Debt Dynamics in Europe:
A Network General Equilibrium GVAR Approach

Panayotis G. Michaelides

Systemic Risk Centre, London School of Economics, United Kingdom
National Technical University of Athens, Greece

Efthymios G. Tsionas

Lancaster University Management School, United Kingdom
Athens University of Economics and Business, Greece

Konstantinos N. Konstantakis

National Technical University of Athens

Contact Author: Dr. Panayotis G. Michaelides, Associate Professor; Laboratory of Theoretical & Applied Economics; School of Applied Mathematics & Physics; National Technical University of Athens, Greece; Heroon Polytechnéiou 9, 157.80, Zografou Campus; Tel: 302107721624, fax: +302107721618, email: pmichael@central.ntua.gr.
Debt Dynamics in Europe: 
A Network General Equilibrium GVAR Approach

ABSTRACT: In this work, we investigate the dynamic interdependencies among the EU12 economies using a competitive general equilibrium network system representation. Additionally, using Bayesian techniques, we estimate the autoregressive scheme that characterizes the equilibrium price system of the network, while characterizing each economy/node in the universe of our network in terms of its degree of pervasiveness. In this context, we unveil the dominant(s) unit(s) in our model and estimate the dynamic linkages between the economies/nodes. Lastly, in terms of robustness analysis, we compare the findings of the degree pervasiveness of each economy against other popular quantitative methods in the literature. According to our findings, the economy of Germany acts as weakly dominant entity in the EU12 economy. Meanwhile, all shocks die out in the short run, without any long lasting effect.

JEL Classification: E1, O5.

Keywords: Bayesian, GVAR, Crisis, Transmission, Debt, EU12

---

1 We would like to thank the Editor and reviewers of the Special Issue for their careful reading of the manuscript and their constructive feedback. The usual disclaimer applies.
1. Introduction

The investigation of the debt dynamics as a crucial macroeconomic variable, both theoretically and empirically, has always been a key research topic for many researchers around the globe. In fact, debt as a key macroeconomic variable, as well as its linkages with other macroeconomic indicators, was first presented in a seminal paper by Fisher (1933). Over the years a vast literature has emerged. See, for instance, Blinder and Solow (1975), Dixit (1976) and Feldstein (1976). Barro (1979) in a seminal contribution developed a debt theory which incorporated the Ricardian invariance theorem.

The related literature on debt dynamics has come a long way, especially over the last years, both empirically and theoretically, mainly due to the increasing globalization of certain markets, as well as due to the formation of the European Monetary Union (EMU). In this context, the steady state of debt under a new Keynesian regime was investigated by Leight and Wrein-Lewis (2006) who found that debt follows a random walk process. Again, Afonso (2007), using data on EU-15, showed that certain countries could face potential debt sustainability problems. In a similar vein, Greiner et al. (2007) investigated the debt sustainability of selected EU economies that exhibited large debt to GDP ratios and/or violated the Maastricht treaty. Their results suggested that all deficits were sustainable. In a prominent work, Arellano (2008) developed a model in a small open economy framework that could predict the relationships between output interest rates and debt that arises in economies that face recession.

The unexpected subprime mortgage crisis in the US that evolved to a global financial and debt crisis, put debt dynamics on the research agenda of many economists. In this context, Reinhart and Rogoff (2010) investigated thoroughly the link between inflation and both government and external debt showing that inflation is not connected to debt in developed countries. For a critique see Herndon et al. (2014). A number of studies have investigated the European debt crisis. See, among others Barrios et al. (2009); Attinasi et al. (2010); Ejsing and Lemke (2011); and Antonini et al. (2013). A comprehensive survey on recent literature on fiscal and monetary policy as well as the dynamics of debt in an economy can be found in Eslava et al.
(2010). Tamakoshi and Hamori (2012) assessed the impacts of the recent sovereign debt crisis on the time-varying correlations of five European financial institutions holding large amounts of Greek sovereign bonds (National Bank of Greece, BNP Paribas, Dexia, Generali, and Commerzbank). According to their findings, the present of significant increases in the correlations between several combinations of the financial institutions' stock returns after the inception of the sovereign debt crisis, indicating contagion effects, was validated. Finally, Blandell-Wignall (2013) investigated the EMU debt crisis as well as the proposed policies in order to exit the crisis and argued that EMU suffers from two distinct crises: debt and financial.

However, inadequate attention has been paid, thus far, to the transmission of the debt crisis among EU12 countries, after the introduction of the Euro currency in 2001 (see inter alia Favero, 2013). In brief, the so-called European debt crisis is an ongoing situation that has made it extremely difficult, or even practically impossible, for some countries in the Euro area to repay their debts. Since then, a number of its periphery members such as Greece, Portugal, Ireland, Spain and Cyprus have been severely hit by the economic crisis and austerity measures have been implemented by the so called “Troika” (ECB/EU/IMF). In this spirit, recently, Antonini et al. (2013) concluded that the debt dynamics in the EU10 are highly complicated, involving important inter-economy interactions and protracted adjustment periods.

In a prominent paper Cipollini et al. (2015) investigated the impact of European Monetary Union (EMU) and of the recent financial and fiscal crisis on the integration of the European sovereign debt market. The results indicated that the elimination of currency risk following the implementation of EMU led to a fundamental and significant one-off increase in integration. In fact, based on their findings, the net impact of fiscal fundamentals was negligible up until 2009 as the markets seemed to be pricing in a potential bailout for member states in crisis and not fully pricing default risk. However, by 2010, the situation of the peripheral economies led the markets to price default risk and heralded a return to segmentation. As a result, the increase in peripheral economy sovereign spreads has exacerbated the problem of fiscal imbalances which pose a major challenge for policy-makers.
The present work builds on the prominent works of Acemoglu et al. (2012), Bailey et al. (2016) and Pesaran and Yang (2016). More specifically, in this work we use the network system structure proposed by Acemoglu et al. (2012) in order to model the interdependencies between the EU-12 economies using a network general equilibrium framework. Additionally, we investigate the pervasiveness of each economy in the network using the δ-value characterization established by Pesaran and Yang (2016) based on Bailey et al. (2016), while extending the modeling choice of Spatial Vector Autoregressive schemes proposed by Pesaran and Yang (2016) by using a GVAR process which acts as a broader infinite approximation of the global factor augmented process. Finally, based on the selection of dominant entities introduced in Tsionas et al. (2016) we provide a robustness analysis for the dominance characterization of each economy (node) in the network, without ignoring -at the same time- the estimation results of the general equilibrium equation that characterizes the network through the estimation of the respective GVAR model as a system of equations.

Based on this approach, we check for the debt dynamics among the EU12 economies tracing the timing pattern and the magnitude of the transmission. In this framework, our work estimates: (a) the dominant characterization of each every economy/node in the universe of our model using a δ-value extremum estimator; (b) the link between output and debt fluctuations in EU12, based on a network system of economies that interact in a general equilibrium framework, using the global variables of trade and finance which act as the transmission channels.

The transmission mechanism that is in place and could be deciphered by the model employed is the following: international financial institutions that operate in different economies are vulnerable to the overall macroeconomic conditions of the respective economy. Therefore, when a specific economy faces excessive deficit, which in turn could affect its overall debt sustainability, then this directly affects the operating risk of these financial institutions. As a result, these institutions affect the subsidiary financial institutions and could thus influence the other economies as well. Hence, this gives the transmission mechanism an “international”
character (see Pesaran et al., 2004). Similarly, this situation could become even more severe if we take into consideration the fact that investors who act in the global market take the same risks when an economy faces debt sustainability problems unexpectedly (like Greece, Ireland, Portugal and Spain).

Of course, the present work builds on previous contributions in the field of GVAR modelling. First, Pesaran and Smith (2006) showed that the VARX* models could be derived as solutions to a DSGE model. Next, Dées et al. (2007b) presented tests for controlling for the long-run restrictions within a GVAR context. Furthermore, Chudik and Pesaran (2011) derived the conditions under which the GVAR approach is applicable in a large system of endogenously determined variables. Lastly, Tsionas et al. (2016) and Cuaresma et al. (2016) were the first papers in the literature that extended GVAR modeling using Bayesian inference.

In comparison to previous contributions, the present work advances the literature in several ways: first, we model by means of a network approach which is based on a general equilibrium framework, the international linkages between the EU12 economies namely: Austria, Belgium, Finland, France, Germany, Greece, Ireland, Italy, Luxembourg, Netherlands, Portugal and Spain, while treating Germany as dominant, as dictated by its degree of pervasiveness in the network structure; second, the paper offers a robustness analysis regarding both the existence and the identification of dominant economies (nodes) in the EU12 using the relevant methodologies introduced in Tsionas et al. (2016); third, the paper studies the period right after the formation of the European Monetary Union (EMU) and extends the estimation period up to the end of 2015, fully capturing the recent global recession, while acknowledging the impact of global crisis through the introduction of the relevant exogenous dummy variables; finally, it is the first study to apply the GVAR approach in a network general equilibrium process for debt issues.

The remainder of the paper is structured as follows: section 2 sets out the methodology; section 3 presents the empirical results; section 4 offers a discussion of our findings; finally, section 5 concludes.
2. Methodology

2.1 The model

Consider a network with \( i = 1, \ldots, N \) nodes where each node represents an economy in an economic system. Each node in this economic network communicates with the rest of the nodes through the edges of the network which can be represented by the input output (IO) Leontief weights. The network evolves in time, i.e. the position of each node (economy) changes over time as a result of a change in the elements of IO weights. In this context, each time stamp \( t \in T \) represents a snapshot of the network in time. For the sake of simplicity, we assume that the number of network nodes remains fixed over time, i.e. no node can neither exit nor enter the network. Following the seminal work of Pesaran and Young (2016), who build on Acemoglu et al. (2012) and Bailey et al. (2016), we assume, without loss of generality, that each node (economy) produces one good whereas the production process is characterized by a Cobb-Douglas production function:

\[
x_{it} = e^{a_{ii}v_{it}} \prod_{j=1}^{m-1} x_{ij,t} \prod_{j=1}^{m-1} \prod_{i=1}^{N} a_{ij} w_{ijt}, \quad i = 1, \ldots, N, \quad t \in T (1)
\]

where: \( x_{it} \) is the produced good of each economy \( i = 1, \ldots, N \), \( a_{ij} \), \( j = 1, \ldots, N \) denote the output elasticities such that \( \sum_{j=1}^{m} a_{ij} = 1 \), i.e. the production of each economy is characterized by constant returns to scale, \( a_{ij} w_{ijt} \geq 0, \forall t \in T \) denotes the share of the \( j - \text{th} \) good used in the production of \( i - \text{th} \) economy (intermediate good) and \( v_{it} \) denotes a productivity shock for economy \( i \in I \), which is composed of an economy specific shock \( \varepsilon_{it} \), and a common technological factor \( f_t \) such that:

\[
v_{it} = \varepsilon_{it} + \gamma_i f_t (2)
\]

where: \( \gamma_i \) is a factor loading which expresses how the common factor influences each economy \( i = 1, \ldots, N \). Following, Pesaran and Yang (2016), we assume that the cross-section
exponent of the factor loadings is $\delta$, such that the following sequence converges to a positive constant i.e.:

$$N^\delta \sum_{i \in I} |y_i| \rightarrow c > 0 \quad (3)$$

In this set up, if $\delta = 1$, then the common factor is pervasive in the sense that it affects all economies (nodes) in the network. Otherwise, if $\delta < 1$, then the common factor is not pervasive, i.e. it does not affect all the economies in the network. Nevertheless, based on the work of Pesaran and Yang (2016) any unit with $\delta_j \geq 0.5$ could be considered as weakly dominant in the network structure. In other words, in the presence of weakly dominant entities a factor can be semi-strong in which case it is still pervasive but does not comply with the extreme case whilst a non-pervasive factor can be one that only has localised effects.

Additionally, we will assume that the economy-specific shocks are cross-sectionally independent with zero mean such that $E(\epsilon_{it}) = 0$ and $Var(\epsilon_{it}) = \sigma_i^2$.

Turning back to the network structure, we will assume that each economy (node) is endowed with one unit of labor, supplied inelastically and has Cobb-Douglas preferences, $u_{it}$, over the $N$ goods produced in the network.

$$u_{it}(c_{1i}, \ldots, c_{Ni}) = A \prod_{i=1}^{N} c_{it}^{1/m}, \quad i = 1, \ldots, N \quad (4)$$

In this set up, the goods produced in the network could be either final goods, $c_{it}$, or intermediate goods, $x_{ijt}$, which are used in the production process of at least one economy (node). Therefore, the amount of final goods in the network is defined as:

$$c_{it} = x_{it} - \sum_{j=1}^{N} x_{ijt} \quad (5)$$

In the presence of general equilibrium, we assume that labor markets, $l_{it}$, clear:

---

We are indebted to an anonymous referee for this insightful comment.
\[ l_t = \sum_{i \in I} l_{it} \quad (6) \]

In this context, the competitive equilibrium solution for a given vector of prices, \( p = (p_1t, ..., p_Nt) \) and a wage rate \( h_t \) is given by:

\[ x_{ijt} = \frac{a_{ij}w_{ijt}p_{jt}}{p_{jt}} \quad (7) \]

and

\[ l_{it} = \frac{a_{it}p_{it}x_{it}}{h_t} \quad (8) \]

Therefore, by substituting in equation (1) the aforementioned expressions and by simplifying we get:

\[ p_{it} = a_{ij} \sum_{j=1}^{M} w_{ijt}p_{jt} + a_{ii}h_t - b_t - a_{ii}(\varepsilon_{it} + \gamma_{ft}) \quad (9) \]

where: \( p_{it} = \ln(P_{it}), h_t = \ln(H_t) \)

and \( b_t = a_{ii} \ln(a_{ii}) + a_{ij} \ln(a_{ij}) + a_{ij} \sum_{i \in I} w_{ij} \ln(w_{ij}) \)

We rewrite equation (9), using matrix notation as:

\[ p_t = a_{ij}Wp_t + a_{ii}h_t1 - (b + a_{ii}\gamma ft + a_{ii}\varepsilon_t) \quad (10) \]

and by solving for the ln-sized price vector we get:

\[ p_t = a_{ii}h_t[I - a_{ij}W]^{-1}1 + a_{ii}[I - a_{ij}W]^{-1}(-a_{ii}^{-1}b + \gamma ft + \varepsilon_t) \quad (11) \]

\[ p_t = a_{ii}h_tIO1 + a_{ii}IOu_t \quad (12) \]

where: \( IO = [I - a_{ij}W]^{-1} \) and \( u_t = -a_{ii}^{-1}b + \gamma ft + \varepsilon_t \)

The price system described by equation (12) characterizes a network system of economies where each economy is represented by a node, whereas the interconnections between the economies, i.e. edges, are represented by the inverse Leontief matrix.
2.2 Estimating the Network General Equilibrium Model

Previous attempts in the literature of Network General Equilibrium, such as Pesaran and Yang (2016), involve writing the price equation in (9) as a Spatial Vector Autoregressive (SAR) scheme of the form:

\[ y_t = a_{ij}W y_t - b(a_{ij}, W) - a_{ij}(y_f + \varepsilon_t) \tag{13} \]

where: \( y_t = p_t - H_t 1 \)

which represents a SAR(1) scheme with an unobserved common factor, where the price specific interests captured by the vector \( b \), depend on the weight matrix \( W \) and on \( a_{ij} \). In this context, \( y_t \) is captured by a GDP measure according to the related literature.

Additionally, in this paper, we propose a more general representation of the price system described by (13) using a Global Vector Autoregressive (GVAR) scheme, so as to directly estimate the impact of each economy (node) in the network to the rest of the economies (nodes). To do so, based on the work of Dees et al. (2007), equation (13) can be represented by a canonical global factor model of the form:

\[ y_{it} = \Gamma_i f_t + \xi_{it}, i = 1, \ldots, N \tag{14} \]

where: \( y_{it} \) denotes the observable variables, \( f_t \) denotes the unobserved common technological factors and other relevant external factors of the network economy, \( \Gamma_i \) is a matrix of factor loadings which is uniformly bounded i.e. \( \| \Gamma_i \| < K < \infty \) and \( \xi_{it} \) is a vector of economy (node) specific shocks, whereas the factors and the economy/node-specific shocks are assumed to follow:

\[ \Delta f_t = \Lambda_f(L)\eta_f, \eta_f \sim IID(0, I) \tag{15} \]

\[ \Delta \xi_{it} = \Sigma_i(L)\omega_{it}, \omega_{it} \sim IID(0, I) \tag{16} \]
where $A_f$ and $\Xi_t$ are uniformly absolute summable, so as to ensure the existence of $\text{Var}(\Delta f_t)$ and $\text{Var}(\Delta \xi_{it})$. Under these assumptions, Dees et al. (2007) showed that the unobserved common factors could be consistently estimated by linear combinations of cross section averages of the observable variables $y_{it}$, given as:

$$y_{it}^* = W_t' y_{it} = \Gamma_t^* f_t + \xi_{it}^* \quad (17)$$

Therefore, they obtained the economy specific VAR augmented models with $y_{it}^*$:

$$\Phi_i(L, L_i)(y_{it} - \tilde{\delta}_i - \tilde{\Gamma}_i y_{it}^*) \approx \omega_{it} \quad (18)$$

where $\Phi_i(L, L_i)$ is the lag polynomial matrix. The above equation corresponds to a conditional VARX model for each economy (node) in the network of the form:

$$y_{i,t} = a_{i0} + \sum_{l=1}^{L_1} y_{i,t-l} A_{il} + \sum_{l=0}^{L_2} y_{i,t-l} B_{il} + \sum_{l=0}^{L_3} g'_{t-l} C_{il} + \xi_{it} \quad (19)$$

where $a_{i0}$ denotes a $(1xm)$ vector of $m$ intercepts, $y_{it}' = [y_{i,t}, \ldots, y_{i,t,m}]$ denotes the transpose of a $(mxt)$ vector $y_{it}$ of $m$ variables for economy $i$, expressing the so-called endogenous variables and $L_1$ denotes the respective lag length, while $A_{il}$ is the matrix of lagged coefficients; $y_{i,t}' = [y_{i,t}, \ldots, y_{i,t,n}]$ denotes the transpose of a $(mxt)$ vector $y_{i,t}'$ of $m$ foreign-specific variables and $L_2$ denotes the respective lag length, while $B_{il}$ is the matrix of lagged coefficients augmented with the contemporaneous effects; and $g'_{t} = [g_{t1}, \ldots, g_{tk}]$ denotes the transpose of a $(kxt)$ vector of $k$ global variables, and $L_3$ denotes the respective lag length, while $C_{il}$ is the matrix of lagged coefficients augmented with the contemporaneous effects. In general, the $m$ and $k$ may be allowed to vary between countries $i$, that is $m_i$ and $k_i$ for each economy $i = 1, \ldots, N$. 
2.3 Network Dominance

Pesaran and Yang (2016), based on Bailey et al. (2016), using formal mathematical derivations characterize the network in terms of “strongly” and “weakly” dominant units, based on the out-degree measure proposed by Acemoglu et al. (2012). In detail, a unit in the network is $\delta_j$ dominant if its weighted out-degree, is of order $N^{\delta_j}$. In other words, if $\delta_j = 1$ the unit is considered to be strongly dominant, otherwise if $\delta_j \in (0,1)$ is considered to be weakly dominant, while non-dominant are the units which exhibit $\delta_j = 0$. In this context, following Pesaran and Yang (2016), we characterize the economies/nodes of the network in terms of their dominance, using the following scheme (out-degrees):

$$d_{it} = \kappa N^{\delta_i} \exp(v_{it}), i = 1, ..., N, t = 1, ..., T \quad (20)$$

$$\kappa = \lim_{N \to \infty} \frac{\exp(-\frac{\sigma^2}{2})}{4 \sum_{i=1}^{N} N^{\delta_i}} \quad (21)$$

Of course, equation (20) that characterizes the dominance of each economy (node) in the network could be consistently estimated using a log transformation. In this paper, we follow the work of Pesaran and Yang (2016) who found that any unit with $\delta_j \geq 0.5$ could be considered as being “weakly” dominant in the network structure, using relevant Data Generating Processes (DGP’s). In other words, in the presence of weakly dominant entities a factor can be semi-strong in which case it is still pervasive but does not comply with the extreme case, whilst a non-pervasive factor can be one that only has localized effects.\(^3\)

In what follows, we summarize the Bayesian estimation of the GVAR scheme that characterizes the general equilibrium solution of the network economy constructed, following Tsionas et al. (2016).

---

\(^3\) We would like to thank an anonymous referee for pointing this out.
2.4 Bayesian estimation

The GVAR model presented in (19) can be written in the form:

\[ y'_{it} = z'_{it} \tilde{\Gamma}_i + z'_{it} \Delta_i + u'_{it} \] (22)

where \( z'_{it} = [y'_{it}, y'_{it-1}, ..., y'_{it-L_2}] \) expresses the foreign specific variables, and \( \tilde{z}'_{it} = [y'_{it}, y'_{it-1}, ..., y'_{it-L_1}, g_{it}, g_{it-L_2}, ..., g_{it-L_3}] \) represents the own lags and the global variables, while the matrix of coefficients of the foreign specific variables is denoted by \( \tilde{\Gamma}_i = B_{itL_2} \), and the matrix of coefficients of the own lagged and global variables is denoted by \( \Delta_i = [A_{itL_1}, C_{itL_3}] \).

Using matrix notation, the model can be written as:

\[ Y_i = \tilde{Z}_i \tilde{\Gamma}_i + Z_i^* \Delta_i + U_i, \quad i = 1, ..., N, \text{ or } (23) \]

\[ y_i = (I \otimes \tilde{Z}_i) \delta_i + u_i = \tilde{X}_i \tilde{\delta}_i + X_i^* \delta_i + u_i, \quad i = 1, ..., N (24) \]

where \( \tilde{X}_i = I \otimes \tilde{Z}_i \) and \( X_i^* = I \otimes Z_i^* \). Now, the foreign specific variables are given by:

\[ X_{it}^* = \sum_{c=1}^{N} w_{ic} x'_{ct} = w'_{it} X_t (25) \]

where: \( w_i \) represents the vector of input-output weights of economy \( i \) with every economy \( c \neq i = 1, ..., N - 1 \), with \( w_{ii} = 0 \), \( \sum_{c=1}^{N} w_{ic} = 1 \). Thus, in summation we get:

\[ X_t^* = W_i X_t (26) \]

where \( W \) represents the \( N \times N \) matrix of input-output weights, and \( X_t^* \) is an \( N \times m \) matrix whose rows represent the \( m \) foreign – specific variables, for a given observation.

Now, without loss of generality, let:
\[ \Gamma = [\tilde{y}_1, ..., \tilde{y}_N, \delta_1, ..., \delta_N] \] and \[ X = \begin{bmatrix} \bar{X}_1 \\ \vdots \\ \bar{X}_N \\ \hat{X}_1 \\ \vdots \\ \hat{X}_N \end{bmatrix} \quad (27) \]

Then, the likelihood function of the GVAR system\(^4\) is:

\[
L(\gamma, \Omega) = |\Omega|^{-T/2} \exp \left\{ -\frac{1}{2} \text{tr}(Y - X\Gamma)'\Omega^{-1}(Y - X\Gamma) \right\} \times \\
|\Omega|^{-T/2} \exp \left\{ -\frac{1}{2} (\gamma - \hat{\gamma})'(\Omega^{-1} \otimes XX')(\gamma - \hat{\gamma}) \right\} |\Omega|^{(T-p)/2} \exp \left\{ -\frac{1}{2} \text{tr} \Omega^{-1}(Y - X\Gamma)'(Y - X\Gamma) \right\} \\
N(\gamma | \hat{\gamma}, \Omega \otimes (XX')^{-1}) \times \text{IW}\left( \Omega \mid (Y - X\Gamma)'(Y - X\Gamma), NTm - (Kx + m + 1) \right),
\]

where IW is the inverted Wishart.

This is used in a Bayesian context to impose priors in the context of Bayesian vector autoregressions. Koop (2013, pp. 197-199) describes a procedure, which has the standard decomposition \( \Sigma^{-1} = \Psi \Psi' \) and \( \Psi \) is upper-triangular. For the diagonal elements, he assumes independent gamma priors of the form \( \psi^2 \sim G(1, 1) \) if data are standardized. For the off-diagonal elements he proposes an SSVS prior which is essentially \( N(0,1) \) or \( N(0,0.1) \) with equal probabilities \( 1/2 \).

In this work, the potential existence of non-dominant entities in the network system dictates the use of sparse matrices in order to capture the covariances among the non-dominant nodes of the system. Sparse matrices are arrays in which most of the elements of the main diagonal could be considered as being negligible, as they are close to zero. More specifically, in our case, we have matrices with at least \( M \) non-zero elements in each line, which in turns corresponds to 0-sparse matrices (El Karoui, 2008, p. 2722).

In this context, we follow the seminal work of Huang and Wand (2013) who proposed a prior for large sparse positive definite matrices with many elements, where control is allowed over the standard deviations and the correlation coefficients:

$$\sum_{(K' \times K')} |a_1 ... a_p| \sim IW(\overline{A}, \nu + K' - 1) \quad (28)$$

where $$\overline{A} = 2\nu \text{diag}(a_1^{-1}, ..., a_K^{-1})$$, and the overall dimension is $$K = K \times m$$

$$a_k \sim IG\left(\frac{1}{2}, \frac{1}{A_k}\right), k = 1, ..., K' \quad (29)$$

where the density of the Wishart $$W(k, S)$$ is:

$$p(\Sigma) \propto |S|^{k/2} \exp\left\{ -\frac{1}{2} tr \Sigma^{-1} \right\}, \ k > 0 \quad (30)$$

and $$\Sigma, S$$ are positive definite matrices.

Large values of $$A_1, ..., A_p$$ imply weakly informative priors on the standard deviations, while the choice $$\nu = 2$$ leads to uniform priors on the correlation coefficients. The explicit form of the prior is:

$$p(\Sigma) \propto |\Sigma|^{-(\nu+2K')/2} \prod_{k=1}^{K'} \left[ 1 + \nu \left( \Sigma^{-1} \right)_{kk} \right]^{-(\nu+K')/2} \quad (31)$$

The marginal distribution of each correlation coefficient is:

$$p(\rho_{ij}) \propto (1 - \rho_{ij}^2)^{\nu-1}, \ -1 < \rho_{ij} < 1 \quad (32)$$

Also, the marginal distribution of each standard deviation follows a half-$$t$$ distribution with parameters $$\nu, A_k$$, that is:

$$\sigma^2 \mid a_i \sim IW\left(\nu, \frac{2\nu}{A_k}\right), \text{ and independently } a_i \sim IG\left(\frac{1}{2}, \frac{1}{A_k}\right), i = 1, ..., K' \quad (33)$$
The important property is that its conditional distribution is still inverse Wishart and the posterior conditionals of $\boldsymbol{a}, \boldsymbol{s}$ are inverse-Gamma distributions (Huang and Wand, 2013, p. 7). Therefore, Gibbs sampling can be implemented easily.

Moreover, the posterior conditional distributions of weights in $W_i$ can be drawn *en bloc*, using a Gibbs sampler update relying on the Kalman filter. This procedure reduced considerably the autocorrelation inherent in MCMC and, in lags of order 50, it was negligible.

In detail, in this work the model consists of twelve (12) major economic entities (nodes) namely Austria, Belgium, Finland, France, Germany, Greece, Ireland, Italy, Luxembourg, Netherlands, Portugal and Spain. Each economy $i$, $i = 1, \ldots, 12$, follows a VAR scheme, augmented by the exogenous variables of global trade ($T$) and global stocks traded ($S$), expressing the transmission channels of trade and finance, respectively.

The endogenous variables $y_{it}$ denote a $12 \times 1$ vector of macroeconomic variables belonging to each economy $i$, $i = 1, \ldots, 12$, consisting of Gross Domestic Product (GDP) and Debt (D) that can perfectly capture the price system of the general equilibrium equation, and are regressed: on an intercept $a_{i0}$, on their lags up to the order $L_1$, the contemporaneous and lagged up to the order $L_2$ foreign variables $y_{i,t}^*$, and some contemporaneous and lagged up to the order $L_3$ common global factors $g_t$. The error term $\zeta_{it}'$ is assumed to be normally distributed $\zeta_{it}' \sim \mathcal{N}(0, \Sigma)$.

The foreign variables $y_{i,t}^*$ represent a weighted average of the other economies’ variables. Thus, the VARX model for each economy, using the notation presented earlier is as follows:

$$
y_{it}' = a_{i} + \Phi_i(L, L_1)y_{it}' + A_i(L, L_2)y_{i,t}^* + \Psi_i(L, L_3)g_t + \zeta_{it}' \quad (34)$$
For $i = 1, \ldots, 12$ and $t = 1, \ldots, T$; where $\Phi_i(L, L_1)$, $\Psi_i(L, L_2)$ and $\Lambda_i(L, L_3)$ are the matrixes of the lag polynomial of the associated coefficients of the economy-specific, of the foreign, and of the global variables, respectively. In this work, matrix $W_i$ is a $12 \times 12$ dimensional matrix of weights that defines $k_i=12$ economy-specific cross section averages of foreign variables. Lastly, $\zeta_{it}$ is a vector of idiosyncratic, serially uncorrelated economy-specific shocks with $\zeta_{it} \sim N(0, \Sigma)$.

The dynamic characteristic of the model are examined through the so-called Generalized Impulse Response Functions (GIRFs), following Koop et al. (1996) and Pesaran and Shin (1998). A basic advantage of this approach is that the GIRFs are invariant to the ordering of the equations. The (Generalized) Impulse Response Function (GIRF) can be expressed as follows:

$$I_j(n) = \sigma_{jj}^{-1/2} + B_n \Sigma e_j \forall n = 1, 2, \ldots (35)$$

where $I_j(n)$ is the Impulse Response Function $n$ periods after a positive standard error unit shock; $\sigma_{jj}$ is the $j$th row and $j$th column element of the variance–covariance matrix $\Sigma$ of the lower Cholesky decomposition matrix of the error term which is assumed to be normally distributed; $B$ is the coefficients’ matrix when inversely expressing the VAR model as an equivalent MA process and $e_j$ is the column vector of a unity matrix. See Koop et al. (1996) and Pesaran and Shin (1998). Simulation from their posterior distribution is straightforward.

In what follows, we provide a robustness analysis in terms of characterization of the dominant units of the network.

---

As in GVAR applications, we prefer GIRFs over more standard orthogonalized impulse responses (OIRs), which would require the definition of an ordering of the variables in the reduced form VAR (see *inter alia* Dovern and Roye, 2014).
3.3 Robustness Analysis

Number of dominant entities in the network

We investigate the eigenvalue distribution of the weight matrix that comes directly by the Input Output weight matrix which is publically available at the World Input-Output Database (WIOD). The eigenvalue distribution of the IO weight matrix expresses the dynamic behavior of all the EU12 economies that enter our analysis (Brody 1997). Let \( \lambda(i) \), denote the eigenvalues of the weight matrix that characterizes the interconnections of the network and let \( \lambda(\text{pf}) = \lambda(1) \) denote the dominant or so-called Perron–Frobenius (P–F) eigenvalue of the \( n \times n \) matrix \( W \). We divide each eigenvalue's modulus with the P–F eigenvalue's modulus to get the normalized eigenvalue: \( \rho(i) = |\lambda(i)| \cdot |\lambda(\text{pf})|^{-1} \), \( i=1,\ldots,12 \). The normalized eigenvalues: \( \rho(i), i=2,\ldots,12 \) are the so-called non-dominant eigenvalues, since \( \rho(\text{pf})=\rho(1)=1 \), is the dominant one.

The number of dominant economies implied by the economy's structure is equal to \( i^* \), for which \( \rho(i^*) > 0.4-0.3 \) approximately, since values of \( \rho(i^*) \) less than 0.40–0.30 might be considered negligible from a practical point of view (Brody, 1997; Mariolis and Tsoulfidis, 2014).

Next, based on the concept of centrality (Freeman 1979), we examine which economies are dominant by using two important vertex theory measures, namely: (i) degree centrality and (ii) eigenvector centrality.

(i) The degree centrality of a node indicates how connected a node is to the other nodes in the graph (see, among others, Ying et al. 2014; Bates et al. 2014). The centrality, \( c_i \), of each node is given by the following formula:

\[
    c_i = d(i) \sum_{j=1}^{N} z_{ij} \tag{36}
\]

where \( d(i) \) is the degree of each node, i.e. the number of ties with the rest of the nodes (Fagiolo et al. 2008). In this context, the dominant economies are those which exhibit the largest centrality. Hence, the largest \( c_i \) corresponds to the dominant economy, the second largest \( c_i \) to the second-dominant economy, and so on.
However, degree centrality does not take into consideration how the neighbors of each node interact with the rest of the nodes of the vertex. In this context, we take into consideration an additional measure of node centrality namely, eigenvector centrality (Bonacich and Lloyd, 2001).

(ii) **Eigenvector centrality** of a node, $i$, was developed by Bonacich (1987) and can identify the centrality power of a node according to the distant neighbors of the specific node. It is given by the following formula:

$$ EC_i = \lambda^{-1} \sum_{j=1}^{N} A_{ij} e_j $$

where: $\lambda^{-1}$ is the inverse of the Perron-Frobenious eigenvalue of the adjacent matrix, $e_j$ the respective eigenvector, $A_{ij} = [z_{ij}], i, j \in \{1, ..., N\}$ is the adjacency matrix. Apparently, dominant economies are those with the largest values of eigenvector centrality.

4. **Empirical Results**

4.1 **Data and Variables**

The data come from IMF, are quarterly, and cover the period 2001–2015 after the introduction of the common currency, i.e. the Euro, fully capturing the recent recession. In order to consistently estimate the general equilibrium price equation of the network system of economies, we make use of two (2) economy-specific variables for each economy: GDP and Debt, which can fully capture the log difference of prices and wages in an economy, following the general spirit of Long and Plosser (1983). In this context, the variable of Debt is an aggregation of various Debt forms, i.e. banks’ debt, government debt and monetary authorities’ debt. Regarding the global variables, we use the aggregate values of (i) **Worldwide Total Trade** and (ii) **Worldwide Total Stocks Traded**, both in millions of dollars, which were obtained in constant 2005 prices from the World Data Bank. All variables under investigation were transformed to constant 2005 prices in billions of dollars. Whenever quarterly data were missing, quarterly series were interpolated from the annual series following Dees et al. (2005).
of dollars using the GDP deflator for every economy in the universe of our model, whereas all quantitative variables were also transformed according to the logarithmic transformation. Additionally, in order to avoid any structural instability we incorporated in every VARX model the dummy variable of Global Crisis (2006Q4-2009Q2) as well as the European Debt Crisis (2009Q1-2012Q3). Additionally, dummy variables for the presence of local crises were also employed in the VARX models of Greece (2010Q1-2015 Q4), Portugal (2010Q2-2014Q2), Spain (2009Q1-2014Q3) and Ireland (2009Q1-2013Q3). The timeline (periodization) of the various crises comes from bbc.com.

As discussed earlier, the transmission mechanism that is in place according to our model is as follows: International financial institutions are vulnerable to unexpected macroeconomic shocks of the economy they operate. As a result, when the respective economy faces unexpected fiscal deficit problems that lead to the deterioration of the overall debt sustainability of the economy, then these shocks influence the smooth operation of the financial institutions, which in turn, influence the operation of their subsidiaries in other countries. As discussed earlier, this situation, could in turn, influence the overall macroeconomic conditions of the other economies as well. Hence, this gives the transmission mechanism an “international” character (see Pesaran et al., 2004). Similarly, this situation is intensified by investors who act in the global market who take the same risks when an economy faces debt sustainability problems unexpectedly (like Greece, Ireland, Portugal and Spain).

4.2 The Network

Figure 1 below presents the EU12 weighted network as set out earlier. In this context, the data used for the construction of the network are the Input-Output weights of the EU12 economies, which correspond to the in- and out- degree of the network system, respectively.
The network’s structure is cyclical since all nodes interconnect with each other. As we can see, the economies of Germany, Spain, France and Italy are the largest economies in our network with respect to the weight out degrees of the network.

4.3 Degree of Pervasiveness

Following Pesaran and Yang (2016) we characterize each economy (node) in the network in terms of its pervasiveness based on its $\delta$-value$^7$.

$^7$ Note that in our empirical application the cross section dimension remains fixed due to data availability, as is also the case in the empirical application of Pesaran and Yang (2016). Nonetheless, based on Pesaran and Yang (2016, p. 21), the time dimension in our application is adequate so as to ensure convergence of the estimator, since the estimator converges with $\sqrt{T}$ rate.
<table>
<thead>
<tr>
<th>Economies (Nodes)</th>
<th>Estimated $\delta$-value of pervasiveness</th>
<th>Rank based on $\delta$-value Pervasiveness</th>
</tr>
</thead>
<tbody>
<tr>
<td>AUT</td>
<td>0.12</td>
<td>7</td>
</tr>
<tr>
<td>BEL</td>
<td>0.28</td>
<td>6</td>
</tr>
<tr>
<td>DEU</td>
<td>0.89</td>
<td>1</td>
</tr>
<tr>
<td>ESP</td>
<td>0.31</td>
<td>5</td>
</tr>
<tr>
<td>FIN</td>
<td>0.08</td>
<td>10</td>
</tr>
<tr>
<td>FRA</td>
<td>0.39</td>
<td>2</td>
</tr>
<tr>
<td>GRC</td>
<td>0.03</td>
<td>12</td>
</tr>
<tr>
<td>IRL</td>
<td>0.11</td>
<td>8</td>
</tr>
<tr>
<td>ITA</td>
<td>0.36</td>
<td>4</td>
</tr>
<tr>
<td>LUX</td>
<td>0.04</td>
<td>11</td>
</tr>
<tr>
<td>NLD</td>
<td>0.37</td>
<td>3</td>
</tr>
<tr>
<td>PRT</td>
<td>0.09</td>
<td>9</td>
</tr>
</tbody>
</table>

Following the works of Acemoglu et al. (2012), Bailey et al. (2016) and Pesaran and Yang (2016), the threshold for an economy to be considered as weakly dominant is 0.5, since any unit with an estimate of below 0.5 will not have any network effects (see Remark 3, Pesaran and Yang, 2016, p. 15). In this context, the economy of Germany is the only one with a $\delta$-value that exceeds the aforementioned threshold, which in turn implies that Germany could be considered as being the only weakly dominant economy in our network structure. Nevertheless, the fact that the rest of the economies are non-dominant implies that the rest of the economies can have only localized effects in the network structure in the sense that are unable to affect each and every node in the network.

Following the methodologies described earlier, we investigate the eigenvalue distribution of the Input-Output matrix, in order to verify the existence of a dominant entity. We begin by investigating the existence of a dominant economy in the data set. In this context, Table 2 below presents the normalized eigenvalues of the weight matrix $W$ for 2005.
Table 2: Normalized Eigenvalues of $W$ (2005)

<table>
<thead>
<tr>
<th>Eigenvalue</th>
<th>$\rho_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.041</td>
</tr>
<tr>
<td>2</td>
<td>0.031</td>
</tr>
<tr>
<td>3</td>
<td>0.027</td>
</tr>
<tr>
<td>4</td>
<td>0.001</td>
</tr>
<tr>
<td>5</td>
<td>0.001</td>
</tr>
<tr>
<td>6</td>
<td>0.004</td>
</tr>
<tr>
<td>7</td>
<td>0.000</td>
</tr>
<tr>
<td>8</td>
<td>0.001</td>
</tr>
<tr>
<td>9</td>
<td>0.003</td>
</tr>
<tr>
<td>10</td>
<td>0.002</td>
</tr>
<tr>
<td>11</td>
<td>0.002</td>
</tr>
<tr>
<td>12</td>
<td>0.002</td>
</tr>
</tbody>
</table>

The results imply the existence of one dominant economy in the EU12, since values of $\rho(i)$ less than 0.40–0.30 are considered negligible from a practical point of view, as we have seen earlier (Brody, 1997; Mariolis and Tsoulfidis, 2014).

We proceed by investigating the centrality measures.

Table 3: Centrality measures based on the average matrix $W$

<table>
<thead>
<tr>
<th>Economies</th>
<th>Degree centrality</th>
<th>Eigenvector Centrality</th>
</tr>
</thead>
<tbody>
<tr>
<td>AUT</td>
<td>0.0056</td>
<td>0.015</td>
</tr>
<tr>
<td>BEL</td>
<td>0.0119</td>
<td>0.027</td>
</tr>
<tr>
<td>DEU</td>
<td>0.0299</td>
<td>0.053</td>
</tr>
<tr>
<td>ESP</td>
<td>0.0119</td>
<td>0.025</td>
</tr>
<tr>
<td>FIN</td>
<td>0.0019</td>
<td>0.004</td>
</tr>
<tr>
<td>FRA</td>
<td>0.0183</td>
<td>0.041</td>
</tr>
<tr>
<td>GRC</td>
<td>0.0016</td>
<td>0.004</td>
</tr>
<tr>
<td>IRL</td>
<td>0.0029</td>
<td>0.006</td>
</tr>
<tr>
<td>ITA</td>
<td>0.0133</td>
<td>0.030</td>
</tr>
<tr>
<td>LUX</td>
<td>0.0019</td>
<td>0.004</td>
</tr>
<tr>
<td>NLD</td>
<td>0.0155</td>
<td>0.035</td>
</tr>
<tr>
<td>PRT</td>
<td>0.0027</td>
<td>0.005</td>
</tr>
</tbody>
</table>
According to the results in Table 3, for both centrality measures, the German economy is dominant in our model, since it exhibits the largest values of degree and eigenvector centrality. Of course, the selection of Germany as the dominant economy in our dataset can also be easily justified by economic intuition based on the latest economic and political developments as of 2013 since: (a) it is the largest economy in terms of output produced, as well as (b) the largest economy in terms of output exchanged.

In fact, the EU economy contains about 500 million people and is the largest trading area in the world. Within this economic entity, Germany has the largest population and the largest economy in the EU. In the world, the German economy ranks 4th in terms of nominal GDP and is the world’s 2nd largest trader (CIA, 2013) in terms of imports and exports, close to the spirit of the traditional GVAR model. As is known, the most important driving forces in the German economy are primarily the industrial and banking sectors that have allowed the local economy to dominate the vehicles, machinery and equipment industries, globally.

In the EU market, currently, the German economy is undoubtedly dominant, a fact which is largely the product of stable growth export-oriented productive industries, a relatively big and powerful public sector with considerable private sector partnership, where the workers’ unions play a role in management. It is also characterized by a well-known aversion to high indebtedness often viewed as being synonymous with economic rationality.

All things considered, the robustness analysis for the dominant economy in the network verifies the findings based on the $\delta$-value of Pesaran and Young (2016). It is worth noticing, that the $\delta$-value characterization of each economy coincides with the results obtained by degree centrality measure.
4.4 Weights

We consider time varying weights, which are based in a raw benchmark set of weights $\tilde{w}_{k,t}$ and assume the following process:

$$w_{k,t} = \rho_{k} w_{k,t-1} + \alpha_{k} \tilde{w}_{k,t} + \epsilon_{k,t}$$

Posterior weights for Germany, using the proposed approach, are presented in Figure 2. The posterior distribution of the weights is characteristically bimodal reflecting the combination of information from the data and evidence through the calibrated prior.

---

**Figure 2: Posterior Trade weights for Germany**

---

8 All credible intervals for GIRFs are computed using the set of draws, thinning every other 10th draw. Similar posteriors were computed for every country in the model, but we do not report the results due to space limitations. Of course, the results are available upon request by the authors.
Given that the VARs contain a large number of parameters, principled priors have to be introduced on the parameters, especially in relatively small data sets. Here, we follow Tsionas et al. (2016). The forecasting performance of the models is examined in the hold-out sample and the model with the smallest mean-squared-forecast-error is selected. Our implementation of the Metropolis-Hastings algorithm relies on: (i) a component-wise update from the conditional posterior distribution of each parameter in \( \mathbf{C} \), (ii) a multivariate normal proposal for all other parameters\(^9\) using 10,000+\( B \) draws the first \( B \) of which are discarded to mitigate the impact of start-up effects. \( B \) is chosen according to Geweke’s (1992) convergence diagnostics.

The number of lags \((L_1, L_2, L_3)\) is chosen randomly from the prior, which is not very different from conditioning on values of these lags and performing posterior analysis for the given values. The proposal for each MCMC update of the parameters is a uniform distribution in an interval of the form \([a,b]\) which is updated during the transient phase to achieve acceptance rates between 20% and 30%. In our application, \( M=10,000 \) models are examined in total. Typically, the value of \( B \) ranged between 2,500 and 5,000, depending on the model\(^{10}\).

We have computed Generalized Impulse Response Functions (GIRFs)\(^{11}\) for the models that performed best. The final GIRFs were computed using model averaging, where the weights are computed from the marginal likelihood of each model. The marginal likelihood is computed, for each model, using the candidate’s formula with a normal approximation to the exact posterior of the parameters, following DiCiccio et al. (1997). This procedure is fast and easy to apply, which is important in this context where repeated MCMC simulations have to be considered. Standard errors of the GIRFs are computed in standard fashion using the posterior draws for the

---

\(^9\)All other parameters are regression-like parameters in the VAR. The multivariate normal proposal was crafted using least squares quantities and its scaled covariance matrix, where the scaling constant is adapted during the transient phase.

\(^{10}\)MCMC procedures performed very well and convergence was fast.

\(^{11}\)The method avoids the drawback of Cholesky decomposition see Koop, Pesaran and Potter (1996).
parameters\textsuperscript{12} and the subsequent computation of GIRFs for each draw, after thinning every other 10\textsuperscript{th} draw to mitigate inherent autocorrelation induced by MCMC.

Now, we base our analysis of Generalized Impulse Response Function (GIRFs) on the Bayes confidence bounds rather than the point estimates in order to avoid any possible structural instability. In this context, a GIRF diverges significantly, if zero does not belong to the confidence interval. Finally, we will need to ensure the robustness of our GIRFs results to the weights.

Each GIRF shows the dynamic response of the output of each region to unit shocks to each EU12 economy’s: (i) Debt and (ii) GDP of up to 16 periods, i.e. 4 years. In the exposition of the results, the reader can focus on the first two years following the shock, which is a reasonable time horizon over which the model presents credible results (Dees et al. 2007a). However, according to the same authors (Dees et al. 2007a), in what follows we provide an analysis of the results over a period of four years, since visual inspection of the results help us with the analysis of the proposed model’s convergence properties (see, among others, Dovern and Roye, 2014). Figures 1-12 show the posterior mean estimates of the GIRFs, as well as their associated 95\% Bayes intervals, regarding the response of every economy’s GDP to an impact on the GDPs and Debts of the rest of the countries. In this context, GDP is significantly affected when the 95\% Bayes interval does not include zero.

In order to avoid complex notation, we made use of the following code numbers instead of economy names. See Table 4.

\textsuperscript{12} We use a Newey-West HAC estimator with 10 lags applied to the draws for GIRFs.
Table 4: Economy code numbers

<table>
<thead>
<tr>
<th>Code Number</th>
<th>Economy/Node</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>AUSTRIA</td>
</tr>
<tr>
<td>2</td>
<td>BELGIUM</td>
</tr>
<tr>
<td>3</td>
<td>FINLAND</td>
</tr>
<tr>
<td>4</td>
<td>FRANCE</td>
</tr>
<tr>
<td>5</td>
<td>GERMANY</td>
</tr>
<tr>
<td>6</td>
<td>GREECE</td>
</tr>
<tr>
<td>7</td>
<td>IRELAND</td>
</tr>
<tr>
<td>8</td>
<td>ITALY</td>
</tr>
<tr>
<td>9</td>
<td>LUXEMBOURG</td>
</tr>
<tr>
<td>10</td>
<td>NETHERLANDS</td>
</tr>
<tr>
<td>11</td>
<td>PORTUGAL</td>
</tr>
<tr>
<td>12</td>
<td>SPAIN</td>
</tr>
</tbody>
</table>

5. Discussion

Figure A1 (Appendix) suggests that the Austrian GDP is significantly affected, in the short run by a shock in the Debt of France, whereas it is affected by the GDPs of EU12 economies, with the exception of the GDPs of Greece and Ireland. The significant impact of the shock in the Debt of France could be attributed to the high degree of financial integration between the two economies, since a number of French Banks have an active role in the economy of Austria.

Next, turning to Figure A2 (Appendix), the results suggest that the GDP of Belgium is significantly affected, in the short run, i.e. less than four (4) quarters by a shock in the Debt of Austria, Finland, France and Italy, whereas its GDP is significantly affected by the majority of the EU12 GDPs, with the exceptions of Greece and Ireland. The relations of Belgian GDP with the rest of the EU12 economies could be attributed to the strong trade relationships or to the financial integration between them. An interesting result, is the absence of relationship between Belgium and Greece or Ireland which are in the EU periphery.

Figure A3 (Appendix) suggests that the GDP of Finland is significantly affected, in the short run, by a shock in the Debt of Italy and Luxembourg, while it is also affected by a shock in the majority of the EU12 GDPs with the sole exception of the Greek GDP. Once again, a
A striking finding is that a shock in the Greek GDP or Debt does not seem to have any effect on the GDP of Finland, probably due to the fact that the two countries do not have any significant trade relationships.

Figure A4 (Appendix) suggests that the GDP of France is significantly affected, in the short run, by a shock in the Debt of Austria, Belgium, Finland, Greece, Italy, Luxembourg and Netherlands, while it is also affected by a shock in the majority of the EU12 GDPs with the exception of Germany, Greece, Luxembourg and Portugal GDPs. The wide connectivity of the French GDP with the rest of the EU12 economies could be attributed to French Banking sector that has penetrated in the EU12 economy, which in consistent, among others, with the work of Dees and Zorell (2012) who found increased business cycles synchronization among EU countries that shared significant trade and financial linkages.

According to Figure A5 (Appendix), the GDP of Germany, which is the dominant economy in our model, is significantly affected in the short run, i.e. less than four (4) quarters, by a shock in the Debts of Belgium, Finland and Italy, whereas it is not affected by the GDPs of Greece, Luxembourg and Portugal. An interesting finding is that the German economy is not dependent of the economies of both Greece and Portugal who are the first victims of the ongoing recession, whereas its GDP is affected by a shock in the Italian Debt, probably due to their very strong trade relationships and is evidence of limited synchronization of the EMU periphery to the core countries, including a noted clustering into small and large economies (see among others Artis and Zhang 1997; and Artis et al. 2003).

Turning to Figure A6 (Appendix), the Greek GDP is significantly affected, in the short run, only by a shock in the Debt of Germany and the GDPs of Belgium, Italy and Netherlands. The interconnection between the German Debt and Greek GDP could be attributed to the strong correlation between the lending spread of the two economies, since the German lending spread acts as the basis of the Greek one. On the other hand, interconnection of the Greek GDP with those of Netherlands, Italy and Belgium is, in general terms, in line with the work, among others, of Gouveia and Correia (2008), and Camacho et al. (2006).
Figure A7 (Appendix) suggests that the GDP of Ireland is significantly affected, in the short run, by a shock in the Debt of Germany and the GDPs of Finland, France and Italy. Once again, the effect of the German Debt on Irish GDP could be attributed to the lending spreads, as in the case of Greece.

According to Figure A8 (Appendix), the GDP of Italy is significantly affected, in the short run, i.e. in less than four (4) quarters by a shock in the Debts of, both, the Greek and the German economies, and the GDPs of Belgium, France, Ireland, Luxembourg, Portugal and Spain. The relationship between the Italian GDP and the Greek Debt could be attributed to the fact that both economies suffer from similar structural debt deficiencies; therefore, a link between the two countries seems to be in place. On the other hand, the German Debts affects the Italian GDP, since it affects its external lending rate.

According to Figure A9 (Appendix), the GDP of Netherlands is significantly affected by a shock in the majority of EU12 Debts with the exception of the Debts of Germany, Greece and Ireland, while being significantly affected by all the EU12 GDPs. The connection between both the Greek and Irish debt with the GDP of Netherlands seems to be dictated by the fact that Netherlands suffers from enormous household debt, which, according to macroeconomic theory, along with the government's debt act as twin deficits. In fact, there is an increasing number of studies in the literature suggesting that deterioration of public finances could result to a debt crisis (see, among others, Haugh et al. 2009, Borgy et al. 2011, Ejsing and Lemke 2011).

Figure A10 (Appendix) suggests that the GDP of Portugal is significantly affected in the short run, i.e. less than four (4) quarters, by a shock in the Debts of Belgium, Finland, France, Ireland, Italy, and Spain, while it is also affected by the majority of EU12 GDPs with the sole exception of Austria, which is in line with the work of Furceri and Karras (2007) that suggest a strong, statistically significant and negative relationship between economy size and business cycle volatility, implying that smaller countries are subject to more volatile business cycles than larger ones.
Next, according to Figure A11 (Appendix), the Spanish GDP is significantly affected, in the short run, by a shock in the Debt of Germany and the majority of GDPs of the EU12 economies with the exceptions of the Belgian and Portuguse GDPs.

Finally, turning to the GDP of Luxembourg, in Figure A12 (Appendix), we witness that it is significantly affected by a shock in the Debts of Finland, France, Germany, Greece and Italy while it is also affected by the GDPs of the majority of the EU12 economies with the exception of Austria, Finland Germany, Greece and Italy.

Moreover, it is worth noticing that the German economy which was found to be dominant economy in the model significantly affects the GDPs of all the EU12 economies, either directly, in the sense that the German GDP affects the GDP of another economy, or indirectly in the sense that the German Debt affects the GDP of another economy. In this context, we witness that the Southern European economies, such as Greece, Italy and Spain that face either Debt issues or Structural issues often due to their inefficient banking system, are primarily affected by the German Debt, as opposed to the rest of the economies that are affected mainly by the German GDP. This could be attributed to the role of the German economy as the locomotive of the overall Debt sustainability in the EMU, since historical data regarding the spreads of external financing of EMU countries clearly indicate that after the EMU formation the German economy benefited by the lowest spreads in the EMU area.

Another interesting finding of our analysis is the fact that the economy of France, which acts as the second primary pillar in the EU12 economies behind the German economy, is primarily affected by unexpected shocks in the Debt level of the so called “core” economies of EMU. This in turn, could be attributed to the fact that the French banking system has penetrated the banking markets of most of the EMU economies, either directly, through subsidiary bank branches, or indirectly, through market investments in the “core” economies. Of course, the vulnerability of the French economy to unexpected shocks gives credit to the view that despite its size, France, unlike Germany, is not immunized to external shocks, probably due to its
dependence on the overall macroeconomic conditions in the EMU, due to the lack of a
globalized exports policy that would diversify its risk of dependence on the EMU markets.

To sum up, based on our findings, the EMU economies seem to be partly divided into
“core” and “periphery” economies, based on their dependence on unexpected shocks in either
Debt or GDP of the rest of the economies. In this context, the “core” economies of EMU,
which correspond to the central and north European economies, seem to be primarily affected
by the GDP of the Germany economy which is the dominant entity in EMU. On the other hand,
the economies of the “periphery” that correspond to the South European economies,
seem to be affected primarily by the German Debt, which of course dictates the lending spreads of each
economy in the EMU. This, in turn, is validated by the increasing investments of the various
financial institutions on the German 10-year bonds that since the begging of the EMU crisis have
yielded very low spreads.

In general, the GIRFs results show that the responses of all variables to the shocks do not
exhibit sizeable effects, which are, on average, equal to less than 1-1.5%. All shocks take place in
the short run, i.e. less than four (4) quarters and die out in the medium run i.e. two years or eight
(8) quarters becoming statistically non-significant. Nevertheless, none of these shocks has a long
lasting effect, since the GDPs of all countries return back to their initial equilibrium positions.13

6. Concluding Remarks

The main point of departure in this work has been the characterization of economic networks in
terms of their degree of pervasiveness, which is considered to be a measure of dominance. To
this end, using the network economy described by Acemoglou et al. (2012) as well as the
generalization of pervasiveness, which is described in Pesaran and Yang (2016), based on Bailey
et al. (2016), we have constructed a GVAR scheme, which is capable of perfectly characterizing

13 Similar results were obtained based on the Debt GIRFs, which are available upon request by the authors, due to
space limitations.
the general equilibrium price equation of the network model. In this context, we expressed the EU-12 economies as a network system, and using data on the GDP and Debt of these economies, we estimated the respective price equations for each economy in a general equilibrium framework. Also, we conducted further robustness analysis and examined the degree of pervasiveness of each economy, which is associated with the existence of dominant(s) entity in the GVAR model.

In this framework, the (macro-)econometric model that has been developed can be used to examine the propagation of fluctuations across economies that face high debt deficits. In fact, it can be easily used for analyzing a number of transmission mechanisms, contagion effects and network interdependencies in a global as well as domestic setting. As we know, financial institutions are increasingly vulnerable to the fluctuations in the economies in which they are exposed. Hence, the risk analyses of a financial institution's activities need to take into consideration domestic as well as international economic conditions of regions that directly or even indirectly influence the institution loan’s portfolio, without neglecting the dominant role of certain economies, such as the German economy.

Hence, our focus has been on developing a compact and robust general equilibrium representation of the complex interactions across factors. The proposed model allows for direct dependence of the financial and macro factors on: (i) the their domestic parts and their lags, (ii) dependence of common global variables such as stocks traded and trade and (iii) certain degree of dependence of idiosyncratic shocks across regions captured via the cross-region covariances (e.g. Pesaran et al. 2004). For example, the proposed model is able to account for linkages between the various debt deficits among the EMU economies. Also, the use of a regional weighting scheme with dominant economies allows for efficient use of all available data.

More specifically, in this work, using a network general equilibrium framework, we studied the transmission of shocks and more specifically of the debt crisis between the EU12 economies, namely Austria, Belgium, Finland, France, Germany, Greece, Ireland, Italy, Luxembourg, Netherlands, Portugal and Spain, after the introduction of the Euro currency.
According to our findings the German economy was found to be the dominant economy in the model using, both, the δ-value of pervasiveness as well as other network theory measures and of course the eigenvalue distribution of the Input-Output weight matrix. As we have seen, this finding is fully consistent with the literature, and the recent developments in the socio-economic situation in Europe.

Next, our work estimated the link between output and debt fluctuations in EU12, based on the global variables of trade and finance, which act as the transmission channels that have been documented in the literature as being most significant. Our results confirm the fact that the role of trade volumes and the volume of stocks traded are of great importance in the transmission of fluctuations, in accordance with Frankel and Rose (1998), Imbs (2004, 2006), Chiquiar and Ramos-Francia (2005), Calderon et al. (2007) and Artis and Okubo (2011). It is exactly in this line of thinking that Stock and Watson (2005, abstract), have argued that: “Had the common international shocks in the 1980s and 1990s been as large as they were in the 1960s and 1970s, G7 business cycles would have been substantially more volatile and more highly synchronized than they actually were” implying that the transmission channels through which the different spillover effects between countries are activated, have been enormously strengthened lately because of globalization.

A main finding is that the shocks die out in the medium run, namely in less than eight (8) quarters, i.e. 2 years, and cannot affect the EU12 economies in the long run. However, our analysis also showed that the German economy has a significant impact on the rest of the EU12 economies either directly, i.e. through its GDP, or indirectly, i.e. through its Debt. An interesting finding of our investigation is the fact that the Southern European economies such as Greece, Italy and Spain that face either Debt issues or structural issues, mainly because of their banking systems, are primarily affected by the German Debt, as opposed to the rest of the economies that are affected mainly by the German GDP. This could be attributed to the role of the German economy as a locomotive of the overall Debt sustainability of the EMU that has benefited by the lowest spreads in the EMU area. Our findings are, in general terms, also consistent with the

Undoubtedly, future and more extended research on the subject seems to be necessary focusing on additional potential transmission channels, such as foreign direct investment, or even more importantly, bank lending and monetary policy. Of course, the proposed analysis could also be extended to account for additional variables, which have often proved to be relevant. Hence, the proposed approach could be routinely extended empirically to include other major economic regions such as USA, China, Russia, etc that would help further explain global imbalances. Of course, an additional gain from the potential inclusion of more regions in the network structure will involve a faster convergence of the $\delta$-value measure of pervasiveness, based on the evidence provided by Pesaran and Yang (2016).
REFERENCES


Bussiere Matthieu and Chudik Alexander and Mehl Arnaud, (2013), "How have global shocks impacted the real effective exchange rates of individual euro area countries since the euro’s creation?," The B.E. Journal of Macroeconomics, 13(1):.1-48.


CIA (2013), World Factbook.


Appendix A.

Figure A1: GIRFs, Response of GDP Austria posterior s.d. appear as bands

Note that the code enumeration for each economy is as follows: 1 - Austria, 2 - Belgium, 3 - Finland, 4 - France, 5 - Germany, 6 - Greece, 7 - Ireland, 8 - Italy, 9 - Luxembourg, 10 - Netherlands, 11 - Portugal, 12 - Spain.
Note that the code enumeration for each economy is as follows: 1 - Austria, 2 - Belgium, 3 - Finland, 4 - France, 5 - Germany, 6 - Greece, 7 - Ireland, 8 - Italy, 9 - Luxembourg, 10 - Netherlands, 11 - Portugal, 12 - Spain.
Figure A2: GIRFs, Response of GDP Belgium posterior s.d. appear as bands

Note that the code enumeration for each economy is as follows: 1-Austria, 2-Belgium, 3-Finland, 4-France, 5-Germany, 6-Greece, 7-Ireland, 8-Italy, 9-Luxemburg, 10-Netherlands, 11-Portugal, 12-Spain.
Note that the code enumeration for each economy is as follows: 1-Austria, 2-Belgium, 3-Finland, 4-France, 5-Germany, 6-Greece, 7-Ireland, 8-Italy, 9-Luxembourg, 10-Netherlands, 11-Portugal, 12-Spain.
Figure A3: GIRFs, Response of GDP Finland posterior s.d. appear as bands

Note that the code enumeration for each economy is as follows: 1-Austria, 2-Belgium, 3-Finland, 4-France, 5-Germany, 6-Greece, 7-Ireland, 8-Italy, 9-Luxemburg, 10-Netherlands, 11-Portugal, 12-Spain.
Note that the code enumeration for each economy is as follows: 1-Austria, 2-Belgium, 3-Finland, 4-France, 5-Germany, 6-Greece, 7-Ireland, 8-Italy, 9-Luxemburg, 10-Nethrelands, 11-Portugal, 12-Spain.
Note that the code enumeration for each economy is as follows: 1-Austria, 2-Belgium, 3-Finland, 4-France, 5-Germany, 6-Greece, 7-Ireland, 8-Italy, 9-Luxembourg, 10-Netherlands, 11-Portugal, 12-Spain.
Note that the code enumeration for each economy is as follows: 1 - Austria, 2 - Belgium, 3 - Finland, 4 - France, 5 - Germany, 6 - Greece, 7 - Ireland, 8 - Italy, 9 - Luxembourg, 10 - Netherlands, 11 - Portugal, 12 - Spain.
Figure A5: GIRFs, Response of GDP Germany posterior s.d. appear as bands

Note that the code enumeration for each economy is as follows: 1 - Austria, 2 - Belgium, 3 - Finland, 4 - France, 5 - Germany, 6 - Greece, 7 - Ireland, 8 - Italy, 9 - Luxembourg, 10 - Netherlands, 11 - Portugal, 12 - Spain.
Note that the code enumeration for each economy is as follows: 1-Austria, 2-Belgium, 3-Finland, 4-France, 5-Germany, 6-Greece, 7-Ireland, 8-Italy, 9-Luxemburg, 10-Netherlands, 11-Portugal, 12-Spain.
Figure A6: GIRFs, Response of GDP Greece posterior s.d. appear as bands

Note that the code enumeration for each economy is as follows: 1-Austria, 2-Belgium, 3-Finland, 4-France, 5-Germany, 6-Greece, 7-Ireland, 8-Italy, 9-Luxemburg, 10-Netherlands, 11-Portugal, 12-Spain.
Note that the code enumeration for each economy is as follows: 1-Austria, 2-Belgium, 3-Finland, 4-France, 5-Germany, 6-Greece, 7-Ireland, 8-Italy, 9-Luxemburg, 10-Netherlands, 11-Portugal, 12-Spain.
Figure A7: GIRFs, Response of GDP Ireland posterior s.d. appear as bands

Note that the code enumeration for each economy is as follows: 1-Austria, 2-Belgium, 3-Finland, 4-France, 5-Germany, 6-Greece, 7-Ireland, 8-Italy, 9-Luxembourg, 10-Netherlands, 11-Portugal, 12-Spain.
Note that the code enumeration for each economy is as follows: 1-Austria, 2-Belgium, 3-Finnland, 4-France, 5-Germany, 6-Greece, 7-Ireland, 8-Italy, 9-Luxemburg, 10-Netherlands, 11-Portugal, 12-Spain.
Figure A8: GIRFs, Response of GDP Italy posterior s.d. appear as bands

Note that the code enumeration for each economy is as follows: 1-Austria, 2-Belgium, 3-Finland, 4-France, 5-Germany, 6-Greece, 7-Ireland, 8-Italy, 9-Luxemburg, 10-Netherlands, 11-Portugal, 12-Spain.
Note that the code enumeration for each economy is as follows: 1-Austria, 2-Belgium, 3-Finland, 4-France, 5-Germany, 6-Greece, 7-Ireland, 8-Italy, 9-Luxemburg, 10-Netherlands, 11-Portugal, 12-Spain.
Figure A9: GIRFs, Response of GDP Luxembourg posterior s.d. appear as bands

Note that the code enumeration for each economy is as follows: 1 - Austria, 2 - Belgium, 3 - Finland, 4 - France, 5 - Germany, 6 - Greece, 7 - Ireland, 8 - Italy, 9 - Luxembourg, 10 - Netherlands, 11 - Portugal, 12 - Spain.
Note that the code enumeration for each economy is as follows: 1-Austria, 2-Belgium, 3-Finland, 4-France, 5-Germany, 6-Greece, 7-Ireland, 8-Italy, 9-Luxemburg, 10-Netherlands, 11-Portugal, 12-Spain.
Figure A10: GIRFs, Response of GDP Netherlands posterior s.d. appear as bands

Note that the code enumeration for each economy is as follows: 1-Austria, 2-Belgium, 3-Finland, 4-France, 5-Germany, 6-Greece, 7-Ireland, 8-Italy, 9-Luxemburg, 10-Netherlands, 11-Portugal, 12-Spain.
Note that the code enumeration for each economy is as follows: 1 - Austria, 2 - Belgium, 3 - Finland, 4 - France, 5 - Germany, 6 - Greece, 7 - Ireland, 8 - Italy, 9 - Luxemburg, 10 - Netherlands, 11 - Portugal, 12 - Spain.
Figure A11: GIRFs, Response of GDP Portugal posterior s.d. appear as bands

Note that the code enumeration for each economy is as follows: 1-Austria, 2-Belgium, 3-Finland, 4-France, 5-Germany, 6-Greece, 7-Ireland, 8-Italy, 9-Luxembourg, 10-Netherlands, 11-Portugal, 12-Spain.
Note that the code enumeration for each economy is as follows: 1-Austria, 2-Belgium, 3-Finland, 4-France, 5-Germany, 6-Greece, 7-Ireland, 8-Italy, 9-Luxembourg, 10-Netherlands, 11-Portugal, 12-Spain.
Figure A12: GIRFs, Response of GDP Spain posterior s.d. appear as bands

Note that the code enumeration for each economy is as follows: 1- Austria, 2-Belgium, 3-Finland, 4-France, 5-Germany, 6-Greece, 7-Ireland, 8-Italy, 9-Luxemburg, 10-Netherlands, 11-Portugal, 12-Spain.
Note that the code enumeration for each economy is as follows: 1-Austria, 2-Belgium, 3-Finland, 4-France, 5-Germany, 6-Greece, 7-Ireland, 8-Italy, 9-Luxemburg, 10-Netherlands, 11-Portugal, 12-Spain.