Limits on gravitational wave emission from selected pulsars using LIGO data

We place direct upper limits on the amplitude of gravitational waves from 28 isolated radio pulsars by a coherent multi-detector analysis of the data collected during the second science run of the LIGO interferometric detectors. These are the first direct upper limits for 26 of the 28 pulsars. We use coordinated radio observations for the first time to build radio-guided phase templates for the expected gravitational wave signals. The unprecedented sensitivity of the detectors allow us to set strain upper limits as low as a few times $10^{-24}$. These strain limits translate into limits on the equatorial ellipticities of the pulsars, which are smaller than $10^{-5}$ for the four closest pulsars.

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A worldwide effort is underway to detect gravitational waves (GWs) and thus test a fundamental prediction of General Relativity. In preparation for long-term operations, the LIGO and GEO experiments conducted their first science run (S1) during 17 days in 2002. The detectors and the analyses of the S1 data are described in Refs. 1 and 2-3, respectively. LIGO’s second science run (S2) was carried out from 14 Feb to 14 April 2003, with dramatically improved sensitivity compared to S1. During S2 the GEO detector was not operating.

A spinning neutron star is expected to emit GWs if it is not perfectly symmetric about its rotation axis. The strain amplitude $h_0$ of the emitted signal is proportional to the star’s deformation as measured by its ellipticity $\epsilon$. Using data from S2, this paper reports direct observational limits on the GW emission and corresponding ellipticities from the 28 most rapidly rotating isolated pulsars for which radio data is complete enough to guide the phase of our filters with sufficient precision. These are the first such limits for 26 of the pulsars. We concentrate on isolated pulsars with known phase evolutions and sky positions to ensure that our targeted search requires relatively few unknown parameters.

The limits reported here are still well above the indirect limits inferred from observed pulsar spindown, where available (Fig. 1). However, fourteen of our pulsars are in globular clusters, where local gravitational accelerations produce Doppler effects that mask the intrinsic pulsar spindown, sometimes even producing apparent spinup. For these pulsars our observations therefore place the first limits that are inherently independent of cluster dynamics, albeit at levels well above what one would expect if all globular cluster pulsars are similar to field pulsars.

Our most stringent ellipticity upper limit is $4.5 \times 10^{-6}$. While still above the maximum expected from conventional models of nuclear matter, distortions of this size would be permitted within at least one exotic theory of neutron star structure.

**Detectors.**—LIGO comprises three detectors. Each detector is a power-recycled Michelson interferometer, with Fabry-Perot cavities in the long arms. A passing GW produces a time-varying differential strain in these arms, and the resulting differential displacement of the cavity test mass mirrors is sensed interferometrically. Two detectors, the 4 km-arm H1 and the 2 km-arm H2 detectors, are collocated in Hanford WA. The 4 km-arm L1 detector is situated in Livingston Parish LA. Improvements in noise performance between S1 and S2 were approximately an order of magnitude over a broad frequency range. Modifications that were made between S1 and S2 to aid in noise reduction and improve stability include: i) increased laser power to reduce high-frequency noise, ii) better angular control of the mirrors of the interferometer and iii) the use of lower noise digital test mass suspension controllers in all detectors.

During S2, the LIGO detectors’ noise performance in the band 40-2000 Hz was better than any previous interferometer. The best strain sensitivity, achieved by L1, was $\sim 3 \times 10^{-22} \text{Hz}^{-1/2}$ near 200 Hz (where it translates through Eq. (2.2) of 2 into a detectable amplitude for a continuous signal of about $3 \times 10^{-24}$, as shown in Fig. 3). The relative timing stability between the interferometers was also significantly improved. Monitored with GPS-synchronized clocks to be better than 10 μs over S2, it allowed the coherent combination of the strain data of all three detectors to form joint upper limits.

**Analysis method.**—In 2 a search for gravitational waves from the millisecond pulsar J1939+2134 using S1 data was presented. In that work, two different data analysis methods were used, one in the time domain and the other in the frequency domain. Here we extend the former method 2, 6 and apply it to 28 isolated pulsars.

Following 2 we model the sources as non-precessing triaxial neutron stars showing the same rotational phase evolution as is present in the radio signal and perform a complex heterodyne of the strain data from each detector at the instantaneous frequency of the expected gravitational wave signal, which is twice the observed radio rotation frequency. These data are then down-sampled to 1/60 Hz and will be referred to as $B_k$. Any gravitational signal in the data would show a residual time evolution reflecting the antenna pattern of the detector, varying over the day as the source moved through the pattern, but with a functional form that depended on several other source-observer parameters: the antenna responses to plus and cross polarisations, the amplitude of the gravitational wave $h_0$, the angle between the line-of-sight to the pulsar and its spin axis $i$, the polarisation angle of the gravitational radiation $\psi$ (all defined in 4) and the phase $\phi_0$ of the gravitational wave signal at some fiducial time $t_0$. Let $\alpha$ be a vector in parameter space,
with components \((h_0, \iota, \psi, \phi_0)\).

The analysis proceeds by determining the posterior probability distribution function (pdf) of \(a\) given the data \(B_k\) and the signal model:

\[
p(a|\{B_k\}) \propto p(a)p(\{B_k\}|a),
\]

where \(p(\{B_k\}|a)\) is the likelihood and \(p(a)\) the prior pdf we assign to the model parameters. We have used a uniform prior for \(\cos \iota, \phi_0, \psi \) and \(h_0\) \((h_0 > 0)\), in common with \([2]\). A uniform prior for \(h_0\) has been chosen for its simplicity and so that our results can readily be compared with other observations. This prior favors high values of \(h_0\) (which comprise the majority of the parameter space) and therefore generates a somewhat conservative upper limit for its value. Indeed the reader might prefer to regard our resulting posterior pdfs for \(h_0\) as marginalised likelihoods rather than probabilities for \(h_0\) — these are functionally identical using our priors.

As in \([2]\) we use a Gaussian joint likelihood for \(p(\{B_k\}|a)\). In \([2]\) the S1 noise floor was estimated over a 60 s period from a 4 Hz band about the expected signal frequency. This gave a reliable point estimate for the noise level but was sensitive to spectral contamination within the band, as demonstrated in the analysis of the GEO S1 data. In this paper we exploit the improved stationarity of the instruments that make it reasonable to assume the noise floor is constant over periods of 30 min \([2]\). In addition we restrict the bandwidth to 1/60 Hz, which makes it possible to search for signals from pulsars at frequencies close to strong spectral disturbances. However, the noise level now determined is less certain as the estimate relies on fewer data. We take account of this increased uncertainty by explicitly marginalising with a Jeffreys prior over the constant but unknown noise level for each 30 min period of data \([2]\). The likelihood for this analysis is then the combined likelihood for all the 30 min stretches of data, labeled by \(j\) in Eq. \((2)\), taken as independent:

\[
p(\{B_k\}|a) \propto \prod_j p(\{B_k\}_j|a), \tag{2}
\]

\[
p(\{B_k\}_j|a) \propto \left(\sum_{k=k_{1(j)}}^{k_{2(j)}} |B_k - y_k|^2\right)^{-m}, \tag{3}
\]

where \(y_k\) is the signal model given by Eq. (4.10) in \([2]\) and \(m = k_{2(j)} - k_{1(j)} + 1 = 30\) is the number of \(B_k\) data points in a 30 min segment.

In principle the period over which the data are assumed stationary need not be fixed, and can be adjusted dynamically to reflect instrumental performance over the run. We have limited our analysis to continuous 30 min stretches of data, which included more than 88% of the S2 science data set. Inclusion of shorter data sections would at best have resulted in a \sim 6\% improvement on the strain upper limits reported here.

Validation by hardware injections.—All the software used in this analysis is available in the
The pulsar signals were injected for 12 h at frequencies of 1 279.123 Hz (P1) and 1 288.901 Hz (P2) with frequency derivatives of zero and $-10^{-8}$ Hz s$^{-1}$ respectively, and strain amplitudes of $2 \times 10^{-21}$. This gives signal-to-noise ratios (as defined by Eq. (79) of [6]) of 26 and 40 for P1 in H1 and L1 respectively and of 38 and 34 for P2. The signals were modulated and Doppler shifted to for P1 in H1 and L1 respectively and of 38 and 34 for P2. The signals were modulated and Doppler shifted to simulate sources at fixed positions on the sky with $\cos \theta = 0$, $\cos \iota = 0$ and $\phi_0 = 0$. To illustrate, posterior pdfs for the recovered P1 signal are shown in Fig. 2. The results derived from the different detectors are in broad statistical agreement, confirming that the relative calibrations are consistent and that the assessments of uncertainty (expressed in the posterior widths) are reasonable. Results for P2 were very similar to these.

The phase stability of the detectors in S2 allowed us to implement a joint coherent analysis based on data from all three participating instruments. This technique was noted in [2], but could not be performed on the S1 data because of timing uncertainties that existed when those observations were performed. The black lines in Fig. 2 show marginalisations of the joint posterior from H1, H2 and L1, i.e.,

$$p(a|H1, H2, L1) \propto p(a) p(H1|a) p(H2|a) p(L1|a).$$

(4)

In an ideal case of three detectors of similar sensitivities and operational times these coherent results would be approximately $\sqrt{3}$ times tighter than the individual results. The posteriors for $\phi_0$ clearly highlight the relative coherence between the instruments and verify that similar joint methods can be used to set upper limits on our target pulsars.

**Results.**—From the ATNF pulsar catalogue ([www.atnf.csiro.au/research/pulsar/psrcat/](http://www.atnf.csiro.au/research/pulsar/psrcat/)) we selected 28 isolated pulsars with rotational frequencies greater than 20 Hz and for which sufficiently good timing data were available (Table I). For 18 of these, we obtained updated timing solutions from regular timing observations made at the Jodrell Bank Observatory using the Lovell and the Parkes telescopes, adjusted for a reference epoch centred on the epoch of the S2 run (stared pulsars in Table I). Details of the techniques that were used to do this can be found in [10]. We also checked that none of these pulsars exhibited a glitch during this period.

The list includes globular cluster pulsars (including isolated pulsars in 47 Tuc and NGC6752), the S1 target millisecond pulsar (J1939+2134) and the Crab pulsar (B0531+21). Although Table I only shows approximate pulsar frequencies and frequency derivatives, further phase corrections were made for pulsars with measured second derivatives of frequency. Timing solutions for the Crab were taken from the Jodrell Bank online ephemeris [11], and adjustments were made to its phase over the period of S2 using the method of [12].

The analysis used 910 hours of data from H1, 691 hours from H2, and 342 hours from L1. There was no evidence of strong spectral contamination in any of the bands investigated, such as might be caused by an instrumental feature or a potentially detectable pulsar signal. A strong gravitational signal would generate a parameter pdf prominently peaked off zero with respect to its width, as for the hardware injections. Such a pdf would trigger
a more detailed investigation of the pulsar in question. No such triggers occurred in the analysis of these data, and we therefore present upper limits.

The upper limits are presented as the value of $h_0$ bounding 95% of the cumulative probability of the marginalised strain pdf from $h_0 = 0$. The joint upper limit $h_0^{95\%}$ therefore satisfies

$$0.95 = \int_{h_0=0}^{h_0^{95\%}} \int\int\int p(a|H1, H2, L1) \, da \, d\psi \, d\phi_0,$$

consistent with $h_0 = 2$. The uncertainty in the noise floor estimate is already included, as outlined above.

The remaining uncertainties in the upper limit values of Table I stem from the calibration of the instrument and from the accuracy of the pulsar timing models. For L1 and H2, the amplitude calibration uncertainties are conservatively estimated to be 10% and 8%, respectively. For H1, the maximum calibration uncertainty is 18%, with typical values at the 6% level. Phase calibration uncertainties are negligible in comparison: less than 10$^\circ$ in all detectors. Biases due to pulsar timing errors are estimated to be 3% or less for J0030+0451, and 1% or less for the remaining pulsars (see [2] for a discussion of the effect of these uncertainties).

Discussion.—The improved sensitivity of the LIGO interferometers is clear from the strain upper limit for PSR J1939+2134, which is more than a factor of ten lower than that achieved with the S1 data [2]. In this analysis the lowest limit is achieved for PSR J1910−5959D at the level of $1.7 \times 10^{-24}$, largely reflecting the lower noise floor around 200 Hz.

Table I also gives approximate limits to the ellipticities of these pulsars from the simple quadrupole model

$$\epsilon \simeq 0.237 \frac{h_0}{10^{-24} \text{ kpc}} \frac{r}{1 \text{ kpc}} \frac{1 \text{ Hz}^2 \times 10^{45} \text{ g cm}^2}{I_{zz}}$$

where $r$ is the pulsar’s distance, which we take as the dispersion measure distance using the model of Taylor and Cordes [13], and $I_{zz}$ its principal moment of inertia about the rotation axis, which we take as $10^{45} \text{ g cm}^2$.

As expected, none of these upper limits improves on those inferred from simple arguments based on the gravitational luminosities achievable from the observed loss of pulsar rotational kinetic energy. However, as discussed in the introduction, for pulsars in globular clusters such arguments are complicated by cluster dynamics, which the direct limits presented here avoid.

The result for the Crab pulsar (B0531+21) is within a factor of about 30 of the spin down limit and over an order of magnitude better than the previous direct upper limit of [14]. The equatorial ellipticities of the four closest pulsars (J0030+0451, J2124+3358, J1024−0719, and J1744−1134) are constrained to be less than $10^{-5}$.

Once the detectors operate at design sensitivity for a year, the observational upper limits will improve by more than an order of magnitude. The present analysis will also be extended to include pulsars in binary systems, significantly increasing the population of objects under inspection.

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