Lagrangian Large Signal Model for Double Corrugated Waveguide

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Abstract: *A novel code based on non-linear Lagrangian formulation of beam wave interaction in traveling wave tubes is presented, modified for non-rotationally symmetric slow wave structures, such as the double corrugated waveguide. By including axial variations in the electric field along the period of the structure, as well as radial and angular variations and a model for the space charge forces in the double corrugated waveguide, a code capable of describing the behavior of the double corrugated waveguide is written.*

Keywords: Lagrangian; Simulation; millimeter wave; THz; traveling wave tube

Introduction

In the sub-terahertz frequency regime, applications such as high data rate communications and radar, and imaging techniques, have been demonstrated [1]. These emerging technologies often require high power amplifiers for full exploitation. Traveling wave tubes (TWTs) are presently the only devices able to produce high output power over a wide frequency band. However, the traditional helically based slow wave structure is not feasible from a manufacturing point of view when the wavelengths begin to approach the millimeter wave range. Novel structures have been proposed for the purpose, but most of them, such as the double corrugated waveguide, do not have the cylindrical symmetry as the helix. In case of helix TWT, fast codes based on the Lagrangian method are available to avoid the long computational time of commercial 3D electromagnetic simulation codes. However, the Lagrangian method is based on a cylindrical symmetry of the structure. The purpose of the paper is to present a code based upon the Lagrangian formulation [2, 3] modified to model not-cylindrical symmetry. In the following, the approach to model the differing effects of space charge and field distribution in the double corrugated waveguide (DCW)-TWT are described. A comparison of the result with Ka-Band DCW TWT simulated by 3D Particle in Cell code will be shown [4].

Double Corrugated Waveguide model

The modifications to the Lagrangian model, aim to provide more accuracy by the calculation of the field values representative of the axial component of the electric field distribution in the structure. By this method, space charge weighting functions and interaction impedance weighting functions are calculated based upon position in the periodic structure.

Space Charge: The space charge fields supported by the double corrugated waveguide differ from those of the helix. Whilst the helix has invariant space charge forces with respect to angular position, this is not the case for the DCW, where the distance from the electron beam to the wall of the waveguide varies significantly. This determines a not symmetric distribution of the space charge forces. A method was derived for the derivation of the space charge forces for different parts of the electron beam, dependent on its angular position in the waveguide.

The model of the DCW to compute the space charge forces in shown in Fig.1. Using the transcendental equations derived in [5] and the method for deriving the distance of

$$r'(\theta) = \begin{cases} (b-(h-\delta))(1+tan(\theta))^{1/2} \; for\ \theta < \alpha \\ \left(\frac{a^2}{4} + (b-(h-\delta))^2\right)^{\frac{1}{2}} for\ \theta = \alpha \\ \frac{a}{2}(1+cot(\theta))^{1/2} \; for\ \theta > \alpha \end{cases} \quad (1)$$

$$r'(\theta) = \begin{cases} \frac{t/2}{\cos(\theta - \pi/2)} \quad for\ \theta < \epsilon \\ \frac{h-\delta}{\sin(\theta - \frac{\pi}{2})} \quad for\ \theta > \epsilon \end{cases} \quad (2)$$

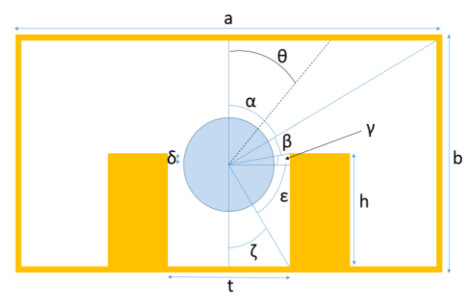

Figure 1. Diagram depicting the angles used in the construction of the *r'*, the effective radius of the DCW with respect to the beam center.

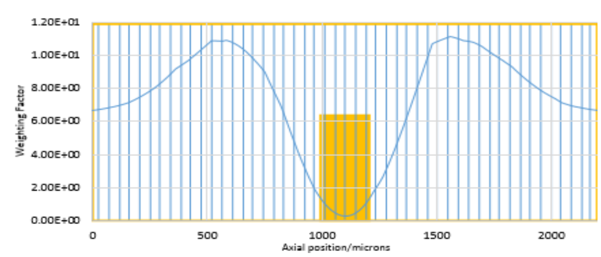

Figure 2. Weighting factor superimposed on a drawing of the DCW for 34 GHz DCW TWT

the electron beam to the nearest metal surface, with the new radii of the DCW, is given by equations (1) and (2). *Electric Field Variance:* While the helix waveguide is axially invariant with respect to the field along the structure, the electric field distribution of the DCW varies axially due to the shape of the structure being axially varying. As the interaction impedance is dependent on the magnitude of the axial component of the electric field, this value, too, varies along the structure. As such, it is useful to have a dynamic weighting function to account for this varying strength of interaction. An equation was derived to accurately locate each particle's position in the field axially, such that the appropriate weighting function can be assigned. This is done for each particle on each iteration. This is an important step, as previously, the weighting of the field along the axis was averaged, at the cost of accuracy as the nonlinearity of the interaction was lost. The particle position is given as

$$P_{e_n}(\phi_{0,n}, r_{0,n}, \theta_{0,n}) = \frac{\phi(\phi_{0,n}, r_{0,n}, \theta_{0,n})}{\beta} + \frac{(N-1)dz}{4} \quad (3)$$

where $\phi_{0,n}$ is the initial phase of a charge group with respect to the rf wave, and $r_{0,n}$ and $\theta_{0,n}$ are the initial radial and angular position of the charge groups. β is the propagation constant of the rf wave, N is the iteration number and dz is the step length of each iteration. This equation permits to take into account the dynamic beam-wave coupling along the structure.

Fig. 2 shows how the weighting function changes over one period of the DCW. Although the average interaction impedance across the entire period is ~1 Ohm, the interaction impedance differs by an order of magnitude along the structure, which drastically affects the simulated interaction.

The weighting functions are extracted from simulations by CST eigenmode solver [6]. The weighting functions are normalized with respect to the average interaction impedance, which is the interaction impedance used as input for the code.

When allocating the phase velocity values, great care must be taken in the calculation as a small change in phase velocity for a given frequency can drastically alter the calculated output, especially at the fringe of the band.

Results

The proposed model has been compared with CST-PS [6]. The results for a Ka-Band DCW-TWT [4] are considered, where the model describes well the beam wave interaction for most of the frequency band. The comparison of the output power (Fig. 3a) and gain (Fig. 3b) between the Lagrangian model and the CST-PS simulations shows a good agreement. The lower side of the band is not yet well modelled. It could be due to the edge of the synchronous region where the difference in beam velocity and wave phase velocity become too different. A further investigation is in progress.

Conclusions

A method has been described for the conversion of a code for the beam wave interaction in a helical TWT to one suitable for application in non-rotationally symmetric structures, and applied to a DCW TWT. Good accuracy is found most of the frequency band, with the model breaking down near the lower edge of the band.

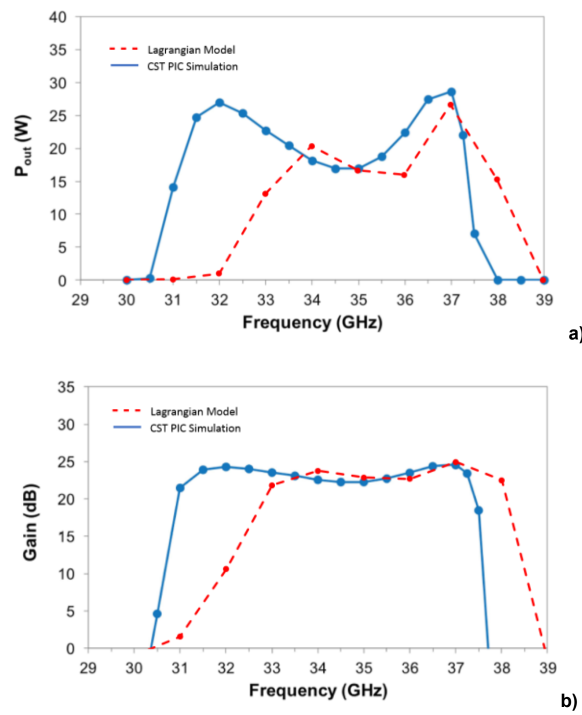

Figure 3. Comparison Lagrangian model with PIC simulations for the Ka-Band DCW-TWT [5]

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