Sequential Choice of Sharing Rules in Collective Contests*

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Abstract

Groups competing for a prize need to determine how to distribute it among their members in case of victory. Considering competition between two groups of different size, we show that the small group's sharing rule is a strategic complement to the large group's sharing rule in the sense that if the small group chooses a more meritocratic sharing rule, the large group wishes to choose a more meritocratic rule as well. On the contrary, the large group's sharing rule is a strategic substitute to the small group's sharing rule, hence the timing of choice is crucial. For sufficiently private prizes, a switch from a simultaneous choice to the small group being the leader consists in a Pareto improvement and reduces aggregate effort. On the contrary, when the large group is the leader aggregate effort increases. As a result, the equilibrium timing is such that the small group chooses its sharing rule first. If the prize is not private enough, the small group retires from the competition and switching from a simultaneous to a sequential timing may reverse the results in terms of aggregate effort. The sequential timing also guarantees that the small group never outperforms the large one.

Keywords: collective rent seeking, sequential, group size paradox, sharing rules, strategic complements, strategic substitutes

JEL classification: C72, D23, D72, D74

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1 Introduction

Contests among organizations and groups of individuals are widespread. Examples include research and development races, pre-electoral campaigns, procurement contests, or even sport and art contests. In all the above, groups' performance depends on individual contributions of their members, which implies that groups need to coordinate and establish some rules regarding their internal organization. As the literature on collective contests has pointed out, a key element of groups' organization is related to the allocation of the prize among the winning group members.[1]

While contests among groups are generally thought as simultaneous, the timing in which contenders organize and implement their internal rules need not necessarily be so. Indeed, there is no *a priori* reason to believe that before the actual competition takes place, the timing in which the involved organizations decide upon their governing rules coincides. Differences in the size or in the informational advantage of organizations, as well as the existence of some established organizations challenged by an entrant, may result in the sequential determination of their governing rules. Consequently, groups involved in a simultaneous competition may well be deciding upon their internal rules in a sequential fashion. Similar to previous literature, we find that if the order of moves is determined endogenously, the equilibrium timing of the game is a sequential one, and this very fact constitutes a strong justification for departing from the simultaneity assumption (Baik and Shogren, 1992; Leininger, 1993; Morgan, 2003).[2]

In this paper, we contribute to the literature on collective contests by showing how different timings with respect to groups' choice of sharing rules alter individual incentives and thus group performance. In our two-group model, most of our results arise from the following observations: The small group's sharing rule is a strategic complement to the large group's sharing rule in the sense that if the small group chooses a more meritocratic sharing rule, the large group wishes to choose a more meritocratic rule as well, while the large group's sharing rule is a strategic substitute to the small group's sharing rule.[3] When

[1]Starting with Nitzan (1991), the literature has considered both exogenous and endogenous sharing rules. For a recent survey on prize-sharing rules in collective rent seeking, see Flamand and Troumpounis (2015). For surveys of the literature on individual contests in general, see among others Corchón (2007), Konrad (2009), Long (2013), Corchón and Serena (2016) and also Dechenaux et al. (2015) for the related experimental evidence. On collective contests, see the recent survey by Kolmar (2013), and also Sheremeta (2017) for a recent review of experimental evidence.

[2]Hoffmann and Rota-Graziosi (2012) consider a general two-player contest model and they qualify previous results by showing that a sequential structure does not always result if the order of moves is endogenized. Most importantly, they assume that the prize to be awarded in the contest may depend on the chosen efforts. This assumption introduces an additional strategic effect, potentially leading to different results regarding the order of moves.

[3]Gal-Or (1985) is an early paper studying incentives and payoffs in sequential games. In a model with homogenous players, she already indicated that the results crucially depend on whether reaction functions are upward or downward sloping.

both groups are active, the large group acting as the leader behaves in a similar manner as in a standard Stackelberg duopoly model. That is, the large group commits to a more meritocratic sharing rule than in the simultaneous game (hence effort increases), while the small group selects a less meritocratic rule than in the simultaneous game (hence effort decreases). On the contrary, the small group acting as the leader follows a very different strategy. It commits to a less meritocratic rule than in the simultaneous game, thereby weakening competition between groups. In turn, the large group also responds with a less meritocratic rule than in the simultaneous game. The latter finding is reminiscent of the main result in Kolmar and Wagener (2013). They consider a group contest and investigate whether groups have an incentive to implement a costless mechanism that solves a group's free-rider problem with respect to the group members' effort choices. Surprisingly, they find that the weaker (e.g., smaller or less productive) group sometimes wishes to abstain from implementing the mechanism. Adopting the mechanism may be problematic for the weaker group because the stronger group may react by competing more aggressively, implying that the contest may escalate. The main difference between our paper and the one by Kolmar and Wagener (2013) is the following. We assume that groups decide sequentially about their sharing rules, whereas the groups in their paper decide simultaneously about the adoption of the mechanism. Furthermore, by changing the sharing rule in our paper a group can affect the efforts of its members continuously; this is different from the paper by Kolmar and Wagener (2013) in which the adoption of the mechanism leads to a discrete change in efforts.

In contrast to the simultaneous timing (Balart et al., 2016), the small group never outperforms the large one in terms of winning probabilities. In other words, Olson (1965)'s celebrated group size paradox (GSP) vanishes regardless of the leader's size. The exact nature of the prize is also key to group performance. Indeed, we show that when the prize is not private enough and the sharing rules allow for transfers, the sequential choice of sharing rules leads to monopolization, a situation in which one group retires from the competition (Ueda, 2002). Interestingly, the large group takes advantage of its leadership by preventing the small group from being active for a greater range of privateness of the prize compared to the simultaneous case (Balart et al., 2016), thereby making monopolization more likely. However, this is not true when it is the small group that moves first, as in this case monopolization occurs in the same instances as in the simultaneous game.

Our results clearly have implications regarding aggregate effort expenditures. When the large group is the leader, and provided that the prize is private enough so that both groups are active, total rent-seeking expenditures increase with respect to the simultaneous case. Conversely, when the small group is the leader, and again provided that both groups are active, aggregate effort is lower compared to the simultaneous case. Situations are conceivable in which a designer has an impact on certain parameters of the contest like the order of moves. Thus, if from the designer's point of view effort is valuable, he should possibly oblige the two groups to choose their sharing rules simultaneously, or even force the large group to move first. If effort is instead considered as wasteful, the designer should opt for the design where the small group is the leader. In this latter case, in fact, the designer should not intervene in the game since the unique equilibrium of the game with an endogenous choice of the timing structure gives the leadership to the small group whenever both groups are active. Notice that in contrast to what happens when both groups are active, when the small group is inactive switching from a simultaneous to a sequential timing may reverse the results in terms of aggregate effort. Thus, the degree of privateness of the prize is a critical feature that should be taken into account to understand the implications of different timing arrangements.

Litigation is one example that our model can be applied to. Since parties involved in a legal battle spend irretrievable resources to prevail in court, litigation has often been modeled as a rent-seeking contest (Baye et al., 2005; Farmer and Pecorino, 1999; Gürtler and Kräkel, 2010). Many law firms make use of incentive pay, conditioning lawyers' compensation on their individual performance (for example, through bonus payments), which is typically measured by the number of billable hours. The structure of incentive pay differs among firms, some firms require lawyers to bill at least 1,600 hours a year, others demand much higher numbers. The size of bonuses also differs among firms.

In the US, many law firms provide publicly accessible information about their compensation practices in the NALP Directory of Legal Employers.[4] Among other things, these firms publicly state whether they pay bonuses to eligible lawyers, what factors a possible bonus payment is based on, and whether a minimum exists for billable hours. Hence, law firms, by simply checking the NALP Directory of Legal Employers, have a very good idea about compensation practices at their competitors, meaning that incentive schemes are publicly known. Finally, not all law firms provide the NALP Directory of Legal Employers with information about their compensation practices at the same time. Accordingly, sequential determination of compensation rules is conceivable if a firm gathers information about compensation practices at competing firms, and then chooses and discloses its own compensation practice.

To have a concrete example at hand, suppose that there is a legal battle, and the involved parties are represented by Baker & McKenzie and Shearman & Sterling, respectively, two law firms that are located in New York City. Baker & McKenzie reports in the

[4]Compensation practices for this example can be found at http://www.nalpdirectory.com/index.cfm. In a similar spirit, disclosure is often a feature of executive pay of CEOs and board of directors. In the UK, for instance, regulations that involve disclosure of remuneration were implemented in 2013 (Gupta et al., 2016).

NALP Directory of Legal Employers that both base salaries and bonuses depend on the number of billable hours and that lawyers are expected to bill at least 2,000 hours a year. In contrast, Shearman & Sterling reports that base salaries depend only on seniority and emphasizes that a minimum billable hour requirement does not exist. This means that at Baker & McKenzie firm profits (that typically depend on how successful the firm is doing in court) are more likely to be shared according to individual performance, whereas at Shearman & Sterling individual performance is relatively less important. In other words, the choice of different compensation contracts can be understood as a choice of different sharing rules.

2 The Model

There are two groups $i = A, B$ with $n_i \in \mathbb{N}$ members and let us refer to A as the large group (i.e., $n_A > n_B > 1$).[5] Each member $k = 1, 2, ..., n_i$ of group i chooses his individual level of effort $e_{ki} \geq 0$ whose cost is linear. The valuation of the prize is the same for all individuals, and is denoted by V.[6] The probability that group i wins the between-group competition is given by

$$P_i = \begin{cases} \frac{1}{2} & \text{for } E_A = E_B = 0 \\ \frac{E_i}{E_A + E_B} & \text{otherwise} \end{cases}$$

where $E_i = \sum_{k=1}^{n_i} e_{ki}$ is the total effort of group i.[7] Individuals are risk neutral, and the expected utility of member $k = 1, ..., n_i$ of group i is given by

$$EU_{ki} = \begin{cases} \left\{ \left[\alpha_i \frac{e_{ki}}{E_i} + (1 - \alpha_i) \frac{1}{n_i} \right] p + (1 - p) \right\} \frac{E_i}{E_A + E_B} V - e_{ki} & \text{for } E_i > 0, E_j \geq 0 \\ \left(\frac{1}{n_i} p + 1 - p \right) \frac{1}{2} V & \text{for } E_i = E_j = 0 \\ 0 & \text{for } E_i = 0, E_j > 0 \end{cases} \tag{1}$$

for $i = A, B$ and $i \neq j$, and where α_i is group i's sharing rule and measures the level of meritocracy. The parameter $p \in (0, 1]$ denotes the degree of privateness of the prize ($p = 0$ would correspond to the case of a pure public good while $p = 1$ corresponds to the

[5]If $n_A = n_B$ the timing of choices of group sharing rules is irrelevant (see Footnote 15).

[6]The assumption of symmetric valuations is the standard one in the literature on endogenous sharing rules, mainly for tractability reasons. In order to analyze the choice of selective incentives in the presence of heterogeneity within groups, Nitzan and Ueda (2016) assume that the contested prize is a public good, and focus on cost sharing rules rather than prize sharing rules, which allows them to tackle the problem analytically. They show that intra-group heterogeneity prevents the effective use of selective incentive mechanisms. For recent approaches with asymmetric valuations see Heijnen and Schoonbeek (2017) and references therein.

[7]This type of contest success function is widely used in the literature and has been axiomatized by Skaperdas (1996). Alternatively, many authors have used all-pay auctions to model competition, which have been analyzed by Baye et al. (1996).

case of a pure private good).[8] Indeed, an important feature of collective contests is that the prize sought by competing groups may often be interpreted as a mixture between a public and a private good. Local governments competing for funds typically devote them to the provision of both monetary transfers and local public goods. Similarly, prizes in research and development races involve both reputational and monetary benefits for the winning team. More generally, any prize sought by competing groups may be interpreted as a mixture between a public and a private good insofar as the winners derive some benefits in terms of status, reputation, or satisfaction following a victory.

The sharing rule $\alpha_i \geq 0$ represents how tied the allocation of the private part of the prize is on individual contributions (i.e., meritocracy) as opposed to egalitarianism in group i. Previous literature on sharing rules in collective rent seeking has considered both the cases of $\alpha_i \in [0, 1]$ (Baik, 1994; Lee, 1995; Noh, 1999; Ueda, 2002) and $\alpha_i \in [0, \infty)$ (Baik and Shogren, 1995; Baik and Lee, 1997, 2001; Lee and Kang, 1998; Balart et al., 2016). If $\alpha_i > 1$, the sharing rule of group i allows for transfers among its members, as in Hillman and Riley (1989).[9] In such case, group i collects $-(1-\alpha_i)\frac{p}{n_i}\frac{E_i}{(E_A+E_B)}V$ from each of its members and allocates $\alpha_i p\frac{E_i}{(E_A+E_B)}V$ according to their relative contributions.[10]

We consider a multi-stage game where sharing rules are chosen prior to the choice of individual efforts. Both groups choose their sharing rule by maximizing their aggregate welfare (i.e., $\max \sum_{k \in i} EU_{ki}$).[11] The equilibrium concept is subgame perfection in pure strategies.[12]

[8]Notice that we exclude the possibility of a pure public good, in which case sharing rules are irrelevant. For the analysis of these cases refer to the multiple equilibria results in Katz et al. (1990). For more recent approaches see also Baik (2016) and references therein.

[9]Cost-sharing in collective contests for purely public prizes can also be interpreted in terms of within-group transfers (Nitzan and Ueda, 2016; Vazquez-Sedano, 2018).

[10]When the sharing rules do not allow for transfers among individuals (i.e., $\alpha_i \in [0, 1]$ for $i = A, B$), the equilibrium is identical regardless of the particular timing of the game. This occurs because the constraints on the sharing rules are binding irrespective of the timing of the interaction, and thus allowing one group to choose its sharing rule first does not alter the strength of competition compared to the simultaneous case (formal results are available upon request).

[11]As the aggregate welfare of group i only depends on aggregate effort, and as in equilibrium aggregate effort is unique for any α_i, it follows that the sharing rule α_i that maximizes the expected utility of the representative individual in a within-group symmetric equilibrium also maximizes $\sum_{k \in i} EU_{ki}$. The sharing rules may thus be chosen by a benevolent leader maximizing the welfare of the group, which can be justified by the fact that such leader cannot do well without the support of the group members, that is, without taking care of their welfare at least partially (Nitzan and Ueda, 2011).

[12]Equilibria in mixed strategies may exist when monopolization arises and there is a continuum of equilibria in the first stage of the game. For example, when the large group moves first and plays a unique pure strategy, there is a continuum of meritocracy levels that result in a zero payoff for the small group and hence the small group randomizing over the continuum is also an equilibrium.

2.1 Effort Stage and Simultaneous Sharing Rules

The simultaneous game consists in two stages: In stage one each group $i = A, B$ chooses its sharing rule α_i^S, while in stage two individuals choose their level of effort simultaneously and independently.

At the effort stage, and given the chosen sharing rules, groups' members choose their level of effort by maximizing (1) subject to efforts being non-negative. The equilibrium of this stage is solved in Balart et al. (2016) who generalize the results of Ueda (2002) and Davis and Reilly (1999) by allowing for a mixed public-private prize, thereby leading to the possibility of monopolization. More precisely, group i retires from the contest (i.e., $e_{ki} = \tilde{e}_i = 0$ for all $k \in i$) whenever

$$\alpha_i \leq \frac{n_i \left[\alpha_j p(n_j - 1) - n_j(1-p)\right] - n_j p}{(n_i - 1)n_j p} \tag{2}$$

with $i = A, B$ and $j \neq i$. As can be seen from (2), the less (more) meritocratic the sharing rule of group i (j), the more likely that monopolization occurs, so that group i is inactive. To understand the intuition for this result, consider a member of group i. This player's incentive to exert effort comes from two different sources. First, by exerting effort, the player's group becomes more likely to win the prize. Second, conditional on winning the prize, by exerting effort the player secures a larger share of the prize for himself. A high α_j induces the members of group j to choose high effort, in which case group i is unlikely to win the contest. Then the considered player's effort has little impact on the contest outcome, moderating the first effect. A low α_i implies that efforts do not have a strong impact on the distribution of the prize among group i's members, thereby mitigating the second effect. Taken together, when α_i is low and α_j is high, members of group i have little incentive to exert effort so that it is likely that the group is inactive. In turn, when group i is inactive, the members of group j compete in a standard n_j-players Tullock contest for a prize of valuation $\alpha_j pV$ (i.e., the private part of the prize that is allocated according to relative effort) and thus exert effort

$$e_{kj} = \tilde{e}_j = \frac{\alpha_j p(n_j - 1)}{n_j^2} V \ , \ \forall k. \tag{3}$$

Conversely, if both groups are active, i.e., if (2) is violated for $i = A, B$, individual effort in the symmetric equilibrium is given by

$$e_{ki} = \hat{e}_i = \frac{\{\rho_j[\rho_i + \alpha_i(1-\rho_i)] + (1-\rho_j)\alpha_j\rho_i\}\{n_j p(1-\alpha_i) + n_i[n_j(1 - p(1 - \alpha_i + \alpha_j)) + p\alpha_j]\}}{n_j[n_i(\rho_i + \rho_j)]^2} \tag{4}$$

where $\rho_i = \frac{p}{n_i} + (1-p)$.[13] Note that the above expression is a strictly convex second degree polynomial in terms of α_i. Violation of condition (2), which, as explained above, imposes a lower bound on α_i, guarantees that individual effort is increasing in own group's meritocracy level (α_i). On the contrary, individual effort is decreasing in the other group's meritocracy level (α_j).

Let us define $\alpha_{i3}(\alpha_j)$ as the value of α_i that satisfies (2) with equality, representing the maximum value of α_i that guarantees that group i's members are inactive given any α_j. By switching the subscripts in (2), we can also define $\alpha_{i2}(\alpha_j)$ as the minimum value of α_i guaranteeing that group j's members are inactive given any α_j. As $\alpha_{i2}(\alpha_j) > \alpha_{i3}(\alpha_j)$ we can write group i's expected utility as follows:

$$EU_i(\alpha_i) = \begin{cases} 0 & \text{if } 0 \leq \alpha_i \leq \alpha_{i3}(\alpha_j) \\ \hat{EU}_i(\alpha_A, \alpha_B) & \text{if } \alpha_{i3}(\alpha_j) < \alpha_i < \alpha_{i2}(\alpha_j) \\ \tilde{EU}_i(\alpha_i) & \text{if } \alpha_i \geq \alpha_{i2}(\alpha_j) \end{cases}$$

where $\hat{EU}_i(\alpha_A, \alpha_B)$ denotes individual payoff in group i when both groups are active, and $\tilde{EU}_i(\alpha_A, \alpha_B)$ denotes individual payoff in group i when only this group is active.[14] One can now obtain the best response correspondence for the simultaneous choice of sharing rules prior to the effort stage:

$$\alpha_i(\alpha_j) = \begin{cases} \alpha_{i2}(\alpha_j) & \text{for } \alpha_j \leq \tilde{\alpha}_j \\ \alpha_{i1}(\alpha_j) & \text{for } \tilde{\alpha}_j < \alpha_j < \hat{\alpha}_j \\ \dot{\alpha}_i \in [0, \alpha_{i3}(\alpha_j)] & \text{for } \alpha_j \geq \hat{\alpha}_j \end{cases} \tag{5}$$

Intuitively, the best response suggests that if group j selects a very "meritocratic" rule (i.e., $\alpha_j \geq \hat{\alpha}_j$) then it is not worth being active for group i, so that it selects any $\dot{\alpha}_i \in [0, \alpha_{i3}(\alpha_j)]$ guaranteeing its inactivity. Conversely, if group j selects a very egalitarian rule (i.e., $\alpha_j \leq \tilde{\alpha}_j$), group i maximizes its payoff by preventing group j from being active in the "cheapest" way (i.e., by choosing $\alpha_{i2}(\alpha_j)$). For intermediate values of α_j group i selects $\alpha_{i1}(\alpha_j)$, both groups are active and $\alpha_{i1}(\alpha_j)$ is obtained from the first order condition when maximizing individual payoff with both groups active (i.e., $\hat{EU}_i(\alpha_A, \alpha_B)$).

Given the above best responses, Balart et al. (2016) show that when groups choose their sharing rules simultaneously, there exists a threshold $p_1 = \frac{n_B(n_A - n_B - 1)}{1 + n_B(n_A - n_B - 1)}$ determining the occurrence of monopolization:

[13]The symmetric equilibrium presented is unique if $\alpha_i > 0$ for both $i = A, B$. In case of purely egalitarian sharing rules, our setup is equivalent to the case of a pure public prize and multiple equilibria may arise (see Balart et al. 2016 for a detailed characterization).

[14]In the Preliminaries of the Appendix we present a formal derivation of the best responses as well as the analytical expressions $\alpha_{i1}(\alpha_j), \alpha_{i2}(\alpha_j), \alpha_{i3}(\alpha_j), \tilde{\alpha}_j, \hat{\alpha}_j, \hat{EU}_i(\alpha_A, \alpha_B), \tilde{EU}_i(\alpha_A, \alpha_B)$.

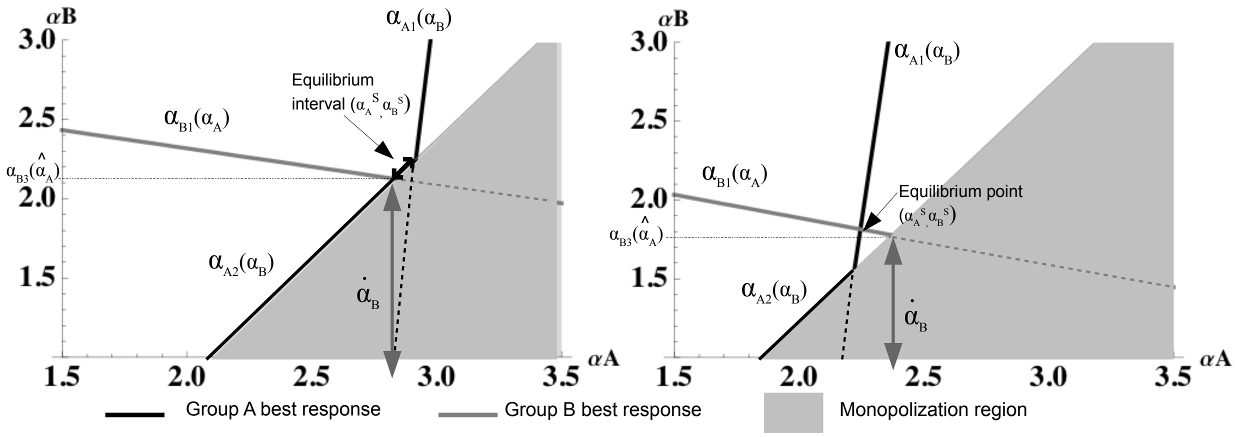

Figure 1: Simultaneous choice of sharing rules for $p = 0.64$ (left) and $p = 0.72$ (right) and $n_A = 4$, $n_B = 2$.

- If $p > p_1$, both groups are active and the sharing rule for $i = A, B$ in the unique subgame perfect equilibrium is given by

$$\alpha_i^S = \frac{n_i}{(n_i - 1)} \frac{2n_i n_j (n_i - 1) + C_i p + D_i p^2}{p\,[2n_i n_j - p(n_i(2n_j - 1) - n_j)]}$$

  where $C_i = n_i n_j (9 - 4n_i) - n_j(n_j + 2) - 2n_i$ and $D_i = n_i n_j (2n_i - 7) + n_j(n_j + 3) + 3n_i - 2$.

- If $0 < p \leq p_1$, only group A is active and the continuum of sharing rules in the subgame perfect equilibrium is given by
  - $\alpha_A^S = \frac{n_A}{n_A - 1}\left[\frac{1-p}{p} + \frac{(n_B-1)\alpha_B + 1}{n_B}\right]$
  - $\alpha_B^S \in [\ \frac{n_B(1-p)+p}{p}, \frac{n_B[n_A(1-p)+3p-2]-2p}{(n_B-1)p}]$

Figure 1 summarizes the best responses, the equilibrium sharing rules and the potential occurrence of monopolization. Observe that in both panels, the slope of a group's best response changes depending on the value of the other group's sharing rule (as summarized in (5)). Notice also that the occurrence of monopolization depends on p. In the right panel where $p > p_1$, the interior best responses $\alpha_{i1}(\alpha_j)$ intersect outside of the monopolization region, so that in the depicted equilibrium both groups are active. In the left panel where $p \leq p_1$, the interior best responses $\alpha_{i1}(\alpha_j)$ intersect within the monopolization region, yielding multiple equilibria where only the large group is active.

The intuition for the results on monopolization is as follows. The larger group A has an advantage compared to the smaller group B in that more members contribute effort to win the prize. At the same time, being larger is also detrimental since, upon winning, the prize must be shared among a greater number of members. This latter effect, however, only refers to the private, but not the public part of the prize. Thus, if and only if the private part of the prize is sufficiently low, the size advantage of the larger group is sufficiently important so that it can successfully deter the smaller group from actively entering the contest.

3 Results

Our first result is essential to understand the implications of a sequential choice of sharing rules:

Proposition 1. *If both groups are active, the small group's sharing rule is a strategic complement to the large group's sharing rule in the sense that if the small group chooses a more meritocratic sharing rule, the large group wishes to choose a more meritocratic rule as well. On the contrary, the large group's sharing rule is a strategic substitute to the small group's sharing rule.*

The interior best response $\alpha_{i1}(\alpha_j)$ is linear and has a positive (negative) slope for the large (small) group. In other words, the small (large) group's sharing rule is a strategic complement (substitute) to the large (small) group's sharing rule irrespective of the specific level at which we evaluate the best response. This, in turn, will determine how each group chooses its sharing rule when given the opportunity to move first. More specifically, each group being the leader precommits to a sharing rule such that the other group (i.e., the follower) is induced to decrease its level of meritocracy compared to the simultaneous game. That is, the large (small) group precommits to higher (lower) levels of meritocracy, thereby increasing (reducing) the strength of competition.

This result has some similitudes as well as some interesting differences with respect to the case of precommitment in effort levels by two players studied by Dixit (1987). He finds that cross derivatives of equilibrium efforts are zero whenever the players are symmetric and thus the timing of the game does not alter the precommitted effort levels. This is also the case regarding the choice of sharing rules provided the two groups have the same size (i.e., $n_A = n_B = n$).[15] Further, Dixit (1987) shows that in a two-player asymmetric contest the effort of the underdog is a strategic complement to the effort of the advantaged player, while the advantaged player's effort is a strategic substitute to the effort of the underdog. Similarly, in our context the small group's sharing rule is a

[15] If the two groups have the same size (i.e., $n_A = n_B = n$), the equilibrium sharing rules are given by $\alpha_A = \alpha_B = 1 + n\frac{1-p}{p}$ regardless of the order of moves, as in this case best responses are independent of the other group's sharing rule.

strategic complement to the large group's sharing rule, whereas the large group's sharing rule is a strategic substitute to the small group's.[16] The large group has an advantage in the sense that it always wins with higher probability conditional on the members of the two groups exerting the same level of individual effort. However, there is an important difference with respect to the strategic implications found by Dixit (1987). In his case, marginal returns to effort are increasing (decreasing) in the other player's effort if it is small (large) enough. This is not the case with sharing rules, for which strategic behaviors only depend on group size.

The strategic responses depicted in the above Proposition hold for any combination of sharing rules that make the two groups active and directly arise from the linearity of the best responses. If we remove the linearity assumption in the cost of effort, the best responses are no longer linear and the whole exercise proves intractable. However, one could expect the strategic responses found in Proposition 1 to carry through if one were to consider a convex cost function. To illustrate that, recall that Proposition 1 in essence highlights a more "aggressive" choice of sharing rules by the large group due to its size advantage, hence this should also be true if such size advantage is further exacerbated by the presence of convex costs.

The differences in groups' strategic behavior due to their size may be better understood by focusing on a) the size deterrence effect, and b) the aggregate effort effect. The size deterrence effect (Nitzan, 1991) penalizes the large group: a more numerous group implies that the private part of the prize has to be divided among more individuals. The aggregate effort effect (Balart et al., 2016), in contrast, is favorable to the large group: if the two groups exert the same level of aggregate effort, the individual cost of effort will be smaller for the members of the large group than for the members of the small group. In other words, since there is a larger number of potential contributors in the large group, the cost of effort is divided among more individuals. Notice now that the case of convex costs would exacerbate the large group's benefits from the aggregate effort effect as dividing the cost of effort among more contributors would be more advantageous in that case.[17] At the same time, a convex cost function should not affect the size deterrence effect since the latter is only affected by the degree of privateness of the prize. Combining the fact that convex costs exacerbate the advantage for the large group from the aggregate effort effect, while not affecting its disadvantage from the size deterrence effect, it follows that the larger group enjoys a higher advantage with convex costs than with linear costs. But

[16] For an application where reaction functions at the Nash equilibrium have slopes with opposite signs relative to Dixit (1987), see Skaperdas et al. (2016).

[17] This feature is precisely the one that eliminates the occurrence of the group size paradox in Esteban and Ray (2001). In their words, *"if the marginal cost of effort rises sufficiently with respect to resources contributed, then larger groups will have a higher win probability, even if the prize is purely private."*

given that under linear costs of effort, the larger group is reacting more "aggressively" than the small group for any degree of privateness of the prize, one should expect that this is also true when the relative cost of effort of its members decreases. Symmetric arguments can also be made for the small group, overall arguing that the strategic effects found under the assumption of liner costs would still hold if the cost of effort is a convex function.[18]

3.1 Sequential Timing

In the following subsections where the game is sequential, we consider the following three stages: In stage one the leader (group $i \in \{A, B\}$) chooses its sharing rule α_i^L. In stage two the follower (group $j \neq i$) chooses its sharing rule α_j^F. In stage three the members of the two groups simultaneously and individually choose their effort levels (e_i^L, e_j^F). In what follows, we present the equilibrium choices of sharing rules (α_i^L, α_j^F) in our formal results. Effort levels (e_i^L, e_j^F) chosen in stage three can be derived directly from expressions (3) and (4).

3.1.1 Large group A is the leader

Proposition 2. *Let* $p_1' = \frac{n_A(n_A-n_B-1)}{1+n_A(n_A-n_B-1)}$ *and group A be the leader:*

- *If* $p > p_1'$*, both groups are active and the sharing rules in the unique subgame perfect equilibrium are given by*
  - $\alpha_A^L = \frac{\rho_A n_A}{p}$
  - $\alpha_B^F = \frac{n_B\rho_B[n_A(2n_B\rho_B-2+p)+(n_B-2)p]-n_A(1-\rho_A)(n_A-n_B)\rho_A}{2n_A n_B(1-\rho_B)\rho_A}$
- *If* $0 < p \leq p_1'$*, only group A is active and the continuum of sharing rules in the subgame perfect equilibrium is given by*
  - $\alpha_A^L = \frac{\rho_B n_B}{1-\rho_A}$
  - $\alpha_B^F \in \left[0, \frac{\rho_B n_B}{p}\right]$

When the large group is the leader and both groups are active, we are in a situation that is very similar to a standard Stackelberg duopoly model: Once the large group commits to a more meritocratic rule than in the simultaneous case ($\alpha_A^L > \alpha_A^S$), the follower reacts by selecting a less meritocratic rule than in the simultaneous case ($\alpha_B^F < \alpha_B^S$).

[18]In contrast, these arguments may not carry through if one were to consider a concave cost function given that the latter would tend to favor the small group and therefore work "against" the previously discussed logic where convex costs exacerbate the large group's size advantage.

Indeed, recall that the large group's sharing rule is a strategic substitute to the small group's sharing rule (Proposition 1). Furthermore, having the leadership advantage allows the large group to induce the small group to retire from the contestfor a larger range of p than in the simultaneous case (i.e., $p'_1 > p_1$). In other words, the large group acting as the leader is capable of tying the small group's hands and oblige the latter to react with a sharing rule leading to its own inactivity, and this for a larger range of p. In turn, the small group is indifferent between choosing any of the sharing rules belonging to the interval such that it retires from the contest. Observe that being the first mover enables the large group to select its preferred equilibrium (i.e., the lowest α_A) out of the continuum arising in the simultaneous case. In other words, monopolization is achieved in the "cheapest" way for the large group.

The following result will be useful in order to understand the implications of the sequential interaction:

Proposition 3. *When the large group A is the leader:*

- *If $p > p_1$ then aggregate effort is strictly greater than in the simultaneous case.*
- *If $p \leq p_1$ then aggregate effort is smaller than in the simultaneous case.*

If p is such that both groups are active (i.e., $p > p'_1$), we saw that the large group increases the level of meritocracy of its sharing rule compared to the simultaneous case, which induces the small group to do the opposite. Given that the large group chooses to strengthen competition between groups, these equilibrium sharing rules are such that aggregate rent-seeking expenditures are strictly greater than when the groups select their sharing rules simultaneously. If p is such that monopolization occurs in both the simultaneous and sequential cases (i.e., $p \leq p_1$), however, the above results are reversed. That is, the large group reduces the level of meritocracy of its sharing rule with respect to the simultaneous case, so that aggregate effort is smaller. Finally, recall that when $p_1 < p < p'_1$ both groups are active in the simultaneous case while only the large one is active when being the leader. Yet, the total effort that the large group's members exert in order to induce the small group to retire from the competition exceeds the sum of both groups' efforts in the simultaneous case.

The next proposition addresses the GSP. The GSP refers to a situation where the small group outperforms the large group in terms of winning probabilities (Olson, 1965).

Proposition 4. *If the large group is the leader and for all values of p including full privateness, the GSP never arises.*

When the large group chooses its sharing rule first, and contrary to the simultaneous case, the GSP never takes place regardless of the degree of privateness of the prize. As the large group acting as the leader selects a more meritocratic sharing rule than in the simultaneous case, and as the small group reacts with a less meritocratic one, it follows that aggregate effort increases in the large group, while it decreases in the small group. As it turns out, these effort variations are large enough to eliminate the GSP.

3.1.2 Small group B is the leader

Our results are significantly altered by giving the leadership to the small group, which again is a direct consequence of Proposition 1:

Proposition 5. *Let group B be the leader:*

- *If $p > p_1$, both groups are active and the sharing rules in the unique subgame perfect equilibrium are given by*
  - $\alpha_A^F = \frac{n_A\rho_A[n_B(2n_A\rho_A-2+p)+(n_A-2)p]-n_B(1-\rho_B)(n_A-n_B)\rho_B}{2n_An_B(1-\rho_A)\rho_B}$
  - $\alpha_B^L = \frac{\rho_B n_B}{p}$
- *If $0 < p \leq p_1$, only group A is active and the continuum of sharing rules in the subgame perfect equilibrium is given by*
  - $\alpha_A^F = \frac{\rho_B+(1-\rho_B)\alpha_B^L}{1-\rho_A}$
  - $\alpha_B^L \in \left[0, \frac{n_A\rho_A-2\rho_B}{1-\rho_B}\right]$

For all degrees of privateness of the prize such that both groups are active, and in stark contrast to the case in which the large group is the leader, it turns out that when the small group has the leadership advantage it strategically chooses to weaken competition by selecting a less meritocratic rule than in the simultaneous game ($\alpha_B^L < \alpha_B^S$). In turn, the large group also reacts with a less meritocratic rule than the one it selects when the game is simultaneous ($\alpha_A^F < \alpha_A^S$). Indeed, recall that the small group's sharing rule is a strategic complement to the large group's sharing rule (Proposition 1).

For sufficiently low degrees of privateness ($p \leq p_1$), the small group is not able to take advantage of its leadership. Regardless of the choice of the small group in stage one, in stage two the large group selects a highly meritocratic sharing rule so that the small group retires from the contest in stage three. Observe that the threshold level of privateness that determines whether the small group is active or inactive is identical to the one obtained in the simultaneous case, that is, the small group cannot take advantage of its leadership for any $p \leq p_1$.

Proposition 6. *When the small group B is the leader:*

- *If $p > p_1$ then aggregate effort is strictly smaller than in the simultaneous case.*
- *If $p \leq p_1$ then aggregate effort is strictly smaller than in the simultaneous case for $\alpha_B^L < \frac{n_B(1-p)+p}{p}$, while it can be either greater or smaller than in the simultaneous case for $\alpha_B^L \geq \frac{n_B(1-p)+p}{p}$.*

When p is such that both groups are active, both groups adopt relatively more egalitarian rules than in the simultaneous game, their members exert a lower level of effort, hence aggregate effort is smaller. As when the large group is the leader, when p is such that the small group is inactive in both the simultaneous and sequential cases (i.e., $p \leq p_1$), the above results may be reversed. More specifically, depending on which particular sharing rule is chosen by the small group, aggregate effort may be greater in the simultaneous than in the sequential case.

The next proposition again refers to the GSP. We demonstrated before that the GSP does not arise if the large group is the leader. Interestingly, the same is true if the small group is the leader, implying that the GSP never arises if group sharing rules are determined sequentially.

Proposition 7. *If the small group is the leader and for all values of p including full privateness, the GSP never arises.*

Observe that here, the GSP does not arise for a very different reason than when the large group is the leader. The small group selects a sharing rule such that its members are inactive when the prize is public enough (but not purely public), which clearly prevents the occurrence of the GSP as in the previous cases. Then, if $p > p_1$ the small group acting as the leader strategically chooses to weaken competition by selecting a less meritocratic rule than in the simultaneous game, which in turn induces the large group to adopt a similar behavior. However, the reduction in meritocracy adopted by the small group is relatively larger than the corresponding reduction in the large group, and is such that the GSP vanishes.

Figure 2 summarizes our findings regarding the sequential choice of sharing rules and the occurrence of monopolization and the GSP. As we commented, one of the differences that arise when the large group moves first is that monopolization may occur for a wider range of p. A related question concerns the way relative group size affects monopolization. By doing that exercise, it can be shown that increasing the relative size of the small group unambiguously reduces the likelihood of monopolization when the large group is the leader, whereas this relationship is non-monotonic when the small group is the leader

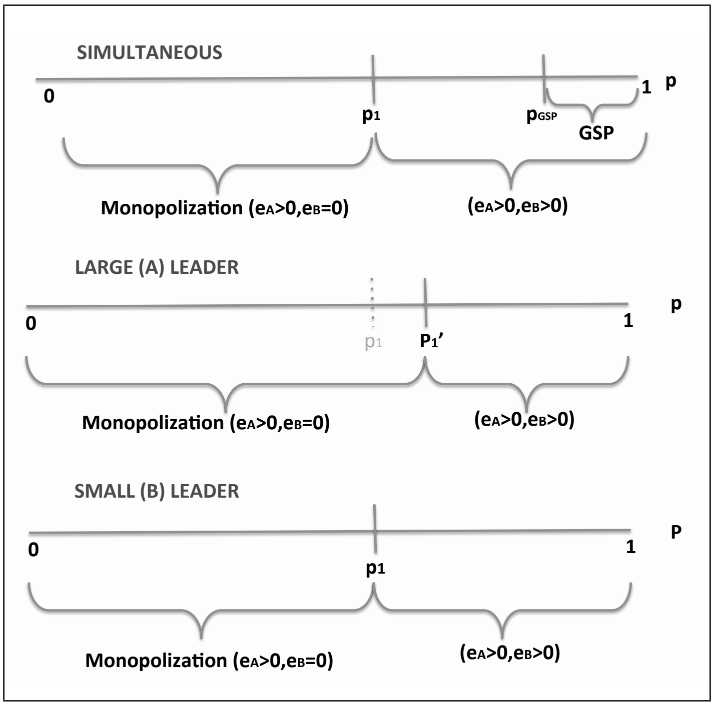

Figure 2: Simultaneous versus sequential choice of sharing rules.

(or in the simultaneous case). This is illustrated in Figure 3, which plots the minimum value of n_A that makes group B inactive as a function of n_B for $p = 0.99$ when the large (black curve) or small (gray curve) group is the leader, respectively.[19] When the small group is the leader and $n_B = 2$, monopolization occurs if and only if $n_A > 52$. However, when $n_B = 10$, monopolization occurs if and only if $n_A > 20$ (i.e., monopolization is more likely). For higher values of n_B the relationship between the threshold and the size of the small group is monotonic and increasing. For $n_B > 20$, monopolization requires a difference of (approximately) no more than four individuals between the two teams, irrespective of which group moves first.

Further, as can be seen in the figure, and as we already concluded from the finding that $p_1 < p_1'$, monopolization is more likely when the large group moves first. Indeed, the smaller n_B, the greater the difference between the two curves.

In Figure 4 we plot the actual best responses as well as isoprofits for different combinations of meritocracy to obtain the intuition behind the equilibrium choices. In the left panel, we display the equilibrium sharing rules when the large group is the leader and compare them with the ones arising under a simultaneous timing, while in the right panel, we do the same when the small group is the leader (both for $n_A = 4$, $n_B = 2$ and $p = 1$). Again, observe that the slope of the best response is positive for the large group and negative for the small one. The simultaneous case is represented by the in-

[19]We do not present the formal results but they are available upon request.

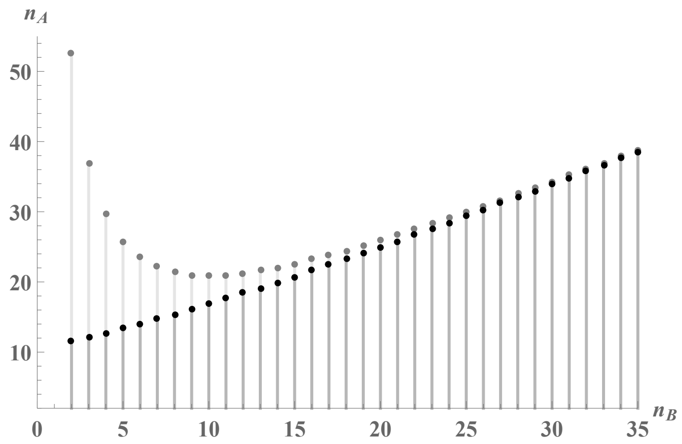

Figure 3: Minimum n_A yielding monopolization when group A (black) or B (gray) moves first, for $p = 0.99$ and different values of n_B.

tersection of the two best responses. The sequential move equilibria are determined by the tangency point between the isoprofit of the first mover and the best response of the follower. When the large group is the leader, its equilibrium sharing rule is higher than in the simultaneous case, and the one of the small group is lower. That is, the large group chooses to strengthen competition, thereby inducing the small group to reduce the meritocracy of its sharing rule. As a result, the large group moves to a higher level isoprofit curve, while the opposite is true for the small group. We also observe that, when the large group is the leader, the equilibrium moves along the best response of group B and towards the region in which it becomes inactive. This illustrates why the range of p for which monopolization occurs expands when the large group moves first.[20] When the small group is the leader, equilibrium takes place at lower levels of meritocracy. When allowed to move first, the small group chooses to weaken competition, which induces the large group to adopt a similar behavior. As a result, both groups enjoy a greater payoff than in the simultaneous case.

3.2 Endogenous Timing

Although the size of the leader does not matter in terms of the GSP, it does have some implications regarding the expected utility of groups' members and aggregate effort. Consider the case where both groups are active for any particular timing. When the large group is the leader, all its members clearly have higher expected utility than in the simultaneous version of the game, while all the members of the small group (the follower) are worse off. Therefore, a transition from simultaneous to sequential competition where

[20] An analogous movement along the follower's best response is responsible for monopolization in Dixit's (1979) seminal entrance model.

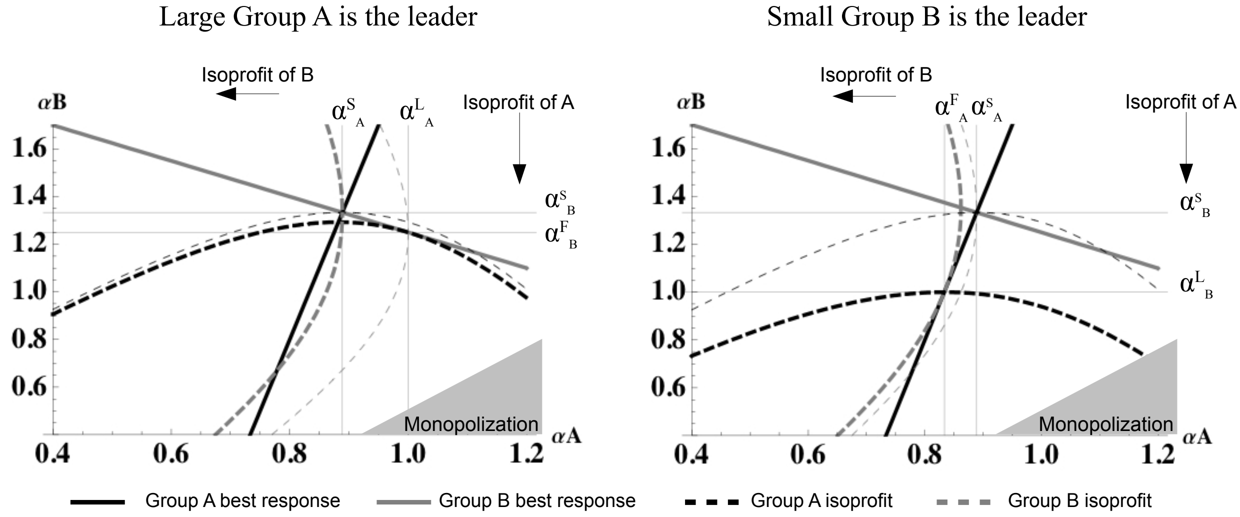

Figure 4: Simultaneous versus sequential choice of sharing rules with $p = 1$ (for $n_A = 4$, $n_B = 2$).

the large group is the leader never consists in a Pareto improvement, and increases aggregate effort. Conversely, when the small group is the leader, the members of both groups exert less effort than in the simultaneous game, and they achieve higher expected utility. Thus, switching from simultaneous to sequential competition where the small group is the leader consists in a Pareto improvement and reduces aggregate effort.

Given these implications, a natural question on which our model can shed light is the following: If groups were to also choose the timing of the game, which sequence would prevail? To answer this question, let us assume that prior to the choice of sharing rules, groups are able to declare their intention to be the leader or the follower of the game. Suppose the two groups have the possibility of choosing between two dates to declare their sharing rules. If they choose the same date, sharing rules are chosen simultaneously. If they choose different dates, sharing rules are chosen sequentially. The results are summarized in the following proposition:

Proposition 8.

- *If $p > p_1$ the unique equilibrium of the timing game is such that the small group is the leader.*
- *If $p \leq p_1$ the payoff-dominant equilibrium of the timing game is such that the small group is the leader.*

Suppose that the prize is private enough so that both groups are active in the simultaneous setup. If the small group selects its sharing rule at date one, the large group prefers to wait, whereas if the small group selects its sharing rule at date two, the large group takes the lead. Hence the large group always prefers the sequential version of the game. On the contrary, if the large group selects its sharing rule at date one, so does the small group, whereas if the large group moves at date two, the small group takes the lead. Therefore, the unique equilibrium of the game with an endogenous choice of the timing structure gives the leadership to the small group. This is also the case under monopolization, provided that we select the payoff-dominant equilibrium among all the possible equilibria.[21] Further, with an endogenous timing selection previous to the rent-seeking activity, the GSP should never take place.

Again, the fact that the sequential game arises endogenously constitutes a strong justification for departing from the simultaneity assumption. This result is in line with the one in contests among individuals where the sequential timing also arises endogenously (Baik and Shogren, 1992; Leininger, 1993; Morgan, 2003).[22] In the presence of asymmetric players, the higher valuation player —whose best-response function is increasing at the Nash equilibrium of the simultaneous game— gives the leadership to the underdog —whose best-response function is decreasing at the Nash equilibrium of the simultaneous game (Baik and Shogren, 1992; Leininger, 1993). We provide an analogous result in the context of group sharing rules: The large group —whose best-response function is increasing— gives the leadership to the small group —whose best-response function is decreasing, resulting in weaker competition between the groups.[23] As indicated in the introduction, the finding by Kolmar and Wagener (2013) that weak groups sometimes decide not to adopt a mechanism that would solve the group's free-rider problem with respect to the members' effort choices is related to this finding. By refraining from adopting the mechanism, a weak group deters the stronger group from competing intensely, thereby weakening competition.

[21] Given the multiplicity of equilibria when $p \leq p_1$, any timing can arise endogenously as an equilibrium depending on the particular sharing rule that group B selects out of the continuum. A reasonable selection criterion which allows a comparison across the three versions of the game is payoff dominance. Given that the small group B is inactive regardless of the particular timing (and thus obtains zero profits), payoff dominance simply requires that monopolization occurs with the small group selecting the smallest α_B out of the continuum. Further, we also use payoff dominance to select among the multiple Nash equilibria of the timing game.

[22] The industrial organization literature has also extensively considered endogenous timing. See for instance Amir and Grilo (1999), Deneckere and Kovenock (1992), Hamilton and Slutsky (1990) and Van Damme and Hurkens (1999) in the context of a duopoly model.

[23] Notice that, in the papers on individual contests, the best-response functions refer to an effort choice, whereas in our model they refer to the choice of a group sharing rule.

4 Discussion

Collective contests have drawn a lot of attention given their wide range of applications. In this paper, we contribute to this literature by studying the case in which groups are allowed to determine their sharing rules sequentially. We believe such an analysis is relevant for several reasons. First, we show that the sequential choice of sharing rules emerges endogenously when the groups are allowed to decide upon the timing of the game. Second, we provide real life examples in which the compensations of organizations' members are public information and are chosen in a sequential fashion. Thus, our framework fits well with compensation schemes in litigation firms or with the debate on the regulation of executive compensation disclosure. Third, we show that the timing according to which organizations of different sizes choose their sharing rules has several consequences of interest from the perspective of a contest designer. Despite being highly stylized, our model provides interesting insights regarding the interaction between groups' size, the nature of the prize and the timing of choices.

The relative size of the first mover is key to our results. The large group is more aggressive when allowed to move first than in the simultaneous contest, while the small group acting as the leader follows the opposite strategy. This is a direct consequence of the fact that the small group's sharing rule is a strategic complement to the large group's sharing rule, while the large group's sharing rule is a strategic substitute to the small group's. The public-private nature of the prize also plays an important role. When the large group moves first and organizations compete for a prize that is close to the definition of a pure private good, aggregate effort is greater than in the simultaneous case. However, the opposite is true when the public component of the contested-prize is sufficiently high, or when the small group moves first and the contested prize is sufficiently private.

We also provide interesting insights regarding the GSP and monopolization, two situations that can arise when the choice of sharing rules is simultaneous. First, the large group takes advantage of its leadership by excluding the small group from the competition for a larger set of parameters than in the simultaneous case. Yet, this is not the case for the small group, as having a first mover advantage does not allow its members to reduce the set of parameters for which monopolization occurs. Regarding the occurrence of monopolization, the GSP never arises under a sequential timing regardless of the leader's size. When allowed to move first, the large group increases the degree of meritocracy of its sharing rule, thereby increasing the effort of its members and preventing the small group to outperform. On the contrary, the small group acting as the leader does exactly the opposite, which reduces its members' effort thus making it easier for the large group to outperform. Overall, our work extends knowledge on the occurrence of the GSP in group contests and therefore complements previous literature summarized in Kolmar (2013). In particular, we show that the timing of the sharing rules choice proves an additional determinant of the GSP along with known results on the technology of conflict through the impact or cost function (Esteban and Ray, 2001; Pecorino and Temimi, 2008; Kolmar and Rommeswinkel, 2011, 2013; Hwang, 2017), the nature of the contested prize (Balart et al., 2016), and group identity (Kolmar and Wagener, 2011) among others.

5 Appendix

Preliminaries

Before proceeding with the proofs of our results and following the arguments by Balart et al. (2016) we sketch how groups' best responses are determined. Although these are not new results their replication is helpful in understanding the rest of our arguments.

Given the equilibrium effort levels if both groups are active as presented in equation (4), the expected utility for the representative individual in group $i = A, B$ is given by

$$\hat{EU}_i(\alpha_A, \alpha_B) = \frac{[n_i n_j + ((\alpha_i - 1)(n_i - 1)n_j + \alpha_j n_i(1 - n_j))p]V}{n_i [n_j p + n_i(2n_j(1-p) + p)]^2} \left[n_i(2n_i - 1)n_j - Cp + D(n_i - 1)p^2\right]$$

where

$$C = n_i(1 - \alpha_j - n_i) + n_j(1 - \alpha_i) + n_i(4n_i + \alpha_i + \alpha_j - 5)n_j$$

$$D = \alpha_i(n_j - 1) + \alpha_j(n_j - 1) + (n_i - 1)(2n_j - 1)$$

Given the equilibrium effort levels if only group i is active as presented in equation (3), the expected utility for the representative individual in group i is given by

$$\tilde{EU}_i(\alpha_i) = V - \frac{(n_i - 1)(n_i + \alpha_i)}{n_i^2} pV$$

Now let us define $\chi_i(\alpha_A, \alpha_B) = (1 - \alpha_i)p(n_i n_j - n_j) - (1 - \alpha_j)p(n_i n_j - n_i) - n_i n_j(1 - p) - pn_i$ with $i = A, B$ and $j \neq i$. Condition (2) for the occurrence of monopolization is equivalent to $\chi_i(\alpha_i, \alpha_j) \geqslant 0$. Notice that $\chi_i(\alpha_i, \alpha_j) + \chi_j(\alpha_i, \alpha_j) = -n_A [2n_B(1 - p) + p] - n_B p < 0$. Hence if $\chi_i(\alpha_i, \alpha_j) \geqslant 0$ then $\chi_j(\alpha_i, \alpha_j) < 0$, meaning that if i is inactive j must be active.

Solving $\chi_j(\alpha_{i2}(\alpha_j), \alpha_j) = 0$ we define $\alpha_{i2}(\alpha_j) = \frac{n_i}{n_i - 1}\left[\frac{1-p}{p} + \frac{(n_j - 1)\alpha_j + 1}{n_j}\right]$. Given that $\chi_j(\alpha_i, \alpha_j)$ is strictly increasing in α_i, $\alpha_{i2}(\alpha_j)$ is the minimum value of α_i which guarantees that members of group j are inactive in the effort stage for a given α_j.

Solving $\chi_i(\alpha_{i3}(\alpha_j), \alpha_j) = 0$ we define $\alpha_{i3}(\alpha_j) = \frac{n_j p + n_i[n_j + \alpha_j p - (\alpha_j + 1)n_j p]}{(1 - n_i)n_j p}$. Given that $\chi_i(\alpha_i, \alpha_j)$ is strictly decreasing in α_i then for all $\dot{\alpha}_i \in [0, \alpha_{i3}(\alpha_j)]$ it also holds that $\chi_i(\dot{\alpha}_i, \alpha_j) \geqslant 0$ and thus group i is inactive.

Given that $\alpha_{i2}(\alpha_j) > \alpha_{i3}(\alpha_j)$ we can write the expected utility as follows:

$$EU_i(\alpha_i) = \begin{cases} 0 & \text{if } 0 \leq \alpha_i \leq \alpha_{i3}(\alpha_j) \\ \hat{EU}_i(\alpha_A, \alpha_B) & \text{if } \alpha_{i3}(\alpha_j) < \alpha_i < \alpha_{i2}(\alpha_j) \\ \tilde{EU}_i(\alpha_i) & \text{if } \alpha_i \geq \alpha_{i2}(\alpha_j) \end{cases}$$

Notice that $EU_i(\alpha_i)$ is a continuous function since from the above expressions

$$\lim_{\alpha_i \to \alpha_{i3}(\alpha_j)} \hat{EU}_i(\alpha_A, \alpha_B) = 0 \text{ and } \lim_{\alpha_i \to \alpha_{i2}(\alpha_j)} \hat{EU}_i(\alpha_A, \alpha_B) = \tilde{EU}_i(\alpha_i)$$

Notice that $\tilde{EU}_i(\alpha_i)$ is strictly decreasing in α_i, while $\hat{EU}_i(\alpha_A, \alpha_B)$ is a strictly concave function (the second derivative is $\frac{2n_B(n_A-1)^2[n_B(p-1)-p]p^2V}{n_A[n_Bp+n_A(2n_B(1-p)+p)]^2} < 0$). Thus, in the unrestricted domain $\alpha_i \in (-\infty, +\infty)$, $\hat{EU}_i(\alpha_A, \alpha_B)$ attains a maximum at

$$\alpha_{i1}(\alpha_j) = \frac{n_j\left[n_i(1-p)+p\right]\left[n_j(2-2n_i(1-p)-3p)-(n_i-2)p\right]-(n_j-1)(n_i-n_j)p^2\alpha_j}{2(n_i-1)n_j\left[n_j(p-1)-p\right]p} \tag{6}$$

If $\alpha_{i1}(\alpha_j) \leq \alpha_{i3}(\alpha_j)$, it follows from the above observations that $EU_i(\alpha_i)$ is strictly decreasing in α_i for all $\alpha_i > \alpha_{i3}(\alpha_j)$ and therefore negative. In that case, $max_{\alpha_i} EU_i(\alpha_i) = 0$ which is attained at any $\dot{\alpha}_i \in [0, \alpha_{i3}(\alpha_j)]$. Comparing the expressions for $\alpha_{i1}(\alpha_j)$ and $\alpha_{i3}(\alpha_j)$, it holds that $\alpha_{i1}(\alpha_j) \leq \alpha_{i3}(\alpha_j)$ if and only if

$$\alpha_j \geq \frac{n_j}{n_j-1}\frac{n_i(1-p)+p}{p} = \hat{\alpha}_j$$

If $\alpha_{i1}(\alpha_j) \geq \alpha_{i2}(\alpha_j)$, it follows from strict concavity of $\hat{EU}_i(\alpha_A, \alpha_B)$ that $max_{\alpha_i} EU_i(\alpha_i) = max_{\alpha_i} \tilde{EU}_i(\alpha_i)$. Since, $\tilde{EU}_i(\alpha_i)$ is strictly decreasing in α_i, then $EU_i(\alpha_i)$ has a unique global maximum at $\alpha_{i2}(\alpha_j)$. Comparing, $\alpha_{i1}(\alpha_j)$ and $\alpha_{i2}(\alpha_j)$, it holds that $\alpha_{i1}(\alpha_j) \geq \alpha_{i2}(\alpha_j)$ if and only if

$$\alpha_j \leq \frac{n_j[n_i(1-p)+3p-2]-2p}{(n_j-1)p} = \tilde{\alpha}_j$$

Finally, $EU_i(\alpha_i)$ has a unique global maximum at $\alpha_{i1}(\alpha_j)$ if $\alpha_{i3}(\alpha_j) < \alpha_{i1}(\alpha_j) < \alpha_{i2}(\alpha_j)$. Using the above thresholds, the best response of group i can be summarized as

$$\alpha_i(\alpha_j) = \begin{cases} \alpha_{i2}(\alpha_j) & \text{for } \alpha_j \leq \tilde{\alpha}_j \\ \alpha_{i1}(\alpha_j) & \text{for } \tilde{\alpha}_j < \alpha_j < \hat{\alpha}_j \\ \dot{\alpha}_i \in [0, \alpha_{i3}(\alpha_j)] & \text{for } \alpha_j \geq \hat{\alpha}_j \end{cases} \tag{7}$$

If group j selects a very “meritocratic” rule (i.e., $\alpha_j > \hat{\alpha}_j$) then it is not worth for group i to be active, so that it selects $\dot{\alpha}_i$. If on the contrary group j selects a very “egalitarian” rule (i.e., $\alpha_j \leq \tilde{\alpha}_j$), it is then group i that prevents group j from being active. For intermediate values of α_j, group i selects $\alpha_{i1}(\alpha_j)$ and thus both groups are active.

Proof of Proposition 1. The proof relies on the best responses derived in the *Preliminaries* of this appendix. Taking derivatives of $\alpha_{A1}(\alpha_B)$ and $\alpha_{B1}(\alpha_B)$ as presented in (6) we directly obtain the result:
$\frac{\partial \alpha_{A1}(\alpha_B)}{\partial \alpha_B} = \frac{(n_B-1)(n_A-n_B)p}{2(n_A-1)n_B[n_B(1-p)+p]} > 0$, hence α_B is a strategic complement for group A.
$\frac{\partial \alpha_{B1}(\alpha_A)}{\partial \alpha_A} = -\frac{(n_A-1)(n_A-n_B)p}{2n_A(n_B-1)[n_A(1-p)+p]} < 0$, hence α_A is a strategic substitute for group B.

□

Proof of Proposition 2. Maximizing the expected utility of the representative individual of group i in the within-group symmetric equilibrium also maximizes the aggregate welfare of group i. Therefore, in this proof and the subsequent ones, we focus on the sharing rule α_i that maximizes the expected utility of the representative individual in group i, which we denote by $EU_i(\alpha_A, \alpha_B)$.

The best response of group B as presented in (7) is:

$$\alpha_B(\alpha_A) = \begin{cases} \alpha_{B2}(\alpha_A) & \text{for } \alpha_A \leq \tilde{\alpha}_A \\ \alpha_{B1}(\alpha_A) & \text{for } \tilde{\alpha}_A < \alpha_A < \hat{\alpha}_A \\ \dot{\alpha}_B \in [0, \alpha_{B3}(\alpha_A)] & \text{for } \alpha_A \geq \hat{\alpha}_A \end{cases}$$

The expected utility of group A being the leader is given by:

$$EU_A(\alpha_A) = \begin{cases} 0 & \text{if } \alpha_A \leq \tilde{\alpha}_A \\ \hat{EU}_A(\alpha_A, \alpha_{B1}(\alpha_A)) & \text{if } \tilde{\alpha}_A < \alpha_A < \hat{\alpha}_A \\ \tilde{EU}_A(\alpha_A) & \text{if } \alpha_A \geq \hat{\alpha}_A \end{cases}$$

Notice that $EU_A(\alpha_A)$ is a continuous function since

$$\lim_{\alpha_A \to \tilde{\alpha}_A} \hat{EU}_A(\alpha_A, \alpha_{B1}(\alpha_A)) = 0 \text{ and } \lim_{\alpha_A \to \hat{\alpha}_A} \hat{EU}_A(\alpha_A, \alpha_{B1}(\alpha_A)) = \tilde{EU}_A(\hat{\alpha}_A)$$

Moreover, the second derivative of $\hat{EU}_A(\alpha_A, \alpha_{B1}(\alpha_A))$ is $\frac{[p(n_A-1)]^2 V}{2n_A^2[n_A(p-1)-p]} < 0$. Thus, $\hat{EU}_A(\alpha_A, \alpha_{B1}(\alpha_A))$ is a strictly concave function in the unrestricted domain $\alpha_A \in (-\infty, +\infty)$, and obtains a global maximum at $\alpha_A^L = 1 + n_A \frac{1-p}{p}$, where α_A^L is the solution of the first-order condition $\frac{\partial \hat{EU}_A(\alpha_A, \alpha_{B1}(\alpha_A))}{\partial \alpha_A} = 0$. Given that $\tilde{EU}_A(\alpha_A)$ is strictly decreasing with respect to α_A, $\hat{\alpha}_A$ strictly dominates any $\alpha_A > \hat{\alpha}_A$. As $\alpha_A^L > \tilde{\alpha}_A$, it follows that $EU_A(\alpha_A)$ is maximized at $\min\{\alpha_A^L, \hat{\alpha}_A\}$ and thus the corresponding expected utility is strictly positive. Observe then that $\alpha_A^L < \hat{\alpha}_A$ if and only if $p > \frac{n_A(n_A-n_B-1)}{1+n_A(n_A-n_B-1)} = p_1'$. Therefore,

- If $p \leq p_1'$, group A selects $\hat{\alpha}_A = \frac{n_A}{(n_A-1)} \frac{n_B(1-p)+p}{p}$ leading to the inactivity of group B, and group B selects any $\alpha_B \in [0, \frac{n_B(1-p)+p}{p}]$, with the upper bound being the solution of $\alpha_B = \alpha_{B3}(\hat{\alpha}_A)$ guaranteeing the inactivity of group B.

- If $p > p'_1$ there exists a unique equilibrium such that both groups are active. Group A selects $\alpha_A^L = 1 + n_A \frac{1-p}{p}$ and group B selects $\alpha_{B1}(\alpha_A^L)$, given by

$$\alpha_{B1}(\alpha_A^L) = \frac{n_A[n_B(1-p)+p][n_A(2n_B(1-p)-2+3p)+(n_B-2)p]-(n_A-1)(n_A-n_B)p[n_A(1-p)+p]}{2n_A(n_B-1)[n_A(1-p)+p]p} = \alpha_B^F$$

Denoting $\rho_i = \frac{p}{n_i} + (1-p)$ we can rewrite the equilibrium sharing rules as presented in the main text. □

Proof of Proposition 3.

- If $p \leq p_1$ monopolization arises both in the simultaneous and in the sequential cases and there is a continuum of equilibria. By comparing the equilibrium supports of α_B^S and α_B^F we can see that the lower bound of the former is equal to the upper bound of the latter which guarantees that $\alpha_B^F \leq \alpha_B^S$. Similarly, α_A^L is equal to the lowest possible value in the support of α_A^S, which guarantees that $\alpha_A^L \leq \alpha_A^S$. Given that only group A is active, equilibrium aggregate effort is equal to $n_A \frac{\alpha_A p(n_A-1)}{n_A^2} V$, which is strictly increasing in α_A, hence aggregate effort is greater in the simultaneous case.
- If $p_1 < p \leq p'_1$ monopolization only arises in the sequential case. Equilibrium aggregate effort in the simultaneous case where both groups are active is given by

$$E_A^S + E_B^S = \frac{n_A n_B(1-n_A-n_B)+\left[n_A+n_B+2(n_A-3)n_A n_B+2n_A n_B^2\right]p-[2n_B-1+n_A(2+n_B(n_A+n_B-5))]p^2}{n_A[2n_B(p-1)-p]-n_B p} V$$

while equilibrium aggregate effort in the sequential case where group B is inactive is given by $E_A^L = (n_B+p-n_B p)V$. Therefore, equilibrium aggregate effort is greater in the simultaneous case if and only if $\frac{[n_A(p-1)-p][(n_A-n_B-1)n_B(1-p)-p]V}{n_A[2n_B(1-p)+p]+n_B p} \leq 0$. Notice that the denominator is always positive and the first term of the nominator is always negative. The second term of the numerator is positive if and only if $p \leq p_1$ which is not in the analyzed interval. Hence, equilibrium aggregate effort is greater in the sequential case.

- If $p > p'_1$ both groups are active in both the simultaneous and sequential cases. Thus we now have that equilibrium aggregate effort in the sequential case where the large group is the leader is given by $E_A^L + E_B^F = \frac{n_A(n_A+n_B-1)-p-n_A(n_A+n_B-3)p}{2n_A} V$. Therefore, equilibrium aggregate effort is greater in the sequential case if and only if $\frac{(n_A-n_B)[n_A(1+n_A-n_B)(1-p)+p]pV}{2n_A[n_A(2n_B(1-p)+p)+n_B p]} > 0$ which is always true for any $p > 0$.

□

Proof of Proposition 4. Clearly, if the small group is inactive, the GSP cannot occur. If $p > p'_1$, the GSP arises if and only if

$$\alpha_A^L < \frac{n_A - n_B + 2\alpha_B^F n_A(n_B-1)}{2n_B(n_A-1)}$$

Substituting for the equilibrium value of α_A^L and α_B^F from Proposition 1, the above condition reduces to

$$\frac{n_A(n_A-n_B)(p-1)\left[n_A(2n_B(p-1)-p)-n_Bp\right]}{2(n_A-1)n_Bp\left[n_A(1-p)+p\right]} < 0$$

As the denominator is positive, the condition holds if and only if the numerator is negative, which requires $p > 1$. Hence we reach a contradiction.

□

Proof of Proposition 5. The best response of group B as presented in (7) is:

$$\alpha_A(\alpha_B) = \begin{cases} \alpha_{A2}(\alpha_B) & \text{for } \alpha_B \leq \tilde{\alpha}_B \\ \alpha_{A1}(\alpha_B) & \text{for } \tilde{\alpha}_B < \alpha_B < \hat{\alpha}_B \\ \dot{\alpha}_A \in [0, \alpha_{A3}(\alpha_B)] & \text{for } \alpha_B \geq \hat{\alpha}_B \end{cases}$$

The expected utility of group B being the leader is given by:

$$EU_B(\alpha_B) = \begin{cases} 0 & \text{if } \alpha_B \leq \tilde{\alpha}_B \\ \hat{EU}_B(\alpha_{A1}(\alpha_B), \alpha_B) & \text{if } \tilde{\alpha}_B < \alpha_B < \hat{\alpha}_B \\ \tilde{EU}_B(\alpha_B) & \text{if } \alpha_B \geq \hat{\alpha}_B \end{cases}$$

Notice that $EU_B(\alpha_B)$ is a continuous function since

$\lim_{\alpha_B \to \tilde{\alpha}_B} \hat{EU}_B(\alpha_{A1}(\alpha_B), \alpha_B) = 0$ and $\lim_{\alpha_B \to \hat{\alpha}_B} \hat{EU}_B(\alpha_{A1}(\alpha_B), \alpha_B) = \tilde{EU}_B(\hat{\alpha}_B)$

Moreover, the second derivative of $\hat{EU}_B(\alpha_{A1}(\alpha_B), \alpha_B)$ is equal to $\frac{[p(n_B-1)]^2 V}{2n_B^2[n_B(p-1)-p]} < 0$. Thus, $\hat{EU}_B(\alpha_{A1}(\alpha_B), \alpha_B)$ is a strictly concave function in the unrestricted domain $\alpha_B \in (-\infty, +\infty)$, and obtains a global maximum at $\alpha_B^L = 1 + n_B\frac{1-p}{p}$, where α_B^L is the solution of the first-order condition $\frac{\partial \hat{EU}_B(\alpha_{A1}(\alpha_B), \alpha_B)}{\partial \alpha_B} = 0$. Given that $\tilde{EU}_B(\alpha_B)$ is strictly decreasing with respect to α_B, $\hat{\alpha}_B$ strictly dominates any $\alpha_B > \hat{\alpha}_B$. Observe that $\alpha_B^L < \hat{\alpha}_B$ always holds, while $\alpha_B^L \leq \tilde{\alpha}_B$ if and only if $p \leq p_1$. Therefore,

- If $p \leq p_1$, group B selects any $\alpha_B \in \left[0, \frac{n_B[n_A(1-p)+3p-2]-2p}{(n_B-1)p}\right]$ so that it remains inactive, and group A selects $\alpha_{A2}(\alpha_B) = \frac{n_A}{n_A-1}\left[\frac{1-p}{p} + \frac{(n_B-1)\alpha_B+1}{n_B}\right]$.
- If $p > p_1$ there exists a unique equilibrium such that both groups are active. Group B selects $\alpha_B^L = 1 + n_B\frac{(1-p)}{p}$ and group A selects $\alpha_{A1}(\alpha_B^L)$, given by

$$\alpha_{A1}(\alpha_B^L) = \frac{n_B[n_A(1-p)+p][n_B(2n_A(1-p)-2+3p)+(n_A-2)p]+(n_B-1)(n_A-n_B)p[n_B(1-p)+p]}{2(n_A-1)n_B[n_B(1-p)+p]p} = \alpha_A^F$$

Denoting $\rho_i = \frac{p}{n_i} + (1-p)$ we can rewrite the equilibrium sharing rules as presented in the main text.

$\square$

Proof of Proposition 6.

- If $p \leq p_1$ monopolization arises both in the simultaneous and in the sequential cases and there is a continuum of equilibria. By comparing the equilibrium supports of $\alpha_B^L \in [0, \frac{n_B[n_A(1-p)+3p-2]-2p}{(n_B-1)p}]$ and $\alpha_B^S \in [\frac{n_B(1-p)+p}{p}, \frac{n_B[n_A(1-p)+3p-2]-2p}{(n_B-1)p}]$ we can see that their upper bounds coincide. Therefore, $\alpha_B^L < \alpha_B^S$ for all $\alpha_B^L < \frac{n_B(1-p)+p}{p}$. As under monopolization group A's equilibrium sharing rule is strictly increasing in α_B, it follows that $\alpha_B^L < \alpha_B^S$ implies $\alpha_A^F < \alpha_A^S$. Given that only group A is active, equilibrium aggregate effort is equal to $n_A \frac{\alpha_A p(n_A-1)}{n_A^2} V$ which is strictly increasing in α_A, hence aggregate effort is strictly greater in the simultaneous case for any $p \leq p_1$ and $\alpha_B^L < \frac{n_B(1-p)+p}{p}$.

  When the supports of the two equilibria overlap (i.e., for $\alpha_B^L \geq \frac{n_B(1-p)+p}{p}$), α_B^L can be either greater or smaller than α_B^S. Following the same reasoning as before, we have that aggregate effort in the sequential case can be either greater or smaller than in the simultaneous case depending on the specific value of α_B one considers in each game.

- If $p > p_1$ both groups are active in both the simultaneous and sequential cases. Equilibrium aggregate effort in the simultaneous case is given by

$$E_A^S + E_B^S = \frac{n_A n_B(1-n_A-n_B)+\left[n_A+n_B+2(n_A-3)n_A n_B+2n_A n_B^2\right]p-[2n_B-1+n_A(2+n_B(n_A+n_B-5))]p^2}{n_A[2n_B(p-1)-p]-n_B p} V$$

  while equilibrium aggregate effort in the sequential case where the small group is the leader is given by $E_A^F + E_B^L = \frac{n_B(n_A+n_B-1)-p-n_B(n_A+n_B-3)p}{2n_B} V$. Therefore, equilibrium aggregate effort is strictly greater in the simultaneous case if and only if $\frac{(n_A-n_B)p[(n_A-n_B-1)n_B(p-1)+p]V}{2n_B[n_A(2n_B(1-p)+p)+n_B p]} > 0$. Given that the denominator is positive, this requires that the numerator is also positive, hence that $[(n_A-n_B-1)n_B(p-1)+p] > 0$, which is true if and only if $p > p_1$. Consequently, aggregate effort in the sequential case is strictly smaller than in the simultaneous case for any $p > p_1$.

$\square$

Proof of Proposition 7. Clearly, if the small group is inactive, the GSP cannot occur. If $p > p_1$, the GSP arises if and only if

$$\alpha_A^F < \frac{n_A - n_B + 2\alpha_B^L n_A(n_B-1)}{2n_B(n_A-1)}$$

Substituting for the equilibrium value of α_A^F and α_B^L from Proposition 3, the above condition reduces to

$$\frac{(n_A - n_B)(p-1)\left[n_A(2n_B(p-1)-p) - n_B p\right]}{2(n_A-1)p\left[n_B(1-p)+p\right]} < 0$$

As the denominator is positive, the condition holds if and only if the numerator is negative, which requires $p > 1$. Hence we reach a contradiction.

□

Proof of Proposition 8.

- $p \leq p_1$

We are in the case where monopolization occurs regardless of the particular timing of the game. As we are looking for the payoff-dominant equilibrium of the timing game, we shall assume that group B selects the minimum sharing rule out of the continuum of equilibria for each possible timing. Then we have that under the simultaneous case

$$\alpha_A^S = \tfrac{n_A[n_B(1-p)+p]}{(n_A-1)p} \text{ and } \alpha_B^S = \tfrac{n_B(1-p)+p}{p}$$

If the large group A is the leader we have

$$\alpha_A^L = \tfrac{n_A[n_B(1-p)+p]}{(n_A-1)p} \text{ and } \alpha_B^F = 0$$

If the large group B is the leader we have

$$\alpha_A^F = \tfrac{n_A[n_B(1-p)+p]}{(n_A-1)n_B p} \text{ and } \alpha_B^L = 0$$

Comparing utility levels across the different timings for the two groups yields

$$EU_A^S = EU_A^L \text{ and } EU_A^S - EU_A^F = \tfrac{(n_B-1)[n_B(p-1)-p]V}{n_A n_B} < 0$$

$$EU_B^S = EU_B^L = E_B^F = 0$$

Let (i, j) denote a Nash equilibria of the timing game, where i (j) is the action of group A (B), that is, $i, j = L, F$. Given the above, there will be three Nash equilibria of the timing game: (L, F),(F, L) and (F, F). Out of these three equilibria, the one where group A achieves the highest payoff is (F, L) (i.e., B is the leader). As we assumed that group B selects the smallest α_B out of the equilibrium support for any possible timing, the equilibrium (F, L) is clearly the payoff-dominant one (i.e., the one such that A achieves the highest payoff).

- $p_1 < p \leq p_1'$

We are in the case where monopolization occurs when the large group A is the leader. Equilibrium sharing rules under the simultaneous game are given by

$$\alpha_i^S = \frac{n_i}{(n_i-1)} \frac{2n_i n_j(n_i-1) + C_i p + D_i p^2}{p\left[2n_i n_j - p(n_i(2n_j-1)-n_j)\right]}$$

where $C_i = n_i n_j(9-4n_i) - n_j(n_j+2) - 2n_i$ and $D_i = n_i n_j(2n_i-7) + n_j(n_j+3) + 3n_i - 2$. If the large group A is the leader, we know that $\alpha_A^L = \frac{n_A[n_B(1-p)+p]}{(n_A-1)p}$ for any value of α_B^F. Finally, if the small group B is the leader, we have

$$\alpha_A^F = \frac{n_B\left[n_A(1-p)+p\right]\left[n_B(2-2n_A(1-p)-3p)-(n_A-2)p\right]-(n_B-1)(n_A-n_B)p\left[n_B(1-p)+p\right]}{2(n_A-1)n_B\left[n_B(p-1)-p\right]p}$$

$$\alpha_B^L = \frac{n_B(1-p)+p}{p}$$

Given that $\alpha_i^L \neq \alpha_i^S$ for $i = A, B$, it follows directly that $EU_i^L > EU_i^S$ for $i = A, B$. We then have that

$$EU_A^S - EU_A^F = \frac{\left\{\left[n_B(n_B-n_A-1)(p-1)+p\right]^2 - \frac{4n_B^2\left[n_A(1+n_A-n_B)(1-p)+p\right]^2(n_B+p-n_Bp)^2}{\left[n_Bp+n_A(2n_B(1-p)+p)\right]^2}\right\}V}{4n_A n_B\left[n_B(p-1)-p\right]}$$

Isolating p in the previous expression, $EU_A^S - EU_A^F \geq 0$ can be written as $p \leq p_1$, hence $EU_A^S < EU_A^F$ for any $p > p_1$.

$$EU_B^S - EU_B^F = \frac{n_A\left[n_A(1-p)+p\right]\left[(n_A-n_B-1)n_B(p-1)+p\right]^2 V}{n_B\left[n_Bp+n_A(2n_B(1-p)+p)\right]^2} > 0$$

Given the above, the unique Nash equilibrium of the timing game is (F, L), that is, B is the leader.

- $p > p_1'$

We are in the case where both groups are active regardless of the particular timing of the game. With respect to the previous case where $p \in (p_1, p_1']$, the only difference is that if the large group A is the leader, we now have

$$\alpha_A^L = \frac{n_A(1-p)+p}{p}$$

$$\alpha_B^F = \frac{n_A[n_B(1-p)+p][n_A(2n_B(1-p)-2+3p)+(n_B-2)p]-(n_A-1)(n_A-n_B)p[n_A(1-p)+p]}{2n_A(n_B-1)[n_A(1-p)+p]p}$$

Given that $\alpha_i^L \neq \alpha_i^S$ for $i = A, B$, it follows directly that $EU_i^L > EU_i^S$ for $i = A, B$.

We then have that

$$EU_A^S - EU_A^F = -\frac{\left\{ [n_B(n_B - n_A - 1)(p-1)+p]^2 - \frac{4n_B^2[n_A(1+n_A-n_B)(1-p)+p]^2(n_B+p-n_Bp)^2}{[n_Bp+n_A(2n_B(1-p)+p)]^2} \right\} V}{4n_An_B\left[n_B(1-p)+p\right]}$$

Isolating p in the previous expression, $EU_A^S - EU_A^F \geq 0$ can be written as $p \leq p_1$. Given that $p_1 < p_1'$, it holds that $EU_A^S < EU_A^F$ for any $p > p_1'$. Finally, we have that

$$EU_B^S - EU_B^F = -\frac{\left\{ [n_A(n_A - n_B - 1)(p-1)+p]^2 - \frac{4n_A^2[(n_A-n_B-1)n_B(p-1)+p]^2(n_A+p-n_Ap)^2}{[n_Bp+n_A(2n_B(1-p)+p)]^2} \right\} V}{4n_An_B\left[n_A(1-p)+p\right]}$$

Isolating p in the previous expression, $EU_B^S - EU_B^F \leq 0$ can be written as

$$p \leq \frac{1}{2}\left\{ \frac{n_A[n_B(5+3n_B)+n_A^2(8n_B-1)+n_A(3-2n_B(5+4n_B))]}{n_A[n_B(5+3n_B)-3+n_A^2(4n_B-1)+n_A(3-2n_B(3+2n_B))]-n_B} - \sqrt{\frac{n_A^2(n_A-n_B)^2[9+n_A^2+6n_A(n_B-1)+n_B(14+9n_B)]}{[n_B+n_A(3+(n_A-3)n_A-5n_B+2(3-2n_A)n_An_B+(4n_A-3)n_B^2)]^2}} \right\} = p^M$$

As $p^M < p_1'$, it must be the case that $EU_B^S > EU_B^F$ for any $p > p_1'$.

Given the above, the unique Nash equilibrium of the timing game is (F, L), that is, B is the leader. □

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