change of the technology shock σ_z in regime 2 is five times larger than in regime 1. The standard deviations of the world output shock σ_{y^*} and the rate change of the terms of trade shock σ_q are three times larger than in regime 1. The standard deviations of the monetary policy shock σ_R and the foreign inflation shock σ_{π^*} almost double in regime 2. The calculated log marginal density is -468.588 calculated from the equation (4.14) for the model one with the Markov-switching variances, which is much bigger compared to the log marginal density produced from the constant parameter model with the best data fitting(-501.064) from the equation (3.1).

Table 4.3: Model One with Markov-Switching Variances(UK)

	Regime 1: Low volatility		Regime 2: High volatility	
parameter	mean	90% interval	mean	90% interval
τ	0.272	[0.159,0.409]		
κ	0.557	[0.270,0.967]		
ϕ_π	1.969	[1.393,2.691]		
ϕ_y	0.162	[0.093,0.246]		
$\phi_{\triangle y}$	0.134	[0.083,0.189]		
ρ_R	0.780	[0.705,0.844]		
ρ_z	0.636	[0.540,0.728]		
ρ_q	0.131	[0.028,0.277]		
ρ_{π^*}	0.555	[0.390,0.728]		
ρ_{y^*}	0.935	[0.884,0.974]		
α	0.131	[0.081,0.188]		
$r^{(A)}$	3.439	[3.069,3.806]		
$\pi^{(A)}$	1.869	[1.148,2.567]		
$\gamma^{(A)}$	0.703	[0.669,0.734]		
σ_R	0.141	[0.113,0.176]	0.213	[0.133,0.364]
σ_z	0.326	[0.203,0.501]	1.653	[0.822,2.960]
σ_{y^*}	0.710	[0.322,1.303]	2.569	[1.011,5.469]
σ_{π^*}	2.288	[1.898,2.712]	4.125	[2.623 ,6.147]
σ_q	0.927	[0.782,1.110]	3.506	[1.951,6.647]
Q_{12}	0.064	[0.028,0.110]		
Q_{21}	0.090	[0.034,0.168]		

Note:The table reports posterior means and 90% probability interval of the model one for the UK.

Numerical Solution and Simulation Results for the UK in the Model One

There are two regimes in the model one with switching variances, and accordingly, there are two solutions for the same model. Table 4.4 provides the numerical solution to regime 1 representing low volatility of the model one, and table 4.5 provides the numerical solution to regime 2 representing high volatility of the same model.

Table 4.4 brings in the information to compute the impulse response functions of the four endogenous variables including $\tilde{yy_t}$,π_t,$\tilde{r}_t$ and $\triangle\tilde{e}_t$ in regime 1:

$$\begin{pmatrix} \tilde{yy_t} \\ \pi_t \\ \tilde{r}_t \\ \triangle\tilde{e}_t \end{pmatrix} = \begin{pmatrix} -0.470 & 0.200 & -0.530 & 0 & 0.081 & 0.005 \\ -0.930 & 0.322 & 0.214 & 0 & 0.160 & -0.007 \\ 0.297 & 0.173 & 0.003 & 0 & -0.051 & -0.002 \\ -0.930 & 0.322 & 0.214 & -0.555 & 0.160 & -0.121 \end{pmatrix} \begin{pmatrix} \tilde{r_{t-1}} \\ z_{t-1} \\ \tilde{yy^*_{t-1}} \\ \pi^*_{t-1} \\ \tilde{yy_{t-1}} \\ \triangle\tilde{q_{t-1}} \end{pmatrix}$$

$$+ \begin{pmatrix} -0.085 & 0.102 & 0.035 & -0.402 & 0 \\ -0.168 & 0.165 & -0.049 & 0.163 & 0 \\ 0.054 & 0.089 & -0.015 & 0.002 & 0 \\ -0.168 & 0.165 & -0.855 & 0.163 & -2.288 \end{pmatrix} \begin{pmatrix} \xi_t^R \\ \xi_t^z \\ \xi_t^q \\ \xi_t^{y_t^*} \\ \xi_t^{\pi_t^*} \end{pmatrix}, \quad (4.15)$$

where the second matrix in the above equation embodies the impact of the structural shocks on the real output, inflation rate, nominal interest rate and depreciation exchange rate in regime 1. Figure 4.1 depicts the impulse responses of the four mentioned endogenous variables to one unit structural shock of regime 1 with solid and blue lines. One major difference from chapter 3 is that the estimated standard deviations of the structural shocks have already entered into the second coefficient matrix above and thus, the standard deviations in figure 4.1 are all assumed to be one.

Table 4.5 provides information to compute the impulse response functions of the four endogenous variables including $\tilde{yy_t}$,π_t,$\tilde{r_t}$ and $\triangle\tilde{e_t}$ in regime 2:

$$\begin{pmatrix} \tilde{yy_t} \\ \pi_t \\ \tilde{r_t} \\ \triangle\tilde{e_t} \end{pmatrix} = \begin{pmatrix} -0.470 & 0.200 & -0.530 & 0 & 0.081 & 0.005 \\ -0.930 & 0.322 & 0.214 & 0 & 0.160 & -0.007 \\ 0.297 & 0.173 & 0.003 & 0 & -0.051 & -0.002 \\ -0.930 & 0.322 & 0.214 & -0.555 & 0.160 & -0.121 \end{pmatrix} \begin{pmatrix} \tilde{r_{t-1}} \\ z_{t-1} \\ \tilde{yy^*_{t-1}} \\ \pi^*_{t-1} \\ \tilde{yy_{t-1}} \\ \triangle\tilde{q_{t-1}} \end{pmatrix}$$

$$+ \begin{pmatrix} -0.128 & 0.519 & 0.132 & -1.456 & 0 \\ -0.254 & 0.837 & -0.186 & 0.589 & 0 \\ 0.081 & 0.451 & -0.058 & 0.008 & 0 \\ -0.254 & 0.837 & -3.233 & 0.589 & -4.125 \end{pmatrix} \begin{pmatrix} \xi_t^R \\ \xi_t^z \\ \xi_t^q \\ \xi_t^{y_t^*} \\ \xi_t^{\pi_t^*} \end{pmatrix}, \quad (4.16)$$

where the second matrix in the above equation represents the impact of the structural shocks on the real output, inflation rate, nominal interest rate and depreciation exchange rate in regime 2. Figure 4.1 depicts the impulse responses of the four mentioned endogenous variables to one unit structural shock of regime 2 with dashed and red lines.

Figure 4.1 compares the impulse response functions of the two regimes in the model one. Holding everything else constant, a unit of the standard deviation of the monetary policy shock will exert a -0.085 impact on the real output deviation in regime 1 while -0.128 in the regime 2; a -0.168 impact on the inflation in regime 1 while -0.254 in regime 2;a 0.054 impact on the nominal interest rate deviation in regime 1 while 0.081 in regime 2; a -0.168 impact on the nominal exchange rate depreciation in regime 1 while -0.254 in regime 2.

Holding everything else constant, a unit of the standard deviation of the shock to the change rate of the terms of trade will exert a 0.035 impact on the real output deviation in regime 1 while 0.132 in regime 2; a -0.049 impact on the inflation in regime 1 while -0.186 in regime 2;a -0.015 impact on the nominal interest rate deviation in regime 1 while -0.058 in regime 2; a -0.855 impact on the nominal exchange rate depreciation in regime 1 while -3.233 in regime 2.

Holding everything else constant, a unit of the standard deviation of the shock to the change rate of the technology will exert a 0.102 impact on the real output deviation in regime 1 while 0.519 in regime 2; a 0.165 impact on inflation in regime 1 while 0.837 in regime 2;a 0.089 impact on the nominal interest rate deviation in regime 1 while 0.451 in regime 2; a 0.165 impact on the nominal exchange rate depreciation in regime 1 while 0.837 in regime 2.

Holding everything else constant, a unit of the standard deviation of the shock to the world output deviation will exert a -0.402 impact on the real output deviation in regime 1 while -1.456 in regime 2; a 0.163 impact on inflation in regime 1 while 0.589 in regime 2;a 0.002 impact on the nominal interest rate deviation in regime 1 while 0.008 impact regime 2; a 0.163 impact on the nominal exchange rate depreciation in regime 1 while 0.589 in regime 2.

Holding everything else constant, a unit of the standard deviation of the shock to the world inflation will only exert a -2.288 impact on the nominal exchange rate depreciation in regime 1 while -4.125 in the regime 2.

Table 4.4: The Numerical Solution to Regime 1 of the Model One for the UK

		static		backward looking				mixed		forward looking	
		$\tilde{yy}_{t,n}$	$\triangle\tilde{e}_t$	$\tilde{r}_t$	z_t	$\tilde{yy}^*_t$	π^*_t	$\triangle\tilde{q}_t$	$\tilde{yy}_t$	π_t	$\triangle\tilde{yy}^*_t$
endogenous variables	$\tilde{r}_{t-1}$		-0.930	0.297					-0.470	-0.930	
	z_{t-1}		0.322	0.173	0.636				0.200	0.322	
	$\tilde{yy}^*_{t-1}$	-0.613	0.214	0.003		0.935			-0.530	0.214	-0.065
	π^*_{t-1}		-0.555				0.555				
	$\tilde{yy}_{t-1}$		0.160	-0.051					0.081	0.160	
	$\triangle\tilde{q}_{t-1}$		-0.121	-0.002				0.131	0.005	-0.007	
exogenous variables	ξ^R_t		-0.168	0.054					-0.085	-0.168	
	ξ^z_t		0.165	0.089	0.326				0.102	0.165	
	ξ^q_t		-0.855	-0.015				0.927	0.035	-0.049	
	$\xi^{y^*_t}_t$	-0.465	0.163	0.002		0.710			-0.402	0.163	0.710
	$\xi^{\pi^*_t}_t$		-2.288				2.288				

Note: This table reports the numerical solutions to regime 1 of the model one for the UK. Regime 1 considers a low level of the volatility.

Table 4.5: The Numerical Solution to Regime 2 of the Model One for the UK

		static		backward looking				mixed		forward looking	
		$y\tilde{y}_{t,n}$	$\triangle\tilde{e}_t$	$\tilde{r}_t$	z_t	$y\tilde{y}^*_t$	π^*_t	$\triangle\tilde{q}_t$	$y\tilde{y}_t$	π_t	$\triangle y\tilde{y}^*_t$
endogenous variables	$\tilde{r}_{t-1}$		-0.930	0.297					-0.470	-0.930	
	z_{t-1}		0.322	0.173	0.636				0.200	0.322	
	$y\tilde{y}^*_{t-1}$	-0.613	0.214	0.003		0.935			-0.530	0.214	-0.065
	π^*_{t-1}		-0.555				0.555				
	$y\tilde{y}_{t-1}$		0.160	-0.051					0.081	0.160	
	$\triangle\tilde{q}_{t-1}$		-0.121	-0.002				0.131	0.005	-0.007	
exogenous variables	ξ^R_t		-0.254	0.081					-0.128	-0.254	
	ξ^z_t		0.837	0.451	1.653				0.519	0.837	
	ξ^q_t		-3.233	-0.058				3.506	0.132	-0.186	
	$\xi^{y^*_t}_t$	-1.684	0.589	0.008		2.569			-1.456	0.589	2.569
	$\xi^{\pi^*_t}_t$		-4.125				4.125				

Note: This table reports the numerical solutions to regime 2 of the model one for the UK. Regime 2 represents a high level of the volatility.

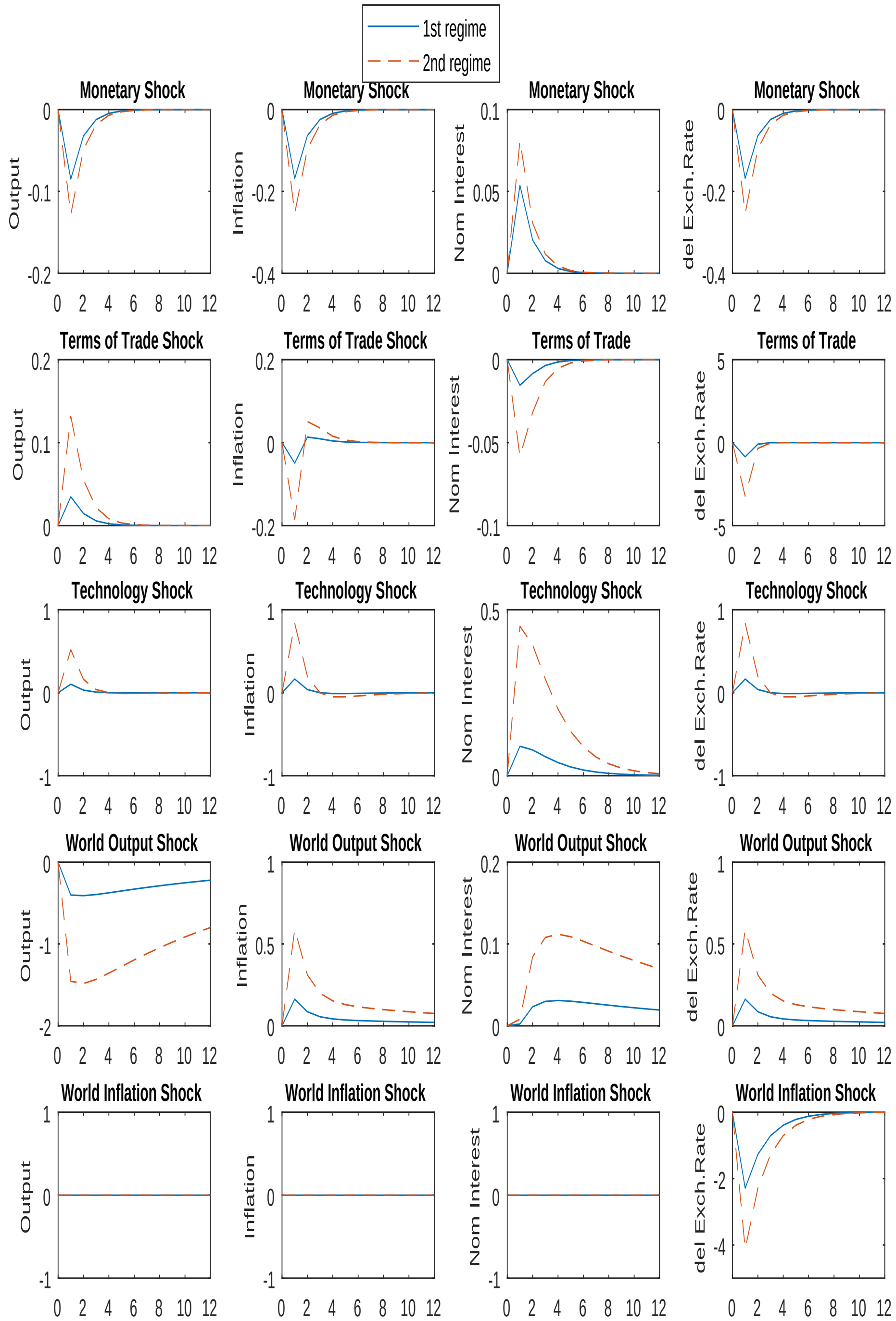

Figure 4.1: Impulse responses,UK(Model One). Note:The figure depicts the impulses responses of real output, inflation rate, nominal interest rate and depreciation exchange rate to one unit structural shock of regime 1 representing the low volatility(solid and blue lines)and regime 2 representing the high volatility(dashed and red lines).

Variance Decomposition of the Model One for the UK

Table 4.6 reports the variance decomposition of the model one with the switching variances for the UK. It summarise the major contributions to the variation of the four endogenous variables $\tilde{yy}_t$,π_t,$\tilde{r}_t$ and $\triangle\tilde{e}_t$ as follows. First, the world output shock $\xi_t^{y_t^*}$ contributes most to the variation of the output deviation $\tilde{yy}_t$ in both of the two regimes. Second, the world output shock $\xi_t^{y_t^*}$ contributes most to the variation of the inflation π_t in regime 1 while the change rate of technology shock ξ_t^z contributes most in regime 2. Third, the change rate of the technology shock ξ_t^z contributes most to the variation of the interest rate $\tilde{r}_t$ in both of the two regimes. Finally, the world inflation shock $\xi_t^{\pi_t^*}$ contributes most to the variation of nominal exchange rate depreciation $\triangle\tilde{e}_t$ in both of the two regimes.

Table 4.6 also compares the contributions of the same structural shock in different regimes. The policy shock ξ_t^R contributes 0.51% to the variation of the output deviation $\tilde{yy}_t$ in regime 1 while 0.09% in regime 2,29.93% to the variation of inflation π_t in the regime 1 while only 5.07% in regime 2,10.05% to the variation of interest rate $\tilde{r}_t$ in regime 1 while only 1.17% in regime 2,0.39% to the variation of exchange rate depreciation $\triangle\tilde{e}_t$ in regime 1 while 0.21% in regime 2.

The terms of trade shock ξ_t^q contributes 0.09% to the variation of the output deviation $\tilde{yy}_t$ in regime 1 while 0.10% in regime 2,2.41% to the variation of inflation π_t in regime 1 while 2.61% in regime 2,0.96% to the variation of interest rate $\tilde{r}_t$ in regime 1 while only 0.71% in regime 2,8.79% to the variation of exchange rate depreciation $\triangle\tilde{e}_t$ in regime 1 while 28.88% in regime 2.

The change rate of the technology shock ξ_t^z contributes 0.70% to the variation of the output deviation $\tilde{yy}_t$ in regime 1 while 1.38% in regime 2,25.81% to the variation of inflation π_t in regime 1 while 50.24% in regime 2,59.54% to the variation of interest rate $\tilde{r}_t$ in regime 1 while 78.38% in regime 2,0.34% to the variation of exchange rate depreciation $\triangle\tilde{e}_t$ in regime 1 while 2.04% in regime 2.

The world output shock $\xi_t^{y_t^*}$ contributes 98.69% to the variation of the output deviation $y\tilde{y}_t$ in regime 1 while 98.44% in regime 2,42.45% to the variation of inflation π_t in the regime 1 while 42.08% in regime 2,29.45% to the variation of interest rate $\tilde{r_t}$ in regime 1 while 19.74% in regime two,0.57% to the variation of exchange rate depreciation $\triangle\tilde{e_t}$ in regime 1 while 1.71% in regime 2.

The world inflation shock $\xi_t^{\pi_t^*}$ contributes 89.91% to the variation of exchange rate depreciation $\triangle\tilde{e_t}$ in regime 1 while 67.17% in regime 2. It contributes nothing to the variation of the other three endogenous variables.

Table 4.6: Variance Decomposition of the Model One for the UK

Regime 1	Output	Inflation	interest rate	Exchange rate depreciation
Policy	0.51	29.33	10.05	0.39
Terms of trade	0.09	2.41	0.96	8.79
Technology	0.70	25.81	59.54	0.34
World output	98.69	42.45	29.45	0.57
World inflation	0.00	0.00	0.00	89.91

Regime 2	Output	Inflation	interest rate	Exchange rate depreciation
Policy	0.09	5.07	1.17	0.21
Terms of trade	0.10	2.61	0.71	28.88
Technology	1.38	50.24	78.38	2.04
World output	98.44	42.08	19.74	1.71
World inflation	0.00	0.00	0.00	67.17

Note:(a)The table reports the variance decomposition of model one for the UK.

(b)Regime 1 is characterized as low volatility compared to Regime 2.

4.3.2 UK: the Model Two with the Switching Taylor Rule

Table 4.7 presents the estimated results of the model two for the UK. This model allows the vector of the policy parameters $[\phi_\pi, \phi_y, \phi_{\triangle y}, \rho_R]$ to shift between two regimes. The posterior mean of the transition probability from regime 1 representing more strict inflation targeting to regime 2 representing less strict inflation targeting (P_{12}) is 0.101, while from regime 2 to regime 1 (P_{21}) is 0.042. The asymmetric transition indicates that it is more likely to hold less strict inflation targeting persistently in the UK within the sample period.

More specifically, regime 1 stands for more strict inflation targeting with the posterior mean of ϕ_π being 2.093 compared to 1.641 in regime 2. Next, the posterior mean of ρ_R in regime 1 is just 0.387 compared to 0.83 in regime 2. Also, the posterior mean of the coefficients of the output deviations from steady states ϕ_y in regime 1 is three times smaller than it in regime 2. Finally, the difference between the coefficient of the rate change of the output deviations $\phi_{\triangle y}$ is less significant across the two regimes. The calculated marginal density is -488.018 for the model with the Markov-switching policy parameters, which is bigger compared to the constant parameter model with the best data fitting (-501.064).

Table 4.7: Model Two with Markov-Switching Policy Parameters(UK)

	Regime 1: more strict inflation targetting		Regime 2: less strict inflation targeting	
parameter	mean	90% interval	mean	90% interval
τ	0.308	[0.175,0.457]		
κ	0.176	[0.082,0.310]		
ϕ_π	2.093	[1.470,2.894]	1.641	[1.195,2.140]
ϕ_y	0.047	[0.022,0.084]	0.159	[0.093,0.238]
$\phi_{\triangle y}$	0.129	[0.062,0.220]	0.137	[0.088,0.193]
ρ_R	0.387	[0.075,0.867]	0.830	[0.770,0.881]
ρ_z	0.603	[0.484,0.711]		
ρ_q	0.094	[0.018,0.212]		
ρ_{π^*}	0.598	[0.435,0.763]		
ρ_{y^*}	0.950	[0.904,0.983]		
α	0.110	[0.077,0.150]		
$r^{(A)}$	3.640	[3.181,4.081]		
$\pi^{(A)}$	1.248	[0.291,2.204]		
$\gamma^{(A)}$	0.693	[0.664,0.721]		
σ_R	0.120	[0.099,0.145]		
σ_z	0.611	[0.333,1.050]		
σ_{y^*}	1.599	[0.667,2.998]		
σ_{π^*}	2.519	[2.174,2.921]		
σ_q	1.213	[1.046,1.409]		
P_{12}	0.101	[0.035,0.196]		
P_{21}	0.042	[0.016,0.080]		

Note:The table reports posterior means and 90% probability interval of the model two for the UK.

Numerical Solution and Simulation Results for the UK in the Model Two

There are two regimes in the model two with the Markov-switching policy parameters. Table 4.8 presents the numerical solution to regime 1 representing more strict inflation targeting of the model two, and table 4.9 brings the numerical solution to regime 2 representing less strict inflation targeting of the same model.

Table 4.8 contributes to computing the impulse response functions for the four endogenous variables including $\tilde{yy_t}$,π_t,$\tilde{r_t}$ and $\triangle\tilde{e_t}$ in the regime 1:

$$\begin{pmatrix} \tilde{yy_t} \\ \pi_t \\ \tilde{r_t} \\ \triangle\tilde{e_t} \end{pmatrix} = \begin{pmatrix} -0.181 & 0.185 & -0.415 & 0 & 0.060 & 0.007 \\ -0.119 & 0.120 & 0.045 & 0 & 0.040 & -0.006 \\ 0.205 & 0.184 & -0.008 & 0 & -0.068 & -0.006 \\ -0.119 & 0.120 & 0.045 & -0.598 & 0.040 & -0.089 \end{pmatrix} \begin{pmatrix} \tilde{r_{t-1}} \\ z_{t-1} \\ \tilde{yy^*_{t-1}} \\ \pi^*_{t-1} \\ \tilde{yy_{t-1}} \\ \triangle\tilde{q_{t-1}} \end{pmatrix}$$

$$+ \begin{pmatrix} -0.056 & 0.188 & 0.084 & -0.699 & 0 \\ -0.037 & 0.122 & -0.075 & 0.075 & 0 \\ 0.064 & 0.186 & -0.083 & -0.013 & 0 \\ -0.037 & 0.122 & -1.155 & 0.075 & -2.519 \end{pmatrix} \begin{pmatrix} \xi^R_t \\ \xi^z_t \\ \xi^q_t \\ \xi^{y^*_t}_t \\ \xi^{\pi^*_t}_t \end{pmatrix}, \quad (4.17)$$

where the second matrix in the above equation incorporates the impact of the structural shocks on the real output, inflation rate, nominal interest rate and depreciation exchange rate regime 1. Figure 4.2 depicts the impulse responses of the four endogenous variables to one unit structural shock in regime 1 with solid and blue lines.

Table 4.9 provides information to compute the impulse response functions of the four endogenous variables including $\tilde{yy_t}$,π_t,$\tilde{r_t}$ and $\triangle\tilde{e_t}$ in regime 2:

$$\begin{pmatrix} \tilde{yy_t} \\ \pi_t \\ \tilde{r_t} \\ \triangle\tilde{e_t} \end{pmatrix} = \begin{pmatrix} -0.970 & 0.268 & -0.315 & 0 & 0.160 & 0.004 \\ -0.866 & 0.164 & 0.156 & 0 & 0.143 & -0.007 \\ 0.429 & 0.090 & -0.008 & 0 & -0.071 & -0.001 \\ -0.866 & 0.164 & 0.156 & -0.598 & 0.143 & -0.091 \end{pmatrix} \begin{pmatrix} \tilde{r_{t-1}} \\ z_{t-1} \\ \tilde{yy^*_{t-1}} \\ \pi^*_{t-1} \\ \tilde{yy_{t-1}} \\ \triangle\tilde{q_{t-1}} \end{pmatrix}$$

$$+ \begin{pmatrix} -0.140 & 0.272 & 0.047 & -0.530 & 0 \\ -0.125 & 0.166 & -0.090 & 0.263 & 0 \\ 0.062 & 0.091 & -0.017 & -0.003 & 0 \\ -0.125 & 0.166 & -1.170 & 0.263 & -2.519 \end{pmatrix} \begin{pmatrix} \xi^R_t \\ \xi^z_t \\ \xi^q_t \\ \xi^{y^*_t}_t \\ \xi^{\pi^*_t}_t \end{pmatrix}, \quad (4.18)$$

where the second matrix in the above equation embodies the impact of the structural shocks on the real output, inflation rate, nominal interest rate and depreciation exchange rate in regime 2. Figure 4.2 depicts the impulse responses of the four endogenous variables to one unit structural shock of regime 2 with dashed and red lines.

Figure 4.2 compares the impulse response functions of the two regimes in the model two with the Markov-switching policy parameters. Holding everything else constant, a unit of the standard deviation of monetary policy shock will exert a -0.056 impact on the real output deviation in regime 1 while -0.140 in regime 2; a -0.037 impact on inflation in regime 1 while -0.125 in regime 2;a 0.064 impact on the nominal interest rate deviation in regime 1 while 0.062 in regime 2; a -0.037 impact on the nominal exchange rate depreciation in regime 1 while -0.125 in regime 2.

Holding everything else constant, a unit of the standard deviation of the shock to the change rate of terms of trade will exert a 0.084 impact on the real output deviation in regime 1 while 0.047 in regime 2; a -0.075 impact on inflation in regime 1 while -0.090 in regime 2;a -0.083 impact on the nominal interest rate deviation in regime 1 while -0.017 in regime 2; a -1.155 impact on the nominal exchange rate depreciation in regime 1 while -1.170 regime 2.

Holding everything else constant, a unit of the standard deviation of the shock to the change rate of technology will exert a 0.188 impact on the real output deviation in regime 1 while 0.272 in regime 2; a 0.122 impact on inflation in regime 1 while 0.166 in regime 2;a 0.186 impact on the nominal interest rate deviation in regime 1 while 0.091 in regime 2; a 0.122 impact on the nominal exchange rate depreciation in regime 1 while 0.166 in regime 2.

Holding everything else constant, a unit of the standard deviation of the shock to the world output deviation will exert a -0.699 impact on the real output deviation in regime 1 while -0.530 in regime 2; a 0.075 impact on inflation in regime 1 while 0.263 in regime 2;a -0.013 impact on the nominal interest rate deviation in both regime 1 and regime 2; a 0.075 impact on the nominal exchange rate depreciation in regime 1 while 0.263 in regime 2.

Holding everything else constant, a unit of the standard deviation of the shock to the world inflation will only exert a -2.519 impact on the nominal exchange rate depreciation in both regime 1 and regime 2.

Table 4.8: The Numerical Solution to Regime 1 of the Model Two for the UK

		static		backward looking				mixed		forward looking	
		$\tilde{yy}_{t,n}$	$\triangle\tilde{e}_t$	$\tilde{r}_t$	z_t	$\tilde{yy}^*_t$	π^*_t	$\triangle\tilde{q}_t$	$\tilde{yy}_t$	π_t	$\triangle\tilde{yy}^*_t$
endogenous variables	$\tilde{r}_{t-1}$		-0.119	0.205					-0.181	-0.119	
	z_{t-1}		0.120	0.184	0.603				0.185	0.120	
	$\tilde{yy}^*_{t-1}$	-0.444	0.045	-0.008		0.950			-0.415	0.045	-0.050
	π^*_{t-1}		-0.598				0.598				
	$\tilde{yy}_{t-1}$		0.040	-0.068					0.060	0.040	
	$\triangle\tilde{q}_{t-1}$		-0.089	-0.006				0.094	0.007	-0.006	
exogenous variables	ξ^R_t		-0.037	0.064					-0.056	-0.037	
	ξ^z_t		0.122	0.186	0.611				0.188	0.122	
	ξ^q_t		-1.155	-0.083				1.213	0.084	-0.075	
	$\xi^{y^*_t}_t$	-0.747	0.075	-0.013		1.599			-0.699	0.075	1.599
	$\xi^{\pi^*_t}_t$		-2.519				2.519				

Note: This table reports the numerical solutions of regime 1 of the model two for the UK. Regime 1 considers a high level of the coefficient of inflation in Taylor rule.

Table 4.9: The Numerical Solution to Regime 2 of the Model Two for the UK

		static		backward looking				mixed		forward looking	
		$\tilde{yy}_{t,n}$	$\triangle\tilde{e}_t$	$\tilde{r}_t$	z_t	$\tilde{yy}^*_t$	π^*_t	$\triangle\tilde{q}_t$	$\tilde{yy}_t$	π_t	$\triangle\tilde{yy}^*_t$
endogenous variables	$\tilde{r}_{t-1}$		-0.866	0.429					-0.970	-0.866	
	z_{t-1}		0.164	0.090	0.603				0.268	0.164	
	$\tilde{yy}^*_{t-1}$	-0.444	0.156	-0.008		0.950			-0.315	0.156	-0.050
	π^*_{t-1}		-0.598				0.598				
	$\tilde{yy}_{t-1}$		0.143	-0.071					0.160	0.143	
	$\triangle\tilde{q}_{t-1}$		-0.091	-0.001				0.094	0.004	-0.007	
exogenous variables	ξ^R_t		-0.125	0.062					-0.140	-0.125	
	ξ^z_t		0.166	0.091	0.611				0.272	0.166	
	ξ^q_t		-1.170	-0.017				1.213	0.047	-0.090	
	$\xi^{y^*_t}_t$	-0.747	0.263	-0.013		1.599			-0.530	0.263	1.599
	$\xi^{\pi^*_t}_t$		-2.519				2.519				

Note: This table reports the numerical solutions of regime 2 of the model two for the UK. Regime 2 considers a low level of the coefficient of inflation in Taylor rule.

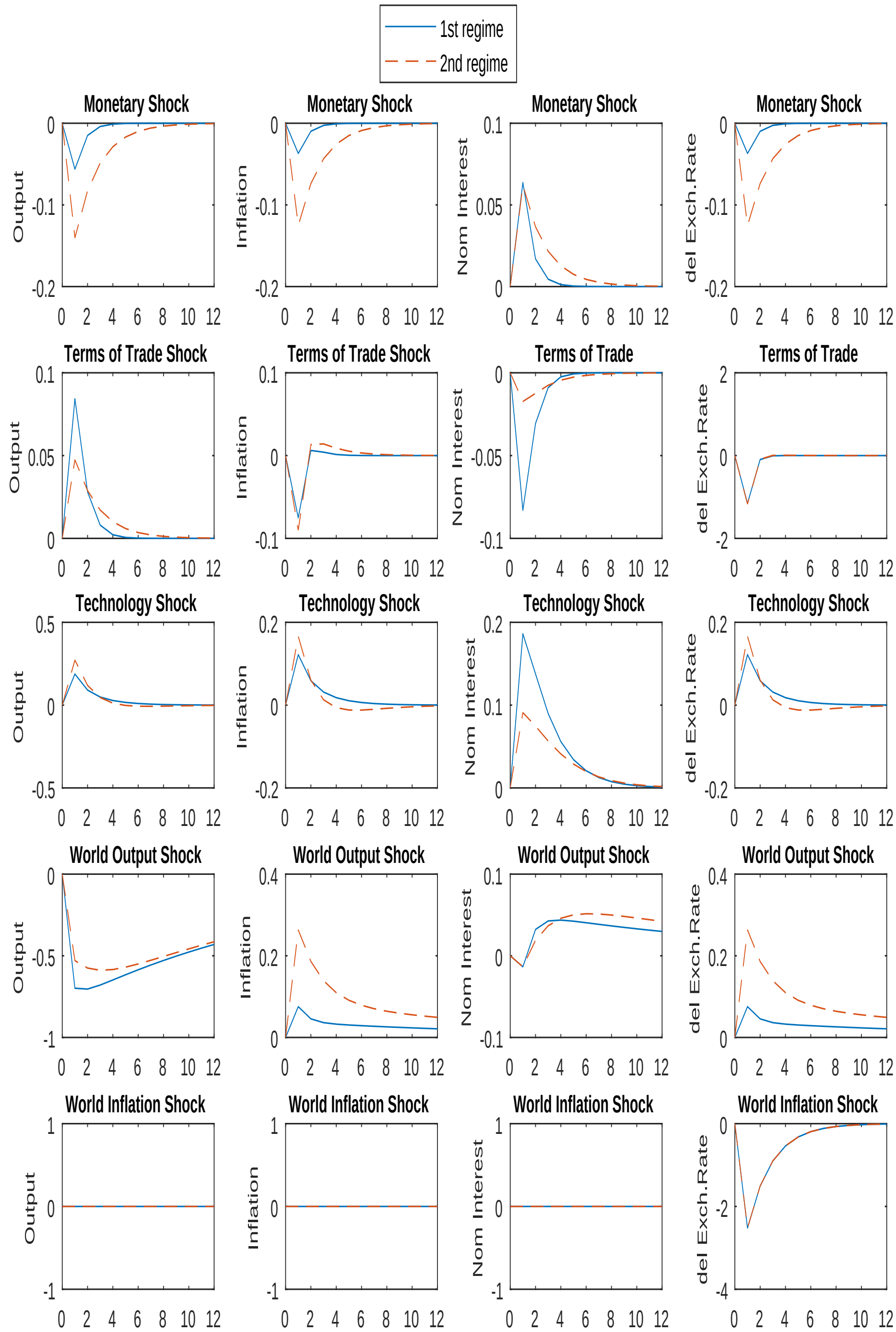

Figure 4.2: Impulse responses,UK(Model Two). Note:The figure depicts the impulses responses of real output, inflation rate, nominal interest rate and depreciation exchange rate to one unit structural shock of regime 1 representing more strict inflation targeting(solid and blue lines)and regime 2 representing less strict inflation targeting(dashed and red lines).

Variance Decomposition of the Model Two for the UK

Table 4.10 presents the variance decomposition of the model two with the Markov-switching policy parameters for the UK. It summarises the major contributions to the variation of the four endogenous variables $y\tilde{y}_t$,π_t,$\tilde{r}_t$ and $\triangle\tilde{e}_t$ as follows. First, the world output shock $\xi_t^{y_t^*}$ contributes most to the variation of the output deviation $y\tilde{y}_t$ in both of the two regimes. Second, the change rate of technology shock ξ_t^z contributes most to the variation of the inflation π_t in regime 1 while the world output shock $\xi_t^{y_t^*}$ contributes most in regime 2. Third, the change rate of the technology shock ξ_t^z contributes most to the variation of the interest rate $\tilde{r}_t$ in regime 1 while the world output shock $\xi_t^{y_t^*}$ contributes most in regime 2. Finally, the world inflation shock $\xi_t^{\pi_t^*}$ contributes most to the variation of nominal exchange rate depreciation $\triangle\tilde{e}_t$ in both of the two regimes.

The policy shock ξ_t^R contributes 0.06% to the variation of the output deviation $y\tilde{y}_t$ in regime 1 while 0.61% in regime 2,3.16% to the variation of inflation π_t in regime 1 while only 9.34% in regime 2,4.25% to the variation of interest rate $\tilde{r}_t$ in regime 1 while 8.96% in regime 2,0.01% to the variation of exchange rate depreciation $\triangle\tilde{e}_t$ in regime 1 while 0.21% in regime 2.

The change of the terms of trade shock ξ_t^q contributes 0.14% to the variation of the output deviation $y\tilde{y}_t$ in regime 1 while 0.07% in regime 2,12.22% to the variation of inflation π_t in the regime 1 while only 3.34% in regime 2,7.72% to the variation of interest rate $\tilde{r}_t$ in regime 1 while only 0.82% in regime 2,11.92% to the variation of exchange rate depreciation $\triangle\tilde{e}_t$ in regime 1 while 11.96% in regime 2.

The change rate of the technology shock ξ_t^z contributes 0.81% to the variation of the output deviation $y\tilde{y}_t$ in regime 1 while 1.82% in regime 2,42.60% to the variation of inflation π_t in regime 1 while 12.42% in regime 2,65.00% to the variation of interest rate $\tilde{r}_t$ in regime 1 while 30.78% in regime 2,0.18% to the variation of exchange rate depreciation $\triangle\tilde{e}_t$ in regime 1 while 0.28% in regime 2.

The world output shock $\xi_t^{y_t^*}$ contributes 99.00% to the variation of the output deviation $y\tilde{y}_t$ in regime 1 while 97.50% in regime 2,42.02% to the variation of inflation π_t in regime 1 while 74.89% in regime 2,23.03% to the variation of interest rate $\tilde{r_t}$ in regime 1 while 59.43% in regime 2,0.17% to the variation of exchange rate depreciation $\triangle\tilde{e_t}$ in regime 1 while 1.67% in regime 2.

The world inflation shock $\xi_t^{\pi_t^*}$ contributes 87.72% to the variation of exchange rate depreciation $\triangle\tilde{e_t}$ in regime 1 while 85.88% in regime 2. It contributes nothing to the variation of the other three endogenous variables.

Table 4.10: Variance decomposition of the Model Two for the UK

Regime 1	Output	Inflation	interest rate	Exchange rate depreciation
Policy	0.06	3.16	4.25	0.01
Terms of trade	0.14	12.22	7.72	11.92
Technology	0.81	42.60	65.00	0.18
World output	99.00	42.02	23.03	0.17
World inflation	0.00	0.00	0.00	87.72

Regime 2	Output	Inflation	interest rate	Exchange rate depreciation
Policy	0.61	9.34	8.96	0.21
Terms of trade	0.07	3.34	0.82	11.96
Technology	1.82	12.42	30.78	0.28
World output	97.50	74.89	59.43	1.67
World inflation	0.00	0.00	0.00	85.88

Note:(a)The table reports the variance decomposition of the model two for the UK.

(b)Regime 1 is characterized as more strict inflation targeting compared to Regime 2.

4.3.3 UK: the Model Three with Switching Variances and Switching Policy Parameters

Table 4.11 reports the estimated results of the model three with Markov-switching variances and Markov-switching policy parameters for the UK. This model enables the vector of the standard deviations of exogenous shocks $[\sigma_R, \sigma_z, \sigma_{y^*}, \sigma_{\pi^*}, \sigma_q]$ and the vector of the coefficients of Taylor rule $[\phi_\pi, \phi_y, \phi_{\triangle y}, \rho_R]$ to follow two independent Markov chains respectively and shift between two regimes in each of the chain. The posterior mean of the transition probability from regime 1 to regime 2 (Q_{12}) for the first Markov chain is 0.070 while from regime 2 to regime 1 (Q_{21}) is 0.080. The posterior mean of the transition probability for the second Markov chain from regime 1 to regime 2 (P_{12}) is 0.075 while from regime 2 to regime 1 (P_{21}) is 0.134.

The combinations of the two regimes in each Markov chain will lead to four regimes. Regime 1 stands for low volatility and more strict inflation targeting. Regime 2 represents high volatility and more strict inflation targeting. Regime 3 serves as low volatility and less strict inflation targeting. Regime 4 substitutes for high volatility and less strict inflation targeting. Table 4.11 directly provides empirical results for regime 1 and regime 4. Exchanging the vector of coefficients of Taylor rule yields regime 3 in the place of regime 1 and regime 2 in the place of regime 4.

More specifically, regime 1 and regime 2 represent more strict inflation targeting with the posterior mean of ϕ_π being 2.540 compared to 1.339 in regime 3 and regime 4. Next, the posterior mean of the coefficient of output deviation ϕ_y in regime 1 and regime 2 is 0.155 compared to 0.194 in regime 3 and regime 4. Furthermore, the posterior mean of the rate change of output $\phi_{\triangle y}$ in regime 1 and regime 2 is two times larger than it in regime 3 and regime 4. Finally, the posterior mean of the persistence ratio ρ_R in regime 1 and regime 2 is 0.707 compared to 0.963 in regime 3 and regime 4.

Moreover, regime 1 and regime 3 substitute for the low level of volatility with the posterior means of the above standard deviations being $[0.195, 0.654, 0.784, 2.144, 0.922]$ compared to $[0.322, 3.993, 2.082, 4.260, 2.849]$ in regime 2 and regime 4. The differences between the two vectors are significant. The standard deviation of the rate change of the technology shock σ_z in regime 2 and regime 4 is six times larger than in regime 1 and regime 3. The standard deviations of the world output shock σ_{y^*} and the rate change of the terms of trade shock σ_q are three times larger than in regime 1 and regime 3. The standard deviations of nominal interest rate shock σ_R and foreign inflation shock σ_{π^*} almost double in regime 2 and regime 4.

The calculated marginal density is -469.735 for model three with the two independent Markov chains, which is much bigger compared to the constant parameter model (-501.064).

Table 4.11: Model Three with 2 Markov Chains(UK)

	Regime 1:		Regime4:	
parameter	mean	90% interval	mean	90% interval
τ	0.286	[0.187,0.400]		
κ	0.646	[0.272,1.341]		
ϕ_π	2.540	[1.800,3.353]	1.339	[1.026,1.896]
ϕ_y	0.155	[0.084,0.256]	0.194	[0.088,0.336]
$\phi_{\triangle y}$	0.175	[0.097,0.249]	0.084	[0.034,0.155]
ρ_R	0.707	[0.578,0.801]	0.967	[0.778,0.994]
ρ_z	0.545	[0.464,0.665]		
ρ_q	0.061	[0.014,0.149]		
ρ_{π^*}	0.516	[0.356,0.689]		
ρ_{y^*}	0.966	[0.926,0.989]		
α	0.139	[0.074,0.199]		
$r^{(A)}$	3.373	[3.027,3.735]		
$\pi^{(A)}$	2.319	[1.520,3.057]		
$\gamma^{(A)}$	0.697	[0.659,0.734]		
σ_R	0.195	[0.143,0.248]	0.322	[0.187,0.516]
σ_z	0.654	[0.275,0.969]	3.993	[1.221,5.918]
σ_{y^*}	0.784	[0.336,1.452]	2.082	[0.884,3.601]
σ_{π^*}	2.144	[1.805,2.553]	4.260	[2.628,6.168]
σ_q	0.922	[0.785,1.130]	2.849	[1.737,4.190]
P_{12}	0.075	[0.036,0.135]		
P_{21}	0.134	[0.051,0.238]		
Q_{12}	0.070	[0.035,0.119]		
Q_{21}	0.080	[0.029,0.141]		

Note:The table reports posterior means and 90% probability interval of the structural parameters in the model three for the UK.

Numerical Solution and Simulation Results for the UK in the Model Three

There are four regimes in the model three with the Markov-switching variances and the Markov-switching policy parameters. Table 4.12 provides the numerical solution to regime 1, representing a low level of volatility and more strict inflation targeting. Table 4.13 presents the numerical solution to regime 2, representing a high level of volatility and more strict inflation targeting. Table 4.14 presents the numerical solution to regime 3, representing a low level of volatility and less strict inflation targeting. Table 4.15 produces the numerical solution to regime 4, representing a high level of volatility and less strict inflation targeting.

Table 4.12 contributes to computing the impulse response functions of the four endogenous variables including $y\tilde{y}_t$,π_t,$\tilde{r}_t$ and $\triangle\tilde{e}_t$ in regime 1:

$$\begin{pmatrix} y\tilde{y}_t \\ \pi_t \\ \tilde{r}_t \\ \triangle\tilde{e}_t \end{pmatrix} = \begin{pmatrix} -0.300 & 0.094 & -0.559 & 0 & 0.074 & 0.003 \\ -0.670 & 0.088 & 0.189 & 0 & 0.166 & -0.002 \\ 0.142 & 0.086 & 0.017 & 0 & -0.035 & -0.001 \\ -0.670 & 0.088 & 0.189 & -0.516 & 0.166 & -0.055 \end{pmatrix} \begin{pmatrix} r\tilde{_{t-1}} \\ z_{t-1} \\ yy\tilde{^*_{t-1}} \\ \pi^*_{t-1} \\ yy\tilde{_{t-1}} \\ \triangle q\tilde{_{t-1}} \end{pmatrix}$$

$$+ \begin{pmatrix} -0.083 & 0.113 & 0.041 & -0.454 & 0 \\ -0.185 & 0.105 & -0.034 & 0.153 & 0 \\ 0.039 & 0.103 & -0.017 & 0.014 & 0 \\ -0.185 & 0.105 & -0.828 & 0.153 & -2.144 \end{pmatrix} \begin{pmatrix} \xi^R_t \\ \xi^z_t \\ \xi^q_t \\ \xi^{y^*_t}_t \\ \xi^{\pi^*_t}_t \end{pmatrix}, \quad (4.19)$$

where the second matrix in the above equation introduces the impact of the structural shocks on the real output, inflation rate, nominal interest rate and depreciation exchange rate in regime 1.

Table 4.13 provides numerical information to compute the impulse response func-

tions of the four endogenous variables including $\tilde{yy_t}$,π_t,$\tilde{r_t}$ and $\triangle\tilde{e_t}$ in regime 2:

$$\begin{pmatrix} \tilde{yy_t} \\ \pi_t \\ \tilde{r_t} \\ \triangle\tilde{e_t} \end{pmatrix} = \begin{pmatrix} -0.300 & 0.094 & -0.559 & 0 & 0.074 & 0.003 \\ -0.670 & 0.088 & 0.189 & 0 & 0.166 & -0.002 \\ 0.142 & 0.086 & 0.017 & 0 & -0.035 & -0.001 \\ -0.670 & 0.088 & 0.189 & -0.516 & 0.166 & -0.055 \end{pmatrix} \begin{pmatrix} \tilde{r_{t-1}} \\ z_{t-1} \\ \tilde{yy^*_{t-1}} \\ \pi^*_{t-1} \\ \tilde{yy_{t-1}} \\ \triangle\tilde{q_{t-1}} \end{pmatrix}$$

$$+ \begin{pmatrix} -0.137 & 0.688 & 0.128 & -1.206 & 0 \\ -0.305 & 0.643 & -0.106 & 0.407 & 0 \\ 0.065 & 0.630 & -0.051 & 0.037 & 0 \\ -0.305 & 0.643 & -2.560 & 0.407 & -4.260 \end{pmatrix} \begin{pmatrix} \xi_t^R \\ \xi_t^z \\ \xi_t^q \\ \xi_t^{y_t^*} \\ \xi_t^{\pi_t^*} \end{pmatrix}, \quad (4.20)$$

where the second matrix in the above equation incorporates the impact of the structural shocks on the real output, inflation rate, nominal interest rate and depreciation exchange rate in regime 2.

Table 4.14 offers information to compute the impulse response functions of the four endogenous variables including $\tilde{yy_t}$,π_t,$\tilde{r_t}$ and $\triangle\tilde{e_t}$ in regime 3:

$$\begin{pmatrix} \tilde{yy_t} \\ \pi_t \\ \tilde{r_t} \\ \triangle\tilde{e_t} \end{pmatrix} = \begin{pmatrix} -4.034 & 0.189 & -0.203 & 0 & 0.350 & 0.002 \\ -10.028 & 0.286 & 1.090 & 0 & 0.871 & -0.004 \\ 0.159 & 0.030 & 0.030 & 0 & -0.014 & 0 \\ -10.028 & 0.286 & 1.090 & -0.516 & 0.871 & -0.057 \end{pmatrix} \begin{pmatrix} \tilde{r_{t-1}} \\ z_{t-1} \\ \tilde{yy^*_{t-1}} \\ \pi^*_{t-1} \\ \tilde{yy_{t-1}} \\ \triangle\tilde{q_{t-1}} \end{pmatrix}$$

$$+ \begin{pmatrix} -0.813 & 0.227 & 0.025 & -0.165 & 0 \\ -2.022 & 0.344 & -0.065 & 0.884 & 0 \\ 0.032 & 0.036 & -0.001 & 0.024 & 0 \\ -2.022 & 0.344 & -0.859 & 0.884 & -2.144 \end{pmatrix} \begin{pmatrix} \xi_t^R \\ \xi_t^z \\ \xi_t^q \\ \xi_t^{y_t^*} \\ \xi_t^{\pi_t^*} \end{pmatrix}, \quad (4.21)$$

where the second matrix in the above equation embodies the impact of the structural shocks on the real output, inflation rate, nominal interest rate and depreciation exchange rate in regime 3.

Table 4.15 brings in numerical information to compute the impulse response functions of the four endogenous variables including $\tilde{yy_t}$,π_t,$\tilde{r_t}$ and $\triangle\tilde{e_t}$ in the regime 4:

$$\begin{pmatrix} \tilde{yy_t} \\ \pi_t \\ \tilde{r_t} \\ \triangle\tilde{e_t} \end{pmatrix} = \begin{pmatrix} -4.034 & 0.189 & -0.203 & 0 & 0.350 & 0.002 \\ -10.028 & 0.286 & 1.090 & 0 & 0.871 & -0.004 \\ 0.159 & 0.030 & 0.030 & 0 & -0.014 & 0 \\ -10.028 & 0.286 & 1.090 & -0.516 & 0.871 & -0.057 \end{pmatrix} \begin{pmatrix} \tilde{r_{t-1}} \\ z_{t-1} \\ \tilde{yy^*_{t-1}} \\ \pi^*_{t-1} \\ \tilde{yy_{t-1}} \\ \triangle\tilde{q_{t-1}} \end{pmatrix}$$
$$+ \begin{pmatrix} -1.343 & 1.384 & 0.079 & -0.438 & 0 \\ -3.339 & 2.098 & -0.200 & 2.349 & 0 \\ 0.053 & 0.218 & -0.002 & 0.064 & 0 \\ -3.339 & 2.098 & -2.653 & 2.349 & -4.260 \end{pmatrix} \begin{pmatrix} \xi_t^R \\ \xi_t^z \\ \xi_t^q \\ \xi_t^{y_t^*} \\ \xi_t^{\pi_t^*} \end{pmatrix}, \quad (4.22)$$

where the second matrix in the above equation includes the impact of the structural shocks on the real output, inflation rate, nominal interest rate and depreciation exchange rate in regime 4.

Figure 4.3 compares the impulse response functions between regime 3 and regime 4 in model three with two independent Markov chains. In other words, It compares the impulse response functions between the high and the low level of the volatility given the same low coefficient of the inflation rate representing less strict inflation targeting. It depicts the impulse response functions of a low level of the volatility with the solid and blue lines, and a high level of the volatility with the dashed and red lines.

Holding everything else constant, a unit of the standard deviation of the monetary policy shock will exert a -0.813 impact on the real output deviation in regime 3 while -1.343 in regime 4; a -2.022 impact on inflation in regime 3 while -3.339 in regime 4;a 0.032 impact on the nominal interest rate deviation in regime 3 while 0.053 in regime 4; a -2.022 impact on the nominal exchange rate depreciation in regime 3 while -3.339 in regime 4.

Holding everything else constant, a unit of the standard deviation of the shock to the change rate of the terms of trade will exert a 0.025 impact on the real output deviation in regime 3 while 0.079 in regime 4; a -0.065 impact on inflation in regime 3 while -0.200 in regime 4;a -0.001 impact on the nominal interest rate deviation in regime 3 while -0.002 in regime 4; a -0.859 impact on the nominal exchange rate depreciation in regime 3 while -2.653 in regime 4.

Holding everything else constant, a unit of the standard deviation of the shock to the change rate of the technology will exert a 0.227 impact on the real output deviation in regime 3 while 1.384 in regime 4; a 0.344 impact on inflation in regime 3 while 2.098 in regime 4;a 0.036 impact on the nominal interest rate deviation in regime 3 while 0.218 in regime 4; a 0.344 impact on the nominal exchange rate depreciation in regime 3 while 2.098 in regime 4.

Holding everything else constant, a unit of the standard deviation of the shock to the world output deviation will exert a -0.165 impact on the real output deviation in regime 3 while -0.438 in regime 4; a 0.884 impact on inflation in regime 3 while 2.349 in regime 4;a 0.024 impact on the nominal interest rate deviation in regime 3 while 0.064 impact regime 4; a 0.884 impact on the nominal exchange rate depreciation in regime 3 while 2.349 in regime 4.

Holding everything else constant, a unit of the standard deviation of the shock to the world inflation will only exert a -2.144 impact on the exchange rate depreciation in regime 3 while −4.260 in regime 4.

Figure 4.4 compares the impulse response functions between regime 2 and regime 4 in model three with two independent Markov chains. It compares the impulse response functions between the high and the low level of the coefficient of inflation given the same high level of volatility. It depicts the impulse response functions of a high level of the coefficient of inflation with solid and blue lines, and a low level of the coefficient of inflation with dashed and red lines.

Holding everything else constant, a unit of the standard deviation of the monetary policy shock will exert a −0.137 impact on the real output deviation in regime 2 while −1.343 in regime 4; a −0.3052 impact on inflation in regime 2 while −3.339 in regime 4;a 0.065 impact on the nominal interest rate deviation in regime 2 while 0.053 regime 4; a −0.305 impact on the nominal exchange rate depreciation in regime 2 while −3.339 in regime 4.

Holding everything else constant, a unit of the standard deviation of the shock to the change rate of the terms of trade will exert a 0.128 impact on the real output deviation in regime 2 while 0.079 in regime 4; a −0.106 impact on inflation in regime 2 while −0.200 in regime 4;a −0.051 impact on the nominal interest rate deviation in regime 2 while −0.002 in regime 4; a −2.560 impact on the nominal exchange rate depreciation in regime 2 while −2.653 in regime 4.

Holding everything else constant, a unit of the standard deviation of the shock to the change rate of the technology will exert a 0.688 impact on the real output deviation in regime 2 while 1.384 in regime 4; a 0.643 impact on inflation in regime 2 while 2.098 in regime 4;a 0.630 impact on the nominal interest rate deviation in regime 2 while 0.218 in regime 4; a 0.643 impact on the nominal exchange rate depreciation in regime 2 while 2.098 in regime 4.

Holding everything else constant, a unit of the standard deviation of the shock to the world output deviation will exert a -1.206 impact on the real output deviation in regime 2 while -0.438 in regime 4; a 0.407 impact on inflation in regime 2 while 2.349 in regime 4;a 0.037 impact on the nominal interest rate deviation regime 2 while 0.064 impact in regime 4; a 0.407 impact on the nominal exchange rate depreciation in regime 2 while 2.349 in regime 4.

Holding everything else constant, a unit of the standard deviation of the shock to the world inflation will only exert a -4.260 impact on the nominal exchange rate depreciation in regime 2 and the regime 4.

Table 4.12: The Numerical Solution to Regime 1 of the Model Three for the UK

		static		backward looking				mixed		forward looking	
		$y\tilde{y}_{t,n}$	$\triangle\tilde{e}_t$	$\tilde{r}_t$	z_t	$\tilde{yy}^*_t$	π^*_t	$\triangle\tilde{q}_t$	$y\tilde{y}_t$	π_t	$\triangle\tilde{yy}^*_t$
endogenous variables	$\tilde{r}_{t-1}$		-0.670	0.142					-0.300	-0.670	
	z_{t-1}		0.088	0.086	0.545				0.094	0.088	
	$\tilde{yy}^*_{t-1}$	-0.624	0.189	0.017		0.966			-0.559	0.189	-0.034
	π^*_{t-1}		-0.516				0.516				
	$y\tilde{y}_{t-1}$		0.166	-0.035					0.074	0.166	
	$\triangle\tilde{q}_{t-1}$		-0.055	-0.001				0.061	0.003	-0.002	
exogenous variables	ξ^R_t		-0.185	0.039					-0.083	-0.185	
	ξ^z_t		0.105	0.103	0.654				0.113	0.105	
	ξ^q_t		-0.828	-0.017				0.922	0.041	-0.034	
	$\xi^{y^*_t}_t$	-0.506	0.153	0.014		0.784			-0.454	0.153	0.784
	$\xi^{\pi^*_t}_t$		-2.144				2.144				

Note: This table reports the numerical solutions of regime 1 in the model three for the UK. Regime 1 considers a low level of the volatility and a high level of the coefficient of inflation.

Table 4.13: The Numerical Solution to Regime 2 of the Model Three for the UK

		static		backward looking				mixed		forward looking	
		$\tilde{yy}_{t,n}$	$\triangle\tilde{e}_t$	$\tilde{r}_t$	z_t	$\tilde{yy}^*_t$	π^*_t	$\triangle\tilde{q}_t$	$\tilde{yy}_t$	π_t	$\triangle\tilde{yy}^*_t$
endogenous variables	$\tilde{r}_{t-1}$		-0.670	0.142					-0.300	-0.670	
	z_{t-1}		0.088	0.086	0.545				0.094	0.088	
	$\tilde{yy}^*_{t-1}$	-0.624	0.189	0.017		0.966			-0.559	0.189	-0.034
	π^*_{t-1}		-0.516				0.516				
	$\tilde{yy}_{t-1}$		0.166	-0.035					0.074	0.166	
	$\triangle\tilde{q}_{t-1}$		-0.055	-0.001				0.061	0.003	-0.002	
exogenous variables	ξ^R_t		-0.305	0.065					-0.137	-0.305	
	ξ^z_t		0.643	0.630	3.993				0.688	0.643	
	ξ^q_t		-2.560	-0.051				2.849	0.128	-0.106	
	$\xi^{y^*_t}_t$	-1.345	0.407	0.037		2.082			-1.206	0.407	2.082
	$\xi^{\pi^*_t}_t$		-4.260				4.260				

Note: This table reports the numerical solutions to regime 2 in the model three for the UK. Regime 2 considers high levels of the volatility and the coefficient of inflation.

Table 4.14: The Numerical Solution to Regime 3 of the Model Three for the UK

		static		backward looking				mixed		forward looking	
		$y\tilde{y}_{t,n}$	$\triangle\tilde{e}_t$	$\tilde{r}_t$	z_t	$\tilde{yy}_t^*$	π_t^*	$\triangle\tilde{q}_t$	$y\tilde{y}_t$	π_t	$\triangle\tilde{yy}_t^*$
endogenous variables	$\tilde{r_{t-1}}$		-10.028	0.159					-4.034	-10.028	
	z_{t-1}		0.286	0.030	0.545				0.189	0.286	
	$\tilde{yy}_{t-1}^*$	-0.624	1.090	0.030		0.966			-0.203	1.090	-0.034
	π_{t-1}^*		-0.516				0.516				
	$\tilde{yy_{t-1}}$		0.871	-0.014					0.350	0.871	
	$\triangle\tilde{q_{t-1}}$		-0.057	0.000				0.061	0.002	-0.004	
exogenous variables	ξ_t^R		-2.022	0.032					-0.813	-2.022	
	ξ_t^z		0.344	0.036	0.654				0.227	0.344	
	ξ_t^q		-0.859	-0.001				0.922	0.025	-0.065	
	$\xi_t^{y_t^*}$	-0.506	0.884	0.024		0.784			-0.165	0.884	0.784
	$\xi_t^{\pi_t^*}$		-2.144				2.144				

Note: This table reports the numerical solutions to regime 3 in the model three for the UK. Regime 3 considers low levels of the volatility and the coefficient of inflation.

Table 4.15: The Numerical Solution to Regime 4 of the Model Three for the UK

		static		backward looking				mixed		forward looking	
		$\tilde{yy}_{t,n}$	$\triangle\tilde{e}_t$	$\tilde{r}_t$	z_t	$\tilde{yy}^*_t$	π^*_t	$\triangle\tilde{q}_t$	$\tilde{yy}_t$	π_t	$\triangle\tilde{yy}^*_t$
endogenous variables	$\tilde{r}_{t-1}$		-10.028	0.159					-4.034	-10.028	
	z_{t-1}		0.286	0.030	0.545				0.189	0.286	
	$\tilde{yy}^*_{t-1}$	-0.624	1.090	0.030		0.966			-0.203	1.090	-0.034
	π^*_{t-1}		-0.516				0.516				
	$\tilde{yy}_{t-1}$		0.871	-0.014					0.350	0.871	
	$\triangle\tilde{q}_{t-1}$		-0.057	0.000				0.061	0.002	-0.004	
exogenous variables	ξ^R_t		-3.339	0.053					-1.343	-3.339	
	ξ^z_t		2.098	0.218	3.993				1.384	2.098	
	ξ^q_t		-2.653	-0.002				2.849	0.079	-0.200	
	$\xi^{y^*_t}_t$	-1.345	2.349	0.064		2.082			-0.438	2.349	2.082
	$\xi^{\pi^*_t}_t$		-4.260				4.260				

Note: This table reports the numerical solutions to regime 4 in the model three for the UK. Regime 4 considers a high level of the volatility and a low level of the coefficient of inflation.

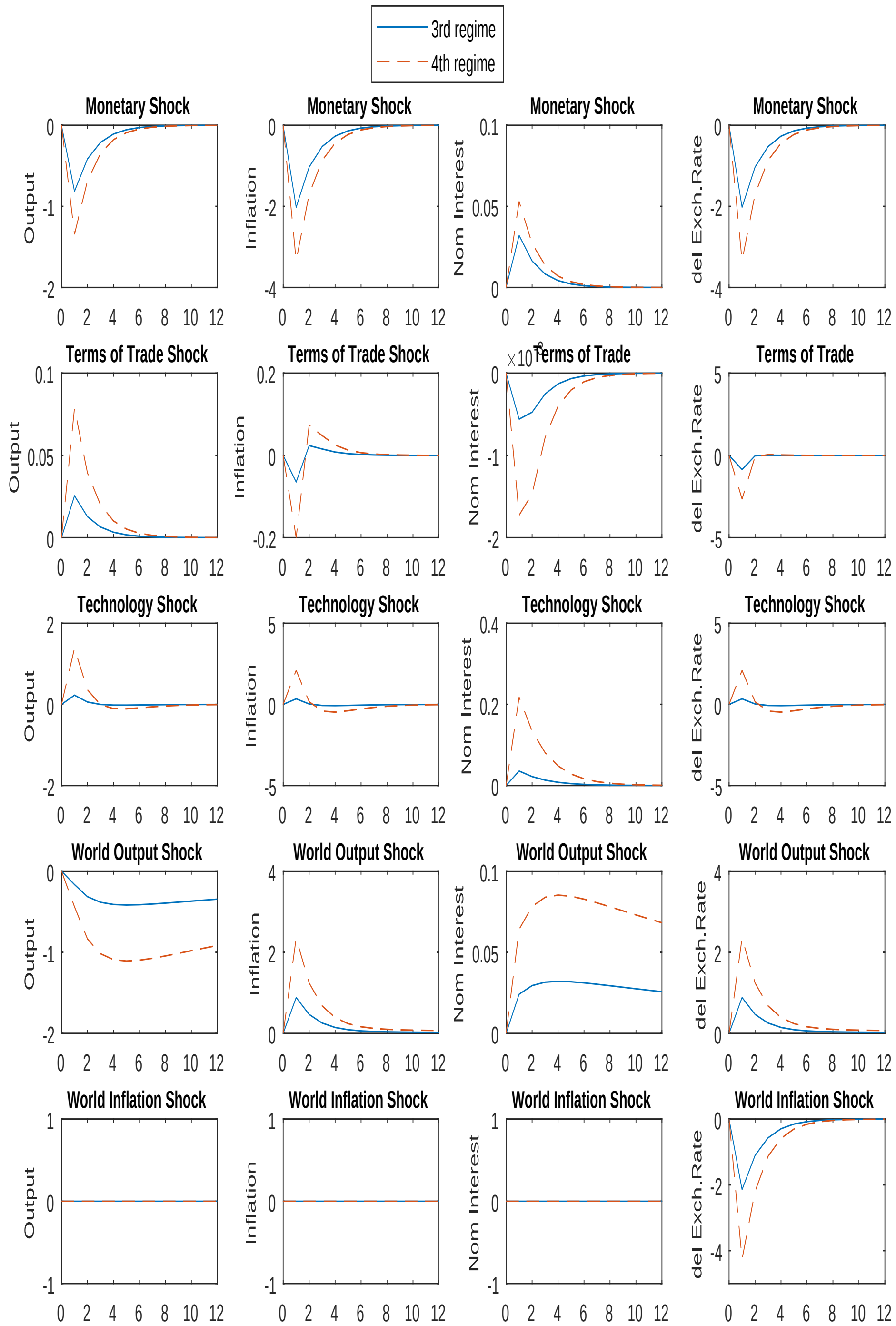

Figure 4.3: Impulse responses,UK(Switching Variance of Model Three). Note:The figure depicts the impulses responses of real output, inflation rate, nominal interest rate and depreciation exchange rate to one unit structural shock of regime 3 representing a low level of the volatility(solid lines)and regime 4 representing a high level of the volatility(dashed lines) given less strict inflation targeting.

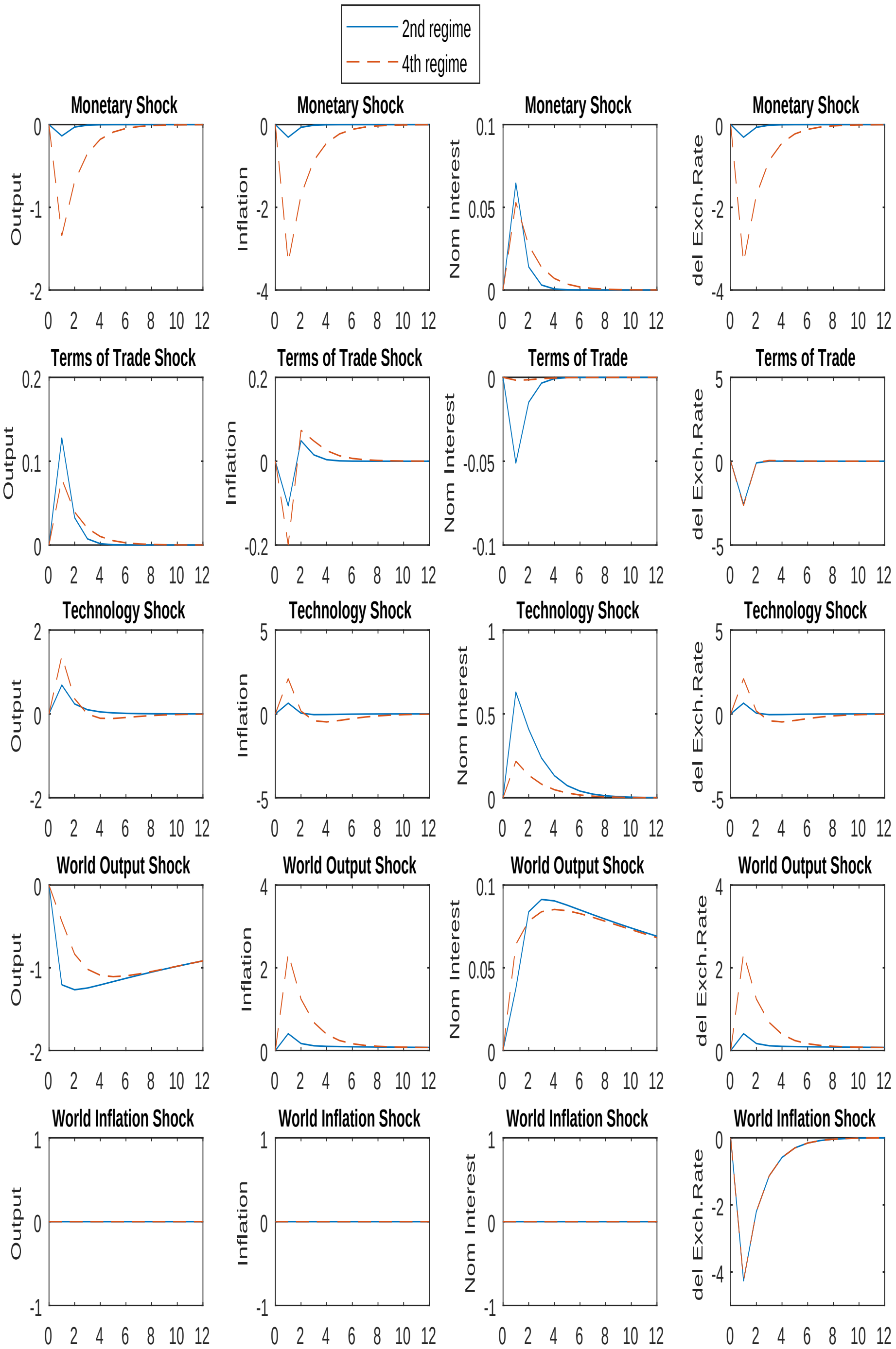

Figure 4.4: Impulse responses,UK(Switching Taylor Rules of Model Three). Note:The figure depicts the impulses responses of real output, inflation rate, nominal interest rate and depreciation exchange rate to one unit structural shock of regime 2 representing a high level of the coefficient of inflation(solid and blue lines)and regime 4 representing a low level of the coefficient of inflation (dashed and red lines) given high levels of volatility.

Variance Decomposition of the Model Three for the UK

Table 4.16 reports the variance decomposition of the model three with two independent Markov chains for the UK. It summarises the major contributions to the variation of the four endogenous variables $y\tilde{y}_t$,π_t,$\tilde{r}_t$ and $\triangle\tilde{e}_t$ as follows. First, the world output shock $\xi_t^{y_t^*}$ contributes most to the variation of the output $y\tilde{y}_t$ in each of the four regimes. Second, the world output shock $\xi_t^{y_t^*}$ contributes most to the variation of the inflation π_t in regime 1, the change rate of the technology shock ξ_t^z contributes most in regime 2 and the policy shock ξ_t^R contributes most in regime 3 and regime 4. Furthermore, the world output shock $\xi_t^{y_t^*}$ contributes most to the variation of the interest rate $\tilde{r}_t$ in all the regimes except for regime 2, where the change rate of the technology shock ξ_t^z contributes most. Finally, the world inflation shock $\xi_t^{\pi_t^*}$ contributes most to the variation of the nominal exchange rate depreciation $\triangle\tilde{e}_t$ in all the regimes.

The policy shock ξ_t^R contributes 0.19% to the variation of the output deviation $y\tilde{y}_t$ in regime 1, 0.07% in regime 2, 21.04% in regime 3 and 8.77% in regime 4. Next, it contributes 36.65% to the variation of the inflation π_t in regime 1, 11.13% in regime 2, 81.44% in regime 3 and 53.61% in regime 4. Moreover, it contributes 4.10% to the variation of the interest rate $\tilde{r}_t$ in regime 1, 0.55% in regime 2, 6.10% in regime 3 and 1.76% in regime 4. Finally, it contributes 0.51% to the variation of the exchange rate depreciation $\triangle\tilde{e}_t$ in regime 1, 0.30% in regime 2, 40.09% in regime 3 and 25.18% in regime 4.

The change rate of the terms of trade shock ξ_t^q contributes 0.05% to the variation of the output deviation $y\tilde{y}_t$ in regime 1, 0.06% in regime 2,0.02% in regime 3 and 0.03% in regime 4. Next, it contributes 1.50% to the variation of the inflation π_t in regime 1, 1.59% in regime 2,0.08% in regime 3 and 0.17% in regime 4. Moreover, it contributes 0.76% to the variation of the interest rate $\tilde{r}_t$ in regime 1, 0.36% in regime 2, and nothing in regime 3 and regime 4. Finally, it contributes 9.75% to the variation of the exchange rate depreciation $\triangle\tilde{e}_t$ in regime 1, 20.40%

in regime 2. 5.35% in the regime 3 and 11.78% in regime 4.

The change rate of the technology shock ξ_t^z contributes 0.39% to the variation of the output deviation $y\tilde{y}_t$ in regime 1, 2.01% in regime 2, 1.31% in regime 3 and 7.49% in regime 4. Next, it contributes 11.49% to the variation of the inflation π_t in regime 1, 47.70% in regime 2, 2.02% in regime 3 and 18.19% in regime 4. Furthermore, it contributes 44.16% to the variation of the interest rate $\tilde{r}_t$ in regime 1, 81.33% in regime 2, 8.89% in regime 3 and 34.96% in regime 4. Finally, it contributes 0.16% to the variation of the exchange rate depreciation $\triangle\tilde{e}_t$ in regime 1, 1.30% in regime 2,0.99% in regime 3 and 8.54% in regime 4.

The world output shock $\xi_t^{y_t^*}$ contributes 99.38% to the variation of the output deviation $y\tilde{y}_t$ in regime 1, 97.86% in regime 2, 77.63% in regime 3 and 83.71% in regime 4. Moreover, it contributes 50.37% to the variation of the inflation π_t in regime 1, 39.58% in regime 2,16.47% in regime 3 and 28.04% in regime 4. Next, it contributes 50.97% to the variation of the interest rate $\tilde{r}_t$ in regime 1, 17.76% in regime 2, 85.01% in regime 3 and 63.28% in regime 4. Finally, it contributes 0.70% to the variation of the exchange rate depreciation $\triangle\tilde{e}_t$ in regime 1, 1.08% in regime 2,8.11% in regime 3 and 13,17% in regime 4.

The world inflation shock $\xi_t^{\pi_t^*}$ contributes 88.88% to the variation of the exchange rate depreciation $\triangle\tilde{e}_t$ in regime 1, 76.91% in regime 2, 45.46% in regime 3 and 41.34% in regime 4. It contributes nothing to the variation of the other three endogenous variables.

Table 4.16: Variance Decomposition of the Model Three for the UK

Regime 1	Output	Inflation	interest rate	Exchange rate depreciation
Policy	0.19	36.65	4.10	0.51
Terms of trade	0.05	1.50	0.76	9.75
Technology	0.39	11.49	44.16	0.16
World output	99.38	50.37	50.97	0.70
World inflation	0.00	0.00	0.00	88.88
Regime 2	**Output**	**Inflation**	**interest rate**	**Exchange rate depreciation**
Policy	0.07	11.13	0.55	0.30
Terms of trade	0.06	1.59	0.36	20.40
Technology	2.01	47.70	81.33	1.30
World output	97.86	39.58	17.76	1.08
World inflation	0.00	0.00	0.00	76.91
Regime 3	**Output**	**Inflation**	**interest rate**	**Exchange rate depreciation**
Policy	21.04	81.44	6.10	40.09
Terms of trade	0.02	0.08	0.00	5.35
Technology	1.31	2.02	8.89	0.99
World output	77.63	16.47	85.01	8.11
World inflation	0.00	0.00	0.00	45.46
Regime 4	**Output**	**Inflation**	**interest rate**	**Exchange rate depreciation**
Policy	8.77	53.61	1.76	25.18
Terms of trade	0.03	0.17	0.00	11.78
Technology	7.49	18.19	34.96	8.54
World output	83.71	28.04	63.28	13.17
World inflation	0.00	0.00	0.00	41.34

Note:The table reports the variance decomposition of the model three for the UK. Regime 1(2) is characterised as a low (high) volatility and a high level of coefficient of inflation. Regime 3(4) is characterised as a low (high) volatility and a low level of coefficient of inflation.

4.3.4 The Model Comparison and Data Analysis for the UK

Table 4.17 presents the model comparison at the second stage for the UK. It ranks the models from best to worst in terms of data fitting. As mentioned in chapter 3, the log marginal data densities lead to the posterior odds ratios, thereby evaluating the relative performance of models. When the ratio is above than 3 or smaller than $\frac{1}{3}$, it shows that one mode outperforms or underperforms others significantly in terms of data fitting.

More specifically, the UK data is firmly in favour of the Markov-switching models compared to the benchmark model with the best data fitting in chapter 3. The model one with switching variances ranks first in terms of data fitting, the model three with two independent Markov chains ranks second, and the model two with switching policy parameters ranks third. The benchmark model without Markov-switching parameters ranks last. Thus, introducing either or both of the Markov chains can improve the performance of the simplified model on data fitting for the UK.

Figure 4.5 plots the smoothed probability (blue and solid) of regime 2 in the model one with Markov-switching variances, representing a high level of volatility, against the actual time series data (red and dashed) in the UK. It scales the smoothed probability by ten times, except for the first panel, to compare the probability and data more explicitly. Figure 4.5 shows that regime 2 dominates at the very beginning and the end of the sample period, which captures the burst of the currency crisis and the most recent financial crisis, respectively. During that period, the UK experience domestic shocks ξ_t^R, ξ_t^z and foreign shocks $\xi_t^q, \xi_t^{y_t^*}, \xi_t^{\pi_t^*}$ with significantly large variances. Regime 1, representing a low level of volatility, dominates persistently from early 1992 until late 2007. During that period, the UK experience domestic shocks and foreign shocks with smaller variances.

Figure 4.6 plots the smoothed probability (blue and solid) of regime 1 in the model two with Markov-switching policy parameters, representing more strict inflation targeting, against the actual time series data (red and dashed) in the UK. Likewise, it scales the smoothed probability by ten times, except for the first panel, to compare the probability and data more explicitly. Figure 4.5 shows that regime 1 begins to dominate the sample period after the most recent financial crisis, which implies the central bank of England abruptly switches to cut nominal interest rate aggressively in response to the falling output and the falling expected inflation rate before zero lower bound. Regime 1, representing less strict inflation targeting, dominates the majority periods of the sample until 2008: Q3.

The model comparison at the second stage indicates that the model one with the Markov-switching variances best fit the UK data. Figure 4.7 plots the historical decomposition of the UK data given the contributions of the structural shocks generated from the model one.

Regime 1 dominates from since early 1993 until the most recent financial crisis. First, the world output shock (yellow) contributes most to the variation of the output growth. Second, the world output shock (yellow), policy shock (green) and the change rate of the technology shock (purple) offer a major contribution to the variation of the inflation rate. Next, the change rate of the technology shock (purple), world output shock (yellow) and the policy shock (green) dominate in the variation of the nominal interest rate. Finally, the world inflation shock (red) and the change of the terms of trade shock (blue) dominate in the variation of the movements of the nominal exchange rate.

Regime 2 prevails in early 1992 and after the most recent financial crisis. First, The world output shock (yellow) contributes most to the variation of the output growth. Second, the change rate of the technology shock (purple) and the world output shock (yellow) offer a major contribution to the variation of the inflation rate. Next, the change rate of the technology shock (purple) and the world output shock (yellow) offer a major contribution to the variation of the nominal interest rate. Last but not least, the world inflation shock (red) and the change rate of the terms of trade shock (blue) dominates in the variation of the movements of the nominal exchange rate.

Table 4.17: Log Marginal Data Densities and Ranks of the Models for the UK

Models	Log MDD	Rank of data fitting
Benchmark Model:Constant parameters model	-501.064	4
Model 1:Markov-switching in volatility of shocks	-468.588	1
Model 2:Markov-switching monetary policy	-488.018	3
Model 3:The model with two Markov chains	-469.735	2

Note:The table reports the log marginal data densities and ranks for the models in the second stage of the model comparison. The model one with Markov-switching variances ranks first in terms of data fitting.

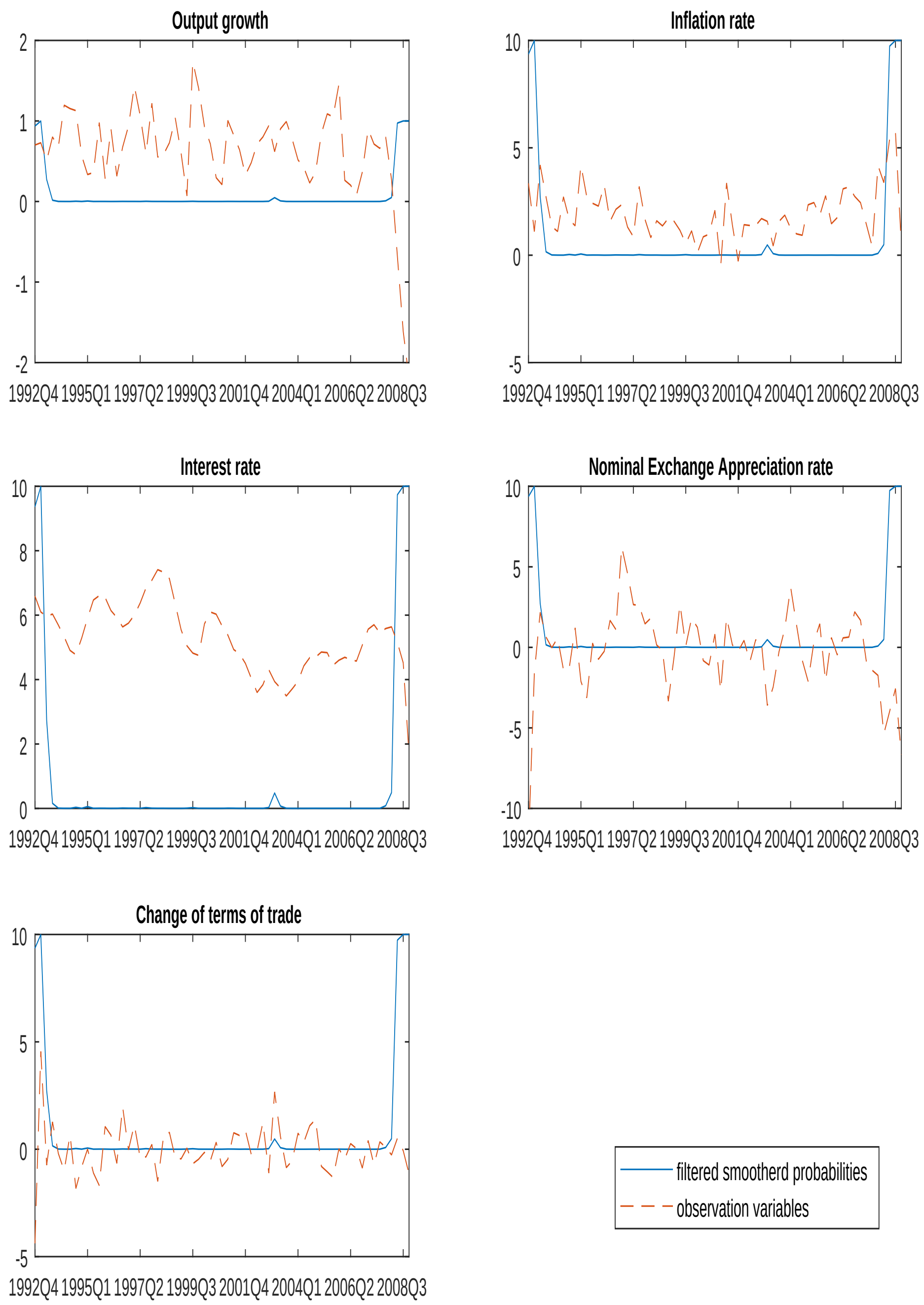

Figure 4.5: Smoothed probability of high volatility from model one. Note:The figure depicts the smoother probability of high volatility against the data from the UK covering sample period 1992:Q4-2008:Q4.

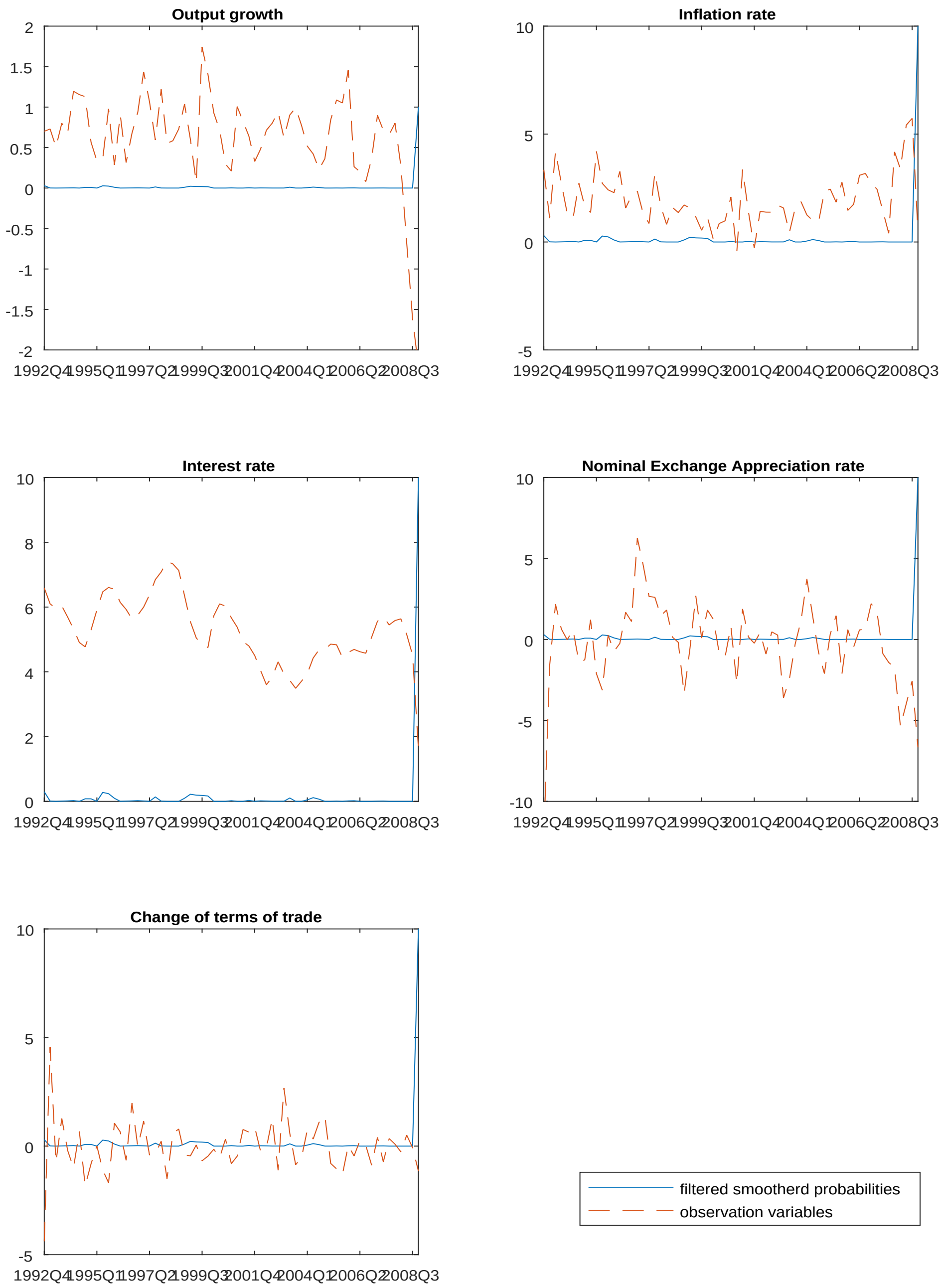

Figure 4.6: Smoothed probability of strict inflation from model two. Note:The figure depicts the smoother probability of more strict inflation targeting against the data from the UK covering the sample period 1992: Q4-2008: Q4.

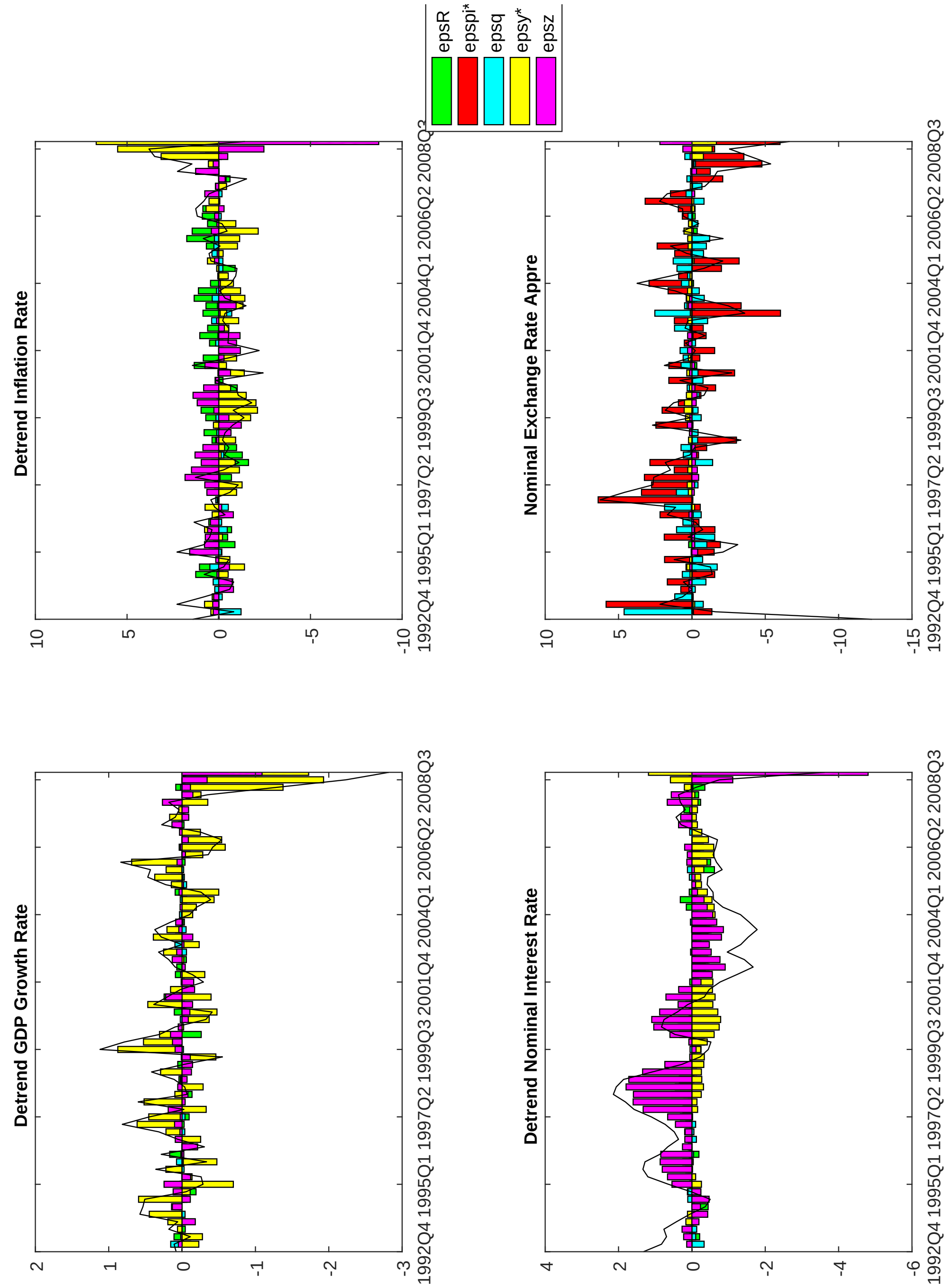

Figure 4.7: Historical Decomposition Using Model One for the UK. Note:The figure depicts the historical decomposition of output growth, inflation rate, nominal interest rate and the movement of the nominal exchange rate in the UK over the sample period 1992: Q4-2008: Q4.

4.4 Estimated Markov Switching DSGE Models for Canada

This section estimates three kinds of Markov-switching DSGE models for Canada, which includes four components, too. The first component offers the estimated results, numerical solutions and variance decomposition of the model one with the switching variances. The second component provides the estimated results, numerical solutions and variance decomposition of the model two with the switching monetary policy parameters. The third component provides the estimated results, numerical solutions and variance decomposition of the model three with switching variances and switching policy parameters. The final component presents an overall model comparison and offers a general analysis of the Canadian data based on the best data-fitting model.

Table 4.18 offers the prior distribution of the parameters for Canada, prepared for the Bayesian estimation of the Markov-switching parameters. Likewise, the structure parameters for Canada share identical prior distributions for chapter 3 and chapter 4. Also, there are two independent Markov chains for Canada. The first Markov chain Q controls the shifts of the standard deviations across two regimes representing low and high volatility, respectively. The second Markov chain P controls the shifts of the coefficients of monetary policy reaction functions across two regimes representing more strict and less strict inflation targeting. The prior means of the transition probability across any two regimes of each chain are still 0.1. Overall, it is still helpful to notice that the standard deviations of the exogenous shocks are permitted to shift in the model one and the model three, while the policy parameters are allowed to shift in the model 2 and the model 3.

Table 4.18: Prior Distributions of the Structural Parameters for Canada

Parameters	Domain	Density	Para(1)	Para(2)	Model Spec
τ	[0,1)	Beta	0.5	0.2	
κ	R$\hat{+}$	Gamma	0.3	0.2	
ϕ_π	R$\hat{+}$	Gamma	1.5	0.5	Model 2 & 3
ϕ_y	R$\hat{+}$	Gamma	0.125	0.05	Model 2 & 3
$\phi_{\triangle e}$	R$\hat{+}$	Gamma	0.125	0.05	Model 2 & 3
$\phi_{\triangle y}$	R$\hat{+}$	Gamma	0.125	0.05	Model 2 & 3
ρ_R	[0,1)	Beta	0.5	0.25	Model 3 & 4
ρ_z	[0,1)	Beta	0.2	0.05	
ρ_q	[0,1)	Beta	0.4	0.2	
ρ_{π^*}	[0,1)	Beta	0.8	0.1	
ρ_{y^*}	[0,1)	Beta	0.9	0.05	
α	[0,1)	Beta	0.2	0.05	
$r^{(A)}$	R$\hat{+}$	Normal	2.47	1	
$\pi^{(A)}$	R$\hat{+}$	Normal	1.62	1	
$\gamma^{(A)}$	R$\hat{+}$	Gamma	0.73	0.2	
σ_R	R$\hat{+}$	Inverse Gamma	0.5	4	Model 1 & 3
σ_z	R$\hat{+}$	Inverse Gamma	1	4	Model 1 & 3
σ_{y^*}	R$\hat{+}$	Inverse Gamma	1.5	4	Model 1 & 3
σ_{π^*}	R$\hat{+}$	Inverse Gamma	0.55	4	Model 1 & 3
σ_q	R$\hat{+}$	Inverse Gamma	1.5	4	Model 1 & 3
P_{12}	[0,1)	Beta	0.1	0.05	Model 2 & 3
P_{21}	[0,1)	Beta	0.1	0.05	Model 2 & 3
Q_{12}	[0,1)	Beta	0.1	0.05	Model 1 & 3
Q_{21}	[0,1)	Beta	0.1	0.05	Model 1 & 3

Note: The table reports the prior distributions of the structural parameters for Canada. Para(1) and Para(2) are the means and the standard deviations for the relevant distributions,respectively.

4.4.1 Canada: the Model One with Switching Variances

Table 4.19 reports the estimation results of the model one for Canada. This model allows the vector of the standard deviations of exogenous shocks $[\sigma_R, \sigma_z, \sigma_{y^*}, \sigma_{\pi^*}, \sigma_q]$ to shift between two regimes. The posterior mean of the transition probability from regime 1 representing low volatility to regime 2 representing high volatility Q_{12} is 0.102, while the transition probability from regime 2 to regime 1 (Q_{21}) is 0.145. The asymmetric transition reflects that it is possible for Canada to experience more periods of high volatility shocks compared to the UK.

More specifically, regime 1 represents the low level of volatilities with the posterior means being $[0.227, 1.138, 0.838, 1.775, 1.087]$ compared to $[0.460, 1.741, 1.293, 3.524$ $, 3.253]$ in regime 2. The differences between the two vectors are still significant. The standard deviation of the rate change of the terms of trade shock σ_q in regime 2 is almost three times larger than in regime 1. The standard deviations of nominal interest rate shock σ_R and world inflation shock σ_{π^*} almost double in regime 2. The standard deviation of the rate change of the technology shock σ_z and the world output shock σ_{y^*} in regime 2 are one and a half times larger than in regime 1. The calculated marginal density is -569.826 for the model one with the Markov-switching variances, which is much bigger compared to the marginal density calculated from the constant parameter model with the best data fitting (-582.077).

Table 4.19: Model One with Markov-Switching Variances(Canada)

	Regime 1: Low volatility		Regime 2: High volatility	
parameter	mean	90% interval	mean	90% interval
τ	0.309	[0.184,0.454]		
κ	0.723	[0.441,1.090]		
ϕ_{π}	2.086	[1.386,2.978]		
ϕ_{y}	0.064	[0.029,0.115]		
$\phi_{\triangle e}$	0.131	[0.070,0.206]		
$\phi_{\triangle y}$	0.140	[0.076,0.216]		
ρ_{R}	0.746	[0.607,0.838]		
ρ_{z}	0.430	[0.331,0.524]		
ρ_{q}	0.512	[0.397,0.627]		
ρ_{π^*}	0.420	[0.298,0.548]		
ρ_{y^*}	0.950	[0.909,0.982]		
α	0.146	[0.088,0.210]		
$r^{(A)}$	2.044	[1.492,2.501]		
$\pi^{(A)}$	1.944	[1.674,2.393]		
$\gamma^{(A)}$	0.748	[0.673,0.820]		
σ_{R}	0.227	[0.174,0.294]	0.460	[0.311,0.659]
σ_{z}	1.138	[0.631,1.844]	1.741	[0.856,2.864]
σ_{y^*}	0.838	[0.396,1.693]	1.293	[0.580,2.300]
σ_{π^*}	1.775	[1.411,2.172]	3.524	[2.587,4.692]
σ_{q}	1.087	[0.838,1.357]	3.253	[2.435,4.402]
Q_{12}	0.102	[0.049,0.168]		
Q_{21}	0.145	[0.073,0.232]		

Note:The table reports posterior means and 90% probability interval of the model one for Canada.

Numerical Solution and Simulation Results for Canada in the Model One

There are two regimes in the model one for Canada, and thus, there is a unique solution for each regime. Table 4.20 offers the numerical solution to regime 1 representing low volatility of the model one, and table 4.21 produces the numerical solution to regime 2 representing high volatility of the same model.

Table 4.20 brings in the numerical information to compute the impulse response functions of the four endogenous variables including $\tilde{yy_t}$,π_t,$\tilde{r_t}$ and $\triangle\tilde{e_t}$ in regime 1:

$$\begin{pmatrix} \tilde{yy_t} \\ \pi_t \\ \tilde{r_t} \\ \triangle\tilde{e_t} \end{pmatrix} = \begin{pmatrix} -0.364 & 0.068 & -0.528 & 0.009 & 0.068 & 0.039 \\ -0.778 & 0.100 & 0.113 & 0.024 & 0.146 & 0.035 \\ 0.251 & 0.067 & -0.019 & 0.001 & -0.047 & 0.012 \\ -0.778 & 0.100 & 0.113 & -0.396 & 0.146 & -0.402 \end{pmatrix} \begin{pmatrix} \tilde{r_{t-1}} \\ z_{t-1} \\ \tilde{yy^*_{t-1}} \\ \pi^*_{t-1} \\ \tilde{yy_{t-1}} \\ \triangle\tilde{q_{t-1}} \end{pmatrix}$$

$$+ \begin{pmatrix} -0.111 & 0.181 & 0.083 & -0.466 & 0.038 \\ -0.237 & 0.266 & 0.075 & 0.100 & 0.100 \\ 0.076 & 0.178 & 0.024 & -0.016 & 0.003 \\ -0.237 & 0.266 & -0.853 & 0.100 & -1.675 \end{pmatrix} \begin{pmatrix} \xi_t^R \\ \xi_t^z \\ \xi_t^q \\ \xi_t^{y_t^*} \\ \xi_t^{\pi_t^*} \end{pmatrix}, \quad (4.23)$$

where the second matrix in the above equation incorporates the impact of the structural shocks on the real output, inflation rate, nominal interest rate and depreciation exchange rate in regime 1. Figure 4.8 depicts the impulse responses of the four endogenous variables to one unit structural shock of regime 1 with solid and blue lines.

Table 4.21 introduces the information to compute the impulse response functions of the four endogenous variables including $\tilde{yy_t}$,π_t,$\tilde{r_t}$ and $\triangle\tilde{e_t}$ in regime 2:

$$\begin{pmatrix} \tilde{yy_t} \\ \pi_t \\ \tilde{r_t} \\ \triangle\tilde{e_t} \end{pmatrix} = \begin{pmatrix} -0.364 & 0.068 & -0.528 & 0.009 & 0.068 & 0.039 \\ -0.778 & 0.100 & 0.113 & 0.024 & 0.146 & 0.035 \\ 0.251 & 0.067 & -0.019 & 0.001 & -0.047 & 0.012 \\ -0.778 & 0.100 & 0.113 & -0.396 & 0.146 & -0.402 \end{pmatrix} \begin{pmatrix} \tilde{r_{t-1}} \\ z_{t-1} \\ \tilde{yy^*_{t-1}} \\ \pi^*_{t-1} \\ \tilde{yy_{t-1}} \\ \triangle\tilde{q_{t-1}} \end{pmatrix}$$

$$+ \begin{pmatrix} -0.225 & 0.277 & 0.249 & -0.719 & 0.076 \\ -0.480 & 0.407 & 0.225 & 0.154 & 0.199 \\ 0.155 & 0.272 & 0.073 & -0.025 & 0.007 \\ -0.480 & 0.407 & -2.553 & 0.154 & -3.325 \end{pmatrix} \begin{pmatrix} \xi^R_t \\ \xi^z_t \\ \xi^q_t \\ \xi^{y^*_t}_t \\ \xi^{\pi^*_t}_t \end{pmatrix}. \quad (4.24)$$

Where the second matrix in the above equation embodies the impact of the structural shocks on the real output, inflation rate, nominal interest rate and depreciation exchange rate in regime 2. Figure 4.8 also depicts the impulse responses of the four endogenous variables to one unit structural shock of regime 2 with dashed and red lines.

Ultimately, figure 4.8 compares the impulse response functions of the two regimes in the model one together. Holding everything else constant, a unit of the standard deviation of the monetary policy shock will exert a -0.111 impact on the real output deviation in regime 1 while -0.225 in regime 2, a -0.237 impact on the inflation in regime 1 while -0.480 in regime 2,a 0.076 impact on the nominal interest rate deviation in regime 1 while 0.155 in regime 2, a -0.237 impact on the movement of the nominal exchange rate in regime 1 while -0.480 in regime 2.

Holding everything else constant, a unit of standard deviation of the shock to the change rate of the terms of trade will exert a 0.083 impact on the real output deviation in regime 1 while 0.249 in regime 2, a 0.075 impact on the inflation in regime 1 while 0.225 in regime 2,a 0.024 impact on the nominal interest rate deviation in regime 1 while 0.073 in regime 2, a -0.853 impact on the nominal exchange rate depreciation in regime 1 while -2.553 in regime 2.

Holding everything else constant, a unit of the standard deviation of the shock to the change rate of the technology will exert a 0.181 impact on the real output deviation in regime 1 while 0.277 in regime 2, a 0.266 impact on the inflation in regime 1 while 0.407 in regime 2,a 0.178 impact on the nominal interest rate deviation in regime 1 while 0.272 in regime 2, a 0.266 impact on the nominal exchange rate depreciation in regime 1 while 0.407 in regime 2.

Holding everything else constant, a unit of the standard deviation of the shock to the world output deviation will exert a -0.466 impact on the real output deviation in regime 1 while -0.719 in regime 2, a 0.100 impact on inflation in regime 1 while 0.154 in regime 2,a -0.016 impact on the nominal interest rate deviation in regime 1 while -0.025 impact in regime 2, a 0.100 impact on the nominal exchange rate depreciation in regime 1 while 0.154 in regime 2.

Holding everything else constant, a unit of the standard deviation of the shock to the world inflation will exert a 0.038 impact on the real output deviation in regime 1 while 0.076 in regime 2, a 0.100 impact on the inflation in regime 1 while 0.199 in regime 2,a 0.003 impact on the nominal interest rate deviation in regime 1 while 0.007 in regime 2, a -1.675 impact on the nominal exchange rate depreciation in regime 1 while -3.325 in regime 2.

Table 4.20: The Numerical Solution to Regime 1 of the Model One for Canada

		static		backward looking				mixed		forward looking	
		$\tilde{yy}_{t,n}$	$\triangle\tilde{e}_t$	$\tilde{r}_t$	z_t	$\tilde{yy}^*_t$	π^*_t	$\triangle\tilde{q}_t$	$\tilde{yy}_t$	π_t	$\triangle\tilde{yy}^*_t$
endogenous variables	$\tilde{r}_{t-1}$		-0.778	0.251					-0.364	-0.778	
	z_{t-1}		0.100	0.067	0.430				0.068	0.100	
	$\tilde{yy}^*_{t-1}$	-0.575	0.113	-0.019		0.950			-0.528	0.113	-0.050
	π^*_{t-1}		-0.396	0.001			0.420		0.009	0.024	
	$\tilde{yy}_{t-1}$		0.146	-0.047					0.068	0.146	
	$\triangle\tilde{q}_{t-1}$		-0.402	0.012				0.512	0.039	0.035	
exogenous variables	ξ^R_t		-0.237	0.076					-0.111	-0.237	
	ξ^z_t		0.266	0.178	1.138				0.181	0.266	
	ξ^q_t		-0.853	0.024				1.087	0.083	0.075	
	$\xi^{y^*_t}_t$	-0.507	0.100	-0.016		0.838			-0.466	0.100	0.838
	$\xi^{\pi^*_t}_t$		-1.675	0.003			1.775		0.038	0.100	

Note: This table reports the numerical solution to regime 1 of the model one for Canada. Regime 1 considers a low level of the volatility.

Table 4.21: The Numerical Solution to regime 2 of the Model One for Canada

		static		backward looking				mixed		forward looking	
		$\tilde{yy}_{t,n}$	$\triangle\tilde{e}_t$	$\tilde{r}_t$	z_t	$\tilde{yy}^*_t$	π^*_t	$\triangle\tilde{q}_t$	$\tilde{yy}_t$	π_t	$\triangle\tilde{yy}^*_t$
endogenous variables	$\tilde{r}_{t-1}$		-0.778	0.251					-0.364	-0.778	
	z_{t-1}		0.100	0.067	0.430				0.068	0.100	
	$\tilde{yy}^*_{t-1}$	-0.575	0.113	-0.019		0.950			-0.528	0.113	-0.050
	π^*_{t-1}		-0.396	0.001			0.420		0.009	0.024	
	$\tilde{yy}_{t-1}$		0.146	-0.047					0.068	0.146	
	$\triangle\tilde{q}_{t-1}$		-0.402	0.012				0.512	0.039	0.035	
exogenous variables	ξ^R_t		-0.480	0.155					-0.225	-0.480	
	ξ^z_t		0.407	0.272	1.741				0.277	0.407	
	ξ^q_t		-2.553	0.073				3.253	0.249	0.225	
	$\xi^{y^*_t}_t$	-0.783	0.154	-0.025		1.293			-0.719	0.154	1.293
	$\xi^{\pi^*_t}_t$		-3.325	0.007			3.524		0.076	0.199	

Note: This table reports the numerical solutions to regime 2 of the model one for Canada. Regime 2 represents a high level of the volatility.

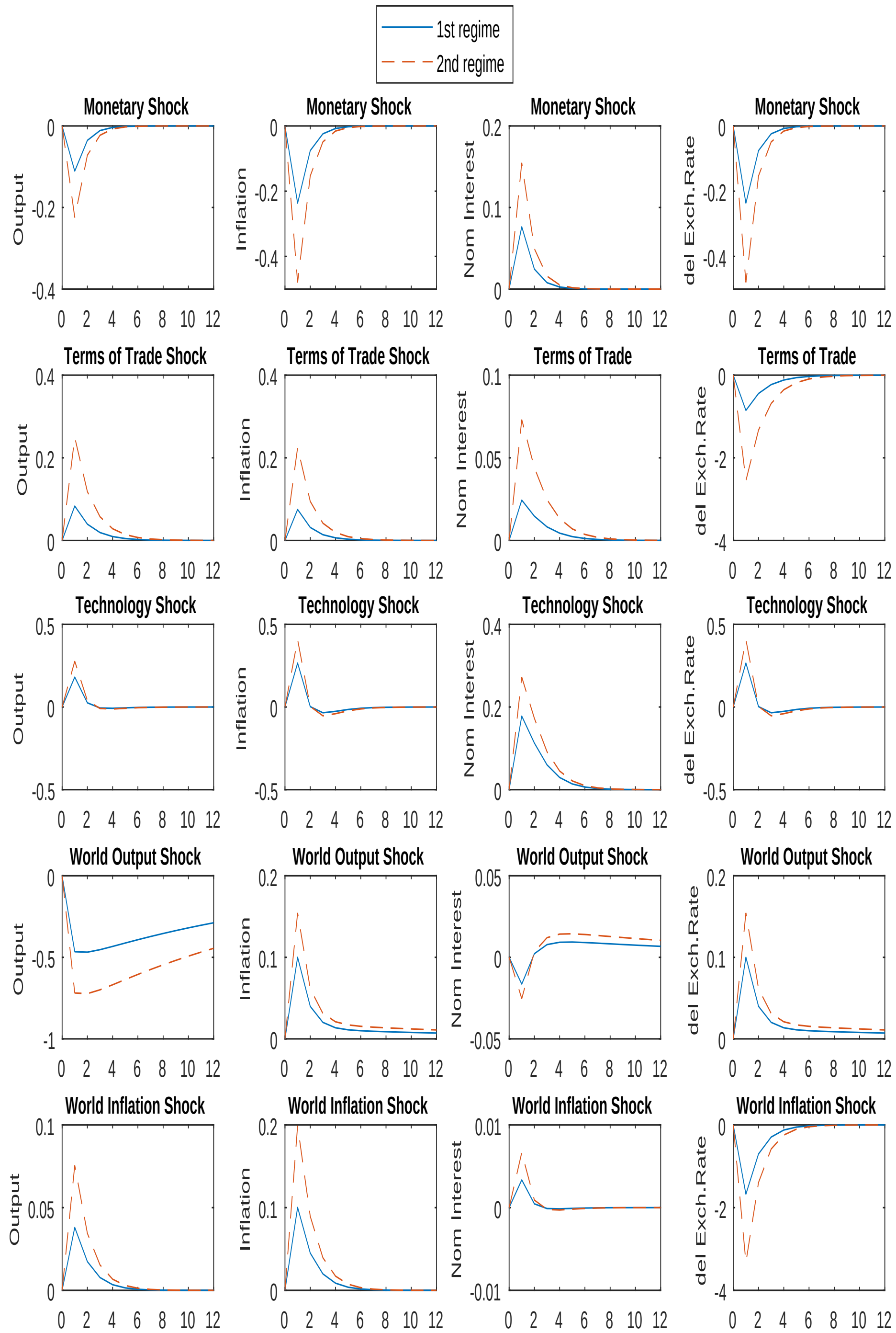

Figure 4.8: Impulse responses,Canada(Model One). Note:The figure depicts the impulses responses of real output, inflation rate, nominal interest rate and depreciation exchange rate to one unit structural shock of regime 1 representing the low volatility(solid and blue lines)and regime 2 representing the high volatility(dashed and red lines).

Variance Decomposition of the Model One for Canada

Table 4.22 reports the variance decomposition of the model one with the Marov-switching variances for Canada. It draws a conclusion to the major contributions to the variation of the four endogenous variables $y\tilde{y}_t$,π_t,$\tilde{r}_t$ and $\triangle\tilde{e}_t$ as follows. First, the world output shock $\xi_t^{y_t^*}$ contributes most to the variation of the output deviation $y\tilde{y}_t$ in both of the two regimes. Second, the change rate of the technology shock ξ_t^z contributes most to the variation of the inflation π_t in the regime 1 and the policy shock ξ_t^R contributes most in regime 2. Furthermore, the change rate of the technology shock ξ_t^z contributes most to the variation of the interest rate $\tilde{r}_t$ in both of the two regimes. Finally, the world inflation shock $\xi_t^{\pi_t^*}$ contributes most to the variation of the movement of the nominal exchange rate $\triangle\tilde{e}_t$ in both of the two regimes.

Additionally, table 4.22 also compares the contributions of the same structural shock in different regimes. The policy shock ξ_t^R contributes 0.52% to the variation of the output deviation $y\tilde{y}_t$ in regime 1 while 0.88% in regime 2, 37.12% to the variation of the inflation π_t in regime 1 while only 44.96% in regime 2, 11.23% to the variation of the interest rate $\tilde{r}_t$ in regime 1 while 17.44% in regime 2,1.37% to the variation of the nominal exchange rate depreciation $\triangle\tilde{e}_t$ in regime 1 while 1.12% in regime 2.

The change rate of the terms of trade shock ξ_t^q contributes 0.34% to the variation of the output deviation $y\tilde{y}_t$ in regime 1 while 1.27% in regime 2,4.10% to the variation of the inflation π_t in regime 1 while 10.83% in regime 2,1.57% to the variation of the interest rate $\tilde{r}_t$ in regime 1 while only 5.31% in regime 2, 21.92% to the variation of exchange rate depreciation $\triangle\tilde{e}_t$ in regime 1 while 39.17% in regime 2.

The change rate of the technology shock ξ_t^z contributes 1.27% to the variation of the output deviation $y\tilde{y}_t$ in regime 1 while 1.23% in regime 2, 43.49% to the variation of the inflation π_t in regime 1 while 29.96% in regime 2,84.82% to the variation of the interest rate $\tilde{r}_t$ in regime 1 while 75.09% in regime 2, 1.60% to the variation of the exchange rate depreciation $\triangle\tilde{e}_t$ in regime 1 while 0.75% in regime 2.

The world output shock $\xi_t^{y_t^*}$ contributes 97.80% to the variation of the output deviation $y\tilde{y}_t$ in regime 1 while 96.50% in regime 2,7.91% to the variation of the inflation π_t in regime 1 while 5.55% in regime 2, 2.36% to the variation of interest rate $\tilde{r}_t$ in regime 1 while 2.13% in regime 2, 0.29% to the variation of the nominal exchange rate depreciation $\triangle\tilde{e}_t$ in regime 1 while 0.14% in regime 2.

The world inflation shock $\xi_t^{\pi_t^*}$ contributes 0.07% to the variation of the output deviation $y\tilde{y}_t$ in regime 1 while 0.11% in regime 2, 7.48% to the variation of the inflation π_t in regime 1 while 8.70% in regime 2, 0.02% to the variation of the interest rate $\tilde{r}_t$ in regime 1 while 0.03% in regime 2, 74.81% to the variation of the nominal exchange rate depreciation $\triangle\tilde{e}_t$ in regime 1 while 58.82% in regime 2.

Table 4.22: Variance Decomposition of the Model One for Canada

Regime 1	Output	Inflation	interest rate	Exchange rate depreciation
Policy	0.52	37.12	11.23	1.37
Terms of trade	0.34	4.10	1.57	21.92
Technology	1.27	43.39	84.82	1.60
World output	97.80	7.91	2.36	0.29
World inflation	0.07	7.48	0.02	74.81

Regime 2	Output	Inflation	interest rate	Exchange rate depreciation
Policy	0.88	44.96	17.44	1.12
Terms of trade	1.27	10.83	5.31	39.17
Technology	1.23	29.96	75.09	0.75
World output	96.50	5.55	2.13	0.14
World inflation	0.11	8.70	0.03	58.82

Note:(a)The table reports the variance decomposition of the model one for Canada.

(b)Regime 1 is characterized as low volatility compared to Regime 2.

4.4.2 Canada: the Model Two with the Switching Taylor Rule

Table 4.23 presents the estimated results of the model two for Canada. This model allows the vector of the coefficients of the policy parameters $[\phi_\pi, \phi_y, \phi_{\triangle e}, \phi_{\triangle y}, \rho_R]$ to shift between two regimes. The posterior mean of the transition probability from regime 1 representing more strict inflation targeting to regime 2 representing less inflation targeting (P_{12}) is 0.098 while from regime 2 to regime 1 (P_{21}) is 0.114. The asymmetric transition probability marks that Canada experiences more periods of very strict inflation targeting compared to the UK.

More specifically, regime 1 represents more strict inflation targeting with the posterior mean of ϕ_π being 2.140 compared to 1.503 in regime 2. Next, the posterior mean of the coefficient of the output ϕ_y in regime 1 is two times smaller than it in regime 2. Also, the posterior mean of the rate change of the output $\phi_{\triangle y}$ in regime 1 is 0.128 compared to 0.160 in regime 2. Besides, the posterior mean of ρ_R in regime 1 is 0.626 compared to 0.821 in regime 2. Finally, the difference between the coefficients of the nominal exchange rate depreciation $\phi_{\triangle e}$ is less significant across the two regimes. The calculated marginal density is -575.813 for the model with the Markov-switching policy parameters, which is bigger compared to the constant parameter model with the best data fitting (-582.077).

Table 4.23: Model Two with Markov-Switching Policy Parameters(Canada)

	Regime 1: more strict inflation targeting		Regime 2: less strict inflation targeting	
parameter	mean	90% interval	mean	90% interval
τ	0.297	[0.157,0.499]		
κ	0.655	[0.381,1.023]		
ϕ_π	2.140	[1.559,3.127]	1.503	[1.069,2.070]
ϕ_y	0.052	[0.024,0.091]	0.118	[0.065,0.180]
$\phi_{\triangle e}$	0.118	[0.064,0.177]	0.119	[0.063,0.189]
$\phi_{\triangle y}$	0.128	[0.063,0.223]	0.160	[0.083,0.251]
ρ_R	0.626	[0.483,0.757]	0.821	[0.751,0.879]
ρ_z	0.338	[0.243,0.439]		
ρ_q	0.538	[0.396,0.655]		
ρ_{π^*}	0.448	[0.323,0.578]		
ρ_{y^*}	0.962	[0.934,0.984]		
α	0.140	[0.098,0.190]		
$r^{(A)}$	2.272	[1.779,2.711]		
$\pi^{(A)}$	2.134	[1.686,2.626]		
$\gamma^{(A)}$	0.725	[0.676,0.772]		
σ_R	0.263	[0.206,0.341]		
σ_z	1.898	[1.033,3.071]		
σ_{y^*}	0.905	[0.394,1.963]		
σ_{π^*}	2.370	[2.062,2.748]		
σ_q	1.901	[1.654,2.163]		
P_{12}	0.098	[0.039,0.189]		
P_{21}	0.114	[0.051,0.186]		

Note:The table reports posterior means and 90% probability interval of the model two for Canada.

Numerical Solution and Simulation Results for Canada in the Model Two

There are two regimes in the model two with the Markov-switching policy parameters. Table 4.24 presents the numerical solution to regime 1, representing more strict inflation targeting, and table 4.25 produces the numerical solution to regime 2, representing less strict inflation targeting of the same model.

Table 4.24 contributes to computing the impulse response functions of the four endogenous variables including $\tilde{yy_t}$,π_t,$\tilde{r_t}$ and $\triangle\tilde{e_t}$ in regime 1:

$$\begin{pmatrix} \tilde{yy_t} \\ \pi_t \\ \tilde{r_t} \\ \triangle\tilde{e_t} \end{pmatrix} = \begin{pmatrix} -0.234 & 0.035 & -0.559 & 0.009 & 0.048 & 0.040 \\ -0.458 & 0.044 & 0.094 & 0.023 & 0.094 & 0.035 \\ 0.204 & 0.042 & -0.003 & 0.001 & -0.042 & 0.015 \\ -0.458 & 0.044 & 0.094 & -0.425 & 0.094 & -0.428 \end{pmatrix} \begin{pmatrix} \tilde{r_{t-1}} \\ z_{t-1} \\ \tilde{yy^*_{t-1}} \\ \pi^*_{t-1} \\ \tilde{yy_{t-1}} \\ \triangle\tilde{q_{t-1}} \end{pmatrix}$$

$$+ \begin{pmatrix} -0.098 & 0.197 & 0.067 & -0.526 & 0.048 \\ -0.193 & 0.247 & 0.058 & 0.089 & 0.123 \\ 0.086 & 0.237 & 0.025 & -0.003 & 0.007 \\ -0.193 & 0.247 & -0.720 & 0.089 & -2.247 \end{pmatrix} \begin{pmatrix} \xi^R_t \\ \xi^z_t \\ \xi^q_t \\ \xi^{y^*_t}_t \\ \xi^{\pi^*_t}_t \end{pmatrix}. \quad (4.25)$$

Where the second matrix in the above equation includes the impact of the structural shocks on the real output, inflation rate, nominal interest rate and depreciation exchange rate in regime 1. Figure 4.9 depicts the impulse responses of the four mentioned endogenous variables to one unit structural shock of regime 1 with solid and blue lines.

Table 4.25 provides numerical information to compute the impulse response func-

tions of the four endogenous variables including $\tilde{yy_t}$,π_t,$\tilde{r_t}$ and $\triangle\tilde{e_t}$ in regime 2:

$$\begin{pmatrix} \tilde{yy_t} \\ \pi_t \\ \tilde{r_t} \\ \triangle\tilde{e_t} \end{pmatrix} = \begin{pmatrix} -0.651 & 0.046 & -0.482 & 0.012 & 0.127 & 0.046 \\ -1.432 & 0.060 & 0.287 & 0.031 & 0.279 & 0.047 \\ 0.287 & 0.026 & -0.004 & 0.001 & -0.056 & 0.012 \\ -1.432 & 0.060 & 0.287 & -0.417 & 0.279 & -0.415 \end{pmatrix} \begin{pmatrix} \tilde{r_{t-1}} \\ z_{t-1} \\ \tilde{yy^*_{t-1}} \\ \pi^*_{t-1} \\ \tilde{yy_{t-1}} \\ \triangle\tilde{q_{t-1}} \end{pmatrix}$$

$$+ \begin{pmatrix} -0.208 & 0.259 & 0.077 & -0.453 & 0.063 \\ -0.459 & 0.340 & 0.080 & 0.270 & 0.162 \\ 0.092 & 0.146 & 0.021 & -0.004 & 0.008 \\ -0.459 & 0.340 & -0.698 & 0.270 & -2.208 \end{pmatrix} \begin{pmatrix} \xi^R_t \\ \xi^z_t \\ \xi^q_t \\ \xi^{y^*_t}_t \\ \xi^{\pi^*_t}_t \end{pmatrix}. \quad (4.26)$$

Where the second matrix in the above equation embodies the impact of the structural shocks on the real output, inflation rate, nominal interest rate and depreciation exchange rate in the second regime. Figure 4.9 depicts the impulse responses of the four mentioned endogenous variables to one unit structural shock of regime 2 with dashed and red lines.

Figure 4.9 compares the impulse response functions of the two regimes in the model two with the Markov-switching policy parameters. Holding everything else constant, a unit of the standard deviation of the monetary policy shock will exert a -0.098 impact on the real output deviation in regime 1 while -0.208 in regime 2, a -0.193 impact on the inflation in regime 1 while -0.459 in regime 2,a 0.086 impact on the nominal interest rate deviation in regime 1 while 0.092 in regime 2, a -0.193 impact on the nominal exchange rate depreciation in regime 1 while -0.459 in regime 2.

Holding everything else constant, a unit of the standard deviation of the shock to the change rate of terms of trade will exert a 0.067 impact on the real output deviation in regime 1 while 0.077 in regime 2, a 0.058 impact on the inflation in regime 1 while 0.080 in regime 2,a 0.025 impact on the nominal interest rate deviation in regime 1 while 0.021 in regime 2, a -0.720 impact on the nominal exchange rate depreciation in regime 1 while -0.698 in regime 2.

Holding everything else constant, a unit of the standard deviation of the shock to the change rate of the technology will exert a 0.197 impact on the real output deviation in regime 1 while 0.259 in regme 2, a 0.247 impact on the inflation in regime 1 while 0.340 in regime 2,a 0.237 impact on the nominal interest rate deviation in regime 1 while 0.146 in regime 2, a 0.247 impact on the nominal exchange rate depreciation in regime 1 while 0.340 in regime 2.

Holding everything else constant, a unit of the standard deviation of the shock to the world output will exert a -0.526 impact on the real output deviation in regime 1 while -0.453 in regime 2, a 0.089 impact on the inflation in regime 1 while 0.270 in regime 2,a -0.003 impact on the nominal interest rate deviation in regime 1 while -0.004 in regime 2, a 0.089 impact on the nominal exchange rate depreciation in regime 1 while 0.270 in regime 2.

Holding everything else constant, a unit of the standard deviation of the shock to the world inflation will exert a 0.048 impact on the real output deviation in regime 1 while 0.063 in regime 2, a 0.123 impact on the inflation in regime 1 while 0.162 in regime 2,a 0.007 impact on the nominal interest rate deviation in regime 1 while 0.008 in regime 2, a -2.247 impact on the nominal exchange rate depreciation in regime 1 while -2.208 in regime 2.

Table 4.24: The Numerical Solution to Regime 1 of the Model Two for Canada

		static		backward looking				mixed		forward looking	
		$y\tilde{y}_{t,n}$	$\triangle\tilde{e}_t$	$\tilde{r}_t$	z_t	$y\tilde{y}^*_t$	π^*_t	$\triangle\tilde{q}_t$	$y\tilde{y}_t$	π_t	$\triangle y\tilde{y}^*_t$
endogenous variables	$r\tilde{}_{t-1}$		-0.458	0.204					-0.234	-0.458	
	z_{t-1}		0.044	0.042	0.338				0.035	0.044	
	$y\tilde{y}^*_{t-1}$	-0.593	0.094	-0.003		0.962			-0.559	0.094	-0.038
	π^*_{t-1}		-0.425	0.001			0.448		0.009	0.023	
	$y\tilde{y}_{t-1}$		0.094	-0.042					0.048	0.094	
	$\triangle q\tilde{}_{t-1}$		-0.428	0.015				0.538	0.040	0.035	
exogenous variables	ξ^R_t		-0.193	0.086					-0.098	-0.193	
	ξ^z_t		0.247	0.237	1.898				0.197	0.247	
	ξ^q_t		-0.720	0.025				0.905	0.067	0.058	
	$\xi^{y^*_t}_t$	-0.558	0.089	-0.003		0.905			-0.526	0.089	0.905
	$\xi^{\pi^*_t}_t$		-2.247	0.007			2.370		0.048	0.123	

Note:This table reports the numerical solution of regime 1 of the model two for Canada. Regime 1 considers a high level of the coefficient of the inflation in Taylor rule.

Table 4.25: The Numerical Solution to Regime 2 of the Model Two for Canada

		static		backward looking				mixed		forward looking	
		$\tilde{yy}_{t,n}$	$\triangle\tilde{e}_t$	$\tilde{r}_t$	z_t	$\tilde{yy}^*_t$	π^*_t	$\triangle\tilde{q}_t$	$\tilde{yy}_t$	π_t	$\triangle\tilde{yy}^*_t$
endogenous variables	$\tilde{r}_{t-1}$		-1.432	0.287					-0.651	-1.432	
	z_{t-1}		0.060	0.026	0.338				0.046	0.060	
	$\tilde{yy}^*_{t-1}$	-0.593	0.287	-0.004		0.962			-0.482	0.287	-0.038
	π^*_{t-1}		-0.417	0.001			0.448		0.012	0.031	
	$\tilde{yy}_{t-1}$		0.279	-0.056					0.127	0.279	
	$\triangle\tilde{q}_{t-1}$		-0.415	0.012				0.538	0.046	0.047	
exogenous variables	ξ^R_t		-0.459	0.092					-0.208	-0.459	
	ξ^z_t		0.340	0.146	1.898				0.259	0.340	
	ξ^q_t		-0.698	0.021				0.905	0.077	0.080	
	$\xi^{y^*_t}_t$	-0.558	0.270	-0.004		0.905			-0.453	0.270	0.905
	$\xi^{\pi^*_t}_t$		-2.208	0.008			2.370		0.063	0.162	

Note: This table reports the numerical solution of regime 2 of the model two for Canada. Regime 2 considers a low level of the coefficient of the inflation in Taylor rule.

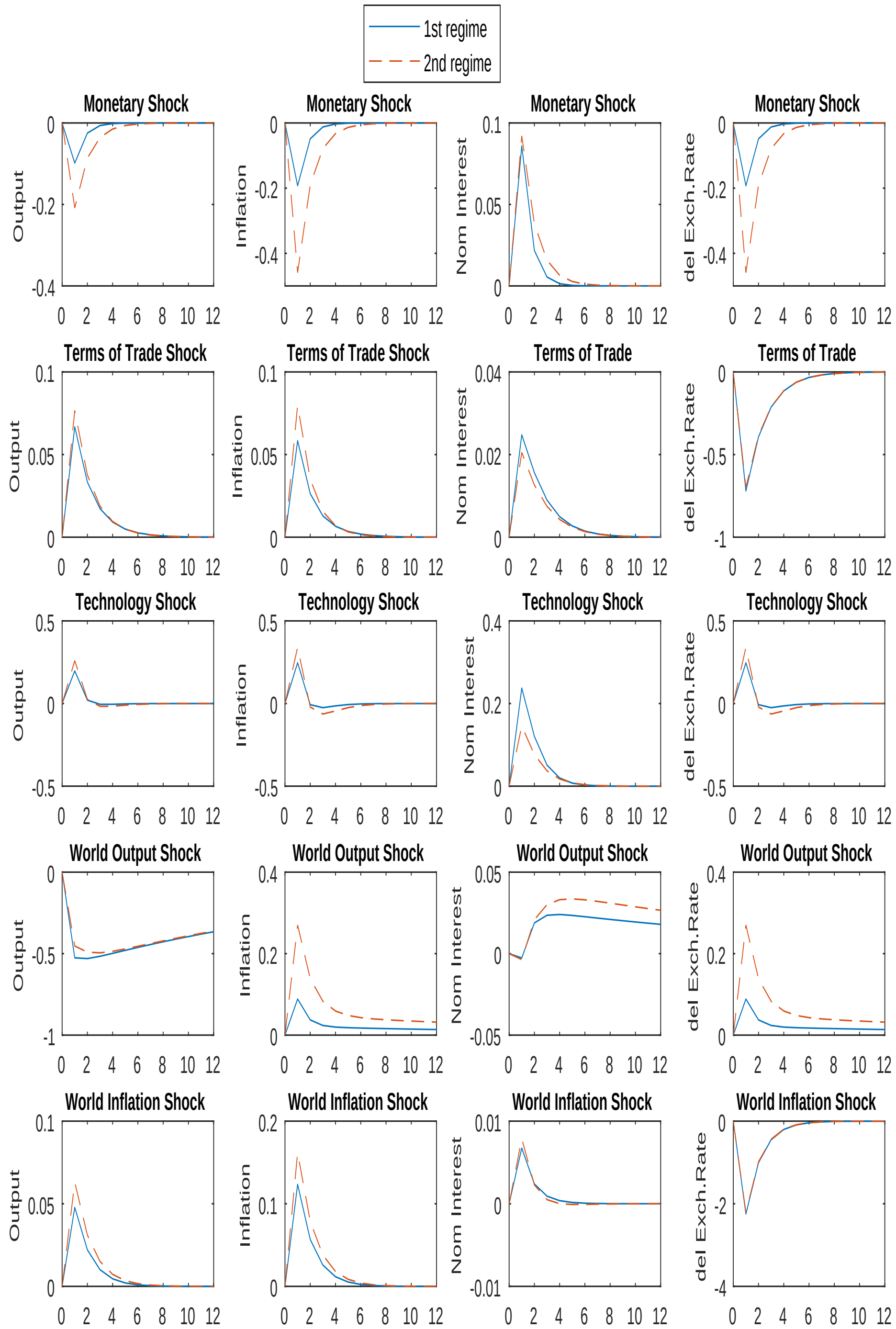

Figure 4.9: Impulse responses,Canada(Model Two). Note:The figure depicts the impulses responses of real output, inflation rate, nominal interest rate and depreciation exchange rate to one unit structural shock of regime 1 representing more strict inflation targeting (solid and blue lines)and regime 2 representing less strict inflation targeting(dashed and red lines).

Variance Decomposition of the Model Two for Canada

Table 4.26 reports the variance decomposition of the model two with the Markov-switching policy parameters for Canada. It summarises the major contributions to the variation of the four endogenous variables $y\tilde{y}_t$,π_t,$\tilde{r}_t$ and $\triangle\tilde{e}_t$ as follows. First, the world output shock $\xi_t^{y_t^*}$ contributes most to the variation of the output deviation $y\tilde{y}_t$ in both of the two regimes. Second, the change rate of the technology shock ξ_t^z contributes most to the variation of the inflation π_t in regime 1 while the policy shock ξ_t^R contributes most in regime 2. Third, the change rate of the technology shock ξ_t^z contributes most to the variation of the interest rate $\tilde{r}_t$ in both of the two regimes. Finally, the world inflation shock $\xi_t^{\pi_t^*}$ contributes most to the variation of the nominal exchange rate depreciation $\triangle\tilde{e}_t$ in both of the two regimes.

The policy shock ξ_t^R contributes 0.25% to the variation of the output deviation $y\tilde{y}_t$ in regime 1 while 1.29% in regime 2,28.39% to the variation of the inflation π_t in regime 1 while only 46.56% in regime 2,8.58% to the variation of the interest rate $\tilde{r}_t$ in regime 1 while 17.52% in regime 2,0.55% to the variation of the exchange rate depreciation $\triangle\tilde{e}_t$ in regime 1 while 3.49% in regime 2.

The change rate of the terms of trade shock ξ_t^q contributes 0.14% to the variation of the output deviation $y\tilde{y}_t$ in regime 1 while 0.19% in rgime 2,3.10% to the variation of the inflation π_t in regime 1 while 1.45% in regime 2,1.06% to the variation of the interest rate $\tilde{r}_t$ in regime 1 while 1.13% in regime 2,10.28% to the variation of the exchange rate depreciation $\triangle\tilde{e}_t$ in regime 1 while 9.58% in regime 2.

The change rate of the technology shock ξ_t^z contributes 0.94% to the variation of the output deviation $y\tilde{y}_t$ in regime 1 while 1.68% in regime 2,44.27% to the variation of inflation π_t in regime 1 while 22.47% in regime 2,80.56% to the variation of the interest rate $\tilde{r}_t$ in regime 1 while 49.33% in regime 2,0.86% to the variation of the exchange rate depreciation $\triangle\tilde{e}_t$ in regime 1 while 1.68% in regime 2.

The world output shock $\xi_t^{y_t^*}$ contributes 98.61% to the variation of the output deviation $y\tilde{y}_t$ in regime 1 while 96.71% in regime 2,10.41% to the variation of the inflation π_t in regime 1 while 23.25% in regime 2,9.74% to the variation of the interest rate $\tilde{r}_t$ in regime 1 while 31.91% in regime 2,0.20% to the variation of the exchange rate depreciation $\triangle\tilde{e}_t$ in regime 1 while 1.74% in regime 2.

The world inflation shock $\xi_t^{\pi_t^*}$ contributes 0.07% to the variation of the output deviation $y\tilde{y}_t$ in regime 1 while 0.13% in regime 2,13.84% to the variation of the inflation π_t in regime 1 while 6.27% in regime 2,0.06% to the variation of the interest rate $\tilde{r}_t$ in regime 1 while 0.12% in regime 2,88.11% to the variation of the exchange rate depreciation $\triangle\tilde{e}_t$ in regime 1 while 83.51% in regime 2.

Table 4.26: Variance Decomposition of the Model Two for Canada

Regime 1	Output	Inflation	interest rate	Exchange rate depreciation
Policy	0.25	28.39	8.58	0.55
Terms of trade	0.14	3.10	1.06	10.28
Technology	0.94	44.27	80.56	0.86
World output	98.61	10.41	9.74	0.20
World inflation	0.07	13.84	0.06	88.11

Regime 2	Output	Inflation	interest rate	Exchange rate depreciation
Policy	1.29	46.56	17.52	3.49
Terms of trade	0.19	1.45	1.13	9.58
Technology	1.68	22.47	49.33	1.68
World output	96.71	23.25	31.91	1.74
World inflation	0.13	6.27	0.12	83.51

Note:(a)The table reports the variance decomposition of the model two for Canada.

(b)Regime 1 is characterized as more strict inflation targeting compared to Regime 2.

4.4.3 Canada: the Model Three with Switching Variances and Switching Policy Parameters

Table 4.27 reports the estimated results of the model three with the Markov-switching variances and the Markov-switching policy parameters for Canada. This model permits the vector of the standard deviations of exogenous shocks $[\sigma_R, \sigma_z, \sigma_{y^*}, \sigma_{\pi^*}, \sigma_q]$ and the vector of the coefficients of Taylor rule $[\phi_\pi, \phi_y, \phi_{\triangle e}, \phi_{\triangle y}, \rho_R]$ to follow two independent Markov chains individually and shift between two regimes of each Markov chain. The posterior mean of the transition probability from regime 1 to regime 2 (Q_{12}) for the first Markov chain is 0.060 while from regime 2 to regime 1 (Q_{21}) is 0.169. The posterior mean of the transition probability for the second Markov chain from regime 1 to regime 2 (P_{12}) is 0.063 while from regime 2 to regime 1 (P_{21}) is 0.082.

The combinations of the two regimes in each Markov chain produces four regimes. Regime 1 represents low volatility and more strict inflation targeting. Regime 2 stans for high volatility and more strict inflation targeting. Regime 3 marks low volatility and less strict inflation targeting. Regime 4 indicates high volatility and less strict inflation targeting. Table 4.27 provides empirical results for regime 1 and regime 4. As mentioned in the case of the UK, exchanging the vector of policy parameters yields regime 3 in the place of regime 1 and regime 2 in the place of regime 4.

More specifically, regime 1 and regime 2 mark more strict inflation targeting with the posterior mean of ϕ_π being 1.621 compared to 1.311 in regime 3 and regime 4. Next, the posterior mean of the coefficient of output ϕ_y in regime 1 and regime 2 is just 0.032 compared to 0.138 in regime 3 and regime 4. Moreover, the posterior mean of the coefficient of nominal exchange rate depreciation $\phi_{\triangle e}$ in regime 1 and regime 2 is 0.104 compared to 0.113 in regime 3 and regime 4. Furthermore, the posterior mean of the rate change of output $\phi_{\triangle y}$ in regime 1 and regime 2 is 0.125 compared to 0.106 in regime 3 and regime 4. Finally, the posterior mean of the persistence ratio ρ_R in regime 1 and regime 3 is 0.594 compared to 0.796 in regime 3 and regime 4.

Additionally, regime 1 and regime 3 stand for low volatility with the posterior means of the above standard deviations being $[0.253, 1.371, 0.581, 1.864, 1.150]$ compared to $[0.325, 2.090, 0.954, 3.600, 4.551]$ in regime 2 and regime 4. The differences between the two vectors are significant. The standard deviation of the rate change of the terms of trade shock σ_q in regime 2 and regime 4 is almost four times larger than in regime 1 and regime 3. The standard deviation of the foreign inflation shock σ_{π^*} almost doubles in regime 2 and regime 4. The standard deviation of the rate change of the technology shock σ_z and the world output shock σ_{y^*} in regime 2 and regime 4 are approximately one and a half times larger than in regime 1 and regime 3. The standard deviation of the nominal interest rate shock σ_R is 0.253 in regime 1 and regime 3 compared to 0.325 in regime 2 and regime 4.

The calculated marginal density is -595.267 for the model three with two independent Markov chains, which is quite smaller compared to the constant parameter model with the best data fitting (-582.077). This finding, which is very different from the case of the UK, implies the Canadian time series data in the sample is not in favour of the assumption of the two independent Markov chains.

Table 4.27: Model Three with 2 Markov Chains(Canada)

	Regime 1:		Regime 4:	
parameter	mean	90% interval	mean	90% interval
τ	0.218	[0.157,0.280]		
κ	0.584	[0.396,0.760]		
ϕ_π	1.621	[1.426,1.847]	1.311	[1.050,1.522]
ϕ_y	0.032	[0.016,0.056]	0.138	[0.085,0.180]
$\phi_{\triangle e}$	0.104	[0.060,0.157]	0.113	[0.070,0.151]
$\phi_{\triangle y}$	0.125	[0.074,0.180]	0.106	[0.047,0.199]
ρ_R	0.594	[0.502,0.690]	0.796	[0.743,0.840]
ρ_z	0.358	[0.308,0.419]		
ρ_q	0.508	[0.450,0.565]		
ρ_{π^*}	0.450	[0.323,0.566]		
ρ_{y^*}	0.926	[0.891,0.964]		
α	0.131	[0.105,0.158]		
$r^{(A)}$	2.698	[2.392,2.946]		
$\pi^{(A)}$	1.866	[1.677,2.050]		
$\gamma^{(A)}$	0.757	[0.720,0.800]		
σ_R	0.253	[0.220,0.283]	0.325	[0.249,0.443]
σ_z	1.371	[0.985,1.712]	2.090	[1.454,2.759]
σ_{y^*}	0.581	[0.362,0.855]	0.954	[0.494,1.463]
σ_{π^*}	1.864	[1.587,2.219]	3.600	[2.815,4.268]
σ_q	1.150	[0.976,1.341]	4.551	[3.201,5.524]
P_{12}	0.063	[0.022,0.110]		
P_{21}	0.082	[0.042,0.126]		
Q_{12}	0.060	[0.028,0.090]		
Q_{21}	0.169	[0.086,0.236]		

Note:The table reports posterior means and 90% probability interval of the structural parameters in the model three for Canada

Numerical Solution and Simulation Results for Canada in the Model Three

There are four regimes in the model three with the Markov-switching variances and the Markov-switching policy parameters. Table 4.28 offers the numerical solution to regime 1, representing low volatility and more strict inflation targeting. Table 4.29 provides the numerical solution to regime 2, representing high volatility and more strict inflation targeting. Table 4.30 presents the numerical solution to regime 3, representing low volatility and less strict inflation targeting. Table 4.31 produces the numerical solution to regime 4, representing high volatility and less strict inflation targeting.

Table 4.28 contributes to computing the impulse response functions of the four endogenous variables including $\tilde{yy_t}$,π_t,$\tilde{r_t}$ and $\triangle\tilde{e_t}$ in regime 1:

$$\begin{pmatrix} \tilde{yy_t} \\ \pi_t \\ \tilde{r_t} \\ \triangle\tilde{e_t} \end{pmatrix} = \begin{pmatrix} -0.226 & 0.039 & -0.768 & 0.010 & 0.048 & 0.034 \\ -0.467 & 0.057 & 0.121 & 0.026 & 0.098 & 0.031 \\ 0.236 & 0.045 & -0.021 & 0.001 & -0.050 & 0.008 \\ -0.467 & 0.057 & 0.121 & -0.424 & 0.098 & -0.410 \end{pmatrix} \begin{pmatrix} \tilde{r_{t-1}} \\ z_{t-1} \\ \tilde{yy^*_{t-1}} \\ \pi^*_{t-1} \\ \tilde{yy_{t-1}} \\ \triangle\tilde{q_{t-1}} \end{pmatrix}$$

$$+ \begin{pmatrix} -0.096 & 0.150 & 0.077 & -0.482 & 0.040 \\ -0.199 & 0.219 & 0.070 & 0.076 & 0.108 \\ 0.100 & 0.174 & 0.018 & -0.013 & 0.003 \\ -0.199 & 0.219 & -0.929 & 0.076 & -1.756 \end{pmatrix} \begin{pmatrix} \xi^R_t \\ \xi^z_t \\ \xi^q_t \\ \xi^{y^*_t}_t \\ \xi^{\pi^*_t}_t \end{pmatrix}. \quad (4.27)$$

Where the second matrix in the above equation includes the impact of the structural shocks on the real output, inflation rate, nominal interest rate and depreciation exchange rate in regime 1.

Table 4.29 provides numerical information to compute the impulse response func-

tions of the four endogenous variables including $\tilde{yy}_t$,π_t,$\tilde{r}_t$ and $\triangle\tilde{e}_t$ in regime 2:

$$\begin{pmatrix} \tilde{yy}_t \\ \pi_t \\ \tilde{r}_t \\ \triangle\tilde{e}_t \end{pmatrix} = \begin{pmatrix} -0.226 & 0.039 & -0.768 & 0.010 & 0.048 & 0.034 \\ -0.467 & 0.057 & 0.121 & 0.026 & 0.098 & 0.031 \\ 0.236 & 0.045 & -0.021 & 0.001 & -0.050 & 0.008 \\ -0.467 & 0.057 & 0.121 & -0.424 & 0.098 & -0.410 \end{pmatrix} \begin{pmatrix} \tilde{r}_{t-1} \\ z_{t-1} \\ \tilde{yy}^*_{t-1} \\ \pi^*_{t-1} \\ \tilde{yy}_{t-1} \\ \triangle\tilde{q}_{t-1} \end{pmatrix}$$

$$+ \begin{pmatrix} -0.124 & 0.229 & 0.306 & -0.791 & 0.077 \\ -0.256 & 0.333 & 0.278 & 0.125 & 0.209 \\ 0.129 & 0.265 & 0.070 & -0.022 & 0.005 \\ -0.256 & 0.333 & -3.667 & 0.125 & -3.391 \end{pmatrix} \begin{pmatrix} \xi_t^R \\ \xi_t^z \\ \xi_t^q \\ \xi_t^{y_t^*} \\ \xi_t^{\pi_t^*} \end{pmatrix}, \quad (4.28)$$

where the second matrix in the above equation embodies the impact of the structural shocks on the real output, inflation rate, nominal interest rate and depreciation exchange rate in regime 2.

Table 4.30 offers numerical information to compute the impulse response functions of the four endogenous variables including $\tilde{yy}_t$,π_t,$\tilde{r}_t$ and $\triangle\tilde{e}_t$ in regime 3:

$$\begin{pmatrix} \tilde{yy}_t \\ \pi_t \\ \tilde{r}_t \\ \triangle\tilde{e}_t \end{pmatrix} = \begin{pmatrix} -0.571 & 0.048 & -0.695 & 0.012 & 0.076 & 0.037 \\ -1.328 & 0.067 & 0.340 & 0.032 & 0.177 & 0.038 \\ 0.333 & 0.026 & 0.006 & 0.001 & -0.044 & 0.006 \\ -1.328 & 0.067 & 0.340 & -0.418 & 0.177 & -0.404 \end{pmatrix} \begin{pmatrix} \tilde{r}_{t-1} \\ z_{t-1} \\ \tilde{yy}^*_{t-1} \\ \pi^*_{t-1} \\ \tilde{yy}_{t-1} \\ \triangle\tilde{q}_{t-1} \end{pmatrix}$$

$$+ \begin{pmatrix} -0.182 & 0.183 & 0.084 & -0.436 & 0.049 \\ -0.422 & 0.258 & 0.086 & 0.213 & 0.134 \\ 0.106 & 0.100 & 0.013 & 0.003 & 0.003 \\ -0.422 & 0.258 & -0.914 & 0.213 & -1.730 \end{pmatrix} \begin{pmatrix} \xi_t^R \\ \xi_t^z \\ \xi_t^q \\ \xi_t^{y_t^*} \\ \xi_t^{\pi_t^*} \end{pmatrix}, \quad (4.29)$$

where the second matrix in the above equation incorporates the impact of the structural shocks on the real output, inflation rate, nominal interest rate and depreciation exchange rate in regime 3.

Table 4.31 produces numerical information to compute the impulse response functions of the four endogenous variables including $\tilde{yy_t}$,π_t,$\tilde{r_t}$ and $\triangle\tilde{e_t}$ in regime 4:

$$\begin{pmatrix} \tilde{yy_t} \\ \pi_t \\ \tilde{r_t} \\ \triangle\tilde{e_t} \end{pmatrix} = \begin{pmatrix} -0.571 & 0.048 & -0.695 & 0.012 & 0.076 & 0.037 \\ -1.328 & 0.067 & 0.340 & 0.032 & 0.177 & 0.038 \\ 0.333 & 0.026 & 0.006 & 0.001 & -0.044 & 0.006 \\ -1.328 & 0.067 & 0.340 & -0.418 & 0.177 & -0.404 \end{pmatrix} \begin{pmatrix} \tilde{r_{t-1}} \\ z_{t-1} \\ \tilde{yy^*_{t-1}} \\ \pi^*_{t-1} \\ \tilde{yy_{t-1}} \\ \triangle\tilde{q_{t-1}} \end{pmatrix}$$
$$+ \begin{pmatrix} -0.233 & 0.279 & 0.333 & -0.716 & 0.095 \\ -0.542 & 0.394 & 0.339 & 0.350 & 0.260 \\ 0.136 & 0.152 & 0.052 & 0.006 & 0.005 \\ -0.542 & 0.394 & -3.615 & 0.350 & -3.340 \end{pmatrix} \begin{pmatrix} \xi^R_t \\ \xi^z_t \\ \xi^q_t \\ \xi^{y^*_t}_t \\ \xi^{\pi^*_t}_t \end{pmatrix}, \quad (4.30)$$

where the second matrix in the above equation embodies the impact of the structural shocks on the real output, inflation rate, nominal interest rate and depreciation exchange rate in regime 4.

Figure 4.10 compares the impulse response functions between regime 3 and regime 4 in model three with two independent Markov Chains. In other words, It compares the impulse response functions between the high and the low volatility given the same less strict inflation targeting. It draws the impulse response functions of low volatility with solid and blue lines, and high volatility with dashed and red lines.

Holding everything else constant, a unit of the standard deviation of the monetary policy shock will exert a -0.182 impact on the real output deviation in regime 3 while -0.233 in regime 4, a -0.422 impact on the inflation in regime 3 while -0.542 in regime 4,a 0.106 impact on the nominal interest rate deviation in regime 3 while 0.136 in regime 4, a -0.422 impact on the nominal exchange rate depreciation in regime 3 while -0.542 in regime 4.

Holding everything else constant, a unit of the standard deviation of the shock to the change rate of the terms of trade will exert a 0.084 impact on the real output deviation in regme 3 while 0.333 in regime 4, a 0.086 impact on inflation in regime 3 while 0.339 in regime 4,a 0.013 impact on the nominal interest rate deviation in regime 3 while 0.052 in regime 4, a -0.914 impact on the nominal exchange rate depreciation in regime 3 while -3.615 in regime 4.

Holding everything else constant, a unit of the standard deviation of the shock to the change rate of the technology will exert a 0.183 impact on the real output deviation in regime 3 while 0.279 in regime 4, a 0.258 impact on the inflation in regime 3 while 0.394 in regime 4,a 0.100 impact on the nominal interest rate deviation in regime 3 while 0.152 in regime 4, a 0.258 impact on the nominal exchange rate depreciation in regime 3 while 0.394 in regime 4.

Holding everything else constant, a unit of the standard deviation of the shock to the world output deviation will exert a -0.436 impact on the real output deviation in regime 3 while -0.716 in regime 4, a 0.213 impact on the inflation in regime 3 while 0.350 in regime 4,a 0.003 impact on the nominal interest rate deviation in regime 3 while 0.006 impact in regime 4, a 0.213 impact on the nominal exchange rate depreciation in regime 3 while 0.350 in regime 4.

Holding everything else constant, a unit of the standard deviation of the shock to the world inflation will exert a 0.049 impact on the real output deviation in regime 3 while 0.095 in regime 4, a 0.134 impact on the inflation in regime 3 while 0.260 in regime 4,a 0.003 impact on the nominal interest rate deviation in regime

3 while 0.005 impact in regime 4, a -1.730 impact on the nominal exchange rate depreciation in regime 3 while -3.340 in regime 4.

Figure 4.11 compares the impulse response functions between regime 2 and regime 4 in model three with two independent Markov chains. It compares the impulse response functions between the more strict and the less strict inflation targeting given the same high volatility. It depicts the impulse response functions of more strict inflation targeting with solid and blue lines, and less strict inflation targeting with dashed and red lines.

Holding everything else constant, a unit of the standard deviation of monetary policy shock will exert a -0.124 impact on the real output deviation in regime 2 while -0.233 in regime 4, a -0.256 impact on the inflation in regime 2 while -0.542 in regime 4,a 0.129 impact on the nominal interest rate deviation in regime 2 while 0.136 in regime 4, a -0.256 impact on the nominal exchange rate depreciation in regime 2 while -0.542 in regime 4.

Holding everything else constant, a unit of the standard deviation of the shock to the change rate of the terms of trade will exert a 0.306 impact on the real output deviation in regme 2 while 0.333 in regime 4, a 0.278 impact on the inflation in regime 2 while 0.339 in regime 4,a 0.070 impact on the nominal interest rate deviation in regime 2 while 0.052 in regime 4, a -3.677 impact on the nominal exchange rate depreciation in regime 2 while -3.615 in regime 4.

Holding everything else constant, a unit of the standard deviation of the shock to the change rate of the technology will exert a 0.229 impact on the real output deviation in regime 2 while 0.279 in regime 4, a 0.333 impact on the inflation in regime 2 while 0.394 in regime 4,a 0.265 impact on the nominal interest rate deviation in regime 2 while 0.152 in regime 4, a 0.333 impact on the nominal exchange rate depreciation in regime 2 while 0.394 in regime 4.

Holding everything else constant, a unit of the standard deviation of the shock to the world output deviation will exert a -0.791 impact on the real output deviation in regime 2 while -0.716 in regime 4, a 0.125 impact on the inflation in regime 2 while 0.350 in regime 4,a -0.022 impact on the nominal interest rate deviation in regime 2 while 0.006 impact in regime 4, a 0.125 impact on the nominal exchange rate depreciation in regime 2 while 0.350 in regime 4.

Holding everything else constant, a unit of the standard deviation of the shock to the world inflation will exert a 0.077 impact on the real output deviation in regime 2 while 0.095 in regime 4, a 0.209 impact on the inflation in regime 2 while 0.260 in regime 4,a 0.005 impact on the nominal interest rate deviation in both of the two regimes, a -3.391 impact on the nominal exchange rate depreciation in regime 3 while -3.340 in regime 4.

Table 4.28: The Numerical Solution to Regime 1 of the Model Three for Canada

		static		backward looking				mixed		forward looking	
		$\tilde{yy}_{t,n}$	$\triangle\tilde{e}_t$	$\tilde{r}_t$	z_t	$\tilde{yy}^*_t$	π^*_t	$\triangle\tilde{q}_t$	$\tilde{yy}_t$	π_t	$\triangle\tilde{yy}^*_t$
endogenous variables	$\tilde{r}_{t-1}$		-0.467	0.236					-0.226	-0.467	
	z_{t-1}		0.057	0.045	0.358				0.039	0.057	
	$\tilde{yy}^*_{t-1}$	-0.813	0.121	-0.021		0.926			-0.768	0.121	-0.074
	π^*_{t-1}		-0.424	0.001			0.450		0.010	0.026	
	$\tilde{yy}_{t-1}$		0.098	-0.050					0.048	0.098	
	$\triangle\tilde{q}_{t-1}$		-0.410	0.008				0.508	0.034	0.031	
exogenous variables	ξ^R_t		-0.199	0.100					-0.096	-0.199	
	ξ^z_t		0.219	0.174	1.371				0.150	0.219	
	ξ^q_t		-0.929	0.018				1.150	0.077	0.070	
	$\xi^{y^*_t}_t$	-0.510	0.076	-0.013		0.581			-0.482	0.076	0.581
	$\xi^{\pi^*_t}_t$		-1.756	0.003			1.864		0.040	0.108	

Note: This table reports the numerical solutions of regime 1 in the model three for Canada. Regime 1 considers low volatility and more strict inflation targeting.

Table 4.29: The Numerical Solution to Regime 2 of the Model Three for Canada

		static		backward looking				mixed		forward looking	
		$y\tilde{y}_{t,n}$	$\triangle\tilde{e}_t$	$\tilde{r}_t$	z_t	$y\tilde{y}^*_t$	π^*_t	$\triangle\tilde{q}_t$	$y\tilde{y}_t$	π_t	$\triangle y\tilde{y}^*_t$
endogenous variables	$\tilde{r}_{t-1}$		-0.467	0.236					-0.226	-0.467	
	z_{t-1}		0.057	0.045	0.358				0.039	0.057	
	$y\tilde{y}^*_{t-1}$	-0.813	0.121	-0.021		0.926			-0.768	0.121	-0.074
	π^*_{t-1}		-0.424	0.001			0.450		0.010	0.026	
	$y\tilde{y}_{t-1}$		0.098	-0.050					0.048	0.098	
	$\triangle \tilde{q}_{t-1}$		-0.410	0.008				0.508	0.034	0.031	
exogenous variables	ξ^R_t		-0.256	0.129					-0.124	-0.256	
	ξ^z_t		0.333	0.265	2.090				0.229	0.333	
	ξ^q_t		-3.677	0.070				4.551	0.306	0.278	
	$\xi^{y^*_t}_t$	-0.838	0.125	-0.022		0.954			-0.791	0.125	0.954
	$\xi^{\pi^*_t}_t$		-3.391	0.005			3.600		0.077	0.209	

Note: This table reports the numerical solutions of regime 2 in the model three for Canada. Regime 2 considers high volatility and more strict inflation targeting.

Table 4.30: The Numerical Solution to Regime 3 of the Model Three for Canada

		static		backward looking				mixed		forward looking	
		$\tilde{yy}_{t,n}$	$\triangle\tilde{e}_t$	$\tilde{r}_t$	z_t	$\tilde{yy}^*_t$	π^*_t	$\triangle\tilde{q}_t$	$\tilde{yy}_t$	π_t	$\triangle\tilde{yy}^*_t$
endogenous variables	$\tilde{r}_{t-1}$		-1.328	0.333					-0.571	-1.328	
	z_{t-1}		0.067	0.026	0.358				0.048	0.067	
	$\tilde{yy}^*_{t-1}$	-0.813	0.340	0.006		0.926			-0.695	0.340	-0.074
	π^*_{t-1}		-0.418	0.001			0.450		0.012	0.032	
	$\tilde{yy}_{t-1}$		0.177	-0.044					0.076	0.177	
	$\triangle\tilde{q}_{t-1}$		-0.404	0.006				0.508	0.037	0.038	
exogenous variables	ξ^R_t		-0.422	0.106					-0.182	-0.422	
	ξ^z_t		0.258	0.100	1.371				0.183	0.258	
	ξ^q_t		-0.914	0.013				1.150	0.084	0.086	
	$\xi^{y^*_t}_t$	-0.510	0.213	0.003		0.581			-0.436	0.213	0.581
	$\xi^{\pi^*_t}_t$		-1.730	0.003			1.864		0.049	0.134	

Note: This table reports the numerical solutions of regime 3 in the model three for Canada. Regime 3 considers low volatility and less strict inflation targeting.

Table 4.31: The Numerical Solution to Regime 4 of the Model Three for Canada

		static		backward looking				mixed		forward looking	
		$y\tilde{y}_{t,n}$	$\triangle\tilde{e}_t$	$\tilde{r}_t$	z_t	$\tilde{yy}^*_t$	π^*_t	$\triangle\tilde{q}_t$	$y\tilde{y}_t$	π_t	$\triangle\tilde{yy}^*_t$
endogenous variables	$\tilde{r}_{t-1}$		-1.328	0.333					-0.571	-1.328	
	z_{t-1}		0.067	0.026	0.358				0.048	0.067	
	$\tilde{yy}^*_{t-1}$	-0.813	0.340	0.006		0.926			-0.695	0.340	-0.074
	π^*_{t-1}		-0.418	0.001			0.450		0.012	0.032	
	$y\tilde{y}_{t-1}$		0.177	-0.044					0.076	0.177	
	$\triangle\tilde{q}_{t-1}$		-0.404	0.006				0.508	0.037	0.038	
exogenous variables	ξ^R_t		-0.542	0.136					-0.233	-0.542	
	ξ^z_t		0.394	0.152	2.090				0.279	0.394	
	ξ^q_t		-3.615	0.052				4.551	0.333	0.339	
	$\xi^{y^*_t}_t$	-0.838	0.350	0.006		0.954			-0.716	0.350	0.954
	$\xi^{\pi^*_t}_t$		-3.340	0.005			3.600		0.095	0.260	

Note: This table reports the numerical solutions of regime 4 in the model three for Canada. Regime 4 considers high volatility and less strict inflation targeting.

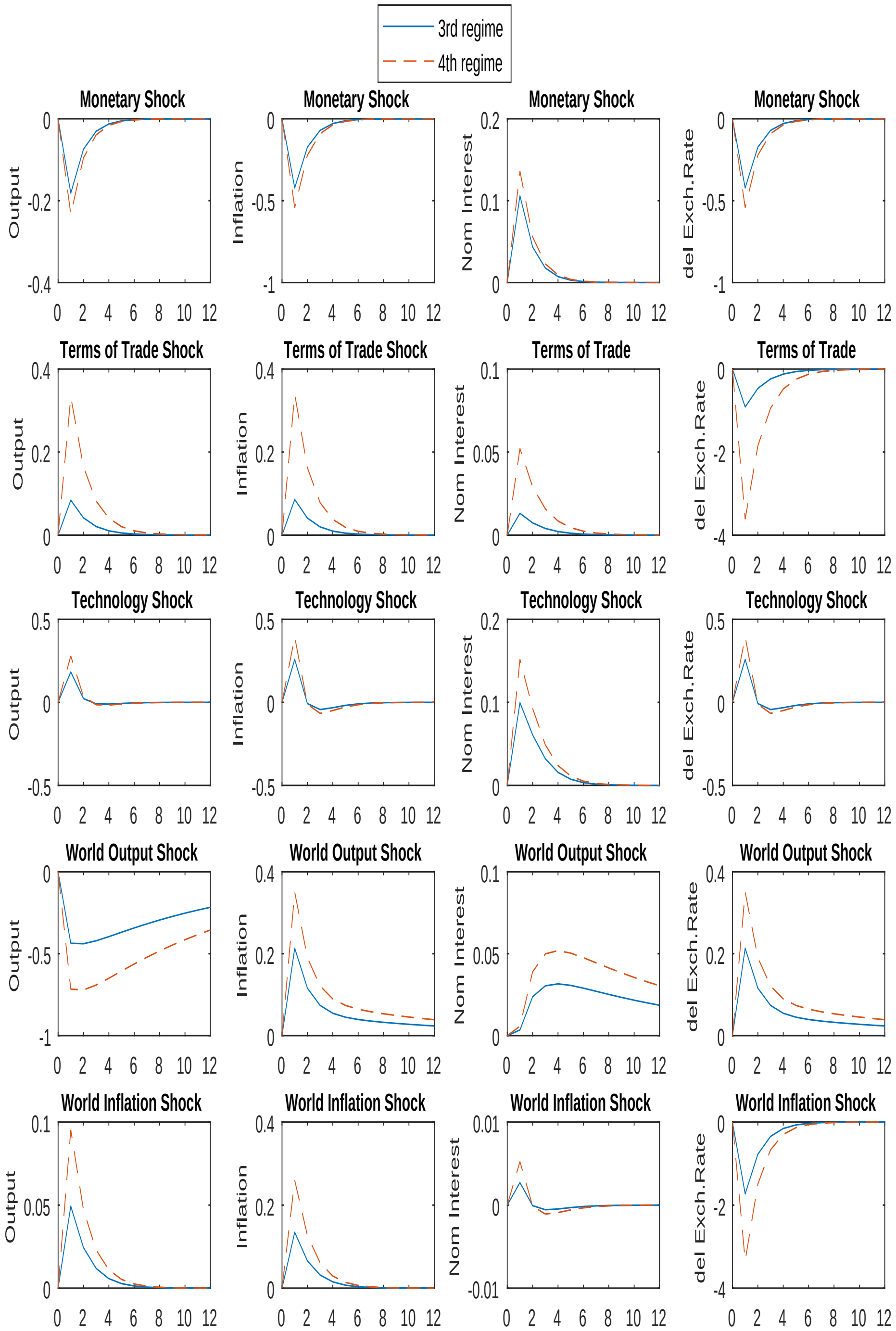

Figure 4.10: Impulse responses,Canada(Switching Variances of Model Three). Note:The figure depicts the impulses responses of real output, inflation rate, nominal interest rate and depreciation exchange rate to one unit structural shock of regime 3 representing low volatility(solid and blue lines)and regime 4 representing high volatility(dashed and red lines), given the same less strict inflation targeting.

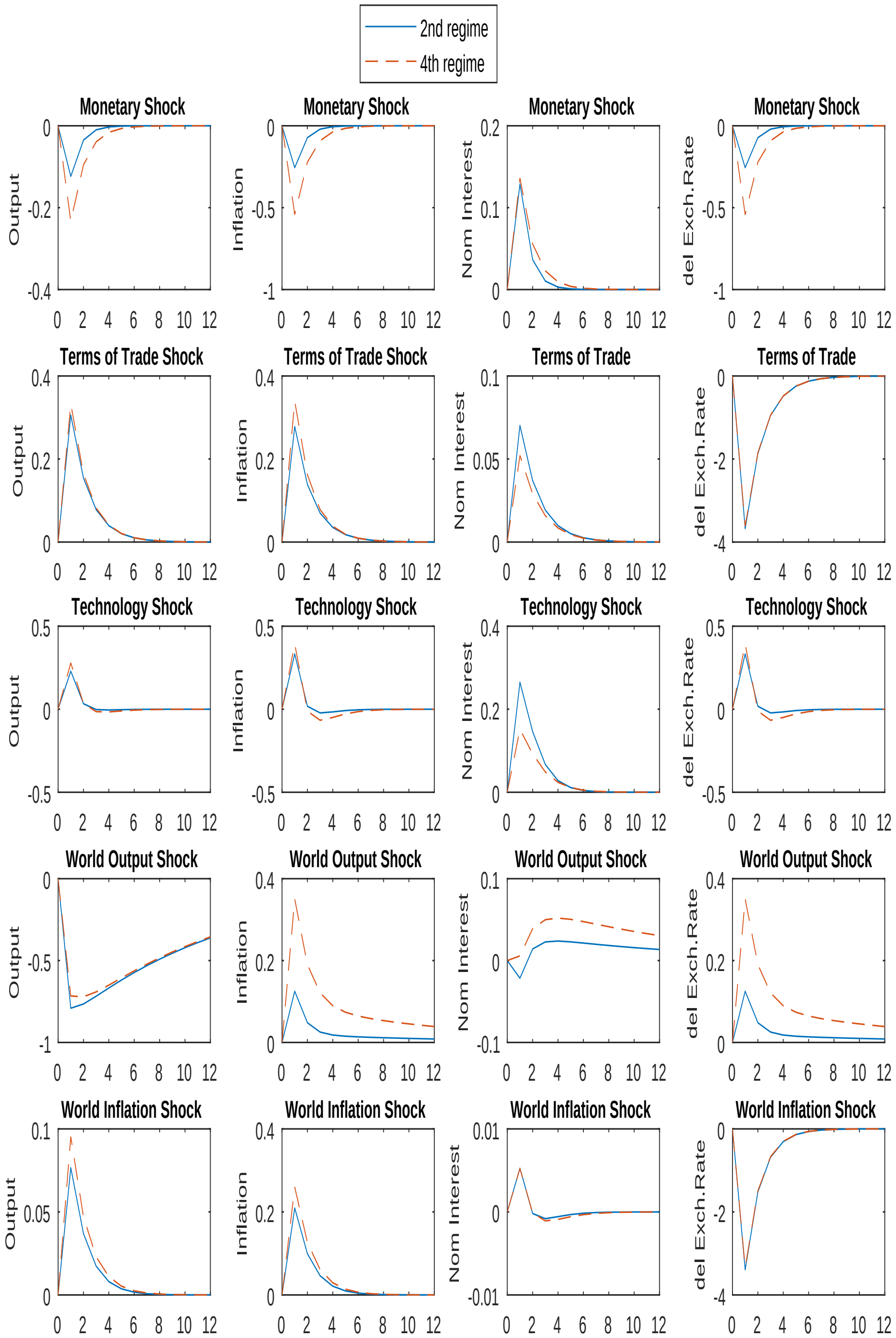

Figure 4.11: Impulse responses,Canada(Switching Taylor Rules of Model Three). Note:The figure depicts the impulses responses of real output, inflation rate, nominal interest rate and depreciation exchange rate to one unit structural shock of regime 2 representing more strict inflation targeting(solid and blue lines)and regime 4 representing less strict inflation targeting (dashed and red lines), given the same high volatility.

Variance Decomposition of Model Three for Canada

Table 4.32 reports the variance decomposition of the model three with two independent Markov Chains for Canada. It summarises the major contributions to the variation of the four endogenous variables $y\tilde{y}_t$,π_t,$\tilde{r}_t$ and $\triangle\tilde{e}_t$ as follows. First, the world output shock $\xi_t^{y_t^*}$ contributes most to the variation of $y\tilde{y}_t$ in each of the four regimes. Second, the change rate of the technology shock ξ_t^z contributes most to the variation of the inflation π_t in regime 1 and regime 2 while the policy shock ξ_t^R contributes most in regime 3 and regime 4. Moreover, the change rate of the technology shock ξ_t^z contributes most to the variation of the interest rate $\tilde{r}_t$ in each of the four regimes. Finally, the world inflation shock $\xi_t^{\pi_t^*}$ contributes most to the variation of the nominal exchange rate depreciation $\triangle\tilde{e}_t$ in each of the four regimes.

The policy shock ξ_t^R contributes 0.54% to the variation of the output deviation $y\tilde{y}_t$ in regime 1, 0.33% in regime 2, 2.24% in regime 3 and 1.35% in regime 4. Next, it contributes 35.70% to the variation of the inflation π_t in regime 1, 19.56% in regime 2, 53.94% in regime 3 and 36.48% in regime 4. Moreover, it contributes 19.86% to the variation of the interest rate $\tilde{r}_t$ in regime 1, 14.19% in regime 2, 35.36% in regime 3 and 25.75% in regime 4. Finally, it contributes 0.84% to the variation of the exchange rate depreciation $\triangle\tilde{e}_t$ in regime 1, 0.22% in regime 2, 4.09% in regime 3 and 1.09% in regime 4.

The change rate of the terms of trade shock ξ_t^q contributes 0.43% to the variation of the output deviation $y\tilde{y}_t$ in regime 1, 2.48% in regime 2,0.53% in regime 3 and 3.04% in regime 4. Next, it contributes 5.47% to the variation of the inflation π_t in regime 1, 28.42% in regime 2,2.40% in regime 3 and 15.43% in regime 4. Moreover, it contributes 0.79% to the variation of the interest rate $\tilde{r}_t$ in regime 1, 5.34% in regime 2, 0.65% in regime 3 and 4.52% in regime 4. Finally, it contributes 22.73% to the variation of the nominal exchange rate depreciation $\triangle\tilde{e}_t$ in regime 1, 55.54% in regie 2,21.58% in regime 3 and 56.45% in regime 4.

The change rate of the technology shock ξ_t^z contributes 1.25% to the variation of the output deviation $y\tilde{y}_t$ in regime 1, 1.06% in regime 2, 1.94% in regime 3 and 1.65% in regime 4. Next, it contribute 40.01% to the variation of the inflation π_t in regime 1, 30.87% in regime 2,17.68% in regime 3 and 16.84% in regime 4. Furthermore, it contributes 75.63% to the variation of the interest rate $\tilde{r}_t$ in regime 1, 76.12% in regime 2, 39.16% in regime 3 and 40.16% in regime 4. Finally, it contributes 0.94% to the variation of the exchange rate depreciation $\triangle\tilde{e}_t$ in regime 1, 0.34% in regime 2,1.34% in regime 3 and 0.5% in regime 4.

The world output shock $\xi_t^{y_t^*}$ contributes 97.66% to the variation of the output deviation $y\tilde{y}_t$ in regime 1, 95.98% in regime 2, 95.10% in regime 3 and 93.71% in regime 4. Moreover, it contributes 6.28% to the variation of the inflation π_t in regime 1, 5.62% in regime 2,20.02% in regime 3 and 22.12% in regime 4. Next, it contributes 3.70% to the variation of the interest rate $\tilde{r}_t$ in regime 1, 4.33% in regime 2,24.81% in regime 3 and 29.53% in regime 4. Finally, it contributes 0.15% to the variation of the exchange rate depreciation $\triangle\tilde{e}_t$ in regime 1, 0.06% in regime 2,1.52% in regime 3 and 0.66% in regime 4.

The world inflation shock $\xi_t^{\pi_t^*}$ contributes 0.11% to the variation of the output deviation $y\tilde{y}_t$ in regime 1, 0.15% in regime 2, 0.18% in regime 3 and 0.25% in regime 4. Moreover, it contributes 12.54% to the variation of the inflation π_t in regime 1, 15.53% in regime 2,5.96% in regime 3 and 9.12% in regime 4. Next, it contributes 0.01% to the variation of the interest rate $\tilde{r}_t$ in regime 1, 0.02% in regime 2 and regime 3, and 0.03% in regime 4. Finally, it contributes 75.34% to the variation of the exchange rate depreciation $\triangle\tilde{e}_t$ in regime 1, 43.84% in regime 2,71.46% in regime 3 and 43.10% in regime 4.

Table 4.32: Variance Decomposition of the Model Three for Canada

Regime 1	Output	Inflation	interest rate	Exchange rate depreciation
Policy	0.54	35.70	19.86	0.84
Terms of trade	0.43	5.47	0.79	22.73
Technology	1.25	40.01	75.63	0.94
World output	97.66	6.28	3.70	0.15
World inflation	0.11	12.54	0.01	75.34
Regime 2	**Output**	**Inflation**	**interest rate**	**Exchange rate depreciation**
Policy	0.33	19.56	14.19	0.22
Terms of trade	2.48	28.42	5.34	55.54
Technology	1.06	30.87	76.12	0.34
World output	95.98	5.62	4.33	0.06
World inflation	0.15	15.53	0.02	43.84

Regime 3	Output	Inflation	interest rate	Exchange rate depreciation
Policy	2.24	53.94	35.36	4.09
Terms of trade	0.53	2.40	0.65	21.58
Technology	1.94	17.68	39.16	1.34
World output	95.10	20.02	24.81	1.52
World inflation	0.18	5.96	0.02	71.46
Regime 4	**Output**	**Inflation**	**interest rate**	**Exchange rate depreciation**
Policy	1.35	36.48	25.75	1.09
Terms of trade	3.04	15.43	4.52	54.65
Technology	1.65	16.84	40.16	0.50
World output	93.71	22.12	29.53	0.66
World inflation	0.25	9.12	0.03	43.10

Note:The table reports the variance decomposition of the model three for Canada. Regime 1 (2) is characterized as low (high) volatility and more strict inflation targeting. Regime 3 (4) is characterized as low (high) volatility and less strict inflation targeting.

4.4.4 Model Comparison and Data Analysis for Canada

Table 4.33 provides the model comparison at the second stage for Canada. It ranks the models from the best to the worst in terms of data fitting. Likewise, the log marginal data densities yield the posterior odds ratios, which are important indicators of the model comparison.

More specifically, the Canadian data is not always in favour of the Markov-switching models compared to the benchmark model with the best data fitting in the previous chapter. The model one with switching variances still ranks first in terms of data fitting, the model two with switching policy parameters ranks second, and the benchmark model without any Markov-switching parameters ranks third. The model three with two independent Markov chains ranks last. Thus, it is beneficial to introduce only one Markov chain to the benchmark model to improve the performance of the data fitting. However, considering two kinds of Markov chain together, there is a potential fitting loss compared to the benchmark model.

Figure 4.12 plots the smoothed probability (blue and solid) of regime 2 in the model one with the Markov-switching variances, representing high volatility, against the actual time series data (red and dashed) in Canada. It scales the smoothed probability by ten times except for the first panel. It is obvious to see Canada experiences domestic shocks ξ_t^R, ξ_t^z and foreign shocks $\xi_t^q, \xi_t^{y_t^*}, \xi_t^{\pi_t^*}$ with significantly large variances more frequently, including the periods of 1993-1995, 2001-2004,2005-2006 and the most recent financial crisis.

Figure 4.13 plots the smoothed probability (blue and solid) of regime 1 in the model two with Markov-switching policy parameters, representing more strict inflation targeting, against the actual time series data (red and dashed) in Canada. Regime 1 dominates at 1992: Q4 and lasts between 1995: Q1 and 2001: Q4 and reappears during the period from 2005 until the beginning of the most recent financial crisis. Regime 2 dominates during the period of the most recent financial crisis, which may imply there is a cautious cut on the nominal interest rate, thereby saving the ammo for the future cut.

The model comparison at the second stage marks the model one with the Markov-switching variances best fit the Canadian data. Figure 4.14 plots the historical decomposition of the Canadian data given the shock contributions generated from the model one.

Regime 2, representing high volatility, dominates from 1993 to 1995, 2001-2004, 2005-2006 and the most recent financial crisis. First, The world output shock (yellow) contributes most to the variation of the output deviation. Second, the policy shock (green), the change rate of technology shock (purple), the change rate of the terms of trade shock (blue) and the world inflation shock (red) contribute most to the variation of the inflation rate. Next, the change rate of the technology shock (purple) and the policy shock (green) dominate in the variation of the nominal interest rate. At last, the world inflation shock (red) and the change rate of the terms of trade shock (blue) contribute most to the variation of the nominal exchange rate depreciation.

During the rest of the sample period, regime 1 representing low volatility dominates. First, the world output shock (yellow) still contributes most to the variation of the output deviation. Second, the change rate of technology shock (purple) and the policy shock (green) dominates in the variation of the inflation rate. Furthermore, the change rate of the technology (purple) and the policy shock (green) also offer a major contribution to the variation of the interest rate. Finally, the world inflation shock (red) and the change rate of the terms of trade shock (blue) dominate in the variation of nominal exchange rate depreciation.

Table 4.33: Log Marginal Data Densities and Ranks of the Models for Canada

Models	Log MDD	Rank of data fitting
Benchmark Model: Constant parameters model	-582.077	3
Model 1:Markov-switching in volatility of shocks	-569.826	1
Model 2:Markov-switching monetary policy	-575.813	2
Model 3:The model with two Markov chains	-595.267	4

Note:The table reports the log marginal data densities and ranks for the models at the second stage of the model comparison. The model one with Markov-switching variances ranks first in terms of data fitting.

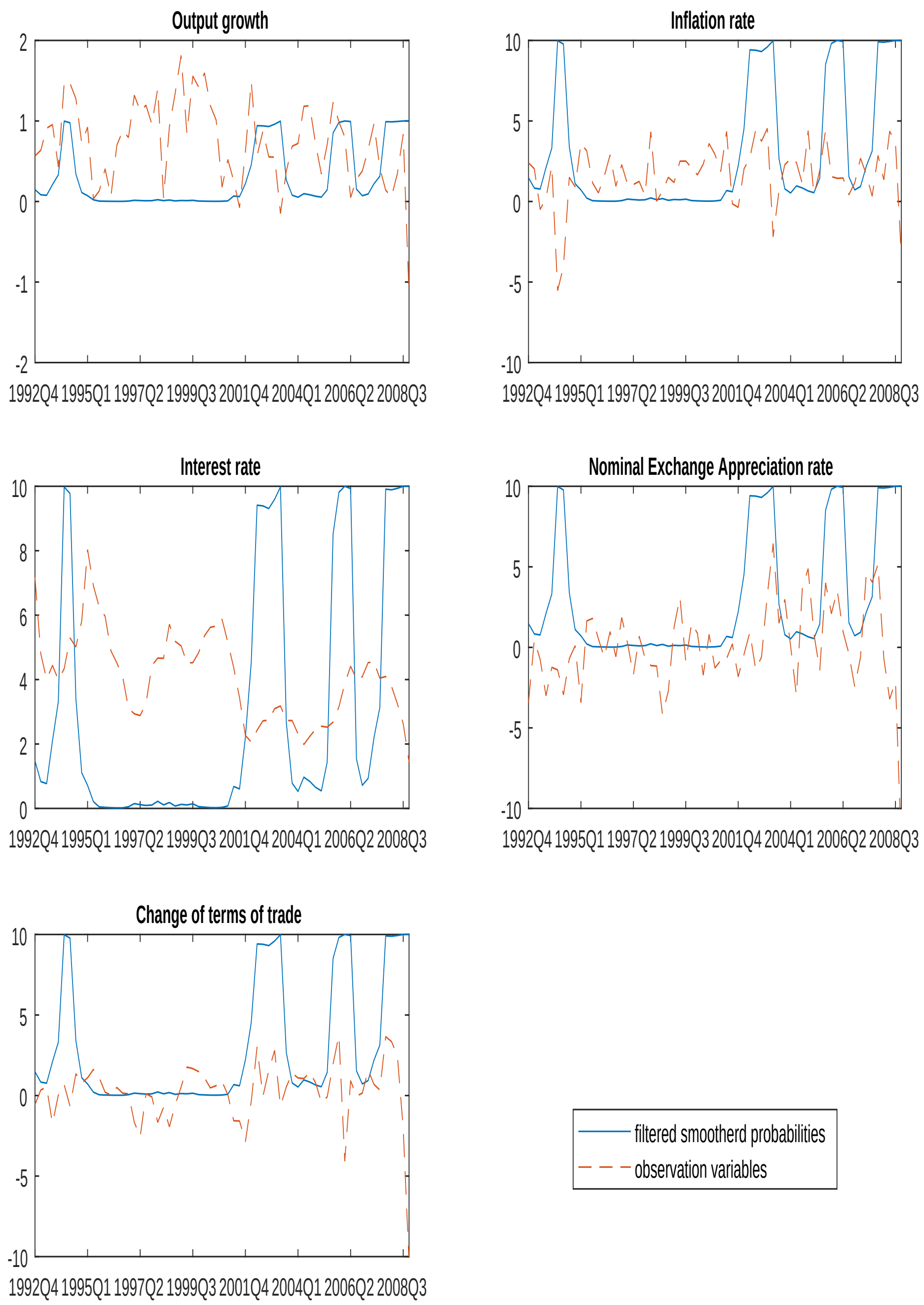

Figure 4.12: Smoothed probability of high volatility from model one. Note:The figure depicts the smoother probability of high volatility against the data from Canada covering the sample period of 1992:Q4-2008:Q4.

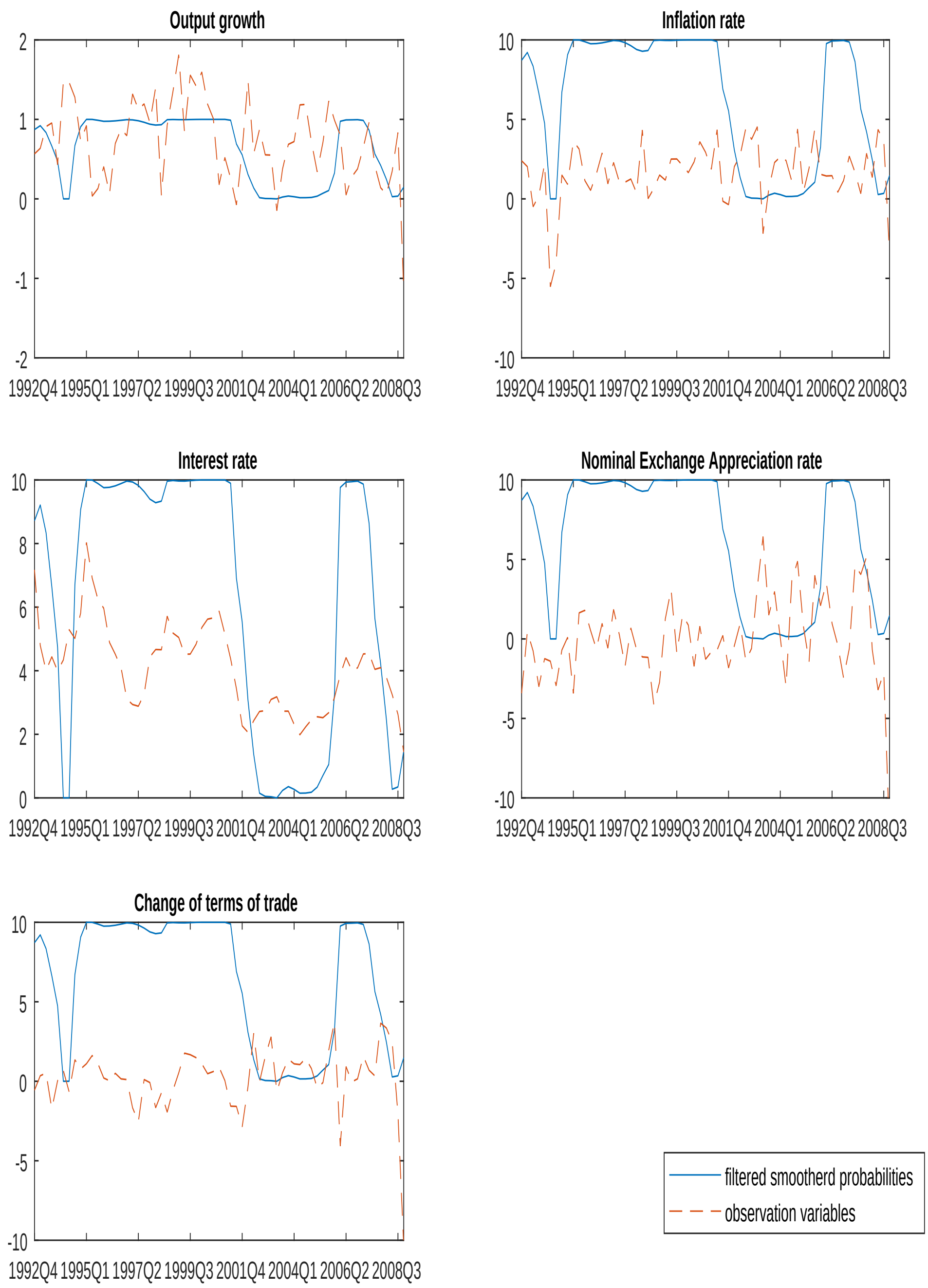

Figure 4.13: Smoothed probability of strict inflation from model two. Note:The figure depicts the smoother probability of strict inflation targeting against the data from Canada covering the sample period 1992:Q4-2008:Q4.

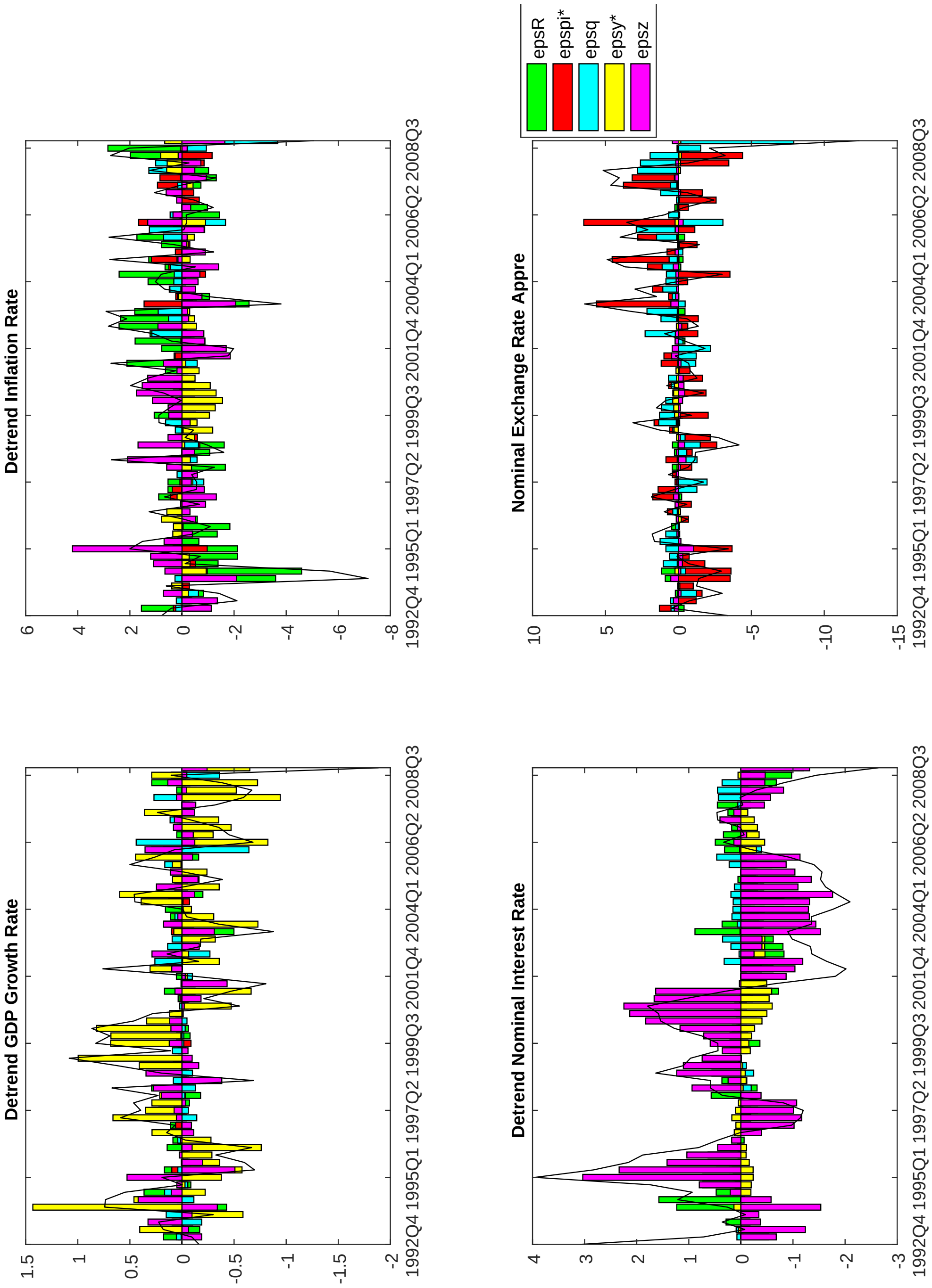

Figure 4.14: Historical Decomposition Using Model One for Canada. Note:The figure depicts the historical decomposition of output growth, inflation rate, nominal interest rate and the movement of nominal exchange rate in Canada over the sample period of 1992:Q4-2008:Q4.

4.5 Conclusion

Chapter 4 introduces two kinds of Markov-switching parameters to the constant parameter models with the best data fitting in chapter 3. The estimation of the Markov-switching DSGE models based on the data collected from the UK and Canada provides the fundamentals o the model comparisons at the second stage.

Model comparisons suggest the model one with the switching variances provide the best empirical fit to the UK and Canadian data, which implies the uncertainties regarding the economic environment play more dominated roles in modelling the economy compared to the uncertainties regarding the behaviours of policymakers in the model two with switching policy parameters. Moreover, the model three with two independent Markov chains behave utterly different in the UK and Canda. That is to say, the combination between the uncertainties regarding the environment and the behaviours of policymakers sometimes cannot provide better data fitting compared to the benchmark model without any structural changes.

Apart from the model comparison, chapter 4 decompose the time series data for the UK and Canada with the model one which outperforms other models at the second stage of model comparison. Here are the main results from the data decomposition. First, the world output shock dominates in the variation of the British and Canadian output in both high and low volatility regimes. Second, when the volatility is low, the world output shock contributes most to the variation of the UK inflation, and when the volatility is high, the change rate of the technology shock contributes most. Compared to the UK, when the volatility is low, the change rate of the technology shock contributes most to the variation of Canadian inflation, and the policy shock contributes most when the volatility is high. Third, the change rate of the technology shock in both of the two countries contribute most to the variation of the nominal interest rate regardless of the high or low volatility. At last, the world inflation shock only affects nominal exchange rate depreciation in the UK since the term of exchange rate depreciation is omitted from the monetary policy reaction function. In contrast, the world inflation shock dominates in the variation of Canadian nominal exchange rate depreciation and also influence Canadian inflation to a large extent.

Chapter 5

General Conclusion

5.1 Main Findings

The small open DSGE model with Markov switching variances offers the best data fitting for the UK and Canada during the sample period 1992: Q4-2008: Q4. More specifically, the UK data supports that the monetary policy function should not include the movement of the nominal exchange rate while the data of Canada supports that the policy function should include the movement of the nominal exchange rates.

For the UK, the regime of high volatility initially dominates during the period of 1992:Q4-1993:Q2, as a result of the currency crisis in the early 1990s. In response to the decline in the GDP growth rate and the inflation rate, the bank of England cuts the nominal interest rate from 6.82 on 1992:Q4 to 5.79 on 1993:Q2. The same type of regime then happens during the period of 2007:Q3-2008:Q4, as a result of the most recent financial crisis. Due to the massive decline of the GDP growth rate and the inflation rate, the bank of England cuts the nominal interest rate dramatically from 5.82 on 2007:Q3 to 1.65 on 2008:Q4. In the regime of high volatility, the change of the technology shock and the world output shock mainly contribute to the fluctuation of the nominal interest rate, while the world inflation shock contributes little. This result is consistent with the policy specifications for the UK. The bank of England determines monetary policy based on the domestic output, affected by the world output shock, and the inflation rate, affected by the change of the technology shock and the world output shock. The feedback rule of the policy rate does not incorporate the movement of the nominal exchange rate, majorly affected by the world inflation shock.

For Canada, the regime of high volatility initially dominates during the period of 1993:Q2-1995:Q1, as a result of the severe budget deficits and the inflationary excess in the early 1990s. The central bank increases the nominal interest rate from 4.29 on 1993:Q2 to 8.06 on 1995:Q1. The central bank of Canada firmly establishes a low-inflation environment in the early 1990s. The regime of high volatility then happens during the period of 2000:Q3-2004:Q2, when the collapse of the high tech companies hit the stock market. The central bank cuts the nominal interest rate from 5.80 on 2000:Q3 to 2.00 on 2004:Q2. The same type of regime plays a role again during the period of 2005:Q1-2006:Q3, after the enormous expansion in 2004 due to the increasing personal expenditure. In response to the most substantial annual increase since 2000, the central bank raises the nominal interest rate from 2.48 on 2005:Q1 to 4.25 on 2006:Q3. The regime finally happens during the period of 2007:Q2-2008:Q4, as a result of the most recent financial crisis. The central bank of Canada cuts the nominal interest rate aggressively from 4.24 on 2007:Q2 to 1.50 on 2008:Q4. In the regime of high volatility, the change of the technology shock and the monetary policy shock contribute most to the fluctuation of the policy rate. This result is also consistent with the policy specifications for Canada. The central bank of Canada mainly determines monetary policy based on the inflation rate, affected by the change of the technology shock and the world output shock. The feedback rule of the policy rate also incorporates the output, affected by the world output shock, and the movement of nominal exchange rate, majorly affected by the world inflation shock. However, the world output shock and the world inflation shock contributes very little to the fluctuations of the nominal interest rate within the sample period.

The UK and Canada both adopt strict inflation targeting strategy in the early 1990s. However, the regimes change less frequently in the UK than in Canada. In addition to the most recent financial crisis, Canada faces two more challenges brought by the exogenous shocks of high variances after 2000. Moreover, the high volatility environment always motivates the bank of England to adopt an expansionary monetary policy while it sometimes urges the central bank of Canada to switch back and forth on the expansionary and restrictive policy.

5.2 Limitations and Directions of Further Research

The primary purpose of the thesis is to use Bayesian estimation likelihood approach to compare the small open DSGE models with different specifications of monetary policy reaction functions. Moreover, it also compares the models with two kinds of Markov Chains, including the switching variances and the switching coefficients of Taylor rule. The two steps of model comparisons can ultimately lead to a model offering the best data fitting over the sample period for each of the two countries. However, it is still necessary to discuss some limitations and possible directions for further research.

First, although it can enhance the data fitting of the small open DSGE model significantly due to an appropriate specification of monetary policy function and the introduced Markov chains, the model itself inevitably suffers from the misspecification problem to some extent. It needs to go back to Gali and Monacelli's framework to check the derivation process of the main equations in the model. For instance, it can incorporate the habit information into the model, as suggested by Justiniano and Preston (2010[48]). It can also consider the capital accumulation process with investment adjustment costs to relax the assumption that decisions made by firms today will not affect future profits (Christiano, Eichenbaum and Evans,2004[78]). After changing the main structure of the model, one can repeat the procedures of this thesis to evaluate the data fitting of the new model and identify whether there is an improvement compared to the current stage.

Second, the kinds of Markov chains in the current stage are very fundamental. In addition to the variance and the coefficients of policy reaction functions, some other parameters can also be allowed to change. For instance, a Markov-Switching Philip coefficient κ, reflecting the degree of price stickiness, and its combinations between the current two chains can be estimated separately to identify whether there is an improvement in terms of data fitting.

Next, the data sample can be extended to cover the period, including the regime of zero lower bounds. The restriction of the current sample period can avoid the possible misspecification problem in the monetary policy reaction function, but also departs from one of the most popular topics after the financial crisis. After obtaining the empirical results just before the nominal interest rate arrives at 0.5%, the prior information of the Bayesian approach is well prepared for the extended size of the sample. To solve and estimate the model with the possible binding on the nominal interest rate, it can borrow the methodology proposed by Iiboshi(2016[44]) or Holden (2017[43]).

Last but not least, the thesis focuses on identifying the monetary policy within the DSGE model, offering the best data fitting, which is different from looking for an optimal monetary policy. The design of the optimal monetary policy requires to minimise the welfare loss functions derived from the consumer utilities, which generally fixes some parameters at specific values for mathematical simplifications. For the practical purpose, the thesis evaluates all the parameters given the actual time series data and ignores its requirements for mathematical simplification, so it cannot apply the derived welfare loss function directly. Having said this, however, further research can fix a subset of the whole parameter space at specific values which fit the mathematical requirements and evaluate the monetary policy with the remaining estimated parameters in this thesis.

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Appendix

Structure of the DSGE Model

IS curve

$$[A.1]\quad \tilde{yy}_t = E_t \tilde{yy}_{t+1} - (\tau+\lambda)(\tilde{r}_t - E_t\pi_{t+1} - E_t z_{t+1}) + \alpha(\tau+\lambda)E_t\triangle\tilde{q}^*_{t+1} + \frac{\lambda}{\tau}E_t\triangle\tilde{yy}^*_{t+1}$$

where $\lambda = \alpha(1-\tau)(2-\alpha)$.

The change rate of world output:

$$[A.2]\qquad \triangle\tilde{yy}^*_t = \tilde{yy}^*_t - \tilde{yy}^*_{t-1}$$

Philips Curve

$$[A.3]\qquad \pi_t = \beta E_t\pi_{t+1} + \alpha\beta E_t\triangle\tilde{q}^*_{t+1} - \alpha\triangle\tilde{q}^*_t + \frac{\kappa}{\tau+\lambda}(\tilde{yy}_t - \tilde{yy}_{t,n})$$

where $\beta = e^{-\frac{r^A}{400}}$.

Potential output

$$[A.4]\qquad \tilde{yy}_{t,n} = \alpha\Psi\tilde{yy}^*_t = -\alpha\frac{(1-\tau)(2-\alpha)}{\tau}\tilde{yy}^*_t$$

CPI

$$[A.5]\qquad \pi_t = \pi^*_t + \triangle\tilde{e}_t + (1-\alpha)\triangle\tilde{q}^*_t$$

Monetary Policy

$$[A.6]\qquad \tilde{r}_t = \rho_R\tilde{r}_{t-1} + (1-\rho_R)[\phi_\pi\pi_t + \phi_y\tilde{yy}_t + \phi_{\triangle e}\triangle\tilde{e}_t] + \xi^R_t, \xi^R_t \sim NID(0,\sigma^2_R)$$

AR(1) of change rate of terms of trade

$$[A.7]\qquad \triangle\tilde{q}^*_t = \rho_q\triangle\tilde{q}^*_{t-1} + \xi^q_t, \xi^q_t \sim NID(0,\sigma^2_q)$$

AR(1) of change rate of technology

$$[A.8] \qquad z_t = \rho_z z_{t-1} + \xi_t^z, \xi_t^z \sim NID(0, \sigma_z^2)$$

AR(1) of world output

$$[A.9] \qquad \tilde{yy}_t^* = \rho_{y^*} \tilde{yy}_{t-1}^* + \xi_t^{y_t^*}, \xi_t^{y_t^*} \sim NID(0, \sigma_{y^*}^2)$$

AR(1) of world inflation

$$[A.10] \qquad \pi_t^* = \rho_{\pi^*} \pi_{t-1}^* + \xi_t^{\pi_t^*}, \xi_t^{\pi_t^*} \sim NID(0, \sigma_{\pi^*}^2)$$

Measurement Equations of the Observed Data

measurement equations of change of real output

$$[A.11] \qquad \triangle y_t^{obs} = \gamma^Q + \triangle y\tilde{y}_t$$

measurement equations of observable inflation

$$[A.12] \qquad \pi_t^{obs} = \pi^A + 4\pi_t$$

measurement equations of observable nominal interest rate

$$[A.14] \qquad r_t^{obs} = r^A + \pi^A + 4\tilde{r}_t$$

measurement equations of observable nominal exchange rate depreciation

$$[A.14] \qquad \triangle e_t^{obs} = -\triangle \tilde{e}_t$$

measurement equations of observable change rate of terms of trade

$$[A.15] \qquad \triangle q_t^{obs} = \triangle \tilde{q}_t^*$$