“Multi-objective, multi-level, multi-stakeholder considerations for airport slot allocation”

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As a requirement for the module:

MSCI650 - MRes in Management Science Dissertation

Lancaster University Management School

2017 - 2018

Acknowledgements

I would first like to thank my supervisor Professor Konstantinos G. Zografos. His door was always open for me whenever I faced challenges with my research. He allowed this thesis to be my own work but kept me in the right track by providing valuable comments, ideas and directions.

I would also like to acknowledge Dr. Jamie Fairbrother for granting me access to his single airport optimisation framework and providing feedback on the algorithmic parts of my work. Without his input, the quality of this document would not be the same.

I owe a big thanks to my colleague Ramin Raeesi for our endless discussions on various research topics and ideas, and his insightful comments on the writing of this thesis.

Finally, I must express my profound gratitude to my parents Thanasis and Katerina and my girlfriend Maria for tolerating my stressful behaviour and encouraging me during the research and composition of this thesis. Through our talks, my sleepless nights and anxieties faded away. Thank you.

My studentship and the work reported in this thesis have been supported by UK's Engineering and Physical Sciences Research Council (EPSRC) and Lancaster University Management School through the Programme Grant EP/MO20258/1 "*Mathematical models and algorithms for allocating scarce airport resources (OR MASTER)*".

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Abstract

Airport slot scheduling has attracted the attention of researchers as a capacity management tool at congested airports. In an attempt to better grasp the demands of the problem, recent research work has employed multi-objective optimisation (MOO) approaches. However, the multiple stakeholders (e.g. airlines, coordinators, aviation and local authorities), their numerous or even conflicting objectives and the complexity of the decision-process (rules and slot priorities), have rendered the holistic modelling of the slot allocation problem a demanding and yet incomplete task. Through a rigorous review of the policy rules and the identification of the modelling gaps in the MOO airport slot allocation literature, this study aims to contribute to the field by proposing novel modelling considerations and solution approaches which accommodate additional characteristics of the real-world decision context. In detail, by building on previous research efforts, we propose a tri-objective slot allocation model (TOSAM), which jointly considers schedule delays, maximum displacement and demand-based fairness. We further proved that multi-level, game-theoretic-based considerations are suitable to capture the interactions among the different slot priorities, leading to enhanced airport slot schedules. To address the incurring complexity, we introduced the notion of inter-level tolerance and solved the TOSAM with systematic multi-level interactions for a medium sized airport. Our computational results suggest that by tolerating small objective function sacrifices at the upper decision levels, the resulting Pareto frontiers are of greater cardinality and quality in comparison to existing solution methods. Finally, we propose and illustrate two alternative bi-stage solution methods that exemplify the potential synergies between the MOO and multi-attribute decision-making literature.

Contents

1. Introduction

IATA's forecast has revealed that air passenger demand will double by 2036 reaching almost 7.8 billion passengers (IATA, 2018a). At the same time, airport capacity remains as one of the main hazards to European air connectivity that will lead to a supply-demand imbalance gap of more than 160 million passengers in 2040 (EUROCONTROL, 2018). In addition, the expansion of existing infrastructure is subject to geospatial, political and financial constraints, which render its implementation rather slow. Therefore, current airport capacity has to be optimally managed to minimise airline and passenger disruption caused by congestion. To do so, IATA has developed a comprehensive policy framework, which provides to congested airports a common tool to distribute airport capacity. This framework is expressed through the worldwide slot guidelines (WSG) as described in IATA (2018b). The current practice is mainly carried out by schedule coordinators, who make use of expert-systems software (e.g. Condor and Score GDC). However, researchers addressing the problem from the economics and the operations research standpoint have highlighted that current practice can be further ameliorated.

The research on airport demand-management mechanisms may be classified into two main categories: (a) pure economic and (b) administrative. (a) mainly focuses on the creation of economic tools (e.g. slot auctions and congestion pricing schemes) while (b) addresses the problem through mathematical programming mechanisms which attempt to model the current regulatory framework and slot priorities. For more information, the reader may refer to the review papers of Gillen et al. (2016) and Zografos et al. (2017b).

Administrative airport slot scheduling has been acknowledged by researchers as an effective airport congestion mitigation technique (Gillen et al., 2016; Zografos et al., 2017b). By taking into account the complex decision process, the literature in the slot allocation has recently adopted a multi-objective decision-making direction, as well as some cunning modelling possibilities. Such modelling considerations allow the inclusion of various rules and priorities when solving for the optimal airport slot schedule. Yet, several policy sections and problem characteristics remain unaddressed.

1.1. Motivation and contributions

The first contribution of this document stems from the analysis of IATA's WSG, coupled with the critical review of the existing literature (Sections 2 and 3). This step allowed the identification of research gaps that are not currently considered in the literature.

Secondly, the models presented in Section 4 and the solution approach of Section 5 manage to consider some policy rules that have been neglected in previous research attempts. For example, alternative weighted schedule displacement cost functions may incorporate punctuality and year-round operations considerations as specified in IATA's rules [Section 8.3.6. of IATA (2018b)]. In addition, simple constraint expressions can address section 8.3.5.4. of IATA's WSG by introducing an upper bound to the maximum displacement occurring for new entrants' requests.

In continuation, the introduction of efficient constraints, which are based on the *Chebyshev decomposition* technique (Ferguson, 1958) can help the definition of the maximum displacement and fairness objectives, hence substituting the non-linear objective functions via computationally efficient linear alternatives. An additional contribution of the document derives from the number of considered objectives. By capitalising on previous modelling attempts, using and contributing to a suitable slot optimisation-programming framework, we present a tri-objective administrative mathematical tool, which considers simultaneously three objectives and provides trade-off analysis among them. In brief, the proposed model builds on previous research and formulations so as to propose a tri-objective approach which minimises schedule and maximum displacement while simultaneously considering demand-based fairness as described by Fairbrother and Zografos (2018a). This extends the objective considerations of previous research attempts as none of them has solved for three objectives in parallel.

Concerning the solution approach (Section 5), this paper moves beyond the modelling phase as it proposes a solution approach that is coherent to the current slot guidelines and objectives ensuring schedule acceptability among the different slot priorities. The notion of inter-level tolerance is also introduced since our solution algorithm allows weakly dominated or dominated solutions at the upper slot priorities (e.g. historic) in order to reap better results at the lower levels (new entrants, others). The idea for this approach is based on Stackelberg games and multi-level programming and proves that existing solution approaches (e.g. hierarchical) do not report the most beneficial set of equally efficient schedule alternatives. Given the decision-planning horizon (tactical-strategic), the proposed solution algorithm takes advantage of the value range and the nature of the objectives to provide trade-off solutions, satisfying the operational needs of airlines and some basic IATA policy rules.

Finally, we highlight the potential synergies between MOO and multi-attribute decision making (MADM) techniques by proposing two bi-stage solution approaches which can enhance the current models by allowing the consideration of additional objectives (e.g. CO2 and noise emissions). We highlight our arguments by providing two illustrative examples.

The remainder of the paper is organised in 7 sections. Section 2 presents an overview of the slot allocation process as defined in IATA (2018b). In Section 3, the reader may find a compact, critical literature review that pinpoints modelling gaps in the existing multiple objective slot allocation applications. In addition, Sections 4 and 5 include the proposed models and the description of the prescribed solution approach. Section 6 provides an exploratory data analysis on the case study's data and discusses the computational results and their potential impact on current practice. In Section 7, we discuss through some simple examples the potential synergies of multi-attribute and multi-objective decision-making techniques in the context of airport slot scheduling. Finally, Section 8 summarises the findings of this work and indicates valuable paths for future work. The document comes with an acronym table (Table 1) explaining some of the most frequent terminology abbreviations used in this paper.

Acronym	Explanation
AHP	Analytical Hierarchy Process
IATA	International Air Travel Association
MADM	Multiple Attribute Decision Making
MOO	Multiple Objective Optimisation
PSO	Public Service Obligations
SCC	Schedule Coordination Conference
SSIM	Standard Schedules Information Message
WSG	World Schedule Guidelines

Table 1*: Useful acronyms*

2. IATA's slot allocation process

For the past 40 years, the worldwide slot guidelines have been used to alleviate the shortage of global airport capacity in a fair and transparent way. The slot allocation process is the dominant airport demand management mechanism being applied by more than 200 airports of whom more than half are situated in Europe (IATA, 2018c). To further understand the complex decision process, as well as the benefit of our model, this section includes an overview of IATA's slot allocation process.

In the European Union, each member state is responsible for the airports that lie within their borders. Under this specification, each member state carries out an annual capacity assessment so as to determine whether its airports are subject to capacity shortages. If there is no short-term solution, then the airport may be characterised either as a *Level 2* or as a *Level 3*. Level 2 airports (*schedule facilitated*) may experience occasional congestion during some operational days, which can be resolved by mutual schedule adjustments by the appointed facilitator and the carriers [Section 4 of IATA (2018b)]. However, in Level 3 airports (*schedule coordinated*) the situation is more complicated because demand for airport infrastructure is greater than airport capacity in the given period and mutual resolution attempts cannot be made [Section 5.1 of IATA (2018b)]. Once an airport is characterised as "coordinated", the national aviation authority has to appoint a slot coordinator who should enact independently, neutrally and transparently [Section 5.2 of IATA (2018b)]. The main duties of the coordinator are to [Section 5.5 of IATA (2018b)]:

i) allocate slots to carriers based on the scheduling parameters (e.g. declared capacity) and the slot coordination guidelines and criteria;
ii) communicate to the interested parties the coordination parameters (e.g. coordination time interval), the local guidelines and regulations as well as any additional criteria;
iii) inform each airline about their allocated slots and the list of the remaining slots at the airport;
iv) monitor cancellations on historic[1] slots and the planned versus the actual use of slots for the application of the *use-it-or-lose-it rule*[2] [Section 8.6 of IATA (2018b)]; and
v) identify slot misusing.

It is obvious that the scheduling and coordination parameters are significant for the designation of the initial slot pool as they define the airport capacity and the number of movements that may be scheduled during each coordination interval, hence defining the initial slot pool. Once finalised, the coordinator has to communicate the initial slot pool to the airlines. Respectively, the airlines based on their commercial interests submit their requests for the next scheduling period. The requests are submitted bi-annually before the summer and winter Schedule Coordination Conferences (SCC) [Section 2.2 of IATA (2018b)] using the IATA Standard Schedules Information Manual (SSIM) message format.

Airline requests may fall into two main typologies: *series of slots* and *individual slots*. If an airline intends to operate a slot more than five times per scheduling period, then it should submit *a series of slots* request. A series request is characterised by the effective period of operation and the time and day of operation. For example, from the 15th of July to the 31st of September, every Tuesday at 14:05. Once all the requests are received by the coordinator, then he is responsible to carry out the *initial slot allocation* and distribute the slot pool to the airlines

[1] The Historics baseline date as well as the use-it-or lose-it-rule determine historic precedence. Ad hoc operations are not eligible for historic precedence [Section 8.7 of IATA (2018b)].
[2] According to the *use-it-or-lose-it rule*, a series of slots is characterised as historic only if it was operated more than 80% of the planned/ requested time.

according to the slot allocation principles described in sections 8 and 9 of IATA (2018b). At this stage, the only slot typology that is considered is slot-series (Zografos et al., 2012). On the contrary, *individual slots* may be requested up to a few days before the actual day of the operations subject to the approval of the coordinator.

Following the initial slot allocation, during the SCC the interested stakeholders (e.g. coordinators, airport and airline representatives, coordination committee etc.) meet and discuss beneficial adjustments to the draft schedule prepared by each coordinator. Such adjustments mainly serve the resolution of timing conflicts of connecting flights. In the post conference activity, the carriers should decide whether it is appropriate to operate each slot and may retain, return or modify it. In case new or modified requests cannot be accommodated, then the coordinator should offer the available slot that is closer to the requested [Section 9.13 of IATA (2018b)].

The models presented in the following sections, concern the initial slot allocation, which largely defines the effectiveness of the SCC and the overall effectiveness of the airport schedule. The guidelines and the criteria of the initial slot allocation process are discussed and presented in the following section.

2.1. The initial slot allocation criteria and guidelines

The rules included in this section can be mainly found in section 8 of IATA (2018b). The principles, priorities and criteria described, aim to serve the interests of the travelling public, the airlines and the other participating actors. In addition, they ensure a fair and transparent treatment of all airlines boosting competition and ensuring airport connectivity (IATA, 2018c).

Principles and general priorities

The coordinators should firstly take into account the slot allocation principles [Section 8.1.1 of IATA (2018b)], which provide definitions of historic precedence, series of slots etc. In continuation, the general priorities apply [Section 8.2 of IATA (2018b)]. At this point, the series of slots have a higher priority than ad hoc services and other operations. This part of the document is inherently incorporated to the initial slot allocation process, as it is only the series of slots that are considered in this stage.

Primary criteria

The primary criteria [Section 8.3 of IATA (2018b)], define the slot priorities, i.e. *historic, changes to historic, new entrants' and other requests*. According to this section of the document, the first slots to be allocated are those that are entitled with historic precedence. Once allocated, requests amending historic slots are considered. The remaining capacity is used to form the *slot pool*. Up to 50% of the slot pool is devoted for the accommodation of new entrants' requests while the remaining slots are given to the rest of the requests. At this point, new entrants' requests should be considered under the additional criteria of section 8.4 and the priority of year round operations [Section 8.3.6 of IATA (2018b)]. New entrants who get slots within one hour of their request, should accept the slots otherwise they won't be entitled for the new entrant status [Section 8.3.5.4 of IATA (2018b)]. Overall, within each category, the continuation of an existing operation should have priority over new requests, while also flexibility has to be considered when addressing the needs of short haul and long haul year round operations [Sections 8.3.6.1-2 of IATA (2018b)].

Additional criteria

Following the initial criteria, the additional rules ensure other operational objectives such as flight connectivity between airports, competition as well as the requirements of the travelling public and the local community [Sections 8.4.1.a-e of IATA (2018b)]. In brief, slots with a larger effective period of operation should have priority (8.4.1.a.) while the type of service and market should be prioritised based on the interests of the airport and the local community (8.4.1.b.). In addition, coordinators should take into account competitive factors (when rejecting slot requests) and curfews at other airports (8.4.1.c.d.). Finally, the requirements of the shippers and passengers must be met as far as possible (8.4.1.e.).

Displacement criteria

Given that all the additional criteria are considered, and the time requested by the airline is not available, the coordinators should displace the airline's slots based on the principles described in sections 9.9.3.a-f of IATA (2018b). According to this part of the document, new offers should not be made placing the carrier in a less favourable position than the one currently held. Therefore, offers should be made either within the requested and the historic time (9.9.3.a.), or within the specified flexibility limits (9.9.3.b.). However, the disclosure of flexibility preference data should not place carriers in a disadvantageous position (9.9.3.d.). Carriers may also communicate their willingness to accept counter offers if the requested time is not available (9.9.3.c.). The last two parts of this section dictate that frequent services should not get different service times unless allowed by the airlines (9.9.3.e) and that for paired requests (having an arrival and departure time requested) the turnaround time has to be respected avoiding additional ground times (9.9.3.f).

Punctuality and performance

Even though historic slot rights are granted based on the use-it-or-lose-it rule (usage above 80%), the performance and usage of all slot types is assessed by the Slot Performance Committee and it is monitored at the SCCs bi-annually. As specified in section 8.9 of IATA's WSG, intentional slot misuse may result in lower priority for future slot requests for each carrier, while sanctions may also apply. In any case, the allocation of the slots happens after the consultation of the Coordination Committee or the Slot Performance Committee, which determine for each series of slots the percentage of slots that were operated in a benevolent way.

Local guidelines - Public service obligations (PSO)

In addition to the aforementioned slot allocation guidelines, IATA (2018b) mentions that the initial slot allocation process should also take into account the local or regional regulations and guidelines that apply to each airport. One of the most eminent set of regional regulations that greatly affects the global airport network applies within the European Union. A key example of this regional set of regulations is the Public Service Obligations (PSO). As per Article 9 of the Council Regulation No 95/93 (1993), member states may retain certain slots for domestic or regional operations either to guarantee the development of the region where the airport is located or in routes where PSO are imposed. The routes where PSO apply are constantly under review and are published in the PSO inventory table. By considering PSO and IATA's WSG simultaneously, those routes constitute an additional priority, which prevails IATA's slot priorities. For the better understanding of the PSO regulatory framework, we refer the reader to the work of Bråthen and Eriksen (2018). Obviously, similar regulatory frameworks may exist in other areas of the world[3].

<u>Decision horizon</u>

A point which is currently overlooked, is the decision horizon in which the slots are allocated. Currently, it is believed to be six months. Conversely, by closely examining the deadlines disclosed in the first pages of IATA (2018b), we understand that the time that the coordinator has to allocate the slots is far less than that. Namely, in IATA (2018b) it is mentioned that airlines must submit their requests 33 days before the SCC (19th -21st of June 2018). Once airlines have submitted their requests (e.g. no later than 17th of May), the coordinators have to carry out the initial slot coordination up to 12 days before the SCC (e.g. no later than 7th of June 2018). That means that the coordinators have 22 days at their disposal to draft the initial slot schedule. Therefore, the decision timeframe is of tactical rather than strategic nature. Henceforth, this factor may act as a constraint to the timeframe of the solution approaches considering IATA's WSG.

Having provided an analytical overview to the regulatory framework of the initial slot allocation process, we may now present an explanation of the SSIM format and action codes in which slots are submitted during initial slot submission deadline (-33 days from the SCC).

2.2. The SSIM slot request format

The most crucial input in the initial slot allocation process is the slot requests themselves. To better understand the factors that are taken into account during the decision process, there is need to provide the information that the coordinator has in his disposal when drafting the slot schedule. To do so, we will provide a compact guide on the standard SSIM format in which requests are submitted.

Under the SSIM protocol, each airline sends a message to the slot coordinator with all the requests that it wants to submit to the current airport. The message is composed by three main parts: the *message header*, the *message body* (flight detailed request) and the *footnotes*. The header contains four pieces of information in the following format:

- The first three characters are the type of request (e.g. SCR: slot clearance request/reply);
- Another three characters represent the scheduling season indicator (e.g. S19: Summer 2019);
- The following five characters are the day that the message was sent (12AUG :12/08); and
- The last three characters define the airport to which the airline sends the message (e.g. LHR: London Heathrow)

Consequently, an example SSIM header would be: *SCR S19 12AUG LHR*.

The second component of the SSIM is the flight detail lines which is the main body of the request. Each line represents a slot request and may have multiple pieces of information. Requests can be submitted as arrivals, departures or as paired requests having both an arrival and a departure slot. Once all airlines send their requests to the airport, the coordinator extracts the flight detail lines and creates a table with all the slot requests submitted to the airport. In addition, in the 'REQUEST' column the coordinator includes action codes characterising the

[3] The equivalent of PSO in the U.S is expressed via the Essential Air Service (EAS).

priority type of each request (column 29). An example of such a table as well as an explanation of each column is given in Table 2. Likewise, the dataset that was used for our computational study (Section 6) is of similar form to Table 2.

The action codes (column 29 of Table 2) used by the coordinator to classify the requests may take the following values:

- A: acceptance of an offer – no further improvement desired;
- B: new entrant request;
- F: historic request;
- L: change to historic which will accept only the requested or the historic slot;
- N: new request which is not entitled of new entrant status;
- R: change to historic request which will accept any slot between the requested and the historic slot times;
- I: change to historic slot extending to a year-round operation;
- Y: new slot request willing to operate as a year-round operation; and
- V: new entrant request extending a year-round operation.

Finally, the footnotes of the SSIM contain general or special information regarding the request.

ID	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**
Title	AC	ANU	DC	DNU	HF	MHF	HT	MHT	M	T	W	H	F	S	U
1	A1	1111	A1	1112	16	JUN	13	OCT	0	2	0	0	0	0	0
	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
449	A2	9998	A2	9999	30	MAY	03	OCT	0	0	0	0	0	6	0

ID	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**
Title	SEN	TYP	AFR	BFR	AH	AM	DH	DM	V	ADE	BDE	FY	Q	REQUEST
1	231	737	PRG	PRG	12	55	13	55	-	PRG	PRG	JJ	3	F
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮		⋮	⋮
449	167	321	BLL	BLL	08	40	09	40	-	BLL	BLL	CC	1	N
Notes:	Arrival / Departure Company (*AC/DC*), Arrival/ Departure Number (*ANU/DNU*), first/ last day of operations (*HF/HT*), first/ last month of operations (*MHF/MHT*), Monday (*M*), Tuesday (*T*), Wednesday (*W*), Thursday(*H*), Friday (*F*), Saturday (*S*), Sunday (*U*), Seats Expected (*SEN*), type of aircraft (*TYP*), airport of origin (*AFR*), last stopover airport (*BFR*), Arrival/Departure Hour (*AH/DH*), Arrival/ Departure Minute (*AM/DM*), Overnight indicator (can be 1 or 2 if the aircraft will depart after one or two days and '-' if it departs the same day) (*V*), next stopover airport (*ADE*), destination airport (*BDE*), Service codes for the arrival and departure flights (*FY* where *J/F*: schedule passenger/ cargo flight, *C/H*: chartered passenger/cargo flight, *P*: positional, *X*: technical, *D*: general or private, *N*: Business aviation/ air taxi), frequency indicator (*Q*), request priority action code (*R*).													

Table 2: *The SSIM request format for all slots received by an airport*

2.3. Summary

The aforementioned guidelines compose a structured sequence of criteria and priorities that is depictured in Figures 1 and 2. Figure 1 groups the criteria at a higher level while Figure 2 presents a more detailed picture. In detail, Figure 2 is a schematic representation of the rules and priorities ordered hierarchically based on the verbal expressions given in the updated version of IATA's WSG.

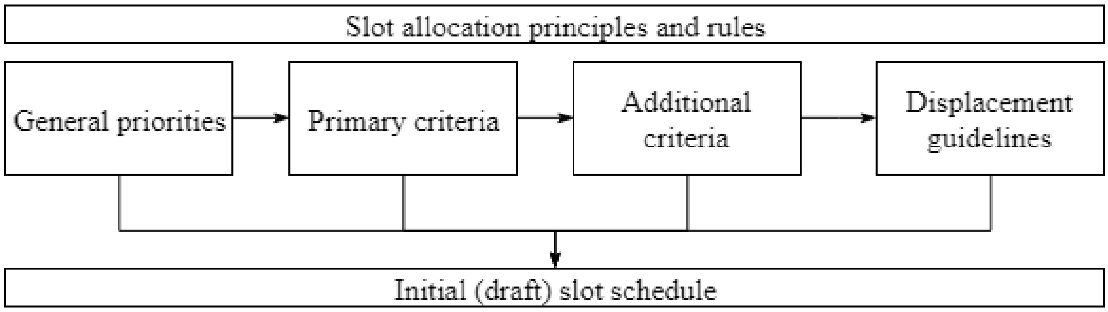

Figure 1: *Overview of the initial slot allocation rules*

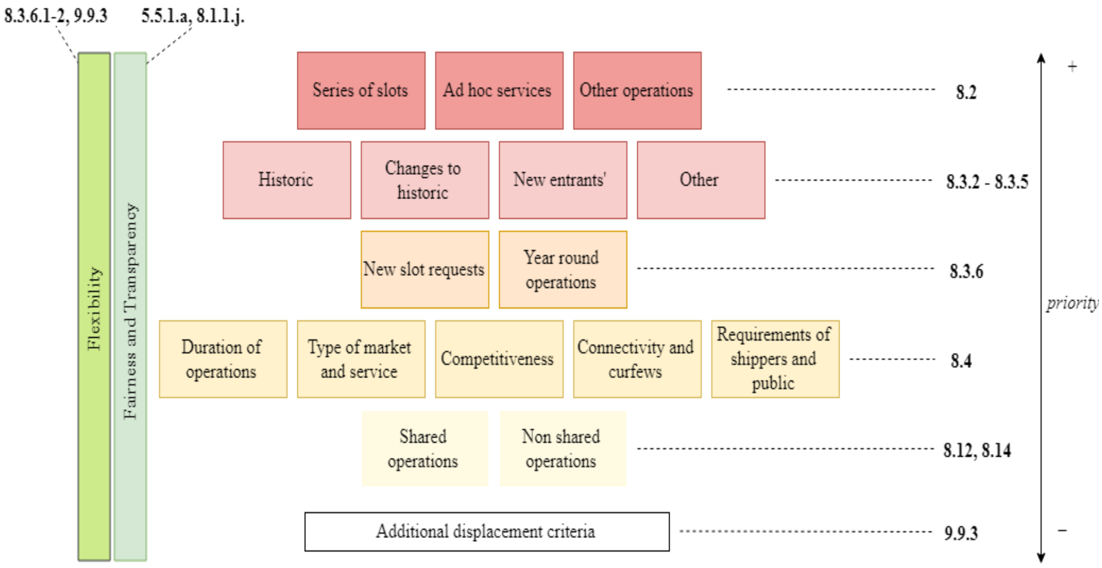

Figure 2: *A detailed schematic representation of the regulatory framework*

From both graphs, we understand that the complexity of the focal decision context as well as the inherent hierarchies that lie within the framework itself demand the consideration of all the rules and priorities when attempting to prescribe a mathematical model aiding in the current slot allocation process.

Moreover, even though PSO route priority is not explicitly included in IATA's WSG, it has to be considered because local or regional guidelines prevail IATA's rules. Therefore, such routes constitute an additional priority which is hierarchically superior to historic slots and has to be allocated before them [Article 9 of the Council Regulation No 95/93 (1993)].

3. Previous related work

This section includes a critical discussion on the multi-objective slot scheduling models that consider the IATA's regulatory framework (Figure 2). As mentioned before, the majority of the most congested airports follow IATA's world schedule guidelines (WSG) to manage airline demand for their resources. In contrast, the airports lying within the U.S [except for John F. Kennedy International Airport (JFK), LaGuardia Airport (LGA), and Ronald Reagan Washington National Airport (DCA)] do not follow this set of rules. In this work, we will only examine multi-objective models, which consider airport operations under IATA's WSG.

Administrative slot scheduling was firstly addressed via integer programming in Zografos et al. (2012). Since then, researchers have contributed to this stream of research by considering multiple objectives and modelling additional rules and specificities of the regulations. The existing MOO models may be divided based on the decision context that they consider. The majority of MOO formulations consider IATA's WSG while a few of them address the U.S decision-making context (Jacquillat and Odoni, 2015; Jacquillat and Vaze, 2018; Pyrgiotis and Odoni, 2015). IATA based models attempt to solve the slot allocation problem for the whole scheduling period while considering series of slot requests and as many of IATA's WSG as the can. Nevertheless, the US-based models consider individual slot requests and only solve for a single day of airport operations. At this point, it is worth mentioning that the former model typology has attracted greater research effort because of the modelling difficulties (slot priorities, series of slots etc.) and its widespread usage by the majority of the most congested airports. A more general review of the total corpus of the MOO slot scheduling models was presented by Katsigiannis (2018a).

The models presented and discussed in this work, take into account IATA's WSG decision context. Through the cross-tabulation of the relevant literature (Table 3), we understand that models with such policy considerations are rather recent and limited (Fairbrother and Zografos, 2018a, 2018b; Ribeiro et al., 2018; Zografos and Jiang, 2016; Zografos et al., 2017a). By examining Table 3, we conclude that there are still exist some sections of the policy framework that have not been addressed either because of the modelling and computational hazards or because of the lack of information, data and systematic consideration of the policy framework.

To provide a brief discussion on the relevant models, we will analyse the existing literature on a chronological order. Firstly, the paper of Zografos and Jiang (2016) proposed two bi-objective models which minimise two different schedule displacement metrics along with a fairness index. In detail, they introduce a measure of fairness, which is expressed via the proportion of displacement that each airline experiences in relation to the number of requests that it submits. If the fairness index is different from one, then the airline experiences disproportional schedule delay (higher or lower) to the number of requests that it makes. In the second formulation that they provide, they introduced a weighted displacement metric capturing the size of the operating aircraft and the flight distance (short haul, long haul). The incorporation of such weights allows the model to address the needs of the travelling public as flights with longer distance and more passengers receive a relative priority on the schedule displacement objective.

In a more recent attempt, Zografos et al. (2017) formulated two additional bi-objective models, which considered schedule and maximum displacement, and the number of violated slot assignments along with schedule displacement accordingly. The authors also considered different declared capacity scenarios to propose scenario-based trade-offs between the objectives. Both aforementioned papers solve the slot allocation problem hierarchically for each slot priority, i.e. considering historic, new entrant and other requests sequentially. To do so, they update the slot pool once all the slots in each priority level are allocated. Their solution approach generates the set of efficient solutions at each level and proceed to the lower levels based on that setting.

By attempting to address more of the IATA's rules and priorities, Ribeiro et al. (2018) provided a more realistic formulation considering additional criteria. In brief, they explicitly considered the "*changes to historic slots*" as a different priority, providing a more accurate definition of the *slot-pool* and the following slot types (new entrant and other requests). In addition, they modelled the displacement criteria described in section 9.9.3 of IATA's WSG. As for the considered objectives, they formulated the problem as a quart-objective weighted cost function minimising the number of rejected slots, the maximum and schedule displacement objectives and the number of violated slot assignments. The weights of the objective function mimic the lexicographic optimisation method, thus prioritising the objectives. Their model was tested on data obtained from two medium-sized Portuguese airports. However, the case study did not allow the consideration of the rejected slots objective as the requests and the capacity did not result in slot rejections.

Regarding the solution approach, the paper of Ribeiro et al. (2018) employs a different method than previous research. Instead of restricting the remaining capacity, they solve the problem for each priority level lexicographically. This approach is better suited to the nature of the problem as it captures to a certain extent the multi-level, hierarchical interdependencies existing between the different types of slots. Albeit, a major drawback of their approach is that the objective function is not suitable for the commensurable nature of the criteria. This argument stems from two major points. Firstly, the criteria are not expressed with the same measurement scales (e.g. maximum and total displacement) and the weights are not normalised (they are all significantly greater than one). Secondly, the lexicographic-like approach explicitly considers the preferences of the decision maker(s) when providing the weights for the objectives. At the same time, it does not provide a systematic way of exploring the total set of efficient solutions. Finally, during the analysis of their results, the authors only provide bi-objective trade-off analyses rather than a quart or tri-objective approach.

As we have schematically demonstrated in Figure 2, fairness, transparency and non-discriminatory considerations are omnipresent in the slot allocation decision process. In this context, Fairbrother and Zografos (2018a) built on the work of Zografos and Jiang (2016) and introduced a demand based fairness index which considers the peak requests of each of the airlines. Therefore, this index is equal to one when an airline receives proportional delays to the peak requests that it makes. The authors moved beyond the proposal of this objective and constructed a budget mechanism that allows airlines to distribute the total displacement of their flights based on their own preferences. This attempt is one step closer to the operational needs of airlines but is limited by the absence of preference data.

This work was more recently extended in Fairbrother and Zografos (2018b). In particular, they introduced to their previous model flexibility considerations. Flexibility, constitutes a promising modelling enhancement since it is a property specified in many of IATA's WSG sections [Sections 9.9.3.c,b,d., 8.3.6.1.,2. and 9.7.3.b. of IATA (2018b)]. In brief, in accordance to the Timing Flexibility Range and section 9.7.3 of IATA's WSG the authors modelled flexibility as upper and lower timing deviation bounds to the requested times. To achieve this modelling enhancement, they added an extra dimension to their decision variables (accounting for days) to allow differentiated slot timings for each day. Yet, the different times can deviate only within the specified flexibility bounds, offering coherent slot times. The drawbacks of this model mainly fall into three main categories. First is the complexity stemming from the increased number of decision variables and constraints. Second are the flexibility range parameters which are applied uniformly to all requests. This is something that contradicts the specifications of the IATA guidelines as each airline should provide its own flexibility range for each of their slots. Finally, it is not clear how schedule displacement is calculated since flexibility acts like an allowable slack which is not counted in displacement cost functions. In any case the disclosure of flexibility information should not put airlines in an unfavourable market condition. Hence, differentiated flexibility bounds for each airline and linear functions of disutility could account for this policy requirement.

At this point it is worth mentioning that there is another stream of research which addresses airport slot scheduling at the airport-network level, hence capturing slot complementarities among inter-connected airports. In this stream of research, two multi-objective approaches exist (Corolli et al., 2014; Pellegrini et al., 2017). The model of Corolli et al. (2014) minimises the schedule displacement as well as the expected queuing delays, while the model of Pellegrini et al. (2017) has cost functions minimising the costs of violated slots and the displacement of coupled or independent slots accordingly. Even though this stream of research is of great interest, the models above are not tabulated in Table 3 since the scope of this work focuses on the single airport level.

To summarise, the existing attempts capture many modelling aspects of the focal problem. Albeit, there still exist research gaps and opportunities, which can be addressed that fall into four main categories:

i) Modelling of the unaddressed regulations and priorities;
ii) Suggesting additional or alternative cost functions and constraints that are more accurate, efficient or model additional aspects of the problem;
iii) Simultaneous optimisation of more objectives; and
iv) Devising solution approaches that can capture the interdependencies between the different decision levels (slot hierarchies).

Having in mind those research directions, in the following sections we propose: a) a tri-objective formulation considering fairness, total and maximum displacement metrics; b) two alternative weighting indexes that can provide alternative total displacement cost functions, modelling unconsidered policy aspects; and c) a multi-level modelling alternative to the slot allocation that better captures the interactions among the slot hierarchies.

	Primary criteria			**Duration**	**Additional criteria**					**Displacement criteria**						**Fairness**	**Flexibility**	**PSO routes**
Model	8.3.2	8.3.3	8.3.4, 8.3.5	8.3.6	8.4.1.a	8.4.1.b.	8.4.1.c.	8.4.1.d, 9.7.3.d	8.4.1.e	9.9.3.a	9.9.3.b	9.9.3.c	9.9.3.d	9.9.3.e	9.9.3.f	5.5.1.a, 8.1.1.j	8.3.6.1-2, 9.7.3.b	CR 95/93 (1993)
Zografos and Jiang (2016)	✓	✓	✓	✗	✗	✓*	✗	✗	✓	✗	✗	✗	✗	✓*	✓	✓	✗	✗
Zografos et al. (2017)	✓	✓	✓	✗	✗	✗	✗	✗	✗	✗	✗	✗	✗	✓*	✓	✗	✗	✗
Ribeiro et al. (2018)	✓	☑	☑	✗	✗	✗	✗	✗	✗	✓	✓*	✓	✗	✓*	✓	✗	✗	✗
Fairbrother and Zografos (2018a)	✗	✗	✗	✗	✗	✗	✗	✗	✗	✗	✗	✗	✗	✓*	✓	✓	✗	✗
Fairbrother and Zografos (2018b)	✓	✓	✓	✗	✗	✗	✗	✗	✗	✗	✓*	✗	✓*	✓	✓	✓	✓*	✗
Addressed	✓	✓	✓	✗	✗	✓*	✗	✗	✓	✓	✓	✓	✓*	✓	✓	✓	✓*	✗

Notes: Historic slot requests (*8.3.2.*), Changes to historic slots (*8.3.3.*), New entrants rules (*8.3.4.*, *8.3.5.*), Year round operations (*8.3.6.*), Effective period of operation (*8.4.1.a.*), Type of service and market (*8.4.1.b.*), Competitive factors when rejecting slots (*8.4.1.c.*), Curfews (*8.4.1.d.*, 9.7.3.d.), Requirements of shippers and travellers (*8.4.1.e.*), offers shall not place airlines in less favourable conditions than the ones held (*9.9.3.a.*), acceptable/ unacceptable offers (*9.9.3.c.*), consistent turnaround times (*9.9.3.f.*), flexibility sections (*9.9.3.b.*, *9.9.3.d.* *9.9.3.e.*), addressed/ accurately addressed (✓/☑), not addressed (✗), * partially considered, Fairness stands for transparency and non-discrimination of airlines.

Table 3: *Modelling considerations of existing literature based on IATA's WSG*

4. Slot allocation models with multiple objectives and multi-level considerations.

In this section, we present a tri-objective slot allocation model that considers efficiency (schedule and maximum displacement) and fairness objectives. Our model builds on previous research by incorporating the modelling capabilities of various models already existing in the literature. For instance, the base problem is adapted from the formulation of Zografos et al. (2012), while the fairness cost function that we consider has already been defined in the literature by Fairbrother and Zografos (2018a). Yet the following formulation is the first to model all the objectives simultaneously, thus being able to provide trade-off analysis among them. Moreover, in 4.2 we propose and prove that the tri-objective slot allocation problem (TOSAM) can be modelled as a multi-level problem. The originality of this approach stems from the fact that no other research considers the slot allocation problem in a multi-level manner. This consideration extends and formalises current modelling and solution approaches and may potentially sparkle new research trends and interests.

4.1. A Tri-Objective Slot Allocation Model (TOSAM)

Models addressing the IATA's WSG decision context have to account for series-of-slots requests as per Section 2. Moreover, the decision-planning horizon is set to be the whole scheduling period, rather than a single day. The backbone of the presented model is a modified version of the work of Zografos et al. (2012), however some modifications are made serving computational efficiency and the incorporation of the multi-objective considerations. In the following subsections we present the notation required (input data sets, parameters, decision variables) so as to better understand the main body of the model (equations 4.1.- 4.8.).

4.1.1. Formulation

Input data sets

A: set of airlines denoted by a;
M: set of request series denoted by m ;
M_a: set of request series of airline a;
$M^{Arr(Dep)}$: $M^{Arr} \cup M^{Dep} = M^{Total}$, set of arrival (departure) series;
$P \subseteq M \times M$: set of paired requests (m_{Arr}, m_{Dep}) indexed by p;
D: set of days in scheduling season denoted by d;
D_m: set of days that slot m is to operate;
C: $\{5, 15, 60\}$ set of capacity time scales indexed by c;
T_c: $\{1,2, \dots n\}$ set of time intervals per day based on scale c indexed by t, s; and
K: $\{Arr, Dep, Total\}$ set of movement types denoted by k.

Input parameters

- t_m: requested time for slot series m;

- $v_{t,m}^{d} = \begin{cases} 1, if\ request\ m\ belongs\ to\ a\ peak\ time\ t\ on\ day\ d \\ 0, otherwise \end{cases}$, as peak times Fairbrother and Zografos (2018) define periods of duration c where airline demand exceeds airport capacity;

- $T_{max,p}, T_{min,p}$: maximum and minimum turnaround times of paired request p;

- $u^k_{d,s,c}$: capacity for movement k for period $[s, s+c]$ on day d based on time scale c;

- $a_{d,m} = \begin{cases} 1, if \ series \ m \ is \ requested \ on \ day \ d \\ 0, otherwise \end{cases}$; and

- $\rho_\alpha = \left(\sum_{m \in M_a} \sum_{d \in D} v^d_{tm}\right) / \left(\sum_{m \in M} \sum_{d \in D} v^d_{tm}\right)$, the proportion[4] of peak requests of airline a.

Decision variables

- $x_{t,m} = \begin{cases} 1, if \ series \ m \ is \ allocated \ to \ time \ t \\ 0, otherwise \end{cases}$

Base Constraints

$$\sum_{t \in T} x_{m,t} = 1, \forall \ m \in M, d \in D \tag{4.1}$$

$$\sum_{m \in M^k} \sum_{t \in [s, s+c-1]} a_{d,m} x_{t,m} \leq \ u^k_{d,s,c} \ , \forall \ k \in K, d \ \in D \tag{4.2}$$

$$T_{min,p} \leq \ \sum_{t \in T} x_{m_{Dep}} t - \sum_{t \in T} x_{m_{Arr}} t \leq T_{max,p} \ , \forall \ p \in P \tag{4.3}$$

Constraints (4.1) ensure that each of the slots will be allocated to a time. Moreover, constraints (4.2) are rolling capacity constraints for each type of movement i.e. arrival, departures, or total movements. The last type of constraints (4.3) are turnaround time constraints which define that the time difference between two paired requests should not be less than the initially requested difference between them (minimum turnaround time) and larger than a specified limit. In the absence of preferences regarding the maximum turnaround time, its value can be set equal to an operationally viable value or infinity.

Please note that the precedence constraints defined in Zografos et al. (2017, 2012) are not required as the programmatic parsing of the input data renders them redundant.

Cost functions

$$\min(Z_1, Z_2, Z_3) \tag{4.4}$$

Where:

$$Z_1 = \sum_{m \in M} \sum_{t \in T} |t - t_m| \, x_{t,m} \tag{4.5}$$

$$Z_2 = \max_{\forall m \in M} |t - t_m| \tag{4.6}$$

$$Z_3 = \max_{\forall a \in A} |\mu_\alpha - 1| \tag{4.7}$$

,and;

$$\mu_\alpha = \frac{\left(\sum_{m \in M_\alpha} \sum_{t \in T} |t - t_m| x_{t,m}\right) / \left(\sum_{m \in M} \sum_{t \in T} |t - t_m| x_{t,m}\right)}{\rho_\alpha} \tag{4.8}$$

Expression (4.4) states that all objectives are minimised. Equations (4.5) and (4.6) define the schedule and maximum displacement cost functions. Equation (4.7) defines the third objective

[4] Fairbrother and Zografos (2018) refer to this index as contribution of airline a to congestion.

of the proposed model expressing the maximum deviation of the fairness index (μ_α) from 1. When the fairness index is equal to one, then the proportion of displacement allocated to airline a is completely analogous to its contribution to congestion. If equation (4.8) is less than one, then airline a is experiencing less displacement in relation to the peak requests that it has submitted. Obviously, for values of μ_α above one, the displacement that the airline will experience is greater than the proportion of its requests at peak times. Therefore, objective function Z_3 is minimised as we would like expression (4.8) to take values close to zero.

<u>ε-type constraints</u>

Scalar approaches may result in reduced computational times, but they cannot generate satisfactory portions of the efficient solutions' set. In addition, commensurable objectives which are not expressed in the same units should receive either normalised or justifiable weights (Takama and Loucks, 1981). Therefore, there is need to modify the base problem described in above to incorporate those two problem characteristics.

The state of the art in tri-objective solution algorithms employ expanded, generalised variants of the *ε-constraint* method (Haimes, 1971). Especially, they constrain the objective values of two of the three cost functions by using a list of upper bounds while optimising the third objective (Boland et al., 2017). In order to employ such solution algorithms two of the objectives have to be expressed in the form of linear constraints. By taking advantage of the scales of the objectives, we chose to re-model cost functions Z_2 and Z_3 . That is because the width of their efficient solution values is smaller than that of the total displacement. For example, fairness stops being a binding objective for values above 1.5 (where the carriers receive 2.5 times less or more displacement than their peak requests), while maximum displacement cannot exceed the total duration of the number of time intervals within a day. On the contrary, the range of Z_1 is by far greater even by the product of the widths of the other two objectives.

Given all the above, objectives Z_2 and Z_3 can be written linearly with the use of expressions (4.9 – 4.12).

$$t - t_m \; x_{t,m} \leq \varepsilon_{Z_2}, \forall \; t \in T, m \in M \tag{4.9}$$

$$t_m - t \; x_{t,m} \leq \varepsilon_{Z_2}, \forall \; t \in T, m \in M \tag{4.10}$$

$$\left(\sum_{m \in M_\alpha} \sum_{t \in T} |t - t_m| \, x_{t,m} \right) - \rho_\alpha \left(\sum_{m \in M} \sum_{t \in T} |t - t_m| \, x_{t,m} \right) (1 + \varepsilon_{Z_3}) \leq 0 \; , \forall \; a \in A \tag{4.11}$$

$$\rho_\alpha \left(\sum_{m \in M} \sum_{t \in T} |t - t_m| \, x_{t,m} \right) (1 + \varepsilon_{Z_3}) - \left(\sum_{m \in M_\alpha} \sum_{t \in T} |t - t_m| \, x_{t,m} \right) \leq 0, \forall \; a \in A \tag{4.12}$$

Constraints (4.9 - 4.10)[5] ease the definition of the maximum displacement objective (Z_2) by limiting it under ε_{Z_2} while constraints (4.11- 4.12) help bound the fairness objective (Z_3) under ε_{Z_3}.

---

[5] It is easier for pre-solving to compute tighter bounds on the variables that are in the constraints. With the formulations that currently exist in the literature, solvers may struggle to find solutions.

Having provided the formulation of our base model, we may now suggest some additional modelling considerations that capture some of IATA's WSG requirements, which have not yet been addressed.

4.1.2. Alternative weighted schedule displacement functions

In this subsection, we introduce two novel weighted models that manage to model performance and punctuality [Sections 8.4.1.b and 8.9. of IATA (2018b)] and prioritise year-round operations [Section 8.3.6. of IATA (2018b)].

4.1.2.1. Introducing punctuality and performance considerations

Performance and slot misuse have yet to be addressed in the literature. As described in Section 2.1, airlines' ill slot performance and slot misuse may result in sanctions and lower slot priorities. Slot monitoring is conducted in a long-term manner in order to give to airlines the opportunity to correct their performance regarding each slot in following seasons. The irregularities that the committees responsible detect, mainly fall into the following three categories (COHOR, 2018):

- Flights that are operated without allocated slots;
- Flights operated in different conditions than those specified in the allocated slot; and
- Unused slots which deprive the airport of valuable capacity resources.

To consider the latter two, we may take advantage of the slot utilisation ratio ($h_{m,j}$) provided by the Slot Performance Committee that monitors and determines slot usage before the SCCs. At the same time the punctual flight operations serve the interest of the travelling public and the shippers [Section 8.4.1.b of IATA (2018b)].

Therefore, given that there are historic records of airline requests, let:

- $J_{m,k}^{a}$: $\{r, r+1, \dots, k\}$ be the set of scheduling seasons up to the current scheduling season (k) that airline a holds slot m, starting from season r and it is indexed by j;

- $h_{m,j}$: be the utilisation ratio of slot m at scheduling season j;

- $\xi_{a,m,j} = \begin{cases} 1, \text{if airline } a, \text{operated slot } m \text{ at period } j \\ 0, \text{otherwise} \end{cases}$; and

- $M^j and\ M_a^j \subseteq M^j$: be the set of requests at period j and the set of requests by airline a submitted at period j.

Then the *performance index* of each airline at the current scheduling period (k) is:

$$I_a^k = \sum_{m \in M_a^k} \left[\frac{\sum_{j \in J_m^a} (h_{m,j} \xi_{a,m,j})}{J_{m,k}^{\ a}} \right] \tag{4.13}$$

Then and the *relative performance index* for airline a may be calculated as follows:

$$IR_a^k = \frac{I_a^k}{\left(\sum_{a \in A} I_\alpha^k\right)/|A|} \tag{4.14}$$

The practical meaning of equation (4.13) is that for all the slots that airline a is currently holding, (M_a^k) we calculate the historical performance and we divide it by the number of periods that the airline operated the slot $\{\ J_{m,k}^{\ a}\ \}$. As for equation (4.14), we divide the historical performance of the airline by the average performance of all airlines operating in the airport. Therefore IR_a^k may take values based on the following expression:

$$IR_a^k = \begin{cases} < 1, \text{if airline } a \text{ performs worse than the other airlines} \\ = 1, \text{if airline } a \text{ performs equally to other airlines} \\ \geq 1, \text{if airline } a \text{ performs better than the airlines} \end{cases} \quad (4.15)$$

Due to the fact that equation (4.14) is a relative measure, it takes into account the overall historical slot usage in the current airport, accounting for disruptions occurring to all airlines operating in the airport. Hence, it penalises airlines that faced internal disruptions that were mainly caused by ill managerial or operational planning (e.g. poor maintenance). In addition, the index may capture inter-seasonal slot misuse and penalise airlines that constantly underperform. Please note that the provided formulation may even account for shared operations as $\xi_{a,m,j}$ can be equal to one for more than one values of a.

Given those updated modelling considerations, the objective function for total displacement is modified as per (4.16):

$$Z_1' = \sum_{a \in A} \left[IR_a \sum_{m \in M_a} \sum_{t \in T} |t - t_m| \, x_{t,m} \right] \quad (4.16)$$

This formulation attempts to facility an airline efficiency-based airport resource allocation that takes into account historic slot usage.

4.1.2.2. Introducing year-round and effective period priority considerations

Another modelling gap stems from the absence of prioritisation for year-round requests and services that have larger effective periods. Section 8.3.6. of IATA (2018b) states that year-round requests of all priority types should get priority over other requests of the same priority. Moreover, requests covering longer periods of operations should receive larger priority [Section 8.4.1.a. of IATA (2018b)].

To capture those two specifications, we have to introduce the *service continuity index* denoted by the Greek letter σ (sigma), described in expression (4.17) along with some extra input sets and parameters.

Let:

- D_m: $\{p_m, \dots, \tau_m\}$ be the set of days that slot m is to operate in the scheduling season having $|D_m|$ days and $\tau_m - p_m$ being the effective period that the slot will operate.

- β is the number of weeks in the scheduling season; and

- $y_m = \begin{cases} 1, \text{if slot } m \text{ is caracterised as a year round operation} \\ 0, \text{otherwise} \end{cases}$.

Then the *service continuity index* for slot m is:

$$\sigma_m = (1 + y_m) \frac{(\tau_m - p_m) div 7}{\beta} \quad (4.17)$$

To explain (4.17) in non-technical terms, if the request is due for a year-round operation $y_m = 1$ then it receives two times the weight that a single period request of the same duration would receive. The right part of the index is the *effective period index*. This sub-index is a fraction consisting of the number of weeks that a slot is requested for $[(\tau_m - p_m) div 7]$ divided by the number of weeks of the scheduling period (β).

An updated version of the schedule displacement objective accounting for service continuity would be:

$$Z_1'' = \sum_{m \in M} \sum_{t \in T} |t - t_m| \, x_{t,m} \sigma_m \tag{4.18}$$

4.2. A multi-level optimisation framework in tri-objective slot allocation modelling

Multilevel decision-making is inspired by game theory and especially by the Stackelberg leadership model (Stackelberg, 2011). Multi-level programming techniques are used in order to address the compromises that are needed to be made between the interactive decision entities which compose a hierarchical organisation or decision process (Lu et al., 2016). The compromises and the tolerance of the upper decision entities increase the welfare output of the system since the aggregate benefit of the individual cost functions is larger than optimising the upper decision levels without foreseeing the benefit of the lower decision entities (Tesoriere, 2017).

To provide some insights and definitions from multi-level decision-making, we will refer to the decision entities of the upper and lower levels as *leaders* and *followers*. The decisions of the different entities are applied sequentially with each of them optimising their respective cost functions independently. The architecture of this decision process implies that the leader has priority when determining his own decision, while the followers react and constrain their decision having *perfect information* (full knowledge) of the decisions of the leaders. Even though leaders' movements and decisions greatly affect followers' decision space, the decisions of the followers' do not explicitly affect the feasible decision space of the leaders. The hierarchical decision structure described above, appears in various real-world management problems. A potential application of multi-level programming would be to model the airport slot allocation with IATA's hierarchies as a multi-level problem.

CLAIM 1.

> The airport slot allocation decision-making process defined by IATA's slot priorities is a multi-level problem and can be modelled as one.

Proof.

> *(1)* The decision process is composed by interacting decision making units with a hierarchical structure, i.e. slot priorities;
> *(2)* The decisions executed by the following levels are defined after and only after the decisions of the leading levels;
> *(3)* For each level, the objectives are optimised independently without considering following levels' actions, yet following levels are constrained by the decisions of the leaders; and
> *(4)* The influence of the decisions of the leading levels is reflected in the feasible and decision spaces of the lower levels.

By accepting that Claim 1 holds, we may continue by providing a multi-level formulation of the focal problem. We already mentioned that the requests submitted for each slot-scheduling season might fall into four main priority types: historic, changes to historic, new entrant and other requests. Therefore, the set of the requests is a union of all the requests belonging to those priorities. Following the notation in section 4.1 let:

- Γ: $\{H, CH, NE, Oth\}$ be the set of priorities denoted by γ where H, CH, NE and Oth stand for historic, changes to historic, new entrant and other series of requests respectively. Then, the whole set of requests can be expressed as: $M = \bigcup_{\gamma \in \Gamma} M^{\gamma}$;

- L^{γ}: $\{\gamma - q, \dots, \gamma - 1, \gamma\}$ is the set of q levels, which are leaders to γ including γ.
- Φ^{γ}: $\{\gamma, \gamma + 1, \dots, \gamma + \varphi\}$ is the set of φ levels, which are followers to γ including γ.
- $x_{\gamma} \in \mathbb{E}^{|M^{\gamma}|}$ be a vector of decision variables defined at level γ, where $|M^{\gamma}|$ is the cardinality of the set of requests of priority γ and $\mathbb{E}$ the feasible space of the variables;
- $F_{\gamma}(x_{\gamma}, \dots, x_{\gamma+\varphi}) = \{Z_1(x_{\gamma}, \dots, x_{\gamma+\varphi}), Z_2(x_{\gamma}, \dots, x_{\gamma+\varphi}), Z_3(x_{\gamma}, \dots, x_{\gamma+\varphi})\}$ be the vector of the cost functions of level γ;
-The expression of the vector-valued cost functions' criterion space for all four levels is F_H, F_{CH}, F_{NE}, F_O: $\mathbb{Z}^{M^H} \times \mathbb{Z}^{M^{CH}} \times \mathbb{Z}^{M^{NE}} \times \mathbb{Z}^{M^{Oth}} \rightarrow \mathbb{R}^3$ with $\mathbb{Z}^{|M^{\gamma}|}$ being the feasible criterion space $\forall\ \gamma \in \Gamma$;
-$g_{\gamma}(x_{\gamma-q}, \dots, x_{\gamma})$ is the set of constraints of level $\gamma : \gamma\ \in L^{\gamma}$
-g_{Γ}: $\mathbb{G}^{M^H} \times \mathbb{G}^{M^{CH}} \times \mathbb{G}^{M^{NE}} \times \mathbb{G}^{M^{Oth}} \rightarrow \mathbb{G}^{\kappa}$ are the constraint conditions of the four levels with κ being the number of constraints for all four levels; and
- $X^{\gamma} \subseteq \mathbb{E}^{|M^{\gamma}|}$ be the restricted feasible design space of the decision variables of level γ based on additional constraints on the decision variables, such as upper and lower bounds on the ε-type constraints of Section 4.1.

Given the above notations, we may now provide a general definition of the ML-TOSAM.

DEFINITION 1.
For $x_H \in X^H \subseteq \mathbb{E}^{M^H}, x_{CH} \in X^{CH} \subseteq \mathbb{E}^{M^{CH}}, x_{NE} \in X^{NE} \subseteq \mathbb{E}^{M^{NE}}, x_{Oth} \in X^{Oth} \subseteq \mathbb{E}^{M^{Oth}}$ the ML-TOSAM is defined as follows:

$$\min_{x \in x_H} F_H(x_H, x_{CH}, x_{NE}, x_{Oth}) \quad \textit{(historic level)} \tag{4.19.a}$$

$$\text{Subject to: } g_H\ x_H\ \leq 0 \tag{4.19.b}$$

Where for each x_H given by the historic level, x_{CH}, x_{NE}, x_{Oth} solve the problems of the second, third and fourth following priorities.

$$\min_{x \in x_{CH}} F_{CH}(x_{CH}, x_{NE}, x_{Oth}) \quad \textit{(changes to historic level)} \tag{4.19.c}$$

$$\text{Subject to: } g_{CH}\ x_H, x_{CH}\ \leq 0 \tag{4.19.d}$$

Where for each x_H, x_{CH} given by the historic and changes to historic level, x_{NE}, x_{Oth} solve the problems of the third and fourth following levels.

$$\min_{x \in x_{NE}} F_{NE}(x_{NE}, x_{Oth}) \quad \textit{(new entrant level)} \tag{4.19.e}$$

$$\text{Subject to: } g_{CH}\ x_H, x_{CH}, x_{NE}\ \leq 0 \tag{4.19.f}$$

Where for each x_H, x_{CH}, x_{NE} given by the historic, changes to historic and the new entrants level, x_{Oth} solve the problems of the third and fourth following levels.

$$\min_{x \in x_{Oth}} F_{Oth}(x_{Oth}) \quad \textit{(other level)} \tag{4.19.g}$$

$$\text{Subject to: } g_{Oth}\ x_H, x_{CH}, x_{NE}, x_{Oth}\ \leq 0 \tag{4.19.h}$$

Please note that the hierarchical structure in Definition 1 is typical of nested multi-level programming. ML-TOSAM can be also written in a condensed, recursive algorithmic manner (Algorithm 1).

```
input: Γ, F
output: Λ list of multi-level trade-offs
Define: recursive ML-TOSAM (γ)
    Λ ⟵ (); # List of solutions is initialised as empty
    if γ ∉ Γ then
        Stop; # termination of Algorithm 1
    else:
        for each x_γ ∈ X^γ repeat:
            solution ← { min_{x∈X^γ} F_γ(x_γ, …, x_{γ+φ}) | g_γ(x_{γ−q}, …, x_γ) ≤ 0 } ;
            Λ.add(solution);
            recursive ML-TOSAM  γ = γ.next() ; # move to the next level
return  Λ;
Λ ⟵ recursive ML-TOSAM (γ = H);
Notes: # stands for comments on the algorithmic process; in bold are common algorithmic
       functions
```

Algorithm 1: *The recursive ML-TOSAM*

The generic definition of the ML-TOSAM allows us to define the solution concepts that occur from its recursive formulation in Algorithm 1. The definitions and solution concepts are an attempt to extend a similar tri-level single-objective formulation proposed by Lu et al. (2012).

DEFINITION 2.
The solution concepts of the ML-TOSAM are defined as per *(i)-(x)*.

(i) The constraint region of the quart-level problem is:

$$S = \{ x_H, x_{CH}, x_{NE}, x_{Oth} \in X^H \times X^{CH} \times X^{NE} \times X^O : g_\gamma(x_{\gamma-q}, \dots, x_\gamma) \leq 0, \forall \gamma \in \Gamma \}$$

(ii) The feasible decision space of level γ which has φ followers and q leaders is generally defined:

$$S\ \gamma\ = \{(x_{\gamma+1}, \dots, x_{\gamma+\varphi}) \in X^{\gamma+1} \times \dots \times X^{\gamma+\varphi} : g_i(x_{i-q}, \dots, x_i) \leq 0, \forall\ i \in\ \Phi^\gamma\}$$

(iii) The feasible decision space of the changes to historic level (second) for each set of decision variables given by the historic level x_H is:

$$S\ x_H\ = \{ x_{CH}, x_{NE}, x_{Oth} \in X^{CH} \times X^{NE} \times X^O : g_\gamma(x_{\gamma-q}, \dots, x_\gamma) \leq 0, \forall\ \gamma \in\ \Phi^{CH}\}$$

(iv) The feasible set of decision variables of the new entrant level (third) for each set of decision variables given by the historic and changes to historic levels x_H, x_{CH} is:

$$S\ x_H, x_{CH}\ = \{ x_{NE}, x_{Oth} \in X^{NE} \times X^O : g_\gamma(x_{\gamma-q}, \dots, x_\gamma) \leq 0, \forall\ \gamma \in\ \Phi^{NE}\}$$

(v) The feasible set of the other level (fourth) for each set of decision variables given by the historic, changes to historic levels and new entrant levels x_H, x_{CH}, x_{NE} is:

$$S\ x_H, x_{CH}, x_{NE}\ = \{x_{Oth} \in X^O: g_{Oth}\ x_H, x_{CH}, x_{NE}, x_{Oth}\ \leq 0\}$$

(vi) The rational reaction of the other (fourth) level:

$$\Theta\ x_H, x_{CH}, x_{NE} = \{x_{Oth} \in X^O: x_{Oth} \in argmin\ F_{Oth}\ x_{Oth}\ : x_{Oth} \in S\ x_H, x_{CH}, x_{NE}\ \}$$

Which practically means that the only reaction that others may have is the minimisation of the values of their objectives based on the feasible decisions of the leading levels.

(vii) The rational reaction of the new entrants (third) level is:

$$\Theta\ x_H, x_{CH}\ = \{\ x_{NE}, x_{Oth}\ \in X^{NE} \times X^O:\ x_{NE}, x_{Oth} \in argmin\ F_{NE}(x_{NE}, x_{Oth}):\ x_{NE}, x_{Oth}\ \in S\ x_H, x_{CH}\ , x_{Oth} \in \Theta\ x_H, x_{CH}, x_{NE}\ \}$$

Equivalently, the reaction of the model at the new entrants level is to minimise its own objective function based on the leading levels' (historic, changes to historic) decision variables and the rational reaction set of the lower levels (other).

(viii) The rational reaction of the changes to historic (second) level is defined accordingly:

$$\Theta\ x_H\ = \{\ x_{CH}, x_{NE}, x_{Oth}\ \in X^{CH} \times X^{NE} \times X^O:\ x_{CH}, x_{NE}, x_{Oth} \in argmin\ F_{CH}\ x_{CH}, x_{NE}, x_{Oth}\ :\ x_{CH}, x_{NE}, x_{Oth}\ \in S\ x_H\ , x_{NE} \in \Theta\ x_H, x_{CH}\ , x_{Oth} \in \Theta\ x_H, x_{CH}, x_{NE}\ \}$$

Where the reaction of the model at the changes to historic level is to minimise its own objective function subject to its feasible space, based on the leading level's (historic) decision variables and the rational reaction set of the lower levels (other, new entrants).

(ix) The inducible region (Chang and Luh, 1982) of the quart-level problem is:

$$R = \{\ x_H, x_{CH}, x_{NE}, x_{Oth}\ :\ x_H, x_{CH}, x_{NE}, x_{Oth}\ \in S,\ x_{CH}, x_{NE}, x_{Oth} \in \Theta\ x_H\ \}$$

The leader can only assume (induce) the followers' behaviour since he has limited (implicit) information. The inducible region of the ML-TOSAM is the values x_H (inducible values) that belong to the rational reaction set of the changes to historic level and result to feasible decision variable values for all following levels (feasible for the bottom level). The definition of the inducible region is essential in order to yield the necessary and sufficient conditions for the optimal solution.

(x) The efficient trade-off set for the quart-level problem is:

$$\Lambda = \{\ x_H, x_{CH}, x_{NE}, x_{Oth}\ :\ x_H, x_{CH}, x_{NE}, x_{Oth} \in argmin[F_H(x_H, x_{CH}, x_{NE}, x_{Oth}):\ x_H, x_{CH}, x_{NE}, x_{Oth}\ \in R]\}$$

In layman's terms, Definition 2.(x) states that if $x_H, x_{CH}, x_{NE}, x_{Oth}$ belong to the inducible region and provides a Pareto optimal solution for the historic level, then it is a Pareto optimal solution for the quart-level problem (Chang and Luh, 1982; Lu et al., 2016).

4.2.1. Discussion

The multi-level formulation provided in (4.2) indicates that in order to find the most beneficial schedules, there is need to consider all the feasible combinations of the decision variables that yield minimum values for the objectives of each level. Moreover, we chose to address explicitly the historic slot priority to provide a robust modelling framework. We mention that because in cases where the capacity of the airport is inferior to the capacity during the previous scheduling season, it will be meaningful to solve for this priority level as well. Obviously, in the general case where the capacity remains the same, the historic slots will be allocated having zero displacement. In any case, our modelling framework allows the consideration of both options.

Multilevel optimisation models are not malleable by standard mixed integer programming techniques and software. Moreover, there are no universal efficient algorithms that can aid in their solution (Cappanera and Scaparra, 2010). The exploration of all feasible points even for the single objective problem is a rather demanding task in terms of computational demands having a complexity of $O(|X^H||X^{CH}||X^{NE}||X^O|)$. In the literature, quart-level, multi-objective models are treated via fuzzy, heuristic or goal programming solution approaches (Sakawa and Nishizaki, 2012), which can be also applied for the solution of the ML-TOSAM. However, in the future, with the technological progression and the upgrades of standard computational machines, it would be interesting to see exact methods implementing and solving the ML-TOSAM.

In the next section, we present a recursive search algorithm along with some hybrid quasi-exact solution approaches which constrain the feasible space of each decision level, thus resulting in reduced computational times and complexity abiding by the decision horizon's requirements. The proposed solution approaches exploit the slot hierarchies in order to reduce the number of follower's strategies to be evaluated.

5. Solution approach

Multi-level approaches explore greater proportions of the feasible decision space but come with increased computational complexity. This issue can be resolved by restricting the feasible space of each level with sensible constraints according to the operational requirements of the real-world problem (e.g. upper bound to the maximum displacement objective, or a threshold to fairness). Yet, one issue renders the solution generation more difficult; in multi-level approaches the generation of Pareto optimal solutions is more complex.

To tackle this issue, in the following sections we will try to respond to the following questions: How to produce efficient solutions that capture the compromises of the leading decision entities? Can we produce multi-level, multi-objective efficient solutions? How can we reduce the feasible space of the decision levels without losing meaningful solutions?

5.1. Preface and definitions

We first have to provide some notions and their definitions. To the best of our knowledge, the only work discussing Pareto optimality in Stackelberg games is the paper of Migdalas (1995). Yet, in order to illustrate the validity of our models, the definitions for Pareto optimality provided in this paper have to extended and modified since they only consider continuous, single objective, bi-level problems.

DEFINITION 3.
The TOSAM of decision level γ is $\min_{x_\gamma \varepsilon X^\gamma} F_\gamma(x_\gamma)$, where $X^\gamma \subseteq \mathbb{E}^{|M^\gamma|}$ is restricted by a set of constraints $g_\gamma(x_{\gamma-q}, \dots, x_\gamma)$ and $F_\gamma(x_\gamma)$ is a vector of three linear functions such that $F_\gamma(x_\gamma) = \{Z_1(x_\gamma), Z_2(x_\gamma), Z_3(x_\gamma)\}$ with $\mathbb{Z}^{|M^\gamma|} \to \mathbb{R}^3$ being the feasible set in the criterion space.

Please observe that the objective functions are now defined only based on the decision variables of the current level. That is because in the solution approaches that we will present (except for Algorithm 2), we allocate only the slots of the current decision level, who are considered fixed in the following decision levels. That is a hybrid approach between the hierarchical solution algorithm of Zografos et al. (2012) and the multi-level formulation of Section (4.2). To apply a genuine multi-level solution approach, for each level we should allocate all slots, yet calculating the objective values only for the slots belonging to the current priority level. Based on this setting, by adjusting the notion of Pareto (non-dominated) efficiency to our problem typology we get:

DEFINITION 4.
We will refer to solutions $x'_\gamma \varepsilon X^\gamma$ as *weakly efficient*, if there is no other $x_\gamma \varepsilon X^\gamma$ such that $Z_i(x_\gamma) < Z_i({x'}_\gamma), \forall\ i \in \{1,2,3\}$. Given that x_γ is weakly efficient, then $F_\gamma({x'}_\gamma)$ is a *level-based weakly nondominated* point.

DEFINITION 5.
For solutions $x^*_\gamma \varepsilon X^\gamma$ that there is no other point $x_\gamma \varepsilon X^\gamma$ such that $Z_i(x_\gamma) \leq\ Z_i({x^*}_\gamma), \forall\ i \in \{1,2,3\}\ and\ F_\gamma(x_\gamma) \neq\ F_\gamma({x^*}_\gamma)$, we will refer to them as *level-based Pareto optimal* (or efficient or non-dominated). Obviously if x^*_γ is Pareto optimal then $F_\gamma({x^*}_\gamma)$ is a *nondominated* point.

PROPOSITION 1.

> For a level γ, if $\Phi^\gamma \neq \emptyset$, and $F_\gamma({x^*}_\gamma)$ is nondominated point then all the aggregate solutions given by lower levels based on this point, are *nondominated points based on level* γ.

Proof. Follows immediately by Definitions 4 and 5.

DEFINITION 6.
For each rolling solution of Γ levels, $\forall\ \gamma\ \exists\ x^*_\gamma \varepsilon X^\gamma$ that is a nondominated solution based on level γ, then the schedule-wide solution is *nondominated* based on level γ.

PROPOSITION 2.

> For a level γ, if all the solutions given by $x_\gamma \varepsilon X^\gamma$, $\forall\ \gamma \in \Phi^\gamma$ are dominated points then all schedule-wide solutions containing $F_\gamma(x_\gamma)$ are level-based dominated.

Proof. Proposition 2 is proved by contradiction to Definition 6.

PROPOSITION 3.

> For a level γ, if $\Phi^\gamma = \emptyset$, and $F_\gamma({x^*}_\gamma)$ is a nondominated point and all $x_\gamma, \gamma \in L^\gamma$ are dominated, then all schedule-wide solutions containing this point are *nondominated points based on level* γ.

Proof. Follows immediately by Definition 6.

The arguments concluded by Definitions 3-6 and Propositions 1-3 can be non-technically summarised in Table 4, framing the analysis of the trade-offs between the objectives of the different decision entities. The table concludes that the only occasion to have a dominated level-based schedule-wide solution is to have dominated solutions for each of the levels (Proposition 2). Meanwhile, all other scenarios where there is at least one nondominated level, yield scheduling solutions which are level-based nondominated. To explain, even if schedule-wide objective values are not Pareto optimal, we do not know the priority that the decision maker(s) assign to each decision entity. Under this prism, even dominated solutions may be meaningful if the objective values of a level abide by the stakeholders' needs.

Level	**Slot Priority**	**Cases**						
		1	2	3	4	…	15	16
1	H	D	ND	ND	ND	…	D	D
2	CH	D	ND	D	ND	…	D	ND
3	NE	D	ND	D	D	…	ND	ND
4	Oth	D	ND	D	D	…	ND	ND
All	Level based optimality	D	ND	ND	ND	…	ND	ND
All	Schedule-wide optimality	D	ND	D/ND	D/ND	…	D/ND	D/ND
Notes:	*Dominated or Weakly dominated/ Nondominated* (D/ND), *Aggregate slot schedule considering all levels* (Schedule-wide)							

Table 4: *Summary of solution optimality cases*

However, given that the slot typologies are already prioritised by the hierarchical allocation, we may assume that the goal of the problem is the generation of all schedule-wide Pareto efficient solutions no matter what the Pareto status of the levels is. By building on previous arguments, we address this setting in Proposition 4.

PROPOSITION 4.

If a schedule-wide solution has for all $\gamma \in \Gamma$ nondominated points $F_\gamma(x^*{}_\gamma)$, then it is a *nondominated aggregate (or schedule-wide) solution*.

Proof. The result follows immediately by Definition 6 and Proposition 3.

Obviously, if a schedule-wide solution results for some $\gamma \in \Gamma$, in level-based nondominated points $F_\gamma(x^*{}_\gamma)$, then it is not necessarily a Pareto optimal schedule-wide solution as there may exist other solutions resulting in the same or better objective values. Therefore, the set of level-based non-dominated solutions is a superset of the schedule-wide nondominated solutions. As a result, in order to parse the whole Pareto frontier, we have to generate the level-based Pareto frontier and then filter out dominated and weakly dominated schedule-wide (aggregate) solutions. This idea sparkled the conceptualisation of the solution algorithms devised in Section 5.2.

5.2. Solution algorithms

The solution approaches that we propose in this section are quasi-exact in the sense that they do not examine the entire decision space. They do so by taking advantage of the multi-level modelling in order to introduce additional bounds to the objectives of each level, hence considering additional policy rules. Moreover, they are hybrid as they employ a hierarchical approach similar to current practice (Ribeiro et al., 2018; Zografos et al., 2012), yet they allow compromise at the upper decision entities.

input: Γ, u
output: Y *# set of efficient multi-level trade-offs*

$\Lambda \leftarrow ()$; *# data structure with Pareto solutions of all levels*
$FO \leftarrow ()$; *# data structure with utopian and negative utopian solutions of all levels for each objective*
$Y \leftarrow ()$;
for $\gamma \in \Gamma$ **repeat**:
  $\Lambda.add(\Lambda_\gamma)$; *# Add the nondominated points of γ to Λ*
for $\gamma \in \Gamma$ **repeat**:
  for $i \in [1,2,3]$ **repeat:**
    $\min_\gamma Z_i \leftarrow \infty, \max_\gamma Z_i \leftarrow 0$;
    for $\lambda \in \Lambda$ **repeat**:
      if $\lambda_\gamma^{Z_i} \leq \min_\gamma Z_i$ **then:** *#$\lambda_\gamma^{Z_i}$ is the value of Z_i for level γ*
        $\min_\gamma Z_i \leftarrow \lambda_\gamma^{Z_i}$;
      if $\lambda_\gamma^{Z_i} \geq \max_\gamma Z_i$
        $\max_\gamma Z_i \leftarrow \lambda_\gamma^{Z_i}$;
for $\gamma \in \Gamma$ **repeat**:
  $FO_\gamma^-.add\,(\max_\gamma Z_1, \max_\gamma Z_2, \max_\gamma Z_3)$; *# maximum values of objectives for level γ*
  $FO_\gamma^+.add\,(\min_\gamma Z_1, \min_\gamma Z_2, \min_\gamma Z_3)$; *# minimum values of objectives for level γ*
  $FO_\gamma \leftarrow (FO_\gamma^+, FO_\gamma^-)$
Define: *recursive ML-search* (γ, FO_γ, u)
  if $\gamma \notin \Gamma$ **then**:
    Stop; *# termination of Algorithm 2*
  else:
    for $\varepsilon_{Z_1} \in \left[\min_\gamma Z_1, \max_\gamma Z_1\right]$ **repeat**:
      for $\varepsilon_{Z_2} \in \left[\min_\gamma Z_2, \max_\gamma Z_2\right]$ **repeat**:
        for $\varepsilon_{Z_3} \in \left[\min_\gamma Z_3, \max_\gamma Z_3\right]$ **repeat**:
          $s \leftarrow TOSAM(cap = cap, reqs = M_\gamma, Z_1 = \varepsilon_{Z_1}, Z_2 = \varepsilon_{Z_2}, Z_3 = \varepsilon_{Z_3})$;
          $cap.update(s)$;
          if $\gamma = Oth$ **then**:
            $cap \leftarrow u$;
          if s is feasible **then**:
            $Y.add\ s$;
  return Y;
  recursive ML-search $(\gamma.next(), FO_{\gamma.next()}, u$= cap);
$Y \leftarrow$*recursive ML-search* $(\gamma = H, FO_H, u = u)$

Notes: # stands for comments on the algorithmic process; in **bold** are common algorithmic functions; solution of the tri-objective slot allocation model with rolling capacity u, priority requests γ, and equality constraints for the three objectives $\varepsilon_{Z_i}, i = 1,2,3$ respectively $\{TOSAM(cap = u, reqs = M_\gamma, Z_1 = \varepsilon_{Z_1}, Z_2 = \varepsilon_{Z_2}, Z_3 = \varepsilon_{Z_3})\}$.

Algorithm 2: *An exhaustive multilevel algorithm for the ML-TOSAM*

Algorithm 2 can employ any efficient tri-objective solution algorithm to find the non-dominated points for each level (Λ_γ) by allocating all the slots while considering in the objective functions only the requests belonging to the current level (Stage 1). Then, from the set of the nondominated points, it reports the minimum and maximum values of the objectives for each level (lines 6-9). Finally (Stage 2), it iterates through all the combinations of the objectives in order to populate the list of the non-dominated multi-level trade-offs. Given that we use an efficient tri-objective solution algorithm, we should expect a maximum complexity of $O(3ND_{\gamma})$ for each level at the first stage (Boland et al., 2017) (where ND_{γ} is the set of non-dominated points in level γ when solving for this specific level independently). While for the second stage, we would have $O(W^{\gamma}_{Z_1} \times W^{\gamma}_{Z_2} \times W^{\gamma}_{Z_2})$ for each level with $W^{\gamma}_{Z_i}, \forall\ i = 1{,}2{,}3$ being the width of objective Z_i for level γ such that $W^{\gamma}_{Z_i} = \max_{\gamma} Z_i - \min_{\gamma} Z_i$. Therefore the total complexity would be $O\left(3(ND_H + ND_{CH} + ND_{NE} + ND_{Oth}) + \prod_{\gamma \in \Gamma}\left(W^{\gamma}_{Z_1} \times W^{\gamma}_{Z_2} \times W^{\gamma}_{Z_2}\right)\right)$. Algorithm 2 is uneconomic as the number of integer programs to be solved is impractical. In the bibliography the computational hazard highlighted above is addressed with the use of fuzzy sets and heuristic approaches (Lu et al., 2016).

In response to the complexity of Algorithm 2, we tailored a hybrid solution algorithm (Algorithm 3[6]) which is based on the hierarchical solution approach currently followed by the main corpus of the literature. However, based on the level-based point efficiency (Definitions 4,5), instead of searching for Pareto optimal solutions at each level, it filters out dominated and weakly dominated solutions only after solving the bottom decision entity. In essence, we report additional nondominated points that would not be explored with the pure sequential and lexicographic approaches. This solution algorithm is inspired by the definitions and propositions of Section 5.1, introducing the notion *of inter-level tolerance*.

To elaborate on this statement, the leading levels tolerate losses on their objective functions' values if they reap better solutions at the lower levels. As per Definition 6, if one of the following levels gets an efficient solution then the aggregate schedule is nondominated. In more detail, for each level $\gamma \in \Gamma$ and each discretised level of fairness $\delta \in \Delta$, Algorithm 3 calculates the range of efficient values of the maximum displacement objective while keeping fairness less or equal to δ. Then for each of the efficient values it minimises schedule displacement by restricting the values of the other two objectives by the current fairness and maximum displacement levels. This approach is inspired by the Quadrant Shrinking Method (QSM) of Boland et al. (2017).

QSM is based on a nondominated two-dimensional point search (2-DNP-Search) which is composed by two steps. Firstly, it minimises the value of one of the objectives by specifying an upper bound constraining the other two objectives, thus resulting in weakly dominated solutions (*intermediate point*). Then, by minimising each of the objectives bounded by the objective function values of the intermediate point, it guarantees that each feasible solution point is a nondominated point. However, by applying the proposed principle of inter-level tolerance to our algorithm, we only solve the first part of the 2-DNP-Search and we filter out dominated aggregate solutions at the last level of the solution process.

The complexity of Algorithm 3 is significantly less than the one of Algorithm 2 $\left(O\left(\prod_{\delta \in \Delta}\prod_{\gamma \in \Gamma}\left[\left(UB^{\gamma,\delta}_{Z_2} - LB^{\gamma,\delta}_{Z_2}\right)\right]\right)\right)$. Yet, even this complexity may be intractable for large datasets. The algorithm can be further relaxed by setting $Z_3 = None$ in lines 6 and 7. Under this

[6] In line 8, the algorithm sets the upper bound of the maximum displacement objective to be either one hour or equal to the lower bound. This command allows the consideration of section 8.3.5.4 of IATA's WSG (see Section 2.1.) enhancing schedule acceptability for the new entrant slot priority.

option, we calculate the efficient bounds of maximum displacement without taking into account fairness considerations. Then, we check if the current maximum displacement value is attainable under the current fairness level. This relaxed version of Algorithm 3 prioritises the maximum displacement objective generating the set of efficient points based only at the maximum displacement objective. The set of points within the $\left[LB_{Z_2}^{\gamma,\delta}, UB_{Z_2}^{\gamma,\delta}\right]$ with fairness considerations is a superset of $\left[LB_{Z_2}^{\gamma}, UB_{Z_2}^{\gamma}\right]$ having $UB_{Z_2}^{\gamma,f} \gg UB_{Z_2}^{\gamma}$ $\left(\left[LB_{Z_2}^{\gamma,\delta}, UB_{Z_2}^{\gamma,\delta}\right] \supseteq \left[LB_{Z_2}^{\gamma}, UB_{Z_2}^{\gamma}\right]\right)$. The complexity of this variant of Algorithm 3 (variant 1) is now reduced to $O\left(\prod_{\gamma\in\Gamma} \Delta^4\left[\left(UB_{Z_2}^{\gamma} - LB_{Z_2}^{\gamma}\right)\right]\right)$. This algorithm takes into account the needs of airlines as it iterates on the maximum displacement values that are Pareto optimal, lower and operationally viable for them. This consideration is coherent with current practice and the priority given to maximum displacement in the lexicographic model of Ribeiro et al. (2018).

	input: Γ, u, Δ *# levels to be considered, capacity, discretized fairness levels accordingly* **output**: Y *# reduced set of efficient multi-level trade-offs*
1	**Define**: *heuristic ML-TOSAM*(γ, u, Δ):
2	**if** $\gamma \notin \Gamma$ **then**:
3	**Stop**; *# termination of Algorithm 3*
4	**else**:
5	**for** $\delta \in \Delta$ **repeat**: *# for each fairness level that is to be parsed*
6	$LB_{Z_2}^{\gamma,\delta} \leftarrow TOSAM(Z_2, cap = u,\ reqs = M_\gamma, Z_1 = None, Z_2 = None, Z_3 = \delta).Z_2;$
7	**if** $\gamma = NE$ **then**:
8	$UB_{Z_2}^{\gamma,\delta} \leftarrow max(1\ hour, LB_{Z_2}^{\gamma,\delta});$
9	**else**:
10	$UB_{Z_2}^{\gamma,\delta} \leftarrow TOSAM(Z_1, cap = u,\ reqs = M_\gamma, Z_1 = None, Z_2 = None, Z_3 = \delta).Z_2;$
11	**for** $i \in (UB_{z_2}, LB_{Z_2})$ **repeat**:
12	$s \leftarrow TOSAM(Z_1, cap = u, reqs = M_\gamma, Z_1 = None, Z_2 = i, Z_3 = \delta);$
13	**if** s is feasible **then**:
14	**if** $\gamma = Oth$ **then**:
15	$cap \leftarrow u;$
16	**if** $i \neq s.Z_2$ **then**: *# it means that is i not binding the value of Z_2*
17	**Break**; *# stop loop*
18	**else if** s is not dominated **then:** *# according to Definition 6*
19	$Y.add(s);$
20	**else**:
21	$Y.add(s);$
22	$cap.update(s);$
23	**return** *Y;*
24	*heuristic ML-TOSAM*($\gamma.next(), u\ = cap, \Delta$);
25	$\mathrm{Y} \leftarrow heuristic\ ML - TOSAM(\gamma = H, u, \Delta);$
Notes:	# stands for comments on the algorithmic process; in **bold** are common algorithmic functions; solution of the tri-objective slot allocation model minimising objective Z_1 with rolling capacity u, priority requests γ, and equality constraints for the three objectives $\varepsilon_{Z_i}, i = 1,2,3$ respectively $\{TOSAM(Z_1, cap = u, reqs = M_\gamma, Z_1 = \varepsilon_{Z_1}, Z_2 = \varepsilon_{Z_2}, Z_3 = \varepsilon_{Z_3})\}$.

Algorithm 3: *A tolerance-based algorithm for the ML-TOSAM*

We may further reduce the complexity of the algorithm by imposing uniform fairness considerations among all levels. This can be effectuated in Algorithm 3 by removing line 5 and altering Δ input argument to a single fairness value (δ) rather than a list. Then by iterating for all discretised fairness values in Δ, we get a modified version of Algorithm 3 (variant 2) with complexity $O\left(\Delta \prod_{\gamma\in\Gamma}\left[\left(UB_{Z_2}^{\gamma} - LB_{Z_2}^{\gamma}\right)\right]\right)$. Again, this option may be sensible as it ensures that all requests are treated in a non-discriminatory manner since they are allocated based on a single fairness threshold. The characteristics of solution approaches presented in this section can be found summarised in the Table 5.

Algorithm	Assumptions	Complexity
2	Exhaustive with multi-level interactions	$3ND_H + 3ND_{CH} + 3ND_{NE} + 3ND_{Oth} + \prod_{\gamma\in\Gamma}\left(W_{Z_1}^{\gamma} \times W_{Z_2}^{\gamma} \times W_{Z_2}^{\gamma}\right)$
3	Hybrid, tolerance-based sequential allocation ([a])	$\prod_{\gamma\in\Gamma}\prod_{\delta\in\Delta}\left(UB_{Z_2}^{\gamma,\delta} - LB_{Z_2}^{\gamma,\delta}\right)$
3 (*variant 1*)	[a] + maximum displacement prioritisation ([b])	$\prod_{\gamma\in\Gamma}\Delta\left[\left(UB_{Z_2}^{\gamma} - LB_{Z_2}^{\gamma}\right)\right]$
3 (*variant 2*)	[a] + [b] with uniform fairness	$\Delta\prod_{\gamma\in\Gamma}\left[\left(UB_{Z_2}^{\gamma} - LB_{Z_2}^{\gamma}\right)\right]$

Table 5: *Summary of proposed algorithms*

The multi-level modelling of section 4.2 allows Γ to differentiate between PSO, year round operations and single period requests such that $\Gamma: \{PSO, H^{YR}, H - H^{YR}, CH^{YR}, CH - CH^{YR}$, $NE^{YR}, NE - NE^{YR},\ Oth^{YR}, Oth - Oth^{YR}\}$ (with γ^{YR} being the requests of priority γ that are to operate for the whole year and PSO being the slots concerning PSO routes). Such level differentiations introduce an absolute priority for routes with public service obligations and year-round operations as they are allocated before historic and single period requests accordingly. However, this approach leads to great computational burdens due to the introduction of additional hierarchical entities.

A more tractable but more arbitrary alternative to this method would be to consider the index proposed in Section 4.1.2.2 which assigns more priority to year-round operations. The difference between the two alternatives lies on the comprehension of the regulatory framework. If the priority for year-round operations is absolute, then the introduction of additional levels must be preferred over the utilisation of the index. In the opposite case, the service continuity index is a simple yet effective weighting approach.

Given the fact that the latter variant is less complex but in accordance with the regulatory framework, current practice and solution approaches; we chose to base our computational efforts and validate our multi-level considerations based on this approach. Section 6 includes a brief presentation of the dataset and discussion on the output of the selected algorithmic approach.

6. Case study

The solution algorithms that we solve, are implemented and tested in Python using Gurobi 8.0.1 (Gurobi Optimization, 2018) as our integer programming solver. The reported computational experiments were conducted on a computer having a 2.5-gigahertz Intel® i7-4710MQ central processing unit and 15.8 gigabyte of RAM, running on the home edition of Windows 10.

6.1. Data

For benchmarking reasons, the datasets used in this case study are identical to those used in previous works (Fairbrother and Zografos, 2018a; Zografos et al., 2017a). The datasets concern the summer scheduling season of 2009 (from the 29th of March to the 26th of October) at a medium sized regional European airport. The declared capacity of the airport is known for both the 15-minute and the 1-hour capacity scales and can be seen in Table 6.

Runway movement type	15 min	60 min
Arrivals	-	4
Departures	-	6
Total	3	10

Table 6: *Airport declared capacity*

In the focal airport, the declared capacity imposes that within an hour timeframe (e.g. 10:55 – 11:55) no more than 10 movements can be scheduled in total, while for the 15-minute time scale, no more than 3. The declared capacity also sets the upper bound for the arrivals and the departures that can be effectuated per hour (four and six accordingly).

Priority	Code	Request series		Individual requests		Average slots per request
		#	%	#	%	
Historic	F	126	28.1%	4304	28.0%	34.2
Other	N	222	49.5%	7412	48.1%	33.4
Changes to historic	R	55	12.2%	2264	14.7%	41.2
	L	22	4.9%	748	4.9%	34
New entrant	B	24	5.3%	660	4.3%	27.5
All	Total	449	100%	15388	100%	34.3
Notes:	Changes to historic requests that are willing to accept slot times between the historic or the requested time (R), changes to historic requests that will only accept the historic slot if the requested time is not available, percentage (%), number (#)					

Table 7: *Requests per priority*

The distribution of the requests per priority type is given in Table 7, where the historic requests account for 28%, the changes to historic 17%, the new entrants 5.3% and the remaining requests reach almost half of the total number of requests. It is interesting that airlines with historic requests falling into category R (willing to accept times between the historic and the requested slot) tend to request more single slots per request on average (41.2) than any other slot priority. Moreover, the new entrants' requests are the fewest (4.3%) among all priority classes and they request the smallest number of individual slots per request (27.5).

The absence of year-round requests (action codes I, Y, V) indicates that the focal airport is schedule coordinated (Level 3 airport based on IATA's WSG) only during the summer period. This is also explained by the fact that the airport is in a remote touristic island that is characterised by summer seasonality. Moreover, the airport is not part of any PSO routes. Hence, we can't consider the year-round and the PSO priorities specified earlier in the paper.

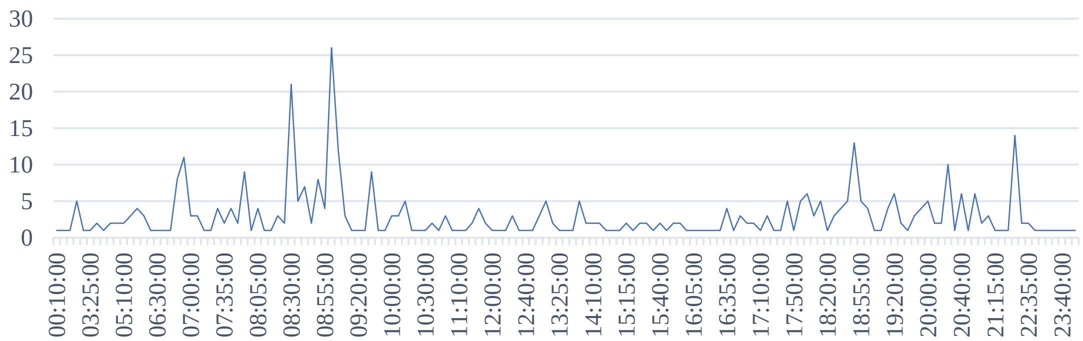

Figure 1: *Distribution of requests per time interval*

In Figure 3, we may see the frequency of requests throughout the complete scheduling period. By observing this frequency chart, most of the supply-demand imbalances will occur during periods with numerous requests (e.g. 08:30:00 – 09:20:00).

Having the characteristics of the set of requests and the airport in mind, we may proceed with the presentation and analysis of our computational results.

6.2. Computational results

In order to test the notion of inter-level tolerance, we solved for the second variant of Algorithm 3. This algorithm treats all slot priorities with the same fairness threshold guaranteeing equal treatment among all requests. Moreover, it prioritises the maximum displacement objective over fairness to ensure slot acceptability and reduce the algorithmic complexity.

For comparison purposes we solved the selected algorithm both with and without inter-level tolerance considerations. Hereafter, we will refer to the variant without tolerance considerations as "*no-tolerance*". The reported solutions for each variant can be found in tables A1 and A2 of the appendix. The no-tolerance approach is equivalent to current practice as it proceeds to the solution of lower levels only if all leading levels yield Pareto optimal solutions.

A general observation occurring by examining Tables A1 and A2 is that the allocation of historic slots could not be effectuated without displacement. By observing the action codes of the requests and comparing the number of historic requests to the declared capacity, we found out that there are several periods where the number of historic requests exceeds the declared capacity of the airport.

For both approaches, we considered a range (Δ) of discretised fairness levels ranging from 0 to 2 with a step of 0.1. The priority levels that we considered are $\Gamma: \{H, CH, NE, Oth\}$. However, by observing tables A1 and A2, we can see that both approaches resulted in feasible solutions for $Z_3 \geq 0.9$. In detail, for $Z_3 = 0.1$ and $Z_3 = 0.2$ the H level could not yield feasible solutions while for $Z_3 \in [0.3, 0{,}9$ the CH level could not solve to optimality. That is because CH level is composed by a small number of requests most of whom are on peak times. Due to the fractional nature of the fairness index (μ_a), if an airline has a small number of requests within a priority, the threshold value for Z_3 cannot be easily attained. For instance, if airline a has submitted only one peak request in slot priority γ, then for this level γ the proportion of its peak requests is a positive float close to zero $0 \leq \rho_a \ll 1$. That means, that in order to have $\mu_a \leq \varepsilon_{Z_3} = 0.9$, the proportion of the displacement that airline a gets should be close to zero $\left[0.9\rho_\alpha \left(\sum_{m\in M}\sum_{t\in T}|t - t_m|\, x_{t,m}\right) \geq \left(\sum_{m\in M_\alpha}\sum_{t\in T}|t - t_m|\, x_{t,m}\right)\right]$, which is something that is not feasible for all airlines simultaneously. To overcome this problem, we may aggregate the H and CH into a single level [similar to Fairbrother and Zografos (2018a)].

Under this setting, some preliminary results of ours have shown that we may get feasible solutions of the TOSAM for $Z_3 \geq 0.2$. Yet, merging priority levels does not abide by IATA's slot prioritisation. Another way to address this problem is to consider differentiated fairness thresholds for each priority level (1st variant of Algorithm 3). Again, we have demonstrated that this alternative is of great computational complexity while it does not satisfy the non-discriminatory treatment of the requests. As a result, in the remainder of this section we will analyse our results based on the second variant of Algorithm 3.

The approach considering inter-level tolerance reported 25 non-dominated schedule-wide solutions. To generate this set of nondominated points, the algorithm examined 958 feasible airport slot schedules of whom 557 where level-based non-dominated. The runtime of the solution process totalled almost 99.1 hours (4.13 days). On the other hand, the no-tolerance approach examined 394 level-based, non-dominated feasible solutions, reporting 22 efficient schedules. Its total runtime was 39.5 hours (1.65 days).

Tolerance		Measure		Z_1	Z_2	Z_3
No		Mean		18634.5	15.59	1.1
		95% CI	LB	17269	14.76	1.03
			UB	20000	16.42	1.16
		Median		17164	15.5	1.1
		Min		16850	13	0.9
		Max		25272	19	1.4
		Range		8422	6	0.5
Yes		Mean		18514.52	15.08	1.11
		95% CI	LB	17328.87	14.33	1.05
			UB	19700.17	15.83	1.17
		Median		17234	15	1.1
		Min		16850	13	0.9
		Max		25272	18	1.4
		Range		8422	5	0.5

Notes: 95% confidence interval (95% CI), Upper/ Lower Bound (UB/ LB)

Table 8: *Descriptive statistics of the results of each approach*

By observing the reported computational times, it is obvious that the method that we proposed is significantly more complex and time consuming. However, the decision horizon of the problem (22 days) dictates that this is a bearable cost. In addition, the increased computational times of our method, are acceptable as the reported nondominated points are on average better that those given by the no-tolerance method (Table 8).

The method with tolerance considerations yielded on average lower values for both objectives Z_1(1.7% reduction) and Z_2 (3.3% reduction), while for Z_3 it resulted in an imperceptible increase of 0.9 %. The average quantities that we report are also more robust for the tolerance-based approaches due to the increased number of reported solutions. Moreover, the value range of tolerance-based Z_2 has a lower upper bound (18) than that of the no-tolerance solution (19). The above observations are summarised in boxplots in Figure 4.

Figure 5 is a more detailed comparison between the two frontiers. Please observe that most of the points yield similar values. However, the no-tolerance efficient points (small markers), for the same levels of fairness and maximum displacement, result in subtly worse schedule displacement values than their tolerance-based counterparts (large markers). Therefore, we may conclude that the solutions produced by the tolerance-based algorithm dominate those produced by the no-tolerance approach. These findings indicate the need of inter-level compromises between the objectives of the different slot priorities. By allowing further sacrifices by the upper levels, we may reap better aggregate results, hence ameliorating the welfare offered by the nondominated solutions independently or as a set. More intensive solution algorithms (e.g. Algorithm 2) can serve the inter-level philosophy better but with an increased cost in terms of temporal and computational resources. Alternatively, the preferences of the stakeholders regarding the demanded sacrifices can be addressed through goal programming and fuzzy solution methods.

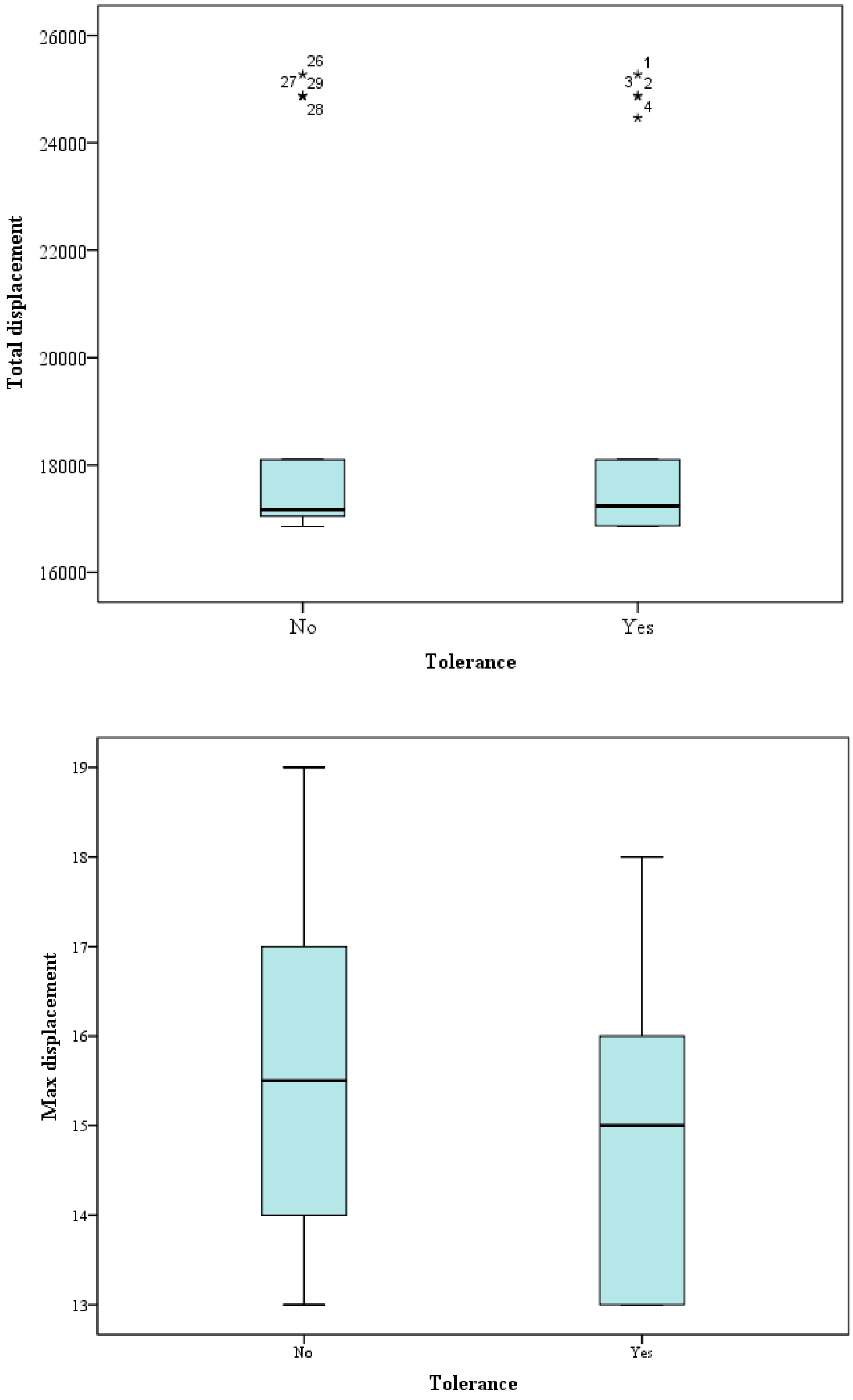

Figure 4: *Comparative boxplots for objectives Z_1 and Z_2*

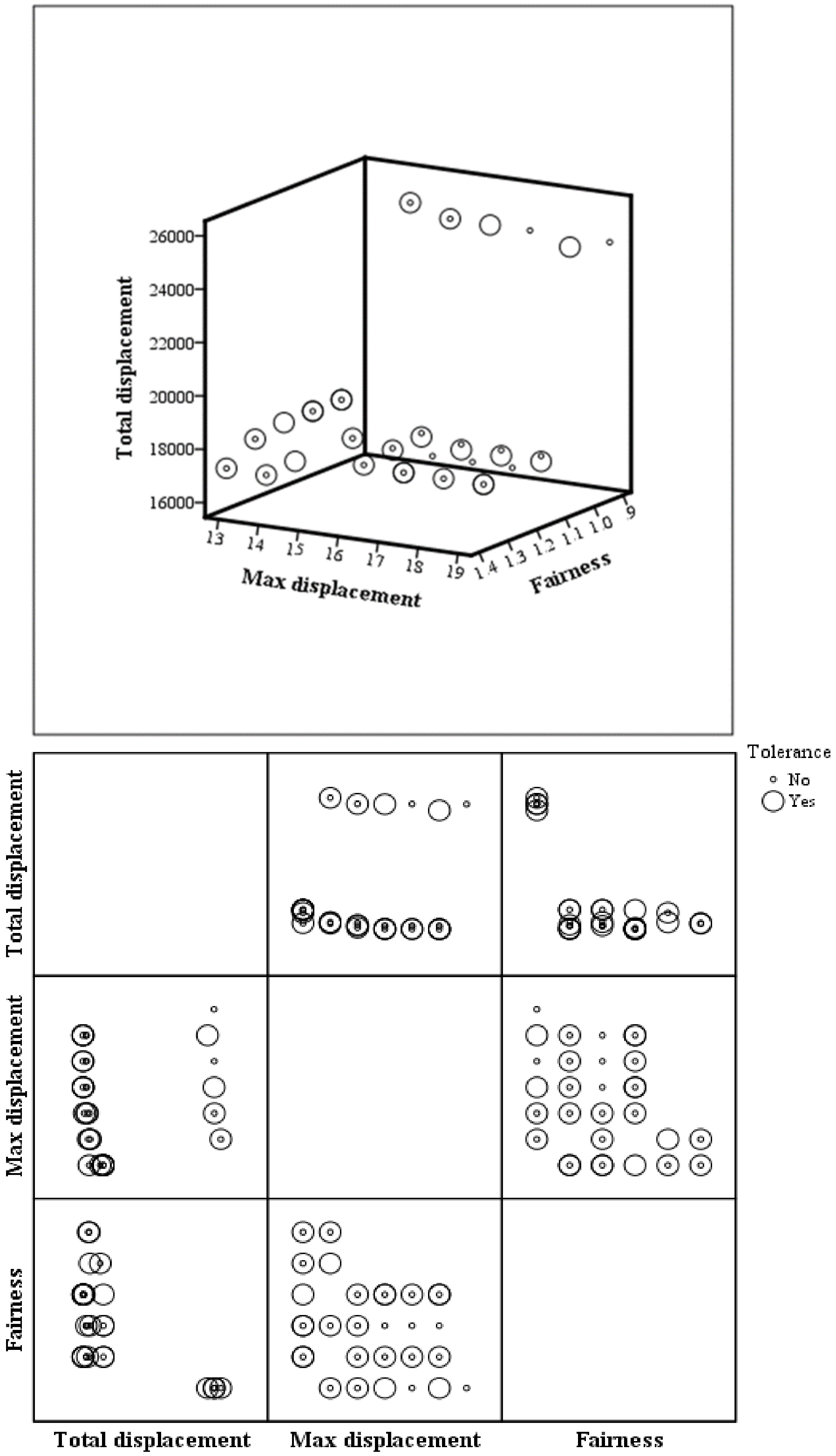

Figure 5: *3D and pairwise 2D comparisons of the two approaches*

Now that we have proved that tolerance considerations yield better slot scheduling solutions, we can analyse in further detail the resulting Pareto set. In Figure 6 we present in a value path chart the normalised trade-offs among the schedule-wide objective values of the efficient solution set.

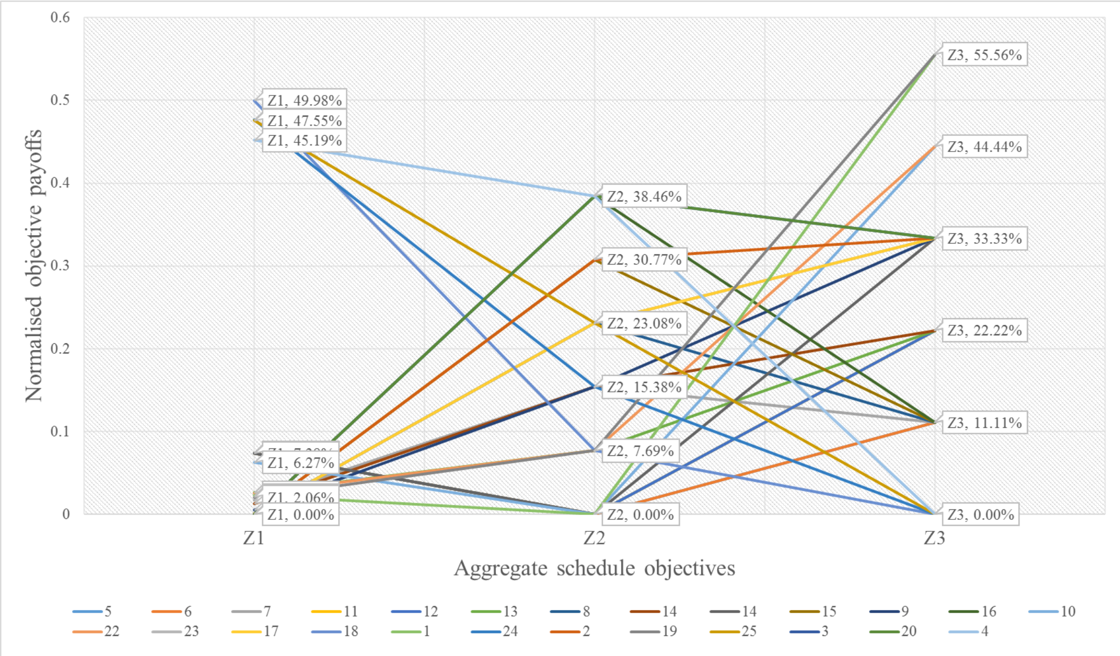

Figure 6: *Schedule-wide value path for tolerance-based approach*

Observe that schedules with low values for one or two objectives result in huge sacrifices in terms of the third objective. For instance, schedule 13 (light green line) has low Z_1 and Z_2 values but results in significantly higher Z_3. Similarly, schedule 16 (olive green line) has low values for Z_1 and Z_3 but high Z_2. Such trade-offs render the selection of a single slot schedule a non-trivial task.

A step towards a more detailed analysis of the trade-offs is to consider the additive sacrifices that each solution yields for the objectives. In Table 9, we have ranked the efficient schedules based on their additive normalised deviation from the minimum values of each objective. Moreover, we have included colour indications (red and green) representing the distance of each table cell from the minimum value of each column. Overall, the solutions that have the least deviation from the minimum values of the each column are schedules 5 and 6. Figure 6 also validates this result.

These two schedules have similar deviations in most of the levels' (CH, NE, and Oth) and aggregate objectives. However, in the historic level, they differentiate significantly in terms of the values of objectives Z_1 and Z_2. If we consider that stakeholders may assign different importance to each objective or individual level, the selection of a commonly accepted solution gets more difficult.

Given the above, the heterogeneity between the Pareto optimal schedules rousts a last series of questions. How to select the most beneficial solution? What is the least disruptive or the most commonly accepted trade-off? How each stakeholder evaluates the importance of each level and objective? We attempt to address this series of questions in the following section, where we present two synergetic approaches between the MADM and MOO literature.

γ	H		CH		NE		Oth		Schedule-wide			Additive deviation
ID	Z_1	Z_2	Z_1	Z_2	Z_1	Z_2	Z_1	Z_2	Z_1	Z_2	Z_3	
5	0.43%	0.00%	0.13%	66.67%	0.00%	14.29%	8.27%	0.00%	7.40%	0.00%	11.11%	18.51%
6	0.00%	50.00%	0.13%	66.67%	0.00%	14.29%	8.28%	0.00%	7.40%	0.00%	11.11%	18.51%
7	0.00%	100.00%	0.13%	66.67%	0.00%	14.29%	1.87%	15.38%	1.67%	15.38%	11.11%	28.16%
11	0.43%	0.00%	0.00%	66.67%	0.00%	14.29%	8.27%	0.00%	7.39%	0.00%	22.22%	29.62%
12	0.00%	50.00%	0.00%	66.67%	0.00%	14.29%	8.28%	0.00%	7.39%	0.00%	22.22%	29.62%
13	0.43%	0.00%	0.00%	66.67%	0.00%	14.29%	2.96%	7.69%	2.65%	7.69%	22.22%	32.56%
8	0.00%	100.00%	0.13%	66.67%	0.00%	14.29%	0.09%	23.08%	0.08%	23.08%	11.11%	34.27%
14	0.00%	100.00%	0.13%	66.67%	0.00%	14.29%	1.43%	15.38%	1.28%	15.38%	22.22%	38.89%
15	0.00%	100.00%	0.00%	66.67%	0.00%	14.29%	8.26%	0.00%	7.38%	0.00%	33.33%	40.71%
9	0.00%	100.00%	0.13%	66.67%	0.00%	14.29%	0.04%	30.77%	0.04%	30.77%	11.11%	41.92%
16	0.00%	50.00%	0.00%	66.67%	0.00%	14.29%	0.59%	15.38%	0.52%	15.38%	33.33%	49.24%
10	0.00%	100.00%	0.13%	66.67%	0.00%	14.29%	0.01%	38.46%	0.01%	38.46%	11.11%	49.58%
22	0.43%	0.00%	0.00%	66.67%	0.00%	14.29%	7.01%	0.00%	6.27%	0.00%	44.44%	50.71%
23	0.00%	100.00%	0.00%	66.67%	0.00%	14.29%	2.90%	7.69%	2.59%	7.69%	44.44%	54.72%
17	0.43%	0.00%	0.00%	66.67%	0.00%	14.29%	0.08%	23.08%	0.07%	23.08%	33.33%	56.48%
18	0.00%	50.00%	0.00%	66.67%	0.00%	14.29%	0.09%	23.08%	0.07%	23.08%	33.33%	56.48%
1	142.31%	100.00%	557.72%	33.33%	23.92%	42.86%	24.17%	7.69%	49.98%	7.69%	0.00%	57.67%
24	0.43%	0.00%	0.00%	66.67%	0.00%	14.29%	2.55%	0.00%	2.28%	0.00%	55.56%	57.83%
2	142.31%	100.00%	557.72%	33.33%	46.95%	0.00%	20.37%	15.38%	47.66%	15.38%	0.00%	63.05%
19	0.43%	0.00%	0.00%	66.67%	0.00%	14.29%	0.03%	30.77%	0.02%	30.77%	33.33%	64.13%
25	0.43%	0.00%	1.70%	0.00%	0.00%	14.29%	2.22%	7.69%	2.06%	7.69%	55.56%	65.31%
3	142.31%	100.00%	557.72%	33.33%	46.95%	28.57%	20.25%	23.08%	47.55%	23.08%	0.00%	70.63%
20	0.00%	50.00%	0.00%	66.67%	0.00%	14.29%	0.01%	38.46%	0.00%	38.46%	33.33%	71.79%
21	0.43%	0.00%	0.00%	66.67%	0.00%	14.29%	0.00%	38.46%	0.00%	38.46%	33.33%	71.79%
4	142.31%	100.00%	561.13%	133.33%	16.28%	42.86%	19.03%	38.46%	45.19%	38.46%	0.00%	83.65%

Notes: *The content of each cell is the normalised deviation of the schedule from the minimum value of each column; Green colour means less sacrifice while red means larger distance from the minimum value of the column; with the term "Additive deviation" we define the simple addition of the normalised deviation of all objectives at the schedule-wide level.*

Table 9: *A heat-map of the normalised trade-offs of the tolerance-based efficient solutions*

7. Synergies of MOO and MADM in airport slot allocation

In previous sections, we have presented and discussed the practical implications of MOO models in airport slot scheduling. The computational results of section 6.2 indicate that such methods produce a set of efficient (nondominated) solutions that provide trade-offs among the considered objectives. In addition, there are still some dimensions of the decision environment, i.e. rules, objectives and relationships, which are not – and maybe cannot be – taken into account with MOO. In contrast, in current practice, the draft schedules composed by the coordinators of each airport, are amended and finalised through the activities that take place in the bi-annual SCCs. In the SCC, all the interested parties participate in order to reach a commonly accepted solution (consensus), always abiding by IATA's WSG. This solution is devised after the consideration of the needs, the objectives and the constraints imposed by the different stakeholders. Through this brief reference on current decision processes, we understand that current MOO slot allocation models are lagging in terms of managerial applicability because of four main reasons.

Firstly, instead of a single solution, they produce multiple equally efficient solutions (1). This is something that perplexes the decision process and does not abide with the real problem's needs. Secondly, due to mathematical, modelling and computational burdens, they do not fully incorporate all of IATA's guidelines (2). This inefficiency means that the proposed schedules may not comply with the regulatory and operational needs of the real world. Third is that current mathematical models cannot consider multiple (more than 3) objectives simultaneously as their complexity would increase dramatically (3). Finally, because there are based on objective metrics, they do not take into account the inherent relationships of the stakeholders and their subjective judgements (4).

Such issues have been addressed in similar real-world problems via the use of MADM methods. A recent literature review indicated that, even though MADM methods are broadly used in the field of air transport, there are currently no applications on the airport slot scheduling problem (Katsigiannis, 2018a). In the remainder of this section, we introduce two novel alternative paradigms making use of one of the most prominent MADM techniques.

7.1. Two alternative approaches

Based on the existing modelling gaps, we were able to devise two two-stage solution approaches that illustrate the potential synergies between current MOO models and the Analytical Hierarchy Process (AHP) (Saaty, 1989). The main advantage of the AHP stems from the fact that it satisfies the requirements of air transport decision-making. Namely, it is transparent, understandable and it may act as a straightforward group decision-making technique. For further details on the suitability of the AHP for the slot allocation problem, the reader may refer to the work of (Katsigiannis, 2018a).

Given the eligibility of AHP we may now propose two alternative hierarchies which address the focal problem from two different aspects. In brief, the first (Section 7.1.1) takes into account the characteristics of the slots of each priority and weights them based on the experience and the judgements of the coordinator (Stage 1). Then, the weights can be used as coefficients in the considered objective functions of the MOO model and produce results which adhere to the expertise of the coordinator and the regulations (Stage 2). The second solution approach that we propose acts as a group decision-making mechanism that ranks the nondominated schedule alternatives (Stage 2) which were given by the used MOO model (Stage 1) based on the views of the multiple participating stakeholders regarding multiple – even unconsidered – objectives

(Section 7.1.2). A brief overview of the main steps of AHP is given in Table A3 of the Appendix.

7.1.1. Using AHP as a weighting method (pre-optimisation)

We have underlined that the incorporation of additional rules in the MOO models will increase their modelling and computational complexity. Nevertheless, we may benefit from the inherent hierarchies that lie within the regulatory framework in order to employ AHP as a weighting mechanism. This will provide relative priority indexes for each slot, which can be then used to determine their significance. Through the review of existing rules, policy documents, research articles and the identification of modelling gaps, we were able to identify the factors that can be used to construct an indicative hierarchy determining the importance of each slot belonging to the set of requests submitted to an airport (Figure 7).

The proposed hierarchical tree is composed by four levels. The upper level represents the objective of the hierarchy. In this case, the objective is to yield the relative weights that each slot gets by considering the criteria and sub-criteria of the lower levels. Let us refer to this objective as "slot priorities" (s). In the second level of the tree are the *level 1 criteria* (identified with s.1.1 – s.1.8). Those entities are defined based on the additional criteria of IATA's WSG [Sections 8.4.1.a-e of IATA (2018b)] most of whom are not currently considered in any of the existing MOO models. Accordingly, the third level includes *level 2 criteria* which are the indicators (or the sub-criteria) of level 1 criteria. This level of criteria (identified with s.1.1 – s.1.1.8) was extracted through clustering the *level 3 criteria* (s.1.1.1 – s.1.1.23). Level 3 criteria where identified by the comprehensive understanding of current practice, and communication formats (SSIM) as well as the review of the policy, and research literature (Sections 2 and 3 of the current document). Finally, at the lower level we may find the slots that belong to each slot priority (e.g. historic). The indicative links connecting the boxed entities are generic, based on logical assumptions and may change for each studied airport. As a result, they will have to be re-validated and tailored by the experts of each airport (e.g. coordinators) to see if they are meaningful or not. Having described the general structure of the tree, below we define[7] each criterion and sub-criterion and provide the rationale behind the depictured links.

<u>s.1- Market type:</u> The type of market is one of the additional slot allocation principles described in section 8.4.1 of IATA's WSG. Based on the SSIM format (column 27 of Table 2), each slot is characterised by a code representing whether it is a passenger/ cargo (s.1.1.4/ s.1.1.2), scheduled/ charter (s.1.1.3 / s.1.1.1) or military (s.1.1.5).

<u>s.2 - Curfews:</u> The same section of IATA's WSG mentions that slots which are subject to curfews in either the origin (s.1.17) or destination (s.1.1.6) airport, may be entitled for additional priority. In addition, a potential indicator for s.1.2 may be the distance of the flight in terms of service, i.e. short haul (regional) or long haul (s.1.1.14 or s.1.1.15) and the type of the route (s.1.18, 9).

<u>s.3 - Type of operations:</u> Shared or non-shared operations may also have a differentiated importance for a specific airport [Section 8.14 of IATA (2018b)]. This is included by adding

[7] Providing the definition of each hierarchical entity is crucial for leading the judgement of the stakeholders since for different definitions they may assign to the same indicator different importance. This is something that can be explained through the theory of semiotics where a definition for an object or notion is a subjective referent in the mind of the interpreter (in our case the interpreter is the coordinator)(Reichling, 1993).

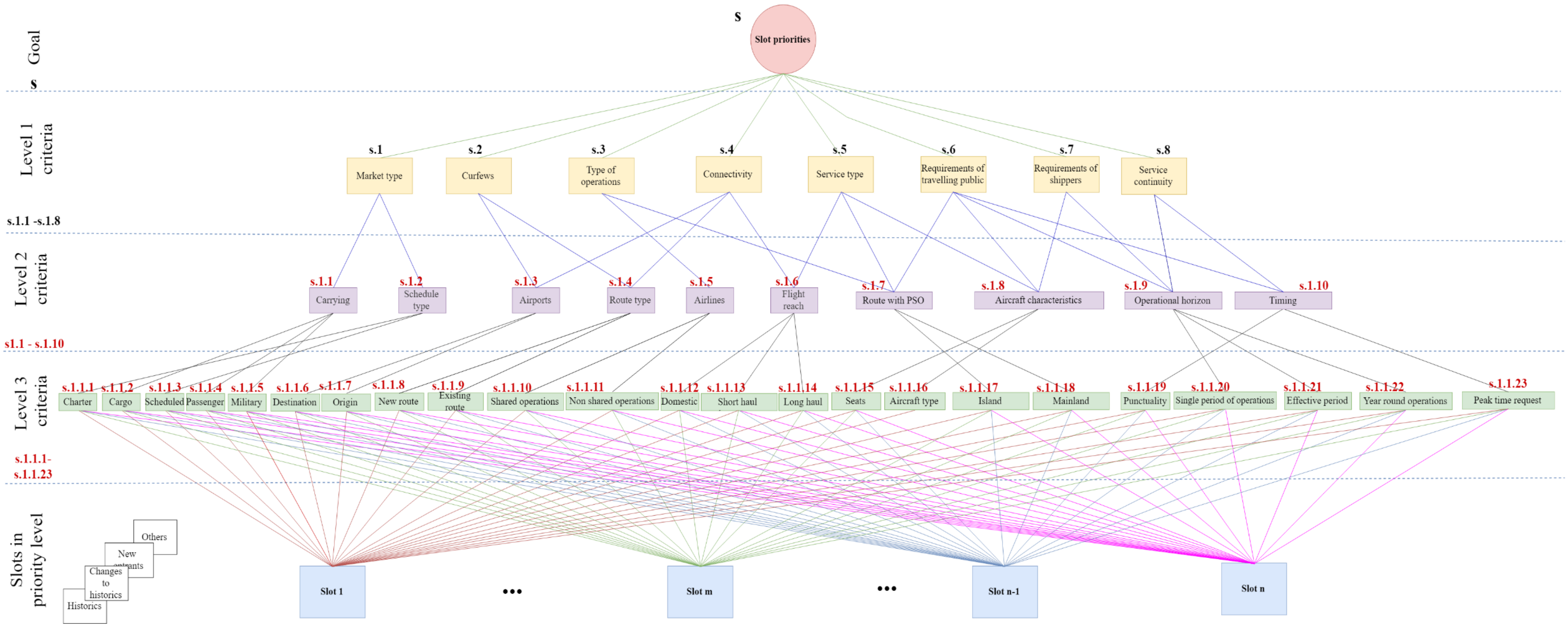

Figure 7: *Hierarchy determining the relative weight of each slot*

boxes s.1.1.10 and s.1.1.11 which represent the way that airlines will operate the request. However, based on the perception of the PSO routes (s.1.1.13, 14), s.1.1.7 may also be a determinant of the operation of a slot.

s.4 - Connectivity: Again, connectivity is an important additional criterion that has to be considered during slot allocation [Section 8.4.1. of IATA (2018b)]. Based on SSIM format (column 29 of Table 2), each request receives an action code "N" if it is a new schedule (s.1.1.8). It is clear that new routes may be more significant to connectivity than existing routes and schedules (s.1.1.9). The connectivity of an airport determines its success and its development, hence being one of the most crucial airport objectives (Egeland and Smale, 2017). However, its notion is complicated and can be expressed through various indicators (Burghouwt and Redondi, 2013). For instance, it may be of temporal nature (year-round vs. single period connectivity) and therefore it would demand to be linked with indicators (s.1.9-s.1.10). Connectivity can be also perceived as of intra-network nature, thus requiring the consideration of PSO routes (s.1.1.7). In any case, the links of the connectivity criterion will have to be re-validated by the views and the opinions of the interested stakeholders and experts.

s.5 – Service type: This criterion is also mentioned in section 8.4.1 of IATA's WSG. The service type of a slot is defined by the geographic distance of the flight and the airports that a slot request concerns. Therefore, slots serving domestic (s.1.1.13), short-haul (s.1.1.14), long haul (s.1.1.15) or PSO routes (s.1.7) may have different importance regarding criterion s. 5.

s.6, s.7 – Requirements of shippers and travelling public: These criteria are mentioned in section 8.4.1 of IATA's WSG. Aircraft characteristics (s.1.8) influence the number of passengers transferred while timing (s.1.10) and the horizon of the operations (s.1.9) guarantee schedule reliability and consistency. However, in remote areas, routes with PSO may be of great importance as well. Punctuality can be measured through the index that we proposed in section 4.1.2.1. while the size of the aircraft can be measured by its type and its capacity in terms of passengers (columns 16 and 17 of Table 2). Aircraft type (s.1.1.16) is employed since it may affect declared capacity in different ways.

s.8 – Service continuity: This criterion is one of the most significant as per IATA's WSG. It is specified in sections 8.3.6 and 8.4.1 of IATA's WSG where the priorities for slots for year-round operations (1.1.22) and more durable effective periods (1.1.21) are defined. For the measurement of the effective period, we may capitalise on the formulation provided in section 4.1.2.2. of this paper. As for the priority regarding year-round requests, there again exists the need of understanding coordinator's opinions and expertise. To clarify, the coordinators may translate the preference for indicator 1.1.22 as an absolute priority over single-period requests (1.1.20). In this case, weighting each slot would not represent the real-world's requirements. Alternatively, we make use of the multi-level, sequential solution approach (see page 33) which will however be computationally complex (the same dilemma would occur for the priority of routes with PSO). Apart from the operational horizon, s.8 can be measured via timing metrics such as the historic operational punctuality (section 4.1.2.1) and the timing of the request.

Now that the components of the tree are defined, one question remains. How inclusive can this hierarchy be? By observing Figure 4, we understand that the hierarchy could contain more criteria and indicators. For instance, s.1.7 can be subdivided into several sub-indicators according to the PSO inventory table (European Commission, 2018). Those indicators can be:

- the geography of the route (e.g. mainland, inland, outermost territory etc.);
- the number of PSO routes affected by the same call for tender;
- market access (restricted or open);

- the number of airlines operating;
- the existence of economic compensation or not; and
- the origin of the carrier (member state or not).

Even though the inclusion of all factors would lead to a more detailed hierarchy, the complexity and the number of pairwise comparisons would significantly increase. Therefore, there is always the need to consider the trade-off between detail and complexity. Approaches that are more detailed lead to better problem representation, but higher complexity and numbers of pairwise comparisons. Equally, for judgement consistency purposes, the number of attributes that are compared has to be relatively small (not more than 9) (Saaty, 1990). Large numbers of sub-criteria and decision levels, would increase the number of pairwise comparisons dramatically and disorient the focus of the information providers (Keeney, 1996), while at the same time they would render the questionnaires impractically big.

To summarise, this hierarchy is designed to provide relative aspect ratios to the slots of each slot priority. In accordance to the formulation provided in section 4.1, let the ratio of each slot to be denoted as π_m. Then a modified version of the total displacement objective function would be:

$$Z_1''' = \sum_{m \in M} \sum_{t \in T} |t - t_m| \, x_{t,m} \, \pi_m \tag{7.1}$$

Expression (7.1) is similar to the currently used and proposed displacement metrics (Z_1, Z_1'') and does not contribute to the complexity of the mixed integer program itself.

7.1.2. Using AHP as a schedule selection method (post-optimisation)

The second usage paradigm that we propose, is a hierarchical tree where AHP is used as an evaluation method for Pareto efficient schedules (Figure 8). In this hierarchy, we employ AHP as a group decision-making technique aggregating the opinions of the interested group members (level a.1). The lower level of the tree consists of Pareto optimal airport slot schedules supplied by a previously solved multi-objective slot allocation model (e.g. TOSAM/ ML-TOSAM). The goal of this hierarchy (denoted in Figure 5 by a) is to rank the alternatives based on the criteria belonging to levels 1 (a.1.1 – a.1.7) and 2. (a.1.1.4 – a.1.1.14). The criteria, sub-criteria and the stakeholders considered were again identified through the rigorous review of current policies and literature conducted in earlier sections of this document.

The opinion aggregation level (a.1), balances the opinions of the different groups of the stakeholders identified in IATA's WSG. At this point, it will be required to indicate a suitable opinion aggregation function. The selected function should be able to assign different importance to the judgements of each group of stakeholders, in order to adhere to the dynamics of the decision environment. The most prominent functions satisfying this prerequisite is the weighted geometric mean method (WGMM) (Saaty, 1989) and its summative variant (Ramanathan and Ganesh, 1994). Moving to the analysis of the criteria, there is again need to examine the rationale behind the selection of the depictured entities.

<u>a.1.1 – Environmental objectives:</u> Air traffic management models have solved for environmental objectives addressing the regulations imposed by national or regional authorities (Katsigiannis, 2018a). Such objectives may be associated with the slot allocation problem since different airport schedules result in different levels of carbon dioxide (a.1.1.1) and noise (a.1.1.2) pollution.

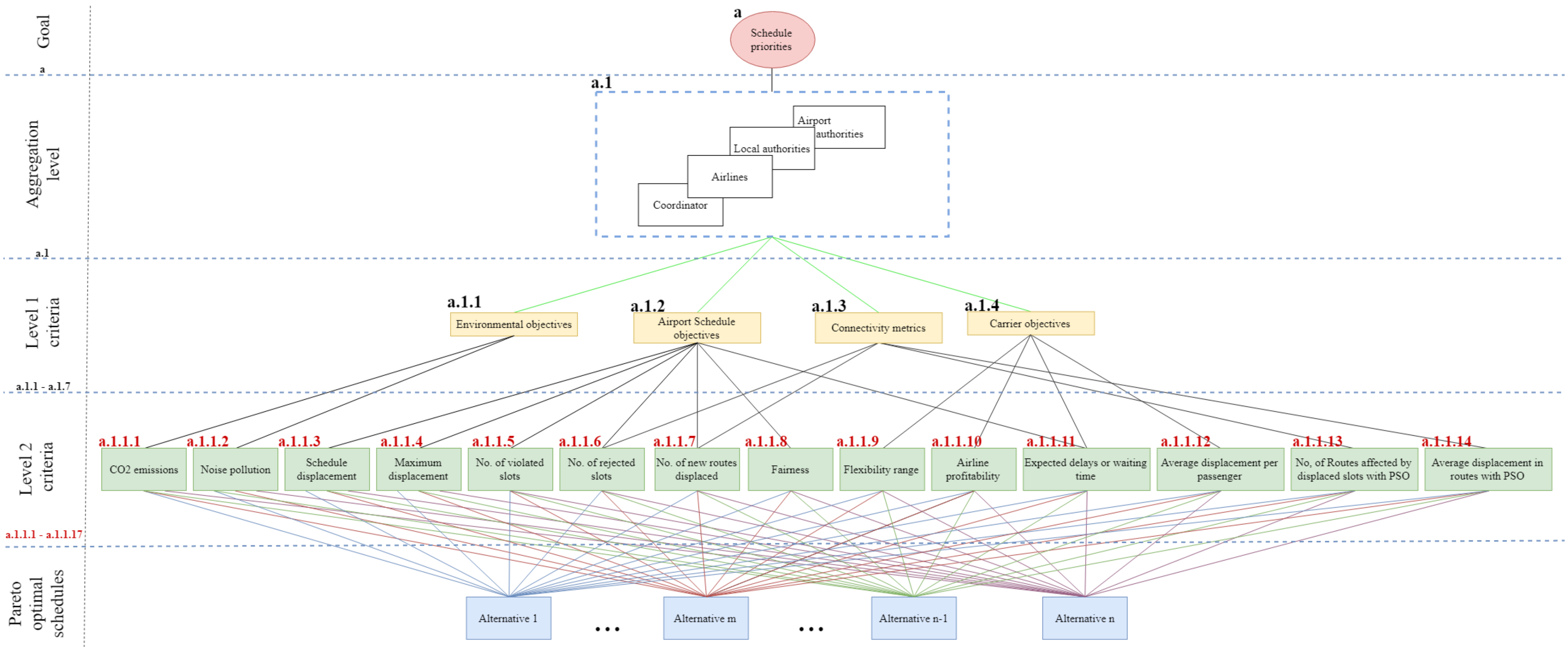

Figure 8: *Hierarchy ranking efficient schedule alternatives*

a.1.2 – Airport schedule objectives: This cluster groups airport schedule metrics as proposed in the literature (see Section 3). For each level 2 indicator (a.1.1.3 – a.1.1.8), AHP allows the consideration of multiple alternative cost functions.

a.1.3 – Connectivity metrics: The importance of connectivity has been highlighted in the previous section. However, for this hierarchical structure we should consider metrics assessing connectivity at the airport schedule level. Some simple and sensible indicators for this criterion are a.1.1.6 and a.1.1.7. Additionally, PSO considerations can be introduced so as to ensure the connectivity of remote areas. In occasions where slots are rejected, additional connectivity indicators can be employed such as those reviewed by Burghouwt and Redondi (2013).

a.1.4 – Carrier objectives: The satisfaction of the objectives of the airlines serving the airport is crucial for the amelioration of the system's welfare. The extent that passengers are disrupted (a.1.1.12), average delays or waiting time (a.1.1.11) and profitability (a.1.1.10) are some of the main factors that airlines consider when scheduling their flights (Katsigiannis, 2018b). Additionally, the flexibility ranges as proposed by Fairbrother and Zografos (2018) can be considered as a metric for schedule consistency.

7.2. Examples and discussion

To better understand the use of the proposed hierarchies, in the following sections we provide two illustrational examples while discussing future actions and calibration for their full-scale utilisation. The implementation of the results was conducted on an AHP spreadsheet template with group decision aggregation and multiple input capabilities (Goepel, 2012).

7.2.1. A slot-weighting example

We will exemplify the first hierarchy with a simplified version of Figure 7. Then for the extraction of the pairwise comparisons, we will assume two respondents (2 coordinators) whose opinions are aggregated.

For the purposes of this example, there will be four different slots belonging to the same slot hierarchy (e.g new entrant) having different characteristics:

- *Slot 1* (curfews at the origin, new route, domestic, 132 seats, type 320, PSO island)
- *Slot 2* (no curfews, existing route, short haul, 196 seats, type 320)
- *Slot 3* (curfews at the destination, existing route, long haul, 300 seats, type 770 capacity constraining)
- *Slot 4* (curfews at the origin, existing route, domestic, 132 seats, type E170, PSO mainland)

The hierarchical tree is depictured in the following figure (Figure 9), where we assume that the

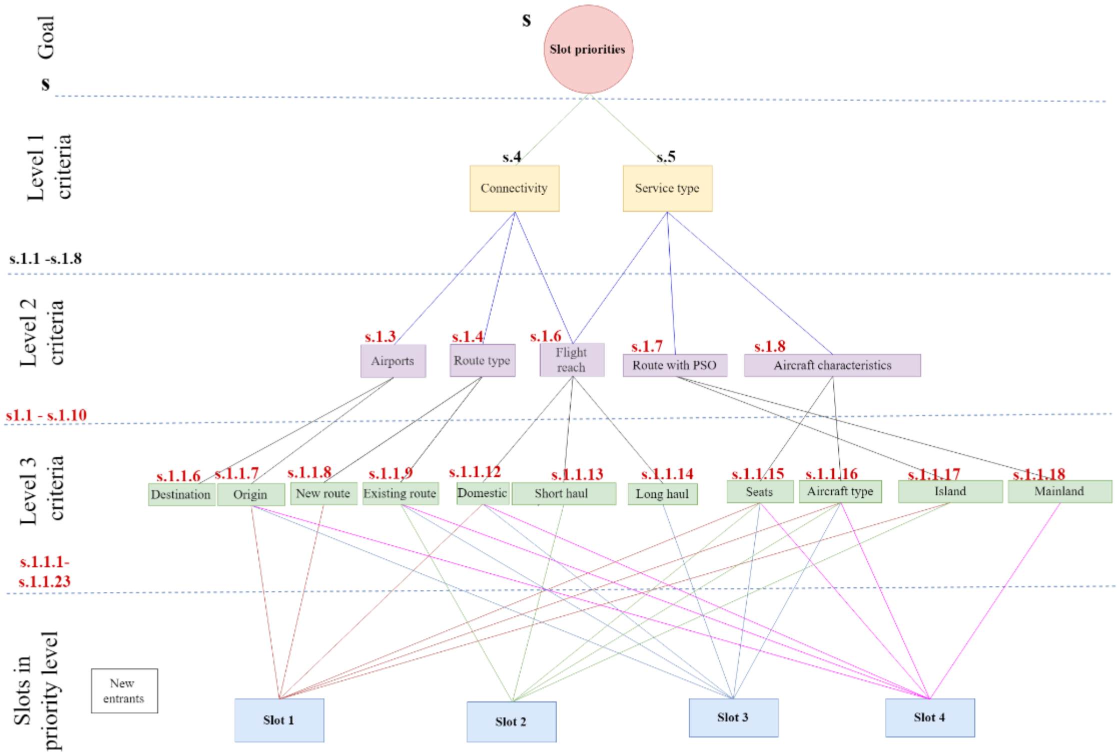

Figure 9: *Example hierarchy for slot prioritisation*

criteria that matter for the focal airport are only s.1.3 – s.1.8

In continuation, in order to extract the pairwise preferences of the coordinators there is need to ask questions of the following type:

> *What is the relative importance of indicator X over indicator Y regarding the upper level criterion Z /goal S?*

This general type of question determines the pairwise importance of level 1,2 or 3 criteria (X,Y) regarding an upper level criterion (Z). In order to avoid number crunching, Saaty (1990) proposes that those comparisons should be conducted based on the fundamental scale of absolute numbers ranging from 1 to 9, with 1 standing for equal importance and 9 for extreme importance.

Suppose that we address the following question[8] to one of the respondents: "*What is the relative importance of slots concerning new routes over slots for existing routes regarding the route type criterion?"* and the respondent answers 7. Then, new route slots have a strong importance of 7 over existing operations for the coordinator that participates in the survey. Accordingly, existing routes' significance only equals to the 1/7 of new routes' importance.

Now, let us assume that two coordinators (coordinator 1, coordinator 2) give the following answers to our survey:

- For level 1 criteria:

- Connectivity over Service type (*3, 5*)

[8] The question is assuming that new routes are more important than existing ones. In practice, the questionnaire would have scale values for both criteria on the same axis.

- For level 2 criteria regarding associated level 1 criteria:
  - as for Connectivity: Route type over Airports (5, 4); Flight reach over Route type (2, 1/2); Flight reach over Airports (7, 4)
  - as for Service type: Route with PSO over Aircraft characteristics (7, 4); Route with PSO over Flight reach (2, 3); Flight reach over Aircraft characteristics (8, 3)

- For level 3 criteria regarding linked level 2 criteria:
  - as for Airports: Destination over origin (1, 2);
  - as for Route Type: New route over existing route (3, 5);
  - as for Flight reach: Domestic over Long haul (4, 1); Long Haul over Short haul (2, 3); Domestic over Short haul (8, 5)
  - as for Aircraft characteristics: Aircraft type over seats (5, 1)
  - as for Route with PSO: Island over mainland (9, 5)

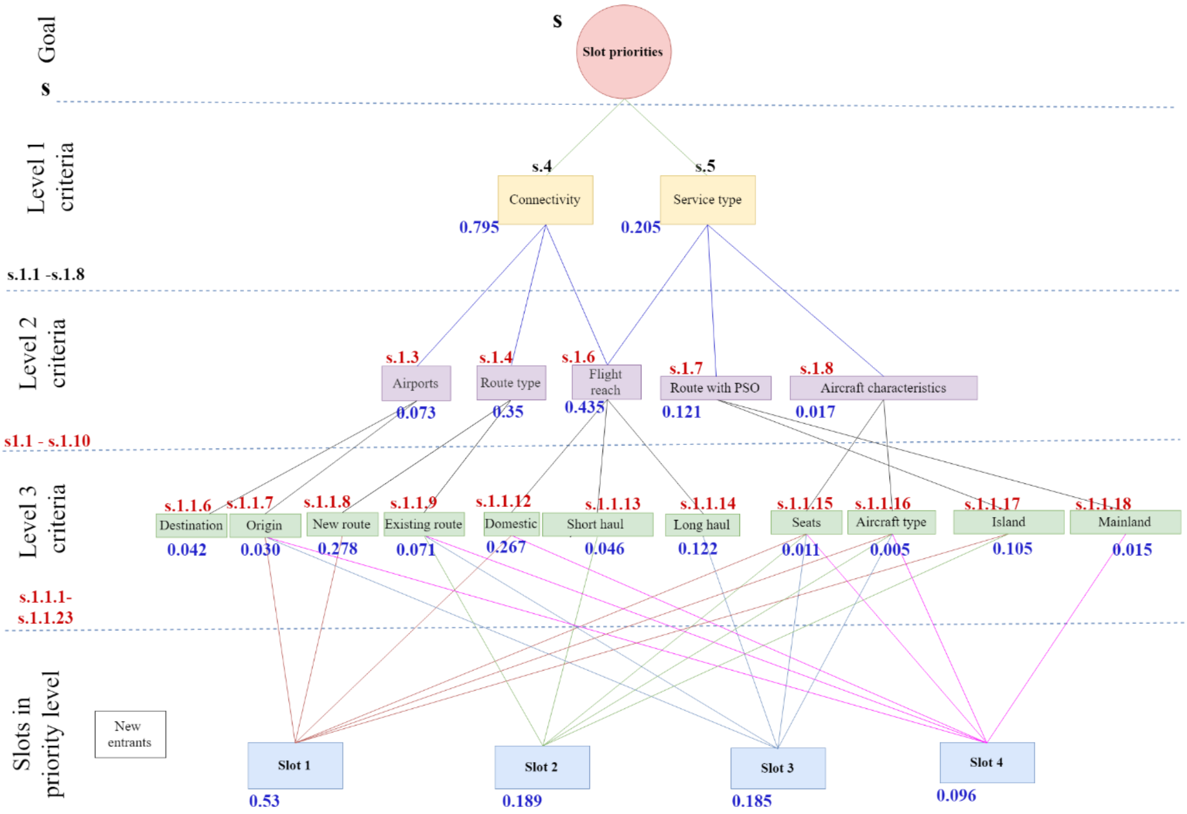

Figure 10: *Example hierarchy for slot prioritisation (solved)*

Please note that we constructed the opinions of the two respondents to be heterogeneous or even contradicting. Nonetheless, their views can be aggregated by using equal or differentiated weights based on their experience or the number of slots that they manage.

By applying the AHP having as input the pairwise preferences of the two respondents (equally weighted), we get the following weights (values rounded to three decimals):

- Level 1 criteria: Connectivity (0.795), Service type (0.205)
- Level 2 criteria given the importance of level 1 criteria: Airports ($0.093 \times 0.795 = 0.073$), Route type ($0.441 \times 0.795 = 0.35$), Flight reach ($0.466 \times 0.795 + 0.319 \times$

$0.205 = 0.435$), Route with PSO $(0.595 \times 0.205 = 0.121)$, Aircraft characteristics $(0.086 \times 0.205 = 0.017)$.

- Level 3 criteria given the relative importance of level 2 criteria: Destination $(0.073 \times 0.586 = 0.042)$, Origin $(0.073 \times 0.414 = 0.030)$, New route $(0.35 \times 0.795 = 0.278)$, Existing Route $(0.205 \times 0.35 = 0.071)$, Domestic $(0.435 \times 0.613 = 0.267)$, Long Haul $(0.435 \times 0.281 = 0.122)$, Short Haul $(0.435 \times 0.106 = 0.046)$, Island (0.121*0.87= 0.105), Mainland $(0.121 \times 0.13 = 0.015)$, Seats $(0.017 \times 0.691 = 0.011)$, Type $(0.017 \times 0.309 = 0.005)$

The results of the calculations above are summarised diagrammatically in the weighted hierarchical tree (Figure 10). The blue coloured numbers are the resulting weight of each node. The weights of the lower level are used to rank the slots based on their attributes as these were submitted using the SSIM format fields. The importance of each slot (g_m) can be calculated based on the values of the slots' characteristics[9]. Then, to provide a normalised weight (π_m) which abides by the AHP theoretical foundation we may divide the importance of each slot (m) by the total importance of all slots $\left(\pi_m = g_m / \sum_{m \in M} g_m\right)$. By doing so, slot 1 ($\pi_1 = 0.53$) is ranked first with slots 4 ($\pi_4 = 0.189$), 3 ($\pi_3 = 0.185$), and 2 ($\pi_2 = 0.096$) following accordingly.

7.2.2 A schedule selection example

In this example, from a given set of equally acceptable airport slot schedules, we will try to elicit the one that minimises the conflicts between the interests of the stakeholders. The set of the alternatives that is considered for this example consists of the five most beneficial schedules (in terms of simple additive deviation from the minimum value of each objective) reported in Section 6.2, i.e. schedules 5, 6, 7, 11 and 12 of Table A1. For this example, the schedules will be assessed based on two air connectivity (a.1.1.6, a.1.1.7) and all airport schedule (a.1.2) metrics while we will consider 4 stakeholder entities (coordinator, aggregated opinions of the airlines, local authority representative and airport representative) which are interested in the slot allocation process whose opinions are equally weighted. Then the hierarchy of the example is shaped as per Figure 11.

The five considered schedules and their hypothesized (with **bold** are the values reported by the case study of section 6) attributes are:

- *Schedule 5:* (a.1.1.3: **18097**, a.1.1.4: **13**, a.1.1.5: 260, a.1.1.6: **0**, a.1.1.7: 15, a.1.1.8: **1,** a.1.1.11: 20 mins)
- *Schedule 6:* (a.1.1.3: **18097**, a.1.1.4: **13**, a.1.1.5: 253, a.1.1.6: **0**, a.1.1.7: 12, a.1.1.8: **1,** a.1.1.11: 19 mins)
- *Schedule 7*: (a.1.1.3: **17131**, a.1.1.4: **15**, a.1.1.5: 260, a.1.1.6: **0**, a.1.1.7: 14, a.1.1.8: **1,** a.1.1.11: 17 mins)
- *Schedule 11*: (a.1.1.3: **18096**, a.1.1.4: **13**, a.1.1.5: 240, a.1.1.6: **0**, a.1.1.7: 25, a.1.1.8: **1.1,** a.1.1.11: 18 mins)

[9] The importance (g_m) of each slot (m) is calculated based on its weighted additive performance which accounts for each level 3 criterion j belonging to the set of criteria (J) and is described mathematically: $g_m = \sum_{j \in J} s_{mj} w_j$. s_{ij} is the value of attribute j for slot m and w_j is the upper level weight regarding attribute j.

- *Schedule 12*: (a.1.1.3: **18096**, a.1.1.4: **13**, a.1.1.5: 260, a.1.1.6: **0**, a.1.1.7: 15, a.1.1.8: **1.1,** a.1.1.11: 14 mins)

Now, let us assume that we receive the answers from the stakeholders in the form: *(coordinator, airlines, local authorities, airport authorities)*.

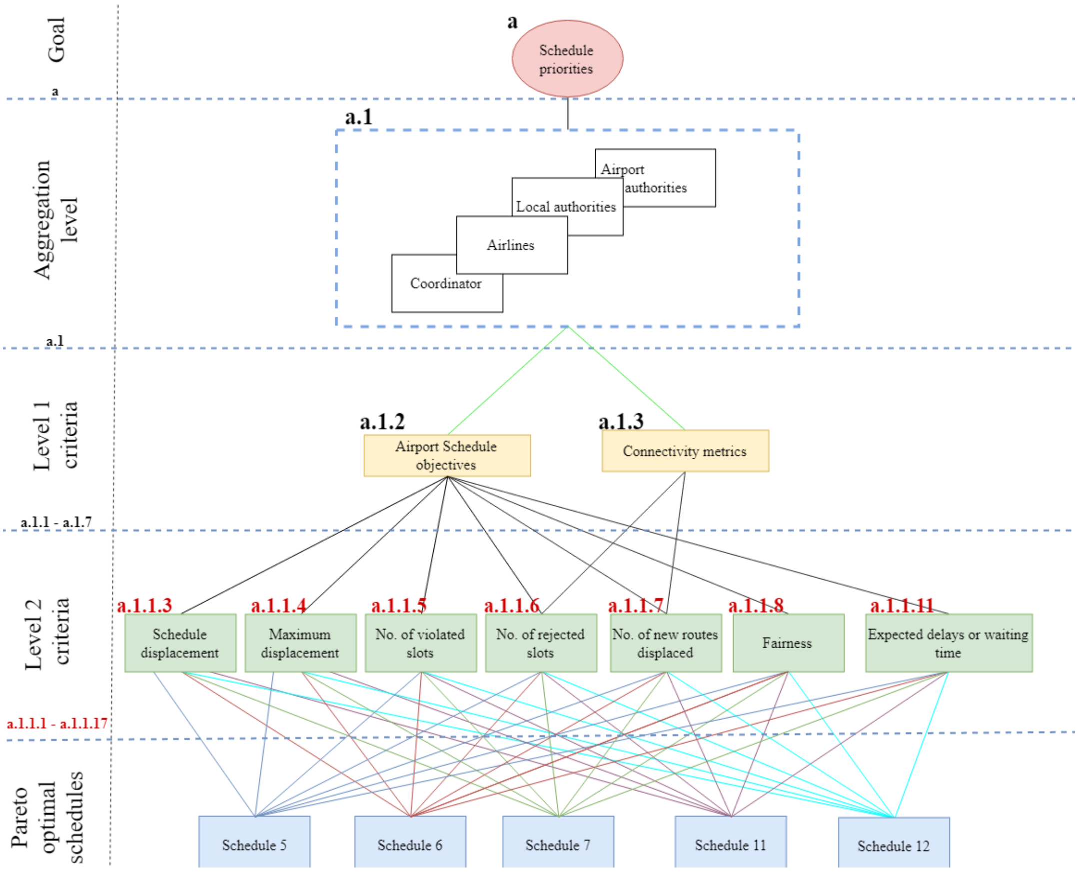

Figure 11: *Example hierarchy for schedule selection*

In addition, we receive the following pairwise preferences:

- For level 1 criteria:

- Airport schedule objectives over Connectivity (*7, 2, 1/3 , 1*)

- For level 2 criteria regarding associated level 1:

- Pairwise comparisons of the level 2 criteria regarding a.1.2 are given in Table 10.

Criteria	a.1.1.3	a.1.1.4	a.1.1.5	a.1.1.6	a.1.1.7	a.1.1.8	a.1.1.11
a.1.1.3	-	1/2, 1/2, 1, 2	5, 7, 2, 4	1/9, 1/7, 1/3, 1/6	2, 1, 1/2, 1/2	2, 2, 1, 1	1, 1/3, 1/2, 1/3
a.1.1.4		-	4, 3, 1, 1	1/2, 1/4, 1, 1/2	6, 4, 1/2, 1/3	2, 2, 3, 1/2	1, 1/4, 1/2, 1/2
a.1.1.5			-	1/9, 1/7, 1/2, 1/7	1, 1/2, 1/6. 1	1/2, 1/2, 1, 1	1/3, 1/6, 1, 1/7
a.1.1.6				-	7, 8, 2, 7	9, 8, 6, 9	2, 4, 2, 5
a.1.1.7					-	1, 1, 5, 4	2, 1/2, 2, 2
a.1.1.8						-	1, 1/2, 1/4, 1
a.1.1.11							-
Notes:	*Blank cells get the inverted values of the symmetric cells.*						

Table 10: *Summary of example answers for airport schedule objectives*

- as for Connectivity metrics: a.1.1.6 over a.1.1.7 (3, 6, 2, 3)

The example responses can be aggregated based on different weights accounting for the institutional influence of each of the actors. In this example, for simplicity purposes, we will assume equal weights for the opinions of the stakeholders.

By applying the AHP considering the pairwise preferences, we get the following weights (valued rounded to three decimals):

- Level 1 criteria: Airport schedule (0.595), Connectivity (0.405) metrics
- Level 2 criteria given the importance of level 1 criteria: Schedule displacement ($0.092 \times 0.595 = 0.055$), Maximum Displacement ($0.117 \times 0.595 = 0.069$), No. of violated slots ($0.051 \times 0.595 = 0.030$), No. of rejected slots ($0.414 \times 0.595 + 0.763 \times 0.405 = 0.555$), No. of new routes displaced ($0.114 \times 0.595 + 0.237 \times 0.405 = 0.164$), Fairness ($0.065 \times 0.595 = 0.038$), Expected waiting time ($0.147 \times 0.595 = 0.087$)

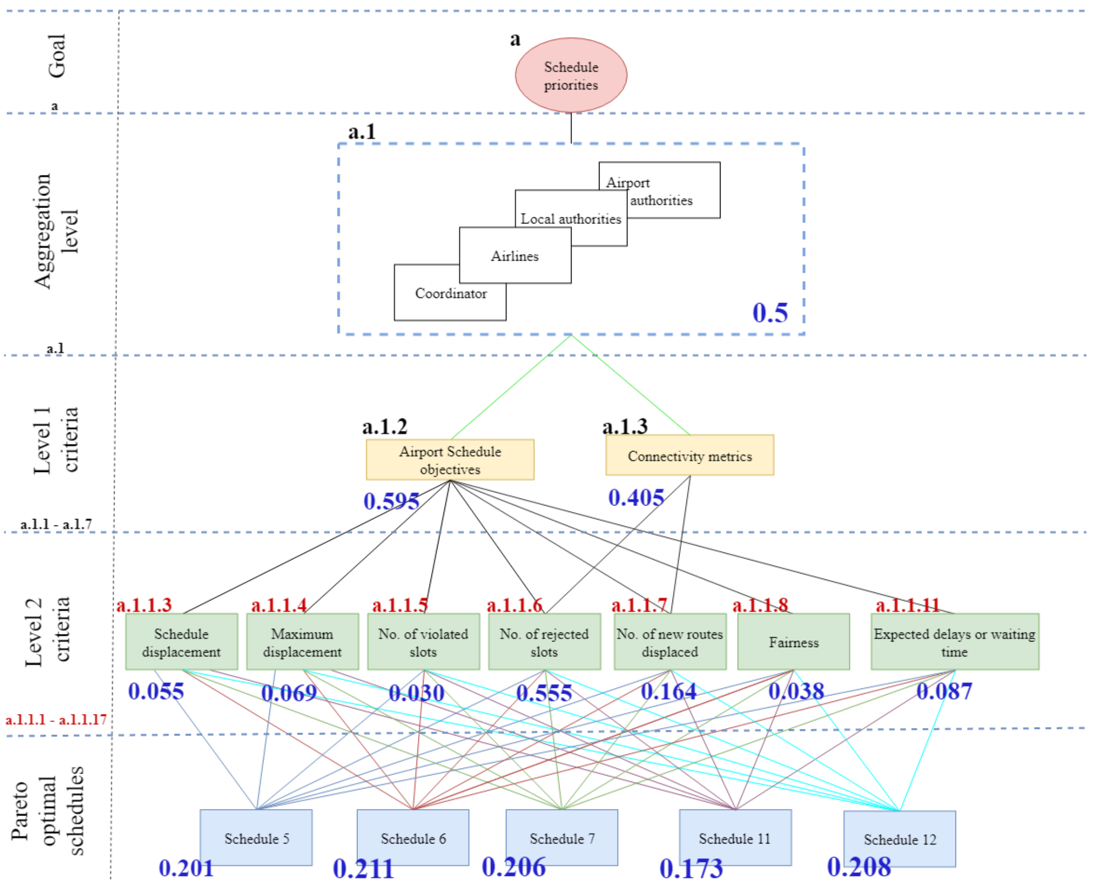

Figure 12: *Example hierarchy for schedule selection (solved)*

The results for this hierarchy are summarised in a weighted tree (Figure 12). The blue coloured numbers are the resulting weights for each entity. The importance of the alternatives is calculated based on the normalised deviation of each attribute from the minimum attribute value in the set of the alternatives[10]. The final ranking for the alternative schedules after the

[10] Assume a set of attributes ($\mathcal{A}$) indexed by j and a set of alternatives ($\mathcal{E}$) indexed by i and k, with $\mathcal{A}_i$ being the set of attributes measured for alternative i. If we assume that the weight of each level 2 criterion is ℓ_j, then the score of each alternative ($score_i$) is $score_i = \sum_{j \in \mathcal{A}_i} \ell_j \left[1 - \left(\left(v_j^i - \min_{i \in \mathcal{E}} v_j\right) / \left(\min_{i \in \mathcal{E}} v_j + 1\right)\right)\right]$ where v_j^i is the value of attribute j for alternative i and $\min_{i \in \mathcal{E}} v_j$ the minimum value of criterion j among

aggregation of the stakeholders' perceptions regarding airport schedule and connectivity metrics indicates that Schedule 6 is the best compromise solution ($SCORE_6 = 0.211$). Please note that for the extraction of the alternatives' scores some objectives that we took into account were not considered during the MOO phase. On the other hand, fairness, total and maximum displacement that were optimised in order to produce equally efficient trade-off solutions, are weighted in the AHP model based on the preferences of the respondents. This two-stage approach is a representative example of how the AHP can enhance MOO modelling considerations.

7.2.3 Discussion

Throughout this section, we highlighted two alternative uses of the AHP facilitating the airport slot allocation problem. The first one acts as a pre-optimisation technique providing the importance of each slot based on its SSIM attributes. The second can be used as a selection method indicating a consensus solution from the set of efficient schedules given by a previously solved MOO model. For the full-scale application of the proposed hierarchies, there is need to validate the significance of the considered attributes (criteria) and obtain pairwise preference data by the stakeholders.

Moreover, from a modelling aspect it is mandatory to keep in mind the trade-offs between complexity and inclusivity. We have seen in both examples that even for a small number of criteria (n), the number of pairwise comparisons raises quickly $[n(n-1)/2]$. Therefore, the inclusion of numerous hierarchical entities may increase the detail of the model but it would also lead to inconsistent judgements and impractical questionnaire sizes. In addition, there is need to test the structure of the hierarchy so as to avoid the threat of rank reversal (Schenkerman, 1994).

Finally, for the creation of a meaningful optimisation solution approach, there is need to understand whether the stakeholders assign absolute or relative priority to each of the criteria (e.g. PSO routes, round or military operations).

8. Concluding remarks

In this paper, we extend existing literature towards several directions. We firstly conducted a thorough review of the regulations and policies defining the airport slot allocation decision environment. This process allowed the identification of an additional slot priority that concerns routes with public service obligations. Even though the number of such routes is relatively small, the requests associated with them should always be allocated before all other request types [Article 9 of the Council Regulation No 95/93 (1993)]. Moreover, the comprehensive analysis of IATA's WSG allowed us to represent the regulatory framework in a compact hierarchical manner that is easier to comprehend.

In addition, by cross tabulating the regulations with the existing MOO models we managed to identify some promising literature gaps and future research directions. In brief, future modelling attempts should elaborate on devising alternative objective functions, addressing additional policy rules and considering multiple objective functions simultaneously. From a regulatory aspect, prospective research attempts should grasp the priority of year-round operations over single period requests and consider the effective duration of each request and the PSO route priority. Another stream of research should focus on the solution approach. In detail, current

all alternatives. Then, the normalised score ($SCORE_i$) of each alternative i is expressed as follows: $SCORE_i = score_i / \sum_{k \in \mathcal{E}} score_k$.

solution algorithms (hierarchical, lexicographic), do not capture the interactions between the different slot priorities and their results on aggregate objective functions. Hence, future research should work on capturing the inter-level interactions, which inherently reside within the slot allocation decision context.

Having these findings in mind, by capitalising on existing modelling attempts, we formulated a tri-objective slot allocation model that considers fairness, minimum and maximum displacement objectives. In detail, by taking advantage of the nature of the objectives, we linearized the maximum displacement and fairness cost functions through the construction of an efficient set of Chebyshev decomposition constraints. Furthermore, we proposed two alternative weighted schedule displacement functions that introduce punctuality and service continuity considerations. Admittedly, another modelling contribution of the current document is the multi-level game-theoretic formulation of the airport slot allocation problem. To the best of our knowledge, it is the first attempt to introduce multi-level considerations and theoretical support in the field. The proposed quart-level approach stands as a finer alternative to existing solution methods, which however comes with an increased computational cost. The flexibility of the proposed multi-level modelling framework coupled with the tailored solution algorithms, allowed us to model some additional policy rules enhancing new entrants' schedule acceptability.

By acknowledging the absence of commonly accepted efficient solution algorithms, we proposed a series of algorithmic processes providing trade-offs between computational complexity and multi-level modelling detailing. As a result, we introduced the notion of inter-level tolerance as a proxy of multi-level interactions. To clarify, we allowed the upper decision levels to tolerate losses on their objective functions by accepting dominated and weakly dominated multi-objective solutions. Such sacrifices are only accepted if they lead to level based or program-wide nondominated solutions.

Our assumptions were tested on a medium sized regional European airport. The computational results suggest that the hybrid hierarchical solution method with inter-level tolerance considerations may result in a larger and higher quality population of efficient solutions than the simple hierarchical solve. On average, we observed that the reduction on the values of the objective functions might exceed 3% while the cardinality of the Pareto set was by 12% larger. Such computational results prove that the slot allocation literature can benefit from the introduction of multi-level modelling. Therefore, another potential future research direction should focus on enhancing the multi-level interactions between slot priorities. To tackle the complexity of such models, fuzzy and goal programming methods could be used to propose a single solution that accounts for the stakeholders' preferences and the real world needs of the problem.

In Section 7, we illustrated the potential synergies between MOO airport slot allocation models and MADM methods. By observing the corpus of MOO models in the field, we understood that the simultaneous consideration of multiple objectives and policy rules is not practical due to the computational complexity that it incurs. The combined use of MOO and MADM methods and especially the AHP can overcome this limitation. For this reason, we suggest and illustrate two alternative two-stage hierarchies that can capture simultaneously various requirements of the problem. Nevertheless, those indicative hierarchies must be validated by the participating decision makers as their construction was solely based on policy and research documents.

To conclude, the research output of this work was constrained by the limited time that we had in our disposal. Yet, our work is original and extends current literature considerations. Our future efforts will focus on the enhancement of the multi-level, multi-objective considerations of airport slot allocation. In parallel, the full-scale application of the proposed AHP approaches requires the collection of pairwise preference data and validation as per the stakeholders' personal views. For this reason, integrated multi-actor decision-making-validation methods such as MAMCA (Macharis et al., 2012) can be employed ensuring decision sustainability. Another significant research direction that we would like to work on is the introduction of schedule consistency objectives. Especially, the flexibility considerations of Fairbrother and Zografos (2018b) result in differentiated arrival and departure times for individual slots belonging to the same series of slots. This is something that is not always acceptable by airlines or passengers and contradicts IATA's specifications [Section 9.9.3.e of IATA (2018b)]. Therefore, schedule consistency objectives (e.g. the minimisation of the number of individual slots allocated to different times) could be incorporated in order to address the schedule requirements described within IATA's WSG and PSO specifications. Besides, by capitalising on the methodological work of Korhonen and Syrjänen (2004), the allocation of scarce airport resources can be also bundled with airline efficiency analyses. Ultimately, a firm comparison of expert systems software facilitating the airport slot allocation and the existing mathematical methods could reveal additional problem characteristics that have yet to be captured in the MOO airport slot scheduling (e.g. passenger flow and parking constraints).

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Appendix

ID	Level based objective values										Schedule-wide		
	Z_3	$Z_1(x_H)$	$Z_2(x_H)$	$Z_1(x_{CH})$	$Z_2(x_{CH})$	$Z_1(x_{NE})$	$Z_2(x_{NE})$	$Z_1(x_{Oth})$	$Z_2(x_{Oth})$	$Z_1\ x_{Oth}\ '$	Z_1	Z_2	Z_3
1	0.9	567	4	5025	4	974	10	18706	14	16992	25272	14	0.9
2	0.9	567	4	5025	4	1155	7	18134	15	17625	24881	15	0.9
3	0.9	567	4	5025	4	1155	9	18116	16	17539	24863	16	0.9
4	0.9	567	4	5051	7	914	10	17932	18	17454	24464	18	0.9
5	1	235	2	765	5	786	8	16311	13	15243	18097	13	1
6	1	234	3	765	5	786	8	16312	13	15244	18097	13	1
7	1	234	4	765	5	786	8	15346	15	15127	17131	15	1
8	1	234	4	765	5	786	8	15079	16	15059	16864	16	1
9	1	234	4	765	5	786	8	15071	17	15051	16856	17	1
10	1	234	4	765	5	786	8	15067	18	15047	16852	18	1
11	1.1	235	2	764	5	786	8	16311	13	15244	18096	13	1.1
12	1.1	234	3	764	5	786	8	16312	13	15244	18096	13	1.1
13	1.1	235	2	764	5	786	8	15511	14	15243	17296	14	1.1
14	1.1	234	4	765	5	786	8	15281	15	15127	17066	15	1.1
15	1.2	234	4	764	5	786	8	16309	13	15245	18093	13	1.2
16	1.2	234	3	764	5	786	8	15154	15	15126	16938	15	1.2
17	1.2	235	2	764	5	786	8	15077	16	15057	16862	16	1.2
18	1.2	234	3	764	5	786	8	15078	16	15058	16862	16	1.2
19	1.2	235	2	764	5	786	8	15069	17	15049	16854	17	1.2
20	1.2	234	3	764	5	786	8	15066	18	15046	16850	18	1.2
21	1.2	235	2	764	5	786	8	15065	18	15045	16850	18	1.2
22	1.3	235	2	764	5	786	8	16121	13	15244	17906	13	1.3
23	1.3	234	4	764	5	786	8	15502	14	15245	17286	14	1.3
24	1.4	235	2	764	5	786	8	15449	13	15244	17234	13	1.4
25	1.4	235	2	777	3	786	8	15399	14	15243	17197	14	1.4
Notes:	Total displacement objective value for others' level without fairness considerations $[Z_1\ x_{Oth}\ ']$												

Table A1: *Nondominated solutions for tolerance-based algorithm*

	Level based objective values										Schedule-wide		
ID	Z_3	$Z_1(x_H)$	$Z_2(x_H)$	$Z_1(x_{CH})$	$Z_2(x_{CH})$	$Z_1(x_{NE})$	$Z_2(x_{NE})$	$Z_1(x_{Oth})$	$Z_2(x_{Oth})$	$Z_1\ x_{Oth}\ '$	Z_1	Z_2	Z_3
1	0.9	567	4	5025	4	974	10	18706	14	16992	25272	14	0.9
2	0.9	567	4	5025	4	1155	7	18134	15	17625	24881	15	0.9
3	0.9	567	4	5025	4	1155	7	18128	17	17503	24875	17	0.9
4	0.9	567	4	5025	4	1155	7	18116	19	17417	24863	19	0.9
5	1	235	2	765	5	786	8	16311	13	15243	18097	13	1
6	1	235	2	779	3	786	8	15472	15	15125	17272	15	1
7	1	235	2	779	3	786	8	15270	16	15057	17070	16	1
8	1	235	2	779	3	786	8	15262	17	15049	17062	17	1
9	1	235	2	779	3	786	8	15258	18	15045	17058	18	1
10	1.1	235	2	764	5	786	8	16311	13	15244	18096	13	1.1
11	1.1	235	2	764	5	786	8	15511	14	15243	17296	14	1.1
12	1.1	235	2	764	5	786	8	15346	15	15125	17131	15	1.1
13	1.1	235	2	764	5	786	8	15270	16	15057	17055	16	1.1
14	1.1	235	2	764	5	786	8	15262	17	15049	17047	17	1.1
15	1.1	234	3	764	5	786	8	15259	18	15046	17043	18	1.1
16	1.2	235	2	764	5	786	8	15153	15	15125	16938	15	1.2
17	1.2	235	2	764	5	786	8	15077	16	15057	16862	16	1.2
18	1.2	235	2	764	5	786	8	15069	17	15049	16854	17	1.2
19	1.2	235	2	764	5	786	8	15065	18	15045	16850	18	1.2
20	1.3	235	2	764	5	786	8	16121	13	15244	17906	13	1.3
21	1.4	235	2	764	5	786	8	15449	13	15244	17234	13	1.4
22	1.4	235	2	777	3	786	8	15399	14	15243	17197	14	1.4
Notes:	Total displacement objective value for others' level without fairness considerations $[Z_1\ x_{Oth}\ ']$												

Table A2: *Nondominated solutions for no-tolerance algorithm*

Step 1: Decomposition of the problem into criteria, subproblems and alternatives;

Step 2: Collection of pairwise preference data according to the fundamental scale of absolute numbers;

Step 3: Generation of the pairwise comparisons of the alternatives with respect to different criteria and the criteria themselves (square matrix of size n)

- ✓ *the diagonal elements of the matrix are equal to one;*
- ✓ *if the element of the i^{th} row is better than the one in the j^{th} column then the value of cell (i, j) is more than one and less than in the opposite occasion;*

Step 4: Data normalisation

- ✓ *computation of the division of each entry towards the sum of each column for each element (w_{ij});*

Step 5: Priority extraction (*eigenvectors*) for each alternative under each criterion by adding the normalised values given in Step 4 per row and dividing this summation with the number of alternatives;

Step 6: Calculation of the consistency ratio (CR): $CR = \frac{Consistency\ index\ \ CI}{Random\ index\ (RI)}$,where:

- ✓ $CI = (Max.\,eigen\ value - n)/(n-1)$*; and*
- ✓ $RI = CI\ (randomly\ generated\ matrix)$*;*
- ✓ Saaty (2005*) proposes that CI should be more than 0.1 in order to have consistent judgements.*

Step 7: The rating of each alternative is multiplied by the weights of the criteria and the sub-criteria;

Step 8: Report of the final scores for each criterion and alternative.

Table A3: *Basic steps of the AHP [adapted from Katsigiannis (2018a)]*