Introducing flexibility and demand-based fairness in slot scheduling decisions

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1 Introduction

In many airports around the world, demand by airlines to use airport infrastructure exceeds the available capacity, leading to congestion problems. Outside of the US, demand at airports is managed through an administrative scheme known as the IATA Worldwide Slot Guidelines (WSG) [1]. Under this scheme, congested airports are designated as *coordinated*, and airlines must obtain *slots*, which is a time interval during which an aircraft can use the airport for landing or take-off. To obtain slots, airlines biannually make requests for slots and series of slots to the *coordinator* of an airport for a six-month season. A series of slots is a request for five or more slots at the same time and on the same day of the week. The coordinator then constructs an initial allocation of slots to airlines which matches the requests as far as possible while satisfying airport capacity and aircraft turnaround constraints. In addition to these logistical constraints, the coordinator must also allocate slots according to priority classes.

Due the complexity of assigning slots, many optimization models have been formulated to solve this problem. These models typically aim in some way to minimize the *schedule displacement*, that is the time difference between requested and allocated slots. More recent models [2, 3] have incorporated fairness, that is equitably distributing schedule delay among the airlines. Although the requirement of fairness will necessarily increase the overall total displacement, it will ensure that the resulting schedule is more acceptable to the airlines.

The aim of this paper is to develop a new multiobjective optimization model which enhances previous models in three directions. Firstly, our model allows for flexibility in the slot times assigned to a series of slots. Secondly, we propose constraints to fairly allocate rejected requests, that is requests which cannot be allocated any slot. Finally, to fairly allocated schedule displacement we propose a new demand-based fairness measure.

2 Modelling extensions

The model we develop is an extension of that in [4]. The main differences being that the decision variables controlling the time of the slot assigned to a request is now additionally indexed by the day of the season, and we include an extra set of variables to indicate whether request are rejected, which occur if for some day the total number of slots is exceeded by demand for slots. The notation used in the model is shown in Table 1 and the model itself is in equations (1)–(11).

Objective, capacity and turnaround constraints The objective of the model in (1) is the lexicographic minimization of rejected requests, followed by total displacement. We minimize in this order as there will be a much greater aversion to rejections than to displacements. Constraints (3) ensure that each request is assigned a slot, or is explicitly rejected; constraints (4) ensure that the allocation complies with rolling capacity constraints, that is constraints which limit the number of movements scheduled within a given amount of time of each other; finally constraints (5) ensure there is sufficient turnaround for arrival-departure pairs of flights.

Flexibility Previous slot allocation models have allocated slots for a series of slots request at the same time. However, the WSG allow for some flexibility in how these are assigned. By indexing the assignment of a slot by the day, we are allowing each day for which a request is made to be assigned a slot at a different time. Generally, it is preferable to assign series of slots which are as close to each other as possible. For each request we therefore place a bound (8) on the range of slot times that one can assign to a request. Constraints (6) and (7) are logical constraints which define the earliest and latest slot times assigned to a request. Allowing flexibility while constraining slot range significantly increases the size of the model and the computational effort required to solve it.

Fair apportionment of rejected requests and schedule displacement For both rejections and schedule displacement we use the proportionality principle of fairness proposed in [3]. Rejections of requests occur when there are too many requests to fulfil in a single day. We allocate a proportion of rejections to an airline a proportional to the proportion of requests p_a made by the airlines. To enforce this, we add maximum deviation from absolute fairness (MDA) constraints (9).

In the case of schedule displacement, the amount a request should be displaced depends on how severe is the demand for the requested time slot. We measure the severity through the marginal cost of a request, denoted by ν_m for $m \in \mathcal{M}$. The marginal costs are calculated in terms of schedule displacement from an exhaustive sensitivity analysis with respect to the model below, without the two sets of fairness constraints. The values can be thought of as a request's contribution to the violation of capacity constraints. We then require the proportion of schedule displacement assigned to an airline to be proportional to a weighted sum of the marginal costs of its requests via MDA constraints (10). By only counting requests which increase total displacement, airlines are not penalised for making requests in off-peak periods, where there is sufficient capacity to meet demand.

$$\text{lexmin} \left(\sum_{m \in \mathcal{M}} z_m, \sum_{m \in \mathcal{M}} s_m \right) \tag{1}$$

$$\text{subject to } s_m \geq \sum_{d \in \mathcal{D}_m} \sum_{t \in T} f_m^t y_m^{td} \tag{2}$$

$$\sum_{t \in T} y_m^{td} + z_m = 1, \; m \in \mathcal{M}, d \in \mathcal{D}_m \tag{3}$$

$$\sum_{m \in \mathcal{M}} \sum_{t \in \mathcal{T}_c^s} a_m^d b_{mc} y_m^{td} \leq u_c^{ds}, \; c \in \mathcal{C}, \; d \in \mathcal{D}, \; s \in \mathcal{T}_c \tag{4}$$

$$\sum_{t \in \mathcal{T}} t y_{m_1}^{td} - \sum_{t \in \mathcal{T}} t y_{m_2}^{td} \geq l_{(m_1, m_2)}, \; (m_1, m_2) \in \mathcal{P}, \; d \in \mathcal{D}_m \tag{5}$$

$$\underline{\tau}_m \leq \sum_{t \in \mathcal{T}} t y_m^{td}, \; d \in \mathcal{D}_m \tag{6}$$

$$\overline{\tau}_m \geq \sum_{t \in \mathcal{T}} t y_m^{td}, \; d \in \mathcal{D}_m \tag{7}$$

$$\overline{\tau}_m - \underline{\tau}_m \leq r_m, \; m \in \mathcal{M} \tag{8}$$

$$\left| \frac{\frac{\sum_{m \in \mathcal{M}_a} z_m}{\sum_{m \in \mathcal{M}} z_m}}{p_a} - 1 \right| \leq \epsilon_1 \qquad \text{for all } a \in \mathcal{A} \tag{9}$$

$$\left| \frac{\frac{\sum_{m \in \mathcal{M}_a} s_m}{\sum_{m \in \mathcal{M}} s_m}}{\frac{\sum_{m \in \mathcal{M}_a} \nu_m}{\sum_{m \in \mathcal{M}} \nu_m}} - 1 \right| \leq \epsilon_2 \qquad \text{for all } a \in \mathcal{A} \tag{10}$$

$$y_m^{td}, z_m \in \{0, 1\} \tag{11}$$

3 Numerical experiments

This new model will be tested for real slot request data for a medium-sized airport. Three aspects of the new model will be studied in particular. Firstly, the sensitivity of the model with respect to the range constraints (8). Secondly, the price of fairly distributing rejections. Finally, the price of fairly distributing schedule displacement. Compared to previous fairness measures not based on demand, we expect there to be an improved trade-off between fairness and total schedule displacement. The models will be solved using a branch-and-cut scheme with a commercial MILP solver.

Sets	
$\mathcal{A}$	set of airlines
$\mathcal{D}$ $(\mathcal{D}_m)$	set of days (for which movement m requested)
$\mathcal{M}(\mathcal{M}_a)$	set of movement requests (by airline a)
$\mathcal{P} \subset \mathcal{M} \times \mathcal{M}$	set of arrival-departure pairs (m_a, m_d)
$\mathcal{C}$	set of airport capacity constraints
$\mathcal{T} = \{1, \ldots, T\}$	set of coordination time intervals
Parameters	
t_m	requested time for movement m
δ_c	duration of constraint c
p_a	proportion of flights requested by airline a
f_m^t	displacement cost for assigning slot t to movement m
l_p	turnaround time for movement pair $p \in \mathcal{P}$
a_m^d	indicates whether constraint $c \in \mathcal{C}$ is active on day $d \in \mathcal{D}$
b_{mc}	contribution of movement m to constraint c
u_c^{ds}	capacity for constraint $c \in \mathcal{C}$ on day $d \in \mathcal{D}$ at time period s
r_m	maximum range of slot times for request m
ν_m	marginal cost of request m
Decision variables	
y_m^{td}	indicates whether movement m is assigned slot t on day d
z_m	indicates if request for movement m is rejected
s_m	schedule displacement for request m
$\underline{\tau}_m(\overline{\tau}_m)$	earliest (latest) slot time assigned to m

Table 1: Notation used for single airport slot allocation model

References

[1] International Air Transport Association. Worldwide slot guidelines, 2017. `http://www.iata.org/policy/infrastructure/slots/Pages/slot-guidelines.aspx`, accessed 25 April 2017.

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