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# All-pay competition with captive consumers<sup>\*</sup>

Renaud Foucart<sup>†</sup>    Jana Friedrichsen<sup>‡</sup>

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## Abstract

We study a game in which two firms compete in quality to serve a market consisting of consumers with different initial consideration sets. If both firms invest below a certain quality threshold, they only compete for those consumers already aware of their existence. Above this threshold, a firm is visible to all and the highest quality attracts all consumers. In equilibrium, firms do not choose their investment deterministically but randomize over two disconnected intervals. On the one hand, the existence of initially captive consumers introduces an anti-competitive element: holding fixed the behavior of the rival firm, a firm with a larger captive segment enjoys a higher payoff from not investing at all. On the other hand, the fact that a firm's initially captive consumers can still be attracted by very high quality introduces a pro-competitive element: high quality investments becomes more profitable for the underdog when the captive segment of the dominant firm increases. The share of initially captive consumers therefore has a non-monotonic effect on the investment levels of both firms.

**Keywords:** quality competition, all-pay auction, endogenous prize, consideration sets

**JEL-Code:** D21, D44, M13

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# 1 Introduction

Consumers typically differ in the set of firms that they consider before making a purchasing decision and firms are not able to easily change a consumer's consideration set. When a product is of particularly high quality however, most consumers become aware of its existence, be it through word-of-mouth recommendation, social networks or news reports. In this case, the firm offering this product is in the consideration set of all consumers. In this paper, we study how the co-existence of initially captive market segments and of the possibility of reaching all consumers by providing exceptionally high quality affects competition.

We model this problem as a duopoly in which firms simultaneously choose to invest in the quality of their product. A share of the consumers observe both firms, while the others are "captive" in the sense that they initially observe only a single firm. Captive consumers may however cease to be so if a product outside of their initial consideration set is of *very* high quality. A standard argument is that the existence of captive market segments is anti-competitive because it decreases the incentives for a dominant firm to compete for the rest of the market. In our model, the presence of captive consumers also introduces a pro-competitive element: As the only way to become visible to the captive base of the competitor is to offer a *very* good product, a higher share of the consumers initially considering only one firm only may actually induce firms to offer products of higher quality.

The impact of a dominant firm trying to make consumers unaware of alternatives to its products is at the heart of several major competition cases. Those cases illustrate the regulator's desire to ensure consumers awareness of competing alternatives when one firm already acquired a dominant market position. In 2013, Microsoft was fined by the European Commission for failing to make Windows users aware of competing web browsers despite being committed to do so since a 2009 settlement.<sup>1</sup> Perhaps ironically, this ruling is almost contemporary to Google Chrome overtaking Microsoft Internet Explorer as the market leader, despite Windows still enjoying more than 80% market share at the time, and before mobile phones became a major source of Internet browsing.<sup>2</sup> In June 2017, Google was fined for hiding alternatives to its own comparison shopping service. At the source of the complaint was a website called "Foundem" arguing that Google showed its users the comparison website "Froogle" (now Google

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<sup>1</sup>"Commission fines Microsoft for non-compliance with browser choice commitments," European Commission Press Release, 6 March 2013, IP/13/196.

<sup>2</sup>According to data by traffic analysis website StatCounter, Chrome became market leader in January 2012, when mobile browsing still represented only 8.49% of the traffic. In January 2012, Microsoft Windows had a 82.06% share of the market for Operating Systems (statcounter.com).

shopping) only, hiding the competitors.<sup>3</sup> In 2018, Google was fined again for restricting Android device manufacturers in what they showed to consumers, in particular forcing them to pre-install Google search and Google's Chrome browser.<sup>4</sup>

Our model confirms the regulator's intuition that a dominant firm hiding the existence of its competitor products to its captive consumer base may be anti-competitive. We however show that the impact of a larger segment of the market observing the dominant firm's product only may be to increase the incentives for both firms to invest in the quality of their products. This can happen when the firm with the fewest captive consumers invests to capture the entire market with strictly positive probability. Thus, competition policy should be different depending on whether there exist an underdog with such capacities - such as in the Internet Explorer case -, or if the "captive" segment of the dominant firm really is impossible to reach - such as in the "Foundem" case.

The question of the economic impact of consumers observing different sets of firms has received much attention in economics. Early models of price dispersion (Varian, 1980; Burdett and Judd, 1983) study consumers that are heterogeneous in the number of firms they observe. Among the newer models using the dichotomy between informed and informed buyers, the one by de Corniere and Taylor (2019) is of particular interest to us, as the authors characterize the competitive impact of a dominant search engine guiding consumers towards a specific product. A general characterization of price competition in oligopoly settings with different consideration sets is provided by Armstrong and Vickers (2018). The novelty of our approach is that we allow a firm to directly enter the consideration set of all consumers by providing a sufficiently high quality product.

By construction, our setting is similar to an all pay auction (Baye *et al.*, 1996) but in contrast to the standard model, prizes are endogenous. The unique equilibrium is in mixed strategies. The intuition is similar to that of the standard all-pay auction: for every given level of investment of the winning firm, the competitor could win instead by choosing a marginally higher level. A crucial difference is that investments below the threshold can only win a share of the "prize", not-including the captive segment of the competitor. When the threshold is very high, no firm ever tries to take the captive segment of the other: the two firms randomize over the same interval of investments in equilibrium, and make an expected profit equal to the value of their captive segment. When the threshold is exactly zero, both firms always compete at high intensity, randomize over the same interval of investments in equilibrium, and make zero profit

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<sup>3</sup>"Commission fines Google €2.42 billion for abusing dominance as search engine by giving illegal advantage to own comparison shopping service" European Commission Press release, IP/17/1784, 27 June, 2017.

<sup>4</sup>"Commission fines Google €4.34 billion for illegal practices regarding Android mobile devices to strengthen dominance of Google's search engine" European Commission Press release, IP/18/4581, 18 July, 2018.

in expectation. Whenever the threshold is not too high but strictly positive, equilibrium bidding strategies are asymmetric. Firms choose investments below and above the threshold and therefore expect to sometimes keep their captive consumers even if they invest less than the competitor. Hence, even by not investing anything, each firm makes a strictly positive expected profit. The higher is the threshold, the higher is the probability mass that the firms put on investments below the threshold, and the probability of one firm obtaining a monopoly position decreases. The support of the equilibrium mixed strategy exhibits a gap just below the minimum investment necessary to attract the captive consumers of the competitor. This is because choosing an investment just above this threshold does not only increase the probability of winning, but also the prize of winning, which is then the whole population instead of just a share of it.

In our model, the level of investments endogenously determines the prizes for which the firms compete. If investment is above a certain threshold, there is only one prize to be won, but there are two prizes for smaller investments. We do not consider the optimal design of the prize structure. Moldovanu and Sela (2001) study the optimal allocation of prizes in a setting of imperfect information where the highest bidder gets the first prize, and the second bidder the second prize. Siegel (2009) studies asymmetric players competing for a fixed number of prizes. Another approach to multiple prize all-pay auction is the Colonel Blotto game (Roberson, 2006), where consumers bid separately for different prizes.

The endogeneity of the prizes in our setup comes from the well-known fact that consumers sometimes are biased in favour of certain options. Consumers may experience switching costs (Klemperer, 1987), or they may inspect competing firms in a certain order while bearing a search cost to observe an additional option (Arbatskaya, 2007, Armstrong *et al.*, 2009). In the presence of switching costs, it is well known that if firms cannot commit to prices they will offer “bargains-then-ripoffs” (Farrell and Klemperer, 2007): assuming homogeneous consumers, firms compete fiercely for market share in the first period (offering bargains) but extract all surplus from their attached users in the second period (ripoffs). In an overlapping generations model where undecided consumers arrive in each period, Farrell and Shapiro (1988) find that the firms alternate in selling to unattached users because they can exploit them once they are attached. In these models, consumers are *ex ante* identical and firms cannot price-discriminate based on consumer behavior. If firms can use behavior-based price discrimination, firms can poach their competitors’ consumers by offering them lower prices without losing on their own attached users (Fudenberg and Tirole, 2000). Following this approach, Gehrig and Stenbacka (2004) show that firms have an incentive to induce maximal switching costs because profits from poaching are increasing in the level of switching costs. In our model, a firm that has a larger segment of captive (at-

tached) consumers will offer on average lower quality, and this mirrors the finding that it will offer a higher (ripoff) price because it relies on its captive segment.

We proceed as follows: we introduce the model and derive important properties of the equilibrium strategies in the simultaneous investment game in Section 2. We then derive the equilibrium in Section 3. We discuss the impact of the different parameters on equilibrium behavior in Section 4. We conclude in Section 5. For those results that do not follow directly from the text, formal proofs are collected in Appendix A.

## 2 The Model

### 2.1 Setup

There are two firms, 1 and 2, that compete by investing in the quality of an otherwise homogenous product. Investment in quality is a fixed cost and the marginal production cost is set to zero. There is a mass one of consumers willing to buy exactly one unit of the product from the firm offering the highest quality in their consideration set. Before the game starts, some consumers have a single firm in their consideration set, while others have both. By investing above a certain quality threshold, a firm can become prominent and enter the consideration set of all consumers.

Firm  $i \in \{1, 2\}$  incurs a cost of  $c(k_i)$  for each unit it invests into the quality of its product, with  $c$  continuous,  $c' \geq 0$  and  $c'' \geq 0$ . In order to obtain closed form solutions, we use the linear function  $c(k_i) = ck_i$  when solving for bidding strategies in Section 3. A firm having invested a total of  $k_i$  at cost  $c(k_i)$  therefore offers a product of quality  $k_i \geq 0$ . We interpret the cost of quality as the cost of generating a given revenue per consumer, and we normalize this revenue to 1. This implies that we make two major assumptions. First, the firm incurs a fixed cost to be able to offer a certain level of quality to its users and this cost only depends on the quality level but it is independent of the number of users. Second, the quality does not influence the per-user revenue.<sup>5</sup>

Building on the specification of Armstrong and Vickers (2018) we assume that a fraction  $\alpha_i \in (0, 1)$  of consumers initially only has firm  $i \in \{1, 2\}$  in their consideration set. The remaining consumers  $\alpha_{12}$ , which we also refer to as the contested segment, have both firms in their consideration set.

The novelty of our model is that a firm becomes visible to all consumers if it chooses an investment in quality above a certain threshold  $\bar{k}$ . If firm  $i$  chooses an investment  $k_i < \bar{k}$ , it is not considered by captive consumers of firm  $j \neq i$ . But if firm  $i$  chooses an investment  $k_i \geq \bar{k}$ , firm  $i$  is considered not only by its own captive consumers and

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<sup>5</sup>While our modeling of costly investment in consumer revenue is close to de Corniere and Taylor (2019), it differs in the sense that we take per-user revenue as given. Thus, there is no impact of per-user revenue on consumer payoff.

Table 1: Firm payoffs for different investment choices.

	level of investment	
	$k_i, k_j < \bar{k}$	$k_i \geq \bar{k}$ or $k_j \geq \bar{k}$
$k_i > k_j$	$(l_i(k_i), o_j(k_j))$	$(w_i(k_i), 0)$
$k_j > k_i$	$(o_i(k_i), l_j(k_j))$	$(0, w_j(k_j))$

the contested segment but also by the captive consumers of firm  $j$ . We are interested in the set of Nash equilibria of this game. The structure of the game and frequencies of types are common knowledge. In reality, it is unlikely that a quality threshold such as  $\bar{k}$  determines with certainty which firm enters each consideration set. This simple modification of an otherwise standard setting allows us to represent the idea that investment can be either incremental or radical, and that only a radical investment can make a firm visible to a consumer who is not yet aware of this firm's product.<sup>6</sup>

Firm 1 and firm 2 simultaneously choose their investment levels. Firm  $i$  receives a payoff of

$$o_i(k_i) = \alpha_i \text{Prob}(k_j < \bar{k}) - c(k_i)$$

if it invests below the threshold ( $k_i < \bar{k}$ ) and less than its competitor ( $k_i < k_j$ ). We refer to the particular case where a firm does not invest  $o_i(0)$  as firm  $i$ 's *outside option*. The outside option of firm  $i$  depends on the probability with which firm  $j$  chooses an investment below  $\bar{k}$ , in which case captive consumers do not consider firm  $j$ 's investment. Denote by

$$l_i(k_i) = 1 - \alpha_j - c(k_i)$$

the payoff of firm  $i$  if it invests below the threshold ( $k_i < \bar{k}$ ) and more than its competitor ( $k_i > k_j$ ). If firm  $i$  invests above the threshold  $k_i \geq \bar{k}$ , captive consumers consider both firms and the outside option of firm  $j$  drops to zero. In this case, firm  $i$  competes for all segments of the market and earns at best a payoff

$$w_i(k_i) = 1 - c(k_i).$$

We assume that each relevant consumer segment is shared equally by both firms in case of a tie. If at least one of the two firms invests  $\bar{k}$  or more, the market becomes a monopoly because the highest quality firm then enters the consideration set of all consumers. Thus, for investments above  $\bar{k}$ , the market becomes a winner-take-all market.

We use this notation to summarize the corresponding payoffs for all combinations of investments in table 1.

<sup>6</sup>To complement the analysis, we provide results from a simple probabilistic setting in Appendix B.

## 2.2 Properties

We now derive some general properties of the equilibrium investments. The game faced by the two firms resembles an all-pay auction where the bids are given by the investment levels and prizes are given by the market shares. If the investment of a firm exceeds the threshold  $\bar{k}$ , the market share of the winning firm increases discontinuously as compared to a winning bid just below the threshold because at this point the investment is just high enough to attract the competitor's captive segment in addition to the contested segment.

We first define the maximum possible level of investment at equilibrium as solving  $c(k^{max}) = 1$ . It is never a best response for either firm to provide an investment greater than  $k^{max}$  because the cost would then exceed the highest possible revenue. As can be expected from the literature, the game does not have a pure-strategy equilibrium. The intuition is that it is profitable to marginally outbid any deterministic investment of the competitor because this only marginally increases costs but ensures winning the entire market.

**Lemma 1.** *The game does not have an equilibrium in pure strategies.*

Furthermore, even if the mixed strategies may contain mass points in equilibrium, these cannot be at the same investment levels for both firms. The intuition is that mass points in one firm's investment strategy imply that the expected profit of the other firm from certain investment levels changes discretely at the respective investment level and overbidding is profitable.

**Lemma 2.** *In equilibrium, at most one firm's investment strategy has a mass point at any given investment level.*

We continue to show that apart from any mass points, the densities of both firms equilibrium investment strategies coincide.

**Lemma 3.** *There exists  $0 < \delta < \bar{k}$  and  $\bar{K} < k^{max}$  such that the densities of both firms' equilibrium investment strategies are continuous and identical on the intervals  $(0, \delta)$  and  $(\bar{k}, \bar{K})$ .*

This result develops from the standard idea that the support of the equilibrium mixed strategy in an all-pay auction is connected and the density of equilibrium investments over this interval is constant and is identical across contestants. The difference with the existing literature comes from the possibility that the support of the investment strategy consists of two disconnected intervals. This result is driven by the discontinuity in the prize of winning at the threshold  $\bar{k}$ . For investments below  $\bar{k}$ , a firm can be better off by choosing exactly  $\bar{k}$  and capturing the entire market than by outbidding the competitor at the margin and winning only the contested segment.



This tradeoff between marginally overbidding and bidding exactly  $\bar{k}$  induces an endogenous gap between the highest “low intensity” investment  $\delta$  and the lowest “high intensity” investment  $\bar{k}$ .

Finally, we show that the equilibrium investment strategies of both firms admit mass points only at zero and at the threshold beyond which the consideration of captive consumers is reached.

**Lemma 4.** *If a firm bids a level of investment  $k$  with strictly positive probability, then  $k \in \{0, \bar{k}\}$ .*

The intuition behind this property is the following. Each firm generally prefers bidding marginally above  $k'$  to bidding marginally below  $k'$  if the competitor invests  $k'$  with strictly positive probability (see Lemma 1 and 2). Therefore, any investment  $k'$  that is chosen with a strictly positive probability in equilibrium must be at the lower bound of an interval of investments in the support of the mixed strategy. By Lemma 3, this implies that only investments of 0 or  $\bar{k}$  can be chosen with strictly positive probability.

A corollary of the above results is that as long as the other firm invests below  $\bar{k}$  with strictly positive probability, a firm has a positive reservation utility, from not providing any investment at all. The reservation utility is equal to the utility from the share of captive consumers multiplied by the probability that the competitor chooses an investment below  $\bar{k}$ :  $o_1(0) = \text{Prob}(k_2 < \bar{k})\alpha_1$  for firm 1 and  $o_2(0) = \text{Prob}(k_1 < \bar{k})\alpha_2$  for firm 2. This implies that firms do not choose investments up to the level at which they just break even. Instead at the maximum investment, the expected profit conditional on this investment is equal to the reservation utility in form of the expected profit from not investing at all.

### 3 Equilibrium

We now characterize the equilibrium of the game for different levels of the threshold  $\bar{k}$ . Let us assume without loss of generality that firm 1 enjoys a larger captive segment than firm 2,  $\alpha_1 > \alpha_2$ . We are looking for closed-form equilibrium solutions and focus on the linear cost function  $c(k_i) = ck_i$ .

We consider three distinct cases. First, the threshold  $\bar{k}$  that conditions the effectiveness of a firm’s investment with respect to captive consumers may be very low. In this case, competition is intense and the market outcome is a monopoly with high probability (Proposition 1). Second, for intermediate levels of  $\bar{k}$ , firm 1, enjoying a larger captive segment, only engages in competition for the contested segment. Firm 2, however, gambles for a monopoly position by choosing investments of  $\bar{k}$  with strictly positive probability (Proposition 2). Third, if  $\bar{k}$  is very high, it is prohibitively costly for

either firm to attract the competitor's captive consumers. Thus, both firms only choose investments below  $\bar{k}$  and compete for the contested segment only (Proposition 3). We now turn to the analysis of these separate cases.

Consider first the case where the threshold is low, specifically,  $\bar{k} < \frac{1}{c} \frac{\alpha_{12} + \alpha_1^2}{1 - \alpha_2}$ . Then, it is relatively easy to enter the consideration set of consumers in the competitor's captive segment. Moreover, both firms are in principle willing to choose investments high enough to do so. We show that in equilibrium both firms randomize over two disconnected intervals, one below and one above the threshold  $\bar{k}$ . In this equilibrium, firm 1 chooses to invest nothing with strictly positive probability because its larger share of captive consumers makes it compete less aggressively.

**Proposition 1.** *If  $\bar{k} < \frac{1}{c} \frac{\alpha_{12} + \alpha_1^2}{\alpha_{12} + \alpha_1} = \bar{k}_l$  there exists a unique equilibrium in which both firms randomize uniformly over two disconnected intervals: a lower interval  $(0, \delta]$  and an upper interval  $(\bar{k}, \frac{1}{c} - \bar{k} \frac{\alpha_1}{1 - \alpha_2})$  where  $\delta = \bar{k} \frac{(1 - \alpha_1)\alpha_{12}}{\alpha_{12} + \alpha_1^2} < \bar{k}$ .*

*The cumulative distribution functions are given by*

$$F_1(k) = \begin{cases} \frac{c}{1 - \alpha_1 - \alpha_2} k + \frac{c(\alpha_1 - \alpha_2)\bar{k}}{(1 - \alpha_1 - \alpha_2 + \alpha_1^2)} & \text{if } k \in [0, \delta] \\ \frac{c(1 - \alpha_2)\bar{k}}{1 - \alpha_1 - \alpha_2 + \alpha_1^2} & \text{if } \delta < k \leq \bar{k} \\ ck + \frac{c(1 - \alpha_1)\alpha_1\bar{k}}{1 - \alpha_1 - \alpha_2 + \alpha_1^2} & \text{if } \bar{k} < k \leq \bar{K} \\ 1 & \text{if } k > \bar{K} \end{cases}$$

$$F_2(k) = \begin{cases} k \frac{c}{1 - \alpha_1 - \alpha_2} & \text{if } k \in (0, \delta] \\ \frac{c(1 - \alpha_1)\bar{k}}{1 - \alpha_1 - \alpha_2 + \alpha_1^2} & \text{if } \delta < k < \bar{k} \\ ck + \frac{c(1 - \alpha_1)\alpha_1\bar{k}}{1 - \alpha_1 - \alpha_2 + \alpha_1^2} & \text{if } \bar{k} \leq k \leq \bar{K} \\ 1 & \text{if } k > \bar{K} \end{cases}$$

The cumulative distribution functions that characterize this equilibrium are such that, (i) at any interior point of both interval, both firms investment strategies have the same density and (ii) both firms investment strategies exhibit higher density on a given investment in the lower interval than in the upper interval. Moreover, firm 2 invests exactly  $\bar{k}$  with strictly positive probability, while firm 1 invests exactly 0 with strictly positive probability. We represent the equilibrium strategies in Figure 1 and derive the following results with respect to profit and market leadership.

**Corollary 1.** *In the equilibrium characterized by Proposition 1, firm 2 invests more in expectation and becomes a market leader with higher probability than firm 1. Both firms make an expected profit of  $\Pi = F_2(\delta)\alpha_1 = F_1(\delta)\alpha_2 > 0$ .*

For low values of the threshold  $\bar{k}$ , firms have an incentive to sometimes invest aggressively to enter the consideration set of all consumers. They can then attract the

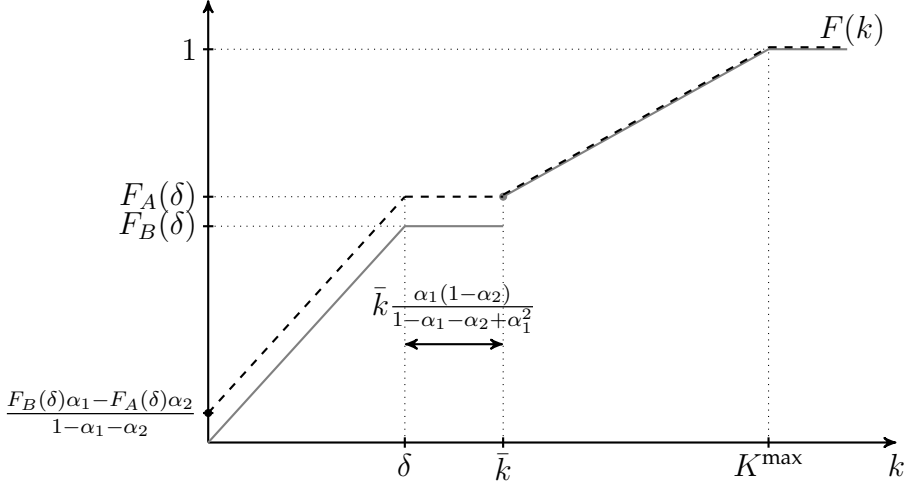


Figure 1: Proposition 1: Cumulative distribution functions if  $\bar{k} < \frac{1}{c} \frac{1-\alpha_1-\alpha_2+\alpha_1^2}{1-\alpha_2}$ .  $\bar{K} = \frac{1}{c} - \bar{k} \frac{1-\alpha_1}{1-\alpha_1-\alpha_2+\alpha_1^2}$ . Dashed: firm 1, Gray solid: firm 2.

contested segment as well as the competitor's captive segment in case of winning. Competition is however never perfect and both firms always make a strictly positive profit in equilibrium. This results from the fact that they can rely on the competitor being less aggressive with positive probability. In such a case the captive consumers remain unaware of the competitor's offer and thus buy from the unique firm in their consideration set, even if this firm did not invest in its quality.

Both firms' strategies are symmetric, except for the level of investments that they play with strictly positive probability. For each strictly positive investment below the threshold beyond which captive consumers come into play, it must hold that

$$(1) \quad f_1(k) = f_2(k) = \frac{c}{\alpha_{12}}.$$

This is, by overbidding at a marginal cost  $c$ , a firm increases its probability of winning the share  $\alpha_{12}$  of indifferent consumers by  $\frac{c}{\alpha_{12}}$ , so that the marginal benefit of overbidding is also equal to  $c$  and the firm is indifferent between all levels of investment in the support. In the range of investments that attract also captive consumers, it must hold that

$$(2) \quad f_1(k) = f_2(k) = c.$$

This is because, for investments above  $\bar{k}$ , having the highest investment is rewarded by serving the entire market of size 1, including all captive consumers and the contested segment. The marginal benefit of overbidding must therefore equal the cost  $c$ . The preceding arguments imply that the slopes of the cumulative distribution functions  $F_i$  are steeper below  $\bar{k}$  than above  $\bar{k}$  for both firms  $i = 1, 2$ .

A consequence of these equilibrium investment strategies is that firm 2, having the smallest segment of captive consumers, invests more aggressively and becomes a market leader more often in expectation. This is not simply a curiosity deriving from the mixed strategy equilibrium but results from the larger captive segment making firm 1 more complacent. We find qualitatively similar results in an environment where investments have probabilistic returns in terms of market size and the equilibrium is in pure strategies (see Appendix B for details).

For the parameter values corresponding to Proposition 1, both firms make the same expected profit, even if firm 1 appears to be the favored one due to its larger captive segment. To understand the logic, it is helpful to consider the problem of firm 1: to maximize its profit, it must be the case that no “obvious” overbidding strategy is available to firm 2. Hence, firm 1 wants to make firm 2 indifferent between all options in the support of the mixed strategy investment. In order to do so, firm 2 must believe that there is a sufficiently high probability  $P'$  that firm 1 invest some amount below  $\bar{k}$ . Similarly, firm 2 wants firm 1 to be indifferent between all options in the support of the mixed strategy investment. For firm 1 to be indifferent between investments above and below  $\bar{k}$ , it must believe that firm 2 invests some amount below  $\bar{k}$  with a sufficiently high probability  $P''$ . However, as  $\alpha_1 > \alpha_2$  and expected profits are determined by the outside options of both firms,  $E[\Pi_1] = o_1(0) = \alpha_1 P'' = \alpha_2 P' = o_2(0) = E[\Pi_2]$ , it must hold that  $P'' < P'$ . Thus, the mixed strategy of firm 1 must be less aggressive than that of firm 2 in order to make firm 2 indifferent between low and high investments. Firm 1 is thus trapped into less aggressive behavior by firm 2's small captive segment.

At equilibrium, by definition, both firm 1 and firm 2 are indifferent between all investment levels in the support of the mixed strategy. Moreover, even if one of the two firms could commit ex-ante to a mixed strategy (using a randomization device), the one that would maximize each firm's expected surplus is the equilibrium one.

This result implies that the larger (dominant) firm is less likely to invest sufficient amounts into quality to enter all consumers' consideration sets. Instead, it counts on its large captive segment remaining unaware of the competitor and abstains from competition for the competitor's small captive segment.

Consider second the case, where the threshold is sufficiently high for firm 1 not to find it worthwhile to attract the consideration of firm 2's captive share but firm 2 may still want to attract firm 1's captive segment. This asymmetry arises because firm 1 is more content with its larger captive segment, and 2 is more eager to escape its initially inferior market position.

**Proposition 2.** *If  $\bar{k}_l < \bar{k} < \frac{\alpha_{12} + \alpha_1}{c} = \bar{k}_h$ , there exists a unique equilibrium in which both firms randomize uniformly over a single interval  $(0, \delta)$ , where  $\delta = \bar{k} - \frac{\alpha_1}{c}$ .*

The cumulative distribution functions are given by

$$F_1(k) = \begin{cases} \frac{c}{\alpha_{12}}k + \frac{1-\alpha_2-c\bar{k}}{\alpha_{12}} & \text{if } k \in [0, \delta] \\ 1 & \text{if } k \geq \delta \end{cases}$$

$$F_2(k) = \begin{cases} \frac{c}{\alpha_{12}}k & \text{if } k \in (0, \delta] \\ \frac{c}{\alpha_{12}}k & \text{if } \delta \leq k \leq \bar{k} \\ 1 & \text{if } k \geq \bar{k} \end{cases}$$

The distribution functions are such that, at any point of the interval, both firms invest with the same density. Moreover, Firm 2 invests exactly  $\bar{k}$  with strictly positive probability, while Firm 1 invests exactly 0 with strictly positive probability. We immediately obtain the following corollary regarding market outcomes.

**Corollary 2.** *In the equilibrium characterized by Proposition 2, firm 2 invests more in expectation than firm 1 and becomes a market leader more often. The expected profit of firm 2 is  $1 - c\bar{k}$  and the expected profit of firm 1 is  $\alpha_1 > 1 - c\bar{k}$ .*

The result from Proposition 2 is similar to that from Proposition 1 but here it is too costly for firm 1 to attract the captive segment of firm 2. Thus, firm 1 does not choose investments equal to or above  $\bar{k}$  at all. As in the preceding arguments for investments below  $\bar{k}$ , it is still the case that both density functions satisfy

$$(3) \quad f_1(k) = f_2(k) = \frac{c}{\alpha_{12}}.$$

At or above  $\bar{k}$ , only firm 2 invests. As it does not face competition at or above  $\bar{k}$ , firm 2 chooses an investment exactly equal to  $\bar{k}$  with strictly positive probability and does never choose any strictly higher investment. This level is sufficient not only to outbid firm 1 but to also attract firm 1's captive segment with certainty. Firm 1 in contrast decides not to invest at all with a strictly positive probability and otherwise randomizes over relatively low investment levels.

Corollary 2 shows that, in this equilibrium, expected profits of both firms differ. Firm 1 benefits from its larger base, invests less and makes a higher expected profit than firm 2. The asymmetry in this equilibrium is twofold: firm 2 invests more aggressively and wins the market more often, but firm 1 actually makes the highest profit in expectation.

We now turn to the third case, where  $\bar{k}$  is so high that neither firm finds it profitable to compete for the consideration of its competitor's captive segment. Thus, both firms refrain from full competition. The winner of this softened competition enjoys a dominant market position but never achieves a monopoly. Even though the two firms have

differently sized captive segments, they behave identically and end up dominating the market with equal probability.

**Proposition 3.** *If  $\bar{k} > \bar{k}_h$ , there exists a unique equilibrium in which both firms randomize uniformly over a single interval  $[0, \frac{\alpha_{12}}{c}]$ .*

*The cumulative distribution functions are given by:*

$$F_i(k) = \begin{cases} \frac{c}{\alpha_{12}}k & \text{for all } k \in [0, \frac{\alpha_{12}}{c}] \\ 1 & \text{for } k \geq \frac{\alpha_{12}}{c} \end{cases} \text{ for } i = 1, 2$$

The distribution functions are such that, at any point, both firms invest with the same density. No firm invest at any point with strictly positive probability. No firm ever chooses an investment that would attract the consideration of the competitor's captive segment. Therefore, expected payoffs must equal the firms' outside options and market leadership is reached with equal probability.

**Corollary 3.** *In the equilibrium characterized by Proposition 3, both firms invest the same amount in expectation and become a market leader with equal probability. The expected profit of firm 2 is  $\alpha_2$  and the expected profit of firm 1 is  $\alpha_1 > \alpha_2$ .*

While it is obvious that competition for the entire population is not profitable for  $\bar{k} > k^{max} = \frac{1}{c}$ , it is the case that  $\bar{k}_h < k^{max}$ . *A priori*, high investments can be profitable for  $\bar{k}_h < \bar{k} < k^{max}$  if the success probability is high enough. However, in equilibrium this is not the case so that neither firm chooses investments equal to or above  $\bar{k}$ . As a consequence,  $\lim_{\varepsilon \rightarrow 0} F(\bar{k} - \varepsilon) = 1$  and captive consumers do not consider the competitor's offer.

Having characterized these three equilibria, we now establish that this characterization is complete.

**Proposition 4.** *The equilibrium of the investment game is unique and is characterized by Proposition 1, 2 or 3 for low, intermediate, and high levels of  $\bar{k}$ , respectively.*

The uniqueness result closely relates to the equilibrium properties outlined in Lemmas 1 to 4. An equilibrium can either be a mixed strategy over two intervals or over a single connected interval. Proposition 1 characterizes an equilibrium over two intervals. Propositions 2 and 3 characterize two candidate equilibria over a single interval. The formal proof shows that for neither of the three cases, an alternative equilibrium exists. As the existence condition the three existing equilibria are mutually exclusive, the equilibrium is indeed unique. This equilibrium is "stable" in the sense that best responses to any small perturbation to the equilibrium probabilities would bring the game back to equilibrium.<sup>7</sup>

<sup>7</sup>Consider a level of investment  $k' < \bar{k}$  that is chosen by both firms with density  $f(k') = \frac{c}{\alpha_{12}}$  in equilibrium. Suppose firm 1 instead chose to put slightly more density at  $k'$ , say  $f(k') = \phi > \frac{c}{\alpha_{12}}$ .

## 4 Implications

In this section, we discuss the major policy implications of our equilibrium results, mostly focusing on the case where both firms are in the position of competing for the whole market, described in Proposition 1. First, we show that the size of the contested segment,  $\alpha_{12}$  has a non-trivial effect on competition. Second, the size of the dominant firm's captive segment  $\alpha_1$  has a non-monotonic effect on both firms' levels of investment. Third, we study the effect of changing the difficulty of reaching the competitor's captive consumers' consideration sets captured by our threshold  $\bar{k}$ .

### 4.1 The share of consumers initially observing both firms has an ambiguous impact on investment

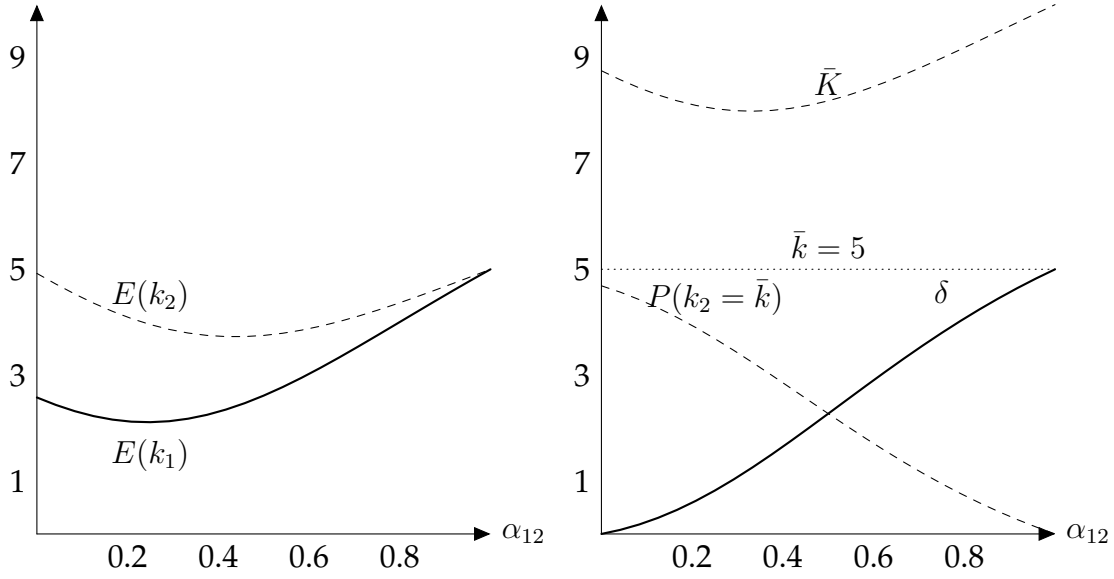
The marginal effect of increasing the share of ex-ante indifferent consumers on expected investments is ambiguous and depends on the conjunction of three effects, that we illustrate in Figure 2 for parameter values corresponding to Proposition 1. The aggregate effect of  $\alpha_{12}$  on expected investments is shown in the panel on the left-hand side (Figure 2a). The expected investment of the firm with the smallest captive segment, firm 2, initially decreases with  $\alpha_{12}$ , and only reaches its initial level when the share of the contested segment approaches 1. For the firm with the larger captive segment, firm 1, the effect of an increase in the contested segment is also negative in the beginning but becomes positive as  $\alpha_{12}$  surpasses  $\frac{1}{2}$ .

We plot the influence of the share of the contested segment  $\alpha_{12}$  on three crucial aspects of the firms' investment strategies on the right-hand side (Figure 2b). First,  $\delta$ , the upper bound of the lower part of the investment strategies' support, monotonically increases to  $\bar{k}$  as the share of the contested segment goes to 1. This effect is driven by the fact that investing at or above  $\bar{k}$  becomes increasingly irrelevant to a firm as the captive segment of the competitor vanishes.

Second,  $\alpha_{12}$  has the opposite effect on the probability that firm 2 chooses the highest investment level. The probability  $P(k_2 = \bar{k})$  of firm 2 investing exactly  $\bar{k}$  decreases as  $\alpha_{12}$  increases because its investments are spread over a larger interval with a higher  $\delta$ . When investments are more spread out, less probability mass can be allocated to  $\bar{k}$ . As  $P(k_2 = \bar{k}) = P(k_1 = 0)$ , this implies that the probability of firm 1 not investing at all also decreases. Consequently, an increase in the share of contested consumers makes the firm with a smaller (larger) captive segment invest less (more) aggressively.

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Then, firm 2 would want to put more weight on the investment level marginally above  $k'$ , as marginally outbidding an investment of  $k'$  would yield an expected benefit of  $\phi\alpha_{12} > c$ . This change in firm 2's investment strategy, however, would induce a strict decrease in the expected profit of firm 1. Thus, firm 1 is better off by sticking to the proposed equilibrium strategy.



(a) Expected investments in equilibrium. (b) Elements of equilibrium bidding strategies.

Figure 2: Comparative statics with respect to the share of ex-ante indifferent consumers  $\alpha_{12}$  within the equilibrium of Proposition 1. Illustration with  $c = 0.1$ ,  $\bar{k} = 5$  and  $\alpha_1 = 3\alpha_2$ .

This result is driven by the fact that the absolute level of asymmetry between the two firms' captive segments shrinks as the contested segment becomes larger.

Third, the maximum bid  $\bar{K}$  initially decreases with  $\alpha_{12}$  because the presence of more contested consumers reduces the additional expected profit from investing at or above  $\bar{k}$ . It then increases when competition for the growing contested segment becomes fiercer.

## 4.2 The share of captive consumers of the dominant firm has a non-monotonic effect on both firms' investments

While the above discussion is useful to disentangle the different effects, it does not provide a tractable result, as further assumptions are needed on how the different subgroups of captive consumers evolve if the contested segment increases. In this section, we focus on the perhaps most policy-relevant question of a dominant firm using its market power to increase the share of its own captive consumers, who initially observe only this firm's product but are unaware of the competitor.

We make the assumption that the smallest firm has no captive consumers ( $\alpha_2 = 0$ ) and study the impact of an increase in  $\alpha_1$ . We further assume the threshold  $\bar{k}$  is sufficiently small for the equilibrium to be characterized by Proposition 1.

**Proposition 5.** *The share of captive consumers of firm 1 has a non-monotonic effect on both firms' investment. An increase in  $\alpha_1$  has a negative effect for  $\alpha_1$  small and a positive effect for  $\alpha_1$  large.*



When the share of firm 1's captive consumers, who initially consider only the product of the dominant firm 1, increases, this has two effects. First, we observe an anti-competitive effect because the dominant firm 1 invests less as  $\alpha_1$  increases. The reason is that firm 1's outside option increase, that is its payoff when not investing at all. As investments are strategic complements, firm 2 also invests less. Second, there is a pro-competitive effect because an increasing share of captive consumers implies that an increasing share of the market can be reached only by investing above the threshold. Thus, these high investments become more profitable for the underdog firm 2 as  $\alpha_1$  increases, and due to complementarity both firms choose these investments more often.

The first effect dominates when few consumers are captive to firm 1, whereas the second effect dominates when already most consumers are captive to firm 1. As long as we stay in the realm of Proposition 1 where both firms are in position to compete, when the dominant firm becomes too dominant, it leaves no other option to the other firm but to compete very aggressively for the entire market. Again, investments are strategic complements so that this results in both firms investing more. When the dominant firm 1 is not too dominant, however, a higher share of captive consumers makes competition softer by segmenting the market.

### 4.3 Comparing the different equilibria

We now relax the assumption that the threshold  $\bar{k}$  is low enough for the equilibrium in proposition 1 to hold and that  $\alpha_2 = 0$ , and compare the three possible equilibria. We illustrate the effect of  $\bar{k}$  on expected investments in Figure 3. The vertical dotted lines represent the values of  $\bar{k}$  that delimit the zones corresponding to Propositions 1 to 3.

When the investment threshold  $\bar{k}$  is small but strictly positive (part (i) of Figure 3, Proposition 1) both firms compete for the captive segment of their competitor with positive probability but not with certainty. Whenever both firms choose investments below  $\bar{k}$ , competition is softened because it is restricted to the contested segment. Both firms can then serve their captive segments even at an investment of zero. As a consequence, both firms include zero investment in their investment strategy and make strictly positive profits in expectation. Competition is dampened if  $\bar{k}$  increases and firm 1 puts increasingly more mass on not investing at all. As a result, both  $E(k_1)$  and  $E(k_2)$  decrease with  $\bar{k}$ , and the gap between the two increases with  $\bar{k}$ . Thus, while part (i) applies, both firms invest less in quality as it gets harder to enter the consideration set of the competitor's captive consumers.

In part (ii) of Figure 3 (Proposition 2), the impact of  $\bar{k}$  on expected investments is ambiguous. For these intermediate values, firm 1 that has the highest initial captive segment invests much more aggressively than for the low values of  $\bar{k}$ . This is because

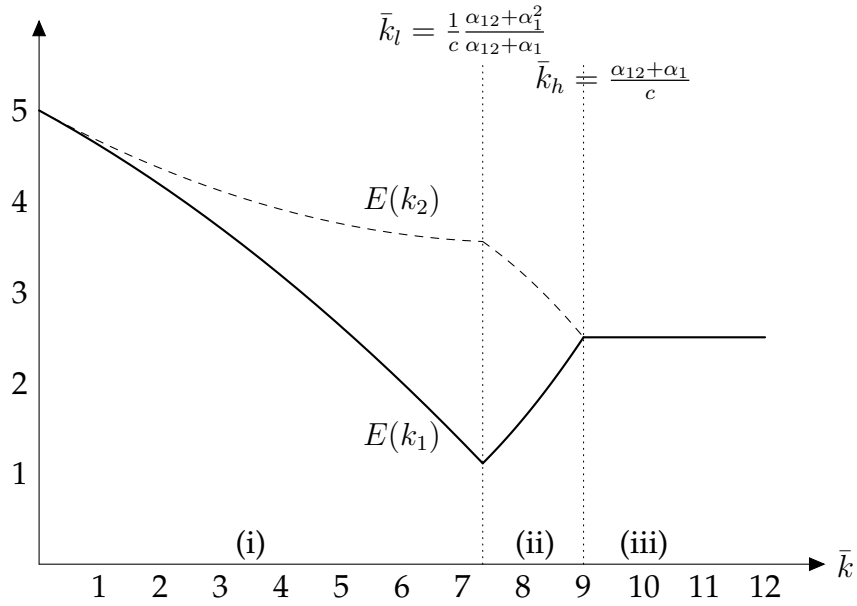


Figure 3: Expected equilibrium investment in the equilibria corresponding to Propositions 1 in part (i), 2 in (ii), and 3 in (iii). Illustration with  $\alpha_1 = 0.4$ ,  $\alpha_2 = 0.1$  and  $c = 0.1$ .

for these values, and as opposed to Proposition 1, the probability mass allocated to boundary points,  $P(k_1 = 0) = P(k_2 = \bar{k})$ , decreases with the threshold  $\bar{k}$ . This implies that the investment strategies of firms become more and more symmetric, whereas the profits become more and more asymmetric as the profit effect of the asymmetric captive segments kicks in.

When  $\bar{k}$  reaches the level at which firms decide to only compete for ex-ante indifferent consumers (part (iii) of Figure 3, Proposition 3), the expected investments in both types of equilibrium are the same and the expected level of investment remains constant for further increases in  $\bar{k}$ . Firms compete only for the contested segment of consumers who anyway consider both firms. Therefore, investment behavior is independent of  $\bar{k}$ .

## 5 Conclusion

When two players bid to win a discrete prize and both winning and losing bids are forfeited, it is well known that in equilibrium players randomize continuously over a connected interval of bids and make zero profit in expectation. In this paper, we analyze a game between two firms that resembles this classic all-pay auction but differs in the following way. We assume that each of the two competing firms may serve a segment of captive consumers who can only be served by the competitor if it invests above a certain threshold. If the investment falls short of this threshold, captive consumers do not consider the product of the competing firm. We find that the existence

of these captive segments induces asymmetries in the probabilities of one or the other firm dominating the market. The reason is that the outside option of serving only its own captive segment in a shared market is less attractive for a firm with a small captive segment than it is for the competitor with a larger one.

The effect of the share of captive consumers on total expected investments is non-monotone: if the underdog has sufficient resources to target the whole market, a higher share of captive consumers of the dominant firm may lead to higher investments. This may well have been the case for Internet Explorer competing with Google Chrome. When Microsoft chose to breach its 2009 promise to make Windows users aware of the existence of competing browsers to Internet Explorer, Google was already big enough to compete aggressively, and eventually overtook most of the market before regulators forced Microsoft to place Google Chrome such that it would enter the consideration set of Windows users. Hence, a key question for a regulator is to identify whether a competitor with the potential to overtake the whole market exists. Facing a weak underdog, a regulator may do well to prevent the dominant firm from keeping consumers unaware of alternatives to its products. Facing a strong underdog, the case is much more balanced.

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# Appendix

## A Proofs

### Proof of Lemma 1

*Proof.* The proof is by contradiction. Suppose there is pure strategy equilibrium in which firms 1 and 2 choose investments  $k_1, k_2 < k^{max}$  with certainty. Suppose first that  $k_1 = k_2$ . Obviously, each firm could profitably deviate to marginally overbidding the other as this would only marginally increase cost but discretely increases the chance of winning. Thus, this cannot be an equilibrium. Suppose instead  $k_1 < k_2$ . We distinguish two cases. If  $k_2 \geq \bar{k}$ , firm 1 can profitably deviate to investing just marginally above  $k_2$  which would imply winning the entire market with certainty and by  $k_2 < k^{max}$  would yield a positive profit. Now consider the case  $k_2 < \bar{k}$ . This can only be an equilibrium if firm 1 chooses  $k_1 = 0$  because it loses in any case. But then, firm 2 would want to just marginally overbid 0 which could in turn be profitably outbid by firm 1. Thus, these investments do not constitute an equilibrium either. The analogous arguments hold if we exchange subscripts 1 and 2. Therefore, the equilibrium must be in mixed strategies.  $\square$

### Proof of Lemma 2

*Proof.* The proof is by contradiction. Suppose firm 1 invests according to an equilibrium investment strategy  $F_1$  and as part of it chooses  $k_1 = k'$  with strictly positive probability,  $P_1(k') > 0$ . Suppose further that also firm 2 chooses  $k'$  with positive probability, i.e.,  $P_2(k') > 0$  is part of firm 2's equilibrium strategy  $F_2$ . Denote by  $E[\Pi_2]$  the expected profit of firm 2 given this strategy. Note that firm 2 can discretely increase its expected profit by switching to a mixed strategy  $F'_2$  that differs from  $F_2$  only in that firm 2 reallocates probability mass from  $k'$  to an investment of  $k_2 = k' + \epsilon$  for any  $\epsilon > 0$  small enough. Thus, both firms will not allocate positive probability to the same investment.  $\square$

### Proof of Lemma 3

*Proof.* (i) We first show that, if there is a gap in the support of the mixed strategy of a firm, the gap must be an interval containing  $\bar{k}$ . Suppose there is a gap  $[k', k'']$  in the support of firm  $i$ 's strategy between with  $k', k'' \in (0, \bar{k})$ ,  $k' < k''$ , and  $F_i(k') = F_i(k'')$ . Note that firm  $j$  then strictly prefers investing  $k'$  over investing  $k''$  because the expected profit is the same but the expected cost is lower

for the lower investment. Thus, firm  $j$  prefers to marginally overbid firm  $i$  at  $k'$  over any higher investment and in particular over investing  $k''$ . This in turn implies that firm  $i$  also strictly prefers to invest marginally above firm  $j$ 's investment over investing  $k''$  because  $k''$  is more expensive but does not increase the chance of winning. Thus, the condition that a firm has the same expected profit over the support of her mixed strategy would be violated. The same reasoning applies to any pair  $k', k'' > \bar{k}$ . Thus, if there is a gap, it must be the case that  $k' < \bar{k}$  and  $k'' \geq \bar{k}$ .

- (ii) It follows that any gap must have as an upper bound  $\bar{k}$ . Else, following the same logic as above, a firm would strictly prefer bidding  $\bar{k}$  over some  $k'' > \bar{k}$ . Hence, if there is a gap, it must be in some interval  $[\delta, \bar{k}]$ .
- (iii) The fact that firms randomize over the same intervals is a standard property: if an investment  $k$  is part of only one firm's mixed strategy support, this firm would be better off investing less (at the top of the other firm's support). The fact that the bottom of the lower investment support is zero is also standard: else investing zero with strictly positive probability would be a profitable deviation.

□

## Proof of Lemma 4

*Proof.* For a mass point to be an equilibrium strategy, it must satisfy two properties. First, by Lemma 2, a firm does not invest  $k$  with strictly positive probability in equilibrium if  $k$  is in the support of the other firm's strategy. Else, the other firm would be better off marginally outbidding  $k$  than bidding just below it. Second, the same holds if a value marginally below  $k$  is in the support of the other firm, for a similar reason. There needs to be a gap in the support of the mixed strategy of player  $i$  below an investment  $k$  for firm  $i$  to invest  $k$  with strictly positive probability in any equilibrium. This leaves only two possibilities:  $k = 0$  (as no one invests below 0) and  $k = \bar{k}$  (if there is a gap in the support of the investment strategy of the other firm below  $\bar{k}$ ). □

## Proof of Proposition 1

*Proof.* We first construct the equilibrium, then verify that indeed neither firm has an incentive to deviate from the proposed investment strategy, and finally show that no other equilibrium exists.

**Characterization:** For every investment of firm 2 below  $\bar{k}$  which is contained in the support of the equilibrium strategy, the following condition has to hold (with  $\epsilon$  arbi-

trarily small):

$$(4) \quad F_2(k)\alpha_{12} + \lim_{\varepsilon \rightarrow 0} F_2(\bar{k} - \varepsilon)\alpha_1 - ck = \lim_{\varepsilon \rightarrow 0} F_2(\bar{k} - \varepsilon)\alpha_1 \quad \Rightarrow \quad F_2(k) = \frac{c}{\alpha_{12}}k$$

and for every investment equal to or above  $\bar{k}$

$$(5) \quad F_2(k) - ck = \lim_{\varepsilon \rightarrow 0} F_2(\bar{k} - \varepsilon)\alpha_1 \quad \Rightarrow \quad F_2(k) = ck + \lim_{\varepsilon \rightarrow 0} F_2(\bar{k} - \varepsilon)\alpha_1$$

If firm 1 chooses zero with positive probability, firm 2's mixed strategy must not contain an atom at zero. However, firm 2 must also be indifferent between all investment levels in the support of its equilibrium mixed strategy. Denote firm 2's expected profit by  $E[\Pi_2]$ . Then, for all  $k < \bar{k}$

$$(6) \quad \begin{aligned} & F_1(k)\alpha_{12} + \lim_{\varepsilon \rightarrow 0} F_1(\bar{k} - \varepsilon)\alpha_2 - ck = E[\Pi_2] \\ \Rightarrow & F_1(k) = \frac{c}{\alpha_{12}}k + \frac{E[\Pi_2] - \lim_{\varepsilon \rightarrow 0} F_1(\bar{k} - \varepsilon)\alpha_2}{\alpha_{12}} \end{aligned}$$

For every investment at  $\bar{k}$  or above having a lower investment than the competitor implies also losing their share of captive consumers.

$$(7) \quad F_1(k) - ck = E[\Pi_2] \quad \Rightarrow \quad F_1(k) = ck + E[\Pi_2]$$

From lines (4) to (7) it follows that firm 1's and firm 2's distribution functions have the same slopes. This is true in both the low and the high investment range. Since the slope is higher for investments below  $\bar{k}$  than for investments above  $\bar{k}$ , there exists  $\delta \in (0, \bar{k})$  such that for both firms

$$(8) \quad F_1(k) = F_1(\delta) \text{ and } F_2(k) = F_2(\delta) \text{ for all } k \in [\delta, \bar{k}]$$

and therefore  $\lim_{\varepsilon \rightarrow 0} F_1(\bar{k} - \varepsilon) = F_1(\delta)$  and  $\lim_{\varepsilon \rightarrow 0} F_2(\bar{k} - \varepsilon) = F_2(\delta)$ .

Neither firm has an incentive to strictly exceed the maximum investment of the other. This would increase the cost but not the probability of winning. Thus, there exists a unique  $\bar{K}$  such that  $F_1(\bar{K}) = F_2(\bar{K}) = 1$  and for all  $\varepsilon > 0$ ,  $F_1(\bar{K} - \varepsilon) < 1$  and  $F_2(\bar{K} - \varepsilon) < 1$ . Since the distribution functions of firms 1 and 2 also have identical slopes for  $k \geq \bar{k}$ , the distribution functions of both firms are identical for  $k \geq \bar{k}$ :

$$(9) \quad F_1(k) = F_2(k) \text{ for all } k \geq \bar{k}$$

Combining Equations (5), (7), and (9) yields  $E[\Pi_2] = F_2(\delta)\alpha_1$ . Starting with Line (6) and plugging in yields for  $k < \bar{k}$

$$(10) \quad F_1(k) = \frac{c}{\alpha_{12}}k + \frac{F_2(\delta)\alpha_1}{\alpha_{12}} - \frac{F_1(\delta)\alpha_2}{\alpha_{12}}.$$

We solve (10) for  $F_2(\delta)$  at  $k = \delta$  and obtain

$$F_2(\delta) = F_1(\delta)\frac{\alpha_{12} + \alpha_2}{\alpha_1} - \frac{c}{\alpha_1}\delta.$$

We plug in from line (4) and solve for  $F_1(\delta)$  to obtain

$$(11) \quad F_1(\delta) = c\delta \left( \frac{\alpha_1}{\alpha_{12}(\alpha_{12} + \alpha_2)} + \frac{1}{\alpha_{12} + \alpha_2} \right).$$

The flat part in the distribution functions (equation (8)) implies together with the different shares of captive consumers that firm 2 chooses an investment equal to  $\bar{k}$  with a positive probability while firm 1's strategy has an atom at zero. Since the two firms cannot have an atom at the same investment level (Lemma 2), and since neither firm chooses  $\delta$  with positive probability in equilibrium (Lemma 4), the distribution function of firm 1 must be continuous in  $\delta$  and  $\bar{k}$ . In addition, at  $\bar{k}$  the distribution functions of both firms take identical values. Thus, the following holds

$$(12) \quad F_1(\delta) = F_1(\bar{k}) = F_2(\bar{k})$$

We can rewrite (5) using (4) as

$$(13) \quad F_2(\bar{k}) = c\bar{k} + \frac{c}{\alpha_{12}}\delta\alpha_1.$$

Taking line (12) and plugging in from line (11) on the left-hand side and from line (13) on the right-hand side, we arrive at

$$(14) \quad \begin{aligned} c\delta \left( \frac{\alpha_1}{\alpha_{12}(\alpha_{12} + \alpha_2)} + \frac{1}{\alpha_{12} + \alpha_2} \right) &= c\bar{k} + \frac{c}{\alpha_{12}}\delta\alpha_1 \\ \Leftrightarrow \delta &= \bar{k} \frac{\alpha_{12}(\alpha_{12} + \alpha_2)}{\alpha_{12} + \alpha_1 - \alpha_1(\alpha_{12} + \alpha_2)} = \bar{k} \frac{(1 - \alpha_1)\alpha_{12}}{\alpha_{12} + \alpha_1^2}. \end{aligned}$$

It is easily verified that

$$(1 - \alpha_1)\alpha_{12} < \alpha_{12} + \alpha_1^2 \Rightarrow \delta < \bar{k}.$$

Finally, we derive the maximum investment levels. Suppose  $K > \bar{k}$ . Since the distribution functions stay constant at one for all investment levels above the maximum



level chosen, we obtain the following condition

$$(15) \quad c\bar{K} + F_2(\delta)\alpha_1 = 1 \Leftrightarrow c\bar{K} = 1 - \frac{\alpha_1 c}{\alpha_{12}}\delta = 1 - \alpha_1 c\bar{k} \frac{(1 - \alpha_1)\alpha_{12}}{\alpha_{12}(\alpha_{12} + \alpha_1^2)}$$

where  $\delta$  has been derived in Equation (14). Rewriting (15) yields the maximum investment level

$$\bar{K} = \frac{1}{c} - \alpha_1 \bar{k} \frac{1 - \alpha_1}{\alpha_{12} + \alpha_1^2}.$$

As by assumption  $\alpha_1 + \alpha_2 + \alpha_{12} = 1$ , we replace in the above results to state Proposition 1.

For the derivation of the maximum investment, we have assumed  $\bar{K} > \bar{k}$ . This is indeed true if

$$(16) \quad \frac{1}{c} - \alpha_1 \bar{k} \frac{1 - \alpha_1}{\alpha_{12} + \alpha_1^2} > \bar{k} \Leftrightarrow \bar{k} < \frac{1}{c} \frac{\alpha_{12} + \alpha_1^2}{1 - \alpha_2}.$$

**Equilibrium verification:** The above computations establish that both firms are indifferent between all levels of investment in their support such that it does not pay to reshuffle probability mass within interior investments. Hence, it suffices to show that there is no strictly profitable deviation for either firm to investments outside the support or at the boundaries. Note that by construction no firm has an incentive to deviate to an investment in the gap or above  $\bar{K}$ , as this would yield strictly lower expected profit. Note further that firm 1 would be strictly worse off to invest  $\bar{k}$  with strictly positive probability than what she already gets by investing marginally above  $\bar{k}$  (as firm 2 invests exactly  $\bar{k}$  with strictly positive probability). The same holds for firm 2 investing exactly 0, as it would get strictly lower profit than by investing just above 0.

**Uniqueness:** By the above construction, the slopes of the distributions over the two intervals and the value of  $\delta$  are the only ones satisfying the condition of equal profit over the intervals. We also know from Lemmas 2 and 4 that the only other possibility in terms of a mass point satisfying the condition that both firms need to invest with total probability of 1 would be to have firm 1 investing  $\bar{k}$  with strictly positive probability and firm 2 investing 0 with strictly positive probability. However, in any equilibrium over two intervals, with  $\bar{K}$  the upper bound of the upper interval, it must hold by Lemma 3 that  $F_1(\bar{K}) = F_2(\bar{K}) = 1$ , the profit of both firms must be identical. This does not hold if  $\alpha_1 > \alpha_2$ ,  $F_2(0) > 0$  and  $F_1(0) = 0$ . Hence, Proposition characterizes the unique equilibrium in which firms randomize over two disconnected intervals. Furthermore, the indifference conditions prohibit any equilibrium with a connected equilibrium support for low  $\bar{k}$ . Thus, the equilibrium is unique.

## Proof of Corollary 1

*Proof.* Using the distribution functions from Proposition 1, we observe that  $F_1(\delta) > F_2(\delta)$  so that firm 2 has a higher investment than firm 1 more often than the reverse. We compute expected investments as

$$\begin{aligned}
E[k_1] &= \int_0^\delta \frac{c}{\alpha_{12}} x dx + \int_{\bar{k}}^{\bar{K}} c x dx \\
&= \frac{c(1-\alpha_1)^2 \alpha_{12} \bar{k}^2}{2(\alpha_{12} + \alpha_1^2)^2} + \frac{1}{2} c \left( \frac{(\alpha_{12} - \alpha_1((1+\alpha_1)c\bar{k} - \alpha_1))^2}{c^2(\alpha_{12} + \alpha_1^2)^2} - \bar{k}^2 \right) \\
E[k_2] &= \int_0^\delta \frac{c}{\alpha_{12}} x dx + \int_{\bar{k}}^{\bar{K}} c x dx + \text{Prob}(k_2 = \bar{k}) \bar{k} \\
&= \frac{c(1-\alpha_1)^2 \alpha_{12} \bar{k}^2}{2(\alpha_{12} + \alpha_1^2)^2} + \frac{1}{2} c \left( \frac{(\alpha_{12} - \alpha_1((1+\alpha_1)c\bar{k} - \alpha_1))^2}{c^2(\alpha_{12} + \alpha_1^2)^2} - \bar{k}^2 \right) + \frac{c(\alpha_1 - \alpha_2) \bar{k}^2}{\alpha_{12} + \alpha_1^2}
\end{aligned}$$

It is easily verified that  $E[k_1] < E[k_2]$ . By the properties of the mixed strategy equilibrium, the expected profit of each firm  $i = 1, 2$  equals its expected profit conditional on investing zero. This corresponds to its outside option  $o_i(0)$  which is the value of its captive segment multiplied with the probability of the competitor investing below  $\bar{k}$ .  $\square$

### A.1 Proof of Proposition 2

*Proof.* We first construct the equilibrium and then verify that indeed neither firm has an incentive to deviate from the proposed investment strategy, and finally show that no other equilibrium exists.

**Characterization:** Suppose that both firms randomize over  $(0, \delta)$  for some  $\delta \in (0, \bar{k})$ . Suppose further that firm 1 chooses zero with positive probability and firm 2 chooses  $\bar{k}$  with positive probability. Finally, suppose that firm 1 chooses investments below or equal to  $\delta$  with certainty (we verify this later), i.e.,  $F_1(\delta)$  whereas firm 2 also chooses  $\bar{k}$  such that  $F_2(\delta) < 1$ . We now derive the value for  $\delta \in (0, \bar{k})$ .

As firm 1 invests only below  $\bar{k}$ , firm 2 could ensure profit  $1 - c\bar{k}$  by investing  $\bar{k}$  with certainty. Thus, the distribution function of firm 1 must fulfill for all  $k \leq \delta$

$$(17) \quad F_1(k) \alpha_{12} + \alpha_2 - ck = 1 - c\bar{k} \Rightarrow F_1(k) = \frac{c}{\alpha_{12}} k + \frac{1 - \alpha_2 - c\bar{k}}{\alpha_{12}}$$

By assumption  $\bar{k} < \frac{1 - \alpha_2}{c}$  and thus  $\frac{1 - \alpha_2 - c\bar{k}}{\alpha_{12}} > 0$ . Note that choosing  $\bar{k}$  also yields an expected profit equal to  $1 - c\bar{k}$  for firm 2.

Firm 1 obtains an expected profit equal to its outside option  $o_1(0)$  which is given by its captive segment multiplied by the probability that firm 2 invests less than  $\bar{k}$ ,

$F_2(\delta)\alpha_1$ . For the distribution function of firm 2 and investments  $k \leq \delta$  the following must hold:

$$F_2(k)\alpha_{12} + F_2(\delta)\alpha_1 - ck = F_2(\delta)\alpha_1 \Leftrightarrow F_2(k) = \frac{c}{\alpha_{12}}k$$

The investment level  $\delta$  is such that the distribution function of firm 1 just reaches 1 at this level

$$(18) \quad \frac{c}{\alpha_{12}}\delta + \frac{1 - \alpha_2 - c\bar{k}}{\alpha_{12}} = 1 \Leftrightarrow \delta = \bar{k} - \frac{\alpha_1}{c}$$

If  $\bar{k} < \frac{1-\alpha_2}{c}$ , then  $\delta < \frac{\alpha_{12}}{c}$ .

Finally, we derive the probability with which firm 2 chooses  $\bar{k}$ .

$$\text{Prob}(k_2 = \bar{k}) = 1 - \frac{c}{\alpha_{12}}\delta = 1 - 1 + \frac{1 - \alpha_2}{\alpha_{12}} - \frac{c}{\alpha_{12}}\bar{k} = \frac{1 - \alpha_2}{\alpha_{12}} - \frac{c}{\alpha_{12}}\bar{k}$$

From line (17) also

$$\text{Prob}(k_1 = 0) = \frac{1 - \alpha_2}{\alpha_{12}} - \frac{c}{\alpha_{12}}\bar{k} = \text{Prob}(k_2 = \bar{k})$$

By  $\bar{k} < \frac{1-\alpha_2}{c}$ , it holds that  $\text{Prob}(k_2 = \bar{k}) > 0$ . Moreover,

$$\begin{aligned} \alpha_{12} > 0 \Rightarrow \alpha_{12} + \alpha_2 > \alpha_2 \Rightarrow (1 - \alpha_1)^2 > \alpha_2(1 - \alpha_1) \Rightarrow \alpha_{12} + \alpha_1^2 > \alpha_1 - \alpha_1\alpha_2 \\ \Rightarrow \frac{\alpha_{12} + \alpha_1^2}{1 - \alpha_2} > \alpha_1 \end{aligned}$$

and therefore

$$\bar{k} > \frac{1}{c} \frac{\alpha_{12} + \alpha_1^2}{1 - \alpha_2} \Rightarrow \bar{k} > \frac{\alpha_1}{c}$$

so that  $\delta < \bar{k}$ .

By  $\bar{k} > \frac{1}{c} \frac{\alpha_{12} + \alpha_1^2}{1 - \alpha_2}$  firm 1 does indeed not want to deviate to choosing  $\bar{k}$ :

$$\bar{k} > \frac{1}{c} \frac{\alpha_{12} + \alpha_1^2}{1 - \alpha_2} \Rightarrow c\bar{k}(\alpha_{12} + \alpha_1) > \alpha_{12} + \alpha_1^2 \Leftrightarrow -\frac{\alpha_1^2}{\alpha_{12}} + \frac{c}{\alpha_{12}}\bar{k}\alpha_1 > 1 - c\bar{k} \Leftrightarrow F_2(\delta)\alpha_1 > 1 - c\bar{k}$$

**Equilibrium verification:** From the above derivations, both firms are indifferent between all levels of investment in the support of their equilibrium investment strategies by construction. Hence, it suffices to show that there is no strictly profitable deviation for either firm. Note that no firm has an incentive to deviate to an investment strictly above  $\bar{k}$ , as this would yield strictly lower expected profit. Firm 1 would be strictly worse off to invest  $\bar{k}$  as it would get the same expected profit as firm 2, lower than what it gets at equilibrium. Further, firm 2 cannot gain from investing exactly 0, as it would then get strictly lower profit than by investing just above 0.

**Uniqueness:** By the above construction, the cumulative distribution functions and the value of  $\delta$  are the only ones satisfying the indifference condition for randomization of investments over a single connected interval. We also know from Lemmas 2 4 that the only other possibility in terms of mass points satisfying the condition that both firms need to invest with total probability of 1 would be to have firm 1 invest  $\bar{k}$  with strictly positive probability and firm 2 invest 0 with strictly positive probability. However, for such a mixed strategy to be an equilibrium it must also be true that firm 1 is indifferent between investing just above 0 and exactly  $\bar{k}$ ,

$$\alpha_1 + F_2(0) = 1 - c\bar{k},$$

and that firm 2 weakly prefers to invest 0 over investing  $\bar{k}$ ,

$$F_1(\delta)\alpha_2 \geq 1 - c\bar{k}.$$

As  $\alpha_1 > \alpha_2$ ,  $F_1(\delta) < 1$  and  $F_2(0) > 0$ , this leads to a contradiction. Hence, the above equilibrium is the unique one where both firms randomized over the same connected interval, when the total probability mass allocated below  $\delta$  (remember there is only one possible slope for the distribution at equilibrium) is strictly below 1. Note further that there cannot be an equilibrium where firms randomize over two disconnected intervals as the one described in Proposition 1 is the only one that exists but the condition on  $\bar{k}$  is not fulfilled here. Thus, the equilibrium we characterized here is unique for the set range of  $\bar{k}$ . □

## Proof of Corollary 2

*Proof.* Using the distribution functions from Proposition 2, we observe that  $F_1(\delta) > F_2(\delta)$ , and we compute expected investments as

$$\begin{aligned} E[k_1] &= \int_0^\delta \frac{c}{\alpha_{12}} x dx = \frac{c(\frac{\alpha_1}{c} - \bar{k})^2}{2\alpha_{12}} \\ E[k_2] &= \int_0^\delta \frac{c}{\alpha_{12}} x dx + \text{Prob}(k_2 = \bar{k})\bar{k} = \frac{c(\frac{\alpha_1}{c} - \bar{k})^2}{2\alpha_{12}} + \bar{k} \frac{\alpha_1 + \alpha_{12} - c\bar{k}}{\alpha_{12}} \end{aligned}$$

where obviously  $E[k_1] < E[k_2]$ .

By the properties of the mixed strategy equilibrium, the expected profit of each firm  $i = 1, 2$  equals its expected profit conditional on investing zero which is its outside option  $o_i(0)$  which is given by its captive segment multiplied with the probability of the competitor investing below  $\bar{k}$ . □

## A.2 Proof of Proposition 3

*Proof.* Let  $\bar{k} > \frac{1-\alpha_2}{c}$ . We first construct the equilibrium, verify that neither firm has an incentive to deviate, and finally show that no other equilibrium exists.

**Characterization:** Suppose that neither firm chooses an investment high enough to steal captive consumers from its competitor. The outside option of firm  $i = 1, 2$  is to keep its captive segment and receive a profit of  $o_i(0) = \alpha_i$ . The prize of winning is then the value of additionally attracting the contested segment  $\alpha_{12}$ . This observation implies that both firms are symmetric at the margin. Moreover, the captive segments can be disregarded since they are not at stake. Both firms compete until their expected profits from competition are zero, in which case their expected profit is determined only by their captive segment. Thus, in equilibrium, the following must hold for all  $k_i < \bar{k}$  for  $i = 1, 2$  and  $j \neq i$ :

$$(19) \quad F_j(k_i)\alpha_{12} - ck_i = 0 \Leftrightarrow F_j(k_i) = \frac{c}{\alpha_{12}}k_i.$$

As the distribution function of investments cannot exceed 1, both firms randomize continuously over  $[0, \frac{\alpha_{12}}{c}]$  and do not invest any higher amounts. The cumulative distribution function is as follows for firm  $i = 1, 2$ :

$$F_i(k) = \begin{cases} \frac{c}{\alpha_{12}}k & \text{for all } k \in [0, \frac{\alpha_{12}}{c}] \\ 1 & \text{for } k \geq \frac{\alpha_{12}}{c} \end{cases}$$

Each firm must be indifferent at equilibrium between all investments in  $[0, \frac{\alpha_{12}}{c}]$ , and none of the two firms chooses zero with strictly positive probability because this would not be consistent with the indifference condition in (19). The expected payoff of firm 1 and 2 is equal to its outside option,  $E[\Pi_1] = o_1(0) = \alpha_1$  and  $E[\Pi_2] = o_2(0) = \alpha_2$ , respectively.

**Equilibrium verification:** By the above, both firms are indifferent between all levels of investment in their support by construction. Hence, it suffices to show that there is no strictly profitable deviation for either firm. Suppose firm  $i$  considered deviating to an investment at  $\bar{k}$ , sufficient to capture the entire population. Then, firm  $i$  would make an expected profit of  $F(\frac{\alpha_{12}}{c}) - c\bar{k} = 1 - c\bar{k} < 1 - (1 - \alpha_2) = \alpha_2 < \alpha_1$  such that this deviation is not profitable for firm  $i = 1, 2$ . As a consequence, no investment level at or above  $\bar{k}$  forms part of the equilibrium mixed strategy. We also know that any investment between  $\frac{\alpha_{12}}{c}$  and  $\bar{k}$  would yield strictly lower profit, hence there is no profitable deviation.

**Uniqueness:** We have shown above that the slopes of the distributions and the value of  $\delta$  as specified in the equilibrium characterization are the unique ones satisfying the condition of equal profit for randomization over a single connected interval. Hence, the above equilibrium is the unique one on a single interval when the total probability mass allocated below  $\delta$  is equal to 1. Note further that there cannot be an equilibrium where firms randomize over two disconnected intervals as the one described in Proposition 1 is the only one that exists but the condition on  $\bar{k}$  is not fulfilled here. According to Lemmas 1 to 4, no further equilibrium types are admissible. Thus, the equilibrium we characterized here is unique for the set range of  $\bar{k}$ .  $\square$

### Proof of Corollary 3

*Proof.* Using the distribution functions from Proposition 3, the expected investment in equilibrium equals

$$E[k_i] = \int_0^{\frac{\alpha_{12}}{c}} \frac{c}{\alpha_{12}} x dx = \frac{1}{2} \frac{\alpha_{12}}{c} \text{ for } i = 1, 2$$

per firm. In total, the two firms invest  $\frac{\alpha_{12}}{c}$ . Since equilibrium mixed strategies and investments are identical, both firms have the same probability of winning of  $\frac{1}{2}$ . The expected profit of each firm equals its expected profit conditional on investing zero which is the value of its captive segment.  $\square$

### A.3 Proof of Proposition 4

*Proof.* Propositions 2 and 3 each characterized an equilibrium that is unique for the range of  $\bar{k}$  to which the respective proposition applies. The proof is completed by noting that the ranges of  $\bar{k}$  given in Propositions 1 to 3 constitute a partition of the admissible range for  $\bar{k}$ .  $\square$

### A.4 Proof of Proposition 5

Using the investment levels found in Proposition 1, replacing  $\alpha_2$  by 0 and  $\alpha_{12}$  by  $1 - \alpha_1$  and taking the derivative with respect to  $\alpha_1$  we find

$$\frac{dE(k_1)}{d\alpha_1} = \frac{\bar{k}(\alpha_1(4 + \alpha_1 c\bar{k}) - c\bar{k} - 2)}{2((\alpha_1 - 1)\alpha_1 + 1)^2}$$

and

$$\frac{dE(k_2)}{d\alpha_1} = \frac{\bar{k}(\alpha_1(4 - \alpha_1 c\bar{k}) - c\bar{k} - 2)}{2((\alpha_1 - 1)\alpha_1 + 1)^2}.$$

Taking  $\alpha_1 \rightarrow 0$  we find

$$\lim_{\alpha_1 \rightarrow 0} \frac{dE(k_1)}{d\alpha_1} = -\frac{1}{2}\bar{k}(c\bar{k} + 2) < 0$$

and

$$\lim_{\alpha_1 \rightarrow 0} \frac{dE(k_2)}{d\alpha_1} = -\frac{1}{2}\bar{k}(2 - c\bar{k}) < 0,$$

as we have assumed  $c\bar{k} < 1$ . Similarly, we find

$$\lim_{\alpha_1 \rightarrow 1} \frac{dE(k_1)}{d\alpha_1} = \lim_{\alpha_1 \rightarrow 1} \frac{dE(k_2)}{d\alpha_1} = \bar{k} > 1.$$

## B Probabilistic setting

In this section, we show that the fact that investment is deterministic with a discrete threshold  $\bar{k}$  is not crucial to our results. Consider two firms, 1 and 2 choosing a level of investment  $e_i$ , with  $i \in \{1, 2\}$ , at cost  $c(e_i)$  with  $c' > 0$ ,  $c'' > 0$  and  $c(0) = 0$ . Firms compete for consumers from a population of mass one. This population consists of three types of consumers,  $t_1, t_2$ , and  $t_u$ . Types  $t_1$  and  $t_2$  occur with frequency  $\alpha_1$  and  $\alpha_2$ , respectively, in the population and the remaining part are of type  $t_u$ ,  $\alpha_{12}$ . The structure of the game and frequencies of types are common knowledge.

Different from the main part of the text, we assume captive consumers of firm  $i$  (types  $t_1$  and  $t_2$ ) bear a switching cost  $\bar{k}$  if they join the other firm. Hence, the utility of a consumer visiting a firm  $i$  is equal to  $e_i$ , minus the switching cost when it applies. Consumers of type  $t_u$  are ex ante undecided and do not experience switching costs.

Suppose all types of customers intend to join the firm that maximizes their utility but may make mistakes and join the “wrong” firm. We employ the commonly used ratio-form contest success function which imposes that the probability of choosing one firm over the other equals its share in total investments.<sup>8</sup>

The ex-ante indifferent consumers choose firm  $i$  with a probability

$$(20) \quad p_{t_u}^i(e_i, e_j) = \frac{e_i}{e_i + e_j}.$$

The captive consumers of type  $t_i$  choose the firm  $i$  with a probability

$$(21) \quad p_{t_i}^i(e_i, e_j) = \frac{e_i + \bar{k}}{e_i + e_j + \bar{k}}.$$

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<sup>8</sup>Jia (2008) shows how such a contest success function can be derived from a model where the realized benefits from given investments are subject to stochastic shocks which are drawn independently from an inverse exponential distribution.

Therefore, the captive consumers of type  $t_j$  choose firm  $i$  with a probability

$$(22) \quad p_{t_j}^i(e_i, e_j) = 1 - p_i^i(e_i, e_j) = \frac{e_i}{e_i + e_j + \bar{k}}.$$

Firm 1 chooses the level of investment that maximizes her expected profit

$$(23) \quad E(\Pi_1) = \alpha_{12}p_{t_u}^1(e_1, e_2) + \alpha_1p_{t_1}^1(e_1, e_2) + \alpha_2p_{t_2}^1(e_1, e_2) - c(e_1).$$

Solving the first-order condition of the profit maximization with respect to  $e_a$  yields

$$(24) \quad c'(e_1) = \frac{\alpha_{12}e_2}{(e_1 + e_2)^2} + \frac{e_2(\alpha_1 + \alpha_2) + \alpha_2\bar{k}}{(e_1 + e_2 + \bar{k})^2}.$$

Solving the same way for firm  $b$  yields

$$(25) \quad c'(e_2) = \frac{\alpha_{12}e_1}{(e_1 + e_2)^2} + \frac{e_1(\alpha_1 + \alpha_2) + \alpha_1\bar{k}}{(e_1 + e_2 + \bar{k})^2}.$$

We immediately observe that:

- (i) The equilibrium level of investment decreases in the cost-efficiency (the  $c$  function).
- (ii) The firm that invests the most in equilibrium is the firm with the smallest captive segment.
- (iii) Assuming no firm has a majority of captive consumers, the largest firm is, in expectation, the one with the smallest captive segment.

□