On the Contribution of the Markowitz Model of Utility to Explain Risky Choice in Experimental Research*

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November 14, 2018

Abstract

It is becoming increasingly common to accept that heterogeneity of preferences is an appropriate approach to describe aggregate experimental data on risky choice. We propose a parametric form of utility consistent with Markowitz's (1952) hypotheses as a useful model to consider. This value function exhibits the fourfold attitude to risk and can also capture different combinations of risk attitudes and higher-order preferences. Moreover, it can be combined with probability weighting functions as well as with other value functions as part of mixture models that capture heterogeneity of preferences. We employ data from three recent experimental studies and show that this model can contribute to the explanation of their findings.

Keywords: decision making under risk, local risk seeking, fourfold pattern of risk preferences, Markowitz model, experiments

JEL codes: D81, E21

*We are particularly thankful to Sebastian Ebert, Rustam Ibrahimov, Harris Schlesinger and the participants of the Fifth International Symposium in Computational Economics and Finance for their helpful comments on earlier versions of the paper. We are grateful to Adrian Bruhin, Thorsten Pachur and Marc Scholten for providing us with their experimental data.

1 Introduction

Experimental research over the last four decades has provided a wealth of evidence about individuals' behaviour that has stimulated the development of theories to describe choice under risk and uncertainty (see e.g. Hey and Orme, 1994; and Starmer, 2000). Over this period of time many alternative theories have been put forward to further adjust or modify Expected Utility Theory (EUT). By 1997 there were already around twenty generalizations of EUT (Hey, 1997). The advances in the available econometrics methods, starting with Harless and Camerer (1994) and Hey and Orme (1994), allowed the horse-race type of comparisons between competitive models. Both the aforementioned studies and subsequent ones (e.g. Carbone and Hey 1994, Gonzales and Wu, 1999) concluded that the two models that best explain behaviour are EUT and Prospect Theory (or Ranked-Dependent Utility if the domain is constrained to gains).[1] Some of the reasons that have made Cumulative Prospect Theory (CPT) successful are its ability to explain the Allais Paradox, gambling behaviour, and the fact that exhibits loss aversion for which there is lots of experimental evidence (e.g. Rieger et al., 2015). Barberis (2013, p. 173), in his review of prospect theory, states "*More than 30 years later, prospect theory is still widely viewed as the best available description of how people evaluate risk in experimental settings. Kahneman and Tversky's papers on prospect theory have been cited tens of thousands of times and were decisive in awarding Kahneman the Nobel Prize in economic sciences in 2002.*" Furthermore, Kothiyal et al. (2014) have recently extended this result (the power of Prospect Theory as an alternative model to Expected Utility) in the field of choice under ambiguity.

However, despite the fact that EUT and CPT remain as pre-eminent in model building and data description, the difficulty of finding one single preference functional that fits any data exactly has recently been addressed by considering mixture models (see Fehr-Duda et al., 2010; and Conte et al., 2011).

In this paper we show that the properties of a value function inspired by the classical paper of Markowitz (1952) can contribute to an explanation of experimental findings on risky choice. This contribution could be as a utility function for money, but could also be used in conjunction with probability weighting functions, or, more generally, as part of a mixture model that allows

[1] Hey (2014) and Conte et al. (2011) provide an overview of the history of fitting models of choice under risk, and conclude that "Two remain pre-eminent: Expected Utility and Rank Dependent Expected Utility."

for heterogeneity of preferences. In that sense, this model offers a way to improve both the EUT framework and the CPT framework.

Markowitz assumed that, from an agent's customary or normal level of wealth, her reference point, the agent was initially risk loving then risk averse over gains, whilst initially risk averse then risk seeking over losses, and that the value function is bounded from above and below. Markowitz also assumed his representative agent is loss averse,[2] and that individuals did not exhibit probability distortion, although he did not rule out that possibility.[3] The Markowitz model predicts the fourfold attitude to risky choice exhibited in numerous experimental studies on risky choice (see Scholten and Read (2014) for recent evidence). We highlight below further empirical findings that are difficult to reconcile with one single of the pre-eminent theories of choice under risk and where we propose the Markowitz model can contribute to their explanation.[4]

First, it is reported in experimental research that a high proportion of choices exhibit risk-seeking preferences when given the choice between a risky option with 0.5 probability of the payoffs and the certain expected value (e.g. Hershey and Schoemaker, 1985; Battalio et al., 1990; Cox and Vjollca, 2010; and Vieider et al., 2015).[5] Choice of the risky option in such lotteries is not easy to reconcile with models of EUT or with alternative theories that incorporate probability weighting to concave value functions. We note that in those latter theories the individual is typically reported to underweight probabilities above around 0.33 (prominent recent examples are Stott 2006; Conte et al. 2011; and Bruhin et al., 2010). As a consequence, the representative individual underweights

[2] "Generally people avoid symmetric bets. This suggests that the curve falls faster to the left of the origin than it rises to the right. We may assume that $\mid U(-X) \mid > U(X), X > 0$ where $X = 0$ is customary wealth." (Markowitz 1952 p. 155). This was the same definition as subsequently employed by Kahneman and Tversky (1979). Conlisk (1993) already noted that economists had initially neglected the Markowitz hypothesis while psychologists, such as Kahneman and Tversky, considered it as basic in their theories.

[3] "I shall only consider situations wherein there are objective odds. This is because we are concerned with a hypothesis about the utility function and do not want to get involved in questions concerning subjective probability beliefs. It may be hoped, however, that a utility function which is successful in explaining behavior in the face of known odds (risk) will also prove useful in the explanation of behavior under uncertainty." (Markowitz 1952 p. 155)

[4] Part of the rationale for the Markowitz model of utility was to present a new model that removed the counterfactual implications of the Friedman and Savage (1948) model of expected utility that introduced a convex segment into an otherwise standard expected utility model. These counterfactual implications include that when in the risk-seeking segment of the Friedman and Savage utility function an individual would be willing to extend insurance at an expected loss to their self. Also, that a wealthy person is willing to risk a large fraction of their wealth at actuarially unfair odds.

[5] Game show data also reveals that a substantial proportion of contestants act as risk seeking (see Post et al. (2008), Roos and Sarafidis (2010), and Deck et al. (2008)). We also note that in experimental research correlating higher-order preferences with risk preferences, Maier and Rüger (2012) report up to 55% of choices exhibit risk-seeking preferences, Noussair et al. (2014) report that 19.4% of choices are risk-seeking, and Deck and Schlesinger (2010) report up to 26% of males' choices and 21% of females' choices are risk seeking.

probabilities of 0.5 which, in conjunction with the assumed risk aversion of the value function over gains, implies choice of the safe option.[6]

The property of local risk seeking present in the Markowitz model is perhaps a particulary interesting addition to the literature in this context since analysis of risk-seeking agents has been relatively neglected. For instance, Deck and Schlesinger (2014, p.1914) note that

> "Across a wide array of settings a majority of people have been found to exhibit risk aversion; but the minority who are risk loving often only receive passing attention. Except for the occasional attempt to explain risk-loving behavior, an abundance of papers simply include an assumption of risk aversion."

Second, there is substantial evidence showing that relative risk aversion increases with stake size (e.g. Hogarth and Einhorn, 1990; Kuehberger et al. 1999; Weber and Chapman 2005), and that this effect is even more distinct when real money is at stake (Holt and Laury, 2002) and the experiment involves substantial monetary incentives (Binswanger, 1981; and Kachelmeier and Shehata, 1992). As pointed out by Fehr-Duda et al. (2010), this evidence seems to confirm Markowitz's conjecture that risk preferences are likely to reverse from risk seeking over small stakes to risk aversion over high stakes.

The third issue relates to the evidence concerning agents' higher-order preferences. Economic theory demonstrates that numerous economic decisions depend on higher-order risk preferences such as prudence and temperance. Recent experimental research on the apportion of risk consistent with the higher order preference of prudence, employing the lottery preference definitions of Eeckhoudt and Schlesinger (2006), report that the majority lottery choices are prudent (see Trautmann and van de Kuilen (2018) for a comprehensive review on this area). Although the majority choices in the experimental research is the prudent choice, the proportion is typically much lower than the hundred percent predicted by value functions typically employed in EUT and CPT, and hard to explain solely on the basis of stochastic error. In addition to this, the correlation between risk aversion and higher-order preferences is weak, and it is often observed that the same experimental subjects switch between prudent and imprudent, sometimes up to 14% of them, depending on the

[6]For example, Ebert and Wiesen (2014) fit the parametric model of Tversky and Kahneman (1992) to higher-order-risk data, and their estimates imply that the average agent in their sample had a subjective probability of 0.2962 when the objective probability is 0.5. We also note that a few studies report overweighting of probabilities of 0.5 (Harrison et al., 2010; Bouchouicha and Vieider, 2017).

size of the lottery payoffs (see Maier and Rüger, 2012; Noussair et al. 2014). The Markowitz value function is a triply inflected function and allows the Markowitz agent to exhibit different combinations of higher order preferences. In particular, the Markowitz individual unlike EUT or CPT can exhibit prudent or imprudent preferences depending on payoff sizes.

Finally, some researchers report that other types of empirical evidence is consistent with the Markowitz model of utility (see e.g. Pennings and Smidts, 2000; Pennings and Smidts, 2003; and Post and Levy, 2005). For example, Post and Levy write

> "Other than considering utility functions that are concave over gains and convex over losses, we also, for completeness, consider utility functions that are convex over gains and concave over losses. Interestingly, Markowitz (1952) already suggested this type of utility function." (Post and Levy, 2005, p. 932)
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> "Finally, we hope that our results provide a stimulus for further research based on Markowitz type utility functions (and non-concave utility functions in general)." (Post and Levy, 2005, p. 950)

We contribute to this line of research and estimate an expo-power value function consistent with Markowitz's hypotheses for three prominent risky-choice data sets from recent studies, namely, by Fehr-Duda et al. (2010), Scholten and Read (2014), and Pachur et al. (2018) (the dataset from Pachur et al. (2018) is also employed in Murphy and ten Brincke (2017)). These papers examine issues related to risky choices mentioned above: increasing relative risk aversion over both gains and losses and the fourfold pattern of risk preferences. We compare and combine the fit of the Markowitz model with four other specifications of CPT widely used in experimental research (e.g. Stott, 2006; Conte et al. 2011; and Bouchouicha and Vieider, 2017). Those CPT specifications include two different value functions –expo-power (nesting the power function) and logarithmic–, and two alternative probability weighting functions –Tversky and Kahneman (1992) and Prelec (1998)–. We consider mixture models of the expo-power value function with those four different CPT parameterisations.

Our results show that the Markowitz model can be a valuable addition to the set of value functions explaining risky choice. For example, it parsimoniously explains the risky choices of between twenty three and forty eight percent of mixture models in the Fehr-Duda et al. (2010), Scholten and Read (2014), and Pachur et al. (2018) datasets.

The rest of the paper is structured as follows. In section 2 we set out the Markowitz model of utility. In section 3 we describe the methodology employed to estimate the models for the three different data sets, and section 4 discusses the results. The final section presents the main conclusions.

2 The Markowitz model of utility

To illustrate the properties of the Markowitz model we employ the parametric form of the value function based on the expo-power specification of Saha (1993) which, as noted by Abdellaoui et al. (2007), captures the hypotheses of the Markowitz model. This value function defined over variable gains, denoted here as g, is given by the following expression

$$u = 1 - e^{-\alpha g^{\eta}}, \tag{1}$$

where α and η are positive constant parameters, and what characterises this value function as consistent with Markowitz's conjectures is that parameter η is above unity.[7] With $\eta > 1$, the agent is risk seeking over gains when $\eta - 1 - \alpha\eta g^{\eta} > 0$, risk averse when $\eta - 1 - \alpha\eta g^{\eta} < 0$, and risk-neutral when $\eta - 1 - \alpha\eta g^{\eta} = 0$. We note that, as α approaches zero, the expo-power value function (1) approximates the power value function g^{η}. Given this property, parametric estimates of risky choice data based on power utility or value functions when the power exponents reported are significantly greater than unity could be interpreted as approximations to the Markowitz value function, and evidence in favor of that model. In fact, there are a number of studies that report estimates of the power exponents which are greater than unity (e.g. Bruhin et al. (2010), Vieider (2012), and Abdellaoui and Bleichrodt (2007)). We also note that when $\eta = 1$ the expo-power value function becomes that of an EUT maximiser with exponential utility. With exponential utility, EUT maximisers risky choices are not determined by the individuals' wealth.[8] Accordingly,

[7]We are aware of two other functional forms that capture the Markowitz value or utility function. The first one is the double expo-power proposed by Peel (2013), $u = 1 - e^{-\alpha g^{\eta}} - \alpha g^{\eta} e^{-\alpha g^{\eta}}$. We note that, in this case, when $\eta > 0.5$ the agent is initially risk-seeking then risk-averse. The double expo-power function approximates the power function $g^{2\eta}$ as $\alpha \to 0$. The second alternative specification of the Markowitz value function was proposed by Cain et al. (2003), $u = 1 - e^{-\alpha g} - \alpha g e^{-\alpha g}$. In this case, the agent is risk seeking over gains when $\alpha g < 1$ and risk averse for $\alpha g > 1$. This value function approximates g^2 as $\alpha \to 0$.

[8]Furthermore, Quiggin's (1982) Rank Dependent Utility with exponential utility function can be nested in our

if we obtain estimates of $\eta > 1$ we are rejecting EUT in favour of the Markowitz model.

The value function defined over the variable losses, denoted here as L, is given by

$$u = -\lambda(1 - e^{-\beta L^{\eta}}), \tag{2}$$

where λ and β are positive constants.[9] The agent is risk averse over losses when $\eta - 1 - \beta\eta L^{\eta} > 0$ and risk-seeking when $\eta - 1 - \beta\eta L^{\eta} < 0$. We note immediately from the conditions to be risk seeking/risk averse over gains and losses that, since $\alpha \geq \beta$ (see Appendix A), an agent who is locally risk seeking over gains will necessarily be locally risk averse over losses for a gain and loss of the same amount. However, if they are risk averse over gains, they could be either risk seeking or risk averse over losses of the same amount.

Another appealing feature of the expo-power value function (1) is that it allows economic agents to exhibit different risk attitudes depending on parameter values and the size of the payoffs. This is illustrated by looking at the first four derivatives over gains

$$\frac{\partial^2 u}{\partial g^2} = \alpha\eta g^{\eta-2} e^{-\alpha g^{\eta}}(\eta - 1 - \alpha\eta g^{\eta})$$
$$\frac{\partial^3 u}{\partial g^3} = \alpha\eta g^{\eta-3} e^{-\alpha g^{\eta}}(\eta^2(1 + \alpha^2 g^{2\eta} - 3\alpha g^{\eta}) + \eta(3\alpha g^{\eta} - 3) + 2)$$
$$\frac{\partial^4 u}{\partial g^4} = -\alpha\eta g^{\eta-4} e^{-\alpha g^{\eta}}(\eta^3(7\alpha g^{\eta} + \alpha^3 g^{3\eta} - 6\alpha^2 g^{2\eta} - 1) + \eta^2(6\alpha^2 g^{2\eta} + 6 - 18\alpha g^{\eta}) + \eta(11\alpha g^{\eta} - 11) + 6).$$

It is worth noting that when in the risk-seeking (or risk-averse) segment of the expo-power value function the higher-order derivatives of a given order can exhibit multiple changes in sign for any given value function and changing values of g, the stake size.[10] This implies that Markowitz agents

estimates if parameter $\eta = 1$ in conjunction with Prelec's weighting function, to be defined in the next section, where $w(p) = 0.5 = p$, i.e. no probability distortion at $p = 0.5$ but overweight or underweight for $p \neq 0.5$. Quiggin was drawn to this specification partly because it explains the early violation of EUT and partly because it has the appealing property that $50 - 50$ bets will be undistorted by probability weighting.

[9] In this representation of the Markowitz model we constrain parameter η to be equal over gains, expression (1), and over losses, expression (2). If exponents differ, then loss aversion can go to zero which in not apppropriate, although this constraint can be relaxed in empirical work. In Appendix A we discuss in more detail the issue of loss aversion, and in section 4 we discuss the constraint of equal parameter η in the empirical estimates.

[10] Since g^{η} can be written as a function of any arbitrary fixed value g_0, such that $g^{\eta} \equiv g_0^{\eta} c^{\eta}$, and since α can be written as a function of any arbitrary fixed value α_0, such that $\alpha \equiv \alpha_0 c^{\eta}$, the value function can be written as $1 - e^{-\alpha g^{\eta}} \equiv 1 - e^{-\alpha(g_0 * c)^{\eta}} \equiv 1 - e^{-(\alpha_0 c^{\eta}) g^{\eta}}$. It therefore follows that we can compute the values of the derivatives by fixing a value of g and to observe how the sign of the derivatives changes depending on the value of α for any given η, or alternatively by fixing α and η and to observe how the sign changes depending on the value of g.

can exhibit prudent or imprudent choices, and temperate or intemperate choices, in the gain or loss domain, or different combinations of these choices, depending on the size of the payoffs and the parameters of the value function.[11]

All the features of the Markowitz model described above are illustrated in Figure 1 with a plot of the expo-power function (1)-(2) for parameter values $\eta = 2, \alpha = 0.0006, \beta = 0.0003, \lambda = 1.5$, with λ being the loss aversion parameter, and a range of gains and losses of up to 100. In this particular case, the agent would be risk seeking for gains up to 29 and risk averse for gains larger than 29 units, while she would be risk averse for losses of up to 40 and risk seeking for losses larger than 40. The curvature of this value function makes apparent how the agent switches preferences, between risk averse and risk seeking, and how relative risk aversion increases with stake size. The agent with this particular parameterisation could over gains exhibit either prudent or imprudent behaviour, and she could make temperate or intemperate choices. This latter feature of the model is illustrated in Figure 2, which displays the values of the second, third and fourth derivative of this particular expo-power parameterisation over gains. In this figure we have also included the values of the derivatives for two other alternative values of α, $\alpha = 0.001$, and 0.006 that, together with $\alpha = 0.0006$, represent the values of this parameter for the three best-fit models obtained in our empirical analysis explained in subsequent sections. It is interesting to note at this point that, as expected, as parameter α increases in value the range of payoffs corresponding to risk seeking, imprudent and intemperate behaviour is reduced.

In the next section we describe the methodologies employed to estimate the competing, or complementary, behavioural models for three recent datasets. Each of those datasets have been employed in the experimental literature to examine different issues on risky choice, which we propose the Markowitz model, either in its basic specification or in combination with probability weighting, or as part of a more general mixture model, can contribute to their explanation.

3 Methodology

We assume that the representative economic agent's utility or value functions are parsimoniously captured by the expo-power form described in the previous section. Given this assumption, three

[11]Eeckhoudt and Schlesinger (2006) discuss the relationship between risk apportionments of n^{th} grade with the signing of the n^{th} derivative of the utility function.

prominent models of risky choice are nested in our estimates. Linear and nonlinear probability weighting and estimates of the exponent of the expo-power function (1) which are significantly larger than unity provide evidence for the Markowitz model of utility. Significant departures from linear probability weighting, with over-weighting of smaller probabilities, and estimates of the exponent of the expo-power function which are significantly less than unity, or not different from unity, are evidence for CPT. We assume two specifications of the probability weighting functions. These are, first, the one-parameter form first proposed by Quiggin (1982) and later employed by Tversky and Kahnemann (1992), and second, the two-parameter form proposed by Prelec (1998). Finally, standard expected utility theory is captured by linear probability weighting and estimates of the exponent in the expo-power utility function that are less or equal to unity. We estimate these three models separately and evaluate their relative statistical fits to our risky choice data sets employing various statistical criteria.

We will, however, not restrict our empirical analysis to single models of risky choice. It is now widely accepted that one model is unlikely to provide a parsimonious fit to risky choice data sets (see e.g. Conte et al, 2011). We examine this possibility and capture any heterogeneity in the preference functionals of individuals by estimating finite mixture models.

Appendix B includes a brief description of the experimental design and reports the stimuli employed in the three papers we examine. The analyses takes place using the pooled dataset which means that the mean parameter values of the population are estimated, compared to the individual subject ones. The three studies ask a sufficiently large number of questions to the participants to guarantee the stability of the estimated parameters (at individual level). Given that we estimate models at the aggregate level, the combination of many subjects with a large number of questions per subject, overcomes any sample issues. Both Fehr-Duda et al. (2010) and Pachur et al. (2018) follow standard payment methods and pay one of the tasks at random at the end of the experiment. Scholten and Read (2014) do not provide any payment related to the task as the amounts that they investigate are forbidding (they only provide a flat payment). The importance of incentives in experimental research is discussed in Bracha and Brown (2012). They develop a dual-process theory of choice under risk and uncertainty (affective decision making) which is the result of two processes, a rational one, which chooses an action, and an emotional one, which forms perception. The rational process coincides with the expected utility model. The emotional process selects an optimal risk perception that balances two contradictory impulses: the affective motivation and the taste for accuracy. This kind of process may lead to optimism bias, the tendency to overestimate the likelihood favourable outcomes and underestimate the likelihood of unfavourable ones. The lack of substantial financial incentives during the experiment may not provide the suitable motivation to the subject to form optimistic beliefs and it may therefore affect the tradeoff between the rational choice and the psychologically based belief utility. While there may be some issues with the lack of monetary payoffs (lack of interest, higher levels of risk seeking behaviour), the evidence from the literature does not seem to be conclusive (see Bardsley et al, 2010; Etchart-Vincent and L'Haridon, 2011).

The first data set we employ to estimate the models is the one in Fehr-Duda et al. (2010), whose experiment involved the elicitation of certainty equivalents of risky choice lotteries, over gains and losses, with both low and high stakes. The experiment involved the elicitation of certainty equivalents from 56 two-outcome lotteries over a wide range of outcomes and probabilities. The lotteries were of the following form $\mathcal{L} = \{x_1, p; x_2, 1-p\}$ with $x_1 > x_2$ and p being the probability of the prospect x_1. Given this, in order to fit the various specifications of risky choice we need to calculate the optimal certainty equivalents, CE^*, for each of the lotteries, based on a given set of behavioural parameters. Since the certainty equivalent is the amount that a subject should receive so as to be indifferent between this amount and a given lottery $\mathcal{L}$, we can define the CE for the simplest case of Markowitz (M) or Expected Utility Theory (EUT) as:

$$u(CE) = pu(x_1) + (1-p)u(x_2). \tag{3}$$

For a lottery in the gains domain, and assuming an expo-power utility function, solving for CE gives:

$$CE = \left[-\frac{\ln(1-Q)}{\alpha}\right]^{1/\eta}, \tag{4}$$

where, for the case of M and EUT, $Q = p(1-e^{-\alpha x_1{}^{\eta}}) + (1-p)(1-e^{-\alpha x_2{}^{\eta}})$. For the case of CPT, $Q = w(p)(1-e^{-\alpha x_1{}^{\eta}}) + (1-w(p))(1-e^{-\alpha x_2{}^{\eta}})$, where $w(p)$ is the probability weighting function, and α, η can differ in each specification.[12] However, in the case that $\eta > 1$, $Q = w(p)(1-e^{-\alpha x_1{}^{\eta}}) + (1-w(p))(1-e^{-\alpha x_2{}^{\eta}})$, would correspond to Markowitz agents. In a similar way, we solve for the

[12]We follow the same procedure for the calculation of CE for the case of a logarithmic value function.

CE for gambles in the losses domain.

We employ two parametric forms of the probability weighting functions with possibly different parameter values over gains and losses. The first one is the form first proposed by Quiggin (1982) and subsequently used in Tversky and Kahnemann (1992):

$$w(p) = \begin{cases} \frac{p^{\gamma}}{(p^{\gamma}+(1-p)^{\gamma})^{1/\gamma}} & \text{if } x \geq 0 \\ \frac{p^{\delta}}{(p^{\delta}+(1-p)^{\delta})^{1/\delta}} & \text{if } x < 0 \end{cases} \tag{5}$$

and the second one is the two-parameter Prelec (1998) weighting function of the form:

$$w(p) = \begin{cases} (exp(-(-log(p))^{\gamma}))^{\gamma_2} & \text{if } x \geq 0 \\ (exp(-(-log(p))^{\delta}))^{\delta_2} & \text{if } x < 0 \end{cases} \tag{6}$$

Finally, deviations from the predicted CE of the deterministic models are captured by an additive, domain-specific, error term ϵ, such that $CE = CE^* + \epsilon$, which is assumed to be normally distributed with mean zero and standard deviation which is a function of the lottery's outcome range $|x_1 - x_2|$. The standard deviation is given by $\sigma_g = s_g|x_1 - x_2|$ for gains, and $\sigma_L = s_L|x_1 - x_2|$ for losses, with s_g and s_L the error parameters to be estimated.

The estimation for the average values of all the behavioural parameters stated above is done using Maximum Likelihood Estimation techniques.[13] To this end, we pool all the data together and we need to define the likelihood function that is going to be maximised. Based on the assumption of normal distribution of the errors, the likelihood function is defined as:

$$L = f(ce, \mathcal{L}, \theta, s) = \prod_{1}^{N} \prod_{l=1}^{\mathcal{L}} \frac{1}{\sigma_l} \phi \left(\frac{ce_l - ce_l^*(\mathcal{L}, \theta)}{\sigma_l} \right), \tag{7}$$

where $\mathcal{L}$ is the set of lotteries, θ is the vector of the behavioural parameters of interest regarding the utility and the weighting function, s is the domain specific error term[14], $\phi(.)$ denotes the

[13]The MLE method we are using assumes the observations are independent. One could object on the grounds that subjects learn during the experiment which may affect their choices. Nevertheless, the incentive mechanism that is applied in the three experiments under consideration excludes the provision of feedback during the experiment. The subjects complete the required number of trials, without receiving any feedback and without any realization of the outcomes. At the end, one of the problems is randomly chosen, and this problem is played for real with the aid of a physical randomisation device (e.g. dice, numbered tokens, etc.). In that case, subjects do not have the opportunity to learn by experience and each task differs from another both in the dimension of payoffs and the dimension of probabilities. Therefore, the independence between tasks assumption may not be so unrealistic.

[14]Fehr-Duda et al. (2010) introduce a subject specific error term in order to capture heterogeneity between subjects.

standard normal distribution, N is the total number of subjects, ce_l is the actual certainty equivalent as stated by the subject for a given lottery l.[15] The objective is then to find the maximum likelihood estimates for the vector θ that maximise the value of the following log-likelihood function:

$$\ln L = \ln f(ce, \mathcal{L}, \theta, s).$$

Still with the Fehr-Duda et al. dataset, we now relax the assumption of a representative agent and homogeneity regarding the preference functional, and we instead assume that there are two types of agents (decision makers), namely, a Markowitz (M) or expected utility (EUT) maximiser, and a CPT maximiser. To this end, we use the choice data to estimate finite mixture models which assign each subject to one of the two distinct behavioural types. Each type is characterised by a distinct vector of parameters θ_M and θ_{CPT}. The estimation procedure yields estimates for the relative sizes of the different groups, where π^M denotes the proportion of subjects characterised by Markowitz and EUT preferences, as well as the group-specific parameters θ. The likelihood function is in this case written as:

$$\ln L = \sum_{1}^{N} \ln(\pi^M \times \ f^M(ce_M, \mathcal{L}, \theta_{EUT}, s^M) + (1 - \pi^M) \times \ f^{CPT}(ce_{CPT}, \mathcal{L}, \theta_{CPT}, s^{CPT})),$$

where

$$f^M = \prod_{l=1}^{\mathcal{L}} \frac{1}{\sigma_{M,l}} \phi \left(\frac{ce_{M,l} - ce^*_{M,l}(\mathcal{L}, \theta_M)}{\sigma_{M,l}} \right),$$

and similarly defined for CPT.

For the second data set, the one of Scholten and Read (2014), we need to employ a different method of estimation as the experimental design did not involve the statement of certainty equivalents. To this end, we use a a modification of the method employed by Scholten and Read (2014) in order to accommodate a finite mixture model. Subjects were asked to choose between a safe amount of money x (Option S) and a risky option (Option R) of the form $\mathcal{L} = \{10\ x, 0.1; 0, 0.9\}$

This adds 153 extra parameters for gains, and another 153 extra parameters for losses. We estimated our econometric models following both the Fehr-Duda et al. (2010) method and the one assuming only two common, domain-specific, error terms for all subjects. Since the results do not differ substantially, for parsimony reasons, we estimate only two additional parameters for each specification in all subsequent analyses.

[15] Notice that we do not distinguish between high-stake and low-stake lotteries. We follow Fehr-Duda et al. (2010) and we estimate the parameters based on the full set of lotteries. In addition, we dropped the dummies for 'high' that Fehr-Duda et al. (2010) estimate in their analysis for parsimony reasons, because omitting them does not qualitatively affect the results.

with x increasing geometrically from \$0.25 to \$25,000.[16]

Under Luce's (1959) choice axiom, the predicted probability of choosing the gamble R is

$$\hat{P}^k = \frac{1}{1 + 1/\hat{\Omega}},$$

where $\hat{\Omega} = \frac{V(R)^{1/s}}{V(S)^{1/s}}$ and $k \in \{M, CPT\}$ denoting whether the model is evaluated assuming a Markowitz-EUT or a CPT specification. $V(R)$ and $V(S)$ are the expected utility values if $k = M$(or the non-expected values in the case of $k = CPT$) of the two options, and ϵ is a noise parameter. For a lottery choice pair l, the likelihood conditional on M being the true model $f_l^M(\mathcal{L}, \theta, s)$ is given by $f_l^M = \hat{P}^M$ if the risky option has been chosen, otherwise $f_l'^M = 1 - \hat{P}^M$ (the likelihood f_l^{CPT} for the CPT model is defined in a similar way). The specification of the mixture model is based on the estimation strategy employed in Harrison and Rustrom (2009) where it is assumed that any one observation can be generated by both models according to the latent probabilities $\pi^M, 1 - \pi^M$, with π^M the probability that the M model is the correct one.[17] The mixture likelihood can be written as a weighted average of the conditional likelihoods $\ln L = \sum_{\mathcal{L}} \left[q \times \ln(\pi^M \times f_l^M + (1 - \pi^M) \times f_l^{CPT}) + (N - q) \times \ln(\pi^M \times f_l'^M + (1 - \pi^M) \times f_l'^{CPT}) \right]$ where q is the number of subjects who chose the risky choice in lottery l and $N - q$ those who chose the safe option. For the estimation, maximum-likelihood techniques are used to obtain estimates for the parameters of the utility functions, the weighting function, as well as the noise parameters which are assumed to be domain-specific. Using Maximum Likelihood Estimation techniques, we find values of the parameters that maximise the log-likelihood function above.

The Pachur et al. (2018) dataset consists of 91 binary lotteries defined in the gains, losses and mixed domains.[18] Each option of a lottery pair has two possible outcomes ranging from -100 to 100, that may occur with probability p and $1 - p$ respectively, with p taking values in the interval [0,1]. We again fit a finite mixture model between M and CPT, adopting the same stochastic model as Pachur et al. (2018) do. The likelihood function is written as a mixture of the conditional

[16]Scholten and Read also include the data set from the original Hershey and Shoemaker (1980) experiment where they asked subjects to choose between a safe amount of money x and a risky option of the form $\mathcal{L} = \{100\ x, 0.01; 0, 0.99\}$ with x increasing geometrically from \$1 to \$10,000.

[17]We resort to this estimation approach due to the available data being in a form of percentages rather than subjects' individual choices (e.g. q% of the subjects preferred the risky lottery). This form of data allows to classify individual observations to latent models, but not subjects themselves.

[18]As the mixed gambles require the estimation of a loss aversion parameter, we drop them from our analysis in order to make the results between the three datasets comparable.

likelihood of a subject being classified as M or CPT. The function to maximise can be written as:

$$\ln L = \sum_{N} \sum_{\mathcal{L}} \ln(\pi^M \times f_l^M + (1 - \pi^M) \times f_l^{CPT}),$$

where $f_l^k = \frac{\exp(sV(A))}{\exp(sV(A))+\exp(sV(B))}$ if the subject picks lottery A in lottery pair l and $f_l^k = 1 - \frac{\exp(sV(A))}{\exp(sV(A))+\exp(sV(B))}$ otherwise, with $k \in [M, CPT]$. $V(A)$ and $V(B)$ are defined as before. s is a domain-specific error term. N is the number of subjects and $\mathcal{L}$ is the set of lotteries. Using Maximum Likelihood Estimation techniques, we obtain estimates for the behavioural parameters of the preference functionals, the stochastic error terms and the mixture probability π^M.[19]

4 Empirical Results

Tables 1, 2 and 3 report the estimates for the four mixture models described above using the Fehr-Duda et al. (2010), Scholten and Read (2014) and Pachur et al. (2018) datasets, respectively.[20] Two of those models are the combination of the Markowitz model with expo-power value function with probability weighting of either the Tversky-Kahneman or the Prelec functional form. The other two mixture models are the combination of the Markowitz model with a logarithmic value function with probability weighting of either the Tversky-Kahneman or the Prelec functional form. In addition to the parameter estimates for gains and losses, we report three measures of *goodness of fit*, namely, the maximised log-likelihood, the Akaike Information Criterion (AIC) and the Bayesian Information Criterion (BIC). The best-fit model is the one with the highest value of the maximised log-likelihood (the least negative value), while for the BIC and AIC criteria, a lower value indicates a better fit of the data.

The empirical results reported in these three tables reveal that the Markowitz model of utility,

[19] The estimation was conducted using the *R* programming language for statistical computing (The *R* Manuals, version 3.4.4. Available at: http://www.r-project.org/). To ensure that the solution is not trapped to a local optimum, and that we instead reach a global one, we use a genetic algorithm that combines evolutionary search algorithms with derivative-based methods. For the genetic algorithm, the package *genoud* (Mebane and Sekhon, 2011) was used. The estimation codes are available upon request.

[20] We already point out in Appendix A that, from a theoretical point of view, it is appropriate to present the Markowitz model specified in expressions (1) and (2) with parameter η constrained to be the same across gains and losses. The empirical results presented in Tables 1-3 are obtained with such constraint imposed. As a robustness check, we have, however, also carried out the empirical analysis employing an unconstrained version of the model. The results, not reported here but available upon request, provide similar qualitative conclusions. The best-fit model specification coincides in both the constrained and unconstrained cases, and the proportion of Markowitz agents presents higher variation but it is similar in most cases.

captured by the expo-power value function with an exponent η significantly greater than unity, plays an important role in the explanation of the risky choice data.[21] The estimated proportion of choices consistent with Markowitz's specification in the mixture models varies with the experiment under consideration. The best fitting mixture model in each of the three datasets assign a proportion of between twenty three to forty eight percent of choices to the Markowitz specification. However, the majority choice in each dataset exhibits probability distortion.

The results also reveal that the different specifications of the probability weighting function make a crucial difference to the estimates and fit of the mixture models. The most parsimonious fit of the mixture model in all three datasets is achieved employing the Prelec weighting functions in the CPT model. In the Scholten and Read (2014) and Pachur et al. (2018) datasets the Prelec weighting function is combined with logarithmic value functions while combined with expo-power value functions in the Fehr-Duda et al. (2010) dataset.[22] To further investigate the reason why the Prelec probability weighting function leads to a different interpretation of the mixture model results to that implied by the Tversky-Kahneman probability weighting function, in Figure 3 we plot the implied degree of probability distortion of each weighting function, $w(p) - p$, against the objective probability, p, for both gains and losses. The plots reveal a much greater degree of probability distortion in the estimates of the Prelec probability weighting function than that proposed by Tversky and Kahneman for any objective probability of less than 0.4. *Ceteris paribus*, the greater the positive (negative) difference between the subjective probability, $w(p)$, and the objective probability, p, for any given objective probability, the more likely the individual to behave as risk-seeking (risk-averse) in experimental research on risky choices. Clearly the risk-seeking choices over smaller probabilities are better captured by the Markowitz value function than the Tversky and Kahneman probability weighting function in this data set.

In any case, it is important to note that the fact that we find a significant proportion of Markowitz agents in a mixture model that includes the best specification obtained in the three papers considered here further endorses the importance of acknowledging heterogeneity of prefer-

[21] We have also produced estimates solely for each representative model for these three datasets. The results, available upon request, confirm that the Markowitz expo-power specification produces estimates of the parameter η significantly greater than unity. However, it is typically a CPT specification with Prelec probability weighting function that exhibits the best fit.

[22] It is interesting to note that, for the data sets of Fehr-Duda et al. and Scholten and Read, an expo-power value function combined with the Tversky-Kahnemann probability weighting function implies all agents are consistent with the Markowitz model, i.e. $\eta > 1$, with a proportion π^M of them exhibiting no probability distortion.

ences in experimental work. Furthermore, it illustrates the fact that the preferences of some agents can be consistent with risk loving behaviour without the need to impose probability distortion. More generally, and unlike previous findings, the introduction of a Markowitz model embedded in a mixture model allows the researcher to capture the four-fold pattern of risk preferences over both gains and losses without necessarily assuming probability distortion.

The difference between the predictions of CPT and Markowitz will be most apparent for higher probabilities. For example, in the Fehr Duda et al. data set, certainty equivalents for 56 two-outcome lotteries over a wide range of outcomes and probabilities were elicited. This included the certainty equivalents of the two lottery choices $\mathcal{L} = \{15, 0.95; 4, 0.05\}$ and $\mathcal{L} = \{320, 0.95; 130, 0.05\}$. The expected values of these two lotteries are 14.45 and 310.5, respectively. Employing the estimates of the expo-power for the Markowitz individual in Table 1, we find predicted certainty equivalents of 14.8 and 308.8. So that the Markowitz individual would be risk-seeking over the lower payoff but risk-averse over the higher payoffs. In both cases, the underweighting of probabilities of 0.95 associated with the higher payoff (weighted first in CPT) gives a risk-averse choice for both lotteries for the estimates for the representative CPT preferences.[23]

In summary, the Markowitz model of utility offers an alternative explanation of risk-seeking choices and the fourfold attitude to risk to that of CPT. The estimated mixture models employing three prominent experimental data sets reveal that a significant proportion of risky choices can be explained by the Markowitz model though the majority choice was captured by a CPT specification.

5 Conclusions

Although theories of risky choice such as EUT and CPT remain as the prominent choices in the economics literature for describing agents behaviour, it is becoming increasingly common to accept

[23] Whilst both Markowitz and CPT can explain gambling on low probability outcomes at actuarially unfair long odds they also exhibit important differences. An individual with Markowitz preferences will not wager on more than one outcome in an event where all possible outcomes have negative expected return to a unit stake such as roulette since they are locally risk-seeking. However, betting on more than one outcome in an event or betting each-way is popular with punters according to Betfair. The CPT model can predict wagering on one or more than one outcome in an event such as roulette or a horse race as the risk-aversion over gains in conjunction with the high subjective expected returns to wagering on the long shot permits conditions for optimal bet diversification (e.g. Peel (2017)). Similarly, individuals with Markowitz preferences would optimally prefer one-prize lotteries whilst national or state lotteries typically have many prizes. Although heterogeneity in Markowitz preferences can justify more than one prize it appears difficult to explain why lotteries would have more than two prizes on the basis of heterogeneity of Markowitz preferences (e.g. Peel (2013)). On the other hand, CPT can more readily explain the multiple prize structure of National lotteries.

that heterogeneity of preferences is an appropriate approach to describe aggregate experimental data. To date the mixture models have combined EUT with CPT.

The purpose in this paper has been to examine whether a parametric form of utility consistent with Markowitz's (1952) hypotheses can help provide an explanation of experimental results on risky choice. We illustrate how this utility function exhibits the fourfold attitude to risk, and it is flexible enough to allow different combinations of risk attitudes and higher-order preferences. Moreover, this value function could also be used in conjunction with probability weighting. We empirically test the Markowitz model by including it in estimates of mixture models employing three datasets from recent studies that focus their analyses on different behavioural issues. In particular, the one of Fehr-Duda et al. (2010) examines increasing relative risk aversion over both gains and losses, whilst those of Scholten and Read (2014) and Pachur et al. (2018) investigate the fourfold pattern of risk preferences. We compare and combine the estimates of the Markowitz model (which nests EUT with exponential utility) with four CPT specifications which include two different value functions and two alternative probability weighting functions.

The empirical results reveal that the Markowitz model can be a useful addition to the set of preference functionals employed to explain risky choices in experimental research. In particular, our estimated mixture models suggest that between twenty three and forty eight percent of individuals make risky choices consistent with the Markowitz model, while the other, majority of choices, are consistent with CPT models with flexible probability weighting functions such as that proposed by Prelec.

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Table 1: Parameter estimates mixture models (data from Fehr Duda et al., 2010)

	Parameter	M & EP/TK	M & EP/PRL	M & LOG/TK	M& LOG/PRL
	α	0.0010934	0.0012041	0.0007631	0.0011239
		0.000	0.000	0.000	0.000
MODEL 1	η	1.12225	1.1321653	1.1896054	1.1424607
M		0.065	0.063	0.082	0.066
	β	0.0001212	0.0001129	0.0000100	0.0001355
		0.000	0.000	0.000	0.000
	α	0.0000002	0.0116962	0.0000000	0.0438867
		0.000	0.001	0.000	0.014
	η	2.323822	0.811606	-	-
		0.000	0.000	-	-
MODEL 2	γ	0.4198914	0.2550274	0.5331487	0.2594438
CPT		0.003	0.008	0.004	0.008
	γ_2	-	0.7478619	-	0.7330843
		-	0.010	-	0.015
	β	0.0000000	0.0000000	0.0000000	3.6981751
		0.000	0.000	0.000	5.573
	δ	0.4653005	0.3079249	0.5822907	0.3199964
		0.000	0.001	0.004	0.008
	δ_2	-	0.7541779	-	0.5610389
		-	0.001	-	0.008
	s_g	0.1785405	0.169802	0.1926512	0.169523
		0.002	0.002	0.002	0.002
	s_L	0.15032	0.1475023	0.1746842	0.150795
		0.002	0.002	0.002	0.002
	π^M	0.2112265	0.2319692	0.1708266	0.2362684
		0.034	0.035	0.032	0.035
	LL	-28215.08	-27923.40	-29171.14	-28010.74
	AIC	56452.16	55872.80	58362.28	56045.48
	BIC	56473.43	55897.93	58381.61	56068.68
	Parameters	11	13	10	12
	Obs	8568	8568	8568	8568

Notes: The Table reports the estimates of four mixture models. For each column, the specification on the upper panel (MODEL 1) is a Markowitz model (M), with expo-power utility function, while on the lower panel (MODEL 2), the specification is a CPT model with corresponding utility and weighting function. EP stands for the expo-power utility function, LOG for the logarithmic utility, TK is the Tversky-Kahneman weighting function and PRL is the Prelec weighting function with 2 parameters. For model 2 (CPT), when the estimate of parameter η is significantly larger than unity, all agents would be classified as M and π^M would in this case denote the proportion of M agents that exhibit no probability distortion. Standard errors are rounded to three digits. All parameters close to zero are confirmed to be statistically significant.

Table 2: Parameter estimates mixture models (data from Scholten and Read, 2014)

	Parameter	M & EP/TK	M & EP/PRL	M & LOG/TK	M& LOG/PRL
	α	0.0029094	0.0068537	0.0051434	0.006166
		0.001	0.001	0.001	0.001
MODEL 1	η	2.4057663	1.7805835	2.2771875	1.2404174
M		0.144	0.110	0.137	0.033
	β	0.0000497	0.0003899	0.000053	0.0000001
		0.000	0.000	0.000	0.000
	α	0.0001907	0.0093531	0.0034564	0.0086267
		0.000	0.003	0.001	0.004
	η	1.2801343	0.7834122	-	-
		0.041	0.024	-	-
MODEL 2	γ	0.2700006	0.0000001	0.3800542	0.271152
CPT		0.016	0.123	0.023	0.088
	γ_2	-	1.1284413	-	0.9846459
		-	0.154	-	0.114
	β	0.0000003	0.0000636	0.0000185	0.0254093
		0.000	0.000	0.002	0.017
	δ	1.0487638	3.5777632	0.8374058	2.7709825
		0.069	0.023	0.042	0.309
	δ_2	-	0.014109	-	0.05545
		-	0.040	-	0.027
	s_g	1.4373579	0.8829167	0.921063	0.7133355
		0.064	0.046	0.003	0.040
	s_L	5.0094524	2.9559209	4.4161233	1.8828997
		0.467	0.2306032	0.012	0.096
	π^M	0.4612979	0.375577	0.4036325	0.4828455
		0.020	0.0190309	0.022	0.0288842
	LL	-7649.31	-7602.08	-7637.45	-7585.70
	AIC	15320.62	15230.16	15294.89	15195.40
	BIC	15343.70	15257.43	15315.87	15220.57
	Parameters	11	13	10	12
	Obs	12518	12518	12518	12518

Notes: The Table reports the estimates of four mixture models. For each column, the specification on the upper panel (MODEL 1) is a Markowitz model (M), with expo-power utility function, while on the lower panel (MODEL 2), the specification is a CPT model with corresponding utility and weighting function. EP stands for the expo-power utility function, LOG for the logarithmic utility, TK is the Tversky-Kahneman weighting function and PRL is the Prelec weighting function with 2 parameters. For model 2 (CPT), when the estimate of parameter η is significantly larger than unity, all agents would be classified as M and π^M would in this case denote the proportion of M agents that exhibit no probability distortion. Standard errors are rounded to three digits. All parameters close to zero are confirmed to be statistically significant.

Table 3: Parameter estimates mixture models (data from Pachur et al., 2018)

	Parameter	M & EP/TK	M & EP/PRL	M & LOG/TK	M& LOG/PRL
	α	0.000448	0.007504	0.000558	0.0006222
		0.000	0.001	0.000	0.000
MODEL 1	η	1.263951	0.994885	1.3897595	1.4999956
M		0.050	0.061	0.026	0.022
	β	0.00013	0.00203	0.0000654	0.0003444
		0.000	0.000	0.000	0.000
	α	0.009674	0.015	0.0185917	0.0824894
		0.002	0.005	0.003	0.024
	η	0.938423	0.694874	-	-
		0.043	0.055	-	-
MODEL 2	γ	0.699084	0.493011	0.7227931	0.6833249
CPT		0.023	0.036	0.024	0.033
	γ_2	-	0.756522	-	0.6644438
		-	0.055	-	0.059
	β	0.001341	0.005254	0.0009445	0.0112901
		0.000	0.001	0.000	0.002
	δ	0.788473	0.479603	0.7806999	0.7543283
		0.031	0.079	0.030	0.054
	δ_2	-	0.58974	-	0.6935879
		-	0.042	-	0.032
	s_g	0.067437	0.054029	0.1387111	0.2415318
		0.015	0.013	0.016	0.021
	s_L	0.014589	0.021549	0.0135117	0.1048963
		0.003	0.005	0.001	0.015
	π^M	0.249348	0.313366	0.2488866	0.2412276
		0.062	0.056	0.046	0.046
	LL	-2856.20	-2832.41	-2850.56	-2833.25
	AIC	5734.39	5690.82	5721.13	5690.50
	BIC	5753.45	5713.34	5738.45	5711.29
	Parameters	11	13	10	12
	Obs	5400	5400	5400	5400

Notes: The Table reports the estimates of four mixture models. For each column, the specification on the upper panel (MODEL 1) is a Markowitz model (M), with expo-power utility function, while on the lower panel (MODEL 2), the specification is a CPT model with corresponding utility and weighting function. EP stands for the expo-power utility function, LOG for the logarithmic utility, TK is the Tversky-Kahneman weighting function and PRL is the Prelec weighting function with 2 parameters. For model 2 (CPT), when the estimate of parameter η is significantly larger than unity, all agents would be classified as M and π^M would in this case denote the proportion of M agents that exhibit no probability distortion. Standard errors are rounded to three digits. All parameters close to zero are confirmed to be statistically significant.

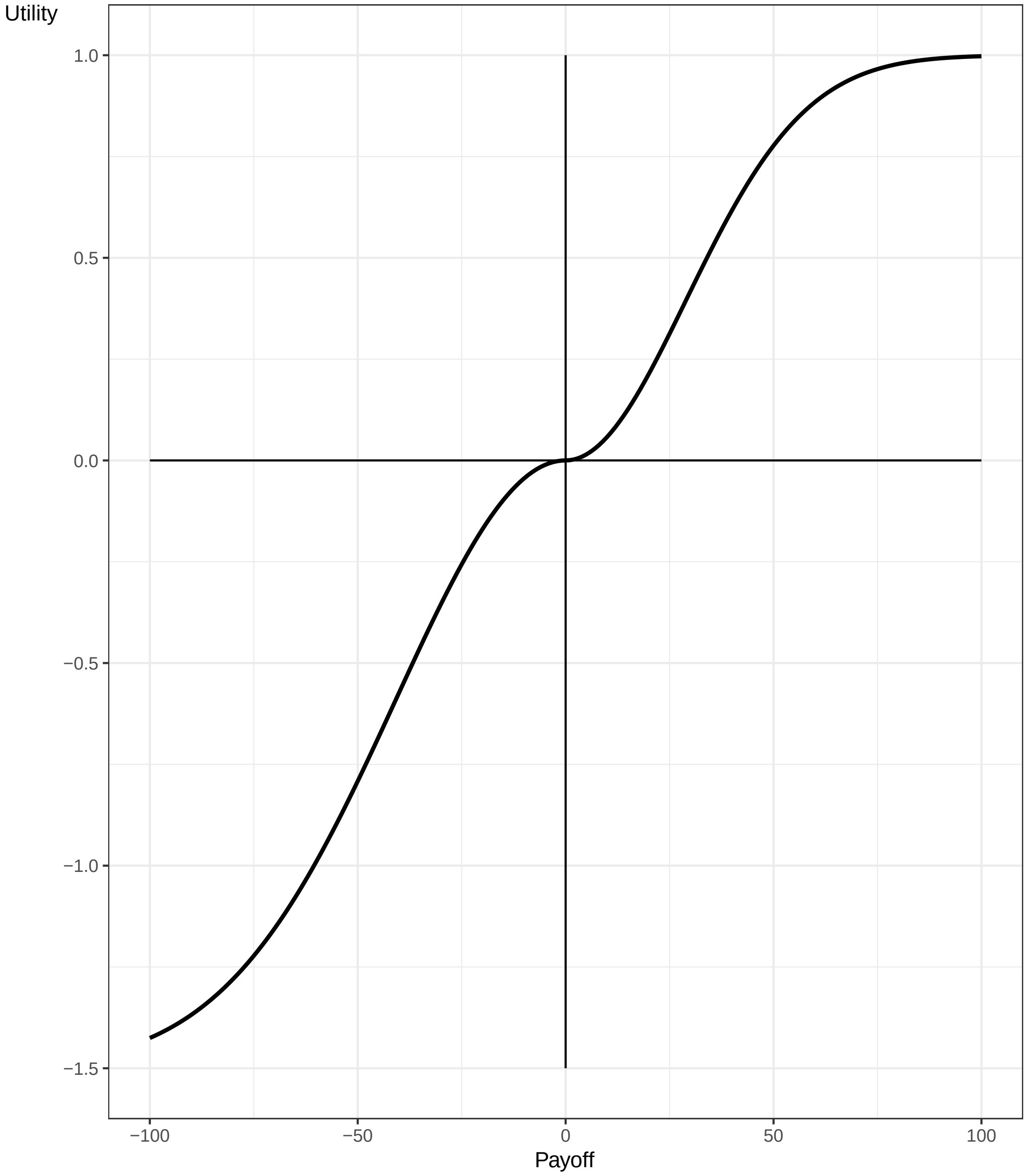

Figure 1: Expo-power utility function for $\eta = 2$, $\alpha = 0.0006, \beta = 0.0003, \lambda = 1.5$.

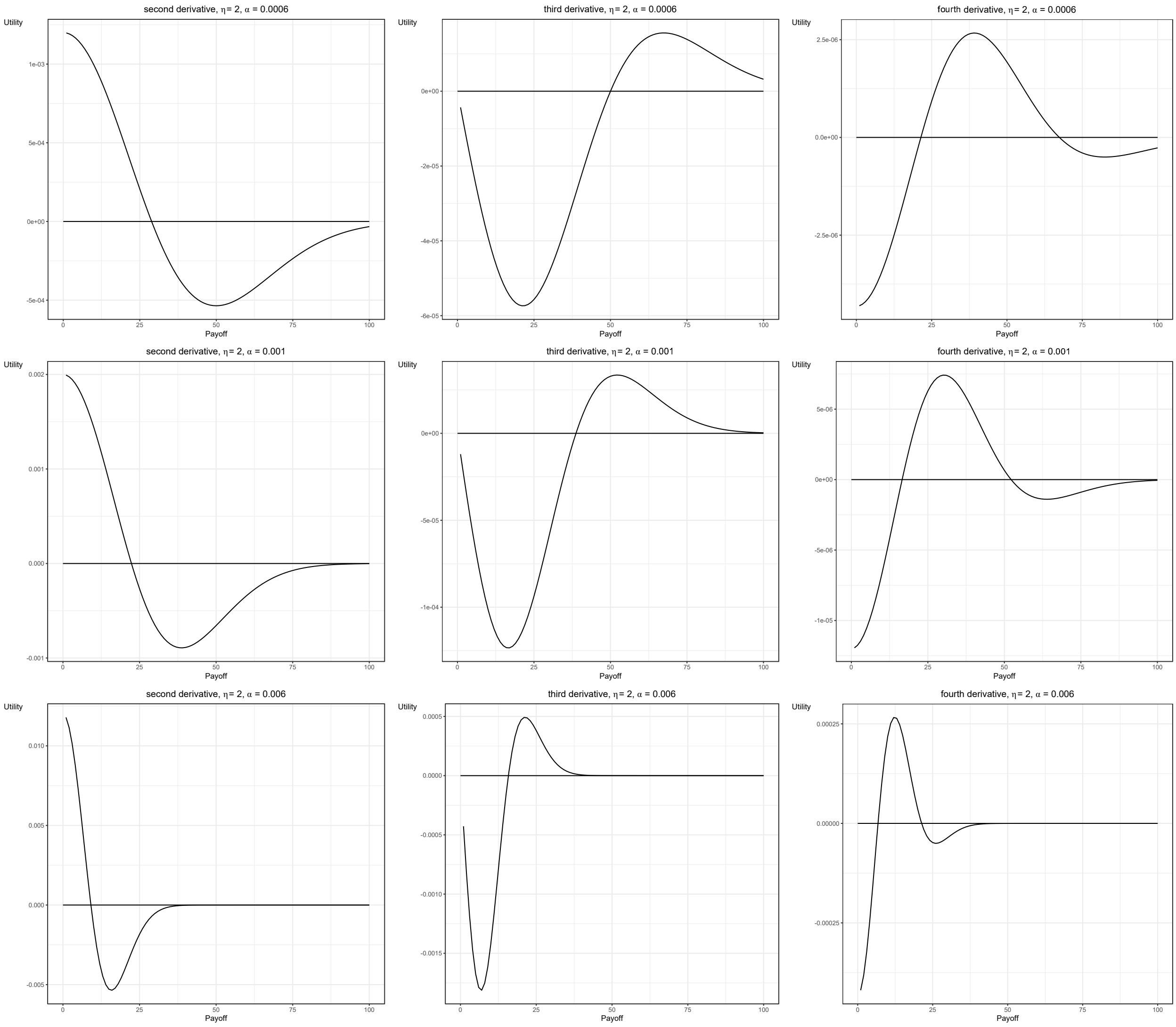

Figure 2: Second, third and fourth derivative of the expo-power utility function over gains.

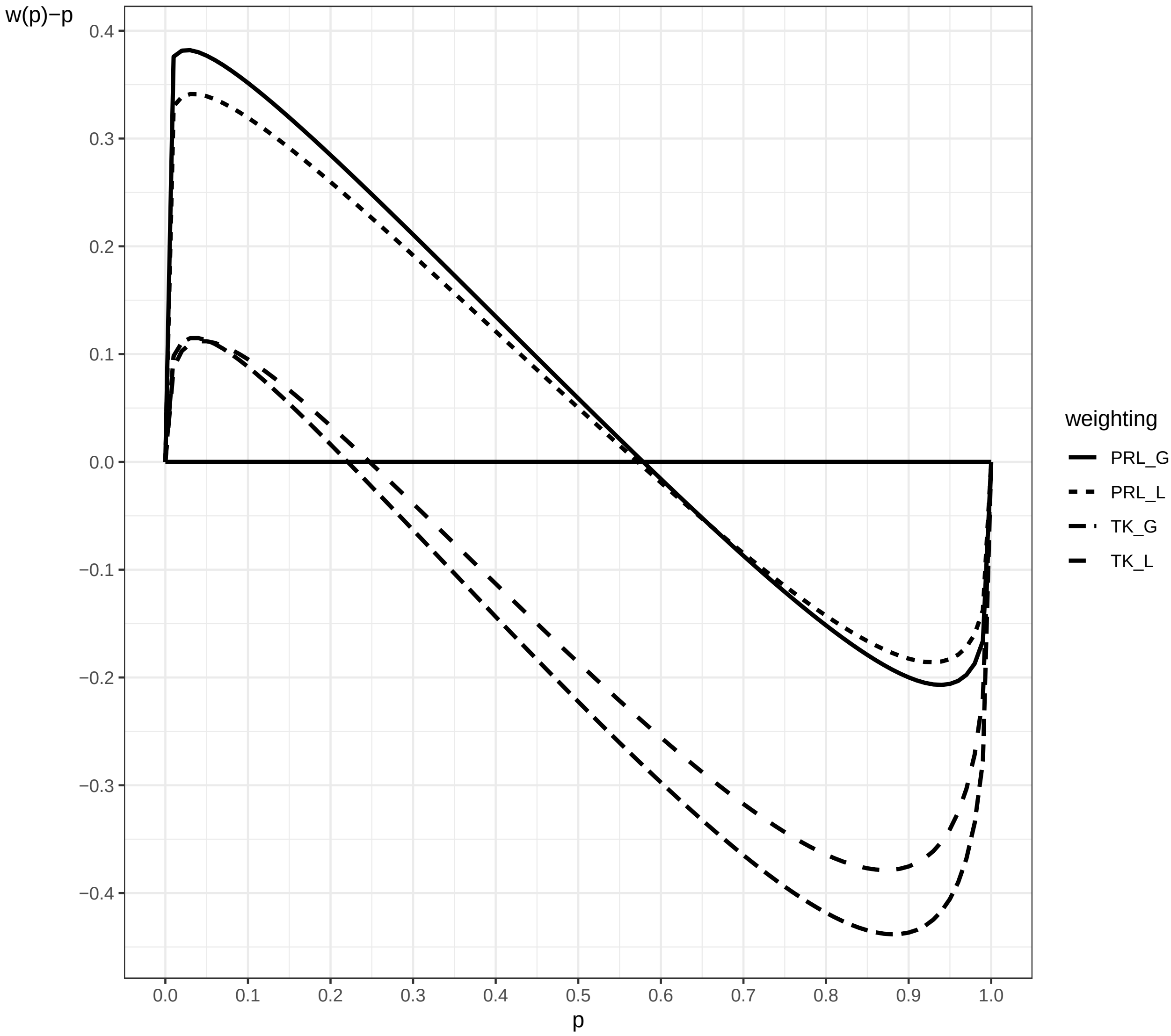

Figure 3: Difference of $w(p) - p$ for the two weighting functions, for both gains and losses based on our estimates of the mixture model (Fehr-Duda et al. 2010) reported in Table 1.

Appendix A Loss Aversion

To facilitate the discussion about loss aversion in the Markowitz model of utility let us first rewrite the expo-power value function in a more general way where parameter η is initially set differently over gains and over losses, and where α is replaced for convenience by $\alpha = \rho * \beta$

$$\begin{aligned} u(g) =& 1 - e^{-\rho*\beta*g^{\eta_g}} \\ u(L) =& -\lambda(1 - e^{-\beta*L^{\eta_L}}), \end{aligned}$$

where $\lambda, \beta > 0, \rho > 1, \eta_g, \eta_L > 1$. For a symmetric gain and loss, $g = L$, the ratio $\frac{u(L)}{u(g)}$ is a measure of loss aversion as $g = L$ goes to zero. However, we note that if the model allows different parameters η_g and η_L, this measure of loss aversion can go to zero which is not appropriate.

$$\frac{u(L)}{u(g)} \simeq \frac{\lambda * \eta_L * L^{\eta_L - \eta_g}}{\eta_g \rho} e^{-\beta(L^{\eta_L} - \rho L^{\eta_g})}$$

$$\lim_{L \to 0} \frac{\lambda * \eta_L * L^{\eta_L - \eta_g}}{\eta_g \rho} e^{-\beta(L^{\eta_L} - \rho L^{\eta_g})} = \frac{\lambda * \eta_L * L^{\eta_L - \eta_g}}{\eta_g \rho}.$$

On the other hand, if parameters η_g and η_L are constrained to be equal (e.g. η), the ratio $\frac{u(L)}{u(g)}$ as $g = L$ goes to zero, a measure of loss aversion, becomes $\frac{\lambda}{\rho}$,

$$\frac{u(L)}{u(g)} \simeq \frac{\lambda}{\rho} e^{-\beta(1-\rho)L^{\eta}}$$

$$\lim_{L \to 0} \frac{\lambda}{\rho} e^{-\beta(1-\rho)L^{\eta}} = \frac{\lambda}{\rho}.$$

We note that while loss aversion is typically discussed in the context of small gains/losses, large-loss-aversion (LLAD), a measure introduced by He and Zhou (2011), characterises loss aversion with respect to large payoffs. This can be linked to the issue of whether agents would potentially take infinite leverage in a portfolio choice setup. Employing the notation by He and Zhou, LLAD can be defined as

$$\tilde{\lambda} := \lim_{x \to \infty} \frac{u_-(x)}{u_+(x)},$$

where u_- and u_+ both mapping from $\Re^+$ to $\Re^+$, measure gains and losses respectively. u_- is in fact the disutility of losses ($u_+ := u(x)$, and $u_- := -u(-x)$ whenever $x \geq 0$), and

$$u_+(z) = 1 - e^{-\rho * \beta * x^\eta}$$
$$u_-(z) = \lambda(1 - e^{-\beta * x^\eta}).$$

In our case, $\tilde{\lambda} = \lambda$. For practical purposes this may be better considered as a local approximation over gains and losses of a moderate amount. The issue of a limiting case for loss aversion has also been recently raised by Kahneman (2011). He hypothesised that loss aversion probably tends to increase (although not dramatically) as stakes rise. Loss aversion could in principle become infinite when possible loss is potentially ruinous. See also Theorem 1 in He and Zhou for limiting cases of λ (0 and $+\infty$) which applies to our case. It is worth noting that, from an empirical perspective and for the range of payoffs typically employed in experimental work, it seems appropriate to assume λ is constant.

We note that, with the expo-power value function, loss aversion is bounded between $\frac{\lambda}{\rho}$ and λ. Estimates of λ will reflect the highest degree of loss aversion observed in the experiment. Consequently, a method of reflecting varying loss aversion as recently suggested by Kahneman (2011).

Appendix B Data sets and experimental stimuli

Fehr-Duda et al. (2010)

The main research question of this study is whether risk aversion increases with stake size, taking into consideration both the domain of gains and losses. To this end, Fehr-Duda et al. elicit certainty equivalents (CE) using 56 two-outcome lotteries of the form $\mathcal{L} = \{x_1, p; x_2, 1-p\}$ with $|x_1| > |x_2|$ and p the probability of the higher gain, over a wide range of stakes and probabilities. Twenty eight lotteries use low stakes, and the rest use high stakes. There are stakes both in the gains and the losses domain and a specific endowment is provided in the loss domain in order to cover potential losses. To incentivise decisions, subjects are paid for two randomly chosen lotteries, one from each set of lottery stakes (low, high). To elicit CEs, they asked subjects, for each lottery, to choose between the lottery and a list of 20 equally spaced certain outcomes, ranging from the lottery's maximum payoff to its minimum payoff. The CE is then calculated as the average of the smallest certain amount preferred to the lottery and the subsequent certain amount on the list. In total, 153 subjects participated in the experiment.

Scholten and Read (2014)

The authors in this study show how Prospect Theory can accommodate the fourfold pattern by combining an overweighting of low probabilities with a decreasingly elastic value function. The choice task is a decision between the sure monetary payoff x and a gamble that yields outcome $10x$ with probability 0.1 and nothing otherwise $(10x, 0.1; 0, 0.9)$. For their estimations they use two different datasets. The first comes from Hershey and Schoemaker (1980) experiment with 41 participants, which consists of 10 pairwise choices of the form x for sure or $100x$ with probability $p = 0.01$, otherwise 0. The second dataset consists of 12 pairwise choices from the experiment that Scholten and Read (2014) conducted of the form x for sure or $10x$ with probability $p = 0.1$, otherwise 0.

569 subjects participated to the second experiment. As there is a disparity between the number of subjects in the two studies, they assumed that all data from the first study were collected from 569 subjects assuming that the data from the 41 subjects would be replicable over 528 additional participants. None of the experiments include real losses, instead, they use random payment of some of the subjects with a lump sum, claiming that incentive compatible payment schedules are

prohibitive for this kind of Markowitzian choice series.

Pachur et al. (2018)

This experiment includes 90 subjects, making choices in a set of 91 risky binary lotteries. Objective of the study is to test whether psychological constructs in CPT, such as loss aversion and outcome and probability sensitivity, can be interpreted in terms of attention allocation. Each binary task includes two lotteries $\mathcal{L}_\mathcal{A}$ and $\mathcal{L}_\mathcal{B}$. Each lottery $\mathcal{L}_i$ has two possible outcomes that occur with known probabilities of the form $\mathcal{L}_\rangle = \{x_{i1}, p_{i1}; x_{i2}, 1 - p_{i1}\}$. The outcomes range from -100 to 100 and there are four types of lotteries, namely (1) only gains (35 lotteries); (2) only losses (25 loteries); (3) mixed lotteries with both gains and losses (25 lotteries); and (4) mixed zero lotteries with one gain and one loss against a zero outcome (6 lotteries). To make comparison easy between all datasets employed in our paper, we fit the various specifications using the lotteries from Pachur et al. dropping the mixed domain lotteries. Subjects participated to the experiment twice, with an approximately 2-week gap between the sessions. At each session, subjects were asked the same 91 lotteries. We perform the analysis by using the data from the first session.

Table 4: Stimuli from Fehr-Duda et al. (2010)

p	x_1	x_2	p	x_1	x_2	p	x_1	x_2
0.05	15	4	0.25	250	65	0.75	250	65
0.05	20	7	0.25	320	130	0.75	320	130
0.05	55	20	0.50	7	4	0.90	7	4
0.05	250	65	0.50	15	4	0.90	130	65
0.05	320	130	0.50	20	7	0.95	15	4
0.05	950	320	0.50	130	65	0.95	20	7
0.10	7	4	0.50	250	65	0.95	250	65
0.10	130	65	0.50	320	130	0.95	320	130
0.25	15	4	0.75	15	4			
0.25	20	7	0.75	20	7			

Notes: The Table presents the 28 gain lotteries $(x_1, p; x_2)$ which were used to elicit certainty equivalents. The losses lotteries follow the same structure.

Table 5: Stimuli from Scholten and Read (2014)

Sample	x_S	p	x_R
HS	1	0.01	100
HS	10	0.01	1,000
HS	100	0.01	10,000
HS	1,000	0.01	100,000
HS	10,000	0.01	1,000,000
HS	1	0.01	100
HS	10	0.01	1,000
HS	100	0.01	10,000
HS	1,000	0.01	100,000
HS	10,000	0.01	1,000,000
SR	0.25	0.10	2.50
SR	2.50	0.10	25
SR	25	0.10	250
SR	250	0.10	2,500
SR	2,500	0.10	25,000
SR	25,000	0.10	250,000
SR	0.25	0.10	2.50
SR	2.50	0.10	25
SR	25	0.10	250
SR	250	0.10	2,500
SR	2,500	0.10	25,000
SR	25,000	0.10	250,000

The Table presents the gambles used in the experiment. x_S represents the safe amount while x_R represent the amount of the gamble $(x_R, p; 0)$. *HS* stands for the stimuli used in the Hershey and Shoemaker (1980) experiment, while *SR* stands for the sample from Scholten and Read (2014).

Table 6: Stimuli Pachur et al. (2018)

task	pa	A1	1-pa	A2	pb	B1	1-pb	B2	session 1	session 2	type
1	0.34	24	0.66	59	0.42	47	0.58	64	20	16	G
2	0.88	79	0.12	82	0.2	57	0.8	94	67	62	G
3	0.74	62	0.26	0	0.44	23	0.56	31	87	80	G
4	0.05	56	0.95	72	0.95	68	0.05	95	72	65	G
5	0.25	84	0.75	43	0.43	7	0.57	97	94	98	G
6	0.28	7	0.72	74	0.71	55	0.29	63	45	54	G
7	0.09	56	0.91	19	0.76	13	0.24	90	28	36	G
8	0.63	41	0.37	18	0.98	56	0.02	8	16	14	G

Table 6: Stimuli Pachur et al. (2018)

task	pa	A1	1-pa	A2	pb	B1	1-pb	B2	session 1	session 2	type
9	0.88	72	0.12	29	0.39	67	0.61	63	68	68	G
10	0.61	37	0.39	50	0.6	6	0.4	45	136	135	G
11	0.08	54	0.92	31	0.15	44	0.85	29	112	115	G
12	0.92	63	0.08	5	0.63	43	0.37	53	85	101	G
13	0.78	32	0.22	99	0.32	39	0.68	56	85	89	G
14	0.16	66	0.84	23	0.79	15	0.21	29	125	131	G
15	0.12	52	0.88	73	0.98	92	0.02	19	16	26	G
16	0.29	88	0.71	78	0.29	53	0.71	91	75	62	G
17	0.31	39	0.69	51	0.84	16	0.16	91	109	104	G
18	0.17	70	0.83	65	0.35	100	0.65	50	40	40	G
19	0.91	80	0.09	19	0.64	37	0.36	65	124	121	G
20	0.09	83	0.91	67	0.48	77	0.52	6	132	132	G
21	0.44	14	0.56	72	0.21	9	0.79	31	121	124	G
22	0.68	41	0.32	65	0.85	100	0.15	2	28	28	G
23	0.38	40	0.62	55	0.14	26	0.86	96	16	16	G
24	0.62	1	0.38	83	0.41	37	0.59	24	50	43	G
25	0.49	15	0.51	50	0.94	64	0.06	14	18	10	G
26	0.16	-15	0.84	-67	0.72	-56	0.28	-83	109	107	L
27	0.13	-19	0.87	-56	0.7	-32	0.3	-37	21	24	L
28	0.29	-67	0.71	-28	0.05	-46	0.95	-44	102	101	L
29	0.82	-40	0.18	-90	0.17	-46	0.83	-64	80	82	L
30	0.29	-25	0.71	-86	0.76	-38	0.24	-99	62	58	L
31	0.6	-46	0.4	-21	0.42	-99	0.58	-37	136	131	L
32	0.48	-15	0.52	-91	0.28	-48	0.72	-74	99	97	L
33	0.53	-93	0.47	-26	0.8	-52	0.2	-93	65	71	L
34	0.49	-1	0.51	-54	0.77	-33	0.23	-30	104	102	L
35	0.99	-24	0.01	-13	0.44	-15	0.56	-62	112	119	L

Table 6: Stimuli Pachur et al. (2018)

task	pa	A1	1-pa	A2	pb	B1	1-pb	B2	session 1	session 2	type
36	0.79	-67	0.21	-37	0.46	0	0.54	-97	48	53	L
37	0.56	-58	0.44	-80	0.86	-58	0.14	-97	61	61	L
38	0.63	-96	0.37	-38	0.17	-12	0.83	-69	28	16	L
39	0.59	-55	0.41	-77	0.47	-30	0.53	-61	16	11	L
40	0.13	-29	0.87	-76	0.55	-100	0.45	-28	94	101	L
41	0.84	-57	0.16	-90	0.25	-63	0.75	-30	18	10	L
42	0.86	-29	0.14	-30	0.26	-17	0.74	-43	112	105	L
43	0.66	-8	0.34	-95	0.93	-42	0.07	-30	77	71	L
44	0.39	-35	0.61	-72	0.76	-57	0.24	-28	26	33	L
45	0.51	-26	0.49	-76	0.77	-48	0.23	-34	50	43	L
46	0.73	-73	0.27	-54	0.17	-42	0.83	-70	58	54	L
47	0.49	-66	0.51	-92	0.78	-97	0.22	-34	78	82	L
48	0.56	-9	0.44	-56	0.64	-15	0.36	-80	112	122	L
49	0.96	-61	0.04	-56	0.34	-7	0.66	-63	16	14	L
50	0.56	-4	0.44	-80	0.04	-46	0.96	-58	108	105	L
51	0.43	-91	0.57	63	0.27	-83	0.73	24	44	48	M
52	0.06	-82	0.94	54	0.91	38	0.09	-73	121	121	M
53	0.79	-70	0.21	98	0.65	-85	0.35	93	53	50	M
54	0.37	-8	0.63	52	0.87	23	0.13	-39	124	116	M
55	0.61	96	0.39	-67	0.5	71	0.5	-26	70	74	M
56	0.43	-47	0.57	63	0.02	-69	0.98	14	54	55	M
57	0.39	-70	0.61	19	0.3	8	0.7	-37	91	87	M
58	0.59	-100	0.41	81	0.47	-73	0.53	15	51	65	M
59	0.92	-73	0.08	96	0.11	16	0.89	-48	41	50	M
60	0.89	-31	0.11	27	0.36	26	0.64	-48	44	53	M
61	0.86	-39	0.14	83	0.8	8	0.2	-88	62	62	M
62	0.74	77	0.26	-23	0.67	75	0.33	-7	48	57	M

Table 6: Stimuli Pachur et al. (2018)

task	pa	A1	1-pa	A2	pb	B1	1-pb	B2	session 1	session 2	type
63	0.91	-33	0.09	28	0.27	9	0.73	-67	102	102	M
64	0.93	75	0.07	-90	0.87	96	0.13	-89	68	53	M
65	0.99	67	0.01	-3	0.68	74	0.32	-2	124	121	M
66	0.48	58	0.52	-5	0.4	-40	0.6	96	60	68	M
67	0.07	-55	0.93	95	0.48	-13	0.52	99	107	109	M
68	0.97	-51	0.03	30	0.68	-89	0.32	46	33	43	M
69	0.86	-26	0.14	82	0.6	-39	0.4	31	70	71	M
70	0.88	-90	0.12	88	0.8	-86	0.2	14	82	89	M
71	0.87	-78	0.13	45	0.88	-69	0.12	83	18	11	M
72	0.96	17	0.04	-48	0.49	-60	0.51	84	87	95	M
73	0.38	-49	0.62	2	0.22	19	0.78	-18	38	43	M
74	0.28	-59	0.72	96	0.04	-4	0.96	63	28	24	M
75	0.5	98	0.5	-24	0.14	-76	0.86	46	95	89	M
76	0.5	-20	0.5	60	0.5	0	0.5	0	104	104	M-ZERO
77	0.5	-30	0.5	60	0.5	0	0.5	0	101	91	M-ZERO
78	0.5	-40	0.5	60	0.5	0	0.5	0	99	78	M-ZERO
79	0.5	-50	0.5	60	0.5	0	0.5	0	87	74	M-ZERO
80	0.5	-60	0.5	60	0.5	0	0.5	0	68	62	M-ZERO
81	0.5	-70	0.5	60	0.5	0	0.5	0	53	50	M-ZERO
82	0.1	40	0.9	32	0.1	77	0.9	2	121	124	G
83	0.2	40	0.8	32	0.2	77	0.8	2	122	116	G
84	0.3	40	0.7	32	0.3	77	0.7	2	119	114	G
85	0.4	40	0.6	32	0.4	77	0.6	2	107	105	G
86	0.5	40	0.5	32	0.5	77	0.5	2	91	92	G
87	0.6	40	0.4	32	0.6	77	0.4	2	85	75	G
88	0.7	40	0.3	32	0.7	77	0.3	2	60	50	G
89	0.8	40	0.2	32	0.8	77	0.2	2	38	30	G

Table 6: Stimuli Pachur et al. (2018)

task	pa	A1	1-pa	A2	pb	B1	1-pb	B2	session 1	session 2	type
90	0.9	40	0.1	32	0.9	77	0.1	2	27	14	G
91	1	40	0	32	1	77	0	2	10	6	G