

An Optimization-Simulation Framework for Vehicle and Personnel Relocations of One-Way Electric Car-Sharing Systems with Reservations

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ABSTRACT

One-way electric vehicle carsharing systems are receiving increasing attention due to their mobility, environmental, and societal benefits. One of the major issues faced by the operators of these systems is the optimization of the relocation operations of personnel and vehicles. These relocation operations are essential in order to ensure that vehicles are available for use at the right place at the right time. Vehicle availability is a key indicator expressing the level of service offered to customers. However, the relocation operations, that ensure this availability, constitute a major cost component for the provision of these services. In this paper we are developing, solving, and applying, in a real world context, an integrated multi-objective mixed integer linear programming (MMILP) optimization and discrete event simulation framework to optimize operational decisions for vehicle and personnel relocation. We are using a clustering procedure to cope with the dimensionality of the operational problem without compromising on the quality of the obtained results. The optimization framework involves three mathematical models: (i) station clustering, (ii) operations optimization and (iii) personnel flow. The output of the optimization is used by the simulation in order to test the feasibility of the optimization outcome in terms of vehicle recharging requirements. The optimization model is solved iteratively considering the new constraints restricting the vehicles that require further charging to stay in the station until the results of the simulation are feasible. The application of the proposed framework using data from a real world system operating in Nice France, suggests that these systems are quite complex to manage without advanced relocation procedures.

INTRODUCTION

The emergence of carsharing as a new transportation mean between public and private transport is quite recent. People benefiting from this service have to register and then can use vehicles distributed all over the operating. Sharing transportation means is an approach to maximize utilization while decreasing the cost encountered to each user. In addition to users' benefits, carsharing systems produce broader societal and environmental benefits such as congestion, air pollution, and urban space for parking use reduction.

Carsharing systems can be classified into different categories depending on the rental conditions. *Free-floating* systems allow people to park the vehicles anywhere in the covered area whereas *non-floating* impose to users to park them at stations with limited number of allowed spots. *Round-trip* systems force the user to return the car to the location where it was picked-up whereas *one-way* systems allow drop-off at any station. The type of vehicle (combustion, electric, etc.) affects also the system's use. We focus in this paper on *non-floating one-way electric carsharing systems* because of their increased flexibility and their eco-friendly characteristics. In particular, we develop and apply an integrated framework for optimizing operational decisions related to vehicle and relocation personnel relocation decisions.

In such systems, rental operations naturally induce imbalances in the spatial and temporal distribution of vehicles. To maximize the demand served, vehicle distribution needs then to be corrected by performing relocations to maximize vehicle availability. The operations management of a carsharing system is complex because of demand characteristics, imbalance issues and limited information about the future availability of vehicles. The operator of such a system has to manage it in a way that maximizes the use of the system while at the same time all operational and business constraints are satisfied.

Carsharing systems have been studied intensively in recent years. The existing body of literature addresses issues related to carsharing systems characteristics and types, assessment of carsharing system impacts(1), and modelling of strategic, tactical (2) and operational (3) decisions. For a more comprehensive literature review the reader refers to [Jorge and Correia \(4\)](#). We limit our literature review to papers that are mostly relevant to the modelling of operational decisions for one-way carsharing systems. These decisions relate to the allocation and re-allocation of carsharing vehicles and relocation personnel to stations. Optimization and/or simulation are the methodological approaches that have been used to support one-way carsharing operational decisions.

[Barth et al. \(5\)](#) developed a queuing based discrete event simulation model to evaluate operational decisions for a shared vehicle system of a resort community in Southern California. [Kek et al. \(6\)](#), introduced a time-stepping simulation model for assessing relocation operations using shortest time and inventory balancing criteria.

[Kek et al. \(7\)](#), used a three-phase optimization-trend-simulation (OTS) approach to develop a Decision Support System (DSS), for relocation operations. The optimization phase is based on a Mixed Integer Programming (MIP) formulation which is used to determine the resources needed to operate the carsharing system at minimum cost. In the second phase the output of the optimization phase is "filtered" to produce the parameters needed to simulate the operation of the carsharing system. The simulator developed in [Kek et al. \(6\)](#) is used in the third step of the proposed DSS.

[Nair and Miller-Hooks \(8\)](#), proposed a stochastic MIP model to generate minimum cost vehicle redistribution plans to satisfy a proportion of the near-term stochastic-demand of the car-sharing system. The model was demonstrated on a real-world carsharing system in Singapore.

Bruglieri et al. (9), addressed the vehicle relocation problem for an one way electric car-sharing system in Milan. A MIP model was developed to determine the optimum sequence of pick-up and delivery operations that should be performed by vehicle-relocation personnel.

Nourinejad and Roorda (10), introduced an optimization-simulation framework for supporting decisions for one way carsharing systems. The proposed framework includes a static benchmark module and a dynamic module. The static component determines the optimum number of vehicles needed to serve the entire demand for the system. The dynamic relocation component involves a vehicle relocation (VRO) and a parking inventory model (PIO).

The model proposed in this paper differs in many respects from the papers mentioned above and complements the existing literature. In particular, in this paper we are introducing an integrated optimization-simulation framework for vehicle and personnel relocations for electric one-way carsharing system with reservation. The characteristics that differentiate the proposed framework from existing models are as follows:

- It considers multiple objectives.
- It determines the “optimal” spatial and temporal re-allocation of vehicles at a detailed operational, as opposed to aggregate strategic level.
- It determines the “optimal” spatial and temporal re-allocation of personnel by generating appropriate rosters for the relocation-personnel per shift.
- It takes into account the charging level of the battery of the electric vehicle in determining the feasibility of its use.

The remainder of the paper is structured as follows: Next section provides the methodological framework, it describes the assumptions and the system characteristics and the problem formulation, consisting of a personnel model, a vehicle model, a station clustering approach and a simulation module. The following section presents the results of an application in a case study for the city of Nice, France.

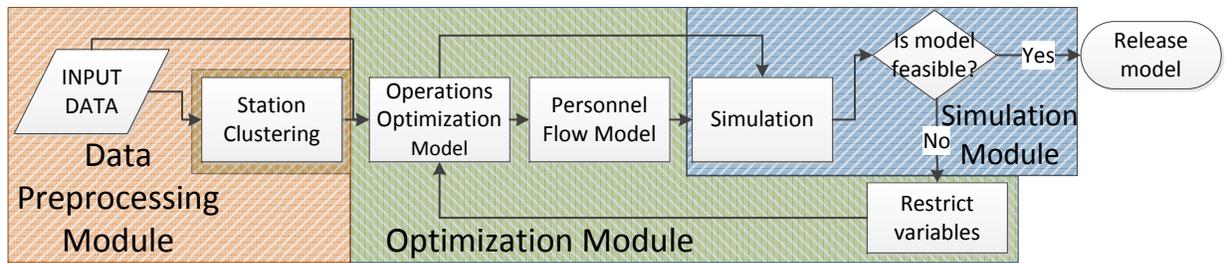
METHODOLOGICAL FRAMEWORK

The proposed framework consists of the following three modules: (i) data preprocessing, (ii) optimization, and (iii) simulation. The data preprocessing module prepares the data inputs for the subsequent optimization and simulation modules. It also creates clusters of stations which are used at the optimization module for relocation flows.

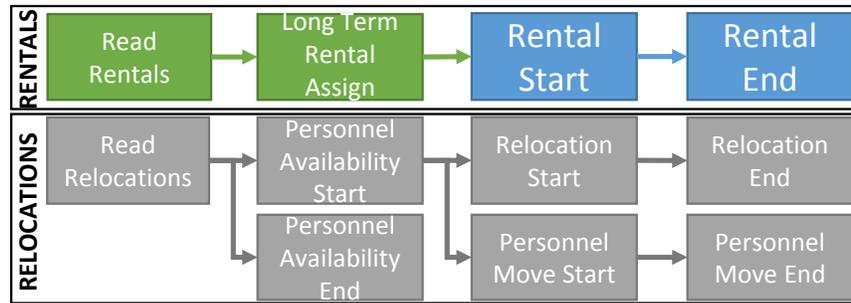
The optimization module (Fig. 1a) involves three mathematical models: (i) station clustering, (ii) operations optimization and (iii) personnel flow. The output of the optimization module is used by the simulation module (Fig. 1b) in order to test the feasibility of the optimization outcome in terms of vehicle recharging requirements. The optimization model is solved iteratively considering the new constraints restricting the vehicles that require further charging to stay in the station until the results of the simulation are feasible. A detailed description of the simulation is presented at a subsequent section of this paper.

Assumptions

The assumptions underlying the operations of the system are:



(a) Flow chart for the solution framework



(b) Simplified structure of events and their links to one another in the simulator.

FIGURE 1: The structure of the solution framework and the simulation module.

- Every trip and relocation with/without vehicle starts (ends) at the beginning (end -not necessarily the same) of a time interval.
- Electric vehicles are allowed to be rented without being fully charged. A threshold value defining acceptable charge levels is used.
- Each vehicle and staff have only one state at the first and second half of each time interval: parked, under rental or under relocation; and available, driving or moving respectively.

Concepts

The following concepts are used in the proposed modelling framework:

- The carsharing system is modelled as a network of stations.
- The operating time of the carsharing system is divided into equal time intervals (15 minutes). A time augmented network is generated including all station-time interval pairs. In addition to the time augmented network of cars, a separate network is created for each personnel shift.
- In the time augmented networks, each node is defined with two attributes, i.e. station and time interval, and represents the state of the station, i.e. number of vehicles or personnel depending on the network, for given time interval (Fig. 2). Operations start at the beginning and ends at the end of each time interval, i.e. each node has two values.

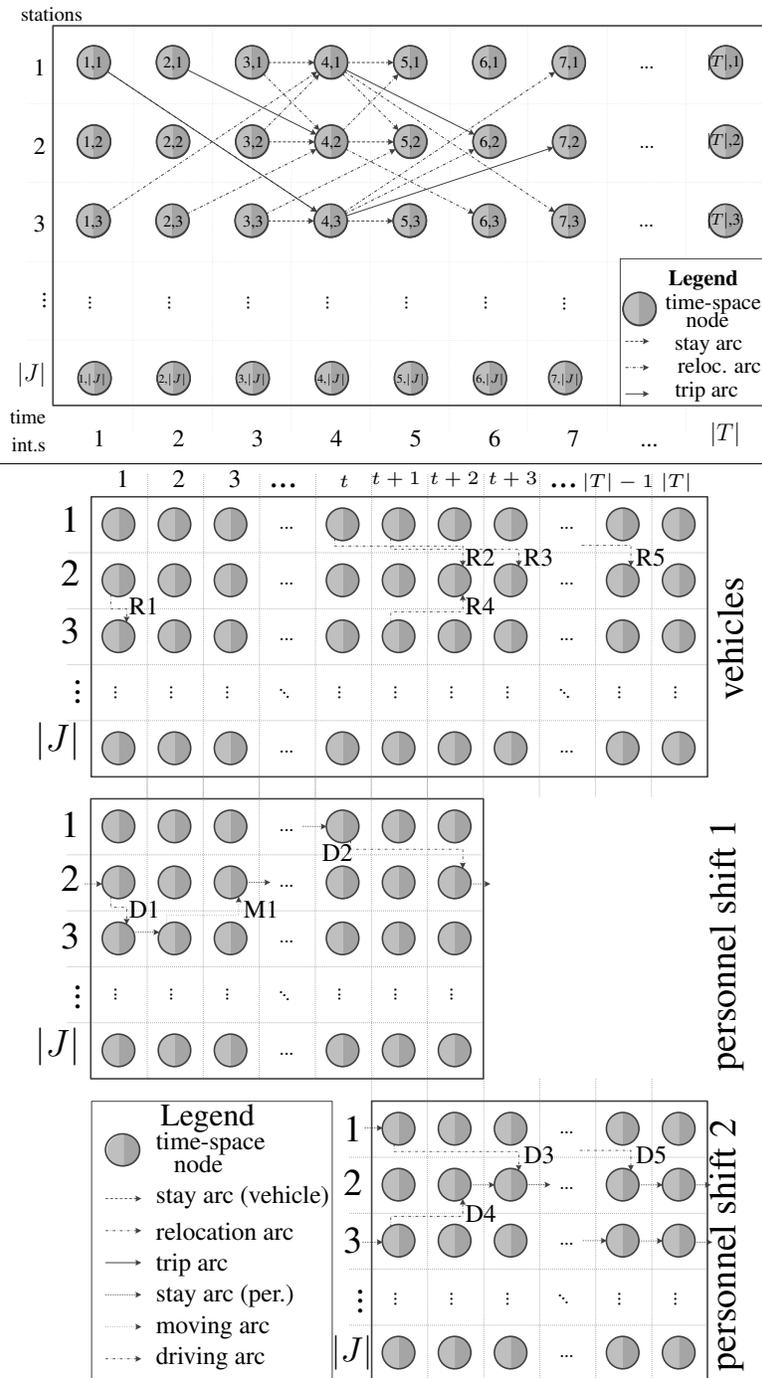


FIGURE 2: Time augmented station network (top figure) for a set of stations (J) and time intervals (T). Different types of arcs, i.e. stay, relocation (D and M) and trip (R), are shown starting and ending from the nodes at time interval 4. Nodes are coloured with two different tones of grey to emphasize each node has two values: number of vehicles (or personnel) at the beginning and end of time interval t in node j . (bottom figure) Note that time augmented networks of personnel contains only nodes of their working time intervals.

- When a relocation is executed for a vehicle at station j to l , from time interval t to u , a unit flow is sent both in the vehicle network and personnel network from nodes (j, t^-) to (l, u^+) . In Fig. 2, 5 relocations are shown with arcs R1-R5 in vehicle network which are executed by personnels from shift 1 (R1-R2) and shift 2 (R3-R5). Note that, there are driving arcs exactly starting from the same nodes in the personnel networks by whom the relocation is executed.
- In addition to driving, relocation personnel can change station by moving. This type of operation is labelled with M1 in Fig. 2. When a member of staff from shift s is moving from station j to l from time interval t to u , a unit flow is sent from (j, t^-) to (l, u^+) in personnel shift s' network. If a staff member from shift s does not change its location at time t and stays in station j , a unit flow is sent from stay arc from (j, t^-) to (j, t^+) .
- In the exact model of operations optimization model, all possible relocation, driving and moving arcs are created. Duration of all these operations are parameters of the model. For any given relocation, driving and moving arc from station j to station l starting at time interval t , there is only one end time interval u .
- To decrease the number of variables in the operations optimization model (see Fig. 1a), stations are clustered for driving and moving operations. A second layer is added for both driving and moving operations to handle them at cluster level. In the operations optimization model, when a driving (moving) is done from station j to station l from time interval t to u this operation is done in three artificial states: (i) from origin station j to origin station's cluster b at time interval t , (ii) from origin station's cluster b to destination station's cluster d starting at time interval t and ending at time interval u and (iii) from destination station's cluster d to destination station l at time interval u . Any relocation between a cluster and a station is named *intra-relocation* and any relocation between two clusters is named *inter-relocation*.
- Clusters of stations are created using an algorithm similar to k -Medoid algorithm (11). Since clustering depends only on the network, the clustering algorithm is the very first algorithm to be executed. The clustering algorithm is run for driving and moving arcs separately since different speeds are used in these two operations (see Fig. 1a).
- After solving the operations optimization model, personnel flow model is used to create personnel assignments from flows between clusters and between clusters and stations.
- Charging levels of vehicles are controlled with a simulator. Operations optimization model, personnel flow model and simulation are run iteratively until a charging level feasible solution is created. If there is an infeasibility in the charging level of any vehicles, the vehicle is forced to stay and charged until it is assigned for its next relocation or trip.

Operations Optimization Module

Sets and Indices

$i \in I$: trips

j and $l \in J$: stations

t, u and $w \in T$: time intervals

$s \in S$: working shifts

b and $d \in B$: station clusters

Parameters

start/end(s) : start/end time intervals of working shift s

start/end(i) : start/end time intervals of trip i

origin/dest(i) : origin/destination stations of trip i

c.end(i) : the last interval that vehicle of trip i can be charged

d.end/m.end(t, b, d) : the time interval that driving/moving from cluster b to d ends if it starts at time interval t

PC _{s} : cost of relocation personnel for working shift s

RC _{j} : inter-relocation cost from a station in cluster b to a station in cluster d

$\widetilde{\mathbf{RC}}_{bd}$: intra-relocation cost from/to station j

Variables

z_i : binary variable indicating if trip i is served or not

\tilde{z}_i^t : binary variable indicating if the vehicle of trip i is charged at its destination station at time interval t

v_s : number of staff member used from shift s

n_j^t (\bar{n}_j^t) : number of vehicles in station j at time interval t before (after) any relocation and trip leaving the station

\tilde{n}_j^t : number of vehicles in station j not fully charged at time interval t before relocations and trips from

m_{sj}^t (\bar{m}_{sj}^t) : number of staff member from shift s in station j at time interval t before (after) any relocation with/without vehicle leaving the station

r_{sj}^t (\bar{r}_{sj}^t) : number of vehicles relocated by personnel of shift s from station j starting (finishing) at the beginning (end) of time interval t

p_{sj}^t (\bar{p}_{sj}^t) : number of staff member of shift s moved from station j starting (finishing) at the beginning (end) of time interval t

$\tilde{r}_{sbd}^{t(u)}$: number of vehicles relocated by personnel of shift s from a station in cluster b to a station from cluster d started at time interval t and ending at time interval u

$\tilde{p}_{sbd}^{t(u)}$: number of staff member of shift s moved from a station in cluster b to a station from cluster d started at time interval t and ending at time interval u

Operations Optimization Model

$$\max \sum_i z_i \quad (1)$$

$$\min \underbrace{\sum_{b,d} \overline{RC}_{bd} \tilde{r}_{sbd}^{t(u)}}_{\text{inter-relocation cost}} + \underbrace{\sum_{j,t} \overline{RC}_j (r_{sj}^t + \bar{r}_{sj}^t)}_{\text{intra-relocation cost}} + \underbrace{\sum_s \overline{PC}_s v_s}_{\text{personnel cost}} \quad (2)$$

$$\max \sum_{j,t} \tilde{z}_i^t \quad (3)$$

$$\text{s.t. } \bar{n}_j^t = n_j^t - \sum_{\substack{i:\text{start}(i)=t \\ \text{origin}(i)=j}} z_i - \sum_s \bar{r}_{sj}^t \quad \forall j, t \quad (4)$$

$$n_j^{t+1} = \bar{n}_j^t + \sum_{\substack{i:\text{end}(i)=t \\ \text{dest}(i)=j}} z_i + \sum_s r_{sj}^t \quad \forall j, t \setminus t_{\text{last}} \quad (5)$$

$$\sum_j m_{sj}^{\text{start}(s)} = v_s \quad \sum_j m_{sj}^{\text{end}(s)} = v_s \quad \forall s \quad (6)$$

$$\bar{m}_{sj}^t = m_{sj}^t - \bar{p}_{sj}^t - \bar{r}_{sj}^t \quad \forall j, s, t \quad (7)$$

$$m_{sj}^{t+1} = \bar{m}_{sj}^t + p_{sj}^t + r_{sj}^t \quad \forall j, s, t \setminus t_{\text{last}} \quad (8)$$

$$\sum_d \tilde{r}_{sbd}^{t(u)} = \sum_{j \in J_b} \bar{r}_{sj}^t \quad \sum_d \tilde{r}_{sdb}^{u(t)} = \sum_{j \in J_b} r_{sj}^t \quad \forall s, b, t \quad (9)$$

$$\sum_d \tilde{p}_{sbd}^{t(u)} = \sum_{j \in J_b^s} \bar{p}_{sj}^t \quad \sum_d \tilde{p}_{sdb}^{u(t)} = \sum_{j \in J_b^s} p_{sj}^t \quad \forall s, b, t \quad (10)$$

$$z_i \geq \tilde{z}_i^{\text{end}_i+1} \quad \tilde{z}_i^t \geq \tilde{z}_i^{t+1} \quad \forall i, t \quad (11)$$

$$\bar{n}_j^t = \sum_{i:\text{dest}(i)=j} \tilde{z}_i^t \quad n_j^t \leq \text{CAP}_j^t \quad \forall j, t \quad (12)$$

$$z_i, \tilde{z}_i^t \in \{0, 1\}; n_j^t, \bar{n}_j^t, \tilde{n}_j^t, v_s \in \mathcal{N} \quad \forall i, j, t, s \quad (13)$$

$$m_{sj}^t, \bar{m}_{sj}^t, r_{sj}^t, \bar{r}_{sj}^t, p_{sj}^t, \bar{p}_{sj}^t, \tilde{r}_{sbd}^{t(u)}, \tilde{p}_{sbd}^{t(u)} \in \mathcal{N} \quad \forall s, j, t, b \quad (14)$$

The proposed optimization model is a multi-objective mixed integer linear programming problem (MILP) with hierarchical objective functions. The model, involves in order of importance, the following objectives: (i) maximize number of trips served, (ii) minimize relocation cost and (iii) maximize charging after rental.

The primary objective function of the model (1) is to maximize the number of trip requests served. Demand served is prioritized over profit because the problem we deal with is based on reserved demand and we assume that the requested trip has been already granted to the customer.

The secondary objective function of the model (2) aims to minimize total relocation cost. It has three components:

- Inter-relocation cost includes the fuel cost associated with relocation operations between clusters.

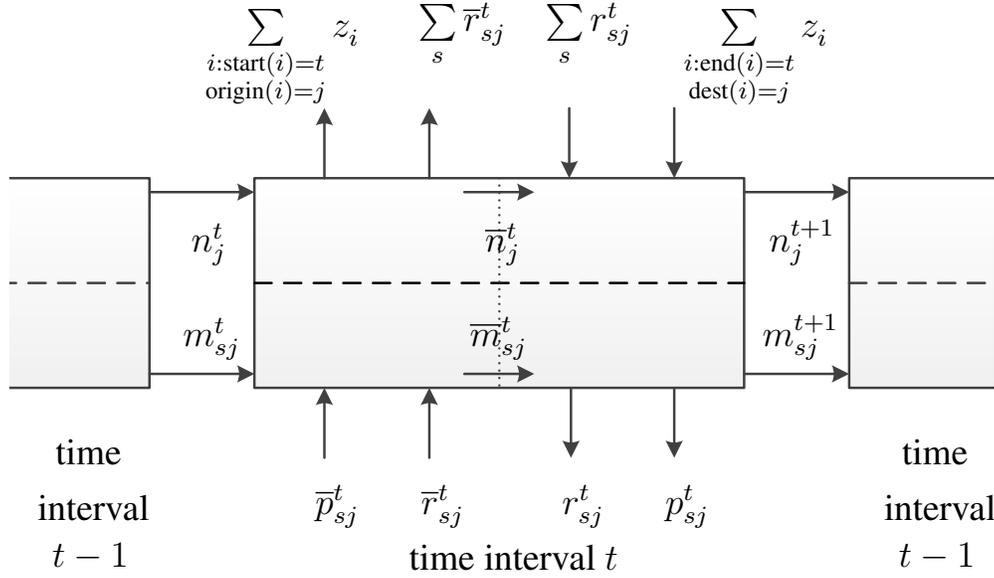


FIGURE 3: Flow diagram of vehicles (top) and personnel shift s (bottom) in station j at time interval t .

- Intra-relocation cost includes the fuel cost associated with relocation operations between stations and clusters.
- Personnel cost is the total cost of personnel hiring. We have limited number of designated shifts and the model determines the number of employees needed per shift.

The tertiary objective function of the model (3) aims to maximize the charging levels of each vehicle before they can be rented after returning from a trip. In our previous work (2), we assumed each vehicle should become available to be rented when they are only fully charged after returning from a trip. This sharply decreases vehicle utilization since there are two peaks of demand during the day (morning and evening) and low demand during most of the day. In this new model, we let vehicles to be rented even if they are not fully charged. When possible vehicles are still kept at the stations to be charged after their trips.

Constraints 4 and 5 are the flow conservation equations for vehicles for each station at the first and second half of each time interval respectively. Flow on the top left and right rectangle at time interval t of Fig. 3 represents this relationship.

Constraints 6 keep track of the number personnel hired for each shift. In the first (second) set of constraints, for all shifts, the total number of personnel started (finished) working at different stations at the beginning (end) of the start (finish) time interval of shift s should be equal to the number of personnel hired for the same shift.

Constraints 7 and 8 are flow conservation equations for each shift, each station at the first and second half of each time interval respectively. Flow on the bottom left and right rectangle at time interval t of Fig. 3 illustrates this relationship.

Constraints 9 and 10 are used to preserve flow conservation between clusters and the stations of clusters for driving and moving operations respectively.

Constraints 11 are used to keep track of charging vehicles after rentals. In the first set of constraints, it is requested that, for all trips, if trip i is not served then there is no vehicle to be

charged. In the second set of constraints, for all trips and time intervals, if the vehicle of trip i left the station without being charged at time interval t , then it cannot be charged at time interval $t + 1$ at the destination node.

The first part of constraints 12 are used to find the number of vehicles not fully charged at station j at time interval t . For all stations and time intervals, the total number of vehicles charging at station j at time interval t should be equal to the number of vehicles of trips destined to the same station and being charged at the same time interval. The second part of constraints 12 set the capacity restrictions for each station j at each time interval t . Note that, the same constraint is not needed for variables \bar{n}_j^t since it is assumed that in the first half of each time interval vehicles only leave stations.

Constraints 13-14 are used to define the domains of each variable: z_i and \tilde{z}_i^t are restricted to be binary variables, the rest of them are positive integers.

Personnel Flow Module

For each working shift s , the following model is solved with the values acquired from the operations optimization model 15-22.

Sets and Indices

$k \in K$: staff member

$j \in J$: stations

b and $d \in B$: clusters

$t \in T$: time intervals

$f \in F$: flows

Parameters

m_{sj}^t (\bar{m}_{sj}^t) : number of staff member from shift s in station j at time interval t before (after) any relocation with/without vehicle leaving the station

r_{sj}^t (\bar{r}_{sj}^t) : number of staff member of shift s driven from (to) station j starting (finishing) at the beginning (end) of time interval t

p_{sj}^t (\bar{p}_{sj}^t) : number of staff member of shift s moved from (to) station j starting (finishing) at the beginning (end) of time interval t

$\tilde{r}_{sbd}^{t(u)}$ ($\tilde{p}_{sbd}^{t(u)}$) : number of staff member of shift s driven (moved) from a station in cluster b to a station from cluster d started at time interval t and ending at time interval u

v_s : number of staff member used from shift s

start/end(f) : start/end time interval of flow f

origin/dest.(f) : origin/destination stations of flow f

Variables

x_{jk}^t (\bar{x}_{jk}^t) : binary variable indicating if staff member k is in node j at the beginning (end) of time interval t or not

y_{fk} : binary variable indicating if staff member k follows flow f or not

Personnel Flow Model

Before starting to solve the model, all possible flows are created by iterating $r_{sj}^t, \bar{r}_{sj}^t, \tilde{r}_{sbd}^{t(u)}, p_{sj}^t, \bar{p}_{sj}^t$ and $\tilde{p}_{sbd}^{t(u)}$. A flow contains two intra-relocations and one inter-relocation of the same type, i.e. driving or moving. In other words, in order to generate a flow from two intra-relocations and one inter-relocation, (i) the first inter-relocation's time interval should be the starting time interval of the intra-relocation, (ii) the station of the first intra-relocation should be element of the origin cluster of the intra-relocation, (iii) the second inter-relocation's time interval should be ending time interval of the intra-relocation, and (iv) the station of the second intra-relocation should be element of the destination cluster of the intra-relocation. Then all these generated flows are put into set F . In order to simplify representation, drivings (r) and movings (p) are represented with q .

$$\min \sum_{f,k} \text{dist}(f) y_{fk} \quad (15)$$

$$\text{s.t.} \sum_k x_{jk}^t = m_{sj}^t \quad \sum_k \bar{x}_{jk}^t = \bar{m}_{sj}^t \quad \forall j, k, t \quad (16)$$

$$\sum_j x_{jk}^t \leq 1 \quad \sum_j \bar{x}_{jk}^t \leq 1 \quad \forall j, k, t \quad (17)$$

$$\bar{x}_{jk}^t = x_{jk}^t - \sum_{\substack{f:\text{start}(f)=t \\ \text{origin}(f)=j}} y_{fk} \quad \forall j, k, t \quad (18)$$

$$x_{jk}^{t+1} = \bar{x}_{jk}^t + \sum_{\substack{f:\text{end}(f)=t \\ \text{dest}(f)=j}} y_{fk} \quad \forall j, k, t \quad (19)$$

$$\sum_{\substack{f:\text{start}(f)=t \\ \text{origin}(f)=j}} y_{fk} = \bar{q}_{sj}^t \quad \sum_{\substack{f:\text{end}(f)=t \\ \text{dest}(f)=j}} y_{fk} = q_{sj}^t \quad \forall j, t \quad (20)$$

$$\sum_{\substack{f:\text{start}(f)=t, \text{origin}(f) \in J_b \\ \text{end}(f)=u, \text{dest}(f) \in J_d}} y_{fk} = \tilde{q}_{sbd}^{t(u)} \quad \forall b, d, t \quad (21)$$

$$x_{jk}^t, \bar{x}_{jk}^t, y_{fk} \in \{0, 1\} \quad \forall f, j, k, t \quad (22)$$

The objective of the personnel flow model (15) aims to minimize total distance travelled. We can minimize the total difference between real relocation time and assumed relocation time with clustering.

Constraints 16 counts the number of staff member from shift s at each station j at each time interval t and sets this number to the number to the value calculated in operations optimization model, i.e. x_{jk}^t and \bar{x}_{jk}^t .

With constraints 17, every staff member can be at most in one station at each time interval t . Note that, the LHS of this constraint does not necessarily exactly equal to 1 since when there may be driving and moving operations which takes more than one time interval. Constraints 18 and 19 are for flow conservation equations for individual staff members.

Constraints 20 and 21 are used to cover the same relocation operations found by operations optimization model in the previous step. Constraints 20 and 21 ensure that, the total number of flows assigned to all personnel from shift s should exactly cover all intra-relocations with/without vehicle from and to and all inter-relocations respectively.

Clustering Algorithm

We are using clustering to find clusters that minimize the error created by aggregating stations for the given number of clusters. Since our model works with time intervals, our objective is to minimize the sum of squares of the deviation of relocation time of every pair of different stations when clustering is used. If $T(j, l)$ is defined as the travel time from station j to station l and $[T_{\min}^B(j, l), T_{\max}^B(j, l)]$ is the interval of the duration of relocation from cluster of station j to cluster of station l ; positive (dev_B^+) and negative (dev_B^-) deviations' calculations can be formulated as Eqs. 23-24. Note that if travel time from origin to destination station is in the interval of the duration of relocation from the cluster of origin to the cluster of destination station, both positive and negative deviations are zero.

$$\text{dev}_B^+(j, l) = \max \{T(j, l) - T_{\max}^B(j, l), 0\} \quad (23)$$

$$\text{dev}_B^-(j, l) = \max \{T_{\min}^B(j, l) - T(j, l), 0\} \quad (24)$$

The algorithm developed for clustering is similar to k -Medoid algorithm (11). The algorithm starts with an initial solution and iterates until the stopping criterion is met.

Simulation Module

Although, for the carsharing systems with combustion vehicles, the solutions generated by the optimization module are always feasible, it is not the case for systems with electric vehicles. The charging level of each vehicle at each time interval should be controlled in order to satisfy charging level feasibility of the solution. For this purpose, a simulation module is added to the solution framework.

The purpose of the simulation framework is to check the feasibility of the solution generated by the optimization module. If the solution is feasible, it is released. If it is not feasible, the simulation module returns the infeasible vehicle with its assigned rentals. The optimization module puts a charging restriction for this vehicle and forces charging of this vehicle after its last rental without forced charging. Note that, forced charging constraint is active only if a demand for a trip served. If the demand with forced charging is not served, charging does not have to be applied.

These additional charging restrictions may result in suboptimal schedules. In order to find the optimal charging strategy, the model needs to test all possible charging restrictions which needs exponential number of runs. It is worth noting our strategy terminates within limited number of iterations. The maximum number of iterations is not more than the total number of requested trip. Furthermore, the strategy always returns a feasible solution. The duration of the charging is just enough to fulfil the consumed electric energy in the last trip. In the worst case, there might be a

request for charging after every trip. In such a case, any solution would be charging level feasible since this will be a model with all forced charging restrictions. A system with a restriction of fully charging after every trip would be a carsharing system which only allows fully charged vehicles to be rented. This strategy is not preferred at the very first step because it sharply decreases the capacity of the system.

The simulation we use here is a discrete event simulation. The flow diagram for the events are shown in Fig. 1b. Rentals and relocations are fed to the simulator (see Read Rentals and Read Relocations events) as parameters. Every rental and relocation of vehicles, and movements of personnel are simulated with the help of this simulator.

MODEL APPLICATION

The model presented with the flow chart (see Fig. 1a) was applied to generate operational decisions for the one-way electric carsharing system Auto Bleue in Nice, France. With its 66 stations, it has been operating a round-trip system since 2011 and a one-way system since 2014. Auto Bleue provided rental and station location data to test the algorithm. The demand from individual days are aggregated. For different demand levels, subsets with different number of elements are generated randomly from this aggregated set.

The whole model including the simulator, clustering algorithm and mathematical models are implemented in C# .NET environment. IBM ILOG Cplex Version 12.6 with Concert Technology is used for solving MILPs. The time interval lengths are set to 15 minutes. We assume relocation personnel move between stations either with a car (if a vehicle is relocated) or with a bike (if a vehicle is not relocated). The driving (30km/h) and cycling (15km/h) distances were estimated using Google Maps API (12).

Clustering Stations

First, clusters for driving and cycling are created. The aim of the clustering is to decrease the total computation time with an acceptable sacrifice from the accuracy of the relocations. The difference between the durations of relocations with and without vehicles results in two separate clusters.

In the runs, we assume that the travel time between two stations is equal to the longest distance of the travel time between the pairs of stations of the two clusters. With this assumption, using clustering always produces feasible relocation operations. We prefer slightly to overestimate relocation durations instead of ending up with infeasible solutions underestimating relocation durations.

There is a trade-off between the efficiency and the accuracy in selecting the number of clusters to be used for relocations with/without vehicles. Higher (lower) number of clusters results in less (more) overestimation in relocation durations but increases (decreases) the number of variables and as a result run times. To decide the most suitable cluster sizes, the clustering algorithm is run for various number of clusters. We have selected 6 and 10 clusters for relocations with and without vehicles respectively.

After all experiments, the error of using 6 and 10 clusters for each run is also calculated. On average, the duration of relocations with vehicles is overestimated 5.97% and 10.93% more for relocations with and without vehicles respectively. Regarding the problem size and the efficiency of the method, these errors are acceptable. Note that, this error could be decreased if more clusters were used. However this results in longer run times.

Settings

Demand samples with rentals 100 to 300 with increments of 50 were created to test level of demand on the system. Each sample is tested with two different running configuration:

Type 1: The entire demand was fed to the model all at once and solved all in a single step.

Type 2: The demand is received one by one and accepted or rejected after each iteration without reconsidering the past decisions. Every rental request is accepted or rejected as soon as it is received.

In all scenarios, each station assumed to have three parking spots and an available fully charged vehicle parked at one of them. All scenarios were started at 6:00 and ended at 22:00. We set three different working shifts throughout the day: (1) morning (6:00-12:00), (2) afternoon (11:00-17:00) and (3) evening (16:00-22:00). Personnel cost was set to €108 per shift. Range of a vehicle with a full battery is 120 kilometres. It takes 8 hours to fully charge an empty battery. Relocation fuel cost is set to 0.02 €/km. Relocation speed with and without vehicle are set to 30 and 15 km/h respectively.

Results

The comparison of performance measures for runs with different request counts and run types is shown in Fig. 4. Each set of parameters (i.e. demand level and run type) was repeated with 50 different samples. In all figures, the red dots show individual runs. At each box-and-whisker-plots, upper and lower whiskers show the boundaries for the 95th and the 5th percentiles whereas the box shows the values between the 25th and the 75th percentiles. Median is shown with a line in the box and mean with a blue diamond. x -axes show the run type and the level of demand. y -axes show counts (in Fig 4b), percentages (in Figs 4a and 4d) or ratios (in Fig 4c).

Fig. 4a shows the percentage of lost demand to total demand for different runs. Type 1 and type 2 runs gave almost the same results for demand levels less than 250. If the same set of demand requests is accepted to be served by the two algorithms, the solution of both runs should be the same. However, since for most of the instances, accepted demand sets are different for the demand level 300, the two running configurations have different results. Having the information of all demand requests before deciding on them gives type 1 algorithm freedom to choose rental requests matching better with each other. Since type 1 runs have additional information about the requests, type 2 runs cannot outperform type 1 runs if all the other settings are the same. For type 2 runs, the accepted requests to be served in previous iterations effect the decision to serve or not to serve an additional demand. On the other hand, type 1 runs try to find the largest subset of demands that can be served with the current system. Because of that, as the system is more stressed with increased demand levels, type 1 runs perform slightly better than type 2 runs.

The effect of demand levels and run types on the number of relocations is shown in Fig. 4b. In this figure, we see a trend of increase with the increase in demand. The relationship between the number of relocations and the number of requests show almost a linear relationship: For every 8-10 rental requests, 3 relocation operations are needed. More relocation is needed with the increase in demand. As the system becomes congested, more resources are used for relocations. We also observe some differences between two run types. As expected, type 1 runs requested less number of relocations compared to type 2 runs for the same level of demand requests. Type 2 runs usually serve less number of demands in high demand levels. The difference in the number of relocations

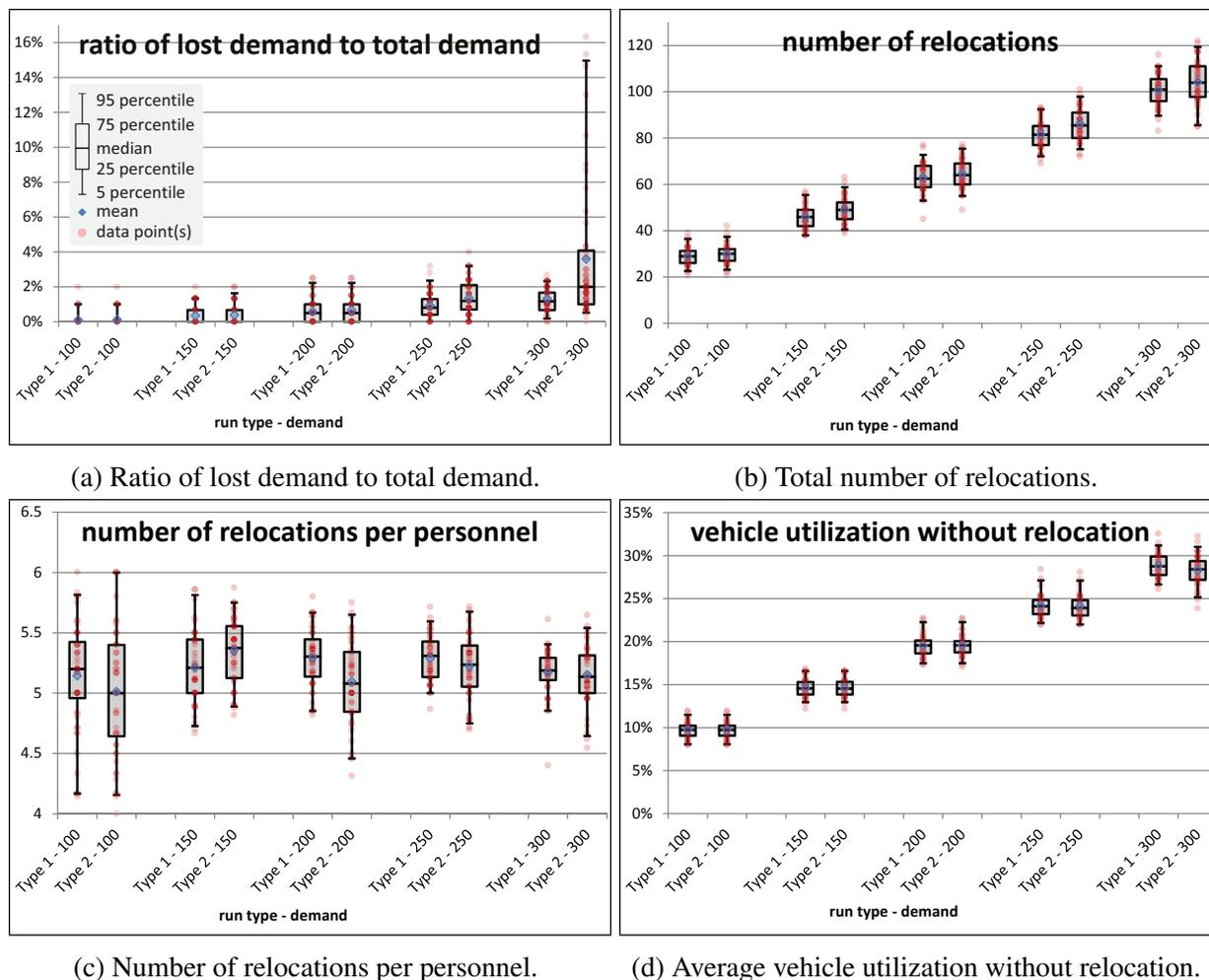


FIGURE 4: Some important performance measures of runs with different request counts (100, 150, 200, 250, 300) and run types (type 1 and type 2.)

between run types shows the effect of selecting better subsets of requests: If the requests are selected with more information, more demand can be served with less number of relocations.

In Fig. 4c, we see the average number of relocations per personnel in different scenarios. The individual runs and box-and-whisker-plots show, the variation of relocation to personnel ratio decreases in different runs with the increase in demand. With increase in demand, we end up with samples better representing population which makes different samples more alike.

In the last figure, Fig. 4d, utilization of vehicles under rental are shown respectively: Vehicles spend 8-34% of the day under rental depending on the demand level. As expected, the increase in the number of demand requests increases the vehicle utilization under rental. It is worth again to compare the utilization differences between two run types. The difference between utilization for rental operations becomes obvious with the demand level 250 and 300. But this difference is not as severe as Fig. 4a. This may be because of the way we selected the first objective function (see Obj. 1) in the operations minimization model (see Sec. 3.3.4). In our first objective function, the duration of the requests was not considered in the objective. Demand requests with longer duration was not superior to shorter ones. As a result, type 1 runs are inclined to select shorter requests in

case of high demand levels to serve more demand with the same system resources.

The remaining analysis is done to a specific instance with 300 demand. In Fig. 5, the movements of vehicles between stations is shown. In Figs. 6a-6b, two separate scenarios are applied on the same problem instance with additional constraints.

In Fig. 5, the two leftmost vertical numbers show the station and the spot of this station. The uppermost horizontal numbers are the starting hour and minute of the intervals. Each colour shows a different state for the spot. The legend for colour coding is shown in the legend on the same figure. Note that, for all states except “spot available”, the number inside the cell identifies the vehicle effected with this state. For this instance, since only the 0th spot of every station has an available vehicle at the beginning of the day, each vehicle is numbered with the station they were parked at the beginning of the day.

Fig. 5 also enables to see the detailed relocation operations. Each orange line shows the relocations starting/ending time and origin/destination stations. One of the expected results observed from this figure is to see the relocations of vehicles from stations with no/low demand to stations with high demand at the beginning of the day and the opposite movement at the end of the day. Vehicles from stations without demand (i.e. 61, 62, 63, 64 and 65) were relocated to stations with high demand (i.e. 14, 35 and 55 then 42 and 55) at the beginning.

The identification of numbers enable to track the daily assignments of each vehicle. For instance the assignments of vehicle 0 are as follows:

- 8:00-8:15: relocated to the 1st spot of Station 25
- 8:45-9:45: rented and returned to the 0th spot of Station 54
- 12:00-12:30: rented and returned to the 0th spot of Station 30
- 19:15-20:45: rented and returned to the 1st spot of Station 29

In Fig. 6a type 1 and type 2 runs are compared for the same single instance with 300 demand. Different than the previous runs, the maximum number of available relocation personnel at each shift is limited to different values from 0 to 8 (see x -axis). The dark (light) black (gray) lines and darker (lighter) columns on the right (on the left) show results for type 1 (type 2) runs. The left vertical axis shows the number of personnel (columns) and the number of relocations per personnel (lines with triangle markers). The right vertical axis is for the total demand served (lines with square markers) and the demand per personnel (lines with diamond markers).

This difference between the number of demand served with type 1 and type 2 runs shows us how many more demands can a carsharing system serve if it has chance to select from a set of requests. But in real systems, it is not an option easily applicable. Rejecting a customer after reservation, or collecting the entire demand requests and answering them just before the rentals are realised are not sustainable strategies for any carsharing companies. However, this figure shows the importance of controlling demand with different strategies. The other three performance measures show the two runs work with almost the same resources and type 2 runs are never better than type 1.

In Fig. 6b, the type 1 run has given freedom to choose a number of demand to be served (170 to 300 in every 5) from a set of 300 demand. We could not have the same test for the other running configuration since in type 2 runs, the requests become available and are accepted/rejected

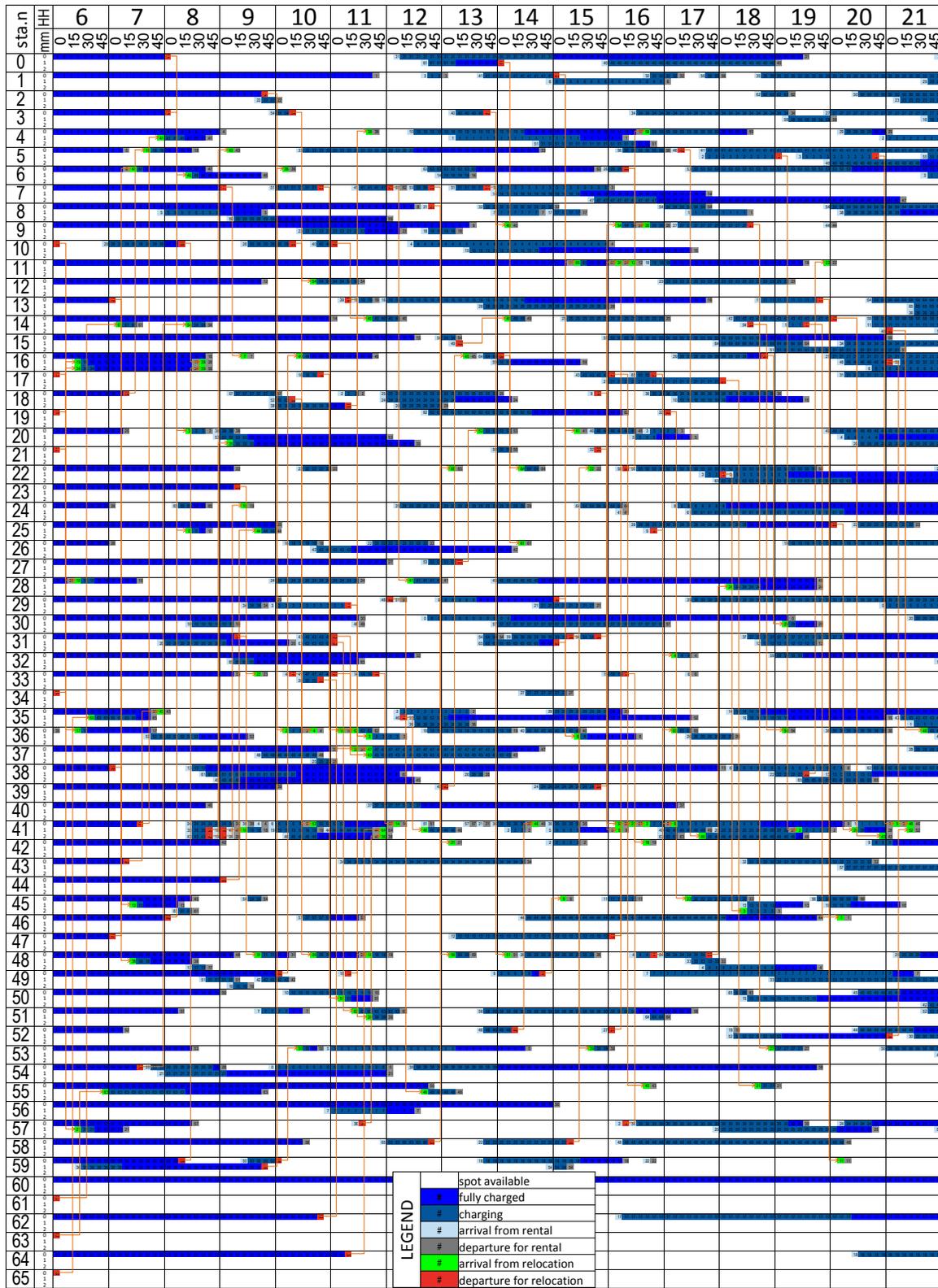
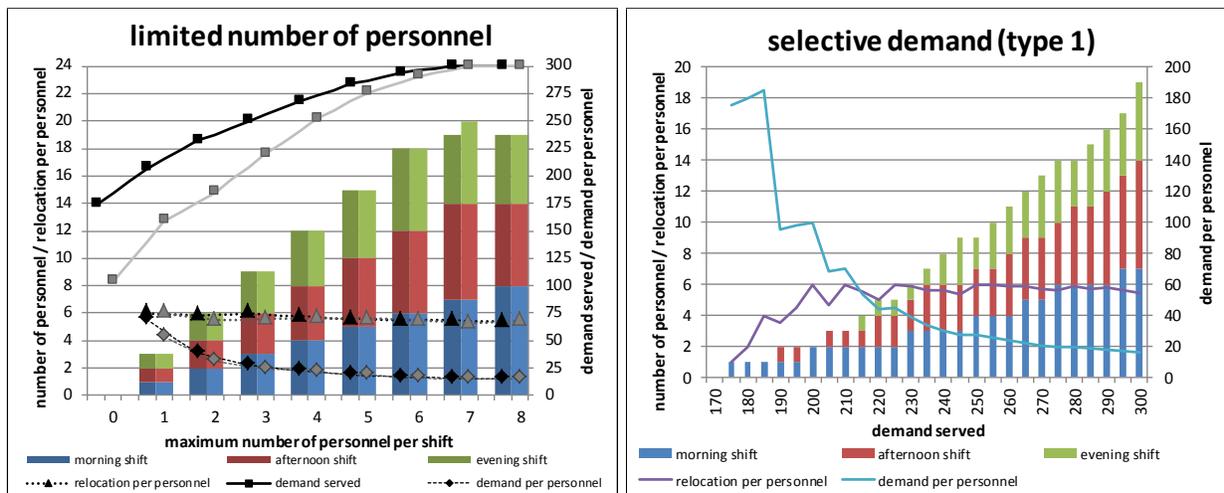
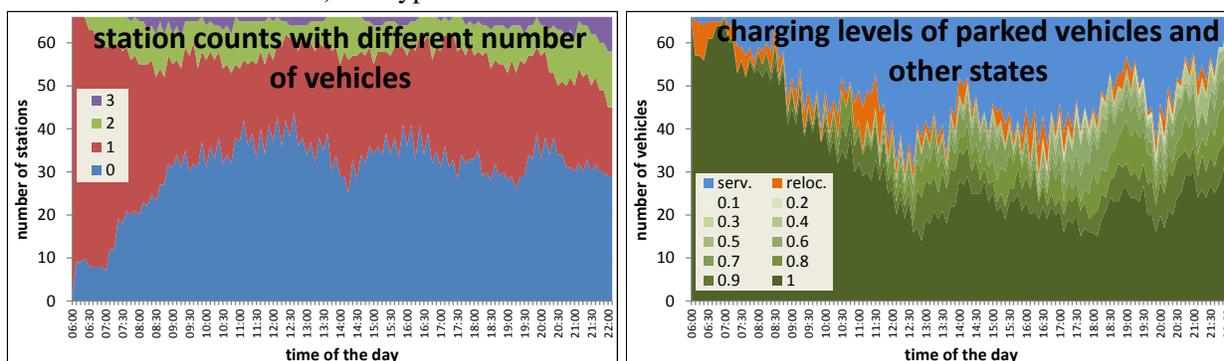


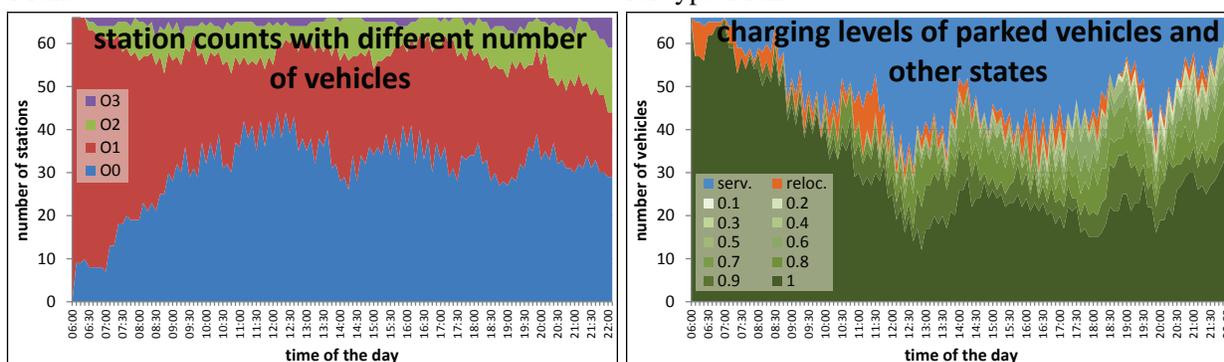
FIGURE 5: Chart showing movements of vehicles and relocation routes for the case with 300 rental requests.



(a) Performance measures if the number of person- (b) Performance measures if the type 1 run has to nel per shift (see x -axis) was limited for type 1 (dark serve a limited number of demand (see x -axis) in- coloured lines and columns) and type 2 runs. stead of all demand.



(c) Stations with different number of vehicles for type (d) Number of vehicles with different charging levels 1 run. for type 1 run.



(e) Stations with different number of vehicles for type (f) Number of vehicles with different charging levels 2 run. for type 2 run.

FIGURE 6: Various performance measures for limited number of personnel (Figs 6a) and limited number of demand served (Fig. 6b) and, comparison of the states of vehicles and stations for two run types for the same selected case with 300 demand (Figs. 6c-6f).

iteratively. In this figure, the horizontal axis shows the number of rental requests that needs to be served among 300 demand. Columns show the number of personnel from different shifts. The purple line shows the number of relocation per personnel. Both the number of personnel and the relocation per personnel are shown on the left vertical axis. The blue line shows the demand per personnel with the values on the right vertical axis.

It is evident that the demand per personnel converges to a specific value. Although, this converged value is highly related to the system parameters, it is worth noting such a value may exist for carsharing systems. This ratio could help in carsharing systems to forecast the number of personnel needed for any demand level.

In Figs. 6c-6f, various system states during the day are illustrated for the two run types. The same 300-demand set is used for both cases. In all figures, x -axes show the time of the day during simulation. y -axes represents either the number of vehicles (Figs. 6d and 6f) or the number of stations (Fig. 6c and 6e).

Figs. 6c and 6e show the stations with different number of parked stations for different times of the day. As it can be seen, in Figs. 6c and 6e, simulations start with one vehicles at each station. Figs. 6d and 6f show the number of vehicles with different battery charging levels parked at any stations. Vehicles that are not parked (under service or under relocation) are also shown in these figures. Note that the two different strategies, even if they have different performance in terms of the objective function, follow similar dynamic trends.

CONCLUSIONS

In this paper, we developed an operational framework to deal with the operational decisions of one-way (electric) carsharing systems with reservations. We developed a clustering algorithm to decrease the computational complexity of the model, an event-based simulator to test the feasibility of charging levels during the optimization procedure and MILP models to handle the operational decisions. The three modules are interacting iteratively to guarantee close-to-optimal feasible solutions. Experimental results showed that, the developed framework is efficient. We have tested two types of runs: Run type 1 is with full information about the system, as it might happen in the case of reservations in advance. In run type 2, requests are appearing one per time and the system does not know the future demand during optimization. We also tested various demand levels to see the performance differences and applicability range for different settings.

Experimental results showed the importance of efficient algorithms for relocation operations. Comparison between two run types also showed that especially in congested systems, forecasting the future demand is quite important to utilize system resources efficiently. To serve higher number of requests without increasing system resources, requests should be diverted with other tools (e.g. pricing).

Ongoing research tries to optimize the initial spatial distribution of vehicles at the beginning of each day (instead of one vehicle per station). It is expected that with such a change, the performance of the system will improve and relocation costs will decrease. A field test to investigate the optimization results in real conditions is also under preparation. Future work is looking to expand these models for systems without reservations.

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