The role of the government support for education attainment and fertility decisions

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Abstract

In this thesis we examine the role of the government policies that target the education and fertility decisions, and their impact on the development of the economy.

In the **Chapter 1**, we provide the motivation for our research in form of literature review that examines the change in the education and fertility choices, and their influence on the social and economic outcomes.

In the **Chapter 2**, we analyse the influence of the flat subsidy rates for education and fertility in the deterministic environment with homogeneous and heterogeneous households. In our analysis, we utilise the OLG model of *de la* Croix and Doepke (2003), which we extend with the government sector. We consider that the government finances its budget with tax on consumption, labour income and capital income. Our results indicate that subsidy for education produces welfare improvement, but at a cost of a lower population size. For the case with subsidy for fertility, we find the opposite. In the case with initial heterogeneity in the human capital of the households, both subsidy policies result in the outcome with complete equality in the human capital among the households. Finally, the economy achieves the largest welfare improvement with subsidy for education financed with tax on capital income, while the largest welfare loss is recorded with the subsidy for fertility and tax on labour income.

In the **Chapter 3**, we continue the examination of the policies that we consider in the chapter 2. We, however, extend the model of *de la* Croix and Doepke (2003) further by introducing uncertainty in the human capital formation process. At the average and aggregate levels, majority of our results are in line with the conclusions of chapter 2. However, due to the stochastic nature, the economy does not reach the complete equality in the level of the human capital – instead, the subsidy for education decreases inequality in the distribution of human capital, while the subsidy for fertility increases this indicator. At the individual level, the considered subsidy for education has been found to be ineffective for improving the education of the households from lowest ability groups. Additionally, with the subsidy for fertility, the households from highest ability groups increase the choice for education and fertility simultaneously, which resolves their parental ‘quality-quantity’ trade-off.

Finally, in the **Chapter 4**, we examine the influence of the state-provides compulsory education and childcare, the progressive labour income taxation, and regressive subsidy rate system. We perform our analysis in the environment that is considered for the chapter 3. We find that compulsory education improves the human capital and fertility, and it decreases the inequality in the distribution of the human capital. However, it reduces the private education investment. The introduction of childcare also improves the fertility choices and diminishes private education investment; but the rest of the outcomes are the opposite to the case of compulsory education. Lastly, in comparison to the flat counterparts, the progressive labour income tax scheme and regressive subsidy rates individually result in a population which is more concentrated among the centre of the original distribution. The economy enjoys a relatively larger welfare but not necessary a relatively larger human capital level.

This thesis is submitted in partial fulfilment of the requirements for the degree Doctor of Philosophy in Economics at Lancaster University.

I declare that the thesis has been composed by myself and that the work has not be submitted for any other degree or professional qualification.

I confirm that appropriate credit has been given within this thesis where reference has been made to the work of others.

I confirm that the work submitted is my own.

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Contents

Introduction

In this thesis, we investigate the immediate and the long-run impacts of the government policies that are designed to improve the education investment and decisions for having children. As a foundation for our analysis, we utilise the model of overlapping generations of *de la* Croix and Doepke (2003) to formalise the behaviour of the household and production sector. In this model economy, the households endogenously choose the education for their children and the number of children to have, which determines the human capital, the size and structure of the population, the utility of the individual households and the welfare in the economy, and the overall level of output that the economy can produce.

Before we consider each of the scenarios that we address in chapter 2, 3 and 4, we provide the motivation for this research and the policies that target the choices for education and fertility through the literature overview presented by the chapter 1. In this chapter, we investigate the changes in the decisions for education and childbearing that have occurred in the developed economies. We also examine and summarise the conclusions for the role of education and fertility on the social and economic outcomes. Lastly, we selectively examine the influence of the policies that were designed to improve the choices associated with the education and fertility. In chapters 2, 3 and 4, however, we provide a more detailed literature overview for the considered government policies.

In the chapter 2, we begin our analysis for the government policies that are introduced to promote the education investment and fertility decisions. To conduct this analysis, we expand the model of *de la* Croix and Doepke (2003) with the government sector. We consider that the government provides the households with flat subsidy rates for education and fertility, and it taxes the consumption, labour income and capital income to finance its budget. Based on our results for the environment with the households that are homogeneous in the human capital, the subsidy for education produces an immediate increase in the education provided in the economy, but there is a decrease in the fertility choices of the households. Furthermore, the first generation that receives the subsidy for education experiences the decrease in the utility and the output produced per adult household. But, as the economy reaches the final steady-state with subsidy for education, the human capital, the welfare and output per adult household all become larger than at the original steady-state. Given the decrease in childbearing, the economy experiences a reduction in the population size. For our analysis with subsidy for fertility, we obtain the results that are completely the opposite to the case with subsidy for education, however.

In our analysis with the households that are heterogeneous in the human capital, our results indicate that the household decisions for education and fertility do not reach an immediate long-run equilibrium level, but rather they follow the relationship that exist between the individual and the average human capital. The rest of conclusions that we obtain for the heterogeneous case, however, are consistent with the results for the homogeneous environment. Additionally, we find that both subsidy for education and fertility that are introduced in the heterogeneous environment would result in the final steady-states with the households that have an identical human capital level. Overall, based on the results for chapter 2, the subsidy for education that is financed with tax on capital income produces the best welfare outcome, while the flat labour income tax rate that finances the subsidy for fertility results in the largest decrease in the welfare. Neither of these policies have been found to resolve the parental 'quality-quantity' trade-off for children on the individual basis. However, with a specific combination of the flat subsidy rates for education and fertility, the economy experiences simultaneous increase in the education investment and fertility.

In the chapter 3, we extend the model economy of *de la* Croix and Doepke (2003) further by introducing an uncertainty to the human capital formation process. We, however, analyse the influence of the same policy mix that we consider in the chapter 2. Overall, at the average and aggregate levels, we find that the results for stochastic environment are consistent with the conclusions for the deterministic case. However, in terms of composition of the population and the changes that occur at the individual ability level, we find the following. Unlike in our deterministic analysis, we find that neither subsidy for education nor subsidy for fertility leads to the outcome with the full equality in the distribution of the human capital across the population. Instead, with uncertainty in the human capital accumulation, the subsidy for education reduces the level of inequality in the distribution of the human capital compared to the original steady-state, whereas the subsidy for fertility creates the opposite. Furthermore, even though the subsidy for education produces an improvement in the individual utility levels across all ability groups when implemented alongside with a tax on consumption or capital income, it is found to be ineffective to stimulate the education provision for low ability households. In turn, the subsidy for fertility leads to welfare improvement of lower ability groups only, who experience the largest increase in fertility and optimally decide to drastically reduce the education provision. Finally, we obtain an unexpected result for the high ability households with the subsidy for fertility in place. We find that these households increase their fertility and education provision simultaneously, which resolves the parental ‘quality-quantity’ trade-off for them.

As a final part of our discussion for the influence of the government support for education and fertility, we analyse the role of the compulsory education and childcare in the chapter 4. Additionally, we re-address our results from the previous chapter by considering alternative policy instruments. First, we examine the case with progressive labour income taxation that finances the flat subsidy rate for education and fertility. We compare these results with our previous conclusions for the flat tax rate on labour income, and tax on consumption and capital income. And second, we analyse the impact of the regressive subsidy rate system which the government finances with flat tax rates on consumption, labour income and capital income; and we compare these outcomes with our previous results for the flat subsidy rate system. Based on our analysis for the model economy from the previous chapter, we find that a state-provided compulsory education increases the average level of human capital and fertility decisions among the households. We also observe, however, a decrease in the private education provision by the households; whereas, the response of the average utility level is determined by the tax option that finances the government budget. With a childcare, we find a considerable increase in fertility decisions, at a cost of lower human capital and welfare, however. Finally, in comparison to the flat counterparts, the progressive labour income tax scheme and regressive subsidy rates individually result in the population which is more concentrated among the centre of the original distribution. Consequently, the economy enjoys a relatively larger average utility but not necessary a relatively larger level of human capital.

Overall, this thesis contributes to the existing literature in the following ways. First, this research merges the studies that often analyse the impact of the policies that are designed to promote education and fertility separately, despite the fact that both of these choices originate in the household sector, and there is an evidence for a connection between these choices. Second, we expand on the literature that analyses the influence of policies that encourage education and fertility in the environment with uncertainty in the human capital formation. To the best of our knowledge, the influence of the subsidy for fertility has not been previously studied for the case with idiosyncratic shocks in the human capital. Finally, we propose that the subsidy for education and subsidy for fertility should be provided together to be able to resolve a parental ‘quality-quantity’ trade-off for children, and that the subsidy system for the education should be regressive in order to promote the education investment among the households from lowest ability groups when there is uncertainty in the human capital development.

Chapter 1

Education and fertility: evidence from the literature

1.1 Introduction

We investigate the evidence presented in the literature for the education attainment and fertility decisions in this chapter. We begin this overview of the literature with an examination of the changes that the developed economies have experience in these two indicators. Additionally, we present the results from the literature that examine the influence of the change in education attainment and childbearing decisions on the social and economic outcomes. Finally, we briefly examine and summarise the results from the studies which investigate the influence of the policies that were designed to improve the education attainment and the fertility choices.

1.2 Education

According to Checchi (2006), the education attainment increased significantly across the world after the Second World War, when it has risen quickly during 1960s and 1970s, but began to slow down during 1980s, and by 1990s majority of the world countries had a full enrolment into primary education. According to the author, the increase in education attainment from primary to secondary and from secondary to tertiary correlates with increasing labour market participation and higher lifetime earnings. According to Morgan and David (1963), the additional benefits of increase in education attainment come from the increase in flexibility of the labour force, the value of informed electorate, improvement in security and stability of labour income, and greater enjoyment of life and culture in general. Brenton (2013) suggest that education positively affects the national income both directly and indirectly. The direct impact of education is viewed in form of improvement of marginal productivity of educated labour force, whereas indirect effect is presented by spill over effects from higher educated individuals on lower educated ones which increases the marginal productivity of a latter group.

At the individual level, through examination of 1979-2010 waves of National Longitudinal Survey of Youth 1979, Braga (2018) reports that more educated workers have lower frequency of job displacement though their careers, and they receive more on-the-job training at the beginning of their career. However, the author reports that more educated individuals suffer larger penalties associated with wage losses due to unemployment spells. From examining the unique longitudinal data set for the first regional school in colonial Benin, Wantchekon, Klasnja and Novta (2015) find significant positive treatment effect of the education on the first generation of students. According to the study, these students enjoy higher living standards; they are less likely to be farmers, and are more likely to be politically active. Furthermore, authors find the evidence for the positive intergenerational effect of education, which in its level is similar to the first generation of education recipients.

Wantchekon, Klasnja and Novta (2015) suggest that parental education has large positive effect on the level of education, living standards and social networks of second generation of descendants. In line with Wantchekon, Klasnja and Novta (2015), Restuccia and Urrutia (2004) suggest that parental level of college education, however, positively influences incentives for investment in early education of children. Moreover, in their analysis for persistence of earnings through the overlapping generation framework of four generations, Restuccia and Urrutia (2004) conclude that investment into early education correspond approximately to one-half of intergenerational correlation in earnings between parental and offspring' generations.

Another evidence for the positive effect of education that takes place between generations is presented by Currie and Moretti (2003). By examining the Visual Statistics Natality data between 1970 and 1999, authors find that amount of maternal education positively influences the health habits of the mother, and it positively affects the infant health, which as mentioned in Gonti, Heckman and Urzua (2010), positively correlates with higher education attainments.

Further evidence of the influence of education on health is depicted by Lleras-Muney (2005). Through examination of the U.S. census data, and changes in the compulsory education laws and child labour laws that occurred from 1915 to 1930 across thirty states, the researcher indicates existence of the causal positive impact of education on health. In particular, according to Lleras-Muney (2005), an additional year of schooling reduces the probability of dying in the next ten years by at least 3.6 percentage points. According to the author, the better educated individuals are more likely to utilise and benefit from advances in medical technologies, they are more likely to think critically in terms of their own health formation, and they have access to the jobs that both generates higher incomes and deprives health by less.

According to Clark and Royer (2013), however, where the analysis utilises the changes in the compulsory schooling laws in Great Britain of 1942 and 1972 to study the presence of causality between education attainment and health outcomes, the authors report that the considered changes in compulsory schooling result in very limited health improvements at best. The authors still find, however, the evidence that increase in compulsory years of schooling causes increase in (further) education attainment and wages. The difference in the conclusions of these two studies can be reasoned with conclusion of Stephens and Yang (2014), where authors state that benefits of education are not global but rather region specific.

In the further consideration for the influence of education on another indicator for well-being of society, Fella and Gallipoli (2014) analyse the impact of education attainment on the level of crime. Based on their analysis which utilises a general equilibrium life-cycle model with agents that are heterogeneous in ability level, inclination for the crime and initial wealth; which also features incomplete markets, and endogenous choices for education and crime, the authors suggest that for the U.S. with the 70 percent of prison population which does not have a high school degree in 1997, the subsidy for education program would increase the high school graduation rates, which would reduce the number of crimes by half, and which would increase the level of welfare. The authors also compare the programme that stimulates education attainment with the policy that increases the length of sentence, and they conclude that only improvement in education attainment results in simultaneous welfare gains and fall in crime levels.

Moving forward, the link between the education and democracy has also been examined, however, according to Acemoglu et al (2005) there is no connection between the two. The study of Spilimbergo (2009), however, shows that foreign students who received their education in the democratic country tend to promote democracy in their domestic country. This result, which has not been previously established in the literature, is obtained from the analysis of a unique panel data set of foreign students starting from 1950s; and according to the author is robust to the choice of estimation techniques and to the definition of democracy.

Finally, even though the evidence depicted in the literature illustrates the real benefits of education on the socio-economic outcomes, in the environments with limited information, education is often viewed as a signalling device. Both, the original publication of Spence (1973), and extension by Swinkels (1999) present the evidence for signalling equilibrium, where the education attainment is utilised as a yardstick for the ability level of individuals.

1.3 Fertility

Opposite to the dynamics for education attainment, the level of fertility is found to be decreasing across OECD countries (Adresa, 2005). According to Bongaarts (1999), during the post-Second World War period of baby-boom, the maximum level of fertility for the developed economies reached 2.8 birth per women on average, with the maximum of 3.7 for North American region and minimum of 2.1 for Japan. After this period in the late 1950s, developed countries experienced a steep decline in the birth rates during 1960s and 1970s, and by 1995 the fertility choices have fallen below the replacement rate (of 2.1) to 1.7 births per women on average.

According to Adresa (2005), however, by the year of 2000, the dynamics for fertility choices in the economically developed regions has diverged. The author reports an increase in the child-bearing decisions in the case of the U.S. which brought the fertility to the population replacement level. For the case of France and Norway, fertility remains at the 1.8 birth per women on average, whereas for Greece, Italy and Spain this demographic indicator has fallen to the level of 1.3. According to Manuelli and Seshadri (2009), who perform the analysis with extended Barro-Becker life-cycle model which features endogenous accumulation of human and health capital, this cross-country difference in the fertility choices is attributed to differences in productivity and taxes.

Strangely enough, however, McDonald (2008) reports that the desired fertility level still remains at the average of 2.5, even when the actual fertility choices decline mainly. Therefore, this difference between the desired and actual fertility levels could suggest the presence socially inefficient outcomes.

To understand the nature of decline in the fertility choices, one could refer to McDonald (2008). According to the author, decrease in the level of fertility can be explained with two major changes. First, increase in social liberalism in form of gender equality raises the opportunity costs of the family formation, for females in particular. And second, the economic restructuring in form of increasing competition in the labour markets creates the additional incentives for risk avoidance among young individuals, who respond by postponement of the family formation. In line with McDonald (2008), Baudin, *de la* Croix and Gobbi (2015) also suggest that the rise in the opportunity costs of having children is responsible for decline in fertility. According to this study which features the framework with two-stage game between women and men, where the individuals choose on marriage, consumption and fertility, and the properties of the framework calibrated to the 1990 U.S. census data, 8.1 percent of women in the U.S. are found to be childless because of high opportunity costs. Additionally, authors indicate that poverty accounts for 2.5 percent of women who suspend the family formation.

The study of Adresa (2005) provides further perspective for the increase in the labour market participation of women and its influence on fertility. According to the author, a rapid increase of the labour force through the large entry of the female demographics into rigid labour market institutions resulted in a relatively high female unemployment rate. According to Adresa (2005), the period without the job creates large negative impact on the life-time income. In particular, unemployment is characterised by the period when the human capital accumulation is dropped, which significantly increases the risk of future unemployment, lowers wages and benefits. Overall, according to the publication, these factors contributed to the initial decline in fertility. Adresa (2005) also suggests that fertility levels remain low because childbearing period is associated with temporal job displacement, and therefore, with discontinuation of human capital accumulation. According to the author, women decide to postpone the birth of child until enough human capital is accumulated that would allow to minimise the negative influence of childbearing on lifetime income.

The different view on the dynamics for fertility choices is depicted by Soares (2005), who develops the model where economic growth is driven by changes in life expectancy. According to the author, with increase in adult longevity, the period over which individuals receive benefits from education is extended. Therefore, with increase in return for education and due to the quality-quantity trade-off that adults experience toward their children, individuals decide on larger education attainment for themselves and their children which leads to the equilibrium of “fewer but better-educated offspring”. Soares (2005) concludes further that with decrease in the child mortality, the benefit of having a large family reduces. Instead, with high child survival rate, it becomes optimal to invest more into the human capital of fewer children.

Leeson (2015), in turn, provides a broader perspective for decrease and postponement of fertility choices. According to the author, the observed decline in birth rates is attributed to change in partnership patterns in form of delay in cohabitation and marriage, change in the gender roles, presence of economic uncertainty, and increase in education and labour market participation of women.

Lutz, Skirbekk and Tesla (2006) additionally suggest that the problem of low liquidity trap could be in place. According to the study, the environment with low fertility rates increases the economic cost of having children substantially, which creates additional disincentives for individuals who consider becoming a parent. Finally, Greenwood, Seshadri and Vandenbroucke (2005) show in their analysis that reduction in economic costs of raising children due to “an atypical burst of technological progress” was responsible for the period of the baby-boom.

With the potential explanations behind the drop in fertility levels for the economically developed regions being considered, the influence of this (demographic) change should be viewed next. According to Lee et al (2014) and Leeson (2015), a decrease in fertility rates below the replacement level combined with increase in the average lifespan leads to rapid aging of the population. Furthermore, for the economically developed regions with relatively low migration, the population decline is also likely, which damages the future labour markets and overall competitiveness of the economy on the global scale (Leeson, 2015). According to Becker (2013), the decrease in the working age population leads to the smaller tax base which, according to Lee et al (2014), negatively affects public finances and standards of living. Becker (2013) indicates that financing of the social security benefits, income of retired individuals and expenditure on the medical care becomes problematic with population that does not replace itself. The author suggests further that the decline in fertility rates negatively affects the rate of innovations, which according to the text “mainly comes from younger individuals”. In conclusion, this potential for innovation, according to Becker (2013), can address the issues raised by Thomas Robert Malthus, who indicates that population growth negatively affects income, world's food supply, standards of living and state of the environment.

1.4 Public policies for education and fertility

In the final part of this literature overview, we selectively examine the evidence for the impact of government policies for education and fertility and its effects on the social and economic outcomes.

Based on the analysis of Jason, Johnson and Persico (2016), where the authors study the impact of the school finance reform that took place in the U.S. in 1970s and 1980s on the long-run outcomes of adults who was born between 1955 and 1985, the authors conclude that 10 percent increase in public spending per pupil, which manifests in "reduction of student-to-teacher ratios, increase in teachers' salaries and longer school years", increases education attainment by 0.31 years. Furthermore, according to this study, it leads to 7 percent higher wages, and to reduction of "annual incidence of adult poverty" by 3.2 percentage points.

Next, based on the research of Todd and Wolpin (2006), the PROGRESSA school subsidy program, which was initiated by the Mexican government, increases education attainment at school by a half of a year. According to the paper, 506 rural villages – 7 out of 31 states in Mexico – participated in the social experiment where the Mexican government provided parents with financial incentives which have been conditional on their children school attendance. The objective of this program was to improve on the human capital accumulation and to reduce a poverty. This analysis of Todd and Wolpin (2006) is conducted with the use of the schooling-fertility theoretical framework that authors develop, and with the data that has been collected by the Mexican government. Despite their conclusion that PROGRESSA subsidy program improves education attainment, the authors suggest that the policy design could be improved. Based on their simulation results, provision of larger subsidies but solely to the households with children "at higher grade levels" would improve the average duration of schooling by 10 percent.

The last paper that is considered in this section for the influence of the government support for education is the study of Duflo, Dupas and Kremer (2015). In this study, the authors analyse the influence of the government subsidy program which reduces the cost of upper primary education, the HIV prevention education, and combination of the two on the school dropout rate, early pregnancy, marriage and contraction of the sexually transmitted infection (STI). This study began in 2003 and has been conducted for seven years in partnership with Kenyan government with the participation of 328 schools of Western Province of the country. Duflo, Dupas and Kremer (2015) find that when only the subsidy for education has been implemented, the school dropout rates for both gender groups reduced. Furthermore, the authors find the evidence for postponement of first pregnancy in marriage. The solo provision of the education subsidy, however, did not cause a reduction in the rate of STI. For the students who participated in the HIV prevention education only, however, the authors do not find the statistically significant evidence for decrease in teenage pregnancies or the risk of STI. Furthermore, the paper suggests an absence of improvement in education attainment. Finally, the authors conclude that with simultaneous provision of subsidy for education and HIV education curriculum, the risk of STI among girls after seven years falls by the greater amount then with subsidy for education being provided alone, but the pregnancy rate decreased by less.

As a concluding point, we consider the influence of government policies on fertility rates, which we begin with the article of Whittingon, Alm and Peters (1990), that according to Crump, Goda and Mumford (2011) is the first publication to “seriously estimate the responsiveness of fertility to child tax benefit changes”. Through the examination of time series data for the U.S. between 1913 and 1984, Whittingon, Alm and Peters (1990) conclude that there is a robust and statistically significant positive effect of child tax benefits in form of ‘personal exemption for dependants’ on fertility rates. In replicating the results of Whittingon, Alm and Peters (1990), Crump, Goda and Mumford (2011) find that $100 increase in the real tax value of personal exemption leads to a 2.1 increase in the fertility rates, which is in line with original publication. Then, Crump, Goda and Mumford (2011) extend the dataset utilised by Whittingon, Alm and Peters (1990) to include 21 additional years of data. The authors note, however, that the share of a personal tax exemption in the total child subsidy has reduced to about 1/3 to a half of the total child subsidy in the U.S. as the time has progressed. As a result, the authors find that a personal tax exemption produces a weaker results in terms of fertility choices, which, furthermore, becomes less robust when more general measures of tax benefits are considered. Crump, Goda and Mumford (2011) still find, however, the short-run positive influence of child tax benefits on fertility.

We consider the study of Domeij and Klein (2013) next, which closes our discussion for government policies toward fertility, where with the use of life-cycle framework the authors analyse the influence of a subsidised childcare for the case of Germany. Domeij and Klein (2013) state that in comparison to the rest of OECD countries, Germany has fewer mothers with young children who are employed. Furthermore, the availability of subsidized day care is limited. According to the authors, the 50 percent subsidy rate of day care maximises the welfare level, and it leads to a 71 percent increase in labour supply of married mothers and 125 percent increase in labour supply of single mothers with young children. Finally, according to the publication, the welfare improvement that results from this policy is equivalent to 0.4 percent increase in consumption, which can be financed with “just 0.1 percentage point increase in the labour income tax schedule”.

1.5 Conclusion

In this chapter, we investigated the role of education and fertility on the social and economic outcomes through the overview of the literature. We also examined the changes that have been taken place in these indicators. Finally, we selectively examined and summarised the results of the previous studies that investigate the role of government policies that aim to improve the education investment and the family planning.

Overall, the objective of this literature overview has been twofold. First, our aim was to illustrate the general importance of decisions for education and childbearing for the social and economic outcomes. And second, through this literature review, we motivate our own research where we examine the influence of various government policy instruments that target education and fertility on the long-run evolution of the economy.

Chapter 2

The role of subsidy for education and fertility in the deterministic overlapping generation economy

2.1 Introduction

We begin our discussion for the influence of the government support for the education attainment and childbearing decisions in this chapter. As a foundation for our analysis, we utilise the model of overlapping generations of *de la* Croix and Doepke (2003). This model economy features the household sector where the choices for education investment and number of children to have are endogenous. These choices define the human capital in the economy, composition of the population and its size, which, overall, determines the capacity of the economy for output and the welfare level.

As we saw from the evidence presented in the chapter 1, both education and fertility are important for the social and economic development. The previous research has concluded that improvement in education attainment produces a positive impact on the labour market participation, lifetime earnings, utility, living standards and national income. Furthermore, it has been indicated that an improvement in education produces a positive intergenerational effect, which results in larger education investment of parents for their children, and, additionally, it could positively influence the health of the generations. For the case of fertility, the literature suggested that public financing, standards of living and potential for innovation all depend on the childbearing decisions especially in the regions with low migration. As we further have indicated in the chapter 1, the potential relevance for the policies that may address the private fertility choices has been expressed by the findings which suggest that the actual level of fertility (in the developed economies) is below the desired level fertility, which could indicate the presence of the socially inefficient outcomes in family planning.

A further motivation for the research that analyses the influence of the government support for education and fertility is presented by the upcoming subsection where we examine the evidence from the literature for these policies. Based on the results from the empirical literature, the government subsidy program for education improves the school enrolment in the developing economies, and it leads to an improvement in a nutritional intake of students. Based on the evidence from the literature that have utilised the intergenerational models, the subsidy for education improves efficiency of education investment, reduces distortions from labour income taxation, improves the distribution of skills across the population and results in an overall welfare improvement. For the subsidy for fertility, the evidence from the empirical literature has suggested the presence of positive response in incentive for having children, which has been found to be heterogeneous in the family size and education level of mothers. The findings from the research that has utilised theoretical modelling have indicated, however, that subsidy for fertility diminishes the human capital, labour market participation, wages, education time per child and per capita growth rate of output.

In spite of a rich evidence in the literature that already exists for the influence of the government programs that subsidise the education and fertility, we believe that the current research contributes in the following ways. First, since the decisions for education provision and fertility originate at the household level and they are believed to be interconnected, both of our analyses investigate the influence of these policies on education attainment and family planning simultaneously. Based on the literature review, however, we find that when the influence of the subsidy for education has been studied previously, the childbearing decisions and their influence on the population formation have not been taken into consideration often. Additionally, based on the evidence presented in the literature, the previous studies (especially in the theoretical field) often focus on the role of subsidy for tertiary education. In our analysis, we investigate the role of the subsidies for the total education that is undertaken by younger generations. Moreover, in this research we investigate both immediate and the long-run impact of the considered policies, while, we believe, that the existing literature primary focuses on the final results without an extensive consideration for the intermediate transitions that occur as a result of the policies for education and fertility. Lastly, based on the existing literature, we believe that the present research offers a larger variety for the considered tax options; and given that the roles of the two subsidy programs are analysed in a single framework, it is relatively easier to compare between the outcomes of these policy options.

In order to conduct our analysis, we expand the model economy of *de la* Croix and Doepke (2003) with the government sector. In our analysis we consider that the government provides the households with subsidy for education and fertility at the exogenously chosen rate[1]. To

[1] In our analysis, we consider that the government provides the subsidy rate of 10%. The behaviour of the economy that we observe with subsidy rate of 10% remains consistent with the changes that take place at a larger subsidy rates

finance its program and to keep the budget balanced, the government levies tax on consumption, labour income and capital income.

In our discussion, first, we consider the case of the households with an identical level of the human capital. Then, we consider the case for the heterogeneous environment, where the population of the households consists of two ability groups. In each of our policy experiments, we analyse the impact of a particular tax option separately from the remaining tax instruments. Additionally, for the majority of our discussion, we investigate the influence of subsidy for education separately from the subsidy for fertility. This is done to obtain a better understanding for the role of a particular government policy instrument that we consider on the evolution of the economy in the long-run.

2.1.1 Literature review

Before we address our results, we examine the influence of the subsidy programs for education and fertility presented in the literature[2]. We begin this overview with an empirical literature that examines the role of subsidy for education.

According to Wang (2018), where the author investigates the influence of education subsidy policy in China for rural households using the difference-in-difference approach for the data drawn from China Health and Nutrition Survey for the period between 2000 and 2006, the subsidy for education increases the school enrolment and nutritional intake of children from poor families. Wang (2018) also notes that the increase in school enrolment is the most prevalent among male students. According to the scholar, in this subsidy program, the central government provided children from poor rural families with textbooks, and the local government provided the poor households with living subsidies. According to Gaddah, Munro and Quartey (2016), however, where the authors examine the influence of subsidy for basic schools on school enrolment in rural Ghana using nested multinomial logit model and data drawn from fifth round of Ghana Living Standard Survey, the direct reduction in cost for public schooling produces only a marginal change in the probability of enrolment of students in rural regions. Unlike in the case of the reform analysed by Wang (2018), according to Gaddah, Munro and

[2] With a perfect foresight for the human capital development

Quartey (2016), the government in Ghana has provided schools with \$3.00 grants for every enrolled student. At a later stage of this subsidy program, the government also provided students with free school uniform and free exercise books. Despite only a marginal improvement in school enrolment, the authors advocate the use of the subsidy program for achieving a goal of universal primary education, which, according to the scholars, would improve the future income levels of the households. Gaddah, Munro and Quartey (2016) note further that in order to promote the education attainment of children from lower income groups, the basic and not tertiary education must be subsidised. Furthermore, the authors indicate that reducing the distance between the schools is crucial for improving the school enrolment in the rural areas. Finally, Gaddah, Munro and Quartey (2016) stress that even when the government subsidises the education in rural poor regions, the quality of education still remains to be an important issue that must be addressed.

Moving from the empirical research to the literature that analyses the influence of subsidy for education from theoretical perspective, we consider the study of Cremer, Gahvari and Pestieau (2011) first. According to the authors, who construct an overlapping generation model with endogenous education and fertility decisions for which the households – from low- and high-ability types – experience a trade-off, the decision for education of both ability types must always be subsidised by the government since the education creates positive externalities. Furthermore, Cremer, Gahvari and Pestieau (2011) indicate that in a first-best environment, the level of subsidy for education should be equal to the level of externality created by the education attainment. Lastly, the authors state that since the cost for education contributes to the total cost of raising children, the subsidy for education helps to reduce the total cost associated with family planning. Next, according to Mochida (2005), where the author develops an overlapping generations model with endogenous growth, uncertain lifetime and pay-as-you-go-pension system, in the environment with aging population where younger working generation faces an increasing fiscal pressure which distorts the parental decisions, the subsidy for education leads to a more efficient education investment of parents for their children. As a result, according to the author, the economy would experience a higher growth rate due to the subsidy for education. According to Armellini and Basu (2010), where the authors construct an overlapping generations model with endogenous growth and parental altruism, the parental altruism is a key determinant for the public education subsidy program of the government in the environment where credit markets are not available to finance the education decisions. According to the researchers, the government would provide a larger level of subsidy for education investment when parents have a low altruism level for their children which manifests in the low level of education investment. Oppositely, in the environment where the parents have a high level of altruism for their children which manifests in sufficiently high level of education investment and human capital development – which extends to a high level of economic growth – the government would provide a lower subsidy level for education investment. Overall, the authors confirm their theoretical predictions for the influence of altruism on the government subsidy level for education based on the cross-country regressions using the World Value Survey for altruism. According to Krueger and Ludwig (2016), where the authors develop quantitative life cycle model and calibrate the model parameters to the U.S. data, subsidy for tertiary education helps to reduce the distortions from the progressive labour income taxation. Additionally, over the long-run, the subsidy for higher education reduces the college wage premium which improves the distribution of skills in the economy. Finally, the authors find that the subsidy for tertiary education leads to a long-run welfare improvement.

Moving forward to a study of Schneider (2010), where author develops an intergenerational framework of human capital investment and analyses the effect of subsidy for education on social mobility and inequality, Schneider (2010) concludes, however, that often there is a trade-off between these two policy targets when the government subsidises education. According to a scholar, in most cases only population of high-skilled households would benefit from subsidy program for education, while the population of low-skilled households would remain indifferent for education investment. In a special case with a large initial population share of high-skilled households, however, Schneider (2010) concludes that subsidy for education leads to a simultaneous improvement of social mobility and decrease in the level of inequality in the economy. Therefore, the author concludes that subsidy for education would improve intra- and intergenerational equity simultaneously only when it is implemented in the developed countries or in the countries with high initial level of university education. Next, in line with Krueger and

Ludwig (2016), Bovenberg and Jacobs (2005) conclude that the subsidy for education helps to reduce the distortion introduced by income taxation on human capital investment, and it helps "to restore efficiency in education choices". In line with Schneider (2010), however, Bovenberg and Jacobs (2005) indicate that the higher income households benefit from subsidy for education more than the rest of the population. Despite this conclusion, however, the authors suggest that when the costs for education influence the work effort, and when the government desires to help lower income households, the government should subside the education more heavily. According to Rey and Lopez-Garcia (2016), where the authors develop an overlapping generation model with endogenous growth, the subsidy for education produces an ambiguous effect on the ratio of physical capital to the human capital which affects the return on investment in physical capital, human capital and overall welfare level of the economy. According to the authors, the subsidy for education increases the education investments which positively affects the human capital level, but it diminishes the ratio of physical capital to human capital. At the same time, the subsidy for education increases the physical capital level though intergenerational transfers, which increases the ratio of physical capital to human capital. Overall, Rey and Lopez-Garcia (2016) conclude that the subsidy for education would improve the welfare and lead to economic growth if the subsidy for education generates a physical capital accumulation.

Next, we examine the evidence presented by Shindo (2010). In this study, the author develops an overlapping generation model with two regions and calibrates the model parameters to reflect the properties of Jiangsu and Liaoning regions in China. In this model economy, each region has closed goods and labour market, and "foreign direct investments and capital from other countries are allowed to flow into the market of each region". Shindo (2010) finds that the subsidy for education improves the human capital development which accelerates the economic growth in both regions. The author concludes, however, that the subsidy for education is not able to converge the rate of economic development of two regions when there is a difference in initial productivity levels, and the growth gap would widen even when the rate of subsidy for education is the same. According to Viaene and Zilcha (2013), where the authors develop an overlapping generations model for a small open economy with heterogeneous agents, the public subsidy for education improves the accumulation of human capital, which, additionally, increases the domestic marginal returns to physical capital and, therefore, it leads to a foreign inflow of physical capital. Consequently, the rate of economic growth improves as a result of subsidy for education. However, Viaene and Zilcha (2013) state the subsidy for education reduces efficiency in the economy and it raises the costs significantly. Based on the argument for economic efficiency, the authors conclude that the government has a dominant policy without the public financing for education investment.

Continuing our discussion for the influence of subsidy for education, we consider the evidence presented by Chaudhuri, Ghost and Banerjee (2018). In this study the authors develop a two-sector specific factor general equilibrium framework for the case of a small open economy with individuals who are heterogeneous in the skill level. According to the authors who perform the analysis for two periods, the subsidy for education, however, is not able to completely eliminate the income inequality between the agents with different skill levels. Furthermore, the authors find that in some cases subsidy for education increases the income inequality. Chaudhuri, Ghost and Banerjee (2018) find that in the first period when subsidy for education is provided, the supply of unskilled labour decreases due to increase in the education investment of the individuals of this skill group, while in the second period of analysis, the supply of high-skilled labour increases. The scholars conclude that if the high-skill sector of production is more capital intensive relative to low-skill sector of production, the subsidy for education increases the income inequality in the first period of analysis, but, in the second period, the income inequality would decrease if a relative capital intensity remains unchanged. Oppositely, the authors indicate that when low-skill sector is relatively more capital intensive, the subsidy for education would reduce the income inequality in the first period, but it would worsen the income inequality in the second period. Chaudhuri, Ghost and Banerjee (2018) conclude by advising the use of subsidy for physical capital accumulation in high-skill production sector together with subsidy for education to the policy makers in the developing economic regions that have a goal of reducing the income inequality between individuals with heterogeneous level of skills. Next, according to García-Peñalosa and Wälde (2000), the policies that involve provision of subsidy for education always suffer from a trade-off between efficiency and equity. According to the authors, when the policy maker provides the subsidy for education at a level that is required to reach the efficient level of human capital, only the individuals from higher income groups would benefit from this policy. The reasoning is twofold. First, due to an introduction of the tax that finances the subsidy program for education and due to an observation that children from higher-income households are more likely to obtain tertiary education, there is a reverse redistribution of the resources from poor to rich individuals. And second, as a result of subsidy level that leads to efficient human capital level, the individual with higher education receive a larger income than the rest of the population which increases the level of life-time income inequality further. On the other hand, García-Peñalosa and Wälde (2000) conclude that if the policy maker provides the level of subsidy that leads to an equitable level of human capital, people would obtain an identical level of higher education which would eliminate the income inequality, but the efficiency in the economy would reduce drastically. The authors state that an overeducation would result in the level of output that is lower than in the equilibrium where some workers remain unskilled. Lastly, according to Burer and Fethke (2016), who develop a framework of public university budgeting where "the legislation is viewed as a leader that can credibly commit to the subsidy structure, and the university is viewed as follower that makes tuition decisions based on student demands and the subsidies provided by the legislature" and calibrate the model parameters to decentralised budget data for University of Iowa and for University of Michigan, a removal of restrictions that prevent non-resident students to receive a subsidy for their higher education improves welfare and overall efficiency. However, similar to García-Peñalosa and Wälde (2000), Burer and Fethke (2016) find that a universally accessible subsidy for education decreases the equity between resident and non-resident students. Overall, Burer and Fethke (2016) conclude that the access to a subsidy scheme that does not depend on the residential status of the students would benefit students, the university and taxpayers.

We conclude this portion of literature overview for the role of the subsidy for education by considering the study of Dur, Teulings and van Rens (2004). In this paper, similar to García-Peñalosa and Wälde (2000), the authors indicate that higher ability students form higher income households benefit the most from the subsidy for education which diminishes the redistributive values of this policy instrument. According Dur, Teulings and van Rens (2004), this problem can be remedied by providing the subsidy for education conditional on parental income.

Overall, the authors find that if the policy authority provides subsidy for education conditional on the income of the parents, the economy would experience a reduction in pre-tax income inequality. Furthermore, the scholars conclude that this subsidy program would result in the most efficient income redistribution, and in combination with income taxation, it would minimise a disincentive on the work effort caused by the tax.

As a concluding part of the literature overview, we examine the evidence for the role of policies that subsidise fertility decisions[3] on the economic development. Similarly to our discussion of subsidy for education, we begin with the results presented in the empirical literature, and then we consider the findings from theoretical research.

According to Milligan (2005), where the author investigates the influence of the Allowance for Newborn Children prenatal transfer policy – that has been available to all residents of Quebec – by employing a difference-in-difference estimation for public-use Canadian Census data, this fertility policy produced a strong, positive and robust effect on decision for having children. Milligan (2005) finds that as a result of this policy, the childbearing decisions in Quebec have increased by 12 percent on average. Furthermore, this policy has produced heterogeneous responses for family planning. According to the scholar, the largest response of 24.7 percent of increase in fertility has been recorded for the families with two or more children, while the smallest response of 9.8 percent has been found for the families without children. Next, according to Erlandsson (2017), where the author examines the role of child home care allowance using the Finish Census Panel data for the ten percent of the population between 1992 and 2007 and by employing a discrete-time event-history analysis, this policy increases the risk of second and third birth. The author notes, however, that the risk of a second child is larger than of a third child. Furthermore, the fertility responses are heterogeneous and they depend on the education level of women. Erlandsson (2017) finds that women with higher education level experience a higher risk of having second and third child as a result of this fertility policy. The scholar notes, however, that the policies that are designed to promote childbearing are often criticised "for strengthening traditional gender roles", reducing the labour market participation, widening gender wage gaps and negatively influencing the future retirement pensions of mothers.

[3] Which is commonly referred as child allowance in the literature

A further criticism for the policies that are designed to reduce a cost of a family planning has been expressed by Fanti and Gori (2011). According to this study, where authors develop a small open economy with overlapping generations, endogenous fertility and public education that forms the human capital, the public provision of child allowances that is financed with a wage tax rate reduces the human capital and the time that individuals spend in the labour market. Furthermore, according to the authors, this prenatal policy does not improve fertility levels in the long-run. In a subsequent research of Fanti and Gori (2012a), where authors consider a closed economy with overlapping generations and endogenous fertility, the scholars conclude that child allowance leads to a reduction of saving which diminishes physical capital accumulation and wages. The authors also state that under a special case when fixed per child allowance is larger than the consumption needed to take care for children, the economy would reach a long-run steady-state of a poverty trap.

In line with a criticism expressed in the literature for a publicly provided child allowance, Yasuoka and Goto (2015) find that it may be optimal to provide a negative child allowance[4] in order to maximise the welfare in the economy. Furthermore, based on the analysis performed for a closed economy with overlapping generations, endogenous fertility choices and pay-as-you-go pension system, the authors conclude that the fertility responses depend on the tax that government uses to finance its budget. For instance, when government taxes consumption to finance the provision of child allowance, the fertility level would always increase, while with the tax on income, the fertility responses are ambiguous and depend on a parametrisation of the model. Despite difference in fertility responses, Yasuoka and Goto (2015) conclude that the economy can always reach the social optimal level of welfare independently of tax option in place. Next, according to Blumkin, Margalioth and Sadka (2015), where authors develop an optimal taxation framework with the assumption of a trade-off between earning abilities and family size, the way in which child allowance is redistributed determines whether it is optimal to tax or to subsidise the fertility decisions. Blumkin, Margalioth and Sadka (2015) conclude that when the child benefits are means tested, it is "unambiguously optimal to tax children on redistributive grounds". However, when child benefits are universally provided, the authors

[4] I.e. to tax fertility

find that it is optimal to subsidise the childbearing.

Moving forward to a discussion presented by Pressman (2011), however, where the author compares between the policies that utilise child allowance and tax exemption, the child allowances have been found to reduce the child poverty, while the tax exemptions have been found to be ineffective for the households from lower income groups. A further evidence for a positive influence of subsidy for fertility is presented by van Groezen, Leers and Meijdam (2003). In this study the authors construct a small open economy with endogenous fertility and pay-as-you-go pension system. According to the authors' results, the introduction of subsidy for fertility helps to overcome an outcome of pay-as-you-go pension system that manifests in small number of children born. Furthermore, van Groezen, Leers and Meijdam (2003) suggest that child allowance helps the society to overcome the negative externalities associated with a larger fertility level, and it further leads to overall pareto welfare improvement.

Next, based on the results of Mochda (2009), where the author develops an overlapping generations model with endogenous growth, uncertain lifetime and pay-as-you-go pension system, the introduction of subsidy program for fertility is found to increase the fertility level in the economy. However, the author also reports a decrease in the time that households contribute to the labour market, a reduction in the education time per child, and a fall in the per capita growth rate of the output due to provision of public child allowance. According to Yasuoka and Miyake (2013), where authors develop an overlapping generations two-period model with endogenous fertility and pension system, the child allowance cannot be used to decrease the level of public debt per unit of GDP. According to the authors, the policy that reduces the costs of fertility decisions improves the fertility level in the economy which increases the income tax revenue. At the same time, however, the child allowance that is financed by debt prevents the capital accumulation which decreases the GDP and, consequently, increases the public debt per unit of GDP. This second effect dominates the increase in the tax revenue, which produces a net effect where the public debt per unit of GDP continues to increase.

As a final point for a discussion of the evidence presented in the literature for the policies that subsidise fertility decisions, we examine how these policies are influences by the properties of the labour markets and of the households. According to Fanti and Gori (2012b), where the authors develop a small open model economy with overlapping generations and labour market unions, the level of competition in the labour market determines whether a policy of child allowance is fertility improving. The authors state that only when the labour market is competitive, the child allowance would improve the fertility level in the economy. Oppositely, when the labour markets are unionised, the child allowance policy has not been found to influence the childbearing decisions. Finally, according to Momota (2000), where the author develops an overlapping generations model with endogenous fertility, the ability of a subsidy for fertility to improve childrearing depends on a fraction of a time that males spend on taking care of their children. According to the author, the subsidy for fertility would increases the number of children born to a larger extend when males spend a significant portion of their time on raising children.

2.1.2 Summary for the results

Based on our results for the environment with the households that are homogeneous in the human capital, the subsidy for education leads to an immediate increase in education provision and decrease in fertility decisions. At the first period of this policy, however, independently of the tax option that the government implements, the economy experiences the decrease in welfare and the output per adult household. However, as the economy reaches the final steady-state with subsidy for education, the human capital, welfare and output per adult household all reach the level that is larger than at the original steady-state. The population size of the economy, however, continues its decrease.

With the case of subsidy for fertility in the environment with the homogeneous households, we observe that at the first period of the government presence the fertility decisions of adult households increase above the original steady-state, but the level of education investment becomes lower than before the government has entered the economy. Both, the welfare and the output per adult household become larger than at the initial equilibrium. However, as the economy reaches the corresponding final steady-states with subsidy for fertility, the human capital, the welfare and the output per adult household become lower than at the original steady-state without the government presence. But, as a result of this program, we observe a continuous increase in the size of the population.

In our final policy experiment for the case of the households with homogeneous human capital, we consider a scenario when the government provides subsidy for education and fertility simultaneously. According to our results[5], when the government sets the subsidy rate for education that is 1.6 times larger than subsidy rate for fertility, the government is able to resolve the trade-off that the households have between education provision and fertility level; and is able to create a simultaneous increase in the education investment, fertility decisions, utility of the households, output per adult household and population size.

In our discussion for the case when the model population consists of two ability groups, we observe that both subsidy for education and subsidy for fertility create the final steady-states where the households have an identical (homogeneous) level of human capital. With the subsidy for education this human capital is larger than at the original steady-state, but with subsidy for fertility this human capital is smaller than at the original equilibrium.

For the majority of indicators in the case with heterogeneous households we observe similar responses to the homogeneous case. However, unlike in the case with homogeneous households, both education and fertility choices of the households do not reach their corresponding final equilibrium levels immediately after an introduction of the government policy. Instead, they follow the relationship that now exists between the human capital level of the households from a particular ability group and the average human capital in the economy.

Finally, independently of the subsidy policy in place, both for the case with homogeneous and heterogeneous households, our results indicate that the tax on the labour income leads to the education investment that is lower than under tax on consumption or capital income, which, through the presence of parental trade-off between quality and quantity for children leads to a relatively higher fertility level when the government finances and balances its budget with tax on labour income. Our results indicate, however, that both the welfare and the output per adult household at the final steady-state with tax on labour income become smaller than when government utilises tax on consumption of capital income. Lastly, based on our results, the economy would experience the best outcome in terms of welfare when the government provides

[5] And based on the current calibration of the model parameters

subsidy for education and levies the tax on the capital income; while the welfare would reduce the most when the government provides subsidy for fertility and implements the labour income taxation.

The remaining part of this chapter has the following structure. In the section 2.2, we formally state the problem of the households, producer and the government. In the section 2.3, we present the calibration for the model parameters and the solution for the original steady-state. In the section 2.4 we perform our analysis for the influence of the publicly financed subsidy for education and fertility in the environment with homogeneous households; and in the section 2.5 we consider the influence of these policies in the environment with heterogeneous households. In the section 2.6 we conclude.

2.2 Theoretical framework

We consider the model economy of overlapping generations developed by *de la* Croix and Doepke (2003). Based on the original assumptions, the household sector consists of individuals who live for three generations. The production sector is presented by a problem of a representative producer. We extend the model economy with the government sector. We assume that the government provides the subsidies for education and fertility to the households, and it taxes consumption, labour income and capital income to keep its budget balanced. Below, we present each of the three sectors, and we begin with the problem of the households.

2.2.1 Household sector

We begin the discussion for the model economy with the household sector. Based on the original assumptions of *de la* Croix and Doepke (2003), the households live for three periods, and they are heterogeneous in the human capital. In the first period of existence, a given member of a household is considered to be a child. This household member receives the education level e_t^i which is optimally chosen by the parents, and which contributes to a human capital formation that is depicted by (2.1). The superscript i refers to an ability group of the households that is defined by the human capital level h_t^i.

$$h_{t+1}^i = \frac{1}{(1+\rho)} B(\theta + e_t^i)^\eta (h_t^i)^\pi (\bar{h}_t)^\kappa \tag{2.1}$$

According to (2.1), the human capital of a child h_{t+1}^i adjusted to the long-run growth rate of the human capital at the steady-state ρ from the household ability group i with parents that have the human capital level h_t^i, increases with the level of education that a child receives, with the human capital of parents, and with the quality of education which is approximated by the average level of human capital $\bar{h}_t$ in the economy at time t when human capital accumulation takes place. Following *de la* Croix and Doepke (2003), B is the efficiency of human capital accumulation, θ is the instrument parameter that ensures non-zero level of human capital for children who optimally receive zero level of education, η is the parameter that shows a relative significance of education for human capital formation, π is the parameter that shows a relative significance of human capital of parents for human capital of children, and κ is the parameter that shows a relative significance of quality of education for human capital attainment.

In the second period of existence, a given member of the household is considered to be an adult. The adult households are defined by the level of human capital h_t^i that they have acquired during childhood. In this model economy, the adult households are only household members that make the economically relevant decisions. The adult households maximise their utility function (2.2) subject to the budget constraint for adulthood which is given by (2.3), subject to the budget constraint for the old age which is given by (2.4), and subject to the human capital formation process for children which we have considered above and which is given by (2.1). Additionally, for our analysis we consider that adult households cannot provide negative education for their children (i.e. $e_t^i \geq 0$), and that $(1 - \phi n_t^i) \geq 0$.

$$\max_{c_t^i, s_t^i, e_t^i, n_t^i} u_t^i = \ln c_t^i + \beta \ln d_{t+1}^i + \gamma \ln n_t^i h_{t+1}^i \tag{2.2}$$

$$(1+\tau_t^c)c_t^i + s_t^i + e_t^i n_t^i w_t \bar{h}_t = (1-\tau_t^l) w_t h_t^i (1-\phi n_t^i) + n_t^i w_t \bar{h}_t (e_t^i sub_t^e + \bar{e}_t sub_t^n) \tag{2.3}$$

$$(1+\tau_{t+1}^c) d_{t+1}^i = \frac{1}{(1+\rho)} \big(1 + r_{t+1}(1-\tau_{t+1}^k)\big) s_t^i \tag{2.4}$$

According to (2.2), the utility of the adult household u_t^i with human capital level h_t^i increases with consumption during adulthood c_t^i, consumption at the old age d_{t+1}^i, and the quality h_{t+1}^i and quantity n_t^i of children. The adult household maximise its utility function by optimally choosing the level of consumption for adulthood c_t^i, saving s_t^i that according to function (2.4) would be one of the determinant for level of consumption during final stage of existence, level of education e_t^i to provide for the children, and number of children to have n_t^i. Following *de la* Croix and Doepke (2003), β is the discount factor of adult households and γ is altruism factor of adult and elderly households toward children.

According to the left hand side of (2.3), the total expenditure of adult household with the human capital h_t^i consists of three components. The first component is the total expenditure of adult household on consumption; the second component is the total saving of adult household; and the final component is the total expenditure of adult household on education investment for its children which is given as the product of the level of education e_t^i that the adult household is optimally choosing for its children, the number of children n_t^i that the household member optimally chooses to have, the real wage rate in the economy w_t that adult household pays to teachers, and the average level of human capital $\bar{h}_t$ which approximates the human capital of teachers (or quality of education). With inclusion of the government sector, we assume that the government can tax consumption at a rate τ_t^c to balance its budget and to be able to provide the households with subsidy for education and fertility.

According to the right hand side of (2.3), the total income of adult household with the human capital level h_t^i consists of two components. The first component, which is given as the product of the real wage rate w_t, the human capital of adult household h_t^i and the free time that adult household have from raising children $(1-\phi n_t^i)$ net of the labour income tax rate τ_t^l, is the labour income of adult household with the human capital h_t^i . The second component on the right hand side of (2.3) is the total subsidy that adult household with human capital level h_t^i receives from the government. This total amount of subsidy is given as a sum between the subsidy for education – which is a product of the number of children that adult household decides to have, real wage rate, the average level of human capital, the level of education chosen by parents for children and the subsidy rate for education sub_t^e; and subsidy for fertility – which is the product of the number of children that adult household decide to have, the real wage rate, the average level of human capital, the average level of education $\bar{e}_t$ in the economy at time t and the subsidy rate for fertility sub_t^n. Following *de la* Croix and Doepke (2003), ϕ is the time-cost parameter for adult household to raise children.

Lastly, according to (2.4), the consumption of the elderly household with the human capital h_t^i adjusted to the long-run growth rate of the human capital at the steady-state positively depends on the real return to saving. According to (2.4), the real return on saving increases with the saving made during adulthood and the real interest rate in the economy r_{t+1}; but it decreases with the tax rate on consumption τ^c and tax rate on capital income τ^k levied by the government to finance the government budget.

In order to obtain the analytical solution for the problem of the adult household from ability group i with the human capital h_t^i, we construct the Lagrangian function (2.5) and derive the first order conditions presented by (2.6) through (2.9) below.

$$\begin{aligned}\mathscr{L}_t^i &= \ln c_t^i + \beta \ln \left[\frac{1}{(1+\rho)} \frac{(1+r_{t+1}(1-\tau_{t+1}^k))}{(1+\tau_{t+1}^c)} \right] + \beta \ln s_t^i + \gamma \ln n_t^i + \gamma \ln B + \ln \left(\frac{1}{(1+\rho)} \right) \\ &+ \gamma\eta \ln(\theta + e_t^i) + \gamma\pi \ln h_t^i + \gamma\kappa \ln \bar{h}_t + \lambda_t^i \Big[(1-\tau_t^l) w_t h_t^i (1-\phi n_t^i) + n_t^i w_t \bar{h}_t (e_t^i sub_t^e + \bar{e}_t sub_t^n) \\ &- (1+\tau_t^c) c_t^i - s_t^i - e_t^i n_t^i w_t \bar{h}_t \Big] \end{aligned} \tag{2.5}$$

where λ_t^i is Lagrange multiplier.

$$\frac{\partial \mathscr{L}_t^i}{\partial c_t^i} = \frac{1}{c_t^i} - \lambda_t^i (1+\tau_t^c) = 0 \tag{2.6}$$

$$\frac{\partial \mathscr{L}_t^i}{\partial s_t^i} = \beta \frac{1}{s_t^i} - \lambda_t^i = 0 \tag{2.7}$$

$$\frac{\partial \mathscr{L}_t^i}{\partial e_t^i} = \gamma\eta \frac{1}{(\theta + e_t^i)} + \lambda_t^i n_t^i w_t \bar{h}_t sub_t^e - \lambda_t^i n_t^i w_t \bar{h}_t = 0 \tag{2.8}$$

$$\frac{\partial \mathscr{L}_t^i}{\partial n_t^i} = \gamma \frac{1}{n_t^i} - \lambda_t^i (1-\tau_t^l) w_t h_t^i \phi + \lambda_t^i w_t \bar{h}_t (e_t sub_t^e + \bar{e}_t sub_t^n) - \lambda_t^i e_t^i w_t \bar{h}_t = 0 \tag{2.9}$$

Based on the results from (2.6) and (2.7), saving of adult household from ability group i

with human capital h_t^i at time t can be expressed in terms of its own consumption given the discount factor β and tax rate on consumption τ_t^c:

$$s_t^i = \beta(1+\tau_t^c)c_t^i \tag{2.10}$$

Next, using (2.10) and the budget constraint of the elderly households (2.4), we obtain an expression (2.11) that shows relationship between consumption during adulthood and at the old age for the household from ability group i with the human capital h_t^i.

$$(1+\tau_{t+1}^c)d_{t+1}^i = \frac{1}{(1+\rho)}\beta(1+\tau_t^c)\big(1+r_{t+1}(1-\tau_{t+1}^k)\big)c_t^i \tag{2.11}$$

Based on the first order conditions (2.6) and (2.9), we obtain the first function that show the existence of a trade-off for adult household from ability group i with human capital h_t^i between number of children, consumption during adulthood and education provision for children. This is expressed by (2.12) below.

$$\gamma(1+\tau_t^c)c_t^i = \phi(1-\tau_t^l)n_t^i w_t h_t^i + n_t^i w_t \bar{h}_t e_t^i(1-sub_t^e) - n_t^i w_t \bar{h}_t \bar{e}_t sub_t^n \tag{2.12}$$

We derive the second trade-off function using the first order conditions (2.6) and (2.8). This expression is summarised by (2.13) below.

$$\gamma\eta(1+\tau_t^c)c_t^i = (\theta+e_t^i)n_t^i w_t \bar{h}_t(1-sub_t^e) \tag{2.13}$$

Using the results summarised in the trade-off functions (2.12) and (2.13), we derive the expression (2.14) according to which the adult households from ability group i with the human capital h_t^i make the optimal decision for education provision for their children at time t.

$$e_t^i = \frac{\eta\phi(1-\tau_t^l)x_t^i - \theta(1-sub_t^e) - \eta\bar{e}_t sub_t^n}{(1-sub_t^e)(1-\eta)} \tag{2.14}$$

where $x_t^i = h_t^i/\bar{h}_t$ is the relative level of human capital for a given household of type i.

According to the results summarised by (2.14), the parental decision for education investment for their children increases with η, which suggests that the adult households would provide their children with a larger education when education contributes more for the human capital development which would positively influence the utility of the adult households. Additionally, due to the presence of the altruism, parents with a larger level of the relative human capital x_t^i and larger fraction of the time that they need to spend for raising children ϕ would provide their children with a greater level of education investment. Furthermore, since the subsidy rate for education sub_t^e decreases the parental cost of education investment, the adult households would increase the education provision when the government provides subsidy at a larger rate. Oppositely, according to (2.14) when the government spends a larger share of GDP on the education – which would increase the value of θ, the adult households would optimally reduce their choice for the education investment for their children. Additionally, when the average level of private education provision in the economy increases, the adult households from a given ability group i would diminish optimal choice for education. Moreover, since the tax on the labour income collected by the government decreases the total income that adult households can distribute between consumption, saving and total expenditure on children, the increase in the labour income tax rate τ_t^l reduces the level of education that parents can provide for their children. Lastly, due to the presence of the trade-off that adult households experience between the 'quality and quantity' of children[6], the parents would provide their children with a lower level of education when the government provides the households with a larger subsidy rate for fertility sub_t^n.

Next, using (2.2), (2.10) and (2.12), we obtain the expression (2.15) which defines the level of consumption of adult households from ability group i with human capital level h_t^i at time t.

$$c_t^i = \frac{(1-\tau_t^l)}{(1+\tau_t^c)} \frac{1}{(1+\beta+\gamma)} w_t h_t^i \tag{2.15}$$

According to (2.15), the consumption of adult households from ability group i at time t increases with the real wage rate w_t and the level of the human capital of these households h_t^i. Our results indicate, however, that consumption of adult households decreases with the tax rate on the labour income τ_t^l, tax rate on consumption τ_t^c, discount factor β and altruism factor of

[6]for more details please refer to the equation (2.18)

adult and elderly households towards children γ.

Moving forward, using (2.10) and (2.15), we derive the expression for saving of adult households from ability group i at time t.

$$s_t^i = (1-\tau_t^l)\frac{\beta}{(1+\beta+\gamma)}w_t h_t^i \tag{2.16}$$

According to (2.16), saving of the households from ability group i at time t increases with the real wage rate w_t and the level of the human capital of adult households h_t^i from this ability group. The results suggest, however, that saving of adult households would diminish with the tax rate on the labour income τ_t^l and with the altruism factor of adult and elderly households towards children γ.

Next, we derive expression for the consumption of the elderly households from ability group i by combining the budget constraint function (2.4) with the results summarised by (2.17).

$$d_{t+1}^i = \frac{1}{(1+\rho)}\frac{(1-\tau_t^l)}{(1+\tau_{t+1}^c)}\frac{\beta}{(1+\beta+\gamma)}\big(1+r_{t+1}(1-\tau_{t+1}^k)\big)w_t h_t^i \tag{2.17}$$

According to (2.17), the consumption of the elderly households from ability group i adjusted to the long-run growth rate of the human capital at the steady-state increases with the real wage rate that adult households have received during adulthood, the level of human capital that elderly households have had during the adulthood, and with the real interest rate. Oppositely, the consumption of the elderly households decreases with the tax rate on labour income that elderly households have faced during adulthood, and with the tax rates on consumption and capital income that elderly households encounter.

Finally, we use (2.12) and (2.15) to derive the expression that determines the optimal number of children that adult households from ability group i with human capital level h_t^i at time t decide to have.

$$n_t^i = \frac{\gamma}{(1+\beta+\gamma)}(1-\tau_t^l)x_t^i\Big[(1-\tau_t^l)\phi x_t^i + e_t^i(1-sub_t^e) - \bar{e}_t sub_t^n\Big]^{-1} \tag{2.18}$$

According to (2.18), the adult households from the ability group i with human capital h_t^i and time t would optimally increase the childbearing if there is an increase in the subsidy rate for education, subsidy rate for fertility, and the average level of education, holding everything else constant. On the other hand, an increase in the time-cost parameter for raising children ϕ diminishes the fertility choices of adult households. Furthermore, according to (2.18), the adult households experience a trade-off between the level of education that they provide for their children and the number of children that they decide to have.

2.2.2 Production sector

Following the original assumptions of *de la* Croix and Doepke (2003), the production sector of the model economy is presented by a single representative firm which maximises its profit function (2.19) subject to the production technology (2.20) by optimally choosing its factor inputs – physical capital stock and effective labour force, both of which are given in ‘per adult household’ level.

$$\max_{K_t, L_t} \Pi_t = Y_t - w_t L_t - (r_t + \delta) K_t \tag{2.19}$$

$$Y_t = A K_t^{\alpha} L_t^{1-\alpha} \tag{2.20}$$

where A is a productivity level, K_t is the physical capital stock per adult household, L_t is the effective labour force per adult household, Y_t is output per adult household, w_t is the real wage rate, r_t is the real interest rate, α is the share of capital income and δ is the depreciation rate of the physical capital.

In order to obtain the analytical solution for the problem of a producer, we maximise (2.19) subject to (2.20) and obtain the first order conditions (2.21) and (2.22).

$$\frac{\partial \Pi_t}{\partial K_t} = \alpha A {K_t}^{\alpha-1} L_t^{1-\alpha} - (r_t + \delta) = 0 \tag{2.21}$$

$$\frac{\partial \Pi_t}{\partial L_t} = (1-\alpha) A {K_t}^{\alpha} L_t^{-\alpha} - w_t = 0 \tag{2.22}$$

Based on these first-order conditions, we derive the demand functions of a producer for the physical capital stock and effective labour force that are given by (2.23) and (2.24), respectively.

$$r_t = \alpha A {K_t}^{\alpha-1} L_t^{1-\alpha} - \delta \tag{2.23}$$

$$w_t = (1-\alpha) A K_t^{\alpha} L_t^{-\alpha} \tag{2.24}$$

According to (2.23) and (2.24) both of the factor inputs are paid their corresponding marginal products.

Finally, we follow *de la* Croix and Doepke (2003) in order to formulate the supply functions for these two factor inputs that are both given in per adult household level by (2.25) and (2.26) below.

$$K_t = \frac{1}{(1+\rho)} \left(\frac{\sum_{i=1}^{I} p_{t-1}^i s_{t-1}^i}{\sum_{i=1}^{I} p_t^i} + (1-\delta) K_{t-1} \right) \tag{2.25}$$

$$L_t = \frac{\sum_{i=1}^{I} p_t^i \left[h_t^i (1-\phi n_t^i) - n_t^i e_t^i \bar{h}_t \right]}{\sum_{i=1}^{I} p_t^i} \tag{2.26}$$

where $p_t^i = p_{t-1}^i n_{t-1}^i$ is the present population size of adult households from ability group i with human capital h_t^i at time t.

According to (2.25) the level of physical capital stock per adult household adjusted to the long-run growth rate of the human capital at the steady-state increases with the past period of the physical capital net of the depreciation, and with the aggregate level of saving made in the previous period of the analysis. The population size of adult households at the present period, however, diminishes the ‘per adult household’ level of the physical capital stock.

Lastly, according to (2.26), the effective labour force per adult household increases with the aggregate level of human capital in the economy and with the aggregate time that adult households can contribute to the labour market net of the time that household must spend for raising children. The effective labour force per adult, however, decreases with the number of teachers[7] required for provision of education, which is given as a product of the total number

[7] In this model economy, teachers are the adult households from all ability groups that are not employed in the production sector, but they are required for the provision of the optimal education chosen by all adult households. Adult households that perform the role of teachers receive the real wage rate w_t from the adult households that are employed in the production sector. Based on (2.26) teachers are not the part of the effective labour force, and if there is an increase in the number of teachers required, the effective labour force would decrease. Oppositely, if there a decrease in the number of teachers required, the effective labour force would increase. Identically to the adult households that are the part of the effective labour force, the adult households that are teachers maximise

of children born, total level of education provision in the economy and the quality of education. Therefore, according to (2.26), if parents decide to increase the education provision for their children, the effective labour force per adult household would decrease because there would be an increase in the number of teachers required, holding everything else constant. In turn, if the households increase their fertility decisions, the effective labour force would decrease because households would have less time for labour market participation and because there would be an increase in the number of teachers required, holding everything else constant.

2.2.3 Government sector

We introduce the government sector into the model of *de la* Croix and Doepke (2003). We consider that the government can provide the households with subsidy for education and subsidy for fertility. However, the government can tax consumption, labour income or capital income to finance and keep its budget balanced. We depict the government budget constraint at time t by (2.27) below, where the left hand side of (2.27) depicts the total tax income of the government and the right hand side of (2.27) depicts the total expenditure of the government.

$$\sum_{i=1}^{I} \tau_t^c p_t^i c_t^i + \sum_{i=1}^{I} \tau_t^c p_{t-1}^i d_t^i + \sum_{i=1}^{I} r_t \tau_t^k p_t^i s_t^i + \sum_{i=1}^{I} \tau_t^l w_t p_t^i h_t^i (1 - \phi n_t^i) = \sum_{i=1}^{I} p_t^i n_t^i w_t \bar{h}_t (e_t^i sub_t^e + \bar{e}_t sub_t^n) \tag{2.27}$$

For the forthcoming policy experiments we consider that the subsidy rates for education and fertility are chosen exogenously. Given these choices for the subsidy rates, the optimal solutions of the households and the firm to their corresponding problems, the government optimally chooses the tax rates in order to keep the budget constraint balanced[8].

2.2.4 Special case: representative agent economy

In the upcoming numerical analysis where we investigate the influence of subsidy for education and subsidy for fertility provided by the government with the tax revenue, we consider a couple of cases, one of which is when the households who are homogeneous in the human capital. In

their utility function by optimally choosing the consumption, saving, number of children to have and level of education for their children.

[8]Please note that the households, the firm and the government make their choices simultaneously

this case of a representative agent economy, there is only one ability group of households in the economy which implies that $i = I = 1$. Furthermore, the individual and the average level of human capital would be identical to one another, i.e. $h_t^i = \bar{h}_t$, which implies that the relative level of human capital in the economy would be equal to one, i.e. $x_t^i = 1$. Finally, the individual level of education attainment would be equal to the average level of education in the economy at time t, i.e. $e_t^i = \bar{e}_t$. Therefore, the analytical solution for the model economy with homogeneous households are presented by expressions (2.28) through (2.38).

From (2.1), human capital of the children at time t is

$$h_{t+1} = \frac{1}{(1+\rho)} B\big(\theta + e_t\big)^{\eta} (h_t)^{(\pi+\kappa)} \tag{2.28}$$

From (2.14), education provision in the economy at time t is

$$e_t = \frac{\eta\phi(1-\tau_t^l) - \theta(1-sub_t^e) - \eta e_t sub_t^n}{(1-sub_t^e)(1-\eta)} \tag{2.29}$$

From (2.15), consumption of adults at time t is

$$c_t = \frac{(1-\tau_t^l)}{(1+\tau_t^c)} \frac{1}{(1+\beta+\gamma)} w_t h_t \tag{2.30}$$

From (2.16), decision of adults for saving at time t is

$$s_t = (1-\tau_t^l) \frac{\beta}{(1+\beta+\gamma)} w_t h_t \tag{2.31}$$

From (2.17), decision for the old-age consumption by adults at time t is

$$d_{t+1} = \frac{1}{(1+\rho)} \frac{(1-\tau_t^l)}{(1+\tau_{t+1}^c)} \frac{\beta}{(1+\beta+\gamma)} \big(1 + r_{t+1}(1-\tau_{t+1}^k)\big) w_t h_t \tag{2.32}$$

From (2.18), decision of adults for the number of children at time t is

$$n_t = \frac{\gamma}{(1+\beta+\gamma)} (1-\tau_t^i) \Big[(1-\tau_t^l)\phi + e_t(1-sub_t^e) - e_t sub_t^n\Big]^{-1} \tag{2.33}$$

From (2.23), demand for physical capital at time t is

$$r_t = \alpha A K_t^{\alpha-1} L_t^{1-\alpha} - \delta \tag{2.34}$$

From (2.24), demand for effective labour force at time t is

$$w_t = (1-\alpha) A K_t^{\alpha} L_t^{-\alpha} \tag{2.35}$$

From (2.25), physical capital stock per adult household at time t is

$$K_t = \frac{1}{(1+\rho)}\left(\frac{p_{t-1}s_{t-1}}{p_t} + (1-\delta)K_{t-1}\right) \tag{2.36}$$

From (2.26) effective labour force per adult household at time t is

$$L_t = \frac{p_t h_t (1-\phi n_t) - p_t n_t e_t h_t}{p_t} \tag{2.37}$$

From (2.27), the budget constraint of the policy maker at time t is

$$\tau_t^c p_t c_t + \tau_t^c p_{t-1} d_t + r_t \tau_t^k p_t s_t + \tau_t^l w_t p_t h_t (1-\phi n_t) = p_t e_t n_t w_t h_t (sub_t^e + sub_t^n) \tag{2.38}$$

2.2.5 Special case: steady-state of representative agent economy

Finally, for the purpose of our computational experiments, we derive the analytical solution for the household sector when the model economy with homogeneous households is at the steady-state. This is given by the expressions (2.39) through (2.44) below. We assume that the economy reaches the steady-state when the human capital between the generations of the households is identical.

From (2.28), the human capital at the steady state is

$$h = \left(\frac{1}{(1+\rho)} B(\theta+e)^{\eta}\right)^{\frac{1}{1-\pi-\kappa}} \tag{2.39}$$

From (2.29), provision of education at the steady state is

$$e = \frac{\eta\phi(1-\tau^l) - \theta(1-sub^e) - \eta esub^n}{(1-sub^e)(1-\eta)} \tag{2.40}$$

From (2.30), consumption of adults at the steady state is

$$c = \frac{(1-\tau^l)}{(1+\tau^c)}\frac{1}{(1+\beta+\gamma)}wh \tag{2.41}$$

From (2.31), decision for saving at the steady state is

$$s = (1-\tau^l)\frac{\beta}{(1+\beta+\gamma)}wh \tag{2.42}$$

From (2.32), consumption of the old age households at the steady state is

$$d = \frac{1}{(1+\rho)}\frac{(1-\tau^l)}{(1+\tau^c)}\frac{\beta}{(1+\beta+\gamma)}\big(1+r(1-\tau^k)\big)wh \tag{2.43}$$

From (2.33), decision for number of children at the steady state is

$$n = \frac{\gamma}{(1+\beta+\gamma)}(1-\tau^l)\left[(1-\tau^l)\phi + e(1-sub^e) - esub^n\right]^{-1} \tag{2.44}$$

2.3 Calibration

To perform the numerical exercises and analyse the influence of the publicly financed subsidy program for education and fertility, we closely follow the original calibration for the model parameters of *de la* Croix and Doepke (2003). Based on this original parametrisation, one period of analysis is equal to one generation which is set to be equal to thirty years. For the purposes of our computational experiments, however, we select the parameters for the productivity level A, the efficiency of the human capital accumulation B and the altruism factor of adult and elderly households towards children γ in order to normalise the real wage rate, the individual and the average level of human capital, and the individual and the average fertility level in the economy at the initial steady-state without the government presence to be equal to 1. Overall, the table

2.1 below reports the calibration and the interpretation for the structural parameters utilised in the model economy[9].

parameter	value	interpretation
A	2.9826	productivity level
B	7.2107	efficiency of human capital accumulation
α	1/3	elasticity of physical capital
β	0.99^{120}	psychological discount factor
γ	0.18766	altruist factor
δ	1	depreciation rate of physical capital stock
η	0.5	relative significance of education for human capital
θ	0.0119	instrument parameter for non-zero human capital of children when education provision is zero
π	0.2	relative significance of human capital of parents for human capital of children
κ	0.1	relative significance of quality of education for human capital
ρ	$1.02^{30}-1$	growth rate of human capital over generations
ϕ	0.075	time-cost parameter to raise children

Table 2.1: Calibration for the structural parameters of the model economy

Given these structural parameters, we obtain the original steady-state for case of representative households without government presence[10]. These results are summarised by table 2.2 below.

variable	value	interpretation
e	0.0512	education attainment/provision
h	1	human capital
n	1	fertility level
p	1	population of adult households
K	0.1111	physical capital stock per adult household
L	0.8737	effective labour force per adult household
r	2.9308	real interest rate
w	1	real wage rate
c	0.6725	consumption of adults households
d	0.4369	consumption of elderly households
s	0.2013	saving
y	1.3106	real output per adult household
u	-0.6448	utility of the households/welfare per adult household

Table 2.2: Original steady-state before government intervention

[9] Please refer to the next chapter for a detailed explanation for the parametrisation procedure

[10] I.e. when the households do not receive the subsidy for education and/or subsidy for fertility, and when the households do not pay taxes on consumption, labour income and/or capital income

2.4 Policy experiments: representative agent economy

We begin our analysis for the influence of the publicly financed subsidy programs for education and fertility in the environment with homogeneous households. In order to be able to carefully analyse the influence of the government intervention and to be able to distinguish between the various government policy instruments considered in this chapter, we consider each subsidy and each tax option separately. Overall, this accounts to six policy experiments[11] that we perform for the representative agent economy. We also consider the case when subsidy for education and fertility are provided simultaneously – which allows to resolve a parental quality-quantity trade-off for children. Overall, according to the results, the subsidies for education that are financed by the tax on capital income are the most welfare improving option for the considered model economy. On the other hand, we find that the subsidies for fertility that are financed with the labour income taxation are the most welfare depriving. We begin our discussion with the case of subsidy for education, at the second period of analysis, when government first enters the economy and it provides subsidy for education and levies taxes to balance its budget[12].

2.4.1 Subsidy for education

We begin our analysis for the environment with homogeneous households with the case of the publicly provided subsidy for education investment. The government has three tax options to finance and balance its budget, which are the tax on consumption, labour income and capital income. As we have mentioned above, we analyse each of the tax option with the subsidy for education separately to be able to distinguish the influence of a particular government policy instrument on the long-run economic development. According to our results, at the second period of analysis when the government enters the economy and provides the households with 10% exogenously chosen subsidy rate for education, the government sets the tax rate on consumption to be equal to 0.53%. When the government uses the tax on labour income instead,

[11](1) Subsidy for education financed by tax on consumption, (2) Subsidy for education financed by tax on labour income, (3) Subsidy for education financed by tax on capital income, (4) Subsidy for fertility financed by tax on consumption, (5) Subsidy for fertility financed by tax on labour income, (6) Subsidy for fertility financed by tax on capital income

[12]Please note that the first period of the analysis is the original steady-state where the human capital between the generations is identical and where government is absent

the tax rate is equal to 0.63%. Lastly, when government utilises the tax on the capital income, the tax rate is equal to 1% at the first period of the government presence.

Based on the analytical solution, only the labour income tax directly diminishes the incentives of the households for education provision. This result takes place because the tax on labour income decreases the total income that adult household distribute between consumption, saving and the total expenditure on children, which, therefore, negatively affects all the choices that adult households make during their utility maximisation. In the case of tax on consumption, however, this tax option reduces the level of consumption of adult and elderly households, but the other choices remain unchanged because the tax on consumption does not reduce the income of the households (directly). With the tax on capital income, only the level of consumption of elderly households is affected, while the choices of adult households for consumption, saving, number of children to have and level of education investment are not influenced (directly) since, again, the tax on capital income does not negatively impact the income of adult households, holding everything else constant.

Therefore, according to our results, when the government provides subsidy for education of 10% and uses either tax on consumption or capital income, the households provide their children with 16.28% larger level of education than at the initial steady-state. However, when the government implements the labour income taxation, the level of education increases by 15.25% at the second period of analysis. Since the human capital of children depends positively on the amount of education received, in the first period of government intervention the human capital of children and, therefore, of next period adult households increases by 6.40% when government taxes consumption or capital income. With the labour income tax, the human capital of children increases by 6.01% above the original steady-state.

Due to the presence of a trade-off that adult households experience between number of children to have and level of education to provide for the children, we observe that at the second period of analysis the childbearing decisions of adult households decrease by 1.85% when government provides subsidy for education at a rate of 10% and it uses either tax on consumption or capital income to finance and balance its budget. With the labour income tax in place instead, the fertility level in the economy decreases 1.12% below the original steady-

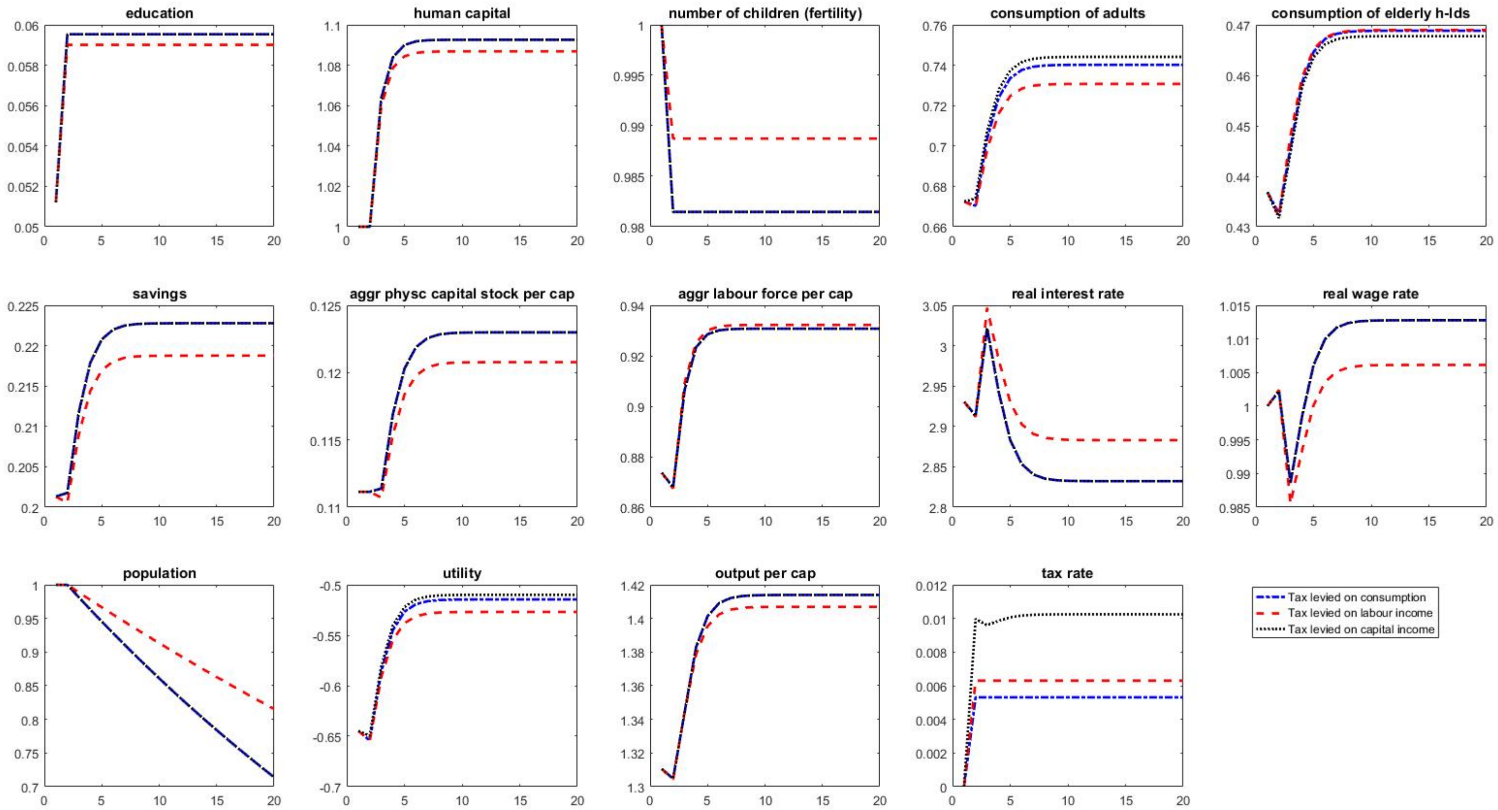

Figure 2.1: Transition path of representative agent economy with subsidy for education.
Note: transition path with tax on consumption is depicted by blue dash-dotted line; transition path with tax on labour income is presented by red dashed line; transition path with capital income tax is shown by black dotted line

state. Due to a relatively smaller increase in education with the tax on labour income, the fertility decisions of adult households decrease at a relatively smaller rate with this tax option in use.

At the first period of the government presence when it subsidises the education attainment, the human capital of the adults is unchanged since it has been formed at the original steady-state. Similarly to the human capital, population size of the adult households is identical to the initial steady-state since it has been formed before the government policy was in place. The combination of absence of change in the level of human capital and the population size of adult households, together with a decrease in the choice for children would increase the size of the effective labour force per adult household, holding everything else constant, since, a decrease in the choice for children results in increase of time that adults can contribute to the labour market participation. Furthermore, holding everything else constant, a decrease in the number of children born would decrease the number of teacher required for education provision, which would further increase the size of the effective labour force. However, since the level of education provision in the economy increases, there is an increase in the number of teachers required, which reduces the supply of effective labour force. Overall, at the second period of analysis, we observe a decrease in effective labour force per adult household by 0.67% when tax on consumption or capital income is in use. With the labour income tax in place instead, the effective labour force per adult household decreases by 0.72% below the original steady-state. Since the supply of effective labour is now reduced and the level of physical capital stock has been determined and is identical to the initial steady-state, the real wage rate increases by 0.22% when government taxes consumption or capital income. With labour income tax in place, at the first period of government presence, the real wage rate rises by 0.24% above the initial equilibrium. In turn, according to our results, the real interest rate at the second period of analysis with subsidy rate for education of 10% and with either tax on consumption or capital income decreases by 0.60%. When government implements the tax on labour income instead, the real interest rate reduces by 0.65%.

Therefore, as a result of increase in the real wage rate, unchanged level of the human capital of adult households and the resulted tax rates at the second period of analysis, we observe, however, 0.31% and 0.39% decrease in the consumption of adult households when government finances its budget with tax on consumption and labour income, respectively. When the government finances and balances its budget with tax on capital income, however, the consumption of adult households becomes 0.22% larger than at the initial steady-state. Next, in the case of tax on labour income and capital income, the household' decisions to save follow the dynamics of consumption of adult households. For the case with tax on consumption, however, adult households increase their saving by 0.22% above the original steady-state. Lastly, given the decrease in the real interest rate, the consumption of the elderly households decreases by 0.97%, 1.11% and 1.18% below the original steady-state when the government taxes consumption, labour income and capital income, respectively.

Due all these changes, our results indicate that the utility of adult households at the second period of analysis decreases by 1.47%, 1.45% and 0.75% when government uses tax on consumption, labour income, or capital income, respectively, to finance the provision of the subsidy rate for education of 10%. Finally, for the first period of government intervention, our results indicate that level of output per adult household reduces by 0.45% with tax on consumption or capital income in place; and it falls by 0.48% with labour income tax in use.

Moving forward to the second period of the government presence, the government adjusts the tax rate on capital income to 0.96% to be able to provide the subsidy rate for education at a rate of 10% and to keep its budget balanced. When the government uses the tax on consumption or labour income, however, the tax rate remains to be identical to the previous period.

Given the past period increase in the level of education investment, we observe that at the third period of analysis the level of human capital of current period adults is 6.40% larger than at the original steady-state when government uses tax on consumption or capital income. With the labour income tax that has led to a relatively lower education attainment at the previous period, the human capital of adult households increases 6.01% above the original steady-state. Due to the past period decrease in fertility, however, the population of current adult households decreases by 1.86% when the government levies the tax on consumption or capital income. With the labour income tax, which, however, resulted in a relatively smaller decrease in fertility level of the past period adult households, the current population of adult households at the second period of the government presence becomes 1.13% lower than at the original steady-state.

In comparison to the previous generation, however, when the government has entered the economy, the level of education provision and the fertility choices of the adult households have not changed, and they will remain at the established levels as long as the government provides the subsidy for education, and as long as in the case with the labour income taxation the tax rate remains unchanged. Furthermore, since here we consider the case of the homogeneous households where the level of individual human capital of an ability group is equal to the average level of human capital in the economy, the level of relative human capital is equal to one and the changes in the human capital of adult households do not change the households' decisions for education and fertility.

Next, given the improvement in the human capital and decrease in the population size of adult households, combined with absence of changes in the choices for education and fertility, at the third period of analysis when the government provides subsidy for education of 10%, and finances and balances its budget with tax on consumption or capital income, the effective labour force per adult household increases by 4.43% from previous period and becomes 3.73% larger than at original steady-state. With the tax on labour income, the effective labour force per adult household increases by 4.82% and becomes 4.06% above initial steady-state.

With a past period increase in saving combined with the decrease in the present period population size of adult households, our results indicate that at the second period of the government presence with tax on consumption or capital income, the physical capital stock per adult household increases by 0.22% above the original steady-state. With the labour income taxation, which decreased saving of the previous generation, the physical capital stock per adult household becomes 0.39% lower than the original equilibrium.

Given these changes in the factor inputs at the third period of analysis, when the government provides subsidy for education of 10% and finances and balances its budget with tax on consumption or capital income, the real wage rate becomes 1.14% lower than at the original steady-state. With the labour income tax in place, this factor price falls 1.45% below the initial equilibrium. For the real interest rate, when the government taxes consumption or capital income, the indicator increases by 3.73% from the previous generation and becomes 3.11% above the original steady-state. When the government implements the labour income taxation instead, the real interest rate increases 4.64% from the previous period and becomes 3.96% larger than at the original equilibrium.

As a net effect of increase in the human capital and decrease in the real wage rate, when the government taxes consumption and provides subsidy for education at 10%, the consumption of adult households increases by 4.95% from the previous period and it becomes 4.63% larger than at the original steady-state. When the government uses tax on the labour income, the consumption of adult households increases by 4.22% and it becomes 3.82% larger than at the initial steady-state. Lastly, with the tax on capital income, the consumption of the adult households increases by 4.95% from the previous period, and it is 5.19% larger than at the first period of analysis without the government presence.

According to our results the dynamics of saving follows the one for consumption when the government implements with tax on labour income and capital income. With the tax on consumption, at the third period of analysis, saving of adult households increase by 4.95% from the previous generation, and it becomes 5.19% larger than at the original steady-state.

Finally, at the second period of the government presence, the consumption of the elderly households increases by 3.01%, 3.70% and 3.03% from the previous period, which becomes 2.01%, 2.55% and 1.81% larger than at the original steady-state when the government provides subsidy for education of 10% and it finances and balances its budget with tax on consumption, labour income and capital income, respectively.

Due to the observed changes at the third period of analysis with publicly provided subsidy for education of 10%, our results indicate that the level of utility of adult households increases by 10.52% and becomes 9.20% larger than at the original steady-state when the government taxes consumption. With the tax on the labour income, the utility of adult households increases by 9.66% from the previous generation and becomes 8.35% larger than at the initial equilibrium. Lastly, with the tax on capital income, the utility of adult households increases by 10.61% from the previous period and becomes 9.94% larger than at the original steady-state.

Finally, with the observed changes in the factor inputs, our results indicate that at second period of the government presence with tax on consumption or capital income, the level of output per adult household becomes 2.548% larger than at the original steady-state. With the labour income tax, this indicator increases by 2.554% above the initial equilibrium.

Moving forward to a third period of the government presence, given the choices that take place, the government increases the tax rate on the capital income to a rate of 0.99% to be able to provide the subsidy rate for education of 10% and to balance the budget. When the government uses tax on consumption or capital income, however, the tax rates remain unchanged.

Driven by the previous period increase in the human capital of adult households that improved the human capital of parents and the quality of education in the economy, all of which has positively influenced the human capital of children in the past generation, we observe an increase in the human capital of adult households at the fourth period of analysis by 1.88% when government uses tax on consumption or capital income to finance and balance its budget. As a result, under these two tax options with subsidy rate for education of 10%, the human capital of adult households becomes 8.39% larger than at the original equilibrium. With the labour income tax in use, the human capital of adult households at the third period of the government presence becomes 1.77% larger than at the previous generation, which becomes 7.88% larger than at the original steady-state. With fertility being at the level below the initial steady-state, however, the population of adult households decreases by 1.85% and becomes 3.18% smaller than at the original steady-state, when the government taxes consumption or capital income. With the labour income tax, the population of adult households at the third period of the government presence decreases by 1.13% and becomes 2.24% lower that at the first period of the analysis.

As a result of these changes in the human capital and the population size, and with absence of change in the education and fertility levels of adult households, at the fourth period of analysis when government taxes consumption or capital income, the effective labour force per adult households increases by 1.88% from the previous generation and becomes 5.68% larger than at the original steady-state. With labour income tax, effective labour force per adult household increases by 1.77% and becomes 5.90% larger than at the original equilibrium.

Due to the previous period increase in saving and present period decrease in the population size of adult households, at the fourth period of analysis when the government implements tax on consumption or capital income, the physical capital stock per adult households increases by 4.95% and becomes 5.19% larger than at the initial steady-state. With the labour income tax in place, the physical capital stock per adult household increases by 4.22% and becomes 3.82% larger than at the first period of analysis.

Consequently, our results indicate that at the fourth period of analysis when the government provides subsidy rate for education at 10% and taxes either consumption or capital income, the real wage rate increases by 0.99% and becomes 0.16% lower than original steady state. The real interest rate, however, decreases by 2.61% and becomes 0.42% larger than at the original steady-state. With the labour income tax, the real wage rate increases by 0.80% and becomes 0.66% lower than at the initial equilibrium; while the real interest rate falls by 2.10% and becomes 1.78% larger than at the initial equilibrium.

Driven by the changes in the real wage rate and in the human capital level, at the fourth period of analysis when the government taxes consumption, the consumption of adult households increases and becomes 7.66% larger than at the original steady-state. With the tax on the labour income, the consumption of adult households increase and become 6.49% larger than at the initial steady-state; while with the tax on capital income, the consumption of adult households rises by 8.22% above the original equilibrium. As before, the dynamics for saving follows the dynamics of consumption of adult households when government finances and balances the budget with tax on labour income and capital income. With the tax on consumption, saving of adult households increase and become 8.22% larger than at the original steady-state.

Finally, given the past the human capital, the wage rate and tax rate on the labour income of the past generation and the real interest rate, tax rate on consumption and capital income at the fourth period of analysis, the consumption of the elderly households increase and become 4.96% larger than at the original steady-state, when government taxes consumption. With tax on labour income, the consumption of the elderly households increase and become 5.20% larger than at the initial steady-state; and with the tax on capital income, the consumption of the elderly households is 4.74% larger than before the government intervention.

As a result of these changes that take place at the third period of the government presence, the utility of adult households increase and become 15.49%, 13.99% and 16.22% larger than at the original steady-state, when government implements the tax on consumption, labour income and capital income, respectively. Finally, with the change in the factor inputs, the level of output per adult household increases and becomes 5.51% larger than at the initial steady-state, when the government taxes consumption or capital income. With the labour income tax, the output per adult household increases and becomes 5.20% larger than at the original equilibrium.

Overall, the dynamics that we observe the for the fourth period of analysis with the government subsidy for education of 10% would continue to exist until the model economy reaches the final steady-state where the human capital between the generation of the households is stable and is equal to one another. At these final steady-states, given the choices that take place, the government continues to provide the subsidy for education at 10% and it levies the tax on consumption of 0.53%. If the government implements the labour income tax instead, the tax rate is set to be equal to 0.63%. Lastly, if the government finances the subsidy for education with tax on capital income, the tax rate is equal to 1.02%.

According to our results, at these final steady-states with subsidy for education of 10%, when government uses the tax on consumption or capital income, the adult households would provide their children with education that is 16.27% larger than at the original steady-state. When the government uses the labour income tax instead, the households provide their children with education that is 15.25% larger than at the initial equilibrium. Given the increase in education investment, the human capital of the households increases 9.26% above the original steady-state when the economy reaches the final steady-state with tax on consumption or capital income. With the labour income tax, the human capital of the households at the final steady-state is 8.69% larger than at the first period of analysis. Due to increase in the education investment, however, the fertility level in the economy decreases 1.85% below the original steady-state when the government taxes consumption or capital income. With the labour income tax, the fertility choices at the second steady-state are 1.12% lower than at the initial equilibrium. Consequently, our results project a continuous decrease in the population size.

Overall, at the final steady-states with the government subsidy for education of 10% and tax levied either on consumption or capital income, the effective labour force per adult household becomes 6.53% larger than at original steady state. The physical capital stock per adult household, in turn, becomes 10.66% larger than at the original steady-state. With the labour income tax financing the subsidy for education instead, the effective labour force per adult household becomes 6.69% larger than at the original equilibrium, and the physical capital stock per adult household is 8.67% larger than at the first period of analysis, when the economy reaches the final steady-state.

With these changes in the factor inputs, the real wage rate becomes 1.28% larger than at the original equilibrium when the economy reaches the final steady-state with tax on consumption or capital income; while with the labour income tax, the real wage rate becomes 0.61% larger than at the original steady-state. The real interest rate, however, becomes 3.36% smaller than at the original steady-state, when the government uses tax on consumption or capital income; whereas, with the labour income tax, the real interest rate decreases 1.63% below the initial steady-state.

As a result, we observe that at the final steady-states with subsidy for education of 10%, the consumption of adult households become 10.08%. 8.67% and 10.66% larger than at the original equilibrium, when the government taxes consumption, labour income and capital income, respectively. As before, the dynamics of saving follows the dynamics for consumption of adult households, when the government taxes labour income and capital income. In the case of a final steady-state with tax on consumption, saving of adult households becomes 10.66% larger than at the initial steady-state. Finally, the consumption of the elderly households becomes 7.32%, 7.35% and 7.07% larger than at the first period of analysis when the economy is at the final steady-state and when the government taxes consumption, labour income and capital income, respectively.

To conclude the summary for the results when the government provides subsidy for education of 10%, our results indicate that at the final steady-states, the utility of the adult households becomes 20.20%, 18.28% and 20.92% larger than at the initial steady-state, when the government finances and balances its budget with tax on consumption, labour income and capital income, respectively. Finally, our results indicate that at the final steady-states with tax on consumption or capital income, the real output per adult households becomes 7.89% larger than at the original steady-state. With the labour income tax, this indicator is 7.35% larger than at the first period without the government presence.

2.4.2 Subsidy for children

For our second policy experiment for the case of representative household, we analyse the influence of publicly provided subsidy for fertility decisions. Similarly to our discussion for the case with subsidy for education, we consider that government provides the adult households with 10% subsidy rate for fertility starting from the second period of the analysis[13]. In order to finance and balance its budget, the government has the option of imposing the tax on consumption, labour income and capital income. As before, we consider each of these tax options separately.

According to our results, given the subsidy rate for fertility of 10% and all choices that take place in the economy at the second period of analysis, the government sets the tax rate on consumption to 0.46% to finance and balance its budget. When the government implements the labour income tax instead, the tax rate is set to be equal to 0.55%. Finally, if the government uses the tax on capital income, the tax rate at the first period of the government presence is equal to 0.86%.

As a result of the subsidy rate for fertility provided by the government, our results indicate that the fertility decisions of the adult households at the second period of analysis increase by 7.96% when government uses tax on consumption or capital income. With the labour income tax financing the subsidy program, the fertility choices increase by 8.66%. This relatively larger increase in the childbearing decisions of the adult households with the tax on the labour income is attributed to the disincentive that this tax policy creates for education investment, which through the parental trade-off between 'quality and quantity' of children would result in a relatively larger fertility choices in comparison to other considered tax options, holding

[13]As before, the first period of the analysis is the steady-state for the economy with representative households without the presence of the government sector

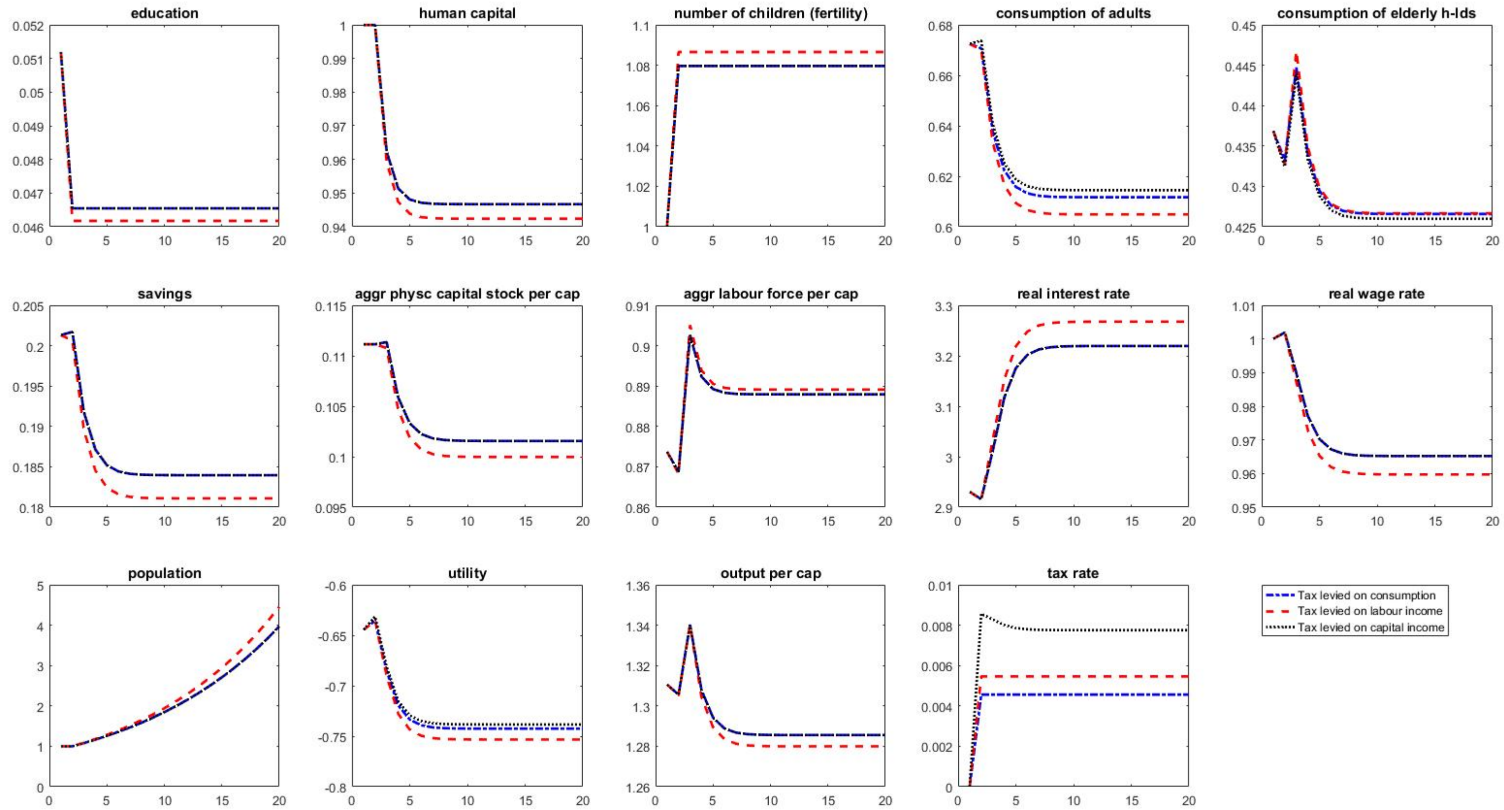

Figure 2.2: Transition path of representative agent economy with subsidy for children.
Note: transition path with tax on consumption is depicted by blue dash-dotted line; transition path with tax on labour income is presented by red dashed line; transition path with capital income tax is shown by black dotted line

everything else constant.

Overall our results indicate that the level of education investment in the economy with subsidy for fertility of 10% and with tax on consumption or capital income decreases by 9.09% below the original steady-state. With the tax on the labour income, instead, the parental choice for education decreases further, and it becomes 9.82% lower than at the original steady-state.

Therefore, based on these results, the immediate consequence of this policy is that children are receiving less education which diminish the human capital of the future adult households by 3.76% with tax on consumption or capital income, and by 4.07% with tax on the labour income. However, there are more children born, which would increase the population size of future adult households.

Due to (1) an increase in the fertility decisions – which increases the demand for teacher and the time that adult households spend on raising children, holding everything else constant; (2) decrease in the households' choice for education – which decreases the number of teacher required, holding everything else constant; and (3) unchanged size of the population of adult households and their level of human capital from the original steady-state, the effective labour force per adult household at the second period of analysis with tax on consumption or capital income becomes 0.58% lower than at the initial equilibrium. With the tax on the labour income, the effective labour force per adult household decreases by 0.63% at the first period of the government presence. The physical capital stock per adult household, however, is identical to the original steady-state, since it is determined by the aggregate saving at the original steady-state and by the present population size of the adult households which has been determined and is identical to the first period of the analysis.

Therefore, at the second period of analysis with the subsidy for fertility of 10% and with tax on consumption or capital income that finances and balances the government budget, we observe that the real wage rate increases by 0.19% and the real interest rate decreases by 0.51% below the original steady-state. With the tax on the labour income, the real wage rate increases by 0.21% and the real interest rate decreases by 0.56% below the initial equilibrium.

Overall, our results indicate that at the first period of the government presence, the consumption of adult households decreases by 0.26% and 0.34% when the tax on consumption and labour income finance the subsidy program, respectively. In the case with tax on capital income, however, the consumption of adult households increases by 0.19% above the original steady-state. The dynamics for saving in the case when the government implements tax on labour income or tax on capital income follows the dynamics of the consumption of adult households at the second period of the analysis. For the case with the tax on consumption, saving of adult households at the first period of the government presence becomes 0.19% larger than at the original steady-state. Finally, the consumption of the elderly households at the second period of analysis with subsidy rate for fertility of 10% becomes 0.84%, 0.96% and 1.02% lower than at the initial equilibrium, when the government taxes consumption labour income, and capital income, respectively.

Due to these observed changes at the first period of the government presence with subsidy for childbearing decisions, our results indicate that the utility of adult households becomes 1.43%, 1.44% and 2.05% larger than at the original steady-state, when government implements tax on consumption, labour income and capital income, respectively[14]. On the other hand, due to decrease in the effective labour force and absence of change in the physical capital stock, the level of output per adult household diminishes by 0.38% below the original steady-state when the government uses tax on consumption or capital income, and it falls by 0.42% when the government utilises tax on labour income instead.

Moving forward to the third period of the analysis, given the individual decisions that take place, the government adjusts the tax rate on the capital income to 0.83% to be able to continue provide the adult households with 10% subsidy rate for fertility and maintain the balanced budget. In the case when the government uses tax on consumption or labour income instead, the tax rates remain unchanged.

As a result of a previous period decrease in the level of education investment, our results indicate that at the second period of the government presence the human capital of adult households decreases 3.76% below the original steady-state when government finances a 10% subsidy rate for fertility with tax on consumption or capital income. With the labour income tax,

[14]This increase in level of utility under all tax options is primary driven by increase in the childbearing decisions

which created further disincentive for the education provision in the past period, the human capital of adult households decreases 4.07% below the original steady-state. However, as a result of the publicly financed subsidy for fertility which increased the fertility level of the past generation, the population of adult households at the third period of analysis becomes 7.96% larger than at the initial equilibrium, when tax on consumption or capital income is in place. When the government uses the labour income tax instead, the population of adult households at the second period of government presence becomes 8.65% larger than at the original steady-state.

Since the current discussion focuses on the case of the homogeneous households, similarly to our results with subsidy for education, the level of education provision and fertility decisions remain identical to the previous period of the analysis until the government continues to provide the subsidy to the households. As we have discussed previously, in the case with representative households, the change in the human capital of adult households would not alter the decision of adult households for education since the average level of human capital is equal to the human capital of the adult households from considered ability group which makes the relative human capital to be equal to one. However, the human capital of current generation of children and future generation of adult households continues to decrease until the economy reaches the final steady-state since the human capital of parents and quality of education both positively relate with the human capital of offspring.

Given the changes in the human capital and the population size of adult households and absence of change in the education and fertility decisions, our results indicate that at the third period of analysis when government provides subsidy for fertility of 10% and finances and balances its budget with tax on consumption or capital income, the effective labour force per adult households increases by 3.91% from the previous generation and becomes 3.31% larger than at the original steady-state. With the labour income tax in place, the effective labour force per adult household increases by 4.24% and becomes 3.59% above the initial equilibrium.

For the physical capital stock per adult household at the third period of analysis with subsidy rate for fertility of 10% and tax on consumption or capital income financing and balancing the government budget, which is determined by the past period aggregate saving and present period population size of adult households, our results indicate 0.19% increase above the original steady-state. With the labour income tax, the physical capital stock per adult household decreases 0.34% below the original steady-state, however.

With these changes in the factor inputs, our results indicate that at the second period of the government presence with subsidy for fertility of 10% and with tax on consumption or capital income, the real wage rate decreases by 1.21% and becomes 1.02% smaller than at the original steady-state; when the real interest rate increases by 3.30% and becomes 2.77% larger than at the initial equilibrium. With the tax on labour income, the real wage rate is found to reduce by 1.49% from the previous period which becomes 1.28% lower than at the original steady-state; but the real interest rate increases by 4.08% and is 3.50% larger than at the first period of the analysis.

Due to these changes, we find that at the third period of analysis when the government provides subsidy for fertility at a rate of 10%, the consumption of adult households decreases by 4.92%, 5.49% and 4.92% from the previous period and becomes 5.17%, 5.81% and 4.74% smaller than at the original steady-state, when government finances and balances its budget with tax on consumption, labour income and capital income, respectively. The dynamics for saving follows the dynamics for consumption of adult households when the government implements the tax on labour income and capital income. With the tax on consumption, at the third period of analysis with subsidy for fertility of 10%, saving decreases by 4.92% from the previous period and becomes 4.74% smaller than at the original steady-state. Lastly, according to our results, the consumption of the elderly households at the third period of analysis increases by 2.65%, 3.25% and 2.67%, and becomes 1.79%, 2.26% and 1.62% larger than at the original steady-state, when government provides subsidy for childbearing at 10% level and finances and balances its budget with tax on consumption, labour income and capital income, respectively.

Overall, with these changes that take place at the second period of the government presence, our results indicate that the utility of adult households decreases by 7.84%, 8.60% and 7.89% from the previous period of analysis, and becomes 6.29%, 7.04% and 5.66% lower than at the original steady-state, when government provides the subsidy for fertility of 10% with tax on consumption, labour income and capital income, respectively. Finally, the level of output per adult household is found to increase by 2.65% from the previous generation, which becomes 2.26% larger than at the original steady-state, when government implements tax on consumption or capital income. With the labour income tax, the output per adult household increases by 2.69% and becomes 2.26% larger than at the original equilibrium.

Moving forward to a fourth period of analysis, the government adjusts the tax rate on capital income to 0.80% in order to continue to provide the subsidy for fertility at a rate of 10% and to keep the budget balanced. If the government finances and balances its budget with the tax on consumption or labour income, however, the tax rates remain unchanged from the previous period and are identical to the first period of the government presence.

With a decrease in the human capital of adult households that we observed in the previous period of analysis that negatively influenced the human capital of children, our results indicate that human capital of the current adult households at the fourth period of analysis decreases by 1.14% and becomes 4.86% smaller than at the original steady-state. With the labour income tax in place instead, the human capital of adult households at the fourth period of analysis becomes 1.24% lower than at the third period, and it is 5.25% lower than at the original equilibrium. However, due to initial increase that we have observed in fertility decisions, our results indicate that when the government finances the provision of subsidy for fertility with tax on consumption or capital income, the population size of adult households at the fourth period of analysis increases further by 7.96% and it becomes 16.55% larger than at the original steady-state. With the labour income tax, the population of adult households at the fourth period of analysis is 7.96% larger than at the third period, and it is 16.55% larger than at the original steady-state.

With absence of change in education and fertility decisions, and with increase in population size and decrease in the human capital of adult households, our results indicate that at the fourth period of analysis when the government provides subsidy for fertility of 10% and finances and balances its budget with tax on consumption or capital income, the effective labour force per adult household decreases by 1.14% and becomes 2.13% larger than at the original steady-state. With the tax on labour income, effective labour force per adult household reduces by 1.24% and it becomes 2.30% larger than at the initial equilibrium.

Due to a decrease in the past period of saving, and increase in the current population of adult households, at the fourth period of analysis when government provides subsidy for fertility at 10% and finances and balances its budget with tax on consumption or capital income, the physical capital stock per adult household decreases by 4.92% and becomes 4.74% lower than at the initial steady-state. With the labour income tax, this indicator decreases by 5.49% from the previous generation and becomes 5.81% below the original steady-state.

As a result of these changes in factor inputs, when the government provides subsidy for fertility of 10% and utilises tax on consumption or capital income, the real wage rate at the fourth period of analysis decreases by 1.29% and becomes 2.29% lower than at the original equilibrium. The real interest rate in this case increases by 3.50% and becomes 6.37% larger than at the initial steady-state. With the tax on labour income in place instead, the real wage rate decreases by 1.46% which becomes 2.72% lower than at the original steady-state; while, the real interest rate increases by 3.96% and becomes 7.60% larger than at the first period of analysis.

With these changes in the factor prices, combined with the resulted level of human capital and tax rates, our results indicate that at the third period of the government presence with the publicly financed subsidy for fertility of 10%, the consumption of adult households decreases by 2.42%, 2.68% and 2.42%, which becomes 7.64%, 8.33% and 7.04% lower than at the original steady-state, when government implements tax on consumption, labour income and capital income, respectively. Our results indicate that the dynamics for saving when government finances and balances its budget with tax on labour income and capital income is identical to the dynamics of consumption of adult households. With the tax on consumption, the saving of households decreases by 2.42% and becomes 7.04% smaller than at the initial steady-state. Finally, the consumption of the elderly households at the fourth period of analysis is found to decrease by 2.41%, 2.68% and 2.40% that becomes 0.67%, 0.48% and 0.82% lower than at the original equilibrium, when the tax on consumption, labour income and capital income are implemented, respectively.

Overall, our results indicate that at the fourth period of analysis, the utility of adult households decreases by 4.69%, 5.45% and 4.98%, that becomes 11.56%, 12.87% and 10.92% lower than at the original steady-state, when government provides subsidy for fertility of 10% and finances and balances its budget with tax on consumption, labour income and capital income, respectively. Finally, our results indicate that at the fourth period of analysis the output per adult household decreases by 2.42% and becomes 0.21% below the original steady-state when the government uses the tax on consumption or capital income. With the tax on labour income, the output per adult household at the third period of the government presence reduces by 2.68% and becomes 0.48% lower than at the initial equilibrium.

According to our results, the dynamics summarised for the four period of analysis continues until the economy reaches the final steady-states where the human capital of the representative households between generations is equal to one another. At the final steady-state, when the government provides the homogeneous households with 10% subsidy rate for fertility and finances and balances the budget with tax on consumption, the tax rate is set to 0.46%. When the government implements the labour income tax instead, the tax rate at the final steady-state is equal to 0.55%. Finally, with the capital income tax in place, the tax rate is equal to 0.78%.

Our results indicate that due to the publicly financed subsidy rate for fertility of 10%, the fertility decisions of adult households become 7.96% than at the original steady-state, when the government employs tax on consumption or capital income. At the final steady-state with tax on labour income, the childbearing decisions of adult households become 8.66% larger than at the initial steady-state. However, our results indicate that the level of education investment at the final steady-state with subsidy for fertility of 10% and with tax on consumption or capital income becomes 9.09% lower than at the original steady-state. This indicator decreases by 9.82% below the original equilibrium when the government utilises the labour income tax instead. As a result of this decrease in the education provision, our results indicate that at the final steady-state with subsidy rate for fertility of 10% and tax on consumption or capital income, the human capital of adult households becomes 5.33% lower than at the original steady-state. With the labour income tax, the human capital of adult households decreases 5.76% below the initial equilibrium, when the economy reaches the final steady-state. The population of adult households, however, continues to increase, with the rate of growth in the size of population of adult households being larger with the tax on labour income than with the tax on consumption or capital income.

With these changes, at the final steady-state with subsidy rate for fertility of 10% that the government finances with tax on consumption or capital income, the effective labour force per adult household becomes 1.63% larger than at the original equilibrium. When the government uses the labour income tax instead, the effective labour force per adult household becomes 1.76% larger than at the first period of analysis.

Given the change in the aggregate level of saving and population of adult households, at the final steady-state with subsidy for fertility of 10% and tax on consumption or capital income, the physical capital stock per adult household becomes 8.62% lower than at the original steady-state. With the labour income tax, this indicator becomes 10.05% lower than at the initial equilibrium, when the economy is at the final steady-state.

Therefore, our results indicate that at the final steady-state with subsidy for fertility of 10% and tax on consumption or capital income financing and balancing the government budget, the real wage rate becomes 3.48% lower than at the original steady-state, and the real interest rate becomes 9.85% larger than at the initial equilibrium. With the labour income tax, the real wage rate at the final steady-state with publicly financed subsidy for fertility of 10% becomes 4.03% lower than at the first period of analysis, and the real interest rate increases 11.49% above the original steady-state.

Consequently, at the final steady-state with subsidy rate for fertility of 10%, the consumption of adult households becomes 9.04%, 10.05% and 8.62% below the initial equilibrium, when government taxes consumption, labour income and capital income, respectively. The rate of change in saving is identical to rate of change in consumption of adult households when the government uses tax on labour income and capital income. With the tax on consumption, saving of households decreases 8.62% below the original steady-state. Lastly, at the final steady-state with subsidy for fertility of 10% in the case of the homogeneous households, the consumption of the elderly households becomes 2.36%, 2.34% and 2.49% below of the initial equilibrium, when the government levies the tax on consumption, labour income and capital income, respectively.

Finally, our results indicate that at the final steady-state with subsidy for fertility of 10%, the utility of households becomes 15.16%, 16.63% and 14.52% lower than at the original steady-state, when the government finances and balances its budget with tax on consumption, labour income and capital income, respectively. Lastly, the level of output per adult households is found to be 1.91% below the original steady-state, when government taxes consumption or capital income. With the labour income tax, the output per adult household becomes 2.34% lower than at the first period of analysis.

2.4.3 Subsidy for both education and children

As a final policy experiment for the case of the representative household, we consider a possible solution to the 'quality-quantity' trade-off of adult households. Based on the current structure of the model economy and calibration for the model parameters, we find that a simultaneous provision of subsidy for education and subsidy for fertility is necessary in order to obtain a simultaneous increase in the education investment and childbearing decisions, which would resolve a trade-off that parents otherwise always have between the 'quality' and 'quantity' of children. According to our results, however, for this dynamics to take place under all considered tax options[15], the subsidy rate for education should be 1.6 times larger than the subsidy rate for fertility (at least). We find, however, that if the subsidy rate for education is less than 1.6 times larger than subsidy for fertility[16], the adult households would increase only the fertility level, but the level of education investment would decline, and the overall conclusions would be similar to the case with just only subsidy for fertility in place considered in the previous section[17]. Consequently, our results indicate that at the final steady state with this proposed policy profile[18] we observe a long-run improvement in the human capital, population size, utility and output produced per adult household. Additionally, we observe that the trade-off of parents for their children now becomes resolved.

[15]I.e. in order to have a simultaneous increase in the education investment and fertility level, which would manifest in simultaneous increase in the human capital and population size, respectively.

[16]This is conditional on the parametrisation of the model and the overall structure of the economy, however.

[17]I.e. the economy would experience an increase in the population size, but the human capital, welfare and output per adult household would diminish as a result of this policy. Furthermore, parents would still experience the trade-off between the choices for education and fertility.

[18]When subsidy rate for education is 1.6 times larger than subsidy rate for fertility.

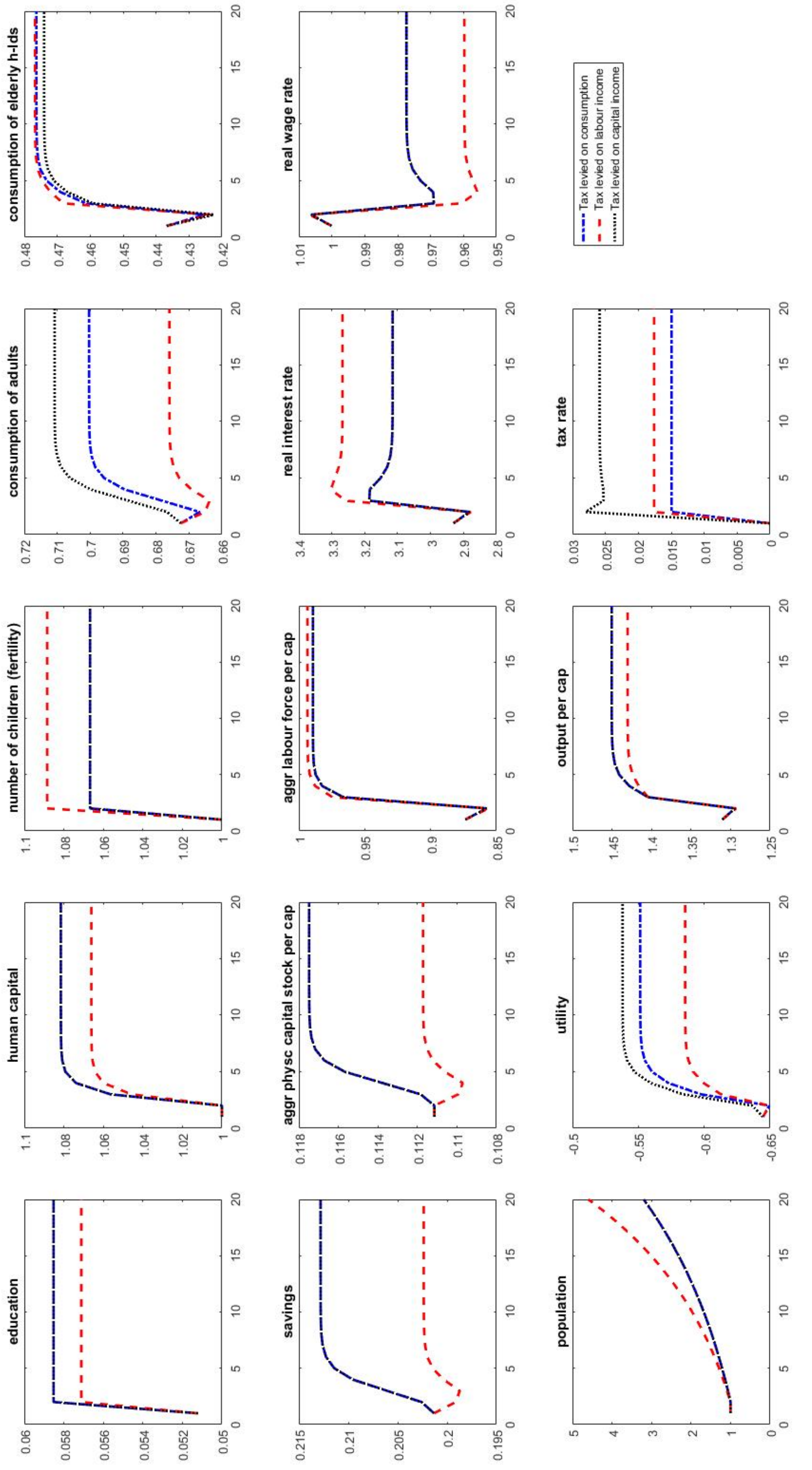

Figure 2.3: Transition path of representative agent economy with subsidy for education of 16% and subsidy for fertility decisions of 10%.
Note: transition path with tax on consumption is depicted by blue dash-dotted line; transition path with tax on labour income is presented by red dashed line; transition path with capital income tax is shown by black dotted line

In our discussion for this section, we consider that the government provides the subsidy for fertility of 10% and the subsidy for education of 16% simultaneously. To be able to provide the households with this subsidy program and to maintain the balanced budget, as before, we consider that the government has three tax options, which are tax on consumption, labour income and capital income. When the government implements the tax on consumption, at the final steady-state with subsidy for education of 16% and subsidy for fertility of 10%, the tax rate is equal to 1.50%. With the labour income tax, the tax rate is 1.76%; while with the tax on capital income, the tax rate is equal to 2.59%.

According to our results, the major difference in the evolution of the economy when the government provides the subsidy for education of 16% and subsidy for fertility of 10% exists when the government finances its budget with tax on labour income and with tax on consumption or capital income. When the government finances and balances its budget with the tax on the labour income, the adult households at the final steady-state provide their children with 2.40% less education than when government implements tax on consumption or capital income. At the same time, however, the adult households at the final steady-state with tax on labour income decide to have 2.03% larger number of children than with tax on consumption or capital income. Therefore, at the final steady-state with 16% subsidy rate for education and 10% subsidy rate for fertility when the government utilises the tax on labour income, the population is larger in its size but the level of the human capital is 1.43% smaller than at the final steady-state with tax on consumption or capital income.

Consequential of this difference in the choices of adult households, our results indicate that at the final steady-state with subsidy for education of 16% and subsidy for fertility of 10% when government implements the tax on labour income, the effective labour force per adult household is 0.42% larger than at the final steady-state with tax on consumption or capital income. However, the level of physical capital stock per adult household at the final steady-state with tax on labour income is 4.91% smaller than at the final steady-state with tax on consumption or capital income.

We observe a smaller level of physical capital stock per adult household with tax on labour income for the following reasons. First, because the labour income taxation reduces the total income, the households optimally decide on a lower level of saving than with tax on consumption or capital income, holding everything else constant. Also, because the households have a lower level of (after tax) income, they optimally decide to provide their children with less education, which makes the long-run level of human capital of adult households with tax on labour income to be less than with tax on consumption or capital income, holding everything else constant. This further makes the individual level of saving with tax on labour income to be below the one with tax on consumption or capital income. Moreover, a larger decrease in the real wage rate that takes place with tax on the labour income than with other tax options (due to the dynamics between the factor inputs) contributes further to the individual level of saving with tax on labour income to be smaller than with tax on consumption or capital income[19]. At the same time, however, due to the disincentive that the labour income taxation creates for education investments, parents optimally decide on a larger number of children, which increase the long-run population size at a larger rate than with other tax options. Therefore, the net effect of a smaller level of saving of adult households and a larger population size when the government implements tax on labour income is a smaller level of aggregate physical capital stock per adult household than in the case with tax on consumption or capital income.

With these relative levels of the factor inputs, our results indicate that at the final steady-state with tax on labour income, the real wage rate is 1.80% lower and the real interest rate is 4.88% higher than at the final steady-state with tax on consumption or capital income.

Overall, according to our results, at the final steady-state with subsidy for education of 16% and subsidy for fertility of 10%, when the government implements tax on consumption or capital income, the level of education provision is 14.30% larger than at the original steady-state, and the fertility decisions of the households are 6.67% larger than at the initial equilibrium. With the labour income tax, the level of education investment becomes 11.55% larger than at the original steady-state, and the fertility level is 8.84% larger than at the first period of analysis. Consequently, under all tax options we observe an increase in the population size and improvement in the human capital level. With the tax on consumption or capital income, the human capital at the final steady-state becomes 8.15% larger than at the original equilibrium; and with

[19]The same reasoning applies for explaining the difference in the consumption of adult households.

tax on the labour income, the human capital increases 6.61% above the initial steady-state.

With these changes[20], our results indicate that at the final steady-state with subsidy for education of 16% and subsidy for fertility of 10% that government finances with tax on consumption or capital income, the effective labour force per adult household becomes 13.23% larger than at the original steady-state, and the physical capital stock per adult household is 5.70% larger than at the initial equilibrium. With the labour income tax in use instead, the effective labour force per adult household becomes 13.70% larger than at the original steady-state, and the physical capital stock per adult household increases by 0.52% above the original period of the analysis. As a result, at the final steady-state with tax on consumption or capital income, the real wage rate becomes 2.27% lower than at the original steady-state, and the real interest rate increases 6.29% above the original equilibrium. With the tax on capital income, the real wage rate becomes 4.02% lower than at the initial steady-state, and the real interest rate is 11.48% larger than at the first period of analysis.

Our results indicate that at the final steady-state with subsidy rate for education of 16% and subsidy rate for fertility of 10%, the consumption of adult households becomes 4.14%, 0.52% and 5.70% larger than at the original steady-state, when the government finances and balances its budget with tax on consumption, labour income and capital income, respectively. The dynamics of saving when the government implements tax on labour income and capital income is identical to the dynamics of consumption of adult households. With the tax on consumption, saving of adult households becomes 5.70% larger than at the original steady-state. Finally, the consumption of the elderly households is found to be 9.03%, 9.13% and 8.50% above the original steady-state, when the government taxes consumption, labour income and capital income, respectively.

In conclusion, our results indicate that when the economy with representative households reaches the final steady-state with publicly provided subsidy for education of 16% and subsidy for fertility of 10%, the utility of adult households becomes 14.18%, 9.19% and 16.55% larger than at the original steady-state, when the government finances and balances its budget with tax on consumption, labour income and capital income, respectively. Lastly, we find that the

[20] Combined with changes in the aggregate level of saving

level of output per adult household at the final steady-state with tax on consumption or capital income becomes 10.66% larger than at the original steady-state; and with the labour income tax, this indicator increases 9.13% above the original equilibrium.

2.5 Policy experiments: economy with heterogeneous households

In the final set of the policy experiments that we consider in this chapter, we investigate the evolution of the model economy with the households that are heterogeneous in the human capital when the government subsidises the education investment and fertility decisions. Similarly to our initial discussion for the case of representative households, we analyse the influence of these two subsidy programs separately from one another. Additionally, as before, we separately consider the influence of the tax options that the government has for financing and balancing its budget, which allows to analyse the influence of particular government policy instruments on the long-run development of the economy more carefully.

In our discussion presented in this section, we assume that at the first period of analysis, the economy is at the original steady-state without the government presence and the households are homogeneous in the human capital. Next, at the second period of the analysis we divide the population of the adult households into two equally-sized ability groups – one with the level of human capital that is 10% larger than at the original steady-state, and the other with the level of human capital that is 10% smaller than at the initial equilibrium. At this second period of analysis we further introduce the government for the first period which provides the adult households with subsidy for education or fertility, and it taxes either consumption, labour income or capital income to finance and balance its budget.

Despite that the policy exercise is identical to the case with homogeneous households, in the heterogeneous environment we observe the additional dynamics that now occurs between the human capital of a particular ability group and the average human capital in the economy[21].

[21] This dynamics does not exist in the case of a representative household since the individual level of human capital is equal to the average human capital when the human capital was considered to be homogeneous across the households

This dynamics, that is summarised by the changes of the relative human capital, now additionally affects the education and fertility decisions of the households, which contributes to the change in the structure of the population and in the overall outcomes of the considered policy options. Due to the presence of this relationship in the case of the heterogeneous households, both education and fertility decisions do not reach their immediate steady-state levels as we have observed for the homogeneous case, but instead they stabilise over time as the economy transitions to the final steady-state and as the difference between average and individual human capital is resolved. Furthermore, due to this additional variation in the level of education investment – and, therefore, in the level of the human capital – it takes the economy longer to reach the final steady-states than in the case with homogeneous households. As the economy reaches the final steady-states, however, both of the ability groups would have the identical level of the human capital, and, therefore, all of their decisions would be the same. However, due to the initial difference in the fertility decisions, our results suggest that at the final steady-states, the majority of the population would consist of the households who have been a part of a lower ability group. The rest of the conclusions for the case with heterogeneous households follows the results from the analysis for the homogeneous environment obtained previously.

We begin our discussion with the case of the subsidy for education, starting, as before, with the second period of analysis when the government is present in the economy for the first period.

2.5.1 Subsidy for education

As in our earlier discussion for the case with homogeneous households, we consider that the government provides the households with 10% subsidy rate for education starting from the first period of analysis. According to our results, when the tax on consumption is used to finance and balance the budget, the government sets the this tax rate to 0.53% at the second period of the analysis. When the government implements the labour income tax instead, the tax rate is equal to 0.63%; while with the capital income tax in place, the tax rate is equal to 1%.

At the second period of analysis, given the exogenously introduced heterogeneity for the human capital of adult households, fifty percent of the population now has a level of human

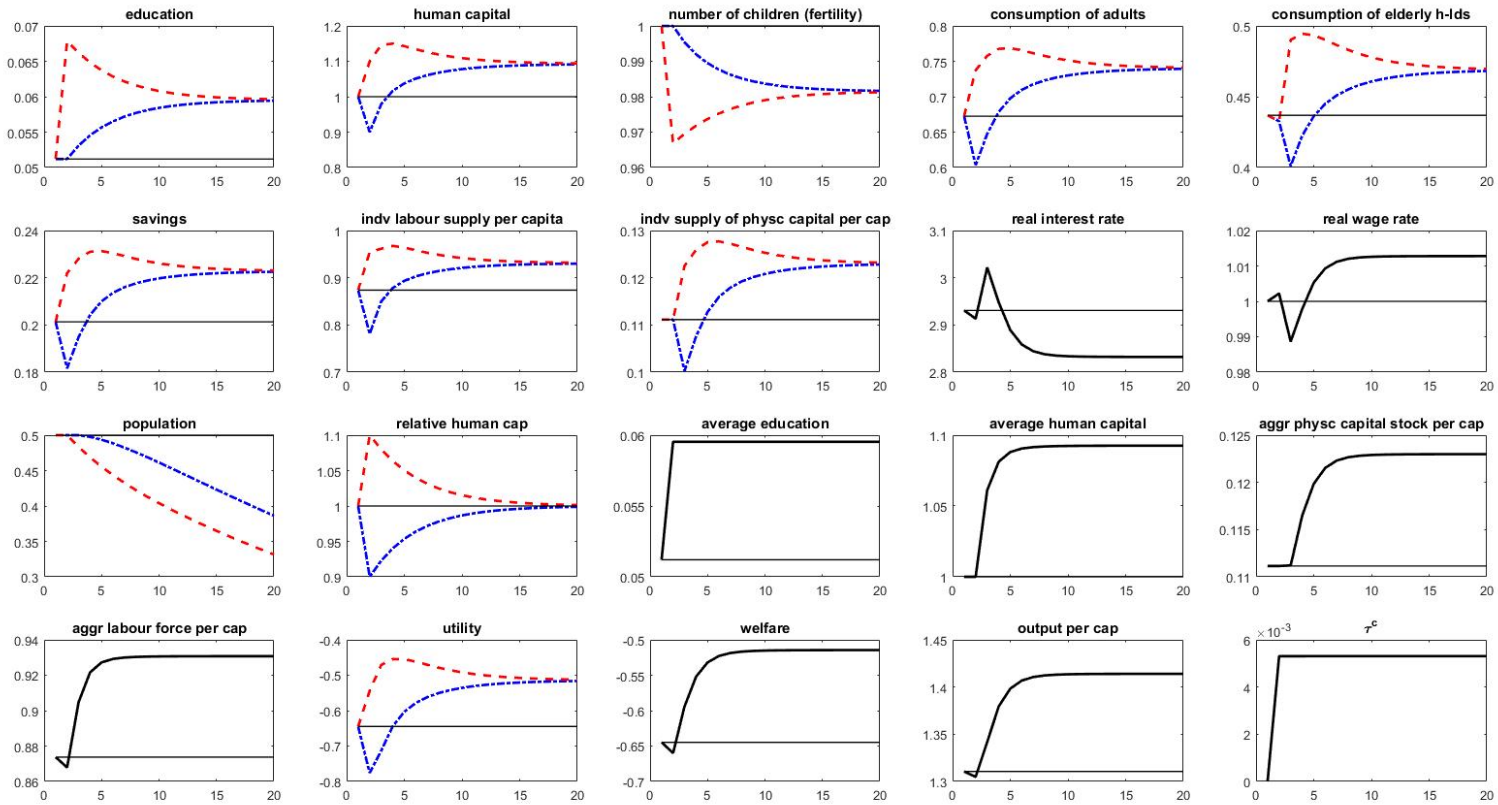

Figure 2.4: Transition path of heterogeneous agent economy with two ability types of households. Government imposes tax on consumption and provides subsidy rate for education of 0.1
Note: High ability type is shown as red dashed line. Low ability type is shown as blue dash-doted line

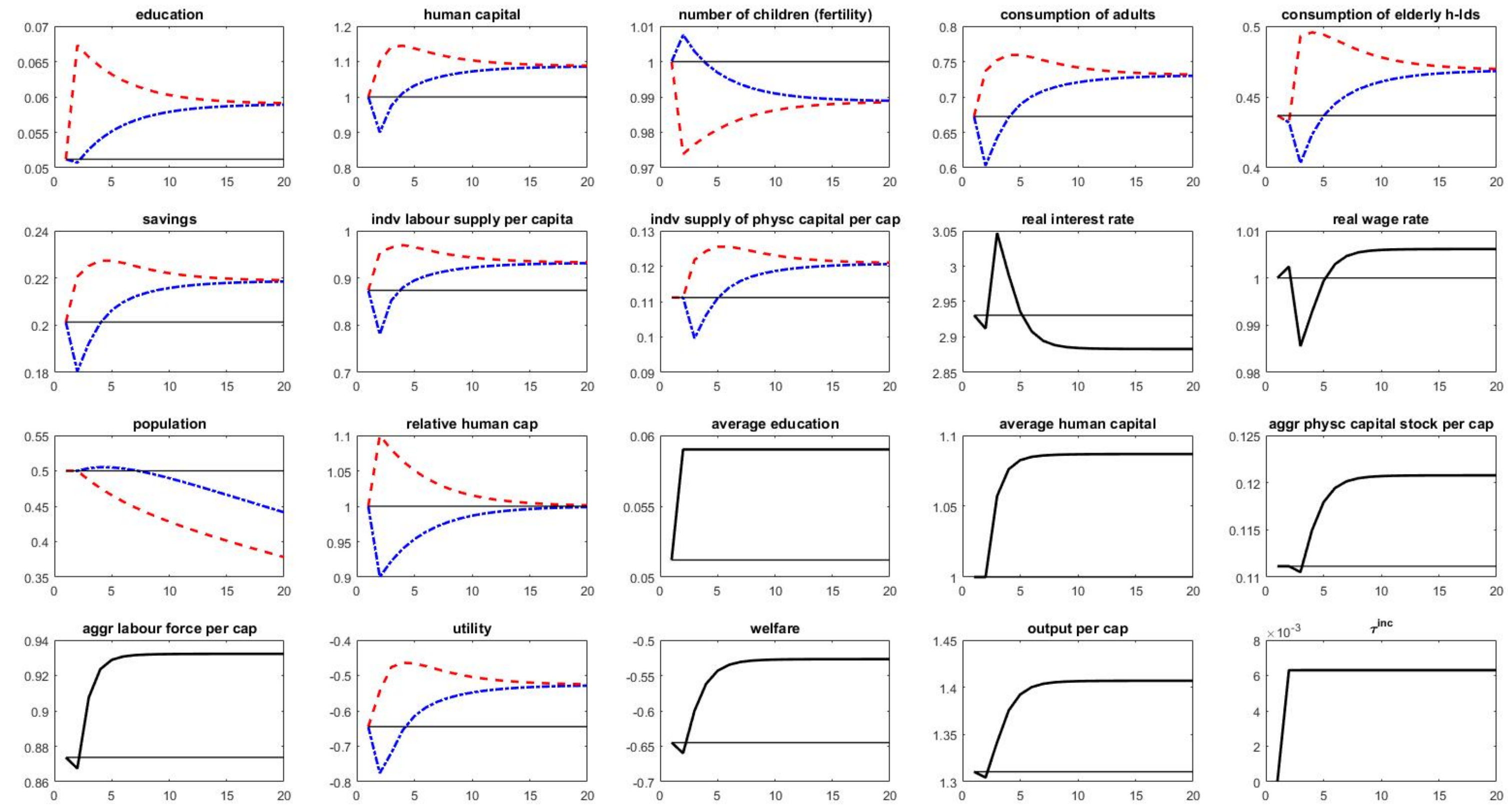

Figure 2.5: Transition path of heterogeneous agent economy with two ability types of households. Government imposes tax on labour income and provides subsidy rate for education of 0.1

Note: High ability type is shown as red dashed line. Low ability type is shown as blue dash-doted line

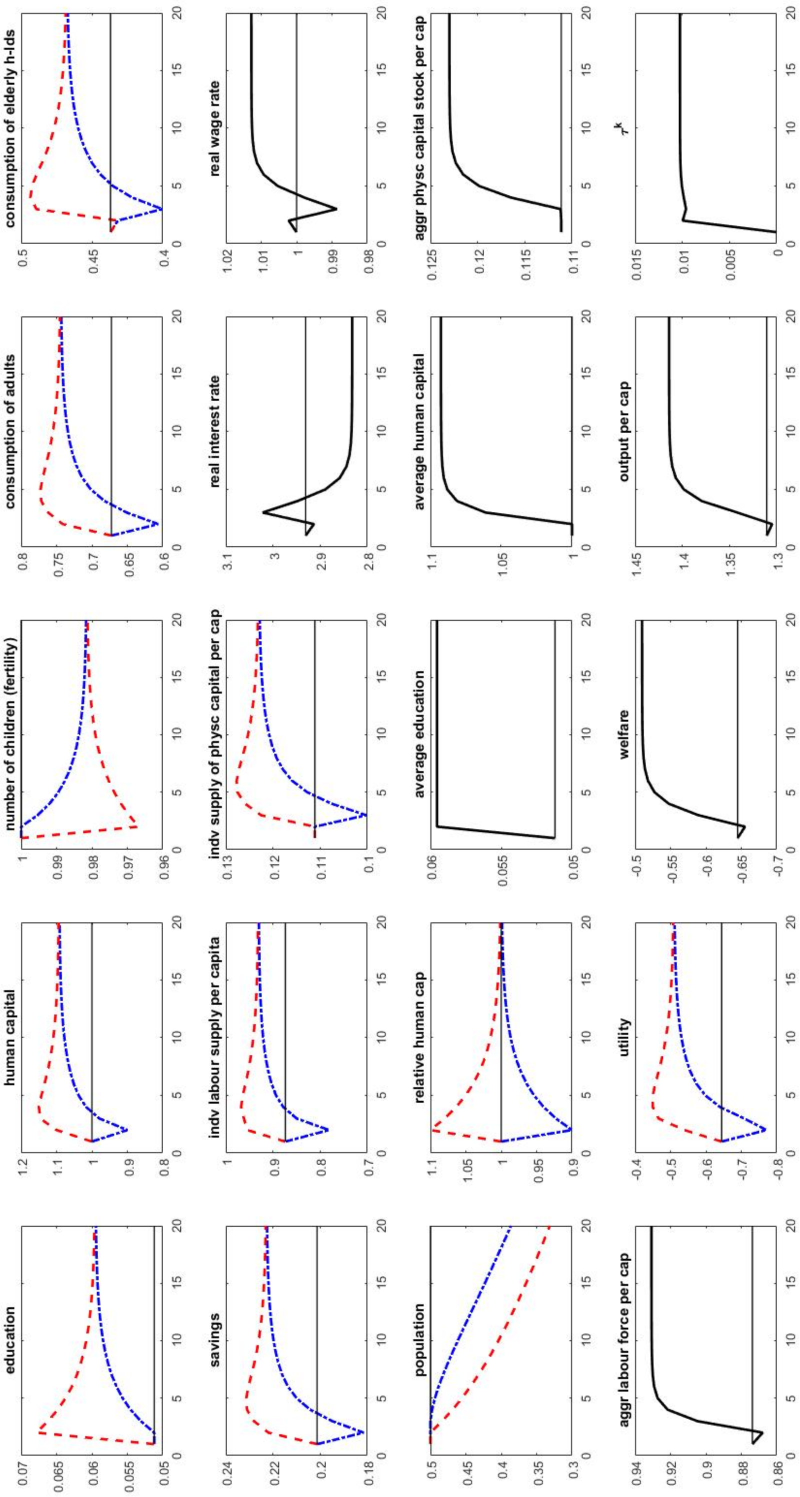

Figure 2.6: Transition path of heterogeneous agent economy with two ability types of households. Government imposes tax on capital income and provides subsidy rate for education of 0.1
Note: High ability type is shown as red dashed line. Low ability type is shown as blue dash-doted line

capital that is 10% larger than at the original steady-state; and we refer to this group as a higher ability households. The rest of the population, however, has the level of human capital that is 10% lower than at the original steady-state; and we refer to this group as a lower ability households. Given the equal distribution of the population of adult households between these two ability groups, however, the average level of the human capital at the second period of analysis remains to be identical to the original equilibrium level. Therefore, our results indicate, that at the second period of analysis, the level of the relative human capital of adult households from a higher ability group becomes 10% larger than at the original steady-state; but the level of relative human capital of the households from lower ability group becomes 10% smaller than at the initial equilibrium. The total population of adult households, however, that has been determined by the fertility decisions at the original steady-state, remains to be identical to the first period of analysis.

Given the change in the relative human capital[22] and the introduction of the subsidy for education of 10%, our results indicate that at the second period of analysis when the government uses tax on consumption or capital income to finance and balance the budget, the education investment of the households from higher ability groups becomes 32.55% larger than at the original steady-state. With the tax on the labour income, this indicator increases by 31.42% above the first period of analysis. For the case of lower ability households, however, at the second period of analysis when the government provides subsidy for education of 10% and implements tax on consumption or capital income, the education provision for children does not change and remains to be identical to the original steady-state. With the labour income tax, which creates additional disincentives for education investment, the level of education that children receive at the first period of the government presence is 0.92% lower than at the original steady-state.

With these changes in the relative human capital and the decisions of adult households for education provision, our results indicate that at the second period of analysis with subsidy for education of 10% that the government finances with tax on consumption or capital income, the fertility decisions of the adult households from higher ability group become 3.32% lower than at

[22] As we found from the analytical solution, the relative human capital level positively influences the decisions of adult households for education provision

the original steady-state. With the labour income tax, the childbearing of adult households from higher ability group decreases by 2.61% below the original equilibrium. For the households from lower ability group, when the government finances and balances its budget with tax on consumption or capital income, the fertility level does not change; while with the labour income tax, this indicator increases 0.75% above the original steady-state.

As a consequence of all these changes, our results indicate that at the first period when the government provides subsidy for education of 10% and levies the tax on consumption or capital income, the effective labour force per adult household becomes 0.67% lower than at the original steady-state. This is the net effect of increase in supply of the effective labour force by higher ability households by 9.25% and 10.59% decrease in the supply of effective labour force by lower ability households[23]. With the labour income tax, the supply of the effective labour force per adult household decreases by 0.72%. This is the net effect of increase in supply of the effective labour force per adult household by higher ability group by 9.17% above the original steady-state, and decrease in supply of the effective labour force per adult household by lower ability group by 10.63%. The physical capital stock per adult household, however, remains unchanged from the original steady-state, since it depends on the aggregate level of saving at the original steady-state and the population size at the second period of analysis, both of which are identical to the initial equilibrium.

Given this dynamics for the factor inputs, at the second period of analysis when the government provides subsidy for education of 10% and taxes consumption or capital income, the real wage rate becomes 0.22% larger than at the original steady-state. With the labour income tax, the real wage rate at the first period of the government presence is 0.24% larger than the original equilibrium. The real interest rate, however, becomes 0.60% smaller than at the first period of analysis when the government implements tax on consumption or capital income. And with the labour income tax, the real interest rate at the second period of analysis is 0.65% smaller than at the original steady-state.

[23]Despite an absence of change in the education provision and fertility decisions of households from lower ability group which would keep the supply of this ability group unchanged, the decrease in the human capital of this ability group below the average level of human capital in the economy diminishes the supply of the effective labour force per adult household of this ability group. Oppositely, for the households from a higher ability group who experience increase in the human capital, increase in the education provision and decrease in the fertility decisions, the net effect is an increase in the supply of the effective labour force per adult household

Driven by changes in the human capital, the factor prices and the tax rates introduced by the government, our results indicate that at the second period of analysis with subsidy for education of 10%, the consumption of adult households from higher ability group becomes 9.66%, 9.57% and 10.25% larger than at the original steady-state, when the government taxes consumption, labour income and capital income, respectively. In turn, the consumption of adult households from lower ability group at the second period of analysis is 10.27%, 10.35% and 9.80% lower than at the first period of analysis, when tax on consumption, labour income and capital income is in place, respectively. The dynamics for saving follows the dynamics for consumption of adult households when the government uses tax on labour income or capita income. With the tax on consumption, saving of higher ability group becomes 10.25% larger than at the original steady-state, but saving of lower ability group becomes 9.80% smaller than at the initial equilibrium. Finally, the consumption of the elderly households[24] at the first period of the government presence with 10% subsidy rate for education becomes 0.97% 1.11% and 1.18% lower than at the original steady-state, when the tax on consumption, labour income and capital income is in use, respectively.

Overall, our results indicate that the level of utility of households from higher ability group becomes 15.65%, 15.66% and 16.37% larger than at the original steady-state, when the tax on consumption, labour income and capital income finance and balance the government budget, respectively. The utility for the lower ability households at the second period of analysis, however, decreases by 20.34%, 20.31% and 19.61% below the initial equilibrium, when tax on consumption, labour income and capital income is in place, respectively. Given these changes in the utility levels of each ability group and the distribution of the population between the ability groups, our results indicate that at the second period of analysis when the government provides subsidy for education of 10%, the level of welfare in the economy becomes 2.34%, 2.32% and 1.62% lower than at the original steady state, when the government levies tax on consumption, labour income and capital income, respectively. Finally, our results indicate that the level of output per adult households at the first period of the government presence becomes 0.45% smaller than at the original steady-state when the government taxes consumption or

[24] Who are still identical in the human capital level at the second period of analysis

capital income. With the labour income tax in place, the output per adult household becomes 0.48% smaller than at the first period of analysis.

Moving forward to a third period of the analysis when the government subsidises the education attainment at 10% level, given the individual choices that take place at this period and combined with the evolution of the model population, our results indicate that when the government uses the tax on consumption to finance and balance its budget, the tax rate is equal to 0.53%. When the government implements the tax on labour income instead, the tax rate at the third period of analysis is equal to 0.63%; while with the capital income tax in use, the tax rate is set to 0.96%.

As a result of the changes of education investment that took place in the previous period, combined with the change in the human capital of parents at the second period of analysis, we observe that the human capital of the households from higher ability group increases by 4.18% and becomes 14.60% larger than at the original steady-state, when government implements tax on consumption or capital income. With the labour income tax, the human capital of adult households from higher ability group at the second period of the government presence increases by 3.80% from the previous period and becomes 14.18% larger than at the original steady-state. For lower ability households at the third period of analysis when the government provides subsidy for education of 10% with tax on consumption or capital income, we observe an increase in the human capital of adult household from this ability group by 8.79% which becomes 2.09% smaller than at the original steady-state. With the tax on labour income, the human capital of adult households increases by 8.39% and becomes 2.45% lower than at the initial steady-state.

This original increase in the human capital of adult households from lower ability group, despite a decrease or absence of change in the education investment of this ability group in the past period, can be attributed to the fact that the economy naturally begins to resolve the heterogeneity and begins a transition to the initial steady-state. Particularly, there are two effects that influence a change in the human capital of past generation of children. First, a lower human capital of parents at the second period of analysis from lower ability group diminishes the human capital adult households at the third period of a simulation. An the same time, however, the average level of human capital in the economy that is larger than the human capital of adult households from lower ability group at the second period of analysis improves the human capital of adult households in the third period. Given the current model parametrisation, the net of these two effects is an increase in the human capital of adult households from lower ability households.

With the fertility decisions that have taken place at the second period of analysis, our results indicate that at the third period when the government provides subsidy for education of 10% and finances and balances its budget with tax on consumption or capital income, the population size of the households from higher ability group decreases by 3.32% below the original steady-state; while the population of lower ability households remains unchanged. With the labour income tax in place instead, the population of higher ability households is 2.61% lower than at the original steady-state, while the population of lower ability households is 0.75% larger than at the original steady-state.

Therefore, with the change in the human capital and the population size of each ability group, our results indicate that at the third period of analysis with subsidy for education of 10% that the government finances with tax on consumption or capital income, the average human capital of adult households increases by 6.12% above the original steady-state. With the labour income tax, the average level of human capital becomes 5.72% larger than at the first period of analysis. Overall, our results indicate that when the government finances its policy with tax on consumption or capital income, the relative human capital level of the household from higher ability group decreases by 1.82% and becomes 7.99% larger than at the original steady-state. For the households from lower ability group, however, the relative human capital increases by 2.52% and becomes 7.73% smaller than at the initial steady-state. For the case when the government provides subsidy for education with tax on labour income, the relative human capital level of higher ability households at the third period of analysis decreases by 1.82% and becomes 8% larger than at the original steady-state. For the households from lower ability group, the relative human capital increases by 2.52% and it is 7.73% larger than at the first period of analysis.

Due to the change in the relative human capital levels and given the government policy in place, we find that the level of education provision by higher ability households at the third period of analysis decreases by 2.46% and becomes 29.29% larger than at the original steady-state, when the government implements tax on consumption or capital income. With the labour income tax, the education investment of the households from higher ability group decreases by 2.46% and becomes 28.19% larger than at the initial equilibrium. For the households from lower ability group, the choice for education increases and becomes 3.70% larger than at the original steady-state when the government implements tax on consumption or capital income. With the labour income tax, the education provision for children of households from lower ability group increases by 3.70% and becomes 2.74% larger than at the first period of analysis.

Next, driven by the change in the levels of relative human capital and in the choices for education provision, our results indicate that at the second period of the government presence with subsidy for education of 10% and with tax on consumption or capital income, the fertility choices of adult households from higher ability group increase by 0.28% and become 3.05% lower than at the original steady-state. With the labour income tax, the fertility level of this ability group rise by 0.28% and become 2.33% smaller than at the initial steady-state. For the households from lower ability group, when the government uses tax on consumption or capital income to subsidise education at 10% level, the fertility decreases and becomes 0.46% smaller than at the original steady-state. With the labour income tax, this indicator reduces by 0.46% and it becomes 0.29% larger than at the original steady-state.

As a result of the changes in the human capital, population size, and the decision of adult households for education provision and childbearing, at the third period of analysis when the government provides subsidy for education of 10% and taxes consumption or capital income, the supply of the effective labour force per adult household of the individuals from higher ability group increases by 0.73% and becomes 10.04% larger that at the original steady-state. With the labour income tax, the supply of the effective labour force per adult household from higher ability groups increases by 1.09% and becomes 10.39% larger than at the initial equilibrium. For the households from lower ability group, when the government finances the subsidy program with tax on consumption or capital income, the supply of the effective labour force per adult household increases by 8.79% and becomes 2.73% smaller than at the original steady-state; and with the tax on the labour income, the supply of the effective labour force per adult household from this ability group increases by 9.20% and becomes 2.41% lower than at the first period of analysis. Overall, according to our results, given the structure of the model population, at the third period of analysis when government provides subsidy for education of 10% and levies the tax either on consumption or capital income, the aggregate effective labour force per adult household increases by 4.25% and becomes 3.55% larger than at the original steady-state. With the labour income tax, this indicator increases by 4.63% and becomes 3.88% larger than at the initial equilibrium.

Next, given the change in the decision for saving that took place at the second period of analysis, and the change in the past and present population size, our results indicate that at the third period of analysis when the government provides subsidy for education of 10% and finances and balances its budget with tax on consumption or capital income, the supply of the physical capital stock per adult household from the households from higher ability group increases and becomes 10.25% larger than at the original steady-state. With the labour income tax, this indicator increases by 9.57% above the original steady-state. The supply of the physical capital stock per adult household from the households of lower ability group, however, decreases and becomes 9.80% smaller than at the first period of analysis, when the government taxes consumption or capital income. With the tax on labour income, the supply of the physical capital stock per adult household from the households of lower ability group reduces and is 10.35% smaller than at the original steady-state. Overall, our results indicate that at the third period of analysis with subsidy for education of 10% and tax on consumption or capital income, the aggregate physical capital stock per adult household increases by 0.05%. With the labour income tax, however, this indicator becomes 0.56% smaller than at the original equilibrium.

Due to these changes in the factor inputs, our results indicate that at the third period of analysis with subsidy for education of 10% and with tax on consumption or capital income, the real wage rate decreases by 1.36% and becomes 1.14% lower than at the original steady-state. The real interest rate, however, increases by 3.73% and becomes 3.11% larger than at the initial equilibrium. With the labour income tax in place instead, the real wage rate decreases by 1.68%

and becomes 1.44% lower than at the first period of analysis, and the real interest rate increases by 4.64% and becomes 3.96% larger than at the original steady-state.

As a result of these alterations that take place at the second period of the government presence with subsidy for education of 10%, the consumption of adult households from higher ability group increases by 2.76%, 2.06% and 2.76% and becomes 12.69%, 11.82% and 13.29% larger than at the original steady-state, when the government taxes consumption, labour income and capital income, respectively. In turn, the consumption of the adult households from lower ability group increases by 7.32%, 6.56% and 7.32% and becomes 3.71%, 4.47% and 3.20% lower than at the original steady-state, when government utilises tax on consumption, labour income and capital income, respectively. Next, the dynamics for saving of households from both ability groups follows the dynamics for consumption of adult households, when the government finances and balances its budget with tax on labour income and capital income. With the tax on consumption in place, saving of the households from higher ability group increases by 2.76% and becomes 13.29% larger than at the original steady-state; and the saving of adult households from lower ability group increases by 7.32% and becomes 3.20% lower than at the original steady-state. Lastly, at the third period of analysis with subsidy for education of 10%, the consumption of the elderly households from higher ability group increases by 13.31%, 14.07% and 13.33%, and becomes 12.20%, 12.81% and 11.99% larger than at the original steady-state, when the government taxes consumption, labour income and capital income, respectively. The consumption of the elderly households from lower ability group is found to decrease by 7.30%, 6.67% and 7.28%, which becomes 8.20%, 7.70% and 8.38% smaller than at the original steady-state, respectively.

Finally, given all these changes that we observe at the third period of analysis when the government provides subsidy for education of 10%, the utility of the households from higher ability group increases by 13.40%, 12.37% and 13.52%, and becomes 27.68%, 26.10% and 26.68% higher than at the original steady state, when the government uses tax on consumption, labour income and capital income to finance and balance the budget, respectively. For the households from lower ability group, the utility is found to increase by 8.16%, 7.37% and 8.16%, which becomes 10.58%, 11.45% and 9.85% smaller than at the original steady-state, when the government uses tax on consumption, labour income and capital income, respectively. Given these changes and the structure of the population at the third period of analysis, the welfare in the economy increases by 9.98%, 9.12% and 10.06%, and becomes 7.87%, 7.01% and 8.60% larger than at the original steady-state. Finally, our results indicate that at the second period of the government presence with subsidy for education of 10% and with tax on consumption or capital income, the output per adult household increases by 2.83% and becomes 2.37% larger than at the original steady-state. And with the labour income tax in use, this indicator increases by 2.87% and becomes 2.38% larger than at the original equilibrium.

The behaviour for the majority of the variables summarised for the second period of the government presence would continue until the economy reaches the corresponding final steady-states. The exceptions are the real interest rate, which will begin to decrease starting from the fourth period of analysis; the real wage rate which will begin to increase starting form the fourth period of analysis; the consumption of the elderly households which begins to increase for the households from lower ability group and decrease for the households from higher ability groups starting from the fourth period of analysis; and the supply of the physical capital stock per adult household by lower ability households which begins to increase starting from the fourth period of analysis.

According to our results, at the final steady-state with subsidy for education that the government finances with tax on consumption, the tax rate is equal to 0.53%. When the economy reaches the final steady-state with the tax on the labour income instead, the tax rate is equal to 0.63%. Finally, when the government implements the tax on capital income, the tax rate is equal to 1.02%.

As a result of the considered government subsidy program, children in the economy receive the education which is 16.28% larger than at the original steady-state, when the government finances and balances its budget with tax on consumption or capital income. With the labour income tax, the level of education investment is 15.25% larger than at the original steady-state.

Overall, according to our results, the economy reaches the final steady-states with the level of the human capital that is identical between the household ability groups. Our results indicate that at the final steady-state with subsidy for education of 10% that the government finances with either tax on consumption or capital income, the human capital of all households becomes 9.27% larger than at the original steady-state. With the labour income tax, the human capital of the households becomes 8.69% larger than at the original equilibrium.

However, as a result of a considered subsidy policy, the households decide to decrease their fertility decisions by 1.85% below the level at the original steady-state when government implements tax on consumption or capital income. With the labour income tax, the fertility decisions become 1.12% lower than at the original steady-state. Despite a unique human capital level that the model population has at the final steady-state, our results indicate that due to an initial difference in the fertility choices between the (formally) higher and lower ability households, the majority of the population at the final steady-state would consist of the households that have been previously a part of lower ability group. Additionally, our results indicate that the total population of the households at the final steady-state with subsidy for education would be lower than at the beginning of the analysis.

Moving forward, according to our results, at the final steady-state with the subsidy rate for education of 10% that the government finances with either tax on consumption or capital income, the (aggregate) effective labour force per adult household becomes 6.53% larger than at the original steady-state. With the labour income tax, this indicator becomes 6.69% larger than at the original equilibrium. Next, at final steady-state with tax on consumption or capital income, the (aggregate) physical capital stock per adult household increases by 10.66% above the initial steady-state. With the tax on labour income, this variable becomes 8.67% larger than at the original steady-state. With these changes in the factor inputs, at the final steady-state with tax on consumption or capital income, the real wage rate becomes 1.27% larger than at the original steady-state, and the real interest rate becomes 3.36% smaller than at the original steady-state. When the government implements the tax on labour income instead, at the final steady-state, the real wage rate increases 0.61% above the original steady-state, and the real interest rate becomes 1.63% lower than at the first period of analysis.

Due to these changes, our results indicate that at the final steady-state with subsidy for education of 10%, the consumption of adult households becomes 10.08%, 8.67% and 10.66%

larger than at the original steady-state, when the government taxes consumption, labour income and capital income. The dynamics for saving follows the dynamics for consumption of adult households when the government finances the subsidy program with tax on labour income or tax on capital income. With the tax on consumption, at the final steady-state with 10% subsidy rate for education, saving of the households becomes 10.66% larger than at the original steady-state. Lastly, at the final steady-state with publicly provided subsidy for education at 10% level, the consumption of the elderly households becomes 7.32%, 7.35% and 7.07% larger than at the original steady-state, when the government taxes consumption, labour income and capital income, respectively.

Finally, our results indicate that at the final steady-state with subsidy for education of 10%, the welfare in the model economy becomes 20.20%, 18.28% and 20.92% larger than at the original steady-state. The level of output per adult household at the final steady-state with tax on consumption or capital income becomes 7.89% larger than at the original steady-state; and, with the case of labour income taxation, the real output per adult household becomes 7.35% larger than at the first period of analysis.

2.5.2 Subsidy for children

In the final policy experiment that we conduct in this chapter, we consider the influence of the subsidy for fertility of 10% that the government provides with either tax on consumption, labour income or capital income in the environment with heterogeneous households. As before, we begin our discussion at the second period of analysis at the time when the heterogeneity in the human capital level of the households is introduced and when the government enters the model economy.

According to our results, at the second period of analysis when the government provides the households with the subsidy rate for fertility of 10% and when the government levies the tax on consumption to finance and balance its budget, given the decisions that take place at the second period of analysis, the tax rate on consumption is set to 0.46%. When the government implements the labour income tax instead, the tax rate is equal to 0.55%; while with the capital income tax in use, the tax rate is set to 0.86%.

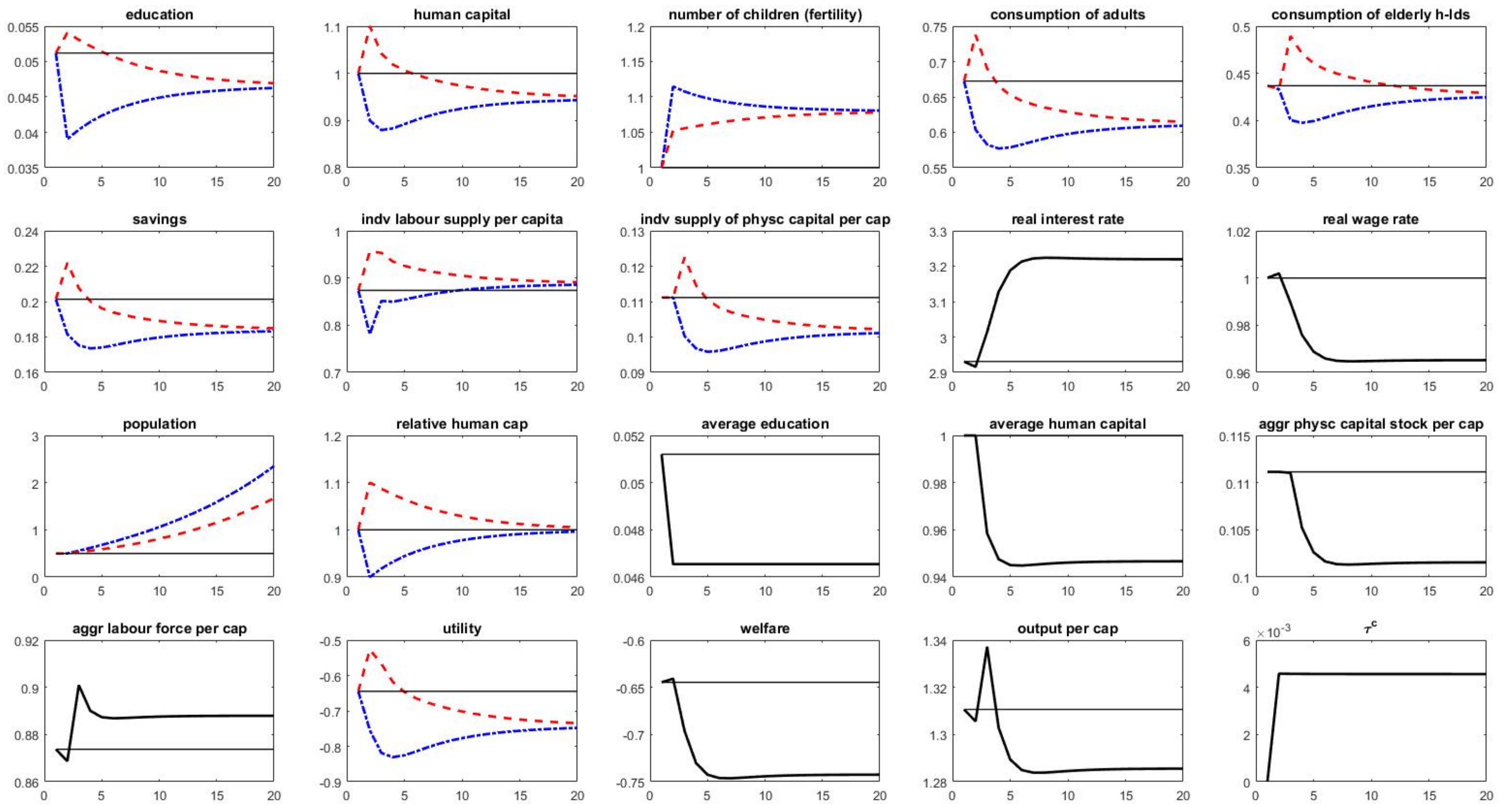

Figure 2.7: Transition path of heterogeneous agent economy with two ability types of households. Government imposes tax on consumption and provides subsidy rate for childbearing decisions of 0.1
Note: High ability type is shown as red dashed line. Low ability type is shown as blue dash-doted line

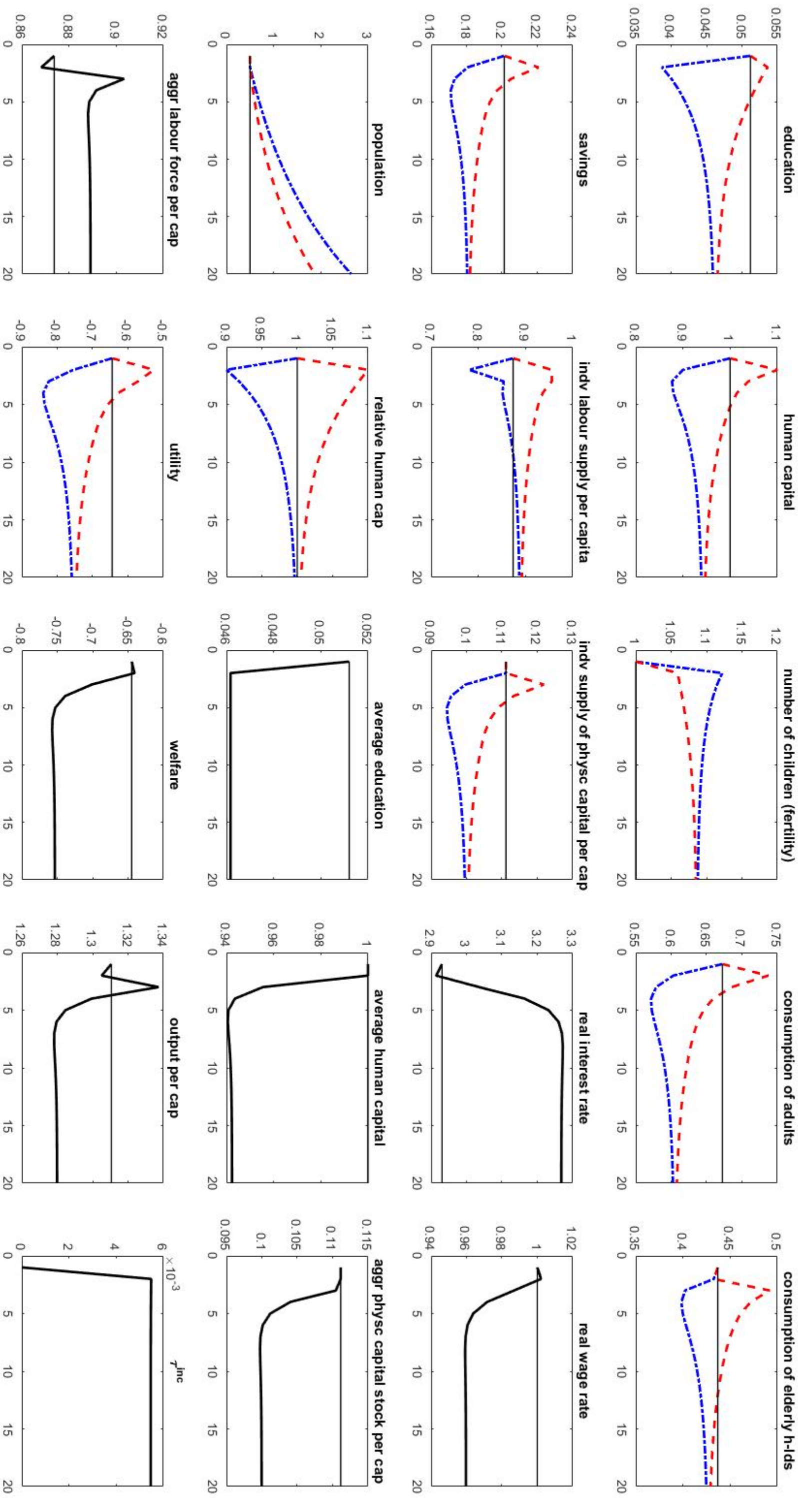

Figure 2.8: Transition path of heterogeneous agent economy with two ability types of households. Government imposes tax on labour income and provides subsidy rate for childbearing decisions of 0.1

Note: High ability type is shown as red dashed line. Low ability type is shown as blue dash-doted line

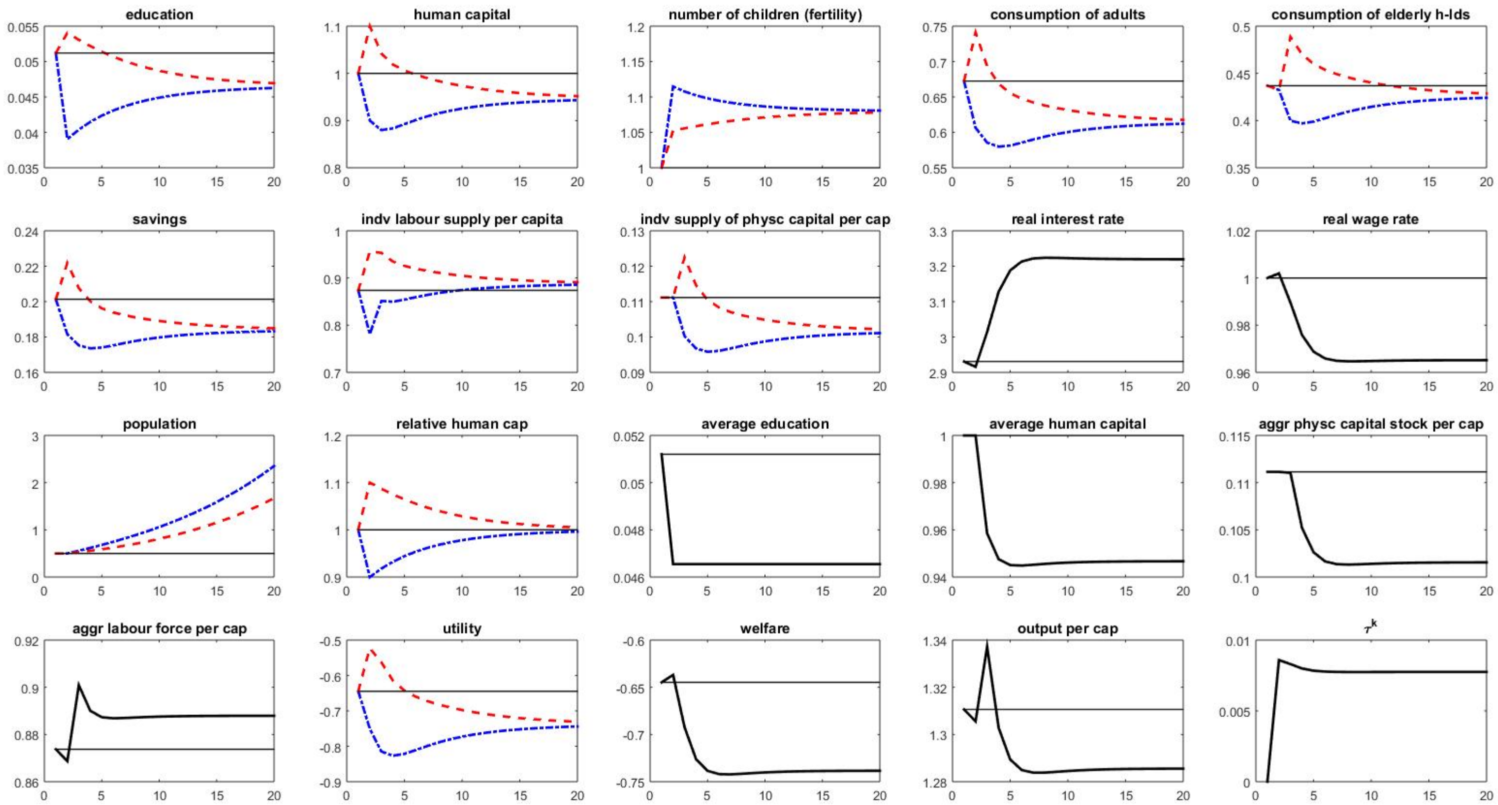

Figure 2.9: Transition path of heterogeneous agent economy with two ability types of households. Government imposes tax on capital income and provides subsidy rate for childbearing decisions of 0.1

Note: High ability type is shown as red dashed line. Low ability type is shown as blue dash-doted line

With the exogenously introduced heterogeneity in the human capital at the second period of analysis, the fifty percent of the population of adult households becomes a part of higher ability group with the level of human capital being 10% larger than at the original steady-state. The remaining half of the population of adult households formulates a lower ability group with the human capital that is 10% smaller than at the original steady-state. Given this even distribution of the adult households between two ability groups, the level of the average human capital in the economy at the first period of the government presence remains to be identical to the original steady-state. Therefore, with absence of change in the average human capital, and with the corresponding changes in the individual human capital levels of adult households, our results indicate that at the second period of analysis, the relative human capital of adult households from higher ability group becomes 10% larger than at the original steady-state, but the relative human capital of the households from lower ability group decreases 10% below the initial equilibrium level.

Therefore, as a result of these changes in the relative human capital combined with the government policy which provides the subsidy for fertility at 10% level, the adult households from higher ability group increase their childbearing decisions by 5.25% above the original steady-state, when the government finances and balances its budget with tax on consumption or capital income. With the labour income tax, at the second period of analysis, the fertility decisions of the households from higher ability group increase by 5.92% above the initial steady-state. For the households from lower ability group, the fertility decisions at the second period of analysis become 11.47% larger than at the original steady-state, when the government utilises the tax on consumption or capital income. With the labour income tax, the fertility level of the households from lower ability group rises by 12.20% above the original steady-state.

Given this change in the fertility decisions[25] and combined with the change in the relative human capital of adult households, our results indicate that at the first period of the government presence with subsidy for fertility of 10% that the government finances with the tax on consumption or capital income, the adult households from higher ability group provide their children with 5.56% larger level of the education than at the original steady-state. With the labour

[25] Please note that, as before, the households' decisions for education and fertility are simultaneous

income tax, the education investment of higher ability households becomes 4.75% larger than at the initial steady-state. For the households from lower ability group, however, the choice for education becomes 23.74% smaller than at the original steady-state, when government finances and balances its budget with either tax on consumption or capital income. With the labour income tax, the choice for education becomes 24.39% lower than at the first period of analysis.

Due to the change in the level of human capital of adult households that arises due to an exogenous introduction of heterogeneity, together with the change in the fertility and education investment decisions of the adult households, our results indicate that at the second period of analysis with subsidy rate for fertility of 10% that the government finances with tax on consumption or capital income, the supply of the effective labour force per adult household by the members of higher ability group becomes 9.44% larger than at the original steady-state. With the labour income tax, the supply of the effective labour force per adult household by the adult households from higher ability group becomes 9.39% larger than at the original steady-state. Oppositely, our results indicate that the supply of effective labour force per adult household by lower ability households at the second period of analysis becomes 10.59% lower than at the original steady-state, when the government finances the subsidy program with tax on consumption or capital income. With the labour income tax, this indicator becomes 10.64% lower than at the first period of analysis. Given that at the first period of the government presence the population of the adult households is distributed between ability groups equally, our results indicate that the aggregate effective labour force per adult household decreases by 0.58% when the government implements tax on consumption or capital income; and with the labour income tax, this indicator becomes 0.63% lower than at the original steady-state.

On the other hand, however, at the second period of analysis, the aggregate physical capital stock per adult household does not change and it is identical to the original steady-state, since both the population and aggregate level of saving have been determined at the first period of analysis before the government presence. Overall, our results indicate that at the second period of analysis when the government provides subsidy for fertility of 10% and taxes consumption or capital income, the real wage rate becomes 0.19% larger than at the original steady-state, but the real interest rate becomes 0.52% smaller than at the first period of analysis. With the labour income tax in place instead, the real wage rate becomes 0.21% larger than at the initial equilibrium, and the real interest rate decreases 0.56% below the original steady-state.

With the change in the human capital and the factor prices, and as a result of the tax rates that finance the government budget in a particular case, our results indicate that at the second period of analysis when the government provides subsidy rate for fertility of 10%, the consumption of the adult households from higher ability group becomes 9.71%, 9.62% and 10.21% larger than at the original steady-state, when the tax on consumption, labour income and capital income is in place, respectively. For the adult households from lower ability group, however, the consumption decreases by 10.24%, 10.31% and 9.83% below the original steady-state, with the tax on consumption, labour income and capital income, respectively. Next, the dynamics for saving follows the dynamics for consumption of adult households when the government finances and balances its budget with tax on labour income and capital income. With the tax on consumption, saving of the households from higher ability group increases by 10.21% from the first period of analysis, while saving of the households from lower ability group becomes 9.83% lower than at the original steady-state. Lastly, the consumption of the elderly households becomes 0.84%, 0.96% and 1.02% smaller than at the original steady-state, when the government taxes consumption, labour income and the capital income, respectively[26].

Finally, according to our results, at the second period of analysis, when the government provides subsidy for fertility of 10%, the utility of the households from higher ability group increases by 18.25%, 18.87% and 18.25% when the government implements tax on consumption, labour income and capital income, respectively. The utility of lower ability households, however, becomes 17.05%, 17.03% and 16.43% smaller than at the original steady-state. Given the distribution of the population, our results indicate that the level of welfare in the economy increases by 0.60%, 0.61% and 1.22% when the tax on consumption, labour income and capital income is in place, respectively. Our results also suggest, however, that at the first period of the government presence, the level of output per adult household decreases by 0.39% with tax on consumption or capital income, and by 0.42% when the labour income tax is implemented

[26]Please note that at the second period of analysis, the elderly households are still homogeneous in the human capital

instead.

Moving forward to a third period of analysis, the government adjusts the tax rate on capital income to 0.83% to provide the subsidy rate for fertility at 10% with the balanced budget. The tax rates on consumption and labour income, however, remain unchanged.

Next, based on the dynamics that we observe in the human capital (of higher ability households), the economy begins its approach to a final steady-state at the third period of analysis. Particularly, the human capital of adult households from higher ability groups decreases 5.28% from the previous period and it becomes 4.20% larger than at the original steady-state, when the government finances and balances its budget with tax on consumption or capital income. With the labour income tax, the human capital of higher adult ability households decreases by 5.57% and becomes 3.38% larger than at the first period of analysis. For the households from a lower ability group, however, the human capital also depreciates due to a sizeable past period decrease in the education attainment. Our results indicate that at the third period of analysis with subsidy for fertility of 10% that the government finances with tax on consumption or capital income, the human capital of adult households from lower ability group decreases by 2.24% and becomes 12.02% lower than at the original steady-state. With the labour income tax, the human capital of adult households from lower ability group decreases by 2.56% and becomes 12.31% larger than at the initial equilibrium.

With an increase in the fertility choices at the second period of analysis of the households from lower ability group being more prominent than of households of higher ability group, our results indicate an increase in the share of the population that is a part of lower ability group and a decrease in the share of the population that is a part of a higher ability group starting from a third period of analysis. Therefore, given this structural change in the population of adult households, combined with the change in the human capital of each ability group, our results indicate that at the second period of the government presence with subsidy for fertility of 10%, the average level of human capital of adult households decreases by 4.14% when the government implements tax on consumption or capital income. With the labour income tax, the average human capital of adult households becomes 4.45% lower than at the original steady-state.

Consequently, our results indicate that at the third period of analysis with subsidy for fertility of 10%, the relative human capital of adult households from higher ability group decreases by 1.18% and becomes 8.70% larger than at the original steady-state, when the government taxes consumption or capital income. With the labour income tax, this indicator decreases by 1.17% and becomes 8.71% larger than at the first period of analysis. For the households from lower ability group, the relative human capital increases by 1.98% and becomes 8.22% lower than at the original steady-state when the government uses any of the tax options considered.

As a result, at the third period of analysis with subsidy for fertility of 10% that government finances with tax on consumption or capital income, the decision for education provision of the households from higher ability group decreases by 1.80% and becomes 3.66% larger than at the original steady-state, and the education investment of the households from lower ability group increases by 3.43% and becomes 21.13% lower than at the original steady-state. When the government implements the tax on labour income instead, the decision for education of the households from higher ability group decreases by 1.79% and becomes 2.87% smaller than at the original steady-state, while the education provision of lower ability households increases by 3.43% and becomes 21.80% smaller than at the initial equilibrium.

The fertility choices of the households from both ability groups adjust as well. According to our results, at the third period of analysis with subsidy for fertility of 10%, the childbearing decision of the households from higher ability group increases by 0.30% and becomes 5.57% larger than at the initial equilibrium, when the government taxes consumption or capital income. With the labour income tax, the fertility level of the households from higher ability group increases by 0.30% and becomes 6.24% larger than at the original steady-state. For the households from lower ability group the decision for children decreases by 0.63% and becomes 10.77% larger than at the original steady-state, when the government finances and balances its budget with tax on consumption or capital income. With the labour income tax in use, the fertility level of the households from lower ability group decreases by 0.63% and becomes 11.50% larger than at the original steady-state.

With these changes in the human capital, population size and decisions for education investment and fertility, our results indicate that at the third period of the analysis with 10% subsidy rate for fertility, the supply of the effective labour force per adult household from the higher ability group decreases by 0.61% and becomes 9.10% larger than at the original steady-state, when the tax on consumption or capital income is in place. With the labour income tax, however, the supply of the effective labour force per adult household from higher ability group increases by 0.01% and becomes 9.40% larger than at the initial equilibrium. Meanwhile, the supply of the effective labour force per adult household from lower ability group increases by 8.99% and becomes 2.56% lower than at the original steady-state, when the tax on consumption or capital income is levied. With the labour income tax, this indicator increases by 9.35% and becomes 2.29% smaller than at the first period of the analysis. Given these changes in the supply of the effective labour force per adult household of both ability groups, and combined with the change in the composition of the population, our results indicate that at the third period of analysis the aggregate effective labour force per adult household increases by 3.71% and becomes 3.11% larger than at the original steady-state when the government taxes consumption or capital income; and with the labour income tax it increases by 4.04% and becomes 3.39% larger than at the initial equilibrium.

Next, as a result of the change in saving of past generation, and combined with the past and present change in the population of adult households, according to our results, at the third period of analysis with publicly provided subsidy for fertility of 10%, the aggregate physical capital stock per adult household becomes 0.09% lower than at the original steady-state, when the government implements tax on consumption or capital income. With the labour income tax in use, this indicator becomes 0.63% lower than at the original steady-state.

Therefore, our results indicate that at second period of the government presence with subsidy for fertility of 10% and with either tax on consumption or capital income, the real wage rate decreases by 1.23% and becomes 1.05% lower than at the original steady-state; and the real interest rate increases by 3.38% and becomes 2.85% larger than at the original equilibrium. With the tax on labour income, the real wage rate is found to reduce by 1.52% to become 1.31% lower than at the original steady-state; while the real interest rate increases by 4.17% and becomes 3.59% larger than at the first period of analysis.

With the change in the factor prices, and driven by the variation in the human capital, and the resulted tax rates, we find that at the third period of analysis with subsidy for fertility of 10%, the consumption of adult households from higher ability group decreases by 6.45%, 7.01% and 6.45%, and it becomes 2.64%, 1.95% and 3.11% larger than at original steady state, when the government finances and balances its budget with tax on consumption, labour income and capital income, respectively. For the adult households from lower ability group, the consumption decreases by 3.45%, 4.04% and 3.45%, and it becomes 13.34%, 13.93% and 12.94% below the original steady-state when the government taxes consumption, labour income and capital income, respectively. The dynamics for saving follows the dynamics for consumption of adult households when the government finances and balances its budget with tax on the labour income and capital income. With the tax on consumption, saving of the households from higher ability group decreases by 6.45% and becomes 3.11% larger than at the original steady-state; while saving of households from lower ability group decreases by 3.45% and becomes 12.94% smaller than at the original equilibrium. Lastly, the consumption of the elderly households from higher ability group increases by 12.99%, 13.65% and 13.01%, and it becomes 12.04%, 12.56%, 11.85% larger than at the original steady-state, when the tax on consumption, labour income and capital income is in place, respectively. For the households from lower ability group, however, the consumption of the elderly households decreases by 7.55%, 7.01% and 7.54%, and becomes 8.33%, 7.91% and 8.48% lower than at the first period of analysis, when the government finances and balances its budget with tax on consumption, labour income and capital income, respectively.

As a result of these changes that take place at the third period of analysis with the publicly provided subsidy for fertility of 10%, our results indicate that the level of utility of the households from higher ability group decreases by 7.53%, 8.45%, 7.58%, and becomes 12.09%, 11.35%, 12.72% larger than at the original steady-state, when the government taxes consumption, labour income and capital income, respectively. For the households from lower ability group the utility decreases by 8.49%, 9.15%, 8.53%, and becomes 26.99%, 27.75%, 26.36% lower than at the original steady-state, when the tax on consumption, labour income and capital income is place, respectively. Overall, at the second period of the government presence, given the composition of the population, the welfare reduces by 8.66%, 9.43% and 8.70%, and it becomes 8.01%, 8.76% and 7.38% lower than the original steady-state, when the government finances and balances its budget with tax on consumption, labour income and capital income, respectively. Finally, our results indicate that the output per adult household increases by 2.42% which becomes 2.03% larger than at the original steady-state, with tax on consumption or capital income in place. When the government utilises the tax on the labour income instead, the output per adult household increases by 2.46% and it is 2.03% larger than at the original steady-state.

The behaviour for the majority of the indicators depicted for the second period of the government presence with 10% subsidy rate for fertility would continue until the economy reaches the corresponding final steady-states. However, our results indicate that since the supply of the physical capital follows the decisions of the households to save with the lag of one generation, starting with the fourth period of analysis, given the change in the composition and size of the model population, the aggregate physical capital stock per adult household would begin its decline. The level of effective labour force per adult household would also decrease due to the loss in the human capital, increase in fertility and rise in the population size. As a result of the fall in both factor inputs, starting from the fourth period of analysis, the output per adult household decreases below the original equilibrium.

When the economy reaches the final steady-states, the government sets the tax rate on consumption, labour income and capital income to 0.46%, 0.55% and 0.78%, respectively, to be able to continue a provision of the subsidy rate for fertility of 10%.

Overall, at the final steady-states with these policy instruments in place, the human capital of the households between the ability groups becomes again equal to one another, and according to our results, with tax on consumption or capital income, the human capital in the economy becomes 5.33% lower than at the original steady-state; while with the labour income tax, it decreases 5.76% below the first period of analysis. Population of the households, that at the final steady-state consists dominantly of the households from a previously considered lower ability group, increases in the size, however.

Our results indicate that the education provision at the final steady-state with tax on consumption or capital income becomes 9.09% lower than at the original steady-state. With the labour income tax, the education investment becomes 9.82% lower than at the initial equilibrium. The fertility decisions, however, become 7.96% larger than at the original steady-state, when the government finances and balances its budget with tax on consumption or capital income. With the labour income tax, the childbearing at the final steady-state becomes 8.66% larger than at the initial equilibrium.

Moving forward, at the final steady-state with subsidy for fertility of 10% and tax on consumption or capital income, the aggregate effective labour force per adult household becomes 1.63% larger than at the original steady-state; but the aggregate physical capital stock per adult household becomes 8.62% lower than at the first period of analysis. With the labour income tax, the aggregate effective labour force per adult household becomes 1.76% larger than at the original steady-state; while the aggregate physical capital stock per adult household decreases 10.05% below the original equilibrium.

With these changes in the factor inputs, our results indicate that the real wage rate decreases by 3.48% with the tax on consumption or capital income, and it becomes 4.03% lower than at the original steady-state when the labour income tax finances and balances the government budget. Meanwhile, the real interest rate becomes 9.85% larger than at the original steady-state, when the government implements tax on consumption or capital income. With the labour income tax, this indicator increases 11.49% above the original steady-state.

Therefore, according to our results, at the final steady-state with subsidy for fertility of 10%, the consumption of adult households becomes 9.04%, 10.05% and 8.62% smaller than at the original steady-state, when the tax on consumption, labour income and capital income is in place, respectively. The dynamics for saving follows the one for consumption of adult households when the government implements the tax on labour income and capital income. With the tax on consumption, saving of the households at the final steady-state becomes 8.62% smaller than at the original steady-state. Finally, the consumption of the elderly households is 2.36%, 2.34% and 2.49% lower than at the original steady-state.

In conclusion, at the final steady-state with subsidy rate for fertility of 10%, the welfare in the model economy becomes 15.16%, 16.83% and 14.52% lower than at the original steady-state, when the government imposes the tax on consumption, labour income and capital income, respectively. Lastly, our results indicate that with the tax on consumption or capital income, the output per adult household becomes 1.91% lower than at the original steady-state; and with the tax on labour income, it decreases 2.34% below the first period of analysis.

2.6 Conclusion

In this chapter, we analysed the influence of the publicly provided subsidy for education and fertility on the long-run development of the economy. For our analysis we utilised the model of overlapping generations of *de la* Croix and Doepke (2003), which we expanded with the government sector. We began our analysis with the environment of the households that are homogeneous in the human capital. In the second part of our policy experiments, however, we considered a case where the model population is heterogeneous and consists of two ability groups. All of our policy experiments in this chapter have been conducted in the deterministic environment.

Based on our analysis with the households that are both homogeneous and heterogeneous in the human capital, the subsidy for education has been found to produce the long-run improvement in the private education attainment, human capital, physical capital stock and effective labour force per adult household, output per adult household and the welfare in the economy. The immediate effect of this policy, however, has been a decrease in the level of welfare due to the associated fall in the consumption of adult and elderly households. With the case of subsidy for fertility, we have found the opposite results. Immediately after the subsidy for fertility has been introduced, the level of the welfare in the economy has increased above the original steady-state level due to the initial rise in the consumption of adult households and their decision for children; however, as the economy approached the final steady-states, the education, human capital, physical capital stock per adult household, output per adult household and the welfare all became lower than at the original steady-state. Opposite to the case with subsidy for education, subsidy for fertility increased the decisions of the households for number of children to have which resulted in a (considerably) larger population size than at the original steady-state.

In spite of the completely opposite outcomes that we have obtained with subsidy for education and fertility, based on our results, both of these subsidy options have produced the final steady-states with complete equality in the human capital between the households that originally had a heterogeneous human capital level. This result is obtained because the average human capital in the economy is one of the variables that defines the human capital of a child, and since the level of private education investment depends on the relative human capital which is a fraction between the human capital of adult households and the average level of human capital – all of which contributes to the final steady-state with homogeneity in the human capital[27].

Despite that both subsidy for education and subsidy for fertility individually have been able to remove the inequality in the human capital, neither of these individual subsidy options have been able to remove the trade-off between the education provision and childbearing that adult households experience. As a potential solution, we analysed the influence of the policy mix which included a simultaneous provision of subsidy for education and subsidy for fertility. Based on the current model structure and parametrisation, the subsidy rate for education provided by the government should be at least 1.6 times larger than the subsidy rate for fertility in order to be able to resolve this parental trade-off under all considered tax options. For any other subsidy rates, the conclusions remained in line with the previous policy experiments where we only analysed the role of subsidy for fertility.

Finally, based on our results, the flat tax rate on the labour income that financed and balanced the government budget resulted in the lowest levels of welfare independently from the subsidy option. Additionally, the labour income taxation has been found to produce disincentive for education investment that resulted in a lower level of human capital in comparison to the other considered tax options. Due to the parental 'quality-quantity' trad-off, however, when the government implemented the tax on labour income, a lower level of education chosen by the parents manifested in a larger level of fertility and overall population size. Lastly, based on our results, the tax on capital income that financed the provision of subsidy for education produced the largest welfare improvement, whereas, the labour income taxation and subsidy for fertility reduced the welfare the most.

[27]Another crucial factor which leads to the equality in the human capital between different ability groups is that the analysis in this chapter has been conducted in the deterministic environment without the presence of any uncertainty, which is opposite to chapter 3 and 4

Chapter 3

Subsidy for education and fertility: long-run evolution of the economy with heterogeneous households and idiosyncratic shocks to the human capital

3.1 Introduction

We continue the discussion of the long-run effects of the government support for education and fertility in this paper. In our previous analysis we utilised the model of overlapping generations of *de la* Croix and Doepke (2003) which we have extended with the government sector. We introduced the government in form of authority that can provide the subsidy for education and fertility which it can finance with a tax on consumption, labour income or capital income. According to our results from the second chapter, in the deterministic environment, a subsidy for education and fertility both led to the final steady-states with complete equality in the human capital distribution across the population. (We found, however, that resulted population from these two subsidy programs are completely different). With a subsidy for education, we observed an increase in the optimal education provision across the population, which improved the human capital levels in comparison to the original steady-state. This improvement, however, has been obtained at a cost of a decrease in fertility decisions which negatively influenced the population size. With a subsidy for fertility however, our results indicated the opposite: an improvement in the population size but deprivation of the human capital, which arose from a parental 'quality-quantity' trade-off for children.

In the present paper, we extend the model economy that has been utilised in our previous deterministic analysis further. We assume that the young households receive idiosyncratic shocks during their human capital accumulation. Therefore, the objective of this paper is to examine the influence of the government policies that subsidise education decisions and family planning through a tax on consumption, labour income or capital income on the long-run economic development in the environment where human capital formation contains uncertainty.

According to the evidence presented in the literature[1], the issue of the unexpected variations in the human capital has been a long-lasting topic for the discussion in the academic work in Economics and Psychology. Based on the earlier and recent conclusions, the family environment seems to be one of the main causes for explaining the variations in the realised human capital.

Based on the recent evidence, the presence of uncertainty in the human capital development

[1] Please refer to the Literature review section for more details

introduces the additional challenges in forecasting the returns to schooling and it results in the inefficient education investments. Furthermore, there is a failure of insurance markets in the case with the risks in the human capital due to the moral hazard problem. In attempt to minimise this uncertainty and to provide the coverage against the risk in the human capital, the various policy options have been proposed and studied, one of which has been a provision of subsidies for education. According to the literature that has utilised theoretical modelling, the subsidy for education results in an improvement in the education attainment and welfare in the environment with uncertainty in the human capital formation. Additionally, it reduces the inequality in the consumption and wealth, improves intergenerational income mobility, and provides a partial social insurance against the unobserved risks in the human capital development. The alternative policies that have been suggested due to the presence of trade-off between efficiency and equity in the case of subsidy for education are the pure loan schemes provided by the government, graduate tax schemes, income contingent loans issued by the government, and a possibility for a discharge of student loans.

Independently of the policy instruments that previous research has considered for the case with uncertainty in the human capital development, similarly to the deterministic case, the fertility decisions have often been absent from the analysis, however, despite that the choices for education and childbearing both originate at the household level and they are often considered to be interconnected. Moreover, as we would see in the results from our policy experiments the joint responses for education and fertility define the properties of the resulted population.

Furthermore, to the best of our knowledge, the influence of the subsidies for fertility has not been considered previously in the environment with idiosyncratic shocks to the human capital. Again, due to the interdependency between the households' choices for education and fertility, the analysis for this subsidy option offers additional point of view for the dynamics in the human capital development, population formation and the distribution of the human capital between the members of the households.

3.1.1 Literature review

Before we discuss our results, however, we examine the evidence presented in the literature. First, we consider the potential reasons for the idiosyncratic shocks in the human capital, and their influence on the returns to education. We also examine the influence of tax- and subsidy-driven policies in the environments with risk in the human capital. Lastly, we present the arguments for the policies that do not involve the subsidy for education, but rather address this uncertainty though different means. We must note, however, that to the best of our knowledge, the influence of subsidies for childbearing decisions in the environment with risk in the human capital has not been studied previously.

According to Cunha and Heckman (2009), where the authors provide an extensive summary for the reasons of inequality in the human capital development drawn from the various areas of research in Economics and Psychology, one of the first scholars who have addressed this issue was Alfred Marshal. In his work of 1890, Marshal stated that the care provided by the mothers is crucial for understanding the reasons for the innate human capital variations. According to Cunha and Heckman (2009), this conclusion of Marshal corresponds to the explanations presented in the modern research where the family environment is viewed as one of the factors that explain the presence of heterogeneity in the human capital development. The authors state that after a second grade the family environment, and not a formal schooling, becomes a major determinant for reducing the gaps in cognitive and noncognitive abilities of children[2,3], which themselves are important indicators for explaining heterogeneity in the outcomes of the future adults across various socio-economic groups. Additionally, Cunha and Heckman (2009) indicate that a family environment is important for a successful manifestation of the natural abilities of children determined by their genes. Furthermore, according to Cunha and Heckman (2009), the family's credit constraint creates a major and lasting influence on the schooling outcomes of children and their further ability and willingness to undertake a college education, all of which contribute to the differences in human capital development. Cunha and Heckman

[2] According to Cunha and Heckman (2009), noncognitive abilities include such traits as perseverance, motivation, self-esteem, self-control, conscientiousness, and forward looking behaviour

[3] According to Cunha and Heckman (2009), the crucial period for forming cognitive abilities is before age of 10. Cunha and Heckman (2009) indicate, however, that due to a slower development of prefrontal cortex, "noncognitive abilities are more malleable until later ages"

(2009) conclude, however, that the optimal policy for reducing the negative factors that are out of individual control when the human capital accumulation takes place does not exist, and it depends on a life-cycle stage of a disadvantaged individual. For the disadvantaged children, “it is efficient to invest [into the human capital] relatively more in the later years of childhood than in early years”, while for the disadvantaged adults, “subsidising work and welfare may be a better response for alleviating poverty than investment in their skills”.

As a result of heterogeneity in the human capital development that contribute to uncertainty in returns for education, Cuncha, Heckman and Navarro (2005) conclude that approximately thirty percent of people find their education decisions ineffective because at a time when decision for education has been made, the individuals experienced uncertainty for their future realised human capital level. Furthermore, due to uncertainty in the human capital formation, Cuncha, Heckman and Navarro (2005) find that only “about sixty percent of variability in returns to schooling is forecastable”.

The additional problem that arises with uncertainty in the human capital formation is the failure of insurance markets. According to Sinn (1996), the private insurance markets are “not available because of the moral hazards, adverse selection, and various legal limitations in trading claims on human capital”. According to Jacobs, Schindler and Yang (2012), however, the public insurance policies “can overcome adverse selection and legal problems by providing mandatory social insurance against human capital risk”, but the problem of the moral hazard would still persist.

We begin the overview for the government policies in the environment with uncertainty in the human capital formation with the study of Krebs (2003). In this paper, the author features the incomplete-market model of economic growth with individuals’ investment decisions for risk-free physical capital and risky human capital. According to Krebs (2003), the policies that would reduce the risk associated with human capital investment produce a substantial improvement in the welfare, level of output produced and level of investment into the human capital. Furthermore, if the government would provide the disadvantaged individuals with severance payments, the economy would experience a further improvement in welfare and level of output produced, even when this policy option is financed with distortionary income taxation. Next, according to Hogan and Walker (2007), where authors analyse the impact of public policy on choices for education attainment under the uncertainty in returns for education investment by utilising the real option price theory, the environment with higher risk creates the additional incentives to stay in education longer which causes a greater accumulation of the human capital. The authors state further that increase in the labour income taxation in the environment with a risk in education investment would decrease the benefits of staying in education. At the same time, a larger labour income taxation decreases the benefits of leaving education by a larger rate, however, which produces a positive net effect of labour income taxation on human capital accumulation in the environment with uncertainty in returns for education investment. According to Jacobs, Schindler and Yang (2012), where the authors employ a two-period life-cycle model with risk in return for human capital investment, the linear income taxation can be used to provide a partial social insurance against risks in human capital which would result in a welfare improvement. The authors conclude further that a combination of the optimal linear subsidy for education and optimal linear income tax would allow for a greater social insurance against the risk in human capital; however, Jacobs, Schindler and Yang (2012) state that a full social insurance coverage is not attainable due to a moral hazard problem. Moving forward to the study of Kapiĉa and Neira (2015), where the authors investigate the optimal tax policies in the environment with risk in human capital by utilising the life-cycle model economy in which the ability and learning effort is the private information of the individuals, the authors find that an implementation of progressive income and capital taxation leads to a welfare improvement. According to the scholars, these taxes provide insurance against the shocks in the human capital, and they also provide the incentives for investment in human capital and for labour market participation. Next, according to Grochulski and Piskorski (2010), where the authors construct a life-cycle Mirrilees economy with endogenous distribution of skills and investment into the human capital containing a risk, the optimal tax system that is based on taxation of capital must defer a tax burden. According to the authors, a deferred taxation until the realisation for the human capital takes place in a latter life-cycle would "provide incentives to prevent high-skilled agents from pretending to be low-skilled". Based on this tax system, the individuals would face a negative expected capital taxes early in life-cycle and positive expected taxes in later life-cycle, which, according to Grochulski and Piskorski (2010), would improve the incentives for human capital investment across the ability spectrum when the return to human capital investment contains an uncertainty. Lastly, according to Bandyopadhyay, King and Tang (2019), where the authors construct an overlapping generations economy with uncertainty in the human capital development which creates the problem of misallocation of resources and negatively influences the total factor productivity, the increase of "income-weighted average marginal tax and transfer rate" to the optimal level – which is above the current status quo level utilised in the U.S. – would help to evaluate the problem of uncertainty in human capital development. According to the authors, at the final steady-state with increased tax and transfer rates, the economy would enjoy an increase in welfare, total factor productivity and decrease in inequality in the population. The scholars note, however, that this policy would result in lower GDP level.

Next, we consider the studies that specifically analyse the influence of subsidy for education in the environment with uncertainty in the human capital development. According to Akyol and Athreya (2004), where the authors develop a dynamic heterogeneous agent model with uncertainty in returns for education investment, the subsidy for tertiary education increases the higher education attainment as a result of decrease in the cost of education investment and decrease in the risks associated with investment into the human capital. The scholars note that these effects of the subsidy for tertiary education are more pronounced for the households with lower wealth. Particularly, the authors find that subsidies for higher education increase the wages for unskilled and lower skilled individuals which reduces the risk of unsuccessful human capital investment. Overall, Akyol and Athreya (2004) conclude that the subsidy for education "help[s] smooth consumption, lower skill premia, increase interest rate as precautionary saving fall, lower inequality of both consumption and wealth, and increase intergenerational income mobility" and it improves the overall level of the welfare in the economy. In a context of a globalisation, however, Andersson and Konrad (2003) reach a different conclusion. The authors find that in the environment with uncertainty in returns for human capital investment where successful individuals are fully mobile between different economies and unsuccessful individuals are immobile, the cost of subsidy for education would shift to less skilled individuals because the mobility of more skilled individuals would allow them to avoid the costs (i.e. taxation) of the subsidy program. Andersson and Konrad (2003) note further that the difference in the income between individuals with successful and unsuccessful human capital investments would increase even further as a result of subsidy for education when the human capital contains a risk and only higher-skilled individual are mobile.

As a continuation of our literature review for the policies in the environment with risk in human capital formation, we present the results obtained by the studies which consider alternative policy options to a more traditional tax-subsidy systems. According to García-Peñalosa and Wälde (2000), the traditional tax-subsidy system cannot reach the target of efficiency and equity simultaneously. The authors explain that with a traditional tax-subsidy system, there is an increase in the inequality in the lifetime income between skilled and unskilled individuals. Furthermore, the scholars state that the increase in subsidy rate would increase the human capital investment across all of population which would reduce the lifetime income inequality gradually, but due to the labour force being overeducated, the economy would experience a loss in potential output. According to García-Peñalosa and Wälde (2000), a more appropriate policy option would involve either a pure loan scheme provided by the government or a graduate tax scheme. In addition, according to the authors, a graduate tax system – where "the payment of education cost is contingent on whether or not the individual succeeds in education" – does provide individuals with partial insurance against a risk in human capital investment. Next, according to Stantcheva (2017), where the author develops a dynamic life-cycle model with risky human capital and with the government facing uncertainty about the abilities and labour supply decisions of the individuals, the government can maximise a social welfare by introducing the income-contingent loans where the repayment schedule "depends on the full history of human capital investments and earnings". Additionally, the author suggests that "if the shocks to ability are independently and identically distributed, a deferred deductibility scheme, in which part of current human capital expenses can be deducted from future years' incomes" can lead to a further welfare improvement. Finally, according to Ionescu (2011), where the author develops a heterogeneous life-cycle model with individuals who experience human capital shocks and are differentiated in the access to financial assets, the possibility for a discharge of student loans under a liquidation regime would reduce the risk of human capital investment. According to the author, this option would improve the choice for higher education of students with "low level of assets, high ability, and medium level of human capital". Additionally, the scholar concludes that a possibility for a discharge of student loans would allow these individuals to invest more in the human capital after a graduation from a college.

As a final point in our overview for the policies in the environment with risk in the human capital, we address two studies that discuss whether the education investment should be privately or publicly financed. According to Wildasin (2000), where the author develops a general-equilibrium model with interjurisdictional mobility of skilled labour and unskilled labour being immobile, a combination of the integration of markets for skilled labour with privately financed investment for the human capital results in overall welfare improvement. According to the author, this policy increases the returns for unskilled workers, equilibrates the net income of skilled and unskilled individuals, and improves the welfare of all ability types. In contrast, when the government integrates the markets for skilled labour and when the human capital investment is publicly financed, the economy reaches the equilibrium where the tax burden shifts from higher ability to lower ability individuals, and where the inequality between individuals from different human capital groups increases which reduces a welfare of unskilled agents. According to Kindermann (2012), where the author constructs an overlapping generations model with risky human capital investment, borrowing constraint and intergenerational transmission of abilities, the privately financed college education makes college graduates better-off due to increase in their wage premium. The author states, however, that a switch from publicly to privately financed higher education decreases the number of college graduates and it removes the insurance against human capital investment risk which was provided by the publicly financed education. Finally, Kindermann (2012) finds that with privately financed higher education, the individuals experience an increase in liquidity constraint during their time of college education; and the overall efficiency in the economy is reduced as a consequence of this shift from publicly to privately financed college system.

3.1.2 Summary for the results

According to our results, the government program which provides the subsidy for education in the environment with idiosyncratic shocks to the human capital increases the average education attainment which increases the average human capital and helps to reduce the impacts of negative shocks during the human capital formation. Due to a parental ‘quality-quantity’ trade-off, however, the subsidy for education reduces the individual choices of adult households for fertility which causes a reduction in the population size. With the government support for education, however, the model population enjoys a higher level of welfare and output per adult household, which is a result of an improvement in abilities of adult households and their children. Upon a visual examination of the distribution functions for the human capital combined with a computation of standard inequality measurements, our results suggest that the subsidy for education decreases the long-run inequality in the distribution of the human capital. According to our results, a reduction in the cost for education attainment through this subsidy program improves the parental incentives for education provision which helps to accumulate further human capital especially to the lower ability groups[4]. This additional level of human capital that children obtain helps them to leave lower ability groups and become a part of higher ability ones when they become adults. This brings population closer together and helps to reduce inequality in distribution of the human capital. Additionally, as mentioned above, an improvement of education provision helps to minimise the impact of negative shocks when human capital is forming, which assists in reduction of inequality in distribution of human capital further. Due to stochastic nature of human capital, however, the population at the second steady-state does not reach complete equality in human capital, and therefore, is not presented by a single ability group with single human capital level as we have observed for the deterministic case. Furthermore, any policy instrument that creates disincentive for education provision is found to cause a higher level of inequality in distribution of human capital. This is the reason why with the labour income taxation, independently from a subsidy of choice, and with subsidy for fertility, independently from a tax option, the second steady-states have higher levels of inequality

[4]With exception for the households from the lowest ability groups who provided their children with zero education at the original steady-state, and continue to do so when economy reaches the second steady-state

in distribution of human capital. Additionally, with the subsidy for fertility, due to parental 'quality-quantity' trade-off for children, the economy reaches the second steady-state that is the opposite to one with subsidy for education.

Lastly, if we would rank the policies that we consider in this paper on the basis of improvement in level of welfare, we find that the best outcome is reached when the government provides subsidy for education and finances it with the capital income tax. The worst outcome in terms of welfare is reached when subsidy for fertility financed with a labour income tax, however. On the other hand, if the ranking criteria is based on ability to reduce the inequality in the distribution of the human capital, the best policy is the subsidy for education that is financed with either tax on consumption or capital income, since both of these tax options affect education, and, therefore, the human capital accumulation, in the same way. Oppositely, the worst policy in term of inequality in distribution of human capital is the subsidy for fertility that is financed with labour income tax. As we have discussed above, any policy instrument which creates additional disincentives for education attainment prevents equality in human capital across members of the population.

The rest of the paper has the following structure. In the section 3.2, we formally state the problem of each sector in the model economy for the stochastic framework and introduce the conditions that must be satisfied to reach the market equilibrium and the steady-states – first without the government presence, and then with various policy options. In the section 3.3, we explain the way in which the random component within the human capital formation function is discretised, and then, in the section 3.4 we present the calibration for the model parameters. Next, we perform the policy experiments for variety of government instruments that we consider. First, we discuss the differences that appear between the original steady-state and the resulted final steady-states, and then we examine the transition paths that the model economy takes. In our discussion, we focus both on the individual decisions of the households and on the average outcomes of the population. Lastly, before we conclude, we compare the resulted levels of inequalities from the different policy options using standard inequality measures as the range, coefficient of variation, Gini coefficient and others.

3.2 The model economy

We utilize the model of overlapping generations presented by *de la* Croix and Doepke (2003) to formalise the behaviour of the households and the representative producer. In the present paper, however, we assume that young households (i.e. children) receive idiosyncratic shocks while the accumulation of human capital takes place. We also introduce the government sector which supports the adult households – who are heterogeneous in the human capital – by reducing the cost of education provision and fertility decisions. We begin the discussion for the model economy with the problem of the households.

3.2.1 Household sector

In line with *de la* Croix and Doepke (2003), the household sector of the model economy is presented by three generations of the households, who are the young households (i.e. children), adult households (i.e. parents), and elderly households (i.e. grandparents). Following the original model assumptions, only the adult households make economically relevant decisions in this environment. We consider that the households are heterogeneous in human capital level. Young households receive the level of education e_t^i optimally chosen by their parents[5]. In the deterministic version of the model, this optimal choice of the parents for their children together with the abilities of parents and quality of education have determined the human capital of the future adult households. For the stochastic version of the model, which is considered by this paper, however, the realised level of the human capital is also influenced by an uncertainty.

To be able to include this stochastic component in our analysis, we modify the original human capital formation function outlined by *de la* Croix and Doepke (2003) to account for the presence of idiosyncratic shocks received during the human capital investment by following Kapička and Neira (2015). As a result, the expected level of the human capital for the future adult households[6] from the i^{th} ability group adjusted to the human capital growth rate over the

[5] Who are members of ability group i from continuum of ability groups I with the human capital h_t^i

[6] I.e.: present period young households who receive education today and enter the labour market in the next period of analysis

long-run is determined by the function (3.1) as following:

$$E_t[h^i_{t+1}] = E_t\left[\frac{1}{(1+\rho)}B(\theta+e^i_t)^{\eta}(h^i_t)^{\pi}(\bar{h}_t)^{\kappa}\times \exp^{\varepsilon}\right] \tag{3.1}$$

where $\frac{1}{(1+\rho)}B(\theta+e^i_t)^{\eta}(h^i_t)^{\pi}(\bar{h}_t)^{\kappa}$ comes from the deterministic version of the human capital formation function and $\exp^{\varepsilon}$ is the component that depicts uncertainty similarly to Kapička and Neira (2015). From the deterministic part, ρ is the long-run growth rate of the human capital, B is the efficiency of human capital accumulation, θ is the instrument parameter that ensures non-zero level of human capital for children who optimally receive zero level of education, η is the parameter that shows a relative significance of education for human capital formation, π is the parameter that shows a relative significance of human capital of parents for human capital of children, and κ is the parameter that shows a relative significance of quality of education for human capital attainment. Therefore, according to (3.1), the human capital of children h^i_{t+1} is increasing with the parental choice for education e^i_t, human capital of parents h^i_t and quality of education which is approximated by the human capital of teachers that is given as the average level of human capital $\bar{h}_t$. From the stochastic component, ε is a random variable which introduces uncertainty in the human capital formation process.

Given the realised level of human capital h^i_t, the adult households maximise their expected utility function (3.2) subject to the resource constraints (3.3) and (3.4), and subject to the human capital formation process depicted by (3.1).

$$\max_{c^i_t, s^i_t, e^i_t, n^i_t} E_t[u^i_t] = E_t\left[\ln c^i_t + \beta \ln d^i_{t+1} + \gamma \ln n^i_t h^i_{t+1}\right] \tag{3.2}$$

$$(1+\tau^c_t)c^i_t + s^i_t + e^i_t n^i_t w_t \bar{h}_t = (1-\tau^l_t)w_t h^i_t(1-\phi n^i_t) + n^i_t w_t \bar{h}_t(e^i_t sub^e_t + \bar{e}_t sub^n_t) \tag{3.3}$$

$$(1+\tau^c_{t+1})d^i_{t+1} = \frac{1}{(1+\rho)}\left(1+r_{t+1}(1-\tau^k_{t+1})\right)s^i_t \tag{3.4}$$

where E_t is the expectation operator, which indicates the expectations of the adult households at time t; c_t^i is the level of consumption for adult households at time t; d_t^i is the level of consumption for elderly households adjusted to the growth rate of the human capital at the steady-state; s_t^i is the growth adjusted level of saving that adult households make to finance their consumption at the elderly stage of existence; e_t^i is the level of education that adult households optimally choose and young households receive at time t; $\bar{e}_t$ is the average level of education in the economy at time t; n_t^i is the number of children chosen by the adult households to have; u_t^i is the level of utility for adult households from the ability type i which increases with consumption during adulthood and at the old age, and with the 'quality and quantity' of children. The coefficient β is the discount factor of adult households, γ is altruism factor of adult and elderly households toward children, ρ is a long-run growth rate of the human capital, and ϕ is a time-cost of parents to raise a single child.

According to the budget constraint (3.3), the total expenditure of adult households is presented by the left-hand-side of the expression and consists of their consumption and saving, and expenditure on the education provision which is a product of the optimal choice for education, the number of children that adult households have and provide the education for, the real wage rate w_t[7], and the human capital of teachers. The right-hand-side of (3.3) contains the expression for the total income of adult household, which is the income from participation in the labour market plus the total amount of subsidies that household receives from the government. According to the original formulation of *de la* Croix and Doepke (2003), the labour income is given as the product of the real wage rate, human capital, and the time that adult households have away from taking care for the children. Given the government policy instruments that appear in (3.3), sub_t^e and sub_t^n indicate the subsidy rates for education and fertility; and τ_t^c and τ_t^l are the tax rates on consumption and labour income, respectively.

Based on the budget constraint for the elderly households (3.4), the growth adjusted consumption at the old age depends on the real return on the financial investment that households make during adulthood, which depends of the real interest rate r, saving made during adulthood, tax rate on consumption and tax rate on the capital income τ^k.

Given this utility maximisation problem, the adult households choose their level of con-

[7] Which, in this case, is the real wage rate that teachers receive

sumption and saving, together with the number of children to have and level of education to provide for their children. These choices, however, are now made in the stochastic environment, where children are subjected to the idiosyncratic shocks ε while human capital accumulation takes place.

3.2.2 Production sector

Following *de la* Croix and Doepke (2003), the production sector is presented by a single firm which produces output by employing the effective labour force and the physical capital stock. This firm maximises its own profit (3.5) subject to the technological constraint (3.6), which forms the demand for both of the factor inputs.

$$\max_{K_t, L_t} \Pi_t = Y_t - w_t L_t - (r_t + \delta) K_t \tag{3.5}$$

$$Y_t = A K_t^{\alpha} L_t^{1-\alpha} \tag{3.6}$$

In this optimisation problem Π_t is the profit of the firm, Y_t is the output level of the firm per adult household, w_t is the real wage paid to the unit of the effective labour force, L_t is the effective labour force per adult household, r_t is the real interest rate (real rental rate of physical capital stock) paid for the unit of the physical capital stock, K_t is the physical capital stock per adult household, δ is the depreciation rate of the physical capital stock, A is a productivity level, and α is the share of the capital income.

Next, the supply of effective labour force per adult household is depicted by (3.7).

$$L_t = \frac{\sum_{i=1}^{I} p_t^i \left[h_t^i (1 - \phi n_t^i) - n_t^i e_t^i \bar{h}_t \right]}{\sum_{i=1}^{I} p_t^i} \tag{3.7}$$

Effective labour force per adult household is the aggregate number of adult households in the model population who are not involved in teaching, with a time $(1 - \phi n_t^i)$ than can be contributed to the labour market, divided by the total population size of adult households, where p_t^i is the population size of adult households from i^{th} ability group with h_t^i level of human capital.

Lastly, the evolution of the physical capital stock per adult household is given by (3.8).

$$K_t = \frac{1}{(1+\rho)} \left(\frac{\sum_{i=1}^{I} p_{t-1}^i s_{t-1}^i}{\sum_{i=1}^{I} p_t^i} + (1-\delta) K_{t-1} \right) \tag{3.8}$$

According to (3.8), the present level physical capital stock per adult household adjusted to the long-run growth rate of the human capital is determined by the past level of physical capital stock per adult household, depreciation rate of the physical capital stock δ, present population size of adult households, and past period aggregate level of saving of adult households from different ability group.

3.2.3 Government

We introduce the government in form of policy maker that provides subsidies for education and fertility to the households. To provide these means of support, we consider that the government can finance its budget with taxes on consumption, labour income and capital income that government collects from the households. These tax rates are optimally chosen by the government in order to satisfy the budget constraint (3.9) given the optimal choices of the households and producer with government policy instruments in place. For our analysis we assume that budget constraint of the government is always balanced. The left-hand-side of (3.9) depicts the total income of the government given the tax option that government implements, and the right-hand-side of (3.9) presents the total expenditure of the government given the subsidy program in place.

$$\sum_{i=1}^{I} \tau_t^c p_t^i c_t^i + \sum_{i=1}^{I} \tau_t^c p_{t-1}^i d_t^i + \sum_{i=1}^{I} r_t \tau_t^k p_t^i s_t^i + \sum_{i=1}^{I} \tau_t^l w_t p_t^i h_t^i (1 - \phi n_t^i) = \sum_{i=1}^{I} p_t^i n_t^i w_t \bar{h}_t (e_t^i sub_t^e + \bar{e}_t sub_t^n) \tag{3.9}$$

3.2.4 Market equilibrium

Since in the present version of analysis, the stochastic process is introduced into the human capital formation function, we rely on the numerical methods to analyse the effects of government policies. We follow Heer and Maussner (2009) and assume that a stationary equilibrium for this model economy takes place when conditions below are satisfied.

1. Individual and aggregate behaviour are consistent. The aggregate variables are equal to the sum of individual variables.

2. The real wage rate and real interest rate solve the firm's optimisation problem by satisfying two conditions below, where real interest rate net of depreciation is equal to the marginal product of the physical capital stock per adult household and real wage rate is equal to the marginal product of effective labour force per adult household.

$$r_t = \alpha A {K_t}^{\alpha-1} L_t^{1-\alpha} - \delta \tag{3.10}$$

$$w_t = (1-\alpha) A K_t^{\alpha} L_t^{-\alpha} \tag{3.11}$$

3. Given the factor prices for physical capital stock and the effective labour force and the government policy in form of (exogenously chosen) subsidy rate(s) for education and/or fertility, the individual choices of adult households for consumption, saving, education and number of children given their endowment of human capital solve the optimisation problem of the households for each corresponding ability group.

4. Given the factor prices, subsidy rates and individual choices of households, the optimal tax rates for consumption, labour income and/or capital income are chosen to keep the government budget balanced.

5. Present population of adult households for each ability group depends on the past period fertility decisions and evolves according to process below.

$$p_t^i = p_{t-1}^i n_{t-1}^i \tag{3.12}$$

3.3 Discretisation of ε

To perform the numerical exercise for the proposed model economy, we discretise the random component $\exp^{\varepsilon}$ within the human capital formation function (3.1) in the following way.

Let us assume that z is a standard normal random variable which has five equally distant nodes (i.e. $J = 5$) with the maximum value for z (i.e. z_j for $j = 5$) being equal to two (i.e. $z_5 = \sqrt{(J-1)}$). Given that the variance of z is equal to one (i.e. $var(z) = (z_5)^2/(J-1)$), the five nodes of z are -2, -1, 0, 1 and 2. In line with Devroye (1986), to approximate the normal distribution for z and to calculate the probability of each node, we use the Binomial distribution and calculate Binomial probability according to (3.13) below.

$$b_j =_{J-1} C_{j-1} P^{j-1} P^{J-j} \tag{3.13}$$

where b_j is the Binomial probability of j's node; j is the index for a particular z's node where $j \in [1,5]$; J is the total number of nodes for standard normal random variable z; ${}_{J-1}C_{j-1}$ is the number of combinations of $J-1$ nodes, taken $j-1$ at a time which is calculated using (3.14); where P is the probability of success on an individual trial which we assume is equal to 0.5.

$${}_{J-1}C_{j-1} = \frac{(J-1)!}{(j-1)!\big((j-1)(J-1)\big)!} \tag{3.14}$$

Therefore, the number of combinations and Binomial probability for each of the nodes of standard normal random variable z is given by the table 3.1 below.

jth node	z_j	$j-1$	$J-1$	${}_{J-1}C_{j-1}$	b_j
1	-2	0	4	1	$\frac{1}{16}$
2	-1	1	4	4	$\frac{4}{16}$
3	0	2	4	6	$\frac{6}{16}$
4	1	3	4	4	$\frac{4}{16}$
5	2	4	4	1	$\frac{1}{16}$

Table 3.1: The number of combinations and Binomial probability for five z nodes

Given the probability of occurrence for the each of these nodes, the mean of z is equal to zero, which indicates that with calculated probabilities for each node, z approximates a standard normal random variable.

Let us consider a new random variable ε which is given as $\varepsilon = \mu + \sigma z$. Since we concluded that z given the probability of each node approximates the standard normal random variable, then ε approximates the standard normal random variable as well with mean of μ and variance of σ^2 (i.e. $\varepsilon \sim N(\mu, \sigma^2)$). Now, let us consider that x is another random variable which is defined as $x = \exp^{\varepsilon}$. As a result, x approximates a log-normal random variable with parameters μ and σ. By normalizing the expected value of x to one (i.e. $E(x) = E(\exp^{\mu + \frac{\sigma^2}{2}}) = 1$), the expression is simplified to $\mu = -\frac{\sigma^2}{2}$.

Finally, we approximate the standard deviation of the idiosyncratic shock to the human capital with the standard deviation in lifetime labour income, which according to Bosworth et al (2000) and Millimet et al (2003) is equal to 20%. Given the value of $\sigma = 0.2$, the mean of ε is equal to -0.02 and five nodes for ε are -0.42, -0.22, -0.02, 0.18 and 0.38. Given these five nodes and given the Binomial probability distribution for each node indicated in table 3.1, we discretise the random process within the human capital formation function.

3.4 Calibration

For the purpose of our policy exercises, we follow the original parametrisation of *de la* Croix and Doepke (2003) closely. We, however, adjust the parameters γ – the altruism factor of adult and elderly households toward children, A – productivity level and B – efficiency of human capital accumulation[8], in order to normalise the average decision of adult households for children, the real wage rate and the average level of human capital to be equal to one at the initial steady-state without government presence. Furthermore, as we have mentioned in the previous section, we calibrate the standard deviation of the shocks in the human capital to approximate the standard deviation in the lifetime labour income, which according to Bosworth et al (2000) Millimet et al (2003) is equal to 20%. Overall, the full list of the structural parameters utilised within the model, and their corresponding values and the interpretation is given by table 3.2

[8] In the original calibration of *de la* Croix and Doepke (2003), A and B parameters both set to one. For γ, the value of the parameter has been set to 0.271, which results in zero growth of population in the balanced growth path.

below[9].

parameter	value	interpretation
A	2.9504	productivity level
B	7.3478	efficiency of human capital accumulation
α	1/3	share of the capital income
β	0.99^{120}	discount factor
γ	0.17957	altruism factor
δ	1	depreciation rate of physical capital stock
η	0.5	relative significance of education for human capital
θ	0.0119	instrument parameter for non-zero human capital of children when education provision is zero
π	0.2	relative significance of human capital of parents for human capital of children
κ	0.1	relative significance of quality of education for human capital
ρ	$1.02^{30}-1$	long-run growth rate of the human capital
σ	0.2	st. deviation of the idiosyncratic shocks to the human capital
ϕ	0.075	time-cost parameter to raise children

Table 3.2: Calibration for the structural parameters of the model economy

According to *de la* Croix and Doepke (2003), these structural parameters have been calibrated in order for the model economy in the balance growth path to follow the properties of the U.S. economy and population under the assumption that one period of analysis is equal to thirty years. To match the empirical evidence, *de la* Croix and Doepke (2003) set the capital income share parameter α to be equal to 1/3. For the discount factor β, the authors indicate that this parameter influences the ratio between human capital and physical capital stock in the balanced growth path, but since this ratio depends on the choice of units, the calibration of β has been found to be inconvenient. Therefore, *de la* Croix and Doepke (2003) have followed the real-business-cycle literature where discount factor for one quarter of analysis is often set to be equal to 0.99, and given that there are 120 quarters in 30 years (i.e. the time frame between one point of the observation and the next), the discount factor β is set to 0.99^{120}. Next, given that it takes thirty years between each period of analysis, *de la* Croix and Doepke (2003) considered the full depreciation of the physical capital stock, and therefore, the δ coefficient is equal to one. For parameter η which identifies the relative significance of education for development

[9]The current analysis is conducted based on the parametrisation presented in the table 3.2. The exercises that involve the robustness check and matching data is a subject for consideration for future work.

of human capital, the authors calibrate this parameter using the maximum fertility differential that they observe in the data set they implement. According to *de la* Croix and Doepke (2003), $\frac{1}{(1-\eta)}$ defines the maximum fertility differential for the model economy, which for the case of Brazil has been reported to be equal to 2.74, which sets η to be equal to 0.635. For the purpose of our computational exercise, however, we normalise the maximum fertility differential to 2, and therefore, the η coefficient in our analysis is equal to 0.5. For θ we rely on the original parametrisation performed by *de la* Croix and Doepke (2003), however. According to the authors, in the balanced growth path for the U.S economy, 7.3 percent of the GDP is dedicated to the total education expenditure, which corresponds to the value of θ of 0.0119. For π which measures a relative significance of the human capital of parents for human capital of children, *de la* Croix and Doepke (2003) calculate the upper limit for the parameter in order to have individual stable dynamics, which according to the authors is equal to 0.246. With review of empirical evidence for the influence of parental education on child school performance, and taking into consideration the long-run nature of the analysis, *de la* Croix and Doepke (2003) set π to be equal to 0.2. In the case for the variable which depicts the relative significance of quality of education for the human capital formation κ, the authors followed the findings of Card and Kruger (1996) and Kruger and Lindahl (2001) and set this parameter to be equal to 0.1 (*de la* Croix and Doepke, 2003). Finally, *de la* Croix and Doepke (2003) set the value for the long-run growth rate of human capital ρ in line with the growth rate of the output in the balanced growth path, which for the case of United States averages at two percent per year; and the time-cost parameter to raise children ϕ is chosen by the authors on the basis of the evidence presented by Haveman and Wolfe (1995) and Knowles (1999) who find that the opportunity cost of a child is equal to 15 percent of parental time endowment, and with assumption placed by *de la* Croix and Doepke (2003) that children would spend 15 year out of 30 living with parents with “overall 50 percent of the time cost per year”, the value for ϕ is set to 0.075.

3.5 Policy experiments

In order to perform the policy experiments and analyse the influence of the government support for education and fertility, we adapt the computational algorithm presented by Heer and Maussner (2009). First, we establish the stable distribution of adult households given the presence of uncertainty in the human capital accumulation in the economy without the government. For a duration of the following discussion, we refer to this point as 'initial' or 'original' steady-state. After we complete the discussion for the initial steady-state, we introduce the government into the stochastic environment. Given the presence of the government policy, the model economy reaches the second steady-state[10] which we analyse alongside with the transition path that the model economy takes to reach this equilibrium. In the upcoming discussion, we examine the case of subsidy for education and fertility separately. For each version of the analysis, the government finances its budget with three tax options – tax on consumption, tax on labour income and tax on capital income. In our discussion, we focus on potential differences that result in the model economy from implementation of different tax options.

3.5.1 Initial steady-state

As a beginning point for our analysis, we discretise the model population of adult households into fifteen ability groups (i.e. $i \in [1, 15]$ where $I = 15$). We normalise the population size to one. At this initial point we consider that the population is uniformly distributed among fifteen ability groups and, therefore, the share of adult households from each ability group is equal to $\frac{1}{15}$ before uncertainty is introduced. Given this uniform distribution of adult households, we introduce the process of human capital accumulation with idiosyncratic shocks. With the exogenously chosen (marginal) level of human capital for each ability group, and with the process of extrapolation and interpolation which is shown below, we obtain the stable distribution of adult households for the original-steady state where the human capital accumulation contains uncertainty and where the government is absent. This original steady-state is depicted by table 3.3 and figure 3.1 below.

[10]We define the second steady-state as the equilibrium at which the model population with government policies in place has a stable distribution

	Y	K	L	r	w			
	1.2757	0.1118	0.8504	2.8049	1.0000			
	$\bar{h}$	tot.pop.	$\bar{e}$	$\bar{n}$	$\bar{c}$	$\bar{s}$	$\bar{d}$	$\bar{u}$
	1.0000	1.0000	0.0512	1.0000	0.6762	0.2024	0.4252	-0.7502
i	h_i	pop.share	e_i	n_i	c_i	s_i	d_i	u_i
1	0.2019	0.0002	0.0000	1.6189	0.1365	0.0409	0.0858	-2.9271
2	0.2466	0.0015	0.0000	1.6189	0.1667	0.0499	0.1049	-2.6314
3	0.3012	0.0054	0.0000	1.6189	0.2037	0.0610	0.1281	-2.3356
4	0.3679	0.0141	0.0038	1.4234	0.2487	0.0745	0.1564	-2.0629
5	0.4493	0.0326	0.0099	1.2513	0.3038	0.0910	0.1911	-1.7902
6	0.5488	0.0692	0.0174	1.1386	0.3711	0.1111	0.2334	-1.5114
7	0.6703	0.1289	0.0265	1.0605	0.4532	0.1357	0.2850	-1.2284
8	0.8187	0.1948	0.0376	1.0040	0.5536	0.1657	0.3481	-0.9424
9	1.0000	0.2200	0.0512	0.9621	0.6762	0.2024	0.4252	-0.6543
10	1.2214	0.1772	0.0678	0.9303	0.8259	0.2472	0.5194	-0.3645
11	1.4918	0.1010	0.0881	0.9058	1.0087	0.3020	0.6343	-0.0735
12	1.8221	0.0412	0.1129	0.8867	1.2320	0.3688	0.7748	0.2184
13	2.2255	0.0117	0.1431	0.8716	1.5048	0.4505	0.9463	0.5111
14	2.7182	0.0020	0.1801	0.8596	1.8380	0.5502	1.1558	0.8044
15	3.3201	0.0001	0.2252	0.8501	2.2449	0.6721	1.4117	1.0982

Table 3.3: Original steady-state with presence of idiosyncratic shocks in the human capital and without presence of the government policies

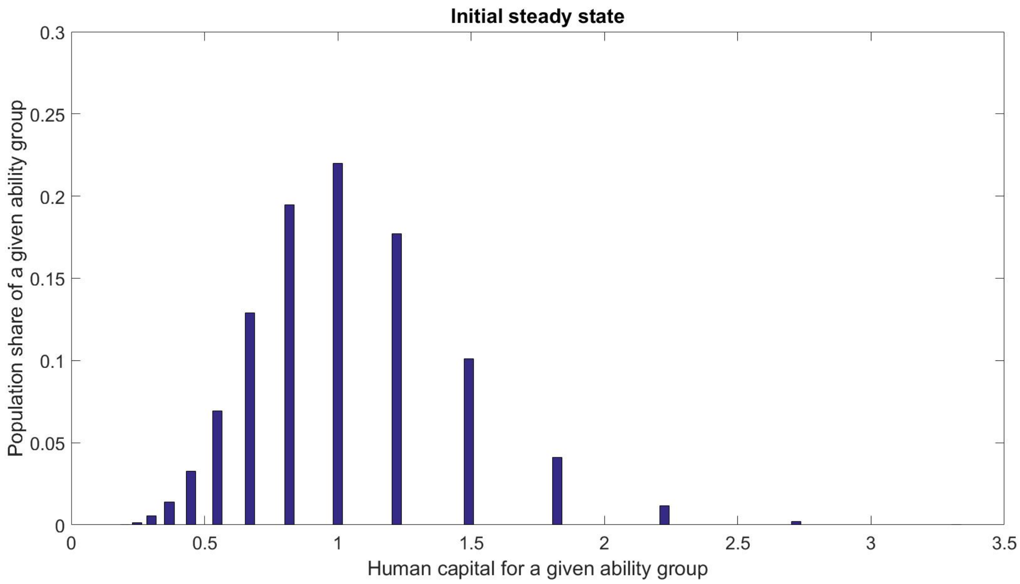

Figure 3.1: Distribution for the population of adult households at the initial steady-state. Note: the (marginal) levels of the human capital are depicted on the horizontal axis; population share for each ability group is depicted on the vertical axis

According to our results, at the initial steady-state, the first three ability groups optimally

<table>
<tr><td colspan="2">if $0\leq h^i_{t,j} < h^1$</td></tr>
<tr><td>if $(h^2-h^1) > (h^1-h^i_{t,j})$
$p^1_t = \frac{(h^2-h^1)}{(h^2-h^i_{t,j})} p^i_{t-1} b^i_j + p^1_t$
$p^2_t = \frac{(h^1-h^i_{t,j})}{(h^2-h^i_{t,j})} p^i_{t-1} b^i_j + p^2_t$</td><td>if $(h^1-h^i_{t,j}) > (h^2-h^1)$
$p^1_t = \frac{(h^1-h^i_{t,j}))}{(h^2-h^i_{t,j})} p^i_{t-1} b^i_j + p^1_t$
$p^2_t = \frac{(h^2-h^1}{(h^2-h^i_{t,j})} p^i_{t-1} b^i_j + p^2_t$</td></tr>
<tr><td colspan="2">if $h^L\leq h^i_{t,j}\leq h^H$</td></tr>
<tr><td colspan="2">$p^L_t = \frac{(h^H-h^i_{t,j})}{(h^H-h^L)} p^i_{t-1} b^i_j + p^L_t$
$p^H_t = \frac{(h^i_{t,j}-h^L)}{(h^H-h^L)} p^i_{t-1} b^i_j + p^H_t$</td></tr>
<tr><td colspan="2">if $h^i_{t,j} > h^{15}$</td></tr>
<tr><td>if $(h^{15}-h^{14}) > (h^i_{t,j}-h^{15})$
$p^{14}_t = \frac{(h^i_{t,j}-h^{15})}{(h^i_{t,j}-h^{14})} p^i_{t-1} b^i_j + p^{14}_t$
$p^{15}_t = \frac{(h^{15}-h^{14})}{(h^i_{t,j}-h^{14})} p^i_{t-1} b^i_j + p^{15}_t$</td><td>if $(h^i_{t,j}-h^{15}) > (h^{15}-h^{14})$
$p^{14}_t = \frac{(h^{15}-h^{14})}{(h^i_{t,j}-h^{14})} p^i_{t-1} b^i_j + p^{14}_t$
$p^{15}_t = \frac{(h^i_{t,j}-h^{15})}{(h^i_{t,j}-h^{14})} p^i_{t-1} b^i_j + p^{15}_t$</td></tr>
</table>

Table 3.4: Extrapolation and interpolation procedure

decide to have a maximum number of children and provide their children with zero level of education. These optimal choices are formed due to a lower level of relative human capital of these ability groups in comparison to the rest of the population. On the other hand, the households with larger level of relative human capital decide to have less children but provide their children with higher level of education. Finally, we can observe a way in which the realised level of human capital affects the consumption and saving decisions of the households, and their utility level overall. Additionally, we observe the way in which the allocations at the individual level affect the average and aggregate values for the economy overall. For more details on the relationship between the variables in this model, please refer to appendix A1 section, where we present the analytical solution for the problem of the households in the deterministic environment.

Extrapolation and Interpolation We adopt the process of (linear) extrapolation and interpolation presented by Judd (1998a and 1998b) to be able to sort the households across the ability groups when the realisation of the human capital takes place. We summarise this procedure in table 3.4 below, where $h^i_{t,j}$ is the realised level of human capital for adult from ability group i at a time t who has received the shock j during human capital accumulation process at the childhood; p^i_{t-1} is the past period population size of adult households from ability group i; b^i_j is the (binomial) probability of a shock from a node j to influence the human capital for adult household from ability group i. We consider that the realised level of human capital for adult household who receives shock from a node j nests between the two predetermined (marginal) levels of the human capital – h^L and h^H. Therefore, the closer the realised value of human capital to a particular node, the larger population size that population group would have. We denote p^L_t as the resulted population size of the group with the (marginal) level of human capital corresponding to L (i.e. $i = L$), and p^H_t as the resulted population size of the group with the (marginal) level of human capital corresponding to H (i.e. $i = H$).

3.5.2 Subsidy for education

We discuss the results for the case of government support for education in this section. We consider that the government provides the subsidy rate of ten percent to every adult household in the model population established in the previous section. We present the results for the second steady-state that takes place in environment with idiosyncratic shocks to the human capital and with the government support for education, and we compare it to the initial steady-state. Then, we discuss the transition path that the model economy takes from the original steady-state to the steady-state with government presence from the perspective of the mean (average) household. We also include the discussion for the changes that we observe at the individual level.

Second (final) steady-state

As we have indicated previously, the government can finance its budget with tax on consumption, labour income and capital income. Therefore, we consider three potential steady-states that can exist in the proposed environment with subsidy for education. The summary for these steady-states is given by the tables 3.5, 3.6 and 3.7 below, whereas the distribution of the adult households for each of three cases is depicted by figures 3.2, 3.3 and 3.4.

	Y	K	L	r	w	τ^c		
	1.3783	0.1240	0.9067	2.7048	1.0134	0.0052		
	(8.04%)	(10.96%)	(6.61%)	(-3.57%)	(1.34%)			
	$\bar{h}$	tot.pop.	$\bar{e}$	$\bar{n}$	$\bar{c}$	$\bar{s}$	$\bar{d}$	$\bar{u}$
	1.0950	0.3603	0.0595	0.9739	0.7465	0.2246	0.4571	-0.6122
	(9.50%)	(-63.97%)	(16.25%)	(-2.61%)	(10.39%)	(10.96%)	(7.49%)	(18.39%)
i	h_i	pop.share	e_i	n_i	c_i	s_i	d_i	u_i
1	0.2019	0.0001	0.0000	1.6189	0.1376	0.0414	0.0843	-2.9245
		(-62.81%)	—	(0%)	(0.82%)	(1.34%)	(-1.83%)	(0.09%)
2	0.2466	0.0006	0.0000	1.6189	0.1681	0.0506	0.1029	-2.6287
		(-60.97%)	—	(0%)	(0.82%)	(1.34%)	(-1.83%)	(0.10%)
3	0.3012	0.0023	0.0000	1.6189	0.2053	0.0618	0.1257	-2.3329
		(-57.23%)	—	(0%)	(0.82%)	(1.34%)	(-1.83%)	(0.11%)
4	0.3679	0.0070	0.0042	1.4078	0.2508	0.0755	0.1536	-2.0622
		(-50.50%)	(10.76%)	(-1.09%)	(0.82%)	(1.34%)	(-1.83%)	(0.03%)
5	0.4493	0.0191	0.0104	1.2415	0.3063	0.0922	0.1876	-1.7890
		(-41.35%)	(5.03%)	(-0.79%)	(0.82%)	(1.34%)	(-1.83%)	(0.07%)
6	0.5488	0.0473	0.0180	1.1319	0.3741	0.1126	0.2291	-1.5098
		(-31.68%)	(3.50%)	(-0.59%)	(0.82%)	(1.34%)	(-1.83%)	(0.11%)
7	0.6703	0.1004	0.0272	1.0557	0.4570	0.1375	0.2798	-1.2265
		(-22.09%)	(2.81%)	(-0.45%)	(0.82%)	(1.34%)	(-1.83%)	(0.15%)
8	0.8187	0.1719	0.0385	1.0005	0.5581	0.1680	0.3418	-0.9404
		(-11.76%)	(2.41%)	(-0.35%)	(0.82%)	(1.34%)	(-1.83%)	(0.22%)
9	1.0000	0.2215	0.0523	0.9595	0.6817	0.2051	0.4174	-0.6521
		(0.68%)	(2.17%)	(-0.27%)	(0.82%)	(1.34%)	(-1.83%)	(0.33%)
10	1.2214	0.2059	0.0692	0.9283	0.8326	0.2506	0.5098	-0.3623
		(16.18%)	(2.00%)	(-0.22%)	(0.82%)	(1.34%)	(-1.83%)	(0.62%)
11	1.4918	0.1357	0.0897	0.9042	1.0170	0.3060	0.6227	-0.0712
		(34.33%)	(1.88%)	(-0.17%)	(0.82%)	(1.34%)	(-1.83%)	(3.19%)
12	1.8221	0.0632	0.1149	0.8854	1.2422	0.3738	0.7606	0.2208
		(53.15%)	(1.79%)	(-0.14%)	(0.82%)	(1.34%)	(-1.83%)	(1.10%)
13	2.2255	0.0204	0.1456	0.8706	1.5172	0.4566	0.9290	0.5136
		(74.10%)	(1.72%)	(-0.11%)	(0.82%)	(1.34%)	(-1.83%)	(0.48%)
14	2.7182	0.0042	0.1831	0.8589	1.8531	0.5576	1.1347	0.8069
		(113.29%)	(1.67%)	(-0.09%)	(0.82%)	(1.34%)	(-1.83%)	(0.31%)
15	3.3201	0.0005	0.2289	0.8495	2.2633	0.6811	1.3859	1.1007
		(212.99%)	(1.63%)	(-0.07%)	(0.82%)	(1.34%)	(-1.83%)	(0.23%)

Table 3.5: Second (final) steady-state with presence of idiosyncratic shocks in the human capital and with the government support for education that is financed with the tax on consumption. Note: for each of the individual, average and aggregate variable, the values inside of the parentheses indicate the percentage deviations from the initial steady-state given the government policy

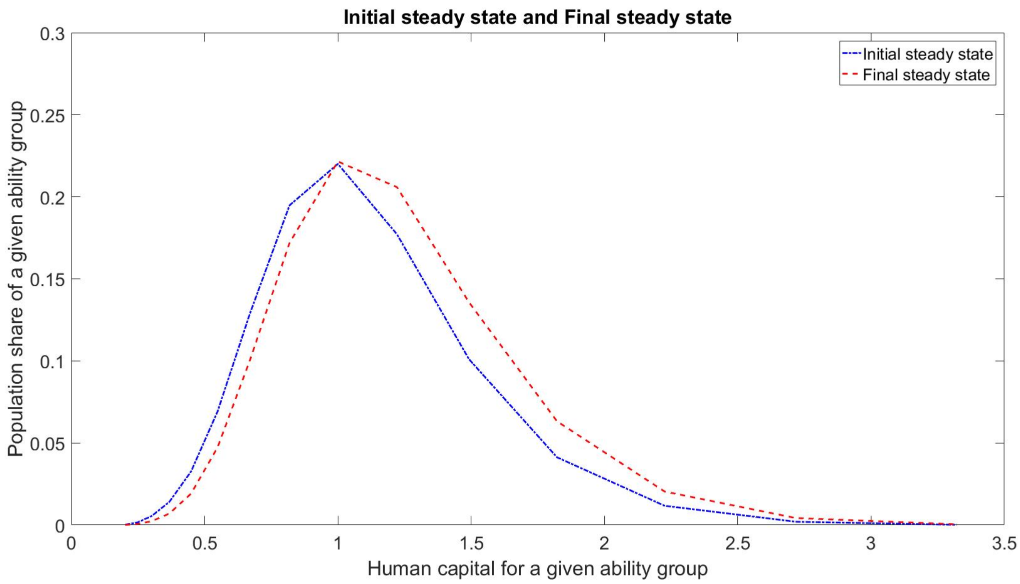

Figure 3.2: Comparison between distribution for the population of adult households at the initial steady-state with the distribution at the second (final) steady-state where government supports education and taxes consumption.
Note: the (marginal) levels of the human capital are depicted on the horizontal axis; population share for each ability group is depicted on the vertical axis; blue dash-dotted line shows the distribution of adult households at the original steady-state; red dashed line illustrates the distribution of adult households at the second (final) steady-state

	Y	K	L	r	w	τ^l		
	1.3716	0.1219	0.9080	2.7520	1.0070	0.0060		
	(7.52%)	(9.04%)	(6.77%)	(-1.89%)	(0.70%)			
	$\bar{h}$	tot.pop.	$\bar{e}$	$\bar{n}$	$\bar{c}$	$\bar{s}$	$\bar{d}$	$\bar{u}$
	1.0893	0.4814	0.0590	0.9812	0.7373	0.2207	0.4572	-0.6244
	(8.93%)	(-51.86%)	(15.27%)	(-1.88%)	(9.04%)	(9.04%)	(7.52%)	(16.77%)
i	h_i	pop.share	e_i	n_i	c_i	s_i	d_i	u_i
1	0.2019	0.0001	0.0000	1.6287	0.1366	0.0409	0.0847	-2.9290
		(-60.45%)	—	(0.60%)	(0.10%)	(0.10%)	(-1.29%)	(-0.06%)
2	0.2466	0.0006	0.0000	1.6287	0.1669	0.0500	0.1035	-2.6332
		(-58.60%)	—	(0.60%)	(0.10%)	(0.10%)	(-1.29%)	(-0.07%)
3	0.3012	0.0024	0.0000	1.6287	0.2039	0.0610	0.1264	-2.3374
		(-54.90%)	—	(0.60%)	(0.10%)	(0.10%)	(-1.29%)	(-0.08%)
4	0.3679	0.0073	0.0042	1.4172	0.2490	0.0745	0.1544	-2.0666
		(-48.29%)	(10.10%)	(-0.43%)	(0.10%)	(0.10%)	(-1.29%)	(-0.18%)
5	0.4493	0.0198	0.0104	1.2495	0.3041	0.0910	0.1886	-1.7934
		(-39.37%)	(4.73%)	(-0.14%)	(0.10%)	(0.10%)	(-1.29%)	(-0.18%)
6	0.5488	0.0484	0.0179	1.1392	0.3714	0.1112	0.2303	-1.5142
		(-30.02%)	(3.29%)	(0.05%)	(0.10%)	(0.10%)	(-1.29%)	(-0.19%)
7	0.6703	0.1021	0.0272	1.0624	0.4537	0.1358	0.2813	-1.2310
		(-20.81%)	(2.64%)	(0.18%)	(0.10%)	(0.10%)	(-1.29%)	(-0.21%)
8	0.8187	0.1735	0.0385	1.0068	0.5541	0.1659	0.3436	-0.9449
		(-10.95%)	(2.27%)	(0.27%)	(0.10%)	(0.10%)	(-1.29%)	(-0.26%)
9	1.0000	0.2218	0.0522	0.9654	0.6768	0.2026	0.4197	-0.6566
		(0.80%)	(2.03%)	(0.35%)	(0.10%)	(0.10%)	(-1.29%)	(-0.35%)
10	1.2214	0.2044	0.0691	0.9340	0.8267	0.2475	0.5126	-0.3668
		(15.34%)	(1.88%)	(0.40%)	(0.10%)	(0.10%)	(-1.29%)	(-0.61%)
11	1.4918	0.1336	0.0896	0.9098	1.0097	0.3023	0.6261	-0.0757
		(32.28%)	(1.76%)	(0.44%)	(0.10%)	(0.10%)	(-1.29%)	(-2.91%)
12	1.8221	0.0618	0.1148	0.8909	1.2332	0.3692	0.7648	0.2163
		(49.72%)	(1.68%)	(0.47%)	(0.10%)	(0.10%)	(-1.29%)	(-0.95%)
13	2.2255	0.0198	0.1454	0.8759	1.5063	0.4509	0.9341	0.5091
		(69.96%)	(1.62%)	(0.50%)	(0.10%)	(0.10%)	(-1.29%)	(-0.40%)
14	2.7182	0.0041	0.1829	0.8641	1.8397	0.5508	1.1409	0.8024
		(104.77%)	(1.57%)	(0.52%)	(0.10%)	(0.10%)	(-1.29%)	(-0.25%)
15	3.3201	0.0004	0.2287	0.8546	2.2471	0.6727	1.3935	1.0962
		(195.46%)	(1.53%)	(0.54%)	(0.10%)	(0.10%)	(-1.29%)	(-0.18%)

Table 3.6: Second (final) steady-state with presence of idiosyncratic shocks in the human capital and with the government support for education that is financed with the tax on labour income.

Note: for each of the individual, average and aggregate variable, the values inside of the parentheses indicate the percentage deviations from the initial steady-state given the government policy

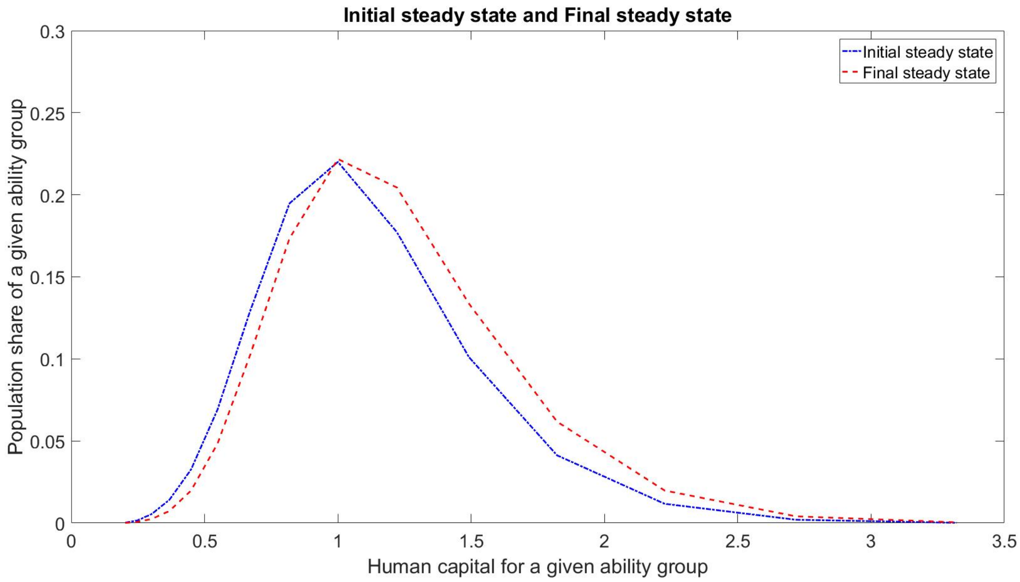

Figure 3.3: Comparison between distribution for the population of adult households at the initial steady-state with the distribution at the second (final) steady-state where government supports education and taxes labour income.

Note: the (marginal) levels of the human capital are depicted on the horizontal axis; population share for each ability group is depicted on the vertical axis; blue dash-dotted line shows the distribution of adult households at the original steady-state; red dashed line illustrates the distribution of adult households at the second (final) steady-state

	Y	K	L	r	w	τ^k		
	1.3783	0.1240	0.9067	2.7048	1.0134	0.0102		
	(8.04%)	(10.96%)	(6.61%)	(-3.57%)	(1.34%)			
	$\bar{h}$	tot.pop.	$\bar{e}$	$\bar{n}$	$\bar{c}$	$\bar{s}$	$\bar{d}$	$\bar{u}$
	1.0950	0.3603	0.0595	0.9739	0.7503	0.2246	0.4560	-0.6078
	(9.50%)	(-63.97%)	(16.25%)	(-2.61%)	(10.96%)	(10.96%)	(7.24%)	(18.98%)
i	h_i	pop.share	e_i	n_i	c_i	s_i	d_i	u_i
1	0.2019	0.0001	0.0000	1.6189	0.1383	0.0414	0.0841	-2.9200
		(-62.81%)	—	(0%)	(1.34%)	(1.34%)	(-2.06%)	(0.24%)
2	0.2466	0.0006	0.0000	1.6189	0.1690	0.0506	0.1027	-2.6243
		(-60.97%)	—	(0%)	(1.34%)	(1.34%)	(-2.06%)	(0.27%)
3	0.3012	0.0023	0.0000	1.6189	0.2064	0.0618	0.1254	-2.3285
		(-57.23%)	—	(0%)	(1.34%)	(1.34%)	(-2.06%)	(0.30%)
4	0.3679	0.0070	0.0042	1.4078	0.2521	0.0755	0.1532	-2.0578
		(-50.50%)	(10.76%)	(-1.09%)	(1.34%)	(1.34%)	(-2.06%)	(0.25%)
5	0.4493	0.0191	0.0104	1.2415	0.3079	0.0922	0.1871	-1.7846
		(-41.35%)	(5.03%)	(-0.79%)	(1.34%)	(1.34%)	(-2.06%)	(0.32%)
6	0.5488	0.0473	0.0180	1.1319	0.3761	0.1126	0.2286	-1.5054
		(-31.68%)	(3.50%)	(-0.59%)	(1.34%)	(1.34%)	(-2.06%)	(0.40%)
7	0.6703	0.1004	0.0272	1.0557	0.4593	0.1375	0.2792	-1.2221
		(-22.09%)	(2.81%)	(-0.45%)	(1.34%)	(1.34%)	(-2.06%)	(0.51%)
8	0.8187	0.1719	0.0385	1.0005	0.5610	0.1680	0.3410	-0.9359
		(-11.76%)	(2.41%)	(-0.35%)	(1.34%)	(1.34%)	(-2.06%)	(0.69%)
9	1.0000	0.2215	0.0523	0.9595	0.6852	0.2051	0.4165	-0.6477
		(0.68%)	(2.17%)	(-0.27%)	(1.34%)	(1.34%)	(-2.06%)	(1.01%)
10	1.2214	0.2059	0.0692	0.9283	0.8369	0.2506	0.5087	-0.3578
		(16.18%)	(2.00%)	(-0.22%)	(1.34%)	(1.34%)	(-2.06%)	(1.84%)
11	1.4918	0.1357	0.0897	0.9042	1.0222	0.3060	0.6213	-0.0668
		(34.33%)	(1.88%)	(-0.17%)	(1.34%)	(1.34%)	(-2.06%)	(9.23%)
12	1.8221	0.0632	0.1149	0.8854	1.2486	0.3738	0.7588	0.2253
		(53.15%)	(1.79%)	(-0.14%)	(1.34%)	(1.34%)	(-2.06%)	(3.14%)
13	2.2255	0.0204	0.1456	0.8706	1.5250	0.4566	0.9268	0.5180
		(74.10%)	(1.72%)	(-0.11%)	(1.34%)	(1.34%)	(-2.06%)	(1.35%)
14	2.7182	0.0042	0.1831	0.8589	1.8626	0.5576	1.1320	0.8114
		(113.29%)	(1.67%)	(-0.09%)	(1.34%)	(1.34%)	(-2.06%)	(0.86%)
15	3.3201	0.0005	0.2289	0.8495	2.2750	0.6811	1.3827	1.1052
		(212.99%)	(1.63%)	(-0.07%)	(1.34%)	(1.34%)	(-2.06%)	(0.63%)

Table 3.7: Second (final) steady-state with presence of idiosyncratic shocks in the human capital and with the government support for education that is financed with the tax on capital income.

Note: for each of the individual, average and aggregate variable, the values inside of the parentheses indicate the percentage deviations from the initial steady-state given the government policy

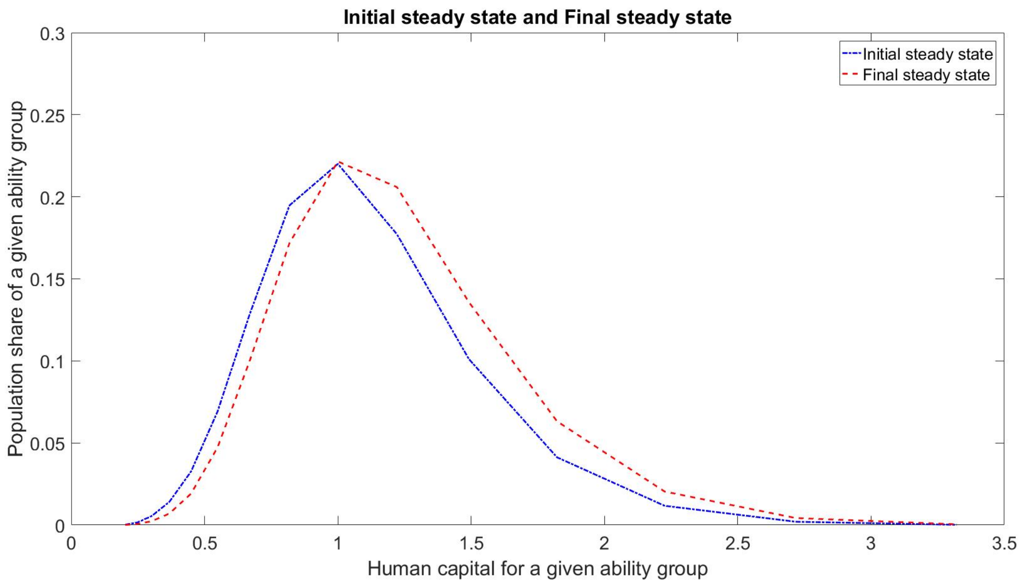

Figure 3.4: Comparison between distribution for the population of adult households at the initial steady-state with the distribution at the second (final) steady-state where government supports education and taxes capital income.
Note: the (marginal) levels of the human capital are depicted on the horizontal axis; population share for each ability group is depicted on the vertical axis; blue dash-dotted line shows the distribution of adult households at the original steady-state; red dashed line illustrates the distribution of adult households at the second (final) steady-state

According to our results, at the second steady-state where the subsidy for education of ten percent is provided, the government has to levy the tax rate on consumption of 0.52% to keep its budget balanced. In the case when government finances its policy with the tax on labour income, however, this tax rate is set at 0.60%. When the capital income tax is utilised instead, the tax rate on the capital income at the second steady-state is chosen at a level of 1.02%. Holding everything else constant, we observe this rise in the tax rates when we shift from tax on consumption to tax on labour income and to tax on capital income because, based on our assumptions, the tax on consumption levies the tax burden on the population of adult and elderly households simultaneously, whereas the tax on labour income is levied on the adult households, and tax on capital income levies the tax burden on elderly households only who receive the return on financial investment.

With the continuous presence of the government support for education in the environment with idiosyncratic shocks to the human capital, we observe that at the second steady-state the level of average education is 16.25% larger than at original steady-state when government utilises the taxes on consumption or capital income, and with the tax on labour income in place instead, the average level of education at the second steady-state is 15.27% larger than at original equilibrium. According to our previous result from the deterministic version of analysis, only the tax on labour income enters directly into the decisions of the adult households for education provision, which reduces the incentives of adult households for education investment for their children.

As we would see from the discussion for the transition path for the dynamics at the individual ability level, the increase in the average education takes place for two reasons. First the provision of subsidy for education reduces the cost for education provision which increases the individual education investment. And second, an increase in education provision increases the human capital at the individual level, which allows children from lower ability groups to become the adult households with larger level of human capital. Due to this increase in the individual human capital levels, population invests into the education more, which raises the average level of education further to the observed levels for the corresponding final steady-states.

Due to increase in the average level of education, the average level of human capital at the second steady-state increases by 9.50% when government levies tax on consumption or capital income. With the labour income tax, the average level of human capital at the second steady-state is 8.93% larger than at original equilibrium. As we have indicated above, the increase in the individual levels of education improves the human capital of the households, which increases the population share of higher ability groups. With a larger population share being a part of the households with a larger level of human capital, the average level of the human capital rises as a result.

The average level of fertility, however, is found to fall by 2.61% when government utilises tax on consumption or capital income, whereas the average fertility at the second steady-state with support for education decreases by 1.88% when the labour income tax is in use. The average level of fertility decreases at a lower rate with tax on the labour income in place, since this tax options crates disincentives for education investment, which through a parental 'quality-quantity' trade-off results in larger fertility decisions at the individual level. Overall, as we would see in our discussion for the dynamics observed at the individual level during the transition of the economy from the original to the corresponding final steady-states, the decrease in the average fertility level occurs due to change in the fertility decisions at the individual ability level due to the presence of the parental 'quality-quantity' trade-off for children, and due to increase in the population share of higher ability households that are characterised by a lower level of fertility.

As a result of a lower fertility decisions at the average level, the population of the adult households becomes 63.97% smaller than at the original steady-state when the economy reaches the final steady-state with subsidy for education of 10% that government finances with tax on consumption or capital income. With the labour income tax, the population size of adult households becomes 51.86% lower than at the original steady-state.

With the lower of average fertility, lower population size of adult households, higher level of average education provision and higher average level of human capital, we observe an increase in the size of effective labour force per adult household. Its level increases by 6.61% when tax on consumption or capital income finance government budget, and by 6.77% when the labour income tax is in use instead. Based on the conclusions from the previous analysis for deterministic case, decrease in the average level of fertility would reduce the average time that parents need to take care for children, which would increase the average time that they can contribute towards the labour market. Decrease in average level of fertility at the second steady-state would also decrease the average number of teachers required, which, in turn, would create an inflow to the effective labour force. Oppositely, increase in the average education would require greater number of teacher on average than at original steady-state to provide the desired level of education. Lastly, increase in the average level of human capital combined with decrease in population size of the adult households would result in smaller but more productive effective labour force[11].

As a result of increase in the effective labour force per adult household at the second steady-state when government subsidies the education, combined with increase in the physical capital stock per adult household, the real wage rate at this second steady-state is 1.34% larger when

[11] At a per adult household level

tax on consumption or capital income is in use, and it becomes 0.70% larger than at original steady-state when labour income tax is in place instead. This, combined with increase in the average level of human capital and presence of taxes, produces an increase in the average level of consumption and average level of saving of adult households at the second steady-state. With the tax on consumption, the average level of consumption increases by 10.39% and the average level of saving rises by 10.96%. With the tax on labour income these average figures both increase by 9.04%. Finally, the average consumption of adult households and their average saving both become 10.96% larger than at original equilibrium if government chooses capital income tax to finance its budget.

Due to increase in the average saving of adult households and decrease in the population of adult households, the physical capital stock per adult household at the second steady-state increases by 10.96% when government uses tax on consumption or capital income, and with the labour income tax in place it rises by 9.04%. With increase in the effective labour force per adult household being smaller than in the physical capital stock per adult household, the marginal product of the physical capital stock per adult household decreases when the economy reaches the second steady-state. Therefore, the real interest rate declines, and it becomes 3.57% lower than at the original steady-state when government uses tax on consumption or capital income, and it falls by 1.89% with labour income tax in place. Consequently, with presence of taxes, higher average level of human capital, higher real wage rate and lower interest rate at the second steady-state, the average consumption of the elderly households increases by 7.49% with presence of tax on consumption, by 7.52% with use of labour income tax, and by 7.24% when capital income tax is in place.

Finally, according to our results, at the second steady-state with the government subsidy for education of ten percent, the average level of utility (i.e. welfare of the economy) increases by 18.39%, 16.77% and 18.98% when tax on consumption, labour income or capital income is in place, respectively. The increase in the level of welfare in the model economy is the net effect of increase in the average level of consumption of adult and elderly households, increase in the average human capital of children, and decrease in the average level of fertility.

Now, when we have a comparison between the original steady-state and the second steady-states for the case of the average (mean) household, we outline the differences that appear at the individual levels. It is still true, however, that when the government budget is financed by the labour income tax, the level of education received by all ability groups is lower than under other tax options considered. Consequently, the individual levels of fertility are higher for all ability groups when labour income tax is in use. Additionally, we still observe that at the individual levels, the consumption of the elderly households at the second steady-state is lower with government budget being financed by capital income tax than with other tax options.

According to our results, at the second steady-state with idiosyncratic shocks to the human capital and with the government support for education, in our model population, the lowest three ability groups of adult households (i.e. $i \in [1,3]$) still provide their children with zero level of education, and they decide to have maximum possible number of children. According to the past analysis for the deterministic environment, the value for the relative human capital is a key variable for the decision on desired level of education. Therefore, with increase in the average level of human capital, due to the change in composition of population that results from the government policy, and with fixed (marginal) levels for human capital, the relative human capital for individual ability group would not increase as we observed for deterministic analysis, but it would decline[12]. As a result, we would either observe a moderate growth in decision for education (as with is the cases for $i \in [4,15]$), or absence of any change (as with the cases for $i \in [1,3]$).

Next, for the cases with tax levied either on consumption or on capital income, we observe an anticipated decline in fertility at the individual-ability-group levels for the second steady-state with government support for education. For the case with the labour income tax, however, the individual fertility choices, in general, are found to be larger at the second steady-state, which, surprisingly, takes place together with increase in individual decisions for education provision. This unexpected result for the case of the labour income tax could be explained as following. First, the tax rate on the labour income distorts the individual choices of the

[12]The level of relative human capital for each individual household, however, would depend on the realised value of the human capital stock for a this particular individual and on the dynamics in the average level of human capital which would depend on the change in the distribution of the population between different ability groups

adult households for education provision, which with the presents of parental ‘quality-quantity’ trade-off for children would result in a marginally larger individual fertility levels than with tax on consumption or capital income. And second, a dampened response of the relative human capital – due to a smaller increase in the average human capital level than under other tax options – would dampen a rise in individual decisions for education, which would minimise the negative response in fertility decisions.

Moving forward, the individual choices for consumption of adult households and their saving are higher at the second steady-state with government support for education. For the individual ability groups, these positive changes occur only due to increase in the real wage rate. Oppositely, the individual consumption for the elderly households across all ability groups is now lower. This occurs because the decrease in the real interest rate is only offset by moderate increase in the real wage rate[13].

Overall, a distorting nature of the labour income tax has been found to decrease the individual utility levels of the adult households at the second steady-state. With other two tax options, however, the individual utility levels are found to increase moderately.

Finally, based on the information for composition of population at the original steady-state and the second steady-state with government policies in place, we observe a marginal narrowing of the distribution of adult households as the economy reaches the second steady-state. Furthermore, with increase in the average level of human capital due to subsidy for education provided by the government, the distribution of adult households shifts to the right. These two changes could suggest that population of the model economy at the second steady-state with government support for education enjoys higher living standards and the human capital is more equally distributed among the population members[14]. As a concluding note, it should be mentioned, however, that even with fixed (marginal) levels of human capital for each ability group, the individual households still experience the changes in their level of human capital which produces the changes in the distribution if the population.

[13] And this moderate increase in the real wage rate is not supported by increase in the individual (marginal) levels of human capital (i.e. what we have observed previously for the deterministic scenario) when we consider the dynamics for the individual ability group.

[14] Please refer to section 3.5.4 where we provide a more detailed discussion for the changes in the inequality in the distribution of the human capital

Transition to the second (final) steady-state

We continue our discussion for the role of the government support for education in the heterogeneous stochastic environment with examination of the transition path that the model economy takes from original steady-state to a second steady-state. We consider the evolution of the model economy from the perspective of average (mean) household together with an overview of dynamics for the individual ability groups. The transition paths of the model economy for average household with the government placing tax on consumption, labour income and capital income are depicted by figure 3.5 where the first period indicates the original steady-state, the final fortieth period represents the second steady-state, and one period of analysis is equivalent to the lifespan of one generation which is considered to be thirty years. We proceed with our discussion starting from a second period.

At the second period of analysis, the government enters into the model economy. In order to provide the subsidy rate for education of ten percent to every heterogeneous adult household and balance its budget, the government levies the tax rate on consumption of 0.51%. In the case when government finances its budget with the tax on labour income, this tax rate is set at 0.60%, whereas, with the capital income tax in use, the tax rate is 0.99%.

With the government support for education, the average level of education received by the children increases by 16.25% from the initial steady-state when government uses tax on consumption or capital income. With labour income tax in place, the average level of education received becomes 15.27% higher compared to the initial equilibrium. At the individual level, the government support for education increases parental choices for education provision for all ability groups with exception for the first three lowest ones (i.e. $i \in [1,3]$). For these three groups, adult households still optimally decide on zero education provision due to low levels of relative human capital, which remains to be identical to the initial steady-state values. An increase in these individual parental choices is proportionally declining with the marginal levels of human capital for individual ability groups, however. For instance, our results indicate, that for the fourth ability group (i.e. $i = 4$), the education attainment increases by 80.88% and 76.02% when government finances its budget with either consumption or capital income tax, and with labour income tax, respectively. This value increases by 16.28% and 15.30% for the

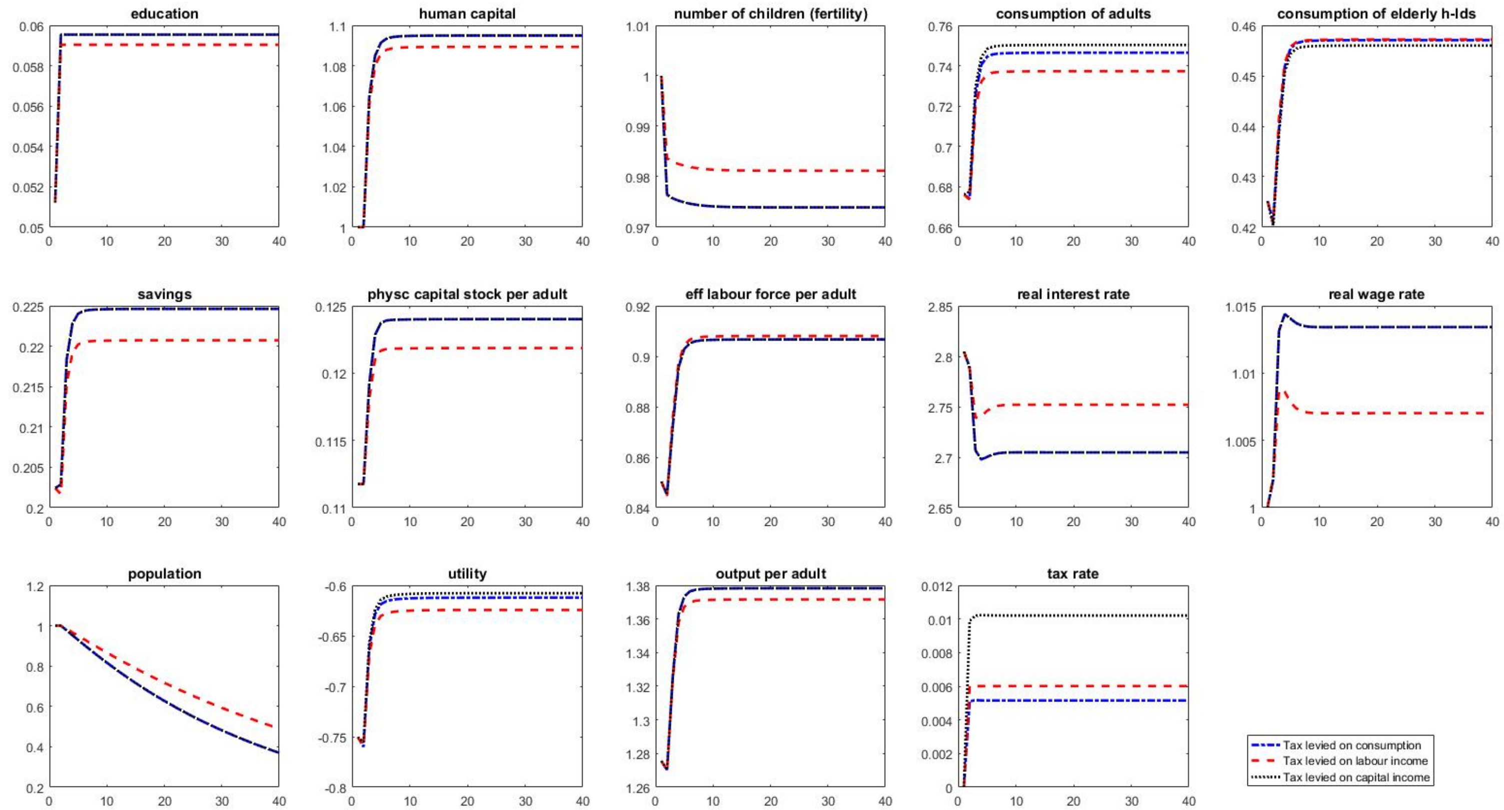

Figure 3.5: Transition path of the model economy with idiosyncratic shocks to the human capital from the perspective of the average household. Government provides subsidy for education of ten percent.

Note: transition path with tax on consumption is depicted by blue dash-dotted line; transition path with tax on labour income is presented by red dashed line; transition path with capital income tax is shown by black dotted line

ninth ability group, and by 12.58% and 11.82% for the fourteenth ability group, with tax on consumption or capital income and with tax on labour income in use, respectively.

These results can be rationalised based on the analytical conclusions from the deterministic version of model economy[15]. In the deterministic version, the individual choice for education is defined by (3.15)

$$e_t^i = \frac{\eta\phi(1-\tau_t^l)x_t^i - \theta(1-sub_t^e) - \eta\bar{e}_t sub_t^n}{(1-\eta)(1-sub_t^e)} \tag{3.15}$$

with the first order partial derivative of the individual choice for education with respect to the subsidy rate for education defined by (3.16)

$$\frac{\partial e_t^i}{\partial sub_t^e} \approx \frac{\eta\phi(1-\tau_t^l)x_t^i - \eta\bar{e}_t sub_t^n}{(1-\eta)(1-sub_t^e)^2} \tag{3.16}$$

and second order partial derivative with respect to the relative human capital defined by (17)

$$\frac{\partial^2 e_t^i}{\partial sub_t^e \partial x_t^i} \approx \frac{\eta\phi(1-\tau_t^l)}{(1-\eta)(1-sub_t^e)^2} \tag{3.17}$$

which is less than one and with the current calibration for the model parameters is equal to 0.0926 for the case of tax on consumption or capital income being used to finance the government budget, and is equal to the 0.0920 when the labour income tax in place instead. This result suggest that with introduction of government support for education and with shift from one ability group to another (i.e. from lower marginal value of relative human capital to higher marginal value of human capital), the rate of increase in the optimal education provision would diminish.

Based on the previous results for the deterministic version of analysis, we would expect to observe an increase in both average and individual levels of human capital for the next generation of adult households, which would begin a change in the composition of the model population. We, however, could not anticipate the complete response of the model economy for the next period of analysis yet due to the presence of the idiosyncratic shocks in the human capital formation process.

[15]Please refer to appendix A1 section

Considering another factor that results in the change of a future composition of the model population and population size overall, we observe that the average level of fertility declines by 2.36% and 1.64% at the first period of the government presence when tax on consumption or capital income, and labour income are in use, respectively. This decline in fertility choices of the adult households is explained by the parental 'trade-off' for children which is an underlying characteristics of the utilised model economy. Furthermore, as we indicated previously, a smaller decline in (the average) fertility under the labour income tax can be explained by the distorting nature of this tax on the parental decisions for education provision. As we observed above, the level of education increases by smaller rate (both at individual and average levels) with the labour income tax financing government budget, which, according to the conclusions from the deterministic version of analysis, would result in a smaller decline in fertility choices compared to the other tax options.

At the individual level, the households from the lower ability groups experience much larger decline in fertility compared to the households from the higher ability groups. For instance, with a presence of tax on consumption or capital income, the adult households from the fourth, eights and top (fifteenth) ability groups reduce their fertility choices by 7.05%, 2.35% and 0.50%, respectively. For the case of the labour income tax, however, due to its distorting nature, for the top two ability groups (i.e. $i = 14, 15$), we even observe an increase in individual fertility choices at the second period of analysis. Overall, the adult households from the fourth and eights ability groups experience decline in fertility by 6.13% and 1.63%, respectively, whereas, the households from the top ability group experience an increase of 0.13% at the first period of government presence when the labour income tax is in use.

In order to address a decrease in a rate of the individual negative responses of adult households for fertility when we shift from lower ability groups to higher ability ones, we utilise conclusions from the discussion for individual choices for education. According to our results above, when the government enters the economy and provides subsidy for education, the lower ability groups with lower levels of relative human capital would increase their choices for education by larger amount than the higher ability households with higher levels of relative human capital. This, combined with the underlying properties for the behaviour of the households which indicate a negative relationship between choices for education and fertility results in a larger decline in fertility for groups who experience a larger increase in education (i.e. lower ability groups). For the households from higher ability groups, the increase in education has been found to be moderate, which translates in moderate decline in fertility as in the case of tax on consumption or capital income, or, even, a moderate increase as for the scenario with the labour income tax. According to our results, however, the only exception for this dynamics is presented by the three lowest ability groups (i.e. $i \in [1,3]$) due to the absence of change in the optimal decisions for education at the second period of analysis.

With this observed dynamics for education attainment and fertility decisions, combined with the absence of a change in the composition of adult households – since it has been formed at the original steady-state before government has entered, our results indicate a decrease in the size of the effective labour force per adult household of 0.63% and 0.68% at the second period of analysis when government uses tax on consumption or capital income and tax on the labour income, respectively. This decline in the size of effective labour force per adult household is the net effect that results from the following changes. First, an increase in the choice for education provision combined with identical level of human capital of adult households to the initial steady-state requires an increase in the number of teachers in the economy which reduces a number of adult households available for participation in the effective labour force. And, second, a wide decrease in the fertility choices creases a decrease in the number of teachers required, and it increases the free time that adult household can contribute to participation in effective labour force instead of taking care for their children.

Opposite to the observed decrease in the size of effective labour force per adult household, the physical capital stock per adult household remains unchanged at the first period of the government presence. This is the case, because according to the underling assumptions of the utilised model economy, the level of the physical capital stock per adult household is defined by the aggregate saving of the past generation of the adult households (considering there is a full depreciation of the physical capital stock) and by the present population size of adult households. In the case of the second period of analysis, both of these variables are defined and identical to the initial steady-state.

Absence of change in the physical capital stock per adult household and observed decrease in the effective labour force per adult household, decreases the value of the marginal product of the former and increases it for the latter. Since both of these factors of production are paid their marginal products, for the second period of analysis we observe a decrease in the real interest rate of 0.57% when government uses tax on consumption or capital income, and it falls by 0.61% with labour income tax in place. For the real wage rate, however, our results indicate 0.21% increase when tax on consumption or capital income is utilised, and it rises by 0.23% when tax on labour income is implemented instead. We observe a marginally larger increase in the real wage rate and marginally smaller decrease in the real interest rate for the case of the labour income tax compared to the other tax options because, according to formulation of the model economy, only labour income tax distorts the decisions of adult households for education provision. As we indicated earlier, the use of the labour income tax results in smaller increase of the education attainment, which results in a smaller decrease in fertility choices, which produces larger decrease in the effective labour force per adult household, which leads to a larger increase in the marginal product of the effective labour force per adult household and greater decrease in the marginal product of the physical capital stock per adult household at the second period of analysis compared to other two taxes.

With increase in the real wage rate and introduction of three potential tax options, our results indicate a decrease in the consumption of the adult households at the second period of analysis when government uses tax on consumption and labour income. With tax levied on consumption, the consumption of the adult households for all ability groups decreases by 0.30%, while with the labour income tax it falls by 0.37%. With the capital income tax in place, however, which does not affect the consumption decisions of the adult households, the consumption of the adult households from every ability group increases by 0.21%. Next, saving of the adult households, which is discouraged by the presence of the labour income tax only and increase with the rise in the real wage rate, grow by 0.21% for adult households from every ability group when tax on consumption or capital income is in use. With the labour income tax in place, however, saving of the adult households fall by 0.37%. Lastly, according to the conclusions from the deterministic version of the analysis, the consumption of the elderly households is increasing with past real wage rate and present real interest rate, and decreasing with the past tax rate on labour income, and present tax rates on consumption and capital income. Therefore, according to our results, consumption of the elderly households for every ability group falls by 0.93%, 1.05% and 1.15% with the tax on consumption, tax on labour income, and tax on capital income in place, respectively.

As a result of these observed changes in consumption of the adult and elderly households, and in the fertility choices for the first period of the government presence, the level of welfare is decreasing under all tax options. For instance, with the tax on consumption, the level of welfare decreases by 1.32%, and with the tax on labour income and capital income this value is falling by 1.29% and 0.72%, respectively. Furthermore, at the individual level, we observe that households from the lower ability groups suffer a larger decrease in the utility level compared to a higher ability groups, due to larger negative responses of the lower ability groups in their fertility decisions. Finally, due to a reduction in the effective labour force per adult household and absence of change in value of the physical capital stock per adult household, we observe a decrease in the level of the real output per adult household produced at the second period of analysis. Our results indicate 0.42% decrease in this value when the government uses tax on consumption or capital income, and it falls by 0.45% when the labour income tax is utilised instead.

Moving to the next period of analysis, we observe a change in the composition of population for adult households due to a past period government support for education. Our results indicate an increase in the population share for a higher adult households from tenth to fifteenth ability groups (i.e. $i \in [10, 15]$); whereas the share within the total population for the remaining ability groups is declining at the third period of analysis. There are few reasons that explain this change. First, due to the subsidy for education and resulted larger response in the education attainment for lower ability groups compared to a higher ability ones, the lower ability groups end up with the realised level of human capital that is larger than the marginal level of human capital that characterises a particular ability group. As a result, past period young households from the lower ability group now have enough human capital to become a part of higher ability groups. And second, due to a larger negative reaction of the lower ability households in their fertility choices that we observed for the second period of analysis that results from a relatively larger increase in education attainment for these groups, there is a relatively smaller number of adult households from a lower ability groups compared to a higher ability ones that exist at the third period of analysis. Therefore, the population of the adult households at the second period of the government support for education is characterised by the average (mean) level of human capital that is 6.45% and 6.07% larger than at the original steady-state when government uses tax on consumption or capital income, and labour income, respectively. However, due to decrease in the past fertility choices, the total population size of the adult households is decreasing at the third period of analysis, and it becomes 2.35% lower than at original steady-state when government utilises tax on consumption or capital income. With labour income tax in use, the size of the adult households diminishes by 1.64%[16].

In order to keep providing the support for education at ten percent rate and keep the government budget balanced, the government increases the tax rate on consumption up to 0.52% and tax rate on capital income up to 1.02%. The tax rate on the labour income is kept unchanged, however.

With a continuous government support for education and change in the structure of the population of adult households, our results indicate that the average level of education received at the third period of analysis is 16.25% larger than at original steady-state when tax on consumption or capital income is in use, which, however, is 0.0018% lower than at the previous period of analysis. In the case of the labour income tax, the average level of education at the second period of government presence is 15.27% above the original steady-state, which indicates a 0.0012% decrease compared to the previous generation.

This change in the average level of education attainment could be reasoned as following. First, since the marginal levels of the human capital for each ability group are kept unchanged, the increase in the average level of human capital due to past period government support for

[16]This smaller decrease in the population size with the labour income tax in use is explained by the distorting nature of this tax. As it has been indicated earlier, the use of the labour income tax by the government to finance the support for education produces response in education provision that is muted compared to the other tax options. With the parental trade-off in place, a relatively smaller increase in education provision results in a relatively smaller decline in fertility, that produces a relatively smaller decrease in the population size of the future adult households

education decreases the values of the relative human capital for each ability group. As we have discussed it earlier, the relative human capital is a key variable for the decision of adult households in the utilised model economy for education provision. Therefore, according to our results, there is a decrease in the value of the (realised) relative human capital across all ability groups at the third period of analysis which diminishes the optimal decisions of the current adult households for education provision for their children. Our results suggest, however, that the optimal choice for education at the second period of the government intervention is still larger than at the original steady-state. For instance, with the tax on consumption or capital income the education provision of the fourth, ninth and fourteenth ability groups are 31.87%, 6.41% and 4.96% higher than at original steady-state, respectively. With the tax on labour income in use the level of education attainment by these groups is 29.99%, 6.03% and 4.66% larger than at first equilibrium. Furthermore, according to our results, the lower ability groups experience larger decrease in optimal choice for education compared to a higher ability groups. The second reason for a very moderate decrease in the average level of education even when there are sizeable declines within this decision for each ability group is a change in composition of population for the adult households. As we have indicated above, with the government support for education, the individual households from lower ability groups (especially ones who faced positive idiosyncratic shocks during human capital formation process) become a part of the population that is defined by a relatively larger level of the marginal human capital, who are able to provide their children with larger level of education. Since the share of the adult households from the higher ability groups is increasing at the second period of government support for education, the average level of education is kept relatively stable.

With decrease in the optimal choices for education within each ability group, our results indicate the increase in fertility levels. For instance, with the tax on consumption or capital income, the childbearing decisions of the adult households for the fourth, eights and fourteenth ability groups are increasing by 4.27%, 1.38% and 0.36%, and become 3.08%, 1.00% and 0.26% below the original steady-state, respectively. With the labour income tax, the fertility choices for these three ability groups are increasing by 4.05%, 1.31% and 0.34% from the previous period of analysis, and they become 2.33% and 0.34% below the original equilibrium for the fourth and eights ability groups. For the adult households from the fourteenth ability group, the level of fertility becomes 0.36% above the original steady-state, however. Despite these positive changes for the fertility levels for each ability group within the population of adult households (with exception for the bottom ability groups that provide their children with zero education), the change in the overall composition of the model population results in the lower levels of the average fertility for the second period of the government presence. According to our results, the average level of fertility at the third period of analysis is falling by 0.07% and becomes 2.43% below of original steady-state when tax on consumption or capital income is in place. With the labour income tax, the average level of fertility reduces by 0.07% and becomes 1.71% below the original equilibrium.

Given an increase in the average human capital of adult households, the effective labour force becomes more productive compared to the previous generation. Furthermore, an increase in the average level of the human capital at the third period of analysis results in improvement of teachers' ability, which decreases the number of teachers required that results in greater number of adult households participating in the effective labour force. Additionally, with decrease in the average fertility level, the average time that adult households need to spend for taking care of the children is falling, which increases the time that adult households participate in the effective labour force on average. Finally, a decrease in the average fertility level reduces the number of teachers required even further. Consequently, our results indicate that at the second period of the government presence, the effective labour force per adult household is increasing by 3.23% from the previous generation and becomes 2.58% above the original steady-state when tax on consumption or capital income finance the government budget. With the labour income tax, the effective labour force per adult household increases by 3.65% and is 2.94% above the initial steady-state. This larger increase in the level of the effective labour force per adult household in the case of the labour income tax is primarily driven by a smaller decrease in the population of the adult households and larger decrease in the average birth rate, all of which result in a larger number of adult households who can participate in the labour market at the current generation.

Next, the physical capital stock per adult household, which is determined by the aggregate saving of the adult households from the past generation and by the present size of the population of adult households, becomes 6.68% larger than at the original steady-state when the government implements tax on consumption or capital income. With the labour income tax, which has reduced incentives of the adult households for saving, our results indicate that the value for the physical capital stock per adult household at the third period of analysis is increases by 5.68% from original equilibrium. With the labour income tax, the model economy experiences an increase in the per adult household value of this second factor input because the rate of fall in the present population size of the adult households outweighs the rate of decrease in saving decisions of the previous generation.

With each of these two factors of production being paid their corresponding marginal products, relatively larger increase in the physical capital stock per adult household compared to increase in the effective labour force per adult household diminishes the real interest rate and increases the real wage rate by 2.94% and 1.10% that become 3.50% below and 1.31% above the original steady-state, respectively, when tax on consumption or capital income is in use. With tax on labour income, the real interest rate falls by 1.74% and it is 2.35% below the original equilibrium, whereas the real wage rate rises by 0.65% and becomes 0.88% above the initial steady-state.

Given these changes in the factor prices together with revised tax rates, we observe a uniform increase in the consumption and saving of the adult households across all ability groups. Our results indicate that with the tax on consumption, the consumption of adult households increases by 1.095% and becomes 0.79% larger than at the original steady-state, whereas saving of adult households inside of every ability group increases by 1.10% and is 1.31% larger than at initial steady-state. With the labour income tax, both of these indicators for every ability group increase by 0.65% and become 0.27% larger than at initial equilibrium, whereas with capital income tax, these values increase by 1.10% and they are 1.31% larger than at the original equilibrium. The consumption of the elderly households within of each ability group continues its decrease, however, due to the present period decline in the real interest rate. Therefore, the consumption of the elderly households in each ability group decreases by 1.97%, 1.06% and 1.98%, which becomes 2.88%, 2.10% and 3.10% below of original steady-state with tax on consumption, labour income and capital income in use, respectively. However, with the change in the structure of the model population, where households from the lower ability groups become a part of higher ability groups who enjoy higher consumption during adulthood and at the old age together with higher decisions for saving, we observe that the average consumption of the adult households increases by 7.62%, 6.76% and 7.62%, and becomes 7.30%, 6.36% and 7.85% larger than at the original steady-state, when tax on consumption, labour income or capital income is in place. The average saving of the adult households at the third period of analysis increases by 7.62%, 6.76% and 7.62%, and is 7.85%, 6.36% and 7.85% higher than at the initial equilibrium; whereas, the average consumption of the elderly households at the second period of the government presence rises by 4.36%, 4.95% and 4.35%, which is 3.39%, 3.85% and 3.15% larger than at the original steady-state, when tax on consumption, labour income or capital income is utilised, respectively.

With all these changes at the third period of analysis, we observe an increase in the level of welfare by 12.98%, 12.06% and 13.06%, which becomes 11.84%, 10.92% and 12.43% above the original steady-state when government uses tax on consumption, labour income and capital income, respectively. Finally, with increase in both effective labour force per adult household and physical capital stock per adult household at the second period of the government presence with support for education attainment, the real output per adult household rises by 4.36% and becomes 3.92% above the initial equilibrium when the government uses tax on consumption or capital income. With labour income tax in place instead, the real output per adult household increases by 4.32% and it is 3.85% above the initial steady-state.

The dynamics described for the third period of analysis continues for all the variables with exception for consumption of the elderly households within a given ability groups until the economy reaches the second steady-state. At the fourth period of analysis the past period increase in the real wage rate is enough to counteract a fall in the real interest rate rate and (the negative) effect of taxes. As a result, the consumption of the elderly households within each ability group increases, which, together with the change in the composition of the model population, contributes to an increase in the average consumption of the elderly households, level of welfare and individual utility levels for each ability group even further.

3.5.3 Subsidy for children

Moving our discussion forward, we consider the influence of the government support for fertility decisions of the heterogeneous households in the proposed stochastic environment next. We consider that the government provides every adult household in the model population with ten percent subsidy rate for childbearing decisions. Similarly to our previous discussion, the government has three tax options[17] that it can utilise in order to finance the subsidy program and keep its budget balanced. In the upcoming discussion we implement the same order of analysis as before: first, we examine the second steady-state that the model economy reaches with subsidy program for fertility, and next, we analyse the transition path that the model economy takes from the original steady-state to the second steady-state. In our discussion, we focus on the changes that take place both for the mean household and for the households from individual ability groups. The summary for the steady-states with government support for fertility that is financed with tax on consumption, labour income and capital income is presented by the tables 3.8, 3.9 and 3.10, whereas figures 3.6, 3.7 and 3.8 illustrate the initial and resulted distribution of the model population, respectively.

Second (final) steady-state

According to our results, in order to balance its budget and provide the subsidy rate for children of ten percent, the government levies the tax rate on consumption of 0.47% at the second steady-state. Alternately, with the labour income tax in place, the tax rate is set at 0.58%, and with the capital income tax in use, the tax rate balances at 0.84%.

[17] I.e.: tax on consumption, labour income and capital income

	Y	K	L	r	w	τ^c		
	1.2421	0.1000	0.8639	3.1417	0.9585	0.0047		
	(-2.63%)	(-10.55%)	(1.58%)	(12.01%)	(-4.15%)			
	$\bar{h}$	tot.pop.	$\bar{e}$	$\bar{n}$	$\bar{c}$	$\bar{s}$	$\bar{d}$	$\bar{u}$
	0.9333	192.87	0.0467	1.1498	0.6020	0.1811	0.4121	-0.9095
	(-6.68%)		(-8.80%)	(14.98%)	(-10.97%)	(-10.55%)	(-3.09%)	(-21.23%)
i	h_i	pop.share	e_i	n_i	c_i	s_i	d_i	u_i
1	0.2019	0.0010	0.0000	2.2733	0.1302	0.0392	0.0891	-2.9020
		(396.02%)	—	(40.42%)	(-4.60%)	(-4.15%)	(3.84%)	(0.86%)
2	0.2466	0.0077	0.0000	2.1181	0.1591	0.0478	0.1089	-2.6189
		(411.58%)	—	(30.84%)	(-4.60%)	(-4.15%)	(3.84%)	(0.47%)
3	0.3012	0.0236	0.0000	2.0060	0.1943	0.0584	0.1330	-2.3329
		(337.56%)	—	(23.91%)	(-4.60%)	(-4.15%)	(3.84%)	(0.11%)
4	0.3679	0.0426	0.0011	1.8417	0.2373	0.0714	0.1624	-2.0525
		(202.68%)	(-71.14%)	(29.39%)	(-4.60%)	(-4.15%)	(3.84%)	(0.51%)
5	0.4493	0.0601	0.0076	1.4959	0.2898	0.0872	0.1984	-1.7940
		(84.41%)	(-22.83%)	(19.55%)	(-4.60%)	(-4.15%)	(3.84%)	(-0.21%)
6	0.5488	0.0859	0.0156	1.2966	0.3540	0.1065	0.2423	-1.5239
		(24.13%)	(-9.94%)	(13.87%)	(-4.60%)	(-4.15%)	(3.84%)	(-0.83%)
7	0.6703	0.1281	0.0254	1.1691	0.4324	0.1301	0.2960	-1.2467
		(-0.61%)	(-4.06%)	(10.24%)	(-4.60%)	(-4.15%)	(3.84%)	(-1.49%)
8	0.8187	0.1728	0.0373	1.0819	0.5281	0.1589	0.3615	-0.9649
		(-11.31%)	(-0.74%)	(7.76%)	(-4.60%)	(-4.15%)	(3.84%)	(-2.38%)
9	1.0000	0.1860	0.0519	1.0197	0.6450	0.1940	0.4415	-0.6797
		(-15.46%)	(1.36%)	(5.99%)	(-4.60%)	(-4.15%)	(3.84%)	(-3.88%)
10	1.2214	0.1498	0.0697	0.9739	0.7878	0.2370	0.5393	-0.3922
		(-15.50%)	(2.78%)	(4.68%)	(-4.60%)	(-4.15%)	(3.84%)	(-7.58%)
11	1.4918	0.0887	0.0914	0.9393	0.9623	0.2894	0.6587	-0.1029
		(-12.25%)	(3.79%)	(3.70%)	(-4.60%)	(-4.15%)	(3.84%)	(-39.89%)
12	1.8221	0.0388	0.1180	0.9127	1.1753	0.3535	0.8045	0.1878
		(-5.84%)	(4.53%)	(2.94%)	(-4.60%)	(-4.15%)	(3.84%)	(-14.03%)
13	2.2255	0.0123	0.1504	0.8921	1.4355	0.4318	0.9827	0.4794
		(4.65%)	(5.08%)	(2.35%)	(-4.60%)	(-4.15%)	(3.84%)	(-6.20%)
14	2.7182	0.0025	0.1900	0.8759	1.7533	0.5274	1.2002	0.7719
		(25.96%)	(5.51%)	(1.89%)	(-4.60%)	(-4.15%)	(3.84%)	(-4.04%)
15	3.3201	0.0003	0.2383	0.8630	2.1415	0.6442	1.4659	1.0651
		(85.14%)	(5.84%)	(1.53%)	(-4.60%)	(-4.15%)	(3.84%)	(-3.02%)

Table 3.8: Second (final) steady-state with presence of idiosyncratic shocks in the human capital and with the government support for fertility that is financed with the tax on consumption. Note: for each of the individual, average and aggregate variable, the values inside of the parentheses indicate the percentage deviations from the initial steady-state given the government policy

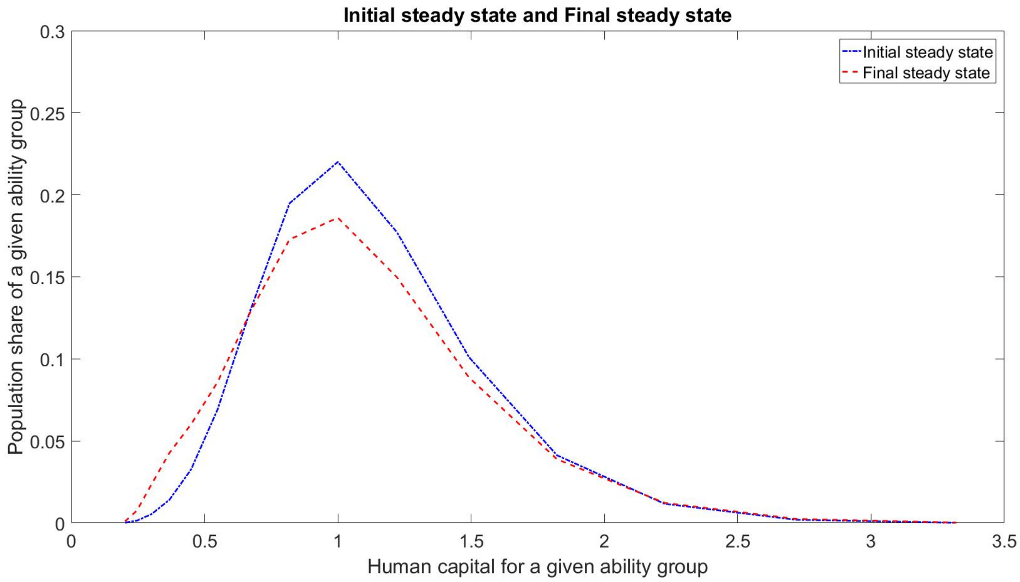

Figure 3.6: Comparison between distribution for the population of adult households at the initial steady-state with the distribution at the second steady-state where government supports fertility decisions and taxes consumption.
Note: the (marginal) levels of the human capital are depicted on the horizontal axis; population share for each ability group is depicted on the vertical axis; blue dash-dotted line shows the distribution of adult households at the original steady-state; red dashed line illustrates the distribution of adult households at the second steady-state

	Y	K	L	r	w	τ^l		
	1.2362	0.0983	0.8652	3.1928	0.9526	0.0058		
	(-3.09%)	(-12.06%)	(1.73%)	(13.83%)	(-4.74%)			
	$\bar{h}$	tot.pop.	$\bar{e}$	$\bar{n}$	$\bar{c}$	$\bar{s}$	$\bar{d}$	$\bar{u}$
	0.9286	255.78	0.0463	1.1583	0.5946	0.1780	0.4121	-0.9216
	(-7.14%)		(-9.57%)	(15.82%)	(-12.06%)	(-12.06%)	(-3.09%)	(-22.84%)
i	h_i	pop.share	e_i	n_i	c_i	s_i	d_i	u_i
1	0.2019	0.0010	0.0000	2.2796	0.1293	0.0387	0.0896	-2.9073
		(408.22%)	—	(40.81%)	(-5.29%)	(-5.29%)	(4.36%)	(0.68%)
2	0.2466	0.0078	0.0000	2.1256	0.1579	0.0473	0.1094	-2.6241
		(422.10%)	—	(31.30%)	(-5.29%)	(-5.29%)	(4.36%)	(0.28%)
3	0.3012	0.0240	0.0000	2.0141	0.1929	0.0577	0.1337	-2.3380
		(345.65%)	—	(24.41%)	(-5.29%)	(-5.29%)	(4.36%)	(-0.10%)
4	0.3679	0.0434	0.0011	1.8489	0.2356	0.0705	0.1633	-2.0575
		(207.89%)	(-70.78%)	(29.90%)	(-5.29%)	(-5.29%)	(4.36%)	(0.26%)
5	0.4493	0.0610	0.0076	1.5028	0.2877	0.0861	0.1994	-1.7990
		(87.30%)	(-22.74%)	(20.09%)	(-5.29%)	(-5.29%)	(4.36%)	(-0.49%)
6	0.5488	0.0870	0.0156	1.3030	0.3514	0.1052	0.2435	-1.5288
		(25.75%)	(-9.93%)	(14.43%)	(-5.29%)	(-5.29%)	(4.36%)	(-1.15%)
7	0.6703	0.1292	0.0254	1.1751	0.4292	0.1285	0.2975	-1.2516
		(0.24%)	(-4.08%)	(10.81%)	(-5.29%)	(-5.29%)	(4.36%)	(-1.89%)
8	0.8187	0.1732	0.0373	1.0877	0.5243	0.1570	0.3633	-0.9697
		(-11.10%)	(-0.78%)	(8.34%)	(-5.29%)	(-5.29%)	(4.36%)	(-2.89%)
9	1.0000	0.1852	0.0519	1.0253	0.6404	0.1917	0.4438	-0.6845
		(-15.82%)	(1.30%)	(6.57%)	(-5.29%)	(-5.29%)	(4.36%)	(-4.61%)
10	1.2214	0.1482	0.0696	0.9793	0.7821	0.2342	0.5420	-0.3969
		(-16.36%)	(2.71%)	(5.26%)	(-5.29%)	(-5.29%)	(4.36%)	(-8.89%)
11	1.4918	0.0873	0.0914	0.9446	0.9553	0.2860	0.6620	-0.1076
		(-13.60%)	(3.71%)	(4.28%)	(-5.29%)	(-5.29%)	(4.36%)	(-46.35%)
12	1.8221	0.0380	0.1179	0.9179	1.1668	0.3493	0.8086	0.1830
		(-7.80%)	(4.45%)	(3.52%)	(-5.29%)	(-5.29%)	(4.36%)	(-16.21%)
13	2.2255	0.0119	0.1503	0.8972	1.4251	0.4267	0.9876	0.4747
		(1.80%)	(5.00%)	(2.94%)	(-5.29%)	(-5.29%)	(4.36%)	(-7.12%)
14	2.7182	0.0024	0.1898	0.8809	1.7406	0.5211	1.2062	0.7672
		(21.40%)	(5.42%)	(2.48%)	(-5.29%)	(-5.29%)	(4.36%)	(-4.63%)
15	3.3201	0.0003	0.2382	0.8680	2.1260	0.6365	1.4733	1.0603
		(75.77%)	(5.75%)	(2.11%)	(-5.29%)	(-5.29%)	(4.36%)	(-3.45%)

Table 3.9: Second (final) steady-state with presence of idiosyncratic shocks in the human capital and with the government support for fertility that is financed with the tax on labour income. Note: for each of the individual, average and aggregate variable, the values inside of the parentheses indicate the percentage deviations from the initial steady-state given the government policy

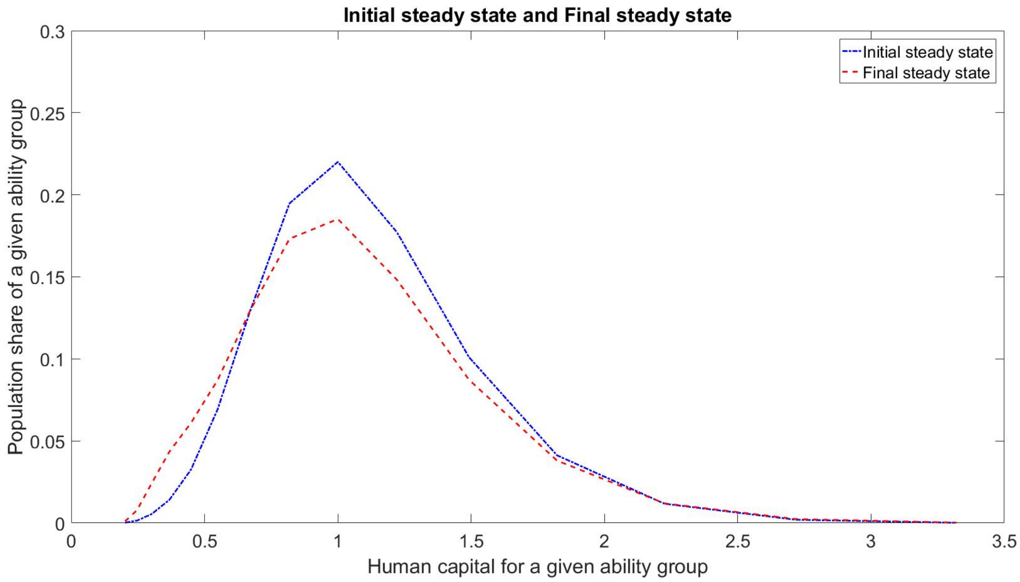

Figure 3.7: Comparison between distribution for the population of adult households at the initial steady-state with the distribution at the second steady-state where government supports fertility decisions and taxes labour income.

Note: the (marginal) levels of the human capital are depicted on the horizontal axis; population share for each ability group is depicted on the vertical axis; blue dash-dotted line shows the distribution of adult households at the original steady-state; red dashed line illustrates the distribution of adult households at the second steady-state

	Y	K	L	r	w	τ^k		
	1.2421	0.1000	0.8639	3.1417	0.9585	0.0084		
	(-2.63%)	(-10.55%)	(1.58%)	(12.01%)	(-4.15%)			
	$\bar{h}$	tot.pop.	$\bar{e}$	$\bar{n}$	$\bar{c}$	$\bar{s}$	$\bar{d}$	$\bar{u}$
	0.9333	192.87	0.0467	1.1498	0.6048	0.1811	0.4114	-0.9052
	(-6.68%)		(-8.80%)	(14.98%)	(-10.55%)	(-10.55%)	(-3.26%)	(-20.66%)
i	h_i	pop.share	e_i	n_i	c_i	s_i	d_i	u_i
1	0.2019	0.0010	0.0000	2.2733	0.1308	0.0392	0.0890	-2.8978
		(396.02%)	—	(40.42%)	(-4.15%)	(-4.15%)	(3.66%)	(1.00%)
2	0.2466	0.0077	0.0000	2.1181	0.1598	0.0478	0.1087	-2.6147
		(411.58%)	—	(30.84%)	(-4.15%)	(-4.15%)	(3.66%)	(0.63%)
3	0.3012	0.0236	0.0000	2.0060	0.1952	0.0584	0.1328	-2.3287
		(337.56%)	—	(23.91%)	(-4.15%)	(-4.15%)	(3.66%)	(0.29%)
4	0.3679	0.0426	0.0011	1.8417	0.2384	0.0714	0.1622	-2.0483
		(202.68%)	(-71.14%)	(29.39%)	(-4.15%)	(-4.15%)	(3.66%)	(0.71%)
5	0.4493	0.0601	0.0076	1.4959	0.2912	0.0872	0.1981	-1.7898
		(84.41%)	(-22.83%)	(19.55%)	(-4.15%)	(-4.15%)	(3.66%)	(0.02%)
6	0.5488	0.0859	0.0156	1.2966	0.3557	0.1065	0.2419	-1.5197
		(24.13%)	(-9.94%)	(13.87%)	(-4.15%)	(-4.15%)	(3.66%)	(-0.55%)
7	0.6703	0.1281	0.0254	1.1691	0.4344	0.1301	0.2955	-1.2425
		(-0.61%)	(-4.06%)	(10.24%)	(-4.15%)	(-4.15%)	(3.66%)	(-1.15%)
8	0.8187	0.1728	0.0373	1.0819	0.5306	0.1589	0.3609	-0.9606
		(-11.31%)	(-0.74%)	(7.76%)	(-4.15%)	(-4.15%)	(3.66%)	(-1.93%)
9	1.0000	0.1860	0.0519	1.0197	0.6481	0.1940	0.4408	-0.6755
		(-15.46%)	(1.36%)	(5.99%)	(-4.15%)	(-4.15%)	(3.66%)	(-3.24%)
10	1.2214	0.1498	0.0697	0.9739	0.7916	0.2370	0.5384	-0.3880
		(-15.50%)	(2.78%)	(4.68%)	(-4.15%)	(-4.15%)	(3.66%)	(-6.42%)
11	1.4918	0.0887	0.0914	0.9393	0.9668	0.2894	0.6576	-0.0987
		(-12.25%)	(3.79%)	(3.70%)	(-4.15%)	(-4.15%)	(3.66%)	(-34.15%)
12	1.8221	0.0388	0.1180	0.9127	1.1809	0.3535	0.8032	0.1920
		(-5.84%)	(4.53%)	(2.94%)	(-4.15%)	(-4.15%)	(3.66%)	(-12.10%)
13	2.2255	0.0123	0.1504	0.8921	1.4423	0.4318	0.9810	0.4837
		(4.65%)	(5.08%)	(2.35%)	(-4.15%)	(-4.15%)	(3.66%)	(-5.37%)
14	2.7182	0.0025	0.1900	0.8759	1.7616	0.5274	1.1982	0.7761
		(25.96%)	(5.51%)	(1.89%)	(-4.15%)	(-4.15%)	(3.66%)	(-3.51%)
15	3.3201	0.0003	0.2383	0.8630	2.1517	0.6442	1.4634	1.0693
		(85.14%)	(5.84%)	(1.53%)	(-4.15%)	(-4.15%)	(3.66%)	(-2.63%)

Table 3.10: Second (final) steady-state with presence of idiosyncratic shocks in the human capital and with the government support for fertility that is financed with the tax on capital income. Note: for each of the individual, average and aggregate variable, the values inside of the parentheses indicate the percentage deviations from the initial steady-state given the government policy

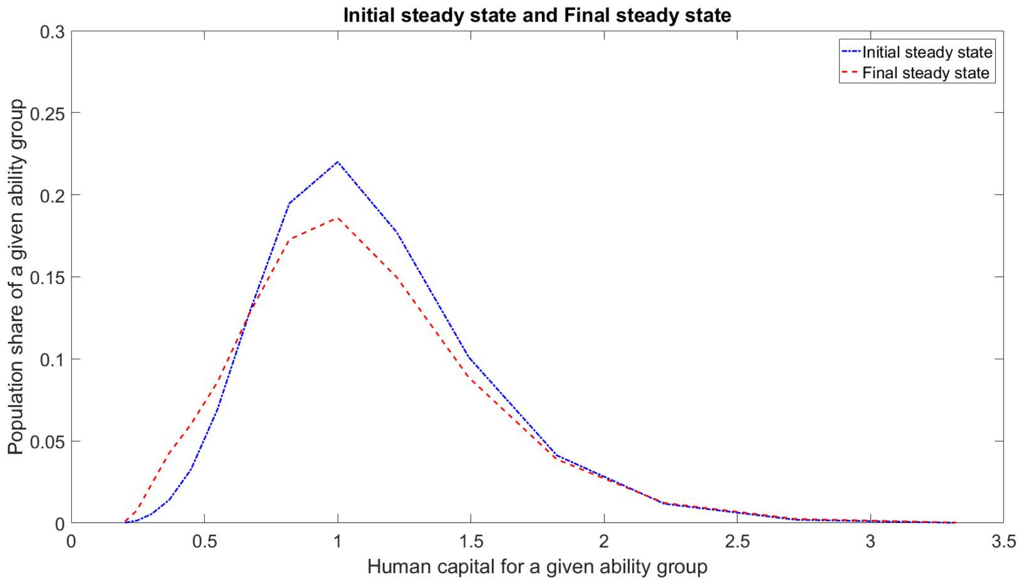

Figure 3.8: Comparison between distribution for the population of adult households at the initial steady-state with the distribution at the second steady-state where government supports fertility decisions and taxes capital income.
Note: the (marginal) levels of the human capital are depicted on the horizontal axis; population share for each ability group is depicted on the vertical axis; blue dash-dotted line shows the distribution of adult households at the original steady-state; red dashed line illustrates the distribution of adult households at the second steady-state

With continuous government support for fertility and with the presence of idiosyncratic shocks to the human capital, we observe that the average childbearing decisions of adult households increase by 14.98% when government implements tax on consumption or capital income, and with the labour income tax, this value is 15.82% larger than at the original equilibrium. Due to parental 'quality-quantity' trade-off and the change in the structure of the population, however, the increase in number of children born reduces the optimal choice of the adult households for education provision. Therefore, with tax on consumption and capital income, the average education attainment at the second steady-state falls by 8.80%, and with the labour income tax in place the level of average education received decreases by 9.57%. This larger rate of response for the case of the labour income tax can be explained by the distorting nature of this tax option on the optimal choice for education provision, which according to the previous analytical conclusions is creating further disincentive for the choice of education that leads to a larger increase in the average level of fertility. Therefore, our results indicate that at the second steady-state with the government support for fertility, the size of the model population is

192.87 times larger than at original equilibrium when government uses tax on consumption or capital income. With the labour income tax, the size of population is 255.78 times larger than at original equilibrium[18]. This increase in the size of the model population, however, comes at a cost of decrease in the average level of human capital of adult households, which is 6.68% lower than at the original steady-state when government uses tax on consumption or capital income, and it falls by 7.14% when tax on labour income is levied instead.

Due to these changes, the level of the effective labour force per adult household for the second steady-state is 1.58% larger than at original equilibrium when government finances its budget with tax on consumption or capital income. With the labour income tax is in place instead, this value rises by 1.73% from the original steady-state. Therefore, at the second steady-state, the effective labour force becomes less skilled, however it is (significantly) larger than at the initial equilibrium due to growth in the size of the model population. Furthermore, this change in the effective labour force is the net effect of decrease in education provision, which reduces number of teachers required and, therefore, increases participation in the effective labour force; and increase in the fertility level, which decreases the time that adult households can spend in the labour market due to associated increase in time required for taking care for the children, and increases number of teachers required to provided the education for increased population of children.

Moving forward, our results indicate 10.55% lower level of the physical capital stock per adult household for the steady-state with government support for fertility when the tax on consumption and capital income is in place. With tax on labour income, the physical capital stock per adult household falls by 12.06% instead. For our policy experiments with the full depreciation of the physical capital stock, the level for this second factor input is fully determined by the level of saving of the adult households (the dynamics for which is discussed below).

With increase in the effective labour force per adult household and decrease in the physical capital stock per adult household, the marginal productivity for the former factor input is decreasing, whereas the marginal productivity of a latter factor input is rising. Since the factor price for each of these factors of production is equivalent to their marginal products, the real

[18]Please note, the population would continue to grow even after the economy reaches the second steady-state

wage rate at the second steady-state is 4.15% below the original equilibrium when government uses tax on consumption or capital income, and it is 4.74% lower than the original steady-state when labour income tax is in use. The real interest rate at the second steady-state is 12.01% above the original equilibrium when tax on consumption or capital income is levied, and with the labour income tax the real interest rate rises 13.83% above the original value.

With each tax option in place combined with lower level of the average human capital, lower real wage rate and higher real interest rate, our results suggest that the average level of consumption at the second steady-state becomes 11.97%, 12.06% and 10.55% lower than original steady-state when tax on consumption, labour income or capital income is in place, respectively. The average saving of the adult households becomes 10.55% lower than at the original steady-state if the government finances the subsidy for fertility with tax on consumption or capital income, and with the labour income tax in use, the average saving of adult households at the second steady-state is 12.06% below the original steady-state. Finally, the average consumption of the elderly households at the second steady-state is 3.0937%, 3.0901% and 3.2584% below the original equilibrium when government places tax on consumption, labour income and capital income, respectively.

As a result of these changes, the level of welfare for the second steady-state is 21.23% lower than at original steady-state, when government taxes consumption. For the case which considers utilisation of the labour income tax, the level of welfare at the second steady-state reduces by 22.84% from original equilibrium; and with capital income tax it falls by 20.66%. Finally, due to a considerable decrease in the physical capital stock per adult household and moderate increase in the effective labour force per adult household, the real output per adult household becomes 2.63% below original equilibrium when government uses tax on consumption or capital income, and with the labour income tax the level of real output per adult household diminishes by 3.09% when the model economy reaches second steady-state with the government support for fertility.

At the individual level, due to decrease in the average level of human capital, which results from the change in the composition of the model population as a consequence of the government support for fertility, combined with the stationary levels of the marginal values of human capital that define each ability group, the value for the (realised) relative human capital becomes larger for all ability groups at the second steady-state. Similarly to our earlier discussion, the increase in the (realised) relative human capital creates a dampening of the responses of adult households to a change in the environment of the economy, which produces the results that are not fully anticipated: the government support for childbearing decisions leads to simultaneous increase in fertility and education provision of the higher ability households who are from ninth to fifteenth ability groups (i.e. $i \in [9,15]$). For the households from fourth to eighth ability groups (i.e. $i \in [4,8]$), the conclusions are more in line with the predictions from the deterministic version of analysis: the subsidy for fertility leads to an increase in childbearing decisions of the households, which due to a parental 'quality-quantity' trade-off results in pronounced decrease in the optimal choices for education. Finally, our results indicate that the households from first to third ability groups (i.e. $i \in [1,3]$) do not change their education choice and they still provide their children with zero education; however, the fertility decisions for these households increase considerably. It must be stressed, however, that these results are true for the ability groups of the households but not necessary for the individual households. Individual adult households (with exception for those who are the part of ability groups that provide their children with zero education) who receives subsidy for fertility increases their optimal choice for children and reduces the education provision for their children. As a result, decrease in the education provision combined with idiosyncratic shocks to the human capital results in the change of composition for the model population of the adult households that becomes more scattered across lower ability groups for the second steady-state.

For the consumption and saving of the adult households at the individual ability level, we observe that the outcome for the second steady-state closely follow dynamics of the real wage considered earlier, given each of the tax options in place. On the other hand, due to increase in the real interest rate, the consumption of the elderly households at the individual ability level is above the original equilibrium.

Given these changes, however, the level of utility at the second steady-state decreases below original equilibrium for the households from fifth to fifteenth ability groups (i.e. $i \in [5,15]$) in the case of a tax being levied on consumption or capital income. With the labour income tax, we additionally observe that the level of utility for a third ability group decreases below the original steady-state as well. Absence or a minor change in education attainment and fertility decisions combined with a pronounced increase in the consumption of the elderly households, results in increase in the level of utility of the households from first to fourth ability groups (i.e. $i \in [1,4]$) when the economy reaches the second steady-state, however, with subsidy for fertility and tax on consumption or capital income. With the labour income tax, the individual utility levels increase only for the first, second and fourth ability groups, however. This difference in the individual utility responses, and generally lower levels of individual utilities with the labour income tax in place, is attributed to the distorting nature that this tax has on the education and consumption choices. Finally, based on the visual examination of the distribution of the human capital across the model population at the second steady-state with government support for fertility, it may be suggested that economy experiences an increase in the level of inequality in the distribution of human capital across the members of the population[19].

Transition to the second (final) steady-state

Continuing with our discussion for the government support of fertility in the environment with uncertainty in the formation of human capital, in the present section we analyse the transition path that the economy takes from the original steady-state to the second steady-states. The figure 3.9 below depicts the evolution of the model economy with tax on consumption, labour income and capital income in place, from the perspective of the average household.

According to our results, in the first period of the government support for the childbearing decisions, the government levies the tax rate on consumption of 0.47% to keep its budget balanced. In the case when government uses the labour income tax, the rate is set at 0.56% to provide the subsidy rate for fertility of ten percent. Finally, if capital income tax is implemented instead, the tax rate is equal to 0.92% at the first period of government presence.

In this second period of analysis with the government support for fertility, the average level of childbearing decisions has been found to increase by 10.29% with the tax on consumption

[19]For a further discussion about the inequality in the distribution of the human capital across the population with subsidy for fertility at a rate of ten percent, please refer to a section 3.5.4

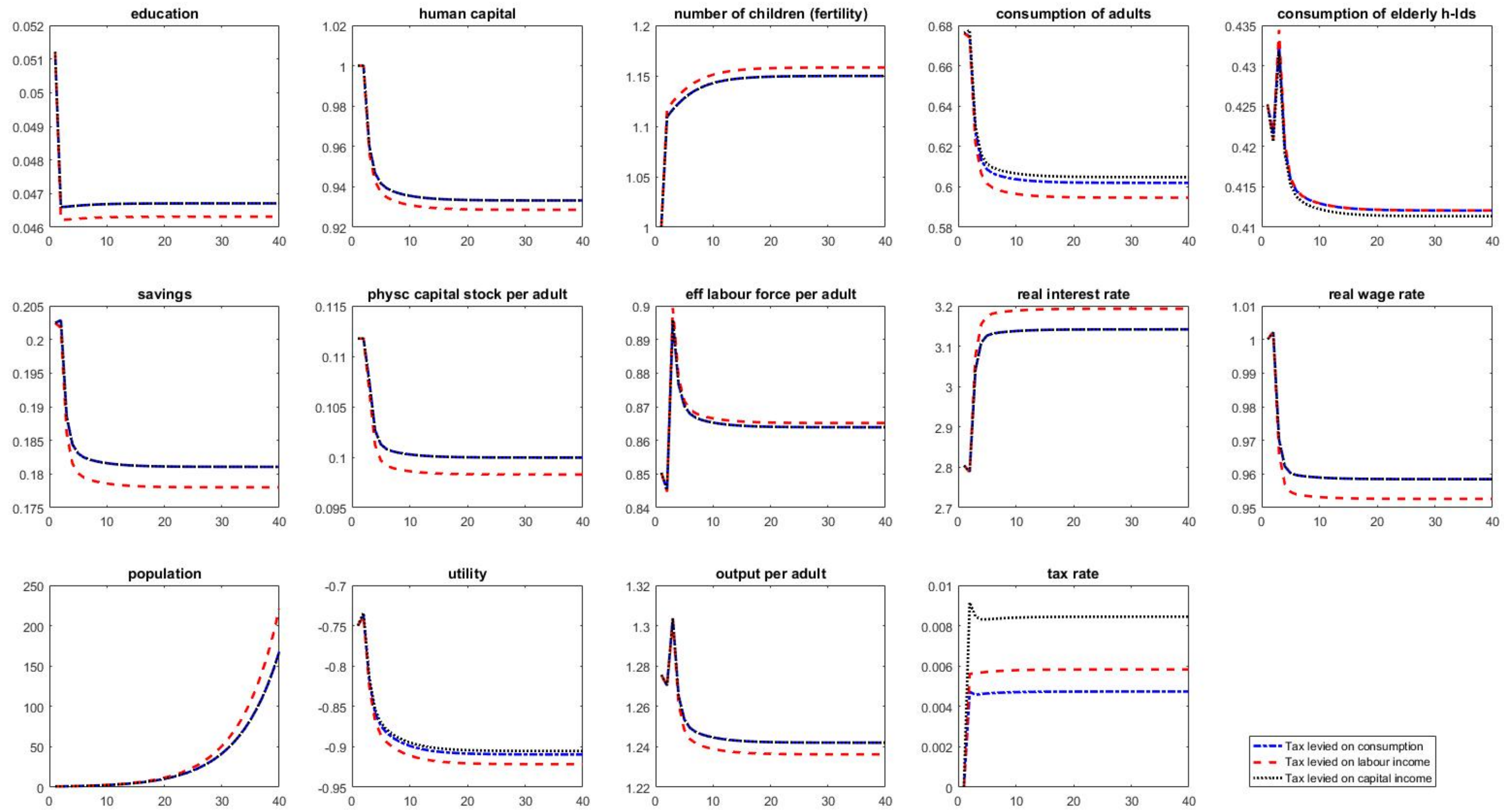

Figure 3.9: Transition path of the model economy with idiosyncratic shocks to the human capital from the perspective of the average household. Government provides subsidy of ten percent for childbearing decisions.

Note: transition path with tax on consumption is depicted by blue dash-dotted line; transition path with tax on labour income is presented by red dashed line; transition path with capital income tax is shown by black dotted line

or capital income financing the government budget. When the government levies tax on labour income instead, the average level of fertility is increasing 11.68% above the original steady-state. This increase in the average level of fertility takes place for few reasons. First, in the first period of the government presence, with absence of a change in the composition of the model population, the subsidy for fertility reduces the cost of having children across all ability groups. And, second, the subsidy for fertility provides disincentives for the education attainment, which with the presence of the 'quality-quantity' trade-off of parents for their children promotes fertility further.

According to our results, the education provision of fourth, fifth, ninth and fourteenth ability groups falls by 100%, 47.08%, 9.10% and 2.59% when government uses tax on consumption or capital income, respectively. As a result, the fertility indicators for these ability groups increase by 36.85%, 27.19%, 7.97% and 2.49%, which raises average level of fertility. The average level of education, however, falls by 9.01% in the first period of government support when tax on consumption or capital income is in place. With use of the labour income tax, the education decisions of fourth, fifth, ninth and fourteenth ability groups decline by 100%, 48.60%, 9.85% and 3.20%, which increases fertility choices of these groups by 37.55%, 28.32%, 8.69% and 3.10%, respectively. Consequently, the average level of fertility increases but the average education attainment falls by 9.75%. As before, according to the results from the deterministic version of analysis, the labour income tax produces direct negative incentives for the parental choice for education, which results in a larger increase in fertility when childbearing decisions are subsidised. Additionally, similar to the case of subsidy for education, the households from the lower three ability groups still optimally choose zero level of education for their children. However, the fertility decisions for these three groups now change and they found to increase substantially.

With increase in fertility and decrease in education provision, we observe 0.62% decrease in the effective labour force per adult household when tax on consumption or capital income is in place. With the labour income tax, the effective labour force per adult household decreases by 0.67% at the first period of government presence. Since the effective labour force is the product of the population of adult households, human capital and time that adult households dedicate to the labour force net of share of the households who are required for education provision and are not utilised by the production sector as factor input, the increase in the fertility affects the effective labour force in two ways. First, due to increase in fertility, the adult households have to spend more time for taking care for children which reduces the time they can spend for the labour market participation. And second, with increase in the number of children born, the number of teacher would follow in order to provide the optimally chosen level of education. Decrease in optimal decision for education, however, diminishes the number of teachers required.

Given a full depreciation of the physical capital stock, the level of this second factor input is completely determined by the aggregate saving of the past generation and by the population size of the present generation. Therefore, for the second period of analysis, the level of physical capital stock per adult household remains unchanged.

With both of these factor inputs being paid their corresponding marginal products, decrease in the size of effective labour force per adult household and absence of change in the physical capital stock per adult household raises the marginal productivity of the effective labour force and decreases marginal productivity of the physical capital. Therefore, the real wage rate is found to increase by 0.21%, whereas, the real interest rate decreases by 0.56% at the first period of government presence when tax on consumption or capital income is implemented. With the tax on labour income, the real wage rate increases by 0.22% and the real interest rate reduces by 0.61%. Consequently, with the tax on consumption, the consumption of adult households from every ability group is decreasing by 0.26% whereas the consumption of elderly households is decreasing by 0.88%. Savings of the adult households increase by 0.21% however. With the labour income tax in use, consumption and saving of adult households are found to diminish by 0.34%. The consumption of the elderly households decreases by 1.01% across all ability groups. Finally, with capital income tax, the consumption and saving of adult households increase by 0.21% at the first period of government presence, but the consumption for the elderly households diminishes by 1.08%.

As a result of these changes that take place at the second period of analysis, however, the level of welfare in the model economy increases by 1.60%, 1.61% and 2.15% when government levies tax on consumption, labour income and capital income to finance the program of support for fertility, respectively. At the individual level, however, due to non-linear responses of individual fertility decisions and decisions for education provision to fertility subsidy, the level of utility for lower ability households is found to increase more than for higher ability groups. Moreover, for the top three ability groups[20], the level of utility is declining at the first period of government presence when tax on consumption or labour income finance the government budget. According to our results, the utility of the households from the fourth and ninth ability groups increases by 2.4743% and 1.2962% when government taxes consumption, respectively. With the labour income tax, the utility level for these two ability groups grows by 2.4648% and 1.3065%; and with the tax on capital income these values rise by 2.6732% and 1.9233%. For the households from fourteenth ability group, however, the utility falls by 0.1098% and 0.1159% with tax levied on consumption and labour income, respectively, but with the tax on capital income the level of utility for this group increases 0.4003% above original steady-state. Finally, with a reduction in size of effective labour force and absence of change in the physical capital stock, our results suggest that at the first period of the government support for fertility in the environment with uncertainty in formation of human capital, the level output per adult household declines by 0.41% when the government uses tax on consumption or capital income, and it falls by 0.45% when the labour income tax is in use instead.

Due to the government support for fertility and reaction of the previous generation of adult households to this policy in form of increase in childbearing decisions and decrease in the education provision, we observe a change in the structure of the model population of the adult households for the third period of analysis. As a result, the average level of human capital for adult households declines by 3.99% at the second period of government presence when the government uses tax on consumption or capital income to finance its budget. With the labour income tax, the average level of human capital for adult households diminishes by 4.31%. The population size of the adult households increases, however, by 10.92% when tax on consumption or capital income is in use, and with the labour income tax the population size of adult

[20] I.e. $i \in [13, 15]$

households increases by 11.68%. With these changes, when the government uses tax on consumption to provide the subsidy rate for fertility of ten percent and to keep the balanced budget, the tax rate is set at 0.46% at the third period of analysis. For the case with tax on capital income income, the tax rate is re-adjusted to 0.85%, whereas, for scenario with the labour income tax, the tax rate is equal to 0.56%.

Given these changes in the composition of adult households and in the tax rates, with the continuous government support for childbearing decisions, our results indicate that the average level of fertility increases by 0.66% and becomes 11.66% above the original equilibrium when government uses tax on consumption or capital income. With the labour income tax, the average level of fertility rises by 0.68% and it is 12.44% above the initial steady-state. For the individual ability groups, however, we observe the results that have been not fully anticipated. For instance, for the fifth, ninth and fourteenth ability groups the childbearing decisions reduce by 3.71%, 1.12% and 0.35%, and become 22.47%, 6.77% and 2.13% above original equilibrium when government implements tax on consumption or capital income, respectively. With the labour income tax, the fertility decisions for these three ability groups fall by 4.01%, 1.20% and 0.38%, and become 23.17%, 7.38% and 2.71% above original steady-state. Furthermore, with a decline in decisions of adult households for having children (relative to the previous period of analysis), and with the presence of parental 'quality-quantity' trade-off for children, the individual decisions for education within the ability groups begin to increase at the second period of government presence. For example, with the tax levied on consumption or capital income, the optimal choice for education of fifth, ninth and fourteenth ability groups increase by 26.73%, 6.70% and 4.83%, and become 32.93% and 3.01% below the original steady-state for fifth and ninth, whereas for fourteenth – the individual education provision increases 2.12% above the original steady-state. With the labour income tax, the choice for education of these three groups increases by 29.60%, 7.26% and 5.23% from the previous generation, and become 33.39% and 3.30% below the original steady-state for the fifth and ninth ability groups, respectively. For the fourteenth ability group, the education provision is 1.86% above the original steady-state. Finally, at the average level, the education attainment increases by 0.02% and becomes 8.99% below the original equilibrium when government uses tax on consumption or capital income. With the labour income tax, the average education provision rises by 0.01%, and it is 9.74% lower than the initial steady-state.

This increase in education attainment for all groups, with exception to the households from first to third ability groups, combined with decrease in fertility decisions among individual ability groups while receiving subsidies for fertility can be explained by the fixed nature of the marginal levels of the human capital that is required for the process of extrapolation and interpolation. With fixed marginal levels of human capital corresponding to each ability group, the government support for fertility reduces average level of human capital which in turn increases the (realised) relative human capital. As we have indicated previously, the change in the (realised) relative human capital level produces an equal-directional change in the education attainment, which in turn changes the fertility in an opposite direction. The increase in education attainments and decrease in fertility decisions at the level of individual ability groups does not, however, result in the same dynamics for the each individual adult households. According to our results, the past period decrease in education provision across all ability groups results in lower realised human capital for adult households (especially for ones who have faced negative shocks during human capital accumulation in a childhood). This would force these individuals to move to lower ability groups that are characterised by having a lower provision of education to their children but by deciding on a larger number of children. The net of these changes for the ability groups and for the composition of adult households results in the reported changes for the average values of education and fertility for the third period of analysis.

With increase in the population of adult households, our result suggest an increase in the effective labour force per adult household by 5.99% when government uses tax on consumption or capital income, and with the labour income tax, the effective labour force per adult household increases by 6.46%. Therefore, at the second period of the government presence, the effective labour force rises 5.33% above the original steady-state with tax levied on consumption or capital income, and it grows by 5.75% above original equilibrium with the government taxing the labour income. Next, with the past period increase in the aggregate saving with the tax levied either on consumption or capital income, the physical capital stock increases. With the labour income tax, however, we observed a decrease in saving of the past generation of adult households, which, therefore, reduced the physical capital stock at the third period of analysis. However, with increase in the population size at the second period of government presence, we observe a decrease in the per adult household value of the physical capital stock under all three tax options. Our results indicate that with the tax on consumption or capital income, the physical capital stock per adult household falls 3.79% below the original steady-state, whereas with the labour income tax in place, this indicator reduces 4.63% below the initial equilibrium. With decrease in the physical capital stock per adult household accompanied with increase in effective labour force per adult household, the real wage rate falls by 3.17% and becomes 2.97% below the first steady-state when government uses tax on consumption or capital income, and with the labour income tax the real wage rate reduces by 3.60% and it is 3.39% below the original equilibrium. The real interest rate, however, rises by 9.06% with tax on consumption or capital income, and it becomes 8.44% larger than the first equilibrium; whereas, with the tax on the labour income, the real interest rate increases by 10.34% and it is 9.67% above the original steady-state.

As a result of decrease in the real wage rate and adjustment in the tax rates, the consumptions of adult households among different ability groups reduces by 3.16% and becomes 3.42% below the originals steady-state when government taxes consumption. With the tax imposed on the labour income, the consumption of adult households among different ability groups reduces by 3.61% and it is 3.93% below first equilibrium. And, finally, with the tax on capital income this value falls by 3.17% and becomes 2.97% lower than at the original steady-state. The saving of adult households among different ability groups reduce by the same amount as consumption when government implements tax on labour income or on capital income. With the tax on consumption, however, the saving of adult households between different ability groups falls by 3.17% and it is 2.97% lower than at the first steady-state. Lastly, with the past period increase in the real wage rate and present period increase in the real interest rate, the consumption of the elderly households between different ability groups increases by 6.90%, 7.85% and 6.93% that becomes 5.94%, 6.74% and 5.75% above the initial steady-state with tax on consumption, labour income and capital income in place, respectively. At the average level, however, the amount by which the consumption and saving of adult households plummets is larger and the rate at which the consumption of the elderly households increases is smaller. This is the case because the composition of adult households has changed where households from higher ability groups become a part of a lower ability groups with lower consumption and saving. Therefore, the average level of consumption of adult households falls by 7.03%, 7.56% and 7.04%, that becomes 7.27%, 8.07% and 6.85% smaller than at the original steady-state, when the tax on consumption, labour income and capital income finance the government budget, respectively. For the tax levied on labour income and capital income, the change in average level of saving is identical to the change in the average level of consumption; whereas, the average saving of the adult households with the tax on consumption reduces by the same amount as in the case with tax on capital income. Finally, the average level of consumption for the elderly households is found to increase by 2.64%, 3.20% and 2.66%, which is 1.73%, 2.17% and 1.54% above the initial steady-state when the government levies tax on consumption, labour income and capital income, respectively.

Finally, at the third period of analysis with the government support for fertility in the environment with uncertainty in the human capital formation, the level of welfare decreases by 10.85%, 11.79% and 10.92%, and it becomes 9.07%, 9.98% and 8.54% lower than at the original steady-state when tax on consumption, labour income and capital income is in place, respectively. Lastly, our results indicate an increase in the level of output produced per adult household by 2.62% that becomes 2.20% larger than at the first steady-state when government implements tax on consumption or capital income. With the labour income tax, the per adult household value of the real output increases by 2.63% and it is 2.17% larger than at the original steady-state.

Moving forward to the fourth period of analysis, our results indicate the decreases in the average level of human capital by 1.39% which becomes 5.32% lower than at the initial steady-state when government uses tax on consumption or capital income. With the labour income tax, the average level of human capital of adult households at the third period of government presence reduces by 1.49% and it is 5.73% lower than at the original equilibrium. The population size of adult households, increases, however, by 11.66% when government uses tax on consumption or capital income, whereas, with the labour income tax, it rises by 12.44% which, therefore, is 23.86% and 25.58% larger than at the original steady-state, respectively. Finally, in order to balance the budget and continue to provide the subsidy for fertility at ten percent, the government sets the tax rate on consumption to 0.46%, on labour income to 0.57%, and on capital income to 0.83% at the fourth period of analysis.

Given a continued decrease in the average level of human capital and fixed values for the marginal levels of human capital for each ability group, the (realised) relative human capital continues to rise. With increase in the (realised) relative human capital for the households across the ability groups, the education attainment continues to increase but the fertility choices continue to fall. Our results indicate that the choices for education of the households from fifth, ninth and fourteenth ability groups increase by 7.42%, 2.21% and 1.62% when government uses tax on consumption or capital income to finance its budget and provide subsidy rate for fertility of ten percent. As a result, the education attainment of the households from fifth and ninth ability groups becomes 27.96% and 0.87% below original steady-state; whereas, for the households from fourteenth ability group the education is 3.78% above the original equilibrium. With the labour income tax, the choices for education of these three groups increase by 8.00%, 2.38% and 1.74%, respectively, which is 28.06% and 1.00% below the original steady-state for the fifth and ninth ones, and it is 3.64% larger than at the original equilibrium for the fourteenth ability group. Due to the large fertility and low (realised) relative level of human capital, however, the households from first to third ability groups still provide their children with zero education. The fertility choices at the third period of the government presence for the fifth, ninth and fourteenth groups decrease by 1.22%, 0.37% and 0.12%, respectively, when government implements tax on consumption or capital income, and with the labour income tax, these values decrease by 1.30%, 0.39% and 0.12%. Therefore, the fertility decisions of the households from fifth, ninth and fourteenth ability groups become 20.98%, 6.37% and 2.01% above original steady-state when government levies tax on consumption or capital income, and they are 21.57%, 6.95% and 2.58% larger than at the original equilibrium when government taxes the labour income. As a result of a persistent change in the composition of the model population, however, our results indicate that the average level of fertility increases by 0.5453% and becomes 12.27% above the original steady-state when tax on consumption or capital income is in place. With the labour income tax in place, the average fertility increases by 0.5493% and it is 13.05% larger than at the original steady-state without government presence. In contrast, the average level of education still continues to be below the initial steady-state. According to our results, the average level of education increases by 0.0381% and becomes 8.96% below the first steady-state when government uses tax on consumption or capital income; and it increases by 0.0341% and is 9.71% lower than initial equilibrium when government implements the labour income taxation.

As a result of these alterations, the level of the effective labour force per adult household decreases by 2.13% and it is 3.09% above original steady-state when tax on consumption or capital income is in place; and it decreases by 2.30% and becomes 3.32% larger than the initial equilibrium when tax on labour income is utilised instead. The per adult household value of the physical capital stock decreases as well at the fourth period of the analysis: it diminishes by 4.52% and it is 8.14% lower than at the original steady-state when the government budget is financed with tax on consumption or capital income. With the labour income tax, the physical capital stock per adult household reduces by 5.04% and it becomes 9.44% below the initial steady-state. This decrease in the effective labour force per adult household is primarily driven by (i) a decrease in the human capital, which reduces the productivity of the labour, (ii) by an increase in the fertility levels which reduces the time available for the labour market participation, and (iii) by increase in the number of teachers required to provide the optimally chosen level of education. In turn, a decrease in the physical capital stock per adult household is explained by a decrease in saving of the past generation , and, by a disproportional increase in the present population size. Overall, we observe, that with the tax on consumption or capital income, the real interest rate increases by 2.21% from the previous generation and it becomes 10.84% larger than at original steady-state. The real wage rate, however, declines by 0.82% and it becomes 3.77% lower than initial steady-state. With the labour income tax, the real interest rate increases by 2.54% and real wage rate falls by 0.95%, and they become 12.46% above and 4.30% below the initial steady-state, respectively.

Given these changes, the consumption of adult households from every ability group diminishes by 0.83%, 0.95% and 0.82%, that becomes 4.21%, 4.84% and 3.77% lower than at the original steady-state when the government finances subsidy for fertility with the tax on consumption, labour income and capital income, respectively. The reduction in saving within every ability group is equivalent to the reduction in consumption when government uses labour income tax or tax on capital income; whereas with the tax on consumption, saving of adult households decreases by 0.82% and it is 3.77% below the original equilibrium. The consumption of the elderly households at the individual ability level is found to decrease as well, which is the net result of considerable decrease in the previous period real wage rate, increase in the present period real interest rate and an adjustment in the tax rates. Therefore, the consumption of the elderly households decreases by 1.57%, 1.76% and 1.56% from the previous period which becomes 4.30%, 4.89% and 4.12% above the initial steady-state with tax levied on consumption, labour income and capital income, respectively. Due to a change in the composition of the model population, however, our results indicate a more pronounced decrease in the average consumption of adult and elderly households and in the average level of saving. According to our results, the average consumption and saving of adult households decreases by 2.4266% and 2.1984%, and they become 10.30% and 8.89% below the original equilibrium when government taxes labour income and capital income, respectively. With the tax on consumption, the average consumption of adult households reduces by 2.2024%, while the average saving fall by 2.1984%, and they become 9.31% and 8.89% below the initial steady-state, respectively. Lastly, the average consumption of the elderly households decreases by 2.9347%, 3.2217% and 2.9241% and which becomes 1.2545%, 1.1249% and 1.4230% lower than at the first steady-state, when the government budget is financed with tax on consumption, labour income and capital income, respectively.

Lastly, our results indicate that at the third period of the government presence with the government support for fertility and with uncertainty in the human capital accumulation, the level of welfare decreases by 4.85%, 5.22% and 4.87% and it is 14.37%, 15.73% and 13.82% below the original steady-state when tax on consumption, labour income and capital income finance the government budget, respectively. Furthermore, when the government uses tax on consumption or capital income, the level of output per adult household decreases by 2.93% and becomes 0.80% lower than at the initial steady-state. With the labour tax, the output per adult household decreases by 3.22% and it is 1.12% lower than at the original equilibrium.

Overall, the dynamics presented for the fourth period of the analysis continues until the model economy reaches the second steady-state.

3.5.4 Inequality

In our discussion, when we compared the original steady-state with the resulted final steady-states under different government policy programs, we have suggested that the level of inequality in the distribution of the human capital increases with the subsidy for fertility and reduces with subsidy for education. To address these propositions more carefully, we calculate the measures of inequality for the distributions of the human capital based on the distribution of the population that we have obtained for each considered policy option. We report these results in the table 3.11 below.

steady state	Gini coefficient	Coefficient of variation	Mean absolute deviation	Kuznets ratios $\leqslant 40\%$	Kuznets ratios $\geqslant 60\%$	The range
original	0.2018	0.3724	0.2818	0.1230	0.3332	3.1190
$sub^e + \tau^c$	0.1968↓	0.3637↓	0.2864↑	0.0764↓	0.4299↑	2.8474↓
$sub^e + \tau^l$	0.1971↓	0.3640↓	0.2866↑	0.0786↓	0.4241↑	2.8622↓
$sub^e + \tau^k$	0.1968↓	0.3637↓	0.2864↑	0.0764↓	0.4299↑	2.8474↓
$sub^n + \tau^c$	0.2389↑	0.4376↑	0.3442↑	0.2209↑	0.2924↓	3.3402↑
$sub^n + \tau^l$	0.2391↑	0.4383↑	0.3449↑	0.2242↑	0.2881↓	3.3580↑
$sub^n + \tau^k$	0.2389↑	0.4376↑	0.3442↑	0.2209↑	0.2924↓	3.3402↑

Table 3.11: Indicators for the inequality in distribution of the human capital for the model population of the original steady-state without government presence and of the second steady-state with various policy options.
Note: ↑ indicates that the level of inequality in distribution of the human capital across the population increases at the second steady-state; ↓ indicates that the level of inequality in distribution of the human capital across the population reduces at the second steady-state

In order to calculate the inequality measures in the distribution of the human capital across the population, we follow Ray (1998) by adopting the standard income inequality measurements. For instance, we calculate the range as the difference between the human capital of the top ability group and the bottom one, divided by the average level of human capital. The Kuznets ratios show the share of the human capital that belongs to the bottom and to the top 40% of the population. We calculate the mean absolute deviation as the total sum of absolute values for the distances between the average level of human capital and the marginal human capital levels for the all ability groups. The coefficient of variation indicates the standard deviation in the distribution of human capital across the population. And, finally, we obtain the Gini coefficient by computing the total absolute differences in the level of human capital between all pairs of ability groups that our model population consists of.

Based on our results, however, we cannot form an unambiguous conclusion for the effect of the considered government policy programs on the level of inequality in the distribution of the human capital across the population. Based on the results for Gini coefficient, coefficient of variation and the range, the government policies that provide the population with subsidy for education result in a lower level of inequality when the economy reaches the final steady-state. In the case of policies that provide the model population with subsidy for fertility, however, the inequality in the distribution of the human capital across the population has been found to increase. Furthermore, with the government policies that have been financed with tax on the labour income, our results suggest that the economy would experience a relatively larger level of inequality.

For the Kuznets ratio that corresponds to the bottom 40% of the population we reach the same conclusions. However, for Kuznets ratio that corresponds to the top 40% of the population, the results are completely opposite. This could be explained as the following. As a result of the subsidy for education, in our earlier discussion we have indicated that the households from bottom ability groups would enjoy a larger realised level of human capital on average. This improvement in the abilities would allow the households from lower ability groups to become a part of higher ability groups, which would increase the population weight of the households in top ability groups, and, therefore, this would increase the Kuznets ration that corresponds to the top 40% of the population. Furthermore, this relative increase in the population weight of households with a larger human capital level could explain the reason for an increase of the mean absolute deviation when the policy with subsidy for education has been considered. Finally, with the subsidy for fertility, we have observed a loss in the realised human capital on average, which deflated the Kuznets ration that corresponds to the top 40% of the population.

3.6 Conclusion

In this paper we have studied the long-run impacts of the government support for education and fertility in the environment with uncertainty in the human capital formation. We utilised the model economy of *de la* Croix and Doepke (2003) to formalise the behaviour of households and producer. In line with chapter 2, we extended this model economy with the government sector, which provides the households with subsidies for education and fertility, and collects taxes on consumption, labour income and capital income. In addition, we considered that the households receive idiosyncratic shocks when the human capital accumulates. In our discussion, we focused on comparison between the initial steady-state with the presence of uncertainty and without the government, with the steady-states that resulted from the government policies. Additionally, we performed the analysis for the transition paths that the model economy has taken under different government policy options. In our discussion we focused on the dynamics that takes place at individual, average and aggregate levels.

According to our results, the subsidy rate for education of 10% produced an initial increase in the choices for education investment of the households across all ability groups with exception for ones that did not provide their children with any education at the original steady-state. The rate of an increase in these initial education choices, however, diminished with the human capital of the adult households. Due to increase in the education choices, and due to the presence of parental trade-off between ‘quantity’ and ‘quality’ for children, the economy experienced an initial decrease in the individual fertility decisions across the majority of the ability groups. However, due to the disincentives that labour income taxation has created for the education decisions, due to a relatively smaller rate of change in the education investment, and due to the presence of parental trade-off, the fertility level of the households from bottom two and top two ability groups has been found to increase at the first period of this policy exercise. As a result of the change in these two indicators, and the positive effect that they had on the average level of the human capital in the economy in the long-run, our results indicated a subsequent increase in the population share of the higher ability households and a decrease in the population share of a lower ability households. Due to an increase in the average level of human capital over the long-run, however, and due to the fixed levels of the (marginal) human capital that define each ability group[21], we have found the level of education investment at the final steady-state to be lower than at the first period with subsidy for education; but, still, these levels have been larger than at the original steady-state without the government in place under all considered tax options for all ability groups that provide their children with above-zero level of education. Consequently, as the economy approached the final steady-states, we have observed an increase in the individual fertility decisions across the ability groups above the levels recorded for the first period with subsidy for education in place. For the population and the policy instruments that we have considered in our analysis, the majority of the ability groups have chosen the fertility levels that have been above the original steady-state. However, due to the change in the structure of the population – increase in the population share of higher ability groups, and decrease of the lower ones – the average level of fertility at the final steady-states with subsidy for education has been lower than before the government policy has been introduced. Finally, based on our results, due to the initial decrease in the consumption of adult households with tax on consumption and labour income, consumption of the elderly households under all tax options, and decrease in the fertility decisions in the majority of cases, we have observed the initial decrease in the individual utility levels of the households across all ability groups and the overall initial reduction in the level of welfare when the subsidy for education has been provided for the first time. However, due to a subsequent improvement in the human capital combined with an increase in the real wage rate, the utility at the individual ability levels increased above the original steady-state with tax on consumption and capital income that financed the subsidy for education. With the labour income tax, as the economy reached the final steady-state, the utility across all ability groups has been found to be below the original steady-state, however. But, due to the change in the structure of the population, the

[21]Which, in combination, produced a decrease in the relative human capital levels in comparison to the first period with subsidy in place.

level of welfare in the economy has been found to increase for the all considered tax options with subsidy for education of 10%. The largest welfare improvement has been recorded for the case with tax on capital income, while the smallest one – for the case with flat labour income taxation.

Based on our analysis for the case with subsidy rate for fertility of 10%, the households from all ability groups have immediately increased their fertility decisions. The rate of an increase in the childbearing decisions, however, has been found to reduce with the level of the human capital of adult households. Due the presence of parental trade-off, a larger number of births has initially diminished the education investment for all households that provided the level of education that has been larger than zero at the original steady-state. Consequently, at the first period with the subsidy rate for fertility, the average level of fertility increased and the average level of education decreased below the original steady-state. Additionally, as a result of these alterations we have observed a change in the composition of the population which has begun at the second period of the government presence. Given this change, starting from a third period of analysis we have observed a fall in the average human capital level. Due to this decrease, the level of the relative human capital has increased for all ability groups, which has improved the level of education investment above the one at the first period of the government presence for all households that provided their children with above-zero level of education at the original steady-state. Furthermore, for the households from the top seven ability groups, the choice for education has become larger than at the original steady-state. This unexpected result[22], which suggests that subsidy for fertility may resolve the parental trade-off for the households from top ability groups could be explained by the observation that these households experienced a relatively smaller initial increase in fertility and decrease in education decisions than the rest of the population, which allowed for an increase in the relative human capital to affect the decisions for education investment at a larger rate, which produced a net result of an increase in these two indicators at the individual level for the households from the top ability groups. Despite this improvement, at the average level the education has continued to be below

[22]Note: this conclusion also has been reached in our analysis for subsidy for education of 10% for the households from sixth to fifteenth ability groups when the government financed and balanced its budget with tax on labour income, due to this tax option creating disincentives for education investment which through parental trade-off resulted in a relatively larger fertility choices than with other policy options.

the original steady-state due to the resulted distribution of the population. Furthermore, the average level of the human capital has continued its decrease, while, the average fertility level has responded in the opposite direction. Overall, the subsidy for fertility of 10% has produced an initial increase in the individual utility levels for all ability groups under the tax on capital income. The rate of increase in the utility at the individual level, however, has been found to decrease with the human capital of adult households due to the relationship between the rate of growth in fertility and the human capital of adult households. With the tax on consumption and labour income, however, the utility of the households from the top three ability groups has been found to reduce due to the effect of these tax options on the consumption of adult and elderly households, and on the level of the education investment. For the rest of the population, the initial utility level has improved under there two tax options. However, as the economy approached the final steady-states with subsidy for fertility, majority of the population have experienced a decrease in the individual utility levels under all tax options due to the dynamics that took place in the consumption of adult households and the human capital of children. The households from the bottom four ability groups, however, enjoyed a larger utility level due to considerable increase in the fertility. However, overall, this considered subsidy option has produced a decrease in the level of welfare as the economy reached the final steady-state due to the dynamics observed in utility at the individual level and due to the change in the composition of the population. Finally, with tax on labour income – that created additional disincentive for education investment – the economy experienced a largest decrease in utility; whereas the tax on either consumption or capital income have produced a reduction in this indicator that has been relatively smaller[23].

With the presence of the idiosyncratic shocks to the human capital, we additionally have observed that the subsidy for education has helped disadvantaged individuals during the human capital accumulation process. These individuals have received a relatively larger level of education and/or the education that has been of a better quality (due to the improvement of the average human capital) which has helped to minimise the effect of negative unexpected

[23]However, the ranking of the policies on the basis of change in the level of welfare with subsidy for fertility remains the same as with subsidy for education. The first best policy option is when the government implements tax on capital income; second best – with tax on consumption; and third best – with (flat) tax on the labour income.

shocks. Consequently, at the final steady-state with subsidy for education the economy enjoyed a smaller level of inequality than at the original equilibrium. With the subsidy for fertility, however, which has created the initial disincentives for education investment for all households, that then persisted only for the households from lower ability groups, the level of inequality in the human capital has been found to increase since children – especially that have received the negative idiosyncratic shocks to the human capital – did not receive the additional support as in the case with subsidy for education, or as in the case of the original steady-state. Overall, the largest level of inequality has been found with the subsidy for fertility that the government financed with tax on the labour income which has created the additional disincentive for education investment; whereas, the smallest level of inequality in the distribution of the human capital has been recorded for the case with subsidy for education and both tax on consumption and capital income, since both of these tax options have affected education, fertility, human capital development and population formation in the same way. Lastly, unlike in our previous analysis for the deterministic environment in chapter 2, neither of the considered policy options have led to the equilibrium with a unique human capital across the model population due to the presence of the idiosyncratic shocks in the human capital formation.

3.7 Appendix

3.7.1 A1: Analytical solution for the problem of the households for the deterministic environment

$$\max_{c_t^i, s_t^i, e_t^i, n_t^i} u_t^i = \ln c_t^i + \beta \ln d_{t+1}^i + \gamma \ln n_t^i h_{t+1}^i$$

$$(1+\tau_t^c)c_t^i + s_t^i + e_t^i n_t^i w_t \bar{h}_t = (1-\tau_t^l) w_t h_t^i (1-\phi n_t^i) + n_t^i w_t \bar{h}_t (e_t^i sub_t^e + \bar{e}_t sub_t^n)$$

$$(1+\tau_{t+1}^c) d_{t+1}^i = \frac{1}{(1+\rho)} \left(1 + r_{t+1}(1-\tau_{t+1}^k)\right) s_t^i$$

$$h_{t+1}^i = \frac{1}{(1+\rho)} B(\theta + e_t^i)^\eta (h_t^i)^\pi \bar{h}_t^\kappa$$

First Order Conditions:

$$\frac{\partial \mathscr{L}_t^i}{\partial c_t^i} = \frac{1}{c_t^i} - \lambda_t^i (1+\tau_t^c) = 0$$

$$\frac{\partial \mathscr{L}_t^i}{\partial s_t^i} = \beta \frac{1}{s_t^i} - \lambda_t^i = 0$$

$$\frac{\partial \mathscr{L}_t^i}{\partial e_t^i} = \gamma \eta \frac{1}{(\theta + e_t^i)} + \lambda_t^i n_t^i w_t \bar{h}_t sub_t^e - \lambda_t^i n_t^i w_t \bar{h}_t = 0$$

$$\frac{\partial \mathscr{L}_t^i}{\partial n_t^i} = \gamma \frac{1}{n_t^i} - \lambda_t^i (1-\tau_t^l) w_t h_t^i \phi + \lambda_t^i w_t \bar{h}_t (e_t^i sub_t^e + \bar{e}_t sub_t^n) - \lambda_t^i e_t^i w_t \bar{h}_t = 0$$

where λ is Lagrange multiplier

Optimal decisions of the households:

$$e_t^i = \frac{\eta \phi (1-\tau_t^l) x_t^i - \theta (1 - sub_t^e) - \eta \bar{e}_t sub_t^n}{(1-sub_t^e)(1-\eta)}$$

$$c_t^i = \frac{(1-\tau_t^l)}{(1+\tau_t^c)} \frac{1}{(1+\beta+\gamma)} w_t h_t^i$$

$$s_t^i = (1-\tau_t^l) \frac{\beta}{(1+\beta+\gamma)} w_t h_t^i$$

$$d_{t+1}^i = \frac{1}{(1+\rho)} \frac{(1-\tau_t^l)}{(1+\tau_{t+1}^c)} \frac{\beta}{(1+\beta+\gamma)} \left(1 + r_{t+1}(1-\tau_{t+1}^k)\right) w_t h_t^i$$

$$n_t^i = \frac{\gamma}{(1+\beta+\gamma)} (1-\tau_t^l) x_t^i \left[(1-\tau_t^l)\phi x_t^i + e_t^i (1 - sub_t^e) - \bar{e}_t sub_t^n \right]^{-1}$$

where x_t^i is the relative human capital and $x_t^i = h_t^i / \bar{h}_t$

Chapter 4

Compulsory education, childcare and progressive tax-subsidy schemes in the OLG economy with heterogeneous households and idiosyncratic shocks to the human capital

4.1 Introduction

As a final part of our discussion for the long-run influence of the government support for education and fertility – which has begun with the analysis for the deterministic environment in chapter two, and has continued with the analysis in stochastic environment in chapter three, where young households received idiosyncratic shocks to the human capital, and, in both cases, the government has provided flat subsidy rate for education and fertility which was financed with flat tax rate on consumption, labour income and capital income – in this chapter, we investigate the role of policies for education and fertility which we have not considered previously. In the first part of our policy excises, we analyse the influence of compulsory education and childcare on the long-run economic development. In the second part of our discussion, we consider the impact of the progressive tax rates on the labour income that finance subsidy for education and subsidy for fertility; and the role of the regressive subsidy rates for education and fertility that government finances with flat tax rate on consumption, labour income and capital income. As in our previous two chapters, we utilise the model of overlapping generations presented by *de la* Croix and Doepke (2003) as a foundation for our model economy, which we extend with the government sector. Finally, for our analysis, we also keep the assumption introduced in the chapter three that young households receive idiosyncratic shocks to the human capital during their human capital accumulation.

Based to the evidence presented in the empirical literature[1], an increase in the compulsory education positively affects the wage and lifetime income. Furthermore, students who at the risk of leaving the education stay in the education longer and obtain their diploma as a result of increase in compulsory schooling. Moreover, a larger level of compulsory education is attributed to a reduction of teenage pregnancies, while the teenage pregnancies are often considered to produce long-lasting negative social and economic outcomes on the adolescent mothers and their children.

According to the evidence from the literature that utilised the models of overlapping generations to investigate the influence of compulsory education, the general results have been as following. The compulsory education has been found to produce a growth in the national

[1] Please refer to the Literature review section for more details

income even when it has been financed through distortionary taxation. Furthermore, in comparison to a system with a private education provision, a publicly provided compulsory education has reduced the income inequality quicker. In addition, the compulsory education has been found to be a second-best policy option for the optimal education investment, and a first-best policy option for escaping a poverty trap. Lastly, the introduction of compulsory education has been found to produce a general welfare improvement and it was necessary to escape the illiterateness trap.

Next, according to the previous studies that investigated the influence of a state provided childcare, the conclusions have been the following. The childcare has been found to produce an increase in the maternal labour market participation. Furthermore, it has been found that childcare helps to reduce the congestion costs which produce negative effect on the fertility decisions and which arise in the living areas with a high density of the population. However, the improvement in fertility has been recorded to be the marginal at best given the high costs of policy implementation. Lastly, the previous research has concluded that children that attended a state-provided childcare experienced an improvement in their school-readiness skills, and over the long-run these children – especially the ones from disadvantaged families – experienced the welfare improvement.

Finally, in spite of the discussion that currently exists for the optimal degree on the progressivity of income taxation, it is generally considered that the progressive labour income taxation is required for a reduction of inequality in income and consumption. Furthermore, the evidence suggests that the progressive income taxation provides a partial social insurance against idiosyncratic and aggregate uninsurable risks that affect the activity in the labour market. Lastly a combination of more progressive labour income taxation with considerable subsidies for education has been suggested to prevent the disincentives in the education investment that are expected to arise due to a more progressive labour income taxation and to obtain an improvement in welfare.

In comparison to the previous studies that analysed the influence of the compulsory education, childcare and progressive tax-subsidy systems, we believe that the current research offers a more detailed version of the analysis for the dynamics that takes place between the generations immediately after the introduction of a considered policy option and its long-run impact on the economy. Furthermore, as before, we believe that the fertility decisions that are crucial for the population formation and for overall understanding of the considered policy instruments has been absent in some of the previous research. Finally, since the foundation of the utilised model economy remains consistent, it allows us to compare the outcomes between the previously considered and presently analysed policy options that the government could utilise for the economic development.

4.1.1 Literature review

Before we present our results, however, we examine the evidence depicted in the literature for the compulsory education, childcare, progressive tax rates on the labour income, and regressive subsidy rates for education and fertility and their corresponding effects on the economy. We begin this summary with the influences of the compulsory education.

According to Angrist and Krueger (1991), who utilise the public use census data for the U.S. and implement the OLS and Two-Stage-Least-Squares estimations, the additional year of schooling, that arose due to the changes in the compulsory schooling laws, results in higher wages. According to the authors, the estimated monetary return for extra year of compulsory education is equal to 7.5 percent. Next, according to Oreopoulos (2006), who examined the historical changes in the compulsory schooling laws in Canada between 1920 and 1990, the additional year of education causes an increase in the annual income between 9 and 15 percent on average. In the subsequent research, Oreopoulos (2007) finds the additional year of education, that takes place due to the changes in the compulsory schooling laws, increases the lifetime wealth by 15 percent across the U.S., Canada and the U.K.. Based on the research of Pischke and von Wachter (2008), however, the authors find no evidence that increase in the compulsory education results in increase in returns for education, despite the research methodology being identical to the rest of the literature. In this research, the authors investigate the influence of the changes in the compulsory schooling law that took place in West Germany after the Second World War. According to Pischke and von Wachter (2008) this result can be explained by the observation that students learn the basic skills needed for the labour market much earlier in Germany than in other countries.

Next, Cabus and De Witte (2011) find that an increase in the school leaving age increases the percentage of the students who decide to stay in education longer beyond the legally allowed school leaving age and obtaining their diploma. The authors reach this conclusion by analysing the influence of the 'Qualification Law' which has been introduced in the Netherlands on the 1^{st} of August in 2007 which increased the legal school leaving age from 17 to 18. Through the difference-in-difference analysis of the data for Amsterdam which included information about 45,0000 students from 2005-06 to 2008-09 academic years, the authors find that the dropout rate has decreased by 2.5 percentage points due to increase in compulsory schooling by one year. According to Clark and Royer (2013), who examined the influence of changes in the compulsory schooling law in Great Britain using the regression discontinuity design, a 1947 increase in the school leaving age from 14 to 15 increases the average years of schooling by 0.5; while an increase in the school leaving age from 15 to 16 that occurred in 1972 increases the average years of schooling by 0.25. Lastly, according to Zhang (2018), who presents the evidence for Taiwan, an increase in compulsory schooling from six years, which has been in effect from 1943 to 1968, to nine years has reduced the likelihood of a dropout from school by 4 percentage points by age of 15, and by 2 percentage points by age of 18. Additionally, Zhang (2018) indicates that this policy had a larger impact on the males than on females.

As a concluding part for a discussion of the effects of compulsory education established in the empirical literature, we consider the research of Black, Devereux and Salvanes (2008) and Silles (2011), where both of these papers investigate the influence of the changes in compulsory education laws on teenage pregnancy, which (i.e. the teenage pregnancy) has been established to produce negative social and economic outcomes for women and their children. According to the study of Black, Devereux and Salvanes (2008), which has been conducted for the U.S. and Norway, and study of Sillies (2011), which utilises the data from Great Britain and Northern Ireland, the additional years of compulsory education reduce cases of teenage pregnancies. To reach this conclusion, Black, Devereux and Salvanes (2008) implemented the probit maximum likelihood estimation, and Sillies (2011) utilised the OLS and instrumental variable approach. In the case of Black, Devereux and Salvanes (2008), the data for the U.S. has been collected from Census of 1940 to 1980, and the data for the Norway has been matched between administrative registers data and Census data of 1960. For the case of Silles (2011), the data for Great Britain was collected from the General Household Survey between 1978 and 2004, and the data for the Northern Ireland was collected from Continuous Household Survey between 1983 and 2004.

To continue the discussion for the influence of the compulsory education on the economic development, we consider the part of the literature which specifically utilises the models of overlapping generations. According to Glomm and Ravikumar (1998), who modify the model of Lucas (1990) to include the government expenditure on education and calibrate the model parameters to the U.S. data, the government expenditure on education financed through distortion taxation is responsible for 2 percent growth of the national income. Furthermore, based on their earlier research which has been conducted with a construction of the model of overlapping generations where agents live for two periods and they are heterogeneous in their skill level and the decision for private and public education are made through majority voting, Glomm and Ravikumar (1992) conclude that income inequality decreases quicker when government provides public compulsory education. Authors also conclude that society would choose the education provided by the government when majority of the population is endowed with income below the average level. Glomm and Ravikumar (1992) conclude, however, that privately provided education results in greater per capita income, but only when the level of income inequality is sufficiently low. Similarly to these results, Eckstein and Zilcha (1994) conclude that the government intervention through a provision of compulsory education results in larger economic growth and more equal distribution of the income between the generations. In their study, the authors construct a model economy based on Diamond (1965); and in their model economy, the agents live for two periods and they are heterogeneous in the preferences of education for their children. Eckstein and Zilcha (1994) conclude, however, that the compulsory education is a second-best policy option for the optimal education investment. The authors state that the first-best policy option for optimal investment into the human capital is the one which contains the intergenerational transfers to education. Based on the research of Bell and Gersbach (2009), the compulsory education is the first-best policy option for escaping the poverty trap. The authors acknowledge, however, that additional lump-sum redistributive taxes may be required “if the education technology is not convex”. In addition to escaping the poverty trap, Kirchsteiger and Sebald (2010) conclude that the compulsory education is required for surmounting the ‘illiterateness trap’, which is defined by the steady-state with low human capital and welfare levels. In their analysis, the authors consider that economic agents are heterogeneous in their level and in their ability to accumulate the human capital. Kirchsteiger and Sebald (2010) also conclude, however, that the subsidies for education that government finances though taxation are able to increase the welfare in the economy further. In contrast to the previous results, however, Lu (2018) concludes that in the environment where agents maximise their social status, which is approximated by the relative level of education, the public education may cause an inefficiency. The author states that an increase in compulsory education when the unit cost of compulsory education is larger than of the education provided by the private sector, the individuals reduce the time dedicated to the higher education, which reduces the human capital accumulation and results in lower economic growth. Lu (2018) also concludes that the government should subsidise people who pursue the education since the public education can lead to inefficient outcomes. Lastly, according to Blankenau and Simpson (2004), a uniform level of expenditure into the public education that would maximise the economic growth does not exist. Instead, the authors conclude that every economy requires a unique level of expenditure for the public education and a unique level of compulsory education in order to maximise the economic development.

Moving our discussion forward, we consider the studies that examine the influence of the policies that provide more affordable public childcare next. We begin this overview with evidence presented in the empirical literature. According to Nollenberger and Rodrìguez-Planas (2015), who examine the influence of a reform which took place in Spain in early 1990s when female labour force participation was low and number of childcare slots was insufficient, a provision of public childcare to 3-year-olds increased the maternal employment by 9.6 percent. With the analysis of the data drawn from Spanish Labour Force Survey between 1987 and 1997, and by employing the difference-in-difference method of analysis, the authors conclude further that this increase in the maternal employment is driven by women who are of 30 years old or older and who have two or more children. Next, according to Martìnez and Perticarà (2017), who presents the evidence for Chile, a programme of afterschool care for children of age between 6 and 13 increases female employment by 5 percent and labour force participation by 7 percent. Moving forward to a study of Lefebvre, Merrigan and Verstraete (2009), where the authors analyse the influence of the policy which took place in Quebec between 1997 and 2004 when the government reduced the age requirement, created new childcare facilities and subsidised 85 percent of the childcare cost, the female participation in the labour market has increased by 5 percentage points as a result of this policy. The authors of this study indicate further that this result is primarily driven by the mothers with absence of post-secondary education. Lefebvre, Merrigan and Verstraete (2009) reach these conclusions through the analysis of data obtained from Statistics Canada' Survey on Labour and Income Dynamics by adopting the difference-in-difference approach. Based on the investigation of the reform that took place in the Netherlands in 2005, Bettendorf, Jongen and Muller (2015) find that as a result of an introduction of 'Law of Childcare' which reduced the average cost of formal childcare by half, the maternal employment rate increased by 3 percent and maternal hours of work increased by 6.2 percent. Authors also report that paternal employment decreased by 0.8 percent as a result of this policy, however. Overall, Bettendorf, Jongen and Muller (2015) reach the conclusion that the marginal cost of this reform exceeds the marginal benefits received by the government due to a modest response of maternal employment to this policy. In their analysis, the scholars used the data from the Labour Force Survey of Statistics Netherlands for the period between 1995 and 2009, and they utilised the difference-in-difference type of regression analysis. Next, according to Yamaguchi, Asai and Kambayashi (2018), where authors employ the marginal treatment effect framework and investigate the influence of the childcare reform that occurred in Japan in early 2000s, this policy has been found to increase the labour market participation, hours of work and earning for mothers with regular employments. For mothers with nonregular employment or who are self-employed, however, the considered policy did not produce any significant effect on the labour market outcomes. In addition, Yamaguchi, Asai and Kambayashi (2018) state that the treatment effect of childcare decreases with the age of the child, and the largest treatment effect of childcare is reported for mothers with infants. According to Givord and Marbot (2015), where authors investigate the impact of a French reform in family allowances of 2004 where childcare has been subsidised at a 50 percent rate, the female labour force participation has increased moderately as a result of this policy. Through the analysis of French fiscal data with difference-in-difference approach, the authors state that the largest impact of this policy on the maternal labour force participation is observed for mothers with one child. According to Havnes and Mogstad (2011a), however, the expansion of a subsidized childcare in Norway which began in 1975 did not influence the female labour in any significant way. The authors reach this conclusion through the analysis of resident population data from 1967 to 2006 with difference-in-difference approach. Havnes and Mogstad (2011a) report, however, that this subsidy program has created significant economic costs due to the crowding out of informal childcare arrangements. Lastly, based on the research of Andrèn (2003), where the author estimates a structural labour supply model with childcare utilization and welfare participation using the data from Swedish Household Income Survey for 1997 and 1998, a decrease in the costs of childcare helps to improve the labour supply of mothers who are already working. However, according to Andrèn (2003), this policy does not encourage the maternal labour market participation for females who have preferred to be not employed before the childcare subsidy program has been introduced.

According to Lee and Lee (2014), where the authors investigate the influence of the pronatal policies that have begun in Japan in early 1990s based on the data for the period between 1971 and 2009; such acts as childcare market deregulation, childcare centre expansions and provision of childcare grants – all were not successful for improving childbearing decisions. According to the authors, even with the government involvement, there is still significant gap between the demand for childcare of 30-44 years-old employed females and the supply of childcare which contributes to avoidance for fertility decisions. Based on the study of Fukai (2017), however, the author finds that the availability of a childcare produces a heterogeneous effect in fertility responses across Japan. Based on the analysis of the municipal level Census and Vital Statistics data for Japan of 2000 to 2010 using a linear probability model, the author concludes that a childcare availability produced a 4 percent increase in fertility rate of 25-39 years-old women who live in the region with high propensity of women to work. For the other regions, however, Fukai (2017) reports that availability of a childcare does not influence the childbearing decisions.

Moving from the empirical research to the analysis which investigates the influence of childcare through theoretical modelling, we consider the literature which utilises a life-cycle framework first. According to Hwang, Park and Shin (2018), where authors construct a life-cycle model of married couple's labour supply and fertility decisions, and calibrate the model parameters to the data from South Korea for the 1960's cohort, in the environment where a paid childcare becomes more substitutable with the maternal time, the negative relationship between the total fertility rate and female labour force participation becomes a positive one. According to Cardia and Gomme (2018), who develop a life-cycle model and estimate the model parameters based on American Time Use Survey from 2003 to 2014, a decrease in the cost of childcare increases the time that young women spend for the labour market participations. Lastly, according to Bick (2016), who analyse the influence of two reforms that took place in Germany which provide a subsidised childcare to full-time and part-time employed mothers with zero to two years old children, the subsidised childcare has been found to improve maternal labour force participation. Bick (2016) concludes that the larger number of mothers would work full-time rather than part-time "if mothers would have a greater access to subsidised childcare". However, the author states that the subsidised childcare was not successful for improving the fertility decisions. Additionally, Bick (2016) indicates that this type of policy causes the decrease in the average level of welfare due to a relatively high costs and relatively low benefits associated with this program.

Next, we consider the publication of Ishida, Oguro and Yasuoka (2018), where authors develop two-region overlapping generations model with migration and land prices. According to the researchers, an accessible and quality childcare helps to reduce the negative effect of the congestion costs on fertility decisions, which originally arise due to high population density. The authors state, however, that each region requires unique level of childcare in order to maximise the welfare. Lastly, according to Casarico and Sommacal (2012), where authors develop three-period overlapping generations model where childcare contributes to formation of the skills during early stages of the development, an inclusion of a childcare into analysis produces more accurate results for the labour income tax policies on the national income growth rate.

We conclude this portion of the literature review by summarising the results presented for the influence of childcare and its quality on the children and family performance. Based on the study of Rege, Solli, Størksen and Votruba (2018), where researchers use the data collected by the Agder-project in 2016 from southern Norway by assessing a five-year-olds who receive childcare, through the estimation of fixed effect and random effect models the authors conclude that the quality of a childcare, measured as the teacher-child ratio, produces large and significant increase in the school readiness skills. Next according to Baker, Gruden and Milligan (2008), where the authors analyse the effects of a reform that occurred in Quebec in the late 1990s, the highly subsidized and universally accessible childcare has produced negative effects on children and their parents in the short-run, however. Based on the analysis which utilises the data from National Longitudinal Survey of Children and Youth which included the waves from 1994 to 2003 and where the authors implement the difference-in-difference estimation, a more accessible childcare has been found to negatively influence the motor and social skills and health of children, and it further led to increase their in anxiety and aggressiveness. In addition, the authors report that this policy "led to more hostile, less consistent parenting, worse parental health, and lower-quality of parental relationships". According to Havnes and Mogstad (2011b), however, an expansion of a subsidised childcare in Norway has produced positive long-term impact on the outcomes of children. Based on the analysis of administrative registers data from Statistics Norway for the period between 1967 to 2006 by adopting the difference-in-difference approach, the authors conclude that a more accessible childcare leads to overall improvement of education attainment of children and their future labour market participation. Additionally, Havnes and Mogstad (2011b) state that children who have received a state childcare as a result of this reform tend to delay their decision to start a family and have children, and they also are less likely to be a welfare recipient. Finally, the authors conclude that children from disadvantaged families – in particular from families with less educated mothers – tend to benefit the most from a more accessible childcare.

We consider the evidence presented in the literature for the influence of the progressive income tax system on the economic activity next. First, we present the general conclusions for progressive income taxation established in the academic research. According to Pechman (1986), a progressive income tax system is designed to reduce the tax burden of the individuals with lower ability to pay taxes by increasing the tax burden for individuals who has higher ability to pay. However, according to the analysis of Carrol and Young (2011), where the authors utilise the theoretical model with heterogeneous households and calibrate their model to replicate the properties of the U.S. economy, a tax structure with a more progressive marginal tax rates on income decreases the incentives of wealthier individuals to earn more, which shifts the tax burden towards groups from a lower income bracket. Additionally, van Ewijk and Tang (2007) state that progressive income taxation may force a more successful workers to leave the country in order to avoid an increase in the tax burden, which would further increase the tax incidence for groups with relatively lower income level. According to Guvenen, Kuruscu and Ozkan (2014), a progressive tax system on income diminishes the returns on more hours of work and on accumulation of the human capital. According to Krueger and Ludwig (2013) this manifests in creating of negative distortions for supply in labour market, and it further negatively influences the decisions for obtaining higher education. Despite these effects, however, Carronell and Llvador (2018) conclude that only the progressive tax system is able to reduce the income inequality. Additionally, Echevarría (2012) indicates that progressive tax schedule reduces the inequality in the consumption between individuals from different income levels, and it further results in smother consumption path over time. Moreover, according to Echevarría (2012), Krueger and Ludwig (2013) and Heathcore, Storesletten and Violante (2017), the progressive income taxation provides a partial social insurance against idiosyncratic and aggregate uninsurable risks that affect the labour market activity and income level in general. In addition, Carrol and Young (2011) suggest that a more progressive tax system on income would result in greater aggregate income and wealth, and it further would produce a more efficient utilisation of the labour force. Despite these results, however, according to Stiglitz (1985) and Heathcote, Storesletten and Violante (2017), the progressive tax programme on income is ineffective for reduction of inequality in the pre-tax wages, which according to Stiglitz (1985) "undermines the original redistributive effect". According to Heathcote, Storesletten and Violante (2017) this occurs because the progressive income taxation creates a "relative scarcity of high skilled workers" which "increases the skill premium in general equilibrium". Conesa and Krueger (2006) also state that progressive income taxation creates disincentives for saving and investment decisions of private economic agents. Lastly, according to Heathcote, Storesletten and Violante (2017), the progressive tax schedule creates a free-riding problem "which increases the social cost of a progressive tax system" when it is used in provision of the public goods by the government.

Due to these mixed responses, the arguments have been made for an implementation of a less progressive income tax schedule. At the same time, however, a more progressive tax system on the income than it is currently in use has been advised as well. We consider both sides of the discussion here, and we begin with a research which suggests the use of a less progressive income tax scheme. According to Heathcote, Storesletten and Violante (2017), where authors develop an analytically tractable equilibrium model with individuals who are heterogeneous in learning ability and in disutility of work effort and experience idiosyncratic labour market shocks that are partially uninsurable, "the utilitarian planner could generate welfare gains by making the U.S. tax and transfer system less progressive". According to the authors, a decrease in income tax progressivity would encourage skill investment and labour supply, which would decrease the inequality in the pre-tax wages – all of which would contribute to a welfare improvement. Additionally, Heathcote, Storesletten and Violante (2017) state that in the environment with less progressive tax scheme on income, the government can finance the high levels of public consumption more easily. In addition, according to Caucutt, İmrohoroğlu and Kumar (2003), where authors construct overlapping generations model with individuals who are heterogeneous in skill level and face a liquidity constraint, and calibrate the model parameters to replicate the properties of the U.S. economy, a less progressive tax system can increase the economic growth, decrease the inequality and provide a "greater mobility for the poor in the long-run". Heathcote, Storesletten and Violante (2017) indicate, however, that more progressive tax system for the case of the U.S. would be required if the utilitarian social planner is "more inequality averse" than considered by the authors, and when poverty creates the barriers for skill investment.

Opposite to Heathcote, Storesletten and Violante (2017), Krueger and Ludwig (2013) suggest that the welfare-maximising policy mix involves a more progressive tax scheme on income than it is currently implemented in the U.S.. Krueger and Ludwig (2013) state, however, that the more progressive tax schedule must be accompanied with the subsidy for education in order to reach the welfare improvement, and mitigate the distortions caused by a more progressive tax system. Based on the analysis of a large-scale overlapping generations model with uninsurable idiosyncratic risks, the combination of a more progressive marginal tax rates on income with subsidy for education leads to decrease in inequality in earnings, consumption and wealth, and to increase in higher education attainment – all of which leads to a welfare improvement. In addition, van Ewijk and Tang (2007) conclude that in the environment where trade unions have a large power to set wages, a more progressive income tax system is required to minimise the distortions caused by trade unions in form of resulted unemployment levels. The authors state, however, that in order to reach a welfare improving outcome, the government must combine an increase in the income tax progressivity with a substantial subsidy for education in order to account for the negative effects of increased marginal tax rates on education investment.

As a final note before presenting a summary for our results, we also have attempted to investigate the evidence from the literature for the influence of regressive subsidy systems for education and fertility on the economic outcomes. Based on our literature review and to the best of our knowledge, however, the role of regressive subsidy systems where individuals receive subsidy rates for education and fertility disproportionally based on their private decisions has not been considered previously. The most closely related literature, however, investigates the possibility that the subsidies for higher education disproportionally affect the individuals from different income groups. According to Hansen and Weisbrod (1969), based on the investigation of the data for the California State between 1964 and 1966, only people from upper income families benefit from the government subsidy program for higher education as only these individuals are eligible to be a subsidy recipient, and only they receive a positive net transfers. As a result, the authors conclude that subsidies for higher education lead to a further increase in inequality between people from different backgrounds. Blöndal, Field and Girouard (2002) reach similar conclusion by investigating the data for the OECD countries between 1998 and 2000. According to the authors, “the beneficiaries of government spending on post-compulsory education tend to come from relatively well-off families and have high income prospects". Furthermore, the researchers suggested that a simultaneous increase in the tuition fees and in student loan availability could distribute the benefits from subsidies for higher education more equally between individuals from various income groups. Lastly, according to Johnson (2006), who investigates the data for the U.S. using National Longitudinal Survey of Youth for a group of young adults born between 1957 and 1964, the people from high-income families receive a greater (economic) benefit from government subsidies for higher education than individuals from lower income groups. Despite this, however, the author states that after taking into consideration the level of taxes paid by the parents and young adults, only people from lower income families receive a positive net transfers from the government as a result of a subsidy, while the individuals from higher income families receive a negative net transfers from the subsidy program due to overall larger level of taxes paid by these families.

4.1.2 Summary for the results

This concludes the part of introduction devoted to overview of the literature. Now, we present the summary for the results from policy experiments considered in this chapter. According to our results, the compulsory education decreases the incentives of households for private education provision, but it increases the incentives for fertility decisions[2]. As a result, in our policy exercise where government provides the compulsory education at a level which is equal to the average education attainment from the original steady-state, the adult households from majority of ability groups completely rely on the government to provide the education for their children. Consequently, children from lower ability groups receive the total level of education which is greater than at the original steady-state, but children from higher ability groups now receive the total level of education which is lower than at the original equilibrium. Overall, conditional on the idiosyncratic shocks to the human capital, we observe considerable change in the distribution of the population, which now clusters around the centre of the original distribution. This illustrates the improvement of the human capital for the lower ability households, but decrease in the human capital of the higher ability households. Additionally, our results indicate a sig-

[2]For analytical results in deterministic environment please refer to appendix A1 section

nificant decrease in inequality in the distribution of human capital across the population as a result of compulsory education[3]. Furthermore, due to increase in fertility decisions, we observe a considerable increase in the population size. Lastly, according to our results, the impact of the considered compulsory education program on welfare depends on the tax option that government chooses to finance its budget. For instance, when government finances the compulsory education with the tax on capital income, our result suggest a welfare improvement; whereas with tax on consumption or with flat tax rate on the labour income, the welfare diminishes below the original equilibrium[4].

Similarly to the case with compulsory education, in our policy experiments when government provides the childcare for fifty percent of time that adult households have to contribute for raising children, we observe an increase in fertility decisions and decrease in education provision across all ability groups. Unlike in the case with compulsory education, however, due to absence of alternative means to accumulate the human capital[5], we observe a loss of the human capital across households from different ability groups as the economy reaches the final steady-state. This, in turn, causes a majority of the population to be concentrated on the left-hand-side of the distribution among lower ability groups, and, therefore, the level of the welfare in the economy decreases. Additionally, due to adjustment of the distribution of population as a result of a state provided childcare, our results indicate an increase in the inequality in distribution of human capital. Lastly, due to increase in fertility decisions, however, we observe the most pronounced increase in the population size out of all policy options that we consider.

For our analysis with progressive labour income tax scheme and flat subsidy rate for education of ten percent, our results indicate that children from lower ability groups now receive marginally larger level of education than at the final steady-state with flat tax rate on labour income[6]. This is the case because the households from lower ability groups generate relatively lower income and they face a relatively lower marginal tax rates on labour income than the rest

[3]Please refer to appendix A2 section for further details

[4]These results are conditional, however, on the level of compulsory education provided by the government

[5]Unlike in Casarico and Sommacal (2012), we assume that childcare does not enter the human capital formation function. We consider the childcare only as an instrument which substitutes a portion of a time that parents have to contribute for raising children.

[6]With exception for the households from the bottom three ability groups who still provide their children with zero education

of the population, which create less disincentive for private education choices. For the households from higher ability groups, however, we observe the opposite. Children from these ability groups receive the education which is lower than under all previously considered tax options. Due to interdependency between education and fertility decisions, our results also indicate that households from lower ability groups now choose relatively lower fertility level, while the fertility decisions of the higher ability groups increase above the final steady-state with flat tax rate on the labour income. Overall, our results indicate that with the progressive tax rates on the labour income and subsidy for education of ten percent, the level of welfare in the economy becomes marginally larger than at the final steady-state with flat tax rate on the labour income; but it is still below the final steady-states with tax on consumption or capital income. At the individual level, we observe that for the first ten out of fifteen ability groups that we consider, the improvement in the utility level takes place; whereas, for the rest (top five) ability groups, the utility becomes lower than at the final steady-state with flat tax on the labour income and subsidy for education. Finally, due to the change in the distribution of the population, our results indicate that when the economy reaches the final steady-state with progressive tax rates on labour income and flat subsidy rate for education, the level of inequality in the distribution of the human capital decreases below the levels obtained for the final steady-states with flat subsidy rate for education. Additionally, based on our results, the combination of flat subsidy rate for education and progressive tax rates on labour income is a third-best policy for reducing an inequality in the distribution of the human capital across the population[7].

For the case with progressive tax rates on the labour income and subsidy rate for fertility of ten percent, we observe similar changes in education and fertility decisions to ones mentioned above. The households from lower ability groups provide their children with relatively larger level of education than at the final steady-state with flat tax rate on the labour income, and they decide on relatively smaller fertility levels. For the households from top ability groups who

[7] If we would rank the policy options on the ability to reduce inequality in the distribution of human capital across the population from best to worst, based on our results, the order would be the following: (1) compulsory education with flat tax system, (2) regressive subsidy rates for education with flat tax system, (3) progressive labour income tax scheduled with flat subsidy rate for education, (4) flat subsidy rate for education with flat tax system, (5) childcare with flat tax rate on labour income, (6) regressive subsidy rates for fertility with flat tax system, (7) childcare with tax on consumption or capital income, (8) progressive tax scheme on labour income with flat subsidy rate for fertility, and (9) flat subsidy rate for fertility and flat tax system. For more details, please refer to appendix A2 section

face relatively larger tax rates on the labour income, it is the opposite. Overall, we observe an increase in population share of lower ability groups and decrease the average level of human capital and the level of the welfare in the economy below the original steady-state. In comparison to the case with flat income tax schedule, however, both welfare and average human capital are larger at the final steady-state with the progressive labour income tax scheme. Finally, at the final steady-state with flat subsidy rate for fertility and progressive tax rates on labour income, our results show that the level of inequality in distribution of human capital across the population becomes smaller than at the final steady-state with flat subsidy rate for fertility and flat tax rate on labour income considered previously.

Next, for the policy exercises with regressive subsidy rates for education – where households who provide their children at the initial steady-state with lower education receive larger subsidy rates as government enters the economy – we observe a sizeable increase in the private education attainment of the households from lower ability groups. However, due to decrease in the level of relative human capital[8] and combined with smaller subsidy rates for education received by the households from higher ability groups when the economy reaches the final steady-state, the education attainment by these ability groups diminishes below the original steady-state. Overall, we observe an increase in the population share of the households from mid-high ability groups. Due to these changes in the distribution of the population and in education provision, we observe that at the final steady-state, the average level of human capital with regressive subsidy rates for education is relatively lower than with the flat subsidy rate given the tax option in place. However, due to considerable increase in education of lower ability groups which allows more households from lower ability groups become a part of higher ability groups, the level of welfare in the economy at the final steady-state with regressive subsidy system for education is found to be larger than with the flat subsidy rate for education, independently of the tax option implemented by the government. Furthermore, based on our results, a combination of regressive subsidy rates for education with a flat tax scheme reduces the level of inequality in the distribution of human capital below the one obtained with the

[8]The relative human capital x_t^i has a positive influence on the incentives for education attainment at the individual ability level, and it is found as the human capital for a particular ability group divided by the average human capital in the economy

flat subsidy rate for education and flat tax scheme on consumption, labour income or capital income or progressive tax scheme on labour income. Our results also indicate, however, that the population size decreases at larger rate when the regressive subsidy rates for education are in place.

Finally, with regressive subsidy rates for fertility – where households with lower fertility decisions receive larger subsidy rates for fertility from the government – we observe that the fertility decisions of the households from higher ability groups increase at a larger rate, while the fertility choices of the households from lower ability groups grow at a smaller rate than with the flat subsidy rates for fertility. Consequently, we observe relatively lower choices for education for higher ability groups and relatively higher education provision by lower ability groups[9]. Overall, our results indicate that, as before with flat subsidy rate for fertility, the welfare in the economy would reduce below the original steady-state when regressive subsidy rates for fertility are in place, but the economy would have relatively larger level of welfare with regressive subsidy rates than with flat subsidy rate for fertility. Additionally, due to a relatively lower fertility choices of the households from lower ability groups, the population growth is found to be smaller when government provides the regressive subsidy rates for fertility versus when the flat subsidy rate for fertility is in place. In conclusion, based on our results, with the regressive subsidy rate for fertility, the economy has a lower level of inequality in distribution of the human capital than with the flat subsidy rate for fertility as it reaches the final steady-state; but the inequality is still larger than at the initial steady-state.

The remaining part of our paper has the following structure. In the section 4.2, we present the theoretical model that we utilise for our analysis, and in the section 4.3, we present the calibration for model parameters. In the section 4.4, we present the detailed analysis for each of the policy experiments that we have summarised above. And, in the section 4.5, we conclude.

[9]With exception for the households from bottom three ability groups who still provide their children with zero education

4.2 Theoretical framework

4.2.1 Household Sector

We begin the presentation of the model economy with the problem for the households. Following *de la* Croix and Doepke (2003), the household sector of the model economy consists of three generations of individuals[10], who are heterogeneous in the level of human capital. In line with *de la* Croix and Doepke (2003), young households receive education which is optimally chosen by their parents that forms the human capital of a future generation. Adult households are main decision makers who optimally choose the level of consumption for adulthood, amount to save, education for their children and fertility. Elderly households consume based on saving made during adulthood.

In line with assumptions introduced in chapter two, the adult households receive subsidies for education and fertility from the government that reduce the costs associated with these two decisions. In turn, the government taxes consumption, labour income and capital income to finance the subsidy program and to keep its budget balanced. Following the chapter three, young households receive idiosyncratic shocks during their human capital accumulation, which introduces an uncertainty into the decisions of the households. Given the topic of present paper, we also consider that the government provides households with compulsory education and childcare, which the government finances through the government budget constraint as before. As mentioned above, we also test the effects of progressive labour income tax rates and regressive subsidy rates for education and fertility on the long-run economic development. For the case with regressive subsidy schemes, we consider that the subsidy rates decrease with parental decisions for education provision and fertility level. Thus, given the original structure of the model economy and previously and presently considered model extensions, the problem of household sector is formally presented as following.

Given the rational expectations of adult households for the realised level of human capital of their children, heterogeneous adult households from ability group i[11] at time t maximise their expected utility function (4.1) subject to the budget constraint for adulthood (4.2), budget

[10] I.e.: young households, adult households, and elderly households

[11] Where ability group i is defined by the level of human capital h_t^i

constraint for elderly stage (4.3), and human capital formation process (4.4), by optimally choosing the level of consumption for adulthood c_t^i, amount of savings s_t^i, level of education to provide for their children e_t^i, and number of children to have n_t^i.

$$\max_{c_t^i, s_t^i, e_t^i, n_t^i} E_t[u_t^i] = E_t\left[\ln c_t^i + \beta \ln d_{t+1}^i + \gamma \ln n_t^i h_{t+1}^i\right] \tag{4.1}$$

As it is depicted by (4.1), the expected utility function of adult households positively depends on the consumption levels during adulthood c_t^i and old age d_{t+1}^i, and quality h_{t+1}^i and quantity n_t^i of children for which adults experience a trade-off. In (4.1), the model parameters β and γ represent the discount factor of adult households and altruism factor of adult and elderly households for children, respectively.

$$(1+\tau_t^c)c_t^i + s_t^i + e_t^i n_t^i w_t \bar{h}_t = (1-\tau_t^{l,i}) w_t h_t^i \big(1-(1-\omega_t)\phi n_t^i\big) + n_t^i w_t \bar{h}_t (e_t^i \chi_t^i sub_t^e + \bar{e}_t \psi_t^i sub_t^n) \tag{4.2}$$

Next, according to (4.2), the total expenditure of any household member during adult stage of the existence consists of consumption, savings and education provision expenses[12], that are the first three terms on the left-hand side of the expression, respectively. On the right-hand side of (4.2) the total income of the adult households can be found, which consists of the labour income and total subsidy amount that adult households receive from the government.

According to (4.2), the labour income net of the tax rate[13] $\tau_t^{l,i}$, increases with the real wage rate, the human capital and free time that adult households have for labour market participation away from taking care for children. Following original assumptions of *de la* Croix and Doepke (2003), the free time that adult households have for labour market participation decreases with the number of children that adult households decide to have at a factor of ϕ, which is the time-cost parameter for raising children. With an introduction of assumption that government provides the childcare, the fraction of the time for which childcare is provided ω_t reduces the time-cost parameter of raising children, and therefore should positively influence the time that

[12] Where τ_t^c is the tax rate on consumption, and the cost of education is a product between the level of (private) education e_t^i that parents decide for their children, number of children n_t^i for whom this education is provided, real wage rate w_t and human capital of teachers approximated by the average level of human capital $\bar{h}_t$

[13] Where the tax rate on labour income can be progressive and it is calculated according to Gouveia-Strauss function (4.12) for each ability group, or flat – in line our with assumptions from previous two chapters

adult households have for labour market participation, holding everything else constant.

The total amount of subsidy received by adult households from the government is presented by the second term on the right-hand side of (4.2), where $\bar{e}_t$ is the average level of education provision chosen by the parents, and sub_t^e and sub_t^n are the subsidy rates for education and fertility, respectively. Variables χ_t^i and ψ_t^i control whether the subsidy program is flat or regressive in nature. If the analysis includes the subsidy program with flat subsidy rates, χ_t^i and ψ_t^i are set to be equal to one. If we analyse the influence of the regressive subsidy system, χ_t^i and ψ_t^i are equal to $\frac{\bar{e}_t}{(\mu+e_t^i)}$ and $\frac{\bar{n}_t}{n_t^i}$, respectively, where μ is an instrument parameter that ensures a finite subsidy rate for the households who provide their children with zero education.

Moving forward, according to (4.3), the consumption of the elderly households adjusted to the growth rate of the human capital at the steady-state ρ net of tax on consumption τ^c is defined by the return on savings made during adulthood, which is determined by the real interest rate r net of capital income tax rate τ^k and savings from adulthood s.

$$(1+\tau_{t+1}^c)d_{t+1}^i = \frac{1}{(1+\rho)}\big(1+r_{t+1}(1-\tau_{t+1}^k)\big)s_t^i \tag{4.3}$$

Lastly, the expected level of human capital for children from ability group i who experience idiosyncratic shocks while the human capital formation takes place is defined by (4.4).

$$E_t[h_{t+1}^i] = E_t\left[\frac{1}{(1+\rho)}B(\theta+e_t^i+\underline{e}_t)^{\eta}(h_t^i)^{\pi}(\bar{h}_t)^{\kappa}\times \exp^{\varepsilon}\right] \tag{4.4}$$

According to the deterministic component of the human capital formation process[14], following original assumptions of *de la* Croix and Doepke (2003), the human capital of children adjusted to the growth rate at the steady-state is defined by the private education investment, human capital of parents, and quality of education which is given as the average level of human capital $\bar{h}_t$. Additionally, as one of the model extensions introduced in this paper, we assume that the government provides compulsory education $\underline{e}_t$ which positively influence the human capital of children; and it enters into the deterministic component of (4.4). Therefore, when the government provides compulsory education, the total education attainment of children from

[14] I.e.: $\frac{1}{(1+\rho)}B(\theta+e_t^i+\underline{e}_t)^{\eta}(h_t^i)^{\pi}(\bar{h}_t)^{\kappa}$

ability group i is equal to $(e_t^i + \underline{e}_t)$, and e_t^i can be interpreted as the private level of education investment that parents decide to provide their children in addition to the compulsory level $\underline{e}_t$. Finally, following assumptions of the chapter 3, the stochastic component is introduced into the human capital formation process through $\exp^{\varepsilon}$, and it is discretised following the steps described previously[15].

The model parameters B, η, π and κ in (4.4) depict the efficiency of the human capital accumulation, relative significance of the (total) education attainment for human capital formation, relative significance of human capital of parents for human capital of children, and relative significance of the quality of education for human capital attainment, respectively. Lastly, θ is an instrument parameter which guaranties a non-zero level for human capital of children when parents optimally decide on zero education provision for their children and when government does not provide households with compulsory education.

4.2.2 Production sector

We introduce the problem for the production sector of the economy in the similar way that we have done previously, which follows the original *de la* Croix and Doepke (2003) setting. We consider that a representative producer maximises its profit, function (4.5), subject to the technological constraint (4.6) by optimally choosing its factor inputs – physical capital stock K_t and effective labour force L_t[16].

$$\max_{K_t, L_t} \Pi_t = Y_t - w_t L_t - (r_t + \delta) K_t \tag{4.5}$$

$$Y_t = A K_t^{\alpha} L_t^{1-\alpha} \tag{4.6}$$

where w_t is the real wage rate, $(r_t + \delta)$ is the nominal interest rate, r_t is the real interest rate, δ is the depreciation rate of the physical capital stock, and Y_t is the level of output[17] which is defined by the factor inputs, productivity level A, share of capital income α and share of labour income $(1 - \alpha)$.

[15] For further details on discritisation procedure please refer to the section 3 of chapter 3

[16] Both of the factor inputs are given at per adult household level

[17] Per adult household

Given the analytical solution for the optimisation problem of a producer, the demand functions for physical capital stock and effective labour force are depicted by (4.7) and (4.8), respectively.

$$r_t = \alpha A {K_t}^{\alpha-1} L_t^{1-\alpha} - \delta \tag{4.7}$$

$$w_t = (1-\alpha) A K_t^{\alpha} L_t^{-\alpha} \tag{4.8}$$

To conclude, with the demand functions for both factor inputs being defined through the optimisation problem for the production sector, given the model extensions that are in place[18], the supply for the effective labour force and physical capital stock is determined by (4.9) and (4.10), respectively.

$$L_t = \frac{\sum_{i=1}^{I} p_t^i \left[h_t^i (1-\phi(1-\omega_t) n_t^i) - n_t^i e_t^i \bar{h}_t - n_t^i \underline{e}_t \bar{h}_t - \omega_t \phi n_t^i \bar{h}_t \right]}{\sum_{i=1}^{I} p_t^i} \tag{4.9}$$

$$K_t = \frac{1}{(1+\rho)} \left(\frac{\sum_{i=1}^{I} p_{t-1}^i s_{t-1}^i}{\sum_{i=1}^{I} p_t^i} + (1-\delta) K_{t-1} \right) \tag{4.10}$$

According to (4.9), the size of the effective labour force per adult household grows with the endowment of human capital of adult household and free time that households can contribute to the labour market and not for taking care for children, which, based on our assumptions, increases with the fraction of a time for which childcare is available[19], holding everything else constant. On the other hand, the effective labour force diminishes with the level of education provision – either optimally chosen by parents e_t^i, or compulsory education provided by the government $\underline{e}_t$[20] – and with the number of children that households optimally decide to have n_t^i. The reason for this decrease comes from requirement for some of the adults to leave the production sector employment and to perform the role of teachers; assuming the population size of adult households, they level of human capital and they time for labour market participation remains unchanged. Furthermore, similarly to education provision, the availability of a childcare would require some of the households to perform the role of teachers which would

[18] In particular, the compulsory education and availability of a childcare

[19] I.e.: it increases with the value of ω_t

[20] As it has been mentioned in the section for the problem of households, when compulsory education $\underline{e}_t$ is in place, e_t^i is interpreted as an additional (private) level of education which children receive when parents view $\underline{e}_t$ to be lower than optimal

prevent their employment in production sector.

Lastly, according to (4.10), the level of physical capital stock per adult household adjusted to the growth rate of the human capital at the steady-state is defined by the aggregate level of savings plus any remaining physical capital stock net of depreciation. In (4.9) and (4.10) p_t^i indicates the population size of ability group i at time t.

4.2.3 Government sector

The final sector that we consider for our analysis is the government. Based on our assumptions, we introduce that government sector and consider that it has four policy options for implementation, which are the subsidies for education and fertility, compulsory education, and the childcare. These four components enter into the government budget constraint (4.11) in the mentioned order on the right-hand side of the expression which represents the aggregate expenditure of government for a given time period.

On the left-hand side of (4.11), we depict the total income that government generates at a given period from taxing consumption, labour income and capital income. As it has been mentioned above, when the government taxes labour income, it can implement either a flat or a progressive tax scheme.

In the case when government implements the flat labour income tax rate, the variable ι_t^i in (4.11) becomes equal to one, and every adult member of the population receives labour income tax rate which is equal to τ_t^l. Oppositely, when government decides to implement a progressive labour income tax schedule, we follow Nishiyama (2015) and ι_t^i is calculated according to Gouveia-Strauss function (4.12), where φ_1 controls the curvature of the tax function and φ_2 adjusts scale of the tax function. In this case τ_t^l is interpreted as the upper-limit for the effective marginal income tax rate when household's labour income $(w_t h_t^i(1-\phi(1-\omega_t)n_t^i)$ goes to infinity, and the product between τ_t^l and ι_t^i returns the marginal tax rates on the labour income

$\tau_t^{l,i}$ for households from i^{th} ability group[21].

$$\sum_{i=1}^{I} p_t^i \tau_t^c c_t^i + \sum_{i=1}^{I} p_{t-1}^i \tau_t^c d_t^i + \sum_{i=1}^{I} p_t^i r_t \tau_t^k s_t^i + \sum_{i=1}^{I} p_t^i \tau_t^l \iota_t^i w_t h_t^i (1-\phi(1-\omega_t)n_t^i) =$$
$$= \sum_{i=1}^{I} p_t^i n_t^i w_t \bar{h}_t (e_t^i \chi_t^i sub_t^e + \bar{e}_t \psi_t^i sub_t^n) + \sum_{i=1}^{I} p_t^i n_t^i w_t \bar{h}_t \underline{e}_t + \sum_{i=1}^{I} p_t^i \omega_t \phi n_t^i w_t \bar{h}_t \tag{4.11}$$

$$\iota_t^i = \tau_t^l \left[\left(w_t h_t^i (1-\phi(1-\omega_t)n_t^i) \right) - \left(\left(w_t h_t^i (1-\phi(1-\omega_t)n_t^i) \right)^{-\varphi_1} + \varphi_2 \right)^{-\frac{1}{\varphi_1}} \right] \tag{4.12}$$

Finally, following our previous discussion in chapter 2 and 3, we assume that the government balances its budget for every period of analysis by setting the tax rates given the decisions and responses to the government program from the household and production sectors.

4.3 Calibration

To conduct our policy experiments, similarly to the previous two chapters, for majority of the model parameters we follow *de la* Croix and Doepke (2003). Namely, for structural parameters α – share of the physical capital income, β – discount factor of the households, δ – depreciation rate of physical capital stock, η – relative significance of education for human capital, θ – instrument parameter for non-zero human capital of children when total education provision is zero, π – relative significance of human capital of parents for human capital of children, κ – relative significance of quality of education for human capital, ρ – long-run growth rate of human capital, and ϕ – time-cost parameter to raise children, the values come directly from *de la* Croix and Doepke (2003)[22]. For the computational purposes, we, however, choose the values for productivity level A, efficiency of human capital accumulation B and altruism factor of adult and elderly households toward children γ in order to normalise the values for the real wage rate, average level of human capital and average fertility level at initial-steady state to be equal to one, respectively. For parameter σ that measures the standard deviation of the idiosyncratic

[21] In our calculation of the marginal tax rates for a different ability groups $\tau_t^{l,i}$, a cross-multiplication between τ_t^l and ι_t^i is necessary to obtain the values for the tax rates which are consistent with the model units and which are comparable to the other considered tax options

[22] Please refer to section 4 in chapter 3 for a discussion of the way in which *de la* Croix and Doepke (2003) performed a calibration for these model parameters

shocks to the human capital, we follow Bosworth, Burtless and Steuerle (2000) and Millimet, Podder, Slottij and Zandvakili (2003), similarly to chapter 3. For the parameters φ_1 and φ_2 inside of Gouveia-Strauss function (12) that controls curvature and adjusts the scale of the tax function, respectively, we follow calibration of Nishiyama (2015). Lastly, we set the instrument parameter μ to be equal to 0.01. Table 4.1 below provides a summary for structural parameters utilised in this model economy, chosen calibration levels and interpretation for them.

parameter	value	interpretation
A	2.9504	productivity level
B	7.3478	efficiency of human capital accumulation
α	1/3	share of physical capital income
β	0.99^{120}	discount factor
γ	0.17957	altruism factor
δ	1	depreciation rate of physical capital stock
η	0.5	relative significance of education for human capital
θ	0.0119	instrument parameter for non-zero human capital of children when education provision is zero
π	0.2	relative significance of human capital of parents for human capital of children
κ	0.1	relative significance of quality of education for human capital
μ	0.01	An instrument parameter that ensures a finite subsidy rate for education for the households who provide their children with zero education when the subsidy system is regressive
ρ	$1.02^{30} - 1$	long-run growth rate of human capital
σ	0.2	st. deviation of the idiosyncratic shocks to the human capital
ϕ	0.075	time-cost parameter to raise children
φ_1	0.3745	parameter that controls curvature of Gouveia-Strauss tax function
φ_2	0.7368	parameter that adjusts the scale of Gouveia-Strauss tax function

Table 4.1: Calibration for the structural parameters of the model economy

4.4 Policy Experiments

4.4.1 Compulsory education

We begin our analysis with the influence of compulsory education on the proposed environment. We consider that government enters into the economy at the second period of analysis[23]

[23] I.e.: at t=2

and it sets the level of compulsory education $\underline{e}_t$ to be equal to the level of average education provision at the initial steady-state[24]. The government has three tax options that it can utilise to finance its budget, which are tax on consumption, labour income[25], and capital income. Tables 4.2 to 4.4 and figure 4.1 below summarise the results for these policy exercises.

	Y	K	L	r	w	τ^c	$\underline{e}$	
	1.6047	0.0929	1.3163	4.7604	0.8127	0.0670	0.0512	
	(25.79%)	(-16.91%)	(54.78%)	(69.72%)	(-18.72%)			
	$\bar{h}$	tot.pop.	$\bar{e}$	$\bar{n}$	$\bar{c}$	$\bar{s}$	$\bar{d}$	$\bar{u}$
	1.0224	1.4203×10^8	0.0000	1.6185	0.5265	0.1682	0.5013	-0.7921
	(2.24%)		(-100%)	(61.84%)	(-22.13%)	(-16.91%)	(17.89)	(-5.59%)
i	h_i	pop.share	e_i	n_i	c_i	s_i	d_i	u_i
1	0.2019	0.0000	0.0000	1.6189	0.1040	0.0332	0.0990	-3.1567
			—	(0%)	(-23.83%)	(-18.73%)	(15.62%)	(-7.84%)
2	0.2466	0.0000	0.0000	1.6189	0.1270	0.0406	0.1209	-2.8609
			—	(0%)	(-23.83%)	(-18.73%)	(15.62%)	(-8.72%)
3	0.3012	0.0000	0.0000	1.6189	0.1551	0.0496	0.1477	-2.5651
			—	(0%)	(-23.83%)	(-18.73%)	(15.62%)	(-9.83%)
4	0.3679	0.0000	0.0000	1.6189	0.1895	0.0605	0.1804	-2.2693
			(-100%)	(13.74%)	(-23.83%)	(-18.73%)	(15.62%)	(-10.01%)
5	0.4493	0.0000	0.0000	1.6189	0.2314	0.0739	0.2203	-1.9736
			(-100%)	(29.37%)	(-23.83%)	(-18.73%)	(15.62%)	(-10.24%)
6	0.5488	0.0054	0.0000	1.6189	0.2826	0.0903	0.2691	-1.6778
			(-100%)	(42.18%)	(-23.83%)	(-18.73%)	(15.62%)	(-11.01%)
7	0.6703	0.0736	0.0000	1.6189	0.3452	0.1103	0.3287	-1.3820
			(-100%)	(52.66%)	(-23.83%)	(-18.73%)	(15.62%)	(-12.50%)
8	0.8187	0.2439	0.0000	1.6189	0.4217	0.1347	0.4015	-1.0862
			(-100%)	(61.24%)	(-23.83%)	(-18.73%)	(15.62%)	(-15.26%)
9	1.0000	0.3543	0.0000	1.6189	0.5150	0.1645	0.4903	-0.7904
			(-100%)	(68.27%)	(-23.83%)	(-18.73%)	(15.62%)	(-20.80%)
10	1.2214	0.2439	0.0000	1.6189	0.6290	0.2009	0.5989	-0.4946
			(-100%)	(74.02%)	(-23.83%)	(-18.73%)	(15.62%)	(-35.68%)
11	1.4918	0.0721	0.0000	1.6189	0.7683	0.2454	0.7315	-0.1988
			(-100%)	(78.73%)	(-23.83%)	(-18.73%)	(15.62%)	(-170.34%)
12	1.8221	0.0050	0.0074	1.5336	0.9384	0.2998	0.8934	0.0872
			(-93.41%)	(72.97%)	(-23.83%)	(-18.73%)	(15.62%)	(-60.06%)
13	2.2255	0.0000	0.0370	1.3196	1.1462	0.3661	1.0913	0.3560
			(-74.13%)	(51.40%)	(-23.83%)	(-18.73%)	(15.62%)	(-30.34%)
14	2.7182	0.0000	0.0732	1.1843	1.3999	0.4472	1.3329	0.6324
			(-59.36%)	(37.77%)	(-23.83%)	(-18.73%)	(15.62%)	(-21.39%)
15	3.3201	0.0000	0.1173	1.0926	1.7099	0.5462	1.6279	0.9137
			(-47.90%)	(28.53%)	(-23.83%)	(-18.73%)	(15.62%)	(-16.80%)

Table 4.2: Summary for the final steady-state when government provides compulsory education $\underline{e} = 0.0512$ and finances its budget with tax on consumption

[24] I.e.: $\underline{e}_t = \bar{e}_1 \, \forall t \in [2, T]$

[25] For the case of compulsory education and upcoming case of childcare, we restrict our analysis to the flat labour income tax rates

	Y	K	L	r	w	τ^l	$\underline{e}$	
	1.5793	0.0751	1.4293	6.0117	0.7366	0.1075	0.0512	
	(23.81%)	(-32.82%)	(62.07%)	(114.33%)	(-26.34%)			
	$\bar{h}$	tot.pop.	$\bar{e}$	$\bar{n}$	$\bar{c}$	$\bar{s}$	$\bar{d}$	$\bar{u}$
	1.0219	1.2171×10^{10}	0.0000	1.8140	0.4543	0.1360	0.5264	-0.9047
	(2.19%)		(-100%)	(81.39%)	(-32.82%)	(-32.82%)	(23.81%)	(-20.59%)
i	h_i	pop.share	e_i	n_i	c_i	s_i	d_i	u_i
1	0.2019	0.0000	0.0000	1.8140	0.0897	0.0269	0.1040	-3.2687
			—	(12.05%)	(-34.26%)	(-34.26%)	(21.15%)	(-11.67%)
2	0.2466	0.0000	0.0000	1.8140	0.1096	0.0328	0.1270	-2.9729
			—	(12.05%)	(-34.26%)	(-34.26%)	(21.15%)	(-12.98%
3	0.3012	0.0000	0.0000	1.8140	0.1339	0.0401	0.1552	-2.6771
			—	(12.05%)	(-34.26%)	(-34.26%)	(21.15%)	(-14.62%)
4	0.3679	0.0000	0.0000	1.8140	0.1635	0.0490	0.1895	-2.3813
			(-100%)	(27.44%)	(-34.26%)	(-34.26%)	(21.15%)	(-15.44%)
5	0.4493	0.0000	0.0000	1.8140	0.1997	0.0598	0.2315	-2.0855
			(-100%)	(44.96%)	(-34.26%)	(-34.26%)	(21.15%)	(-16.49%)
6	0.5488	0.0054	0.0000	1.8140	0.2440	0.0730	0.2827	-1.7897
			(-100%)	(59.31%)	(-34.26%)	(-34.26%)	(21.15%)	(-18.42%)
7	0.6703	0.0737	0.0000	1.8140	0.2980	0.0892	0.3453	-1.4940
			(-100%)	(71.05%)	(-34.26%)	(-34.26%)	(21.15%)	(-21.62%)
8	0.8187	0.2461	0.0000	1.8140	0.3639	0.1090	0.4218	-1.1982
			(-100%)	(80.67%)	(-34.26%)	(-34.26%)	(21.15%)	(-27.14%)
9	1.0000	0.3545	0.0000	1.8140	0.4445	0.1331	0.5152	-0.9024
			(-100%)	(88.54%)	(-34.26%)	(-34.26%)	(21.15%)	(-37.92%)
10	1.2214	0.2437	0.0000	1.8140	0.5429	0.1625	0.6292	-0.6066
			(-100%)	(94.99%)	(-34.26%)	(-34.26%)	(21.15%)	(-66.40%)
11	1.4918	0.0718	0.0000	1.8140	0.6632	0.1985	0.7685	-0.3108
			(-100%)	(100.26%)	(-34.26%)	(-34.26%)	(21.15%)	(-322.59%)
12	1.8221	0.0049	0.0000	1.8140	0.8100	0.2425	0.9387	-0.0150
			(-100%)	(104.58%)	(-34.26%)	(-34.26%)	(21.15%)	(-106.88%)
13	2.2255	0.0000	0.0195	1.5996	0.9893	0.2962	1.1465	0.2582
			(-86.35%)	(83.52%)	(-34.26%)	(-34.26%)	(21.15%)	(-49.49%)
14	2.7182	0.0000	0.0518	1.4051	1.2083	0.3618	1.4003	0.5307
			(-71.23%)	(63.45%)	(-34.26%)	(-34.26%)	(21.15%)	(-34.03%)
15	3.3201	0.0000	0.0912	1.2779	1.4759	0.4418	1.7103	0.8094
			(-59.49%)	(50.32%)	(-34.26%)	(-34.26%)	(21.15%)	(-26.30%)

Table 4.3: Summary for the final steady-state when government provides compulsory education $\underline{e} = 0.0512$ and finances its budget with tax on labour income

	Y	K	L	r	w	τ^k	$\underline{e}$	
	1.6047	0.0929	1.3163	4.7604	0.8127	0.0860	0.0512	
	(25.79%)	(-16.91%)	(54.78%)	(69.72%)	(-18.72%)			
	$\bar{h}$	tot.pop.	$\bar{e}$	$\bar{n}$	$\bar{c}$	$\bar{s}$	$\bar{d}$	$\bar{u}$
	1.0224	1.4203×10^8	0.0000	1.6185	0.5618	0.1682	0.4969	-0.7299
	(2.24%)		(-100%)	(61.84%)	(-16.91%)	(-16.91%)	(16.85%)	(2.70%)
i	h_i	pop.share	e_i	n_i	c_i	s_i	d_i	u_i
1	0.2019	0.0000	0.0000	1.6189	0.1109	0.0332	0.0981	-3.0945
			—	(0%)	(-18.73%)	(-18.73%)	(14.30%)	(-5.72%)
2	0.2466	0.0000	0.0000	1.6189	0.1355	0.0406	0.1198	-2.7987
			—	(0%)	(-18.73%)	(-18.73%)	(14.30%)	(-6.36%)
3	0.3012	0.0000	0.0000	1.6189	0.1655	0.0496	0.1464	-2.5029
			—	(0%)	(-18.73%)	(-18.73%)	(14.30%)	(-7.17%)
4	0.3679	0.0000	0.0000	1.6189	0.2022	0.0605	0.1788	-2.2071
			(-100%)	(13.74%)	(-18.73%)	(-18.73%)	(14.30%)	(-6.99%)
5	0.4493	0.0000	0.0000	1.6189	0.2469	0.0739	0.2184	-1.9114
			(-100%)	(29.37%)	(-18.73%)	(-18.73%)	(14.30%)	(-6.77%)
6	0.5488	0.0054	0.0000	1.6189	0.3016	0.0903	0.2667	-1.6156
			(-100%)	(42.18%)	(-18.73%)	(-18.73%)	(14.30%)	(-6.89%)
7	0.6703	0.0736	0.0000	1.6189	0.3684	0.1103	0.3258	-1.3198
			(-100%)	(52.66%)	(-18.73%)	(-18.73%)	(14.30%)	(-7.44%)
8	0.8187	0.2439	0.0000	1.6189	0.4499	0.1347	0.3979	-1.0240
			(-100%)	(61.24%)	(-18.73%)	(-18.73%)	(14.30%)	(-8.66%)
9	1.0000	0.3543	0.0000	1.6189	0.5495	0.1645	0.4860	-0.7282
			(-100%)	(68.27%)	(-18.73%)	(-18.73%)	(14.30%)	(-11.30%)
10	1.2214	0.2439	0.0000	1.6189	0.6712	0.2009	0.5936	-0.4324
			(-100%)	(74.02%)	(-18.73%)	(-18.73%)	(14.30%)	(-18.62%)
11	1.4918	0.0721	0.0000	1.6189	0.8198	0.2454	0.7250	-0.1366
			(-100%)	(78.73%)	(-18.73%)	(-18.73%)	(14.30%)	(-85.77%)
12	1.8221	0.0050	0.0074	1.5336	1.0013	0.2998	0.8855	0.1494
			(-93.41%)	(72.97%)	(-18.73%)	(-18.73%)	(14.30%)	(-31.58%)
13	2.2255	0.0000	0.0370	1.3196	1.2230	0.3661	1.0816	0.4182
			(-74.13%)	(51.40%)	(-18.73%)	(-18.73%)	(14.30%)	(-18.17%)
14	2.7182	0.0000	0.0732	1.1843	1.4937	0.4472	1.3211	0.6946
			(-59.36%)	(37.77%)	(-18.73%)	(-18.73%)	(14.30%)	(-13.65%)
15	3.3201	0.0000	0.1173	1.0926	1.8245	0.5462	1.6136	0.9759
			(-47.90%)	(28.53%)	(-18.73%)	(-18.73%)	(14.30%)	(-11.14%)

Table 4.4: Summary for the final steady-state when government provides compulsory education $\underline{e} = 0.0512$ and finances its budget with tax on capital income

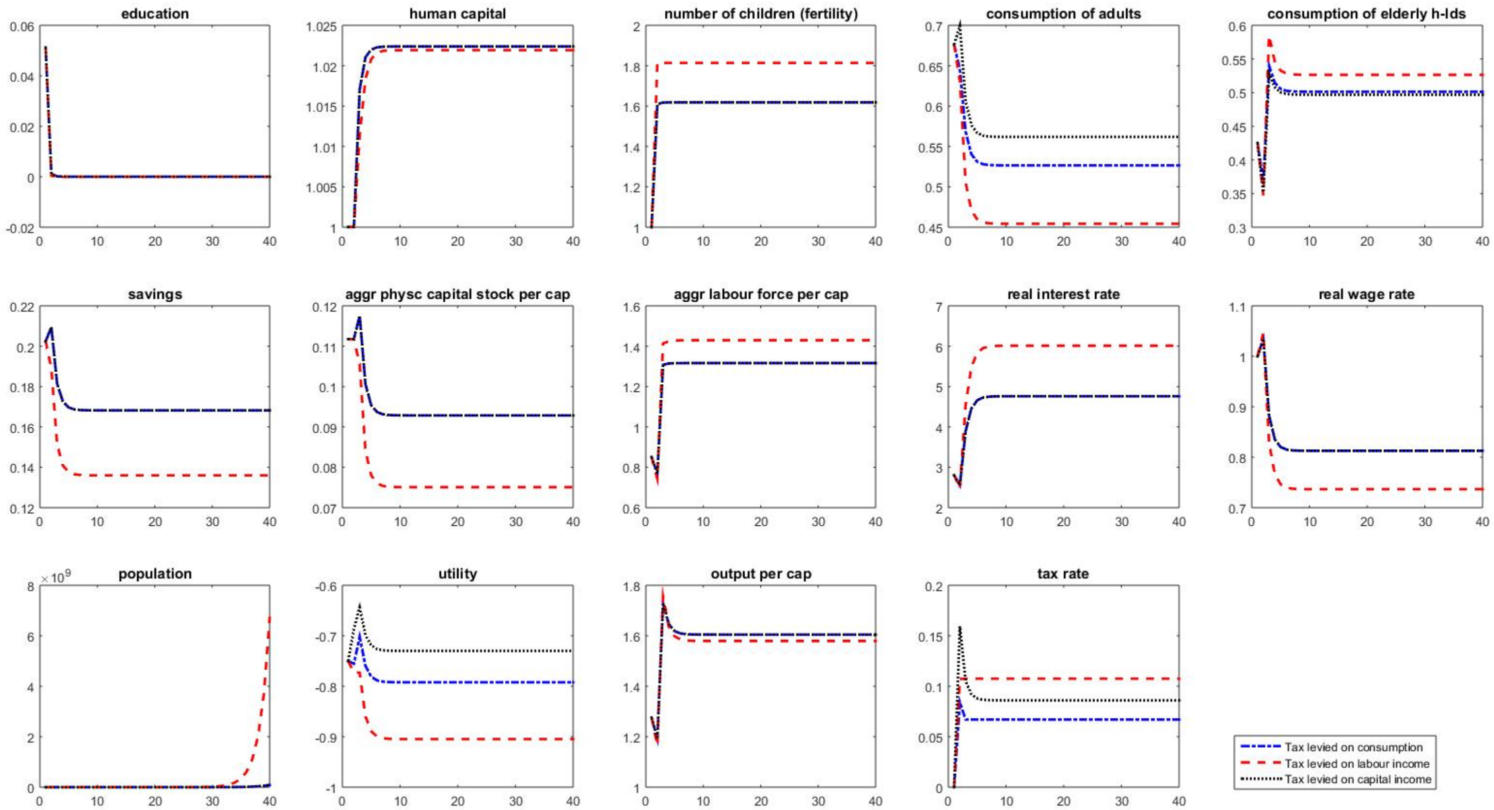

Figure 4.1: Transition path of the model economy with idiosyncratic shocks to the human capital from the perspective of the average household. Government provides compulsory education $\underline{e} = 0.0512$.

Note: transition path with tax on consumption is depicted by blue dash-dotted line; transition path with tax on labour income is presented by red dashed line; transition path with capital income tax is shown by black dotted line

According to our results, at the first period of compulsory education provision by the government, given the structure of population for adult households and the population size[26], together with individual economic choices at the second period of analysis, the government sets the tax rates to be equal to 8.43%, 10.72% and 15.94% when it balances its budget with tax on consumption, labour income and capital income, respectively.

Based on the analytical results for the model economy for deterministic environment[27], the presence of the compulsory education has shown to reduce incentives for private education provision. As a result, with compulsory education in place we observe that parents from majority of ability groups decide to provide their children with zero education and they completely rely on the government for schooling[28]. Consequently, at the first period of compulsory education, the average level of private education provision closely reaches zero level.

From the perspective of human capital accumulation and total level of education attainment, the compulsory education program which supplies every child with average education attainment level from the original steady-state benefits children from lower ability groups. On the other hand, however, the households from higher ability groups who previously individually provided their children with education above the average level, now experience disincentive for private education provision, and therefore, children from these ability groups receive the total level of education which is lower than at the original steady-state. For instance, the total level of education attainment by the top four ability groups[29] decreases by 45.40%, 35.80%, 28.45% and 22.75% when government finances its budget with tax on consumption or capital income. These values fall by 54.63%, 48.30%, 40.59% and 34.60%, respectively, when government implements the labour income taxation instead, due to further disincentives that this tax option creates for private education provision.

With decrease in the private education provision across ability groups, we observe sizeable increase in fertility decisions both at individual and average levels, which takes place due to parental ‘quality-quantity’ trade-off for children and unchanged structure of the population

[26] Both of which have been determined by decisions made at original steady-state before government intervention

[27] Please refer to the appendix A1 section

[28] Exceptions are the members of the ability groups 12, 13, 14 and 15

[29] I.e.: $i = 12, 13, 14, 15$

from the original steady-state. According to our results, when government implements tax on consumption or capital income, the fertility levels for individual groups who reduce their private education provision to zero become equal to the fertility choices of adult households from bottom three ability groups at original steady-state. For the adult households who still provide their children with additional private education, fertility decisions rise as well. Namely, the members of twelfth, thirteenth, fourteenth and fifteenth ability groups increase their childbearing decisions by 69.65%, 49.34%, 36.39% and 27.55%, respectively. Overall, according to our results, these changes at the individual level produce an increase in the average fertility level of 60.94%. For the labour income tax, which provides additional disincentives for private education attainment, the individual fertility choices become even larger. For instance, for households from the first, fourth, seventh, tenth, thirteenth and fifteenth ability groups the fertility levels increase by 12.00%, 27.39%, 70.98%, 94.91%, 80.45% and 48.94%, respectively, and average level for childbearing in the economy becomes 80.95% larger than at the original steady-state.

Given a prominent decrease in private education provisions[30], combined with the presence of compulsory education and sizeable increase in fertility choices[31], given the structure of population of adult households that is identical to the original steady-state, the size of the effective labour force per adult household is found to decrease by 9.70% when government uses tax on consumption or capital income, whereas with labour income tax, it diminishes by 12.55%[32]. The level of physical capital stock per adult household, however, remains unchanged, since it is determined by the savings made at the original steady-state.

With decrease in effective labour force and absence of change in the physical capital stock, the marginal productivity of each factor input adjust accordingly. Based on our results, the marginal productivity of effective labour force, and therefore, the real wage rate increases by 3.46% when government uses tax on consumption or capital income, and it rises by 4.57% with

[30]Which decreases a number of teachers required, and therefore, increases the number of adult household for participation in production sector

[31]Both of which increases a number of teachers required and, therefore, reduces number of adult households available for participation in production sector. Furthermore, increase in fertility levels diminishes the time that adult households have for labour market participation

[32]With the labour income tax we observed a larger increase in fertility choices, which would increase the number of teachers required and decrease the effective labour force per adult household by greater level in comparison to the other tax options

labour income tax in place. The marginal productivity of physical capital stock net of the depreciation rate, and therefore, the real interest rate is found to fall by 8.92% when a considered compulsory education program is financed with taxing consumption or capital income, and under the labour income tax, the factor price for physical capital stock diminishes by 11.60%.

As a result of all these changes, the consumption of adult and elderly households, both at individual and average level, decrease by 4.59% and 13.84%, respectively, when government finances its budget with tax on consumption. The decision to save, however, is found to increase by 3.46% under this tax option. With presence of tax on labour income, our results suggest that consumption and savings of adults decrease by 6.64%, whereas, the level of consumption for elderly households diminishes by 18.36%. Lastly, when government taxes capital income, consumption and savings of adult households increase by 3.46%, while the consumption of old-age households decreases by 17.28% both at individual and the average level.

Consequently, for the first period with compulsory education we observe that level of welfare reduces by 0.70% and 2.93% when government implements tax on consumption and labour income, respectively. With the capital income tax in place, however, the welfare becomes 8.47% larger than at original steady-state. Overall, we observe a relatively larger welfare loss experienced by households from lower ability group than by ones from higher ability groups. Finally, our results indicate that the level of output produced per adult household becomes 6.58% smaller than at original steady-state when government implements tax on consumption and capital income, and with labour income tax in place this indicator diminishes by 8.55%.

Moving our discussion forward to the third period of analysis, we observe a major structural change in the composition of population for adult households. This change takes place because the adults at the third period of analysis are the first generation who have received compulsory education from the government which allows them to change their ability group based on the realised level of human capital. Both, adult households from higher and lower ability levels received the total amount of education which is closer to one another than at original steady-state. As a result, we observe that entire population of adult households now consists of ability groups that are in the centre of the original distribution. In addition, due to the past period increase in childbearing decisions the population of adult households increases by 60.95% when government finances its budget with tax on consumption and capital income, and with the labour income tax in place this level becomes 80.95% larger than at original equilibrium.

Given this change in the composition and size of the population, together with the economic decisions that take place at this third period of analysis, the government balances its budget with the tax rate on consumption of 6.70%. If government implements the tax on labour or capital income instead, these tax rates are set at a rate of 10.75% and 10.50%, respectively.

As a result of compulsory education provided in the previous period and resulted structural change in the composition of the population of adult households in the second period of government presence, we observe an increase in the average level of human capital by 1.70% when government implements the tax on consumption or capital income, while with the labour income tax the average human capital level is found to increase by 1.13%. As we have seen from the analytical solution for the deterministic environment[33], an increase in the average human capital level diminishes the relative level of human capital for households in every ability group, which reduces the incentives for private education provision further. As a result, at the second period of compulsory education provision by the government, the private education provision by twelfth, thirteenths, fourteenth and fifteenth declines by 21.95%, 6.87%, 4.40% and 3.40%, and it becomes 92.79%, 73.53%, 58.78% and 47.34% below original steady-state, when government utilises tax on consumption or capital income. With the tax on labour income, the private education provision by these ability groups reduce by 100%, 86.07%, 42.94% and 30.45%, and it becomes 100%, 85.28%, 70.19% and 58.48% lower than at the original steady-state. For the remainder of the ability groups, the choice for private education provision continues to be at zero level.

Given these changes in the private education provision by higher ability households, we observe increase in the fertility choices of these four ability groups by 1.48%, 1.05%, 0.77% and 0.58%, that becomes 72.16%, 50.91%, 37.44% and 28.29% larger than at original steady-state, when government implements tax on consumption or capital income. With the labour income tax in use, the fertility choices of households in twelfth, thirteenth, fourteenth and

[33]Please refer to appendix A1 section

fifteenth ability groups increase by 0.04%, 0.90%, 0.66% and 0.50%, that is 104.58%, 82.08%, 62.52% and 49.69% above the original equilibrium. With these changes in fertility choices for higher ability households, and with fertility levels for lower ability groups being identical to the previous generation, our results indicate an increase in the average level of fertility at the third period of analysis by 0.49% which becomes 61.74% above original steady-state when the government uses tax on consumption or capital income to finance the compulsory education program. With the labour income tax in use instead, the average fertility level increases by 0.23% and is 81.37% larger than at the first period of analysis.

With structural changes in composition and the size of the population, combined with observed dynamics for private education provision and fertility decisions, our results indicate that at the third period of analysis the size of the effective labour force per adult households increases by 70.24% and is 53.72% above original steady-state, when tax on consumption or capital income is in place. With the labour income tax in use, the level of effective labour force per adult households rises by 90.07% and it becomes 66.21% larger than at original steady-state.

Following the dynamics of household savings observed for the past period, our results suggest that when government implements the tax on consumption or capital income, the physical capital stock per adult households rises and becomes 5.22% above original steady-state. With the labour income tax, however, which led to a decrease in decisions to save of the previous generation of adult households, the level of physical capital stock per adult households at the third period of analysis is found to reduce by 5.58%.

Given these changes in the factor inputs, we observe that the real wage rate at the second period of the government presence decreases by 14.82% and is 11.87% lower than at original steady-state, while the real interest rate increases by 52.62% and is 39.00% above the original steady-state when government uses tax on consumption or capital income. With the labour income tax, the real wage rate falls by 20.80% and it becomes 17.18% below of original equilibrium, when the real interest rate increases by 83.40% and is 62.12% larger than at original steady-state.

Consequently, for the third period of analysis we observe that consumption of adult households across all ability groups decreases by 13.44%, 20.83% and 14.82%, that becomes 17.41%, 26.08% and 11.87% below of original steady-state, when government finances compulsory education provision with tax on consumption, labour income and capital income, respectively. Due to the change in the structure of population and observed changes in consumption of adult households at the individual level, the consumption of adult households at the average level decreases by 11.96%, 19.94% and 13.37%, and it becomes 16.00%, 25.25% and 10.37% lower than at original equilibrium, when tax on consumption, labour income and capital income is implemented, respectively. For the saving of adult households, at the individual level across all ability groups we observe that it decreases by 14.82% and becomes 11.87% lower than original steady-state; while at the average level, the saving of adult households reduces by 13.37% and is 10.37% below the original equilibrium, when government taxes consumption or capital income. With the labour income tax in place instead, saving of adult households across individual ability groups reduces by 20.83% and becomes 26.08% lower than before government presence. At the average level, this indicator diminishes by 19.94% and it is 25.25% lower than at original steady-state. Lastly, due to a sizable increase in the real interest rate, we observe that at the second period of compulsory education provided by the government, the consumption of the elderly households across individual ability groups increases by 44.90%, 66.66% and 47.58%, and becomes 24.84%, 36.07% and 22.08% above the original steady-state, when government finances its budget with the tax on consumption, labour income and capital income, respectively. At the average level, consumption of the elderly households is found to increase by 47.36%, 68.54% and 50.09%, which is 26.97%, 37.61% and 24.16% higher than at original equilibrium, when tax on consumption, labour income and capital income is in use, respectively.

To conclude, at the second period of government provision of compulsory education, we observe that the level of welfare increases by 7.12% and becomes 6.47% larger than the original steady-state when government implement tax on consumption. With the tax on capital income, the level of welfare rises by 6.29% and is 14.23% larger than before government has entered. Under the labour income tax, however, the welfare is found to reduce by 0.14% and it becomes 3.07% smaller than at initial steady-state. Lastly, according to our results, the level of output produced per economically active member of population increases by 45.01% and becomes 35.48% above steady-state when government uses tax on consumption or capital income; while with the labour income tax in place, it increases by 50.54% and is 37.66% above original steady-state.

With behaviour for majority of variables being consistent with one discussed for the third period of analysis[34], the economy reaches the second steady-states where government provides compulsory education and it taxes either consumption, labour income or capital income. At this second equilibrium, the government imposes tax rate on consumption of 6.70%, on labour income of 10.75% and on capital income of 8.60% to keep its budget constraint balanced.

As a result of this policy and consequential change in the individual human capital levels, the population of adult households follows distribution depicted by figure 4.2 and 4.3 below, and consists only of ability groups that are in the centre of original distribution[35], with the households who receive negative idiosyncratic shocks to the human capital being a member of lower ability groups, and for ones who are the subject of positive idiosyncratic shocks to the human capital being a part of higher ability groups. Overall, our results suggest that the average level of human capital increases 2.24% above original steady-state when the economy reaches the final steady-state with the compulsory education which is financed by the tax on consumption or capital income. With the labour income tax, the final steady-state is characterised by the average human capital level that is 2.19% above the initial equilibrium.

[34]The exceptions are: (1) the physical capital stock per economically active member of population which follows the aggregate level of saving with the lag of one period, and which, therefore, begins to decrease after third period of policy experiment; and (2) the level of welfare – both for the case when government taxes either consumption or capital income

[35]For the policy experiment considered here, these groups are indexed as sixth, seventh, eighth, ninth, tenth, eleventh and twelfth

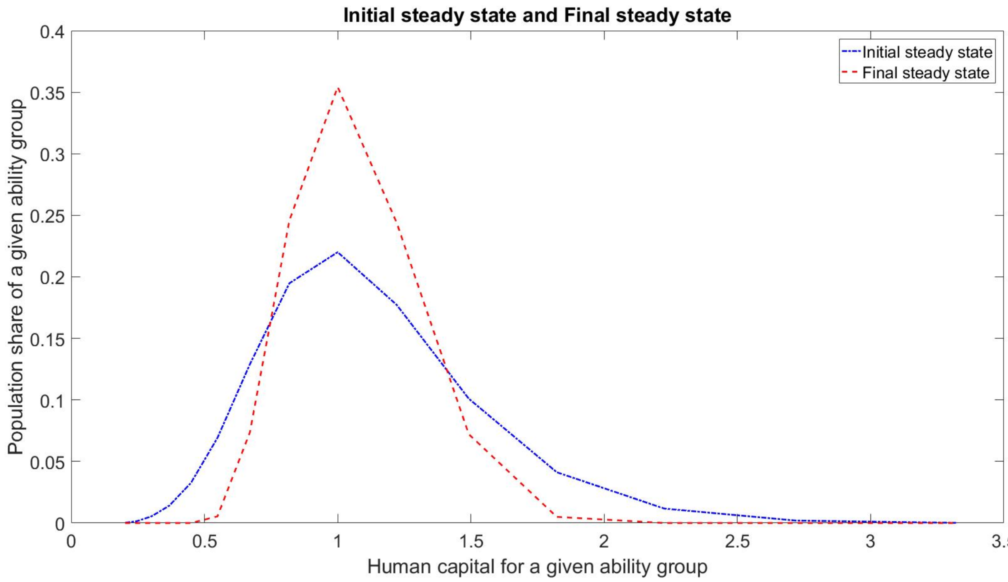

Figure 4.2: Distribution of the population at initial steady-state without government presence (blue dash-dotted line), at the second steady-state where government provides compulsory education $\underline{e} = 0.0512 = \bar{e}_1$ and taxes either consumption or capital income (red dashed line)

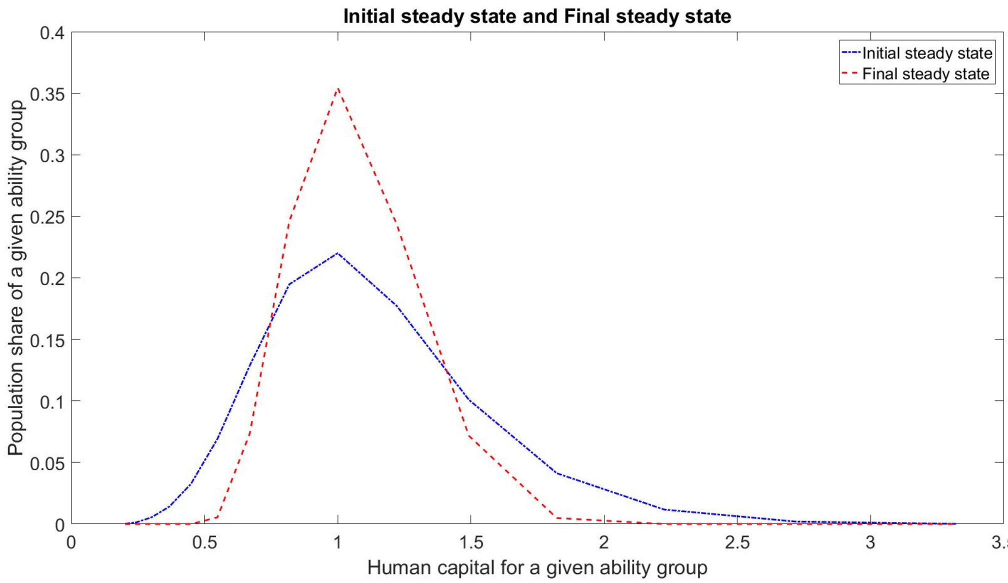

Figure 4.3: Distribution of the population at initial steady-state without government presence (blue dash-dotted line), at the second steady-state where government provides compulsory education $\underline{e} = 0.0512 = \bar{e}_1$ and taxes labour income (red dashed line)

Due to a sizeable increase in the fertility decision of households across entire population, which occurs as a result of decrease in private education provision, the size of the model population sky-rockets. This increase even more pronounced for the case when government taxes labour income, which, as we have discussed, creates further disincentives for private education attainment, which, therefore, increases fertility levels further.

At these final steady-states, we observe that almost entire population decides to fully rely on the government to educate their children, which brings the level of average private education provision to zero. Due the compulsory education program, however, the size of the aggregate effective labour force per adult household becomes 54.78% larger than at original steady-state when government uses tax on consumption or capital income; and it increases by 68.07% when the labour income tax is in place. The size of the physical capital stock per adult household, however, is found to fall by 16.91% with tax on consumption or capital income, and by 32.82% when labour income tax is in use. As a result, we observe that at these final steady-states, the level of real wage rate reduces by 18.73% from original equilibrium, when government finances its budget with the tax on consumption or capital income. With the labour income tax implemented instead, the real wage rate becomes 26.34% below original steady-state. As a result, given the tax option in use, the individual choices of adult households to consume and their decisions to save reduce below original equilibrium levels, and, therefore, the average level of consumption of adults fall by 22.13%, 32.82% and 16.91%, when average decision to save reduce by 16.91%, 32.82% and 16.91% below original steady-state when government uses tax on consumption, labour income and capital income, respectively. For the real interest rate, we observe the opposite, and according to our results, this factor price increases by 69.72% when government taxes consumption or capital income, and by 114.33% when labour income tax is in place. This larger value of the real interest rate at these final steady-states contributes to a growth in the individual levels of consumption of elderly households given the tax option in place, and therefore, the average level of consumption of households at the old age becomes 17.89%, 23.81% and 16.85% above the original steady-state. In conclusion, given all the changes from above, we observe that the level of welfare decreases by 5.59% and 20.59% below original steady-state when government taxes consumption and labour income, respectively; while with the tax on capital income, the level of welfare becomes 2.70% above the original steady-state. Lastly, we observe that at these final steady-states, the level of output per adult household becomes 25.79% above original steady-state when government utilises the tax on consumption or the capital income to finance its budget; whereas, with the labour income tax in place, the output per economically active member of population is found to rise 23.81% above original equilibrium.

4.4.2 Childcare

Continuing our analysis, we consider the influence of the government policy which provides the childcare for fifty percent of the time which adult households have to contribute to raise children[36]. Identically to the case of compulsory education, we assume that government has three tax options which it can use to finance its budget. These options include imposing the flat tax rate either on consumption, labour income or capital income.

[36] I.e.: $\omega = 0.5$

	Y	K	L	r	w	τ^c	ω	
	0.8253	0.0482	0.6742	4.7131	0.8161	0.0793	0.5	
	(-35.30%)	(-56.91%)	(-20.72%)	(68.03%)	(-18.39%)			
	$\bar{h}$	tot.pop.	$\bar{e}$	$\bar{n}$	$\bar{c}$	$\bar{s}$	$\bar{d}$	$\bar{u}$
	0.5280	1.0191×10^{16}	0.0144	2.5750	0.2699	0.0872	0.2549	-1.7869
	(-47.20%)		(-71.90%)	(157.50%)	(-60.08%)	(-56.91%)	(-40.06%)	(-138.19%)
i	h_i	pop.share	e_i	n_i	c_i	s_i	d_i	u_i
1	0.2019	0.0096	0.0000	3.2378	0.1032	0.0334	0.0975	-3.0442
			—	(100%)	(-24.39%)	(-18.39%)	(13.54%)	(-4.00%)
2	0.2466	0.0503	0.0000	3.2378	0.1261	0.0407	0.1191	-2.7484
			—	(100%)	(-24.39%)	(-18.39%)	(13.54%)	(-4.45%)
3	0.3012	0.1189	0.0000	3.2378	0.1540	0.0498	0.1454	-2.4526
			—	(100%)	(-24.39%)	(-18.39%)	(13.54%)	(-5.01%)
4	0.3679	0.1738	0.0023	2.9728	0.1881	0.0608	0.1776	-2.1722
			(-38.56%)	(108.86%)	(-24.39%)	(-18.39%)	(13.54%)	(-5.30%)
5	0.4493	0.1885	0.0081	2.5815	0.2297	0.0742	0.2169	-1.9017
			(-18.04%)	(106.30%)	(-24.39%)	(-18.39%)	(13.54%)	(-6.23%)
6	0.5488	0.1713	0.0152	2.3303	0.2806	0.0907	0.2650	-1.6243
			(-12.56%)	(104.66%)	(-24.39%)	(-18.39%)	(13.54%)	(-7.47%)
7	0.6703	0.1335	0.0238	2.1584	0.3427	0.1107	0.3236	-1.3423
			(-10.06%)	(103.53%)	(-24.39%)	(-18.39%)	(13.54%)	(-9.27%)
8	0.8187	0.0861	0.0343	2.0354	0.4186	0.1353	0.3953	-1.0570
			(-8.65%)	(102.73%)	(-24.39%)	(-18.39%)	(13.54%)	(-12.16%)
9	1.0000	0.0444	0.0472	1.9447	0.5113	0.1652	0.4828	-0.7694
			(-7.76%)	(102.13%)	(-24.39%)	(-18.39%)	(13.54%)	(-17.60%)
10	1.2214	0.0176	0.0629	1.8763	0.6245	0.2018	0.5897	-0.4801
			(-7.16%)	(101.68%)	(-24.39%)	(-18.39%)	(13.54%)	(-31.70%)
11	1.4918	0.0050	0.0822	1.8237	0.7627	0.2464	0.7202	-0.1894
			(-6.73%)	(101.34%)	(-24.39%)	(-18.39%)	(13.54%)	(-157.53%)
12	1.8221	0.0009	0.1056	1.7828	0.9316	0.3010	0.8797	0.1023
			(-6.42%)	(101.07%)	(-24.39%)	(-18.39%)	(13.54%)	(-53.16%)
13	2.2255	0.0001	0.1343	1.7507	1.1378	0.3676	1.0744	0.3948
			(-6.18%)	(100.86%)	(-24.39%)	(-18.39%)	(13.54%)	(-22.75%)
14	2.7182	0.0000	0.1693	1.7252	1.3897	0.4490	1.3123	0.6880
			(-6.00%)	(100.70%)	(-24.39%)	(-18.39%)	(13.54%)	(-14.47%)
15	3.3201	0.0000	0.2120	1.7049	1.6974	0.5485	1.6028	0.9816
			(-5.86%)	(100.56%)	(-24.39%)	(-18.39%)	(13.54%)	(-10.61%)

Table 4.5: Summary for the final steady-state when government provides childcare $\omega = 0.5$ and finances its budget with tax on consumption

	Y	K	L	r	w	τ^l	ω	
	0.6764	0.0300	0.6336	6.5120	0.7117	0.1650	0.5	
	(-46.97%)	(-73.14%)	(-25.49%)	(132.17%)	(-28.83%)			
	$\bar{h}$	tot.pop.	$\bar{e}$	$\bar{n}$	$\bar{c}$	$\bar{s}$	$\bar{d}$	$\bar{u}$
	0.4496	1.4027×10^{20}	0.0090	3.2880	0.1816	0.0544	0.2255	-2.1912
	(-55.04%)		(-82.50%)	(228.79%)	(-73.14%)	(-73.14%)	(-46.97%)	(-192.07%)
i	h_i	pop.share	e_i	n_i	c_i	s_i	d_i	u_i
1	0.2019	0.0196	0.0000	3.8570	0.0816	0.0244	0.1013	-3.2369
			—	(138.25%)	(-40.26%)	(-40.26%)	(17.95%)	(-10.58%)
2	0.2466	0.0908	0.0000	3.8570	0.0996	0.0298	0.1237	-2.9411
			—	(138.25%)	(-40.26%)	(-40.26%)	(17.25%)	(-11.77%)
3	0.3012	0.1839	0.0000	3.8570	0.1217	0.0364	0.1511	-2.6454
			—	(138.25%)	(-40.26%)	(-40.26%)	(17.25%)	(-13.26%)
4	0.3679	0.2194	0.0020	3.5843	0.1486	0.0445	0.1845	-2.3627
			(-48.30%)	(151.82%)	(-40.26%)	(-40.26%)	(17.25%)	(-14.53%)
5	0.4493	0.1873	0.0077	3.1016	0.1815	0.0543	0.2254	-2.0929
			(-22.59%)	(147.87%)	(-40.26%)	(-40.26%)	(17.25%)	(-16.91%)
6	0.5488	0.1355	0.0146	2.7936	0.2217	0.0664	0.2753	-1.8159
			(-15.73%)	(145.34%)	(-40.26%)	(-40.26%)	(17.25%)	(-20.15%)
7	0.6703	0.0871	0.0231	2.5835	0.2708	0.0811	0.3362	-1.5342
			(-12.60%)	(143.62%)	(-40.26%)	(-40.26%)	(17.25%)	(-24.89%)
8	0.8187	0.0472	0.0335	2.4337	0.3307	0.0990	0.4106	-1.2491
			(-10.84%)	(142.39%)	(-40.26%)	(-40.26%)	(17.25%)	(-32.54%)
9	1.0000	0.0205	0.0462	2.3233	0.4040	0.1209	0.5015	-0.9617
			(-9.72%)	(141.49%)	(-40.26%)	(-40.26%)	(17.25%)	(-46.98%)
10	1.2214	0.0068	0.0617	2.2402	0.4934	0.1477	0.6126	-0.6724
			(-8.96%)	(140.80%)	(-40.26%)	(-40.26%)	(17.25%)	(-84.45%)
11	1.4918	0.0015	0.0807	2.1764	0.6026	0.1804	0.7482	-0.3818
			(-8.43%)	(140.28%)	(-40.26%)	(-40.26%)	(17.25%)	(-419.12%)
12	1.8221	0.0002	0.1038	2.1269	0.7361	0.2204	0.9139	-0.0902
			(-8.04%)	(139.87%)	(-40.26%)	(-40.26%)	(17.25%)	(-141.28%)
13	2.2255	0.0000	0.1320	2.0879	0.8990	0.2691	1.1162	0.2023
			(-7.74%)	(139.56%)	(-40.26%)	(-40.26%)	(17.25%)	(-60.42%)
14	2.7182	0.0000	0.1665	2.0571	1.0981	0.3287	1.3633	0.4954
			(-7.51%)	(139.30%)	(-40.26%)	(-40.26%)	(17.25%)	(-38.41%)
15	3.3201	0.0000	0.2087	2.0325	1.3412	0.4015	1.6652	0.7891
			(-7.34%)	(139.10%)	(-40.26%)	(-40.26%)	(17.25%)	(-28.15%)

Table 4.6: Summary for the final steady-state when government provides childcare $\omega = 0.5$ and finances its budget with tax on labour income

	Y	K	L	r	w	τ^k	ω	
	0.8253	0.0482	0.6742	4.7131	0.8161	0.1012	0.5	
	(-35.30%)	(-56.91%)	(-20.72%)	(68.03%)	(-18.39%)			
	$\bar{h}$	tot.pop.	$\bar{e}$	$\bar{n}$	$\bar{c}$	$\bar{s}$	$\bar{d}$	$\bar{u}$
	0.5280	1.0191×10^{16}	0.0144	2.5750	0.2913	0.0872	0.2521	-1.7139
	(-47.20%)		(-71.90%)	(157.50%)	(-56.91%)	(-56.91%)	(-40.71%)	(-128.46%)
i	h_i	pop.share	e_i	n_i	c_i	s_i	d_i	u_i
1	0.2019	0.0096	0.0000	3.2378	0.1114	0.0334	0.0964	-2.9712
			—	(100%)	(-18.39%)	(-18.39%)	(12.31%)	(-1.50%)
2	0.2466	0.0503	0.0000	3.2378	0.1361	0.0407	0.1178	-2.6754
			—	(100%)	(-18.39%)	(-18.39%)	(12.31%)	(-1.67%)
3	0.3012	0.1189	0.0000	3.2378	0.1662	0.0498	0.1438	-2.3796
			—	(100%)	(-18.39%)	(-18.39%)	(12.31%)	(-1.89%)
4	0.3679	0.1738	0.0023	2.9728	0.2030	0.0608	0.1757	-2.0991
			(-38.56%)	(108.86%)	(-18.39%)	(-18.39%)	(12.31%)	(-1.76%)
5	0.4493	0.1885	0.0081	2.5815	0.2479	0.0742	0.2146	-1.8287
			(-18.04%)	(106.30%)	(-18.39%)	(-18.39%)	(12.31%)	(-2.15%)
6	0.5488	0.1713	0.0152	2.3303	0.3028	0.0907	0.2621	-1.5513
			(-12.56%)	(104.66%)	(-18.39%)	(-18.39%)	(12.31%)	(-2.64%)
7	0.6703	0.1335	0.0238	2.1584	0.3699	0.1107	0.3201	-1.2693
			(-10.06%)	(103.53%)	(-18.39%)	(-18.39%)	(12.31%)	(-3.33%)
8	0.8187	0.0861	0.0343	2.0354	0.4518	0.1353	0.3910	-0.9840
			(-8.65%)	(102.73%)	(-18.39%)	(-18.39%)	(12.31%)	(-4.41%)
9	1.0000	0.0444	0.0472	1.9447	0.5518	0.1652	0.4775	-0.6964
			(-7.76%)	(102.13%)	(-18.39%)	(-18.39%)	(12.31%)	(-6.44%)
10	1.2214	0.0176	0.0629	1.8763	0.6740	0.2018	0.5833	-0.4071
			(-7.16%)	(101.68%)	(-18.39%)	(-18.39%)	(12.31%)	(-11.67%)
11	1.4918	0.0050	0.0822	1.8237	0.8232	0.2464	0.7124	-0.1164
			(-6.73%)	(101.34%)	(-18.39%)	(-18.39%)	(12.31%)	(-58.24%)
12	1.8221	0.0009	0.1056	1.7828	1.0054	0.3010	0.8701	0.1753
			(-6.42%)	(101.07%)	(-18.39%)	(-18.39%)	(12.31%)	(-19.72%)
13	2.2255	0.0001	0.1343	1.7507	1.2280	0.3676	1.0628	0.4679
			(-6.18%)	(100.86%)	(-18.39%)	(-18.39%)	(12.31%)	(-8.46%)
14	2.7182	0.0000	0.1693	1.7252	1.4999	0.4490	1.2981	0.7610
			(-6.00%)	(100.70%)	(-18.39%)	(-18.39%)	(12.31%)	(-5.40%)
15	3.3201	0.0000	0.2120	1.7049	1.8320	0.5485	1.5854	1.0547
			(-5.86%)	(100.56%)	(-18.39%)	(-18.39%)	(12.31%)	(-3.96%)

Table 4.7: Summary for the final steady-state when government provides childcare $\omega = 0.5$ and finances its budget with tax on capital income

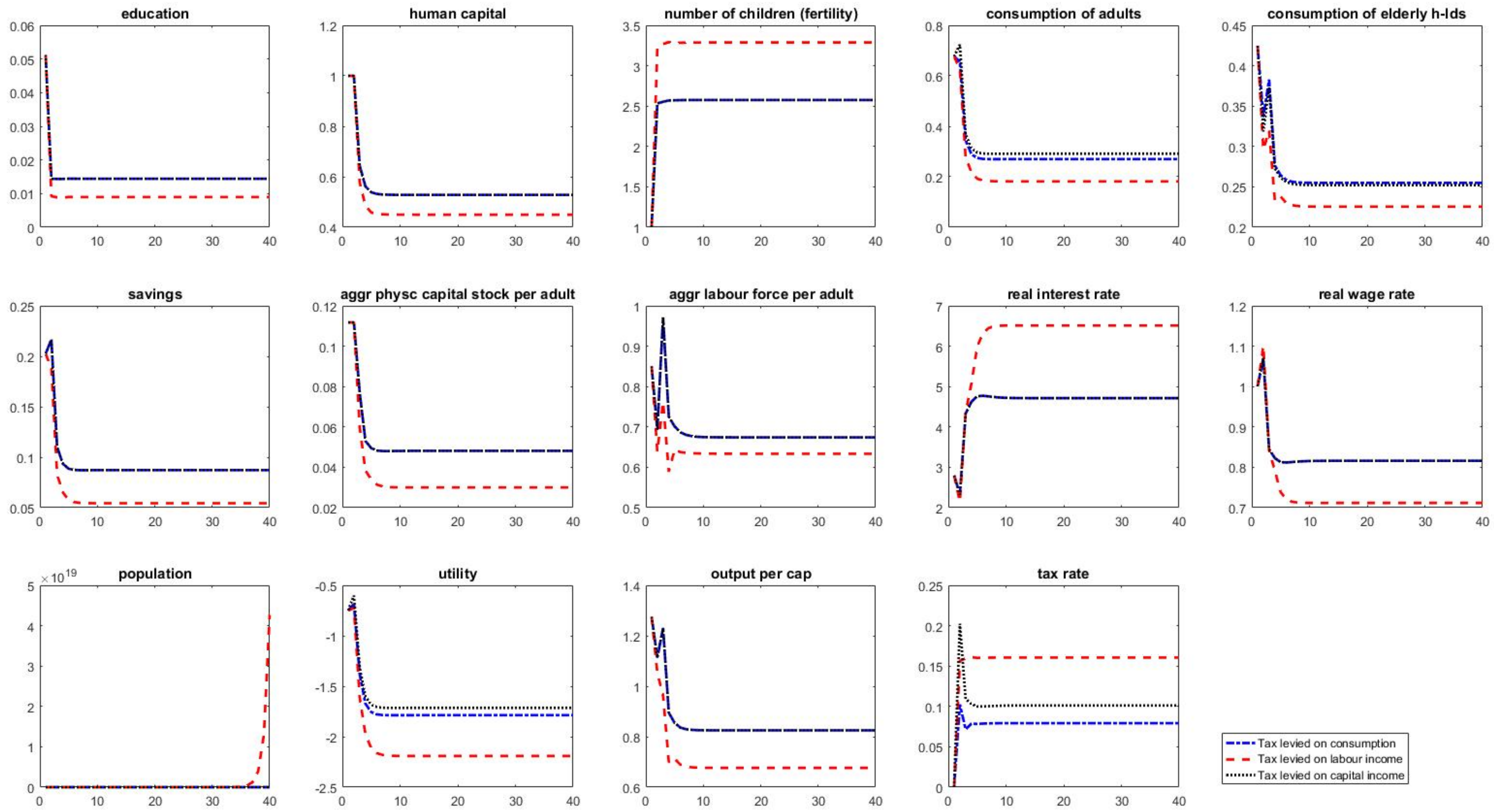

Figure 4.4: Transition path of the model economy with idiosyncratic shocks to the human capital from the perspective of the average household. Government provides childcare $\omega = 0.5$.

Note: transition path with tax on consumption is depicted by blue dash-dotted line; transition path with tax on labour income is presented by red dashed line; transition path with capital income tax is shown by black dotted line

At the second period of our policy experiment, the government enters the model economy and it provides childcare and collects taxes. Given the size and composition of population that has been determined at the original steady-state, and given the individual economic decisions that take place during this second period of analysis, the government levies tax on consumption at a rate of 10.24% to balance its budget. If government implements labour income tax instead, the tax rate is set to be equal to 15.64%; while for the case with tax on capital income, the government sets the rate to 20.23%.

According to the analytical solution for the problem of the households[37], the childcare provides a direct positive incentive for fertility decisions, but also it creates a direct negative incentive for education provision. Furthermore, with the presence of a negative relationship between education and fertility decisions of the households, the positive effect of childcare on fertility and negative on schooling is reinforced even further. According to our results, the fertility decisions for the fourth, seventh, tenth and thirteenth ability groups increase by 127.48%, 189.90%, 135.10% and 116.63% when government implement tax on consumption or capital income; while with the labour income tax, they increase by 169.64%, 261.90%, 198.07% and 164.95%, respectively. The education provision for these ability groups decreases, however, by 100%, 94.95%, 67.55% and 58.32% when government utilises tax on consumption or capital income, respectively; and with tax on labour income, it falls by 100% for third and seventh ability group, and by 78.11% and 67.43% for tenth and thirteenth group, respectively. As a result, given the distribution of adult households which is identical to the original steady-state, the average level of fertility at the second period of analysis rises by 153.32% when government finances its budget with tax on consumption or capital income; and it growth by 221.69% when labour income tax is in use instead. Consequently, the level of average education attainment drops by 71.92% with tax on consumption or capital income; and it reduces by 82.03% when labour income tax is implemented.

Given these changes in the education provision and fertility decision, we observe the decrease in the size of effective labour force per adult household by 18.52% when government levies tax on consumption or capital income[38]. With the labour income tax, the size of this fac-

[37]Please refer to appendix A1 section

[38]The decrease in the size of the effective labour force is the net effect of (1) increase in fertility decisions which,

tor input decreases further, and it becomes 24.93% below original steady-state. The size of the physical capital stock per adult household does not change, however, since it has been formed by decisions that take place at original steady-state. As a result, we observe that real wage rate increases by 7.07% when government uses tax on consumption or capital income, and by 10.03% when labour income tax is in use instead. The real interest rate, however, decreases by 17.31% with tax on consumption or capital income present; while, with the labour income tax in the system, this factor price falls by 23.61%.

Due to these changes in factor prices and taking into consideration the presence of each tax option, we observe that at the first period of government provision of childcare, the consumption of adult households both at individual and average levels decreases by 2.88% and 7.17% when government uses tax on consumption and tax on labour income to finance its budget, respectively. With the capital income tax in place, however, the individual and average consumption of adult households increases by 7.07%. Next, our results indicate that the decisions to save both at individual and average level increase by 7.07% when government implement tax on consumption or capital income; but with the labour income tax, individual and average saving decrease by 7.17%. Lastly, our results indicate a decrease in the average and individual levels of consumption by elderly households by 20.87%, 30.32% and 25.10% with government budget being financed though tax on consumption, labour income and capital income, respectively.

As a result of all these changes that take place at the second period of analysis when government enters the model economy, the level of welfare is found to increase by 8.83%, 3.47% and 19.63%[39] when government taxes consumption, labour income and capital income, respectively. As a final observation for this second period of analysis, due to the fall in the size of effective labour force and absence of change for physical capital stock[40], we observe that

given the education attainment, population size and quality of education, would increase the number of teachers required, and therefore, decrease number of adult households who can participate in the production sector; (2) decrease in education attainment which, given the fertility decisions, population size and quality of education, decreases number of teachers required that increases number of adult households available for production sector; and (3) provision of a childcare

[39]This initial increase in level of welfare is driven by a sizeable increase in fertility levels and by the fact that distribution of adult households is identical to the original steady-state. At subsequent periods, when composition of population changes because of decrease in the education attainment which decreases the level of human capital for each individual household member, the welfare level would follow

[40]Both of these factor inputs are measured at a "per adult household" level

level of output produced per economically active member of the population reduces by 12.76% when government taxes consumption or capital income, and with the labour income tax it falls by 17.40%.

Moving our discussion to the next period of analysis, we observe a beginning of a change in the structure of the model population. Particularly, our results indicate a decrease in the population share of the households in ability groups with level of human capital larger than the average level at original steady-state. For the ability groups that are characterised by the human capital which is below the average level before government intervention, the population share increases, however. The reason for this change is twofold. First, because the lower ability groups experienced larger increase in fertility decisions at the past period, the adult population of these groups increase at a greater rate. And second, since the provision of childcare reduced incentives for education provision across the population at previous generation, the current adult households received less education than at original steady-state which diminished their individual level of human capital. Therefore, given this reduction in the individual human capital levels, the households from the higher ability groups now become a part of population with lower human capital level.

Therefore given the structure and the size of the model population[41], and individual decisions that take place at the second period of government presence, the government sets the tax rate on consumption to be equal to 7.17% to balance its budget. For the case when government utilises the labour income tax instead, the tax rate is set at 15.95%; while, if government implements tax on capital income, the tax rate is equal to 10.92%.

With the change in a composition of population, we observe that the average level of human capital for adult households decreases by 35.67% when government uses tax on consumption or capital income; and it falls by 42.27% when the labour income tax is in place instead. As a result, the level of the relative human capital of households from every ability group[42] increases at the third period of analysis. Based on the results from the analytical solution for

[41]The size for the model population is found to increase by 153.33% when government uses tax on consumption or capital income; and it rises by 221.70% from the original steady-state when the tax on labour income is in place

[42]I.e.: $x_t^i \ \forall i$

the household sector, the relative level of human capital positively influences the decision for education provision; and therefore, the level of education provided by fifth, tenth and fifteenth ability groups increases from the previous period and become 75.82%, 30.09% and 24.62% below original steady-state when government uses tax on consumption or capital income, respectively. For the case with labour income tax, the education provision for these ability groups increases and becomes 92.63%, 36.76% and 30.09% lower than original equilibrium. With this positive dynamics observed for higher ability groups, which, however, is dominated by the change in the structure of population, we find that average level of education attainment at the third period of analysis decreases by 0.21% from the previous generation and becomes 71.98% lower than the original steady-state when the government implements tax on consumption or capital income. With the labour tax, the level of average education attainment falls by 3.74% and is 82.70% below original equilibrium.

For the dynamics in the individual fertility decisions, we observe that at the second period of childcare provision by the government, the childbearing decisions by fifth, tenth and fifteenth ability groups respond to increase in the education provision at the individual level and decrease by 8.37%, 11.13% and 3.63%, and become 137.08%, 108.94% and 102.92% above original steady state, respectively, when government utilises tax on consumption or capital income. With the labour income tax, the fertility decisions of these ability groups reduce by 2.52%, 15.45% and 4.74%, and are 198.96%, 152.02% and 142.51% above original equilibrium. For the ability groups that provide their children with zero education, however, fertility decisions do not change and they remain to be 100% above the original steady-state when government uses tax on consumption or capital income; while with the tax on labour income, childbearing decisions for these ability groups are 137.96% above initial equilibrium. Due to the structural change in composition of population, however, and increase in the population share of lower ability groups that are characterised by larger fertility levels than the rest of the population, the average level of fertility is found to increase by 0.80%, and it becomes 155.36% above original steady-state, when government implements tax on consumption or capital income. With the labour income tax, the average level of fertility increases by 1.58% and is 226.78% above the original steady-state.

With these changes in size and distribution of the population, in decisions of the households for education provision and fertility, and in the average level of human capital, we observe that the size of the effective labour force per adult household increases by 40.18% and is 14.22% above the original steady-state, when government uses tax on consumption or capital income to finance its budget. With the government implementing the labour income tax instead, the size of this factor input increases by 19.35% which becomes 10.40% below original stead-state, however. On the other hand, given the dynamics in decision for saving at the past period and the present size of the population, the physical capital stock per adult household decreases by 31.13% when government uses tax on consumption or capital income to finance its budget. With the tax on labour income, the size of this second factor input falls by 46.41%. Consequently, the real wage rate decreases by 21.09% and becomes 15.52% lower than original steady-state when government taxes consumption or capital income; while, with tax on labour income, the real wage rate reduces by 23.42% and is 15.74% lower than before government presence. The real interest rate, however, increases by 86.74% and becomes 54.42% higher than at original steady-state, when tax on consumption or capital income is in place. With the labour income tax, the real interest rate rises 103.46% and becomes 55.43% larger than at initial steady-state.

As a result of these changes in factor prices and adjustment in the tax rates that take place at the third period of analysis, we observe a decrease in consumption of adult households at individual level across all ability groups by 18.83%, 23.71% and 21.09%, that becomes 21.17%, 29.18% and 15.52% below original steady-state, when government taxes consumption, labour income and capital income, respectively. Due to the change in the structure of population and increase in the population share of lower ability groups that are characterised by a lower consumption of adult households compared to the rest of the population, the average consumption of adult households decreases by 47.79%, 55.96% and 49.24% from the previous period, and it becomes 49.29%, 59.12% and 45.66% lower than original steady-state when government implements tax on consumption, labour income and capital income, respectively. Next, when the government uses tax on consumption or capital income to finance its budget, the dynamics for savings of adult households at the individual and average levels both follow the one mentioned for the case of consumption of adult households when government taxes capital income; while the dynamics for savings with the labour income tax in place follows the one discussed for the consumption of adult households with the labour income tax in the system. Lastly, the consumption of the elderly households at the individual ability levels increases by 76.89%, 86.94% and 82.50%, and becomes 39.98%, 30.27% and 36.70% above the original steady-state, when government implements tax on consumption, labour income and capital income, respectively. At the average level, however, consumption of the elderly households increases by 13.78%, 7.93% and 17.40%, and it is 9.96%, 24.79% and 12.07% below original steady-state.

In conclusion, we observe that at the second period of the government presence, the level of welfare reduces by 101.53%, 122.40% and 118.31%, which becomes 83.74%, 114.68% and 75.46% below original steady state with tax on consumption, labour income and capital income being in use, respectively. With behaviour observed for the factor inputs, our results suggest, however, that the level of output produced per economically active member of population rises by 10.61% and is 3.50% below original steady-state when tax on consumption or capital income is in place. But with the tax on labour income, this indicator decreases by 8.60% from the previous generation and is 24.51% lower than original steady-state.

With increase in education provision and decrease in fertility choices at the individual levels that we have observed for the previous generation, at the fourth period of analysis we witness a minor adjustments in the population weights of each ability group. With these changes and given the size of the model population[43] and individual decisions that take place at this third period of government presence, the tax rates for consumption, labour income and capital income adjust and set to 0.0786, 0.1610 and 0.1030, respectively.

Given the distribution of adult households for this fourth period of analysis, however, the average level of human capital decreases by 12.66% and is 43.82% below the original steady state, when government taxes consumption or capital income. With the labour income tax,

[43] With tax on consumption or capital income financing the government budget, the population size increases by 155.36% from the previous period of analysis and it becomes 546.92% larger than at original steady-state. With the labour income tax, the population increases by 226.79% and is 951.29% above initial equilibrium

the average level of human capital reduces by 16.01% which becomes 51.51% below of initial steady-state. As we have discussed above, this creates positive incentives for education provision for members of all ability groups, and as an example, our results suggest that education provision by fifth, tenth and fifteenth ability groups increases by 158.66%, 21.78% and 16.53%, and becomes 37.44%, 14.86% and 12.16% below original steady-state when government finances its budget with tax on consumption or capital income, respectively. With the labour income tax, the education decision for these three ability groups rises by 633.68%, 29.31% and 21.70%, and they are 45.92%, 18.22% and 14.91% below the initial equilibrium, respectively. For the lower ability groups, who optimally provided their children with zero education at the original steady-state, this decision remains unchanged. Given the composition of the population, however, the average level of education at this fourth period of analysis is found to reduce by 0.07% when government implements tax on consumption or capital income, which becomes 72.00% lower than the original equilibrium. With the tax on labour income, average level of education falls by 0.98% and is 82.797% below the initial steady-state.

As a consequence of the changes that we observe for education provision, our results indicate a decrease in individual fertility levels for ability groups that provide education at above-zero level. For instance, the childbearing decisions of fifth, tenth and fifteenth ability groups reduce by 9.54%, 2.48% and 0.82%, and they become 114.47%, 103.76% and 101.25% above original steady state, when government uses tax on consumption or capital income to finance its budget, respectively. With the labour income tax, the fertility choices of these households fall by 12.85%, 3.16% and 0.93%, and they are 160.56%, 144.06% and 140.26% larger than the original equilibrium. For the ability groups that provide their children with zero education, however, the fertility decisions remains identical to the previous generation. Due to the change in distribution of population, however, our results indicate that the average fertility level at the third period of the government presence increases by 0.61% and becomes 156.93% above original steady-state when government taxes consumption or capital income; while with the labour income tax, it rises by 0.77% and is 229.30% larger than at initial steady-state.

Next, we observe that the size of the effective labour force per adult household decreases by 25.36% and is 14.75% lower than the original steady-state, when tax on consumption or capital income is in place. With the labour income tax in use, the size of this factor input falls by 22.70% and becomes 30.74% lower than before government has entered the economy. This decrease in the size of the effective labour force per adult household can be primarily explained by the change in composition of the population. Overall, population experiences decrease in the human capital which diminishes the effective labour force. Additionally, the increase in the size of population does not compensate for the loss in the human capital, and the 'per adult household' value of this factor input diminishes further. For the physical capital, we observe a similar trend. Due to decrease in the savings of past generation and increase in the population size, our results suggest that the physical capital stock per adult household decreases by 31.09% and is 52.54% lower than at the original steady-state when tax on consumption or capital income is in use. With tax on labour income, this factor input shrinks by 35.93% and becomes 65.66% smaller than at initial equilibrium. Lastly, due to these changes in the factor inputs, the real wage rate decreases by 2.62% and becomes 17.74% smaller than at the original steady-state, when government taxes consumption or capital income; while with the labour income tax, the real wage rate diminishes by 6.06% and is 20.85% below the original equilibrium. The real interest rate, however, is found to increase by 6.72% which becomes 64.80% higher than at the original steady-state, when tax on consumption or capital income is implemented; and with the tax on labour income in use, the real interest rate increases by 16.38% and is 80.89% larger than before government intervention.

Given the changes that take place in the factor prices together with adjustments in the tax rates, our results indicate that the consumption of adult households at the individual level decreases by 3.25%, 6.23% and 2.62%, and becomes 23.73%, 33.60% and 17.74% lower than at the original steady state, when government implements tax on consumption, labour income and capital income to finance its budget, respectively. At the average level, consumption of adult households diminishes by 15.50%, 21.24% and 14.96%, and is 57.15%, 67.80% and 53.78% lower than at initial equilibrium, with tax on consumption, labour income and capital income in place, respectively[44]. Identically to discussion for the previous generation, when the government uses tax on consumption or capital income to finance its budget, the dynamics for

[44] This larger change at the average compared to the individual level arises from the change in the distribution of the population

savings of adult households at the individual and average levels both follow the one mentioned for the case of consumption of adult households when government taxes capital income; while the dynamics for savings with the labour income tax in place follows the one discussed for the consumption of adult households with the labour income tax in the system. Lastly, consumption of the elderly households at the individual level decreases by 17.32%, 13.37% and 16.41%, which becomes 15.73%, 12.85% and 14.27% above the steady-state, when tax on consumption, labour income and capital income is in use, respectively. At the average level, this indicator diminishes by 27.79%, 27.24% and 27.00%, and is 34.98%, 45.28% and 35.81% lower than at initial equilibrium, when government budget is financed through tax on consumption, labour income and capital income, respectively.

As a result of all these changes that take place at the fourth period of analysis, we find that level of welfare in the economy decreases by 21.44% from the previous generation and becomes 123.15% below original steady-state. With the labour income tax in use, the welfare reduces by 22.82% and is 163.65% lower than original equilibrium. Lastly, with the tax on capital income, this indicator drops by 21.72% from the past period of analysis, and is 113.56% below the original equilibrium. Finally, our results also suggest a decrease in the level of output produced per economically active member of population by 27.32%, which becomes 29.87% below original steady-state when government taxes consumption or capital income. With the tax on labour income, this value diminishes by 27.39% and it is 45.18% below initial equilibrium.

Overall, the dynamics that we observe for the fourth period of analysis continues until the economy reaches the final steady-states. Given each of the tax options that we consider, figures 4.5 and 4.6 below present the distribution of the population for the second steady-states with the childcare at fifty percent provided by the government.

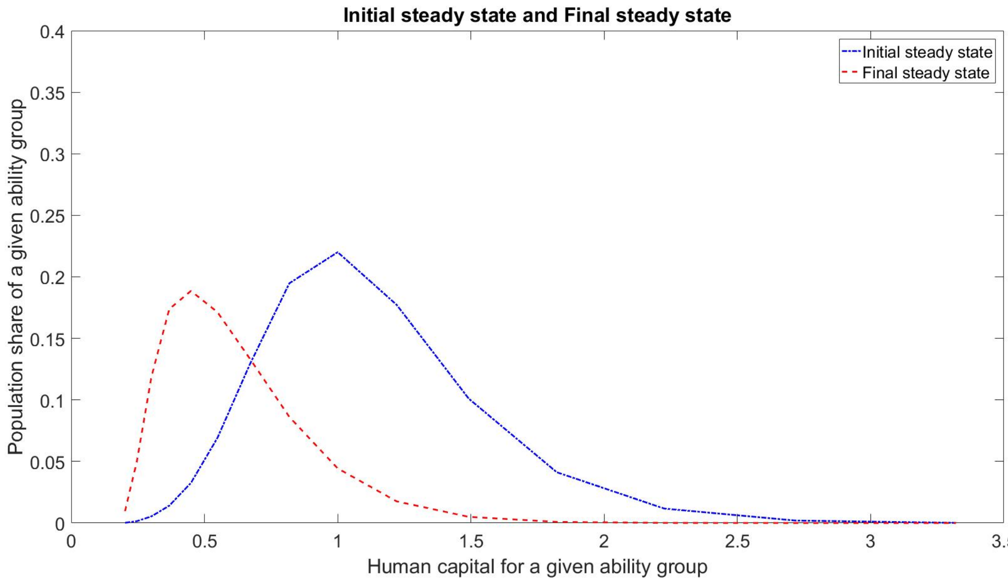

Figure 4.5: Distribution of the population at initial steady-state without government presence (blue dash-dotted line), at the second steady-state where government provides childcare $\omega = 0.5$ and taxes either consumption or capital income (red dashed line)

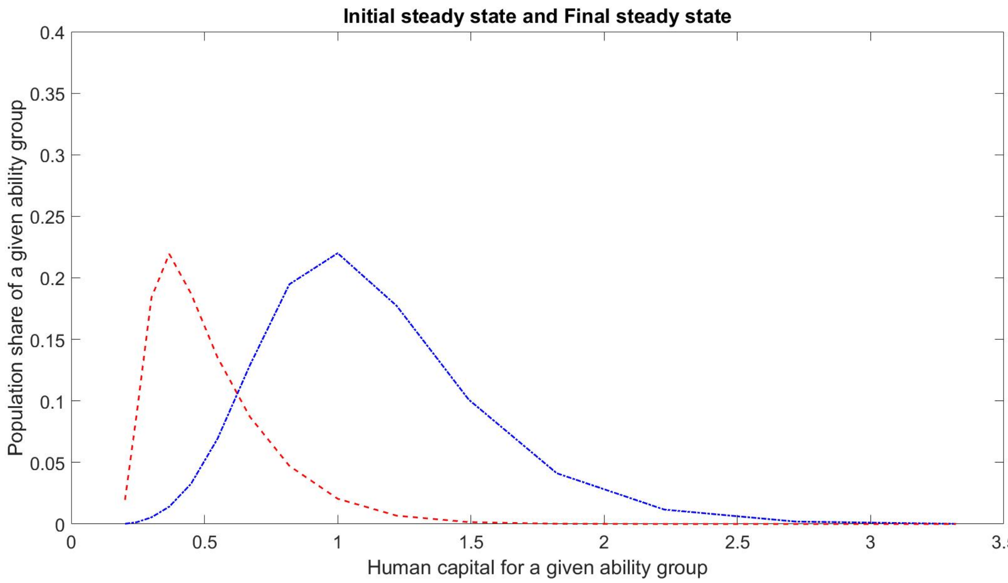

Figure 4.6: Distribution of the population at initial steady-state without government presence (blue dash-dotted line), at the second steady-state where government provides childcare $\omega = 0.5$ and taxes labour income (red dashed line)

At the second steady-state, given the distribution and the size of the population, and economic decisions that take place, when the government taxes consumption to finance its budget and provide childcare to the households, the tax rate is set to be equal to 7.93%. When government implements the labour income tax instead, the tax rate is equal to 16.05%; while with the capital income tax in use, the tax rate is equal to 10.12%.

As a result of the government program, we observe the decrease in individual education attainment for all ability groups who provided their children with above zero education before the government has entered. For instance, the education provision by fourth, seventh, tenth and fifteenth ability groups becomes 38.56%, 10.06%, 7.16% and 5.86% lower than at the original steady-state, when government taxes consumption or capital income, respectively. With the tax on labour income, the education provision for these ability groups falls by 48.30%, 12.60%, 8.96% and 7.34%, respectively. For the ability groups with zero education provision at the original steady-state, this decision does not change when economy reaches each of the corresponding final steady-states. Given this change in decisions for education at the individual level and the change in the distribution of the population, the average education attainment decreases by 71.90% when government taxes consumption or capital income; while with the labour income tax, it decreases by 82.50% as the economy reaches the final steady-state.

With these changes in the education decisions, we observe the fertility choices for each ability group increase. For example, the childbearing decisions of first, fourth, seventh, tenth and fifteenth ability groups rise by 100%, 108.86%, 103.53%, 101.68% and 100.55% when government taxes consumption or capital income, respectively. With the tax on labour income, these choices increase by 138.25%, 151.82%, 143.62%, 140.80% and 139.10% when economy reaches the second steady-state, respectively. Given these changes at the individual level, and the change in the distribution of population, the average fertility level at the second steady-state becomes 157.50% larger than before government presence, when tax on consumption or capital income is in place. With the labour income tax, the average fertility level increase by 228.79%, however.

Given all these changes combined with decrease in the average level of human capital by 47.20% when government taxes consumption or capital income, and by 55.04% when government taxes labour income instead; we observe a decrease in size of the effective labour force per adult household by 20.72% with tax on consumption or capital income financing the government budget, and it reduces by 25.49% when the labour income tax is in place instead. Due to a decrease in savings of the households and increase in the population size, our results suggest that the size of the physical capital stock per adult household reduces by 56.91% when tax on consumption or capital income is in use. With the labour income tax, when economy reaches the second steady-state, the size of this second factor input diminishes by 73.14%. As a consequence of these changes in the factor inputs, the real wage rate decreases by 18.39% with the tax on consumption or capital income in the system; while with the labour income tax, this factor price becomes 28.83% lower than at the original steady-state. The real interest rate, however, is found to increase by 68.03% when government taxes consumption or capital income; and with the labour income tax, it increases by 132.17%.

As a result of these changes in the factor prices, and given the tax rates that balance the government budget, the consumption of adult households at the individual ability level is found to decrease 24.39%, 40.26% and 18.39% below original steady-state, when government taxes consumption, labour income and capital income respectively. At the average level, the consumption of adult households diminishes by 60.08%, 73.14% and 56.91% when economy reaches final steady-state and the tax on consumption, labour income and capital income is implemented, respectively. Next, when the government taxes consumption or capital income, the dynamics for savings both at the individual and average levels follows the one mentioned for the case of consumption for adult households when the tax on capital income is in use. When government taxes labour income instead, the behaviour for savings both at the individual and average level follows the one discussed for consumption of adult households under this tax option. Lastly, the consumption of the elderly households at the individual level is found to increase by 13.54%, 17.95% and 12.31% when tax on consumption, labour income and capital income is in place, respectively. For the average level, however, consumption of the elderly households decreases by 40.06%, 46.97% and 40.71% when economy reaches the second steady-state and government finances its budget through tax on consumption, labour income and capital income, respectively.

Finally, we observe that when government provides a childcare for fifty percent of the time that parents need to take care for their children, the level of welfare becomes 138.19%, 192.07% and 128.46% below the original steady state, when government taxes consumption, labour income and capital income, respectively. Lastly, given the decrease in the factor inputs and increase in the population size, the level of output produced per adult household diminishes by 35.30% from the original steady state when tax on consumption or capital income is in the system; and with the labour income tax, it reduces by 46.97%.

4.4.3 Progressive labour income taxation

In the present and upcoming sections, we return to a discussion of the cases when government provides subsidy for education and fertility. We, however, in comparison to the chapter 3, consider the alternative tax and subsidies schemes – namely, the progressive labour income tax schedule that is subject to a discussion in the present section, and the subsidy scheme with regressive rates that is considered in the next part. Lastly, due to the responses that we observe being similar to ones that we have described in detail in the previous chapter, the present and upcoming sections focuses on the comparison between the final steady-states for newly considered policy options and previously obtained final steady-states with flat tax and subsidy rates.

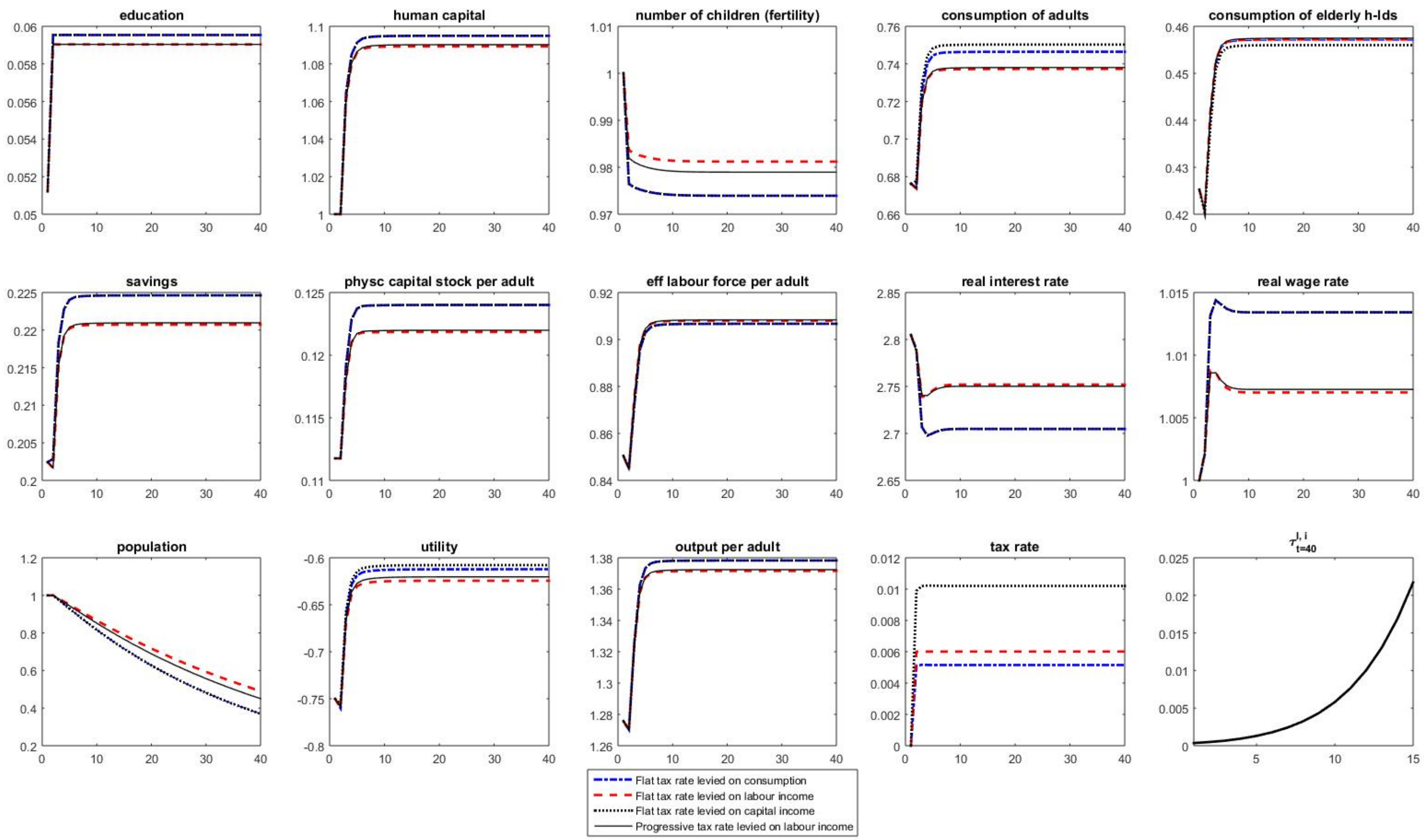

Figure 4.7: Transition path of the model economy with idiosyncratic shocks to the human capital from the perspective of the average household. Government provides subsidy for education at a rate of 10% (i.e. sub^e=0.1).
Note: transition path with tax on consumption is depicted by blue dash-dotted line; transition path with flat tax rate on labour income is presented by red dashed line; transition path with capital income tax is shown by black dotted line; transition path with progressive labour income taxation is illustrated by black solid line

Subsidy for education

		Y	K	L	r	w			
		1.3724	0.1220	0.9083	2.7502	1.0073			
		(7.58%)	(9.15%)	(6.81%)	(-1.95%)	(0.73%)			
		$\bar{h}$	tot.pop.	$\bar{e}$	$\bar{n}$	$\bar{c}$	$\bar{s}$	$\bar{d}$	$\bar{u}$
		1.0902	0.4409	0.0590	0.9789	0.7381	0.2210	0.4575	-0.6203
		(9.02%)	(-55.91%)	(15.27%)	(-2.11%)	(9.15%)	(9.15%)	(7.58%)	(17.32%)
i	τ_i^l	h_i	pop.share.	e_i	n_i	c_i	s_i	d_i	u_i
1	0.00035	0.2019	0.0001	0.0000	1.6195	0.1375	0.0412	0.0852	-2.9225
			(-64.16%)	—	(0.04%)	(0.69%)	(0.69%)	(-0.76%)	(0.16%)
2	0.00049	0.2466	0.0006	0.0000	1.6197	0.1679	0.0503	0.1040	-2.6268
			(-62.70%)	—	(0.05%)	(0.68%)	(0.68%)	(-0.77%)	(0.17%)
3	0.00067	0.3012	0.0022	0.0000	1.6200	0.2050	0.0614	0.1271	-2.3312
			(-58.96%)	—	(0.07%)	(0.66%)	(0.66%)	(-0.79%)	(0.19%)
4	0.00094	0.3679	0.0068	0.0043	1.4056	0.2503	0.0749	0.1552	-2.0613
			(-52.43%)	(13.28%)	(-1.25%)	(0.63%)	(0.63%)	(-0.81%)	(0.08%)
5	0.0013	0.4493	0.0189	0.0105	1.2411	0.3056	0.0915	0.1894	-1.7883
			(-42.43%)	(6.08%)	(-0.82%)	(0.60%)	(0.60%)	(-0.85%)	(0.11%)
6	0.0018	0.5488	0.0473	0.0181	1.1328	0.3731	0.1117	0.2313	-1.5096
			(-32.92%)	(4.12%)	(-0.51%)	(0.55%)	(0.55%)	(-0.90%)	(0.12%)
7	0.0024	0.6703	0.1013	0.0273	1.0576	0.4554	0.1363	0.2823	-1.2269
			(-21.60%)	(3.18%)	(-0.27%)	(0.48%)	(0.48%)	(-0.96%)	(0.12%)
8	0.0033	0.8187	0.1739	0.0386	1.0035	0.5558	0.1664	0.3445	-0.9417
			(-10.79%)	(2.59%)	(-0.05%)	(0.40%)	(0.40%)	(-1.05%)	(0.08%)
9	0.0044	1.0000	0.2235	0.0523	0.9637	0.6781	0.2030	0.4203	-0.6546
			(1.74%)	(2.16%)	(0.16%)	(0.29%)	(0.29%)	(-1.15%)	(-0.04%)
10	0.0058	1.2214	0.2062	0.0690	0.9339	0.8270	0.2476	0.5126	-0.3663
			(16.83%)	(1.79%)	(0.39%)	(0.14%)	(0.14%)	(-1.30%)	(-0.49%)
11	0.0076	1.4918	0.1344	0.0894	0.9115	1.0083	0.3019	0.6249	-0.0773
			(33.84%)	(1.45%)	(0.64%)	(-0.04%)	(-0.04%)	(-1.48%)	(-5.12%)
12	0.0100	1.8221	0.0614	0.1141	0.8949	1.2285	0.3678	0.7615	0.2120
			(50.33%)	(1.09%)	(0.93%)	(-0.28%)	(-0.28%)	(-1.72%)	(-2.91%)
13	0.0130	2.2255	0.0193	0.1441	0.8827	1.4960	0.4479	0.9272	0.5014
			(66.52%)	(0.69%)	(1.28%)	(-0.59%)	(-0.59%)	(-2.02%)	(-1.90%)
14	0.0169	2.7182	0.0038	0.1805	0.8743	1.8201	0.5449	1.1281	0.7904
			(93.99%)	(0.23%)	(1.70%)	(-0.97%)	(-0.97%)	(-2.40%)	(-1.74%)
15	0.0217	3.3201	0.0004	0.2245	0.8691	2.2121	0.6623	1.3711	1.0787
			(156.50%)	(-0.32%)	(2.23%)	(-1.46%)	(-1.46%)	(-2.88%)	(-1.77%)

Table 4.8: Summary for the final steady-state when government provides subsidy for education at a rate of 10% (i.e. $sub^e = 0.1$) and finances its budget with progressive tax rate on labour income

First, let us consider the final steady-state when the government provides the flat subsidy rate for education of 10% which it finances with the progressive labour income taxation, where the tax rate for each ability group is calculated based on the Gouveia-Strauss function (4.12). We report the resulted marginal tax rates in the second column of table 4.8 and in the bottom-right sub-plot of the figure 4.7.

According to our results, the tax rate[45] on the labour income varies from 0.035% for the households from the lowest ability group to 2.17% for the households from the highest ability group when the economy reaches the final steady-state with uniform subsidy rate for education of 10%. In comparison to our previous results, when the government uses the flat tax rate on the labour income, the tax rate for every household has been set to 0.60%; while, with the tax on the capital income – the largest tax rate that was previously required for provision of subsidy for education – the tax rate at the second steady-state has been equal to 1.02%.

Due to the differences in the labour income tax rates levied on the different ability groups, and due to the negative influence of the labour income taxation on education provision, we observe that at the final steady-state the households from fourth to eighth ability groups provide their children with (private) education that is larger by 2.27%, 1.00%, 0.59%, 0.36% and 0.17% than at the final steady-state with tax on consumption or capital income considered previously[46]. Overall, the education provision by these household groups is 13.27%, 6.08%, 4.12%, 3.18%, 2.59% above the original steady-state. For the ninth ability group, we observe that the level of education provision with the subsidy for education being financed with either progressive labour income taxation, or flat tax rate on consumption or capital income is identical, and it is 2.16% above the original steady-state. For the households with larger level of human capital, however, we observe that at the final steady-state due to relatively larger marginal tax rates on the labour income, the education provision falls below the level of all previously considered tax options, with an extreme case for the fifteenth ability group where education provision is 0.32% below the original steady-state. The education attainment of three household groups with the

[45]The level of these tax rates depends on the level of the model variables, and it is primarily influenced by the exogenously chosen realised human capital levels that characterise each ability group. Depending on the differences in the human capital that actually exists in the world economies, these tax rates may or may not be sufficient for the provision of the flat subsidy rate for education of 10% in the real-world application.

[46]In the chapter 3, we have observed that independently of the subsidy option, the case with either tax on consumption or capital income resulted in the largest level for education provision

lowest level of human capital, however, remains at the level of zero, which is identical to the original steady-state and to the final steady-states considered previously.

Overall, given the distribution of the population at the final steady-state with a subsidy for education being financed with a progressive labour income tax scheme, the average level of education is found to be identical to one with the case of flat labour income taxation, which is 15.27% above the original steady-state, but which is 0.85% lower than under the final steady-state with tax on consumption or capital income. The average level of the human capital with the progressive labour income taxation, however, is found to be 9.02% above the original steady-state, which is 0.08% larger than at the final steady-state with flat tax rate on the labour income, but which is 0.43% below of the average human capital level at the final steady-state with either tax on consumption or capital income.

With these responses in the education provision, we observe the following changes in child-bearing decisions. For the households from the first three ability groups, the fertility increases by 0.04%, 0.05% and 0.07% above the initial steady-state, which, however, is 0.57%, 0.55% and 0.53% below the final steady-state with flat tax rate on the labour income. For the households from the fourth to eighth ability groups, the fertility decisions follow the dynamics observed for the case with subsidy for education being financed with tax on consumption or capital income, that become 1.25%, 0.82%, 0.51%, 0.27% and 0.05% below the original equilibrium without government intervention. In comparison to the final steady-state with the flat tax rate on the labour income, the fertility decisions of these five ability groups become 0.82%, 0.68%, 0.56%, 0.44% and 0.32% lower; while comparing these results with the case when tax on consumption or capital income finances the government budget, we find that with the progressive labour income tax system the fertility decisions of fourth and fifth ability groups would become 0.16% and 0.03% lower, when for the sixth to eighth ability groups these choices are 0.08%, 0.18% and 0.30% larger than in the final steady-state with the tax on consumption or capital income. For the remaining part of the population, the fertility choices follow the dynamics observed for the case with flat labour income taxation. Furthermore, when the economy reaches the final steady-state with progressive labour income taxation, the fertility decisions for eleventh to fifteenth ability groups become 0.19%, 0.45%, 0.77%, 1.18% and 1.69% larger than at the final steady-state with flat tax rates on labour income, and 0.81%, 1.07%, 1.39%, 1.80% and 2.31% larger than at the final steady-state with tax on consumption or capital income.

Overall, given the distribution of the population at the final steady-state and the changes in the individual fertility decisions, the average level of fertility is found to decrease by 2.11% below the original steady-state when the economy reaches the steady-state with subsidy for education that is financed with the progressive labour income tax system. In comparison to the previous results, the average level of fertility is 0.23% lower than in the case with the flat tax rates on the labour income, but it is 0.51% larger than at the final steady-state with tax on consumption or capital income. Given all these changes, the population size decreases and at the 40^{th} period of analysis it is 55.91% smaller than at the initial steady-state.

With these changes in the education and fertility decisions of the households, and given the resulted distribution of the population and the population size, we find that with subsidy for education being financed with progressive labour income tax scheme, the effective labour force per adult household becomes 6.81% larger than at the initial steady-state, which is 0.18% larger than at the final steady-state with tax on consumption or capital income, and also 0.03% larger than at the final steady-state with flat tax rates on labour income. For the physical capital stock per adult household member – the dynamics for which follows the changes in the individual decisions for saving, distribution of the population and population size – we find that at the final steady-state with the progressive labour income taxation, this factor input increases by 9.15% above the original steady-state, which is 0.11% above the final steady-state with subsidy for education and flat tax rate on labour income, but it is 1.63% below the final steady-state with tax on consumption or capital income. Given these changes in the factor inputs, the economy reaches the steady-state with progressive tax scheme on labour income where the real wage rate increases 0.73% above the original steady-state; which becomes 0.02% larger than for the case with flat tax rates on the labour income, but it is 0.61% smaller than at the final steady-state with the tax on consumption or capital income. Lastly, according to our results, the real interest rate falls 1.95% below the original steady-state when the economy reaches the final steady-state with progressive labour income tax rates. In comparison to our previous results, at the final steady-state the real interest rate becomes 0.07% lower than in the case with the flat labour income tax rate, but it is 1.68% larger than in a scenario when subsidy for education of 10% is being financed with the tax on consumption or capital income.

Following the individual choices for education and fertility, together with the resulted real wage rate, and combined with the resulted progressive labour income tax rates, we observe that consumption of adult households from first to tenth ability groups becomes larger than at the original steady-state; while for the rest of the population this indicator decreases below the original steady-state due to progressively increasing marginal tax rates on the labour income. For instance, with the progressive labour income taxation, the consumption of the households from the first, fifth and tenth ability groups becomes 0.69%, 0.60% and 0.14% above the original steady-state, which is 0.59%, 0.50% and 0.05% larger than the consumption at the final steady-state with the flat tax rate on the labour income; but it is 0.13%, 0.22% and 0.67% smaller than at the final steady-state with the tax on consumption, and 0.64%, 0.74% and 1.18% smaller than at the final steady-state with tax on capital income, respectively. For the adult households from twelfth and fourteenth ability groups, however, the consumption becomes 0.28% and 0.97% smaller than at the original steady-state when the economy reaches the final steady-state with the progressive labour income taxation. At the final steady-state the consumption for these ability groups becomes 0.38% and 1.07% smaller than at the final steady-state with the flat tax rate on the labour income; 1.10% and 1.78% smaller than at the final steady-state with tax on consumption; and 1.60% and 2.28% smaller than at the final steady-state with the tax on capital income, respectively. Overall, with these changes in the consumption of adult households at the individual level combined with the change in the distribution of the population across different ability groups, the average level of consumption of adult households becomes 9.15% above original steady-state when economy reaches the final steady-state with subsidy for education and progressive tax rates on the labour income. In comparison to the previous results, this average level of consumption is 0.11% larger than at the final steady-state with flat tax rate on the labour income, but it is 1.12% smaller than at the final steady-state with tax on consumption, and 1.63% below the final steady-state with tax on the capital income. The dynamics for saving, both at the individual and the average levels, follows the one described for consumption of adult households.

Next, with the resulted real wage and the real interest rates, combined with the progressive tax rates on the labour income, we observe that at the final steady-state the consumption of the elderly households at the individual level decreases below original steady-state, which is similar to the previously considered tax options. We, however, observe that for the elderly households from first to ninth ability groups, the level of consumption becomes larger than under all previously considered tax options. For instance, at the final steady-state with progressive tax rates on the labour income, the consumption of the elderly households from first, fifth and ninth ability groups become 0.76%, 0.85% and 1.15% below the initial steady-state, which is 0.54%, 0.45% and 0.14% larger than at the final steady-state with flat tax rates on the labour income; 1.09%, 1.00% and 0.69% larger than at the final steady-state with tax on consumption; and 1.33%, 1.23%, 0.92% larger than at the final steady-state with tax on capital income, respectively. For the elderly households from tenth ability group, the consumption under the flat and progressive labour income tax schedules at the final steady-state is identical, and it is 1.30% below the original steady-state, which is 0.54% larger than with the tax on consumption and 0.78% larger than with the tax on the capital income. For the remaining part of the population, the consumption of the elderly households changes as following. For the eleventh and twelfth ability groups, consumption of elderly households becomes 1.48% and 1.72% below the original steady-state, which is 0.19% and 0.43% below the final steady-state with flat tax rates on the labour income, but it is 0.36% and 0.12% above the final steady-state with tax on consumption, and 0.59% and 0.35% above the final steady-state with tax on capital income, respectively. For the thirteenth ability group, the consumption of the elderly households at the final steady-state with progressive labour income taxation becomes 2.02% lower than at the original equilibrium, and it is 0.73% and 0.19% lower than at the final steady-state with flat tax on the labour income and tax on consumption, respectively; but it is 0.04% larger than at the final steady-state with the tax on capital income. Finally, for the fourteenth and fifteenth ability groups, the consumption of the elderly households decreases by 2.40% and 2.88% below the original steady-state when progressive labour income taxation is in place, which becomes lower than under all previously considered cases. For the fourteenth ability group the consumption of the elderly households at the resulted steady-state is 1.12%, 0.58% and 0.35% lower than at the final steady-state with flat tax rate on labour income, consumption and capital income, respectively. For the elderly households from fifteenth ability group, the consumption at the final steady-state with progressive labour income tax rates is 1.60%, 1.07% and 0.83% below the final steady-state with flat tax rates on labour income, consumption and capital income. Overall, this larger decrease in the consumption of the elderly households from ability groups with a marginally larger level of human capital is explained, as before, by a relatively larger tax rates on the labour income that higher ability households have to bear. Finally, with these changes at the individual level, and given the resulted distribution of the population at the final steady-state with the progressive labour income tax rates, the average level of consumption of elderly households becomes 7.58% larger than at the initial steady-state, which is 0.06%, 0.09% and 0.32% larger than at the final steady-state with the flat tax rate on the labour income, consumption and capital income, respectively.

Finally, consequential of all outcomes presented above, we observe that the level of welfare at the final steady-state with the progressive tax rates on the labour income becomes 17.32% larger than at the initial steady-state; and it is 0.66% larger than at the previously considered final steady-state with flat tax rate on the labour income. At the final steady-state with the progressive labour income tax system the level of welfare, however, is 1.32% and 2.06% lower than at the final steady-states with tax on consumption and capital income, respectively. At the individual level, the previously considered capital income taxation still produces the outcomes that are the best in terms of utility level. We, however, observe that for the households from the first to sixth ability groups, the progressive labour income taxation is the second best policy option in terms of utility level when government provides the subsidy rate for education of 10%. For the households from seventh to tenth ability groups, however, the progressive labour income taxation results in the utility which is lower than at the final steady-states with tax on capital income and with tax on consumption. Finally, for the households from eleventh to fifteenth ability groups, the progressive labour income taxation results in the worst outcomes in terms of the utility at the individual ability level. Lastly, due to the change in the factor inputs and the population size, our results indicate that the level of output per adult household increases 7.58% above the original steady-state when government implements the progressive labour income tax schedule to finance the subsidy for education of 10%. Furthermore, this indicator increases 0.06% above the final steady-state with flat tax rate on the labour income, but it is 0.43% below the final steady-state with tax on consumption or capital income.

Subsidy for fertility

Moving our discussion forward, we analyse the final steady-state of the economy where the government provides subsidy for fertility at a rate of 10% and finances the budget with the progressive labour income tax scheme, where the marginal tax rates for every ability group are calculated based on the Gouveia-Strauss function (4.12). We report these tax rates in the second column of the table 4.9 and in the bottom right sub-plot in the figure 4.8. Similarly to the discussion with subsidy for education above, we focus on a comparison between the final steady-state with progressive labour income taxation and the final steady-states with flat tax rate on consumption, labour income and capital income considered previously.

According to our results, at the final steady-state with progressive tax scheme on the labour income and subsidy for fertility of 10%, the marginal tax rates vary from 0.035% for the households from the lowest ability group to 2.45% for the households with the largest level of human capital considered. In comparison to our previous results, the flat tax rate on the labour income at final steady-state with subsidy for fertility has been set to 0.58% to balance the government budget; while with the tax on consumption and capital income, the tax rates have been equal to 0.47% and 0.84% in each of the corresponding steady-state, respectively.

With these tax rates on the labour income combined with our previous conclusions that a relatively larger labour income tax rates result in a disincentive for education provision that manifests in form of relatively larger fertility responses, we observe that at the final steady-state with progressive labour income tax scheme, the lower ability households – who are a subject to a relatively smaller marginal tax rates than under all steady-states previously considered – they decide to provide their children with a marginally larger level of education than under all previously considered cases, which produces a relatively smaller fertility responses of these household groups. For the households with a relatively larger level of the human capital, however, we observe the opposite, since these ability groups are characterised by a larger levels of

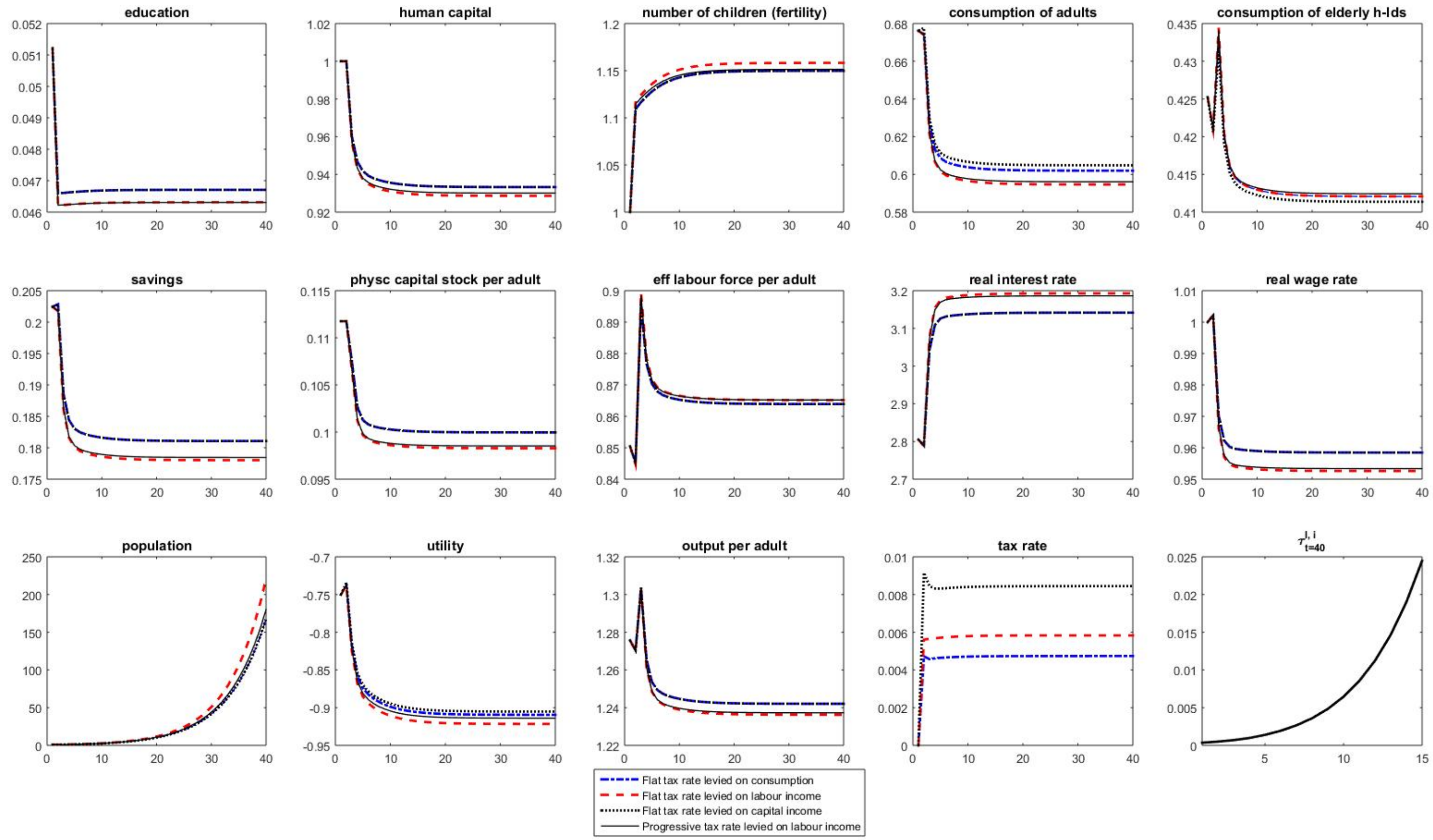

Figure 4.8: Transition path of the model economy with idiosyncratic shocks to the human capital from the perspective of the average household. Government provides subsidy for fertility at a rate of 10% (i.e. sub^n=0.1).
Note: transition path with tax on consumption is depicted by blue dash-dotted line; transition path with flat tax rate on labour income is presented by red dashed line; transition path with capital income tax is shown by black dotted line; transition path with progressive labour income taxation is illustrated by black solid line

		Y	K	L	r	w			
		1.2373	0.0985	0.8652	3.1862	0.9534			
		(-3.01%)	(-11.84%)	(1.74%)	(13.60%)	(-4.66%)			
		$\bar{h}$	tot.pop.	$\bar{e}$	$\bar{n}$	$\bar{c}$	$\bar{s}$	$\bar{d}$	$\bar{u}$
		0.9301	207.29	0.0463	1.1514	0.5961	0.1785	0.4124	-0.9139
		(-7.00%)		(-9.59%)	(15.34%)	(-11.84%)	(-11.84%)	(-3.09%)	(-21.82%)
i	τ_i^l	h_i	pop.share.	e_i	n_i	c_i	s_i	d_i	u_i
1	0.00035	0.2019	0.0010	0.0000	2.2635	0.1301	0.0390	0.0900	-2.9009
			(568.36%)	—	(39.81%)	(-4.70%)	(-4.70%)	(4.85%)	(0.90%)
2	0.00050	0.2466	0.0073	0.0000	2.1117	0.1589	0.0476	0.1099	-2.6177
			(539.96%)	—	(30.44%)	(-4.71%)	(-4.71%)	(4.84%)	(0.52%)
3	0.00069	0.3012	0.0227	0.0000	2.0020	0.1940	0.0581	0.1342	-2.3318
			(419.88%)	—	(23.66%)	(-4.73%)	(-4.73%)	(4.82%)	(0.16%)
4	0.00097	0.3679	0.0414	0.0012	1.8322	0.2369	0.0709	0.1639	-2.0523
			(278.01%)	(-68.17%)	(28.72%)	(-4.76%)	(-4.76%)	(4.79%)	(0.51%)
5	0.0014	0.4493	0.0594	0.0078	1.4924	0.2892	0.0866	0.2001	-1.7939
			(117.37%)	(-21.68%)	(19.26%)	(-4.80%)	(-4.80%)	(4.75%)	(-0.20%)
6	0.0019	0.5488	0.0865	0.0157	1.2961	0.3531	0.1057	0.2443	-1.5241
			(42.28%)	(-9.33%)	(13.82%)	(-4.85%)	(-4.85%)	(4.69%)	(-0.84%)
7	0.0027	0.6703	0.1302	0.0255	1.1705	0.4310	0.1290	0.2982	-1.2476
			(11.48%)	(-3.75%)	(10.38%)	(-4.92%)	(-4.92%)	(4.61%)	(-1.56%)
8	0.0036	0.8187	0.1756	0.0374	1.0850	0.5259	0.1574	0.3638	-0.9666
			(-2.58%)	(-0.66%)	(8.07%)	(-5.01%)	(-5.01%)	(4.51%)	(-2.57%)
9	0.0048	1.0000	0.1880	0.0518	1.0244	0.6415	0.1921	0.4439	-0.6828
			(-9.04%)	(1.21%)	(6.47%)	(-5.12%)	(-5.12%)	(4.38%)	(-4.36%)
10	0.0065	1.2214	0.1498	0.0694	0.9803	0.7823	0.2342	0.5412	-0.3970
			(-10.95%)	(2.40%)	(5.37%)	(-5.28%)	(-5.28%)	(4.21%)	(-8.91%)
11	0.0085	1.4918	0.0873	0.0908	0.9478	0.9535	0.2854	0.6597	-0.1100
			(-9.61%)	(3.13%)	(4.64%)	(-5.48%)	(-5.48%)	(4.00%)	(-49.58%)
12	0.0112	1.8221	0.0373	0.1169	0.9237	1.1614	0.3477	0.8036	0.1776
			(-5.74%)	(3.54%)	(4.18%	(5.73%)	(5.73%)	(3.71%)	(-18.67%)
13	0.0146	2.2255	0.0113	0.1484	0.9062	1.4136	0.4232	0.9781	0.4655
			(0.58%)	(3.70%)	(3.97%)	(-6.06%)	(-6.06%)	(3.36%)	(-8.93%)
14	0.0190	2.7182	0.0022	0.1866	0.8938	1.7190	0.5146	1.1893	0.7530
			(12.81%)	(3.63%)	(3.98%)	(-6.47%)	(-6.47%)	(2.90%)	(-6.39%)
15	0.0245	3.3201	0.0002	0.2327	0.8858	2.0878	0.6251	1.4445	1.0399
			(44.33%)	(3.35%)	(4.21%)	(-7.00%)	(-7.00%)	(2.32%)	(-5.31%)

Table 4.9: Summary for the final steady-state when government provides subsidy for fertility at a rate of 10% (i.e. $sub^n = 0.1$) and finances its budget with progressive tax rate on labour income

the realised human capital and they earn larger labour income and face larger marginal tax rates on the labour income which creates additional disincentive for education provision, which with the presence of parental 'quality-quantity' trade-off for children manifests in a larger fertility decisions than under previously considered flat tax schedules.

According to our results, at the final steady-state with a progressive labour income tax schedule that finance the subsidy for fertility of 10%, the fertility choices of adult households from the first to sixth ability groups become 39.81%, 30.44%, 23.66%, 28.72%, 19.26% and 13.82% larger than at the initial steady-state, respectively. In comparison to the previous results, however, the fertility decisions for these six ability groups are 0.71%, 0.65%, 0.60%, 0.91%, 0.69% and 0.53% smaller than at the final steady-state with a flat tax rate on the labour income; and 0.43%, 0.30%, 0.20%, 0.52%, 0.24% and 0.04% smaller than at the final steady-state with tax on consumption or capital income. Since these lower ability households are characterised by a lower level of the realised human capital than the rest of the population, they earn a relatively lower labour income and face lower tax rates on the labour income since the tax schedule is progressive. These relatively lower tax rates in comparison to the rest of the population and to the previously considered final steady-states, result in a lower disincentive for the optimal choice for education investment and, through the parental trade-off, in a relatively lower fertility decisions.

For the households from seventh to ninth ability groups, the fertility decisions increase by 10.38%, 8.07% and 6.47% above the original steady-state, which become 0.12%, 0.29% and 0.46% larger than at the final steady-state with the tax on consumption or capital income; but the fertility decisions for these three ability groups are 0.39%, 0.25% and 0.09% smaller than at the final steady-state with the flat tax rate on the labour income. Finally for the households from tenth to fifteenth ability groups, the fertility decisions increase 5.37%, 4.64%, 4.18%, 3.97%, 3.98% and 4.21% above the initial equilibrium, that become 0.11%, 0.34%, 0.63%, 1.00%, 1.47% and 2.05% larger than at the final steady-state with a flat tax scheme on the labour income; and 0.66%, 0.91%, 1.21%, 1.58%, 2.05% and 2.64% larger than at the final steady-state with the tax on consumption or capital income. Oppositely to the households from lower ability groups, the households with a larger level of realised human capital pay larger marginal tax rates on the labour income which creates greater disincentive for the education investment, which through the parental trade-off manifests in a relatively larger fertility levels than in the previously considered cases.

Overall, given these changes in the individual fertility decisions and in the distribution of the population at the final steady-state[47], the average level of fertility in the economy increases by 15.34% above the original steady-state when the government provides subsidy for fertility of 10% and finances its budget with a progressive labour income tax system. In comparison to our previous results, this average level of fertility is 0.14% larger than at the final steady-state with tax on consumption or capital income, but it is 0.59% smaller than at the final steady-state with a flat tax rate on the labour income. Finally, as a result of these changes in the fertility decisions and in the distribution of the population, the population size of the economy with subsidy for fertility and progressive labour income tax rates becomes 207.29 times larger than at the initial steady-state. In comparison to our previous conclusions, the population size is 7.33% larger than in the case with tax on consumption or capital income, but it is 18.48% smaller than in a scenario with a flat labour income tax schedule.

Next, with the subsidy for fertility, progressive labour income tax schedule and resulted fertility decisions, we find that the education provision changes as following. First, when the economy reaches the final steady-state with progressive labour income tax scheme, for the households from fourth to eighth ability groups, the choices for private education diminish 68.17%, 21.68%, 9.33%, 3.75% and 0.66% below the initial steady-state. In comparison to the previous results, however, these levels are 10.29%, 1.49%, 0.68%, 0.32% and 0.08% larger than at the final steady-state with tax on consumption or capital income; and 8.92%, 1.38%, 0.67%, 0.35% and 0.12% larger than at the final steady-state with the flat tax rate on the labour income. For the households from ninth to fifteenth ability groups, however, the education provision increases 1.21%, 2.40%, 3.13%, 3.54%, 3.70%, 3.63% and 3.35% above the initial steady-state. However, because the progressive labour income tax scheme causes a progressively increasing disincentive for education investment, our results indicate that the education choices for

[47] In comparison to the previously considered final steady-states with flat tax options, at the final steady-state with a progressive labour income taxation and with flat subsidy rate for fertility of 10%, the population consists of a greater share of the households that are in centre of the original distribution, and of a smaller share of the households that are from the tails of the original distribution.

the households from these top seven ability groups are 0.14%, 0.37%, 0.63%, 0.94%, 1.31%, 1.78% and 2.35% lower the final steady-state with tax on consumption or capital income; and they are 0.09%, 0.31%, 0.56%, 0.86%, 1.24%, 1.70% and 2.27% below the final steady-state with flat tax rate on the labour income, respectively. Finally, for the households from the bottom three ability groups, the level of education does not change and it remains to be equal to zero. This result is identical to all previously considered steady-states.

As a result of the changes in the private education investment and in the distribution of the population, we find that at the final steady-state with the subsidy for fertility of 10% and considered progressive labour income tax scheme, the average level of education becomes 9.59% lower than at the initial steady-state. This result is 0.02% lower than at the final steady-state with the flat tax rate on the labour income, and is 0.86% lower than at the final steady-state with tax on consumption or capital income. Furthermore, we find that the average level of human capital becomes 7.00% below the initial steady-state when the economy reaches the final steady-state with the progressive labour income taxation; which, however, is 0.16% larger than at the final steady-state with the flat tax rate on the labour income, but it is 0.34% lower than at the final steady-state with the tax on consumption or capital income.

Consequently, at the final steady-state with the subsidy for fertility of 10% which is financed with the progressive labour income tax scheme, we find that the effective labour force per adult household – which is determined by the human capital and population of adult households, their free time for the labour market participation, education provision and fertility levels that jointly determine the number of teachers required – increases 1.74% above the initial steady-state, which is 0.0044% larger than at the final steady-state with the flat tax rate on the labour income, and 0.15% larger than at the final steady-state with the tax on consumption or capital income considered previously. The level of the physical capital stock per adult household member – which follows the changes in the individual decisions to save and population size of the adult households – is found to decrease 11.84% below the initial steady-state; and it is 1.44% below the final steady-state with the tax on consumption or capital income, but it is 0.24% above the final steady-state with the flat tax rate on the labour income. As a result of these changes in the factor inputs, at the final steady-state with progressive labour income taxation, the real wage rate decreases and becomes 4.66% lower than at the initial steady-state, which is 0.08% larger than at the final steady-state with flat tax rate on the labour income, but is 0.53% lower than at the final steady-state with the tax on consumption or capital income. For the real interest rate we observe an increase of 13.60% above the original steady-state when the economy reaches the final steady-state with progressive labour income taxation, which is 1.41% larger than the final-steady state with tax on consumption or capital income, but it is 0.21% lower than at the final steady-state with the flat tax rate on the labour income.

Overall, given the decrease in the real wage rate combined with the progressive tax system on the labour income, we observe that consumption of adult households for all ability groups falls below the original steady-state when the economy reaches the final steady-state. For instance, the consumption of the adult households from first to tenth ability groups reduce by 4.70%, 4.71%, 4.73%, 4.76%, 4.80%, 4.85%, 4.92%, 5.01% and 5.12% below the original steady-state when the progressive tax system on the labour income is in place, which is 0.63%, 0.62%, 0.60%, 0.59%, 0.53%, 0.47%, 0.40%, 0.30%, 0.18% and 0.02% larger than at the final steady-state with subsidy for fertility being financed with flat tax rate on the labour income. In comparison to the final steady-state with tax on consumption, however, the consumption of adult households from these ten ability groups with progressive labour income taxation in place is 0.10%, 0.11%, 0.13%, 0.16%, 0.20%, 0.26%, 0.33%, 0.42%, 0.55% and 0.71% lower. Also, the consumption of adult households from the first to tenth ability groups at the final steady-state with progressive labour income taxation is 0.57%, 0.58%, 0.60%, 0.63%, 0.67%, 0.73%, 0.80%, 0.89%, 1.01% and 1.17% lower than at the final steady-state with the tax on the capital income. For the households from eleventh to fifteenth ability groups, when the economy reaches the final steady-state with progressive labour income tax scheme and subsidy rate for fertility of 10%, the consumption of adult households decreases even further and becomes 5.48%, 5.73%, 6.06%, 6.47% and 7.00% lower than at the initial steady-state. The consumption of adult households from these five ability groups becomes 0.19%, 0.46%, 0.81%, 1.25% and 1.80% smaller than at the final steady-state with flat tax rate on the labour income; 0.92%, 1.18%, 1.53%, 1.96% and 2.51% smaller than at the final steady-state with the tax on consumption; and 1.38%, 1.65%, 1.99%, 2.42% and 2.97% smaller than at the final steady-state with the tax on capital income. As a result of these changes at the individual ability level combined with the resulted distribution of the population at the final steady-state, the average level of consumption for adult households becomes 11.84% smaller than at the initial steady-state, when the government finances the subsidy for fertility of 10% with progressive labour income taxation. In comparison to the previous results, the average level of consumption of adult households with progressive labour income tax system, however, is 0.24% larger than at the final steady-state with flat tax rate on the labour income, but it is 0.98% and 1.44% smaller than at the final steady-state with the tax on consumption and tax on capital income, respectively. The dynamics for saving both at the individual and average levels precisely follows the changes depicted for consumption of adult households.

With the changes in the factor prices and resulted tax rates on the labour income, we observe that consumption of the elderly households becomes larger than at the initial steady state. For instance, the consumption of the elderly households from first to ninth ability groups increases 4.85%, 4.84%, 4.82%, 4.79%, 4.75%, 4.69%, 4.61%, 4.51% and 4.38% above the initial equilibrium when the government provides subsidy for fertility at 10% and finances its budget with progressive labour income tax system. In comparison to the final steady-state with flat tax rate on the labour income, the consumption of the elderly households for these nine ability groups with the progressive tax rates on the labour income becomes 0.47%, 0.46%, 0.44%, 0.41%, 0.37%, 0.31%, 0.24%, 0.15% and 0.02% larger. Additionally, at the final steady-state with progressive tax rates on the labour income, the consumption of the elderly households from first to ninth ability groups is 0.98%, 0.96%, 0.94%, 0.91%, 0.87%, 0.82%, 0.74%, 0.65% and 0.52% larger than at the final steady-state with the tax on consumption; and 1.15%, 1.13%, 1.11%, 1.09%, 1.04%, 0.99%, 0.92%, 0.82% and 0.69% larger than at the final steady-state with tax on capital income. For the elderly households from tenth and eleventh ability groups, the consumption of the elderly households with progressive tax rates on the labour income increases by 4.21% and 4.00% above the original steady-state, respectively. The consumption of the elderly households from these two ability groups, however, is 0.14% and 0.35% lower than at the final steady-state with flat tax rate on the labour income; but it is 0.36% and 0.15% larger than at the final steady-state with tax on consumption, and 0.53% and 0.32% larger than at the final steady-state with tax on the capital income, respectively. For the elderly households from twelfth ability group, when the economy reaches the final steady-state with the progressive labour income tax rates, the consumption increases 3.71% above the initial steady-state, which is 0.05% larger than at the final steady-state with tax on capital income; but it is 0.12% and 0.62% smaller than at the final steady-state with tax on consumption and flat tax rate on the labour income, respectively. Finally, the consumption of the elderly households from thirteenth to fifteenth ability groups increases 3.36%, 2.90% and 2.32% above the initial steady-state when the economy reaches the final steady-state with the progressive tax rates on the labour income. In comparison to our previous results, the consumption of the elderly households from these three ability groups is 0.30%, 0.74% and 1.29% lower than at the final steady-state with tax on capital income; 0.47%, 0.91% and 1.46% lower than at the final steady-state with tax on consumption; and 0.97%, 1.40% and 1.95% lower than at the final steady-state with flat tax rate on the labour income. Given the distribution of the population and the change at the individual ability level, the average level of consumption of the elderly households decreases 3.09% below the initial steady-state when the economy reaches the final steady-state with the subsidy for fertility of 10% which government finances through progressive tax scheme on the labour income. In comparison to our previous results, the average consumption of the elderly households becomes 0.08%, 0.09% and 0.26% larger than at the final steady-state with flat tax rate on labour income, tax on consumption and tax on capital income, respectively.

Finally, with these outcomes that take place at the final steady-state with progressive labour income tax schedule, our results indicate that the households from lower ability groups that face relatively lower tax rates on the labour income are better-off, while the households from higher ability groups that receive larger marginal tax rates in the labour income are worse-off in comparison to the outcomes that we have previously obtained for the case with flat tax rate on the labour income. Particularly, the utility of the households from first to fourth ability groups increases by 0.90%, 0.52%, 0.16% and 0.51% above the original steady-state; which becomes 0.040%, 0.046%, 0.49% and 0.01% larger than at the final steady-state with tax on consumption; and 0.22%, 0.24%, 0.264% and 0.255% larger than at the final steady-state with flat tax rate on the labour income; but 0.11%, 0.12%, 0.13% and 0.20% smaller than at the final steady-state with tax on capital income. For the households from the fifth ability group, however, the utility decreases 0.20% below the original steady-state when the final steady-state is reached with the progressive tax rate on the labour income; which is 0.01% and 0.28% larger than at the final steady-state with tax on consumption and flat tax rate on the labour income; but it is 0.23% below the final steady-state with the tax on the capital income. For the households from sixth to ninth ability groups, the utility decreases by 0.84%, 1.56%, 2.57% and 4.36% below the initial steady-state, when the final steady-state is reached with the progressive tax rates on the labour income; which is 0.01%, 0.07%, 0.18% and 0.45% below the final steady-state with tax on consumption, and 0.29%, 0.41%, 0.62% and 1.08% below the final steady-state with tax on capital income; but it is 0.31%, 0.32%, 0.31% and 0.25% larger than at the final steady-state with flat tax rate on the labour income. For the remaining households from tenth to fifteenth ability groups, the utility decreases by 8.91%, 49.58%, 18.67%, 8.93%, 6.39% and 5.31% below the initial steady-state when the economy reaches the final steady-state with progressive tax rates on the labour income. As a result the utility for these six ability groups becomes 0.02%, 2.21%, 2.94%, 1.94%, 1.84% and 1.92% smaller than at the final steady-state with the flat tax rate on the labour income; 1.23%, 6.93%, 5.39%, 2.91%, 2.45% and 2.36% smaller than at the final steady-state with tax on consumption; and 2.33%, 11.50%, 7.47%, 3.76%, 2.98% and 2.74% smaller than at the final steady-state with the tax on capital income. Overall given these changes in the individual utility levels and the distribution of the population, our results indicate the at the final steady-state with subsidy for fertility at 10% and progressive labour income taxation, the level of welfare decreases by 21.82% below the initial equilibrium. In comparison to our previous results, at the final steady-state with progressive labour income tax scheme, the welfare becomes 0.83% larger than at the final steady-state with flat tax rate on the labour income; but it is 0.49% and 0.96% smaller than at the final steady-state with tax on consumption and capital income, respectively. Finally, with the change in the factor inputs that we have reported earlier, at the final steady-state with the progressive tax rates on the labour income and subsidy for fertility at 10%, the level of output per population of adult households decreases 3.01% below the initial steady-state; and it is 0.38% lower than at the final steady-state with tax on consumption or capital income; but it is 0.08% larger than at the final steady-state with a flat tax rate on the labour income.

4.4.4 Regressive subsidy rate system

In the final part of our discussion, we focus on the influence of the regressive subsidy rates for education and fertility on the long-run economic development. As in our discussion above, however, due to similarity between the transition paths with flat and regressive subsidy rates, we focus on the comparison between the final steady-states that have been considered previously in the chapter three and newly obtained final steady-state with regressive subsidy rates. We begin our discussion with three cases where the flat tax rates on consumption, labour income and capital income finance the regressive subsidy rates for education. We consider that in this subsidy scheme, the households who provide their children with the smallest level of education would receive the largest subsidy rate from the government.

Subsidy for education

At the final steady-state, the subsidy rates for education for each ability group – that are reported by the second column of table 4.10, 4.11 and 4.12, and by the bottom-right sub-plots in the figure 4.9, 4.10 and 4.11 – range between 0.0265, 0.0326 and 0.0401 for fifteenth, fourteenth and thirteenth ability groups, 0.0495, 0.0612 and 0.0758 for twelfth, eleventh and tenth ability groups, 0.0941, 0.1170 and 0.1456 for ninth, eighth and seventh ability groups, 0.1810, 0.2245 and 0.2763 for sixth, fifth and fourth ability groups, and 0.3361, 0.4019 and 0.4706 for third, second and first ability groups when government finances its budget with tax on consumption or capital income, respectively. Given the individual choices that take place when government finances its budget with flat tax rate on the labour income instead, the subsidy rates for education range between 0.0263, 0.0324 and 0.0399 for the fifteenth, fourteenth and thirteenth ability groups, 0.0492, 0.0608 and 0.0754 for twelfth, eleventh and tenth ability groups, 0.0936, 0.1163 and 0.1448 for ninth, eighth and seventh ability groups, 0.1802, 0.2235 and 0.2753 for sixth, fifth and fourth ability groups, and 0.3350, 0.4009 and 0.4697 for third, second and first ability groups, respectively.

To be able to provide these regressive subsidy rates for education and to have the balanced

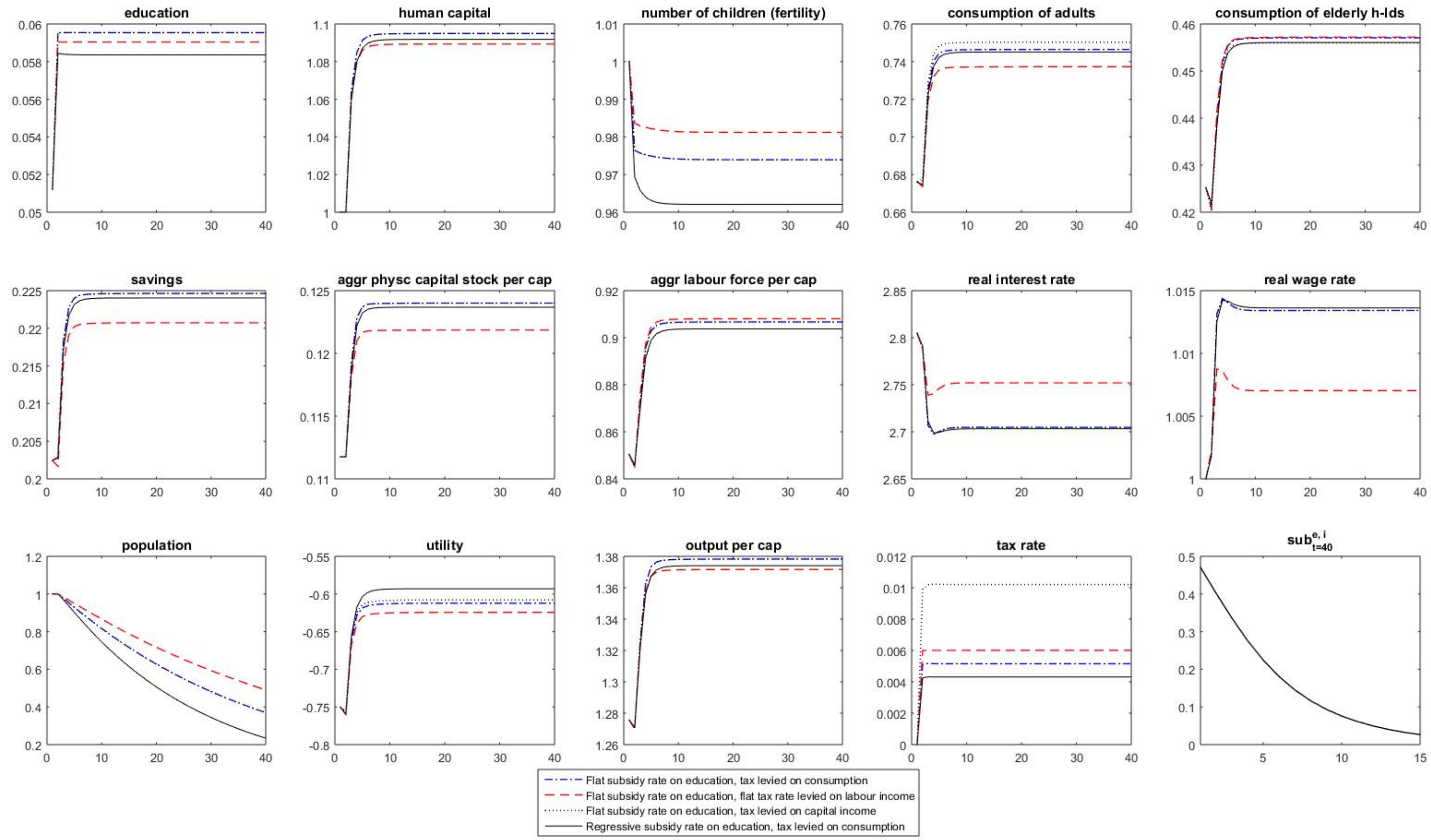

Figure 4.9: Transition path of the model economy with idiosyncratic shocks to the human capital from the perspective of the average household. Government provides subsidy for education and finances its budget with tax on consumption. The subsidy rates are regressive and computed for every ability groups as $sub_t^{e,i} = \chi_t^i * sub^e$ where $\chi_t^i = \bar{e}_t/(\mu + e_t^i)$ and $sub_t^e = 0.1$.

Note: transition path with tax on consumption and flat subsidy rate for education is depicted by blue dash-dotted line; transition path with flat tax rate on labour income and flat subsidy rate for education is presented by red dashed line; transition path with capital income tax and flat subsidy rate for education is shown by black dotted line; transition path with tax on consumption and regressive subsidy rate system for education is illustrated by black solid line

		Y	K	L	r	w	τ^c		
		1.3741	0.1237	0.9037	2.7033	1.0136	0.0043		
		(7.71%)	(10.67%)	(6.27%)	(-3.62%)	(1.36%)			
		$\bar{h}$	tot.pop.	$\bar{e}$	$\bar{n}$	$\bar{c}$	$\bar{s}$	$\bar{d}$	$\bar{u}$
		1.0918	0.2338	0.0584	0.9621	0.7451	0.2240	0.4561	-0.5931
		(9.18%)		(13.94%)	(-3.79%)	(10.19%)	(10.67%)	(7.25%)	(20.95%)
i	sub^e_i	h_i	pop.share.	e_i	n_i	c_i	s_i	d_i	u_i
1	0.4706	0.2019	0.0000	0.0024	1.4831	0.1378	0.0414	0.0843	-2.9390
			(-100%)		(-8.39%)	(0.93%)	(1.36%)	(-1.77%)	(-0.40%)
2	0.4019	0.2466	0.0000	0.0045	1.3960	0.1683	0.0506	0.1030	-2.6541
			(-100%)		(-13.77%)	(0.93%)	(1.36%)	(-1.77%)	(-0.86%)
3	0.3361	0.3012	0.0000	0.0074	1.3095	0.2055	0.0618	0.1258	-2.3698
			(-100%)		(-19.11%)	(0.93%)	(1.36%)	(-1.77%)	(-1.46%)
4	0.2763	0.3679	0.0007	0.0111	1.2279	0.2511	0.0755	0.1537	-2.0855
			(-95.15%)	(193.35%)	(-13.73%)	(0.93%)	(1.36%)	(-1.77%)	(-1.10%)
5	0.2245	0.4493	0.0068	0.0160	1.1547	0.3066	0.0922	0.1877	-1.8008
			(-79.02%)	(61.62%)	(-7.72%)	(0.93%)	(1.36%)	(-1.77%)	(-0.59%)
6	0.1810	0.5488	0.0326	0.0222	1.0916	0.3745	0.1126	0.2292	-1.5151
			(-52.80%)	(28.07%)	(-4.13%)	(0.93%)	(1.36%)	(-1.77%)	(-0.24%)
7	0.1456	0.6703	0.0966	0.0301	1.0388	0.4574	0.1375	0.2800	-1.2282
			(-25.02%)	(13.66%)	(-2.04%)	(0.93%)	(1.36%)	(-1.77%)	(0.02%)
8	0.1170	0.8187	0.1902	0.0399	0.9954	0.5587	0.1680	0.3420	-0.9401
			(-2.35%)	(6.08%)	(-0.86%)	(0.93%)	(1.36%)	(-1.77%)	(0.25%)
9	0.0941	1.0000	0.2511	0.0520	0.9601	0.6824	0.2052	0.4177	-0.6508
			(14.11%)	(1.62%)	(-0.21%)	(0.93%)	(1.36%)	(-1.77%)	(0.54%)
10	0.0758	1.2214	0.2221	0.0670	0.9316	0.8335	0.2506	0.5102	-0.3604
			(25.35%)	(-1.21%)	(0.14%)	(0.93%)	(1.36%)	(-1.77%)	(1.13%)
11	0.0612	1.4918	0.1319	0.0854	0.9085	1.0181	0.3061	0.6231	-0.0691
			(30.55%)	(-3.10%)	(0.30%)	(0.93%)	(1.36%)	(-1.77%)	(6.02%)
12	0.0495	1.8221	0.0524	0.1079	0.8899	1.2434	0.3739	0.7611	0.2229
			(27.05%)	(-4.41%)	(0.36%)	(0.93%)	(1.36%)	(-1.77%)	(2.08%)
13	0.0401	2.2255	0.0134	0.1355	0.8748	1.5187	0.4566	0.9296	0.5157
			(14.74%)	(-5.34%)	(0.37%)	(0.93%)	(1.36%)	(-1.77%)	(0.89%)
14	0.0326	2.7182	0.0019	0.1692	0.8626	1.8550	0.5577	1.1354	0.8089
			(-3.59%)	(-6.03%)	(0.35%)	(0.93%)	(1.36%)	(-1.77%)	(0.56%)
15	0.0265	3.3201	0.0001	0.2105	0.8528	2.2657	0.6812	1.3868	1.1027
			(-37.58%)	(-6.54%)	(0.32%)	(0.93%)	(1.36%)	(-1.77%)	(0.41%)

Table 4.10: Summary for the final steady-state when government provides subsidy for education and finances its budget with tax on consumption. The subsidy rates are regressive and computed for every ability groups as $sub_t^{e,i} = \chi_t^i * sub^e$ where $\chi_t^i = \bar{e}_t/(\mu + e_t^i)$ and $sub_t^e = 0.1$

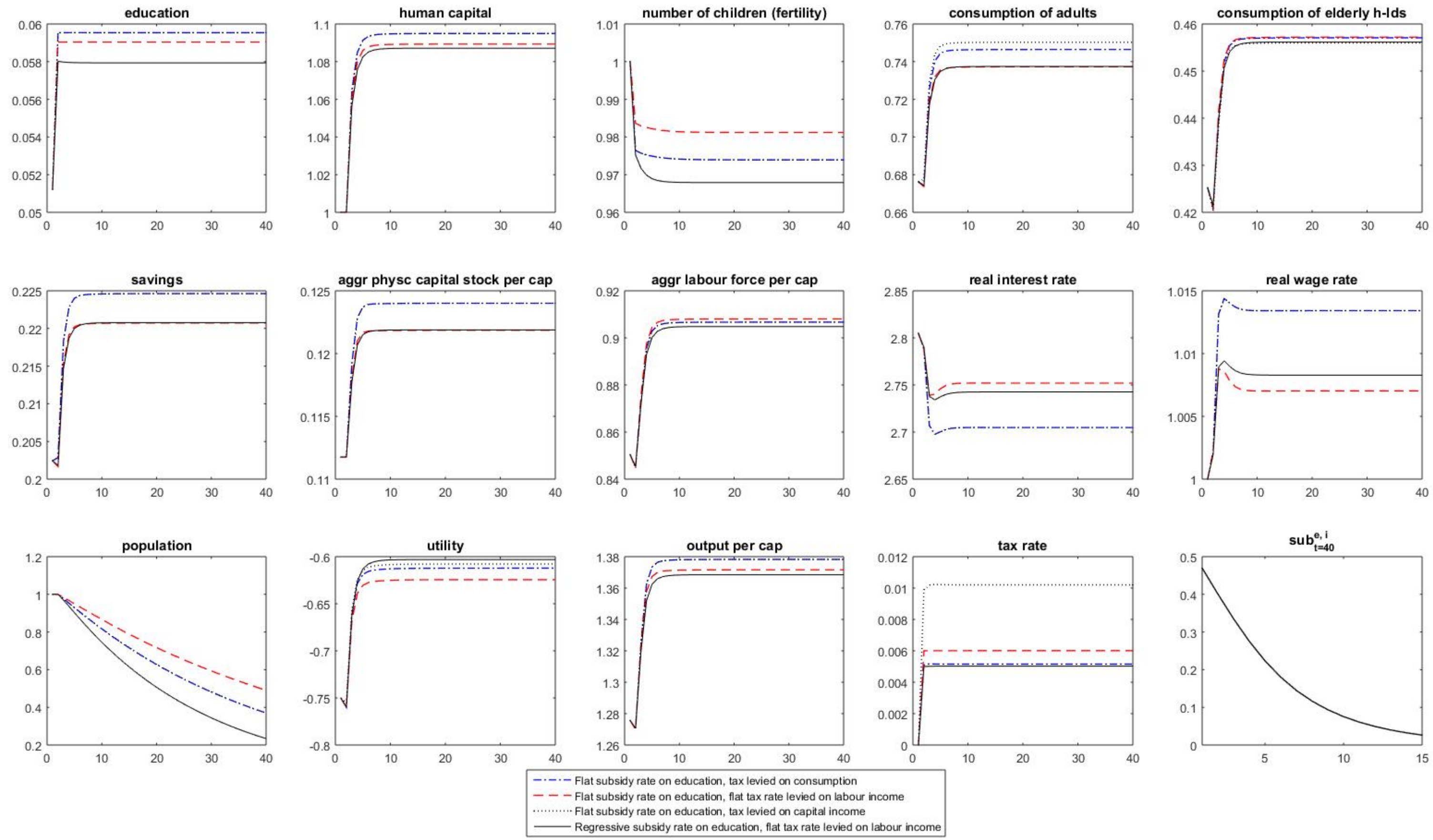

Figure 4.10: Transition path of the model economy with idiosyncratic shocks to the human capital from the perspective of the average household. Government provides subsidy for education and finances its budget with flat tax rate on the labour income. The subsidy rates are regressive and computed for every ability groups as $sub_t^{e,i} = \chi_t^i * sub^e$ where $\chi_t^i = \bar{e}_t/(\mu + e_t^i)$ and $sub_t^e = 0.1$.
Note: transition path with tax on consumption and flat subsidy rate for education is depicted by blue dash-dotted line; transition path with flat tax rate on labour income and flat subsidy rate for education is presented by red dashed line; transition path with capital income tax and flat subsidy rate for education is shown by black dotted line; transition path with flat tax rate on the labour income and regressive subsidy rate system for education is illustrated by black solid line

		Y	K	L	r	w	τ^l		
		1.3685	0.1219	0.9048	2.7426	1.0083	0.0050		
		(7.28%)	(9.06%)	(6.39%)	(-2.22%)	(0.83%)			
		$\bar{h}$	tot.pop.	$\bar{e}$	$\bar{n}$	$\bar{c}$	$\bar{s}$	$\bar{d}$	$\bar{u}$
		1.0871	0.2941	0.0579	0.9679	0.7374	0.2208	0.4562	-0.6031
		(8.71%)		(13.12%)	(-3.21%)	(9.06%)	(9.06%)	(7.28%)	(19.61%)
i	sub_i^e	h_i	pop.share.	e_i	n_i	c_i	s_i	d_i	u_i
1	0.4697	0.2019	0.0000	0.0023	1.4936	0.1370	0.0410	0.0847	-2.9424
			(-100%)		(-7.74%)	(0.32%)	(0.32%)	(-1.32%)	(-0.52%)
2	0.4009	0.2466	0.0000	0.0045	1.4055	0.1673	0.0501	0.1035	-2.6575
			(-100%)		(-13.18%)	(0.32%)	(0.32%)	(-1.32%)	(-0.99%)
3	0.3350	0.3012	0.0000	0.0073	1.3180	0.2043	0.0612	0.1264	-2.3733
			(-100%)		(-18.59%)	(0.32%)	(0.32%)	(-1.32%)	(-1.61%)
4	0.2753	0.3679	0.0007	0.0110	1.2355	0.2495	0.0747	0.1544	-2.0891
			(-94.75%)	(191.39%)	(-13.20%)	(0.32%)	(0.32%)	(-1.32%)	(-1.27%)
5	0.2235	0.4493	0.0072	0.0159	1.1615	0.3048	0.0912	0.1885	-1.8044
			(-78.01%)	(60.84%)	(-7.18%)	(0.32%)	(0.32%)	(-1.32%)	(-0.79%)
6	0.1802	0.5488	0.0336	0.0222	1.0978	0.3723	0.1115	0.2303	-1.5187
			(-51.40%)	(27.60%)	(-3.58%)	(0.32%)	(0.32%)	(-1.32%)	(-0.48%)
7	0.1448	0.6703	0.0984	0.0300	1.0446	0.4547	0.1361	0.2813	-1.2319
			(-23.63%)	(13.34%)	(-1.50%)	(0.32%)	(0.32%)	(-1.32%)	(-0.28%)
8	0.1163	0.8187	0.1921	0.0398	1.0008	0.5554	0.1663	0.3435	-0.9438
			(-1.39%)	(5.84%)	(-0.32%)	(0.32%)	(0.32%)	(-1.32%)	(-0.14%)
9	0.0936	1.0000	0.2515	0.0519	0.9652	0.6783	0.2031	0.4196	-0.6545
			(14.30%)	(1.42%)	(0.32%)	(0.32%)	(0.32%)	(-1.32%)	(-0.03%)
10	0.0754	1.2214	0.2206	0.0669	0.9364	0.8285	0.2480	0.5125	-0.3641
			(24.47%)	(-1.37%)	(0.66%)	(0.32%)	(0.32%)	(-1.32%)	(0.11%)
11	0.0608	1.4918	0.1298	0.0852	0.9132	1.0119	0.3030	0.6260	-0.0729
			(28.47%)	(-3.23%)	(0.82%)	(0.32%)	(0.32%)	(-1.32%)	(0.95%)
12	0.0492	1.8221	0.0511	0.1077	0.8944	1.2360	0.3700	0.7646	0.2192
			(23.96%)	(-4.53%)	(0.879%)	(0.32%)	(0.32%)	(-1.32%)	(0.37%)
13	0.0399	2.2255	0.0130	0.1353	0.8793	1.5096	0.4520	0.9338	0.5119
			(11.04%)	(-5.45%)	(0.885%)	(0.32%)	(0.32%)	(-1.32%)	(0.16%)
14	0.0324	2.7182	0.0018	0.1690	0.8670	1.8439	0.5520	1.1406	0.8052
			(-7.68%)	(-6.13%)	(0.86%)	(0.32%)	(0.32%)	(-1.32%)	(0.10%)
15	0.0263	3.3201	0.0001	0.2103	0.8571	2.2521	0.6742	1.3931	1.0989
			(-40.73%)	(-6.64%)	(0.83%)	(0.32%)	(0.32%)	(-1.32%)	(0.07%)

Table 4.11: Summary for the final steady-state when government provides subsidy for education and finances its budget with flat tax rate on labour income. The subsidy rates are regressive and computed for every ability groups as $sub_t^{e,i} = \chi_t^i * sub^e$ where $\chi_t^i = \bar{e}_t/(\mu + e_t^i)$ and $sub_t^e = 0.1$

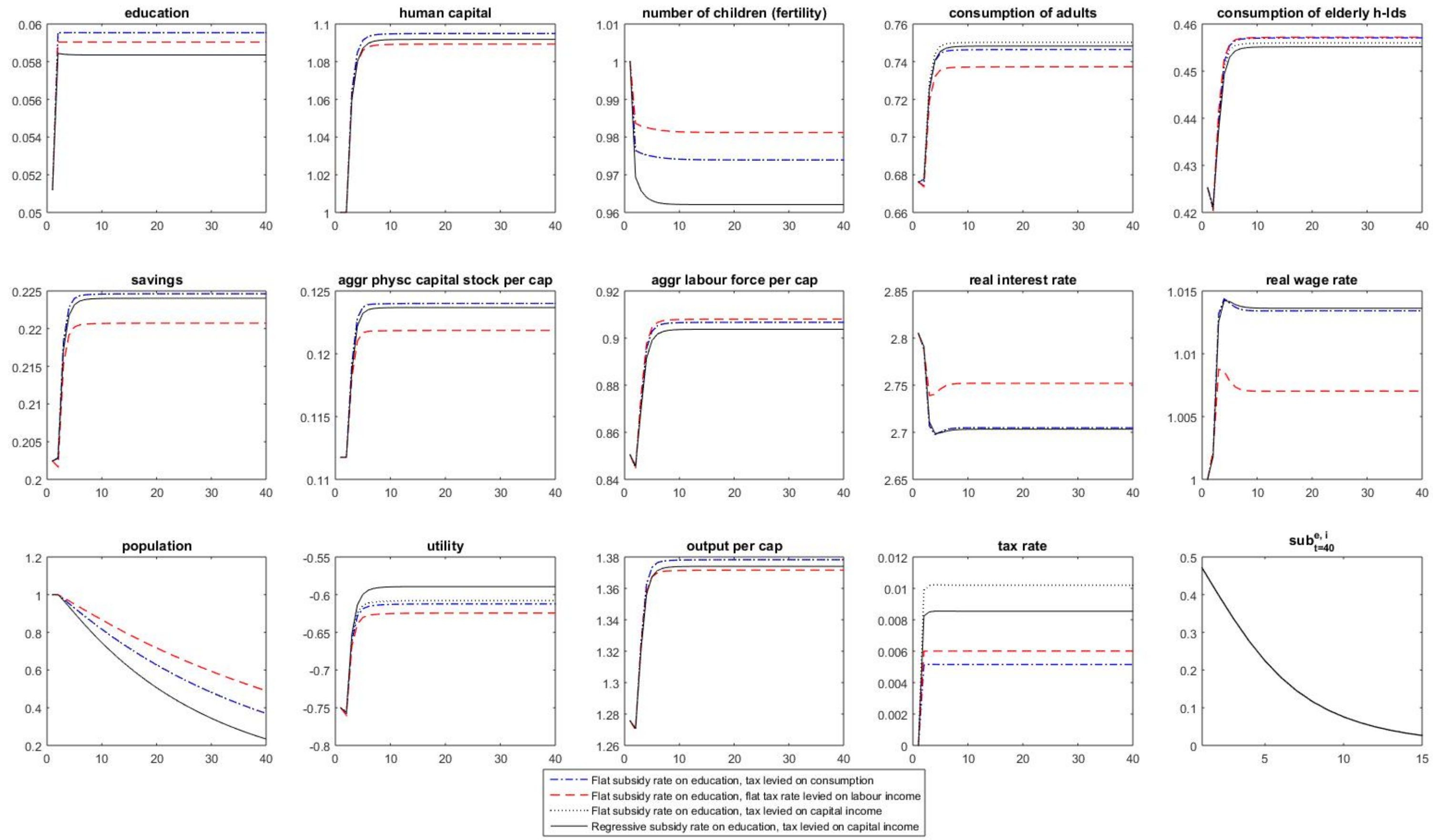

Figure 4.11: Transition path of the model economy with idiosyncratic shocks to the human capital from the perspective of the average household. Government provides subsidy for education and finances its budget with tax on capital income. The subsidy rates are regressive and computed for every ability groups as $sub_t^{e,i} = \chi_t^i * sub^e$ where $\chi_t^i = \bar{e}_t/(\mu + e_t^i)$ and $sub_t^e = 0.1$.

Note: transition path with tax on consumption and flat subsidy rate for education is depicted by blue dash-dotted line; transition path with flat tax rate on labour income and flat subsidy rate for education is presented by red dashed line; transition path with capital income tax and flat subsidy rate for education is shown by black dotted line; transition path with tax on capital income and regressive subsidy rate system for education is illustrated by black solid line

		Y	K	L	r	w	τ^k		
		1.3741	0.1237	0.9037	2.7033	1.0136	0.0085		
		(7.71%)	(10.67%)	(6.27%)	(-3.62%)	(1.36%)			
		$\bar{h}$	tot.pop.	$\bar{e}$	$\bar{n}$	$\bar{c}$	$\bar{s}$	$\bar{d}$	$\bar{u}$
		1.0918	0.2338	0.0584	0.9621	0.7483	0.2240	0.4552	-0.5893
		(9.18%)		(13.94%)	(-3.79%)	(10.67%)	(10.67%)	(7.04%)	(21.44%)
i	sub_i^e	h_i	pop.share.	e_i	n_i	c_i	s_i	d_i	u_i
1	0.4706	0.2019	0.0000	0.0024	1.4831	0.1384	0.0414	0.0842	-2.9353
			(-100%)		(-8.39%)	(1.36%)	(1.36%)	(-1.96%)	(-0.28%)
2	0.4019	0.2466	0.0000	0.0045	1.3960	0.1690	0.0506	0.1028	-2.6503
			(-100%)		(-13.77%)	(1.36%)	(1.36%)	(-1.96%)	(-0.72%)
3	0.3361	0.3012	0.0000	0.0074	1.3095	0.2064	0.0618	0.1256	-2.3661
			(-100%)		(-19.11%)	(1.36%)	(1.36%)	(-1.96%)	(-1.31%)
4	0.2763	0.3679	0.0007	0.0111	1.2279	0.2521	0.0755	0.1534	-2.0818
			(-95.15%)	(193.35%)	(-13.73%)	(1.36%)	(1.36%)	(-1.96%)	(-0.92%)
5	0.2245	0.4493	0.0068	0.0160	1.1547	0.3080	0.0922	0.1873	-1.7971
			(-79.02%)	(61.62%)	(-7.72%)	(1.36%)	(1.36%)	(-1.96%)	(-0.38%)
6	0.1810	0.5488	0.0326	0.0222	1.0916	0.3761	0.1126	0.2288	-1.5114
			(-52.80%)	(28.07%)	(-4.13%)	(1.36%)	(1.36%)	(-1.96%)	(0.002%)
7	0.1456	0.6703	0.0966	0.0301	1.0388	0.4594	0.1375	0.2794	-1.2245
			(-25.02%)	(13.66%)	(-2.04%)	(1.36%)	(1.36%)	(-1.96%)	(0.32%)
8	0.1170	0.8187	0.1902	0.0399	0.9954	0.5611	0.1680	0.3413	-0.9364
			(-2.35%)	(6.08%)	(-0.86%)	(1.36%)	(1.36%)	(-1.96%)	(0.64%)
9	0.0941	1.0000	0.2511	0.0520	0.9601	0.6854	0.2052	0.4169	-0.6471
			(14.11%)	(1.62%)	(-0.21%)	(1.36%)	(1.36%)	(-1.96%)	(1.11%)
10	0.0758	1.2214	0.2221	0.0670	0.9316	0.8371	0.2506	0.5092	-0.3567
			(25.35%)	(-1.21%)	(0.14%)	(1.36%)	(1.36%)	(-1.96%)	(2.15%)
11	0.0612	1.4918	0.1319	0.0854	0.9085	1.0224	0.3061	0.6219	-0.0654
			(30.55%)	(-3.10%)	(0.30%)	(1.36%)	(1.36%)	(-1.96%)	(11.07%)
12	0.0495	1.8221	0.0524	0.1079	0.8899	1.2488	0.3739	0.7596	0.2267
			(27.05%)	(-4.41%)	(0.36%)	(1.36%)	(1.36%)	(-1.96%)	(3.78%)
13	0.0401	2.2255	0.0134	0.1355	0.8748	1.5253	0.4566	0.9278	0.5194
			(14.74%)	(-5.34%)	(0.37%)	(1.36%)	(1.36%)	(-1.96%)	(1.62%)
14	0.0326	2.7182	0.0019	0.1692	0.8626	1.8630	0.5577	1.1332	0.8126
			(-3.59%)	(-6.03%)	(0.35%)	(1.36%)	(1.36%)	(-1.96%)	(1.02%)
15	0.0265	3.3201	0.0001	0.2105	0.8528	2.2755	0.6812	1.3841	1.1064
			(-37.58%)	(-6.54%)	(0.32%)	(1.36%)	(1.36%)	(-1.96%)	(0.74%)

Table 4.12: Summary for the final steady-state when government provides subsidy for education and finances its budget with tax on capital income. The subsidy rates are regressive and computed for every ability groups as $sub_t^{e,i} = \chi_t^i * sub^e$ where $\chi_t^i = \bar{e}_t/(\mu + e_t^i)$ and $sub_t^e = 0.1$

budget, at the final steady-state when the government taxes consumption to finance its budget, the tax rate is set to be equal to 0.0043, which, however, is 16.35% lower than at the previously considered final steady-state with flat subsidy rate for education. When the government finances its budget though the tax labour income instead, the tax rate is equal to 0.0050, which is 16.39% lower than at the final steady-state with flat subsidy rate for education; while, when the government imposes the tax on the capital income, the tax rate is equal to 0.0085 when the economy is at the final steady-state, which is 16.25% lower than at the final steady state with flat subsidy rate for education.

With these regressive subsidy rates and tax level for each corresponding final steady-states, we observe that education attainment from first to third ability groups increases and becomes greater than zero, which previously has only been achieved though provision of the compulsory education and with flat subsidy rate for education of 80% and above, which has been financed with either tax on consumption at a rate of 0.3306 or tax on the capital income at a rate of 0.8242[48]. We still observe, however, that at the final steady-state with tax on consumption or capital income, the education provision of these three ability groups is 2.70%, 1.51% and 0.97% larger than at the final steady-state with flat tax rate on the labour income, due to disincentive that labour income tax rate creates for education provision. We also observe that education attainment of the households from fourth to eighth ability groups becomes 193.35%, 61.62%, 28.07%, 13.66% and 6.08% larger than at initial steady-state when the economy reaches the final steady-state with regressive subsidy rates for education and with tax on consumption or capital income in place. Furthermore, these education provision levels are 62.24%, 35.01%, 19.18%, 9.55% and 3.46% larger than at the final steady-state with flat subsidy rate for education and with the tax on consumption or capital income. For the rest of the population, however, due to increase in the level of relative human capital combined with the subsidy rates for education being below of previously considered level of 0.1, we observe that the with tax on consumption or capital income which finances the regressive subsidy rate program for education, the level of education for the households from tenth to fifteenth ability groups decreases

[48]With the flat tax rate on the labour income, however, due to disincentives that this tax option creates for private choices for education, the education provision by the households from first to third ability groups remained zero for all subsidy rates.

1.21%, 3.10%, 4.41%, 5.34%, 6.03% and 6.54% below original steady-state; and is 3.25%, 5.13%, 6.48%, 7.46%, 8.20% and 8.75% below the final steady-state with flat subsidy rate for education being financed with either tax on consumption or capital income, and 3.12%, 5.01%, 6.37%, 7.35%, 8.09% and 8.64% lower than at the final steady-state with flat subsidy rate for education being financed with labour income taxation. For the households from ninth ability group, however, the education attainment becomes 1.62% larger than at initial steady-state when the government provides regressive subsidy rates for education and finances its budget with tax on consumption or capital income; which, however, is 0.54% smaller than at the final steady-state with flat subsidy rate for education and with tax on consumption or capital income in use, and it is 0.41% smaller than at the final steady-state with flat subsidy rate for education and flat tax rate on the labour income previously considered.

For the case with regressive subsidy rates for education that is financed though labour income taxation, we observe that the level of education of the households from fourth to eighth ability groups increases 191.39%, 60.84%, 27.60%, 13.34% and 5.84% above the original steady-state. Due to disincentive that labour income tax rate creates for education provision, however, these levels are 0.67%, 0.49%, 0.37%, 0.28% and 0.23% lower that at the final steady-state with regressive subsidy rates for education that are financed through taxation on consumption or capital income considered above. In comparison to our previous results, when the government provides regressive subsidy rates for education and taxes the labour income, the education attainment levels of households from fourth to eighth ability groups are 61.99%, 34.70%, 18.89%, 9.29% and 3.24% larger than at the final steady-state with flat subsidy rate for education and taxes on consumption or capital income; and 62.21%, 34.89%, 19.05%, 9.45% and 3.38% larger than at the final steady-state with flat subsidy rate for education and tax levied on the labour income. For the households from the ninth ability group, when the economy reaches the final steady-state and the government provides the regressive subsidy rates for education and finances its budget with flat tax rate on the labour income, the education becomes 1.42% larger than at the original steady-state, which, however, is 0.19% below the final steady-state level with regressive subsidy rate for education and with tax on consumption or capital income in place. Furthermore, at the final steady-state with regressive subsidy rates for education and flat tax rate on the labour income, the education attainment of the households from ninth ability group is 0.60% smaller than at the final steady-state with flat subsidy rate for education and flat tax rate on the labour income; and it is 0.73% smaller than at the final steady-state with flat subsidy rate for education which is financed with tax on consumption or capital income. Lastly, at the final steady-state with regressive subsidy rates for education and flat tax rates on the labour income, the choices for education of the households from tenth to fifteenth ability groups become 1.37%, 3.23%, 4.53%, 5.45%, 6.13% and 6.64% lower than at the initial steady-state without government presence. In comparison to the final steady-state with regressive subsidy rates for education that are financed with either tax on consumption or capital income, the education attainment for the households from these six ability groups is 3.41%, 5.28%, 6.62%, 7.59%, 8.31% and 8.86% lower when the economy reaches the final steady-state with flat tax rate on labour income and regressive subsidy rates for education. Furthermore, at the final steady-state with regressive subsidy rates for education and with the flat tax rate on the labour income, the education provision of the households from tenth to fifteenth ability groups is 3.29%, 5.16%, 6.50%, 7.48%, 8.20% and 8.75% lower than at the previously considered final steady-state with flat subsidy rate for education and flat tax rate on the labour income.

Given these changes in the individual decisions of the households for the education together with the resulted distribution of the population among different ability groups, our results indicate that the average level of education increases 13.94% above the original steady-state when the economy reaches the final steady-state with regressive subsidy rate system for education and when the government finances its budget with tax on consumption or capital income. This result, however, is 1.17% below the average education level at the final steady-state with flat subsidy rate for education that government finances with the flat labour income tax scheme; and it is 2.03% below the final steady-state with flat subsidy rate for education that government finances with taxes imposed either on consumption or capital income. For the final steady-state with regressive subsidy rates for education and flat tax rate on the labour income, the average level of education is 13.12% larger than at the initial steady-state; but it is 0.72% lower than at the final steady-state with regressive subsidy rates for education that government finances with either tax on consumption or capital income. Additionally, in comparison to the final steady-state with flat subsidy rate for education and flat tax rate on the labour income, the average level of education is 1.90% lower when the economy reaches the final steady-state with regressive subsidy rate for education and flat tax rate on the labour income. Finally, the average education level at the final steady-state with regressive subsidy rates for education and flat tax rate on the labour income is 2.77% lower than at the final steady-state with flat subsidy rate for education that government finances with either tax on consumption or tax on capital income.

With these changes in the individual decisions for education combined with the resulted distribution of the population (given the presence of the idiosyncratic shocks to the human capital), our results indicate that the average level of human capital at the final steady-state with regressive subsidy rates for education and tax on consumption or capital income is 9.18% larger than at the initial steady-state without government presence, and it is 0.23% larger than at the final steady-state with flat subsidy rate for education and flat tax rate on the labour income. This resulted level of the average human capital, however, is 0.29% smaller than at the final steady-state with flat subsidy rate for education that government finances with tax on consumption or capital income. In turn, for the final steady-state with regressive subsidy rate for education and flat tax rate on labour income, the average level of human capital is 8.71% above the initial steady-state; but it is 0.20% below the average human capital level at the final steady-state with flat subsidy rate for education and flat tax rate on the labour income. Furthermore this result is 0.43% below the average level of human capital at the final steady-state with regressive subsidy rates for education that government finances with tax on consumption or capital income; and it is 0.72% lower than the average human capital at the final steady-state with flat subsidy rate for education and with government taxing consumption or capital income.

Next, given the subsidy rates for education that households from every ability group receive, and combined with the tax rates that government set at each corresponding steady-state, together with the distribution of the population, the evolution of the human capital and the change in the education provision, our results indicate that the fertility choices of the households from first to eighth ability groups decrease and become 8.39%, 13.77%, 19.11%, 13.73%, 7.72%, 4.13%, 2.04% and 0.86% lower than at the original steady-state, when the economy reaches the final steady-state with regressive subsidy rates for education and taxes imposed either on consumption or capital income. Furthermore, in comparison to our previous results where the economy reaches the final steady-state with flat subsidy rate for education that is financed with the tax on consumption or capital income, the fertility decisions for the households from these eight ability groups are 9.16%, 15.96%, 23.63%, 14.65%, 7.51%, 3.69%, 1.62% and 0.51% lower. Additionally, the fertility decisions of the households from these eight ability groups when the economy reaches the final steady-state with regressive subsidy rates for education and tax levied either on consumption or capital income are 9.82%, 16.66%, 24.38%, 15.42%, 8.21%, 4.35%, 2.26% and 1.14% lower than at the final steady-state with flat subsidy rate for education and flat tax rate on the labour income considered previously. For the households from ninth ability group, when the economy reaches the final steady-state with regressive subsidy rates for education that are financed with tax on consumption or capital income, the fertility decisions become 0.21% lower than at initial steady-state without government intervention. Furthermore, this result, is 0.55% lower than fertility level of the households from ninth ability group at the final steady-state with flat subsidy rate for education and flat tax rate on the labour income; but it is 0.07% above the fertility decisions that this ninth ability group makes at the final steady-state with flat subsidy rate for education that government finances with either tax on consumption or capital income. Finally, for the households from tenth to fifteenth ability groups, when the economy reaches the final steady-state with regressive subsidy rates for education and with tax either on consumption or capital income, the fertility decisions increase 0.14%, 0.30%, 0.36%, 0.37%, 0.35% and 0.32% above the original steady-state. In comparison to the final steady-state where government provides flat subsidy rate for education with tax on consumption or capital income, the fertility choices of the households from these six ability groups are 0.35%, 0.47%, 0.50%, 0.48%, 0.44% and 0.39% larger. However, in comparison to the final steady-state with flat subsidy rate for education and flat tax rate on the labour income, the fertility levels of the households from tenth to fifteenth ability groups are 0.26%, 0.14%, 0.11%, 0.13%, 0.17% and 0.22% lower.

For the final steady state with regressive subsidy rates for education and flat tax rate on the labour income, we observe the fertility responses of lower ability households become relatively larger, when for the higher ability households the fertility choices decrease at a relatively smaller rate than at the final steady-state with regressive subsidy rates for education and with taxation of consumption or capital income. This, as before, is due to a disincentive that tax rate on the labour income creates for education provision, which manifests in relatively larger fertility choices. For instance, our results indicate the fertility levels for the households from first to seventh ability groups decrease 7.74%, 13.18%, 18.59%, 13.20%, 7.18%, 3.58% and 1.50% below the original steady-state, when government provides regressive subsidy rates for education and finances its budget with flat tax rate on the labour income. In comparison to our previous results, the fertility choices for the households from these seven ability groups become 8.39%, 15.18%, 22.83%, 13.95%, 6.88%, 3.11% and 1.07% lower than at the final steady-state with flat subsidy rate for education and tax on consumption or capital income. Also, these results are 9.04%, 15.88%, 23.58%, 14.71%, 7.58%, 3.77% and 1.70% lower than at the final steady-state with flat subsidy rate for education and flat tax rate on the labour income. For the households from eighth and ninth ability groups, the fertility level decreases by 0.32% below– and increases by 0.32% above the initial steady-state, respectively, when the economy reaches the final steady-state with regressive subsidy rates for education and flat tax rate on the labour income. These results are 0.60% and 0.02% lower than at the final steady-state with flat subsidy rate for education and flat tax rate on the labour income; but, they are 0.03% and 0.59% larger than at the final steady-state with flat subsidy rate for education that government finances with either tax on consumption or capital income. For the remaining households from tenth to fifteenth ability groups, we find the fertility choices increase by 0.66%, 0.82%, 0.88%, 0.89%, 0.086% and 0.83% above the initial steady-state without the government presence. In comparison to our previous results, the childbearing decisions of the households from these six ability groups become 0.26%, 0.37%, 0.40%, 0.38%, 0.34% and 0.29% larger than at the final steady-state with flat subsidy rate for education and flat tax rate on the labour income; and 0.87%, 0.98%, 1.01%, 0.99%, 0.94% and 0.89% larger than at the final steady-state with flat subsidy rate for education and tax on consumption or capital income. Finally, in comparison to the final steady-state with regressive subsidy rates for education that government finances with tax on consumption or capital income, the fertility levels for the households from first to fifteenth ability groups are 0.71%, 0.68%, 0.64%, 0.61%, 0.59%, 0.56%, 0.55%, 0.54%, 0.53%, 0.52%, 0.52%, 0.51%, 0.51%, 0.51% and 0.51% larger when government provides regressive subsidy rates for education that it finances with flat tax rate on labour income.

With the changes that we observe in the distribution of the population and in the individual fertility decisions, our results indicate that the average fertility level in the economy at final steady-state with regressive subsidy rates for education that government finances with tax on consumption or capital income becomes 3.79% lower than at the original steady-state without government presence. Furthermore, in comparison to our previous results this average fertility level is 1.23% lower than at the final steady-state with flat subsidy rate for education that government finances with either tax on consumption or capital income; and it is 1.99% smaller than at the final steady-state with flat subsidy rate for education and flat tax rate on the labour income. For the case with regressive subsidy rates for education that the government finances with flat tax rate on the labour income, our results indicate that the average fertility level decreases 3.21% below the original steady-state when the economy reaches the final steady-state. In comparison to our previous results, this indicator is 0.62% below the final steady-state when government provides flat subsidy rate for education and finances its budget with tax on consumption or capital income; and it is 1.37% lower than at the final steady-state with flat subsidy rate for education and flat tax rate on the labour income. Finally, according to our results, the average fertility level at the final steady-state with regressive subsidy rates for education and flat tax rate on the labour income is 0.60% larger than at the final steady-state with regressive subsidy rates for education that government finances with either tax on consumption or capital income.

Additionally, due to observed changes in the distribution of the population and fertility levels among members of different ability groups, when the government provides regressive subsidy rates for education with tax on consumption or capital income, at the fortieth period of analysis, when the economy is at the final steady-state, the population becomes 327.69% smaller than at the original steady-state. The resulted population size is 58.23% smaller than with flat subsidy rate for education and tax on consumption or capital income; and it is 109.85% smaller than with the flat subsidy rate for education and flat tax rate on the labour income considered previously. For the case with regressive subsidy rates for education and flat tax rate on the labour income, we find that the population size at the fortieth period of analysis decreases 240.00% below the initial steady-state, which in comparison to the previous cases is 25.79% lower than at the final steady-state with flat subsidy rate for education which government finances with either tax on consumption or capital income; and it is 66.82% smaller than at the final steady-state with flat subsidy rate for education and flat tax rate on the labour income. It should be noted, however, that even when the economy is at its final steady-state given the government policy option in place, the population would continue to decrease in its size. Finally, at the fortieth period of analysis, the population size with regressive subsidy rates for education and flat tax rate on the labour income is 20.50% larger than with the regressive subsidy rates for education that government finances with either tax on consumption or capital income.

Given the changes in the individual decisions of the adult households for education provisions and fertility, together with the changes in the distribution of the population between different ability groups, in the individual and average human capital, and in the population size, we observe that the level of effective labour force per adult households becomes 6.27% larger than at the original steady-state, when the government provides regressive subsidy rates for education and finances its budget with tax on consumption or capital income. In comparison to the previous result, however, when the economy reaches the final steady-state with regressive subsidy rate for education and with tax either on consumption or capital income, the size of effective labour force per adult households is 0.33% smaller than at the final steady-state with flat subsidy rate for education that government finances with tax on consumption or capital income; and it is 0.48% smaller than at the final steady-state with flat subsidy rate on education and flat tax rate on the labour income. Next, for the case with regressive subsidy rates for education and flat tax rate on the labour income, we observe that as the economy reaches the final steady-state, the level of the effective labour per adult households becomes 6.39% larger than at the original steady-state without the government presence. In comparison to the previous results, however, with the regressive subsidy rates for education and flat tax rate on the labour income, the effective labour force per adult households is 0.20% smaller than at the final steady-state with flat subsidy rate for education and with taxes imposed either on consumption or capital income; and it is 0.35% smaller than at the final steady-state with flat subsidy rate for education and flat tax rate on the labour income. Finally, our results show that at the final steady-state the size of the effective labour force per adult households with regressive subsidy rates for education and flat tax rate on the labour income is 0.12% larger than with regressive subsidy rates for education and tax on either consumption or capital income.

For the size of the physical capital stock per adult households, which follows the aggregate level of saving of the households and the population size, when the economy reaches the final steady-state with regressive subsidy rates for education and government taxing either consumption or capital income, this indicator becomes 10.67% larger than at the initial steady-state. Furthermore, this level is 1.47% larger than at the final steady-state with flat subsidy rate for education and flat tax rate on labour income. However, the physical capital stock per adult households at the final steady-state with regressive subsidy rates for education and tax levied on either consumption or capital income is 0.24% below the final steady-state level with flat subsidy rate for education and with government taxing either consumption or capital income. Moving forward, for the final steady-state with regressive subsidy rates for education and flat tax rate on the labour income, we observe that the physical capital stock per adult households becomes 9.06% larger than at the original steady-state. In comparison to the previous results, this factor input is 0.02% larger that at the final steady-state with flat subsidy rate on education and flat tax rate on the labour income; but it is 1.74% smaller than at the final steady-state with flat subsidy rate for education and tax on consumption or capital income. Finally, due to disincentive that labour income tax rate creates for individual decisions of the households to save[49], the size of the physical capital stock per adult households at the final steady-state with regressive subsidy rates for education and with tax either on consumption or capital income is found to be 1.47% larger than at the final steady-state with regressive subsidy rates for education and flat tax rate on the labour income.

With these changes in the factor inputs, we observe that the real wage rate increases by 1.36% above the original steady-state, when the economy reaches the final steady-state with regressive subsidy rates for education and with tax on consumption or capital income. In com-

[49]Combined with the differences in the distribution of the population and fertility levels

parison to our previous results, the real wage rate becomes 0.02% larger than at the final steady-state with flat subsidy rate for education and tax on consumption or capital income; and it is also 0.65% larger than at the final steady-state with flat subsidy rate for education and flat tax rate on the labour income. For the final steady-state with the regressive subsidy rates for education and flat tax rate on the labour income, our results indicate that the wage rate increases 0.83% above the original steady-state, and it becomes 0.12% larger than at the final steady-state with flat subsidy rate for education and flat tax rate on labour income previously considered. However, in comparison to the final steady-state with flat subsidy rate for education and taxes being levied either on consumption or capital income, the real wage rate is found to be 0.51% lower. Lastly, our results show that the real wage rate at the final steady-state with regressive subsidy rates for education and tax introduced either for consumption or capital income is 0.53% larger than at the final steady-state with regressive subsidy rates for education and flat tax rate on the labour income.

For the real interest rate, as the economy reaches the final steady-state with regressive subsidy rates for education and government budget financed either with tax on consumption or capital income, it becomes 3.62% lower than at the original steady-state. This level for the real interest rate is 0.05% lower than at the final steady-state with flat subsidy rate for education and tax levied either on consumption or capital income; and also it is 1.80% lower than at the final steady-state with flat subsidy rate for education and flat tax rate on the labour income previously considered. In the case with regressive subsidy rates for education that government finances with the flat tax rate on the labour income, the real interest rate decreases 2.22% below the original equilibrium when the economy reaches the final steady-state. Furthermore, the real interest rate becomes 0.34% lower than at the final steady-state with flat subsidy rate for education and flat tax rate on the labour income; but it is 1.38% larger than at the final steady-state with flat subsidy rate for education and flat tax rate on the labour income. Overall, our results suggest that at the final steady-state with regressive subsidy rates for education flat tax rate on the labour income, the real interest rate is 1.43% larger than at the final steady-state with regressive subsidy rates for education that government finances with tax on consumption or capital income.

With the change in the factor prices, and given the resulted tax rates and the distribution of the population among different ability groups, our results indicate that when the economy reaches the final steady-state with regressive subsidy rates for education and tax imposed on consumption, the consumption of adult households at individual ability levels becomes 0.93% larger than at the original steady-state; and it is 0.10% larger than at the final steady-state with flat subsidy rate for education and tax on consumption and 0.82% larger than at the final steady-state with flat subsidy rate for education and flat tax rate on the labour income, but this level is 0.41% smaller than at the final steady-state with flat subsidy rate for education and tax on capital income considered previously. Additionally, in comparison to other final steady-states with regressive subsidy rates for education, the consumption of adult households at the individual ability levels with tax on consumption in place is 0.60% larger than at the final steady-state with flat tax rate on the labour income, but it is 0.43% smaller than at the final steady-state with tax on capital income. For the final steady-state with regressive subsidy rates for education and flat tax rate on the labour income, the consumption of adult households at the individual ability levels increases 0.32% above the original steady-state; and it becomes 0.22% larger than at the final steady-state with flat subsidy rate for education at flat tax rate on the labour income, but it is 0.50% and 1.02% smaller than at the final steady-state with regressive subsidy rates for education that government finances with tax on the labour income and the tax on the capital income, respectively. Finally, when the economy reaches the final steady-state with regressive subsidy rates for education and tax on capital income, the consumption of adult households at the individual ability levels becomes 1.36% larger than at the original steady-state without the government presence. Furthermore, this level is 0.02%, 0.53% and 1.25% larger than at the final steady-state with government providing the flat subsidy rate for education of 10% and taxing capital income, consumption and labour income, respectively.

Thereby, with these changes in consumption of adult households at the individual ability levels and given the resulted distribution of the population, the average consumption of adult households becomes 10.19% larger than at the original steady-state when the economy reaches the final steady-state with regressive subsidy rates for education and tax on consumption. In addition, in comparison to our previous results, this indicator becomes 1.05% larger than at the final steady-state with flat subsidy rate for education and flat tax rate on the labour income. However, in comparison to our previous results with flat subsidy rates for education that government finances with tax on consumption, the average consumption of adults households at the final steady-state with regressive subsidy rates for education and tax on consumption is 0.18% smaller. Furthermore, it is 0.70% smaller than at the final steady-state with flat subsidy rate for education and tax on capital income. Moving forward, our results indicate that at the final steady-state with regressive subsidy rates for education and flat tax rate on the labour income, the average consumption of the adult households becomes 9.06% larger than at the original steady-state without the government presence. According to our results, this indicator is 0.02% larger than at the final steady-state with flat subsidy rate for education and flat tax rate on the labour income; but it is 1.22% and 1.74% smaller than at the final steady-state where government provides flat subsidy rate for education and finances its budget with the tax on consumption and with the tax on capital income, respectively. Next, for the final steady-state with regressive subsidy rates for education and tax on the capital income, the average consumption of adult households is 10.67% larger than at the initial steady-state; and it is 0.25% and 1.47% larger than at the previously considered final steady-state with flat subsidy rate for education and tax on consumption and tax on capital income, respectively. However, this indicator is 0.27% smaller than at the final steady-state with flat subsidy rate for education and tax on capital income. Additionally, based on our results, the average consumption of adult households at the final steady-state with regressive subsidy rates for education and tax on capital income is 0.43% and 1.45% larger than at the final steady-state where government provides regressive subsidy rates for education and finances its budget with tax on consumption and with tax on labour income, respectively. Finally, the dynamics for saving of households both at the individual and average levels with regressive subsidy rates for education that government finances with either tax on consumption or capital income is identical to the behaviour of consumption when government provides regressive subsidy rates for education and taxes capital income. For the case with regressive subsidy rates for education and tax on the labour income, the dynamics of savings both at the individual and average levels follow the one for consumption of the adult households.

In addition, with the resulted factor prices, tax rates and distribution of the population among different ability groups, our results indicate that at the final steady-state with regressive subsidy rates for education and tax on consumption, the consumption of the elderly households at the individual ability level becomes 1.77% lower than at the original steady-state without government presence. In comparison to our previous results, we find that this indicator is 0.06% and 0.30% larger than at the final steady-state with flat subsidy rate for education and when government taxes consumption and capital income, respectively. In comparison to the final steady-state with flat subsidy rate for education and flat tax rate on the labour income, however, this value is 0.48% lower. Next, when government provides regressive subsidy rates for education and taxes labour income, at the final steady-state the consumption of the elderly households at the individual level becomes 1.32% lower than at the original steady state. In comparison to our previous results, this level is found to be 0.52% and 0.75% larger than at the final steady-state with flat subsidy rate for education when government taxes consumption and capital income, respectively. This indicator, however, is 0.03% lower than at the final steady-state with flat subsidy rate for education and tax on the labour income. Finally, our result suggest that at the final steady-state with regressive subsidy rates for education and tax on capital income, the consumption of the elderly households at the individual ability level becomes 1.96% smaller than at the original steady-state. This result, however, is 0.10% larger than at the final steady-state with flat subsidy rate for education and flat tax on capital income; but it is 0.13% and 0.68% lower at the final steady-state with flat subsidy rate for education when government taxes consumption and labour income, respectively. Overall, our results indicate that consumption of the elderly households at the individual ability level at the final steady state with regressive subsidy rates for education and tax on capital income is 0.20% and 0.65% below the final steady state with regressive subsidy rates for education when government levies the tax on consumption and labour income, respectively.

Despite the decrease in the consumption of the elderly households at the individual ability level, due to the change in the distribution of the population among different ability groups as a result of the government policy, the average consumption of the elderly households increases 7.25% above the original steady-state when economy reaches the final steady-state with regressive subsidy rates for education and tax on consumption. In comparison to our previous results, this indicator is 0.01% larger than at the final steady-state with flat subsidy rate for education and tax on capital income, but it is 0.22% and 0.25% smaller than at the final steady-state with flat subsidy rate for education which government finances with tax on consumption and labour income, respectively. Next, when the economy reaches the final steady-state with regressive subsidy rate for education and flat tax rate on the labour income, the average consumption of the elderly households increases 7.28% above the original steady-state, and becomes 0.04% larger than at the final steady-state with flat subsidy rate for education and tax on the capital income considered previously. This indicator, however, is 0.20% and 0.23% smaller that the average consumption of the elderly households at the final steady-state with flat subsidy rate for education with tax on consumption and tax on labour income, respectively. Finally, the average consumption of the elderly households at the final steady-state with regressive subsidy rates for education and tax on capital income becomes 7.04% larger than at the original steady-state, but it is 0.18%, 0.42% and 0.45% lower than at the final steady-state with flat subsidy rate for education and tax levied on capital income, consumption and labour income, respectively. Overall, our results indicate that at the final steady-state with regressive subsidy rates for education and tax on capital income, the average consumption of the elderly households is 0.20% and 0.22% lower than at the final steady-state with regressive subsidy rates for education that government finances with tax on consumption and labour income, respectively.

In conclusion, we observe that at the final steady-state with regressive subsidy rates for education and tax on consumption, the utility of the households from first to sixth ability groups decreases by 0.40%, 0.86%, 1.46%, 1.10%, 0.59% and 0.24% below the original steady-state. In comparison to the previous results, the utility for these six ability groups is 0.34%, 0.79%, 1.37%, 0.91%, 0.41% and 0.06% lower than at the final steady-state with flat subsidy rate for education and flat tax rate on the labour income; 0.49%, 0.96%, 1.56%, 1.12%, 0.65% and 0.35% lower than at the final steady-state with flat subsidy rate for education and tax on consumption; and 0.64%, 1.12%, 1.74%, 1.33%, 0.90% and 0.64% lower than at the final steady-state with flat subsidy rate for education and tax on capital income. For the households from the seventh ability group, the utility level increases 0.02% above the original steady-state when the economy reaches the final steady-state with regressive subsidy rates for education and with tax on consumption. This utility level is 0.23% larger than at the final steady-state with flat subsidy rate for education and tax on the labour income; but it is 0.14% and 0.50% lower than at the final steady-state with flat subsidy rate for education that government finances with tax on consumption and tax on the capital income, respectively. For the households from eighth ability group, the utility level increases 0.25% above the original steady-state when the economy reaches the final steady-state with regressive subsidy rates for education and tax on consumption. Furthermore, the utility of this ability group is 0.51% and 0.03% larger than at the final steady-state when government provides flat subsidy rate for education (of 10%) and finances its budget with tax on labour income and tax on consumption, respectively. In comparison to our previous results for the final steady-state with flat subsidy rate for education and tax on capital income, however, the utility level of the households from eighth ability groups is found to be 0.44% lower. Moving forward, as the economy reaches the final steady-state with regressive subsidy rates for education and tax on consumption, the utility for the households from ninth to fifteenth ability groups becomes 0.54%, 1.13%, 6.02%, 2.08%, 0.89%, 0.56% and 0.41% larger than at the original steady-state without the government presence. In comparison to our previous results, the utility for households from these seven ability groups is 0.90%, 1.76%, 9.50%, 2.97%, 1.28%, 0.81% and 0.58% larger than at the final steady-state with flat subsidy rate for education and flat tax rate on the labour income; and 0.21%, 0.52%, 3.00%, 0.95%, 0.41%, 0.25% and 0.18% larger than at the final steady-state with flat subsidy rate for education and tax on consumption; but, these results are 0.47%, 0.71%, 3.42%, 1.04%, 0.45%, 0.30% and 0.23% lower than at the final steady-state with flat subsidy rate for education and tax on capital income.

Next, given the choices that take place at the final steady-state with regressive subsidy rates for education that government finances with flat tax rate on the labour income, we observe the following changes in the utility levels of the households at the individual ability level. First, for the households from first to seventh ability groups, the utility becomes 0.52%, 0.99%, 1.61%, 1.27%, 0.79%, 0.48% and 0.28% lower than at the original steady-state when the economy reaches the final steady-state. Furthermore, in comparison to our previous results, the utility of the households from these seven ability groups is 0.45%, 0.91%, 1.51%, 1.08%, 0.61%, 0.29% and 0.07% lower that at the final steady-state with flat subsidy rate for education and flat tax rate on the labour income; 0.61%, 1.08%, 1.70%, 1.29%, 0.85%, 0.59% and 0.43% lower than at the final steady-state with flat subsidy rate for education and tax on consumption; and 0.76%, 1.25%, 1.89%, 1.50%, 1.10%, 0.88% and 0.79% lower than at the final steady-state with flat subsidy rate for education and tax on the capital income.

For the households from eighth and ninth ability groups, when the economy reaches the final steady-state with regressive subsidy rates for education and flat tax rate on the labour income, the utility is found to decrease by 0.14% and 0.03% below the original steady-state; which in comparison to our previous results is 0.358% and 0.360% below the final steady-state with flat subsidy rate for education and tax rate on consumption, and 0.83% and 1.04% below the final steady-state with flat subsidy rate for education and tax on capital income, but 0.12% and 0.32% above the final steady rate with flat subsidy rate for education and flat tax rate on the labour income, respectively.

For the households from tenth to fifteenth ability groups, the level of utility at the final steady-state with regressive subsidy rates for education and flat tax rate on the labour income becomes 0.11%, 0.95%, 0.37%, 0.16%, 0.10% and 0.07% above the original steady-state. As a result, the utility for the households from these six ability groups becomes 0.72%, 3.89%, 1.32%, 0.56%, 0.34% and 0.24% larger than at the final steady-state with flat subsidy rate for education and flat tax rate on the labour income; but it is 0.51%, 2.27%, 0.73%, 0.32%, 0.21% and 0.17% lower than at the final steady-state with flat subsidy rate for education and tax on consumption; and 1.73%, 8.36%, 2.76%, 1.19%, 0.77% and 0.57% below the final steady-state with flat subsidy rate for education and tax on the capital income.

For the final steady-state with regressive subsidy rates for education and tax on capital income, the level of utility for the households from first to fifth ability groups is found to decrease by 0.28%, 0.72%, 1.31%, 0.92%, and 0.38% below the original steady-state. In relation to our previous results, we find that the utility level for these five ability groups at the final steady-state with regressive subsidy rates for education and tax on capital income is 0.21%, 0.65%, 1.21%, 0.73% and 0.20% smaller than at the final steady-state with flat tax rate for education and flat tax rate on the labour income; it is 0.37%, 0.82%, 1.40%, 0.94% and 0.45% smaller than at the final steady-state with flat subsidy rate for education and tax on consumption; and 0.52%, 0.98%, 1.59%, 1.16% and 0.70% smaller than at the final steady-state with flat subsidy rate for education and tax on the capital income.

Next, for the households from sixth ability group, as the economy reaches the steady-state with regressive subsidy rates for education and tax on the capital income, the utility is found to increase by 0.002% above the original steady-state. In comparison to the previous results, the utility for the households from the sixth ability group at the final steady-state with regressive subsidy rates for education and with tax on the capital income is 0.19% larger than at the final steady-state with flat subsidy rate for education and tax on the labour income; but it is 0.10% and 0.40% smaller than at the final steady-state when government provides flat subsidy rate for education (of 10%) and taxes consumption and capital income, respectively.

In the case of the households from seventh and eighth ability groups, at the final steady-state with regressive subsidy rates for education and tax on the capital income, the utility becomes 0.32% and 0.64% larger than at the original steady-state, respectively. In comparison to the final steady-state with flat subsidy rate for education and tax on the labour income, the utility level for these two ability groups becomes 0.53% and 0.91% larger when government provides regressive subsidy rates for education and finances its budget with tax on the capital income. The utility for the households from seventh and eighth ability groups is also 0.17% and 0.43% larger than at the final steady-state with flat subsidy rate for education and tax on consumption; but it is 0.19% and 0.04% smaller than at the final steady-state with flat subsidy rate for education and tax on the capital income.

Lastly, for the remaining households from ninth to fifteenth ability groups, the utility increases by 1.11%, 2.15%, 11.07%, 3.78%, 1.62%, 1.02% and 0.74% above the original steady-state when the economy reaches the final steady-state with regressive subsidy rates for education and tax on capital income. Furthermore, in comparison to our previous results, at the final steady-state with regressive subsidy rates for education and tax on capital income, the utility for the households from these seven ability groups is 1.47%, 2.82%, 15.71%, 4.56%, 1.98%, 1.26% and 0.92% larger than at the final steady-state with flat subsidy rate for education and flat tax rate on labour income; 0.78%, 1.56%, 8.85%, 2.58%, 1.12%, 0.71% and 0.51% larger than at the final steady-state with flat subsidy rate for education and tax on consumption; and 0.10%, 0.32%, 2.07%, 0.62%, 0.26%, 0.16% and 0.11% larger than at the final steady-state with flat subsidy rate for education and tax on capital income.

Finally, our results also indicate that at the final steady-state with regressive subsidy rates for education and tax on the capital income the utility for the households from first to fifteenth ability group is 0.24%, 0.27%, 0.30%, 0.35%, 0.41%, 0.49%, 0.60%, 0.79%, 1.15%, 2.08%, 11.38%, 3.29%, 1.43%, 0.92% and 0.67% larger than at the final steady-state with regressive subsidy rates for education and flat tax rate on the labour income; and 0.13%, 0.14%, 0.16%, 0.18%, 0.21%, 0.25%, 0.30%, 0.40%, 0.57%, 1.04%, 5.68%, 1.64%, 0.72%, 0.46% and 0.34% larger that at the final steady-state with regressive subsidy rates for education and tax on consumption.

With these changes in the utility at the individual ability levels combined with the change in the distribution of the population, our results indicate that when the economy reaches the final steady-state with regressive subsidy rates for education and tax on consumption, the level of welfare – which is given as the average utility level in the economy – increases 20.95% above the original steady-state; and becomes 5.29%, 3.23% and 2.49% larger than at the final steady-state with flat subsidy rate for education (of 10%) that government finances with tax on labour income, tax on consumption and tax on the capital income, respectively. When the economy reaches the final steady-state with regressive subsidy rates for education and flat tax rate on the labour income, instead, the level of welfare becomes 19.61% larger than at the original steady-state; and it is 3.54%, 1.52% and 0.78% larger than at the final steady-state with flat subsidy rate for education and tax on labour income, consumption and capital income, respectively. Finally, when the economy reaches the final steady-state with regressive subsidy rates for education and tax on the capital income, the level of welfare increases 21.44% above the original steady-state, and becomes 5.95%, 3.88% and 3.13% larger than at the final steady state where government provides flat subsidy rate for education and taxes labour income, consumption and capital income, respectively. We also observe that the level of welfare at the final steady-state with regressive subsidy rates for education and tax on capital income is

0.63% larger than at the final steady-state with regressive subsidy rates for education and tax on consumption, and 2.33% larger than at the final steady-state with regressive subsidy rates for education and flat tax rate on the labour income.

Finally, given the resulted levels of factor inputs per adult household, our results indicate that at the final steady-state with regressive subsidy rates for education that government finances with either tax on consumption or capital income, the level of output per adult household becomes 7.71% larger than at initial steady-state. This indicator is also 0.18% larger than at the final steady-state with flat subsidy rate for education and flat tax rate on the labour income; but it is 0.31% smaller than at the final steady-state with flat subsidy rate for education that government provides with either tax on consumption or tax on capital income. For the final steady-state with regressive subsidy rates for education and flat tax rate on the labour income, the level of output per adult household becomes 7.28% larger than at the original steady-state. In comparison to the previously considered results, however, this indicator is 0.23% below the final steady-state with flat subsidy rate for education and flat tax rate on the labour income; and 0.82% below the final steady-state with flat subsidy rate for education that government finances with either tax on consumption or tax on capital income. In conclusion, according to our results, the level of output per adult household at the final steady-state with regressive subsidy rates for education and with tax on consumption or capital income is found to be 0.41% larger than at the final steady-state with regressive subsidy rates for education and flat tax rate on the labour income.

Subsidy for fertility

As a final part of our discussion, we consider the influence of the regressive subsidy rates for fertility[50] which government finances with either tax on consumption, labour income or capital income. Identically to our discussion above with regressive subsidy rates for education, due to similarity in the transition paths that the economy experiences with flat and regressive subsidy rates for fertility, we focus our discussion on the comparison between the resulted and previously considered final steady-states with government support for childbearing decisions.

[50] Under this subsidy program, the households who decide to have less children receive larger subsidy rate for fertility

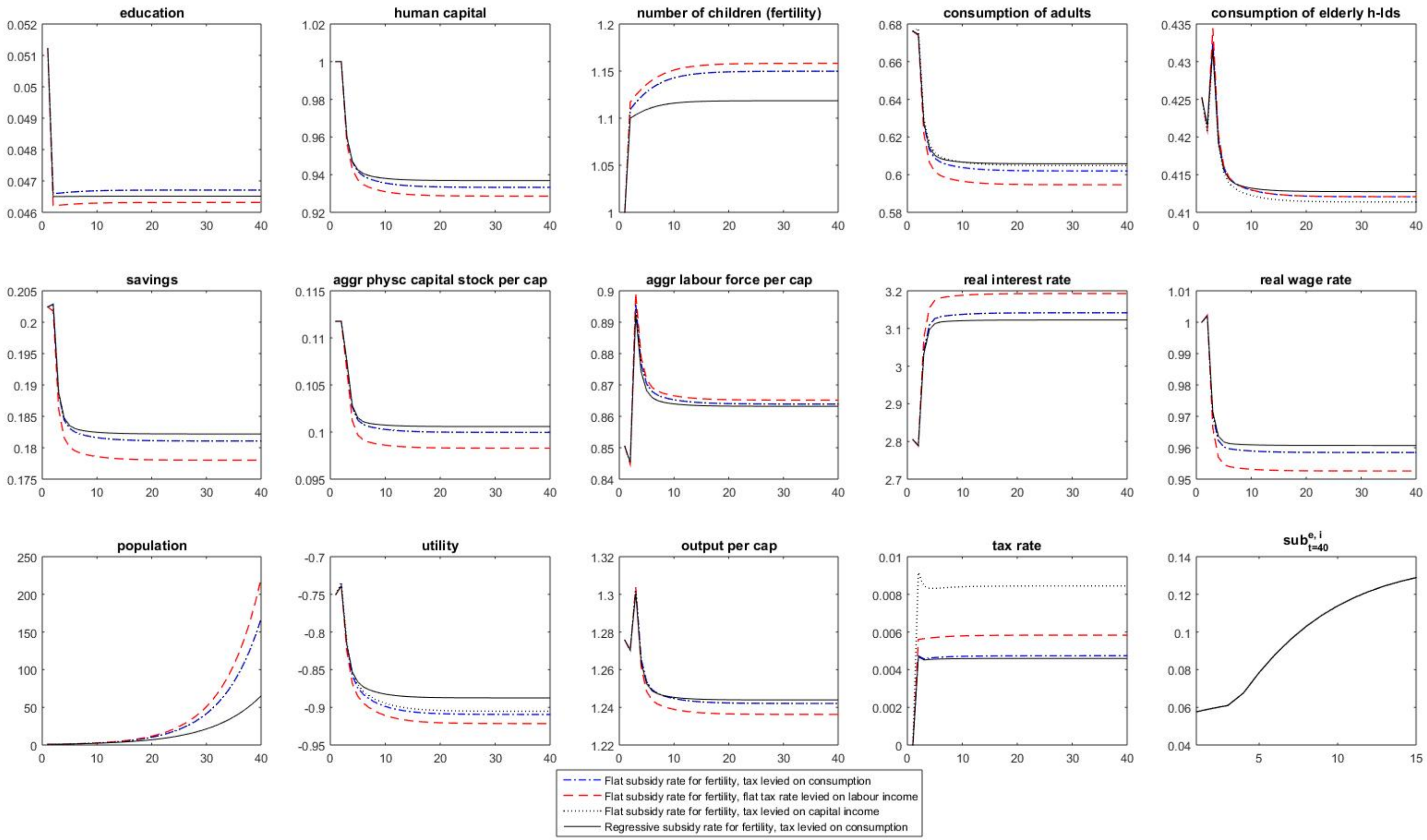

Figure 4.12: Transition path of the model economy with idiosyncratic shocks to the human capital from the perspective of the average household. Government provides subsidy for fertility and finances its budget with tax on consumption. The subsidy rates are regressive and computed for every ability groups as $sub_t^{n,i} = \psi_t^i * sub^e$ where $\psi_t^i = \bar{n}_t / n_t^i$ and $sub_t^n = 0.1$.
Note: transition path with tax on consumption and flat subsidy rate for fertility is depicted by blue dash-dotted line; transition path with flat tax rate on labour income and flat subsidy rate for fertility is presented by red dashed line; transition path with capital income tax and flat subsidy rate for fertility is shown by black dotted line; transition path with tax on consumption and regressive subsidy rate system for fertility is illustrated by black solid line

		Y	K	L	r	w	τ^c		
		1.2439	0.1006	0.8632	3.1224	0.9607	0.0046		
		(-2.49%)	(-10.00%)	(1.50%)	(11.32%)	(-3.93%)			
		$\bar{h}$	tot.pop.	$\bar{e}$	$\bar{n}$	$\bar{c}$	$\bar{s}$	$\bar{d}$	$\bar{u}$
		0.9368	65.25	0.0465	1.1185	0.6058	0.1822	0.4127	-0.8875
		(-6.32%)		(-9.18%)	(11.85%)	(-10.41%)	(-10.00%)	(-2.93%)	(-18.29%)
i	sub_i^n	h_i	pop.share.	e_i	n_i	c_i	s_i	d_i	u_i
1	0.0576	0.2019	0.0006	0.0000	1.9408	0.1306	0.0393	0.0890	-2.9286
			(208.43%)	—	(19.88%)	(-4.37%)	(-3.93%)	(3.61%)	(-0.05%)
2	0.0594	0.2466	0.0046	0.0000	1.8824	0.1595	0.0480	0.1086	-2.6383
			(207.51%)	—	(16.28%)	(-4.37%)	(-3.93%)	(3.61%)	(-0.26%)
3	0.0610	0.3012	0.0150	0.0000	1.8347	0.1948	0.0586	0.1327	-2.3471
			(178.01%)	—	(13.33%)	(-4.37%)	(-3.93%)	(3.61%)	(-0.50%)
4	0.0676	0.3679	0.0314	0.0025	1.6547	0.2379	0.0715	0.1621	-2.0699
			(123.25%)	(-33.86%)	(16.25%)	(-4.37%)	(-3.93%)	(3.61%)	(-0.34%)
5	0.0785	0.4493	0.0545	0.0085	1.4257	0.2905	0.0874	0.1980	-1.8008
			(67.17%)	(-13.90%)	(13.94%)	(-4.37%)	(-3.93%)	(3.61%)	(-0.59%)
6	0.0879	0.5488	0.0902	0.0160	1.2725	0.3549	0.1067	0.2418	-1.5255
			(30.40%)	(-7.56%)	(11.76%)	(-4.37%)	(-3.93%)	(3.61%)	(-0.93%)
7	0.0960	0.6703	0.1403	0.0254	1.1647	0.4334	0.1304	0.2953	-1.2456
			(8.89%)	(-4.07%)	(9.83%)	(-4.37%)	(-3.93%)	(3.61%)	(-1.40%)
8	0.1030	0.8187	0.1862	0.0370	1.0860	0.5294	0.1592	0.3607	-0.9624
			(-4.39%)	(-1.73%)	(8.16%)	(-4.37%)	(-3.93%)	(3.61%)	(-2.12%)
9	0.1089	1.0000	0.1933	0.0512	1.0271	0.6466	0.1945	0.4406	-0.6766
			(-12.15%)	(-0.01%)	(6.76%)	(-4.37%)	(-3.93%)	(3.61%)	(3.41%)
10	0.1139	1.2214	0.1495	0.0687	0.9822	0.7898	0.2375	0.5381	-0.3888
			(-15.63%)	(1.30%)	(5.58%)	(-4.37%)	(-3.93%)	(3.61%)	(-6.66%)
11	0.1181	1.4918	0.0852	0.0901	0.9474	0.9646	0.2901	0.6573	-0.0995
			(-15.67%)	(2.33%)	(4.60%)	(-4.37%)	(-3.93%)	(3.61%)	(-35.31%)
12	0.1216	1.8221	0.0359	0.1164	0.9202	1.1782	0.3544	0.8028	0.1910
			(-12.93%)	(3.16%)	(3.78%)	(-4.37%)	(-3.93%)	(3.61%)	(-12.53%)
13	0.1245	2.2255	0.0109	0.1486	0.8987	1.4391	0.4328	0.9805	0.4826
			(-7.40%)	(3.82%)	(3.11%)	(-4.37%)	(-3.93%)	(3.61%)	(-5.58%)
14	0.1269	2.7182	0.0021	0.1879	0.8816	1.7577	0.5286	1.1976	0.7749
			(5.03%)	(4.36%)	(2.55%)	(-4.37%)	(-3.93%)	(3.61%)	(-3.67%)
15	0.1289	3.3201	0.0002	0.2360	0.8679	2.1468	0.6457	1.4627	1.0679
			(40.57%)	(4.80%)	(2.09%)	(-4.37%)	(-3.93%)	(3.61%)	(-2.76%)

Table 4.13: Summary for the final steady-state when government provides subsidy for fertility and finances its budget with tax on consumption. The subsidy rates are regressive and computed for every ability groups as $sub_t^{n,i} = \chi_t^i * sub^n$ where $\chi_t^i = \bar{n}_t / n_t^i$ and $sub_t^n = 0.1$

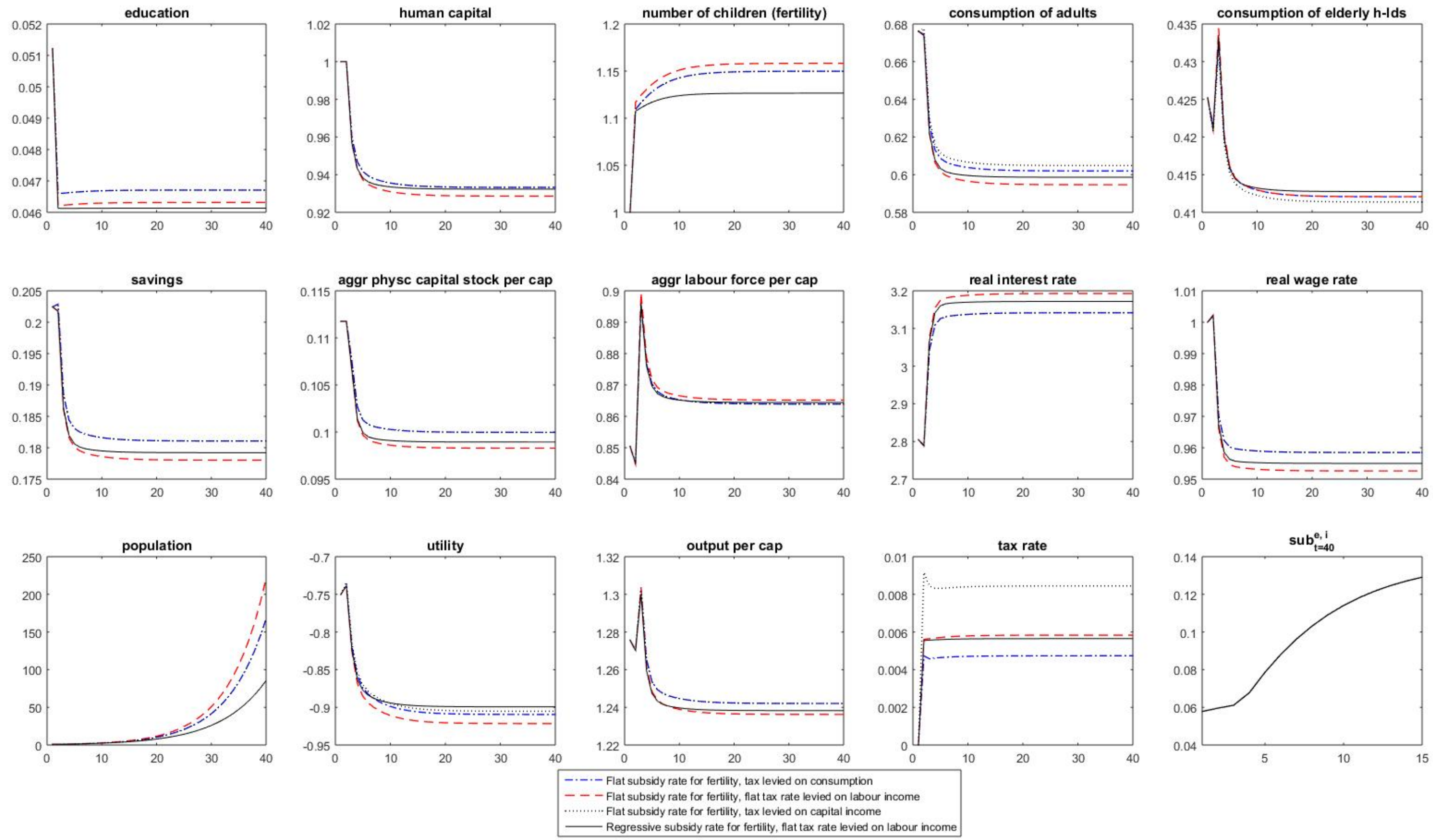

Figure 4.13: Transition path of the model economy with idiosyncratic shocks to the human capital from the perspective of the average household. Government provides subsidy for fertility and finances its budget with flat tax rate on the labour income. The subsidy rates are regressive and computed for every ability groups as $sub_t^{n,i} = \psi_t^i * sub^e$ where $\psi_t^i = \bar{n}_t / n_t^i$ and $sub_t^n = 0.1$.
Note: transition path with tax on consumption and flat subsidy rate for fertility is depicted by blue dash-dotted line; transition path with flat tax rate on labour income and flat subsidy rate for fertility is presented by red dashed line; transition path with capital income tax and flat subsidy rate for fertility is shown by black dotted line; transition path with flat tax rate on the labour income and regressive subsidy rate system for fertility is illustrated by black solid line

		Y	K	L	r	w	τ^l		
		1.2383	0.0989	0.8644	3.1718	0.9550	0.0056		
		(-2.93%)	(-11.47%)	(1.64%)	(13.08%)	(-4.50%)			
		$\bar{h}$	tot.pop.	$\bar{e}$	$\bar{n}$	$\bar{c}$	$\bar{s}$	$\bar{d}$	$\bar{u}$
		0.9323	85.50	0.0461	1.1265	0.5986	0.1792	0.4128	-0.8993
		(-6.77%)		(-9.93%)	(12.65%)	(-11.47%)	(-11.47%)	(-2.93%)	(-19.87%)
i	sub_i^n	h_i	pop.share.	e_i	n_i	c_i	s_i	d_i	u_i
1	0.0578	0.2019	0.0006	0.0000	1.9499	0.1296	0.0388	0.0894	-2.9333
			(218.47%)	—	(20.44%)	(-5.04%)	(-5.04%)	(4.12%)	(-0.21%)
2	0.0596	0.2466	0.0047	0.0000	1.8915	0.1583	0.0474	0.1092	-2.6430
			(216.50%)	—	(16.84%)	(-5.04%)	(-5.04%)	(4.12%)	(-0.44%)
3	0.0611	0.3012	0.0154	0.0000	1.8438	0.1934	0.0579	0.1333	-2.3518
			(185.31%)	—	(13.89%)	(-5.04%)	(-5.04%)	(4.12%)	(-0.70%)
4	0.0677	0.3679	0.0322	0.0025	1.6632	0.2362	0.0707	0.1629	-2.0745
			(128.33%)	(-33.97%)	(16.85%)	(-5.04%)	(-5.04%)	(4.12%)	(-0.56%)
5	0.0786	0.4493	0.0555	0.0085	1.4331	0.2885	0.0864	0.1989	-1.8055
			(70.22%)	(-13.97%)	(14.53%)	(-5.04%)	(-5.04%)	(4.12%)	(-0.85%)
6	0.0881	0.5488	0.0914	0.0160	1.2792	0.3524	0.1055	0.2430	-1.5301
			(32.16%)	(-7.62%)	(12.34%)	(-5.04%)	(-5.04%)	(4.12%)	(-1.24%)
7	0.0962	0.6703	0.1415	0.0254	1.1708	0.4304	0.1289	0.2968	-1.2502
			(9.79%)	(-4.12%)	(10.40%)	(-5.04%)	(-5.04%)	(4.12%)	(-1.78%)
8	0.1032	0.8187	0.1866	0.0369	1.0918	0.5257	0.1574	0.3625	-0.9670
			(-4.21%)	(-1.79%)	(8.74%)	(-5.04%)	(-5.04%)	(4.12%)	(-2.61%)
9	0.1091	1.0000	0.1924	0.0512	1.0326	0.6421	0.1922	0.4427	-0.6812
			(-12.56%)	(-0.08%)	(7.33%)	(-5.04%)	(-5.04%)	(4.12%)	(-4.11%)
10	0.1141	1.2214	0.1479	0.0686	0.9875	0.7842	0.2348	0.5407	-0.3934
			(-16.53%)	(1.23%)	(6.15%)	(-5.04%)	(-5.04%)	(4.12%)	(-7.93%)
11	0.1183	1.4918	0.0839	0.0901	0.9526	0.9579	0.2868	0.6605	-0.1041
			(-17.00%)	(2.26%)	(5.17%)	(-5.04%)	(-5.04%)	(4.12%)	(-41.56%)
12	0.1218	1.8221	0.0352	0.1163	0.9252	1.1700	0.3503	0.8067	0.1864
			(-14.76%)	(3.09%)	(4.35%)	(-5.04%)	(-5.04%)	(4.12%)	(-14.63%)
13	0.1247	2.2255	0.0106	0.1485	0.9036	1.4290	0.4278	0.9853	0.4780
			(-9.92%)	(3.74%)	(3.68%)	(-5.04%)	(-5.04%)	(4.12%)	(-6.48%)
14	0.1271	2.7182	0.0020	0.1878	0.8865	1.7454	0.5225	1.2034	0.7703
			(1.20%)	(4.28%)	(3.12%)	(-5.04%)	(-5.04%)	(4.12%)	(-4.24%)
15	0.1291	3.3201	0.0002	0.2358	0.8727	2.1318	0.6382	1.4699	1.0633
			(33.20%)	(4.72%)	(2.66%)	(-5.04%)	(-5.04%)	(4.12%)	(-3.18%)

Table 4.14: Summary for the final steady-state when government provides subsidy for fertility and finances its budget with the flat tax rate on the labour income. The subsidy rates are regressive and computed for every ability groups as $sub_t^{n,i} = \chi_t^i * sub^n$ where $\chi_t^i = \bar{n}_t / n_t^i$ and $sub_t^n = 0.1$

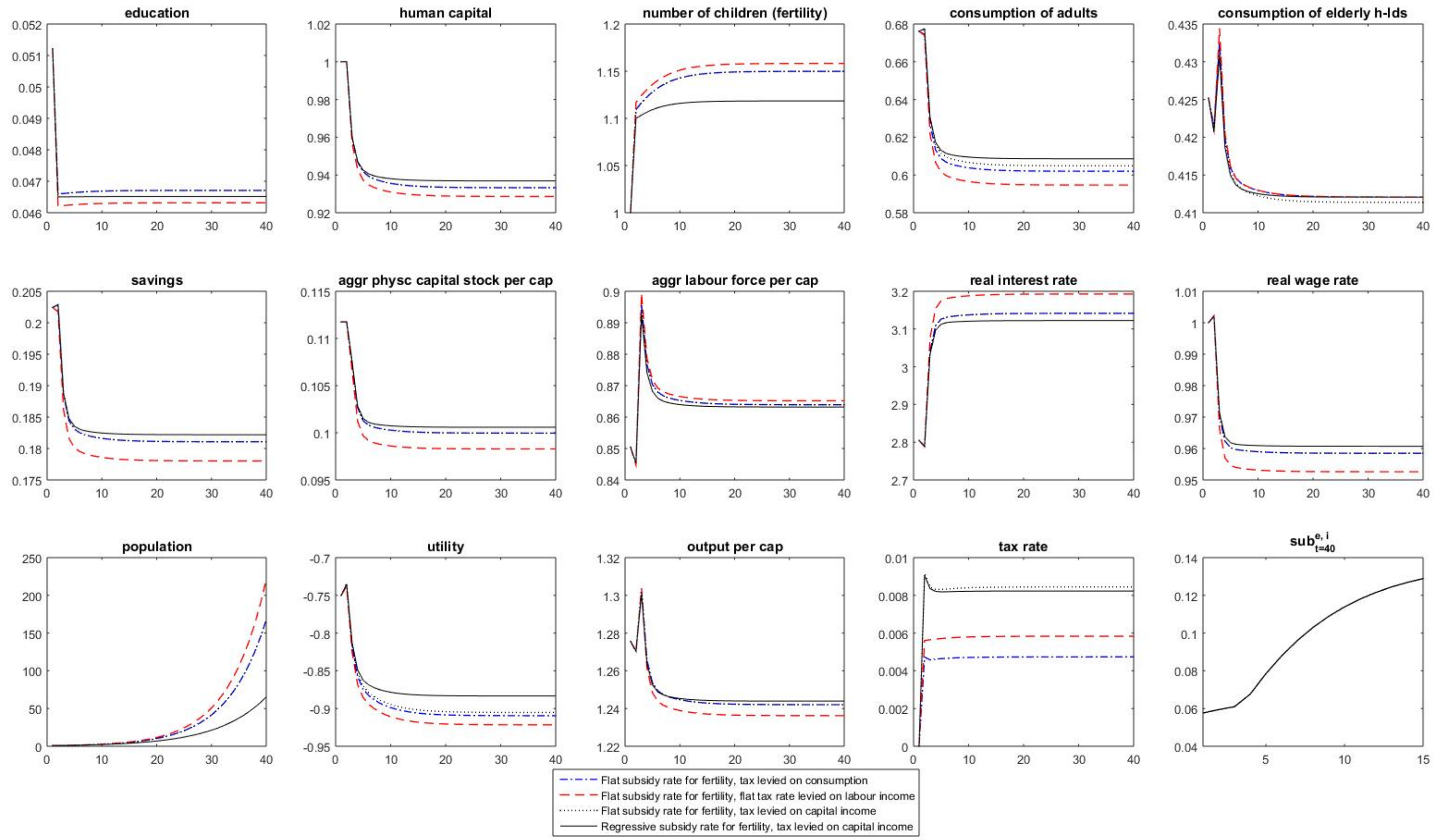

Figure 4.14: Transition path of the model economy with idiosyncratic shocks to the human capital from the perspective of the average household. Government provides subsidy for fertility and finances its budget with tax on capital income. The subsidy rates are regressive and computed for every ability groups as $sub_t^{n,i} = \psi_t^i * sub^e$ where $\psi_t^i = \bar{n}_t / n_t^i$ and $sub_t^n = 0.1$.
Note: transition path with tax on consumption and flat subsidy rate for fertility is depicted by blue dash-dotted line; transition path with flat tax rate on labour income and flat subsidy rate for fertility is presented by red dashed line; transition path with capital income tax and flat subsidy rate for fertility is shown by black dotted line; transition path with tax on capital income and regressive subsidy rate system for fertility is illustrated by black solid line

		Y	K	L	r	w	τ^k		
		1.2439	0.1006	0.8632	3.1224	0.9607	0.0082		
		(-2.49%)	(-10.00%)	(1.50%)	(11.32%)	(-3.93%)			
		$\bar{h}$	tot.pop.	$\bar{e}$	$\bar{n}$	$\bar{c}$	$\bar{s}$	$\bar{d}$	$\bar{u}$
		0.9368	65.25	0.0465	1.1185	0.6086	0.1822	0.4121	-0.8834
		(-6.32%)		(-9.18%)	(11.85%)	(-10.00%)	(-10.00%)	(-3.10%)	(-17.75%)
i	sub_i^n	h_i	pop.share.	e_i	n_i	c_i	s_i	d_i	u_i
1	0.0576	0.2019	0.0006	0.0000	1.9408	0.1312	0.0393	0.0888	-2.9245
			(208.43%)	—	(19.88%)	(-3.93%)	(-3.93%)	(3.44%)	(0.09%)
2	0.0594	0.2466	0.0046	0.0000	1.8824	0.1602	0.0480	0.1085	-2.6342
			(207.51%)	—	(16.28%)	(-3.93%)	(-3.93%)	(3.44%)	(-0.11%)
3	0.0610	0.3012	0.0150	0.0000	1.8347	0.1957	0.0586	0.1325	-2.3431
			(178.01%)	—	(13.33%)	(-3.93%)	(-3.93%)	(3.44%)	(-0.32%)
4	0.0676	0.3679	0.0314	0.0025	1.6547	0.2390	0.0715	0.1618	-2.0658
			(123.25%)	(-33.86%)	(16.25%)	(-3.93%)	(-3.93%)	(3.44%)	(-0.14%)
5	0.0785	0.4493	0.0545	0.0085	1.4257	0.2919	0.0874	0.1976	-1.7968
			(67.17%)	(-13.90%)	(13.94%)	(-3.93%)	(-3.93%)	(3.44%)	(-0.36%)
6	0.0879	0.5488	0.0902	0.0160	1.2725	0.3565	0.1067	0.2414	-1.5214
			(30.40%)	(-7.56%)	(11.76%)	(-3.93%)	(-3.93%)	(3.44%)	(-0.66%)
7	0.0960	0.6703	0.1403	0.0254	1.1647	0.4354	0.1304	0.2948	-1.2415
			(8.89%)	(-4.07%)	(9.83%)	(-3.93%)	(-3.93%)	(3.44%)	(-1.07%)
8	0.1030	0.8187	0.1862	0.0370	1.0860	0.5318	0.1592	0.3601	-0.9583
			(-4.39%)	(-1.73%)	(8.16%)	(-3.93%)	(-3.93%)	(3.44%)	(-1.68%)
9	0.1089	1.0000	0.1933	0.0512	1.0271	0.6496	0.1945	0.4398	-0.6725
			(-12.15%)	(-0.01%)	(6.76%)	(-3.93%)	(-3.93%)	(3.44%)	(-2.78%)
10	0.1139	1.2214	0.1495	0.0687	0.9822	0.7934	0.2375	0.5372	-0.3847
			(-15.63%)	(1.30%)	(5.58%)	(-3.93%)	(-3.93%)	(3.44%)	(-5.54%)
11	0.1181	1.4918	0.0852	0.0901	0.9474	0.9691	0.2901	0.6562	-0.0954
			(-15.67%)	(2.33%)	(4.60%)	(-3.93%)	(-3.93%)	(3.44%)	(-29.75%)
12	0.1216	1.8221	0.0359	0.1164	0.9202	1.1836	0.3544	0.8014	0.1951
			(-12.93%)	(3.16%)	(3.78%)	(-3.93%)	(-3.93%)	(3.44%)	(-10.66%)
13	0.1245	2.2255	0.0109	0.1486	0.8987	1.4457	0.4328	0.9789	0.4867
			(-7.40%)	(3.82%)	(3.11%)	(-3.93%)	(-3.93%)	(3.44%)	(-4.78%)
14	0.1269	2.7182	0.0021	0.1879	0.8816	1.7658	0.5286	1.1956	0.7790
			(5.03%)	(4.36%)	(2.55%)	(-3.93%)	(-3.93%)	(3.44%)	(-3.16%)
15	0.1289	3.3201	0.0002	0.2360	0.8679	2.1567	0.6457	1.4603	1.0720
			(40.57%)	(4.80%)	(2.09%)	(-3.93%)	(-3.93%)	(3.44%)	(-2.39%)

Table 4.15: Summary for the final steady-state when government provides subsidy for fertility and finances its budget with tax on capital income. The subsidy rates are regressive and computed for every ability groups as $sub_t^{n,i} = \chi_t^i * sub^n$ where $\chi_t^i = \bar{n}_t / n_t^i$ and $sub_t^n = 0.1$

According to our results, given the individual fertility decisions, distribution of the population and the resulted average fertility level in the economy, at the final steady-state with tax on consumption or capital income, the households from first to fifteenth ability group receive the subsidy rates for fertility of 0.0576, 0.0594, 0.0610, 0.0676, 0.0785, 0.0879, 0.0960, 0.1030, 0.1089, 0.1139, 0.1181, 0.1216, 0.1245, 0.1269 and 0.1289, respectively, as it is indicated in the second column of the table 4.13 and 4.15 and as it is illustrated by the bottom-right sub-plot in the figure 4.12 and 4.14. At the final steady-state with flat tax rate on the labour income, the subsidy rates for fertility for the households from these fifteenth ability groups are equal to 0.0578, 0.0596, 0.0611, 0.0677, 0.0786, 0.0881, 0.0962, 0.1032, 0.1091, 0.1141, 0.1183, 0.1218, 0.1247, 0.1271 and 0.1291, which is summarised by the second column of table 4.14 and by the bottom-right sub-plot in the figure 4.13.

Given the distribution of the population, population size, decisions of the households and resulted subsidy rates for fertility, when the economy reaches the final steady-state with tax on consumption, the tax rate is equal to 0.0046 in order to have a balanced government budget. In comparison to the previously considered final steady-state with flat subsidy rate for fertility of 10%, this tax rate is 3.05% lower, however. At the final steady-state with regressive subsidy rates for fertility and flat tax rate on the labour income, the tax rate is set to be equal to 0.0056, which is 3.27% lower than at the final steady-state with flat subsidy rate for fertility of 10%. Finally, at the final steady-state with regressive subsidy rates for fertility that government finances with tax on capital income, the tax rate is equal to 0.0082, which is 2.60% lower than at the final steady-state with flat subsidy rate for fertility.

As a result of these subsidy rates, tax rates, distribution of the population and the average level of the human capital, our results indicate that at the final steady-state with regressive subsidy rates for fertility and tax on consumption or capital income, the fertility decisions of the households from first to seventh ability groups increase 19.88%, 16.28%, 13.33%, 13.25%, 13.94%, 11.76% and 9.83% above the original steady-state. In comparison to our previous results with flat subsidy rate for fertility, however, due to lower subsidy rates that these households receive under the regressive program, the fertility choices of the households from first to seventh ability groups are 17.13%, 12.52%, 9.34%, 11.30%, 4.92%, 1.89% and 0.38% lower than at the final steady-state with tax on consumption or capital income; and 17.46%, 12.92%, 9.78%, 11.74%, 5.40%, 2.40% and 0.90% lower than at the final steady-state with flat tax rate on labour income. For the households from eighth and fifteenth ability groups, when the government provides support for fertility decisions with regressive subsidy rates and finances its budget with tax on consumption or capital income, the fertility choices for these households become 8.16% and 2.09% larger than at the original steady-state, respectively. Furthermore, the fertility level of the households from these two ability group is 0.37% and 0.56% larger than at the final steady-state with flat subsidy rate for fertility of 10% that government finances with either tax on consumption or capital income; but they are 0.16% and 0.02% lower than at the final steady-state with flat subsidy rate for fertility and flat tax rate on the labour income. Finally, for the households from ninth to fourteenth ability groups, the fertility choices become 6.76%, 5.58%, 4.60%, 3.78%, 3.11% and 2.55% larger than at the original steady-state, when the economy reaches the final steady-state with regressive subsidy rates for fertility and tax on consumption or capital income. In addition, these fertility choices are 0.18%, 0.298%, 0.302%, 0.25%, 0.16% and 0.07% larger than at the final steady-state with flat subsidy rate for fertility and flat tax rate on the labour income; and 0.72%, 0.85%, 0.86%, 0.81%, 0.73% and 0.64% larger than at the final steady-state with flat subsidy rate for fertility that government finances with either tax on consumption or capital income.

For the final steady-state that the economy reaches with regressive subsidy rates for fertility that government finances with flat tax rate on the labour income, due to additional disincentives that this tax option creates for education provision, the fertility choices for the households from first to sixth ability groups become 0.47%, 0.48%, 0.50%, 0.512%, 0.515% and 0.519% larger than at the final steady-state with regressive subsidy rates for fertility that government finances with tax on consumption or capital income; and they are 20.44%, 16.84%, 13.89%, 16.85%, 14.53% and 12.34% larger than at the original steady-state without the government presence. However, in comparison to our previous results, due to a relatively lower subsidy rates that households from lower ability groups now receive, the fertility levels of the households from these six ability groups are 16.59%, 11.98%, 8.80%, 10.73%, 4.38% and 1.36% lower than at the final steady-state with flat subsidy rate for fertility that government provides with tax on consumption or capital income; and 16.91%, 12.37%, 9.24%, 11.17%, 4.86% and 1.86% lower than at the final steady-state with flat subsidy rate for fertility and flat tax rate on the labour income. Next, for the households from seventh ability group, the fertility choices become 10.40% larger than at the original steady-state, 0.52% larger than at the final steady-state with regressive subsidy rates for fertility and tax on consumption or capital income, and 0.15% larger than at the final steady-state with flat subsidy rate for fertility of 10% and tax on consumption or capital income considered previously. This result, however, is 0.37% lower than at the final steady-state with flat subsidy rate for fertility and flat tax rate on the labour income. Lastly, for the households from eighth to fifteenth ability groups, the fertility level becomes 8.74%, 7.33%, 6.15%, 5.17%, 4.35%, 3.68%, 3.12% and 2.66% larger than at the original steady-state. Additionally, with regressive subsidy rates for fertility and flat tax rate on the labour income, the fertility for the households from these eight ability groups becomes 0.90%, 1.25%, 1.38%, 1.40%, 1.35%, 1.28%, 1.19% and 1.11% larger than at the final steady-state with flat subsidy rate for fertility and tax on consumption or capital income; and 0.37%, 0.71%, 0.835%, 0.844%, 0.79%, 0.71%, 0.63% and 0.54% larger than at the final steady-state with flat subsidy rate for fertility and flat tax rate on the labour income. Finally, at the final steady-state with regressive subsidy rates for fertility and flat tax rate on the labour income our results also indicate that fertility levels for the households from these eight ability groups are 0.530%, 0.535%, 0.539%, 0.543%, 0.547%, 0.550%, 0.552% and 0.555% larger than at the final steady-state with regressive subsidy rates for fertility that government finances with either tax on consumption or capital income.

With the change in the distribution of the population combined with the fluctuation in the individual fertility decisions, when the economy reaches the final steady-state with regressive subsidy rates for fertility and with tax on consumption or capital income, the average fertility level in the economy becomes 11.85% larger than at the original steady-state without government presence. In comparison to the final steady-states with flat subsidy for fertility, however, the average fertility level is 2.80% lower than at final steady-state with tax on consumption or capital income, and 3.55% lower than at the final steady-state with flat tax rate on the labour income. At the final steady-state with regressive subsidy rates for fertility and flat tax rate on the labour income, the average fertility level increases further and becomes 12.65% larger than at the initial steady-state. This result, however, is 2.07% lower than at the final steady-state with flat subsidy rate for fertility and tax on consumption or capital income, and 2.81% lower than at the final steady-state with flat subsidy rate for fertility and flat tax rate on the labour income. In comparison to the final steady-state with regressive subsidy rates for fertility that government finances with either tax on consumption or capital income, however, the average fertility level is 0.71% larger when the economy reaches the final steady-state with regressive subsidy rates for fertility and flat tax rate on the labour income.

Additionally, we also observe that at the fortieth period of analysis[51], with regressive subsidy rates for fertility and tax on consumption or capital income, the population size becomes 65.25 times larger than at the first period of analysis. This, however, is 157.06% smaller than at the fortieth period of analysis with the flat subsidy rate for fertility and tax on consumption or capital income; and 238.43% smaller than at the fortieth period of analysis with flat subsidy rate for fertility and flat tax rate on the labour income. For the case with regressive subsidy rates for fertility and flat tax rate on the labour income, the population size at the fortieth period of analysis is 85.50 times larger than at the first period of analysis without government presence. This result, however, is 96.18% smaller than at the fortieth period of analysis with flat subsidy rate for fertility and tax on consumption or capital income, and 158.27% smaller than at the fortieth period of analysis with flat subsidy rate for fertility and flat tax rate on the labour income; but it is 23.68% larger than the with regressive subsidy rates for fertility which government finances with tax on consumption or tax on capital income.

Moving forward, with the government policies in place and with the change in the distribution of the population and in the individual fertility decisions, we observe that at the final steady-state with regressive subsidy rates for fertility and tax on consumption or capital income, the education of the households from fourth to sixth ability groups becomes 33.86%, 13.90% and 7.56% smaller than at the original steady-state. In comparison to our previous results, however, the education of the households from these three ability groups is 56.37%, 10.36% and 2.57% larger than at the final steady-state with flat subsidy rate for fertility that government

[51] When the economy reaches the final steady-state

finances with either tax on consumption or capital income; and 55.82%, 10.27% and 2.57% larger than at the final steady-state with flat subsidy rate for fertility and flat tax rate on the labour income. For the households from seventh ability group, when the economy reaches the final steady-state with regressive subsidy rates for fertility and tax on consumption or capital income, the education level becomes 4.07% smaller than at the original steady-state; and it is 0.01% smaller than at the final steady-state with flat subsidy rate for fertility that government finances with tax on consumption or capital income, but 0.02% larger than at the final steady-state with flat subsidy rate for fertility and flat tax rate on the labour income. For the households from eighth and ninth ability groups, the education decreases 1.73% and 0.01% below the initial steady-state, respectively. In comparison to the previous results, the education for these two ability groups is 1.01% and 1.37% lower than at the final steady-state with flat subsidy rate for fertility and tax on consumption or capital income; and 0.96% and 1.31% lower than at the final steady-state with flat subsidy rate for fertility and flat tax rate on the labour income. For the households from tenth to fifteenth ability groups, however, at the final steady-state with regressive subsidy rates for fertility and tax on consumption or capital income, the education becomes 1.30%, 2.33%, 3.16%, 3.82%, 4.36% and 4.80% larger than at the original steady state. But, in comparison to our previous results, the education for these six ability groups is 1.46%, 1.42%, 1.33%, 1.21%, 1.10% and 0.99% lower than at the final steady-state with flat subsidy rate for fertility and tax on consumption or capital income; and 1.39%, 1.35%, 1.25%, 1.14%, 1.02% and 0.91% smaller than at the final steady-state with flat subsidy rate for fertility and tax on labour income. Finally, identically to the original steady-state and previously considered final steady-states with flat subsidy rate for fertility, the education provision for the households from first to third ability groups at the final steady-state with regressive subsidy rates for fertility and tax on consumption or capital income remains at zero level.

For the final steady-state with regressive subsidy rates for fertility and flat tax rate on the labour income, for the households from first to third ability groups, the education remains at zero level. For the households from fourth to sixth ability groups, the education decreases 33.97%, 13.97% and 7.62% below original steady-state when economy reaches the final steady-state with regressive subsidy rates for fertility and flat tax rates on the labour income. In relation to our previous results, however, the education for the households from these three ability groups becomes 56.30%, 10.30% and 2.52% larger than at the final steady-state with flat subsidy rate for fertility and tax on consumption or capital income; and 55.75%, 10.20% and 2.51% larger than at the final steady-state with flat subsidy rate for fertility and flat tax rate on the labour income. But, in comparison to the results for the final steady-states with regressive subsidy rates for fertility and government taxing either consumption or capital income, the education for the households from fourth to sixth ability groups is 0.16%, 0.07% and 0.06% lower when government finances provision of regressive subsidy rates for fertility with flat tax rate on the labour income. Next, when the economy reaches the final steady-state with regressive subsidy rates for fertility and flat tax rate on the labour income, for the households from seventh ability group, education becomes 4.12% lower than at the original steady-state without government presence. Furthermore, this result is 0.07% lower than at the final steady-state with flat subsidy rate for fertility and tax on consumption or capital income; 0.04% lower than at the final steady-state with flat subsidy rate for fertility and flat tax rate on the labour income; and 0.06% lower than at the final steady-state with regressive subsidy rates for fertility and tax on consumption or capital income. Moving forward to the households from eighth to ninth ability groups, as the economy reaches the final steady-state with regressive subsidy rates for fertility, the education becomes 1.79% and 0.08% lower than at the initial steady-state; which is 1.07% and 1.44% lower than at the final steady-state with flat subsidy rate for fertility and tax on consumption or capital income, 1.02 and 1.38% lower than at the final steady-state with flat subsidy rate for fertility and tax on labour income, and 0.0597% and 0.619% lower than at the final steady-state with regressive subsidy rates for fertility and tax on consumption or capital income, respectively. Lastly, at the final steady-state with regressive subsidy rates for fertility and flat tax rate on the labour income, the education for the households from tenth to fifteenth ability groups becomes 1.23%, 2.26%, 3.09%, 3.74%, 4.28% and 4.72% larger than at the original steady-state. However, in comparison to our previous results, the education for the households from these six ability groups is 1.53%, 1.49%, 1.40%, 1.29%, 1.17% and 1.07% smaller than at the final steady-state with flat subsidy rate for fertility and tax on consumption or capital income; and 1.46%, 1.42%, 1.32%, 1.21%, 1.09% and 0.99% lower than at the final steady-state with flat subsidy rate for fertility and flat tax rate on the labour income. Finally, at the final steady-state with regressive subsidy rates for fertility and flat tax rate on the labour income, the education for the households from tenth to fifteenth ability groups is 0.064%, 0.067%, 0.069%, 0.071%, 0.072% and 0.074% lower than at final steady-state with regressive subsidy rates for fertility and tax on consumption or capital income.

As a result of these changes in the education for the households at the individual ability level, combined with the change in the distribution of the population among different ability groups, we observe that the average education level becomes 9.18% lower than at the original steady-state when the economy reaches the final steady-state with regressive subsidy rates for fertility and tax on consumption or capital income. In relation to our previous results, the average education level at the final steady-state with regressive subsidy rates for fertility and tax on consumption or capital income is 0.43% larger than at the final steady-state with flat subsidy rate for fertility and flat tax rate on the labour income; but 0.42% smaller than at the final steady-state with flat subsidy rate for fertility and tax on consumption or capital income. For the final steady-state with regressive subsidy rates for fertility and tax on the labour income, the average education becomes 9.93% lower than at the original steady-state; 1.25% lower than at the final steady-state with flat subsidy rate for fertility and tax on consumption or capital income; and 0.39% lower than at the final steady-state with flat subsidy rate for fertility and flat tax rate on the labour income. Additionally, according to our results, the average education at the final steady-state with regressive subsidy rates for fertility and flat tax rate on the labour income is 0.83% lower than at the final steady-state with regressive subsidy rates for fertility that government finances with tax on consumption or capital income.

In addition to the change in the average education, we also observe that the average level of human capital becomes 6.32% lower than at the original steady-state when the economy reaches the final steady-state with regressive subsidy rates for fertility and tax on consumption or capital income. This result, however, is 0.38% larger than at the final steady-state with flat subsidy rate for fertility and tax on consumption or capital income, and 0.88% larger than at the final steady-state with flat subsidy rate for fertility and flat tax rate on the labour income considered previously. For the final steady-state with regressive subsidy rates for fertility and flat tax rate on the labour income, however, the average level of human capital becomes 6.77% lower than at the original steady-state; and it is 0.10% lower than at the final steady-state with flat subsidy rate for fertility that government finances with either tax on consumption or capital income, but 0.40% larger than at the final steady-state with flat subsidy rate for fertility and flat tax rate on the labour income. Finally, our results suggest that at the final steady-state with regressive subsidy rates for fertility and flat tax rate on the labour income, the average level of the human capital is 0.49% smaller than at the final steady-state with regressive subsidy rates for fertility and tax on consumption or capital income.

With the levels that we observe for education attainment, fertility decisions, human capital and the population size – which contribute to a change in the time that adult households have for labour market participation, number of teacher required for education provision and skill level of the households – as the economy reaches the final steady-state with regressive subsidy rates for fertility and tax on consumption or capital income, the effective labour force per adult households becomes 1.50% larger than at the initial steady-state without the government presence. In comparison to our previous results, however, the effective labour force per adult household is 0.08% and 0.23% smaller than at the final steady-states with flat subsidy rate for fertility that the government finances with tax on consumption or capital income and with the tax on the labour income, respectively. For the case with regressive subsidy rates that government finances with flat tax rate on the labour income, the effective labour force per adult household becomes 1.64% larger than at the original steady-state; which is also 0.06% larger than at the final steady-state with flat subsidy rate for fertility and tax on consumption or capital income, but 0.09% smaller than at the final steady-state with flat subsidy rate for fertility and flat tax rate on the labour income considered previously. Additionally, at the final steady-state with regressive subsidy rates for fertility and flat tax rate on the labour income, the effective labour force per adult household is 0.14% larger than at the final steady-state with regressive subsidy rates for fertility that government finances with either tax on consumption or capital income.

Next, with the change in the saving of the households, distribution of the population and the population size, our results indicate that at the final steady-state with regressive subsidy rates for fertility and tax on consumption or capital income, the physical capital stock per adult household decreases 10.00% below the original steady-state. In comparison to our previous results, however, this indicator is 0.62% and 2.29% larger than at the final steady-states with flat subsidy rate for fertility that government finances with tax on consumption or capital income and with the tax on the labour income, respectively. For the final steady-state with regressive subsidy rates for fertility and flat tax rate on the labour income, the physical capital stock per adult household becomes 11.47% smaller that at the original steady-state, which, additionally, is 1.04% smaller than at the final steady-state with flat subsidy rate for fertility and tax on consumption or capital income, but 0.66% larger than at the final steady-state with flat subsidy rate for fertility and flat tax rate on the labour income considered previously. Lastly, our results indicate that at the final steady-state with regressive subsidy rates for fertility that government finances with tax on consumption or capital income, the physical capital stock per adult household is 1.63% larger than at the final steady-state with regressive subsidy rates for fertility and flat tax rate on the labour income.

Given these changes, we observe that as the economy reaches the final steady-state with regressive subsidy rates for fertility and tax on consumption or capital income, the real wage rate decreases 3.93% below the original steady-state; but becomes 0.23% and 0.84% larger than at the previously considered final steady-states with flat subsidy rate for fertility that the government finances with tax on consumption or capital income and with tax on the labour income, respectively. For the final steady-state with regressive subsidy rates for fertility and flat tax rate on the labour income, the real wage rate decreases 4.50% below the initial equilibrium, and becomes 0.36% lower than at the final steady-state with flat subsidy rate for fertility and tax on consumption or capital income. However, in comparison to our previous results with flat subsidy rate for fertility and flat tax rate on the labour income, the real wage rate at the final steady-state with regressive subsidy rates for fertility and flat tax rate on the labour income is 0.25% larger. Lastly, our results indicate that at the final steady-state with regressive subsidy rates for fertility and tax on consumption or capital income, the real wage rate is 0.59% larger than at the final steady-state with regressive subsidy rates for fertility and flat tax rate on the labour income.

Additionally to the fluctuation in the real wage rate, we observe that as the economy reaches the final steady-state with regressive subsidy rates for fertility and tax on consumption or capital income, the real interest rate increases 11.32% above the original steady-state. In relation to our previous results, however, this indicator is 0.62% and 2.26% lower than at the final steady-state with flat subsidy rate for fertility that government finances with tax on consumption or capital income and with flat tax rate on the labour income, respectively. For the case with regressive subsidy rates for fertility and flat tax rate on the labour income, our results indicate that at the final steady-state the real interest rate becomes 13.08% larger than at the original steady-state. Additionally, this result is 0.95% larger than at the final steady-state with flat subsidy rate for fertility and tax on consumption or capital income; but, it is 0.66% smaller than at the final steady-state with flat subsidy rate for fertility and flat tax rate on the labour income. Finally, according to our results, at the final steady-state with regressive subsidy rates for fertility and flat tax rate on the labour income, the real wage rate is 1.56% larger than at the final steady-state with regressive subsidy rates for fertility that government finances with tax on consumption or capital income.

Driven by the change in the factor prices and in the human capital level, and combined with the resulted tax rates, we observe that as the economy reaches the final steady-state with regressive subsidy rates for fertility that government finances with tax on consumption, the consumption of the adult households across the individual ability groups decreases by 4.37% below the original steady-state. In comparison our previous results, however, this indicator is 0.25% and 0.97% above the original steady-state with flat subsidy rate for fertility and tax on consumption and labour income, respectively; but it is 0.23% below the final steady-state with flat subsidy rate for fertility and tax on capital income. Next, for the final steady-state with regressive subsidy rates for fertility and flat tax rate on the labour income, the consumption of adult households for all ability groups becomes 5.04% smaller than at the original steady-state. In relation to our previous results, it is 0.46% and 0.93% lower than at the final steady-state with flat subsidy rate for fertility that government finances with tax on consumption and tax on capital income, respectively; but it is 0.27% larger than at the final steady-state with flat subsidy rate for fertility and flat tax rate on the labour income. For the final steady-state with regressive subsidy rates for fertility and tax on capital income, the consumption of adult households at the individual ability levels is 3.93% lower than at the original steady-state; which, however, is 0.70%, 1.42% and 0.23% larger than at the previously considered final steady-states with flat subsidy rate for fertility that government finances with tax on consumption, labour income and capital income, respectively. Lastly, our results also indicate that at the final steady-state with regressive subsidy rates for fertility and tax on capital income, the consumption of adult households across different ability groups is 0.46% and 1.16% larger than at the final steady-state with regressive subsidy rates for fertility and tax on consumption and capital income, respectively.

With these changes that we observe in the distribution of the population and in consumption of adult households across different ability groups, the average level of consumption of adult households becomes 10.41% lower than at the original steady-state when the economy reaches the final steady-state with regressive subsidy rates for fertility and tax on consumption. In comparison to our previous results, however, this indicator is 0.63%, 1.84% and 0.16% larger than at the final steady-state with flat subsidy rate for fertility that government finances with tax on consumption, labour income and capital income, respectively. For the final steady-state with regressive subsidy rates for fertility and flat tax rate on the labour income, the consumption of adult households at the average level becomes 11.47% lower than at the initial steady-state. Furthermore, this result is 0.56% and 1.04% lower than at the previously considered final steady-states with flat tax subsidy rate for fertility and tax on consumption and capital income, respectively; but it is 0.66% larger than at the final steady-state with flat subsidy rate for fertility and flat tax rate on the labour income. Lastly, our results suggest that at the final steady-state with the regressive subsidy rates for fertility and tax on capital income, the average consumption of adult households is 10.00% below the initial steady-state; which, however, is 1.08%, 2.29% and 0.62% larger than at the final steady-state with flat subsidy rate for fertility and tax on consumption, labour income and capital income, respectively. Our results also indicate that at the final steady-state with regressive subsidy rates for fertility and tax on capital income, the average consumption of adult households is 0.46% and 1.63% larger than at the final steady-states with regressive subsidy rates for fertility that government finances with tax on consumption and labour income, respectively. Finally, we observe that behaviour of the households for saving, both at the individual ability and average levels, is identical to the behaviour for consumption of adult households when the government finances its budget with tax on labour income or capital income. When the government budget is financed with tax on consumption instead, the behaviour for saving is identical to the dynamics in consumption of adult households with tax on capital income in place.

For the dynamics in consumption of the elderly households – which is influenced by the changes in the factor prices, tax rates and human capital – we observe that at the final steady-state with regressive subsidy rates for fertility and tax on consumption, the consumption of the elderly households across different ability groups becomes 3.61% larger than at the original steady-state; that, in comparison to our previous results, however, is 0.22%, 0.72% and 0.05% lower than at the final steady-state with flat subsidy rate for fertility that government finances with tax on consumption, labour income and capital income, respectively. Next, for the final steady-state with regressive subsidy rates for fertility and flat tax rate on the labour income, the consumption of the elderly households across ability groups increases 4.12% above the original steady-state; and it is 0.27% and 0.44% larger than at the final steady-state with flat subsidy rate for fertility and tax on consumption and capital income, respectively, but it is 0.23% below the final steady-state with flat subsidy rate for fertility and tax on the labour income. Finally, when the economy reaches the final steady-state with regressive subsidy rates for fertility and tax on capital income, the consumption of elderly households across individual ability levels increases 3.44% above the original steady-state; but it is 0.39%, 0.89% and 0.22% lower than at the final steady-state with flat subsidy rate for fertility that government finances with tax on consumption, labour income and capital income, respectively. Our results also indicate that at the final steady-state with regressive subsidy rates for fertility and tax on capital income, the consumption of the elderly households between different ability groups is 0.17% and 0.66% smaller than at the final steady-state with regressive subsidy rates for fertility and tax on consumption and labour income, respectively.

With these changes in consumption of the elderly households at the individual ability levels combined with the resulted distribution of the population between different ability groups, we observe that at the final steady-state with regressive subsidy rates for fertility and tax on consumption and on labour income, the average consumption of the elderly households becomes 2.93% lower than at the original steady-state. However, in comparison to our previous results, it is 0.17%, 0.16% and 0.33% larger than at the final steady-state with flat subsidy rate for education that government finances with tax on consumption, labour income and capital income, respectively. Next, at the final steady-state with regressive subsidy rates for fertility and tax on capital income, the average consumption of the elderly households becomes 3.10% lower than at the original steady-state. In relation to our previous results, this indicator is 0.002% and 0.005% lower than at the final steady-state with flat subsidy rate for fertility that government finances with tax on consumption and labour income, respectively; but it is 0.17% larger than at the final steady-state with flat subsidy rate for fertility and tax on capital income. Overall, our results suggest that at the final steady-state with regressive subsidy rate for fertility and tax on the capital income, the average consumption of the elderly households is 0.17% smaller than at the final steady-state with regressive subsidy rates for fertility and tax on consumption or labour income.

Overall, as a result of all changes that are described above when the economy reaches the final steady-state with regressive subsidy rates for fertility, the utility of the households at the individual ability level changes in the following manner. First, at the final steady-state with regressive subsidy rates for fertility and tax on consumption, the utility of the households from first to fifth ability groups decreases by 0.05%, 0.26%, 0.50%, 0.34% and 0.59% below the original steady-state. In comparison to our previous results, the utility for these five ability groups is 0.91%, 0.73%, 0.61%, 0.84% and 0.38% smaller than at the final steady-state with flat subsidy rate for fertility and tax on consumption; 0.73%, 0.54%, 0.39%, 0.60% and 0.10% lower than at the final steady-state with flat subsidy rate for fertility and flat tax rate on the labour income; and 1.05%, 0.89%, 0.79%, 1.05% and 0.61% smaller than at the final steady-state with flat subsidy rate for fertility and tax on capital income. For the households from sixth ability group, when the economy reaches the final steady-state with regressive subsidy rates for fertility and tax on consumption, the utility decreases by 0.93% below the original steady-state; and it becomes 0.10% and 0.38% smaller than at the final steady state with flat subsidy for fertility that government finances with tax on consumption and tax on capital income, respectively; but it is 0.22% larger than at the previously considered final steady-state with flat subsidy rate for fertility and flat tax rate on the labour income. For the remaining households from seventh to fifteenth ability groups, as the economy reaches the final steady-state with regressive subsidy rates for fertility and tax on capital income, the utility becomes 1.40%, 2.12%, 3.41%, 6.66%, 35.31%, 12.53%, 5.58%, 3.67% and 2.76% below the original steady-state. These utility levels, however, are found to be 0.09%, 0.26%, 0.46%, 0.86%, 3.38%, 1.71%, 0.65%, 0.38% and 0.26% larger than at the final steady-state with flat subsidy rate for fertility that government finances with tax on consumption; and 0.48%, 0.76%, 1.17%, 2.09%, 8.16%, 4.20%, 1.63%, 0.99% and 0.71% larger than at the final steady-state with flat subsidy rate for fertility and flat tax rate on the labour income; but 0.25%, 0.18%, 0.16%, 0.22%, 0.86%, 0.49%, 0.22%, 0.16% and 0.13% smaller than at the final steady-state with flat subsidy rate for fertility and tax on capital income.

Next, according to our results, when the economy reaches the final steady-state with regressive subsidy rates for fertility and flat tax rate on the labour income, the utility of the households from first to sixth ability groups becomes 0.21%, 0.44%, 0.70%, 0.56%, 0.85% and 1.24% below the original steady-state. In comparison to our previous results, the utility for the households from these six ability groups becomes 1.07%, 0.91%, 0.80%, 1.06%, 0.63% and 0.40% smaller than at the final steady-state with flat subsidy rate for fertility and tax on consumption; 0.89%, 0.72%, 0.59%, 0.82%, 0.36% and 0.09% smaller than at the final steady-state with flat subsidy rate for fertility and flat tax rate on the labour income; and 1.21%, 1.07%, 0.98%, 1.27%, 0.87% and 0.68% lower than at the final steady-state with flat subsidy rate for fertility and tax on capital income. For the remaining households from seventh to fifteenth ability groups, our results indicate that at the final steady-state with regressive subsidy rates for fertility and flat tax rate on the labour income, the utility is 1.78%, 2.61%, 4.11%, 7.93%, 41.56%, 14.63%, 6.48%, 4.24% and 3.18% lower that at the original steady-state. In relation to our previous results, these indicators are 0.28%, 0.220%, 0.219%, 0.32%, 0.70%, 0.30%, 0.21% and 0.17% below the final steady-state with flat subsidy rate for fertility and tax on consumption; and 0.62%, 0.66%, 0.84%, 1.39%, 5.23%, 2.97%, 1.19%, 0.76% and 0.56% smaller than at the final steady-state with flat subsidy rate for fertility and tax on capital income; but, it is 0.11%, 0.28%, 0.48%, 0.89%, 3.39%, 1.84%, 0.69%, 0.41% and 0.28% larger than at the final steady-state with flat subsidy rate for fertility and flat tax rate on the capital income.

Lastly, for the case with regressive subsidy rates for fertility and tax on the capital income, our results indicate that the utility of the households from the first ability group increases 0.09% above the original steady-state. In comparison to our previous results, however, this indicator is 0.77%, 0.59% and 0.91% lower than at the final steady state with flat subsidy rate for fertility that government finances with tax on consumption, labour income and capital income, respectively. But in relation to the results that we describe above, when the economy reaches the final steady-state with regressive subsidy rates for fertility and tax on capital income, the utility for the households from first ability group is 0.14% and 0.30% larger than at the final steady-state with the regressive subsidy rates for fertility that government finances with tax on consumption and tax on labour income, respectively. Next, for the households from second to fourth ability groups, the utility becomes 0.11%, 0.32% and 0.14% smaller that at the original steady-state when the economy reaches the final steady-state with regressive subsidy rates for fertility and tax on capital income. Additionally, in relation to our previous results, these indicators are 0.58%, 0.43% and 0.65% smaller than at the final steady-state with flat subsidy rate for fertility and tax on consumption; 0.39%, 0.22% and 0.40% smaller than at the final steady-state with flat subsidy rate for fertility and flat tax rate on the labour income; and 0.74%, 0.61% and 0.85% smaller than at the final steady-state with flat subsidy rate for fertility and tax on capital income. At the final steady-state with regressive subsidy rates for fertility and tax on capital income, however, the utility for the households from these three ability groups is 0.16%, 0.17% and 0.20% larger than at the final steady-state with regressive subsidy rates for fertility and tax on consumption; and 0.33%, 0.37% and 0.42% larger than at the final steady-state with regressive subsidy rates for fertility and tax on labour income. Moving forward to the households from the fifth ability groups, we observe that at the final steady-state with regressive subsidy rates for fertility and tax on capital income, the utility level becomes 0.36% lower than at the original steady-state; which, in relation to our previous results is 0.12% larger than at the final steady-state with flat subsidy rate for fertility and flat tax rate on the labour income, but it is 0.15% and 0.39% lower than at the final steady-state with flat subsidy rate for fertility that government finances with tax on consumption and tax on capital income, respectively. In comparison to other tax options, however, our results indicate that at the final steady-state with regressive subsidy rates for fertility and tax on capital income, utility for the households from fifth ability group is 0.23% and 0.49% larger than at the final steady-state with regressive subsidy rates for fertility and tax on consumption and labour income, respectively. For the households from sixth ability group, as the economy reaches the final steady-state with regressive subsidy rates for fertility and tax on capital income, the utility becomes 0.66% lower than at the original steady-state; which, additionally, is 0.11% lower than at the final steady-state with flat subsidy rate for fertility and tax on capital income, but it is 0.17% and 0.49% larger than at the final steady state with flat subsidy rate for fertility that government finances with tax on consumption and labour income, respectively. Our results also indicate that at the final steady-state with regressive subsidy rates for fertility and tax on capital income, the utility for the households from sixth ability group is 0.27% and 0.57% larger than at the final steady-state with regressive subsidy rates for fertility and tax on consumption and labour income, respectively. Lastly, for the households from seventh to fifteenth ability groups, the utility becomes 1.07%, 1.68%, 2.78%, 5.54%, 29.75%, 10.66%, 4.78%, 3.16% and 2.39% larger than at the original steady-state when the economy reaches the final steady-state with regressive subsidy rates for fertility and tax on capital income. In comparison to our previous results, however, the utility for these nine ability groups becomes 0.42%, 0.69%, 1.07%, 1.93%, 7.81%, 3.77%, 1.48%, 0.91% and 0.64% larger than at the final steady-state with flat subsidy rate for fertility and tax on consumption; 0.81%, 1.19%, 1.78%, 3.17%, 12.79%, 6.20%, 2.46%, 1.51% and 1.09% larger than at the final steady-state with flat subsidy rate for fertility and flat tax rate on the labour income; and 0.08%, 0.25%, 0.44%, 0.84%, 3.39%, 1.61%, 0.62%, 0.37% and 0.25% larger than at the final steady-state with flat subsidy rate for fertility and tax on capital income. In addition our results indicate that at the final steady-state with regressive subsidy rates for fertility and tax on capital income, the utility for the households from these nine ability groups is 0.33%, 0.43%, 0.61%, 1.06%, 4.28%, 2.10%, 0.84%, 0.52% and 0.38% larger than at the final steady-state with regressive subsidy rates for fertility and tax on consumption; and

0.70%, 0.91%, 1.29%, 2.26%, 9.10%, 4.45%, 1.78%, 1.11% and 0.81% larger than at the final steady-state with regressive subsidy rates for fertility and flat tax rate on the the labour income.

As a result of these changes in the utility at the individual ability levels, combined with the distribution of the population between different ability groups for each corresponding final steady-state, our results indicate that at the final steady-state with regressive subsidy rates for fertility and tax on consumption, the level of welfare in the economy becomes 18.29% smaller than at the original steady-state. In comparison to our previous results with flat subsidy rates for fertility, however, this indicator is 2.95%, 4.32% and 2.48% larger than at the final steady-state with tax on consumption, labour income and capital income, respectively. Next, for the final steady-state with regressive subsidy rates for fertility and tax on the labour income, the average utility level in the economy becomes 19.87% smaller than at the original steady-state; and it is 1.13%, 2.48% and 0.66% larger than at the previously considered final steady-states with flat subsidy rate for fertility and tax on consumption, labour income and capital income, respectively. Lastly, for the final steady-state with regressive subsidy rates for fertility and tax on capital income, the welfare decreases 17.75% below the initial steady-state; but it is 2.95%, 4.32% and 2.48% larger than at the final steady-states with flat subsidy rate for fertility that government finances with tax on consumption, labour income and capital income, respectively. In addition, our results suggest that at the final steady-state with regressive subsidy rates for fertility and tax on capital income, the welfare is 0.46% and 1.80% larger than at the final steady-state with regressive subsidy rates for fertility and tax on consumption and labour income, respectively.

In conclusion, due to the changes in the factor inputs and population size, our results suggest that at the final steady-state with regressive subsidy rates for fertility and tax on consumption or capital income, the level of output per adult household decreases by 2.49% below the original steady-state. This result, however, is 0.15% larger than at the final steady-state with flat subsidy rate for fertility that government finances with either tax on consumption or capital income; and it is 0.62% larger than at the final steady-state with flat subsidy rate for fertility and flat tax rate on the labour income. For the final steady-state with regressive subsidy rates for fertility and flat tax rate on the labour income, the level of output per adult household becomes 2.93%

smaller than at the original steady-state; and it is 0.31% smaller than at the final steady-state with flat subsidy rate for fertility that government finances with either tax on consumption or capital income. However, this indicator at the final steady-state with regressive subsidy rates for fertility and flat tax rate on the labour income is 0.16% larger than at the final steady-state with flat subsidy rate for fertility and flat tax rate on the labour income. Finally, our results indicate that at the final steady-state with regressive subsidy rates for fertility and tax on consumption or capital income, the level of output per adult household is 0.46% larger than at the final steady-state with regressive subsidy rates for fertility and flat tax rate on the labour income.

4.5 Conclusion

In this chapter we have examined the influence of the compulsory education and childcare on the long-run economic development. We also re-examined our previous results when government provided flat subsidy rates for education and fertility which the government financed with flat tax rate on consumption, labour income and capital income. Namely, we analysed the influence of the progressive tax system on labour income which has been used to provide flat subsidy rates for education and fertility; and we also investigated the role of the regressive subsidy rates for education and fertility which government provided with tax on consumption, flat tax rate on the labour income, and on capital income. Similarly to our previous two chapters, we performed our analysis by extending the model of overlapping generations presented by *de la* Croix and Doepke (2003). Finally, our analysis has been conducted in the stochastic environment where heterogeneous households receive idiosyncratic shocks to the human capital.

According to our results from the analysis for the environment with heterogeneous households that have been receiving idiosyncratic shocks to the human capital, where the government has provided the compulsory education at a level of the average private education attainment from the original steady-state, the immediate optimal response of adult households has been to reduce the level of private education investment. Furthermore, the majority of the population completely relied on the government for education provision, and only the households from top three ability groups provided their children with (additional) private education at an above-zero level[52]. As a result of the considered compulsory education programme and the change in the decisions of the households for the private education investment, children from lower ability groups have received the total level of education that was larger than at the original steady-state, while children from higher ability groups received the total education level that was lower than at the original steady-state. Moreover, with the compulsory education, children from lowest ability groups that received zero education at the original steady-state, have escaped the illiterateness trap, which previously was only possible with flat subsidy rate for education of 80% and larger that was financed with either tax on consumption or capital income (but not with the flat tax rate on the labour income).

Due to a reduction in the private decisions for education that was a result of an introduction of compulsory education, and due to the presence of parental trade-off, the adult households that have provided their children at the original steady-state with any level of education that was greater than zero, have increased their fertility decisions. For the case with flat tax rate on the labour income the rate of increase in childbearing decisions has been larger than for the cases with tax on consumption and capital income. Furthermore, the adult households from the bottom ability groups that did not provide their children with private education investment at the original steady-state have experienced an increase in the fertility level only when the government financed and balanced its budget with flat tax rate on the labour income. The adult households that still provided their children with (additional) private education, experienced an increase in fertility at a smaller rate than the rest of the population due to the presence of a parental trade-off.

Consequently, due to the change in the total education attainment that has influenced the realised levels of the human capital, and as a result of the change in the individual fertility decisions, our results indicated that the population share of the top and bottom ability groups has reduced to zero. At the final steady-state with considered compulsory education programme, the population consisted of the households from the ability groups that have been in the centre of the original distribution. This observation together with the calculated inequality measures indicated that the economy obtained the lowest level of inequality in the distribution of the

[52]Please note that the population share of these households became zero when the economy approached the final steady-state with compulsory education in place

human capital across the population out of all previously and presently considered government policies[53]. Additionally, we have observed that both average level of human capital and average fertility level in the economy increased above the original steady-state with considered compulsory education in place. This increase in the average level of the human capital increased the quality of teachers in the economy which further has helped to the disadvantaged children.

However, as a result of the compulsory education, the economy reaches the final steady-state with the individual utility levels that are lower than at the original steady-state for all considered policy options, primarily, due to the dynamics that we have observed for the consumption of adult households at the individual ability level. With the tax on capital income, however, the adult households experienced a decrease in the utility levels that were smallest in the magnitude, while, with the tax on labour income, the individual utility levels have fallen at a largest rate. As a result of these changes in the utility at the individual ability levels and due to the change in the distribution of the population, we have observed that the level of welfare in the economy with the considered compulsory education only became larger than at the original steady-state with tax on capital income in place. With both tax on consumption and labour income, however, the welfare in the economy at the final steady-state was lower than at the original equilibrium.

Lastly, our results indicated that the level of output per adult household has increased above the original steady-state when the economy reached the final steady-state with the compulsory education under all considered tax options. However, the rate of the increase in this indicator was lower when the government financed its budget with the flat tax rate on the labour income. This is because the tax on the labour income has created additional disincentives for saving of adult households which resulted in a lower level of aggregate physical capital stock per adult household. Additionally, with the labour income taxation, the population of adult households grew at a larger rate than with other tax options.

Next, based on our results for the case when the government provided the childcare for 50% of the time that adult households have to spend on raising children, we have observed a considerable initial decrease in the decisions of adult households for education investment

[53]This conclusion, however, is conditional on the level of compulsory education and level of subsidy rates implemented by the government.

across all ability groups that provided their children with the above-zero level of education at the initial steady-state. At the same time, however, the adult households across all ability groups increased their fertility decisions. Due to this initial change in the decisions of adult households, our results indicated an increase in the population share of lower ability households at the second period of the government program, while, the population share of higher ability households has decreased. Given these changes that took place at the individual ability level, the average level of the human capital at the third period of analysis has decreased below the original steady-state. This decrease in the average human capital, combined with the fixed (marginal) human capital levels that characterise each ability group, has increased the relative human capital levels for the households from all ability groups. In turn, this resulted in an increase in the choices for education and in a decrease in the fertility levels for the households from ability groups that provided their children with the above-zero level of education investment. For the households from bottom ability groups, the level of education provision remained at zero, and the fertility decisions have not changed from the previous period of analysis.

Overall, when the considered model economy has reached the final steady-states with the state provided childcare, at the individual level, the households provided their children with less education but they decided on a larger fertility than at the original steady-state. As a result, the average education and the average human capital became smaller, while the average fertility and the population size became larger than at the original steady-state. Furthermore, the majority of the population at the final steady-states with the considered childcare policy were a part of the lowest ability groups; and the level of inequality in the distribution of the human capital increased above the original steady-state. In addition, when the economy reached the final steady-states, the utility of adult households became lower than at the original steady-state due to decrease in the consumption of adult households and decrease in the human capital of children. Given this change at the individual level and combined with the change in the distribution of the population, our results indicated a decrease in the level of welfare. Furthermore, since both of the factor inputs at a per adult household level became lower than at the original steady-state, the level of output per adult household also diminished below the initial equilibrium. In conclusion, the order of the considered tax options and their influence on the economic indicators has remained in line with the case for compulsory education generally. The inequality measures for the distribution of the human capital have been an exception, however, since the tax on labour income created a relatively larger disincentive for the education investment than other considered tax options, which resulted in a larger population share to be concentrated among the bottom ability groups and a smaller population share to be a part of a higher ability groups.

Moving forward to the case with progressive labour income taxation that financed the flat subsidy rate for education of 10%, the economy reached the final steady-state where the parents from lower ability groups that provided their children with above-zero level of education at the original steady-state have improved their choices for education above the levels of the original steady-state and the previously considered final steady-states with subsidy for education of 10% and flat tax schedules. For the households from higher ability groups, however, that at the final steady-state received larger marginal tax rates on the labour income than the rest of the population, the decisions for education investment became lower than under previously considered tax options, with the households from the top ability group providing their children with education that was lower than at the original steady-state. Due to this dynamic in the decisions for education investment, and as a result of a parental trade-off, at the final steady-state with progressive labour income taxation and flat subsidy rate for education of 10%, the households from lower ability groups decided to have less children than at the final steady-state with flat tax rate on the labour income, while the households from higher ability groups increased their childbearing above the previously determined outcomes for the cases with flat tax system. Given these changes in the education and fertility decisions, our results indicated that at the final steady-state with flat subsidy rate for education and progressive labour income taxation, the population consisted of a larger fraction of the households that has been at a centre of the original distribution, and a smaller fraction of the households that has been either from the bottom or the top ability groups than at the previously considered final steady-states with flat tax options. Lastly, with flat subsidy rate for education of 10% and with progressive labour income taxation, only the households from lower ability groups enjoy an improvement in an individual utility level. For these households, the utility was also above the previously considered levels with flat labour income taxation. For the households from higher ability groups, however, that have payed a higher marginal tax rates on the labour income, enjoyed less consumption during adulthood and old age, and their children had a lower human capital level in comparison to the previously considered final steady-states, the individual utility levels have been found to deteriorate below the previously considered levels.

With these outcomes at the individual level and given the distribution of the population, at the final steady-state with flat subsidy rate for education of 10% and progressive labour income tax schedule, the economy enjoyed the average human capital, welfare and output per adult household that have been larger than at the original and previously considered final steady-state with flat tax rate on the labour income. However, these indicators have been below the levels at the final steady-state with tax on consumption or capital income. But, at the final steady-state with progressive labour income taxation, the level of inequality in the distribution of the human capital has been found to be lower than for the previously considered final steady-states with subsidy for education of 10% and flat tax schedules.

For the case with flat subsidy rate for fertility of 10% and progressive labour income taxation, we have observed that the households from lower ability groups that faced a relatively smaller tax rates on the labour income than the rest of the population, provided their children with a relatively larger level of education than under all previously considered final steady-states with flat tax options. These levels, however, were still smaller than at the original steady-state. With these marginally larger choices for education, and due to the presence of parental trade-off, the adult households from lower ability groups decided on a relatively smaller fertility than under previously considered final steady-states with subsidy for fertility of 10% and flat tax schedules. The adult households from higher ability groups, however, at the final steady-state with progressive income taxation provided their children with a smaller level of education than under all previously considered tax options. These levels with progressive labour income taxation were still larger than at the original steady-state. Consequently, at the final steady-state with subsidy for education of 10% and progressive labour income tax scheme, the adult households from higher ability groups decided on a larger number of children to have than at the previously considered final steady-states with flat tax options. With these changes in the education and fertility decisions, we have observed an increase in both lower and higher ability groups, while the share of the households from the centre of the distribution diminished. However, in comparison to other tax options, the population of the households with progressive labour income taxation that financed and balanced the government budget consisted of relatively larger share of the households that were in the centre of the original distribution, and relatively smaller share of the households from the top and bottom ability groups.

Due to these changes, at the final steady-state with subsidy for fertility of 10% and with progressive labour income tax scheme, the average human capital, welfare and output per adult household all diminished below the original equilibrium, and they were larger than at the final steady-state with flat tax rate on the labour income, but smaller than with either tax on consumption or capital income. The average fertility level and the population size became larger than at the original steady-state, and they were larger than with tax on consumption or capital income, but these indicators were smaller than with the flat labour income taxation. Finally, at the final steady-state with subsidy for fertility of 10% and progressive labour income taxation, the level of inequality in the distribution of the human capital became larger than at the original steady-state, and larger than under previously considered final steady-states with flat tax options.

In our final policy experiments where we considered that the government provides a larger subsidy rate for education to the households that decide on a smaller level of education investment, and a larger subsidy rate for fertility to the households that decide on a smaller number of children to have, we obtain the following results. First, based on our analysis with subsidy for education, where the households from lower ability groups received a larger subsidy rate than the households from higher ability groups, under all tax options at the final steady-state, the households from lower ability groups provided their children with education that was larger-while the households from higher ability groups provided their children with education that was smaller than under previously considered flat subsidy rate for education. Furthermore, the households from the bottom ability groups that provided their children with zero education at the original steady-state, now optimally provided their children with above-zero levels of education. The households from higher ability groups, however, with this disproportional subsidy system have provided their children with education that was lower than at the original steady-state. As a result of these disproportional subsidy rates for education, given the changes in the decisions for education investment, the households from lower ability groups decided on fertility levels that were considerably smaller, while the households from higher ability groups optimally decided on the fertility decisions that were larger than under the original steady-state and previously considered final steady-states with flat subsidy rate for education. With these decisions we observed that the population share of both top and bottom ability groups became lower that with flat subsidy rate for education considered previously; but the population share of the households in the centre of the original distribution increased above the previously considered levels. Finally, at the individual level with regressive subsidy rates for education, the households from lower ability groups were worse-off, while the households from higher ability groups were better-off than at the final steady-state with flat subsidy rate for education for a given tax option that financed and balanced the government budget.

Based on these changes that we have observed, in comparison to the final steady-state with flat subsidy rate for education for a given tax option, the economy at the final steady-state with regressive subsidy rates for education had a lower level of the average human capital, lower level of output per adult household, larger rate of fall in the population, but larger level of welfare. Furthermore, at the final steady-state with disproportional subsidy rates for education, the economy had a lower level of inequality in the distribution of the human capital across the population than at the original steady-state and at the previously considered final steady-states with flat subsidy rate for education. In terms of the results for inequality in the distribution of the human capital, the considered regressive subsidy program for education has produced the second-best outcomes after the considered compulsory education programme.

Lastly, based on our results from the analysis with disproportional subsidy rates for fertility, where the households that decided on a smaller number of children received larger subsidy rates for fertility than the households with larger number of children, we have observed the following. The households from lower ability groups have optimally decided on the fertility levels that were lower than at the final steady-states with flat subsidy rate for fertility, while, the households from higher ability groups increased the childbearing decisions above the previously considered levels at the final steady-states with flat subsidy rate for fertility. The fertility decisions across the population for all ability groups at the final steady-state with regressive subsidy rates for fertility were larger than at the original steady-state, however. Due to this dynamic in the fertility choices, at the final steady-states with the disproportional subsidy rates for fertility, the households from lower ability groups provided their children with relatively larger education, but the households from higher ability groups decided on a relatively lower level of education investment than at the final steady-states with flat subsidy rate for fertility. However, the households that provided their children with zero education at the original steady-state have not changed their choices of education investment when the economy reached the final steady-states with the regressive subsidy system for fertility. Furthermore, it still has been the case that the lower ability households provided their children with education that was lower than at the original steady-state; whereas, the households from higher ability groups provided their children with the education that was larger than at the original equilibrium. With these choices in education and fertility, we observed that, as before, when the economy reached the final steady-state with disproportional subsidy rates for fertility, there has been an increase in the population share of the households from the top and bottom ability groups, but the population share of the households that has been in the centre of the original distribution has diminished. However, in comparison to the final steady-states with flat subsidy rate for fertility, with the regressive subsidy system there was a relatively larger share of the households from the centre of the original distribution, and a relatively smaller share of the households from the two tails of the distribution. Finally, similarly to the case with regressive subsidy rates for education, when the economy has reached the final steady-states with regressive subsidy rates for fertility, for a given tax option, the households from lower ability groups were worse-off, but the households from higher ability groups were better-off – in comparison to the case with flat subsidy for fertility of 10%. Overall, however, the utility at the individual level for entire population diminished below the original steady-state as a result of a considered disproportional subsidy policy for fertility.

With these changes at the individual level and due to the change in the distribution of the population, our results indicated that for a given tax option, when the economy has reached the final steady-states with the regressive subsidy rates for fertility, the average human capital, welfare and output per adult household were larger, and the average fertility, growth rate of the population and inequality in the distribution of the human capital were smaller than at the previously considered final steady-states with flat subsidy rate for fertility. However, at the final steady-state with the considered regressive subsidy rates for fertility, these results for the average human capital, welfare and output per adult household were still smaller, while the outcomes for the average fertility, growth rate of the population and inequality in the distribution of the human capital were still larger than at the original steady-state.

4.6 Appendix

4.6.1 A1: Analytical solution for the problem of the households for the deterministic environment

$$\max_{c_t^i, s_t^i, e_t^i, n_t^i} u_t^i = \ln c_t^i + \beta \ln d_{t+1}^i + \gamma \ln n_t^i h_{t+1}^i$$

$$(1+\tau_t^c)c_t^i + s_t^i + e_t^i n_t^i w_t \bar{h}_t = (1-\tau_t^{l,i}) w_t h_t^i \big(1-(1-\omega_t)\phi n_t^i\big) + n_t^i w_t \bar{h}_t (e_t^i \chi_t^i sub_t^e + \bar{e}_t \psi_t^i sub_t^n)$$

$$(1+\tau_{t+1}^c) d_{t+1}^i = \frac{1}{(1+\rho)}\big(1+r_{t+1}(1-\tau_{t+1}^k)\big)s_t^i$$

$$h_{t+1}^i = \frac{1}{(1+\rho)} B(\theta + e_t^i + \underline{e}_t)^{\eta} (h_t^i)^{\pi} (\bar{h}_t)^{\kappa}$$

First Order Conditions:

$$\frac{\partial \mathscr{L}_t^i}{\partial c_t^i} = \frac{1}{c_t^i} - \lambda_t^i (1+\tau_t^c) = 0$$

$$\frac{\partial \mathscr{L}_t^i}{\partial s_t^i} = \beta \frac{1}{s_t^i} - \lambda_t^i = 0$$

$$\frac{\partial \mathscr{L}_t^i}{\partial e_t^i} = \gamma\eta \frac{1}{(\theta + e_t^i + \underline{e}_t)} + \lambda_t^i n_t^i w_t \bar{h}_t \chi_t^i sub_t^e - \lambda_t^i n_t^i w_t \bar{h}_t = 0$$

$$\frac{\partial \mathscr{L}_t^i}{\partial n_t^i} = \gamma \frac{1}{n_t^i} - \lambda_t^i (1-\tau_t^{l,i}) w_t h_t^i (1-\omega_t)\phi + \lambda_t^i w_t \bar{h}_t (e_t^i \chi_t^i sub_t^e + \bar{e}_t \psi_t^i sub_t^n) - \lambda_t^i e_t^i w_t \bar{h}_t = 0$$

Optimal decisions of the households:

$$e_t^i = \frac{\eta\phi(1-\omega_t)(1-\tau_t^{l,i})x_t^i - \theta(1-\chi_t^i sub_t^e) - \eta \bar{e}_t \psi_t^i sub_t^n - \underline{e}(1-\chi_t^i sub_t^e)}{(1-\chi_t^i sub_t^e)(1-\eta)}$$

$$c_t^i = \frac{1}{(1+\beta+\gamma)} \frac{(1-\tau_t^{l,i})}{(1+\tau_t^c)} w_t h_t^i$$

$$s_t^i = \beta(1+\tau_t^c)c_t^i$$

$$n_t^i = \frac{\gamma}{(1+\beta+\gamma)}(1-\tau_t^{l,i})x_t^i \Big[(1-\tau_t^{l,i})(1-\omega_t)\phi x_t^i + e_t^i(1-\chi_t^i sub_t^e) - \bar{e}_t \psi_t^i sub_t^n\Big]^{-1}$$

4.6.2 A2: Inequality in distribution of the human capital

steady state	Gini coefficient	Coefficient of variation	Mean absolute deviation	Kuznets ratio $\leqslant 40\%$	$\geqslant 60\%$	The Range
original	0.2018	0.3724	0.2818	0.1230	0.3332	3.1190
flat subsidy rates and flat tax rates						
$sub^e + \tau^c$	0.1968	0.3637	0.2864	0.0764	0.4299	2.8474
$sub^e + \tau^l$	0.1971	0.3640	0.2866	0.0786	0.4241	2.8622
$sub^e + \tau^k$	0.1968	0.3637	0.2864	0.0764	0.4399	2.8474
$sub^n + \tau^c$	0.2389	0.4376	0.3442	0.2209	0.2924	3.3402
$sub^n + \tau^l$	0.2391	0.4383	0.3449	0.2242	0.2881	3.3580
$sub^n + \tau^k$	0.2389	0.4376	0.3442	0.2209	0.2924	3.3402
compulsory education and flat tax rates						
$\underline{e} + \tau^c$	0.1175	0.2177	0.1684	0.0054	0.3210	3.0545
$\underline{e} + \tau^l$	0.1176	0.2173	0.1688	0.0054	0.3204	3.0509
$\underline{e} + \tau^k$	0.1175	0.2177	0.1684	0.0054	0.3210	3.0545
childcare and flat tax rates						
$\omega + \tau^c$	0.2261	0.4302	0.3292	0.7124	0.0236	5.9056
$\omega + \tau^l$	0.2128	0.4162	0.3056	0.8329	0.0085	6.9634
$\omega + \tau^k$	0.2261	0.4302	0.3292	0.7124	0.0236	5.9056
flat subsidy rates and progressive tax rates on labour income						
$sub^e + \tau^l_i$	0.1952	0.3604	0.2840	0.0758	0.4255	2.8600
$sub^n + \tau^l_i$	0.2359	0.4317	0.3401	0.2183	0.2881	3.3520
regressive subsidy rates and flat tax rates						
$sub^e_i + \tau^c$	0.1723	0.3197	0.2534	0.0401	0.4218	2.8565
$sub^e_i + \tau^l$	0.1724	0.3198	0.2533	0.0415	0.4164	2.8686
$sub^e_i + \tau^k$	0.1723	0.3197	0.2534	0.0401	0.4218	2.8565
$sub^n_i + \tau^c$	0.2251	0.4148	0.3245	0.1963	0.2838	3.3283
$sub^n_i + \tau^l$	0.2255	0.4154	0.3254	0.1998	0.2798	3.3438
$sub^n_i + \tau^k$	0.2251	0.4148	0.3245	0.1963	0.2838	3.3283

Table 4.16: Measures for inequality in the distribution of human capital across the population at the corresponding steady-states given the government policy in place

Conclusion

In this thesis, we investigated the influence of the government policies that target the education and fertility choices. We began this research, however, with the chapter 1, where we examined the evidence from the literature for the dynamics of education attainment and childbearing, and highlighted the role of these decisions for the social and economic development as it has been indicated by the previous research.

To conduct our research, we utilised the model of overlapping generations of *de la* Croix and Doepke (2003). We expanded this model economy with the government sector; and latter we also introduced an assumption that the human capital contains a stochastic component.

In the chapter 2, we analysed the influence of the flat subsidy rates for education and fertility in the deterministic environment. We considered that the government finances and balances its budget with (flat) tax on consumption, labour income and capital income. Our results have shown that subsidy for education leads to the outcome with higher (private) education attainment, human capital, welfare and the output per adult household, but at a cost of lower fertility and decreasing population size. With subsidy for fertility we observed the opposite. Our results indicated, however, that both of these programs would lead to the final steady-state with unique human capital level and single ability group, when the households are heterogeneous in the human capital. Additionally, the combination of subsidy for education and subsidy for fertility was required to resolve the parental 'quality-quantity' trade-off for children.

In the chapter 3, we analysed the same policy mix as in the chapter 2. However, our analysis in the chapter 3 featured the stochastic environment with uncertainty in the human capital development. We found that the behaviour at the average and aggregate levels in the stochastic environment are consistent with conclusions that we obtained for the deterministic version of analysis. However, at the individual ability level, we found that the considered subsidy for education is not able to improve the education decisions of the households from lowest ability groups, who still provide their children with zero education. Furthermore, the subsidy for fertility has been found to improve the utility of lower ability groups only, while the households from higher ability groups simultaneously increased the education provision and fertility level, which resolved the parental trade-off for them. Additionally, we found that neither subsidy for education nor subsidy for fertility could establish the equilibrium with homogeneous level of human capital when the human capital accumulation contains uncertainty. Instead, our results indicated that the level of inequality would reduce with subsidy for education; and it would increase above the original level with subsidy for fertility.

Lastly, in the chapter 4, we examined the influence of compulsory education, childcare, progressive tax rate on labour income, and the regressive subsidy system for education and fertility. From the analysis that has been conducted in the environment with uncertainty in the human capital, we found that compulsory education improves the average human capital and fertility level in the economy. The private choice for the education with this policy, however, was found to fall. With publicly provided childcare, our results indicated a sizeable improvement in fertility and continuous growth in the population size, with decrease in the private education investment, average human capital and welfare in the economy, however. Finally, in comparison to the flat counterparts, the progressive labour income tax scheme and regressive subsidy rates individually result in population which is more concentrated among the centre of the original distribution. The economy enjoys a relatively larger welfare but not necessary a relatively larger human capital level. Furthermore, with regressive subsidy system and progressive tax rate on the labour income, our results indicate a presence of a relatively smaller level of inequality in the distribution of the human capital than in the case with flat tax rates and flat subsidy rates.

In conclusion our results indicated that the economy reaches the largest level of welfare with tax on the capital income. The tax on consumption produces the second-best outcomes in terms of welfare; while the (flat) tax on the labour income results in a third-best level of welfare. Both tax on consumption and capital income have been found to affect the education, fertility, composition and size of the population, factor inputs, and the level of output per adult households in the same way. The (flat) tax on the labour income, however, has been found to produce disincentives for education investment, which resulted in lower level of human capital, welfare and output per adult household, but larger fertility, population size and inequality in the distribution of the human capital than with other tax options.

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