Hybrid Stochastic Framework for Freeway Traffic Flow Modelling

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Abstract

Traffic flow is an interesting many-particle phenomenon, with nonlinear interactions between the vehicles. This paper studies highway traffic and presents a general framework for traffic modelling as a *stochastic hybrid system* (with continuous and discrete dynamics and some interactions between its components). The freeway is considered as a network of components, each component modelling a different section of the traffic network. A model is developed in order to design stochastic predictors of the flows. The different traffic modes and transitions between them are detected and analyzed in real data, collected by induction loop detectors from Belgian and Dutch freeways.

1 Motivation

A freeway network consists of many interacting components. Traffic flow on each component is a complex system, with highly variable demand patterns, traffic jams, stop-and-go waves and hysteresis phenomena. Experimental studies [6, 7, 9, 10] based on traffic data have shown that traffic flow in different sections of a freeway possesses distinct dynamic modes (phases), such as: *free flow* traffic that is similar to the laminar flow in fluid systems: cars do not interact much with each other and each car approaches its own desired speed; *synchronized traffic flow*, a mode in which drivers move with nearly the same speed on the different lanes of the highway; *congested* mode in which the speed is quite low and can fluctuate very much while flow does not vary significantly; and *jammed* mode, where vehicles almost do not move and the flow is very low. Some changes in the section mode, for example from "free flow" to "congested", are induced by the traffic dynamics themselves, while others are due to external events such as accidents, road works or weather conditions.

We model the freeway traffic as a network of dynamic components, each component representing a different section of the network. The traffic is a *stochastic hybrid system*, i.e. each section is presented as a system with continuous and discrete states, interactions with variables of neighboring sections, and some unexpected abrupt changes in its modes. The continuous variables in a section are the flow through the boundary of the section, the average speed and the average density. Discrete state variables in a section are the number of lanes, traffic modes (free flow, synchronized, congested, jammed) and external conditions like weather. The traffic dynamics is described in this model by *macroscopic* variables, i.e. in terms of the collective vehicle dynamics, by aggregated variables and treating the traffic as a fluid flow. Various macroscopic models have been developed over the last 60 years for the deterministic or stochastic evolution of the three main variables of aggregated traffic: flow, speed, and density (for a recent survey, see [2, 3, 5, 8, 7]). The majority of the proposed in the literature macroscopic models are deterministic. Until now macroscopic models are predominantly used to simulate the traffic dynamics, less for traffic states estimation and control. Compared to the other types of models with higher level of details, such as microscopic (particle-based) and mesoscopic (gas-kinetic) [3], the macroscopic (fluid-dynamic) models are more suitable for the *on-line* description of traffic states in order to make predictions with real data. These predictions are then useful for model based feedback control, which is the ultimate motivation of our work.

The remainder of the paper is organized as follows. Section 2 presents the framework for modelling of the traffic flow as a stochastic hybrid system. The freeway network is divided into components. Each component is described by its dynamic mode and by aggregated traffic variables. Section 3 is focussed on analysis of the transitions between different traffic modes and their on-line detection, based on real data from freeways. Finally, conclusions are given in Section 4.

2 Traffic Flow Considered as a Hybrid System

The network of freeways under study is divided into many sections, as indicated in Fig. 1. Each section corresponds to a stretch of a freeway where the behavior is fairly homogeneous. The traffic vehicular flow on the freeways is described as a *stochastic hybrid system*

$$x_{i,k+1} = f_i(x^d_{i,k}, x_{i-1,k}, x_{i,k}, x_{i+1,k}, d_{i,k}, \eta_{i,k}), \tag{1}$$

where the state vector $x_{i,k} = (\rho_{i,k}, v_{i,k})^T$ contains the average traffic *density* (the number of vehicles per length unit, [*veh/km*]) and the average *speed*, [*km/h*]. The state vector $x_{i,k+1}$ of the i-th section at time $k+1$ is a function of state vectors at time k, from the i-th, and neighbor sections, $i-1$ and $i+1$. $x_{i,k}$ contains the continuously evolving variables at several equidistant points along the stretch of the freeway; $x^d_{i,k} = m$ is the vector of discrete variables (e.g. the number of lanes, the traffic mode, the weather) that describes a particular traffic mode; $d_{i,k}$ is the vector of demands (on-ramp and off-ramp flows); $\eta_{i,k}$ is a disturbance vector, with a known probability distribution $p_{\eta_{i,k}}(\eta_i)$. $\eta_{i,k}$ reflects the fluctuations in the speed selected by different drivers under the same conditions. The traffic mode makes sudden transitions, jumps to a different mode with a rate that depends on $x_{i,k}$, $x_{i+1,k}$. Using traffic modes as discrete state variables allows for much simpler form of f_i (e.g. affine function). In the first and the last zone of a section, $d_{i,k}$ acts as a part of boundary conditions. These first and last zones are special because they describe boundary conditions, including the interaction with upstream and downstream sections, on- and off-ramps, etc.

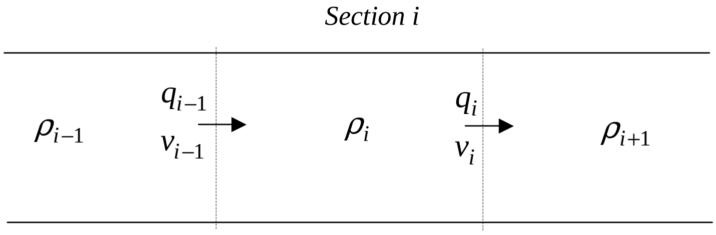


Figure 1: Freeway divided per sections. The flow q_i and the speed v_i are at the outflow boundary, ρ_i is within the section.

The measurement equation is of the form

$$z_{i,k} = h_i(x_{i,k}, \xi_{i,k}), \tag{2}$$

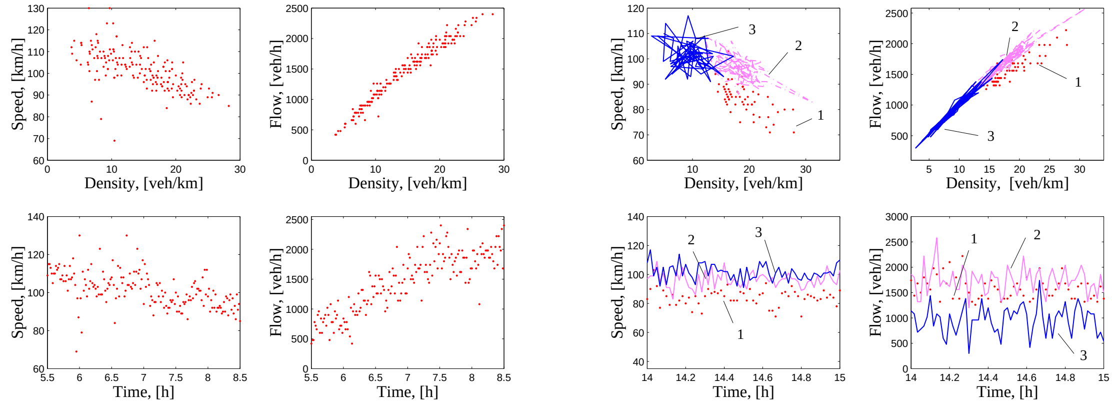
where the measurement vector $z_{i,k} = (q_{i,k}, v_{i,k})^T$ comprises the average flow (the number of vehicles: cars and trucks, passing a specific location in a time unit, $[veh/h]$) and the average speed, measured by induction loop detectors at the boundary between some sections. The data are averaged over an interval of fixed length, e.g. one minute. They are corrupted by noise due to sensor calibration errors, communication link errors, sensor failures. The measurement noise $\xi_{i,k}$ is assumed uncorrelated with $d_{i,k}$ and $\eta_{i,k}$, with a known probability distribution $p_{\xi_{i,k}}(\xi_i)$. Functions f_i and h_i are nonlinear in general.

We are building up efficient *hybrid* traffic models (with continuous and discrete components and some interaction between them) in order to solve the problems of: i) *estimation* of the traffic variables, i.e. in finding of $E(x_k/Z^k)$ by using all available information, $Z^k = \{z_{j,l}, \forall\, j = 1, 2, \ldots, i, \ldots, N,\ l = 1, 2, \ldots, k\}$ from N sections, and ii) *prediction* of a future state, i.e. $E(x_{k+1}/Z^k)$, where x_k stacks all vectors $x_{i,k}$, $E(.)$ denotes the mathematical expectation operator. The posterior probability density function of the state $P(x_k/Z^k)$ is recursively updated from the sensor information, using the state update and the measurement update steps, aiming respectively the computation of the probability density functions $P(x_{k+1}/x_k)$ and $P(Z^k/x_k)$. The recursive Bayesian approach is a suitable framework (e.g. particle filtering [1]). One of the challenges in these tasks is the need for on-line solutions, quickly after a new observations come in.

3 Traffic Analysis and Classification of its Modes

We investigate the transitions between the traffic modes empirically and theoretically. The traffic phenomenon is analyzed taking into account various upstream and downstream conditions and after splitting up the data according to the different phases. The data are collected from Belgian and Dutch freeways, respectively in June 2001 and August 2002. The transitions between the different modes may happen in a very irregular way: for instance the transition between free flow and synchronized traffic is of hysteretic nature, which means that the inverse transition from congested to free flow traffic occurs at a lower density and at a higher speed [4, 7].

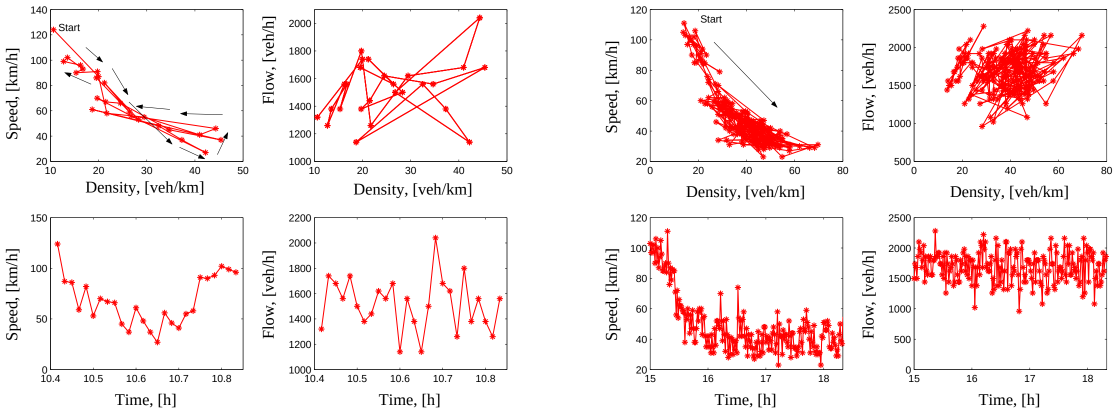
On the basis of our analysis of the traffic data, and in agreement with the results of [7], we distinguish the following major modes: free flow (Fig. 2a), synchronized (Fig. 2b),



a) Free-flow traffic: middle lane, 2 b) Synchronized mode : 1 - the rightmost lane, 2 - the middle one, and 3 - the fastest lane

Figure 2: Traffic on a freeway in Belgium near Antwerpen (Kennedy tunnel)

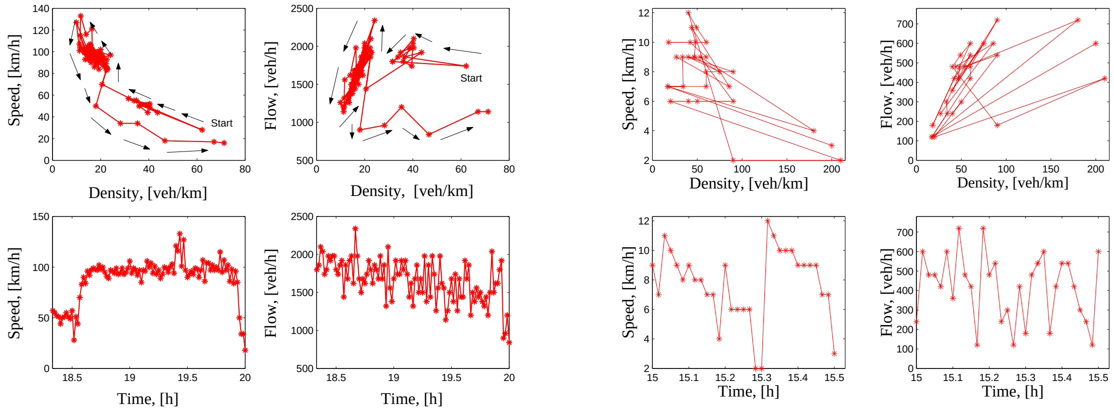
congested (Fig. 3, Fig. 4a), and jammed (Fig. 4b). In this section we present results based on the fundamental diagram (flow-density plot, $q_{i,k}(\rho_{i,k})$), the flow-speed diagram $q_{i,k}(v_{i,k})$, as well as the evolution of the speed $v_{i,k}$ and flow $q_{i,k}$ as a function of time. In a free-flow mode a clearly expressed tendency of linear dependence between the density and flow is present, except few outliers (Fig. 2a). In (Fig. 2b) the synchronization of the speed between the middle and the fastest lanes is obvious. Hysteresis cycles can be seen in the plots of Figs. 3a and 5a both in the speed-density and in the flow-density diagram. In the congested mode a large scattering effect is observed in the flow-density diagram Fig. 3b. In the jammed mode (Fig. 4 b) the vehicles are moving with a speed $v_{i,k} < 15$ $[km/h]$ and the flow is $q_{i,k} < 800$ $[veh/h]$.



a) Transition from a synchronized to a congested mode : middle lane b) Transition from a free-flow to a congested mode: middle lane

Figure 3: Traffic on a freeway in Belgium near Antwerpen (Kennedy tunnel)

The transitions between the traffic modes can be detected on-line by normalized change detection ratios: $I_v = (v_{i,k} + v_{i,k-1})/2v_{max}$, $I_q = (q_{i,k} + q_{i,k-1})/2q_{max}$, where v_{max} and q_{max} are the maximum values of the speed and flow, respectively. As seen from Fig. 5 b, this indicator reflects the transitions of the speed and the flow exactly. Both flow and speed have to be observed, because in some modes only the speed changes abruptly, whereas the flow remains relatively constant, nevertheless the mode is completely different.



a) Transition between free-flow and two congested regions : middle lane

b) Jam : middle lane

Figure 4: Freeway in Belgium near Antwerpen (Kennedy tunnel)

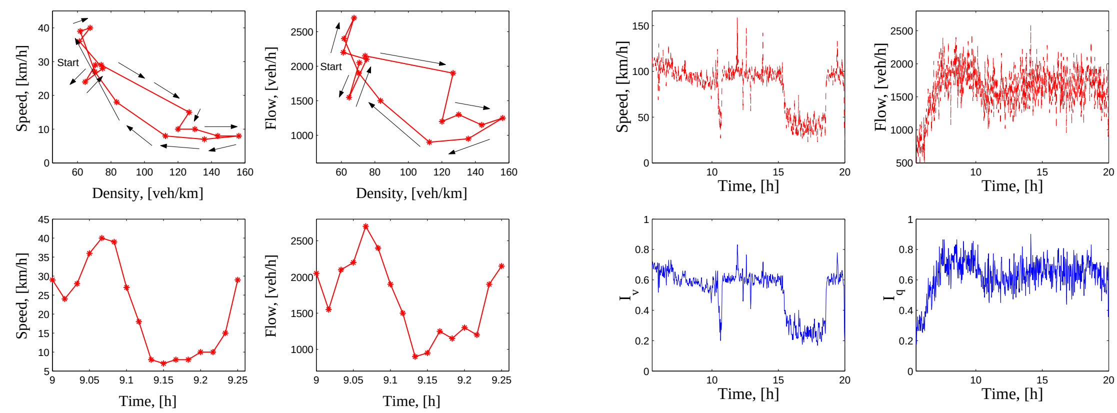


Figure 5: a) Transition from a congested to a jammed mode (data from a freeway A1 in the Netherlands: aggregated data from two lanes) b) Detection of the changes in traffic modes (data from Belgium, Kennedy tunnel): data from the middle lane

Probabilistic mode detection is investigated, as well as the formulation of other criteria for on-line detection of the different traffic modes and the transitions between them. Simulation studies are currently conducted with hybrid macroscopic models of the traffic modes.

4 Conclusions

Experimental investigations of traffic modes and the transitions between them are presented. This paper develops a modelling approach of freeway traffic flow, in order to describe the traffic states with on-line data for the goals of prediction of future traffic states and application to traffic control systems. The freeway traffic is modelled as a hybrid stochastic system with different components.

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Acknowledgments. Financial support by the project DWTC-CP/40 "Sustainability effects of traffic management", Belgium is gratefully acknowledged, as well as the Belgian Programme on Inter-University Poles of Attraction initiated by the Belgian State, Prime Minister's Office for Science, Technology and Culture. We also thank the Antwerpen Traffic Centre, Belgium, and Mr. Frans Middleham from the Transport Research Centre of the Ministry of the Transport, the Netherlands for providing the data used in this study.