Preprints,18th IFAC Symposium on System Identification
July 9-11, 2018. Stockholm, Sweden

Adaptive Neural Prescribed Performance DSC for Non-affine SISO Nonlinear Systems with External Disturbances

Pouria Tooranjipour*, Ramin Vatankhah*, Mohammad Mehdi Arefi, Allahyar Montazeri*****

**School of Mechanical Engineering, Shiraz University, Shiraz, Iran (e-mail: rvatankhah@shirazu.ac.ir).*
***Department of Power and Control Engineering, School of Electrical and Computer Engineering, Shiraz University, Shiraz, Iran (e-mail:arefi@shirazu.ac.ir).*
****Engineering Department, Lancaster University, Lancaster, United Kingdom (e-mail:a.montazeri@lancaster.ac.uk)*

Abstract: Explosion of complexity and undesirable transient response of systems, are two major problems that conventional backstepping methods suffer from it. Furthermore, lack of information about the system and undesirable external disturbances are other problems that have been addressed in this paper. Therefore, an adaptive neural controller is designed to consider the proposed problems in this paper. The presented controller is constructed for the class of single-input, single-output (SISO) non-affine strict feedback systems with unknown gain signs and a neural network is employed to approximate unknown functions. By applying dynamic surface control (DSC) and prescribed performance functions, two major problems of an explosion in terms and the transient response of the system will be solved, respectively. Nussbaum functions are also utilized to address the problem of unknown gain signs. The proposed controller guarantees that all the closed-loop signals are semi-globally, uniformly ultimately bounded (SGUUB). Finally, in order to show the feasibility of this approach, a simulation example is provided.

Keywords: neuro-adaptive control; dynamic surface control; Nussbaum-type function; prescribed performance; non-affine nonlinear systems

1. INTRODUCTION

In the past decades, adaptive control of nonlinear systems with matched and mismatched uncertainties has attracted many attentions (Marino and Tomei, 1995). The main cause of mismatched uncertainties are the unmodeled dynamics, unknown disturbances and time-varying delays (Chen, Li and Miao, 2010; Zhang, Tong and Li, 2014). One of the common and systematic approach to deal with these uncertainties is utilizing backstepping methods for strict-feedback and pure-feedback systems. In this method, unlike to the feedback linearization, the controller is designed without canceling useful terms, but the backstepping technique has a shortcoming named "explosion of complexity". This problem stems from repeating the virtual control differentiations, specifically in the large order systems. Dynamic surface control (DSC) is introduced in (Swaroop *et al.*, 2000) to solve this problem by introducing the first order filter and passing through the virtual controller.

By owing to the universal function approximators, many unknown uncertainties are approximated by these functions such as fuzzy logic and neural networks (Ramezani *et al.*, 2016). Many works have been done to utilize Nussbaum type functions for addressing the problem of unknown gain signs (Ge and Wang, 2002; Chen and Zhang, 2010). In (Wang, Ge and Hong, 2010) time-varying delays are added to these aforementioned schemes.

Recent years have witnessed many attentions into the control of non-affine systems. For example, in (Theodoridis, Boutalis and Christodoulou, 2010) an indirect adaptive control with fuzzy approximator for multi-input, multi-output (MIMO) systems is proposed. In (Ramezani *et al.*, 2016) neuro-backstepping controller is designed for SISO non-affine systems with unknown gain signs. All of the aforementioned methods which contain fuzzy or neural network approximators suffer from updating many parameters of hidden nodes in neural network or adaptive weights in fuzzy methods. In (Arefi, Ramezani and Jahed-Motlagh, 2014) observer-based adaptive robust control has been proposed by using the norm of parameters instead all of them.

It should be mentioned that the practical problems often require satisfying performance indices such as overshoot, transient response and prescribed steady-state response in a finite time. Many noticeable approaches have been done to control the behavior of the systems or satisfy the imposed constraints (Tee *et al.*, 2009). Barrier Lyapunov function (BLF) technique (Tee *et al.*, 2009; Tang, Tee and He, 2013) is one of the common approaches, but a piecewise smooth BLF is needed to establish the stability of the closed-loop system (Han and Lee, 2014). Therefore, the prescribed performance function is introduced by utilizing a transformation function (Bechlioulis *et al.*, 2008; Bechlioulis and Rovithakis, 2010).

Inspired by the preceding discussion, there are few papers that consider the DSC and prescribed performance for non-affine systems with unknown gain signs together. The main contributions of this paper can be summarized as: (I) using a prescribed performance in order to control the transient and steady-state behavior of the system, by introducing performance functions and transformation errors. As a result, all of the surface errors evolve strictly within prescribed bounds. Although the stability analysis shows the bound for the closed-loop signals, the proposed method can define these bounds, and this is a superiority of combining these approaches together; (II) by using just one parameter instead of vector of adaptive parameters can reduce computational burden in the proposed method; (III) by utilizing Nussbaum type functions, the prior knowledge of gain signs is not needed. Furthermore, unlike (Ramezani *et al.*, 2016) DSC method can avoid form “explosion in terms”; (IV) upper bound of disturbances or control direction is not required in adaptation laws or control input. However, the upper bounds of disturbances are used just in the stability analysis. Although uncertainties in these approaches lead to the SGUUB stability, prescribed performance technique can bring the satisfactory performance indices to the system such as overshoot, undershoot and a prescribed transient and steady-state performance in a finite time.

2. SYSTEM DESCRIPTIONS AND PRELIMINARIES

2.1 System descriptions and assumptions

Consider an uncertain non-affine SISO nonlinear system as follows:

$$\begin{cases} \dot{x}_1 = f_1(x_1) + g_1(x_1)x_2 + d_1(t), \\ \vdots \\ \dot{x}_i = f_i(\bar{x}_i) + g_1(\bar{x}_i)x_{i+1} + d_i(t), \\ \vdots \\ \dot{x}_n = f(\bar{x}_n, u(t)) + d_n(t), \\ y = x_1(t), \end{cases} \quad (1)$$

where $\bar{x}_i = [x_1,...,x_i]^T \in R^i$; $x = [x_1,...,x_n]^T \in R^n$, $u \in R$, $y \in R$ and $d_i(t),\ i = 1,...,n;$ are the state variables, the control input, the system output and the external disturbance, respectively. $f_i(.)$, $g_i(.),\ i = 1,...,n-1$ and $f(.)$ are unknown smooth functions. The main goal is to design a controller to ensure

1. The output of the system eventually tracks the desired trajectory y_d, while all the closed-loop signals are semi-globally, uniformly and ultimately bounded.
2. The steady and transient responses of the system are bounded and evolved by the performance functions.

Assumption 1. Functions $g_i(.),\ i = 1,...,n-1$ are non-zero functions, and their signs are unknown, and constants $\underline{g}_i$ and $\bar{g}_i$ exist, which satisfiy $0 < \underline{g}_i \le g_i(.) \le \bar{g}_i$.

Assumption 2. Assume $d_i(t),\ i = 1,\dots,n$ are bounded as $d_i(t) \le \bar{d}_i$, where $\bar{d}_i,\ i = 1,...,n$ are positive unknown constants.

Definition 1. A function $N(\varsigma)$ is called Nussbaum function, if it satisfies the following properties (Nussbaum, 1983)

$$\lim_{z\to\infty} \sup \frac{1}{z}\int_0^z N(\zeta)d\zeta = +\infty, \quad (2)$$

$$\lim_{z\to\infty} \inf \frac{1}{z}\int_0^z N(\zeta)d\zeta = -\infty. \quad (3)$$

In this paper, Nussbaum functions $N_i(\varsigma_i),\ i = 1,...,n$ are implemented, in order to address the problem of unknown signs $g_i(.),\ i = 1,...,n$. Some common Nussbaum functions are $\varsigma^2\cos\varsigma$, $\varsigma^2\sin\varsigma$ and $e^{\varsigma^2}\cos(\frac{\pi}{2}\varsigma)$. In this paper, $\varsigma^2\cos\varsigma$ is employed as an even Nussbaum function.

Lemma 1. (Xudong and Jingping, 1998) Let $V(t) \ge 0$ and $\varsigma(t)$ are smooth functions on the interval $[0,t_f)$, and $N(\varsigma)$ is an even Nussbaum type function. If the following inequality holds

$$V(t) \le c_0 + e^{-c_1 t}\int_0^t [g_n(x(\tau))N(\varsigma) + 1]\dot{\varsigma}e^{c_1\tau}d\tau, \quad (4)$$

where $0 \le t < t_f$, c_1 is a positive constant, and c_0 represents some suitable constants, and $g_n(.)$ is a positive non-zero time-varying parameters, then $V(t)$, $\varsigma(t)$ and $\int_0^t [g_n(x(\tau))N(\varsigma)]\dot{\varsigma}d\tau$ must be bounded on $[0,t_f)$.

By applying mean value theorem (Apostol, 1974) into $f(\bar{x}_n(t), u(t))$, it can be rewritten as

$$f(x,u) = f(x,0) + \left.\frac{\partial f}{\partial u}\right|_{u=\beta} u = f_n(x) + g_n(x)u, \quad (5)$$

where $f_n(x) = f(x,0)$, $g_n(x) = \left.\frac{\partial f}{\partial u}\right|_{u=\beta} > 0$ and $\beta \in (0,u)$. Therefore, the above equation is utilized to transform non-affine system (1) into the affine one.

2.2 Prescribed performance

According to (Bechlioulis and Rovithakis, 2010), the prescribed performance is achieved by bounding the response of the system arbitrary and ensure that each error $s_i(t),\ i = 1,...,n$ evolves within predefined bounds, which are applied by performance functions $\bar{h}_i(t),\ i = 1,...,n$ as

$$\begin{cases} -\delta_i\bar{h}_i(t) < s_i(t) < \bar{h}_i(t), & if\ \ s_i(0) > 0, \\ -\bar{h}_i(t) < s_i(t) < \delta_i\bar{h}_i(t), & if\ \ s_i(0) < 0, \end{cases} \quad (6)$$

where $0 \le \delta_i \le 1$, and $\bar{h}_i(t)$ satisfies following properties,

1) $\bar{h}_i(t)$ is a smooth, positive and decreasing function,

2) $\lim_{t\to\infty}\bar{h}_i(t) = \bar{h}_{i,\infty}(t) > 0$.

For example, $\bar{h}_i(t) = (\bar{h}_{i,0} - \bar{h}_{i,\infty})e^{-n_i t} + \bar{h}_{i,\infty},\ i = 1,...,n$ possess all aforementioned property and can be used as a performance function, where $\bar{h}_{i,0}$, $\bar{h}_{i,\infty}$ and n_i are positive constants.

In order to transform nonlinear system (1) with the constrained in the sense of (6) error behavior into an unconstraint form, the transformed error is introduced as

$$\varepsilon_i = \phi_i(\frac{s_i}{\bar{h}_i}),\ i = 1,...,n, \quad (7)$$

where ε_i, s_i and $\bar{h}_i$ are transformed errors, errors and performance functions, respectively. Furthermore, ϕ_i are

strictly positive, smooth functions which have some properties detailed and defined completely in (Bechlioulis and Rovithakis, 2010).
As (Zhang, Tong and Li, 2014), dynamic errors are obtained as

$$\dot{s}_i = \nu_i \left(\dot{s}_i - \frac{\dot{\bar{h}}_i}{\bar{h}_i} s_i \right) \tag{8}$$

where $\nu_i = \frac{1}{\bar{h}_i} \frac{\partial \phi_i}{\partial (\frac{s_i}{\bar{h}_i})} \geq 0$ calculated as

$$\nu_i = \begin{cases} \frac{1}{\bar{h}_i} \frac{1+\delta_i}{(1-\frac{s_i}{\bar{h}_i})(\delta_i + \frac{s_i}{\bar{h}_i})} & s_i(0) \geq 0 \\ \frac{1}{\bar{h}_i} \frac{1+\delta_i}{(1+\frac{s_i}{\bar{h}_i})(\delta_i - \frac{s_i}{\bar{h}_i})} & s_i(0) \leq 0 \end{cases} \tag{9}$$

2.3 RBF neural approximator for unknown functions

Neural networks as nonlinear approximators are widely used where unknown functions exist, and Gaussian RBF neural networks are employed (Gupta *et al.*, 1994) in this article to approximate continuous function $\bar{f}(Z): R^m \rightarrow R$ as follows:

$$\hat{\bar{f}}(Z) = \theta \xi(Z), \tag{10}$$

where $Z \in R^m$ is the input vector, $\theta = [\theta_1, ..., \theta_k] \in R^k$ is the vector of adjustable parameters where k is the number of nodes, and $\xi(Z) = [\xi_1(Z), ..., \xi_k(Z)]^T \in R^{1\times K}$ is a Gaussian basis function vector, which is defined as

$$\xi_i(Z) = \exp(-\frac{\|Z - \mu_i\|}{\eta_i^2}), \; i = 1, ..., k \tag{11}$$

where $\mu_i = [\mu_{i1}, ..., \mu_{im}] \in R^m$ and η_i ,$i = 1, ..., k$ are centers and width of Gaussian function. By choosing the sufficiently large number of nodes, the neural network can approximate $\bar{f}(Z)$ with desired precision as

$$\bar{f}(Z) = \theta^* \xi(Z) + \delta \qquad \delta \leq \varepsilon \tag{12}$$

where δ is the approximation error, ε is an upper bound of error and the optimal parameter θ^* is defined as

$$\theta^* = \arg\min \left\{ \sup \left| \bar{f}(Z) - \hat{\bar{f}}(Z) \right| \right\}. \tag{13}$$

3. ADAPTIVE NEURAL BACKSTEPPING CONTROL DESIGN AND STABILITY ANALYSIS

The following transformations are defined as

$$s_1 = y - y_d, \quad s_i = x_i - q_i, \; i = 2, ..., n, \tag{14}$$

where s_i is an error surface, q_i is a state variable, which is defined as

$$\zeta_{i+1} \dot{q}_{i+1} + q_{i+1} = \alpha_i, \; i = 1, ..., n-1 \tag{15}$$

where ζ_i is a positive design constant. The Above equations show that q_i is obtained through the first order filter on the virtual control α_i , and the output error of the first order filter X_{i+1} is also defined as

$$X_{i+1} = q_{i+1} - \alpha_i, \; i = 1, ..., n-1. \tag{16}$$

Design the virtual controls and adjustable parameters as follows:

$$\alpha_i = \frac{s_i \nu_i}{2a_i^2} N(\varsigma_i) \hat{\psi}_i, \; i = 1, ..., n-1 \tag{17}$$

$$\dot{\varsigma}_i = \frac{s_i^2 \nu_i^2}{2a_i^2} \hat{\psi}_i, \; i = 1, ..., n \tag{18}$$

$$\dot{\hat{\psi}}_i = \frac{\gamma_i}{2a_i^2} s_i^2 \nu_i^2 - \eta_i \hat{\psi}_i, \; i = 1, ..., n \tag{19}$$

$$u = \frac{s_n \nu_n}{2a_n^2} N(\varsigma_n) \hat{\psi}_n, \tag{20}$$

where a_i, γ_i and η_i, $i = 1, ..., n$ are positive design constants, and $N(\varsigma_i)$ represents Nussbaum type functions, and $\hat{\psi}_i$ is an adjustable parameter which is defined later.

Theorem 1. Under **Assumptions (1-2)**, consider the closed-loop system consisting of the nonlinear system (1). If the control law (20), virtual controls (17) with the adaptive laws (18-20) are applied, then all the closed-loop signals are SGUUB. Furthermore, the transient performance of the system is under control of the prescribed functions defined as (6) at all times.

Step 1: The time derivative of s_1 along with (14) and using $x_2 = s_2 + (X_2 + \alpha_1)$ is

$$\dot{s}_1 = \nu_1 (f_1 + g_1 (s_2 + (X_2 + \alpha_1)) + d_1 - \dot{y}_d - \frac{\dot{\bar{h}}_1}{\bar{h}_1} s_1). \tag{21}$$

Choose the Lyapunov candidate of the first step as

$$V_1 = \frac{1}{2} s_1^2 + \frac{1}{2\gamma_1} \tilde{\psi}_1 \tilde{\psi}_1, \tag{22}$$

where γ_1 is a positive design constant and $\tilde{\psi}_1 = \psi_1^* - \hat{\psi}_1$ is adjustable parameter. The time derivative of V_1 along with (21) is

$$\dot{V}_1 = s_1 \nu_1 \left[f_1(x_1) + g_1(x_1) x_2 + d_1 - \dot{y}_d - \frac{\dot{\bar{h}}_1}{\bar{h}_1} s_1 \right] + \frac{1}{\gamma_1} \tilde{\psi}_1 \dot{\tilde{\psi}}_1. \tag{23}$$

By using Assumption 2, we have

$$\dot{V}_1 \leq s_1 \nu_1 \left[f_1(x_1) - \dot{y}_d - \frac{\dot{\bar{h}}_1}{\bar{h}_1} s_1 \right] + s_1 \nu_1 g_1 (s_2 + X_2 + \alpha_1) + \frac{1}{\gamma_1} \tilde{\psi}_1 \dot{\tilde{\psi}}_1 + \frac{s_1^2 \nu_1^2}{2\rho_1^2} + \frac{1}{2} \rho_1^2 \bar{d}_1^2, \tag{24}$$

where ρ_1 is a positive constant. By applying Young's inequality, we can get

$$\dot{V}_1 \le -s_1^2 v_1^2 \left(c_1 - \frac{1}{2\rho_1^2} \right) + s_1 v_1 (f_1(x_1) - \dot{y}_d - \frac{\dot{\bar{h}}_1}{\bar{h}_1} s_1 + X_2^2 \quad (25)$$
$$\frac{1}{2} s_1 v_1 g_1^2 + c_1 s_1 v_1) + s_1 v_1 g_1 \alpha_1 + \frac{1}{\gamma_1} \tilde{\psi}_1 \dot{\hat{\psi}}_1 + \frac{1}{2} \rho_1^2 \bar{d}_1^2 + s_2^2$$

where c_1 is a positive constant. Let $\bar{f}_1 = f_1(x_1) - \dot{y}_d - \frac{\dot{\bar{h}}_1}{\bar{h}_1} s_1 + \frac{1}{2} s_1 v_1 g_1^2 + c_1 s_1 v_1$, and by utilizing RBF neural network (12), unknown function $\bar{f}_1(Z)$ is approximated as

$$\bar{f}_1(Z_1) = \theta_1^* \xi_1(Z_1) + \delta_1, \qquad \delta_1 \le \varepsilon_1, \quad (26)$$

where $Z_1 = [s_1, \dot{y}_d, \bar{h}_1, \dot{\bar{h}}_1]^T$ and $\varepsilon_1 > 0$ is an upper bound of network error. By using the fact that $\left| \xi_1(Z_1) \xi_1(Z_1)^T \right| \le 1$, substituting (26) into (25), using Young's inequality and considering $\psi_1{}^* = \left\| \theta_1^* \right\|^2$ for convenience and simplicity, we have

$$\dot{V}_1 \le -s_1^2 v_1^2 \left(c_1 - 1 - \frac{1}{2\rho_1^2} \right) + \frac{s_1^2 v_1^2}{2a_1^2} \psi_1^* + \frac{1}{2} a_1^2 \quad (27)$$
$$+ \frac{1}{2} \varepsilon_1^2 + s_1 v_1 g_1 \alpha_1 - \frac{1}{\gamma_1} \tilde{\psi}_1 \dot{\hat{\psi}}_1 + s_2^2 + X_2^2 + \frac{1}{2} \rho_1^2 \bar{d}_1^2$$

let $c_1 \ge 1 + \frac{1}{2\rho_1^2}$. Now, by substituting (17-19), we can get

$$\dot{V}_1 \le -s_1^2 v_1^2 \left(c_1 - 1 - \frac{1}{2\rho_1^2} \right) - \frac{\eta_1}{2\gamma_1} \tilde{\psi}_1 \tilde{\psi}_1 + \quad (28)$$
$$(g_1 N_1(\varsigma_1) + 1) \dot{\varsigma}_1 + s_2^2 + X_2^2 + \Delta_1,$$

where $\Delta_1 = \frac{1}{2} a_1^2 + \frac{1}{2} \varepsilon_1^2 + \frac{\eta_1}{2\gamma_1} \psi_1^{*2} + \frac{1}{2} \rho_1^2 \bar{d}_1^2 > 0$.

Step k ($2 \le k \le n-1$): Similar to the step 1, the time derivative of s_k along with (14) is

$$\dot{s}_k = v_k (f_k + g_k x_{k+1} + d_k - \frac{\dot{\bar{h}}_k}{\bar{h}_k} s_k - \dot{q}_k), \quad (29)$$
$$= v_k (f_k + g_k x_{k+1} + d_k - \frac{\dot{\bar{h}}_k}{\bar{h}_k} s_k + \frac{X_k}{\zeta_k}).$$

Consider the following Lyapunov candidate as

$$V_k = V_{k-1} + \frac{1}{2} s_k^2 + \frac{1}{2\gamma_k} \tilde{\psi}_k \tilde{\psi}_k + \frac{1}{2} X_k^2. \quad (30)$$

where γ_k is a positive design constant and $\tilde{\psi}_k = \psi_k^* - \hat{\psi}_k$ is an adjustable parameter. Similar to the step 1, by getting time derivative from (30), we have

$$\dot{V}_k = \dot{V}_{k-1} + s_k v_k (f_k(\bar{x}_k) + g_k(\bar{x}_k) x_{k+1} + d_k + \quad (31)$$
$$\frac{X_k}{\zeta_k} - \frac{\dot{\bar{h}}_1}{\bar{h}_1} s_1) + \frac{1}{\gamma_1} \tilde{\psi}_1 \dot{\hat{\psi}}_1 + X_k (-\frac{X_k}{\zeta_k} - \dot{\alpha}_{k-1}).$$

Let $\dot{\alpha}_{k-1} = -H_k(s_{k-1}, v_{k-1}, \varsigma_{k-1}, \psi_{k-1})$, where $H_k(.)$ is a continuous function. By using Assumption 2 and Young's inequality, we have

$$\dot{V}_k \le \dot{V}_{k-1} + s_k^2 v_k^2 \left(c_k - \frac{1}{2\rho_k^2} \right) + s_k v_k \bar{f}_k +$$
$$s_k v_k g_k \alpha_k + \frac{1}{\gamma_k} \tilde{\psi}_k \dot{\hat{\psi}}_k + \frac{1}{2} \rho_k^2 \bar{d}_k^2 + s_{k+1}^2 + X_{K+1}^2 \quad (32)$$
$$\frac{-X_k^2}{\zeta_k} + X_k H_k + \frac{s_k v_k X_k}{\zeta_k},$$

where ρ_k and c_k are positive constants, approximate $\bar{f}_k = f_k - \frac{\dot{\bar{h}}_k}{\bar{h}_k} s_k + \frac{1}{2} s_k v_k g_k^2 + c_k s_k v_k$, and let it can be approximated as $\bar{f}_k = \theta_k^* \xi_k(Z_k) + \delta_k$, $\delta_k \le \varepsilon_k$, where ε_k is a positive constant. By utilizing RBF neural network to approximate above unknown function and using the fact that $\psi_k{}^* = \left\| \theta_k^* \right\|^2$ and $\left| \xi_k \xi_k^T \right| \le 1$, it yields to

$$\dot{V}_k \le \dot{V}_{k-1} - s_k^2 v_k^2 \left(c_k - 1 - \frac{1}{2\rho_k^2} \right) + s_k v_k g_k \alpha_k$$
$$+ \frac{1}{\gamma_k} \tilde{\psi}_k \dot{\hat{\psi}}_k + \frac{1}{2} \rho_k^2 \bar{d}_k^2 + s_{k+1}^2 + X_{K+1}^2 + \frac{s_k v_k X_k}{\zeta_k} \quad (33)$$
$$+ \frac{s_k^2 v_k^2}{2a_k^2} \psi_k^* + \frac{1}{2} a_k^2 + \frac{1}{2} \varepsilon_k^2 - \frac{X_k^2}{\zeta_k} + X_k H_k,$$

By substituting (17-19) into (33), we have

$$\dot{V}_k \le -\sum_{i=1}^{k} s_i^2 v_i^2 \left(c_i - 1 - \frac{1}{2\rho_i^2} \right)$$
$$- \sum_{i=1}^{k} \frac{\eta_i}{2\gamma_i} \tilde{\psi}_i \tilde{\psi}_i + \sum_{i=1}^{k} (g_i N(\varsigma_i) + 1) \dot{\varsigma}_i + \sum_{i=1}^{k} \Delta_i \quad (34)$$
$$+ \sum_{i=2}^{k} X_i H_i + \sum_{i=2}^{k+1} (s_i^2 + X_i^2) - \sum_{i=2}^{k} \frac{X_i^2}{\zeta_i},$$

where $\Delta_k = \frac{1}{2} a_k^2 + \frac{1}{2} \varepsilon_k^2 + \frac{\eta_k}{2\gamma_k} \psi_k^{*2} + \frac{1}{2} \rho_k^2 \bar{d}_k^2 > 0.$

Step n : In this final step, control input will be obtained. Similar to the previous steps, the time derivative of s_n according to (14-16) is

$$\dot{s}_n = v_n (f_n + g_n u + d_n - \frac{\dot{\bar{h}}_n}{\bar{h}_n} s_n + \frac{X_k}{\zeta_k}). \quad (35)$$

Consider the final Lyapunov candidate as

$$V_n = V_{n-1} + \frac{1}{2} s_n^2 + \frac{1}{2\gamma_n} \tilde{\psi}_n \tilde{\psi}_n + \frac{1}{2} X_n^2, \quad (36)$$

where γ_n is a positive constant. By getting time derivative of V_n , one can write

$$\dot{V}_n = \dot{V}_{n-1} + s_n v_n [f_n + g_n u + d_n +$$
$$\frac{X_n}{\zeta_n} - \frac{\dot{\bar{h}}_n}{\bar{h}_n} s_n] + \frac{1}{\gamma_n} \tilde{\psi}_n \dot{\hat{\psi}}_n + X_n (-\frac{X_n}{\zeta_n} - \dot{\alpha}_{n-1}) \quad (37)$$
$$\le \dot{V}_{n-1} + s_n^2 v_n^2 \left(c_n - \frac{1}{2\rho_n^2} \right) + s_n v_n \bar{f}_n + s_n v_n g_n u +$$
$$\frac{1}{\gamma_n} \tilde{\psi}_n \dot{\hat{\psi}}_n + \frac{1}{2} \rho_n^2 \bar{d}_n^2 - \frac{X_n^2}{\zeta_n} + X_n H_n + \frac{s_n v_n X_n}{\zeta_n},$$

where $H_n(s_{n-1}, v_{n-1}, \varsigma_{n-1}, \psi_{n-1}) = -\dot{\alpha}_{n-1}$ and ρ_n is a positive constant. RBF neural network is also used in this final stage to approximate $\bar{f}_n$ as $\bar{f}_n = \theta_n^{*}\xi_n(Z_n) + \delta_n$, $\delta_n \le \varepsilon_n$, where $\bar{f}_n = \left[f_n - \frac{\dot{\bar{h}}_n}{\bar{h}_n} s_n + c_n s_n v_n \right]$. By using the fact that $\psi_n^{*} = \left\| \theta_n^{*} \right\|^2$ and substituting (18-20) into (37), and assuming $|H_i| \le M_i$, $i = 1,\dots,n$, where M_i is a positive constant, we have

$$\dot{V}_n \le -\sum_{i=2}^{n} s_i^2 v_i^2 \left(c_i - 1 - \frac{1}{2\rho_i^2} - \frac{1}{v_i^2} \right) - \tag{38}$$

$$s_1^2 v_1^2 \left(c_1 - 1 - \frac{1}{2\rho_1^2} \right) - \sum_{i=1}^{n} \frac{\eta_i}{2\gamma_i} \tilde{\psi}_i \tilde{\psi}_i + \sum_{i=1}^{n} (g_i N(\varsigma_i) + 1)\dot{\varsigma}_i$$

$$+ \sum_{i=1}^{n} \Delta_i - \sum_{i=2}^{n} X_i^2 \left(\frac{1}{\varsigma_i} - 1 - \frac{M_i^2}{2} \right),$$

let $c_i > 1 + \frac{1}{2\rho_i^2} + \frac{1}{v_i^2}$, $\frac{1}{\varsigma_i} > 1 + \frac{M_i^2}{2}$ $i = 2,\dots,n$, and $c_1 > 1 + \frac{1}{2\rho_1^2}$, Rewrite (38) as

$$\dot{V}_n \le -C V_n + \sum_{i=1}^{n} (g_i N(\varsigma_i) + 1)\dot{\varsigma}_i + \Lambda, \tag{39}$$

where $\Lambda = \sum_{i=1}^{n} \Delta_i \ge 0$, and $C = \min(\eta_i, 2(c_i - 1 - \frac{1}{2\rho_i^2} - \frac{1}{v_i^2}),$ $2(\frac{1}{\varsigma_i} - 1 - M_i^2))$, $i = 2,\dots,n$. Multiplying both sides of (39) by e^{-Ct}, then by integrating (39) over $[0,t]$, we have

$$V_n(t) \le c_0 + e^{-Ct} \int_0^t \sum_{i=1}^{n} [g_i(\bar{x}_i) N(\varsigma_i) + 1]\dot{\varsigma}_i e^{C\tau} d\tau, \tag{40}$$

where $c_0 = \frac{\Lambda}{C} + (V_n(0) - \frac{\Lambda}{C})e^{-Ct}$. By applying **Lemma 1**, it can be concluded that $V_n(t)$, $\varsigma(t)$ and $\int_0^t \sum_{i=1}^{n} [g_i(\bar{x}_i) N(\varsigma_i) + 1]\dot{\varsigma}_i d\tau$ must be bounded and SGUUB on $[0,t]$, so it can be shown that all of the closed-loop signals are bounded on $[0,t]$.

4. SIMULATION STUDY

In this section, the following example is given to show the feasibility of this proposed approach. Consider the following second-order non-affine SISO nonlinear system, with unknown external disturbances as

$$\begin{cases} \dot{x}_1 = \frac{1 - e^{-x_1}}{1 + e^{-x_1}} + (2 + \sin x_1) x_2 + d_1(t), \\ \dot{x}_2 = x_1 x_2 + (2 + \sin(x_1 x_2)) \frac{u}{\sqrt{|u| + 0.1}} + d_2(t), \\ y = x_1, \end{cases} \tag{41}$$

Where $d_1(t) = 0.5\cos t$, $d_2(t) = 0.2\sin t$ and the desired trajectory is defined as $y_d = \sin t + \cos(0.5t)$. The Performance functions $\bar{h}_1(t)$ and $\bar{h}_2(t)$ are chosen as

$$\bar{h}_i(t) = (\bar{h}_{i,0} - \bar{h}_{i,\infty}) e^{-n_i t} + \bar{h}_{i,\infty},\ i = 1,2, \tag{42}$$

with the parameters given in Table.1. The design parameters are selected as $a_1 = 0.5$, $a_2 = 0.5$, $\gamma_1 = 20$, $\gamma_2 = 40$, $\eta_1 = 0.1$ and $\eta_2 = 0.1$. The initial values are chosen as $x_1(0) = 0.5$, $x_2(0) = -0.2$, $\psi_1(0) = 0.01$, $\psi_2(0) = 0.01$, $\varsigma_1(0) = 1.9$ and $\varsigma_2(0) = 10$. The tracking performance of the proposed controller is depicted in Fig. 1.

Table. 1 Performance function parameters values

$\bar{h}_{1,0}$	$\bar{h}_{2,\infty}$	$\bar{h}_{1,0}$	$\bar{h}_{2,\infty}$	n_1	n_2	δ_1	δ_2
2	0.2	3	0.2	1	0.5	0.9	0.9

As can be seen in Fig.1, it can be concluded that the system tracks reference signal y_d properly.

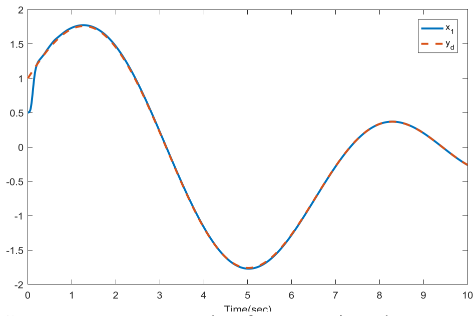

Fig. 1. System output y and reference signal y_d versus time

Figs. 2-3 depict surface errors and prescribed bounds, which show the effectiveness of the proposed method.

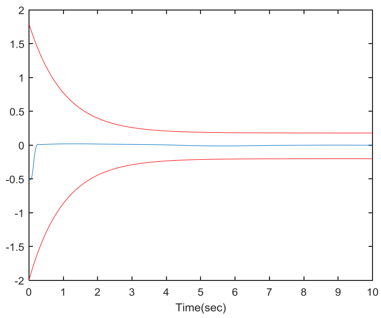

Fig .2. Variation of s_1 and performance bounds versus time.

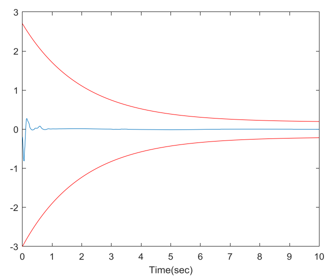

Fig. 3. Variation of s_2 and performance bounds versus time.

As can be seen in Figs. 2-3, the error surfaces s_1 and s_2 evolve strictly within the prescribed performance bounds. Finally, the control input u is illustrated in Fig.4. From this figure, the control signal is bounded and feasible for implementation.

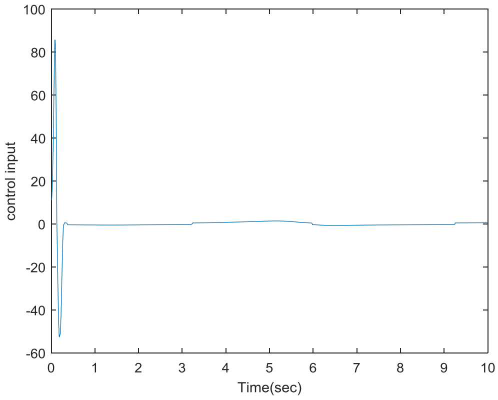

Fig.4 Control input versus time

5. CONCLUSIONS

In this paper, an adaptive neuro-backstepping controller with prescribed performance was designed for a class of SISO non-affine systems with unknown disturbances. In order to avoid complexity in terms caused by derivatives of the virtual controller in each step, DSC method was utilized by using the first order filter. Unknown terms of the system were approximated by RBF neural network, and the prescribed performance was achieved by using the proper performance functions. It was shown that all the closed-loop signals are bounded and the dynamic surface errors converge to the neighborhood of the origin with the prescribed decaying bounds. Finally, the simulation results demonstrated the effectiveness of the proposed method to tackle with unknown disturbances and uncertainties in the non-affine systems. Future research will extend the results of this control approach for a system with an unknown dead-zone nonlinearity and time-varying delays.

REFERENCES

Apostol, T. (1974) '*Mathematical analysis*'. Addison-Wesley, Reading, Mass, USA.

Arefi, M. M., Ramezani, Z. and Jahed-Motlagh, M. R. (2014) 'Observer-based adaptive robust control of nonlinear nonaffine systems with unknown gain sign', *Nonlinear Dynamics*, 78(3), pp. 2185–2194.

Bechlioulis, C. P. *et al.* (2008) 'Robust Adaptive Control of Feedback Linearizable MIMO Nonlinear Systems With Prescribed Performance', *IEEE Transactions on Automatic Control,* 53(9), pp. 2090-2099.

Bechlioulis, C. P. and Rovithakis, G. A. (2010) 'Prescribed Performance Adaptive Control for Multi-Input Multi-Output Affine in the Control Nonlinear Systems', *IEEE Transactions on Automatic Control*, *55*(5), pp.1220-1226

Chen, W., Li, W. and Miao, Q. (2010) 'Backstepping control for periodically time-varying systems using high-order neural network and Fourier series expansion', *ISA Transactions*, 49(3), pp. 283–292.

Chen, W. and Zhang, Z. (2010) 'Globally stable adaptive backstepping fuzzy control for output-feedback systems with unknown high-frequency gain sign', *Fuzzy Sets and Systems*, 161(6), pp. 821–836.

Ge, S. S. and Wang, J. (2002) 'Robust adaptive neural control for a class of perturbed strict feedback nonlinear systems', *IEEE transactions on neural networks / a publication of the IEEE Neural Networks Council*, 13(6), pp. 1409–19.

Gupta, M.M. and Rao, D.H., (1994). 'Neuro-control systems' *Um volume selecionado reeditado. IEEE Neural Networks Council, Sponsor.*

Han, S.-I. and Lee, J.-M. (2014) 'Recurrent fuzzy neural network backstepping control for the prescribed output tracking performance of nonlinear dynamic systems', *ISA Transactions*, 53(1), pp. 33–43.

Nussbaum, RD. (1983) 'Some remarks on a conjecture in parameter adaptive control. *Syst Contr Lett* (3), pp. 243–6.

Marino, R. and Tomei, P., (1996) '*Nonlinear control design: geometric, adaptive and robust*'. Prentice Hall International (UK) Ltd.

Ramezani, Z. *et al.* (2016) 'Neuro-adaptive backstepping control of SISO non-affine systems with unknown gain sign', *ISA Transactions*, 65, pp. 199–209.

Swaroop, D. *et al.* (2000) 'Dynamic Surface Control for a Class of Nonlinear Systems', *IEEE transactions on automatic control*, 45(10), pp. 1893–1899.

Tang, Z. L., Tee, K. P. and He, W. (2013) 'Adaptive Neural Control for Output Feedback Nonlinear Systems Using a Barrier Lyapunov Function', *IFAC Proceedings Volumes (IFAC-PapersOnline)*, 3(PART 1), pp. 449–455.

Tee, K.P., Ge, S.S. and Tay, E.H. (2009) 'Barrier Lyapunov functions for the control of output-constrained nonlinear systems', *Automatica*, *45*(4), pp.918-927.

Tee, K. P. *et al.* (2009) 'Control of Nonlinear Systems with Time-Varying Output Constraints', *2009 IEEE International Conference on Control and Automation, Vols 1-3*, pp. 524–529.

Theodoridis, D., Boutalis, Y. & Christodoulou, M. (2010) 'Indirect adaptive control of unknown multi variable nonlinear systems with parametric and dynamic uncertainties using a new neuro-fuzzy system description', *International Journal of Neural Systems*, *20*(02), 129-148.

Wang, M., Ge, S. S. and Hong, K. S. (2010) 'Approximation-based adaptive tracking control of pure-feedback nonlinear systems with multiple unknown time-varying delays', *IEEE Transactions on Neural Networks*, 21(11), pp. 1804–1816.

Xudong, Y. and Jingping, J. (1998) 'Adaptive Nonlinear Design Without Priori Knowledge Of Control Directions', *IEEE Transactions on Automatic Control*, 43(11), pp. 1617–1621.

Zhang, L., Tong, S. and Li, Y. (2014) 'Dynamic surface error constrained adaptive fuzzy output-feedback control of uncertain nonlinear systems with unmodeled dynamics', *Neurocomputing*, 143, pp. 123–133.