

Revealing Infinite Derivative Gravity’s true potential: The weak-field limit around de Sitter backgrounds

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General Relativity is known to produce singularities in the potential generated by a point source. Our universe can be modelled as a de Sitter (dS) metric and we show that ghost-free Infinite Derivative Gravity (IDG) produces a non-singular potential around a dS background, while returning to the GR prediction at large distances. We also show that although there are an apparently infinite number of coefficients in the theory, only a finite number actually affect the predictions.

By writing the linearised equations of motion in a simplified form, we find that at distances below the Hubble length scale, the difference between the IDG potential around a flat background and around a de Sitter background is negligible.

General Relativity (GR) has been extremely successful in describing gravity at large distances [1], in particular, the recent observation by LIGO & VIRGO of the collision of two neutron stars [2]. However, GR breaks down at short distances because it predicts singularities in both black holes and the cosmological setting [3, 4].

An obvious way to ameliorate these problems was to add higher derivative terms to the GR action, for example Stelle’s 4th derivative theory [5], or higher derivative gravity models [6, 7] did not succeed because of the Ostrogradsky instability [8], which produces an unstable vacuum [9–11]. However this instability is avoided for Infinite Derivative Gravity (IDG). IDG modifies the propagator so that it is exponentially suppressed at high energies, but doesn’t produce ghosts

Infinite derivative actions constructed from the d’Alembertian operator $\square = g^{\mu\nu}\nabla_\mu\nabla_\nu$, were first used in string theory [12], and later employed in gravity [13]. They have been examined around a flat Minkowski background [14] and constantly curved backgrounds [15].

So that the only pole in the graviton propagator is the benign pole found in GR, we can require that any infinite derivative function in the denominator of the propagator is the *exponential of an entire function* which by definition contains no zeroes [13–19]. The mass scale M of the theory determines the length scales below which the infinite derivative terms start to have a significant effect. The prospect of avoiding singularities through IDG has prompted much investigation over recent years [13–61].

Previous work has shown that IDG gives a non-singular potential for a test mass in a flat background, which returns to the observed GR prediction at large distances [14, 24, 26]. Even though this calculation was made using the linearised equations of motion, if the test mass is small enough then the perturbation will still be in the linear regime. This is because the effect of IDG is to stop the potential from growing in size once we go reach the distances where IDG has a significant effect.

We extend the prediction of a non-singular potential, which returns to GR at large distances, to a curved background. We do this by writing down the equations of

motion in a simplified form and noting that in the areas we might wish to test IDG, such as laboratory experiments or solar system tests, we can use the approximation $H^2r^2 \ll 1$ [62].

EQUATIONS OF MOTION

We will investigate the action¹

$$S = \int d^4x \frac{\sqrt{-g}}{2} \left[M_P^2 R + R F_1(\square) R + R_{\mu\nu} F_2(\square) R^{\mu\nu} + \Lambda \right], \quad (1)$$

where $R_{\mu\nu}$ is the Ricci curvature tensor and R is the Ricci scalar. The $F_i(\square)$ are the infinite sums $F_i(\square) = \sum_{n=0}^{\infty} f_{i_n} \frac{\square^n}{M^{2n}}$, where $\{f_{i_0}, f_{i_1}, f_{i_2}, \dots, f_{i_n}\}$ are the dimensionless coefficients of the series and M is the mass scale of the theory. The lower bound on M is 0.004 eV from laboratory experiments [24, 64], while the upper bound is the Planck mass M_P . Statistical analysis showed that the best fit of the IDG prediction matched the data better than the GR prediction by around 2σ [65].

It was shown in [14, 24, 26] that this action generates a non-singular potential around a flat background for a static test mass.

For the action (1), IDG gives a vacuum de Sitter solution with $\Lambda = 3M_P^2 H^2$ [15, 59], where the background Ricci scalar is $\bar{R} = 12H^2$. The linear equations of motion around this de Sitter background can be written as²

$$\frac{1}{M_P^2} T_\nu^\mu = a(\bar{\square}) r_\nu^\mu - \frac{1}{2} \delta_\nu^\mu c(\bar{\square}) r - \frac{1}{2} \nabla^\mu \partial_\nu f(\bar{\square}) r, \quad (2)$$

where the functions $a(\square)$, $c(\square)$, $f(\square)$, given in (15) are combinations of $F_1(\square)$ and $F_2(\square)$ from the action, r_ν^μ

¹ More generally, there is also a Weyl tensor term $C_{\mu\nu\lambda\sigma} F_3(\square) C^{\mu\nu\lambda\sigma}$ but it is possible to set $F_3(\square) = 0$ without loss of generality [58, 59]. This action is the most general torsion-free action which is quadratic in curvature.

² It should be noted that this definition of a , c and f allows the minimum condition for perturbations around an (A)dS background to avoid Penrose singularities in [28] to be extended to include a non-zero $F_2(\square)$.

and r are the perturbed Ricci tensor and Ricci scalar respectively and \square is the background d'Alembertian.

DEGREES OF FREEDOM

If $a(\square) = c(\square)$ then around a flat background, the propagator is given by

$$\Pi_{\text{IDG}} = \frac{\Pi_{\text{GR}}}{a(-k^2)}, \quad (3)$$

where $\square \rightarrow -k^2$ in momentum space. We can choose $a(-k^2) = e^{\gamma(k^2/M^2)}$ where $\gamma(k^2/M^2)$ is an entire function. This choice means that the propagator has no extra poles compared to GR and also reduces to GR in the limit $M \rightarrow \infty$. An entire function can be written as a polynomial, i.e. $\gamma(k^2/M^2) = \sum_{n=1}^{\infty} c_n \frac{k^{2n}}{M^{2n}}$. Therefore the effect of IDG on the potential, which depends on $e^{-\gamma(k^2/M^2)}$ can be written as

$$e^{-\gamma(k^2/M^2)} = e^{-c_1 \frac{k^2}{M^2}} \cdot e^{-c_2 \frac{k^4}{M^4}} \cdot e^{-c_3 \frac{k^6}{M^6}}. \quad (4)$$

This would a priori seem to have an infinite number of degrees of freedom as there is no restriction on the coefficients c_n . It was already shown that as long as $\gamma(k^2/M^2) > 0$ for $k \rightarrow \infty$ that IDG would produce a non-singular solution [14]. However, we can reduce the freedom in the model further by noting that for large n and positive c_n , $\exp\left[-c_n \left(\frac{k^2}{M^2}\right)^n\right]$ forms a rectangle function:

$$e^{-c_n \left(\frac{k^2}{M^2}\right)^n} \approx \begin{cases} 1, & \text{for } \frac{\sqrt[n]{c_n k^2}}{M^2} < 1 \\ 0, & \text{for } \frac{\sqrt[n]{c_n k^2}}{M^2} > 1 \end{cases}. \quad (5)$$

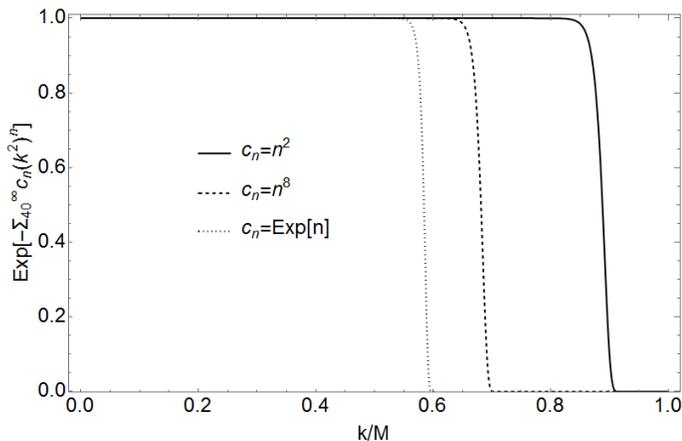


FIG. 1. We plot $\exp\left[-\sum_{n=40}^{\infty} c_n \left(\frac{k^2}{M^2}\right)^n\right]$ for various c_n to show that even if c_n increases very quickly, we can model the higher terms in the polynomial γ as the rectangle function $\text{Rect}(Ck^2/M^2)$ without knowing the exact values of c_n . Here C is simply another constant to be found.

We can combine all of the terms in the exponential which involve large values of n into a rectangle function and thus we do not need to know the value of c_n

for large n as long as it is positive. As an example, we plot $\exp\left[-\sum_{n=40}^{\infty} c_n \left(\frac{k^2}{M^2}\right)^n\right]$ for various values of c_n in Fig. 1 which turns out to still be well approximated by the rectangle function $\text{Rect}(Ck^2/M^2)$ even if c_n increases exponentially. In other words, the potential is well described by the lowest order terms and a single constant representing the higher order terms.³

PERTURBING THE METRIC

Following the example of [62], we write the perturbation to the background de Sitter metric as

$$ds^2 = -\left(1 + 2\Phi(r) - H^2 r^2\right) dt^2 + \left(1 - 2\Psi(r)\right) \left[\left(1 + H^2 r^2\right) dr^2 + r^2 d\Omega^2\right], \quad (6)$$

where $\Phi(r)$ and $\Psi(r)$ are perturbations we want to find. As we want to look at perturbations at relatively short distances such as when we are looking at laboratory or solar system tests of gravity, we can use the approximation $H^2 r^2 \ll 1$ to find the Ricci curvature tensors and Ricci scalar up to linear order in $\Phi(r)$ and $\Psi(r)$

$$R_{tt} \approx \Delta\Phi(r) - 3H^2(1 + 2\Psi(r) - \Phi(r)), \quad (7)$$

$$R_{rr} \approx \frac{2(r\Psi''(r) + \Psi'(r))}{r} - \Phi''(r) + H^2 \left[3 - r\Phi'(r) - 2\Phi(r) + 4\Psi(r)\right], \quad (8)$$

$$R \approx 4\Delta\Psi(r) - 2\Delta\Phi(r) + 12H^2(1 + 3\Psi(r) - \Phi(r)). \quad (9)$$

We note that in the limit $H^2 r^2 \ll 1$, for any scalar $X(r)$ which is a function only of the radial coordinate [62],

$$\square X(r) \approx \Delta X(r), \quad (10)$$

where $\Delta \equiv \eta^{ij} \nabla_i \partial_j = \partial_r^2 + \frac{2}{r} \partial_r$ is the spatial d'Alembertian operator around a flat background. This gives us a tremendous simplification to our equations of motion, as we will see later.

SOLVING THE EQUATIONS OF MOTION

We investigate a perturbation caused by the addition of a static point mass with density $\rho = m \delta^3(\mathbf{x})$ where m is the mass of the source and negligible pressure $P \approx 0$. Using the equations of motion and $T_\mu^\mu = -\rho = T_0^0$, we find Ψ in terms of Φ in (16) and therefore find Φ in terms of our density ρ in (17).

³ We need in the region of $\mathcal{O}(10)$ of the lower order terms depending on how accurate one needs to be.

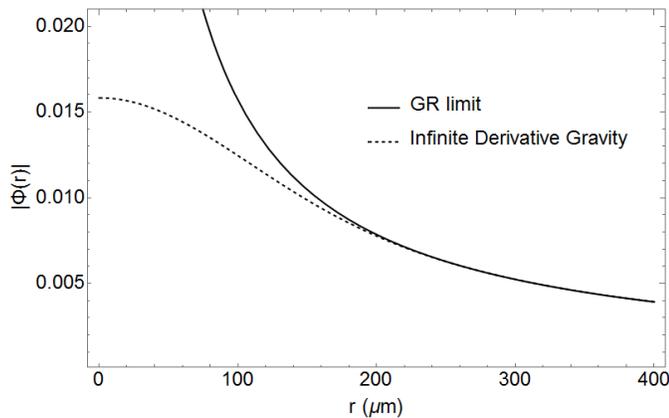


FIG. 2. We plot the perturbation to the metric $\Phi(r)$ versus distance r using (18), giving exactly the same result as for flat backgrounds. We have used the Hubble constant $H = 7.25 \times 10^{-27} \text{ m}^{-1}$, set the Planck mass M_P and the mass of the source equal to one and used the IDG mass scale $M = 1.79 \times 10^4 \text{ m}^{-1}$. We have chosen the coefficients f_n such that around a flat background $a = c = e^{-\square/M^2}$, as in [24]. As for flat backgrounds, we have the familiar prediction $\Phi(r) \propto -\frac{\text{Erf}(r)}{r}$ [14].

Using a similar method to [14] we can go into momentum space to find

$$\Phi(r) = \frac{m}{4\pi^2 M_P^2 r} \int_{-\infty}^{\infty} dk I(k). \quad (11)$$

where $I(k)$, a complicated function of H , $a(\square)$, $c(\square)$ and $f(\square)$ is given in (18).

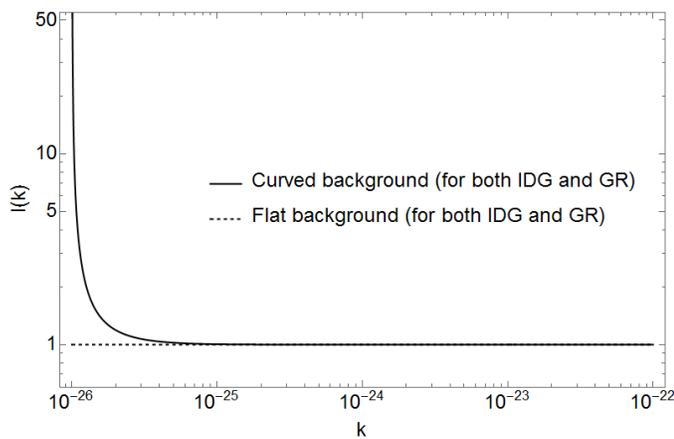


FIG. 3. We plot the difference from a flat background for the integrand $I(k)$ given in (18) for $H = 7.25 \times 10^{-27} \text{ m}^{-1}$ and $r = 1 \text{ m}$. The graph is the same whether we take GR or IDG with $M = 1.79 \times 10^4 \text{ m}^{-1}$. Note that for $k \gg H$, $I(k)$ is similar to if we had a flat background.

For the value of H at the current time $H = H_0 = 7.25 \times 10^{-27} \text{ m}^{-1}$, the result is very similar to the result around a flat background, and we plot this in Fig. 2. Perhaps

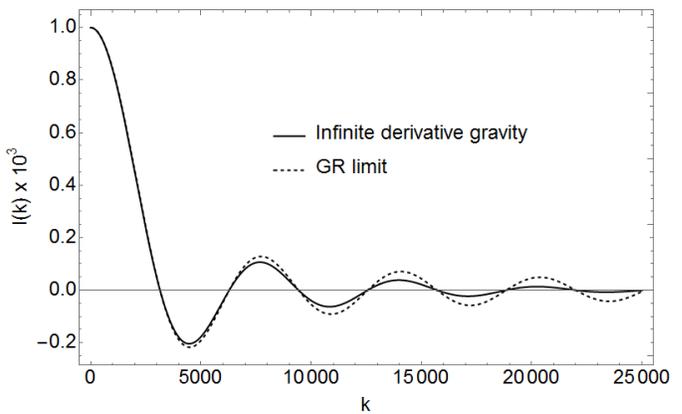


FIG. 4. We plot the the integrand $I(k)$ given in (18) for $H = 7.25 \times 10^{-27} \text{ m}^{-1}$, $M = 1.79 \times 10^4 \text{ m}^{-1}$, $r = 1 \text{ m}$ to show that if $k < 10^3$, the difference from GR is negligible. The chosen value of M is the lower bound found by experiment [24, 64] and coefficients f_n such that around a flat background $a = c = e^{-\square/M^2}$ as in [24]. For higher M , IDG matches GR up to even higher energy scales. The point at IDG stops being a good approximation for the GR prediction of $I(k)$ is unaffected by the choice of r .

this is to be expected - the differences from GR kick in only once we reach $k \approx M$, and our mass scale is at a significantly higher energy than the Hubble parameter, i.e. for large k , the background curvature has a negligible effect - for $H \ll k$ then the integrand is the same as for a flat background. We illustrate this by plotting in Fig. 3 and Fig. 4 the value of the integrand for small and large k , choosing our value of M to be the lower bound given by experiments [24].

Note that increasing r decreases the value of k in the region where it provides the biggest contribution to the integral.⁴ Therefore for distances above the length scale where IDG contributes, around 10^{-5} m , we can use the GR prediction, and for distances at this length scale or below, we can use the IDG prediction around a flat background. Therefore we still obtain a non-singular potential even when we look at a curved background. At distances larger than the mass scale of IDG, the system returns to the standard de Sitter-Schwarzschild metric.

DISCUSSION

It was already shown that IDG gives a non-singular potential for a test mass added to a flat background, which returns to the GR prediction at large distances. Intuitively, one might have predicted this by noting that IDG has a significant effect at much larger energy scales

⁴ Due to the $\sin(kr)$ term, the part of the integrand where $k \ll r^{-1}$ can be neglected.

than the curvature of our de Sitter universe. Here we have shown explicitly that in the region where the background curvature must be taken into account IDG has no effect, and where IDG has an effect the background curvature has no effect.

If we want to examine a system where H is much larger than it is today, so that the background curvature can no longer be discarded, then it is possible to solve (11) numerically, as long as $H^2 r^2 \ll 1$ still holds.

Could our result be extended to more general backgrounds? It is extremely difficult to use the equations of IDG where we encounter derivatives in more than one coordinate, but it is reasonable to guess that if the background curvature is at an energy much smaller than the energy scale where IDG becomes important, then we will see a similar effect.

CONCLUSION

We have shown that the non-singular IDG potential that is found around a flat background also extends to a curved de Sitter background. It still reverts to the GR limit at large distances. We also show that only a finite number of the degrees of freedom in the theory actually contribute to the potential.

IDG is a promising model because it is ghost-free while allowing the possibility of avoiding singularities as well as providing a framework for inflation. This result shows that we can generate a non-singular potential even when we are looking at non-flat backgrounds, and hints that we will see similar effects when looking at more general backgrounds, based on examining the energy scales involved.

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APPENDIX

Commutation relations

We want to write the linear equations of motion around a constantly curved background which is maximally symmetric, i.e. $\bar{R}^\mu{}_{\nu\rho\sigma} = \frac{\bar{R}}{12} (\delta^\mu_\rho g_{\nu\sigma} - \delta^\mu_\sigma g_{\nu\rho})$ and $\bar{R}_{\mu\nu} = \frac{\bar{R}}{4} g_{\mu\nu}$.

We use the relations from the appendix of [39] and also derive that for a scalar S ,

$$\begin{aligned} \nabla^\mu F(\square) \nabla_\nu S &= F(\square - 5H^2) \nabla^\mu \nabla_\nu S \\ &+ \frac{1}{4} (F(\square + 3H^2) - F(\square - 5H^2)) \delta^\mu_\nu \square S, \end{aligned} \quad (12)$$

and that for a symmetric tensor $t_{\mu\nu}$

$$\nabla_\mu F(\square) t^{\mu\nu} = F(\square + 5H^2) \nabla_\mu t^{\mu\nu} - 2H^2 X(\square) \nabla_\nu t^\mu{}_\mu, \quad (13)$$

where we have defined

$$X(\square) \equiv \sum_{n=1}^{\infty} f_n \sum_{m=0}^{n-1} \left[(\square + 5H^2)^m (\square - 3H^2)^{n-1-m} \right]. \quad (14)$$

Full equations of motion

Using the commutation relations above, we can write the equations of motion as (2) with

$$\begin{aligned} a(\square) &\equiv 1 + 24M_p^{-2} H^2 \tilde{f}_{1_0} + (\square - 2H^2) M_p^{-2} F_2(\square), \\ c(\square) &\equiv 1 + M_p^{-2} \left\{ 24H^2 \tilde{f}_{1_0} - 4(\square + 3H^2) F_1(\square) \right. \\ &\quad \left. - \frac{1}{2} F_2(\square + 8H^2) \square - \frac{1}{2} F_2(\square) (\square + 8H^2) + 4H^2 \square F'_2(\square) \right\}, \\ f(\square) &\equiv M_p^{-2} (4F_1(\square) + 2F_2(\square) - 8H^2 X(\square - 5H^2)), \end{aligned} \quad (15)$$

and we can therefore find Ψ in terms of Φ

$$\frac{\Psi(r)}{\Phi(r)} = \frac{a(\Delta)(\Delta + 3H^2) - (3c(\Delta) + \Delta f(\Delta)) (\Delta + 6H^2)}{a(\Delta)(4\Delta + 30H^2) - 2(3c(\Delta) + \Delta f(\Delta)) (\Delta + 9H^2)}, \quad (16)$$

and therefore find Φ in terms of our density ρ

$$\Phi(r) = \frac{\rho (a(\Delta)(2\Delta + 15H^2) - (3c(\Delta) + \Delta f(\Delta)) (\Delta + 9H^2))}{M_P^2 a(\Delta) [2a(\Delta) - 4c(\Delta) - \Delta f(\Delta)] (\Delta^2 + 15H^2 \Delta + 63H^4)}. \quad (17)$$

By going into momentum space, i.e. sending $\Delta \rightarrow -k^2$, taking a Fourier transform and using $\rho = m \delta^3(\mathbf{x})$, we can write $\Phi(r)$ as an integral we can solve

$$\Phi(r) = \frac{m}{4\pi^2 M_P^2 r} \int_{-\infty}^{\infty} dk k \sin(kr) \frac{a(-k^2)(-4k^2 + 30H^2) - 2(3c(-k^2) - k^2 f(-k^2)) (-k^2 - 18H^2)}{a(-k^2)(2a(-k^2) - 4c(-k^2) + k^2 f(-k^2)) (k^4 - 15H^2 k^2 + 63H^4)}. \quad (18)$$

which is written in the main text as (11).

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