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ACTIVE SENSING OF A NONHOLONOMIC WHEELED MOBILE ROBOT

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ABSTRACT

This paper focuses on active sensing of nonholonomic wheeled mobile robots (WMRs). *Active sensing* solves the following problem: given a current knowledge about the robot state and the environment, how to select the next sensing action or sequence of actions. A vehicle is moving autonomously through a static environment gathering information from sensors. The sensor data are used to generate the robot actions in order to move around a reference trajectory with preset initial starting and desired goal configurations and imposed constraints. The paper presents a method for the determination of optimal trajectories based on optimization techniques. A suitable performance criterion is formulated to characterize the uncertainty and the extraction of information from sensor data. Finally results from experiments are given.

1. MOTIVATION

A great deal of attention has been paid to nonholonomic robots during the last years and nevertheless it is still an open area of research where a lot of questions are waiting to find answers. Predominantly tracking and motion generation topics have been treated [1], [2], [3], [4], [5]. Recently attention is given to active sensing, that incorporates in itself tracking and motion planning solutions in the presence of uncertainties. The main question to answer in active sensing is: "Where will the robot move at the next step?". An appropriate criterion is needed for gathering maximum information about the environment and for properly determining the robot motions. Some proposed trajectory generation strategies are based on entropy minimization [3], [2] or consider the robot motion composed of primitives [6]. In [7] the concentration is on information-gathering tasks and the choice 'where to look next' is investigated as a special case of an optimal experiment design. A weighted trace of the estimation error covariance matrix is chosen as a criterion to perform the next motion.

Active sensing for nonholonomic wheeled mobile robots (WMRs) is a challenging goal for various reasons:

- The *nonholonomic character* of the systems. A *nonholonomic* system can be defined as a system subject to kinematic constraints such that the dimension of the admissible controls at each point is less than the dimension of the configuration variables [1], [8]. A consequence of the nonholonomic constraints is that not each path from the admissible configuration space corresponds to a feasible trajectory for the robot.
- The task solution depends on the *optimality criterion*. It should be such that maximum information is extracted from the sensor data and at the same time this information is processed in a computationally efficient way.
- It is related to the *computational load* (time, number of operations). All generated motions are needed to be executed *on-line*.
- The *nonlinear character* of the problem poses questions about the system *controllability*. The majority of existing methods reduce the nonlinear model to a form easier to deal with (chained forms, Goursat normal forms or other *linear* representations with special properties), generate motion with the chosen specific form and then transform the resulting trajectory into the original representation [1], [9].
- *Obstacle* avoidance adds an additional level of difficulty. Steering methods rely on topological properties of the environment [8], or other learning techniques [3], [2].
- Other *uncertainties*, e.g. in the models and sensor data.

So, it can be pointed out that active sensing is an *optimization* problem. This work deals with active sensing of nonholonomic WMRs in the presence of uncertainties. The robot is moving in the Cartesian space starting from a given *initial configuration* to a desired *goal* configuration. Between two points there are an infinite number of possible trajectories. On the basis of the sensor data the robot is taking decisions how to move around a preset reference trajectory. The errors in the sensor data, the inaccuracies in the WMR model, and inaccuracies of other type can be the reason that the robot does not arrive at the desired goal configuration or to arrive with considerable errors. The key idea of the approach proposed here is to use some parameterized family of possible trajectories and thus to reduce the infinite-dimensional problem to a finitely parameterized optimization problem. To characterize the robot motion and to process the sensor information efficiently, an appropriate criterion is introduced. The approach proposed here examines active sensing as a *global optimization problem* subject to constraints.

The paper is organized as follows. The motion and measurement models of the considered nonholonomic WMR are described in Section 2. The proposed approach for active sensing and related criteria are given in Section 3. Section 4 presents simulation results illustrating the effectiveness of the approach. Section 5 summarizes the results.

2. MOTION AND MEASUREMENT MODELS

A WMR is moving in a plane. The environment is supposed known and obstacle free. The WMR motion is in the configuration space starting from a point (x_s, y_s), and it is required to reach a desired goal configuration (x_g, y_g) moving around a reference trajectory. The WMR generates its actions by processing the sensor data.

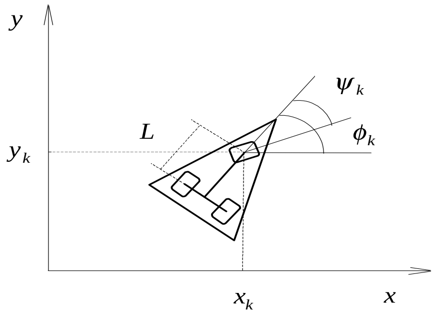

Figure 1: WMR coordinates

The vehicle motion at each time instant k is determined by the kinematic model [5]

$$\begin{pmatrix} x_{k+1} \\ y_{k+1} \\ \phi_{k+1} \end{pmatrix} = \begin{pmatrix} x_k + v_k \Delta T cos(\phi_k + \psi_k) \\ y_k + v_k \Delta T sin(\phi_k + \psi_k) \\ \phi_k + \frac{v_k \Delta T}{L} sin\psi_k \end{pmatrix} + \begin{pmatrix} \eta_{x,k} \\ \eta_{y,k} \\ \eta_{\phi,k} \end{pmatrix}, \quad (1)$$

where x_k and y_k denote the WMR position coordinates in a fixed frame (Fig. 1), ϕ_k is the angle measuring the orientation with respect to the x axis. L is the wheel base line (the distance between the front steering wheel and the axis of the driving wheels), ΔT is the travel time between two time steps, $\boldsymbol{\eta}_k = (\eta_{x,k}, \eta_{x,k}, \eta_{\phi,k})^T$ represents a process noise due to both modeling and discretization errors. The state vector $\boldsymbol{x}_k = (x_k, y_k, \phi_k)^T$ is denoted below with different subscripts: s stands for the *starting* configuration, g refers to the *goal* configuration, r to the *reference* trajectory, and the modified robot trajectory is without any subscript.

The WMR is controlled through a demanded velocity v_k and a direction of travel ψ_k, i.e. the control vector is $\boldsymbol{u}_k = (v_k, \psi_k)^T$. Due to physical constraints, both the velocity v_k, and the angle ψ_k of any WMR cannot exceed boundary values, namely $v_k \in [0, v_{max}]$, $\psi_k \in [-\psi_{max}, \psi_{max}]$ ($\psi_{max} \leq \frac{\pi}{2}$). The WMR can perform only forward motions.

The vehicle is equipped with a sensor that can measure the range r_k and bearing θ_k to a beacon B, located at coordinates, $(x_B, y_B)^T$. The observation equation for the beacon is

$$\begin{pmatrix} r_k \\ \theta_k \end{pmatrix} = \begin{pmatrix} \sqrt{(x_B - x_k)^2 + (y_B - y_k)^2} \\ \arctan(\frac{y_B - y_k}{x_B - x_k}) - \phi_k \end{pmatrix} + \begin{pmatrix} \xi_{r,k} \\ \xi_{\theta,k} \end{pmatrix}, \quad (2)$$

where $\boldsymbol{\xi}_k = (\xi_{r,k}, \xi_{\theta,k})^T$ is the observation noise. The measurement vector is further denoted by $\boldsymbol{z}_k = (r_k,\ \theta_k)^T$. The noise vectors $\boldsymbol{\eta}_k$ and $\boldsymbol{\xi}_k$ are assumed Gaussian, zero mean, mutually uncorrelated, with covariances $\boldsymbol{Q}_k$ and $\boldsymbol{R}_k$, respectively.

3. TRAJECTORY OPTIMIZATION

The robot 'knows' preliminary a reference trajectory, i.e. $\boldsymbol{x}_{r,k} = (x_{r,k},\ y_{r,k}, \phi_{r,k})^T$ is preset at every moment $k = 1, 2, ...$. The control vector of this trajectory is $\boldsymbol{u}_{r,k}$. How to move in the 'best' way according to a formulated criterion, from the starting to the goal configuration?

Let $\mathcal{Q}$ be a class of smooth functions. The problem of determining the 'best' trajectory $\boldsymbol{q}^*$ with respect to an index J can be formulated as

$$\boldsymbol{q}^* = argmin(J) \quad (3)$$

subject to constraints

$$|l_{y,k}| \leq l_{y,max}, \quad (4)$$

$$v_k \leq v_{max}, \quad (5)$$

$$|\psi_k| \leq \psi_{max}, \quad (6)$$

where $l_{y,k}$ is the lateral deviation of the optimal trajectory from the reference one (lateral is called the orthogonal robot motion deviation from the reference trajectory in y direction), $l_{y,max}$ is the maximum allowed lateral deviation value. The experiments are conducted in a way to minimize a measure, characterizing the WMR estimate vector.

The approach proposed here is based on a parametrization of a class $\mathcal{Q}$,

$$\mathcal{Q} = \mathcal{Q}(\boldsymbol{p}),\ \boldsymbol{p} \in \mathcal{P}, \quad (7)$$

of harmonic functions where $\boldsymbol{p}$ is a vector of parameters obeying to preset physical constraints. Given N number of harmonic functions, the new (modified) robot trajectory is generated on the basis of the reference one by a lateral deviation as a linear superposition

$$l_{y,k} = \sum_{i=1}^{N} A_{i,k} sin(i\pi \frac{s_{r,k}}{s_{r,fin}}), \quad (8)$$

of sinusoids, the amplitudes $A_{i,k}$ of which are constants; $s_{r,k}$ is the path length up to instant k, and $s_{r,fin}$ is the whole path length. Clearly, the problem described above can be cast into the problem of trajectory generation of a system described by equations

$$\boldsymbol{x}_{k+1} = f(\boldsymbol{x}_k, \boldsymbol{u}_k, \boldsymbol{\eta}_k) \quad (9)$$

$$\boldsymbol{z}_k = h(\boldsymbol{x}_k, \boldsymbol{\xi}_k) \quad (10)$$

with f and h nonlinear functions. In this formulation active sensing is a *global optimization problem* (on the whole WMR trajectory) with a criterion to be minimized

$$J = \min_{A_{i,k}} \{c_1 \mathcal{I} + c_2 \mathcal{C}\} \quad (11)$$

under *constraints* (4)-(6). The optimization reduces to an optimal choice of amplitudes $A_{i,k}$. The criterion (11) is composed of two terms : $\mathcal{I}$ characterizes the information extraction and accuracy, $\mathcal{C}$ is the cost part. As $\mathcal{I}$ could be chosen the entropy, or a scalar function of the covariance matrix of the estimated states. Here $\mathcal{I}$ is in the form

$$\mathcal{I} = trace(\boldsymbol{W}\boldsymbol{P}), \quad (12)$$

where $\boldsymbol{P}$ is the covariance matrix of the estimated states (at the goal configuration), $\boldsymbol{W}$ is a weighting matrix; $trace(.)$ denotes the matrix trace; c_1 and c_2 above are positive weighting constants. $\mathcal{C}$ accounts the relative time

$$\mathcal{C} = t_{fin}/t_{r,fin} \quad (13)$$

where t_{fin} is the final time for reaching the goal configuration on the modified trajectory versus the respective time $t_{r,fin}$ over the reference trajectory (when the WMR travels at a constant velocity). Minimization of J with respect to parameters of the modified trajectories guarantees trajectories with minimal uncertainty bounds. Within a statistical framework the covariance matrix $\boldsymbol{P}$ represents an information criterion. The weighting matrix $\boldsymbol{W}$ is a product of a normalizing matrix $\boldsymbol{N}$, by a scaling matrix $\boldsymbol{M}$, i.e. $\boldsymbol{W} = \boldsymbol{M}\ \boldsymbol{N}$. The normalizing matrix $\boldsymbol{N} = diag\{1/\sigma_1^2,\ 1/\sigma_2^2,$

$\ldots, \sigma_n^2\}$, (n is the state vector dimension) transforms the criterion into invariant measure to different physical units. σ_i are the standard deviations at the goal configuration on the reference trajectory, and so the extraction of information compared to the reference trajectory is obvious. Depending on the particular task, they could be chosen in another way. The scaling matrix $\boldsymbol{M} = diag\{m_1, \ldots, m_n\}$ gives different weights to the separate terms of the trace. The optimization is conducted by higher impact on these state vector components with higher m_i, whereas the impact of the other states is weakened. It is assumed that $\sum_{i=1}^{n} m_i = n$. The criterion J introduced in this way is a dimensionless scalar. As 'good' are considered trajectories which at the goal configuration have the first term $\mathcal{I}$ within the range $[0, n]$.

The state estimation in the present paper is carried out based on the Unscented Kalman Filter (UKF) [10], [11] for state vector estimation. The UKF is implemented in its form with an augmented state vector (a concatenation of the states and the noises) [11]. The sigma points and their weights are calculated using the scaled Unscented Transform [11]. The WMR and beacon models, (1) and (2), are highly nonlinear, that motivates the use of the UKF as a filtering algorithm. It does not require linearization, nor explicit calculation of Jacobians and Hessians. The solution obtained is optimal within a selected class of harmonic functions, with a fixed, finite number terms. Harmonic signals have been used for other aims, for experimental identification of robot parameters in [12], and [13]. In [13] the problem of robot dynamic calibration is considered and the optimization problem is solved by a genetic algorithm. In [12] experimental robot identification has been performed within a statistical framework.

4. SIMULATION RESULTS

The paper is concluded by some simulation results which show the performance of the developed approach for active sensing.

The covariance matrix $\boldsymbol{P}_k$ of the estimation error $\boldsymbol{x}_k - \hat{\boldsymbol{x}}_k$ defines an uncertainty ellipsoid $(\boldsymbol{x}_k - \hat{\boldsymbol{x}}_k)^T \boldsymbol{P}_k (\boldsymbol{x}_k - \hat{\boldsymbol{x}}_k) = 1$ that with respect to the positions (x_k, y_k) only is converted into a confidence ellipse, characterizing the performance of active sensing.

The *reference trajectory* is a *straight line* with a starting configuration $\boldsymbol{x}_s = (1\,m, 15\,m)$ and a goal configuration : $\boldsymbol{x}_g = (12.84\,m, 15\,m)$. The sampling time is $\Delta T = 0.2\ sec$. The beacon is located in a point with coordinates $x_B = 9\ m$, $y_B = 19\ m$. It is assumed that $v_{max} = 0.2\,m/sec$, $\psi_{max} = 60\,deg$, $l_{y,max} = 3\,m$ and $L = 0.5\ m$.

The UKF is implemented with the following parameters, recommendable for systems with Gaussian noises and of order $n = 3$ (so that $\kappa + n = 3$) [10], [11] : $\alpha = 1$, $\beta = 2$, $\kappa = 0$. The initial state estimate vector and covariance matrix are: $\hat{\boldsymbol{x}}_{0/0} = (1\,m, 15\,m, 0\,deg)^T$, $\boldsymbol{P}_0 = diag\{0.3\,m^2, 0.3\,m^2,\ 0.0025\,deg^2\}$. The noise covariance matrices are: $\boldsymbol{Q}_k = diag\{10^{-6}\ m^2,$ $10^{-6}\ m^2,\ 10^{-4}\ deg^2\}$, $\boldsymbol{R}_k = diag\{0.0004 {d_k}^2 m^2,\ 100\ deg^2\}$, where d_k is the distance from the WMR to the beacon. As in [4], measured distances are used for simulating the measurements, estimates are used in the UKF.

The reference and modified trajectories, generated with different number of sinusoids N, in accordance with (8), together with the uncertainty ellipses are shown on (Figs. 2, 4). The evolution in time of the weighted covariance trace is presented on Fig. 5. The bigger N is, the smaller the value of J is. Better accuracy is provided with bigger N, at the cost of increased computational load.

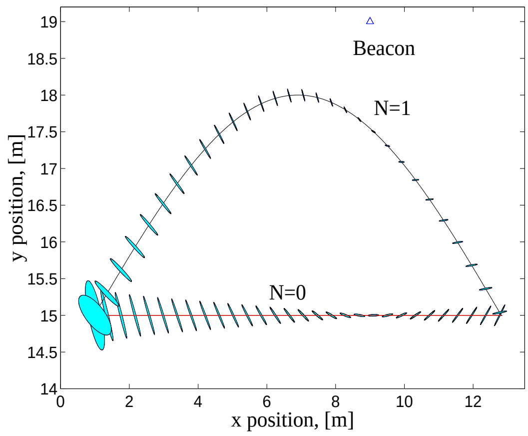

Figure 2: Trajectories: reference ($N = 0$) and modified ($N = 1$)

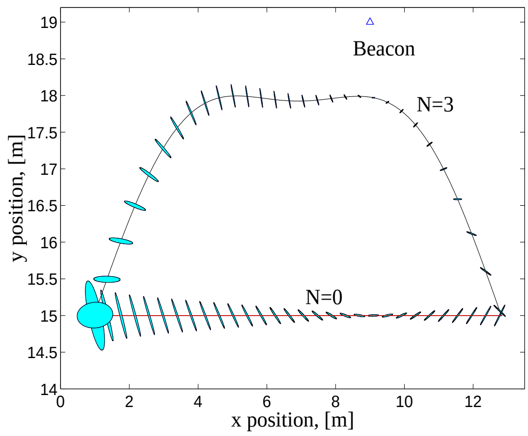

Figure 3: Trajectories: reference ($N = 0$) and modified ($N = 3$)

For comparison, the iteration process in the case of $N = 1$ was stopped after 3 iterations, and for $N = 5$, after 64 iterations.

In the experiment the WMR velocity is changeable, so that it reaches the goal configuration through the different trajectories for the same time. When the robot is moving with a constant velocity, the time t_{fin} is also an important part of the criterion. The cost criterion $\mathcal{C}$ is presented in Table 1, together with the information criterion $\mathcal{I}$, as well as the whole criterion J, computed with $c_1 = 1$, $c_2 = 0.1$. $\mathcal{C}$ is the ratio between the time for traveling over every trajectory versus the time for traveling on the reference trajectory.

The generated optimal trajectory is required to have a smaller value for J than the straight line trajectory. For this reason it is advisable to chosen the elements of $\boldsymbol{N}$ equal to the standard deviations of $\boldsymbol{P}$ on the straight line (at the final time step). The squared standard deviations of $\boldsymbol{N}$, are $\sigma_x^2 = 0.025\ m^2$, $\sigma_y^2 = 0.023\ m^2, \sigma_\phi^2 = 1.5^2\ deg^2$ are received from the straight line reference trajectory at its end point. The scaling matrix $\boldsymbol{M}$ is the identity matrix.

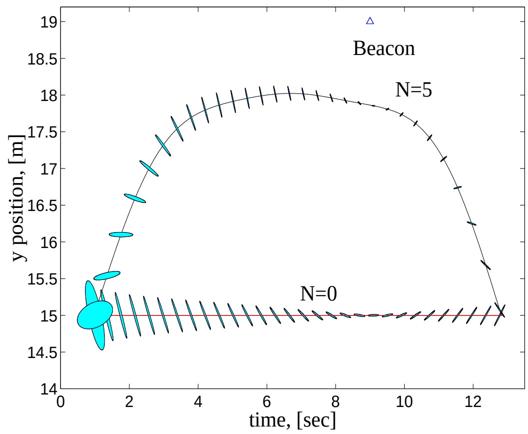

Figure 4: Trajectories: reference ($N = 0$) and modified ($N = 5$)

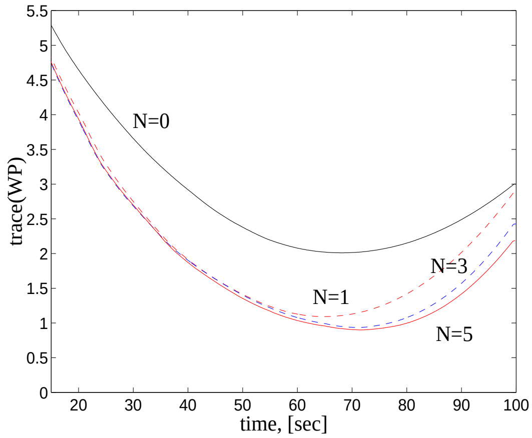

Figure 5: Evolution of $trace(\boldsymbol{WP})$ in time

Table 1: Active sensing results at the final time step.

N	0	1	3	5
$\mathcal{I}$	3.00	2.76	2.42	2.10
$\mathcal{C}$	1.00	1.14	1.20	1.21
J	3.10	2.99	2.54	2.29

5. CONCLUSIONS

This work concentrates on active sensing of a nonholonomic WMR in the presence of uncertainties. The robot is required to move from an initial to a final goal configuration (preset positions). The paper presents an information-based approach for designing optimal trajectories of the nonholonomic WMR. The problem has been examined as a global optimization problem subject to constraints. A relevant performance criterion has been defined taking into account accuracy requirements and the constraints (nonholonomic, physical). The criterion incorporates two parts: information and cost part. Their influence is decoupled. The criterion is invariant to physical units and is formulated in an appropriate way to gain information from the measurements. The optimal trajectory is searched within a preset class of functions, namely those of the harmonic ones and as a linear combination of sinusoidal components.

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