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- A filtration technique that permits the use of the FCVAR model for making inference in systems with I(0) and I(d) variables.
- This technique yields more precise model estimates and superior out-of-sample forecasts for the I(0) variable.
- Results are demonstrated using Monte Carlo simulations.

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Modelling Systems with a Mixture of $I(d)$ and $I(0)$ Variables Using the Fractionally Co-integrated VAR Model

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Abstract

We propose a filtration technique for making inference in systems with $I(0)$ and $I(d)$ variables using the fractionally co-integrated vector autoregressive (FCVAR) model with long memory in the co-integrating residuals. Superior predictions for the $I(0)$ variable are demonstrated using simulations.

Keywords: Long memory; Fractional co-integration; Model predictability.
JEL Classification: C5; C15; C22.

1. Introduction

The fractionally co-integrated vector autoregressive (FCVAR) model was introduced by Johansen (2008) and further developed by Johansen and Nielsen (2012). In serving as a direct model of fractional co-integration, it provides a central tool for the analysis of long-run equilibrium relationships among the $I(d)$ variables. Compared with traditional $I(1)/I(0)$ co-integration,

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fractional co-integration allows linear combinations of $I(d)$ processes to give $I(d-b)$ processes with $d \geq b > 0$ and with d and/or b as fractional numbers.

In addition to the analysis of long-run relationships among the $I(d)$ variables, the FCVAR has also been employed in several studies involving a mixture of $I(d)$ and $I(0)$ variables, see Bollerslev et al. (2013) and Chen, Chiang, and Karl (2018). In their work, the estimation of the FCVAR is simplified by letting $d = b$; i.e. no memory in the co-integrating residuals. According to Definition 2 in Johansen (2008), the FCVAR allows for variation in the integration order of the variables within the system. Consequently, the inclusion of the $I(0)$ variable is natural in the FCVAR, which is similar to the coexistence of the $I(1)$ and $I(0)$ variables in the VECM. However, the case of $d > b$ poses a challenge for the analysis of the FCVAR as the fractional differencing operator Δ^{d-b} is applied, not only to the real $I(d-b)$ co-integrating vectors, but also to the $I(0)$ variable serving as pseudo co-integrating vector. This gives rise to the anti-persistence of the latter. As a result, under the FCVAR model, the representation of the $I(0)$ variable is found to be $I(d-b)$, which may lead to biased parameter estimates.

This paper proposes a filtering procedure for the pre-application of the FCVAR model in a mixture of $I(d)$ and $I(0)$ variables to evade the potential bias arising from the over-differencing of the pseudo co-integrating vector when $d > b$. Specifically, the fractional differencing operator (Δ^{d-b}) is applied to the $I(d)$ variables within the system prior to the estimation of the FCVAR model. This procedure does not alter the representation theorem and the calculation of maximum likelihood estimators of the FCVAR. With this adjustment, the $I(0)$ variable is shown to be correctly represented as an $I(0)$ process.

We illustrate the usefulness of our technique using Monte Carlo simulations containing both stationary and non-stationary fractional co-integration. Our findings show that the pre-filtration tends to reduce the observed bias in the estimates of parameters d, b and co-integrating vectors and that the gains are more evident with the gap between d and b. In the out-of-sample (OOS) forecasts for the $I(0)$ variable, the filtration leads to better predictions across various horizons

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where the forecasting gains tend to be significant over long horizons.

The rest of this paper is organized as follows. Section 2 presents the FCVAR specifications and the proposed filtering procedure. The Monte Carlo study is outlined in Section 3. Section 4 concludes.

2. The Model

The FCVAR model is defined as

$$\Delta^d X_t = \alpha\beta'\Delta^{d-b} L_b X_t + \sum_{c=1}^{k} \Gamma_c \Delta^d L_b^c X_t + \varepsilon_t \tag{1}$$

where $X_t \in I(d)$ contains p elements and ε_t is p-dimensional $i.i.d.(0,\Omega)$. Let $L_b = 1 - \Delta^b$ be the fractional lag operator and Δ^d be the fractional difference operator where $\Delta^d = (1-L)^d$. The error correction term is denoted by $\beta'\Delta^{d-b}X_t$, where β is a $(p \times r)$ matrix consisting of r co-integrating vectors and r is the so-called co-integration rank. The linear combination $\beta' X_t$ is integrated of order $(d-b)$ with $d \geq b > 0$. The matrix α is of order $(p \times r)$ and contains parameters representing the speed of adjustment towards long-run equilibrium. The short-run dynamics are measured by the lag coefficients $(\Gamma_1, \ldots, \Gamma_k)$. As suggested by Johansen (2008), the FCVAR in equation (1) does not require that all components of X_t exhibit the same order of integration. As a result, the representation theorem and the properties of maximum likelihood estimators (MLE) of the FCVAR remain unchanged when the $I(0)$ variables are introduced into the system of fractional variables.

The following section gives an outline of the problem that may arise when the FCVAR in equation (1) is applied to a system containing $I(d)$ and $I(0)$ variables. We assume that there are two $I(d)$ variables, X_{1t}, X_{2t}, that are fractionally co-integrated of order b and one $I(0)$ variable X_{3t} in the system X_t, i.e. $X_t = (X_{1t}, X_{2t}, X_{3t})'$. As a standard method employed in the literature treating an $I(0)$ variable in the VECM, we adopt the idea of a 'pseudo' co-integrating relation.

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Specifically, we involve the extra co-integration vector as a unit vector with unity in the position corresponding to the $I(0)$ variable and zeros elsewhere. We then construct

$$\alpha = \begin{pmatrix} \alpha_1 & \delta_1 \\ \alpha_2 & \delta_2 \\ \alpha_3 & \delta_3 \end{pmatrix} \quad \beta' = \begin{pmatrix} 1 & \beta_1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{2}$$

The FCVAR in equation (1) is no longer appropriate for modelling a system containing a mixture of $I(d)$ and $I(0)$ variables when $d > b$, in which case the term $\beta'\Delta^{d-b}X_t$ contains the anti-persistent error correction term that arises from the presence of the $I(0)$ variable in X_t. The mis-specification problem can also be seen by considering the representation theorem as follows.

Given α and β as defined in equation (2), we obtain

$$\beta_\perp = \begin{pmatrix} -\beta_1 \\ 1 \\ 0 \end{pmatrix} \text{ and } \alpha_\perp = \begin{pmatrix} 1 \\ \frac{\alpha_3\delta_1-\alpha_1\delta_3}{\alpha_2\delta_3-\alpha_3\delta_2} \\ \frac{\alpha_1\delta_2-\alpha_2\delta_1}{\alpha_2\delta_3-\alpha_3\delta_2} \end{pmatrix} \tag{3}$$

With $\Gamma = I - \sum_{c=1}^{k} \Gamma_c$, the matrix $C = \beta_\perp(\alpha'_\perp\Gamma\beta_\perp)^{-1}\alpha'_\perp$ can be computed as

$$C = (\alpha'_\perp\Gamma\beta_\perp)^{-1} \begin{pmatrix} -\beta_1 & -\beta_1\frac{\alpha_3\delta_1-\alpha_1\delta_3}{\alpha_2\delta_3-\alpha_3\delta_2} & -\beta_1\frac{\alpha_1\delta_2-\alpha_2\delta_1}{\alpha_2\delta_3-\alpha_3\delta_2} \\ 1 & \frac{\alpha_3\delta_1-\alpha_1\delta_3}{\alpha_2\delta_3-\alpha_3\delta_2} & \frac{\alpha_1\delta_2-\alpha_2\delta_1}{\alpha_2\delta_3-\alpha_3\delta_2} \\ 0 & 0 & 0 \end{pmatrix} \tag{4}$$

which contains only zeros in the last row corresponding to the $I(0)$ variable X_{3t}. Following the work of Johansen and Nielsen (2012), the FCVAR in equation (1) has the solution

$$X_t = C\Delta_+^{-d}\varepsilon_t + \Delta_+^{-(d-b)}Y_t^+ + \mu_t \tag{5}$$

for $d \geq 1/2$ where the operator Δ_+^{-d} is used to define a nonstationary process and Y_t is fractional of order zero. The solution of the FCVAR model for the $I(0)$ X_{3t} then reduces to

$$X_{3t}^{FCVAR} = e3'\Delta_+^{-(d-b)}Y_t^+ + e3'\mu_t \tag{6}$$

where $e3' = (0, 0, 1)$. It is clear that the X_{3t}^{FCVAR} is integrated of order $(d-b)$, which erroneously

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exhibits long memory if $d > b$ due to the mis-specifications. This problem remains in the case of $d < 1/2$ where the solution of the FCVAR becomes $X_t = C\Delta^{-d}\varepsilon_t + \Delta^{-(d-b)}Y_t$.

To adjust for this problem, we apply the fractional differencing operator Δ^{d-b} to each of the $I(d)$ variables in X_t and construct a new system $X_t^* = (\Delta^{d-b}X_{1t}, \Delta^{d-b}X_{2t}, X_{3t})'$ in the FCVAR as follows

$$\Delta^b X_t^* = \alpha\beta' L_b X_t^* + \sum_{c=1}^{k} \Gamma_c \Delta^b L_b^c X_t^* + \varepsilon_t \tag{7}$$

Here, the model above differs from the FCVAR in (1) only in the way that the fractional $I(d)$ variables have been transformed to $I(b)$ variables. On this basis, the Johansen representation theorem must still hold for the FCVAR with the pre-filtering procedure in (7). We can then demonstrate that, with the adjustments made to the input vector X_t^*, X_{3t} is correctly represented as the following $I(0)$ process

$$X_{3t}^{FCVAR^*} = e3'\Delta_+^{-(b-b)}Y_t^+ + e3'\mu_t \tag{8}$$

3. Simulation Study

To illustrate the gains from the adoption of the filtering procedure, we conduct a simulation study that compares the FCVAR with and without the filtration in terms of the model fit, parameter estimation and predictive power.

3.1. In-sample estimation

We generate X_{1t} and X_{2t} that are fractionally co-integrated of order $CI(d, b)$ and one $I(0)$ process X_{3t} from the FCVAR without including short-run dynamics

$$\begin{aligned} X_{1t} &= \alpha_1\Delta^{-b}L_b(X_{1t} + \beta_1 X_{2t}) + \delta_1\Delta^{-d}L_b X_{3t} + \Delta^{-d}\varepsilon_{1t} \\ X_{2t} &= \alpha_2\Delta^{-b}L_b(X_{1t} + \beta_1 X_{2t}) + \delta_2\Delta^{-d}L_b X_{3t} + \Delta^{-d}\varepsilon_{2t} \\ X_{3t} &= \alpha_3\Delta^{d-b}L_b(X_{1t} + \beta_1 X_{2t}) + \delta_3 L_b X_{3t} + \varepsilon_{3t} \end{aligned} \tag{9}$$

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where ε_{1t}, ε_{2t} and ε_{3t} are randomly created from a trivariate normal distribution with mean 0, variance 1 and correlation equal to 0. The Monte Carlo simulation is based on 5000 replications, with sample sizes $T = (2500, 1000, 500)$. We vary d from 0.4 to 0.8, covering the range commonly seen in empirical studies and consider several cases with the gap between d and b from 0.1 to 0.6. The case of $b = 0.5$ is omitted in our analysis following Assumption 4 in Johansen and Nielsen (2012). Both stationary ($d - b < 1/2$) and non-stationary ($d - b > 1/2$) co-integrating relations are included in our simulation based on the recent extension of the FCVAR made in Johansen and Nielsen (2018). In addition, we let $\beta_1 = -1$ and $\alpha = \begin{pmatrix} -0.5 & -0.1 \\ 0.5 & -0.3 \\ 0.01 & -0.2 \end{pmatrix}$. By setting rank equal to 2, we estimate the FCVAR with $X_t = (X_{1t}, X_{2t}, X_{3t})'$ and the FCVAR with $X_t^* = (\Delta^{d-b} X_{1t}, \Delta^{d-b} X_{2t}, X_{3t})'$. We take the natural normalization of the β matrix as

$$\beta = \begin{pmatrix} 1 & \beta_1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

and report the results of the model estimates in Table 1.

We show that estimates of the model parameters become more precise as the sample size increases, which is in line with the asymptotic results in Johansen and Nielsen (2012). Under the same fractional integration order d, the precision in the estimates improves as b increases. Notably, across different sample sizes, the MLE of d is more precise than b. On the other hand, estimates of β are more dispersed. Results for the estimates of α are not reported for brevity since $\widehat{\alpha}$ is a function of $\widehat{d}$, $\widehat{b}$ and $\widehat{\beta}$ and so is heavily affected by the estimation uncertainty present in the earlier steps. Turning to the comparison between the FCVAR and the pre-filtered FCVAR, we show that the latter achieves a better in-sample fit, i.e. lower BIC, in all cases considered and tends to produce more precise estimates.

The improvements made by adopting the pre-filtering technique are outlined in Table 2, where the gains are measured by the reduction in the values of MSE and BIC of the pre-filtered FCVAR relative to those of the FCVAR. For cases where the gap between d and b is within [0.3, 0.6], we

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	FCVAR								Pre-filtered FCVAR							
d	0.4	0.4	0.5	0.5	0.6	0.6	0.8	0.8	0.4	0.4	0.5	0.5	0.6	0.6	0.8	0.8
b	0.2	0.3	0.3	0.4	0.2	0.3	0.2	0.3	0.2	0.3	0.3	0.4	0.2	0.3	0.2	0.3
T=2500																
MSE_ $\widehat{d}$ (1.0E-3)	[illegible]	1.333	[illegible]	0.414	1.555	0.354	3.894	2.008	1.502	1.429	0.430	0.461	0.269	0.263	0.288	0.237
MSE_ $\widehat{b}$	0.031	[illegible]	[illegible]	[illegible]	0.094	0.031	0.276	0.103	0.004	0.003	0.004	0.003	0.006	0.005	0.007	0.004
MSE_ $\widehat{\beta}_1$	0.114	0.022	[illegible]	[illegible]	[illegible]	0.028	5.799	0.027	0.053	0.017	0.020	0.007	0.025	0.010	0.022	0.006
BIC	21369	21353	21355	21339	[illegible]	[illegible]	21371	21401	21358	21349	21337	21331	21328	21320	21314	21313
T=1000																
MSE_ $\widehat{d}$ (1.0E-3)	4.799	4.046	1.096	1.269	3.093	1.321	4.203	[illegible]	4.507	3.929	1.406	1.336	1.780	0.695	1.697	0.674
MSE_ $\widehat{b}$	0.046	0.020	0.025	0.009	0.114	0.038	0.292	[illegible]	[illegible]	0.005	0.005	0.005	0.018	0.012	0.021	0.010
MSE_ $\widehat{\beta}_1$	0.419	0.061	0.084	0.023	108.480	0.165	524.680	2.673	[illegible]	[illegible]	[illegible]	0.018	0.249	0.036	0.201	0.026
BIC	8565	8558	8562	8550	8554	8559	8554	8572	8561	[illegible]	8555	[illegible]	8541	8543	8535	8538
T=500																
MSE_ $\widehat{d}$ (1.0E-3)	10.815	8.201	3.484	2.883	4.761	3.146	4.812	5.540	10.139	6.884	2.988	2.883	[illegible]	[illegible]	[illegible]	1.926
MSE_ $\widehat{b}$	0.062	0.035	0.041	0.018	0.139	0.055	0.320	0.157	0.046	0.007	0.007	0.006	0.048	0.028	[illegible]	[illegible]
MSE_ $\widehat{\beta}_1$	218.350	0.087	1.052	0.037	241.780	8.589	282.960	592.230	1.180	0.060	0.103	0.030	1.008	0.162	1.517	0.146
BIC	4293	4291	4289	4285	4282	4287	4280	4286	4291	4291	4286	4284	4277	4279	4272	4271

Table 1
Monte Carlo Simulation Results. This table reports the MSE of the model estimates $\widehat{\lambda} = (\widehat{d}, \widehat{b}, \widehat{\beta}_1)$ and the BIC under both the standard FCVAR and the FCVAR coupled with the pre-filtering procedure. The Monte Carlo experiment is based on 5000 replications, with T=(2500, 1000, 500). The values of d and b are provided in the table.

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observe greater gains of the pre-filtered FCVAR in terms of the estimation precision in parameters b and β_1 as the difference between d and b increases. As for cases with the smaller gap between d and b, gains of the pre-filtered FCVAR remain for the estimates of b and β_1 as well as with the in-sample fit but are absent for $\widehat{d}$ under several scenarios. For various sample sizes under analysis, our Monte Carlo results show that the superiority given by the use of the filtration technique is more evident as the gap between d and b grows.

3.2. Out-of-sample forecasts

Better performances of the FCVAR relative to the conventional VAR and AR models in predicting $I(0)$ market returns are well documented in the work of Bollerslev et al. (2013) and Chen, Chiang, and Karl (2018). In our analysis, we further undertake OOS forecasting exercises to demonstrate the superiority of the FCVAR using the filtered long-memory series in predicting the $I(0)$ variable X_{3t}.

The forecasts are based on re-estimating the model parameters for each day with a fixed length rolling window containing the previous $T/2$ days. We consider different forecasting horizons for the $I(0)$ variable by replacing X_{3t} with $\frac{1}{h}\sum_{j=1}^{h} X_{3t+j}$ in the FCVAR (1) and pre-filtered FCVAR (7), where h is set as 1, 5 and 22. Table 3 reports the average relative MSE of the predictions for the $I(0)$ variable X_{3t} from the two models, and this is computed such that values less than one favor the pre-filtered FCVAR model forecasts. Similar to the in-sample analysis, the simulation results are generated based on 5000 replications, in which cases the Diebold and Mariano (DM) test is employed to examine the equal predictive ability. The results in Table 3 clearly favor the pre-filtered FCVAR model forecasts over different sample sizes. Specifically, the pre-filtered FCVAR exerts more superior predictive performance over longer horizons, i.e. $h = 5$ and 22, where the gains in most replications undertaken are significant under the DM test.

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		Gains (%)							
	d	0.4	0.4	0.5	0.5	0.6	0.6	0.8	0.8
	b	0.2	0.3	0.3	0.4	0.2	0.3	0.2	0.3
T=2500	ΔMSE_$\widehat{d}$	-23.169	-7.148	-44.988	-11.405	82.724	25.497	92.602	88.210
	ΔMSE_$\widehat{b}$	55.707	70.739	77.902	46.729	93.183	84.691	97.430	96.230
	ΔMSE_$\widehat{\beta}_1$	53.212	20.755	49.175	25.267	98.958	64.688	99.620	76.910
	ΔBIC	0.051	0.019	0.084	0.037	0.164	0.178	0.267	0.411
T=1000	ΔMSE_$\widehat{d}$	6.072	2.880	-23.361	-5.305	42.445	47.423	59.621	82.688
	ΔMSE_$\widehat{b}$	55.759	73.187	79.120	51.729	83.932	68.374	92.936	91.866
	ΔMSE_$\widehat{\beta}_1$	42.471	45.746	42.144	21.390	99.711	78.484	99.962	99.044
	ΔBIC	0.043	0.016	0.086	0.034	0.145	0.186	0.224	0.388
T=500	ΔMSE_$\widehat{d}$	6.251	16.059	14.231	0.000	19.391	33.810	17.749	65.241
	ΔMSE_$\widehat{b}$	26.521	79.511	83.614	67.367	65.564	50.215	86.660	84.967
	ΔMSE_$\widehat{\beta}_1$	99.460	31.776	90.225	17.731	99.583	98.115	99.464	99.975
	ΔBIC	0.047	0.000	0.070	0.023	0.114	0.173	0.199	0.343

Table 2
Percentage Gains of the MSE and BIC. The gains of the FCVAR implemented with the pre-filtering procedure are computed as ΔMSE_$\widehat{\lambda}$ =[MSE($\widehat{\lambda}_{\text{FCVAR}}$)-MSE($\widehat{\lambda}_{\text{pre-filtered FCVAR}}$)]/MSE($\widehat{\lambda}_{\text{FCVAR}}$) where $\widehat{\lambda} = (\widehat{d}, \widehat{b}, \widehat{\beta}_1)$ and
$\Delta BIC = [BIC_{\text{FCVAR}} - BIC_{\text{pre-filtered FCVAR}}] / BIC_{\text{FCVAR}}$

Relative MSE								
d	0.4	0.4	0.5	0.5	0.6	0.6	0.8	0.8
b	0.2	0.3	0.3	0.4	0.2	0.3	0.2	0.3
T=2500								
h=1	0.994	0.998	0.998	0.999	0.989	0.998	0.999	0.998
h=5	0.857	[illegible]	0.886	0.898	0.903	0.902	0.909	0.952
h=22	0.913	0.935	0.946	0.960	0.947	0.961	0.978	0.976
T=1000								
h=1	0.998	0.990	0.994	0.998	0.990	0.989	0.990	0.996
h=5	0.875	0.880	0.914	0.936	0.889	0.924	0.960	0.985
h=22	0.915	0.900	0.912	0.929	0.961	0.979	0.986	0.978
T=500								
h=1	0.995	0.994	0.996	0.993	0.993	0.995	0.989	0.992
h=5	0.826	0.920	0.908	0.945	0.917	0.981	0.928	0.953
h=22	0.964	0.929	0.978	0.945	0.930	0.995	0.971	0.976

Table 3
Monte Carlo Simulation Results. This table reports the out-of-sample MSE for the I(0) variable under the FCVAR using the pre-filtering relative to the MSE of the standard FCVAR model. The forecast are based on re-estimating the parameters of the different models each day with a fixed length Rolling Window (RW) with the size $\frac{1}{2}T$.

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4. Conclusion

We propose the use of a pre-filtering technique that allows for better inference of the fractionally co-integrated VAR (FCVAR) of Johansen (2008) for modelling systems with $I(0)$ and $I(d)$ variables, where there exists long memory in the co-integrating residuals. The problem occurring particularly in the use of the standard FCVAR with $I(0)$ and $I(d)$ variables is associated with the anti-persistent error correction term when $d > b$, which brings fractional property to the representation for the $I(0)$ variable. Using the FCVAR with the pre-filtering procedure allows for a correct representation of the dynamics underlying the $I(0)$ process. Our Monte Carlo simulations show that this technique generally results in more precise model estimates and better out-of-sample predictions of the FCVAR for the $I(0)$ variable. The gains are realized for various sample sizes and combinations of d and b.

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