The Effects of Sample Size on the Estimation of Regression Mixture Models

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Abstract

Regression mixture models are a statistical approach used for estimating heterogeneity in effects. This study investigates the impact of sample size on regression mixture's ability to produce 'stable' results. Monte Carlo simulations and analysis of resamples from an application dataset were used to illustrate the types of problems that may occur with small samples in real datasets. The results suggest: 1) when class separation is low very large sample sizes may be needed to obtain stable results; 2) it may often be necessary to consider a preponderance of evidence in latent class enumeration; 3) regression mixtures with ordinal outcomes result in even more instability; and 4) with small samples it is possible to obtain spurious results without any clear indication of there being a problem. indicate a substantial impact of small samples (relative to class separation) on both the number of classes supported by the data and estimates of differential effects in those classes. In some cases, there was no indication of invalid results, and yet the reported effects were opposite to those that existed in reality. This concerning finding was related to another: that dramatic differences sometimes appeared between multiple subsamples from the same data as sample size decreased. Overall these results suggest that sample sizes much larger than those typically considered large are needed to assure stable results (500 to 1000 subjects were needed for most analyses in this paper). Great caution is therefore urged in the use of regression mixtures with small samples, and the results highlight the importance of model validation. Because no one simulation can provide comprehensive guidelines for required sample sizes, however, it is recommended that multiple simulations reflecting the structure of the dataset of interest be conducted to understand model stability for a given result.

Keywords: Regression mixture models, sample size, heterogeneous effects

The Effects of Sample Size on the Estimation of Regression Mixture Models

The notion that individuals vary in their response to their environment has been well-accepted across substantive fields. Leading theories in the behavioral, social, and health sciences emphasize the synergistic role of environmental risk in individual development (Bronfenbrenner, 2005; Elder, 1998; Patterson, DeBaryshe, & Ramsey, 1989; Sampson & Laub, 1993) and consequently the search for differential effects – i.e., individual differences in the relationship between a predictor and an outcome – has become of increased salience to applied researchers. Traditional approaches for assessing differential effects involve the inclusion of a multiplicative interaction term into a regression equation. This method is intuitive and useful for testing differential effects which have been hypothesized a priori and involve observed subgroups. An alternative strategy, regression mixture modeling, utilizes a finite mixture model framework to capture unobserved heterogeneity in the effects of predictors on outcomes (Desarbo, Jedidi, & Sinha, 2001). In other words, regression mixture models are an exploratory approach to finding differential effects that do not require their predictors to be measured (Dyer, Pleck, & McBride, 2012; Van Horn et al., 2009).

This paper uses simulations and resamples from applied data to show how sample size impacts regression mixture results with the aim of providing users of this method with a starting point for selecting their samples. ~~As regression mixture modeling is a relatively new method, further work is needed to understand the conditions under which models will provide unbiased and stable estimates of differential effects. The question of what sample size is needed to achieve reliable results is both urgent and difficult to answer. Based on our review of prior research on regression mixtures, we believe the answer to this question~~We aim to show that sample size requirements depends critically on class separation, with both regression-parameter estimation and latent-class enumeration being a function of both sample size and class separation. We therefore hypothesize that, as sample size and/or class separation decrease the likelihood of unstable solutions will increase.

Methodological Overview

Regression mixture models are a specific form of finite mixture model. The latter term refers to a broad class of statistical models that estimate population heterogeneity through a finite set of empirically derived latent classes. Regression mixture models typically aim to identify discrete differences in the effect of a predictor on an outcome. This differs from other more commonly known mixture approaches, such as growth mixture models (B. Muthen, 2006; B. Muthén, Collins, & Sayer, 2001; B. O. Muthen et al., 2002) and semi-parametric models (D. S. Nagin, 2005; Daniel S. Nagin, Farrington, & Moffitt, 1995), in that the latent classes in a regression mixture are defined by bewteen-class differences in the associations between two variables, rather than between-class differences in the means or variances of a single variable (Desarbo et al., 2001; Van Horn et al., 2009; Wedel & Desarbo, 1994). The formulation, estimation, and details around the specification of regression mixtures are already well established (M. L. Van Horn et al., 2015). This paper focuses on helping users of regression mixtures understand the role that sample size and class seperation plays in the stability of regression mixture results.

Sample size in mixture models

Sample-size requirements for finite mixture models can be approached from two perspectives. One is the standard question of power: i.e., for a given sample size what is the probability that some hypothesis will be rejected, given the population values for all the model parameters? (the latter case, the hypothesis concerns latent class enumeration; the typically ... associated with ...) This is a question of obtaining the sampling distribution of a specific parameter; and it can in principle be derived analytically. However, mixture models include many parameters that impact power, and attempts at latent-class enumeration typically rely on comparison of penalized information criteria, such as the Bayesian Information Criterion (BIC), for which there is no known sampling distribution. Thus, power for regression mixtures is typically estimated using Monte Carlo simulation. This paper raises a The second perspective relates to the present paper's hypothesis that a second issue exists with related to finite mixtures in general, and with regression mixtures in particular. Because mixture models (Bauer & Curran, 2003, 2004) and especially regression mixtures (George et al., 2011; Van Horn et al., 2012) rely on strong distributional assumptions for parameter estimation, we hypothesize aim to show that model results will be increasingly unstable with smaller samples to the point that – even under ideal conditions – such models will yield more extreme results than expected – i.e. results may be far outside of the confidence interval suggested by estimated standard errors.

One of the difficulties encountered in estimating finite mixture models in general (without incorporating class-varying regression weights) is that the distribution of each model parameter depends on multiple model- and data- specific factors, including the number of classes estimated, the restrictiveness and complexity of the within-class model, the quality of the covariates, and the reliability of within-class observations (G. H. Lubke & Muthén, 2005; MacCallum, Widaman, Zhang, & Hong, 1999; Marcon, 1993; Nylund, Asparauhov, & Muthen, 2007). {Lubke, 2007 #1084;Nylund, 2007 #1070}Moreover, sample-size considerations must take account of class separation, overall sample size, and the within-class sample size. If the estimated proportion of respondents within a given class is small, then a larger overall sample will likely be required to find a stable solution for that class. This makes it challenging to provide a "rule of thumb" for sample-size requirements. However, proposing such a rule is not our goal.

Rather, this paper uses both simulations of selected scenarios and resampling of a real dataset to raise researchers' awareness of the types of problems that regression mixture modeling is likely to encounter when small samples are used, we focus specifically on the interplay between class separation and sample size while also looking at the proportion of subjects in each class.d.

Much work has looked at latent class enumeration, with some also looking at parameter estimation, with mixture models in general. Of particular note is work which has looked at sample size in factor mixture models (G. Lubke & Muthen, 2007; Nylund et al., 2007), when looking across the other factors this work found that class enumeration and parameter estimates were adequate with sample sizes of 500 or less. Few prior studies have examined the effects of sample-size requirements on regression mixture models specifically. Sarstedt and Schwaiger (2008) examined the use of regression mixture models to model market segmentation in the field of marketing, focusing only on the ability of these models to find the true number of latent classes. They found that while the Akaike's Information Criterion (AIC; Akaike, 1973) performed poorly regardless of sample sizes, the Consistent Akaike's Information criterion (CAIC; Bozdogan, 1994) performed well when samples were as small as n=150 to n=250. However, Sarstedt and Schwaiger's study (2008) was focused on situations with very high class separation resulting from in which there were large differences across classes in both intercepts and multiple regression weights. Across the different classes effect sizes, measured as R-square values ranged from .60 to .98, indicating that in some classes, very little residual variance remained. Under such conditions, there is substantial separation between latent classes, and thus regression mixture models would be expected to perform well even with small samples. Effects in the social sciences are generally much smaller; and when one's interest is in finding differential effects, intercept differences may be small to nonexistent. It should also be noted that Sarstedt and Schwaiger (2008) did not evaluate the precision or stability of parameter estimates.

One other study examined sample size requirements for regression mixtures, this time using a negative binomial model. Park, Lord, and Hart (2010) incorporated design features typically seen in highway crash data into their simulation, examining bias in parameter estimates, and found large bias in the dispersion parameter in samples less than n=2,000 under realistic conditions. They noted unstable solutions with small sample sizes and moderate or low effects, but also found that under conditions of high class separation (i.e., large mean differences between classes), their model was stable for samples as small as n=300. A reason for these discrepant results has to do with how much classes differ; as Park, Lord, and Hart (2010) put it, "the sample size need not be large for well-separated data, but it can be huge for a poorly-separated case." Class separation is at its lowest when differences between latent classes are solely a function of differences in regression weights with no mean differences; and in this case, the multivariate distributions of the data for the different classes overlap almost completely. This is also the point at which regression mixtures fulfill their promise as a method for exploring for differential effects, since they should be capable of finding discrete groups of respondents distinguished primarily by differences in regression weights.

The Current Study

This study aims to demonstrate the consequences of using regression mixtures ~~to find differential effects~~ as sample sizes decrease. Using both simulations and resampling of a real-world dataset, we evaluate the impact of sample size and class balance on latent class enumeration, bias in model parameters, the adequacy of estimated standard errors, and model stability. We are particularly interested in cases where the result of small samples is not low power, but rather parameter estimates which do not represent the population well. Based on the results of previous applied research and simulations, we hypothesize that the use of small samples in regression mixture models will increase the likelihood of extreme results, such that estimates of regression parameters across classes will be biased away from each other, while the confidence intervals of the estimates will be too narrow. ~~We also hypothesize that class enumeration will become more difficult with small samples, and that there will be an increasing number of convergence problems for the model.~~Additional analyses focus on the role of class separation in this relationship.

Ordinal logistic regression mixture models have been found effective for evaluating differential effects in the presence of skewed outcomes (Fagan, Van Horn, Hawkins, & Jaki, 2012; George, Yang, Van Horn, et al., 2013; Van Horn et al., 2012). Therefore, we ~~will~~ also test the hypothesis that the effects of sample size will be stronger on the ordinal logistic model ~~than on other models because they require additional parameters, and~~ because less information is available for analyses with ordinal outcomes.

Methods: Simulation Study

Five Hundred ~~The~~ Monte Carlo simulations ~~phase of our study included 500 simulation~~were runs per condition. Because of our interests in the problems that can occur when latent classes are defined solely by differential effects, our initial simulations were for a two-class model ~~for which~~where the only parameters that differed between the classes were regression weights and residual variances (more complex models are subsequently evaluated). We only consider 2 classes for the true model because we want to illustrate the issue in a relatively simple context. The initial simulations used one predictor, X, and one outcome variable, Y. The regression relationship for class 1 was Y = 0.70X + e, and for class 2, Y = 0.20X + e. ~~In both classes, t~~The predictor and the residuals, e, were drawn from a ~~standard~~ normal distribution with the residuals for Y scaled so that the standard deviation of Y is one. Thus, the slope of the predictor is equal to the correlation of X and Y.

~~To answer our research questions, we examined 18~~Eighteen simulation conditions were examined. For the first 10 conditions, the total number of individuals in the data set (6,000, 3,000, 1,000, 500, and 200) as well as the proportion of the sample in each class (50% in each class and 75% in class 1 and 25% in class 2) was varied~~ across conditions~~. ~~Our five chosen levels of overall sample size were 6,000, 3,000, 1,000, 500, and 200.~~ The largest case, 6,000 individuals in total was chosen based on prior studies (e.g., Van Horn et al., 2009; Van Horn et al., 2012) that suggested this was a sufficient number of individuals to find expected results. ~~The smallest total sample size examined was 200 cases. For each sample size, two different balance designs for latent classes were examined: i.e., 50/50 and 75/25 splits of individuals in class 1 and class 2, respectively.~~

Data were generated in R (R Core Team, 2016), and the models fit using M*plus* version 7 (L. K. Muthén & Muthén, 2008). The true model had two classes, and thus one-, two-, and three-class models were fit ~~for the first 10 conditions~~ to examine how frequently the correct number of classes would be selected based on the BIC and the bootstrap likelihood ratio test (BLRT). However, due to the computational burden, BLRTs were not run for conditions 11-18. We also chose to focus on the BIC because it delivered the most reliable results in previous research with regression mixtures (George et al., 2012; Van Horn et al., 2012). Results for the AIC and adjusted Bayesian Information Criterion (aBIC) were also collected for the ordinal regression mixture model, as these results differed across criteria. The percentage of ~~times~~ for which the two-class model would have been selected over the one-class or the three-class model were reported to better understand failures to select the true two-class model, we also calculated the percentage of times the three-class model is chosen over the two-class model; but importantly, we considered failure of the three-class model to converge as indicating support for the two-class result. This decision was based on our previous experience that over-parameterized models frequently fail to converge to a replicated loglikelihood (LL) value. This assumption changed results dramatically only for the ordinal outcomes model, class enumeration tables without this assumption are available from the authors upon request. Finally, the average size of the smallest class across simulations was recorded for each condition; and when the smallest class is relatively small (e.g., lower than 10% of the overall sample size), it was necessary to give further consideration to whether there was sufficient evidence to support a meaningful additional class, or if the apparent presence of an additional class was due to outliers or violations of the distributional assumption. We note that 10% is an arbitrary number and that it is possible to have true and meaningful classes below this size, given enough information in the data to reliably detect these classes.

All study simulation conditions were evaluated for replicated convergence, model fit, class enumeration and parameter estimation. Replicated-convergence is defined as a simulation run in which 1) a solution was obtained and 2) the log-likelihood value was replicated to the next integer in at least two of the 24 starting values.

Bias in parameter estimates was examined for every replicated solution in which the true two-class model was selected using the BIC. Specifically, we calculated the proportion of individuals in each class, the average across simulations for each parameter estimate and the associated standard error, as well as the parameter coverage, i.e., the percentage of simulations for which the true parameter is contained in the 95% confidence interval. Lastly, we displayed the distribution of slopes across simulations for conditions with smaller sample sizes. This serves two purposes. First, it helps to identify the presence of outliers in the estimated slopes; and, secondly, it helps to assess the robustness of the estimation and underlying sampling distribution. To correct for the problem of label-switching in simulations, classes were sorted such that the class with the stronger effect of X on Y was always class 1 (McLachlan & Peel, 2000; Sperrin, Jaki, & Wit, 2010). In cases where the two classes were not distinct, as evidenced by the distribution of the parameter estimates, average parameter estimates were somewhat biased in favor of the correct solution because of this class sorting.

Results: Simulation Study

Class Enumeration. Table 1 shows for each of the first 10 basic conditions the proportion of 500 replications in which the LL value was replicated ~~Table 1 shows the proportion of 500 replications for each of the first 10 basic conditions in which the LL value was replicated~~ along with average entropy values across these replications. No problems with model estimation were observed for the one-class model in any conditions, or for the two-class model when sample sizes were moderate to large. However, the two-clas model's rate of convergence to a replicated LL value dropped to around 70% for small sample sizes. In the case of the three-class model, only around 60% of the simulations converged to a replicated LL value when sample sizes were large, and when they were small, replicated convergence rates were as low as 38%. In most cases, non-convergence was due to the best-likelihood value not being replicated, rather than to a failure of convergence for all starting values. Further evaluations with 504 starting values (a multiple of 24, the number of processors available), 96 of which were run to convergence, did not improve the percentage of solutions that replicated the best LL value. This suggests that this problem was largely due to model misspecification, i.e., resulted from estimating an incorrect three-class model when there were in fact two classes in the population.

The entropy of the two-class and three-class results exhibits an interesting pattern with entropy being the lowest for the two-class models with large sample sizes. Low entropy values are typical for regression mixture models (Fagan et al., 2012) and can be expected when classes are poorly separated. Because entropy has been used as a criterion for selecting latent-class models (Ramaswamy, Desarbo, Reibstein, & Robinson, 1993), the true entropy values for our models are worth knowing. More specifically, if the true entropy is lower for the true two-class model, it would suggest that entropy should not be used for regression mixture model selection. Accordingly, we estimated the true entropy for these models using a dataset generated from the same population model but with 1,000,000 cases in each class. In these runs, the models with balanced and unbalanced class sizes had entropy values of .13 and .30, respectively. These dDifferences between the balanced and unbalanced designs can be attributed to the construction of entropy: when the highest posterior probabilities are used, individuals are more likely to be classified as being in the larger class, and since this class represents a larger proportion of the data in an unbalanced design, entropy is also higher. Therefore, we take these numbers as indicating that the conditions with the lowest average entropy estimates (i.e., conditions 1, 2, 6, and 7) are reasonably well estimated, whereas those with high entropy values (i.e., those with smaller sample sizes) are biased. The results in Table 1 demonstrated two important features of entropy in regression mixture models: 1) that the true models may have the lowest rather than the highestlower values of entropy, and 2) that estimates of entropy may be upward-biased if as sample size decreases and/or if the model is misspecified as having too many classes. While low entropy values do not discredit a model i.e., it can still be effective for finding differential effects in the population - they do suggest that its performance when classifying individuals will be poor.

Our class-enumeration results are provided in Table 2. For the basic set-up, the BIC criterion usually yielded the correct two-class solution when sample sizes were 3,000 or more (conditions 1, 2, 6, and 7), but none of the criteria performed well when sample sizes were smaller than that. These analyses also looked at the size of the smallest class for both the two- and three-class solutions, and found that the average class size of the smallest class for the three-class solution was always well below 10% of the overall sample size, whereas in all two-class solutions it was over 10%. In practice, it appears that small classes can be an indicator of a spurious class. For these simulations, if an arbitrary criterion of 10% in the smallest class was utilized to exclude a result, the three-class solution would usually be excluded from consideration because of the size of the smallest class. ; a two-class solution would likely be chosen over a one-class solution in cases where the smallest class was moderately large and the other criteria for the one-class and two-class solutions were similar. On the whole, these simulations suggest that for samples of 1,000 or more individuals researchers are reasonably likely to arrive at the correct two-class solution for this data generating scenario, while smaller samples are not if all information is used rather than any one criterion.

We next examined the percentage of the population estimated as being in each class. For conditions 1-5, we expect 50% of the population in each class; but the results showed that on average, when N<1,000, the model classifies more individuals into the class with the higher regression weight. For conditions 6-10, in which 75% of the individuals in the population were actually in class 1, the pattern was somewhat different with bias only at sample sizes 200 or 500.

As shown in Table 3, average model parameters were reasonably well estimated for all conditions in class 1 (with the larger regression weight). However, in class 2, bias in all parameters increased as sample size or class separation decreased, with class means (intercepts) showing an upward bias, and regression weights and variances showing a downward bias. While some of the model-parameter estimates appeared reasonable even with small samples, the coverage probabilities for the parameter estimates –defined as the percentage of simulations for which the true value is inside the 95% confidence interval – revealed serious problems with estimated confidence intervals as sample size decreased. Note that even in conditions with sample sizes over 1000, coverage was slightly less than desirable for the slope parameters. This suggests that estimated standard errors were too small. The very poor coverage estimates observed for sample sizes of 200 and 500 - especially for class 2 - could be a function of model instability as some simulations yielded extreme estimates (~~It should be n~~Note~~d here~~ that, for the residual variances, the 95% confidence interval was not accurate~~,~~ because variances do not follow a t distribution).

We further investigated model instability by examining the distribution of regression weights across simulations. Figure 1 presents histograms of the slopes for both classes mixed, for the conditions with less than 3,000 observations. The conditions with 3,000 and 6,000 observations (not shown) demonstrated a clear separation between estimated slopes with little evidence of any outlying solution. For smaller samples distribution of the estimated slopes became unimodal suggesting that – across simulations – the parameter estimates for the two classes are not reliably distinguished. Of concern is the appearance of many outlying, which indicates that in many simulations the estimated parameters bear little resemblance to the true values in the population. These graphs should show peaks at 0.2 and 0.7, the true values for the regression weights in each class. These peaks were evident in conditions 3 and 8, although both conditions feature some extreme outliers. However, at sample sizes of 500 and 200, the two peaks merge into one and there are many outliers, both above and below the true values.

As sample sizes decrease, we also expect wider confidence intervals and more variation across simulations. However, the extreme results seen in some simulations are not just a function of sampling variability, as the models' estimated standard errors are still relatively low and some of the parameter estimates are more than 15 standard errors from the true value. We then examined individual results from the small samples that showed extreme values, and found that many of the simulation results with extreme regression weights contained quite small classes that in practice would probably not be considered strong evidence for differential effects. However, it was also not uncommon to find results that featured: 1) strong effects in the opposite direction to the true effects with reasonably large class sizes, 2) replicated LL values, and 3) no other evidence that the result was erroneous. Small samples, in other words, could make it extremely difficult to discover that there is a problem with a given finding.

[Figure 1]

Our next set of simulations focused on how identification of the correct number of classes was affected by class separation. With a sample size of 500 in conditions 4 and fewer than 5% of the replications according to the BIC resulted in the correct number of classes being chosen. With increased class separation in conditions 11-14, the proportion of simulations that chose the correct number of classes rose dramatically to over 70% and 95% when between-class intercept differences were 1 and 1.5, respectively. Conditions 15 and 16 replicated condition 2 (with 1,500 observations in each group), but with decreased class separation caused by decreasing the differences in the slopes from 0.2 and 0.7 to 0.4 and 0.7; this ~~resulted~~ reduced the~~in~~ proportion of simulations that correctly identified two latent classes crashing from 87.9% to just 4.2%. Finally, in conditions 17 and 18, (not included in Table 3 because of the additional parameters) we examined the impact of including more information in the regression mixture model by adding an additional predictor. In this condition with a sample size of 500, the BIC found the correct two class solution in more than 97% of the simulations. Parameter estimates from these models were all reasonable, although coverage rates were somewhat less than .95 for the models with strong class separation and far less than .95 for models with weaker class separation.

We also investigated the use of an ordinal logistic model for identifying the correct number of classes (Table 4), which was recommended by Van Horn et al. (2012) and George et al. (2013) as a method for addressing non-normal errors. As in the normally distributed model, there were substantial issues with model convergence for the two-class ordinal logistic models when the sample size fell below 3,000. Further, even with 6,000 observations (the same number as in George et al., 2013), the BIC chose the correct two-class model in only 5% of the simulation replications. The main difference between this result and the previously reported results (George et al, 2013; Van Horn et al., 2012) is that ours hadhere there was no intercept differences. When we added a between-classes intercept difference of .5 standard deviations, we replicated the previous results, choosing the correct two-class solution in 95% of the simulations. With large sample sizes, the BLRT and aBIC had better, though still inadequate results; in the best case scenario with a sample of 3,000 the BLRT found two classes in 74% of simulations. Because the correct number of classes was rarely selected, parameter estimates are not reported.

[Table 4]

Simulation Study: Conclusion

Our initial simulations examined the effects of sample size on regression mixture models when the only feature defining latent classes was the heterogeneous effects of a predictor on an outcome. We deliberately choose chose a simulation scenario that was ideal in terms of distributional assumptions and the number of latent classes, but rendered more difficult by the very weak class separation caused by the lack of mean differences between classes in the outcome and no other predictors with which to separate the latent classes. We showed that, in such circumstances, entropy in the true model is very low and that model convergence to a replicated LL value becomes increasingly unlikely as sample sizes drop to 1,000 or less. None of the model-selection criteria were effective in selecting the true model when samples were less than 3000 although when a preponderance of evidence was used the correct solution could be found , and their performance was merely adequate with samples of 1,000. The problem appears to be not only a lack of power, but also the selection of solutions with superfluous, typically very small, classes. The problem is reduced if solutions with small classes are eliminated from consideration, this leaves open the question of how to find true small classes. We suspect that in this case either substantial class separation or very large sample sizes will be needed. We found that, with ordinal logistic regression model, all the selection criteria were underpowered; and that and no intercept differences it was possible to arrive at the right number of classes, but only if a preponderance of the evidence was used – an approach that implies never choosing solutions with any classes that contain 10% or less of the respondents. When there are no intercept differences between classes, it is quite difficult to arrive at the correct number of classes using the ordinal logistic regression mixtures.We note that a limitation of this study is that we only examined a true model with 2-classes. We hypothesize, but did not test, that adding additional classes without increasing class separation would increase required sample size because of the need to estimate more parameters without having much additional information.

When the correct number of latent classes were found, model-parameter estimates were on average reasonable, except for the very small class sizes of 500 and below. However, this hides an additional issue. With sample sizes this small, there were many cases in which multiple classes were supported and apparently reasonable solutions found, but where the parameter estimates were extreme, or even opposite of the true values. In short, aAlthough regression mixture models work well with large samples, using such models with small samples appears to be a dangerous proposition, as it will never be completely clear that the results are correct, or even how to identify that they are suspect.

To better understand these results we further investigated the effects of class separation on required sample size, showing that increasing class separation led to adequate results with samples of 500 and decreasing class separation resulted in samples of 1000 being inadequate to find differential effects (the correct number of classes). A promising result came from including additional predictors in the model, in this case model performance improved dramatically. This final result calls for more research as we examined only two conditions. Finally, we examined the implications of these results when using ordinal outcomes, finding that this case requires additional class separation if the correct number of classes is to be found.

Applied Example: Introduction

To illustrate the issues that can arise in practice when small samples are used in regression mixture models, we analyzed data from a previously published study that used regression mixtures to examine heterogeneity in the effects of family resources on academic achievement (Van Horn et al., 2009). Specifically, that prior study identified three latent classes:

one defined by low achievement (especially in reading but also in mathematics outcomes); one defined by a strong effect of basic needs (e.g., housing, food, and clothing); and the last being resilient to the effects of a lack of basic needs. Because the latter two classes had similar means for achievement, the class separation between them was weak. Nevertheless, the three classes appeared to be robust, especially with regards to the inclusion of covariates, and the study had a ~~reasonably large~~ sample size of 6,305. This data provides us with an opportunity for assessing what would have happened if a smaller dataset had been used with applied rather than simulated data.~~.~~

Applied Example: Methods

Data ~~for this phase of our research~~ were collected between 1992 and 1997 as part of the National Head Start Transition study: a thirty-site longitudinal intervention study (for a full description see C. T. Ramey, Ramey, & Phillips, 1996; S. L. Ramey et al., 2001). The sample consisted of children who had formerly been in the Head Start program and their peers from the same classrooms. Family resources were assessed using the Family Resource Scale (FRS; Dunst & Leet, 1994; Dunst, Leet, & Trivette, 1988), an instrument designed to measure the resources and needs of families of high-risk children. ~~In terms of family resources specifically, t~~The FRS assesses four aspects: ability to meet basic needs; adequacy of financial resources; amount of time spent together; and amount of time parents have for themselves (Van Horn, Bellis, & Snyder, 2001). ~~The c~~Children's receptive language skills were measured with the Peabody Picture Vocabulary Test-Revised (PPVT, Dunn & Dunn, 1981), a ~~good~~ predictor of school performance among low-income children (McLoyd, 1998). To demonstrate the method, our analyses were run using third grade data only, collected in 1996 and 1997.

Analyses were run on the full dataset that includes 6,305 students. To assess the effects of running regression mixtures on small samples we drew 500 replications without replacement from the full dataset of the same four sizes used in the simulation study described above (i.e., n=200, 500, 1,000, and 3,000). For each sample-size condition, analyses were run for all 500 datasets to evaluate the effect of sample size on class enumeration and parameter estimates, using the same methods as in our simulations. Given that the true population values for the empirical data were not known, we assessed the differences in the model results between the full dataset with 6,305 cases and the subsets of the data with smaller sample sizes. We were especially interested in the between-subsample differences within each condition, as these would indicate the range of results that might arise across many small samples.

Applied Example: Results

The first step in this phase of our analyses was to examine the regression mixture solution for the full sample. The BIC chose a two-class solution in the full sample, the aBIC was more equivocal: with the two and three-class solutions being about the same, but the latter's third class was small, with 8% of the students. We chose to retain the two-class solution. ~~Its~~ The classes were similar in substance to those already published; the first class containing 27% of the students, and defined by a strong positive effect of basic needs (B = 3.93, SE =.71) and a weaker negative effect of time spent with family (B = -1.76, SE = .71), and the second class with 73% of the students, featuring a weak positive effect of money (B = .83, SE = .31) and a weak negative effect of time spent with family (B = -.56, SE = .27). The intercepts for the two classes were quite similar, B = 98.74, SE = .67 in class one and B = 101.07, SE = .27 in Class 2.

Turning to our multiple replications of each smaller subsample, the first interesting result concerns model convergence. In simulated data, there were convergence problems for the two-class model in about 30% of the simulations with sample sizes of 200, and convergence was a problem in most simulations for the three-class model. ~~-~~With the applied data, however, convergence was rarely a problem with a sample of 200, the two-class model converged 96% of the time, and the three-class model converged 94% of the time~~, and convergence was even higher in larger samples~~. This is consistent with previous results in which convergence became a problem when models were over-parameterized with simulated data that was perfectly behaved (M. Lee Van Horn et al., 2015), but convergence is generally not a problem with applied data, which never perfectly meets ~~researchers'~~ model assumptions. While convergence was not a problem with the applied data, replicating the two-class solution was much more difficult. With a sample size of 3,000, only 141 of the 500 replications ~~choose~~ chose the two-class over the one-class and three-class models using the BIC. This fell to 73 out of 500 replications when the sample size was reduced to 1,000, but then went back up again to 154 out of 500 replications when the sample fell further, to 500; and edged up again, to 181 out of 500 replications, with the very lowest sample size, 200. By this criterion alone, then, it appeared that a sample size of 200 yielded the best model performance. We further explored these results by taking the size of the smallest class into account. When classes that contained less than 5% of the students were excluded from consideration, the two-class model was chosen 140 times with a sample of 3,000, less than five times when the sample was 1,000 or 500, and 139 times when it was 200. ~~Using the aBIC increased all these numbers somewhat: with the two-class model being chosen 280, 117, 109, and 71 times with samples sizes of 3,000, 1,000, 500, and 200, respectively. Still, these~~Results indicate that ~~there are often inconsistencies in~~ class enumeration~~, and that these vary as~~ varies greatly as a function of sample size, and that applied data often shows different properties than simulated data.

Finally, we examined parameter estimates across replications within each condition. Here, we focused on the regression weight for the effects of students' basic needs, looking only at those cases where the smallest class contained over 5% of the sample, since cases with smaller classes than that typically had extreme outliers. In other words, we assumed that the analyst would have arrived at the two-class model even if the model-selection criteria did not clearly indicated support for two classes. The number of 500 simulations for which the smallest class in the two-class solution contained more than 5% of the students was 411 when the sample size was 200, 242 when it was 500, 346 when it was 1,000, and 496 when it was 3,000. Figure 2 presents histograms of each condition of regression weights for the effects of basic needs for each condition, and the full model results are included in Appendix A. Classes are not sorted here (since it would clearly be problematic in the small-sample conditions), and thus if the solution from is stable and matches the full dataset- is stable — we should see two relatively normal distributions, with one centered on about 0.2 (the non-significant effect of basic needs in the resilient class) and the other centered on about 3.9. When the sample size was 3,000, the results were almost perfectmirrored this, with nearly complete separation between the different classes. Thus, any 2-class solution with a sample of 3000 would lead to similar results Neither the BIC nor the aBIC was a highly reliable means of identifying the correct number of classes, even with a sample of 3,000; however, when they did identify the correct number, the results reflected the full sample in every case, with only a few outliers. With a sample size of 1,000, the slopes were still reasonable stable most of the time, although their distributions in the two classes now clearly overlap. It is interesting to note that in the smaller class (i.e., of students more affected by basic needs), the average standard error for the effect of basic needs was 1.7 across all replications. The observed sampling distribution for the largest class across all replications was 2.1, was far larger than would be suggested by the estimated standard error~~; and in fact, the standard deviation of the slopes for the largest class across all replications was 2.1, substantially larger than the standard error that it supposedly represents~~. Finally, the model results mostly break down with samples of 500 and 200, which provided vague, general evidence for the existence of the class with no basic-need effects, but rarely replicated the results from the full sample.

Applied Example: Conclusions

Examining small sample sizes by resampling a previously published example dataset ~~yielded some interesting results. First, it c~~confirmed a previous antidotal finding that convergence issues were more common when working with simulated rather than applied data. The reason for this may be that simulated data meets all model assumptions, whereas applied data typically violates assumptions to some degree.~~ Second, t~~These results also showed that in applied situations there may be more variability in the number of classes chosen than in simulated data: ~~the limitations of penalized information criteria for selecting the correct model, since~~ even with sample sizes of 3,000 and ~~even~~ when the model results appear ~~reasonable~~ stable across samples, ~~in nearly every replication, the most common~~ model-selection criteria chose 2 classes~~the correct model~~ only about half the time.~~ This result parallels that of our simulation study that used ordered logistic regression, which likewise implied that penalized information criteria can work for model selection, but are less than ideal in many cases. And third~~Finally, while parameter estimates were reasonable and exhibited little variability when the sample size was 3,000, they were markedly more variable with a sample of 1,000, and became quite poor when the samples were 500 or smaller. In many cases, the practical result of this would be a failure to find differential effects due to a one-class model being selected. In other cases, however, using small samples would not only yield quite inaccurate results, but estimated standard errors that give the researcher a false sense of confidence in such results.

Discussion

One of the most common questions asked at presentations on regression mixture models concerns the sample size required to use this method. Our purpose in this study was to help applied researchers understand the interplay between class separation and sample size when estimating regression mixture models with continuous and ordinal outcomes. Looking across all results of this study suggests: 1) when class separation is low (as is typical in regression mixtures), sample sizes as much as an order of magnitude greater than suggested by previous research may be needed to obtain stable results; 2) there is a direct relationship between class separation and required sample size such that increasing class separation would make most results stable, although potentially at the cost of losing what made a regression mixture useful; 3) regression mixtures with ordinal outcomes result in even more instability; 4) with small samples it is possible to obtain spurious results without any clear indication of there being a problem; 5) very small latent classes may be an indicator of a spurious result (it isn't clear to us how truly small classes can be reliably identified when class separation is low); 6) higher values of entropy are not necessarily indicative of a correct model; and 7) at least within the range of a 25% to 75% split between classes, the effects of class size were less in our study than of sample size.

~~This study provides insight into that question. We specifically focused on cases with very weak class separation, because it is in such cases that regression mixture models are truly defined by differential effects. If there are large differences in the means of the outcomes between classes, then class separation is deemed high and the models are more stable; but this also means that the primary driver of the latent classes is the mean differences rather than effect differences.~~

~~This study found that when there were no mean differences between classes, even when data was generated to be ideal (in the sense that distributional assumptions were met in every class), sample size had a clear effect on both latent-class enumeration and parameter recovery. As sample size decreased, penalized information criteria and the BLRT frequently failed to find the true number of classes; and, when the true number of classes was found, these models struggled to distinguish the true differences in parameters between classes. Classes with less residual error were generally better estimated.~~

~~We conducted a series of additional simulations designed to make the point that sample size requirements are a function of sample size, class separation, and available information. With increased class separation, smaller samples will still lead to replicable results. With decreased class separation, even larger samples are needed. And, when more information – such as additional covariates – is brought into the model, results become more stable. The final point is interesting because it is often fairly easy for investigators to add additional predictors into a study.~~

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Figure Legend:

Figure 1. Histogram of estimated slopes for scenarios with 1,000 or fewer observations.

Figure 2. Histogram of the slope for basic needs as a function of sample size.

Table 1. Convergence and entropy for each simulation condition

Cond ID	Sample Size Balanced Design	True Model Sample Sizes 2 Classes		Estimated Latent Class Model 1 Class	2 Classes		3 Classes	
		N_1	N_2	% Cnvrg	Entropy	% Cnvrg	Entropy	% Cnvrg
1	Balanced (50/50 split)	3000	3000	100.0	0.11	100.0	0.40	58.2
2		1500	1500	100.0	0.14	100.0	0.43	56.8
3		500	500	100.0	0.37	90.2	0.57	50.0
4		250	250	100.0	0.60	77.4	0.69	41.0
5		100	100	100.0	0.75	71.0	0.78	42.2
6	Unbalanced (75/25 split)	4500	1500	99.6	0.27	100.0	0.50	57.4
7		2250	750	98.6	0.28	99.8	0.51	56.4
8		750	250	99.4	0.49	88.2	0.65	51.0
9		375	125	100.0	0.68	74.8	0.74	45.6
10		150	50	100.0	0.79	71.8	0.82	37.6

Note: N_1 is the sample size within class 1 and N_2 is the sample size in class 2. The mean entropy across all simulations is reported. % Cnvrg is the percentage of 500 replications which converged to a replicated solution.

Table 2. Latent class enumeration across simulations.

Cond ID	Equations of Data-Generated Scenarios	Sample size Balance Design	True Model		Estimated Models					
					Selecting 2 over 1 and 3 class			Selecting 3 over 2 class		
			N_1	N_2	% BIC	% BLRT	Smallest Class Size	% BIC	% BLRT	Smallest Class Size
1	*Basic Regression Mixture Set-up:*	Balanced	3000	3000	99.6	95.2	42.5	0.2	4.5	2.4
2	$LC\ 1: Y = 0.2X + e$		1500	1500	87.6	92.8	39.2	0.2	5.8	2.4
3	$LC\ 2: Y = 0.7X + e$		500	500	18.2	40.6	26.9	5.4	1.6	2.5
4			250	250	4.8	10.6	17.5	9.7	1.2	2.3
5			100	100	7.2	6.2	12.3	20.3	1.0	3.4
6		Unbalanced	4500	1500	96.8	94.6	25.8	0.6	5.4	1.9
7			2250	750	84.8	91.8	26.6	1.4	7.4	2.3
8			750	250	21.0	42.2	19.8	6.0	3.8	1.8
9			375	125	9.4	16.8	12.5	12.9	2.2	2.1
10			150	50	10.0	10.4	10.5	15.2	1.2	2.9
11	*Intercept Difference of 1*	Balanced	250	250	72.8	NA	38.6	1.9	NA	6.5
12	$LC\ 1:\ Y = 0 + 0.2X + e$ $LC\ 2:\ Y = 1 + 0.7X + e$	Unbalanced	375	125	86.0	NA	24.7	2.1	NA	4.8
13	*Intercept Difference of 1.5*	Balanced	250	250	97.9	NA	42.8	0.5	NA	7.0
14	$LC\ 1:\ Y = 0 + 0.2X + e$ $LC\ 2:\ Y = 1.5 + 0.7X + e$	Unbalanced	375	125	98.0	NA	25.6	1.8	NA	5.9
15	*Decrease Slope Differences*	Balanced	1500	1500	4.2	NA	25.0	6.7	NA	4.1
16	$LC\ 1:\ Y = 0.4X + e$ $LC\ 2:\ Y = 0.7X + e$	Unbalanced	2250	750	7.4	NA	20.6	0.4	NA	3.2
17	*Uncorrelated Predictors*	Balanced	250	250	98.5	NA	48.2	1.5	NA	8.3
18	$LC\ 1:\ Y = 0.2X_1 + 0.2X_2 + e$ $LC\ 2:\ Y = 0.7X_1 + 0.7X_2 + e$	Unbalanced	375	125	97.4	NA	25.1	2.6	NA	5.4

Note: %BIC is the percentage of simulations selecting this model using the Bayesian information criteria and BLRT is the bootstrap likelihood ratio test. Smallest class size is the average proportion of respondents in the smallest class across all simulations.

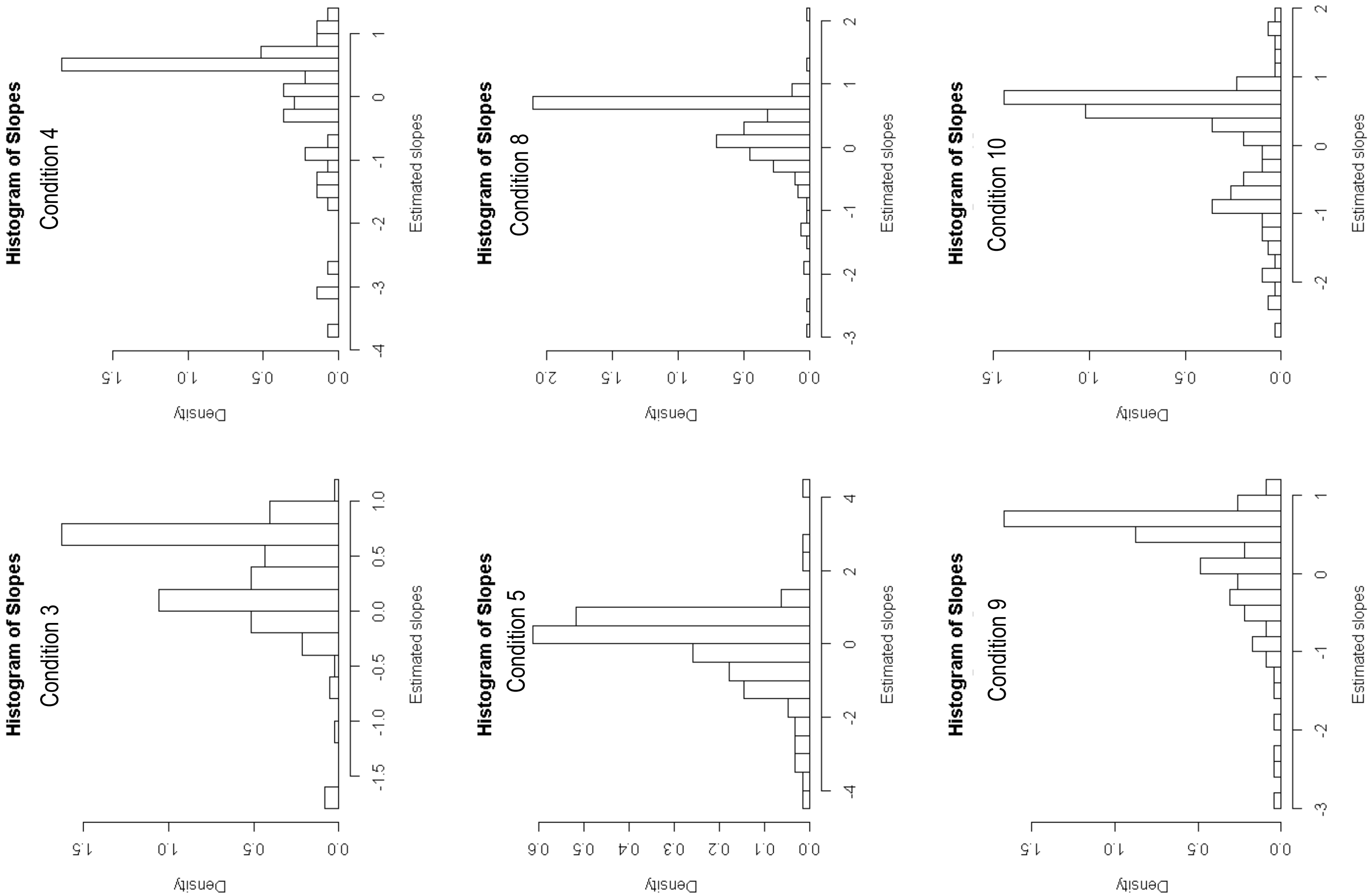
Table 3. Estimated parameter means, standards errors and coverage across all simulations.

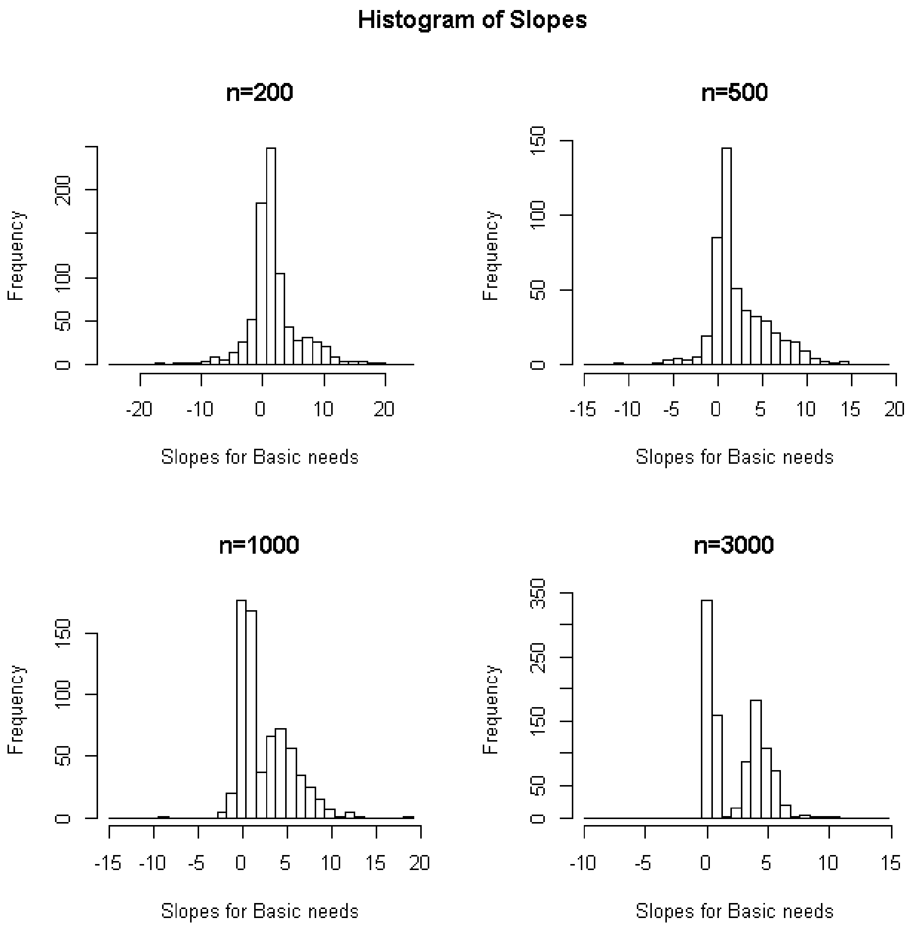
	Class 1									Class 2								
	Intercept			Slope			Residual Variance			Intercept			Slope			Residual Variance		
Condition	Mean	SE	Cover	Mean	SE	CoZver	Mean	SE	Cover	Mean	SE	Cover	Mean	SE	Cover	Mean	SE	Cover
Truth	0.00			0.70			0.51			0.00			0.20			0.96		
1	0.00	(0.03)	0.95	0.70	(0.05)	0.93	0.50	(0.06)	0.94	0.00	(0.03)	0.96	0.20	(0.06)	0.93	0.96	(0.05)	0.92
2	0.00	(0.04)	0.96	0.71	(0.06)	0.90	0.50	(0.08)	0.91	0.00	(0.05)	0.97	0.17	(0.09)	0.93	0.96	(0.07)	0.93
3	0.01	(0.05)	0.89	0.71	(0.07)	0.77	0.49	(0.08)	0.67	0.08	(0.09)	0.84	-0.04	(0.13)	0.68	0.88	(0.12)	0.84
4	-0.11	(0.05)	0.85	0.61	(0.05)	0.29	0.57	(0.06)	0.26	0.37	(0.06)	0.47	-0.69	(0.09)	0.26	0.49	(0.08)	0.32
5	0.14	(0.05)	0.79	0.67	(0.06)	0.11	0.57	(0.06)	0.23	0.40	(0.06)	0.21	-0.77	(0.05)	0.08	0.22	(0.05)	0.08
6	0.00	(0.02)	0.95	0.70	(0.03)	0.94	0.51	(0.03)	0.93	0.00	(0.05)	0.96	0.19	(0.10)	0.93	0.95	(0.08)	0.93
7	0.00	(0.03)	0.96	0.71	(0.04)	0.95	0.50	(0.05)	0.94	-0.01	(0.07)	0.95	0.18	(0.14)	0.91	0.95	(0.11)	0.92
8	0.03	(0.04)	0.88	0.72	(0.05)	0.76	0.49	(0.06)	0.76	0.00	(0.13)	0.80	-0.14	(0.18)	0.67	0.84	(0.17)	0.76
9	-0.01	(0.04)	0.86	0.69	(0.05)	0.53	0.50	(0.05)	0.42	0.44	(0.11)	0.47	-0.31	(0.11)	0.23	0.49	(0.14)	0.37
10	0.01	(0.05)	0.79	0.72	(0.05)	0.62	0.49	(0.05)	0.62	0.44	(0.04)	0.25	-0.48	(0.03)	0.08	0.15	(0.03)	0.09
Increased class separation																		
Truth	1.00			0.70			0.51			0.00			0.20			0.96		
11	0.99	(0.12)	0.88	0.69	(0.09)	0.84	0.49	(0.11)	0.86	-0.09	(0.21)	0.82	0.19	(0.11)	0.92	0.88	(0.16)	0.84
12	1.00	(0.08)	0.92	0.70	(0.06)	0.89	0.49	(0.08)	0.90	-0.13	(0.29)	0.79	0.19	(0.15)	0.88	0.82	(0.21)	0.78
Truth	1.50			0.70			0.51			0.00			0.20			0.96		
13	1.49	(0.11)	0.92	0.71	(0.08)	0.91	0.50	(0.10)	0.90	-0.02	(0.19)	0.89	0.19	(0.09)	0.95	0.93	(0.17)	0.86
14	1.49	(0.07)	0.93	0.71	(0.05)	0.93	0.50	(0.07)	0.92	-0.02	(0.28)	0.83	0.20	(0.13)	0.92	0.91	(0.24)	0.82
Decreased class separation																		
Truth	0.00			0.70			0.51			0.00			0.40			0.84		
15	0.00	(0.06)	0.93	0.70	(0.08)	0.69	0.49	(0.10)	0.68	-0.03	(0.09)	0.83	0.23	(0.12)	0.72	0.71	(0.10)	0.71
16	-0.01	(0.05)	0.93	0.72	(0.06)	0.72	0.48	(0.08)	0.69	0.00	(0.12)	0.85	0.22	(0.15)	0.68	0.68	(0.15)	0.67

Note: For simulations where the 2-class solution was selected by the BIC, the median estimated standard error, and the coverage estimate (percentage of simulations for which the 95% confidence interval included the true value) are reported.

Table 4. Latent class enumeration across simulations for the ordinal regression mixture model.

Balanced Design	True Model		Estimated Models										
			Selecting 2 over 1 and 3 class				Selecting 3 over 2 class				% of runs not converged		
	N_1	N_2	% AIC	% BIC	% aBIC	% BLRT	% AIC	% BIC	% aBIC	% BLRT	1-class	2-class	3-class
Balanced (50/50 split)	3000	3000	76.4	5.0	50.6	73.8	22.8	0.6	0.8	23.2	0.0	1.6	71.6
	1500	1500	60.6	4.2	15.0	39.4	24.1	4.2	4.2	23.6	0.0	17.2	71.4
	500	500	42.0	0.4	9.2	18.6	27.4	10.8	12.6	20.6	0.0	32.2	73.0
	250	250	34.8	0.0	15.4	17.6	27.6	14.3	18.9	19.0	0.0	36.2	69.8
	100	100	36.0	0.4	34.4	20.6	25.6	13.0	23.8	23.4	0.0	30.6	61.2
Unbalanced (75/25 split)	4500	1500	61.8	1.0	22.4	53.8	31.1	3.7	3.7	25.0	0.0	12.2	63.0
	2250	750	52.2	0.0	6.8	29.4	29.3	7.3	7.6	22.0	0.0	24.0	67.4
	750	250	40.6	0.4	6.4	18.2	24.5	10.1	10.9	18.8	0.0	32.4	73.2
	375	125	38.2	0.0	15.0	22.8	28.9	15.0	17.3	22.4	0.0	32.8	65.6
	150	50	37.0	0.4	36.0	21.4	29.4	16.4	26.1	27.2	0.0	32.6	61.0


Histogram of Slopes
Condition 3
Histogram of Slopes
Condition 4
Histogram of Slopes
Condition 5
Histogram of Slopes
Condition 8
Histogram of Slopes
Condition 9
Histogram of Slopes
Condition 10
Density
Estimated slopes


Histogram of Slopes
n=200
n=500
n=1000
n=3000
Frequency
Slopes for Basic needs
0
50
100
150
200
250
350
-20
-15
-10
-5
5
10
15
20

Appendix A. Full results from applied regression mixture models.

Full results for the analyses of the applied dataset with different sample sizes are presented in this appendix. Table 1 presents the class enumeration results using the BIC and aBIC for the full dataset.

Table 1. Class enumeration for the full dataset (n=6305)

BIC				ABIC			
1-class	2-class	3-class	4-class	1-class	2-class	3-class	4-class
42522.8	42478.5	42496.0	42505.2	42494.2	42427.7	42422.9	42409.9

We next examine latent class enumeration for the smaller subsamples of the applied data, meant to simulate what would happen across many smaller subsamples of the data. Results in Table 2 indicate that even when the subsample size is 3000, neither the BIC nor the aBIC do a great job of selecting the same 2-class solution found in the full dataset.

Table 2. Number of simulations where the smallest class was above 5% of the sample selecting the 2, 3, and 4 class solutions with with BIC and aBIC

		BIC				aBIC			
	[a]N>5%	2c over 1c	3c over 1c/2c	4c over 1c/2c/3c	2c over 1c/3c	2c over 1c	3c over 1c/2c	4c over 1c/2c/3c	2c over 1c/3c
n200	411	222	83	16	139	347	294	241	53
n500	242	6	3	2	3	123	105	89	18
n1000	346	10	5	1	5	152	99	74	53
n3000	496	157	17	0	140	360	82	46	278

[a]Number of simulations containing at least 5% of subjects in the smallest class

Finally, we examine the parameter estimates for the full dataset and each of the smaller subsamples. Results in Table 4 indicate that the mean estimates tend to be quite close to those observed in the full sample, but that there is extensive variability across estimates.

This can especially be seen in the difference between the average standard errors and the standard deviation across subsamples in each of the parameters. There is substantially more variability observed than the standard errors suggest should be there. Estimates of the standard errors appear to underestimate the sampling variability at low samples

Table 3. Parameter estimtes for subsets of applied data

	N = 200					
	Basic needs			Resilient		
	Est.	S.E.	S.D.	Est.	S.E.	S.D.
Intercept	98.19	0.64	6.13	101.99	0.53	4.79
Basic needs	4.15	0.82	3.81	-0.66	0.58	2.64
Money	0.93	0.85	4.38	0.69	0.67	3.31
Time-self	-1.31	0.85	4.29	0.17	0.66	2.96
Time-family	-2.24	0.88	4.48	-0.34	0.69	3.47
African American	-2.45	0.38	1.34	-2.45	0.38	1.34
Hispanic	-1.25	0.58	2.26	-1.25	0.58	2.26
White	4.81	0.37	1.34	4.81	0.37	1.34
Residual	22.35	4.19	21.71	24.33	3.84	20.16
Class Proportion	44.18%			55.82%		
	N = 500					
	Basic needs			Resilient		
	Est.	S.E.	S.D.	Est.	S.E.	S.D.
Intercept	97.33	1.63	4.20	101.35	0.89	2.05
Basic needs	4.60	1.79	2.85	0.07	0.70	1.46
Money	1.83	1.84	3.29	0.65	0.96	1.57
Time-self	-1.51	1.86	3.06	0.23	0.93	1.75
Time-family	-2.24	1.71	3.07	-0.64	0.89	1.52
African American	-2.51	0.62	0.70	-2.51	0.62	0.70
Hispanic	-1.41	1.01	1.09	-1.41	1.01	1.09
White	4.73	0.59	0.60	4.73	0.59	0.60
Residual	37.06	12.87	27.60	35.28	6.48	17.75
Class Proportion	31.99%			68.01%		
	N = 1000					
	Basic needs			Resilient		
	Est.	S.E.	S.D.	Est.	S.E.	S.D.
Intercept	97.37	1.43	2.79	101.35	0.67	0.99
Basic needs	4.71	1.65	2.43	0.19	0.56	0.77
Money	1.98	1.57	2.26	0.58	0.75	1.12
Time-self	-1.43	1.55	2.08	-0.05	0.68	0.86
Time-family	-2.63	1.68	2.68	-0.37	0.67	1.04
African American	-2.52	0.44	0.42	-2.52	0.44	0.42
Hispanic	-1.38	0.72	0.65	-1.38	0.72	0.65
White	4.65	0.42	0.39	4.65	0.42	0.39
Residual	51.50	13.58	25.77	36.89	5.65	12.34
Class Proportion	28.50%			71.50%		
	N = 3000					
	Basic needs			Resilient		
	Est.	S.E.	S.D.	Est.	S.E.	S.D.
Intercept	97.77	1.08	1.68	101.14	0.37	0.33
Basic needs	4.37	1.18	1.05	0.30	0.27	0.23
Money	1.82	1.14	1.22	0.76	0.38	0.33
Time-self	-1.51	1.12	0.98	-0.05	0.34	0.26
Time-family	-2.07	1.16	0.91	-0.57	0.37	0.27
African American	-2.52	0.25	0.17	-2.52	0.25	0.17
Hispanic	-1.39	0.41	0.30	-1.39	0.41	0.30
White	4.68	0.24	0.18	4.68	0.24	0.18
Residual	67.48	12.33	17.86	37.19	2.94	3.40
Class Proportion	23.42%			76.58%		
	N = 6305					
	Basic needs			Resilient		
	Est.	S.E.		Est.	S.E.	
Intercept	98.76	0.67		101.07	0.27	
Basic needs	3.93	0.71		0.20	0.17	
Money	1.31	0.86		0.83	0.31	
Time-self	-1.22	0.68		-0.07	0.24	
Time-family	-1.76	0.71		-0.56	0.27	
African American	-2.52	0.17		-2.52	0.17	
Hispanic	-1.37	0.28		-1.37	0.28	
White	4.67	0.16		4.67	0.16	
Residual	77.02	8.19		35.58	1.97	
Class Proportion	26.63%			73.37%		