

# A Generic One-Way Multi-Objective Car-Sharing Problem With Dynamic Relocation

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## 1 Introduction

Environmental, energy, and societal considerations have given rise to the concept of shared-vehicle mobility systems (SMS). This concept postulates that the use of a fleet of vehicles, made available on demand to the general public on a rental basis, can increase the mobility of certain population categories, e.g. individuals who, at a given point in time, do not have access to privately-owned means of transportation, visitors/tourists, public transport users who need to access with private means the public transport system, and/or people that cannot afford on a permanent basis the use of private means of transportation. In addition to mobility enhancement, SMS have the potential to contribute to the sustainability of the transportation system through the decrease in environmental impact, energy and space requirements (Barth et al., 2006; Duncan, 2011).

As a consequence of the promises that SMS hold, numerous systems have been introduced in many cities around the world. On the research front, a body of literature has been developed for planning (Fassi et al., 2012), operating (de Almeida Correia and Antunes, 2012) and evaluating (Fan et al., 2008) SMS. The operational characteristics of the SMS vary according to the transportation means employed to offer the shared mobility services, e.g. bicycles, electric or conventional cars. However, the types of decisions associated with the planning and operation of these systems, and consequently the resulting decision making problems, can be modeled generically.

The objective of this paper is twofold: first, to propose a generic model for supporting strategic and tactical decisions related to SMS facility location and sizing, fleet sizing and allocation, and second to apply the propose model to an SMS system in Nice, France.

## 2 The Proposed Model

The generic vehicle-sharing problem with dynamic relocation can be expressed as follows: Given the spatial and temporal distribution of the demand for one- and/or two-way shared-vehicle trips determine i) the number, location, size and catchment area of shared vehicle stations, ii) the size of the fleet that should be used, and iii) the vehicle relocation strategy throughout an operating period. This would “optimize” both the level of service offered to the system users and the costs incurred by the service provider while satisfying a set of operational constraints.

The following multi-objective “optimization” model has been developed in order to cope with the above problem. Note that space limitations prevented us describing the constraints of the model. In the full paper, the complete model will be provided.

$$\max SW^s \left[ \begin{array}{l} \overbrace{\sum_{(j,l,t,u)} (\text{REV}_{jl}^{stu} + \text{SUB}_{jl}^{stu}) z_{jl}^{stu}}^{\text{total revenue + subsidy}} - \overbrace{\text{VOC}^{G(s)} v^{G(s)}}^{\text{depreciation}} \\ - \left( \underbrace{\sum_{j,l,t} \text{MVC}_{jl}^{G(s)t} r_{jl}^{G(s)t} + \sum_f \text{CPP}_f^{G(s)} h_f^{G(s)}}_{\text{relocation cost}} \right) - \left( \underbrace{\sum_j \text{FIX}_j x_j + \text{VAR}_j n_j^*}_{\text{station opening cost}} \right) \end{array} \right] \quad (1)$$

$$\max SW^s \left[ \begin{array}{l} \underbrace{\sum_{(j,l,t,u)} (\text{UTI}_{jl}^{stu} - \text{REV}_{jl}^{stu}) z_{jl}^{stu}}_{\text{total utility - expense}} - \underbrace{\sum_{(i,j,t)} (\text{ACC}_{ij}^{G(s)t} p_{ij}^{st} + \overline{\text{ACC}}_{ij}^{G(s)t} \overline{p}_{ij}^{st})}_{\text{accessibility cost}} \end{array} \right] \quad (2)$$

Equation 1 is the net revenue of the operator. The objective function contains total revenue and subsidy, depreciation, relocation and station opening costs. Note that the first three items for the operator and all the items for the users are weighted with a constant specific to the scenario ( $SW^s$ ), whereas the station opening cost is not weighted since the number of stations operating should be equal in all the scenarios.

Revenue for a specific trip type from station  $j$  to  $l$  from time interval  $t$  to  $u$  in scenario  $s$  equals the revenue per trip ( $\text{REV}_{jl}^{stu}$ ) times the number of trips undertaken of the same type ( $z_{jl}^{stu}$ ). The total revenue equals the sum of all the revenue of realized trips. Similarly, for each trip undertaken, a subsidy is gained by the operator which is represented by  $\text{SUB}_{jl}^{stu}$  for the same type of trip.

The depreciation is the cost directly proportional to the number of vehicles operating in the system. For scenario  $s$ , this cost equals the multiplication of the depreciation cost per vehicle ( $\text{VOC}^{G(s)}$ ) and the number of vehicles in the system ( $v^{G(s)}$ ) in scenario  $s$ .

The relocation cost has two components. The first is related to the fuel (i.e. electricity) consumption whereas the second is the expenses incurred by the personnel in charge of

relocating vehicles. The total vehicle moving cost equals the expenses of all the relocations. Relocation cost from station  $j$  to  $l$  in time interval  $t$  in scenario  $s$  equals the cost per relocation  $\left(\text{MVC}_{jl}^{G(s)t}\right)$  times the number of relocations  $\left(r_{jl}^{G(s)t}\right)$ . Similarly, the total relocation cost paid to the personnel equals the sum of all personnel costs. The personnel cost for shift  $f$  in scenario  $s$  equals the cost per member of personnel  $\left(\text{CPP}_f^{G(s)}\right)$  times the number of personnel hired for the same shift  $\left(h_f^{G(s)}\right)$ .

The station opening cost is the cost dedicated to the station operations. There is a fixed cost for opening a station  $\left(\text{FIX}_j\right)$  and a variable cost  $\left(\text{VAR}_j\right)$  for each parking spot  $\left(n_j^*\right)$  operating in the given station  $j$ . Note that, both  $\text{FIX}_j$  and  $\text{VAR}_j$  are cost per day.

The users' net benefit is represented by Equation 2. The utility can be defined as the monetarized value (i.e. €) of the benefit gained per realized trip.  $\text{UTI}_{jl}^{stu}$  can be defined as the utility gained per trip from station  $j$  to  $l$  from time interval  $t$  to  $u$  in scenario  $s$ . The total utility can be defined as the sum of the number of trips of each type times this monetarized utility per trip.

The total expense is the sum of money paid to the operator for the rental of vehicles by the users. The value equals to the summation of the multiplications of revenue per trip  $\left(\text{REV}_{jl}^{stu}\right)$  and the number of trips of the same type  $\left(z_{jl}^{stu}\right)$ .

The accessibility cost is the cost associated with reaching (from or to) the center to or from the station at the beginning (or end) of the trip.  $\text{ACC}_{ij}^{G(s)t}$  and  $\overline{\text{ACC}}_{ij}^{G(s)t}$  are the accessing costs from region  $i$  to station  $j$  and station  $j$  to region  $i$  respectively in time interval  $t$  of scenario  $s$  per access.  $p_{ij}^{st}$   $\left(\overline{p}_{ij}^{st}\right)$  is the number of trips requested to start from (or end at) center  $i$  and assigned to station  $j$  in time interval  $t$  of scenario  $s$ .

### 3 Model Application and Results

An instantiation of the proposed generic model, addressing the operational requirements and characteristics of a shared mobility system based on electric cars was used to examine the trade-off between the level of service and the cost of the provided services. The resulting model was applied in Nice (France) for the company *Auto Bleue*, which was operating a two-way car-sharing system with 17 stations in April 2011. In September 2011 they added 12 new stations to their system. In the following instance, we have investigated the performance of these stations with new 12 candidate stations in a one-way car-sharing framework.

Although the current system operating in Nice is a two-way car-sharing system, since their system tracks locations of vehicles location, we managed to create a new set of one-way demand by dividing the trips if the vehicle is not changing its place more than 60 minutes. A total of 18 different scenarios of three seasons (summer, autumn, winter) for different days were generated. In the model, it is also assumed that the number of

operating vehicles and relocation personnel in the same season should be equal which are considered as operational decisions that cannot be changed on a daily basis.

Since the vehicles used in the system are electric, after every operation (rental or relocation) vehicles are not assigned to any other operations and are recharged for at least two hours. In order to simplify demand, we use time intervals. Each operation (rental, relocation, charging) is assumed to start (or end) at the beginning (or end) of time intervals.

The multi-objective MILP model with the objectives stated above (Equation 1-2) is solved by CPLEX 12.2. Models are terminated when the relative optimality gap is less than 1%. The efficient frontier generated by these runs can be seen in Figure 1.

Figure 1 shows the effect of the satisfaction of demand on the operator's net revenues. Although higher levels of demand satisfaction are expected to increase gross revenues for the operator, the net revenues (shown in Figure 1) are decreasing. The reduction of the net revenues is attributed to the fast increase of the costs required to provide additional vehicles, stations, and parking spaces to provide higher levels of demand satisfaction. Furthermore, higher levels of demand satisfaction require more relocation operations which also contribute substantially to the costs encountered for the provision of services. Thus, a 25% increase in the satisfaction of demand (from 831 to 1025) results to a dramatic decrease of the net revenues (from 727.74 to 43.76) this is due to the increase of the number of cars, stations, parking spaces that should be available, and the extra cost of the car relocation. Note, that because of the limitations in the station size, relocating vehicles becomes indispensable for achieving higher levels of demand satisfaction.

## 4 Concluding Remarks and Future Work

The model proposed above considers simultaneous decisions associated with the allocation of strategic assets, i.e. stations and vehicles of car-sharing systems and the allocation of personnel for relocation operations (tactical decision). The model provides the decision makers with ample opportunities to perform a sensitivity analysis for the relevant model parameters, a feature particularly useful for cost values that are difficult to establish empirically, e.g. unit cost of unsatisfied customers, population coverage and station accessibility cost.

Research work under way involves a simulation model that will provide a more realistic representation of the relocation operation costs. Modeling the operational problem and assigning the vehicle rosters while taking their electrical charge level into consideration is another future work directions of this project.

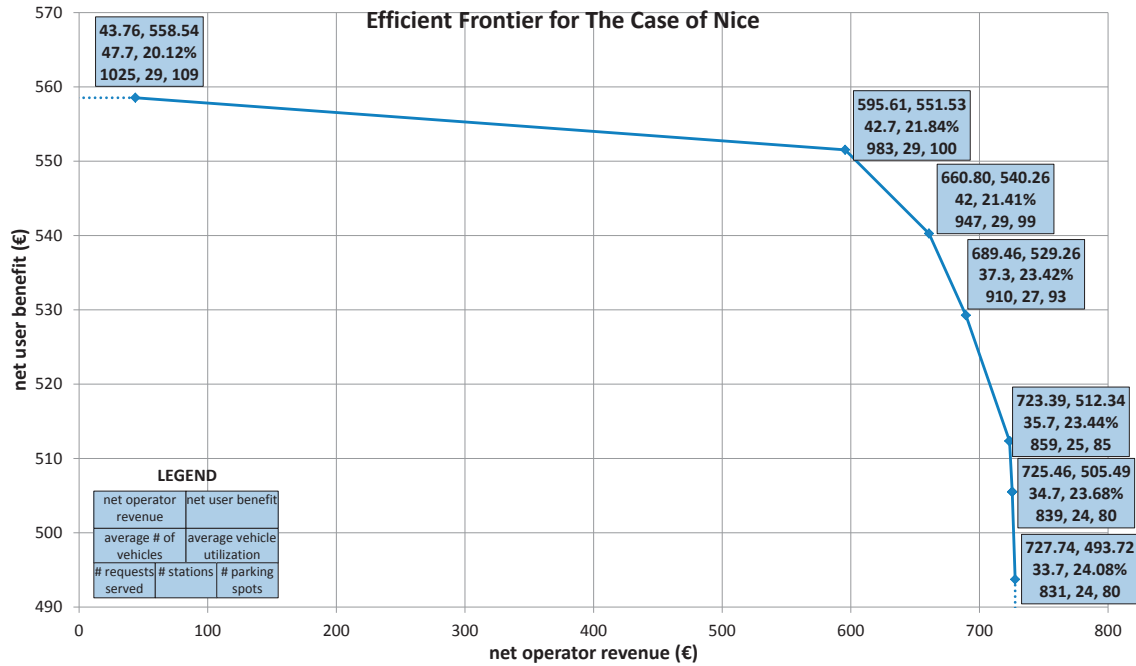


Figure 1: Efficient frontier of the one-way car sharing system framework for Nice, France

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