MEASURING NONFUNDAMENTALNESS FOR STRUCTURAL VARS

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Abstract. As nonfundamental vector moving averages do not have causal VAR representations, standard structural VAR methods are deemed inappropriate for recovering the economic shocks of general equilibrium models with nonfundamental reduced forms. In previous literature it has been pointed out that, despite nonfundamentalness, structural VARs may still be good approximating models. I characterize nonfundamentalness as bias depending on the zeros of moving average filters. However, measuring the nonfundamental bias is not trivial because of the simultaneous occurrence of lag truncation bias. I propose a method to disentangle the bias based on population spectral density and derive a measure for the nonfundamental bias in population. In the application, I find that the SVAR exercises of Sims (2012) are accurate because the nonfundamental bias is mild.

Keywords: Nonfundamentalness, SVAR, DSGE, news shocks.

1. Introduction

Vector autoregressive (VAR) models are the dominant approach to date for the empirical validation of dynamic stochastic general equilibrium (DSGE) models. It is well known that when the structural model is nonfundamental, estimated VARs do not recover the economic shocks. Nonetheless, as shown by Sims (2012) and Beaudry et al. (2015)

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structural VAR (SVAR) methods may still perform well in some applications. In this paper, I show that this is the case when the VAR is affected by a mild *nonfundamental bias*. I provide a population measure of nonfundamentalness by disentangling the nonfundamental bias from the lag truncation bias.

Since their appearence, DSGE models have been extensively validated with SVAR methods. In the last decade of research, the econometric challenges of this approach have received much attention (see e.g. Giacomini, 2013). In this spirit, both the existence of an infinite order VAR representation - see Fernandez-Villaverde et al. (2007) and Franchi and Paruolo (2014) - and its approximation with a finite order VAR - see Chari et al. (2008), Christiano et al. (2007), Erceg et al. (2005) and Poskitt and Yao (2012) - have been addressed. Nevertheless, those have remained two separate literatures and to the best of my knowledge, no study has ever measured the nonfundamentalness in population.

However, given that the nonexistence of an infinite order autoregressive representation (nonfundamentalness) implies the nonexistence of a finite order approximation, nonfundamental models are generally affected by truncation bias. Therefore, measuring nonfundamentalness requires disentangling between nonfundamentalness and lag truncation and it is misleading to evaluate the former without taking the latter into account.

If the set of observables used to estimate a VAR encloses all the relevant information necessary to retrieve the state of the economy, then the fundamentalness is granted, and the econometrician employing structural VAR methods is capable to estimate accurate impulse response functions to economic shocks. On the other hand, if the information available to the econometrician is insufficient, responses are contaminated by the error committed in the estimation of the state of the economy. Forni and Gambetti (2014) test for sufficient information in SVAR by comparing with a dynamic factor model whose estimated factors virtually include all information available acting as a proxy for the state of the economy[1].

Nonetheless, the information used to estimate a VAR, albeit inferior, may be sufficiently close to that of the agents. Sims (2012), Beaudry and Portier (2013) and Beaudry et al. (2015) show that there are applications in which invertibility failures are mild and VARs remain a useful tool. Beaudry et al. (2015) derive a R^2 diagnosis based on the fact that under fundamentalness the innovations to the econometrician information set do not correlate with the past of the factors (and of the innovations to agents' information set). Yet neither does their approach provide a measure of nonfundamentalness in population.

In order to address this problem, I build on the fact that nonfundamentalness is a source of bias depending on the distance between the nonfundamental representation of the data providing the structural shocks and its unique fundamental representation. Population quantities are derived from the time series properties of the observables. Fernandez-Villaverde et al. (2007) provide a condition for nonfundamentalness. I contribute to this literature with a measure of the nonfundamental bias based on the frequency domain. Forni et al. (2016) focus on single shocks rather than the nonfundamentalness of the whole VAR system as I do in this paper.

I first show that the error is a combination of the nonfundamental and lag truncation bias. The measure proposed here is then applied to the news shock model of Sims (2012). I find that the econometrician estimating the VAR of Sims (2012) is faced with little nonfundamental bias. This explains why in this application SVAR methods are found to perform well. I also find that when the DSGE is reduced to a real business cycle (RBC) model with news shocks the lag truncation bias is at least as large as the nonfundamental bias.

While avoiding stochastic singularity in the VAR representation of a DSGE model makes impossible to increase information by adding observables so mitigating nonfundamentalness, the lag truncation bias

[1]There is still an information loss due stationary transformations required for the estimation of the factor model (see Barigozzi et al., 2013).

may in principle be ameliorated by estimating high-order VARs[2]. I find that this advice does not apply to the nonfundamental case.

The structure of the paper is as follows. Given a state-space representation of the DSGE model, the literature reviewed in section 2 provides simple conditions in order to check for nonfundamentalness and the existence of a finite order VAR representation for the observables. In section 3, I illustrate the nonfundamental bias and discuss how it relates to the truncation bias. Section 4 provides a measure of nonfundamentalness obtained by decomposing the bias of estimated VARs with a method based on the spectral density matrix of the data. Section 5 is a brief discussion of the economics of anticipated shocks and their link with nonfundamentalness in the general equilibrium literature. In section 6, I apply the method proposed here to measure the nonfundamentalness in a news shocks model along the lines of Sims (2012). Last section concludes with practical suggestions and discusses future work.

2. Background: invertibility, nonfundamentalness and lag truncation

Typically the approximation to the solution of a DSGE model is cast into the state space form:

$$X_t = A(\theta) X_{t-1} + B(\theta) \varepsilon_t \tag{2.1}$$

$$Y_t = C(\theta) X_{t-1} + D(\theta) \varepsilon_t \tag{2.2}$$

where θ is a vector of deep parameters, Y_t is a $n_y \times 1$ vector of observed variables, X_t is a $n_x \times 1$ vector of endogenous and exogenous state variables, and $\varepsilon_t \sim iid\, N(0, \Sigma)$ a vector of n_ε structural shocks, (2.2) is the measurement equation and (2.1) the state equation.

DSGE models typically have unobserved latent states and the information enclosed in Y_t is limited because avoiding singularity requires

[2]On a fundamental model De Graeve and Westermark (2013) show that extending the order of the estimated VAR above and beyond that suggested by information criteria helps in reducing the truncation bias. Using nonparametric approaches Christiano et al. (2007) and Mertens (2012) find mixed results.

the *square case* - i.e. $n_y = n_\varepsilon$. Assuming that D is nonsingular, from equation (2.2) we get $\varepsilon_t = D^{-1}\left(Y_t - CX_{t-1}\right)$. Plugging this expression for the structural shocks into the state equation (2.1) and rearranging, the mapping between the states and the observables is

$$\left(I_{n_x} - FL\right) X_t \;=\; BD^{-1}Y_t \tag{2.3}$$

where $F := A - BD^{-1}C$.

Proposition 1 (Fernandez-Villaverde et al. (2007) - *Poor man's invertibility condition*)**.** *Y_t admits a $VAR(\infty)$ representation if F is a stable matrix - i.e. all its eigenvalues are less than one in absolute value.*

If the Poor Man's Invertibility condition (PMIC) holds true the linear function (2.3) mapping states into observables is invertible and the infinite order VAR representation is

$$Y_t \;=\; C\sum_{j=1}^{\infty} F^j BD^{-1}Y_{t-j} + u_t \tag{2.4}$$

where $u_t := D\varepsilon_t$.

Definition 2 (Rozanov (1967) - *Fundamentalness*)**.** In the moving average $Y_t = G(L)\,u_t$ the shock u_t is Y_t*-fundamental* if and only if the Hilbert space spanned by $(u_j;\ j = 1,\ldots,t)$ is the same as that spanned by the present and past of the observables $\mathcal{H}_t^Y$. Then $\det G(z) \neq 0, \forall\, |z| < 1$ and $Y_t = G(L)\,u_t$ is the unique fundamental moving average representation of Y_t.

Fundamentalness[3] and invertibility are closely related. In Section 4 I prove the PMIC is a condition for fundamentalness. Whether a root of a polynomial in the lag operator lays inside (nonfundamentalness) or outside (fundamentalness) the unit disk, the inversion of the polynomial is defined over respectively negative or positive powers of the lag

[3]Fundamentalness holds true on the unit disk (see Alessi et al., 2011). Conversely, from an algebraic point of view, any polynomial is invertible unless it has a unit root. Therefore, while invertibility is violated on the unit disk, fundamentalness holds true in presence of a unit root.

operator. Roots into the unit circle correspond to invertibility of the shocks in the future of the observables. Of course, in applied research it is required invertibility in the past of the observables and for this reason nonfundamentalness is a problem.

Proposition 3 (Franchi and Vidotto (2013) - Existence of finite order VAR representation)**.** *A finite order VAR representation exists if F is nilpotent - i.e. all its eigenvalues are equal to zero.* [4]

Therefore lag truncation is a necessary condition for nonfundamentalness and measuring the latter requires disentangling from the former. Even if the econometrician knows the structural shocks, their projection on the (finite) past of the observables does not only measure nonfundamentalness but also lag truncation. This is the reason why root flipping is required to disentangle the two biases.

Under the assumption that the PMIC holds true, the VAR(∞) representation (2.4) of the observables is easily obtained from the state-space system. More generally, the VAR representation of Y_t is:

$$Y_t = C\left(I - FL\right)^{-1} BD^{-1}Y_{t-1} + u_t \tag{2.5}$$

Franchi and Paruolo (2014) point out that the conditions above are sufficient but not necessary because of possible pole cancellations in $C\left(I - Fz\right)^{-1} B$ due to irrelevant states in the ABCD system (2.1) - (2.2). On the other hand, from the minimal[5] ABCD system, whose

[4]Franchi and Vidotto (2013) prove that their condition is equivalent to that of Ravenna (2007) requiring that

$$|I - Az|\, I + C\, adj\left(I - Az\right) BD^{-1}z \qquad \text{is unimodular}$$

i.e. its determinant is a nonzero constant.

[5]An ABCD system is minimal if it is *controllable* and *observable*. Controllability holds if

$$\mathcal{C} = \left[\begin{array}{cccc} B & AB & \ldots & A^{n_x-1}B \end{array}\right]$$

has full row rank, observability holds if

$$\mathcal{O} = \left[\begin{array}{c} C \\ CA \\ \vdots \\ CA^{n_x-1} \end{array}\right]$$

has full column rank.

state vector is of the smallest dimension possible for replicating the dynamic properties of the original system, necessary and sufficient conditions are derived. Letting the minimal system be

$$X_t = A_m(\theta) X_{t-1} + B_m(\theta)\varepsilon_t \tag{2.6}$$

$$Y_t = C_m(\theta) X_{t-1} + D(\theta)\varepsilon_t \tag{2.7}$$

where A_m, B_m, C_m arise from the Kalman decomposition theorem[6] (see Antsaklis and Michel, 2007, Theorem 6.6), and defining $F_m \coloneqq A_m - B_m D^{-1} C_m$ we have the following condition:

Proposition 4 (Franchi and Paruolo (2014) - Necessary and sufficient conditions)**.** *Y_t admits a VAR(∞) representation if and only if F_m is a stable matrix. Moreover the VAR representation is of finite order if and only if F_m is nilpotent.*

It is important to note that the similarity transformation required for the reduction of the ABCD model to its minimal counterpart does not affect the observables and their dynamics. Given that the impulse response functions and the shocks are invariant to this transformation, the economic interpretation is preserved in the minimal system (2.6) - (2.7) (see Antsaklis and Michel, 2007, section 3.5.3). For this reason, in the rest of the paper I refer to the minimal system and to the following VAR representation

$$Y_t = C_m (I - F_m L)^{-1} B_m D^{-1} Y_{t-1} + u_t \tag{2.8}$$

which is also equivalent to the (2.5).

3. The nonfundamental bias

Nonfundamentalness is a source of bias and it should not come as a surprise that it may be small. A nonfundamental moving average (MA) whose roots in the unit circle are sufficiently close to the circle is generally well approximated by its fundamental counterpart. Straightforwardly, the reciprocal of a root which is close to the circle will be

[6]The minimal system (2.6) - (2.7) can be computed with popular control system packages (e.g. the function `minreal` in Matlab).

itself pretty close to the circle. In the same way, a white noise process corresponding to the residuals of a nonfundamental representation of the data will be pretty close to the white noise arising from the fundamental representation of the data when the MA roots of the two representations are sufficiently close to the unit disk.

Suppose that the econometrician wants to estimate the structural MA

$$y_t = \left(1 - \frac{1}{a_1}L\right)\left(1 - \frac{1}{a_2}L\right)\dots\left(1 - \frac{1}{a_p}L\right)u_t = a(L)\,u_t$$

in which for simplicity the roots $a_1, a_2, \dots, a_p$ are all real[7]. Assume that the model is nonfundamental because $\exists\ j,\ 1 \leq j \leq p\ :\ |a_j| < 1$, while $|a_k| > 1,\ k \neq j$. The fundamental MA representation of y_t is:

$$\begin{aligned} y_t &= \underbrace{a(L)\frac{a_j^{-1} - L}{a_j - L}}_{\widetilde{a}(L)}\underbrace{\frac{a_j - L}{a_j^{-1} - L}u_t}_{v_t} \\ &= \widetilde{a}(L)\,v_t \end{aligned}$$

where u_t and v_t are both white noise (see Lippi and Reichlin, 1994) but u_t lies in the past, present and future of y_t while v_t lies in the past of y_t. Therefore:

$$\begin{aligned} |u_t - v_t| &= \left| u_t\left(1 - \frac{a_j - L}{a_j^{-1} - L}\right)\right| \\ &= \left| u_t\left(\frac{a_j^{-1} - a_j}{a_j^{-1} - L}\right)\right| \end{aligned}$$

which goes monotonically to zero as $|a_j| \to 1$.

I measure this distance as

$$d_\infty = \frac{E\left(u_t - v_t\right)^2}{Eu_t^2}$$

(in Section 4 the measure is generalized to the multivariate case and its asymptotic behaviour as a function of MA roots is described).

[7]This assumption is made for illustrative purpose and it is relaxed in the next section.

For example, consider the MA of order 1 $y_t = (1 - \theta L) u_t$. Then $d_\infty = \frac{\left(\theta - \theta^{-1}\right)^2}{1 - \theta^{-2}}$. The top panel of Figure 1 shows that the measure d_∞ increases in θ - the reciprocal of the MA root. The value of d_∞ for MA (1) processes is yet close to 100 percent error when $\theta = 1.387$ and is exactly 125 percent when $\theta = 1.5$ so the root is closer to the center of the unit disk.

Let $v_t^{(p)}$ be the residuals in a VAR (p) representation of Y_t.

Proposition 5. *The distance between the reduced form residuals* u_t *and* $v_t^{(p)}$ *is monotonically decreasing in* p

Proof. in Appendix □

In a similar manner to (4.6) let us measure this distance as: $d(p) = \frac{E\left(u_t - v_t^{(p)}\right)}{Eu_t^2}$. The bottom plot of Figure 1 shows the path $d(p)$ in the MA (1) example. For a sufficiently high order p, the error $d(p)$ asymptotically converges to its limiting value - i.e. the component of the error d_∞ due to the nonfundamentalness bias. The closer the root to the unit circle, the slower the convergence to d_∞.

4. The nonfundamental bias in population

In this section a method to decompose the bias in population is provided. To do so in subsection 4.1 three representations of the data are employed - the structural, the fundamental and the truncated - and their properties are summarised. If the structural model is nonfundamental, subsection 4.2 describes how to get the fundamental vector moving average representation starting from the nonfundamental structural representation. Finally, in subsection 4.3, I propose a method to calculate the bias decomposition in population. This last subsection outlines the decomposition in the covariance matrix of VAR residuals employed in any identification scheme.

4.1. **Alternative representations of the data.** When the PMIC is violated being some eigenvalues of F_m greater than one in absolute value, the VAR (2.8) is noncausal because the term $(I - F_m z)^{-1}$ also maps in negative powers of z and $H(z) := C_m (I - F_m z)^{-1} B_m D^{-1} z$ is two-sided. As Lanne and Saikkonen (2011, 2013) show, a noncausal VAR can be estimated although, contrary to the common practice in DSGE modeling, non-Gaussianity is required for its identification.

The moving average representation[8] associated with the non necessarily causal VAR representation (2.8) is

$$Y_t = G(L)\, u_t \tag{4.1}$$

where $G(z) := (I - H(z))^{-1}$. When the PMIC is violated, the VMA representation (4.1) is nonfundamental and inverts into a noncausal VAR. The fundamental VMA representation of Y_t

$$Y_t = \widetilde{G}(L)\, v_t \tag{4.2}$$

[8]Considering that in minimal systems no cancellation takes place, yet at first sight representation (4.1) reveals the link between F_m's eigenvalues and MA roots. Thus, in this framework root-flipping becomes easy (see Proposition 6).
There are several equivalent ways to write the MA representation. For example, we may consider:

$$Y_t = W(L)\, \varepsilon_t, \qquad W(z) := D + C(I - Az)^{-1} Bz$$

Franchi and Paruolo (2014) show that, due to possible pole cancellations, the roots of $|I - Fz|$, that is the poles of $(I - Fz)^{-1}$, are not necesarily roots of $|C(I - Fz)B|$. Cancelling poles are related to the eigenvalues of A - i.e. reciprocal roots of $I - Az$ - which are also eigenvalues of F. This property is immediately evident writing the MA representation as Forni et al. (2016): $Y_t = DB^{-1}(I - FL)(I - AL)^{-1} B\varepsilon_t$.
The same argument applies to the zeros of $W(z)$. In fact, the system matrix

$$P(z) = \begin{pmatrix} I - Az & B \\ -Cz & D \end{pmatrix}$$

has determinant

$$|P(z)| = |I - Az| \left| C(I - Az)^{-1} Bz + D \right|$$

so that the zeros of $W(z)$ are system zeros which are not zeros of $|I - Az|$. To see this it is enough to write

$$P(z) = \begin{pmatrix} I - Az & 0 \\ -Cz & I \end{pmatrix} \begin{pmatrix} I & (I - Az)^{-1} B \\ 0 & C(I - Az)^{-1} Bz + D \end{pmatrix}$$

is found via multiplying and dividing by a matrix $\mathcal{B}(z)$ that flips the zeros into the unit circle of $G(z)$

$$Y_t = \underbrace{G(L)\,\mathcal{B}(z)}_{\widetilde{G}(L)}\underbrace{\mathcal{B}(z)^{-1}u_t}_{v_t}$$

In practice, the above step for getting the fundamental VMA representation (4.2) is very easy because, according to the following Proposition, it boils down to flipping the appropriate roots of $(I - F_m z)$.

Proposition 6. *The fundamental MA representation of the data is* $Y_t = \widetilde{G}(L)\,v_t$ *with*

$$\widetilde{G}(z) \coloneqq \left(I_{n_y} - C_m\left(I_{n_m} - \widetilde{F}_m z\right)^{-1} B_m D^{-1} z\right)^{-1}$$

where

(1) $\widetilde{F}_m$ *is such that* $\det\left(I - \widetilde{F}_m z\right) = 0 \;\; \forall z = (\widetilde{z}_1, \ldots, \widetilde{z}_{n_m})$ *and*

$$\widetilde{z}_i = \begin{cases} z_i & |z_i| > 1 \\ \overline{z}_i^{-1} & |z_i| < 1 \end{cases}$$

(2) *the residuals* v_t *of the fundamental MA representation are related to those of the nonfundamental MA representation* u_t *according to*

$$v_t = \prod_{i=1}^{n_m} \frac{(L - z_i)}{(L - \widetilde{z}_i)} u_t$$

Proof. in Appendix ☐

Finally, the infinite order causal VAR representation of Y_t is

$$Y_t = \widetilde{H}(L)\,Y_{t-1} + v_t \tag{4.3}$$

where $\widetilde{H}(z) = C_m\left(I_{n_m} - \widetilde{F}_m z\right)^{-1} B_m D^{-1} z$ and the white noise v_t is the Wold innovation for Y_t. In the next subsection the root flipping procedure to find $\widetilde{F}_m$ is outlined.

In the equation (4.6) the nonfundamental bias is evaluated as the distance d_∞ between the true (nonfundamental) residuals u_t and the fundamental residuals v_t of the infinite order VAR representation.

Proposition 7. *The distance between the reduced form residuals* u_t *and* v_t *goes monotonically to zero as the eigenvalues of* F *less than* 1 *in absolute value approach the complex unit disk.*

Proof. in Appendix □

Standard practice is to approximate the infinite order VAR (4.3) with an estimated finite order VAR (p)

$$Y_t = \widetilde{H}^{(p)}(L)\, Y_t + v_t^{(p)} \tag{4.4}$$

where: the coefficients of $\widetilde{H}^{(p)}(z)$ are found by projecting Y_t on its first p lags, and the residuals $v_t^{(p)}$ differ from the fundamental residuals of the infinite order VAR representation v_t by a truncation bias term. Disentangling the nonfundamental bias from the lag truncation bias requires root flipping otherwise $\widetilde{H}(z)$ would be replaced by $\widetilde{H}^{(p)}(z)$ and v_t by $v_t^{(p)}$. As found in the literature on lag truncation, this practice might be somewhat inaccurate.

4.2. **Root flipping.** This subsection describes the method employed to flip the roots in the unit disk of the $n_m \times n_m$ polynomial $F(z) := I - F_m z$ obtaining a new polynomial $\widetilde{F}(z) := I - \widetilde{F}_m z$ which shares the roots of $F(z)$ out of the unit circle and flips the roots of $F(z)$ laying inside the unit circle.

Let n_{NF} be the number of roots $z_1, \ldots, z_{n_{NF}}$ in the unit disk of $F(z)$ with $n_{NF} \leq n_m$. Then $|z_k| < 1$, $k = 1, \ldots, n_{NF}$ and $|F(z_k)| = 0$. The following steps are needed to flip the roots in the disk of $F(z)$.

(1) As I am formally flipping the roots of the equation (2.3) I first need to orthonormalize the right-hand side of

$$B_m D^{-1} Y_t = (I - F_m L)(I - A_m L)^{-1} B_m \varepsilon_t$$

using the variance of X_t. Start with $k = 1$.

(2) The spectral decomposition $F(z_k) = U_k V_k U_k^{-1}$ where $U_k = [\mathcal{U}_{k,1}, \mathcal{U}_{k,2}, \ldots, \mathcal{U}_{k,n_m}]$ collects the eigenvectors $\mathcal{U}_{k,i}$ $(i = 1, \ldots, n_m)$ of $F(z_k)$ and V_k is a block-diagonal matrix whose Jordan blocks

have the corresponding eigenvalues on the diagonal. The decomposition is such that at least one element in the diagonal of V_k is equal to zero.

(3) Let $j \in [1,\, n_m]$ be an integer such that the j-th diagonal element $V_{k,jj} = 0$. The number of such zero valued elements along the diagonal of V_k is equal to the multiplicity of z_k.

(4) Define a matrix $M_k = \left[\begin{array}{cc} \mathcal{U}_{k,j} & \ker\left(\mathcal{U}'_{k,j}\right) \end{array}\right]$. By construction M_k is an orthogonal matrix.

(5) Compute $F^{(k)}(z) = F(z)\, M_k$. Notice that its $(1,1)$ element $f^{(k)}_{11}(z)$ is equal to zero.

(6) Compute

$$\tilde{F}^{(k)}(z) = \left[\begin{array}{cccc} \mathcal{B}_k(z)\, f^{(k)}_{11}(z) & f^{(k)}_{12}(z) & \dots & f^{(k)}_{1n_m}(z) \\ \mathcal{B}_k(z)\, f^{(k)}_{21}(z) & f^{(k)}_{22}(z) & \dots & f^{(k)}_{2n_m}(z) \\ \vdots & \vdots & & \vdots \\ \mathcal{B}_k(z)\, f^{(k)}_{n_m1}(z) & f^{(k)}_{n_m2}(z) & \dots & f^{(k)}_{n_mn_m}(z) \end{array}\right]$$

where

$$\mathcal{B}_k(z) = \frac{1 - z\,\bar{z}_k}{z - z_k}$$

is a factor replacing the nonfundamental root z_k with its reciprocal[9].

(7) Repeat steps 2-5 for the multiplicity of z_k.

(8) By repeating steps 1-6 for $k = 2, \dots, n_{NF}$ I get $F^{\dagger}(z) := \tilde{F}^{(1,\dots,n_{NF})}(z)$.

(9) Finally, note that $F^{\dagger}(z) = F^{\dagger}_0 + F^{\dagger}_1 z$. Straightforwardly, the unique polynomial I am searching for is

$$\widetilde{F}(z) = I - \widetilde{F}_m\, z$$

[9]Note that $\mathcal{B}_k(z)$ is not a Blaschke factor because it has a pole into the unit circle. As in Lippi and Reichlin (1994), a Blaschke matrix flips roots into the unit circle thanks to a Blaschke factor

$$\frac{z - z_k}{1 - \overline{z_k} z}$$

The root flipping problem in this section is carried out the other way around. For a given nonfundamental representation of the data in terms of structural shocks, $\mathcal{B}_k(z)$ is meant to flip roots from inside to outside the circle. Therefore $\mathcal{B}_k(z)$ is the reciprocal of a Blaschke factor with a pole in z_k.

where $\widetilde{F}_m = -F_1^\dagger F_0^{\dagger^{-1}}$.

4.3. **Computing the measure for SVARs.** Let

$$\Sigma_y(\omega) = \frac{1}{2\pi} G\left(e^{-i\omega}\right) \Sigma_u G\left(e^{i\omega}\right)'$$

be the spectral density matrix of the data at frequency ω, with $\Sigma_u = D\Sigma D'$. It is well known that $\Sigma_y(\omega)$ is unique, therefore it can be calculated starting from any moving average representation of Y_t. So it must be that $\Sigma_y(\omega) = \frac{1}{2\pi}\widetilde{G}\left(e^{-i\omega}\right)\Sigma_v\widetilde{G}\left(e^{i\omega}\right)'$, where $\Sigma_v = Ev_tv_t'$.

Exploiting the uniqueness property of the spectral density matrix, the above formula can be inverted to calculate the covariance matrix of any vector of residuals that can be expressed as a moving average of Y_t. Hence, from $u_t - v_t = \left(\widetilde{H}(L) - H(L)\right) Y_t$, the covariance matrix $\Sigma_{u-v} = E\left(u_t - v_t\right)\left(u_t - v_t\right)'$ is obtained as

$$\Sigma_{u-v} = \int_{-\pi}^{\pi} \left(\widetilde{H}\left(e^{-i\omega}\right) - H\left(e^{-i\omega}\right)\right) \Sigma_y(\omega) \left(\widetilde{H}\left(e^{i\omega}\right) - H\left(e^{i\omega}\right)\right)' d\omega \tag{4.5}$$

The measure[10] is:

$$d_\infty = \frac{\|\Sigma_{u-v}\|}{\|\Sigma_u\|} \tag{4.6}$$

5. DSGE, anticipated shocks and nonfundamentalness

Yet Lippi and Reichlin (1993) argue that economic models can lead to nonfundamental representations of the data. As surveyed in Alessi et al. (2011) and Lütkepohl (2012), nonfundamentalness is basically of two kinds: one which is peculiar to the story-telling of the DSGE and the other which arises as an omitted variable problem. In the former case, nonfundamentalness is model-based and the economic shocks may well be nonfundamental with respect to any set of observables (models with imperfect information where the agents are faced with a filtering problem to infer the structural shocks). In the latter (full information), the nonfundamentalness depends on the information available to the econometrician while the agents observe the shocks. For this reason

[10] $\|\cdot\|$ stands for the Euclidean norm of a square matrix and is equal to its largest singular value. Of course, other matrix norms are equally suitable.

this kind of nonfundamentalness is referred to as omitted variables nonfundamentalness.

Model-based nonfundamentalness is essentially an identification issue which is not fatal for structural VARs. More generally, when this kind of nonfundamentalness is present, the vector autoregressive moving average (VARMA) representation of the DSGE model has one or more roots into the unit disk which are known. For example, in models of imperfect information it is known the link between the shocks agents observe and the economic shocks. There will be a fundamental VARMA representation - with no roots in the unit disk - in terms of innovations to agents information set and a nonfundamental VARMA representation - with some roots in the unit disk - in terms of structural shocks. The way imperfect information is modeled does generally reveal the mapping between the structural shocks and the innovations to agents information set and, therefore, the roots in the unit disk of the structural nonfundamental VARMA.

In last decade a branch of empirical macroeconomic research has focused on the (Pigouvian) idea that expectational swings could generate business cycles. In this spirit, standard models have been augmented with anticipated shocks like technology news shocks or fiscal foresight. Even models with anticipated shocks produce model-based nonfundamentalness if information flows are properly modelled. So the econometrician that knows the structural model also knows how to map Wold residuals to structural shocks (see Forni et al., 2013a,b; Mertens and Ravn, 2010; Leeper et al., 2013).

On the other hand, the omitted variable nonfundamentalness studied here is a potentially harmful estimation issue. In this case, the agents anticipate future shocks and the econometrician is challenged by the difficult task to infer a source of randomness which is not mapped into the few observables she is endowed with in the usual way. Observing more variables would be a panacea enabling her to match the information of the agents but the VAR representation of the DSGE model does not allow her to observe more than n_ε variables. In this framework, if the econometrician cannot count on sufficiently forward looking variables needed to make up for the information the agents anticipate, then her analysis based on an inferior information set will be biased.

Early VAR evidence on models with technology news shocks, as that of Beaudry and Portier (2006), has depicted positive comovements between macroeconomic aggregates in response to news shocks. Forni et al. (2014) with a structural factor model obtain very different findings supporting wealth effects and implied negative comovements, as predicted by the standard neoclassical growth model. Barsky and Sims (2011) prove that, once the information enclosed in the analysis is carefully selected, the problem can be solved even in a VAR framework. Exploiting the information of a very forward looking set of observables they show that, at least on a qualitative level, VAR impulse response functions are in the same ballpark of those of Forni et al. (2014).

Are violations of PMIC condition as stated in Proposition 1 implying that impulse responses estimated with structural VAR are inaccurate? Sims (2012) analyses a news shock model in which they are not. In the next section I find that his result is due to small nonfundamental bias.

6. Application: Sims (2012)

I study a simple generalization of the news shock DSGE model of Sims (2012) in which the technology is

$$\ln a_t = g_a + \ln a_{t-1} + \xi_t + \eta_{t-q} \tag{6.1}$$

The news shock $\eta_t \sim iid\,(0, \sigma_\eta)$ is observed by the agents q periods before it affects a_t. The econometrician observing only a_t will have to wait q periods for that information, that's why η_t is mapped into the future of a_t. Nonetheless, as the model features two shocks, the SVAR econometrician can overcome this problem by observing one additional variable. If such variable is sufficiently forward looking then she will be able to retrieve η_t as it will map into the present of the observables. In this sense, the anticipation may drive a wedge between agents and econometrician information sets.

$\xi_t \sim iid\,(0, \sigma_\xi)$ is the traditional unanticipated technology shock and g_a is the growth rate of the TFP. The rest of the model is a standard medium scale DSGE with nominal and real frictions (see Appendix B). I analyze two nested specifications: a frictionless RBC model and a full model with Calvo price stickiness, habit formation in consumption and investment adjustment costs.

Table 1 reports the moduli of the eigenvalues of F_m in the full model and RBC model respectively. When the agents learn the news with at least two period of anticipation all the models turn nonfundamental. Kurmann and Otrok (2011) find that the same result holds for the reduced form of the DSGE model of Smets and Wouters (2007). Table 2 tells us how large is the nonfundamental bias in the two models for $q = 1, \ldots, 8$. Although the full model has a larger nonfundamental bias for any $q \geq 2$, those numbers are relatively small. The maximum value for the nonfundamental bias is around 27 percent and requires 8 periods of anticipation which with quarterly data means news that affect technology with a two-year delay.

Like Sims (2012) the responses to a technology shock and the news shock of the two observed variables - technology a_t and output y_t - are identified according with a short run scheme through Choleski orthogonalization[11] with technology ordered first (the news shock has no contemporaneous effect on the technology) and a VAR (p) is estimated.

Mean absolute percentage errors

$$MAPE^{(m)}(\widehat{IRF}^{i,j,m}) = h^{-1} \sum_{t=1}^{h} \frac{\left| IRF_t^{i,j,m} - \widehat{IRF}_t^{i,j,m} \right|}{\left| IRF_t^{i,j,m} \right|}$$

are computed for variables $i = \{Y_t,\, a_t\}$ and shocks $j = \{\xi_t,\, \eta_t\}$ over a horizon of $h = 40$ periods, where $IRF^{i,j,m}$ are the true response of variable i to the shock j in the model m and $\widehat{IRF}^{i.j,m}$ are VAR estimates. $\widehat{IRF}$'s are averaged across 500 samples in the small sample exercise and 5 samples in the large sample exercise. The performance

[11]As pointed out by Christiano et al. (2007), a short run identification scheme is less affected by lag truncation than a long run scheme involving the estimate of the sum of VAR coefficients.

of each model is further summarized by averaging the four responses estimated, so a single mean absolute percentage error $MAPE^{(m)}$ is obtained for every specification $m = 1, \ldots, n_S$ of the exogenous news process.

In Tables 3 and 4 the MAPE in impulse response functions estimated with VAR of increasing orders for the full and RBC models in finite and large samples are reported. The first results standing out is that, as d_∞ also the MAPE increases with q.

The distance d_∞ between the true residuals u_t and the fundamental residuals v_t implied by the infinite order VAR representation (4.3) is much larger in the full model as compared with the RBC model. Nevertheless, the MAPE in the two models is somewhat close suggesting that in the RBC case there is relatively much more truncation and, at least in this DSGE model, the truncation bias is empirically as relevant as the nonfundamental bias. Consider for example the replication of Sims (2012) - i.e. $q = 3$. In Figure 2 we have that the eigenvalues of F_m are much closer to those of $\widetilde{F}_m$ in the RBC case and $d_\infty = 0.048$ while it is equal to 0.212 in the full model. Then the most accurate VAR is much more parsimonious in the full model (both in small and large samples) and at least in small samples the full model MAPE is even smaller than that of the RBC model.

In large samples when the data generating process is fundamental - i.e. form Table 1 when $q = 1$ in any model - the error goes to zero. This is in line with the results of De Graeve and Westermark (2013) who show that, in the fundamental case, high-order VAR allows to recover the economic shocks. As the anticipation is increased more parsimonious VAR perform better.

Probably due to more truncation bias, in the RBC case the most accurate VARs are higher in order than in the full model which prefers parsimonious VARs. This is specially true in small samples where the most accurate VAR is always the most parsimonious while in large samples such result is achieved for $q > 4$. This does not mean that the *untruncating* strategy advocated by De Graeve and Westermark (2013) of estimating very long VAR also works in the nonfundamental case. Neither in large samples the longest VAR is the most accurate in recovering the impulse responses of the RBC model whose bias is mainly due to truncation.

The reason behind this result is that mitigating the truncation bias does only require a good approximation of the coefficients of $H(L)$ in the positive powers of the lag operator rather than the whole filter. So, while according to the Proposition 5 a good approximation of the residuals u_t requires a long VAR, in practice increasing the order of a one-sided filter does not help in approximating the coefficients of a two-sided filter triggering an identification bias (see Ravenna, 2007) when impulse responses to economic shocks are being estimated.

As different amounts of lag truncation bias affect the two models, the Monte Carlo experiment is repeated in small samples using the BIC information criterion for $q = 3$. In Figure 3 the estimated IRFs from such VARs represented with dashed and dotted lines are compared with those in dashed lines estimated by Sims (2012) with a VAR(8). In the full model the truncation bias is very small so the choice of the lag order does not seem to affect the shape of the responses. Dashed and dashed and dotted lines are in fact very similar in the full model.

In the RBC case the truncation plays a more important role and the conclusions are different. The econometrician employing the BIC criterion would estimate more precisely the responses to the traditional unanticipated technology shock, but she would also get a much worse estimation of the responses to the news shock. The interpretation of these results is straightforward. Being the introduction of news shock in the DSGE the cause of the nonfundamentalness, the BIC criterion seems to do well what it is known for. That is to choose an optimal lag length for retrieving (linear combination of) shocks which are mapped in the present and past of the observables like the unanticipated technology shock.

The observation of Figure 3 also reveals that the surprise technology shock is less accuratelly estimated than the news shock. While the degree of nonfundamentalnes, as measured by d_∞, summarizes the overall accuracy of SVAR models, that is a "global" measure, how single shocks are affected by the nonfundamentalness bias requires further analysis. For this purpose, Forni et al. (2016) develop a "partial" measure of nonfundamentalness.

7. Conclusions and practical suggestions

An extensive literature has studied the accuracy of SVAR techniques in recovering the impulse response functions to the structural shocks of DSGE models. Addressing nonfundamentalness gained attention with the recent interest in news shocks and fiscal foresight. Previous literature noticed that nonfundamentalness is not an either/or proposition but no approach has been developed to determine how severe the problem is in a given application. Being nonfundamentalness sufficient for the nonexistence of a finite order VAR representation, I provide a frequency domain method to measure the bias due to nonfundamentalness disentangling from that due to lag truncation.

Starting from a state-space representation of the DSGE model this measure is very easy to compute, so the suggestion here comes at no additional cost with respect to the advice of Fernandez-Villaverde et al. (2007) and provides the advantage of offering a measure of the nonfundamental bias rather than a condition for its existence. As it employs reduced form quantities it does not depend on identification assumptions. In fact, under nonfundamentalness even SVAR exercises with a correct identification scheme are invalid because no rotation of the Wold innovations can retrieve the economic shocks.

Inference on the measure proposed here can be performed considering the parametric uncertainty in the state-space representation. For example, rather than testing for nonfundamentalness as Forni and Gambetti (2014), the econometrician might find more interesting building a confidence interval for the size of the nonfundamental bias in population. I leave this for future research.

In the application, I find little nonfundamentalness in the model of Sims (2012) which explains his finding that SVAR methods perform well. Similar results are found by Beaudry et al. (2015). This conclusion is not general as models with more sophisticate dynamics might generate larger bias for SVAR analysis. The measure proposed in this paper is a guide for the econometrician addressing this issue.

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FIGURE 1. d_∞ and $d\,(p)$ - MA(1): $y_t = (1 - \theta L)\, u_t$

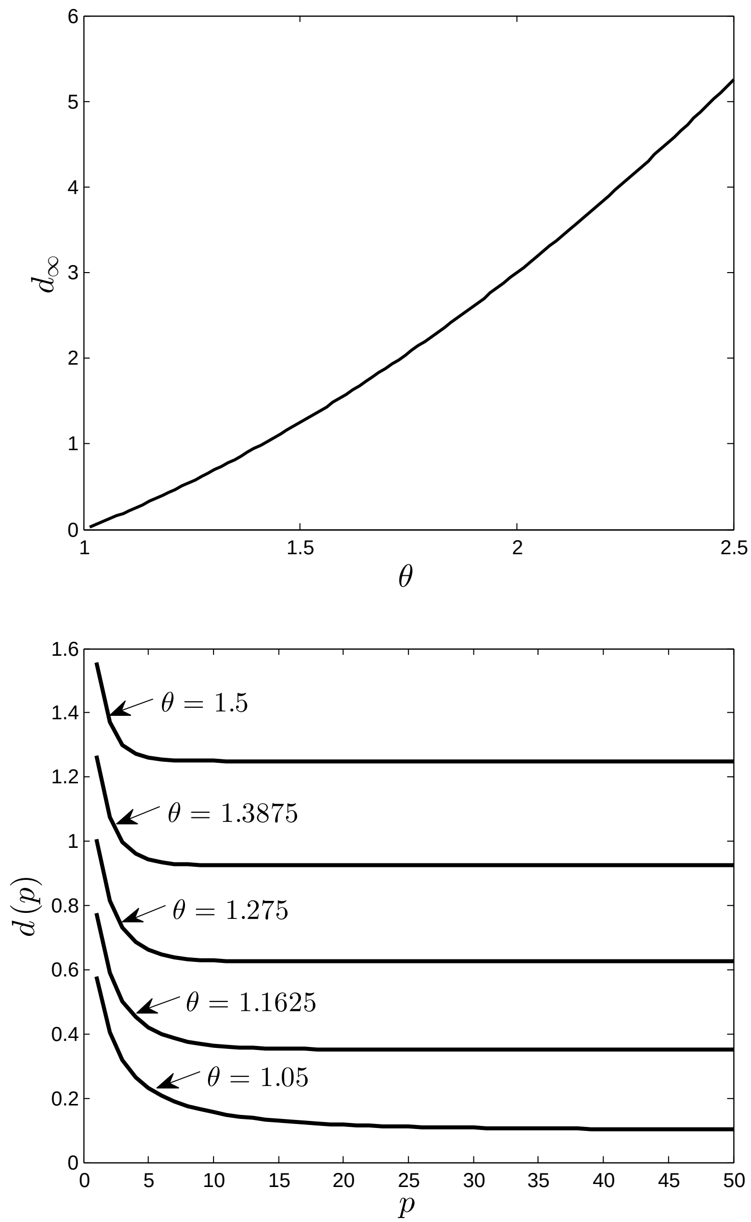


Table 1. Poor man's invertibility condition eigenvalues

q=1	q=2	q=3	q=4	q=5	q=6	q=7	q=8
				full			
0.9411	1.249	1.319	1.415	1.569	1.984	1.98	1.427
0.8043	0.9437	1.319	1.415	1.569	1.984	1.751	1.427
0.7305	0.8049	0.9471	1.375	1.476	1.719	1.678	1.398
0.5391	0.7278	0.8057	0.9515	1.476	1.719	1.678	1.398
0	0.5624	0.7248	0.8065	0.9575	1.666	1.538	1.341
0	0	0.5792	0.7219	0.8074	0.9656	1.538	1.341
0	0	0	0.5911	0.7191	0.8082	0.9773	1.332
	0	0	0	0.5993	0.7167	0.809	0.9965
		0	0	0	0.6047	0.7147	0.8096
			0	0	0	0.608	0.7133
				0	0	0	0.6101
					0	0	0
						0	0
							0
				RBC			
0.8886	1.063	1.063	1.063	1.063	1.063	1.063	1.063
0	0.8886	1.063	1.063	1.063	1.063	1.063	1.063
	0	0.8886	1.063	1.063	1.063	1.063	1.063
		0	0.8886	1.063	1.063	1.063	1.063
			0	0.8886	1.063	1.063	1.063
				0	0.8886	1.063	1.063
					0	0.8886	1.063
						0	0.8886
							0

Note: moduli of the eigenvalues of F_m

Table 2. Measure of nonfundamentalness: d_∞

	q=1	q=2	q=3	q=4	q=5	q=6	q=7	q=8
				d_∞				
full	0	0.1356	0.2123	0.2477	0.2593	0.2620	0.2664	0.2693
RBC	0	0.0266	0.0480	0.0665	0.0827	0.0970	0.1097	0.1208

Table 3. Small sample MAPE

p	q=1	q=2	q=3	q=4	q=5	q=6	q=7	q=8
				full				
5	**0.15543**	**0.15359**	**0.18315**	**0.23313**	**0.29916**	**0.36340**	**0.43924**	**0.54438**
6	0.16221	0.15494	0.18385	0.23417	0.30003	0.37100	0.44767	0.55122
7	0.16635	0.1576	0.18404	0.23745	0.30567	0.37504	0.45505	0.55727
8	0.16925	0.15735	0.18615	0.24052	0.30988	0.38153	0.46037	0.56406
9	0.17179	0.15949	0.18811	0.24167	0.31423	0.38507	0.46335	0.56831
10	0.17610	0.16283	0.18900	0.24587	0.31806	0.38818	0.46650	0.56937
11	0.17804	0.16329	0.19081	0.24694	0.31973	0.39013	0.46745	0.57012
12	0.18028	0.16399	0.19097	0.24944	0.32434	0.39461	0.47086	0.57246
13	0.18537	0.16595	0.19231	0.25060	0.32717	0.39796	0.47351	0.57424
14	0.18701	0.16763	0.19327	0.25295	0.33116	0.40196	0.47487	0.57546
15	0.19167	0.17229	0.19521	0.25668	0.33471	0.40412	0.47926	0.57713
16	0.19740	0.17719	0.19849	0.26013	0.33525	0.40781	0.48137	0.57858
17	0.20016	0.17712	0.20069	0.26300	0.34281	0.41334	0.48410	0.57845
18	0.20226	0.18209	0.20312	0.26531	0.34684	0.41558	0.48437	0.57987
19	0.20857	0.18780	0.20426	0.26661	0.34447	0.41319	0.48377	0.58009
20	0.21404	0.19205	0.20707	0.26722	0.34441	0.41195	0.48355	0.58176
				RBC				
5	0.22236	0.23813	0.25959	0.26810	0.34155	0.38822	0.48410	0.62537
6	0.21431	0.23455	0.25343	0.26447	0.28496	0.36122	0.45494	0.59985
7	0.21087	0.22454	0.23922	0.26128	0.28696	0.32115	0.44579	0.59101
8	0.20884	0.22279	0.23675	0.26082	0.28594	0.32391	0.39349	0.58583
9	0.21078	**0.21901**	0.23790	0.25054	0.28585	0.32465	0.39650	0.50689
10	**0.20857**	0.22001	**0.23220**	0.25127	0.28507	0.32508	0.39962	0.51567
11	0.21185	0.21929	0.23587	0.25085	**0.27596**	0.32569	0.40222	0.51839
12	0.21254	0.22082	0.23512	0.25156	0.27670	0.32468	0.39948	0.52040
13	0.2157	0.22073	0.23315	**0.24843**	0.27803	**0.31578**	0.39901	0.51907
14	0.21747	0.22332	0.23623	0.24902	0.27656	0.31641	0.39847	0.51832
15	0.22137	0.22540	0.23798	0.25249	0.27980	0.31914	**0.38687**	0.52022
16	0.22386	0.22861	0.23782	0.25531	0.27802	0.32113	0.38984	0.51914
17	0.22766	0.23075	0.24093	0.25406	0.28087	0.32207	0.39300	**0.49716**
18	0.23048	0.23335	0.24485	0.25890	0.28342	0.32477	0.39530	0.49995
19	0.23276	0.23701	0.24671	0.26001	0.28507	0.32198	0.39735	0.50081
20	0.23757	0.24174	0.24922	0.26311	0.29035	0.32732	0.40082	0.50280

Note: q is the periods of anticipation, p is the order of the estimated VAR

Table 4. Large sample MAPE

p	q=1	q=2	q=3	q=4	q=5	q=6	q=7	q=8
				full				
5	0.06466	0.06624	0.09350	0.14895	**0.21473**	**0.27426**	**0.34868**	**0.45740**
6	0.05709	0.05743	0.08828	**0.14464**	0.21649	0.28271	0.35520	0.46101
7	0.05083	0.05071	**0.08369**	0.15320	0.22672	0.29638	0.36492	0.46643
8	0.04643	0.04347	0.08390	0.16362	0.24184	0.31257	0.37781	0.47317
9	0.04017	0.03883	0.08946	0.17396	0.25784	0.32869	0.39289	0.48146
10	0.03804	0.03610	0.09587	0.18472	0.26924	0.34119	0.40441	0.48668
11	0.03459	0.03434	0.10453	0.19486	0.28016	0.35400	0.41718	0.49212
12	0.03086	**0.03415**	0.11227	0.20405	0.29154	0.36646	0.42892	0.49786
13	0.02792	0.03549	0.11810	0.21241	0.30159	0.37740	0.43861	0.50135
14	0.02488	0.03786	0.12514	0.22149	0.31212	0.38764	0.44754	0.50624
15	0.02322	0.04149	0.13142	0.22907	0.32043	0.39571	0.45467	0.51073
16	0.02135	0.04551	0.13707	0.23590	0.32733	0.40343	0.46218	0.51246
17	0.01820	0.05079	0.14336	0.24230	0.33522	0.41229	0.46880	0.51732
18	0.01565	0.05438	0.14765	0.24816	0.34213	0.41795	0.47512	0.51845
19	0.01336	0.05751	0.15235	0.25488	0.34786	0.42467	0.47983	0.52021
20	**0.01326**	0.05907	0.15608	0.25794	0.35309	0.42918	0.48315	0.52076
				RBC				
5	0.09916	0.14309	0.19533	0.21502	0.33661	0.38923	0.50078	0.65778
6	0.07837	0.12417	0.16888	0.19173	0.22021	0.35002	0.46421	0.62441
7	0.06347	0.09113	0.12194	0.16666	0.20054	0.24756	0.42666	0.59123
8	0.05132	0.07976	0.10396	0.14686	0.18227	0.23339	0.32734	0.57000
9	0.04466	0.06019	0.09028	0.11766	0.16452	0.21838	0.31674	**0.45575**
10	0.03594	0.05272	0.07420	0.10363	0.14877	**0.21003**	**0.31341**	0.45933
11	0.03119	0.04302	0.06305	0.09197	0.13582	0.21082	0.31676	0.46411
12	0.02728	0.03488	0.05385	0.08713	**0.13461**	0.21283	0.31937	0.46880
13	0.02244	0.02995	0.05079	**0.08468**	0.13762	0.20094	0.32493	0.47798
14	0.01900	0.02573	**0.04691**	0.08799	0.14187	0.20504	0.33431	0.48872
15	0.01706	0.02378	0.04953	0.09246	0.14851	0.21319	0.31524	0.50013
16	0.01497	**0.02021**	0.05254	0.09836	0.14660	0.22189	0.32684	0.50988
17	0.01235	0.02037	0.05665	0.09932	0.15255	0.23042	0.33754	0.48061
18	0.01058	0.02205	0.05997	0.10262	0.15963	0.23752	0.34233	0.48987
19	0.00876	0.02393	0.06118	0.10869	0.16613	0.23011	0.35001	0.49636
20	**0.00747**	0.02522	0.06482	0.11389	0.16932	0.23646	0.35477	0.50141

Note: q is the periods of anticipation, p is the order of the estimated VAR

FIGURE 2. Eigenvalues of F_m and $\widetilde{F}_m$ in the complex unit disk ($q = 3$)

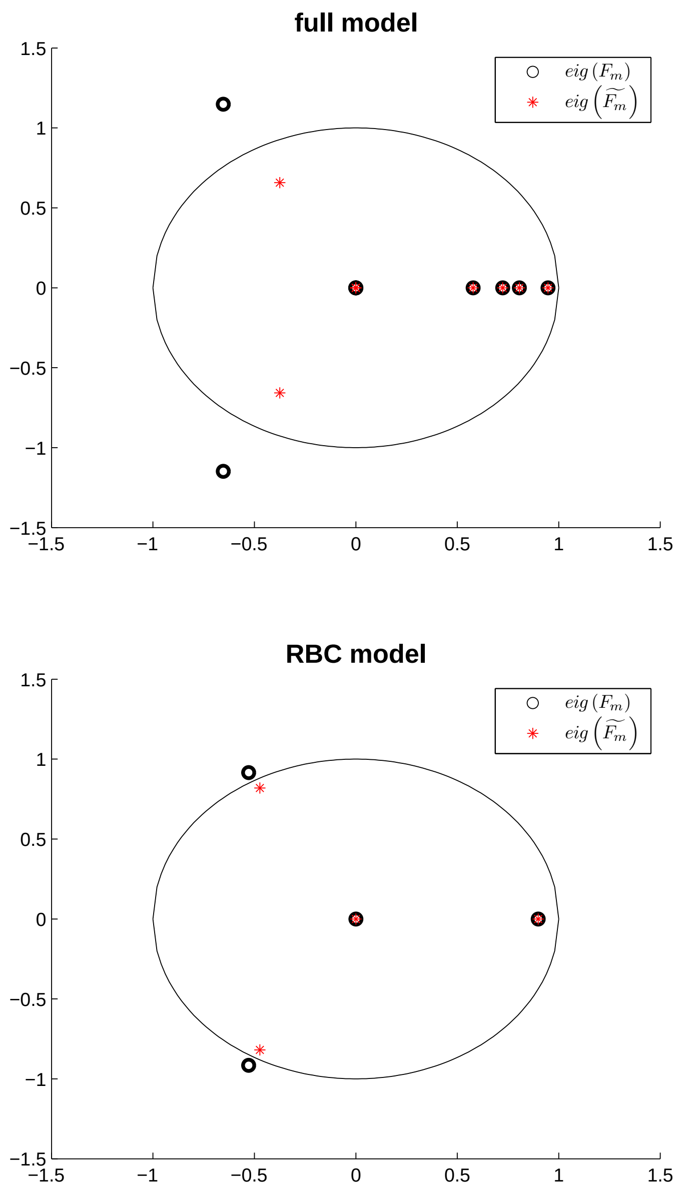
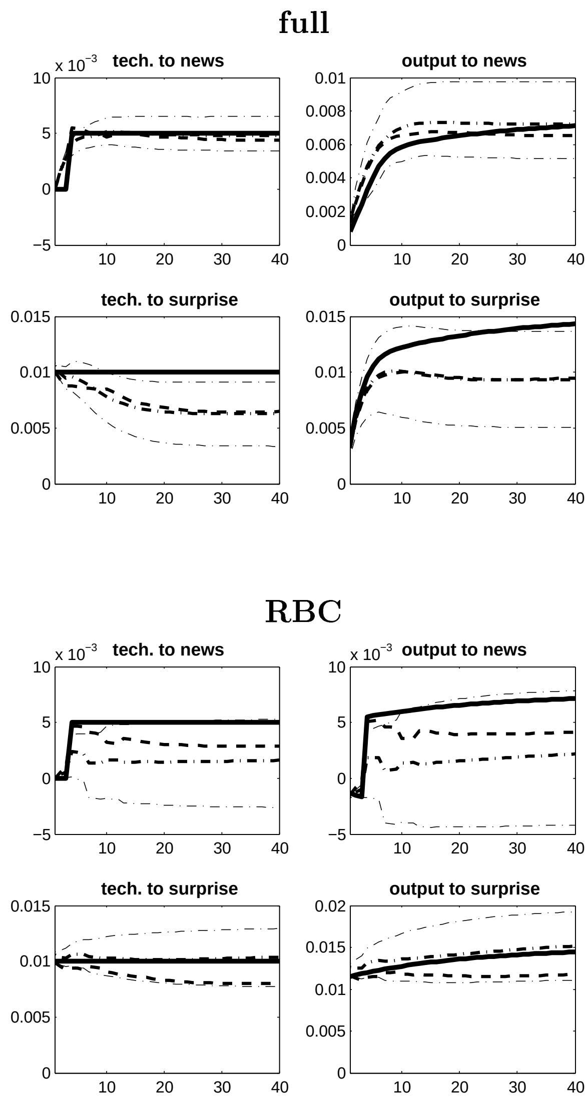


FIGURE 3. Impulse response functions ($q = 3$): VAR(8) and BIC lag length



Note: dashed VAR (8) as in Sims (2012), dashed and dotted BIC lag order, bold true, thin dashed and dotted 68 percent confidence bands around BIC responses.

Appendix A: Proofs

Proposition 5.

Proof. Let x_t be an n-dimensional stationary vector. Consider the projection

$$x_t = A_1^{(p)} x_{t-1} + \cdots + A_p^{(p)} x_{t-p} + v_t^{(p)}$$

and

$$x_t = A_1^{(p+1)} x_{t-1} + \cdots + A_p^{(p+1)} x_{t-p} + A_{p+1}^{(p+1)} x_{t-p-1} + v_t^{(p+1)}. \qquad (A1)$$

Let ξ_t be the residual of the projection of x_{t-p-1} on $x_{t-1}, \cdots, x_{t-p}$. Then (A1) can be rewritten as

$$\begin{aligned} x_t &= proj(x_t | x_{t-1}, \ldots, x_{t-p-1}) + v_t^{(p+1)} \\ &= proj(x_t | x_{t-1}, \ldots, x_{t-p}, \xi_t) + v_t^{(p+1)} \\ &= proj(x_t | x_{t-1}, \ldots, x_{t-p}) + proj(x_t | \xi_t) + v_t^{(p+1)} \\ &= A_1^{(p)} x_{t-1} + \ldots + A_p^{(p)} x_{t-p} + proj(x_t | \xi_t) + v_t^{(p+1)} \end{aligned}$$

Therefore

$$v_t^{(p)} = proj(x_t | \xi_t) + v_t^{(p+1)}. \qquad (A2)$$

Because $v_t^{(p+1)} \perp \xi_t$,

$$\mathrm{cov}(v_t^{(p)}) = \mathrm{cov}(v_t^{(p+1)}) + S, \qquad (A3)$$

where S is non-negative definite. From (A2) and (A3)

$$\mathrm{cov}(v_t^{(p)} - v_t^{(p+1)}) = \mathrm{cov}(v_t^{(p)}) - \mathrm{cov}(v_t^{(p+1)}) = S.$$

In this sense, $v_t^{(p)}$ *decreases* with p. Now suppose that

$$x_t = u_t + B_1 u_{t-1} + \cdots = B(L) u_t$$

not necessarily fundamental. Consider

$$\begin{aligned} v_t^{(p)} - u_t &= [I - A_1^{(p)} L - \ldots - A_p^{(p)} L^p] B(L) u_t - u_t \\ &= [u_t + C_1 u_{t-1} + \ldots] - u_t \\ &= C(L) u_t - u_t \end{aligned}$$

We have $cov(C(L)u_t, u_t) = cov(u_t)$. Therefore

$$cov(v_t^{(p)} - u_t) = cov(v_t^{(p)}) - cov(u_t)$$

so that $cov(v_t^{(p)} - u_t)$ decreases as well. □

Proposition 6.

Proof. We have that $\gamma\left(z, \mathbf{z}\right) \coloneqq \left(z - z_1\right)\left(z - z_2\right)\ldots\left(z - z_{n_m}\right) = \det\left(I - F_m z\right)$ By multiplying and dividing $G\left(z\right)$ by $\det\left(I - F_m z\right)$:

$$G\left(z\right) = \gamma\left(z, \mathbf{z}\right)\left(I_{n_y}\gamma\left(z, \mathbf{z}\right) - C_m \text{adj}\left(I_{n_m} - F_m z\right) B_m D^{-1} z\right)^{-1}$$

whose zeros are exactly those of $\gamma\left(z, \mathbf{z}\right)$ because there is no finite z such that $I_{n_y}\gamma\left(z, \mathbf{z}\right) - C_m \text{adj}\left(I_{n_m} - F_m z\right) B_m D^{-1} z$ is equal to infinity. This, together with the results on minimal systems in Franchi and Paruolo (2014), implies that the roots $z_1, z_2, \ldots, z_{n_m}$ of the VMA representation (4.1) are the reciprocals of the eigenvalues of F_m. Therefore, the matrix required to flip the roots in the unit disk of $G\left(z\right)$ reduces to a product of scalar polynomials flipping the roots in the unit disk of $\gamma\left(z, \mathbf{z}\right)$:

$$G\left(L\right) u_t = \gamma\left(L, \mathbf{z}\right)\left(I\gamma\left(L, \mathbf{z}\right) - C_m \text{adj}\left(I - \widetilde{F}_m L\right) B_m D^{-1} z\right)^{-1} \frac{\gamma\left(L, \widetilde{\mathbf{z}}\right)}{\gamma\left(L, \mathbf{z}\right)} \frac{\gamma\left(L, \mathbf{z}\right)}{\gamma\left(L, \widetilde{\mathbf{z}}\right)} u_t$$

where:

$$\gamma\left(z, \widetilde{\mathbf{z}}\right) = \left(z - \widetilde{z}_1\right)\left(z - \widetilde{z}_1\right)\ldots\left(z - \widetilde{z}_{n_m}\right)$$

replaces the roots z_i with the roots $\widetilde{z}_i$. By construction, $\left(I - \widetilde{F}_m z\right)$ with roots $\widetilde{z}_i$ and determinant $\det\left(I - \widetilde{F}_m z\right) = \prod_{i=1}^{n_m}\left(z - \widetilde{z}_i\right)$ is the solution of this root flipping problem:

$$\begin{aligned} G\left(L\right) u_t &= \widetilde{G}\left(L\right) \frac{\gamma\left(L, \mathbf{z}\right)}{\gamma\left(L, \widetilde{\mathbf{z}}\right)} u_t \\ &= \widetilde{G}\left(L\right) v_t \end{aligned}$$

with the last representation being fundamental as all the roots $\widetilde{z}_1, \widetilde{z}_1, \ldots, \widetilde{z}_{n_m}$ lay outside the unit disk.

□

Proposition 7.

Proof. From Proposition 6:

$$\begin{aligned}
v_t &= \frac{\gamma\left(L,\mathbf{z}\right)}{\gamma\left(L,\widetilde{\mathbf{z}}\right)}u_t \\
&= \prod_{i=1}^{n_m}\frac{L-z_i}{L-\widetilde{z}_i}u_t \\
&= \prod_{i=1}^{n_m}\mathbf{1}\left(\left|z_i\right|<1\right)\frac{\left(L-z_i\right)}{\left(L-\bar{z}_i^{-1}\right)}u_t
\end{aligned}$$

which goes to u_t as the roots in the disk go to 1 because

$$\bar{z}_k^{-1}=\frac{z_k}{\left|z_k\right|^2}\rightarrow z_k$$

as $|z_k|\rightarrow 1$ for all $|z_k|<1$. □

Appendix B: Sims (2012) - DSGE Equilibrium conditions

$$\mathrm{k}_{t+1}=\left(1-\frac{\tau}{2}\left(\frac{I_t}{I_{t-1}}\right)^2\right)T_t+(1-\delta)\,k_t \qquad (B1)$$

$$\lambda_t=\frac{1}{c_t-\gamma c_{t-1}}-\beta\gamma E_t\frac{1}{c_{t+1}-\gamma c_t} \qquad (B2)$$

$$\theta n_t^{\xi}=\lambda_t w_t \qquad (B3)$$

$$\lambda_t=\beta E_t\lambda_{t+1}\left(1+i_t\right)\frac{p_t}{p_{t+1}} \qquad (B4)$$

$$\mu_t=\beta E_t\left(\lambda_{t+1}R_{t+1}+(1-\delta)\,\mu_{t+1}\right) \qquad (B5)$$

$$\chi\left(\frac{M_{t+1}}{p_t}\right)^{-\nu}=\left(\frac{i_t}{1+i_t}\right)\lambda_t \qquad (B6)$$

$$\lambda_t=\mu_t\left(1-\frac{\tau}{2}\left(\frac{I_t}{I_{t-1}}-\Delta_I\right)^2-\tau\left(\frac{I_t}{I_{t-1}}\right)\right)+\beta\tau E_t\mu_{t+1}\left(\frac{I_{t+1}}{I_t}-\Delta_I\right)\left(\frac{I_{t+1}}{I_t}\right)^2 \qquad (B7)$$

$$\mathrm{w}_t=mc_t\,(1-\alpha)\,a_t\left(\frac{k_t}{n_t}\right)^{\alpha} \qquad (B8)$$

$$\mathrm{R}_t=mc_t\ \alpha\ a_t\left(\frac{k_t}{n_t}\right)^{\alpha-1} \qquad (B9)$$

$$\mathrm{p}_t^{\#}=\frac{\epsilon}{\epsilon-1}\frac{\sum_{s=0}^{\infty}(\varphi\beta)^s\left(mc_{t+s}p_{t+s}^{\epsilon}y_{t+s}\right)}{\sum_{s=0}^{\infty}(\varphi\beta)^s\left(p_{t+s}^{\epsilon-1}y_{t+s}\right)} \qquad (B10)$$

$$\mathrm{i}_t=\rho i_{t-1}+(1-\rho)\,\psi_\pi\left(\pi_t-\pi^*\right)+(1-\rho)\,\psi_y\left(\frac{y_t}{y_{t-1}}-\Delta_y\right) \qquad (B11)$$

$$1+\pi_t=\left((1-\varphi)\left(1+\pi_t^{\#}\right)^{1-\epsilon}+\varphi\right)^{\frac{1}{1-\epsilon}} \qquad (B12)$$

$$\mathrm{v}_t=(1-\varphi)\left(\frac{1+\pi_t^{\#}}{1+\pi_t}\right)^{-\epsilon}+\varphi\left(1+\pi_t\right)^{\epsilon}v_{t-1} \qquad (B13)$$

$$\mathrm{y}_t=c_t+I_t \qquad (B14)$$

Equations $(B1)$ to $(B7)$ solve the agent problem, $(B8)$ to $(B10)$ arise from the production sector, $(B11)$ is the Taylor rule, $(B12)$ and $(B13)$ are market clearing conditions and $(B14)$ is the resource constraint. The only exogenous process is equation (6.1) for technology.

The RBC model is obtained by setting $\tau = \gamma = \phi = 0$ and $\epsilon = \infty$. Remaining parameters are set as (Sims, 2012, Table 1).