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ON ENVIRONMENTAL CONTOURS FOR MARINE AND COASTAL DESIGN

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ABSTRACT

Environmental contours are used in structural reliability analysis of marine and coastal structures as an approximate means to locate the boundary of the distribution of environmental variables, and to identify environmental conditions giving rise to extreme structural loads and responses. There are different approaches to estimating environmental contours, some directly linked to methods of structural reliability. Each contouring approach has its pros and cons. Although procedures for applying contours in design have been reported in articles and standards, there is still ambiguity about detail, and the practitioner has considerable flexibility in applying contours. It is not always clear how to estimate environmental contours well. Over four years, DNV-GL, Shell, the University of Oslo and HR Wallingford worked together to review current practice regarding the use of design contours. In this paper, we present a summary of our findings. We overview the motivations for different approaches to contours, and their resulting characteristics. Using different marine applications, we also explore the various sources of uncertainty present, their impact on contour estimates and the estimation of extreme environmental loads and responses.

INTRODUCTION

Different methods are used to establish design criteria for environmental loading and responses of marine structures and coastal facilities. Rigorous comparison of some approaches has been reported in the literature, but there is still uncertainty in the user community regarding the relative merits of different approaches. Within the marine industry, estimation of a joint metocean description has been considered for more than thirty years. It was shown that typically, environmental forces on marine structures may be reduced by 5% to 40% by accounting for the lack of complete dependence between metocean variables (wind, wave, current, etc.) traditionally used in design (e.g. [1], [2]). Development of reliability methods (e.g. [3]) and their implementation by some parts of the industry in the 1980s brought joint probabilities into focus: they are required for a consistent treatment of the loading in Level III reliability analysis and for assessment of the relative importance of various metocean variables during extreme load and response conditions, fatigue damage and at failure.

The environmental contour defines a set of extreme sea state conditions, and can be used to approximate extreme values of long-term structural response extremes by considering only a few short-term metocean conditions. Environmental contours are ap-

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pealing since they can be specified for a given metocean environment independently of any structure; they can also be linked to a well-established approach to structural design, familiar to practitioners. To establish them, joint probabilities of metocean parameters, historically in the metocean community in the form of tables, are needed. The idea behind the method is to define contours in the metocean space (for example, H_S, T_P) along which extreme responses with given return period should lay. It is a simplified and approximate method compared with full long-term response analysis but requires less computational effort.

Structural responses for combinations of environmental conditions lying on the contour can be used to estimate the extreme response due to a sea state with the same return period as the contour. Importantly, only combinations of metocean parameters lying on the contour need be considered. With additional a priori knowledge of the response, it is possible to limit the interval of the contour over which to evaluate structural response, substantially reducing the computational effort for calculating extreme response. An underlying assumption is that the extreme N-year response is governed by sea state conditions on the N-year environmental contour.

Some approaches to estimation of environmental contours (for example IFORM, [4]) make additional explicit assumptions regarding the nature of structural failure surfaces expressed in terms of (potentially transformed) environmental variables. When these assumptions are valid, statements regarding the relative magnitude of the exceedance probability of the N-year environmental contour and the N-year structural failure probability can be made more reasonably. However it is not always clear that the additional assumptions are satisfied for a given application.

Using the environmental contour, an estimate of the extreme response is obtained by searching along the contour for the condition giving maximum characteristic extreme response. The contour method is affected by uncertainties related to metocean data and adopted joint models and has its own limitations which are pointed out by [5] and [6]. It will tend to underestimate extreme response levels because it neglects short term variability of response between different realisations of sea states. Both standards recommend approaches based on [7] to account for this, including (a) increasing the return period corresponding to the contour, and hence *inflating* the environmental contours; (b) replacing the stochastic response by a fixed fractile level higher than the median value; or (c) applying multipliers of the median extreme response estimates, to introduce more conservatism.

In the coastal engineering community, contours of joint exceedance probability of environmental variables are estimated using the JPC method (a) to find design events that form the boundary conditions for numerical and physical models for the purposes of structural design; and (b) to estimate return values of overtopping and overflow rates corresponding to some return period for use in flood mapping and risk analysis ([8, 9]). A series of combinations of values of environmental variables from the contour are tested in order to find the *worst case* value of the response. This worst case value is then assumed to have the same return period as the return period associated with the environmental contour. Since again, without further assumptions, there is no link between environmental contour and structural response, there are obvious short-comings to this approach, which are well recognised (e.g. [10]).

There are a number of more fundamental reviews of environmental contour methods, including excellent recent work by [11]. The performance of different environmental contour methods has also been investigated in several studies, including work by some of the current authors (including [10, 12–14]). After consideration of the fundamental mathematical differences between different contour methods, it is unreasonable in general to expect to find any consistent trends in comparisons of contour methods across different applications. The characteristics of different environmental contour methods must be assessed on an application-by-application basis.

Over the past four years, DNV-GL, Shell, the University of Oslo and HR Wallingford have been working together (on a project called ECSADES: "Environmental Contours for SAfe DEsign of Ships and other marine structures") to review current practice regarding the use of design contours. In the review article [15] we detail our findings. We overview the motivations for different approaches to contours, and their resulting characteristics. Using different marine applications, we also explore the various sources of uncertainty present, their impact on contour estimates and the estimation of extreme environmental loads and responses. We also include the findings of an informal survey of the offshore and coastal engineering community regarding the use of environmental contours.

The objectives of the current conference paper are as follows: (a) to summarise the main findings of the full study reported in [15]; (b) to illustrate the construction of three different environmental contours for a typical North Sea wave environment; and (c) to make recommendations concerning when and how it is reasonable to use environmental contours in design.

SUMMARY OF CONCEPTS

Estimation of return values for a single environmental variable X_1 is relatively straightforward and has been studied extensively. Unfortunately, a "joint" return value for two or more variables $(X_1, X_2, ..., X_p)$ cannot be uniquely defined (e.g. [15, 16]). To specify design values for more than one variable rationally, we need to understand and exploit the joint distribution of the variables. This leads naturally to consideration of joint probabilities, of environmental contours, and of structure variables (such as structural response to environmental loading) which capture the important joint characteristics of (a multivariate) environment in terms of a single "structure" or response variable. Typically, estimation of the joint distribution of random variables is thought of in two parts: (a) marginal characterisation (for each X_j individually) and subsequent transformation to standard marginal scale (corresponding to random variable Z_j say), and then (b) characterisation of the joint dependence of $Z_1, Z_2, ... Z_p$. Estimating the marginal distribution of each X_j in turn is already challenging, especially with respect to specifying thresholds for peaks-over-threshold analysis, and to including the effects of covariates such as direction and season carefully (e.g. [17]). Quantifying uncertainties due to threshold uncertainty and covariate effects is essential for reliable modelling.

Estimating the joint distribution of environmental variables

Given a sample from the joint distribution of p (standardised) environmental variables $\underline{Z}$ $(= (Z_1, Z_2, ..., Z_p))$, a number of different models for the joint distribution have been reported in the literature. Models can be categorised as being parametric (adopting a functional form for the density of the joint distribution) or non-parametric (typically using kernels for density estimation).

One simple form of non-parametric density estimation is kernel density estimation, a typical kernel choice being the multivariate normal density. Kernel density models are suitable in general to describe the body of the joint distribution, and the choice of kernel (and kernel parameters) tends not to be too critical to estimate central characteristics. In contrast, kernel density models are not suitable to describe tails of distributions, since the tail (away from locations of data) is strongly influenced by the choice of kernel and kernel parameters; tail models motivated by extreme value considerations should be preferred. A popular choice of parametric description is a copula model, a multivariate probability distribution for $\underline{Z}$ for which the marginal probability distribution of each variable is uniform. There is a huge literature on copulas (e.g. [18], [19]), and there are many families of copulas (including the Gaussian and Archimedian), and some (so-called *max-stable* or inverted max-stable copulas) more suited to the descriptions of extreme environments (e.g. [20]). Another popular choice is a hierarchical model, which describes the joint distribution of variables in terms of marginal and conditional distributions, and can be visualised as a directed acyclic graph (e.g. [21]). H_S, T_P described in terms of the distribution of H_S and $T_P|H_S$ (e.g. [22]) is a classic example. For the joint tails of distributions, the conditional extremes model [23] is advantageous. This model is motivated by the existence of an asymptotic form for the limiting conditional distribution of one or more conditioned random variables (say $Z_2, Z_3, ..., Z_p$) given a large value of a conditioning variable (say Z_1) for a large class of distributions expressed on a particular standard marginal scale.

Estimating environmental contours

Joint modelling of environmental variables, and the construction of environmental contours, has a long history. [22, 24, 25] present joint models for environmental variables from which environmental contours can be estimated. [4] introduces the IFORM method, motivated by transformation of the joint distribution of environmental variables to standard multivariate Normal using the Rosenblatt transformation. Joint exceedance (e.g. [12]) and the so-called "direct sampling" method [26] estimate joint exceedance contours based on direct Monte Carlo simulation under a model for the joint distribution of environmental variables. Some approaches (e.g. [12, 24]) seek only to find contours which describe the distribution of environmental variables, either in terms of levels of constant exceedance probability or joint (or iso-) probability density. Other methods (e.g. [4, 26]), with extra assumptions, provide a direct link between the characteristics of the environmental contour and structural failure. [11, 13, 14] provide comparisons of different approaches to contour estimation. Other literature (e.g. [10, 27, 28]) discusses how joint models for the environment can be combined with simple models for structural responses given environment, to estimate the characteristics of response directly. The review article [15] gives a mathematical outline of the most popular contouring approaches.

There is no fundamental link between points on an environmental contour and structural response in general, and no reasonable expectation therefore that points on the N-year environmental contour should yield the N-year maximum response. The manner in which an environmental contour relates to extreme response depends on the specifics of the structure. However, for typical H_S-driven structures, empirical evidence suggests the responses generated from points along the environmental contour in (H_S, T_P)-space for a given return period are reasonable estimates of the actual maximum response corresponding to the same return period. In the presence of resonant response and non-extreme values of T_P, using points from the contour near the maximum H_S can be misleading, since the response is not completely H_S-dominated. It is critical therefore that the dominant environmental variables are included in the estimation of environmental contours. It is apparent from physical considerations that extreme occurrences of some structural responses should not coincide with those of extreme environmental variables; the N-year environmental contour is unlikely to provide any guidance regarding the N-year maximum response for such responses. We also note methods to adjust (or inflate) contours to calibrate or correct them for sources of bias (for estimation of extreme response and failure surface) including the effects short-term variability, violation of (marginal and dependence) modelling assumptions, uncertainty in parameter estimates, etc.

Nevertheless, methods such as IFORM and direct sampling are advantageous in that they relate the environment and structure by making assumptions about the characteristics of failure surfaces as a function of the environmental variables. Given these assumptions, it is possible to link the *exceedance probability* associated with a given environmental contour with structural failure probability. Although conditions from an N-year environmental contour need not result exactly in N-year responses, IFORM and direct sampling provide at least some understanding of how an N-year environmental contour is related to the N-year maximum response. Both IFORM and direct sampling approaches assume a linearised failure boundary. The basic difference between the approaches arises from the fact that linearisation for IFORM is performed in a transformed Gaussian space, and in direct sampling approach in the original space of environmental variables (e.g. [13, 14] and references therein). For both IFORM and direct sampling contours, the relationship established is between the *exceedance probability* associated with the contour (on some scale) and the probability of structural failure. This does not guarantee however that searching along an IFORM or direct sampling contour for return period N will isolate the key features of the N-year maximum response; the relative performance of IFORM and direct sampling in estimating extreme responses is application-dependent.

ILLUSTRATIVE APPLICATION

We now seek to quantify how well estimates of extreme responses (in a three-hour sea state, for a particular return period) on a contour compare with estimates obtained by direct simulation of the response. We perform our analysis for four responses, whose relationship to the environment is quantified entirely in terms of H_S and T_P, and is known to us. We are therefore able to simulate from the known distributions to estimate the correct characteristics of response, and hence to quantify the performance of contour-based estimates for maximum response.

Data

For simplicity, we define the environment in terms of a large historical sample of sea-state H_S and T_P for a typical northern North Sea environment for the period 1979-2013, from the NORA10-WAM hindcast ([29]). NORA10 (Norwegian Re-Analysis 10km grid) is a 58-year hindcast that has been developed by the Norwegian Meteorological Institute. It is a regional HIRLAM (atmosphere) and WAM Cycle-4 (wave) hindcast covering Northern European waters. The regional model uses wind and wave boundary conditions from the ERA-40 reanalysis (1958-2002) and is extended using the ERA-Interim reanalysis from 2002 onwards. NORA10 produces three-hourly wave and wind fields at 10km resolution. We isolate storm peak events using the procedure of [30]. We then estimate structural responses using *known* non-linear functions of environmental variables corresponding to each storm event.

A total of four responses R_1, R_2, R_3, R_4 were considered. Two responses correspond to output of a structural response simulator for overturning moment (R_1, for a typical fixed structure) and maximum pitch (R_2, for a floating structure), as a function of H_S and T_P for a three-hour sea state. These response simulators assume that the most probable value of maximum response in a sea state can be written as a closed form expression in terms of a number of sea state variables, including sea state H_S and T_P. The actual value of maximum response is then simulated from a Rayleigh distribution with the most probable maximum response as scale parameter. A further two synthetic responses are defined, which are simple deterministic functions of H_S and T_P, using the following equation

$$R_i = \frac{\alpha_i H_S}{(1+\beta_i(T_p - T_{p0,i})^2)} \text{ for } i = 3,4, \tag{1}$$

where $T_{p0,i}$ (in seconds) is the resonant peak period for response R_i. The values of $\{\alpha_i, \beta_i, T_{p0,i}\}$ are $\{2, 0.007, 7\}$ and $\{2, 0.005, 26\}$ for $i = 3,4$ respectively. These combinations of parameters were chosen to provide large responses at different neighbourhoods of the environmental space, and hence to correspond to different frontier intervals. The distribution of maximum 100-year response for synthetic responses R_3, R_4 was estimated by generating multiple environmental simulations corresponding to periods of 100 years, calculating response per sea state and storing only the maximum response observed and the values of H_S and T_P responsible for it. For responses R_1, R_2, `PPC` (see next sub-section) was used to extend the environmental model to include response; simulation under the model was then again used to accumulate the distribution of maximum response.

Contours

To construct an environmental contour, we require a statistical model for the environment. Here, we achieve this by means of a conditional extremes model for the historical sample, using a penalised piecewise constant (`PPC`) extreme value model [31] and software (outlined in [15]). The `PPC` extreme value model allows the estimation of non-stationary marginal and conditional extremes for peaks over threshold using a simple description of non-stationarity with respect to covariates in marginal and dependence models. We use the `PPC` model to estimate a number of the environmental contours and investigate their characteristics, in particular their relationship to extremes of structural response. Because of its recent popularity, we first consider the direct sampling contour. Later, we also incorporate the joint exceedance contour [12], and an isodensity contour from the conditional extremes model ([15], similar to the approach of [24] recommended in standard [5]).

Estimating the joint exceedance and direct sampling contours requires a (H_S, T_P) sample simulated under the environmental model. Once the contour is estimated, we identify a "frontier" interval of the contour which we think might be informative for estimation of response. Here, the “frontier” corresponds to the whole interval of the environmental contour lying close to pairs of H_S, T_P values present in the sample. Then we consider two possibilities: (a) that only a single combination of H_S and T_P corresponding to the maximum value of H_S on the contour is informative for estimating the maximum 100-year response, and (b) that the whole frontier interval is informative. Then, for both scenarios, we estimate the distribution of the maximum 100-year response.

RESULTS

We note that readers might find it useful to consult the companion paper [15] at this point for discussion of results of related studies.

For each response in turn, the mean maximum 100-year response is plotted in Figure 1, coloured by value. Also plotted in the figure are direct sampling contours corresponding to 40, 100 and 200 years. Note that for each response R_i, only combinations of H_S and T_P giving rise to at least one occurrence of maximum 100-year response appear in the figure. The values of synthetic responses R_3 and R_4 vary smoothly with H_S and T_P, whereas R_1 and R_2 exhibit more complicated dependence on the environment. Compared to R_4, large values of R_3 correspond to lower values of T_P. In all cases, the largest mean responses occur for combinations of H_S and T_P lying beyond the environmental contours. The extent to which the maximum response on the 100-year environmental contour agrees with the actual distribution of the maximum 100-year response from simulation can be assessed by comparing an estimate for the distribution of the true response against that evaluated for conditions on the contour, as illustrated in Figure 2. It shows kernel density estimates for maximum 100-year response estimated by direct simulation (in dashed blue; which can be regarded as “the truth”). The figure also shows corresponding kernel density estimates for response *from all combinations* of (H_S, T_P) lying on the contour frontier (scenario (b), shown in Figure 1), for a range of choices of return period N. The increase of most probable response with contour return period is clear for responses R_3 and R_4. In contrast, for responses R_1 and R_2, the difference in location of response density corresponding to the 40-, 100- and 200-year contours is smaller. The factors Δ by which the $\exp(-1)$ response of the 100-year environmental contour would need to be inflated to give the true $\exp(-1)$ 100-year response in each case, is close to unity.

We next perform a similar comparison of response distributions, this time using only *the single combination* of (H_S, T_P) on the contour with maximum H_S (that is, scenario (a)). The general characteristics of Figure 3 are similar to those of Figure 2. We observe also that the widths of estimated distributions for response are generally narrower in Figure 3.

We extend the study for responses R_1 (maximum overturning momemt) and R_2 (maximum pitch) to make a comparison of direct sampling contours, joint exceedance contours and isodensity contours. For brevity, these approaches are henceforth referred to as “direct sampling”, “joint exceedance” and “empirical density” respectively. Figure 4 shows minima and maxima of the maximum 100-year response from the same 1000 simulations used to generate Figure 1. The colour of each disc in the top row indicates the value of the *minimum* 100-year maximum response seen for that combination of H_S and T_P, using the same algorithm as for Figure 1 to identify near neighbours. The bottom row shows corresponding values of *maximum* 100-year maximum response. It is clear that there is considerable variability in response for a given pair of values for H_S and T_P. 100-year environmental contours from each of the direct sampling, joint exceedance and empirical density methods are also shown in the figure. As shown in Figures 2 and 3, response distributions are relatively well-estimated using points on the 100-year environmental contour. However it is interesting that the (yellow) areas in Figure 4 of largest values of maximum response (over 1000 realisations of 100 years, on the bottom row of the figure) also lay near to the frontier interval of the contours, but somewhat “inside” the contours in these cases. For synthetic response R_3 in Figure 1, the frontier interval is offset (to lower T_P) from that part of the environmental contour corresponding to largest H_S: focussing on an interval of the contour corresponding to largest

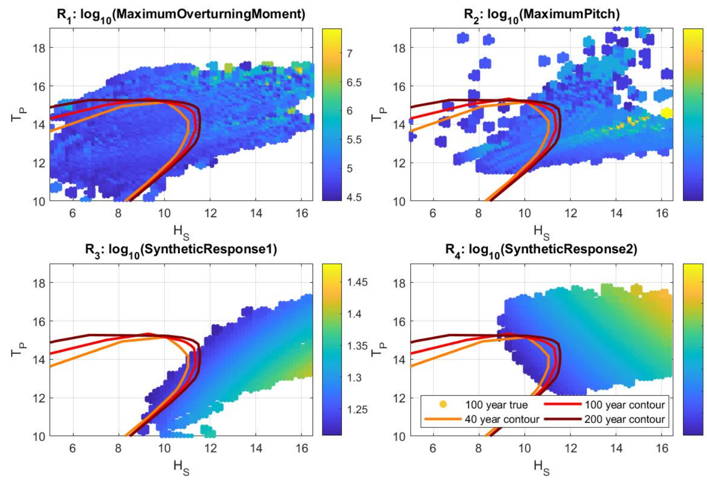

FIGURE 1. Mean 100-year maximum responses as a function of H_S and T_P estimated using 1000 realisations (of length 100 years) of H_S and T_P. Points are coloured by the local mean value of maximum response estimated on a lattice of values for H_S and T_P. Also shown are N-year (H_S, T_P) direct sampling environmental contours for different values of N; contours are coloured yellow to dark brown by return period, in order of $N = \{40, 100, 200\}$ years. Panels on top row correspond to historic responses R_1 (left) and R_2 (right); panels on bottom row correspond to synthetic responses R_3 (left) and R_4 (right).

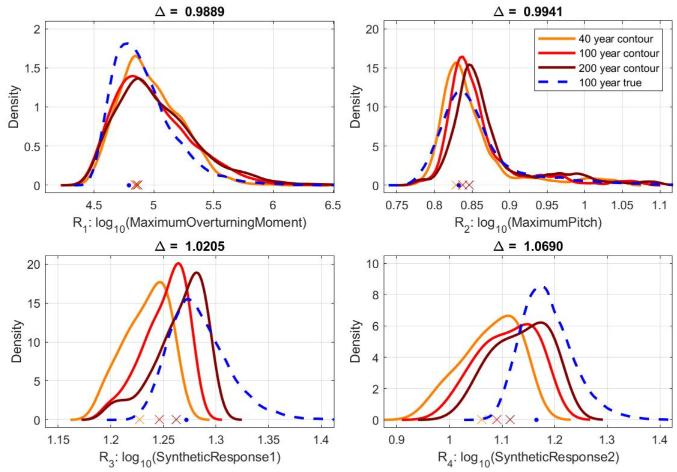

FIGURE 2. Kernel density estimates for maximum 100-year response. Estimates based on direct simulation of response are shown in dashed blue. Other density estimates (solid lines) are calculated from (H_S, T_P) combinations lying near the corresponding N-year direct sampling contour shown in Figure 1, for $N = \{40, 100, 200\}$. Coloured crosses indicate the location of the quantile of the response distribution with non-exceedance probability $\exp(-1)$ along each contour; the blue dot gives the corresponding $\exp(-1)$ "true" response from the blue curve. The factor Δ by which the $\exp(-1)$ response of the 100-year environmental contour would need to be inflated to give the true $\exp(-1)$ 100-year response is given in the title to each panel.

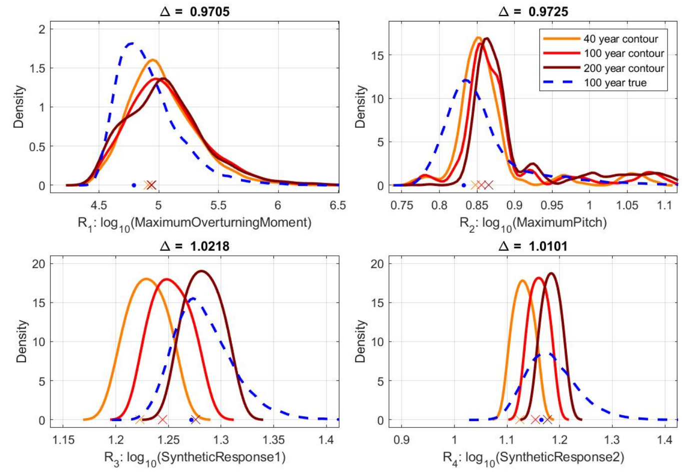

FIGURE 3. Kernel density estimates for maximum 100-year response. Estimates based on direct simulation of response are shown in dashed blue. Other density estimates (solid lines) are calculated from (H_S, T_P) combinations near the point on the corresponding N-year direct sampling contour (Figure 1) *corresponding to maximum* H_S, for $N = \{40, 100, 200\}$. Coloured crosses indicate the location of the quantile of the response distribution with non-exceedance probability $\exp(-1)$ along each contour; the blue dot gives the corresponding $\exp(-1)$ "true" response from the blue curve. The factor Δ by which the $\exp(-1)$ response of the 100-year environmental contour would need to be inflated to give the true $\exp(-1)$ 100-year response is given in the title to each panel.

H_S to estimate maximum response would seem particularly suspect in this case, regardless of the choice of contour method.

DISCUSSION

Environmental contours provide useful characterisations of the joint distribution of environmental variables. Some contour methods assume particular parametric forms for the (conditional) distributions of environmental variables; other methods generate convex contours on particular scales; other contour approaches are only defined on part of the domain of environmental variables. The usual motivation for applying a contour approach in ocean engineering is to find environmental conditions efficiently (for a return period of N years say) which will generate approximately the N-year maximum response. Environmental contours therefore provide a means of reducing the burden of running full long-term response analysis for a wide range of environmental conditions. Different types of environmental contours find favour based on their ability to estimate the N-year maximum response from the N-year environmental contour. An environmental contour is estimated with no regard whatsoever to structural details. Since environmental contours are independent of structural specifics, they can then be used in principle to study different structures in a given environment provided that the underlying assumptions linking environment and structure are not violated. There is no fundamental link between points on an environmental contour and structural response in general. Methods such as IFORM and direct sampling are advantageous in that they provide a bound for the probability of structural failure given the *exceedance probability* for the contour. However, this does not guarantee that searching along an IFORM or direct sampling contour for return period N years will isolate the key features of the N-year maximum response.

The findings of the review [15] can be summarised to give the practising engineer some guidance in deciding when and where to use an environmental contour approach. We emphasise that, in general, environmental contours can only provide approximations to extreme responses. The use of contour approaches may need to be supported in final design by full long-term analysis. We suggest that it is appropriate to consider using environmental contours when: (a) the dominant environmental variables and structural responses are all known; (b) the influence of short-term environmental variability is relatively small; (c) response-based analysis is not possible or feasible; and (d) design is at an outline stage. Once it has been decided that an en-

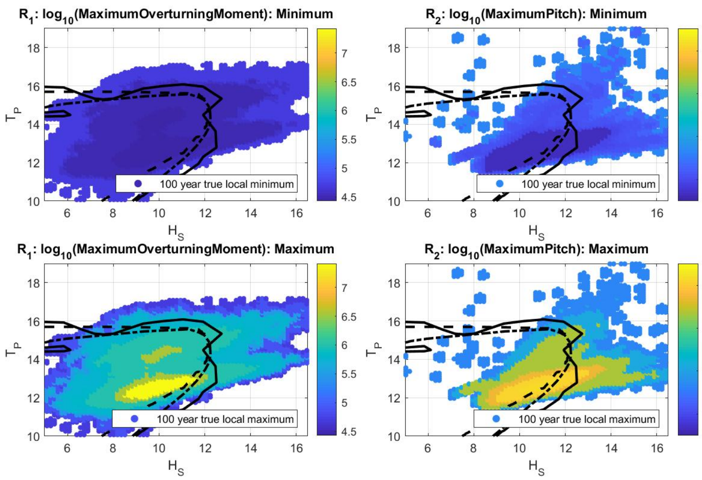

FIGURE 4. Minima (top row) and maxima (bottom row) of the 100-year maximum response, as a function of H_S and T_P estimated using 1000 realisations (of length 100 years) of H_S and T_P for responses R_1 (left) and R_2 (right). Points are coloured by the local minimum (top) or maximum (bottom) value of maximum response estimated on a lattice of values for H_S and T_P. Each panel also shows 100-year environmental contours from each of the direct sampling (black dot-dashed), joint exceedance (black dashed) and CE density (black solid) methods.

vironmental contour approach is suitable, the following are then recommended to ensure that the environmental contour method is used wisely: (e) ensure sufficient environmental data is available, so that statistical models can be well-estimated; (f) estimate more than one environmental model, and consider the sensitivity of the model to arbitrary modelling choices; (g) if unsure which contour to use, estimate more than one type and check their consistency; (h) choose multiple points from the environmental contours for response evaluation; and (i) consider other sources of uncertainty likely to impact contour estimation, such as neglect of covariate effects in environmental models.

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