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Education as a Participation Game: An Explanation for Educational Under-Investment

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Individuals who under-invest in education experience negative outcomes along most measurable dimensions. Although such under-investment is common, it is not adequately explained by existing economic theory. We disaggregate the canonical educational investment decision into a series of incremental educational opportunities, and thereby endogenously separate economic agents into high- and low-participation equilibria. We derive self-productivity in cognitive ability development, and we identify the effects of specific noncognitive skills. Our results suggest that early intervention should focus on children's noncognitive skills, whilst later intervention should not target disadvantaged individuals directly — it should focus instead on specified aspects of their educational provision.

JEL Codes: I24; D91; C73; C61; J24.

Keywords: Education; Human Capital; Repeated Game; Noncognitive Skills; Sophistication.

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I. Introduction

Economic inequality poses a challenge to society. The prevailing level of inequality in developed nations is often considered to be socially unfair, and its links with excess morbidity, mortality, and political unrest are well known (Graham 2007, Ezcurra & Palacios 2016, Jetten et al. 2017). Whilst an extensive literature has established that modern economic inequality is perpetuated through the educational under-investment of disadvantaged children (Cunha et al. 2006, Conti, Heckman & Urzua 2010, Lavecchia, Liu & Oreopoulos 2016), our knowledge as to why and how disadvantaged families under-invest in education remains limited. Without such knowledge the design of interventions is reduced to trial, error, and educated guesswork.

Until recently, economic theory treated educational investment as a single-period decision over years-of-schooling. However, in 2007 Cunha & Heckman proposed an important new model of educational production based upon multiple stages of childhood development. Their model incorporates self-productivity of multi-dimensional ability and dynamic complementarity between accumulated ability stocks and present investment, and it shows that these features can explain six key empirical facts of childhood development. The central implication of their model is that interventions to reduce economic inequality should be weighted toward the first stage of childhood development. Existing economic theory is therefore able to inform the optimal timing of intervention, but it remains silent as to the form that such intervention should take because educational production is still treated as a black box.

In this paper we model the mechanism of educational production. We postulate that educational investment should be founded on the incremental choices that parents and children make on a daily basis. At the earliest ages, such decisions include whether to: talk to the child, play with the child, read with the child, and so forth. As the child develops, she begins to take decisions such as whether to: engage in group activities, attempt classwork tasks, and study for tests. We build a new model of human capital development based upon these 'nano-foundations', and we use that model to extend existing economic theory in four important respects.

First, our model provides an explanation for the striking empirical anomaly of educational under-investment. In the US, for example, high school dropouts are 2.1 times less likely to earn over $25,000 per annum, 2.4 times more likely to be incarcerated, 3.0 times more likely to have a child before the age of twenty, and 1.3 times less likely to report 'good' health outcomes[1] — and yet around 30% of US children drop out of their high school education (Messacar & Oreopoulos 2013). A large body of literature has attempted to reconcile these facts, but it has fallen short of that goal because, for any

[1]These comparative statistics are derived from the General Social Survey; they are not causal estimates, but rather illustrate the extent of social inequality. A detailed derivation is provided in the supplementary materials.

high-level choice over years-of-schooling, the net benefit of completing high school is overwhelmingly positive, even allowing for behavioural adjustments to an adolescent's payoff function (see Section II). By contrast, the educational benefit of each incremental educational opportunity is small enough that behavioural factors, and agents' noncognitive ability to withstand them, can be decisive. We show that our nano-founded model endogenously separates the population into divergent high- and low-participation pathways, whereby an arbitrarily small change in initial conditions can precipitate a life-changing reduction in equilibrium investment.

Second, where previous models take self-productivity, dynamic complementarity, and sensitivity to early investments as primitive assumptions, we derive them as a consequence of our modeling approach. Since Cunha & Heckman (2007) it has been accepted that these characteristics underpin the technology of skill formation, but this paper is the first to provide a theoretical explanation of how they might arise. As a consequence, our theory shares the salient characteristics of the seminal framework proposed by Cunha & Heckman (2007) when its implications are aggregated to the level of developmental stages. We therefore provide a tangible theory of educational investment which can explain the six key empirical facts of childhood development that those authors identify.[2]

Third, we are able to identify the effects of three specific aspects of noncognitive ability on children's educational investment. When participation decisions are considered in aggregate, children and their families are implicitly assumed to possess perfect self-control, perfect forward-planning, and perfect self-knowledge. Our analyses allow each of those assumptions to be relaxed. We demonstrate that greater self-knowledge unambiguously increases participation, but that greater forward-planning increases participation only for agents who also possess self-knowledge. These findings warn of potentially counterproductive outcomes from interventions which focus on teaching young children to plan ahead, without teaching them to anticipate their own future temptations. Where children display bounded self-control they will persevere with educational activities if and only if they experience initial success or enjoyment. This characteristic could be harnessed by parents or educators who wish to increase a child's participation, but it also warns that interventions wherein a child is not supported to achieve some early success could have a negative net effect.

Fourth, our model provides a new and concrete interpretation of educational underinvestment during childhood. Traditionally, educational investment is thought of as years-of-schooling, which only becomes manifest at the school leaving age. However, our results imply that incremental participation decisions taken during childhood represent a vital form of educational investment, because later schooling outcomes arise as an equilibrium

[2]The first five of these facts describe the properties of the divergent developmental pathways that are emerge from our analyses, together with the self-productivity and dynamic complementarity that characterise those pathways; the sixth affirms the central importance of noncognitive skills.

response to those early decisions.

Taken together, our results contribute an explicit mechanism for the observed educational under-investment of disadvantaged children: i) at an early age, the child's participation is essentially determined by the decision-making of her parents, ii) disadvantaged parents are likely to be able to provide less frequent educational activities,[3] and iii) the frequency of early participation in incremental educational activities critically determines the child's educational pathway.

Our findings therefore support the consensus view in the literature that early intervention is vital if the persistence of economic inequality is to be reduced. However, they challenge the prevailing assumption that such intervention should focus on raising the cognitive ability of disadvantaged children. We show that if a disadvantaged child could learn the noncognitive skills of forward-planning and self-knowledge, she would be substantially more likely to participate in incremental educational opportunities thereafter. Such an intervention could thereby materially increase the probability of her escaping a low-participation equilibrium, whilst an intervention which exogenously increased cognitive ability would be unlikely to overcome disadvantageous model parameters.

Our results nevertheless demonstrate that if a child's noncognitive skill deficit is not addressed within a critical time-period, the low-participation pathway would become her equilibrium strategy even with perfect forward-planning and self-knowledge. We therefore conclude that an effective later intervention should target the child's situation, rather than the child herself. The child's situation is that they face incremental educational investment decisions described by parameters which they cannot control. But because those exogenous parameters represent tangible aspects of any educational task, we are able to make specific pedagogical recommendations that would enable educators and parents to manipulate those parameters and thereby increase equilibrium participation. Although this conclusion represents a paradigm shift for some policy-makers, our results suggest that intervening with the situation rather than the child could meaningfully reduce the persistence of economic inequality.

In addition to its practical implications, our model also yields an important moral implication. At any cross-section, a child's observable ability is endogenously co-determined by the their initial ability endowment and by their sequence of educational participation decisions to date. Because current ability influences future participation, those inputs not only interact inseparably within the educational production technology, but ex-post it would be impossible to disentangle their relative contributions due to an initial-conditions problem. However, children can influence neither their genetic endowment nor their early

[3]The literature suggests many potential reasons for this: disadvantaged parents may have reduced levels of noncognitive skills (including self-control, self-knowledge, and forward-planning), reduced time due to a higher incidence of single-parenthood and higher fertility rates, reduced capability due to multi-dimensional poverty, reduced esteem of or knowledge of the educational process, credit constraints impacting the provision of educational play materials, and so forth.

educational participation. Thus, since apparent under-investment in adolescence could be an equilibrium response to early disadvantage, we must conclude that the victims of poverty may not be wholly responsible for their ostensibly poor educational investment decisions.

The paper proceeds as follows. Section II reviews the existing educational investment literature, and finds that the observed extent of under-investment is not yet adequately explained. Section III then presents our model, whereafter Subsections IVA–D characterize its analytic solutions, and Subsections E–F illustrate those solutions numerically for a robust set of functional form assumptions. Section V then discusses the implications of our results, and Section VI concludes.

II. The Existing Literature

Educational investment decisions determine many individual outcomes. A large body of evidence suggests that the financial returns to education appreciably surpass market rates of return (Cahuc, Zylberberg & Carcillo 2014), that those returns may themselves be surpassed by the non-pecuniary benefits of education (Oreopoulos & Salvanes 2011), and that the social returns to education are probably of comparable magnitude to those personal benefits (McMahon 2004). It is therefore an important objetive for economic theory to be able to explain the observation that a substantial minority of individuals drop out of education considerably before it would be optimal for them to do so (Oreopoulos 2007).

Most economic theories of educational investment are built upon the canonical investment model of Becker (1962, 1964). That model yields the elegant and intuitive result that individuals should optimally invest until the marginal cost of further education exceeds its marginal product. This implies that the apparent under-investment of many disadvantaged children could be an optimal response, if they either: possess a particularly low educational productivity, or experience a particularly high participation cost. We assess the evidence for each of these hypotheses in turn.

The first hypothesis lacks empirical support. It was shown as early as Griliches (1977) that the returns to education for observationally less able children are at least as great as those for their more able peers, and that conclusion is now supported by a large body of IV literature in which the LATE for individuals affected by exogenous increases in compulsory schooling often exceeds OLS estimates of the average returns to schooling (Harmon, Oosterbeek & Walker 2000). Thus it is not the case that those children who invest the least in their education do so because of lower productivity.

The second hypothesis has now also been refuted empirically. For an economically rational agent, educational participation costs arise due to credit constraints, however Carneiro & Heckman (2002) determine that such constraints are of minor importance in the developed world, and Jensen (2010) found that they affect only the poorest families in the developing world. These results suggest that the apparent educational under-investment of many disadvantaged individuals in the developed world does indeed represent a normatively suboptimal choice.[4] The challenge is therefore to understand the mechanism behind that choice.

Several economic theories attempt to explain suboptimal educational participation. One possible explanation is that disadvantaged children might under-invest because they underestimate their true returns to education. There is evidence that this may be an important factor in the Dominican Republic (Jensen 2010), but those authors believe that such ignorance is unlikely to be significant in the developed world, and Rouse (2004) finds firm evidence in support of that belief. Nevertheless Lavecchia, Liu & Oreopoulos (2016) survey a large number of nudge-based interventions to find that some succeed in increasing participation by expounding the benefits of post-compulsory eduction, which suggests that incomplete knowledge regarding the returns to education may contribute toward explaining under-investment.

Perhaps the most promising avenue toward explaining educational under-investment is the acknowledgement of behavioral aspects of decision-making. Lavecchia, Liu & Oreopoulos (2016) eloquently articulate the intuition that present-bias could lead to educational under-investment, and studies such as Shoda, Mischel & Peake (1990) have provided convincing experimental corroboration of that hypothesis. Nevertheless, Oreopoulos (2007) estimates the parameters of a standard investment model which incorporates present-bias to find that an implausibly large degree of bias would be necessary to completely explain observed under-investment.

A complementary approach could be to incorporate additional behavioral motivations into the model. For example, Wang & Yang (2003) and Köszegi (2006) include a payoff to self-worth within their agents' objective function, which induces a psychic cost of failure within educational participation decisions and therefore reduces participation. Analogously, Akerlof & Kranton (2002) include a payoff to social identity, and thereby suggest that poorly endowed agents might choose to reduce their educational effort in order to fit in with the 'burnouts'. These approaches each provide useful insights, but once again they seem unlikely to explain the magnitude of observed under-investment, which Cunha & Heckman (2008) estimate to be equivalent to an unobserved cost in the order of $500,000 for U.S. college attendance.

The model presented in Section III does predict severe under-investment in education by a subset of individuals. The model incorporates a combination of present-bias and psychic payoffs to success and failure, but its main driving forces are a modest time-

[4] If this conclusion is correct then one implication is that many early-leavers of education should be expected to later regret that decision. Bridgeland, Dilulio & Morison (2006) corroborate that implication, by establishing that 74% of U.S. high school drop-outs later regret dropping out.

consistent discount rate, and a derived self-productivity in cognitive ability. These attributes are consequential because the canonical one-shot educational investment decision is disaggregated into elemental participation decisions, each of which contributes only an incremental payoff in terms of educational development. Section **??** demonstrates that such disaggregation remains consistent with the canonical investment criterion of Becker (1962).

There is surprisingly little economic theory that examines more than a handful of periods of educational investment. Sjögren & Sällström (2004) and Filippin & Paccagnella (2012) both analyse the many-period case, but neither model incorporates dynamic skill-development. Those papers focus instead on the implications of over- or under-optimism regarding an agent's fixed ability endowment, to reveal that over-optimism leads to greater participation. Some of the most important insights in this area are therefore applications of more general results. For example, Thaler & Shefrin (1981) analyse the conflict between an agent's ex-ante preferences and his extemporary desires, and O'Donoghue & Rabin (1999) analyse the implications of present-bias, both for sophisticated agents who anticipate it, and for naïve agents who only experience it. This paper builds on each of those analyses, to derive the implications of a simple many-period educational investment model, under three contrasting levels of sophistication. In doing so, it also extends the application of participation games to situations where a single agent interacts with nature.

III. The Model

A. *The Game*

Agents face a series of T educational participation decisions. Their (potentially mixed) strategy space is therefore given by $S := \{s_1, s_2, ..., s_T\}$, where s_t is their chosen probability of participating in the period t opportunity.

Each individual decision is presented as an extensive form participation game in Figure 1. The decision utility payoffs relevant to educational participation are:

- $\boldsymbol{d_t}$ the present value of the human capital developed by participating in the task,
- $\boldsymbol{c_t}$ the direct and opportunity cost of effortful task participation,
- $\boldsymbol{p_t^s}$ the psychic payoff to achieving success,
- $\boldsymbol{p_t^f}$ the psychic cost of failure,

where the subscript $t \in \{1, 2, ..., T\}$ denotes period-specific, or equivalently, task-specific variation. We shall refer to the first two items as the material components of the payoff function, and the final two items as the psychic components. Since these payoffs are formally defined up to affine transformation, we may normalize the payoff of task avoidance to be 0, without loss of generality. It is then uncontentious to further assume that

$p_t^s, p_t^f > 0 \ \forall t$ — that is: success is pleasant, and failure unpleasant, ceteris paribus (see, for example: Bénabou & Tirole 2002, Wang & Yang 2003). Although we initially analyse the implications of the model for one representative agent, it will already be evident that individual outcomes must be substantially determined by individual heterogeneity in decision utilities. The implications of such individual heterogeneity are discussed in Section V.

[Figure 1 about here.]

As can be seen in Figure 1, the agent's probability of achieving success at time t is denoted by π_t. π_t is considered to be a draw from Π_t, which is the agent's probability of success distribution across all possible tasks at time t. Π_t will be determined by a spectrum of individual and familial characteristics, but also by the human capital which has been developed as a consequence of educational participation in periods $\tau < t$. We therefore assume that Π_{t+1} stochastically dominates Π_t if the agent attempted task t, and that Π_{t+1} is stochastically dominated by Π_t if the agent avoided task t. In this paper we make the additional simplifying assumption that $\Pi_t(n)$ is uniquely determined by the period, t, and the number of educational tasks thus far attempted, n.[5] That simplifying assumption yields the intuitive and useful lemma that $E(\Pi_t(n))$ is a strictly increasing function of n, a formal proof of which appears in the appendix, as do proofs of all subsequent propositions.

An agent's probability of success distribution Π_t characterizes their stock of cognitive ability in period t. This paper provides a partial equilibrium model of human capital development, in that it allows Π_t to develop dynamically whilst noncognitive abilities, psychic payoffs, and participation costs are modelled as time-invariant traits. This approach allows us to expose the implications of our nano-founded theory whilst identifying the effects of heterogeneity in those traits. Ongoing work extends the present theory to a general equilibrium model in which noncognitive abilities evolve alongside cognitive abilities, and it finds that those more realistic feedback mechanisms reinforce the dynamic implications of the time-invariant traits model.

We nevertheless allow $d_t(n)$ to vary by period t and by prior participation n. To see why, define $V(n)$ as the present value in period $T+1$ of having attained educational level n by the end of compulsory schooling. This value will represent the sum of: expected future remuneration, expected non-pecuniary benefits of education, and the opportunity value of whichever further and higher educational opportunities are accessible to an agent of attainment level n. Without loss of generality, we normalize $V(0) := 0$, recognising that in absolute terms $V(0)$ will be affected by factors such as social security policy.

[5]This amounts to an assumption that educational tasks are perfect substitutes, which greatly improves tractability but costs little in generality, since its relaxation would have an analogous effect to increasing the magnitude of the psychic payoffs. In reality this assumption will be true to the extent that teachers and parents are able to differentiate educational tasks to match the current needs of each child.

Thus, under the quasi-hyperbolic discounting of Laibson (1997), we will have that:

$$d_t(n) = \beta\delta^{T-t+1}\left[V(n+1) - V(n)\right] =: \beta\delta^{T-t+1}\left[V'(n)\right] \tag{1}$$

where $\delta \in [0,1]$ is the agent's per-period discount rate for future payoffs, where $\beta \in [0,1]$ parameterizes present-bias, and where we define $V'(n) := V(n+1) - V(n)$ to be the first difference of V at n.[6] Thus $d_t(n)$ represents the present value to the agent of increasing her current education level by participating in the $n+1^{\text{th}}$ task.

Note that the above derivation of d_t implicitly assumes the educational benefit of task participation to be independent of whether success is achieved. This assumption contrasts with some existing economic models (e.g. Sjögren & Sällström 2004, Filippin & Paccagnella 2012), however it is in line with the educational literature, where it is recognized that we may typically learn at least as much from our mistakes as from flawless task completion (see, for example: Black & Wiliam 1998). The contrasting assumption that only success begets learning would meaningfully change the decision-making of naifs by introducing an incentive to front-load participation, but it would have little qualitative effect on sophisticates who already experience such an incentive. We now formally define these agent archetypes.

B. The Players and their Noncognitive skills

Three specific noncognitive skills constrain an agent's equilibrium actions under the proposed model. In order to calculate the first best solution, an agent would need to posses substantial forward planning ability, and perfect anticipation of her future period payoffs. An agent who possesses such forward-planning and self-knowledge will be described as sophisticated, following the terminology of O'Donoghue & Rabin (1999). We also follow those authors in identifying the specific effects of limited self-knowledge by comparing the sophisticate's solution to that of a naïve agent. A Naif is considered to possess perfect forward-planning, but limited self-knowledge: they do not anticipate that psychic payoffs and present-bias will continue to affect them in future periods. The naif therefore provides an archetype of the most extreme version of the human tendency to discount any visceral influences over our future behavior, as described by Loewenstein (1996). These archetypes are informative, because they bound the continuum of possible levels of self-knowledge.

We similarly identify the effect of limited forward-planning ability by considering the extreme case of a myopic agent. The myope does not consider the existence of any future educational opportunities: he merely maximises his expected utility for each stage game in isolation. The myope is therefore akin to the myopic 'doer' of Thaler & Shefrin (1981),

[6] Analogously, we define $V''(n) := V'(n+1) - V'(n)$ to be the second difference of V at n.

save that he is assumed to internalise the (arguably future) educational benefit of task participation d_t. Nevertheless, under the *as if* interpretation of Expected Utility Theory, he need not be consciously aware of this or any other aspect of the game structure. The Noncognitive skill levels of each agent archetype are summarized in Table 1.

[Table 1 about here.]

To identify the effect of limited self-control, we analyse the behavior of each archetype both with and without commitment. An agent exhibits perfect self-control when she commits to each period's strategy s_t before any information is received as to that period's realized probability of success π_t. The opposite extreme is modelled by allowing the agent to costlessly 'try' each educational task to learn their realized probability of success π_t, without necessarily seeing that task through to completion.[7] In reality, it is unlikely that the child's signal of π_t would be perfect, however we model this extreme case in order to provide an upper bound on the effect of limited self-control. Similarly, we assume throughout that all agents have perfect information regarding their probability of success distribution Π_t, in order to isolate the unique predictions of our model.[8]

Throughout our analyses we will derive Bayesian Nash Equilibria of the participation supergame, which O'Donoghue & Rabin (1999) refer to as 'perception-perfect' equilibria. This is a strong solution concept, since it allows agents to choose their entire strategy S without restriction. It is therefore unsurprising that, with probability 1, there is a unique equilibrium for any given set of parameters, sophistication assumptions, and commitment constraints (see Proposition 3). A weaker solution concept which requires only that each period's strategy s_t should be a best response, holding all other participation decisions s_{-t} constant, would generally produce two markedly different weak equilibria, representing high- and low-participation pathways respectively. Propositions 2 and 5 expose the fact that an arbitrarily fine change in initial conditions could determine which of these two markedly different pathways will be the unique (Bayesian Nash) equilibrium outcome.

IV. Analyses

The objectives of these analyses are threefold. First, we wish to establish the implications of our model for the aggregate technology of skill formation. This is undertaken in

[7] This concept could be termed periodwise commitment. An alternative concept of ex-ante commitment, under which an agent must commit to her entire strategy in period 0, is unjustifiable in this context. For completeness a discussion of ex-ante commitment is included in the supplementary materials – in summary: it is meaningless for the myope, it allows the naif to reproduce the normatively optimal solution, and it is almost identical to periodwise commitment for the sophisticate. The last result is interesting, because for the sophisticate the two concepts differ precisely by the effect of present-bias, and so we establish that present-bias is qualitatively unimportant in our example.

[8] Relaxing these assumptions to allow unbiased noise would merely increase the stochastic element of non-commitment explored in Section F; allowing bias in the signal would replicate the main result of Filippin & Paccagnella (2012) that under- (rsp. over-) optimism regarding one's ability reduces (increases) educational participation.

Subsection A. Second, we wish to establish the implied pattern of participation for each agent archetype. By comparing these, we will be able to establish our third objective of identifying the participation effect of each noncognitive skill. Subsections B–D address these objectives analytically, and Subsections E–F illustrate our results numerically.

A. *The Technology of Skill Production*

We begin by characterizing s_t, the equilibrium strategy of the stage game depicted in Figure 1. Let us denote by $w(\pi_t, \Pi_t)$ the general form of an agent's believed probability of success in period t. The Bayesian Nash Equilibrium of the stage game is therefore a strategy s_t^* that maximizes the expected utility

$$\begin{aligned} U_t &= s_t \left[w(\pi_t, \Pi_t)(d_t + p^s - c) + (1 - w(\pi_t, \Pi_t))(d_t - p^f - c) \right] + (1 - s_t) \left[\, 0 \,\right] \\ &= s_t \left[d_t - c - p^f + w(\pi_t, \Pi_t)(p^s + p^f) \right] \end{aligned} \tag{2}$$

Proposition 1 characterizes such a strategy:

Proposition 1

1. *Any Bayesian Nash equilibrium strategy of the stage game (Figure 1) is, with probability 1, a pure strategy.*
2. *Any agent faced with the stage game will participate whenever $d_t > c + p^f$; avoid the task whenever $d_t + p^s < c$; and otherwise participate if and almost only if her believed probability of success exceeds $(c + p^f - d_t)/(p^s + p^f)$.*

The first of these results formalizes the notion that the nano-founded framework disaggregates educational investment into a series of binary decisions. The second result establishes the intuition that an agent will optimally participate in any incremental opportunity if and only if her probability of success is sufficiently high. That intuition forms the basis of the self-productivity of cognitive ability which generates many of the model's implications. Proposition 2 exposes the source of that self-productivity:

Proposition 2 *In any period t, and for an agent who has no information concerning the realization π_t:*
The psychic component of the stage-game payoff function, $-p^f + w(\Pi_t)(p^s + p^f)$ exhibits increasing returns to previous participation.

The implications of an increasing-returns production technology are well-known: for example Arthur (1989) demonstrates that this characteristic can lead to multiple equilibria, path-dependence, and inefficient outcomes. Thus the participation decision becomes, in general, a complex dynamic problem. That problem is made more tractable by the fact that the results of Proposition 1 generalize to the case of forward-looking agents, as proven in Proposition 3. Proposition 3 also proves the useful result that, absent any signal of π_t, any utility-maximising agent would set $w(\Pi_t) = E(\Pi_t)$. The intuition behind the latter result is that an agent who wishes to maximize her expected utility should, in practice, adopt the best available estimate of its conditional realization.

Proposition 3 *In the finitely repeated game with T periods:*

1. *Any Bayesian Nash Equilibrium strategy $S \in \{0,1\}^T$ with probability 1.*

2. *In any period t, an agent will participate if and almost only if her believed probability of success exceeds some determinate critical value.*

3. *In any period t, an agent who has no information concerning the realization π_t should optimally set $w(\Pi_t) = E(\Pi_t)$.*

Together, these results establish that self-productivity, dynamic complementarity, and sensitivity to early investments are endogenously produced by our model. In any period t an agent is more likely to participate in the present educational opportunity if their stock of (cognitive) ability Π_t is greater (Propositions 3.2; 3.3). Thus, since educational opportunities are precisely those situations which develop ability, we have that ability is self-productive.

To see that there is dynamic complementarity between a child's accumulated ability stock and present period external investment, consider the child's period t participation decision. If the child's ability exceeds the participation threshold given by Proposition 3.2 then no intervention is necessary. If, however, the child has an ability deficit relative to that participation threshold, then they would only participate if some external investment were to intervene to improve their expected participation payoff. Exemplar interventions might therefore aim to support the child's probability of success, to reduce her participation cost, or to reduce her psychic cost of failure. An intervention would be successful if and only if it closes the gap between expected participation costs and payoffs, and that gap is increasing in the size of the child's ability deficit. Thus the chance of any given level of external investment having a positive effect is increasing in the child's current cognitive ability. A corollary to this conclusion is that the probability of present period external investment having a positive effect is also an increasing function of prior investment.

Proposition 2 illuminates the optimal timing of any external investment. Consider a child who would require continuous participation from periods 1 to n in order to develop her cognitive ability to the self-sustaining participation threshold given by Proposition 3.2. Then that child would not participate in the absence of external investment, and so the contrapositive of Proposition 2 implies that a later intervention would have to overcome a greater ability deficit. Thus not only would providing n periods of delayed participation be more costly than the corresponding immediate intervention, but it may also fail to boost the child to the self-sustaining participation threshold. Moreover, if ever that threshold were reached, the child would still achieve a lower final education level on account of the missed opportunities in her early life. Intervention is therefore most effective if received at the earliest possible age; hence period 1 of our model is a sensitive period in the sense of Cunha & Heckman (2007).

B. *The Myopic Solution*

The myope has minimal forward planning ability. His equilibrium strategy S is therefore composed of the sequence $\{s_1^*, s_2^*, ...s_T^*\}$ of actions which each maximise (2), the expected utility of each successive stage game. This strategy is characterised by Propositions 4 and 5:

Proposition 4 *For $V'(n) > 0$, $\beta, \delta \in [0, 1]$, and with commitment:*
A myope will maximally postpone all participation whenever educational opportunities are sufficiently incremental, specifically whenever

$$(p^s + p^f)\big[E(\Pi_{t+1}(n+1)) - E\Pi_t(n)\big] \geq \beta\delta^{T-t}\big[\delta V'(n) - V'(n+1)\big] \;\forall\, n, t : n < t. \qquad (3)$$

Proposition 5 *For $\beta, \delta \in (0, 1)$, $V'(n) \geq 0$, $V(T)$ finite, $E(\Pi_1) = \frac{1}{2}$, and K an arbitrarily large integer; with commitment and as $T \to \infty$:*
Myopes will participate in all of the first K periods if $p^s - p^f > 2c$, else they will avoid participation in all of the first K periods.

Proposition 3 implies that, in general, there are 2^T possible equilibrium strategies, however if the condition of Proposition 4 is satisfied, a myopic agent would adopt one of the $T + 1$ strategies in which his participation is maximally postponed. The conditions of Proposition 4 will often be satisfied. $V'(n) > 0$ is a tautology in that it requires educational activities to provide some educational benefit; and $\beta, \delta \in [0, 1]$ holds by the construction of quasi-hyperbolic discounting. For the final condition it would suffice for the marginal product of educational activities to diminish by at most the discount rate, since then $\big[\delta V'(n) - V'(n+1)\big] \leq 0$, but, since the left hand side of condition (3) is positive, it will in any case be satisfied during a child's early years since then $\beta\delta^{T-t} << 1$.

The conditions of Proposition 5 model the situation faced by a young child who perceives the end of compulsory schooling to be imponderably distant. Its result supports the thesis of Lavecchia, Liu & Oreopoulos (2016) that the participation decision of a young child will be dominated by her present-period payoffs, and it further demonstrates that those present-period payoffs exhibit a profound path dependence, in that an arbitrarily small change in initial conditions could lead to a diametric reversal of outcome. Together with Proposition 4, this conclusion suggests that the equilibrium strategies of a myope can be meaningfully dichotomized into high- and low-participation equilibria, where the former is characterized by full participation, and where the latter is characterized by a substantial period of non-participation, followed by a belated period of full participation as the consequences of underachievement loom large towards the end of compulsory schooling.

C. The Naïve Solution

The naif possesses perfect forward planning ability, but minimal self-awareness. Her equilibrium strategy is therefore generated by a family of T utility maximization problems, each of which maximizes the discounted sum of the remaining stage-game utilities from the perspective of one particular period $\tau \leq T$. These T objective functions each take the form of (4), where a strategy $S = \{s_t\}_{t=1}^{T}$ is considered to include all those decisions which have already been taken in periods preceding τ, as well as all anticipated future decisions.

$$\max_{\{s_t\}_{t=\tau}^{T}} \; s_\tau \left[d_t - c - p^f + w(\Pi_\tau, \pi_\tau)(p^s + p^f)\right] + \sum_{t=\tau+1}^{T} s_t \beta \delta^{t-\tau} \left[\frac{d_t}{\beta} - c - p^f + w(\Pi_t, \pi_t)(p^s + p^f)\right]$$

$$= \max_{\{s_t\}_{t=\tau}^{T}} \; s_\tau \left[w(\Pi_\tau, \pi_\tau)(p^s + p^f) - c - p^f\right] + \sum_{t=\tau+1}^{T} s_t \beta \delta^{t-\tau} \left[w(\Pi_t, \pi_t)(p^s + p^f) - c - p^f\right]$$
$$+ \beta \delta^{T+1-\tau} \left[V\left(\textstyle\sum_{t=1}^{T} s_t\right) - V\left(\textstyle\sum_{t=1}^{\tau-1} s_t\right)\right] \qquad (4)$$

In general, the family of expressions $\{4\}$ represents a complex dynamic problem because of the endogeneity of Π_t. One way to proceed would be to adopt the normative assumption of economic rationality. In the absence of behavioral motivations, that is when $p^s \equiv p^f \equiv 0$ and $\beta \equiv 1$, Lemma 1 shows that our model reduces in aggregation to the canonical investment criterion of Becker (1962): normatively optimal participation should continue until its marginal product no longer exceeds its marginal cost. The present approach therefore provides a nano-foundation for that canonical investment criterion.

Lemma 1 *For $V'(n) > 0$, $V''(n) \leq 0$,*[9] *$\delta \in (0, 1)$, and either with or without commitment:*
A normatively optimal strategy would set $s_t = 0$ for $t \leq T - n^*$, and $s_t = 1$ thereafter, where n^* satisfies:

$$\delta^{n^*} \left[V(n^*) - V(n^* - 1)\right] \geq c > \delta^{n^*+1} \left[V(n^* + 1) - V(n^*)\right]$$

and where we define $V(-1) := -\infty$, and $V(T + 1) := V(T)$.

[9] This standard concavity condition ensures that the normative solution is unique. It is highly plausible, since it states that educational attainment yields a diminishing marginal product. Note that, where behavioral payoffs are non-zero, the result of Proposition 2 demonstrates that our educational production technology could exhibit increasing returns to prior participation, even when educational attainment per se has a diminishing marginal product.

An important application of this Lemma is in the characterization of the naif's equilibrium strategy. Since the naif lacks self-knowledge, they believe that in future they will act normatively, that is they set $p^s \equiv p^f \equiv 0$ and $\beta \equiv 1$ for all future periods. Thus the endogeneity of Π_t in {4} is irrelevant for them also. The equilibrium strategy for a naif is therefore reminiscent of that of the myope:

Proposition 6

1. *Proposition 5 also holds for naïve agents.*

2. *For* $V'(n) > 0,\ \beta, \delta \in (0,1),\ E(\Pi_1) = \frac{1}{2},\ p^s = p^f$, *and with commitment:*
A naif will delay participation until weakly beyond the normatively optimal point.

3. *For* $V'(n) > 0,\ V''(n) < 0, V''(n) \leq 0,\ \beta, \delta \in (0,1)$, *and either with or without commitment:*
In the final period, participation for a naif is just as likely as for a myope, but theretofore participation is strictly less likely for a naif than for a myope.[10]

The first two results imply that the equilibrium strategies of a naif are dichotomised into high- and low-participation pathways in a similar way to those of the myope. The second and third results determine that the combination of perfect forward-planning with minimal self-knowledge results in a level of participation that is generally below both the normatively optimal level, and the level of participation achieved by the myope; even though the latter possesses neither self-knowledge nor forward-planning. Thus, in the absence of self-knowledge, an intervention which teaches children to plan ahead is likely to prove counter-productive.

D. *The Sophisticated Solution*

The sophisticate possesses both perfect forward-planning and complete self-knowledge. She must therefore solve the full periodwise maximization problem {4} by internalizing the endogeneity of Π_t. This makes her utility maximization problem considerably more complex than that of either the myope or the naif. In particular, the sophisticate needs to know (or assume) the functional forms and relative sizes of each constituent part of her payoff function in order to calculate the optimal trade-off between skill accumulation and potentially costly participation. Her behavior can therefore only be characterized relatively loosely without such assumptions.

Proposition 7

[10]Throughout this paper we intend both possible interpretations of the phrase 'strictly less likely': Firstly the set of parameter values for which the naif would participate in any given period is strictly smaller than that for the myope, and secondly the set of realized abilities for which the naif would participate (without commitment) is strictly smaller than that for the myope.

1. *Under commitment, a sophisticate who does not discount future payoffs, that is for whom $\beta = \delta = 1$, would front-load her participation to the maximum possible extent.*

2. *In the contrasting commitment case where discounting is relatively substantial and participation costs are relatively large, specifically whenever $(1 - \beta\delta)\frac{c+p^f}{p^s+p^f} \geq 1$, a sophisticate's participation would always be postponed to the maximum possible extent.*

3. *For $V'(n) > 0$ and under commitment: If ever $\left[E(\Pi_t)(p^s + p^f) - c - p^f\right] \geq 0$, then a sophisticate would participate in period t and all subsequent periods.*

4. *In the final period, participation for a sophisticate is just as likely as for a naif, but theretofore participation is strictly more likely for a sophisticate than for a naif (either with or without commitment).*

Proposition 6.2 showed that, for the naif, the effects of both psychic payoffs and of present-bias were to unambiguously reduce participation. For a sophisticate considering whether to commence participation, those behavioral components still unambiguously reduce her present-period payoff, but the net participation effect of that reduction is uncertain. This is because a sophisticate also internalizes her reduced future participation payoff, and so in some circumstances it may be optimal for her to overcompensate for it. Ceteris paribus, Proposition 7.1 shows that the sophisticate's psychic payoff component would favour the front-loading of any exogenously required participation. Thus a sophisticate's participation pattern will emerge as the net effect of a conflict between the front-loading influence of those psychic payoffs and the postponing influence of intertemporal discounting (Proposition 7.2). The most fortuitous resolution of that conflict occurs if the sophisticate were able to ensure that the net effect of both costs and psychic payoffs could become positive in the reasonably near future — since in that case her equilibrium strategy could well be to participate fully in all educational opportunities (Proposition 7.3).

Proposition 7.4 establishes that the sophisticate's perfect self-knowledge unambiguously increases her participation, whether by moderating the negative effects of p^s, p^f and β, or by reversing them to attain a high-participation equilibrium. Proposition 6, on the other hand, shows that forward-planning without self-knowledge leads to procrastination: the naif always believes that she will act optimally next period, and so she recursively delays participation to avoid negative behavioral payoffs in the present period. These twin results extend the main findings of O'Donoghue & Rabin (1999) to a situation where participation is optional and has dynamic consequences.

The next two subsections explore the above results by quantifying the participation of myopes, naifs, and sophisticates, under two representative model specifications.

E. A Quantitative Illustration of the Results

We now provide a numerical illustration of our results numerically. Although we necessarily sacrifice generality to do this, the supplementary materials demonstrate that the findings presented here are remarkably robust to a comprehensive set of alternative specifications. Table 2 details our preferred specification.

[Table 2 about here.]

The specification detailed in Table 2 is intended to model 'reality' as faithfully as possible. Its key features include: $V' > 0$; $V'' < 0$; Beta-distributed ability which is bell-shaped on the support of $(0, 1)$ and updates intuitively with diminishing returns and cross-returns to participation; $T = 10,000$ periods which represent approximately four participation decisions per day for twelve years of compulsory schooling; $\delta = 0.999$ which represents an annualized discount rate of 0.43 on that time scale, and a maximum schooling benefit $V(T)$ which is approximately five times greater than the total material cost $c \times T$ of full educational participation. The fact that $V(T) >> c \times T$ reflects the common empirical finding that the net benefit of compulsory education far outweighs its cost. A full rationale for this preferred specification is provided in the supplementary material.

The equilibrium actions for myopes and for naifs under commitment are explained by Figure 2. Figure 2 evaluates the respective marginal developmental payoffs d_t^{myope} and d_t^{naif} for commencing participation in each period t, in present-period 'money', and given the condition that participation is maximally postponed. Propositions 4 and 6 establish that this condition accurately determines the points at which both myopes and naifs will optimally commence their participation. d_t^{myope} exceeds d_t^{naif} because education yields a diminishing marginal product, and so the naif's naïve belief that they will participate for the final n^* periods reduces her perceived payoff to present participation.

Each agent archetype will participate once her d_t exceeds the solid black line that represents their total cost $c + p^f - E(\Pi_t)(p^s + p^f)$. That total cost is steadily increasing because it is calculated given the result of Proposition 5 that myopes and naifs do not participate in the initial periods, for any reasonably balanced psychic payoffs. We can therefore see that, for $p^s = p^f = 10$ the myope would participate for the final 1,508 periods and the naif for the final 750 periods, whilst for $p^s = p^f = 1$, the myope would participate for the final 3,177 periods and the naif for the final 1,583 periods. Thus the unique effect of perfect forward-planning is to approximately halve participation for agents who do not also possess self-knowledge.

[Figure 2 about here.]

Figure 2 therefore suggests that a child who is either myopic or naïve would, without external motivation, participate in only around 8-32% of educational activities. It is interesting to contrast such a child's extemporal preference with her ex-post preference. The latter is for full participation, since ex-post the benefits of education are enjoyed at the expense of only sunk costs. This discrepancy could explain why high school drop-outs commonly regret dropping out (see, for example: Bridgeland, Dilulio & Morison 2006). It is also possible to objectively adjudicate between these conflicting preferences using our numerical illustration. We find that the unweighed expected utility sum for full participation exceeds that of even n^*, the normatively efficient low-participation equilibrium, whenever psychic payoffs are non-negligible; specifically whenever $p^f = p^s > 0.1085$. This result supports the consensus view in the literature that a high level of educational investment is optimal.

Figure 3 shows the equilibrium strategy of the sophisticate for all possible situations. Possible situations are the complete set of *Period* $\times$ *Prior Participation* pairs at which a sophisticate could exogenously be placed, and the equilibrium participation decision for each of those cells can be calculated by reverse induction. Educational development pathways would therefore be represented in Figure 3 by a path which starts at $t=1$, $n=0$, and traverses to $t=10,001$ by travelling through 10,000 line segments, each of which would head due East in the case of non-participation, or North-East in the case of participation in that period. Clearly it is impossible for an agents' prior participation to exceed $t-1$ in any period t. Thus we can see that for psychic payoffs $p^s = p^f = 10$ the sophisticate would participate in period 1, and in all periods thereafter, but if $p^s = p^f = 1$ she would only participate for the final 1,622 periods. These results illustrate Proposition 7 by showing that self-knowledge either mitigates the postponing influence of psychic payoffs and present-bias, or reverses it entirely.

[Figure 3 about here.]

To see why a sophisticate would attain a high-participation pathway only if her psychic payoffs are sufficiently large, consider again Figure 2. The sophisticate experiences the same payoffs as the myope in any given period, and so her period 0 expected utility under full participation is given by the integral of $d_t^{myope} - \sum cost|attempt$ over all periods, weighted by each period's cumulative discount factor. In Panel 2A this integral is initially negative, but soon becomes substantially positive, whereas in Panel 2B this integral remains negative for many periods. The sophisticate has the foresight to pay an initial cost, provided that it is smaller than the expected future benefits of attaining the high-participation pathway. By contrast, neither the myope nor the naif would pay that initial cost, because neither would anticipate the positive future psychic payoffs that the high-participation pathway could provide.

Figure 3 shows clearly the threshold between high- and low-participation pathways for $p^s = p^f = 10$. In this case, if a sophisticate were to be exogenously placed[11] at

[11] The exogenous situations described here could be produced by any of: missed early-years devel-

period 509 with no prior participation, then her future participation would follow a low-participation equilibrium pathway in which she participates for only the final 800 periods. Contrastingly, if the same sophisticate were to exogenously arrive at period 508, she would fully participate throughout the remaining 9,493 periods. This illustrates the results of Propositions 2 and 5, in that there is a clear bifurcation between high- and low-participation pathways for a child who possesses both forward-planning and self-knowledge.

This subsection has illustrated that agents' developmental pathways under commitment can be broadly dichotomized into either high- or low-participation equilibria. Of these, the former corresponds to the anecdote of a 'good pupil' who always tries her best, and the latter corresponds to the anecdote of a pupil who essentially gives up on her education due to early disadvantage, before putting in some effort as the consequences of not doing so become apparent towards the end of compulsory schooling. We have seen that, for a range of reasonable psychic payoffs, agents who lack either self-knowledge or forward-planning are likely to become trapped into a low-participation equilibrium, unless some external intervention is provided during their early childhood. Where a child does posses those noncognitive skills, their equilibrium pathway will be determined by the interaction between her ability level and the exogenous (to her) parameters of the model. Subsection VA discusses the ways in which interventions could be designed to improve the child's participation likelihood given a set of exogenous parameters, and Subsection VB discusses the ways in which educators and parents could manipulate those exogenous parameters to create a more supportive environment for any given child.

F. *The Quantitative Effect of Limited Self-Control*

This subsection explores the quantitative implications of limited self-control, for the specification listed in Table 2, and with the intermediate psychic payoffs $p^s = p^f = 5$. Figure 4 shows, for each agent archetype and for every possible situation, the probability ρ that their realized ability π_t will be high enough to induce participation. Since equilibrium participation under commitment will occur whenever $\rho > 0.5$,[12] we can also read agents' commitment solutions directly from Figure 4 as a dichotomization around $\rho > 0.5$. Thus we can see that the qualitative effect of limited self-control is to introduce stochastic variation around the solution under perfect self-control.

[Figure 4 about here.]

The most striking feature of Figure 4 is the similarity between the equilibrium strategies of sophisticates, naifs, and myopes. The broad appearance of each Subfigure reflects

opment opportunities, low initial ability, or less conducive initial parameter values. Moreover, these 'possible pasts' would be observationally equivalent for a cross-sectional empiricist.

[12]This holds because Proposition 3 demonstrates that, at equilibrium: $w(\Pi_t) = E(\Pi_t)$.

the divergence between high- and low-participation equilibria, and their commonality reflects the fact that the equilibria of each agent archetype are characterized by such a divergence.

Low-participation pathways arise because each Subfigure has a large area of very low participation likelihood. If ever an agent enters that region of low prior participation, their equilibrium response would be to follow an almost horizontal (zero-participation) path until they reach the small triangular region of high-participation likelihood towards the end of their compulsory schooling. High-participation pathways arise because each Subfigure has a large area of very high participation likelihood. If ever an agent enters that region of high prior participation, their equilibrium response would be to follow an almost diagonal (full-participation) path for the remainder of their educational journey. Thus we can conclude that low-participation equilibria are possible for individuals of any noncognitive skill level, given sufficiently disadvantageous initial conditions, but also that high-participation equilibria are possible for individuals of any noncognitive skill level, given sufficient social advantage.

Closer inspection of Figure 4 reveals a critical difference between the equilibrium actions of sophisticates and those of either myopes or naifs. In the earliest periods of the model, the participation likelihood of sophisticates is very high (in fact their initial participation probability is 99.5%), whereas the initial participation likelihood of the other agent archetypes is very low (around 33%, which means that the conjunctive probability of their participating in at least three-quarters of the first twenty periods is less than 0.1%). Noncognitive skills therefore have a profound effect on a child's educational outcomes. In the absence of either forward-planning or self-knowledge it is highly unlikely that any child would attain the self-sustaining ability threshold of the high-participation pathway.

Figure 4 also illustrates many of our other analytic results. For example, we can see that the region of postponed participation is reduced when an agent learns to plan ahead without learning self-knowledge. We can also observe that the equilibrium quantity of postponed participation is decreasing in an agent's prior educational level; this will be the case whenever educational attainment has a diminishing marginal product. Several more subtle implications from Figure 7 are discussed in the supplementary material.

In order to better understand the participation patterns that emerge from Figure 4, we now simulate the evolution of 9 myopes' realized ability draws across time. The parametric assumptions are again those of Table 2 with $p^f = p^s = 5$, but here $c = 0$ so that the agents initial conditions lie close to the threshold between high- and low-participation. Figure 5 shows the results of these simulations.

[Figure 5 about here.]

Ex-ante, all of the myopes described by Figure 5 are identical. Each has initial expected ability $E(\Pi_0) = \frac{1}{2}$, and quite a wide dispersion of initial realized ability. As these agents progress through their compulsory education however, their Beta-parameterized ability distributions become much more precise, and develop towards probable success or probable failure according to their individual participation histories. Without commitment, a myope will participate if and only if his realized success probability exceeds the critical value derived in proposition 1. That critical value is shown in Figure 5 as a narrow black line, which begins at approximately $\frac{1}{2}$ in all cases, since in period 0 d_t is negligible due to discounting, and since balanced psychic payoffs $p^s = p^f$ are assumed. The critical value then changes little for agents who frequently participate, due to the assumption of diminishing returns to education, however, for agents who stop participating, the critical value becomes lower as the period $T+1$ consequences of (non)participation become more immediate. The bar across the top of each panel of Figure 5 is shaded black with a density that reflects the realized local participation probability.

Figure 5 shows that, for these parametric assumptions, around half of those individuals who do not plan ahead (1, 2, 4, 6, and 8) experience sufficient early success to obtain a development pathway characterized by mutually-reinforcing increases in ability and participation. By contrast, individuals 3, 5, 7, and 9 experience a slightly poorer draw of initial realized abilities, which precipitates a mutually-reinforcing decline in ability and participation. However, we can also see that those individuals with the most marked decline in ability are the most likely to increase their effort as the consequences of their present low attainment level gain immediacy towards the end of their compulsory education. Further analyses have demonstrated that these few simulated vignettes appear to be representative of population outcomes under the current parametric assumptions. In particular, Figure 6 demonstrates that simulated educational investment outcomes are polarized into either high- or low-participation pathways.

Figure 6 also investigates the effect of altering the current parametric assumptions. Panel 6A shows that, outside of a narrow window around $c = 0$, participation patterns of myopes without commitment vary little from the deterministic outcomes which would be attained with commitment. This is because, for balanced psychic payoffs $p^s = p^f$, the first few periods' utility maximization problems are dominated in expectation by c, and so an improbably fortunate or unfortunate series of realized ability draws would be needed to overcome that influence. The gray dots in Figure 6A represent simulated total participation outcomes for 200 myopes for each c value, and so it can be seen that, outside of the window $c \in [-0.1, 0.1]$ there is very little chance that any myope will escape the influence of their participation cost. Thus, with c=1, essentially all vignettes would resemble that of Panel 5.5, save that the critical ability level would be transposed vertically upward, wherefore the decline in ability through time would be even more marked.

[Figure 6 about here.]

The black diamonds of Panel 6A show the equilibrium outcomes for a myope under commitment. It can be seen that, with commitment, the sign of c generates a stark bifurcation between a complete participation pathway, and a low participation pathway wherein participation would be zero until the final ≈20% of periods. This bifurcation illustrates the predictions of Propositions 1 and 5. We can also understand how variation around those two pathways would arise without commitment. A stochastic reduction from full participation would arise because some initial ability draws would be below expectations, whilst a stochastic increase from low participation would arise because some later ability draws would exceed expectations. Both of these phenomena are exemplified in Figure 5.

Panel 6B shows the effects of variation in the magnitude of the psychic payoffs, for the reasonable participation cost $c = 1$. Since, in expectation, initial participation is not optimal for any moderately positive c, participation under periodwise commitment occurs entirely in the final periods of compulsory education. Thus, as the psychic payoffs increase, that participation becomes further delayed due to the negative expected influence of p^f following many periods of task avoidance. A more interesting dynamic is seen for very large psychic payoffs. Here, as previously, it becomes increasingly likely that some unexpectedly high realized abilities in early periods could overcome the negative influence of c. However the qualitative importance of this effect remains negligible: in additional analyses, all of 10,000 simulated individuals remained on a low-participation pathway with $c = 1$ and $p = 20$.

This subsection has therefore established that imperfect self-control induces stochastic variation around the familiar high- and low-participation pathways. Although the present simulations suggest that the boundary region wherein an agent's level of self control could affect their qualitative pathway may be relatively narrow, it could nevertheless have important implications for pedagogy and for intervention design. Any educational task, whether provided by a parent, an educator, or an intervention, would successfully engage a child who lacks self-control if and only if she were able to achieve success in its initial stages.

V. Discussion with Implications for Policy and Practice

A. *Implications for Intervention Design*

The previous section has established that, under the proposed model, educational outcomes are dichotomized into high- or low-participation pathways. Of these, the high-participation pathway is always optimal from the point of view of society, and from the point of view of the child who ex-post enjoys the benefits of high education at the expense of only sunk costs. When the psychic payoffs to success or failure are non-negligible —

in this specification whenever $p^f = p^s > 0.1085$ — the high-participation pathway would also be optimal from the point of view of the child ex-ante, but we have seen that several factors could prevent her from attaining that pathway. In this Subsection we assess how public policy could intervene to improve the child's cognitive or noncognitive abilities; in the next Subsection we assess how parents and educators could support the child by manipulating the exogenous parameters of the model.

The most direct design of intervention would aim to improve the child's (cognitive) ability. This could be effective, because we have established that there generally exists some threshold level of ability above which high participation would become self sustaining. However we have also established that that threshold ability level swiftly diverges away from the level accumulated by agents on the low-participation pathway (see Figure 4 or 5), and so if such an intervention is not undertaken very early in the life-course it is likely to prove ineffective. Indeed, such an intervention could even prove counterproductive due to the psychic cost of trying, but failing, to catch up with peers whose ability is also steadily improving.

Our results therefore suggest that an indirect intervention design could be more effective. Figure 4 establishes that teaching young children the noncognitive skills of forward-planning and self-knowledge could allow them to decide for themselves to participate fully in educational opportunities. Nevertheless, there will be a critical window (in the sense of Cunha & Heckman 2007) in which to provide such an intervention, whereafter non-participation would become the equilibrium strategy of even a sophisticated child with low prior participation. In such cases our model suggests that the child would need to consciously decide to prioritise their future outcomes over their present behavioral payoffs if they were to escape the low-participation pathway. Unless the child is successfully supported to make that decision, no intervention is likely to have a lasting effect on her participation decisions after it has been removed.

In order to support a child to not only consciously delay their gratification, but also to stick by that decision against unfavourable present-period payoffs; long-term one-one mentoring is likely to be required. However such an intervention would present financial and logistical challenges. It may therefore be more efficient to intervene with the game, rather than with the players who fail to achieve within it.

B. *Implications for Pedagogical Practice*

Although the model's environmental parameters are exogenous for the child, it will generally be possible for parents and teachers to manipulate them. For example: $-c$ could be made positive by the use of sufficiently 'fun' and engaging tasks, or at worst by the imposition of credible sanctions on the outside option. p^s could be increased by agreeing appropriately challenging goals, and by the judicious use of praise and rewards. $-p^f$

could be made less negative, or possibly even positive, by both explicitly teaching and implicitly modeling that failure is positive: because it shows that you are taking on challenges and because it generates learning. Finally, the distant positive payoff of d_t could be made more immediate by emphasising the intrinsic value of developing one's abilities, and the sophisticated extrinsic value that present learning will render future tasks more accessible and therefore more enjoyable. Table 3 maps these specific implications onto existing pedagogical practices, thereby demonstrating that these implications articulate the insights of experienced teaching professionals.

[Table 3 about here.]

The implications discussed in this section support the conclusion of Cunha & Heckman (2007) that early intervention is vital if the educational pathway of disadvantaged children is to be altered. However they also extend that conclusion by establishing that an intervention which naïvely seeks to improve a child's cognitive ability is less likely to succeed than one which focusses on the noncognitive determinants of her daily decisions to under-invest in educational opportunities. Moreover, we have demonstrated that such under-investment could arise as an equilibrium response to initial disadvantage. This suggests that it may not be the players which require intervention from policy-makers, but rather the game itself. This subsection suggests how such an intervention could be undertaken.

VI. Conclusion

This paper has developed a new model of educational investment which is both tractable and intuitively plausible. We propose that educational outcomes might be cumulatively determined by a series of minor participation decisions, rather than pre-determined by an hypothetical one-shot investment decision. Any such one-shot decision would implicitly require perfect forward planning, complete self-knowledge, and complete self-control, none of which is feasible in this context. We identify the specific effects of limitations in each of these noncognitive abilities, both analytically and numerically.

We establish that our model of education as a repeated participation game is consistent with the canonical model, in that it recovers the same solution under normatively optimal assumptions. However, we also establish that when those assumptions are relaxed to admit psychic payoffs for success and failure, a profound path-dependence emerges whereby small changes in initial conditions could lead to divergent educational pathways. That result suggests a mechanism for the observed persistence of economic inequality, which has hitherto lacked a robust theoretical explanation.

By founding aggregate educational investment on incremental participation decisions we are also able to provide a theoretical basis for the six key stylized facts of educational development identified by Cunha & Heckman (2007). Self-productivity of cognitive ability, dynamic complementarity of external investment with present ability stocks, and the sensitivity of educational outcomes to early investment are all endogenously derived within our model. These results explain the empirical finding that early intervention is vital if initial disadvantage is to be overcome (see, for example: Cunha & Heckman 2010, Chetty, Hendren & Katz 2015). However our results also go beyond existing literature in that they are able to inform how, as well as when, to intervene.

A conventional public policy response to educational underinvestment would identify individuals on a low-participation pathway and support them to improve their educational attainment. Our findings suggest that an intervention which directly targets those individuals' cognitive ability is likely to be less effective than one which targets their noncognitive skills of forward-planning and self-knowledge: the latter could achieve a lasting improvement in those individuals' participation decision-making, whilst the former would only raise their educational attainment for a transient period. However, our findings also establish that the main challenge for any individual-focussed intervention is that non-participation could arise as an equilibrium response to earlier disadvantage. This implies that the causes of ongoing non-participation are likely to be exogenous to the individual, and so any attempt to intervene should consider targeting that exogenous situation, rather than the individual trapped within it. Since the parameters of our model describe tangible aspects of a child's educational situation, we have identified specific intervention actions which could enable participation in equilibrium by altering the educational opportunities provided to disadvantaged children. Our results suggest that such pedagogical interventions could contribute toward a meaningful reduction in the persistent economic inequality of modern society.

A. Mathematical Appendix

A. *Proof of Lemma 0:* $E(\Pi_t(n))$ *is a strictly increasing function of* n

Since we have the simplifying assumption that $\Pi_t(n)$ is well-defined, we may, without loss of generality, choose the order in which the implied n periods of participation appear within $t-1$ prior periods. Let us therefore compare $\Pi_t(n)$ with $\Pi_t(m)$, where $m < n < t$, by assigning the first $t-n-1$ periods of both participation sequences to non-participation, and the following m periods to participation. The final $n-m$ periods were therefore either periods of participation to reach $\Pi_t(n)$, or periods of non-participation to reach $\Pi_t(m)$. Thus $\Pi_t(n)$ stochastically dominates $\Pi_t(m)$ by the transitivity of stochastic dominance, hence $E(\Pi_t(n)) > E(\Pi_t(m))$.

B. *Proof of Proposition 1*

In any given period $t \le T$, the Bayesian Nash Equilibrium of Figure 1 is a strategy s_t^* that maximizes the agent's expected utility (2).

The constrained maximization of (2) yields three cases. Firstly, it is possible that the expected value of participation, $\left[d_t - c - p^f + w(\pi_t, \Pi_t)(p^s + p^f)\right]$ could be precisely 0, in which case all values for the decision variable provide identical expected utility. However, provided that at least one of the decision parameters is continuously distributed, this case occurs with probability 0, and so we do not analyse it further. Otherwise, if $\left[d_t - c - p^f + w(\pi_t, \Pi_t)(p^s + p^f)\right] > 0$, then the optimal strategy is to set $s_t = 1$, and conversely, if $\left[d_t - c - p^f + w(\pi_t, \Pi_t)(p^s + p^f)\right] < 0$, then the optimal strategy is to set $s_t = 0$. Thus, with probability 1, the decision problem in period t has a unique equilibrium response of $s_t \in \{0, 1\}$.

For definiteness we may therefore declare, with almost no loss of generality, that agents will participate in a given educational task if and only if:

$$\left[d_t - c - p^f + w(\pi_t, \Pi_t)(p^s + p^f)\right] > 0$$

$$w(\pi_t, \Pi_t) > -\frac{d_t - c - p^f}{p^s + p^f} \tag{5}$$

Finally, note that we assume throughout that $p^s, p^f, c > 0$, hence participation is always optimal if $d_t > c + p^f$, and never optimal if $d_t + p^s < c$, since the subjective success probability $w(\pi_t, \Pi_t)$ must be bounded within $[0, 1]$. $\square$

C. *Proof of Proposition 3*

For the first two items, note that in period T the participation condition is precisely (5). Then, by backward induction we will assume that results 1 and 2 hold for all periods

$t > \tau$, and we will show that they must then hold for period τ. The period τ utility maximization problem is therefore given by:

$$\max_{s_\tau} U_\tau(n) = s_\tau \left[d_\tau(n) - c - p^f + w(\pi_\tau, \Pi_\tau)(p^s + p^f) + \beta\delta U^*_{\tau+1}(n+1) - \beta\delta U^*_{\tau+1}(n)\right] + \beta\delta U^*_{\tau+1}(n)$$

where n is the number of educational tasks thus far attempted, and $U^*_{\tau+1}(n)$ is the period $\tau+1$ value of the maximum payoff that could be achieved throughout periods $t > \tau$, given prior participation n by that period. (Note that this exists and is well-defined because of the induction assumption, and because the intersection of finitely many mathematically certain events is mathematically certain). The solution to this problem yeilds the participation condition

$$w(\pi_t, \Pi_t) > -\frac{d_t - c - p^f + \beta\delta U^*_{\tau+1}(n+1) - \beta\delta U^*_{\tau+1}(n)}{p^s + p^f} \tag{6}$$

with probability 1 provided that at least one of the parameters is continuously distributed. Thus, by induction, the first two results are proven.

Now consider an agent who has no information regarding π_t. We must have that $w(\Pi_t, \pi_t)$ is in fact a function of Π_t alone. Suppose then that $w(\Pi_t) < E(\Pi_t)$ for some ability distribution Π_t. Then there is some value of d_t for which the agent will not participate under $w(\Pi_t)$, but would if $w(\Pi_t)$ were $E(\Pi_t)$. (6) implies that one such d_t is given by

$$d_t = c + p^f - \frac{w(\Pi_t) + E(\Pi_t)}{2}(p^s + p^f) - \beta\delta U^*_{\tau+1}(n+1) + \beta\delta U^*_{\tau+1}(n).$$

However, at this value of d_t the expected payoff to participation is

$$d_t - c - p^f + E(\Pi_t)(p^s + p^f) + \beta\delta U^*_{\tau+1}(n+1) - \beta\delta U^*_{\tau+1}(n) = \frac{E(\Pi_t) - w(\Pi_t)}{2}(p^s + p^f) > 0,$$

and so participation would maximize expected utility. Thus $w(\Pi_t) < E(\Pi_t)$ cannot be optimal in expectation. An analogous argument shows that $w(\Pi_t) > E(\Pi_t)$ cannot be an equilibrium outcome. □

D. *Proof of Proposition 2*

p^f and p^s are assumed to be time-invariant, and $w(\Pi_t) = E(\Pi_t)$ in equilibrium by Proposition 3. Therefore to prove the proposition it suffices to show that $E(\Pi_t(n))$ is a strictly increasing function of n, which was shown in Lemma 0. □

E. *Proof of Proposition 4*

By the principle of induction it suffices to show that participation in any period t implies participation in period $t+1$. From the participation condition of the stage game (2) we have that this will certainly be the case whenever

$$\left[d_{t+1} - c - p^f + w(\pi_{t+1}(n+1), \Pi_{t+1}(n+1)(p^s + p^f)\right] \geq \left[d_t - c - p^f + w(\pi_t(n), \Pi_t(n)(p^s + p^f)\right]$$

for all $n, t : n < t$. Thus, under commitment and by Proposition 3 we require:

$$\begin{aligned} &\left[d_{t+1} - c - p^f + E(\Pi_{t+1})(p^s + p^f)\right] - \left[d_t - c - p^f + E(\Pi_t)(p^s + p^f)\right] \geq 0 \\ &\left[E(\Pi_{t+1}(n+1)) - E(\Pi_t(n))\right](p^s + p^f) - \beta\delta^{T-t}\left[V'(n+1) - \delta V'(n)\right] \geq 0 \ \forall\, n, t : n < t, \end{aligned}$$

where the 2nd line follows from the definition of d_t (1). □

F. *Proof of Proposition 5*

First note that, as $T \to \infty$, we have, by (1) that $d_K \to 0$ for any finite K, provided $\left[V(n_K + 1) - V(n_K)\right]$ is bounded, sufficient conditions for which are: $V(0) = 0$, $V' \geq 0$, and $V(T)$ finite.

Next note that a myope maximises his stage game utility function $U_k = s_k \left[-c - p^f + w(\Pi_k)(p^s + p^f)\right.$ (from 2) in all periods $k \leq K$.

Since in the first period we have $w(\Pi_1) = E(\Pi_1) = \frac{1}{2}$, by Proposition 3, the utility maximization problem yields $s_1 = 1$ if and only if $p^s - p^f > 2c$. Since finite K has been fixed, we may then apply induction, from $k = 1$ to $k = K$, by noting that, given participation was optimal in period k, participation will remain optimal in period $k+1$, since the only change in the payoff function will be that Π_{k+1} now stochastically dominates Π_k, hence $w(\Pi_{k+1})(p^s + p^f) > w(\Pi_k)(p^s + p^f)$, since $w(\Pi_t) = E(\Pi_t)$, and p^s, $p^f > 0$.

The converse holds by an analogous argument. □

G. *Proof of Lemma 1*

First note that, with probability 1, $s_t \in \{0, 1\}\ \forall t$, by Proposition 3. Next, note that since $p^s = p^f = 1$ for a normatively optimal agent (NOA):[13] $w(\Pi_t, \pi_t)$ is irrelevant, and so the solutions with and without commitment are identical.

We now characterize the pattern of participation, for a normatively optimal agent who maximizes the family of utility functions {4}. Suppose for some strategy S with n^s participation periods, there exist ϕ, t and τ, all integers, such that $s_t = 1$ and $s_\tau = 0$, and $\phi \leq t < \tau$. From {4} the expected utility contribution of period t from the point of view of period ϕ is $U_t^\phi = \beta\delta^{t-\phi}\left[d_t - c - p^f + w(\pi_t, \Pi_t)(p^s + p^f)\right]$, which reduces to

[13] As an aside, I note that NOA's biblical namesake also behaved normatively.

$d_t - c$ for NOA.[14] From (1) we have that $d_t = \beta\delta^{T-t+1}\big[V(n^s) - V(n^s-1)\big]$. The Utility contribution of period τ is $U_\tau = 0$, and so the total contribution of periods t and τ is $U_{t,\tau} = \delta^{T-\phi+1}\left[V(n^s) - V(n^s-1)\right] - \delta^{t-\phi}c$. Consider the deviation from this strategy wherein s_t is altered to 0 and s_τ is altered to 1. Then, since n^s is unaltered, we would have $\tilde{U}_{t,\tau} = \delta^{T-\phi+1}\left[V(n^s) - V(n^s-1)\right] - \delta^{\tau-\phi}c$, which exceeds $U_{t,\tau}$ since $\tau > t$ and $\delta \in (0,1)$, and so S cannot be a Nash Equilibrium strategy. Thus the normatively optimal pattern of participation under ex-ante commitment is to postpone participation so far as possible.

It remains to determine n^*, the normatively optimal number of periods of task participation. Given the above, we need only consider strategies, S, characterized by $T - n^s$ periods of non-participation, followed by n^s periods of participation. For an interior solution, the foregoing analysis yields that n^s is optimal if and only if both: $\delta^{n^s}\left[V(n^s) - V(n^s-1)\right] > c$ and $\delta^{n^s+1}\left[V(n^s+1) - V(n^s)\right] < c$. The assumption that $V''(n) \leq 0$ is sufficient to ensure that at most one n^* satisfies this condition, since $\delta < 1$. If no n^* satisfies this condition, then we necessarily have a corner solution, whereby either zero participation is optimal — since $c > \delta\left[V(1) - V(0)\right]$ — or full participation is optimal — since $c < \delta^T\left[V(T) - V(T-1)\right]$. These situations are made compatible with the proposition by defining $V(-1) := -\infty$, and $V(T+1) := V(T)$. Nevertheless, these situations are also somewhat pathological, in the first case because the returns to primary education are known to be high (Psacharopoulos & Patrinos 2004), and in the second because it requires c to become vanishingly small as T becomes large. □

H. Proof of Proposition 6

For the second result, note first that Proposition 3 implies that $w(\Pi_t) = E(\Pi_t)$, and so the assumption of evenly-weighted psychic payoffs: $E(\Pi_1) = \frac{1}{2}$ and $p^s = p^f$, is sufficient to ensure that the first period utility maximization problem of the naif is identical in value to that of her NOA counterpart, except that the value of d_t will be a smaller positive value since $\beta < 1$ and $V'(n) > 0$. Thus if a naif participates in period 1, then her NOA counterpart would do so a fortiori.

We now proceed inductively. Suppose that a naif does not participate in period t. Then in period $t+1$ the psychic component of her participation payoff $-p^f + w(\Pi_t)(p^s + p^f)$ will have decreased (or become more negative), since Π_{t+1} will be stochastically dominated by Π_t, hence $w(\Pi_{t+1}) < w(\Pi_t)$ by Proposition 3. The material component of her payoff, $d_t - c$, will remain below (more negative than) that of her NOA counterpart, since $\beta < 1$. Thus, by the principle of induction, we have that the naif will participate only if her normatively optimal counterpart would do so, up until the naif's first period of participation. Thus the naif does not participate until weakly after NOA. Since NOA

[14] If $t = \phi$ then the factors β should not appear in U_t^ϕ, however since $\beta = 1$ for NOA this is an irrelevant detail.

maximally postpones his participation (Proposition 1), and since the Naif believes that in all future periods she will act as NOA, we also have the useful result that the naif will only participate if she expects that she will continue to do so thereafter.

For the third result, note first that the present-period payoffs of the naif are identical to those of the myope. Thus, in the final period their solutions are also identical. In any period $\tau < T$, either $\tau \geq T - n^*$ or $\tau < T - n^*$.[15] In the former case, the naif believes that she will participate in all periods $t > \tau$, and so the total utility benefit of participation to the naif exceeds that to the myope by

$$\sum_{t=\tau+1}^{T} \beta\delta^{t-\tau}\left[d_t(n_\tau + t - \tau) - c\right] - \sum_{t=\tau+1}^{T} \beta\delta^{t-\tau}\left[d_t(n_\tau + t - \tau - 1) - c\right] \tag{7}$$

provided that the naif's expected future participation wouldn't reduce under present participation. (7) is negative since $V''(n) < 0$ implies that each term in the second summation exceeds its corresponding term in the first. If the naif's optimal future participation has reduced, then this remains true because: until the first such period of non-participation the terms of the second sum exceed that of the first; during the first such period of non-participation the first summand is 0 and so $V'(n) > 0$ implies that it is exceeded by its corresponding term in the second summation; after the fist such period of non-participation the terms are identical (and so there will be no further non-participation in expectation). Thus in all cases the naif's total utility benefit to participation is strictly less than that of the myope, hence she is strictly less likely to participate in any period $\tau \geq T - n^*$. For periods $\tau < T - n^*$ an identical argument holds, with the summations in (7) running instead from $t = n^* \to T$.

Now for the first result, we make use of the fact that the naif believes they will act as NOA in all future periods (that is, they will set $p^f = p^s = 0$, and $\beta = 1$). Thus they do not appreciate that the ability development implications of their period k choice will in any way affect their future payoffs. Thus in any period k their only consideration is to maximise their present-period utility $U_k = s_k\left[d_k - c - p^f + w(\Pi_k)(p^s + p^f)\right]$, where $d_k = \beta\delta^{T-k+1}\big[V'(n + \min\{n^*, T-k\})$ by (1), where n is the number of periods of participation to date, and n^* is the normatively optimal number of participation periods. U_k^{naif} therefore converges to U_k^{myope} as $T \to \infty$, so Proposition 5 holds for the naif also. □

I. *Proof of Proposition 7*

To prove the first three results, denote the sophisticate's optimal strategy (which we know to be well-defined by Proposition 3) by S. Unless participation is maximally front-loaded, S must include, at some point, the strategy sequence $s_t = 0$, $s_\tau = 1$, where $\tau = t+1$. Let $\tilde{S}$

[15] Where n^* is the well-defined number of participation periods for NOA, see Proposition 1.

denote the alternative strategy where participation is swapped between those two periods, such that $s_t = 1$ and $s_\tau = 0$. Then, from the point of view of period t[16], the net increase in utility due to switching from S to $\tilde{S}$, would be $\tilde{U}_{t,\tau} - U_{t,\tau} = \left[w(\Pi_t, \pi_t)(p^s + p^f) - c - p^f\right] - \beta\delta\left[w(\Pi_\tau, \pi_t)(p^s + p^f) - c - p^f\right]$ since the payoffs in all other periods are identical under the reordering and the net developmental payoff is unchanged. For $\beta, \delta \in (0, 1]$ and under commitment we have, by Proposition 3, that a switch to earlier participation would therefore be beneficial if and only if $\left[E(\Pi_t) - \beta\delta E(\Pi_\tau)\right](p^s + p^f) - (1 - \beta\delta)(c + p^f) > 0$. Rearranging yields the equivalent condition: $E(\Pi_t) - \beta\delta E(\Pi_\tau) > (1 - \beta\delta)\frac{c+p^f}{p^s+p^f}$, since $p^s, p^f > 0$. Analogously, the converse of the foregoing argument yields that participation should optimally be postponed if and only if $E(\Pi_t) - \beta\delta E(\Pi_\tau) < (1 - \beta\delta)\frac{c+p^f}{p^s+p^f}$.

Thus, in the special case that $\beta = \delta = 1$, the above conditions collapse such that the participation sequence $\{0, 1\}$ can never be optimal provided $E(\Pi_t) > E(\Pi_\tau)$. The latter is always true, since Π_τ is evaluated for a strategy which is identical to that which gives Π_t, save that the agent does not participate in period t, hence we have that Π_τ is stochastically dominated by Π_t.

Further, under ex-ante commitment, we have that participation should always be postponed whenever $(1 - \beta\delta)\frac{c+p^f}{p^s+p^f} \geq 1$. This is true, since $E(\Pi_t), \beta\delta E(\Pi_\tau) \in (0, 1)$, whereby their difference is $\in (-1, 1)$, and so can never be ≥ 1. Note, however, that as $\beta\delta \to 1$ this condition would require $\frac{c}{p^s} \to \infty$, which is infeasible. As $\beta\delta \to 0$ it becomes sufficient for the direct and opportunity costs to merely exceed the psychic payoff of success, such that $c \geq p^s$. (We might also note the special case of NOA, where postponement is always optimal since $p^s = p^f = 0$).

The proof of the third result is straightforward. $V'(n) > 0$ ensures that $d_t(n) > 0$ for all $n \leq t \leq T$. Proposition 1 then ensures that the complete stage-game payoff (2) is positive whenever $\left[E(\Pi_t)(p^s + p^f) - c - p^f\right] \geq 0$. Moreover, the sophisticate knows that, if she participates in period t, then $E(\Pi_{t+1}) > E(\Pi_t)$, and so $\left[E(\Pi_{t+1})(p^s + p^f) - c - p^f\right] \geq 0$ will be true a fortiori. Thus, inductively, the sophisticate will know that all future period payoffs will be positive if she participates in all future periods. Not participating in any (current or future) period would therefore reduce payoffs for that period. Importantly, it would also reduce payoffs for periods thereafter, since then $E(\Pi_{t+1}) < E(\Pi_t)$, and that reduction could not be compensated for by increased d_t since the final number of participation periods could only be reduced, and since each participation period contributes positively to the total $\sum_{t=1}^{T} \delta^t d_t$ because $V'(n) > 0$. Thus any deviation from the proposed strategy of full participation would necessarily yield a lower total payoff.

To prove the final result, we compare the stream of future utilities for any given period×prior-participation pair for naifs and for sophisticates. First note that the present period payoffs, are identical for both levels of sophistication. Thus, in the final period, each type of agent is equally likely to participate. Next we need attend to periods

[16] t is the operative period in which the decision between S and $\tilde{S}$ would be finalized,

wherein the naif participates under commitment. From the proof of Proposition 6 we have that the naif only participates in any period $\tau < T$ if she expects to keep participating thereafter. It would therefore suffice to show that the sophisticate's expected utility stream from period τ under the constraint of continuous participation thereafter, exceeds that of the naif, since the sophisticate's constrained utility stream must be at least as great as her unconstrained utility stream. But, given continuous future participation, the difference between the sophisticate's total expected utility gain due to participation and that of the naif is $\sum_{t=\tau+1}^{T} \beta\delta^{t-\tau} \left[E(\Pi_t(n+t-\tau))(p^s+p^f)-p^f\right] - \sum_{t=\tau+1}^{T} \beta\delta^{t-\tau} \left[E(\Pi_t(n+t-\tau-1))(p^s+p^f)-p^f\right]$, which is positive since $E(\Pi_t(n))$ is a strictly increasing function of n by Lemma 0. Next we note that, in any period before $T-n^*$ (the period after which the naif expects to participate fully), and constraining the sophisticate to match the expected future participation of the naif, the comparison is identical save that the limits of the summations run instead from $t = n^* \to T$ (as per the proof of Proposition 6.3). Finally, we note that, since the case without commitment is identical to that with commitment except in the current period, and since the current period payoffs are identical for sophisticates and for naifs, this result equally applies without commitment. □

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List of Figures

Figure 1:
A Representative Agent's Participation Decision Process

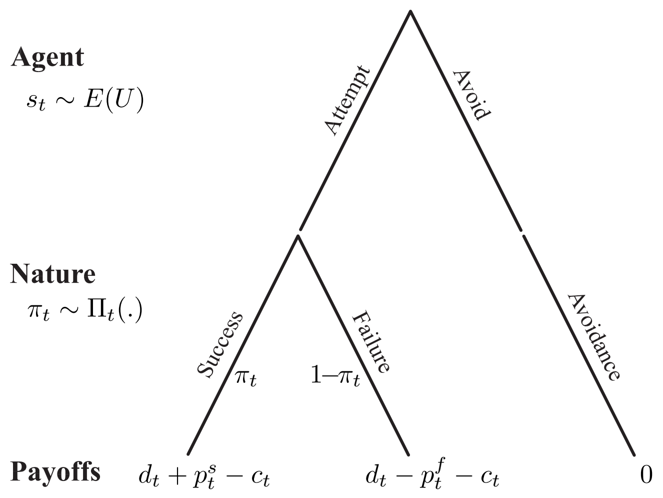
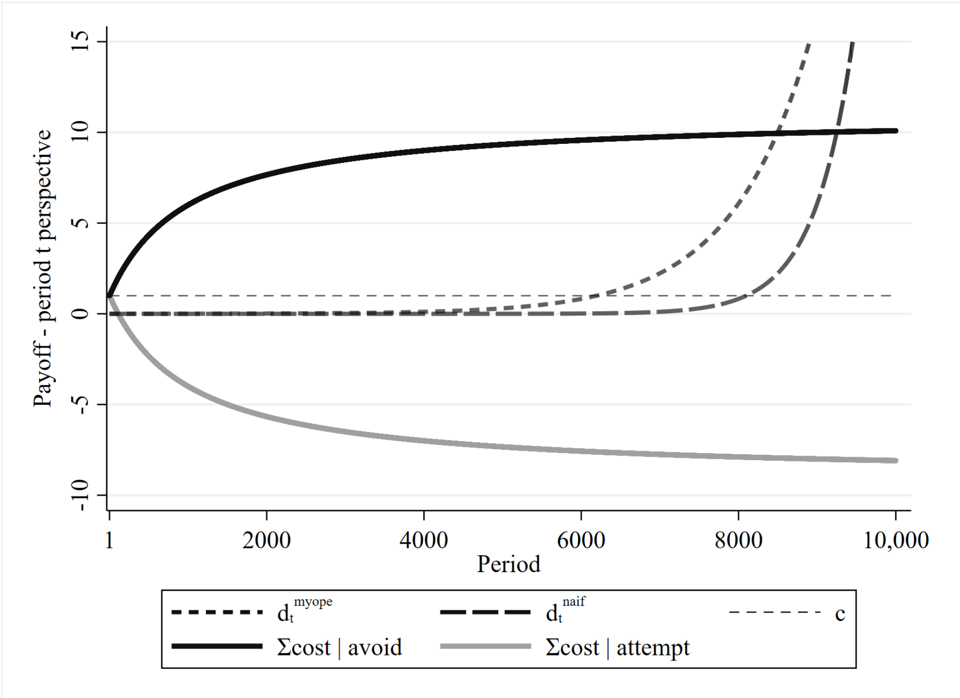
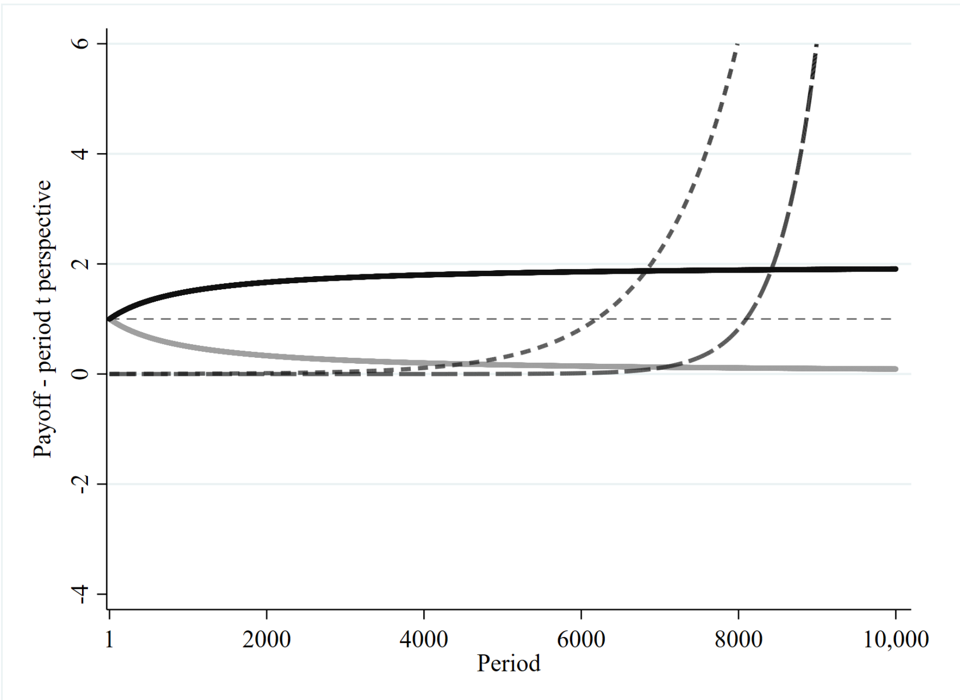


Figure 2:
Payoff Components for Myopes and for Naifs with Commitment



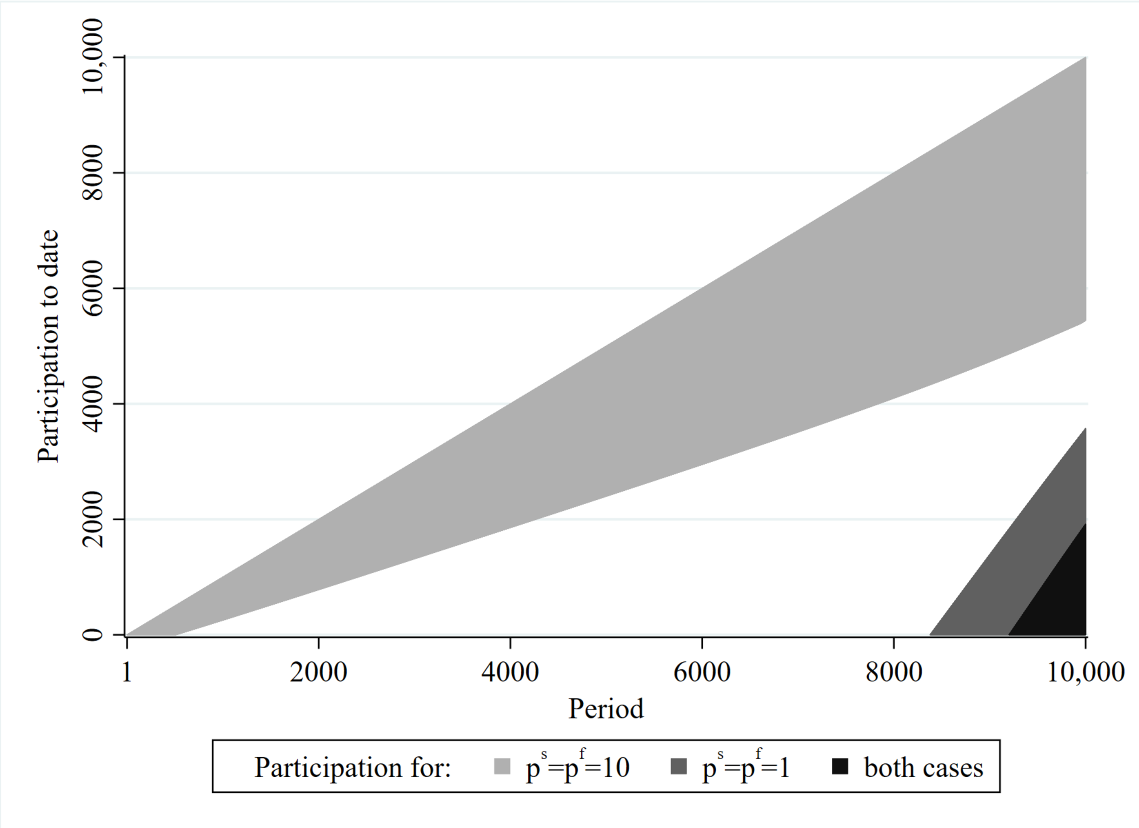
(A) *Periodwise Payoff Components for* $p^s = p^f = 10$.



(B) *Periodwise Payoff Components for* $p^s = p^f = 1$.

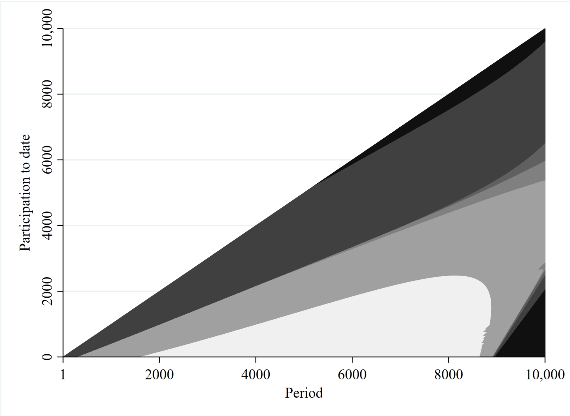
The conditional Expected Utility payoff for each successive stage game, as given in equation (2), partitioned into the developmental payoff d_t^{myope} or d_t^{naif}, and the total participation cost $c + p^f - E(\Pi_t)(p^s + p^f)$. Two conditional realizations of the total cost are shown: that for the case of task avoidance in periods $1, ..., t-1$ and that for the case of task participation throughout those periods. Also shown is c, the constant direct and opportunity cost of participation.

Figure 3:
The Equilibrium Actions of Sophisticates with Commitment

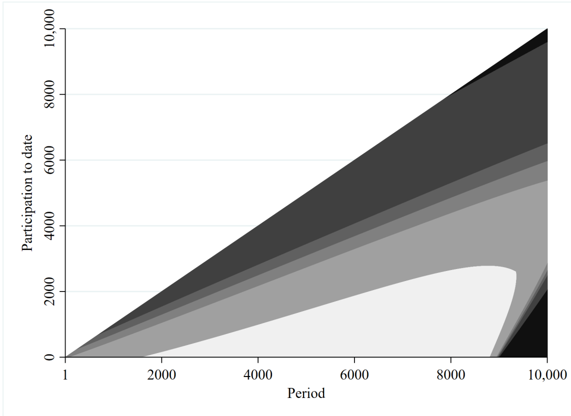


All possible situations are uniquely identified by a $Period \times Prior\,Participation$ pair. Situations which induce equilibrium participation only for $p^s = p^f = 10$ are shaded in light gray, situations which induce equilibrium participation only for $p^s = p^f = 1$ are shaded in dark gray, and the black region indicates situations where both parametrizations induce equilibrium participation.

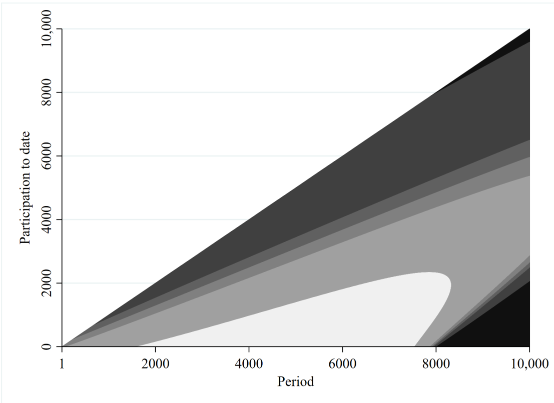
Figure 4:
Participation Probabilities without Commitment, for $p^s = p^f = 5$



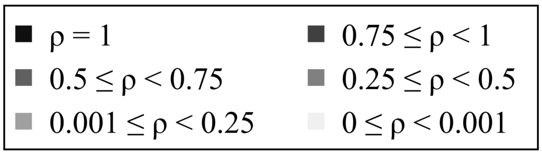
(A) *Sophisticates*



(B) *Naifs*

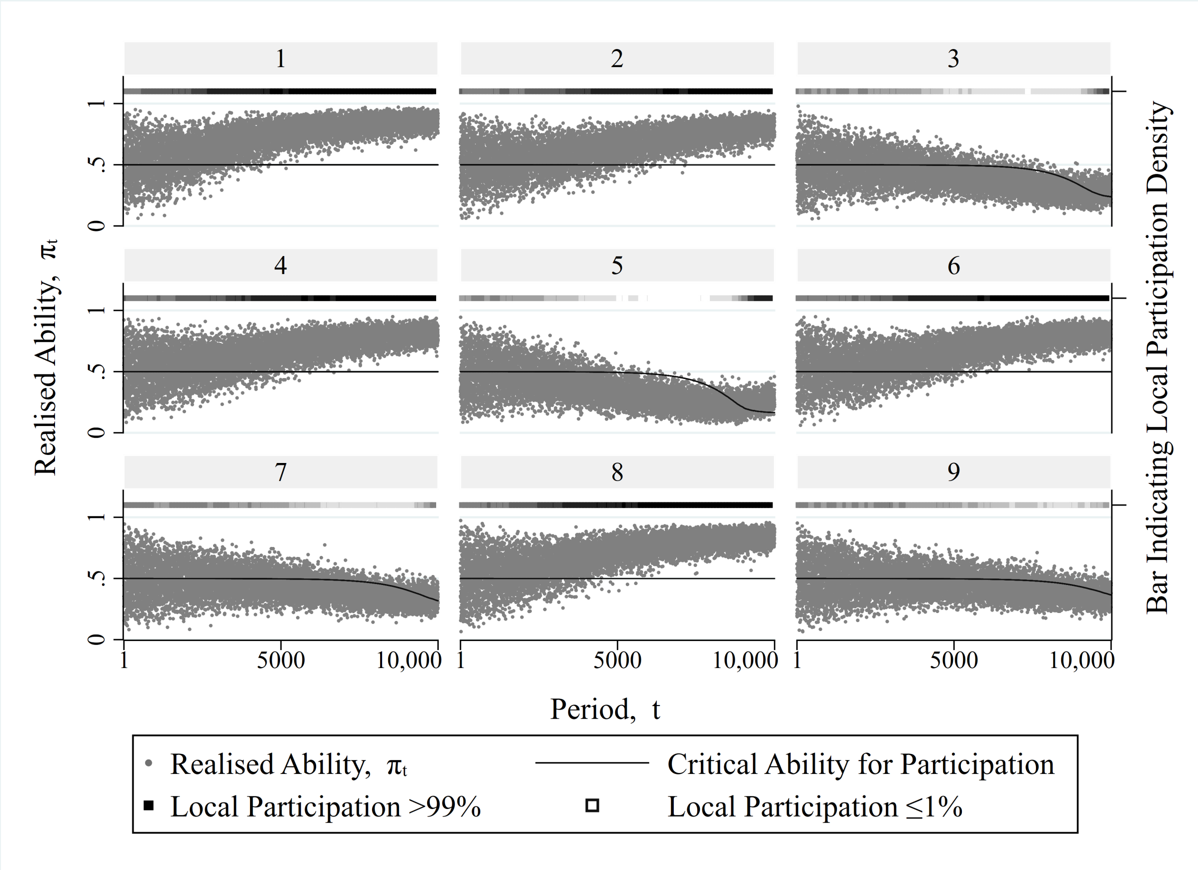


(C) *Myopes*



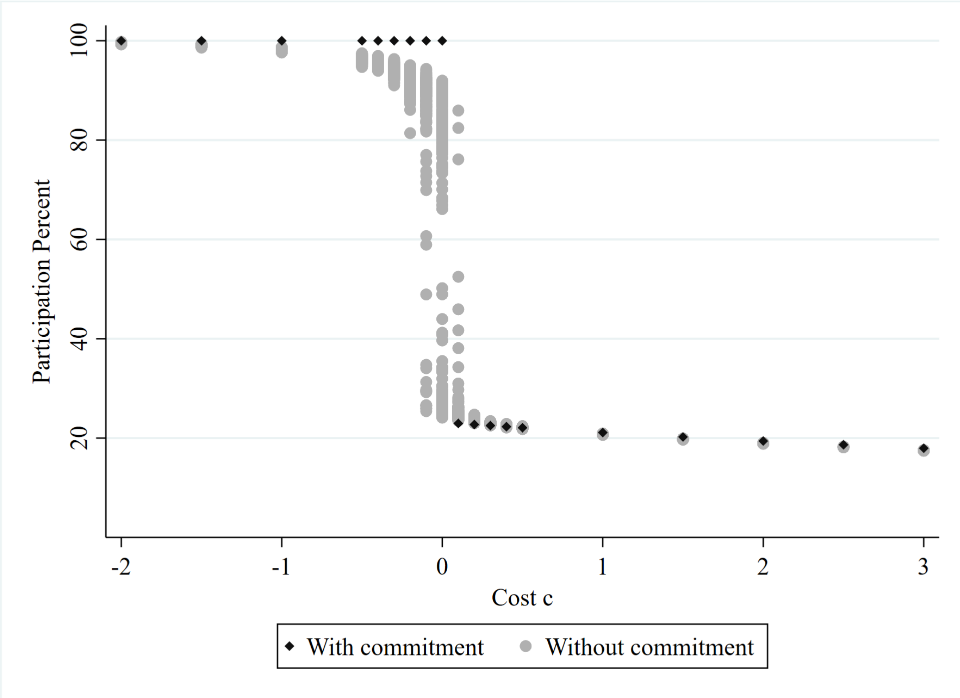
All possible situations are uniquely identified by a *Period* × *Prior Participation* pair. For any such pair, the agent or myope will participate if and only if their realized probability of success π_t is sufficiently high. The probability of this occurring is denoted ρ, and illustrated in these figures.

Figure 5:
Simulated Ability Development for 9 Myopes

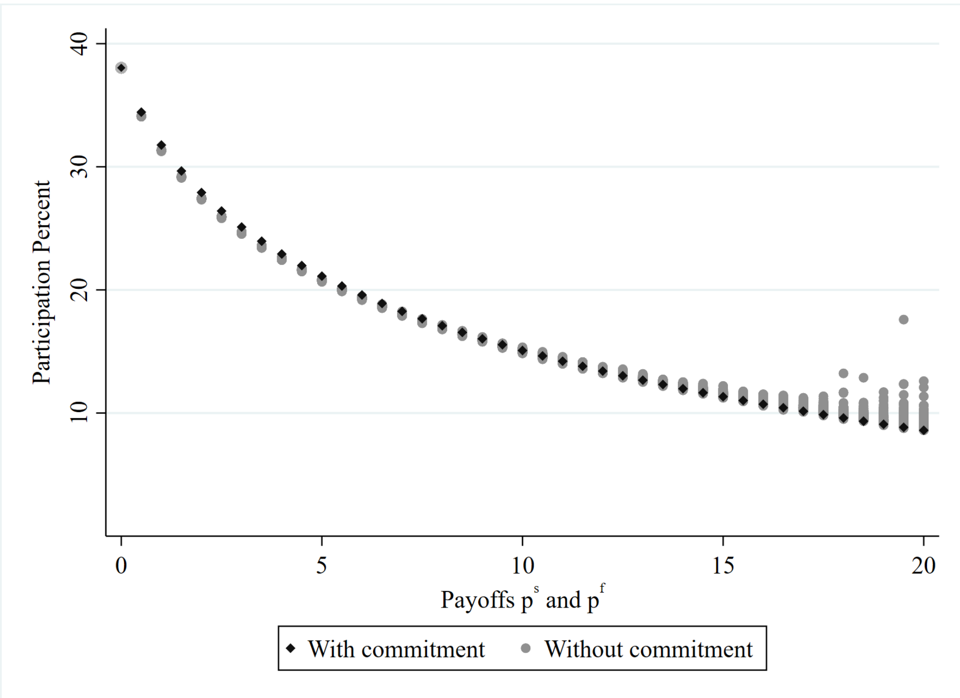


Simulated relative ability development for 9 myopes with initial ability 0.5, $p^f = p^s = 5$, $c = 0$, and other parameters as per Table 2. The bar above each panel is shaded to indicate the local participation density.

Figure 6:
The Effects of p^s, p^f, and c on Myope Participation



(A) *Percent Participation for $p^s = p^f = 5$*



(B) *Percent Participation for $c = 1$*

Gray dots show the simulated participation percentages for each of 200 myopes (for the model without commitment) at each c or $p^s = p^f$, with other parameters as per Table 2. Black Diamonds show the participation percentage of a myope under periodwise commitment, for each of the above cases.

List of Tables

Table 1:
The Sophistication Constraints (Noncognitive skill levels) of each Agent Type

Agent Type	Characteristics
Sophisticate	Perfect forward planning; complete self-knowledge.
Naif	Perfect forward planning, but unaware that she will continue to experience psychic payoffs and present-bias in future periods.
Myope	No forward planning — merely acts *as if* he were maximising the expected utility of the stage game; self-knowledge irrelevant.

Table 2:
The Parametric Assumptions for the Model Solved in this Subsection

Parameter	Assumption	Notes
Number of periods	$T = 10,000$	Robust to parameter variation.
Initial ability distribution	$\Pi_1 \sim Beta[2.5, 2.5]$	As per Filippin & Paccagnella (2012); Robust to truncated normal; Robust to parameter variation.
Π_t update magnitude	$\iota = 0.005$	Robust to parameter variation.
Participation benefit	$V(n) = V(T)\left[1 - \left(\frac{999}{1000}\right)^n\right]$	Robust to parameter variation; Robust to linear benefit accrual.
Maximum participation benefit	$V(T) = 50,000$	Robust to parameter variation.
Psychic payoffs	$p^s = p^f = \begin{cases} 10 & \text{Figures 2a,3} \\ 1 & \text{Figures 2b,3} \\ 5 & \text{Figures 4,5,6} \end{cases}$	Variation examined below; Robust to asymmetric values.
Participation cost	$c = 1$	Robust to parameter variation.
Discount rates	$\beta = 0.9,\ \delta = 0.999$	Reasonable cf. (Benhabib, Bisin & Schotter 2010) and (Frederick, Loewenstein & O'Donoghue 2002); Robust to parameter variation.

A detailed discussion and robustness checks are provided in the Online Appendix.

Table 3:
Mapping the Model's Implications onto Existing Pedagogy

$-p^f$ **more positive**	Fostering grit and resilience (e.g. Duckworth et al. 2007).
p^s **more positive**	Judicious use of praise (e.g. Hart 2010);
	Appropriately challenging goals (e.g. Bandura & Schunk 1981).
$-c$ **more positive**	Tasks should be engaging (e.g. Christenson, Reschly & Wylie 2012);
	Effective use of sanctions (e.g. Emmer, Everston & Anderson 1980).
d_t **more immediate**	Emphasis on the formative use of assessment (e.g. Black & Wiliam 1998);
	Fostering growth mindsets (e.g. Blackwell, Trzesniewski & Dweck 2007).